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JEL Classification: G23, G30

Keywords: Collateral, Credit chains, Secured Lending, intermediation, fragility

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Financial Fragility with Collateral Circulation*

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Abstract

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1 Introduction

Lenders often require borrowers to post some asset as collateral. Collateral protects lenders against default as it ensures some revenue when the borrower fails, and enhances the borrower's incentives to repay. Collateral has another benefit: when lenders themselves need to borrow, they may be able to repledge the collateral or the cash flows of the loan backed by the collateral. Through this process, collateral circulates along credit chains from ultimate borrowers to ultimate lenders.

There are various examples of such collateral circulation. Firms that extended trade credit often use their account receivables or invoices as collateral when they borrow from banks (see Berger and Udell (1990) or Omiccioli (2005)). Through securitization, banks can secure fundings for new loans by pledging the cash flows from other loans. In markets for repurchase agreements (repo), intermediaries protect lenders by repledging financial securities such as T-bills they themselves received as collateral, a practice know as collateral rehypothecation or collateral re-use.[1] In segmented markets, collateral circulation allows funds to flow from ultimate lenders to ultimate borrowers. Gottardi and Kubler (2015) and Gottardi et al. (2019) show that when funding constraints are tight, this circulation can generate additional borrowing and increase welfare.

Financial regulators raised concerns that secured lending based on collateral circulation may generate risks and weaken credit chains.[2] Such concerns about the fragility of a *secured* credit chain are intriguing, because lenders along the chain are supposed to be protected against the default of their debtors, which should mute the contagion of adverse shocks along the chain. However, financial authorities provide little discussion about the potential contagion channels along a secured credit chain. The objective of this work is to provide an answer to the following questions. Can the formation of secured credit chains make financial systems more fragile? What are the underlying mechanisms? How is collateral quality related to fragility in a secured credit chain?

To address these questions, we present a simple model with three risk-neutral agents: a borrower, a lender and an intermediary. Both the borrower and the intermediary have profitable investment opportunities but no funds of their own. The lender has deep

[1] At the macro level, Singh (2011) shows that collateral velocity measured as the ratio between the total collateral pledged to the total collateral available is about 3, so the same piece of collateral is used to secure three loans on average.

[2] For example, the Financial Stability Board (FSB, 2017) remarks that "Collateral re-use can increase the interconnectedness among market participants and potentially contributes to the formation of contagion channels and risks." See ICMA (2019) for a rebuttal of the FSB arguments.

pockets but no investment opportunity. Hence, there are gains from trade but credit is subject to two frictions. First, the intermediary's investment is subject to moral hazard, as in Holmstrom and Tirole (1997). The probability of success of the intermediary's project depends on her unobservable effort level, which is costly. Second, the market is segmented as the borrower can only obtain funding, directly, from the intermediary, but not from the lender. The intermediary can channel funds from the lender to the borrower through a credit chain, by taking a larger debt position that finances her own investment as well as her intermediation activity.

Each loan, from the lender to the intermediary, and from the intermediary to the borrower, must be backed by collateral. The investment returns are pledgeable and can thus be used as collateral. We say the intermediary can repledge collateral when the collateral received from the borrower, or the yields of the loan granted, can be pledged as collateral with the lender. The ability to repledge collateral allows the intermediary to pledge more assets to finance her own investment and her intermediation activity. The intermediary can raise a single large loan secured by both assets or two smaller loans secured by one asset each. These options are equivalent under the assumption that the intermediary cannot ring-fence the pledgeable assets on her balance sheet. Hence, even a loan secured by only one asset as collateral, either the intermediary's own investment or the repledgeable collateral, provides recourse to the other pledgeable assets held by the intermediary. We argue in Section 7 that this recourse feature is relevant to various situations to which our model applies: factoring, repo, and securitization.

The first insight from our analysis is that the ability to repledge collateral is essential for intermediation. To channel funds to the borrower, the intermediary needs to raise a larger debt than when she only finances her own investment. Without repledging, this larger debt is backed only by her own investment as collateral, which undermines her incentives to exert effort, more than offsetting the profits of the intermediation activity. When repledging collateral is possible, the intermediary's larger debt is also backed by the payoff from her loan to the borrower. The additional pledging of a positive NPV loan allows the intermediary to retain a larger claim to the yield of her own project compared to the no-intermediation benchmark. Additional equity in the intermediary's own investment helps sustains her incentives. This skin-in-the-game effect is then the first consequence of repledging collateral. When the repledged collateral is safe enough, we show it is the only effect present. Hence, taking a larger loan to fund the intermediation activity is always profitable and improves the intermediary's incentives, thus decreasing her probability of default, and making the chain less fragile.[3]

The second insight from our analysis is that, when instead the circulating collateral is sufficiently risky, repledging it may induce an increase in fragility, as FSB (2017) suggests. The reason is that repledging risky collateral provides the intermediary some hedging against the failure of her own project. Fixing the expected value of collateral, more risk implies a higher collateral payoff in case of success. Hence, when the risky collateral pays off, the intermediary can still repay her entire debt even though her own project fails. This weakens her incentives to exert effort, thus increasing the default probability of her own investment. This hedging effect is the second consequence of collateral repledging and goes against the skin-in-the-game effect. The hedging effect is compounded by the recourse feature of the loan secured by repledged collateral, as the intermediary's larger debt must be repaid entirely out of her own investment return when the repledged collateral fails.

When collateral risk exceeds a given threshold, the hedging effect dominates the skin-in-the-game effect. Incentives are weaker, and as a result, the probability of default is higher when the intermediary chooses to repledge collateral and borrow a larger amount in order to intermediate funds. In this case, collateral circulation along the credit chain generates fragility. Provided collateral risk is not too high, however, the intermediary still prefers the large loan because the profits from intermediation exceed the losses from the negative effect on her incentives.

Our model is general and can be applied to various environments with intermediation and secured lending. In Section 7 we describe three such applications: trade credit, securitization, repos, and we argue our mechanism for fragility is present. In these markets some intermediary, either a bank or a firm, repledges collateral in the form of assets or loans. In all the three markets considered, loans backed by repledged collateral are typically recourse. As argued above, this feature increases the risk of other investments made by the intermediary when the repledged collateral is risky. In securitization, for instance, sponsors provide guarantees to the creditors of their Special Purpose Vehicle (SPV) that go beyond the value of the loans held by the SPV. Hence, while securitization allows banks to capture intermediation gains, their balance sheet becomes more fragile when the SPV loans are risky. In general, fragility is the price to

[3] For instance, a dealer bank would only enter a reverse repo with a borrower if she can re-use the collateral pledged by the borrower, when she enters a repo with a lender to match her repo book. Singh (2011) argues that dealer banks' ability to re-use collateral is essential to their role as repo intermediaries.

pay for the development of secured credit chains with risky collateral.

We then show in Section 5 the third insight of our work: collateral repledging can further increase fragility through an additional *news channel*. To this end, we extend the model to allow the intermediary to receive some news about the yield of the repledged collateral before she chooses the level of effort for her own investment. The intermediary can then optimally adjust her effort to this information. Intuitively, she will exert less effort when she learns the collateral value is low because she understands the lender will then claim most of the cash flow generated by her own investment. This induces a positive correlation between the cash flow of the borrower's project - the repledged collateral - and the one of the intermediary's own project. This endogenous correlation effect is akin to contagion: the negative shock to the yield of the borrower's investment increases the default probability of the intermediary's project.

But there is more than just correlation: the collateral value is low exactly when the lender needs it, that is, when the intermediary's project fails. As a result, in the presence of news, repledged collateral has a lower value for the lender. He then charges a higher interest rate which in turn induces the intermediary to choose a lower level of effort in expectation. Our analysis shows that the news channel exacerbates the fragility consequences of collateral repledging. In line with the claims of FSB (2017), we find that collateral re-use can amplify a shock about the value of the collateral.

We also endogenize the arrival of news by examining the case when the intermediary can pay a cost to acquire information about the value of the repledged collateral. The decision to acquire information is not observable by the lender. In this situation, the higher collateral risk is, the higher is the propensity of the intermediary to covertly acquire information. Anticipating this, the lender will charge a higher interest rate, as explained above. As a consequence, intermediaries are worse off - ex ante - when the cost of acquiring information is low, which means they prefer an opaque environment where information about collateral is hard to obtain, or to be able to commit not to acquire information.

Finally, we endogenize the quality of the collateral that intermediaries repledge. To this end, we assume borrowers also face a moral hazard problem. Hence, the riskiness of the borrowers' project and of the cash-flow of the loan granted by the intermediary vary with the face value of the loan. The higher the face value is, the lower is the borrower's incentive to exert effort. We show that the ability to repledge collateral provides the intermediary with incentives to sacrifice intermediation profits, by lowering the face value of the loan granted, in order to reduce the riskiness of the collateral acquired with such loan. Hence, intermediaries are willing to pay a re-use premium for safer collateral. Despite this, fragility may still arise in equilibrium.

Literature review

Our paper contributes both to the literature on collateral re-use and securitization, two forms of collateral circulation. The role of collateral re-use in shadow banking is discussed in a series of papers by Singh and Aitken (2010), Singh (2011) and Singh (2013). Some theoretical analyses of collateral re-use and its role in expanding borrowing are Andolfatto et al. (2017), Infante (2019), Bottazzi et al. (2012) and Gottardi et al. (2019). In this latter work we also showed that collateral re-use can explain the formation of intermediation chains. The presence of market segmentation is instead assumed in the current paper, as our focus is on the role of collateral re-use for the fragility of credit chains. Pyramiding shares many features with collateral re-use, except for the fact that the asset repledged is the cash flow of the loan granted rather then the (financial or tangible) asset pledged by the borrower to secure the loan.[4] Similarly to the results cited above on collateral re-use, Gottardi and Kubler (2015) show that pyramiding relaxes the collateral constraints by allowing an efficient use of the existing collateral (see also Geanakoplos and Zame, 2010). We complement this theoretical literature on re-use and pyramiding by showing that greater collateral circulation may undermine the stability of financial intermediation chains.

Securitization of loans is another form of collateral circulation as it entails the full or partial sale of collateralized loans via Special Purpose Vehicles financed with debt. The great financial crisis triggered a debate about the effect of securitization on lenders' incentives to monitor loans in the mortgage market. Several works, including Keys et al. (2010), Purnanandam (2011), Piskorski et al. (2015) and Griffin and Maturana (2016), show that securitization led issuers to apply lax standards for subprime loans.[5] Plantin (2011) shows theoretically that a greater level of securitization, even though it leads to less screening by lenders, needs not generate an inefficient outcome. Relatedly, Chemla and Hennessy (2014) and Vanasco (2017) show that investors purchasing securitized loans face asymmetry of information as intermediaries acquire private information when screening borrowers before selling these loans (see also the empirical analysis by Downing et al. (2008)). All these works show that securitization can reduce the quality of the loans extended by an intermediary. Our model shows instead that securitization

[4] Maurin (2017) shows that pyramiding is a more efficient way to reuse collateral if loans are non-recourse. For a joint analysis of pyramiding and rehypothecation, see Muley (2016).

[5] See Bubb and Kaufman (2014) for a critic of these results.

can make intermediaries' financial situation more fragile by increasing the default risk of other investments on their balance sheet.

More broadly, our paper relates to a large literature on fragility and contagion in credit chains and networks. Kiyotaki and Moore (1997) study the propagation of default along credit chains and Allen and Gale (2000) show that the structure of the network affects the propagation of risk. Subsequent works extended these results by considering either simple interactions in complex networks or richer relationships in simplified networks. Eisenberg and Noe (2001), Acemoglu et al. (2015) and Cabrales et al. (2017) belong to the first category, and analyze the topology of resilient networks. Our work with endogenous lending contracts belongs to the second category, together with Farboodi (2017) and Di Maggio and Tahbaz-Salehi (2015). As in our model, the intermediary in Di Maggio and Tahbaz-Salehi (2015) is subject to a moral hazard problem. However, in their paper, intermediaries have no investment opportunities, while our focus is on the contagion between the intermediary's own investment and her intermediation business. Similar to Farboodi (2017), in our model the intermediary chooses to expose herself to fragility to reap intermediation profits. But unlike in her paper, we model explicitly the role of collateral and study contagion between different loan contracts. With our focus on the spillover between the various activities of intermediaries, our channel for secured funding fragility differs from the role of fire sales, discussed by Brunnermeier and Pedersen (2009), Kuong (2020) or Biais et al. (forthcoming).

At a fundamental level, our analysis highlights a negative effect of increasing asset pledgeability, a theme that is also present in Donaldson et al. (2020). In that paper, the authors show that an increase in pledgeability leads firms to issue secured debt in order to dilute pre-existing unsecured debt. Our effect is different as it relies on the contamination of the borrower's other assets by the repledged collateral, which arises with joint financing, or, equivalently, when debt is recourse. A contamination effect under joint financing is also present in Banal-Estañol et al. (2013) but the mechanism is different, as it relies on default costs, while ours is due to moral hazard.[6] Besides, our focus on an intermediation chain leads to different predictions when we endogenize the quality of collateral.

Finally, we identify a news channel for fragility whereby the access to a technology to produce more accurate information about the value of collateral increases default risk when intermediaries re-use collateral. Our results, suggesting that opacity about

[6] See also Bahaj and Malherbe (2020)

assets' yields may be optimal when such assets are used as collateral, are reminiscent of Dang et al. (2015), Gorton and Ordoñez (2014) and Monnet and Quintin (2017). Our news channel, however, is different from the Hirshleifer (1971) effect at play in these papers. In our model, the information acquired by the intermediary induces her to correlate the effort choice on her own investment with the value of the collateral and this correlation generates fragility.

The rest of the paper is structured as follows. Section 2 presents the model. The benchmark case without collateral re-use is studied in Section 3. Our main results about collateral re-use and fragility are gathered in Section 4. Section 5 shows that fragility worsens in the presence of news about the collateral value. In Section 6, we endogenize the quality of the re-used collateral. Finally, Section 7 discusses applications of our model and Section 8 concludes. All proofs are in the Appendix.

2 Model

2.1 Technology and Preferences

The economy has two dates $t = 0, 1$. There is one good, called cash. There are three risk neutral agents: B, whom we call the borrower, D, the dealer intermediary, and L, the lender (we can equivalently think there is a plurality of agents acting as lenders). The latter has a large initial endowment of cash. B and D have instead no cash, but they are both endowed with a risky project that requires an investment of size 1 in the initial period.[7] The lender may thus be asked to provide 2 units of cash overall.

The project of the borrower matures at date 1 and pays off X_B with probability p_B and 0 otherwise. The project of the intermediary also matures at date 1 and pays off X_D in case of success and 0 otherwise. However, the probability p_D of success of this project is endogenously determined by D's effort choice. More precisely, D chooses p_D at the end of date 0 at a utility cost $\frac{1}{2}X_D p_D^2$. We will refer to p_D both as the effort choice and the probability of success of D's investment.[8]

[7] Although we consider for simplicity the case where the intermediary has no funds of her own, the results extend to the case where the intermediary has some limited funds, so she must still borrow to finance her activities.

[8] The quadratic cost function is used for tractability. All that matters is that the cost is increasing and convex in the probability of success. In Section 6, we extend the analysis to the case in which the borrower's investment is also subject to moral hazard.

2.2 Frictions and Contracting

There are frictions and some restrictions on admissible trades that can limit the gains arising from L lending to agents B and D.

Moral hazard

The intermediary's investment is subject to moral hazard as in Holmstrom and Tirole (1997): she cannot commit ex-ante to an effort choice. The socially optimal level of effort maximizes the expected payoff of $D's$ project net of the effort cost, and is given by $p_D^* = 1$. It is thus optimal that D's project always succeeds, but, when the intermediary finances her project with a loan, she will choose a level of effort $p_D < 1$, due to the moral hazard problem.

Segmented market

The borrower and the lender cannot trade together, while the intermediary can trade directly with both of them. D can thus borrow from L to finance both her own project and the loan she extends to B, playing the role of an intermediary. This assumption reflects institutional settings of the markets we consider as applications in Section 7.

Asset pledgeability and collateral circulation

Both B and D can always pledge the entire cash flow from their own investment as collateral backing their loans. The issue concerns the collateral pledged by B to D. Our benchmark is the case in which the loan from D to B is segregated and cannot be repledged by D, which also means L cannot seize its cash flow. We will contrast this benchmark to the case where D can repledge the loan as collateral. In this latter case, we say that the collateral circulates along the credit chain, and abusing market parlance somewhat, we will sometimes say that the intermediary can re-use the collateral received from the borrower.[9]

Loan terms

Loan contracts specify the amount borrowed, the collateral that is required and the payment due (the face value of the debt). We assume the latter is not state contingent, in particular, it does not depend on the realized yield of the investment projects.[10] Furthermore, loan contracts are recourse, that is, the lender in a contract has access to all pledgeable assets on the borrower's balance sheet. We will argue this feature is

[9] Collateral re-use or rehypothecation usually implies the explicit transfer of an asset or a title (e.g. to a house). Here we employ the term more generally as the intermediary only transfers the promise to repay the cash flow from B's investment. The two specifications are equivalent in the environment considered, as will become clear in what follows.

[10] In the repo market for example, the repurchase price is fixed in advance.

present in all applications considered in Section 7.

Bargaining Power

Given the assumed segmentation of the market, B is only able to fund his project if D chooses to intermediate funds with L. We focus on the case in which the intermediary always has all the bargaining power when dealing with the borrower or the lender. We show in Appendix A that our main results are robust to different specifications of the bargaining power.[11]

Finally, we assume the borrower's investment has a positive net present value but the expected payoff of his investment is lower than the 2 units of funds required to finance both projects. This second requirement ensures that D must always pledge her own investment to secure a loan from L.

Assumption 1. $p_B X_B \in (1, 2)$.

Similar to B's investment, we assume D's project has positive net present value. Accounting for the effort cost and the moral hazard problem, this assumption takes the following form:

Assumption 2. $X_D \geq 4$.

3 No Repledging

In this section we show that when the intermediary is unable to repledge the collateral of the borrower, she may not be willing to intermediate funds between the borrower and the lender. The reason is intuitive: in this case, the intermediary has to pledge such a high fraction of the yield of her own investment to secure a loan of 2 units from the lender that her incentives to provide effort are minimal. We will show that the negative effect on incentives may be so strong to induce the intermediary to borrow only one unit and hence to forgo the profits from intermediation.

At $t = 0$, D has to choose whether to borrow 1 unit from L to fund only the investment in her project, or 2 units to fund also the loan to B. In what follows, we first derive the optimal loan contract for each loan size and then derive the optimal loan size. Let $R_{D,l}$ be the face value of D's debt and $p_{D,l}$ her effort choice when she borrows

[11] For example, in the repo market, there is evidence that dealer banks have market power. As discussed by Infante (2019), dealers earn a spread on their matched repo book by charging higher haircuts as lenders.

$l \in \{1, 2\}$ units from L. Without repledging, the only asset D can pledge as collateral is her own project. Hence the lender only receives a positive payment when this project is successful, that is, with probability $p_{D,l}$. The lender's participation constraint is then

$$p_{D,l} R_{D,l} \geq l. \tag{1}$$

The intermediary has all the bargaining power in both relationships with B and L. As a lender to B, she will then set the face value R_B equal to the whole yield X_B of B's project when successful.[12] As a borrower from L, she will set the face value $R_{D,l}$ so as to maximize her utility subject to L's participation constraint.

Formally, the problem faced by the intermediary in this environment consists in choosing the size of the loan $l \in \{1, 2\}$ and the face value of the debt $R_{D,l}$ so as to maximize her expected utility given by

$$(p_B X_B)\, \mathbb{I}_{l=2} + \max_{p_{D,l}} p_{D,l} \max\{X_D - R_{D,l}, 0\} - \frac{1}{2} X_D p_{D,l}^2 \tag{2}$$

subject to (1), where $\mathbb{I}_{l=2}$ is the indicator function taking value 1 when $l = 2$ and 0 otherwise. If D's project succeeds, she repays her debt and retains the residual cash-flow $X_D - R_{D,l}$.[13] If instead her project fails, she makes no payment to her creditors. When D cannot repledge her loan to B, the lender is also unable to seize any of its cash flow. Hence, D always keeps all the revenue from her intermediation activity, equal to the entire yield of B's project when it succeeds.

The optimal choice of effort we obtain from problem (2) is

$$p_{D,l} = \frac{X_D - R_{D,l}}{X_D}. \tag{3}$$

Effort is decreasing in the face value of the debt $R_{D,l}$ because a higher repayment obligation weakens incentives. It is then easy to verify that the expected utility of the intermediary is decreasing in the face value of her loan $R_{D,l}$. Hence D will choose the lowest value of $R_{D,l}$ that satisfies the participation constraint of the lender which, after

[12] The environment is thus equivalent to one where she owns both investment projects. This equivalence breaks down, however, if B has all the bargaining power with D, a case we analyze in Appendix A to show our results are robust. The equivalence breaks down too when B's project is also subject to moral hazard, a situation considered in Section 6. In both cases, the distinction between a credit chain and a single borrower financing multiple investments becomes important.

[13] In this situation, D will never choose a face value $R_{D,l} > X_D$.

substituting (3), can be rewritten as follows:

$$(X_D - R_{D,l})\, R_{D,l} \geq l X_D \tag{4}$$

From this expression we obtain then:

Proposition 1. *The intermediary is able to borrow* 2 *units from* L*, secured only by her own project, whenever its productivity is sufficiently high that is, when* $X_D \geq 8$*. The optimal face value of the loan of size* $l \in \{1, 2\}$ *is*

$$R_{D,l} = \frac{2l}{1 + \sqrt{1 - \frac{4l}{X_D}}} \tag{5}$$

The intermediary's effort choice and her utility are respectively:

$$p_{D,l} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{l}{X_D}} \tag{6}$$

$$U_{D,l} = p_B X_B \mathbb{I}_{l=2} + \frac{1}{2} p_{D,l}^2 X_D \tag{7}$$

Notice that the interest rate $R_{D,l}/l - 1$ is strictly positive, because the default probability, $1 - p_{D,l}$, is also strictly positive. Furthermore, the interest rate increases with the size of the loan as $D's$ incentives deteriorates and so the probability of default increases with l.

If the intermediary could commit to her optimal effort level $p_D^* = 1$, she would be able to borrow at zero interest rate. In that case, she would prefer to take a large, 2 unit loan from L because the expected yield of the loan to B exceeds the cost of borrowing an extra unit at this rate by Assumption 1. Without commitment, however, the financing cost of an extra unit of loan is higher than one, because, as shown above, the interest rate is increasing in the loan size, due to the weakening of incentives.[14] We show next that this cost may be larger than the intermediation profits. In this case, D prefers to take a small, one unit loan and not intermediate funds between L and B.

[14] In fact, we can show that D would always take the 2-unit loan if she could commit to maintain her effort at the optimal level chosen for the 1-unit loan.

Corollary 1. *When collateral repledging is not possible, the intermediary chooses to borrow only 1 unit from L if $X_D \leq 8$ or if $X_D \geq 8$ and*

$$p_B X_B - 1 \leq \frac{1}{2} X_D \left[\sqrt{\frac{1}{4} - \frac{1}{X_D}} - \sqrt{\frac{1}{4} - \frac{2}{X_D}} \right] - \frac{1}{2} \tag{8}$$

As shown in Proposition 1, when the yield of D's project is too low ($X_D \leq 8$), the intermediary cannot get a 2 unit loan. When instead $X_D \geq 8$, both 1-unit and 2-unit loans are feasible but D still prefers a smaller loan when her intermediation profit ($p_B X_B - 1$) is smaller than the negative effect on $D's$ incentives of a larger loan, captured by the term on the right-hand side of (8).

In what follows, we assume the conditions stated in Corollary 1 always hold, so that the intermediary chooses not to intermediate when repledging collateral is not possible.

Assumption 3. *Either $X_D \leq 8$ holds or $X_D \geq 8$ and (8) hold.*

4 Collateral circulation

In this section we examine the case in which the intermediary is able to repledge the collateral received from the borrower.[15] The intermediary can then use all the assets on her balance sheet to secure her borrowing from the lender.[16] To borrow 2 units, D can either get a single 2-unit loan collateralized by both assets or two distinct 1-unit loans backed by each piece of collateral, even from different lenders. These two approaches are equivalent if lending is recourse, as assumed, because a creditor then holds a claim to the intermediary's balance sheet if the asset he receives as collateral falls short of the promised repayment.[17] To simplify the exposition, in what follows, we adopt the first approach in which D takes a single 2-unit loan from L.

We now derive the optimal debt contract chosen by D when she can repledge collateral. Let R_D^r denote the face value of her debt when she takes a 2-unit loan from L, where the superscript r is for repledge. The associated effort level is then p_D^r. We guess and later verify that $R_D^r \leq X_D$, so D can again fully repay her loan when her

[15] As we showed, the face value of the loan to B is set to $R_B = X_B$. Hence, the cases where D can repledge only the loan extended to B or the whole investment pledged by B are equivalent in our framework. See also footnote 9.

[16] Note that to get a 2-unit loan the intermediary must pledge her own investment. Under Assumption 1, the expected payoff of B's project is lower than 2.

[17] See Appendix B for details. In all the applications considered in Section 7, loans have this recourse feature.

investment pays off. When the intermediary pledges all assets on her balance sheet as collateral, she can also pay back a portion of her debt when her own investment fails if the repledged collateral has a positive yield. More precisely, when D's project fails, lenders seize the repledged collateral and, when it pays off, obtain $\min\{R_D^r, X_B\}$. As a consequence, the lenders' participation constraint becomes:

$$p_D^r R_D^r + (1 - p_D^r) p_B \min\{R_D^r, X_B\} \geq 2 \tag{9}$$

The intermediary chooses the face value of the debt R_D^r to maximize her expected utility, which has now the following expression:

$$\max_{p_D^r} \left\{ p_D^r (X_D + p_B X_B - R_D^r) + (1 - p_D^r) p_B \max\{X_B - R_D^r, 0\} - \frac{1}{2} X_D (p_D^r)^2 \right\} \tag{10}$$

subject to the lenders' participation constraint (9). Solving (10) for the optimal effort choice, given R_D^r, yields

$$p_D^r = \frac{X_D - R_D^r + p_B X_B - p_B \max\{X_B - R_D^r, 0\}}{X_D} \tag{11}$$

Comparing (11) with (3), we see that when D is able to repledge collateral, two new terms appear in the numerator of the right-hand-side of equation (10) for the effort choice. These terms correspond to two opposite effects of repledging on D's incentives.

Relative to its counterpart in equation (3), the first term $p_B X_B - (R_D^r - R_{D,1})$ captures a "skin-in-the-game" effect. It is equal to the additional fraction of her project D can retain when successful with a 2-unit loan and repledging compared to a 1-unit loan without repledging. This term is typically positive and thus strengthens D's incentives. To see this in the clearest way, it is useful to consider the case $R_D^r \geq X_B$, in which the participation constraint of L in (9) can be rewritten as

$$p_B (R_D^r - p_B X_B) \geq 2 - p_B X_B \tag{12}$$

Contrasting equations (12) and (11) when $R_D^r \geq X_B$ with the corresponding equations (1) and (3) in the no re-use case, we see that, with re-use, D effectively borrows $2 - p_B X_B$ units with a net repayment $R_D^r - p_B X_B$ to finance her own investment. Because the intermediation profit, $p_B X_B - 1$ is positive under Assumption 1, re-use allows D to reduce the net borrowing needed to fund her own investment below 1 unit. This reduction in her net debt level means that D has more skin-in-the game in her investment, which strengthens her incentives to exert effort.[18]

When $R_D^r \geq X_B$ the skin-in-the-game is the only effect present and unambiguously raises D's utility for taking a 2-unit loan and intermediating funds when she can repledge collateral. When $X_B > R_D^r$, however, this is no longer true as the second term in (11), equal to $-p_B \max\{X_B - R_D^r, 0\}$, has a strictly negative value. It captures an additional, negative effect of repledging collateral on incentives. We call it a "hedging effect", because, as we can see from the expression of the intermediary's expected utility in (10), the loan to B provides a partial hedge to D against the failure of her own investment project. When D's investment fails, she still obtains a positive payoff whenever B's project succeeds. Naturally this hedge weakens D's incentives.

Furthermore, the hedging effect is more likely to be present, and is stronger, the riskier the repledged collateral is. To see this, consider varying p_B and adjusting X_B so that the expected value of the repledged collateral $p_B X_B$, remains constant. As p_B decreases, both X_B and the variance of the collateral yield increases, and we see from (11) that the hedging effect becomes stronger. We will show that, when the repledged collateral is sufficiently risky, this negative hedging effect on incentives becomes so strong to trump the positive skin-in-the-game effect we described above.

In the proposition below, we characterize the optimal choice of the intermediary, in terms of quantity borrowed, face value of the debt and effort level, for all levels of the risk of the repledged collateral, as described by p_B (keeping again $p_B X_B$ fixed). To determine the optimal loan size we compare D's utility with the optimal 2-unit loan, obtained solving problem (10) subject to (9), with the optimal 1-unit loan, characterized in Proposition 1.[19]

[18] Observe that the same outcome obtains if D could sell the loan extended to B at its market value $p_B X_B$ and use the proceeds to reduce her outstanding loan from L from 2 units to $2 - p_B X_B$ units.

[19] The ability to repledge collateral clearly does not matter when D borrows only 1 unit to fund her project.

Proposition 2. *Consider all positive values of p_B, X_B such that $p_B X_B$ is a given constant satisfying Assumption 1. With collateral repledging, under Assumption 3, there exist thresholds $\underline{p}_B$ and $\bar{p}_B$ with $0 < \underline{p}_B < \bar{p}_B \leq 1$ such that*

1. *when the repledged collateral is very risky ($p_B \leq \underline{p}_B$), D prefers to borrow 1 unit to finance only her investment project,*

2. *when collateral is not too risky ($p_B > \underline{p}_B$), D prefers a 2-unit loan.*
   For intermediate levels of collateral risk ($p_B \in (\underline{p}_B, \bar{p}_B)$), the effort choice p_D^r and the face value R_D^r at the optimal loan contract respectively increases and decreases with p_B, and $R_D^r \geq X_B$.
   When instead collateral is sufficiently safe ($p_B \geq \bar{p}_B$), the effort choice and the face value are constant with p_B, and $R_D^r \leq X_B$. In this case, $p_D^r > p_D$, that is, the intermediary exerts more effort with collateral re-use.

Proposition 2 shows that the ability to repledge collateral may induce the intermediary to expand her borrowing and to choose to intermediate funds, generating a secured credit chain. In particular, when the collateral is rather safe ($p_B \geq \bar{p}_B$), the only effect of re-use is the skin-in-the-game effect, which makes the system more solid, by lowering the probability of failure, that is, $p_D^r \geq p_D$. However, this is not always the case: as collateral gets riskier, that is, when $p_B \in (\underline{p}_B, \bar{p}_B)$, the hedging effect is also present and counteracts the skin-in-the-game effect. This effect becomes stronger, and $D's$ incentives weaker, when collateral risk increases, leading to the interest rate required by lenders to increase. When collateral risk exceeds a given threshold ($p_B \leq \underline{p}_B$), the negative hedging effect is so strong and the interest rate required on a 2-unit loan so large, that D prefers not to re-use collateral and gives up on intermediation profits.

One may think D would prefer a 2-unit loan with repledging only if her effort level is higher than without repledging. However, we show next this is not the case: There is a range of values of collateral risk for which D prefers a 2-unit loan even though this leads to a higher level of default.

Proposition 3. *Under the assumptions of Proposition 2, there exists a range of collateral risk levels $(\underline{p}_B, p_B^*)$, with $p_B^* \in (\underline{p}_B, \bar{p}_B)$, such that for all p_B in that range, the intermediary strictly prefers a 2-unit loan with repledging and exerts less effort than when repledging is not allowed, that is, $p_D^r < p_D$.*

For the levels of collateral risk identified in the proposition, allowing re-use makes the system more fragile, as the probability of default increases. To understand why the

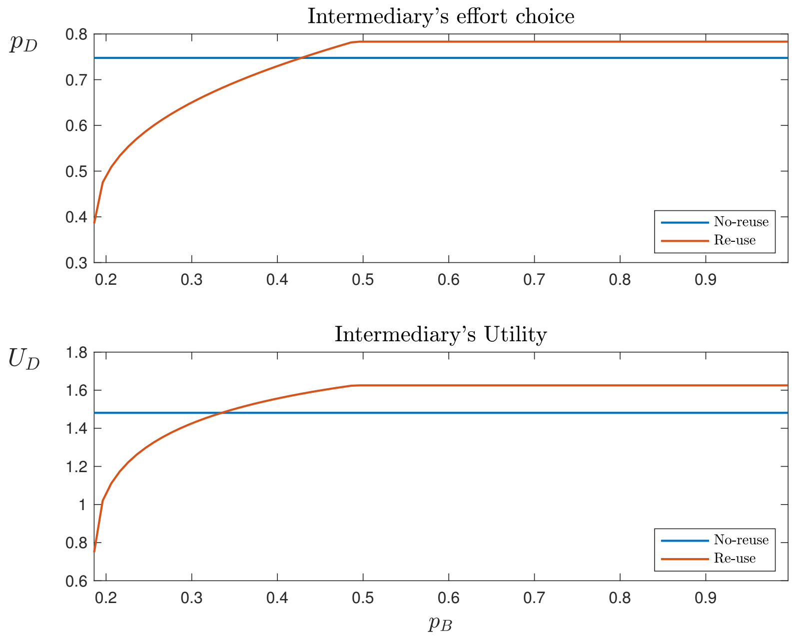

Figure 1: Fragility with collateral re-use. *Threshold values are:* $\underline{p}_B = 0.34$*,* $p_B^* = 0.43$*,* $\bar{p}_B = 0.48$*.*

intermediary may choose to expose herself - and the lenders - to a higher level of default, and the consequent increase in interest rate, it is useful to examine the expression of D's utility at the optimal loan contract. Using equations (10) and (11) we obtain the following expression:

$$U_D^r = \frac{1}{2}(p_D^r)^2 X_D + p_B \max\left\{X_B - R_D^r, 0\right\}. \tag{13}$$

The second term in (13) is D's expected payoff conditional on her own investment failing. When $p_B \leq \bar{p}_B$, it is positive because, in that case, $X_B > R_D^r$, as shown in Proposition 2. Comparing (13) with (7) for the no re-use case, evaluated at the optimal effort level respectively with and without re-use, it is easy to see that U_D^r can be larger than $U_{D,1}$ even when there is a higher default risk with re-use ($p_D^r < p_{D,1}$). The reason is that the intermediation profits D is able to reap with re-use generate a second, strictly positive term in (13) that may dominate the cost of a higher failure rate. So the circulation of risky collateral can generate fragility along the lending chain.

These results are illustrated in Figure 1 for the following parameter values: $X_D = 5.3$, $p_B X_B = 1.1$. The red, solid curve in the top (bottom) panel of Figure 1 is the level of effort (utility) of the intermediary at the optimal 2-unit contract with re-use as a function of the riskiness of the pledgeable collateral, described by p_B. Below the threshold $\bar{p}_B = 0.48$, both the level of effort and the utility increase with p_B, that is, they decrease when the risk of the collateral yield increases. The blue solid clines present the corresponding variables for the optimal 1-unit loan without re-use; they are both straight lines as, in this case collateral risk plays no role. In the top and the bottom panels, the intersections between the blue line and the red line define the thresholds, respectively, p_B^* and $\underline{p}_B$. In Figure 1 we see that fragility arises in the region $[0.34, 0.43]$ where D prefers to re-use collateral despite exerting less effort with re-use.

5 Information and Fragility

In this section we show that collateral re-use can generate additional fragility through a news channel. We extend the analysis to allow the intermediary to acquire, at a cost γ, a signal about the yield of the collateral pledged by the borrower. Without loss of generality, we assume the signal is fully informative and reveals whether $B's$ project succeeded or not. D can acquire the signal at the end of period 0, before choosing her level of effort. In Section 5.1 we show how the arrival of news affects D's incentives and the face value of the loan she contracts. We show that the fragility induced by re-use is amplified when D receives news about the collateral value. In Section 5.2 we then show that D chooses to acquire information at the interim stage, provided the cost γ is not too high.

5.1 Collateral repledging with news

Introducing information has no effect on the analysis of the benchmark case without collateral repledging. When the intermediary takes a 1-unit loan, B's project is not funded and so information about it is irrelevant. But also when D takes a 2-unit loan, her incentives to exert effort are unaffected by news if the loan she gets is only backed by her own investment project. As a result, the trade-off between a small and a large loan is the same as in Section 3, and under Assumption 3, the intermediary still prefers borrowing only 1 unit. With collateral re-use, the arrival of news regarding the repledged collateral matters. The reason is that learning about the realized yield of the collateral pledged by the borrower affects D's incentives: her effort decision at the end of date 0 is now contingent on the signal received.

To analyze this effect more formally, say the news can be either good (g), when B's investment is successful, or bad (b) when B's investment fails. For each news realization $s \in \{b, g\}$, D now chooses her effort $p_{Ds}^{r,n}$ to maximize her expected payoff conditional on the news received. With a slight abuse of notation, we let X_{Bs} denote the expected yield of the repledged collateral when news s arrives, with $X_{Bb} = 0$ and $X_{Bg} = X_B$. Let also $R_D^{r,n}$ denote the repayment due by the intermediary to the lender for a 2-unit loan when she receives information about the collateral value. D's effort choice problem for news realization $s \in \{b, g\}$ is:

$$\max_{p_{D,s}^{r,n}} \left\{ p_{Ds}^{r,n} \left(X_D + X_{Bs} - R_D^{r,n} \right) + \left(1 - p_{Ds}^{r,n} \right) \max \left\{ X_{Bs} - R_D^{r,n}, 0 \right\} - \frac{1}{2} X_D \left(p_{Ds}^{r,n} \right)^2 \right\} \tag{14}$$

As in the previous section, we guess and then verify that, for the contract chosen by the intermediary, the face value of the debt is such that $R_D^{r,n} \leq X_D$, that is, D can always fully repay the lender when her project succeeds. Given this property, D's effort choice when news s arrives is given by

$$p_{Ds}^{r,n} = \frac{X_D + X_{Bs} - R_D^{r,n} - \max \left\{ X_{Bs} - R_D^{r,n}, 0 \right\}}{X_D} \tag{15}$$

The key difference with respect to expression (11) is that, with news, the effort choice of the intermediary is positively correlated with the value of the repledged collateral. When this value is low, a high share of the yield of D's investment will be used to service her debt. This means that D captures a lower share of her investment returns and thus chooses to exert less effort. Hence, bad news regarding the value of repledged collateral induces D to lower her effort level which increases her default probability. So we find news generates contagion.

As news allows the intermediary to tailor her effort choice to the value of the repledged collateral, she enjoys an ex-post – that is, once the debt's face value is set – information rent. To find the value of this rent, we can rewrite the intermediary's utility replacing $p_{D,s}^{r,n}$ with the optimal effort choice derived in equation (15). We obtain so

$$\begin{aligned} U_D^{r,n} &= \frac{1}{2} \mathbb{E} \left[\left(p_{D,s}^{r,n} \right)^2 \right] X_D + p_B \max \left\{ 0, X_B - R_D^{r,n} \right\} \\ &= \frac{1}{2} \left(\mathbb{E} \left[p_{D,s}^{r,n} \right] \right)^2 X_D + \frac{1}{2} \mathbb{V}ar \left[p_{D,s}^{r,n} \right] X_D + p_B \max \left\{ 0, X_B - R_D^{r,n} \right\} \end{aligned} \tag{16}$$

If D did not receive any news, her utility, for the same face value $R_D^{r,n}$ of the debt, would be given by the sum of the first and third terms of equation (16).[20] Hence, the benefit of information for D – her ex-post information rent – is captured by the second term, $\frac{1}{2}\mathbb{V}ar\left[p^{r,n}_{D,s}\right]X_D$. The information rent is proportional to the variance of the effort choice because D uses this information to correlate her effort with the value of the repledged collateral.

Although the intermediary enjoys an ex-post information rent from the arrival of news, it does not mean that she also benefits ex-ante from the arrival of news. The ex-post rent of the intermediary constitutes in fact an ex-post loss for the lender due to the correlation between D's effort choice and the value of the repledged collateral, induced by the arrival of news. To gain some intuition, observe that the lender seizes the re-used collateral when the intermediary's project fails. But this project is more likely to fail when the collateral value is low, because bad news reduces D's incentives to exert effort. Hence, collateral offers a worse protection to lenders precisely when they need it most.[21] Anticipating this effect, the lender will charge a higher interest rate than in the case without news to ensure his participation constraint is still satisfied. This in turn has a negative effect on D's incentives, lowering her *expected* effort and utility.

Our next result shows that the negative effect of a higher debt burden with news trumps the ex-post benefit from information for the intermediary.

Proposition 4. *With collateral repledging, the expected probability of default of the intermediary is always higher and her expected utility lower in the presence of news than without news.*

With news, therefore, risky collateral is never good collateral. This result contrasts with Proposition 2 in which we showed that, below a given level of risk, risky collateral was as good as perfectly safe collateral.

[20]This can be seen using (11) to show that D's effort choice without news is equal to $\mathbb{E}\left[p^{r,n}_{D,s}\right]$ and substituting then this value in the expression (13) of D's utility obtained in the previous section.

[21]To see that lthe lender's ex-post payoff is lower with news, suppose the face value of D's loan were the same with or without news, that is, $R^{r,n}_D = R^r_D$. As discussed above, D's expected level of effort would be the same with or without news, that is, $\mathbb{E}[p^{r,n}_{Ds}] = p^r_D$. Using equation (9) in the case with news, the expected payoff of the lender with news is then:

$$U_L = \mathbb{E}[p^{r,n}_{Ds}]R^r_D + (1 - \mathbb{E}[p^{r,n}_{Ds}])p_B \min\{X_B, R^r_D\} - \mathbb{C}ov\left[p^{r,n}_{Ds}, \min\{X_{Bs}, R^r_D\}\right] \tag{17}$$

The first two terms of this equation give the lender's utility without news. Hence, the positive correlation between D's effort choice, $p^{r,n}_{Ds}$ and the value of the collateral X_{Bs} show that the lender's ex-post utility is lower with news.

We show next that, although the credit chain is even more fragile with news, fragility may still be the optimal choice for the intermediary: she may choose a 2-unit loan so as to reap the intermediation profits, even when she is more likely to default than with a 1-unit loan.

Proposition 5. *In the presence of news there exist thresholds $\underline{p}_B^n > \underline{p}_B$ and $p_B^{*,n} > p_B^*$ such that the intermediary prefers a 2-unit loan with collateral re-use when $p_B \geq \underline{p}_B^n$ and there is fragility if $p_B \in [\underline{p}_B^n, p_B^{*,n}]$. Hence, with news, fragility arises for lower levels of collateral risk.*

The above result shows that the weakening of incentives induced by the arrival of news reduces the region of levels of collateral risk for which collateral re-use occurs. At the same time, with news the region where fragility occurs shifts and includes higher values of p_B, where collateral is relatively safer. As we saw, even without news, the intermediary could choose to expose herself to fragility in order to reap intermediation profits. With news, an additional force (captured by the second term of equation (16)) increases D's tolerance for fragility: as we explained, for a given expected level of effort, D enjoys a higher utility with news thanks to the ex-post information rents.

Figure 2 illustrates these results for the same parameter values we used for Figure 1. The yellow curves describe the probability of success (in the top panel) and the expected utility (in the bottom panel) of the intermediary with collateral re-use in the presence of news. We see that for every value of p_B these curves lie strictly below the red curves, describing the corresponding values without news, reported in Figure 1 and again for convenience in Figure 2. This shows the negative effect of news on incentives and welfare. As a consequence, for values of the collateral risk – described by p_B — lying between 0.34 and 0.61 the intermediary chooses to re-use collateral without news, but refrains from doing so when she receives news, as re-using collateral is less attractive in that case. We also see, in line with Corollary 5 that with news the fragility region shifts to the right, where collateral risk is lower, as effort is lower. Fragility now obtains when p_B lies in the interval $[0.61, 0.70]$ against $[0.34, 0.43]$ with news (see Figure 1).

5.2 Information Acquisition

So far, we took the arrival of information as exogenous. We now consider the case where the intermediary can choose to acquire information at the interim stage. We showed in Proposition 4 that D is unambiguously worse-off in the presence of news. The decision

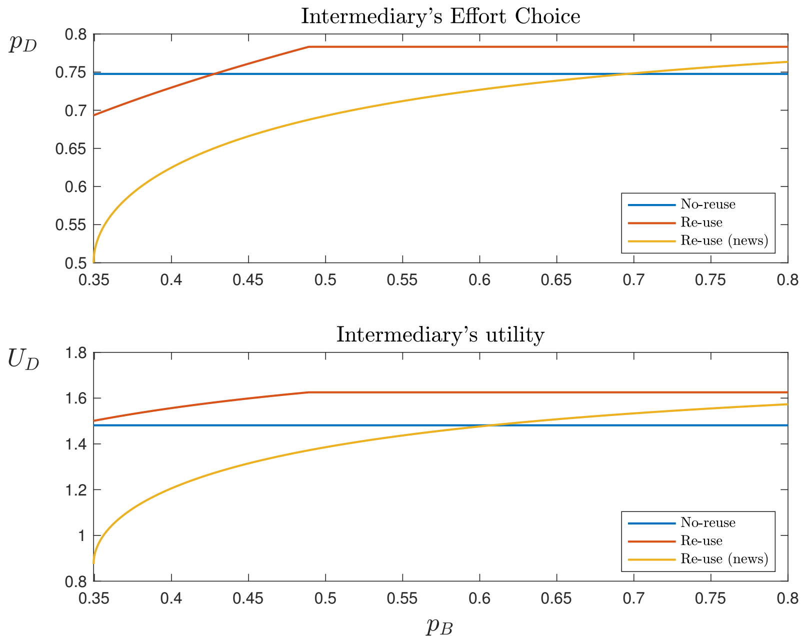

Figure 2: News-driven fragility. Threshold values are: $\underline{p}_B^n = 0.61$, $p_B^{n,*} = 0.70$.

to acquire information, however, takes place after the loan contract with L is signed. The next result then establishes that, even when information is costly, the intermediary acquires it, provided the cost is sufficiently small. In stating the result, we use $\mathbb{V}ar[X_B]$ to denote the variance of the re-used collateral and focus on the case where collateral is sufficiently safe ($p_B \geq \bar{p}_B$), for simplicity.

Proposition 6. *When $p_B \geq \bar{p}_B$, the intermediary chooses to acquire information at the interim stage provided the information cost γ is sufficiently smaller than the variance of the yield of the repledged collateral:*

$$\gamma \leq \frac{\mathbb{V}ar[X_B]}{2X_D}. \tag{18}$$

The result follows directly from our discussion of equation (16) above. As we explained, D's ex-post information rents are captured by the second term of (16), which is proportional to the variance of effort. From equation (15) we see that the variance of effort depends only on the variance of the collateral payoff. Intuitively, the intermediary's willingness to pay for information increases when the collateral payoff is more volatile as the benefits from tailoring her effort level to the collateral value are larger.

Ex-ante, D would like to commit not to acquire information because she anticipates that a rational lender would charge a higher interest rate if she does. However such commitment is not credible unless the cost of information is high. Once the face value $R_D^{r,n}$ of the loan to D has been set, D is always willing to pay the cost if it satisfies condition (18). As the right-hand-side of (18) is proportional to the variance of the collateral payoff, riskier collateral is worse collateral also because it is more likely to induce the intermediary to acquire information.

We thus showed that information production about collateral returns is harmful but it will happen in equilibrium when information costs are low. The result that opacity about collateral payoffs is bliss is reminiscent of the findings in Gorton and Ordoñez (2014) or Dang et al. (2015) who show that information about collateral returns may be detrimental for lending and welfare. However, the mechanism is different. In these papers, when lenders acquire information *ex-ante*, they choose not to lend to positive NPV borrowers with bad collateral, while, under opacity, all borrowers would receive financing. The mechanism is then a variant of Hirshleifer (1971)'s effect. Instead, in our model, information is detrimental because borrowers acquire it *ex-post* to correlate the effort on their investment with the return of the repledged collateral at the expense of lenders. Lenders anticipate this behavior, and charge a higher interest rate.

6 Endogenous Collateral Quality

In this section we endogenize collateral risk by allowing the intermediary to affect the riskiness of the loan to the borrower. To this end, we assume the probability of success of B's project is also the result of some costly, unobservable effort of the borrower. The interest rate set by D in the loan contract offered to B will then affect B's effort choice. Hence, the intermediary indirectly determines the probability of success of B's project and so the risk of her loan to B, which she repledges to L.

To be precise, we assume that B chooses the probability of success of his investment p_B at a cost $\frac{1}{2}c_B p_B^2 X_B$ with $c_B > 1$ and $X_B \in [4c_B, 8c_B]$. The latter condition is the counterpart of Assumption 1 with endogenous risk: the bounds on X_B ensure that lending to B is profitable but not so profitable that D would only need to repledge the loan as collateral to obtain a 2-unit loan from L. The condition $c_B > 1$ implies that the first-best level of effort is lower than one.[22] The set-up is otherwise identical to Section 4 and we focus on the version of the model without news for simplicity.

[22]As we have seen in Proposition 3, fragility arises when p_B is not too high. The condition $c_B > 1$ ensures that the (now endogenous) value of p_B may lie in this fragility region.

We first determine B's effort choice. Let R_B denote the face value of the 1-unit loan granted to B. Proceeding as in Section 3, we find B's choice of effort is

$$p_B = \frac{X_B - R_B}{c_B X_B} \tag{19}$$

If the intermediary sets the face value R_B to maximize the value of the expected payment $p_B R_B$ received from her loan to B, she chooses $R_B^{max} = X_B/2$, so that the induced effort is $p_B^{max} = 1/(2c_B)$ and D's expected revenue is $X_B/(4c_B)$. However, we show in the next proposition that maximizing the revenue generated by this loan is not optimal for D, when she re-uses the collateral to raise financing from the lender. We proved in Section 4 that, for a given expected yield, the riskiness of the repledged entails a cost for the intermediary. As the riskiness of the repledged collateral is now endogenous, the intermediary can reduce the variability in the yield of the collateral by lowering the face value R_B of her loan to B below R_B^{max}. Thus collateral re-use may induce D to sacrifice profits on her loan to B in order to obtain higher quality collateral.

Proposition 7. *With endogenous collateral quality there exists a threshold value $\underline{X}_B < 8c_B$ of the yield of B's project when successful such that, for all $X_B \geq \underline{X}_B$, the intermediary finds it optimal to sacrifice intermediation profits in exchange for collateral safety.*

Proposition 7 shows that intermediaries who re-use collateral are willing to set a lower face value of their loan to B than R_B^{max} so that B's effort level is higher than p_B^{max}. Safer collateral is better because it reduces the harmful consequences of the hedging effect. Therefore, collateral re-use may incentivize intermediaries to source safer assets as collateral, despite their lower returns. This result implies there is an endogenous premium for safe collateral when it circulates along collateral chains. This finding resonates with the evidence that re-used collateral in the swaps and derivatives markets is mostly in the form of highly liquid and safe Treasuries (see e.g. ISDA (2019)).

Interestingly, our results also imply that the borrower's profit is higher when he borrows from an intermediary who re-uses collateral. Suppose instead B could borrow directly from L, leaving all the bargaining power to the lender for symmetry. In such a situation the lender would simply maximize the expected revenue from this loan and thus choose face value R_B^{max}. In contrast D, as we showed in the previous proposition, prefers to set a lower face value thus increasing B's surplus from the transaction. This result is in line with the observation that counterparties who agree to the re-use of their collateral typically enjoy a discount on their borrowing terms (Monnet (2011)).

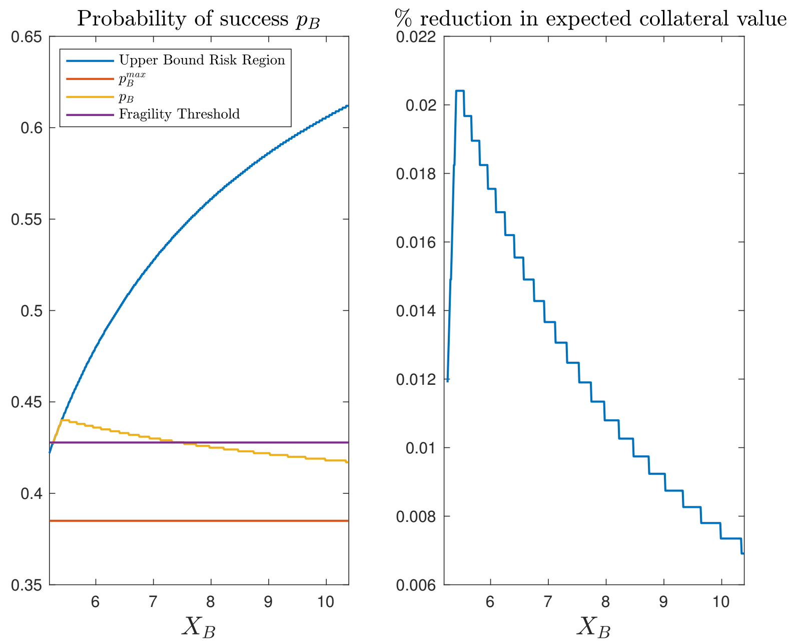

Figure 3: Optimal Collateral Risk.

Despite the intermediary's incentives to source safer collateral, fragility may still arise with endogenous collateral quality. Figure 3 provides a numerical illustration of this claim. The value of $X_D = 5.3$ is the same as in Figures 1 and 2, and we set $c_B = 1.3$. The left panel reports the borrower's probability of success (yellow solid curve) at the loan contract optimally set by D, as a function of X_B, the yield of B's project when successful. The purple line represents the fragility threshold characterized in Proposition 3 for these parameter values.[23] When $X_B \geq 7.53$, the yellow curve lies below the purple line, that is, fragility arises in equilibrium.[24] Fragility arises despite D's incentives to reduce collateral risk. The left panel of Figure 3 shows that these incentives are active for all values of X_B as the chosen level of collateral risk always lies strictly above the benchmark value p_B^{max} (the orange line). The right panel in the figure quantifies how much of the expected value of the collateral D sacrifices in order to reduce collateral risk. Overall, fragility may still arise even if D chooses to mitigate the collateral risk channel by sourcing safer collateral.

[23]This value is the same as in Figure 1 because the fragility threshold p_B^* of Section 4 is independent of X_B and only depends on X_D which has the same value here (see the proof of Proposition 3).

[24]For $X_B \in [5.20,\ 5.26]$, D prefers not to re-use collateral because of the negative hedging effect. For $X_B \in [5.26, 7.53]$, D re-uses collateral and the optimal choice of collateral riskiness is such that p_B exceeds the fragility threshold.

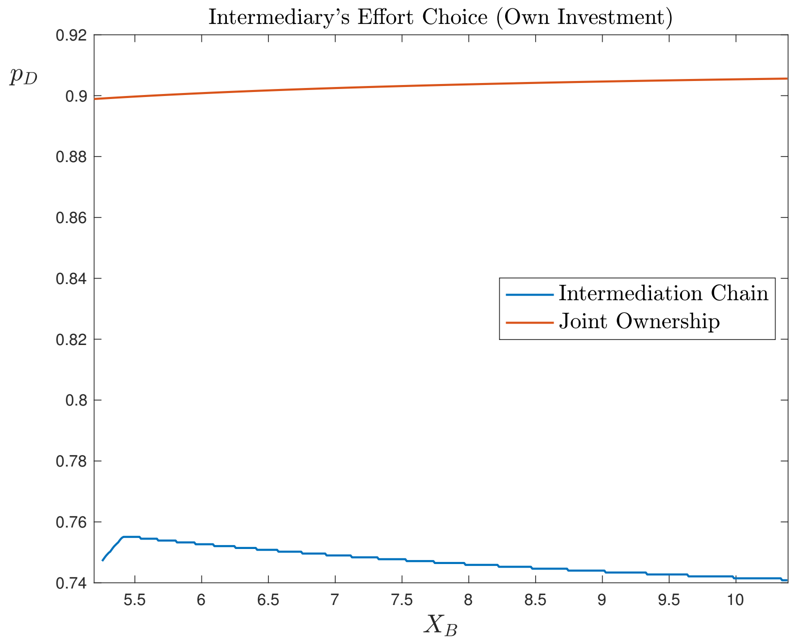

Figure 4: Intermediation Chain vs. Joint Ownership of Investments

As we observed before,[25] if the probability of success of B's project is exogenously given, the environment considered is equivalent to one in which D owns both investment projects. Hence, even though the focus of our analysis is on the circulation of collateral along an intermediation chain, it has also implications for the joint financing of projects. In particular, our results show that $D's$ investment project (subject to moral hazard) can become riskier when financed jointly with another positive NPV investment (B's project) rather than on a standalone basis. While a large literature has identified benefits from joint financing, our results suggest that it is prone to fragility and contagion when investments are risky. This finding is reminiscent of Banal-Estañol et al. (2013), although, in that paper, contamination is caused by default costs rather than moral hazard of the borrower, as in our setup.

Importantly, when the success probability of B's project is also endogenous and subject to moral hazard, as in this section, the intermediation chain is no longer equivalent to the situation in which D owns both investments. The chain should exhibit more fragility. As we saw in Proposition 7 above, intermediation is costly because D, even though she has all the bargaining power, still leaves some rents to the borrower to sustain his incentives in running his project. As a consequence, when B's investment succeeds D's payoff is significantly lower than in the case where she owns and directly

[25]See footnote 12.

chooses the effort for both investments. Given this feature, it is natural to conjecture that in the latter case D would also exert more effort on her project. We show this conjecture is indeed valid in the numerical example studied above. The red solid curve in Figure 4 reports the probability of success of D's own investment at the optimal loan when D also owns B's investment. The blue curve reports then the corresponding value found in Figure 3 with the intermediation chain. We see the probability of success is always strictly lower in the second case, with the chain.

7 Applications

Our model is stylized and is not meant to be a perfect fit for a specific economic application. However, we believe that our analysis highlights some important forces at play in several markets. In this section, we discuss three such applications in detail: securitization, trade credit, and repos.[26] We rely on the specification of the model where the intermediary borrows with repledging using two distinct 1-unit loans, secured respectively by the re-used collateral and D's own investment. As explained at the beginning of Section 4, this specification is equivalent to the one with a single 2-unit loan whenever loans are recourse and provide access to all pledgeable assets in D's balance sheet. The necessary ingredient for fragility is that, when the repledged collateral fails, D is "on the hook" to repay the loan she contracted in order to lend to B. In all three applications, we argue that the creditor whose loan is secured by the repledged collateral has indeed recourse to the balance sheet of D. Hence, the fragility channel we identified is likely to be active.[27] We discuss these applications in more details below.

Trade Credit

Our first application regards chains of trade credit. Trade credit is one of the major sources of funds for corporate firms. Instead of borrowing money from a bank, a firm can obtain the inputs it needs by using trade credit. With this instrument, the firm (the borrower) obtains inputs from a supplier by promising to pay for those inputs

[26] We thank Francisco Urzua for pointing out a fourth potential application: In business groups, holdings often borrow and pledge collateral obtained from subsidiaries where assets are typically located. See for example Ghatak and Kali (2001).

[27] In our model all loans taken by the intermediaries are recourse. As we show in Appendix B, however, the fact that loans not secured by the re-used collateral are recourse is not essential for the fragility result. What matters is that loans secured with re-used collateral provide recourse. In fact, we show that if only these loans have recourse features, fragility is even stronger than in our analysis.

at a later date. The supplier (the intermediary, here) records these loans as "account receivables" on its balance sheet. This has some analogy with the relationship between B and D in the environment we considered. In turn, the supplier may obtain funding from a financial lender (L in our model), by pledging or selling the trade receivable. This practice is sometimes known as factoring when the supplier uses invoices in order to borrow. Factoring can be recourse or non-recourse. With non-recourse factoring, the factoring firm is left empty-handed if the borrower (B) fails to pay. With recourse factoring instead, the supplier is on the hook to repay the factoring firm when the borrower fails. In Europe, while non-recourse factoring is increasing, recourse factoring has been prevalent[28]

The findings by Petersen and Rajan (2015) also suggest that such trade credit chains are a common arrangement. They show that firms with better access to credit from financial institutions offer more trade credit, that is, they may play a role as intermediaries. In our model only the intermediary has access to credit from lenders who can fund the trade credit position extended by D to B. In addition, Berger and Udell (1990) and Omiccioli (2005) show that firms use account receivables to secure borrowing from banks. Once again, this is akin to D repledging the cash flow of the loan she extended to B to secure her own loan from L. Interestingly, Omiccioli (2005) shows that this behavior is concentrated among small and risky firms. Our analysis suggests that the use of account receivables as collateral contributes to making firms riskier, thereby providing an explanation for this finding.

Securitization

With securitization an intermediary (the originator) can park loans off-balance sheet in a Special Purpose Vehicle (SPV) to free some balance sheet space. The SPV funds these loans by selling bonds.[29] The firm who sets up the SPV is called the sponsor and can be the same agent as the loan originator. In this interpretation of our model, D is the intermediary/sponsor of the SPV, the loan to B is held by the SPV while D only keeps her own project on her balance sheet.

As we explained, this arrangement can generate fragility if the creditors of the SPV

[28] Data from Factor Chain International, the industry representative body, show that more than 60% of factoring happens in Europe (see https://fci.nl/en/industry-statistics).

[29] This process often involves the pooling of different loans and their tranching into different debt claims to cater to an heterogeneous investor clientele. These features of securitization are important but they are not relevant to our argument.

have recourse to the balance sheet of the sponsor. In practice, sponsors often extend implicit or explicit guarantees to their SPVs in order to improve the rating of the SPV's debt (see Acharya et al. (2013)). The simplest forms of credit enhancement is an explicit recourse arrangement, whereby the creditors of the SPV would receive a payment directly from the enhancer should the SPV fail to pay. A more common form of credit enhancement is an irrevocable letter of credit. With such credit enhancement, the creditors of the SPV effectively have recourse to the balance sheet of the sponsor. Hence, as in our model, the intermediary is on the hook to repay the SPV creditors.

Viewing credit enhancement as a recourse arrangement, our model sheds light on how securitization can benefit the originator bank, by allowing it to expand its lending activity, while making its balance sheet more risky. We show that cross-subsidization between securitization and other activities, induced by SPV credit enhancements, can generate contagion and fragility, thus formalizing the argument in Acharya et al. (2013). Our mechanism is different from the narrative in Keys et al. (2010) and others who argue that securitization leads to fragility because banks have no incentive to exert due diligence for loans they plan to sell. In our model, the enhancement guarantee puts the balance sheet of the intermediary at stake affecting her incentives to exert due diligence for the assets remaining on her balance sheet and so increasing the probability of default of these assets. Because cross-subsidization sometimes both increases lending and reduces fragility in our model, pure ring-fencing between banks' own trading activities and their intermediation business may not always be efficient though.

Repurchase Agreements

The third application of our model is given by the bilateral repurchase agreement (repo) market. In this market, financial institutions borrow funds, usually short term, by selling assets with the agreement to buy them back at a later date at an agreed price. Essentially, the sale of these assets amounts to borrowing funds collateralized by the assets sold. Risky assets such as MBS or equity can be used as collateral in repos, and dealer banks often act as intermediaries in repo markets.[30] For example Aldasoro and Ehlers (2018) provide evidence that French banks (among others) are intermediating the US dollar funding needs of Japanese banks with onshore US money market funds. Key to this intermediation process is the ability of financial institutions to re-use the asset they obtained in a previous repo. Infante et al. (2018) document high re-use rate

[30]See Julliard et al. (2019) for the UK and Baklanova et al. (2015), for the US.

of collateral even for non-Treasuries among US dealers and FSB (2017) has identified re-use as a key source of risk. Finally, repos are recourse loans, as Gottardi et al. (2019) points out. So a lender in the repo market has an (unsecured) claim to the borrower entire balance sheet in case the collateral value is not high enough to cover the borrower's debt. Hence, our model can explain why secured credit chains in repo markets are cause for concerns.

8 Conclusion

Our paper shows that the ability of intermediaries to repledge collateral can induce a trade-off between a higher level of total borrowing along secured credit chains and greater fragility. We first show that collateral circulation has in fact a stabilizing effect when the repledged collateral is safe: in that case, re-use makes the system less fragile. When instead the repledged collateral is risky, a trade-off arises: intermediaries still choose to re-use collateral in order to reap intermediation profits but then expose themselves and the whole system to fragility. We show that such fragility is exacerbated in the presence of news about the value of the collateral. Hence, our model provides conditions under which collateral re-use can lead to contagion and fragility along credit chains. Furthermore, due to the fragility effect associated with the re-use of risky collateral, intermediaries are willing to pay a premium for safe collateral. Our findings apply both to the explicit re-use of collateral through rehypothecation, as in the repo market, and to the implicit re-use of collateral through securitization and other forms of asset-based financing. Our analysis shows that the formation of credit chains built on safe assets make the financial system both more integrated and more solid. However, the lack of safe assets such as Treasuries can entice market participants to rely increasingly more on risky collateral, which we show can generate fragility.

While our analysis focuses for simplicity on a short intermediation chain, it would be interesting to see how our results change when considering a longer chain of trades or a richer network of credit relationships. As we have seen, the circulation of collateral can generate fragility but also induces intermediaries to source safer collateral, in order to mitigate this fragility. Accounting jointly for these effects, it is thus unclear whether more complex networks or longer intermediation chains lead to more fragility. We also believe our model could provide a basis to compare different market structures. In the present paper we maintained the assumption that borrowers and lenders could only trade through the intermediaries, as in OTC markets. Recent regulatory efforts to reduce the fragility of these markets led to a push toward centralization of trades, via, for instance Central Counterparties (CCP). Market centralization could indeed shorten credit chains and reduce the need to re-use collateral. However, there are also widespread concerns that risk would be concentrated on a single agent rather being spread over a collection of intermediaries.

Appendix

A Bargaining Power to Borrowers

In this section, we consider the case in which borrowers have the bargaining power, that is, B sets the terms when borrowing from D and, as in the main text, D sets the terms when borrowing from L. We denote R_B the face value of the loan from L to B with $R_B \leq X_B$. Unlike in the main text, D's payoff from the loan is not equal to the payoff from B's investment. We assume D can repledge the loan as collateral rather than $B's$ entire investment. Effectively, the loan is an asset with payoff R_B (resp. 0) with probability p_B (resp. $1-p_B$). We focus on the case without news for simplicity.

A.1 No repledging

The analysis of this case is identical to Section 3 despite the different allocation of bargaining power. Intuitively, agent B can promise to repay up to X_B to D, when his project succeeds. If the intermediary is not willing to lend when $R_B = X_B$, as per Assumption 3, she would not be willing to lend for any $R_B \leq X_B$. Hence, under Assumption 3, the equilibrium without repledging is the same when B has the bargaining power.

A.2 Collateral Repledging

We now turn to the case where D can re-use the loan to B as collateral. Let $\underline{R}_B := 1/p_B$ be the break-even rate, which is the minimum face value the intermediary should accept to lend 1 unit to B, given the probability of success p_B. We show below that our main results from Section 4 survive and, in particular, that collateral re-use can generate fragility.

Proposition A.1. *There exists $\bar{p}_{BB} > \underline{p}_B$ with $\underline{p}_B$ defined in Proposition 2, such that*

1. *Agent B takes a 2-unit loan with re-use if and only if $p_B \geq \underline{p}_B$*
2. *Agent B exerts less effort with re-use than without if $p_B \in [\underline{p}_B, \bar{p}_{BB}]$.*

The face value of the loan to B satisfies $R_B > \underline{R}_B$ in the fragility region $[\underline{p}_B, \bar{p}_{BB}]$, and $R_B = \underline{R}_B$ for $p_B \geq \bar{p}_{BB}$.

Proof. With re-use, the objective of B is to minimize the face value of the loan extended by D subject to D's participation constraint. Formally, B's problem writes:

$$\min R_B \qquad \text{subject to} \quad U_D^r(R_B) \geq U_D \tag{A.1}$$

with $U_D^r(R_B)$ the utility of D when repledging the collateral with payoff R_B in case of success. We first determine $U_D^r(R_B)$ and then solve for R_B.

For the first step, observe that the only difference with our analysis in Section 4 is the payoff of the re-pledged collateral in case of success, which is R_B rather than X_B. We can thus use the results in Proposition 2 to characterize $U_D(R_B)$. In particular, for any $R_B \geq \underline{R}_B$, extending the notation of Proposition 2, there exists thresholds $\underline{p}_B(R_B)$ and $\bar{p}_B(R_B)$ such that the statements of Proposition 2 hold, substituting X_B with R_B.

In the second step of the analysis, we determine R_B using (A.1). Conjecture first that R_B is such that $p_B \geq \bar{p}_B(R_B)$ which, by definition, implies that $R_B \leq R_D^r$. The utility of D is given by $U_D(R_B) = \frac{1}{2}\left[p_D^r(R_B)\right]^2 X_D$. Hence, D's participation constraint binds if and only if

$$p_D = p_D^r(R_B) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - p_B R_B}{X_D}}$$

where $p_D^r(R_B)$ is given by equation (C.7) substituting X_B with R_B. It follows immediately that $R_B = \underline{R}_B$ is the solution to problem (A.1). Let thus $\bar{p}_{BB} := \bar{p}_B(\underline{R}_B)$ be the threshold below which $R_D^r \geq \underline{R}_B$ does not hold.

We now turn to the case $p_B \leq \bar{p}_{BB}$ to characterize the threshold below which D prefers a 1-unit loan. Observe that the maximum face value B can set is X_B. This implies that the threshold of interest is given by $\underline{p}_B := \underline{p}_B(X_B)$, introduced in Proposition 2.

We can now show that $p_D(R_B) < p_D$ for all $p_B \in [\underline{p}_B, \bar{p}_{BB}]$. By definition of $\bar{p}_{BB}$, the face value of the loan satisfies $R_B \geq R_D^r$. Using the results from Proposition 2 and the characterization of $U_D^r(R_B)$ in the proof of that result, we obtain

$$U_D^r(R_B) = \frac{1}{2}\left[p_D^r(R_B)\right]^2 X_D + p_D^r(R_B)(R_B - R_D^r(R_B)) = \frac{X_D}{2} + R_B - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_D^r(R_B))^2}{X_D}}} \tag{A.2}$$

where $p_D(R_B)$ and $R_D(R_B)$ are given by equations (C.11) and (C.10) respectively, substituting X_B with R_B. By the participation constraint of agent D, we have $U_D(R_B) = U_D$. Equation (A.2) then implies that $p_D^r < p_D$ when $p_B \in [\underline{p}_B, \bar{p}_{BB})$.

Finally, we show that $R_B > \underline{R}_B$ when $p_B \leq \bar{p}_{BB}$. To see this, suppose by contradiction that $R_B = \underline{R}_B$. From equation (A.2), we can see that U_D^r would be strictly increasing for $p_B \in [\underline{p}_B, \bar{p}_{BB}]$ because $p_D(R_B)$ is strictly increasing with p_B for a given value of R_B. This result contradicts the condition that agent D's participation constraint binds for all p_B.

□

B Recourse Loans and Fragility

In this section, we analyze the version of the model in which D obtains two distinct loans of one unit each. These two loans can equivalently be financed by the same investor or by two different investors. For ease of exposition, we call L_D the creditor secured by D's own investment and L_B the lender secured by D's loan to B. With two distinct loans, an important feature of the lending relationships between the intermediary and lenders is whether loans provides recourse. A lender has recourse if he has an (unsecured) claim to D's other assets when the payoff of the asset he receives as collateral falls short of the promised repayment of the loan.

We first show in Section B.1 that the model with two loans is equivalent to our benchmark model if both creditors have recourse. Motivated by the empirical applications discussed in Section 7, we then show in Section B.2 that fragility is even stronger than in our benchmark model if recourse is only given to creditor L_B whose claim is secured by the repledged collateral.

B.1 Symmetric Recourse

In this case, for $i \in \{B, D\}$, lender L_i has an unsecured claim to the asset pledged by D to creditor L_j with $j \neq i$. We let R_{DB} and R_{DD} denote the face value of the loan secured by the re-used collateral and D's own investment respectively. We guess and verify that the face value of the loans are such that $X_D > R_{DB} + R_{DD}$, that is, D can repay both loans in full using only the cash flow of his own investment when it succeeds. We are left to determine agents' payoff when $D's$ own investment fails but the loan to B succeeds. Lender L_B receives $R_{DB} < X_B$. Lender L_D's payoff is $\min\{X_B - R_{DB}, R_{DD}\}$ and D gets payoff $\max\{0, X_B - R_{DD} - R_{DB}\}$. Lenders L_B and L_D's participation constraint are respectively

$$p_D R_{DB} + (1 - p_D) p_B R_{DB} \geq 1,$$
$$p_D R_{DD} + (1 - p_D) p_B \min\{X_B - R_{DB}, R_{DD}\} \geq 1.$$

Agent D's effort decision is the solution to the following problem:

$$\max_{p_D} p_D (X_D - R_{DD} - R_{DB} + p_B X_B) + (1 - p_D) p_B \max\{0, X_B - R_{DD} - R_{DB}\} - \frac{1}{2} p_D^2 X_D \quad \text{(B.1)}$$

Denoting $R_D = R_{DB} + R_{DD}$, and saturating the lenders' participation constraint, we obtain

$$p_D R_D + (1 - p_D) p_B \min\{X_B, R_D\} = 2$$

and D's effort choice is given by

$$p_D = \frac{X_D + p_B X_B - R_D - p_B \max\{0, X_B - R_D\}}{X_D} \tag{B.2}$$

Observe that R_D and p_D are determined by the same equations as in Section 4 when we assumed a single creditor. Hence, the equilibrium face value of the total debt incurred by D and the equilibrium effort choice are again given by Proposition 2. This observation also implies that our conjecture $X_D > R_D$ is satisfied under the assumptions of the model.

B.2 Asymmetric Recourse

In this case, only the loan extended by L_B is recourse. We guess and verify again that $X_D > R_{DB} + R_{DD}$. In this case, only the participation constraint of lender L_D is different with respect to Section B.1. Because lender L_D receives a payoff of zero when D's investment fails, his participation constraint is now given by:

$$p_D R_{DD} \geq 1.$$

The effort decision of agent D is the solution to the following problem

$$\max_{p_D} p_D(X_D - R_{DD} - R_{DB} + p_B X_B) + (1 - p_D)p_B(X_B - R_{DB}) - \frac{1}{2}p_D^2 X_D \tag{B.3}$$

The second term of (B.3) is different from the second term of (B.1). When D's own investment fails, lender L_D does not have recourse to the payoff of the loan to B. We can then prove the following result.

Proposition B.1. *With asymmetric recourse, the intermediary's default probability with re-use is higher than with symmetric recourse and than without reuse for all values of p_B. Despite the additional fragility due to asymmetric recourse, B prefers to re-use collateral when it is sufficiently safe, that is, when p_B is high enough.*

Proof. To prove the first result, we derive the effort choice of D. Solving for p_D in (B.3) gives

$$p_D = \frac{X_D - R_{DD} - (1 - p_B)R_{DB}}{X_D} \tag{B.4}$$

Comparing equations (B.4) and (B.2), it follows immediately, that for given values R_{DB} and R_{DD}, the effort choice is strictly lower with asymmetric recourse. Comparing now the participation constraints of creditor L_D, for a given effort choice p_D, the face value R_{DD} must be strictly higher with asymmetric recourse. From these two observations, we can conclude that the effort choice of agent D is weakly lower with recourse. A similar argument shows that the effort choice is also lower than without re-use.

To prove the second result, consider the limit case when $p_B \to 1$. Then, comparing equations (B.4) and (6), the effort choice is the same with reuse and asymmetric recourse as without re-use. However, the utility derived by agent D with re-use is given by

$$U_D = \frac{1}{2} p_D^2 X_D + X_B - 1,$$

which is strictly higher than her utility level without re-use. Hence, by continuity, for p_B close enough to 1, agent D prefers to re-use collateral in the asymmetric recourse model.

$\square$

C Proofs

C.1 Proof of Proposition 1

Letting L's participation constraint, equation (4), bind, we obtain

$$R_{D,l}^2 - X_D R_{D,l} + l X_D = 0$$

The value of $R_{D,l}$ is the lowest root of this second-order equation with discriminant $\Delta_l = X_D^2 - 4lX_D$. We have

$$R_{D,l} = \frac{X_D - \sqrt{\Delta_l}}{2}$$

Replacing Δ_l by its value, we obtain equation (5). Expression (6) is obtained by plugging equation (5) in equation (3). Finally, observe that

$$U_{D,l} = p_{D,l}(X_D - R_{D,l}) - \frac{1}{2} X_D p_{D,l}^2 = X_D p_{D,l}^2 - \frac{1}{2} X_D p_{D,l}^2$$

where we used (6) to substitute for $R_{D,l}$. Equation (7) immediately follows.

C.2 Proof of Corollary 1

The result is obvious when $X_D \leq 8$ because, then ,a large loan of 2 units is not feasible. When $X_D \geq 8$, D prefers a small loan if

$$\frac{1}{2} p_{D,1}^2 X_D \geq p_B X_B + \frac{1}{2} p_{D,2}^2 X_D$$
$$\frac{X_D}{2}\left(\frac{1}{2} - \frac{1}{X_D} + \sqrt{\frac{1}{4} - \frac{1}{X_D}} - \frac{1}{2} + \frac{2}{X_D} - \sqrt{\frac{1}{4} - \frac{2}{X_D}}\right) \geq p_B X_B$$
$$\frac{1}{2} X_D \left(\sqrt{\frac{1}{4} - \frac{1}{X_D}} - \sqrt{\frac{1}{4} - \frac{2}{X_D}}\right) + \frac{1}{2} \geq p_B X_B$$

The last inequality is equivalent to condition (8).

C.3 Proof of Proposition 2

In Step 1, we characterize the optimal contract for a 2-unit loan with collateral repledging. In Step 2, we compare this outcome to the the 1-unit loan outcome without repledging characterized in Proposition 1.

Step 1. Equilibrium with re-use.

Case i) Conjecture $R_D^r \in [X_B, X_D]$.

We first solve for the face value R_D^r under this conjecture and then verify it. The effort choice in equation (11) becomes

$$p_D^r = \frac{X_D - R_D^r + p_B X_B}{X_D} \tag{C.5}$$

We can thus rewrite L's participation constraint as a function of R_D^r only. From equation (9) , we get

$$\begin{aligned} p_D^r R_D + (1 - p_D^r) p_B X_B &\geq 2 \\ p_D^r (R_D^r - p_B X_B) &\geq 2 - p_B X_B \\ (X_D - R_D^r + p_B X_B)(R_D^r - p_B X_B) &\geq X_D (2 - p_B X_B) \end{aligned}$$

The variable $\tilde{R}_D^r = R_D^r - p_B X_B$ is thus a solution to the following equation

$$\left(\tilde{R}_D^r\right)^2 - X_D \tilde{R}_D^r + X_D(2 - p_B X_B) = 0$$

Solving for the smallest root of the equation above, we obtain

$$R_D^r = R_B + \frac{1}{2}\left(X_D - \sqrt{X_D^2 - 4X_D(2 - p_B X_B)}\right) \tag{C.6}$$

For the effort choice, let us plug equation (C.6) in (C.5) to obtain

$$p_D^r = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - p_B X_B}{X_D}} \tag{C.7}$$

Comparing equations (6) and (C.7) shows that $p_D^r > p_D$, because $p_B X_B > 1$ under Assumption 1. It is also immediate that p_D^r and R_D^r are independent of p_B in this case, as $p_B X_B$ is assumed to be fixed.

We are left to verify the conjecture $R_D^r \in [X_B, X_D]$ and to characterize the associated threshold $\bar{p}_B$. The condition $R_D^r \leq X_D$ is equivalent to

$$2R_B \leq X_D + \sqrt{X_D^2 - 4X_D(2 - p_B X_B)}$$

which holds under Assumptions 1 and 2. The condition that $R_D^r \geq X_B$ writes

$$\begin{aligned} X_D - \sqrt{X_D^2 - 4X_D(2 - p_B X_B)} &\geq 2(1 - p_B) X_B \\ p_B \geq \bar{p}_B := \frac{2 p_B X_B}{X_D + 2 p_B X_B - \sqrt{X_D^2 - 4X_D(2 - p_B X_B)}} \end{aligned} \tag{C.8}$$

where $\bar{p}_B \leq 1$. This concludes Step 1 of the proof for the case $p_B \geq \bar{p}_B$.

Case ii) $R_D^r \leq \min\{X_B, X_D\}$.

From equation (10), the optimal choice of effort by agent D is

$$p_D^r = \frac{X_D - (1-p_B)R_D^r}{X_D} \tag{C.9}$$

The participation constraint of lender L thus writes

$$\begin{aligned} p_D^r R_D^r + (1-p_D^r)p_B R_D^r &\geq 2 \\ X_D p_B R_D^r + (X_D - (1-p_B)R_D^r)(1-p_B)R_D^r &\geq 2X_D \\ -(1-p_B)^2 \left(R_D^r\right)^2 + X_D R_D^r - 2X_D &\geq 0 \end{aligned}$$

A solution to this equation exists if and only if $X_D \geq 8(1-p_B)^2$, that is, if

$$p_B \geq \hat{p}_B := 1 - \sqrt{\frac{X_D}{8}}.$$

In this case, R_D^r is given by the smallest root of the second order polynomial above, that is

$$R_D^r = \frac{X_D - \sqrt{X_D^2 - 8X_D(1-p_B)^2}}{2(1-p_B)^2} = \frac{4}{1 + \sqrt{1 - \frac{2(1-p_B)^2}{X_D}}} \tag{C.10}$$

The second expression in (C.10) shows that R_D^r is decreasing with p_B. To obtain the equilibrium effort choice, plug in (C.10) in equation (C.9):

$$p_D^r = \frac{1}{2} - \frac{p_B}{2(1-p_B)} + \sqrt{\frac{1}{4(1-p_B)^2} - \frac{2}{X_D}} \tag{C.11}$$

Let us now study the monotonicity of p_D^r as a function of p_B. Differentiating the right-hand-side of (C.11) with respect to p_B we obtain,

$$\frac{\partial p_D^r}{\partial p_B} = -\frac{1}{2(1-p_B)^2} + \frac{1}{4(1-p_B)^3} \frac{1}{\sqrt{\frac{1}{4(1-p_B)^2} - \frac{2}{X_D}}}$$

Hence, p_D^r is increasing with p_B because

$$0 \leq 1 - 2(1-p_B)\sqrt{\frac{1}{4(1-p_B)^2} - \frac{2}{X_D}} = 1 - \sqrt{1 - \frac{8(1-p_B)^2}{X_D}}$$

We must first verify the conjecture $R_D^r \leq \min\{X_B, X_D\}$. The condition $R_D^r \leq X_B$ is equivalent to $p_B \leq \bar{p}_B$. Expression (C.10) shows that $R_D^r \leq 4$ which implies that $R_D^r \leq X_D$ under Assumption 2.

Finally, we must verify that the interval $[\hat{p}_B, \bar{p}_B]$ is not empty to ensure Case ii) arises in equilibrium for some values of p_B. If $X_D \geq 8$, we have $\hat{p}_B \leq 0$, which proves the result because $\bar{p}_B > 0$. Consider thus the case $X_D \in [4, 8]$. The threshold $\bar{p}_B$ is strictly increasing with $p_B X_B$ while $\hat{p}_B$ does not depend on $p_B X_B$. It is thus enough to verify that $\bar{p}_B \geq \hat{p}_B$ for $p_B X_B = 1$. In particular, we have

$$\bar{p}_B(p_B X_B = 1, X_D = 4) = \frac{1}{3} > 1 - \frac{1}{\sqrt{2}} = \hat{p}_B(X_D = 4).$$

As $\bar{p}_B$ is strictly increasing with X_D while $\hat{p}_B$ is strictly decreasing with X_D, we can conclude that the inequality above holds, in fact, for all value of X_D. Hence, the interval $[\hat{p}_B, \bar{p}_B]$ is non-empty. This concludes Step 1 of the proof for the case $p_B \leq \bar{p}_B$.

Step 2. Optimality of re-use

We now prove the existence of the threshold $\underline{p}_B$ with $\underline{p}_B \leq \bar{p}_B$ such that D prefers a 2 unit loan with collateral re-use to a 1-unit loan if and only if $p_B \geq \underline{p}_B$. Our analysis in Step 1 shows that $\underline{p}_B \geq \hat{p}_B$ because a 2-unit loan is not feasible for $p_B \leq \hat{p}_B$. To further characterize $\underline{p}_B$, it is useful to derive D's utility with collateral re-use. Using equations (11) and (10), we obtain equation (13).

We show first $U_D^r > U_D$ when $p_B \geq \underline{p}_B$. From the analysis of Step 1, the condition $p_B \geq \underline{p}_B$ implies $R_D^r \geq X_B$. Comparing equations (7) with $l = 1$ and (13) with $R_D^r \geq X_B$, the result follows from the finding $p_D^r \geq p_D$ derived in Step 1. Hence, it must be that $\underline{p}_B < \bar{p}_B$.

Consider now the case $p_B \leq \bar{p}_B$. We first show that U_D^r decreases with p_B. From (13),

$$\begin{aligned}
U_D^r &= \frac{1}{2}\left(p_D^r\right)^2 X_D + R_B - p_B R_D^r \\
&= \frac{1}{2}\left(1 - \frac{(1-p_B)}{X_D} R_D^r\right)^2 X_D + R_B - p_B R_D^r \\
&= \frac{X_D}{2} + p_B X_B - R_D^r\left(1 - \frac{(1-p_B)^2}{2X_D} R_D^r\right) \\
&= \frac{X_D}{2} + p_B X_B - \frac{R_D^r}{4}\left(4 - 1 + \sqrt{1 - \frac{8(1-p_B)^2}{X_D}}\right) \\
&= \frac{X_D}{2} + p_B X_B - \frac{R_D^r}{2} - \frac{X_D}{8(1-p_B)^2}\left(1 - \sqrt{1 - \frac{8(1-p_B)^2}{X_D}}\right)\left(1 + \sqrt{1 - \frac{8(1-p_B)^2}{X_D}}\right) \\
&= \frac{X_D}{2} + p_B X_B - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_B)^2}{X_D}}} \qquad \text{(C.12)}
\end{aligned}$$

where to derive the second, third, and final line, we used equation (C.9), (C.10) and (C.11) respectively. It follows from (C.12) that U_D^r is strictly increasing with p_B when $p_B \leq \bar{p}_B$.

Two cases are then possible. If $U_D^r(\hat{p}_B) \geq U_D$, define then $\underline{p}_B := \hat{p}_B$. If instead $U_D^r(\hat{p}_B) < U_D$, as U_D^r is strictly increasing with p_B for $p_B \in [\hat{p}_B, \bar{p}_B]$,then, $\underline{p}_B$ is the unique value of $p_B \in [\hat{p}_B, \bar{p}_B]$ implicitly defined by $U_D^r(\underline{p}_B) = U_D$. This concludes the proof.

C.4 Proof of Proposition 3

We first show that, if it exists, the threshold p_B^* belongs to the interval $(\underline{p}_B, \bar{p}_B)$. We then show that the threshold exists.

For the first step, observe from Proposition 2, that, for $p_B \geq \bar{p}_B$, D enjoys a higher utility and exerts more effort with re-use than without. Second, we showed that D prefers a 1-unit loan if $p_B \leq \underline{p}_B$, D prefers a 1-unit loan. Hence, the threshold p_B^*, if it exists, must belong to the interval $(\underline{p}_B, \bar{p}_B)$

For the second step, we showed in Proposition 2 that the effort choice with re-use, p_D^r, is increasing with p_B. To show that p_B^* exists, we are are thus left to show that $p_D^r(\underline{p}_B) < p_D$. Consider the two cases analyzed in the proof of Proposition 2. Suppose first that $\underline{p}_B = \hat{p}_B$, which is the case when $U_D^r(\hat{p}_B) > U_D$. Then, we have

$$p_D^r(\hat{p}_B) = \frac{1}{2} - \frac{\hat{p}_B}{1 - \hat{p}_B} = 1 - \sqrt{\frac{2}{X_D}}$$

Using equation (3), the inequality $p_D^r(\hat{p}_B) < p_D$ holds if and only if

$$\begin{aligned} & \frac{1}{2} - \sqrt{\frac{2}{X_D}} \leq \sqrt{\frac{1}{4} - \frac{1}{X_D}} \\ \Leftrightarrow \qquad & \frac{1}{4} - \sqrt{\frac{2}{X_D}} + \frac{2}{X_D} \leq \frac{1}{4} - \frac{1}{X_D} \end{aligned}$$

The last equation holds because $X_D \geq 4$ by Assumption 2. Consider now the case $\underline{p}_B > \hat{p}_B$, such that $U_D^r(\underline{p}_B) = U_D$. Then,

$$U_D = \frac{1}{2} p_D^2 X_D = U_D^r(\underline{p}_B) = \frac{1}{2}\left(p_D^r\left(\underline{p}_B\right)\right)^2 X_D + \underline{p}_B\left[X_B - R_D^r\left(\underline{p}_B\right)\right]$$

where the expression for U_D^r is given by equation (13). As $X_B > R_D^r$ when $p_B < \bar{p}_B$, this implies that $p_D^r\left(\underline{p}_B\right) < p_D$. Hence, in both cases, there exists $p_B^* \in (\underline{p}_B, \bar{p}_B)$ such that $p_D^r < p_D$ if and only if $p_B \in (\underline{p}_B, p_B^*)$.

We can derive an analytical solution for p_B^* by solving for the equation $p_D = p_D^r(p_B)$. We obtain

$$
\begin{aligned}
\sqrt{\frac{1}{4}-\frac{1}{X_D}} &= -\frac{p_B^*}{2(1-p_B^*)}+\sqrt{\frac{1}{4(1-p_B^*)^2}-\frac{2}{X_D}} \\
\frac{1}{4}-\frac{1}{X_D}+\frac{(p_B^*)^2}{4(1-p_B^*)^2}+\frac{p_B^*}{1-p_B^*}\sqrt{\frac{1}{4}-\frac{1}{X_D}} &= \frac{1}{4(1-p_B^*)^2}-\frac{2}{X_D} \\
\frac{1}{4}+\frac{1}{X_D}+\frac{p_B^*}{1-p_B^*}\sqrt{\frac{1}{4}-\frac{1}{X_D}} &= \frac{1+p_B^*}{4(1-p_B^*)} \\
\Rightarrow \qquad p_B^* &= \frac{2}{X_D+2-\sqrt{X_D^2-4X_D}}
\end{aligned}
$$

C.5 Proof of Proposition 4

The proof is in several steps. We first derive the values of $R_D^{r,n}$ and $p_{Ds}^{r,n}$ under the two different cases $R_D^{r,n} \geq X_B$ and $R_D^{r,n} \leq X_B$. We then compare the expected level of effort and the utility of agent D in the two regions $p_B \geq \bar{p}_B$ and $p_B < \bar{p}_B$, characterized in Proposition 2.

Step 1. *Values of* $R_D^{r,n}$ *and* $p_{Ds}^{r,n}$ when $R_D^{r,n} \geq X_B$

In this case, using equations (15) and (17), the participation constraint of the lender writes

$$
(X_D - R_D^{r,n} + R_B)R_D^{r,n} + \left(R_D^{r,n} - X_B\right) R_B \geq 2X_D
$$

Denoting $\tilde{R}_D^{r,n} = R_D^{r,n} - R_B$, we have

$$
\left(\tilde{R}_D^{r,n}\right)^2 - X_D\tilde{R}_D^{r,n} + X_D(2-R_B) + p_B(1-p_B)X_B^2 = 0
$$

This second order equation has real solutions if and only if

$$
0 \leq X_D^2 - 4X_D(2-R_B) - 4R_B(X_B - R_B)
$$

which is equivalent to

$$
p_B \geq \hat{p}_B := \frac{4R_B^2}{4R_B^2 + X_D^2 - 4X_D(2-R_B)} \tag{C.13}
$$

The loan face value is then the lowest root of the second order polynomial, given by

$$
\begin{aligned}
R_D^{r,n} &= R_B + \frac{1}{2}\left(X_D - \sqrt{X_D^2 - 4X_D(2-R_B) - 4R_B(X_B - R_B)}\right) \qquad \text{(C.14)} \\
&= R_B + 2\frac{2 - R_B + \frac{R_B(X_B - R_B)}{X_D}}{1 + \sqrt{1 - 4\frac{2-R_B}{X_D} - \frac{4R_B(X_B-R_B)}{X_D^2}}}
\end{aligned}
$$

The expression for $\mathbb{E}[p_{Ds}^{r,n}]$ is obtained by plugging the expression for $R_D^{r,n}$ obtained above in equation (15) and taking the average over the states $s \in \{b, g\}$. We obtain

$$\mathbb{E}[p_{Ds}^{rn}] = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{(2 - R_B)}{X_D} - \frac{R_B(X_B - R_B)}{X_D^2}} \tag{C.15}$$

As $\mathbb{E}[p_{Ds}^{rn}]$ is decreasing with X_B and $X_B = R_B/p_B$ where R_B is fixed, it follows that $\mathbb{E}[p_{Ds}^{rn}]$ is decreasing with p_B.

Let us now derive D's utility. From equation (14), we have

$$\begin{aligned}
U_D^{r,n} &= \frac{1}{2}\mathbb{E}\left[\left(p_{Ds}^{r,n}\right)^2\right] X_D \\
&= \frac{1}{2X_D}\mathbb{E}\left[\left(X_D - R_D^{r,n} + X_{Bs}\right)^2\right] \\
&= \frac{1}{2X_D}\left[\left(X_D - R_D^{r,n} + R_B\right)^2 + \mathbb{V}ar[X_{Bs}]\right] \\
&= \frac{1}{8X_D}\left[\left(X_D + \sqrt{X_D^2 - 4X_D(2 - R_B) - 4R_B(X_B - R_B)}\right)^2 + 4R_B(X_B - R_B)\right] \\
&= \frac{1}{8X_D}\left[2X_D^2 - 4X_D(2 - 2p_BX_B) + 2X_D\sqrt{X_D^2 - 4X_D(2 - R_B) - 4R_B(X_B - R_B)}\right] \\
&= \frac{1}{4}\left[X_D + \sqrt{X_D^2 - 4X_D(2 - R_B) - 4R_B(X_B - R_B)} - 2(2 - R_B)\right]
\end{aligned} \tag{C.16}$$

As $U_D^{r,n}$ is decreasing with X_B and $X_B = R_B/p_B$ with R_B fixed, it follows that $U_D^{r,n}$ is increasing with p_B.

Step 2. *Values of* $R_D^{r,n}$ *and* $p_{Ds}^{r,n}$ when $R_D^{r,n} < X_B$

The conjecture $R_D^{r,n} < X_B$ together with equation (15) imply that

$$p_{Dg}^{r,n} = 1, \quad p_{Db}^{r,n} = \frac{X_D - R_D}{X_D}$$

Using again equations (15) and (17) with the effort choices derived above, the participation constraint of the lender now writes

$$\begin{gathered}
p_B R_D^{r,n} + (1 - p_B)p_{Db}^{r,n} R_D^{r,n} \geq 2 \\
-(1 - p_B)\left(R_D^{r,n}\right)^2 + R_D^{r,n} X_D - 2X_D \geq 0
\end{gathered}$$

This second-order equation has a solution if $X_D \geq 8(1 - p_B)$. The solution is the smallest root of the second-order polynomial above, given by

$$R_D^{r,n} = \frac{X_D - \sqrt{X_D^2 - 8X_D(1-p_B)}}{2(1-p_B)} = \frac{4}{1+\sqrt{1-\frac{8(1-p_B)}{X_D}}} \tag{C.17}$$

The expected level of effort in this case is obtained thanks to equation (15):

$$\begin{aligned}
\mathbb{E}[p_{Ds}^{rn}] &= p_B + (1-p_B)\left[1 - \frac{R_D^{r,n}}{X_D}\right] \\
&= 1 - \frac{X_D - \sqrt{X_D^2 - 8X_D(1-p_B)}}{2X_D} \\
&= \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{8(1-p_B)}{X_D}}
\end{aligned} \tag{C.18}$$

The expression above shows that $\mathbb{E}[p_{Ds}^{rn}]$ is also increasing with p_B in this case. Finally, the ex-ante utility of agent D is given by

$$\begin{aligned}
U_D^{r,n} &= p_B\left(\frac{X_D}{2} - R_D^{r,n} + X_B\right) + \frac{1}{2}(1-p_B)\left(p_{D,b}^{r,n}\right)^2 X_D \\
&= p_B\frac{X_D}{2} - p_B R_D^{r,n} + R_B + \frac{1}{2}(1-p_B)\left[X_D - 2R_D^{r,n} + \frac{\left(R_D^{r,n}\right)^2}{X_D}\right] \\
&= \frac{X_D}{2} + R_B - R_D^{r,n}\left(1 - \frac{(1-p_B)R_D^{r,n}}{2X_D}\right) \\
&= \frac{X_D}{2} + R_B - \frac{R_D^{r,n}}{4}\left(4 - 1 + \sqrt{1 - \frac{8(1-p_B)}{X_D}}\right) \\
&= \frac{X_D}{2} + R_B - \frac{R_D^{r,n}}{2} - 1
\end{aligned} \tag{C.19}$$

The utility $U_D^{r,n}$ of agent D is increasing with p_B because $R_D^{r,n}$ is decreasing with p_B, as can be seen from equation (C.17).

Step 3. *Proof that $U_D^{r,n} \leq U_D^r$ and $\mathbb{E}[p_{D,s}^{r,n}] < p_D^r$ for $p_B \geq \bar{p}_B$.*

By definition of $\bar{p}_B$, the face value of the loan in the absence of news satisfies $R_D^r \geq X_B$. Comparing equation (C.6) for R_D^r and equation (C.14) for $R_D^{r,n}$ shows that $R_D^{r,n} \geq R_D^r$. Hence, the conjecture $R_D^{r,n} \geq X_B$ is also satisfied for $p_B \geq \bar{p}_B$. The result that the expected level of effort is lower with news follows directly from the comparison between equations (C.7) and

(C.15). For the comparison between utility levels, observe that

$$U_D^r = \frac{1}{2}(p_D^r)^2 X_D = \frac{1}{2}\left(\frac{1}{2} - \frac{2-R_B}{X_D} + \sqrt{\frac{1}{4} - \frac{2-R_B}{X_D}}\right) X_D$$
$$= \frac{1}{4}\left(X_D - 2(2-R_B) + \sqrt{1-4(2-R_B)X_D}\right)$$
$$= U^{r,n}_{D|p_{B=1}}$$

As $U_D^{r,n}$ is increasing with p_B, $U_D^r \geq U_D^{r,n}$ for all $p_B \geq \bar{p}_B$, with a strict inequality for $p_B < 1$. This concludes the proof for the case $p_B \geq \bar{p}_B$.

Step 4. *Proof that $U_D^{r,n} \leq U_D^r$ and $\mathbb{E}[p_{D,s}^{r,n}] < p_D^r$ for $p_B < \bar{p}_B$*

When $p_B < \bar{p}_B$, the face value of the loan in the absence of news satisfies $R_D^r \leq X_B$ by definition of $\bar{p}_B$. In the model with news, two cases are possible, with, either $R_D^{r,n} \leq X_B$, or $R_D^{r,n} > X_B$. Consider first the case $R_D^{r,n} \leq X_B$. Then the comparison between equations (C.11) and (C.18) shows that $\mathbb{E}[p_{D,s}^{r,n}] < p_D^r$ because $(1-p_B)^2 < (1-p_B)$. The comparison between equations (C.12) and (C.19) shows that $U_D^{r,n} < U_D^r$ for the same reason.

Suppose now that the equilibrium with news is such that $R_D^{r,n} > X_B$. We first show that the expected level of effort is lower than in the model without news. Using equation (15), we obtain

$$\mathbb{E}[p_{D,s}^{r,n}] = \frac{X_D + p_B X_B - R_D^{r,n}}{X_D} \leq \frac{X_D - (1-p_B)R_D^{r,n}}{X_D} \leq \frac{X_D - (1-p_B)R_D^r}{X_D} = p_D^{r,n}$$

where the first inequality follows from $X_B \leq R_D^{r,n}$ and the second inequality is implied by the assumed inequality $R_D^{r,n} \geq X_B \geq R_D^r$.

We are then left to show that $U_D^{r,n} \leq U_D^r$ in this case. For this, suppose agent D could commit to the maximum effort level $\tilde{p}_{Dg} = 1$ in state g. Given the face value $\tilde{R}_D^{r,n}$ that the lender would require, D's effort choice in state b would be given $\tilde{p}_{Db}^{rn} = \frac{X_D - \tilde{R}_{D,n}}{X_D}$. Fixing the face value of the loan $\tilde{R}_D^{r,n}$, these effort levels are the same than in the case analyzed in Step 2. Hence, the fictitious face value $\tilde{R}_D^{r,n}$ and agent B's utility $\tilde{U}_B^{r,n}$ would be given by equation (C.17) and (C.19), respectively. We have shown above that $\tilde{U}_D^{r,n} \leq U_D^r$ for all values of p_B. Because the ability to commit in state g is valuable, we have $U_D^{r,n} \leq \tilde{U}_D^{r,n}$ which implies $U_D^{r,n} \leq U_D^r$ also in the case when $R_D^{r,n} > X_B$. This concludes the proof for the case $p_D < \bar{p}_D$.

C.6 Proof of Proposition 5

The proof of Proposition 4 shows that the utility of agent D with news is increasing with p_B. In addition, $U_D^{r,n} \leq U_D^r$ for all values of p_B with a strict inequality except for $p_B = 1$. By Proposition 2, we also know there exists a threshold $\underline{p}_B$ such that $U_D^r \geq U_D$ if and only if $p_B \geq \underline{p}_B$. Combining these three observations, there exists a threshold $\underline{p}_B^n \geq \underline{p}_B$ such that $U_D^{r,n} \geq U_D$ if and only if $p_B \geq \underline{p}_B^n$.

Because $\mathbb{E}[p_{D,s}^{r,n}]$ is increasing with p_B and always lower than p_D^r, a similar argument establishes that there exists a threshold $p_B^{*,n} \geq p_B^*$ such that $\mathbb{E}[p_{D,s}^{r,n}] \geq p_D$ if and only if $p_B \geq p_B^{*,n}$. We are thus left to show that the fragility region with news, that is, the region $[\underline{p}_B^n, p_B^{*,n}]$ is non-empty. As we proved that $U_D^{r,n}$ is increasing with p_B, it is enough to show that D's utility with re-use (and news) is higher than without re-use for $p_B = p_B^{*,n}$.

Suppose first that $R_D^{r,n} \geq X_B$ for $p_B = p_B^{*,n}$. Then, D's utility is given by equation (C.16). By definition of $p_B^{*,n}$, $\mathbb{E}[p_D^{r,n}] = p_D$ when $p_B = p_B^{*,n}$, and, thus

$$U_D^{r,n} = \frac{1}{2}\mathbb{E}\left[\left(p_{Ds}^{r,n}\right)^2\right] X_D > \frac{1}{2}\left(\mathbb{E}\left[p_{Ds}^{r,n}\right]\right)^2 X_D = \frac{1}{2}p_D^2 X_D = U_D$$

using Jensen's inequality. This proves D strictly prefers re-using collateral for $p_B = p_B^{*,n}$. By continuity, the fragility region $[\underline{p}_B^n, p_B^{*,n}]$ is non-empty.

Suppose now that $R_D^{r,n} < X_B$ for $p_B = p_B^{*,n}$. Then, D's utility is given by equation (C.19), which we can rewrite as

$$\begin{aligned} U_D^{r,n} &= \frac{1}{2}p_B X_D + \frac{1}{2}(1-p_B)\left(p_{D,b}^{r,n}\right)^2 X_D + p_B(X_B - R_D^{r,n}) \\ &= \frac{1}{2}\mathbb{E}\left[\left(p_{D,s}^{r,n}\right)^2\right] X_D + p_B(X_B - R_D^{r,n}) \end{aligned}$$

As the second term of $U_D^{r,n}$ is positive when $R_D^{r,n} < X_B$ and because $\mathbb{E}[p_{D,s}^{r,n}] = p_D$ for $p_B = p_B^{*,n}$, it follows again by Jensen's inequality that $U_D^{r,n} > U_D$ for $p_B = p_B^{*,n}$. This concludes the proof for this case.

C.7 Proof of Proposition 6

Suppose that agent D takes a 2-unit loan and re-uses the collateral of agent B. Let R_D^r be the face value of the loan assuming D does not acquire information. Our discussion in the main text shows that the value of acquiring information is given by the second term of (16). Using equation (14), we have

$$\frac{1}{2}\mathbb{V}ar[p_{Ds}^{r,n}]X_D = \frac{1}{2}\frac{\mathbb{V}ar[X_B]}{X_D}$$

The comparison of the benefit of information above with the cost γ leads to Condition (18). Hence, if γ satisfies condition (18), D will acquire information in equilibrium.

C.8 Proof of Proposition 7

We will show that the threshold $\underline{X}_B$ exists. We need to find conditions such that D reuses collateral and sets $R_B < R_B^{max}$.

We first derive the condition such that $R_B < R_B^{max}$ provided agent D re-uses collateral. Denote $\bar{p}_B(p_B R_B)$ the threshold $\bar{p}_B$ introduced in Proposition 2 where the dependence of $\bar{p}_B$ on the endogenous expected value of the collateral payoff, $p_B R_B$, is emphasized. Suppose first $p_B > \bar{p}_B(p_B R_B)$ holds in equilibrium, with $p_B R_B = p_B X_B(1 - c_B p_B)$ substituting R_B thanks to equation (19). When $p_B > \bar{p}_B$, Case 2 of Proposition 2 applies. Agent D's utility is increasing with the expected value of the collateral and this utility does not depend on other moments of the distribution of the collateral payoff. This implies that the profit-maximizing face value is $R_B = R_B^{max} = \frac{X_B}{2}$ and, we obtain, $p_B = p_B^{max}$. We are left to verify the initial conjecture $p_B > \bar{p}_B(p_B R_B)$ holds. Using equation (C.8), which defines $\bar{p}_B$, this condition writes

$$1 > \frac{X_B}{X_D + \frac{X_B}{2c_B} - \sqrt{X_D^2 - 4X_D\left(2 - \frac{X_B}{4c_B}\right)}} \tag{C.20}$$

The right hand side of this inequality is increasing with X_B and it is equal to $2c_B \geq 2$ for $X_B = 8c_B$. Hence, there exists $\underline{X}_{B,1} < 8c_B$ such that $p_B > \bar{p}_B(R_B)$ holds if and only if $X_B < \underline{X}_{B,1}$.

If instead $X_B \geq \underline{X}_{B,1}$, it must be that $p_B \leq \bar{p}_B(p_B R_B)$. This implies that Case 1 of Proposition 2 applies. We showed in the proof of this Proposition that agent D's utility is given by equation (C.12). Hence, D's optimization problem is given by

$$\max_{p_B} U_D^r(p_B) = \frac{X_D}{2} + p_B(1 - c_B p_B)X_B - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1-p_B)^2}{X_B}}} \tag{C.21}$$

$$\text{subject to } p_B \leq \bar{p}_B\left(p_B(1 - c_B p_B)X_B\right)$$

The constraint ensures that the optimal choice of p_B lies below the threshold $\bar{p}_B$ so that agent D's utility is indeed given by $U_D^r(p_B)$ for any feasible choice p_B. As will be clear shortly, this constraint is redundant. The second term of the objective function is increasing with p_B, which implies agent B chooses $p_B > \frac{1}{2c_B}$. Observe that there is no benefit in increasing p_B beyond $\bar{p}_B$. Indeed, the expected value of the collateral would further decrease without any risk reduction benefit because collateral risk is irrelevant for $p_B \geq \bar{p}_B$. This observation confirms that the constraint is redundant.

Finally, we are left to verify that agent D re-uses collateral in equilibrium. Re-use is preferred if $U_D^r(p_B) > U_D$ with $U_D^r(p_B)$ defined in equation (C.21) and p_B the profit maximizing choice. Because p_B is preferred to $p_B^{max} = \frac{1}{2c_B}$, a sufficient condition for the result is that

$U_D^r(p_B^{max}) \geq U_D$. When $X_B \geq \underline{X}_{B,1}$, using equation (C.21), this condition writes

$$U_D \leq \frac{X_D}{2} + \frac{X_B}{4c_B} - 1 - \frac{2}{1+\sqrt{1-\frac{8(1-\frac{1}{2c_B})^2}{X_D}}}$$

As the left-hand side of the inequality is independent of X_B and the right-hand side is increasing with X_B, this condition defines a lower bound $\underline{X}_{B,2}$ on X_B . It is easy to verify that the condition holds strictly for $X_B = 8c_B$ and thus that $\underline{X}_{B,2} < 8c_B$. Hence, for all $X_B \geq \max\left\{\underline{X}_{B,1}, \underline{X}_{B,2}\right\}$, D prefers to re-use collateral and the optimal level of effort by B satisfies $p_B > p_B^{max}$, which means D sacrifices intermediation rents.

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