DISCUSSION PAPER SERIES

DP15730

Directed Technical Change in Labor and Environmental Economics

David Hémous and morten olsen

MACROECONOMICS AND GROWTH

CEPR

Directed Technical Change in Labor and Environmental Economics

David Hémous and morten olsen

This Discussion Paper is issued under the auspices of the Centre's research programmes:

- Macroeconomics and Growth

Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Directed Technical Change in Labor and Environmental Economics

Abstract

It is increasingly evident that the direction of technological responds to economic incentives. We review the literature on directed technical change in the context of environmental economics and labor economics, and show that these fields have much in common both theoretically and empirically. We emphasize the importance of a balanced growth path. We show that the lack of such a path is closely related to the slow development of green technologies in environmental economics and growing inequality in labor economics. We discuss whether the direction of innovation is efficient.

David Hémous - david.hemous@econ.uzh.ch
University of Zurich and CEPR

morten olsen - mgo@econ.ku.dk
University of Copenhagen

Directed Technical Change in Labor and Environmental Economics

David Hémous and Morten Olsen [*]

January 2021

Abstract

It is increasingly evident that the direction of technological responds to economic incentives. We review the literature on directed technical change in the context of environmental economics and labor economics, and show that these fields have much in common both theoretically and empirically. We emphasize the importance of a balanced growth path. We show that the lack of such a path is closely related to the slow development of green technologies in environmental economics and growing inequality in labor economics. We discuss whether the direction of innovation is efficient.

1 Introduction

Economists have long recognized that the direction of innovation responds to economic incentives (Hicks, 1932; Kennedy, 1964). While early endogenous growth theory models, such as Romer (1990) and Aghion and Howitt (1992) only featured one type of innovation, models of "directed technical change" (DTC) with several types of innovation were quickly developed. The earliest example is Aghion and Howitt (1996) who model separately research and development and analyze researchers' incentives to allocate their effort to one or the other.[1] Closer to the questions of Hicks (1932) and Kennedy (1964), Acemoglu (1998) develops a canonical DTC model where innovation can augment either low- or high- skill labor. Since then, the insights of DTC have been incorporated into several areas of economics, two of which we focus on here: Environmental and Labor Economics.

Despite some differences between these two strands of literature, we show that they have much in common both theoretically and empirically, as demonstrated by frequent cross-fertilization between the two. On the theory side, we emphasize two aspects. First, whether a given model features a Balanced Growth Path (BGP), that is, whether there exists an equilibrium path in which relevant variables grow at equal rates. The lack of such a feature is closely related to the slow development of green technologies in environmental economics and rising inequality in labor economics. Second, we discuss whether the direction of innovation is efficient: Are clean research subsidies necessary to address climate change in the presence of carbon taxation? Is there too much automation? Further, we show that there is overwhelming empirical evidence that the direction of technology responds strongly to economic incentives in the environmental context, and emerging evidence in the labor context.

Section 2 briefly presents a version of the DTC models of Acemoglu (1998, 2002). Section 3 shows how environmental economics has used this framework. Section 4 continues with more recent DTC models that depart from the usual assumption of factor-augmenting technical change to study automation. Finally, Section 5 presents empirical evidence.

---

[1]In their model, research corresponds to the development of a new potential line of products and development to secondary innovations which introduce one of these products.

2 The Canonical Directed Technical Change Model

The last two decades of the 20th century saw a concurrent increase in both the skill ratio and the skill premium. A common explanation for this is that Skill-Biased Technical Change meant that the relative demand for skill outpaced the relative supply (Goldin and Katz, 2008). Acemoglu (1998) notes that this simultaneity is in need of an explanation and seeks to endogenize the rise in skill demand as a technological response to the increase in the skill ratio. Acemoglu (1998, 2002) feature two competitive markets for intermediate goods. These goods are combined using an aggregate CES production function:

$$Y = \left(Y_L^{\frac{\varepsilon-1}{\varepsilon}} + Y_H^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $\varepsilon > 0$ is the elasticity of substitution between the two (we omit time subscripts when they are not necessary). In the original setting, Y_L, is an intermediate input produced using low-skill labor, whereas Y_H is produced using high-skill labor. However, as we will show below, the framework is well-suited for other applications. The intermediate inputs are each produced using a combination of labor and a unique set of machines of measure one. These machines are distinct for each sector, and their productivity evolves endogenously. The level of technology of the most advanced machine, the one employed in equilibrium, is denoted $A_{ji} > 0$ for $j \in \{L, H\}$ and $i \in [0, 1]$. The production functions for the two sectors are:

$$Y_L = \frac{1}{1-\beta}L_L^\beta \int_0^1 A_{Li}^\beta x_{Li}^{1-\beta} di \text{ and } Y_H = \frac{1}{1-\beta}L_H^\beta \int_0^1 A_{Hi}^\beta x_{Hi}^{1-\beta} di, \tag{2}$$

where L_j is the supply of workers of type j. Machines are produced monopolistically with $1-\beta$ units of the final good, which is the numeraire. With a demand elasticity of $1/\beta$, the price of a machine is 1.

Denote p_j the price of the intermediate goods. Use the monopolist's solution to find that output for intermediate input $j = L, H$ obeys

$$Y_j = \frac{1}{1-\beta}p_j^{(1-\beta)/\beta}A_j L_j, \tag{3}$$

where $A_j \equiv \int_0^1 A_{ji} di$ is the aggregate technology in sector j. Profits of a monopolist are given by:

$$\pi_{ji} = \beta p_j^{1/\beta} L_j A_{ji}. \tag{4}$$

2

Combining the final good producer's problem with labor market clearing conditions gives the skill premium:

$$\frac{w_H}{w_L} = \left(\frac{L_H}{L_L}\right)^{-\frac{1}{\sigma}} \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}}, \tag{5}$$

where $\sigma \equiv 1 + \beta(\varepsilon - 1) > 0$ is the derived elasticity of substitution between L and H and $\sigma > 1$ if and only if $\varepsilon > 1$. This mirrors the framework of Goldin and Katz (2008) (building on Katz and Murphy, 1992). They focus on the college skill-premium and reconcile the large rise in college attainment with the substantial increase in the skill-premium since the 1980s by arguing that low-skill and high-skill workers are gross substitutes, $\sigma > 1$, and by inferring a positive secular trend in A_H/A_L. They take this trend in skill-biased technical change as exogenous, whereas Acemoglu (1998) argues that when technology is endogenous, the growth in A_H/A_L can be *driven* by the increase in the skill-ratio, L_H/L_L.

To demonstrate this, we model innovation in a quality-ladder fashion (Aghion and Howitt, 1992). The literature generally models the cost of innovation in terms of the final good or of a limited factor of production, "scientists" (Acemoglu, 2002). To facilitate comparison with Section 3.1, we implement the latter. Time is discrete, and the usual Ramsey setup gives the interest rate r_t. At the beginning of every period, scientists of mass $S = 1$ can work to innovate either in the low-skill or high-skill intensive sector. Given this choice, each scientist is randomly allocated to one machine in their target sector without congestion (this can be rationalized using within sector spillovers).

Inspired by Acemoglu (2002), the probabilities of successful innovation for scientists in the low-skill and the high-skill sector are given by $\eta_L \left(A_{Ht}/A_{Lt}\right)^{(1-\delta)/2}$ and $\eta_H \left(A_{Lt}/A_{Ht}\right)^{(1-\delta)/2}$, respectively.[2] δ is inversely related to the complementary of technologies in the innovation functions. When $\delta = 1$, the innovation possibility frontier is independent of the technology levels. When $\delta < 1$, the productivity of innovation declines with the level of technology but knowledge spillovers from the other sector compensates for this in a way that permits a BGP. Once innovation is complete, the scientist increases the quality of her targeted machine by a factor $1 + \gamma$ and obtains monopoly rights until she is replaced by a future innovator. We impose the inconsequential as-

---

[2]Acemoglu (2002) has an expanding variety framework and the probability of innovation for each scientist in the low-skill sector obeys $\eta_L N_L^{(1+\delta)/2} N_H^{(1-\delta)/2}$, where N_L (N_H) is the mass of low-skill (high-skill) products. This formulation is equivalent to ours since in the expanding variety model, profits for each firm are mechanically diluted with the number of products: they are proportional to $p_L Y_L/N_L$ in the expanding-variety model but to $p_L Y_L$ in the quality ladder model.

sumption that $(1+\gamma) > (1-\beta)^{\frac{\beta-1}{\beta}}$ which ensures that the technological leader charges the unconstrained monopoly price.

We focus on a BGP where the two technologies grow at the same rate and the probability ρ that an incumbent is replaced by an entrant is the same in both sectors and constant. Moreover, the interest rate r and profits for a given technology are also constant. Therefore the value of a firm obeys:

$$V_{ji} = \frac{\pi_{ji}(1+r)}{r+\rho}. \tag{6}$$

Since scientists are randomly allocated within a sector, the expected technology obtained by an innovator in sector j is given by $(1+\gamma) A_{j(t-1)}$. Using (4), (6) and the BGP condition that the two technologies grow at the same rate (such that $A_{Lt}/A_{Ht} = A_{L(t-1)}/A_{H(t-1)}$), we obtain the relative value of innovating in the low-skill versus high-skill sector, Ω, as

$$\Omega_t = \frac{\eta_L}{\eta_H} \left(\frac{A_{Ht}}{A_{Lt}}\right)^{1-\delta} \frac{p_{Lt}Y_{Lt}}{p_{Ht}Y_{Ht}} = \frac{\eta_L}{\eta_H} \underbrace{\left(\frac{p_{Lt}}{p_{Ht}}\right)^{\frac{1}{\beta}}}_{\text{price effect}} \underbrace{\frac{L_L}{L_H}}_{\text{market size effect}} \underbrace{\left(\frac{A_{Lt}}{A_{Ht}}\right)^{\delta}}_{\text{technology effects}}.$$

The first equality emphasizes Kennedy (1964)'s finding that the relative incentive to innovate combines the innovation possibility frontier and the relative factor shares (more specifically intermediate input shares). The second equality emphasizes Acemoglu's decomposition between a price effect, a market size effect, and technology effects. Innovation has higher value in the sector with the more expensive good and the larger labor market. Technology also directly increases the value of innovation, but this effect is diminished by the presence of knowledge spillovers (when $\delta < 1$) across the two types of technology. Solving for the relative price p_L/p_H returns:

$$\Omega_t = \frac{\eta_L}{\eta_H} \left(\frac{L_L}{L_H}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{A_{Lt}}{A_{Ht}}\right)^{\frac{\delta\sigma-1}{\sigma}}. \tag{7}$$

Innovation can only occur in both sectors when $\Omega = 1$. If $\delta\sigma > 1$, the relative incentive to innovate in low-skill products is increasing in A_{Lt}/A_{Ht}, and a BGP is not stable. Therefore, except for knife-edge cases, the economy eventually features innovation in only one sector. Intuitively, the sector with a technological advantage commands a larger revenue share when the elasticity of substitution σ is larger, and low knowledge

spillovers (high δ) make a BGP less likely.

In contrast, if $\delta\sigma < 1$, a stable BGP with innovation in both sectors is possible. Solving for A_{Lt}/A_{Ht} and using the expression for the skill-premium in (5) gives that on a BGP:

$$\frac{w_H}{w_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\frac{\sigma-1}{1-\sigma\delta}} \left(\frac{L_H}{L_L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}.$$

This replicates the "strong induced-bias" hypothesis of Acemoglu (1998): if $\sigma > 2 - \delta$ (and $\delta\sigma < 1$), an increase in the skill-ratio increases the skill premium. Intuitively, an increase in the skill ratio leads to skill-biased technical change: an increase in L_H/L_L decreases Ω which pushes innovation towards the high-skill sector if and only if $\sigma > 1$ (see 7) and a decline in A_L/A_H is high-skill biased if and only if $\sigma > 1$ (see 5). When the two inputs are sufficiently substitute, the technological response is sufficient to overturn the direct supply effect.[3]

Acemoglu (2003) uses an analogous framework to demonstrate that when capital is a reproducible factor, and capital and labor are complements, innovation must be labor-augmenting. This endogenizes one of the assumptions underlying the Uzawa theorem and ensures stable factor shares. An extensive literature has emerged building on the framework of Acemoglu (1998), including Acemoglu and Zilibotti (2001) and Acemoglu, Zilibotti and Gancia (2012). In the following, we focus on applications to environmental economics (Section 3) as well as new DTC models that depart from factor-augmenting technologies (Section 4).[4]

3   Directed Technical Change and the Environment

While policymakers and climate scientists have long argued that overcoming the challenges of climate change requires the development of clean technologies, the economics literature initially focused on models with exogenous technological change (see e.g. Nordhaus, 1994). Meanwhile, a growing empirical literature has shown that innovation responds to energy prices (see Section 5). Several papers added induced technical change

---

[3]Acemoglu (2007) demonstrates that this result holds very generally in models with factor-augmenting technology. See also Loebbing (2020) who demonstrates that an increase in the skill-ratio increases the skill premium if and only if the production function is quasi-convex in factors taking endogenous technology responses to factors into account.

[4]We focus on models of imperfect competition where profits drive innovation effort. There is a small literature on DTC models with perfect competition, but these models are too different in their setup to cover them here (Irmen, 2017, Irmen and Tabaković, 2017, and Casey and Horii, 2019; as well as the references therein).

to computable general equilibrium (CGE) models. Still they did not build on modern growth theory and therefore either ignored knowledge externalities or modeled them in an ad-hoc way: for instance, in Nordhaus (2002) and Popp (2004, 2006) technological progress results from the accumulation of an R&D stock similar to capital.[5] Bovenberg and Smulders (1995, 1996) present the first model of modern endogenous growth theory in an environmental context but only model one type of innovation.

We focus here on DTC models, which build on modern endogenous growth theory and feature two different types of innovations. In the environmental context, these models come in two varieties. Some focus on energy-saving innovation and model energy or a resource as an input complementary to capital or labor (the first example is Smulders and de Nooij, 2003). Others analyze DTC between two substitute inputs where one is cleaner than the other (Acemoglu, Aghion, Bursztyn and Hémous, 2012, henceforth AABH).[6] We start with the substitute case in Section 3.1, move to the complement case in Section 3.2, and present further applications of the DTC framework in Section 3.3 . Our review is not exhaustive and we focus primarily on recent work.[7]

3.1   The substitute case: clean and dirty energy

AABH build on the framework of Section 2, but the two inputs differ in whether they generate greenhouse gas emissions (the dirty input Y_{dt}) or not (the clean input Y_{ct}). The two inputs are assumed to be substitute ($\varepsilon > 1$, see empirical evidence in Papageorgiou, Saam and Schulte, 2017) so that this framework can be used to analyze the choice between renewable (or nuclear) and fossil fuel energy, or the choice between electric and fossil fuel vehicles.[8] Production occurs as in Section 2, except that the labor allocation between the two sectors is endogenous.

CO_2 emissions are directly proportional to the use of the dirty input. Implicitly, using the dirty input requires consuming a freely available fossil fuel with a Leontif technology. As a result, AABH do not model improvements in energy efficiency or

---

[5]See also Goulder and Schneider (1999), Massetti, Carraro, and Nicita (2009) or Sue Wing (2003). Gerlagh and Lise (2005) and Grimaud and Rouge (2008) microfound innovation but still impose ad-hoc relationships between its social and private values.

[6]Instead of building on Acemoglu (1998)'s DTC framework, Hart (2004) and Ricci (2007) present models where innovation either only increases the productivity of an intermediate or increases it by a lower amount while making it cleaner.

[7]For other literature reviews see Popp, Newell and Jaffe (2010) and Fischer and Heutel (2013).

[8]Aghion and Howitt (2009, ch. 16) preempt some of the results of AABH in the case of perfect substitutes.

resource productivity ("thermal efficiency") of power plants or fossil fuel vehicles but focus on other innovations that reduce their effective costs.[9]

Innovation is modeled as in the previous section except that patents only last for 1 period and $\delta = 1$ so that the innovation possibility frontier is independent of the technology levels.[10] The law of motion of input $j \in \{c, d\}$ technology is:

$$A_{jt} = (1 + \gamma \eta_j s_{jt}) A_{jt-1},$$

where s_{jt} is the mass of scientists in sector j, η_j their productivity and γ the innovation size. This innovation set-up features a "building-on-the-shoulders-of-giants" externality since an innovator not only improves the current technology but also enables future innovators to build on her innovation.

As profits still obey (4), the expected profits of a scientist working for sector j are:

$$\Pi_{jt} = \eta_j (1 + \gamma) \beta p_{jt}^{\frac{1}{\beta}} L_{jt} A_{j(t-1)} = \frac{\eta_j \beta p_{jt} Y_{jt}}{1 + \gamma \eta_j s_{jt}}.$$

Scientists target the sector with the highest expected profits which is the clean sector if the following ratio is greater than 1:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c (1 + \gamma \eta_d s_{dt})}{\eta_d (1 + \gamma \eta_c s_{ct})} \frac{p_{ct} Y_{ct}}{p_{dt} Y_{dt}} = \frac{\eta_c}{\eta_d} \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{\beta}}}_{\text{price effect}} \underbrace{\frac{L_{ct}}{L_{dt}}}_{\text{market size effect}} \underbrace{\frac{A_{ct-1}}{A_{dt-1}}}_{\text{direct productivity effect}}. \quad (8)$$

Therefore scientists target the sector with the largest revenue (adjusted with the productivity of the innovation technology). Relative revenues depend on the same forces as above. Yet, there are no cross-sectoral knowledge spillovers and the labor allocation is now endogenous, with the more advanced sector attracting relatively more labor when the inputs are substitute.

---

[9]Such innovation could be included if pollution were proportional to the use of dirty machines, x_{dit} (similar to Gans, 2012). This would not change any of the following results.

[10]The supply of R&D resources is fixed so that clean R&D fully crowds out dirty R&D. This is not an innocuous assumption as a policy which aims at increasing clean innovation also depresses dirty innovation and output growth (see Popp, 2004).

We can then express the relative expected profits from innovation as:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma \eta_c s_{ct}}{1 + \gamma \eta_d s_{dt}}\right)^{\sigma-2} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\sigma-1}. \tag{9}$$

When the two inputs are substitute, the price effect is weaker and innovation tends to be directed toward the most advanced sector: it exhibits *path dependence*, which is the first lesson of the framework.[11] In fact, the solution is "bang-bang" and for a sufficiently low ratio A_{c0}/A_{d0}, all innovation at time 1 occurs in the dirty sector. A_{ct}/A_{dt} further declines and innovation remains locked in dirty technologies. Intuitively, we should not expect much clean innovation in *laissez-faire* because an innovation which aims at improving a component in a solar panel would have a much smaller market than an innovation aimed at improving a component in a fossil fuel power plant. Therefore, while the canonical model of Section 2 focuses on a BGP, we focus here on unbalanced trajectories.[12]

As a result, when fossil fuel technologies are initially ahead, the production of dirty inputs in laissez-faire grows without bound and so do CO_2 emissions. To prevent this, a social planner could implement a carbon tax or research subsidies for clean innovation. A carbon tax imposes a wedge between the producer price of the dirty input and its marginal product in final good production, and decreases the producer price p_{dt} for given technologies in equation (8). A clean research subsidy directly multiplies the right-hand-side.[13] With a sufficiently strong policy intervention, the social planner can redirect innovation away from dirty towards clean technologies. If this intervention is maintained for a sufficiently long time, clean technologies will catch up, and market forces will favor clean innovation. When the two inputs are sufficiently substitute ($\varepsilon > 1/\beta$), a temporary intervention is enough to ensure that emissions decline in the long-run. This intervention, however, has the cost of lower productivity growth during the catch-up phase while innovation is improving the less productive input. Yet, the longer the social planner waits, the larger the gap between clean and dirty technologies before the intervention, and the longer the intervention and the larger the costs. This is the second lesson from the framework: taking endogenous technical change into account

---

[11]For more discussion on path dependence see the review of Aghion, Hepburn, Teytelboym and Zenghelis (2019).

[12]With cross-sectoral knowledge spillovers as in the innovation function of Section 2 where scientists' productivity obeys $\eta_j (A_{(-j)t}/A_{jt})^{(1-\delta)/2}$, there is still path dependence when $\sigma > 2 - \delta$. The difference with the threshold $\sigma > 1/\delta$ given above comes from the endogeneity of the labor allocation.

[13]That is, (9) becomes $\frac{\Pi_{ct}}{\Pi_{dt}} = (1 + q_t)(1 + \tau_t)^\epsilon \frac{\eta_c}{\eta_d} \left(\frac{1 + \gamma \eta_c s_{ct}}{1 + \gamma \eta_d s_{dt}}\right)^{\sigma-2} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\sigma-1}$, where q_t is a clean research subsidy and τ_t is an (add-valorem) carbon tax.

calls for *earlier intervention*. Gerlagh, Kverndokk and Rosendahl (2009) similarly find that endogenous innovation in abatement technology calls for a front-loaded policy.

Finally, AABH study the optimal policy when the representative agent values consumption and is hurt by environmental degradation. They show that this policy can be decentralized using a Pigovian carbon tax and research subsidies to clean innovation (plus a subsidy to remove the monopoly distortion). This is the third lesson from the framework: *a carbon tax is not enough to obtain the first best.* In the optimum, innovation is allocated to the sector with the highest social value. The ratio of social values can be expressed as:

$$\frac{SV_{ct}}{SV_{dt}} = \frac{\eta_c \left(1 + \gamma\eta_d s_{dt}\right) \sum_{\tau \geq t} \lambda_{t,\tau} p_{c\tau}^{\frac{1}{\beta}} L_{c\tau} A_{c\tau}}{\eta_d \left(1 + \gamma\eta_c s_{ct}\right) \sum_{\tau \geq t} \lambda_{t,\tau} p_{d\tau}^{\frac{1}{\beta}} L_{d\tau} A_{d\tau}},\tag{10}$$

where $\lambda_{t,\tau}$ is the discount factor between t and τ. This ratio reflects the environmental value as a higher carbon tax decreases $p_{d\tau}$. However, even with a carbon tax, the market still allocates innovation according to the ratio of equation (8), and will in general not implement the first best without a research subsidy. Intuitively, the social planner allocates innovation according to the discounted benefits that a higher technology brings in every period, while the market only cares about immediate profits.

This intuition extends to the case of patents lasting more than one period, or patents lasting till the following innovation (as mentioned in AABH, see also Greaker, Heggedal and Rosendahl, 2018). Moreover, the private value does not internalize the building-on-the-shoulders-of-giants externality. To see this, consider an extreme case with perpetual patents, such that future innovators would have to pay royalties to the incumbent to compensate them for their profit loss once the new technologies has arrived. In that set-up, the ratio of private values of innovation would obey:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c \left(1 + \gamma\eta_d s_{dt}\right) \sum_{\tau \geq t} \lambda_{t,\tau} p_{c\tau}^{\frac{1}{\beta}} L_{c\tau} A_{ct}}{\eta_d \left(1 + \gamma\eta_c s_{ct}\right) \sum_{\tau \geq t} \lambda_{t,\tau} p_{d\tau}^{\frac{1}{\beta}} L_{d\tau} A_{dt}}.\tag{11}$$

In this setting only the building on the shoulders-of-giants externality is active. The only difference between equations (10) and (11) is that (11) sums over the expected technology of the current innovation, A_{jt}, instead of a future time τ. This corresponds

9

directly to the building-on-the-shoulders-of-giants externality: not only does an innovator improve current technology but all future technology since innovators build on each other's work.[14] Therefore, finite-lived patents, creative destruction, imitation, and the building-on-the-shoulders-of-giants externality all imply that the private value of an innovation tends to be more short-sighted than its social value. The key is that this "short-termism" does not affect clean and dirty technologies equally. Consider a setting in which dirty technologies are initially more advanced, but the clean technologies must dominate in the future in the social planner's allocation. A larger fraction of the social value of dirty innovation is realized in the short-run than for clean innovation. In other words, a high share of the social value from improving a solar panel today comes from the benefits of getting better solar panels in the future, while most of the benefits from improving coal power plants are realized today. Then, the short-termism in the market innovation allocation implies inefficiently low clean innovation relative to dirty even with Pigovian taxation.[15]

To summarize AABH provide three lessons. First, there is path dependence in the development of clean versus dirty technologies. Second, taking into account the endogeneity of innovation calls for earlier action. Third, in addition to Pigovian carbon taxation, the optimal policy includes research subsidies specifically devoted to clean innovation.

Acemoglu, Akcigit, Hanley and Kerr (2016) further build on AABH by calibrating a firm dynamics model with clean and dirty innovation. A final good is produced as a Cobb-Douglas aggregate of a mass 1 of intermediates. Each intermediate can be produced with a clean or a dirty technology, which evolve on their own ladder and are perfect substitutes within a line. A firm is a collection of leading clean or dirty technologies in different lines. Innovation can be incremental, building on each technology separately, or radical, building on the leading technology whether it is clean or dirty. As a result, their model features cross-sectoral spillovers which were absent in AABH and mitigate (without eliminating) path dependence in innovation. As in the "new DTC models" of section 4, technical change in each line is microfounded (similar to the tasks below) instead of immediately taking a factor-augmenting form at the aggregate level. They calibrate their model to the US energy sector. Their conclusions are in line with

---

[14] In contrast, the optimal policy in a model with horizontal innovation and DTC need not feature research subsidies on top of Pigovian taxation.

[15] Gerlagh, Kverndokk and Rosendahl (2014) make a related point in a model with clean innovation only. This contrasts with an earlier literature of integrated assessment models with a constant ratio of social to private value of innovation (Nordhaus, 2002, Popp, 2004, 2006, or Gerlagh and Lise, 2005).

AABH: the optimal policy requires both (large) clean research subsidies and a carbon tax and features a rapid switch from dirty to clean innovation.

3.2 The Complementarity case: Energy-saving Innovation

While AABH focus on the development of clean substitutes to dirty inputs, others have focused on energy- or resource-saving innovations. In our framework, the final good is produced with (1) but the two inputs Y_L and Y_H are replaced by a production input Y_P and an energy-services input Y_E with $\epsilon < 1$. Both inputs are produced similarly to equation (2) with a capital-labor aggregate instead of low-skill labor for Y_P and energy (or a fossil fuel resource) E for Y_E. The papers differ in whether there is a fixed resource flow (Smulders and de Nooj, 2003), a constant resource price (Shanker and Stern, 2018) or an exhaustible resource stock (Di Maria and Simone Valente, 2008, André and Smulders, 2014, and Hassler, Krusell and Olovsson, 2019). This literature seeks to account for stylized facts regarding energy consumption and growth.

In particular, Hassler, Krusell and Olovsson (2019) build a quantitative macroeconomic model. They estimate an elasticity of substitution between energy and other inputs close to 0 and show that energy-saving technical change took off in the 70s with the oil shocks in line with the DTC theory. Their model further predicts that thanks to the innovation response resource scarcity will only lead to a slight increase in the energy share. One of their conclusions is that "subsidies may not be necessary for regulating the direction of technical change." Why are their conclusion so different from AABH? Because energy is a complement to other inputs and consequently a BGP arises more easily.

Within the framework we have sketched and with one period patents, the relative expected profits from labor-augmenting over energy-augmenting innovation obey:

$$\frac{\Pi_{Pt}}{\Pi_{Et}} = \frac{\eta_P \left(1 + \gamma \eta_E s_{Et}\right)}{\eta_E \left(1 + \gamma \eta_P s_{Pt}\right)} \frac{p_{Pt} Y_{Pt}}{p_{Et} Y_{Et}} = \frac{\eta_P}{\eta_E} \left(\frac{p_{Pt}}{p_{Et}}\right)^{\frac{1}{\beta}} \frac{L}{E} \frac{A_{Pt-1}}{A_{Et-1}} = \frac{\eta_P \left(1 + \gamma \eta_E s_{Et}\right)}{\eta_E \left(1 + \gamma \eta_P s_{Pt}\right)} \left(\frac{L A_{Pt}}{E A_{Et}}\right)^{\frac{\sigma-1}{\sigma}}.$$
(12)

Since the two inputs are complement, the price effect dominates. When the resource flow is constant (as in Smulders and de Nooj, 2003), innovation tends to favor the least advanced sector (following the last expression in 12) and in the long-run, the economy converges to a balanced growth path (BGP) with innovation in both sectors ($\Pi_{Pt} = \Pi_{Et}$), equal growth in the two sectors and constant factor shares (following the

11

first equality in 12). When the resource flow decreases (because of resource exhaustion or a growing carbon tax), the resulting increase in the energy share favors energy-saving technical change (again from the last expression in 12).[16] Here as well, the economy converges in the long-run toward a BGP with a constant interior energy share but where energy-saving technical change A_{Et} grows faster than labor-saving technical change A_{Pt} to compensate for the reduction in the resource flow. This follows the same logic as Acemoglu (2003) where labor scarcity leads to labor-augmenting technical change. With a constant resource price, the logic is reversed and innovation in the long run is entirely labor-augmenting.

The social planner solution also features balanced growth and converges toward the same innovation allocation as in the equilibrium provided that energy is optimally priced through a carbon tax. Research subsidies may be necessary in the transition, but their importance is greatly reduced (Hassler, Krusell and Olovsson, 2019, show a case where they are not necessary in the transition either, see also Hart, 2008). The "short-termism" of the market innovation allocation now favors the least advanced technology adjusting for resource availability, which ensures that the economy moves toward a balanced growth path, as called for by the social planner. While public intervention is crucial to the development of clean alternatives to fossil fuel energy, carbon pricing can do the heavy lifting for the development of energy-saving technologies.[17]

A consequence of DTC is that while the short-run elasticity between energy and other inputs is very low, the long-run energy share is constant. One may be tempted to conclude that climate models are not missing much by ignoring energy-saving technical change and simply assuming that energy enters final good production in a Cobb-Douglas way. Casey (2019) shows that this would be misguided. He builds a model similar to Hassler et. al (2019) where energy and the capital-labor aggregate are Leontief for given technologies, but the long-run elasticity is 1 for the same reason as above. He calibrates both his DTC model and a Cobb-Douglas economy to US data, and shows that a given carbon tax is less effective at reducing cumulative emissions in the DTC model. Intuitively, technological adjustment is sluggish and with Leontief technology emissions do not decline as rapidly as in a Cobb-Douglas setting. Since climate damages depend on the stock of emissions, this transition period matters quantitatively.

---

[16]A tax on energy, E, moves innovation toward A_E when $\varepsilon < 1$; a tax on energy-services Y_E moves innovation toward A_P regardless of the value of ε.

[17]This conclusion may not hold in the presence of multiple equilibria as shown by van der Meidjen and Smulders (2017).

3.3 Applying Directed Technical Change to Environmental Questions

In the following, we review papers that use these two DTC frameworks in the context of energy shocks, historical energy transitions, and carbon leakage.

Energy market shocks. Fried (2018) uses the oil shocks of the 1970s to calibrate a DTC model which combines elements of both AABH and Hassler et al. (2019) but features a more detailed representation of the economy. A final good is produced with a production input and energy services, which are themselves an aggregate of local fossil fuel energy, oil imports and green energy. The production input and energy services are highly complement while the different types of energy are substitutes. Innovation can be targeted at local fossil fuel energy, green energy or the production input. As in AABH, emissions are proportional to the quantity of fossil fuel energy. She studies the implementation of a carbon tax, which can cut emissions by 30% in 20 years. Such a carbon tax redirects innovation away from fossil fuel energy toward mostly green energy. DTC reduces the size of the necessary carbon tax by 19.2% compared to a model with exogenous technical change.[18]

Acemoglu, Aghion, Barrage and Hémous (2019) build on AABH to study the shale gas boom, which started in 2009. They show that since then the ratio of renewable patents relative to fossil fuel patents in the electricity sector has declined sharply. To analyze the consequences on emissions, they build a DTC model where electricity can be green or produced with coal or natural gas. Innovation can be targeted at improving the productivity of fossil fuel power plants or green power plants. Following a drop in the natural gas prices (as from the shale gas boom) electricity production shifts toward natural gas. Since natural gas is much cleaner than coal, emissions decrease in the short-run. However, the price decline also increases the market for innovations in fossil-fuel power plant and as a result, green innovation declines. Calibrating their model to the US electricity sector, they find that this innovation effect eventually dominates, so that emissions increase in the medium term following the shale gas boom. They argue that policymakers should react to the shale gas boom by raising subsidies to green innovation.[19]

---

[18]Hart (2019) also calibrates an integrated assessment model with AABH features. The optimal policy includes both a carbon tax and clean research subsidies but the relative importance of research subsidies is diminished particularly because of intersectoral knowledge spillovers.

[19]In a similar spirit, Acemoglu and Rafey (2019) look at the effect of an exogenous shock to geo-engineering technology. They find that when environmental policy is endogenous and commitment is

Historical energy transitions. DTC can also be used to explain historical energy transitions. Stern, Pezzey and Lu (2020) build on Acemoglu (2002) to explain the Industrial Revolution as resulting from the transition from a wood-powered to a coal-powered economy. In their model, the final good is produced with two substitute intermediate inputs; one wood-intensive and the other coal-intensive. Innovation may be directed at either. Wood is in fixed supply each period while coal is supplied at a fixed extraction cost. Constant long-run growth is only possible in a coal-based economy, and their model can generate transitional dynamics akin to the British Industrial Revolution: Initially, the economy relies mostly on wood and grows slowly but with economic development it progressively shifts toward coal which spurs innovation in coal technologies through a market size effect. This leads to a take-off in economic growth.[20]

Lemoine (2018) builds a DTC model where different energy services are produced using two complement inputs, machines and natural resources (as in Acemoglu et al., 2019), and where natural resources are isoelastically supplied. Even though the model generates endogenous energy transitions, a calibration shows that the optimal climate policy still relies on clean research subsidies to accelerate the transition to renewables.

Carbon leakage. The models we have studied so far all consider either a country in isolation or a global solution. In practice, international climate negotiations have stalled and countries have largely conducted climate policy unilaterally. International trade, however, may reduce the scope for unilateral actions as it may lead to "carbon leakage" (a move of the production of polluting goods from regulated to unregulated countries). The DTC literature shows that the elasticity of substitution between traded goods and the pattern of innovation/imitation across countries play a crucial role in determining carbon leakage. Di Maria and Smulders (2004) consider a two-country (North, South), two goods (energy-intensive, non–energy-intensive) trade model where the North innovates while the South imitates exogenously. The implementation of a carbon tax in the North and the ensuing reallocation of energy-intensive production to the South leads to an increase in innovation in the non-energy intensive sector. This reduces carbon leakage when the goods are substitute and amplifies it when they are complements (innovation in the energy-intensive sector is resource-augmenting and therefore resource-saving in

---

impossible, such a shock may decrease clean innovation as it would reduce future environmental taxes. Progress in geoengineering technology may then backfire leading to an increase in emissions.

[20]Similarly, Gars and Olovsson (2019) build a DTC model to explain the 19th century Great Divergence. In their model, a switch from wood-powered to fossil-fuel powered innovation leads to faster economic growth. However, one country switching to fossil fuels raises their world price, which reduces innovation in fossil-fuel technologies elsewhere.

the complement case). Di Maria and van der Werf (2008) start from the same set-up but allow both countries to innovate on a global market. They find that carbon leakage is always reduced by the innovation response to a unilateral cut in emission. Acemoglu, Aghion and Hémous (2014) and van den Bijgaart (2017) focus on endogenous imitation or innovation by the South (the unregulated country) by extending AABH to a two-country set-up. In both cases, the technological response by the South following a unilateral carbon tax by the North amplifies carbon leakage.

Hémous (2016) analyzes which unilateral policy can be successful with endogenous innovation. He also considers a two-country, two-goods (energy-intensive, non–energy-intensive) trade model with unit elasticity between the two, but the energy-intensive good can be produced in a clean or a dirty way (as in AAHB). In each country, innovation can be targeted at the non-energy-intensive sector, or within the energy-intensive sector at clean or dirty technologies. The innovation response from the unregulated country amplifies carbon leakage: the implementation of a unilateral carbon tax displaces the production of the energy-intensive good toward the unregulated country, which increases dirty innovation in that sector when the dirty technology is more advanced than the clean one there. A unilateral carbon tax may then backfire and lead to an increase in global emissions. Instead, a green industrial policy, consisting of green research subsidies and possibly carbon tariffs, can reduce emissions in both countries by directing innovation within the regulated country toward the clean sector, and innovation in the unregulated country toward the non-energy intensive sector and (with strong knowledge spillovers) clean energy. Overall, trade acts a double-edged sword: unilateral carbon taxes are less effective, but an appropriate policy can decrease emissions globally.[21]

Other applications and future research. Therefore DTC theory often provides policy answers that differ from models with exogenous technology and accounts well for historical trends. This calls for further integrating DTC in climate change economics. In particular, microfounded DTC should be more systematically incorporated in Integrated Assessment Models. Dietz and Lanz (2019) is a recent example in a detailed multisectoral model with endogenous population dynamics. Kruse-Andersen (2020) also includes population dynamics into a DTC model. Another important avenue is to expand the 2-country set-ups discussed above for more realistic models of international environ-

---

[21]Witajewski-Baltvilks and Fischer (2019) also build on AABH but with trade in machines so that innovation incentives reflect market conditions in both countries. A unilateral clean research subsidy can redirect innovation toward clean technologies in both countries if the regulated country is large enough. Moreover, it may induce the government of the unregulated country to introduce their own clean research subsidy as long-run growth is higher when the two countries innovate in the same sector.

mental agreements building on game-theoretic contributions such as Barrett (2006) and Harstad, Lancia and Russo (2019). Finally, climate change is a problem riddled with uncertainties about climate dynamics, climate damages but also technological prospects. The models reviewed here are all deterministic but the interaction between technology and uncertainty is a promising avenue for future research (see Heutel, Moreno-Cruz and Shayegh, 2018, who show that geoengineering can be used as an insurance mechanism against climate uncertainty).

4 Automation and new Directed Technical Change models

We have argued that the canonical DTC has provided insights both for the study of income inequality and environmental issues. Nevertheless, a few papers have criticized this framework for being too imprecise in describing the effect of technology on work. Among these, Autor, Levy and Murnane (2003) postulate the "routinization hypothesis" by introducing the notion of tasks: the work inputs required for producing a given output. They argue that since computers are highly capable of performing tasks that can be codified in a computer program—labeled routine tasks—they are disproportionately substitutable to workers performing such tasks. Specifically, they model output in an industry i as:

$$y(i) = (l_R(i) + x(i))^{\beta_i} \, l_N^{1-\beta_i}(i), \tag{13}$$

where l_R denotes the input of routine labor, l_N the input of non-routine labor, x the use of computer capital and $\beta_i \in (0, 1)$ the importance of routine-tasks in a given industry. Autor et al. (2003) formalize technical progress as the continuously declining price of computer capital. When both are employed, the specification above implies that low-skill wages equal the price of computer capital and correspondingly decline. They show empirically that the implementation of computer capital correlates strongly with changes in the use of routine-tasks across industries. Acemoglu and Autor (2011) argue that the canonical model based on factor-augmenting technical change cannot account for several features of the evolution of the income distribution. These include a continuous increase in labor income inequality and absolute declines in low-skill wages (see also Acemoglu and Restrepo, 2020b). Furthermore, the canonical model does not microfound automation as the replacement of workers with machines in the execution of certain tasks. To do so,

16

they extend the Autor et al. (2003) model with exogenous technology and endogenous assignment of skills to tasks.

While Habakkuk (1963) already postulated that labor scarcity encouraged innovation in the US, Zeira (1998) is the first to model automation in a growth model. Output is produced as an aggregate of intermediates, each of which can be produced with either a manual technology or more capital-intensive "automated" technology. Exogenous technological progress in TFP raises the wage so that a growing number of intermediate producers adopt the industrial technology over time. He then focuses on the role of automation in amplifying productivity differences across countries. Acemoglu (2010) shows that Habbakuk's hypothesis can only hold if innovation is labor-saving. Peretto and Seater (2013) build a dynamic model where innovation in automation changes the exponent of an aggregate Cobb-Douglas production function. Yet, none of these papers feature DTC since they only consider one type of innovation.

4.1 Automation and non-balanced growth

A recent literature, starting with Hémous and Olsen (forthcoming; henceforth HO), has explicitly built task-models into DTC frameworks. Their model endogenizes several aspects of the automation process described in Zeira (1998) and provides some answers to the Acemoglu and Autor (2011) critique of the DTC literature. HO build on the expanding-variety model (Romer, 1990) and consider an economy in which a final good is produced as a CES aggregate of a mass N of products:

$$Y = \left(\int_0^N y(i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

with $\sigma > 1$ and $y(i)$ being the use of product i. Each product is produced monopolistically using a generalization of equation (13):

$$y(i) = \left(l(i)^{\frac{\epsilon-1}{\epsilon}} + \alpha(i)x(i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon\beta}{\epsilon-1}} h(i)^{1-\beta}, \tag{14}$$

where $l(i)$ denotes low-skill workers and $h(i)$ high-skill workers. These labor inputs correspond to different tasks, so that each product comes with its own tasks. Automation occurs when machines can be used to (partly) substitute for low-skill labor in a task and $\alpha(i)$ switches from 0 to 1. In the baseline model, machines are produced one-for-one with

the final good.[22] Technology is characterized both by the number of products, N_t, and the share of automated products (or low-skill tasks) G_t. Automation is therefore a secondary innovation which occurs in product lines developed through horizontal innovation. This makes HO closer in spirit to Aghion and Howitt (1996).

Aggregate output can be represented as:

$$Y = N^{\frac{1}{\sigma-1}} \left(\{1-G\}^{\frac{1}{\sigma}} \left\{ \underbrace{(L^N)^\beta (H^{P,N})^{1-\beta}}_{T_1} \right\}^{\frac{\sigma-1}{\sigma}} + G^{\frac{1}{\sigma}} \left\{ \underbrace{[(L^A)^{\frac{\epsilon-1}{\epsilon}} + (X)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon\beta}{\epsilon-1}} (H^{P,A})^{1-\beta}}_{T_2} \right\}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

(15)

where L^A (L^N) denotes the total mass of low-skill workers in automated (non-automated) firms, $H^{P,A}$ $(H^{P,N})$ the total mass of high-skill workers hired in production in automated (non-automated) firms and $X = \int_0^N x(i)di$ total use of machines. The first term T_1 captures the case where production is non-automated. The second term T_2 represents the factors used within automated products and features substitutability between low-skill labor and machines. Equation (15) shows that G is not a factor-augmenting technology but the share parameter of the "automated" products nest. $N^{\frac{1}{\sigma-1}}$ plays the role of a TFP parameter.

HO assume that $\mu = \beta(\sigma-1)/(\epsilon-1) < 1$, which ensures that automation is low-skill labor-saving at the firm level.[23] Yet, this does not necessarily imply that automation is low-skill labor-saving for the aggregate economy. For given N and G the static equilibrium can be described as the intersection of two equations, illustrated in Figure 1, where w_L and w_H denote the wages of low- and high-skill workers, respectively. The *unit isocost* curve draws the cost of producing one unit of final good and is a downward sloping curve in (w_L, w_H) space. The *relative demand* curve is upward sloping.[24] When there is no automation, $G = 0$, the aggregate economy inherits the Cobb-Douglas structure and the relative demand curve is linear. With $G > 0$, the curve bends upwards: automated firms can substitute more toward machines and non-automated firms lose market size when w_L rises. An increase in automation pivots this curve counter-clockwise, which reduces

---

[22]In contrast to Autor et al. (2003) the price of an existing machine is fixed, but there is technological progress insofar as machines can be used in a growing number of tasks.

[23]Automation both lowers the cost of production, which increases demand for low-skill labor, and allows for the substitution towards machines, which lowers it. The latter effect dominates when $\mu < 1$.

[24]The unit iso-cost is given by $\dfrac{\sigma N^{\frac{1}{1-\sigma}} \left(G(1+w_L^{1-\epsilon})^\mu + (1-G)w_L^{\beta(1-\sigma)} \right)^{\frac{1}{1-\sigma}} w_H^{1-\beta}}{(\sigma-1)\beta^\beta(1-\beta)^{1-\beta}} = 1$, and the relative demand curve by $\dfrac{w_H H}{w_L L} = \dfrac{1-\beta}{\beta} \dfrac{G(1+w_L^{\epsilon-1})^\mu + (1-G)}{G(1+w_L^{\epsilon-1})^{\mu-1} + (1-G)}$.

18

low-skill wages, a negative *aggregate substitution effect*. It also increases the productive capabilities of the economy which pushes the isocost curve to the northwest, a positive *aggregate scale effect*. Consequently, while high-skill wages and the skill premium are always increasing in G, the effect on low-skill wages is ambiguous. In particular, for low G, the aggregate scale effect dominates and low-skill wages rise in G, but for higher levels of automation, the effect is negative (an analogous point is made informally in Autor, 2015). Horizontal innovation, an inflow of non-automated products, raises both low- and high-skill wages and, for high enough N, lowers the skill premium. Figure 1 illustrates a central feature of HO. For any paths of technology $[N_t, G_t]_{t=0}^{\infty} \in (0, \mathbb{R}^+) \times (0, 1)$ where $\lim_{t \to \infty} N_t = \infty$ and G_t has a strictly positive limit, low-skill wages must grow at a positive but lower rate than high-skill wages. This will make a BGP with equal growth in low- and high-skill wages impossible.
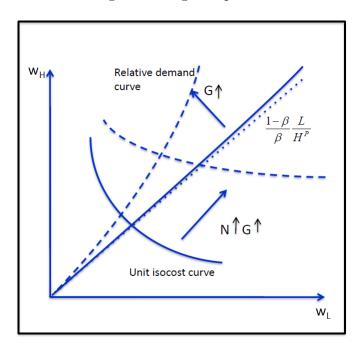

Figure 1: The static equilibrium in Hémous and Olsen (forthcoming)

HO endogenize both N_t and G_t. Horizontal innovation uses high-skill workers: $\dot{N}_t = \gamma N_t H_t^D$. Moreover, a monopolist of a non-automated firm can hire high-skill workers, h_t^A, to automate their production technology with Poisson rate $\eta G_t^{\tilde{\kappa}} \left(N h_t^A \right)^{\kappa}$, where $\kappa \in (0, 1)$ controls the curvature of the innovation function and $\tilde{\kappa} \in [0, \kappa]$ parameterizes a knowledge externality in automation innovation. This gives the law of motion:

$$\dot{G}_t = \eta G_t^{\tilde{\kappa}} (h_t^A N_t)^{\kappa} (1 - G_t) - G_t g_t^N, \tag{16}$$

where $g_t^N = \dot{N}_t/N_t$. Equation (16) has strong similarities with a capital accumulation function and the stock of automated tasks can be seen analogously: Automation of existing tasks accumulates automation stock, and the inflow of (not-yet) automated tasks depreciates the existing stock of automation. Both of these innovation processes respond to economic incentives. The resulting state of technology (N_t, G_t) then determines wages in the economy.

Denote V_t^A and V_t^N the value functions of automated and non-automated firms, respectively. Non-automated firms employ high-skill workers to automate the production process implying the following first order condition for automation innovation:

$$\eta \kappa G_t^{\tilde{\kappa}} \left(N_t h_t^A \right)^{\kappa-1} \left(V_t^A - V_t^N \right) = w_{Ht}/N_t. \tag{17}$$

The number of automation innovations therefore depends on the ratio between the increase in firm value associated with automation and its effective cost:

$$\frac{V_t^A - V_t^N}{w_{Ht}/N_t} \tilde{\propto} \frac{\pi_t^A - \pi_t^N}{G_t \pi_t^A + (1 - G_t)\, \pi_t^N} = \frac{1 - \left(1 + w_{Lt}^{\epsilon-1}\right)^{-\mu}}{G_t + (1 - G_t)\left(1 + w_{Lt}^{\epsilon-1}\right)^{-\mu}}, \tag{18}$$

where "$\tilde{\propto}$" refers to "approximately proportional" and π_t^A and π_t^N are the profits of automated and non-automated firms. Intuitively, with a positive discount rate, $V_t^A - V_t^N$ moves with $\pi_t^A - \pi_t^N$ to a first approximation. Furthermore, since both aggregate profits and high-skill labor compensation are proportional to output, w_{Ht}/N_t is proportional to average profits. The second half of the equation follows from $\pi_t^N/\pi_t^A = (1 + \varphi w_L^{\epsilon-1})^{-\mu}$. This highlights low-skill wages as the key determinant of automation innovations. When $w_{Lt} \approx 0$, there is little advantage to being automated and $\pi_t^A \approx \pi_t^N$ implying little automation innovation. When $w_{Lt} \to \infty$, the right-hand side of equation (18) approaches a constant and with it innovation in automation. In contrast with the classic DTC model with factor-augmenting technical change, the direction of technical change is entirely determined by a price effect with no market size effect (as in Acemoglu and Restrepo, 2018, below). Intuitively, horizontal innovation and automation affect the same market. This price effect bears similarity to Zeira (1998), where the adoption of a labor-saving technology also depends on the price of labor.

HO show that this economy cannot feature a BGP with equal growth in low- and high-skill wages. An economy starting with low N_t and consequently low w_{Lt} and automation is initially nearly on a BGP growing just through horizontal innovation. As low-skill wages grow so does the incentive to automate and the economy endogenously shifts

towards automation innovation. This leads to an endogenous rise in the skill premium and a decline in the labor share as experienced in the US since the 1980s. Eventually, the model features an asymptotic steady state where G_t is constant, all wages grow, but the skill premium continues to grow. HO take an extended version of the model where machines belong to a capital stock to the data. The model can replicate quantitatively (and endogenously) the evolution of the skill premium, the labor share, automation and productivity growth of the United States from 1963 to 2012.

4.2 Automation and balanced growth

Acemoglu and Restrepo (2018, henceforth AR) also consider DTC in a task model but reach sharply different conclusions. In contrast to HO, their model features a unit measure of task and output obeys:

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{19}$$

For this review, we restrict $\sigma > 1$. Therefore, the introduction of a new task N replaces an old, now obsolete task, $N-1$. Tasks are produced monopolistically using the production function:

$$y(i) = \alpha(i)k(i) + \gamma(i)l(i), \tag{20}$$

where $k(i)$ is the use of capital in the production of task i, $l(i)$ is the use of labor, and $\alpha(i) \in \{0,1\}$ is the automation index. $\gamma(i) = e^{Ai}$ is the labor-augmenting productivity of task i, where $A > 0$: new tasks feature higher labor productivity, though once automated, all tasks have the same (capital) productivity.

In the DTC case, automation is costly so that all automated firms use machines. With $\gamma(i)$ increasing in i, there is a threshold I such that tasks below I are automated ($\alpha(i) = 1$) and sold at price $p(i) = \sigma/(\sigma-1)R$, where R is the gross return on capital. In contrast, tasks in $[I, N]$ are non-automated ($\alpha(i) = 0$) and sold at price $p(i) = \sigma/(\sigma-1) \times W/\gamma(i)$, where W is the real wage. Using (19), (20), and factor clearing conditions for K and L, one gets the aggregate production function:

$$Y = \left\{ [I - (N-1)]^{\frac{1}{\sigma}} K^{\frac{\sigma-1}{\sigma}} + \left(\int_{I}^{N} \gamma(i)^{\sigma-1} di \right)^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}. \tag{21}$$

Equation (21) demonstrates that technology, N and I, determine factors shares. Hence,

as in HO, technology is not factor-augmenting.[25]

For given factors K and L, W/R depends positively on the introduction of new non-automated products, N, and negatively on the automation of existing products, I. An increase in N always increases the absolute level of wages. As in HO, an increase in automation has an ambiguous effect on wages due to the combination of a scale effect (referred to as productivity effect in AR) and a substitution effect (referred to as a displacement effect). The latter effect may dominate and leave automation labor-saving. This occurs in particular when $R \approx W/\gamma(i)$, i.e. the cost savings of automation are relatively low, a situation deemed a "so-so" automation in Acemoglu and Restrepo (2019a). Acemoglu and Restrepo (2019a, 2020a,) argue that many modern innovations in automation have this feature and correspondingly are labor-saving.[26/27]

Following this, AR endogenizes capital through a standard Ramsey setting. Innovation is undertaken by scientists who are in fixed supply S and either develop new tasks or automate existing tasks, so that $\dot{N}(t) = \kappa_N S_N(t)$ and $\dot{I}(t) = \kappa_I S_I(t)$ where S_N and S_I are the respective number of scientists. AR assume that innovators must compensate the previous incumbent. Here we focus on BGPs where both types of innovation are active. This gives the value functions of automating a task and introducing a new one, respectively:

$$V_I(t) = Y(t) \int_t^\infty e^{-\int_t^\tau (R(s)-\delta-g_y(s))ds} \left(\pi_I(\tau) - \pi_N(\tau, I(t)) \right) d\tau, \qquad (22)$$

$$V_N(t) = Y(t) \int_t^\infty e^{-\int_t^\tau (R(s)-\delta-g_y(s))ds} \left(\pi_N(\tau, N(t)) - \pi_I(\tau) \right) d\tau. \qquad (23)$$

where $g_y(t) = \dot{Y}(t)/Y(t)$, $R(s) - \delta$ is the return to capital net of depreciation, $\pi_N(t, i) = \sigma^{-1} (W(t)/\gamma(i))^{1-\sigma}$ denote the profits of non-automated tasks/products and $\pi_I(t) = \sigma^{-1} R(t)^{1-\sigma} Y(t)$ those of automated tasks/products. Equation (22) reflects the total discounted value of being a monopolist of an automated task less compensation to the previous non-automated monopolist at the time of automation, t, $I(t)$. Equation (23)

---

[25]In fact, one can write the production function (21) as $Y = \left[(AK)^{\frac{\sigma-1}{\sigma}} + (BL)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$, where $A \equiv (I - N + 1)^{\frac{1}{\sigma-1}}$ and $B \equiv (\int_I^N \gamma(i)^{\sigma-1} di)^{\frac{1}{\sigma-1}}$. When $\sigma > 1$, automation, I, increases A and decreases B and can therefore be seen as a combination of capital-augmenting and labor-depleting technical change.

[26]The aggregate substitution effect of HO is derived for an endogenous use of machines as intermediate inputs, whereas AR hold the stock of capital constant. This distinction is explicit in Acemoglu and Restrepo (2019a) which refers to the endogenous response of capital as a "capital-deepening" effect.

[27]Acemoglu and Restrepo (2020a) find that robotization leads to a decline in employment which suggests that the aggregate substitution effect dominates the scale effect.

reflects the analogous expression for a monopolist developing a new non-automated task. Therefore, the value function of a new automated task, $V_I(t)$, depends positively on the (path of) $W(t)/R(t)$, and the value function of a new non-automated task, $V_N(t)$, negatively.

Combining this insight with the fact that W/R depends positively on $N - I$, AR demonstrate that under appropriate regularity conditions (on ρ and κ_I/κ_N) a BGP exists where $\kappa_I V_I = \kappa_N V_N$, and all variables: $W(t)$, $K(t)$, $Y(t)$, $V_I(t)$ and $V_N(t)$ grow at the same rate while $R(t)$ is constant.[28] Further, N and I grow at the same rate such that the share of products that are automated, $1 - N + I$ as well as the factor shares are constant. In the full paper, AR include heterogeneous effort cost for scientists working in each type of innovation, which ensures that the allocation of scientists varies smoothly. They demonstrate that this BGP is locally stable such that a small perturbation in the stock of automation, I, above its BGP path reduces $W(t)/R(t)$, relatively discourages automation and correspondingly brings the economy back to the BGP level of automation

Therefore, AR demonstrate that one can build an economy with tasks and automation that replicates the features of a model with purely labor-augmenting technical change. In their model, workers' role in the economy remains undiminished despite the continued presence of automation. This is in sharp contrast to HO who find that no BGP is possible and the role of low-skill workers in the economy must diminish as the economy grows. Therefore, the two models present distinct views on the development of the US economy over the past decades. Seen through the lens of AR, the recent decline in the labor share in the United States must come from factors outside of the model. Acemoglu and Restrepo (2019b) compare the United States economy before and after 1987 and argue that the latter period features lower overall productivity growth. They ascribe this shift to tax advantages for equipment compared to labor, the popular focus on automation as well as declining government support for innovation which tends to favor the creation of new tasks. In contrast, seen through the lens of HO, the recent development of the US economy is simply consistent with automation endogenously and gradually increasing as an economy matures. The extent to which the increase in automation innovation reflects the endogenous development of an economy or shocks and

---

[28]The incentive for automation innovation in AR depends on the relative cost of labor and capital, whereas it only depends on low-skill wages in HO. This difference arises from the assumption that machines are intermediate inputs in the baseline HO model but is immaterial. In fact, when HO take the model to the data, machines are a part of the capital stock and the incentive to automate similarly depends on the relative cost of low-skill labor to capital.

policy changes is an important issue for future research.

4.3   Other models of automation

The distinct theoretical predictions of HO and AR arise because of different assumptions on labor-augmenting technical progress in new products. To demonstrate this, consider a combination of the two by replacing equation (20) with:

$$y(i) = [b(i)l(i) + \alpha(i)b(i)^{\varsigma}x(i)]^{\beta} [b(i)h(i)]^{1-\beta}, \qquad (24)$$

where variables are as in HO and $b(i)$ represents a technology level. We are interested in whether low and high-skill wages can grow at the same rate for an economy with an asymptotic BGP. With this in mind, we consider exogenous technical change in which $b(i) = \exp(Bi)$ and $N_t = nt$ for some $n > 0$ and where products are automated at a constant Poisson rate. The parameter $\varsigma \in [0, 1]$ reflects factor-augmenting technical change for machines, where $\varsigma = 0$ ensures only labor-augmenting technical change. Aggregate output continues to be given by a standard CES production function using all products, N_t. The stocks of low-skill and high-skill labor are exogenously given, whereas machines are produced one-for-one with the final good. In the online Appendix, we demonstrate that only for $\varsigma = 0$ will low- and high-skill wages grow at the same rate asymptotically. When $\varsigma > 0$ low-skill wages must grow slower. This result is in the spirit of Uzawa's theorem but differs in so far as it refers to technological progress from one product to another instead of aggregate technological progress

A related result is demonstrated in Ray and Mookherjee (2020). In a general framework with both capital (complementary to labor) and robots (substitute for labor) they demonstrate that under general conditions, an economy which grows through capital accumulation must eventually have a labor share going towards zero, although wages might be growing asymptotically like low-skill wages in HO. They extend their model to include DTC, which permits but does not require technological change to be labor-augmenting as in AR. They show that asymptotically capital-augmenting technical change will be at least as rapid as labor-augmenting technical change and consequently growth in labor income will still be lower than that of the overall economy.

Neither HO nor AR make strong predictions about whether the direction of technology is efficient. More interestingly, Acemoglu and Restrepo (2019b) argue that innovation is inefficiently directed toward automation because the benefits from automation

may be more front-loaded (analogously to dirty innovation in AABH). Acemoglu and Restrepo (2020c) argue that this is particularly the case for AI. This could be easily microfounded in the HO framework by assuming that automation can also be undertaken by entrants. Since the profits realized once a good is automated partly motivate the creation of new intermediates, the incentive for horizontal innovation would be lower if there is a risk that another firm reaps these profits. In this sense, the returns to horizontal innovation would be back-loaded relative to the returns to automation, a situation akin to clean innovation relative to dirty in AABH.

In contrast with the models described above, Aghion, Jones and Jones (2017) model automation in a task framework when the different tasks are complement but ignore other innovations (in equation 19, $\sigma < 1$ and N is fixed). Automation still allows for the use of machines instead of workers in a given task, but with $\sigma < 1$, it is now equivalent to labor-augmenting technical change combined with capital-depleting technical change (see footnote 25). They show that there is a path of automation that is nearly consistent with balanced growth. It would be interesting to see whether this proposition can be reconciled with endogenous innovation. Prettner and Strulik (2020) model automation as additional varieties of machines which are imperfect substitute with labor (so that automation does not directly lead to the replacement of workers in existing tasks).[29]

Further, while both HO and AR focus on automation of the goods production and have models where the asymptotic growth rate is finite and constant, in a second model, Aghion, Jones and Jones (2017) explicitly focus on automation of the idea production function. They show that whether explosive growth happens depends on whether a model features "increasing returns to accumulable factors", ie. the product of the extent to which the product and idea functions scale with the reproducible factors of technology and capital. For instance, in either HO or AR, if any of the tasks currently performed by high-skill workers/scientists in the development of new products/tasks were automatable, the models would feature explosive growth.

Zeira and Nakamura (2018) study the effects of automation on unemployment in a model related to HO but with general processes of automation and horizontal innovation. Automated tasks can be produced solely by capital but to allow for a BGP they require the productivity of capital to be lower for newer tasks. They consider a small open economy with free capital movement and assume that workers who are displaced by

---

[29]In a model with exogenous technical change, Martinez (2019) also derives an aggregate CES production function from a microfoundation of automation. In a cross-industry analysis, he finds evidence that automation was a driving force behind the recent decline of the US labor share.

automation stay unemployed for a fixed period of time before finding a new job. On their (asymptotic) BGP, the fraction of tasks performed by humans automated in every period declines toward zero and with it unemployment. Casey (2018) also develops a model which features technological unemployment in equilibrium. In his model with DTC, innovation might speed up both labor productivity growth and unemployment. This new DTC literature is still in its infancy and more research can and should be done particularly to analyze the policy implications of DTC.

5   Empirical Evidence

5.1   In environmental economics

A large empirical literature has looked for evidence of induced technical change in environmental economics. Popp, Newell and Jaffe (2010) and Popp (2019) provide extensive literature reviews and here we mainly focus on a few recent papers. Newell, Jaffe and Stavins (1999) provide the first example by showing that technical change in air conditioners was biased against energy efficiency in the 1960s when energy prices were low, but that this bias reversed after the energy shocks of the 70s. Most of the early literature uses macro data in contexts where identification is difficult. In a seminal paper, Popp (2002) uses time-series data on US patents and finds a long-run elasticity of energy efficiency innovation on energy prices of 0.35. In a panel of US manufacturing industries, Brunnermeier and Cohen (2003) find that environmental patents increase following an increase in pollution abatement expenditures. In a panel of OECD countries, Johnstone, Haščič and Popp (2010) find that public policies have an effect on innovation in renewable energy with broad policies (such as a renewable mandate) being more effective for technologies closer to competing with fossil fuels (namely wind in their sample). Technologies farther from the market (solar) require more targeted subsidies. Such results are consistent with the AABH framework. See also Verdolini and Galeotti (2011) who include knowledge spillovers and Dechezleprêtre and Glachant (2011) who separate domestic and foreign policies.

Aghion, Dechezleprêtre, Hémous, Martin and van Reenen (2016) go further by presenting firm-level evidence. They focus on the car industry from 1978-2005 and distinguish between clean patents (associated with electric, hybrid and hydrogen engines) and dirty patents (associated with fossil fuel engines). To measure the effect of fuel prices on innovation at the firm level, they take advantage of the fact that innovators in the

car industry are selling in several national markets to build a firm-specific fuel price. This fuel price is computed as a weighted average of country-level fuel prices where the firm-specific weights are computed using a firm's patent history pre-sample (as a proxy for firm's market shares).[30] In the spirit of a shift-share instrument, the effect of fuel price on firms' innovation is identified by cross-country variations in fuel prices or taxes affecting firms differently according to their exposure to different markets. They estimate a large positive effect of fuel prices on clean innovation with an elasticity close to 1 and a negative effect on dirty innovation with an elasticity close to -0.5.[31] Furthermore, they find evidence for path dependence. Through simulations, they show that, in line with AABH, path dependence exacerbates the gap between clean and dirty knowledge in business-as-usual but actually reduces the increase in fuel prices necessary to induce clean technology to catch-up with dirty ones by 2020.

Several papers have used the same method to get variation at the firm level. Noailly and Smeets (2015) study how clean and dirty innovations in electricity production respond to both fuel price and market size where market size is calculated in an analogous manner (see also Lazkano, Nøstbakken and Pelli, 2017 and Lööf, Perez and Baum, 2018). Overall, their results support the DTC hypothesis: an increase in renewable market size or fossil fuel prices increase renewable innovation and a larger fossil fuel market leads to more fossil fuel innovation. Surprisingly, an increase in fossil fuel price also leads to a large increase in fossil fuel innovation but an increase in energy-efficiency innovations drives this. Their results also support path dependency.

Using different identification strategies, other recent papers measure the direct effect of environmental policies on innovation with microdata. Calel and Dechezleprêtre (2016) show that the EU ETS cap-and-trade system has increased low-carbon innovation by 10% in regulated firms. To establish this result, they take advantage of the existence of regulatory thresholds at the plant level and follow a matched difference-in-difference strategy where they compare regulated firms with unregulated firms of the same size. Calel (2020) finds similar results. Dugoua (2020) evaluates the effect of international environmental agreements on innovation. She focuses on the Montreal protocol, which

---

[30]Since a patent only protects an invention in the country in which it is applied for, whether a firm decides to apply for a patent in a given country or not reflects how important this country is for the firm. Coelli, Moxnes and Ulltveit-Moe (forthcoming) show empirically that this is a good proxy for market share.

[31]In line with these results, Knittel (2011) finds that there is a trade-off between improving fuel efficiency and other vehicle attributes, and that technical progress has responded to the implementation of regulatory standards.

has regulated the use of CFC since 1989 and finds that it led to an increase of 4000% in patents pertaining to CFC-substitutes relative to similar molecules. Howell (2017) exploits that the US Department of Energy allocates R&D grants to small businesses through a grading scheme. Using a regression discontinuity analysis, she finds that receiving a grant increases patenting, survival rate and venture capital, with stronger effects for firms likely to be more financially constrained.

Having established the empirical existence of directed technical change from price and market size effects, the literature is moving to study other factors driving technical change as well as interaction effects. For instance, Aghion, Bénabou, Martin and Roulet (2020) extend the set-up of Aghion et al. (2016) to study both the role of consumer value and competition in driving innovation in the car industry. They find that when consumers value the environment more, clean innovation in the car industry increases, particularly when competition is more intense. They estimate that the simultaneous increase in environmental valuation and competition which happened between 1998-2002 and 2008-2012 had the same effect on innovation as a 40% increase in fuel price.

5.2   In labor economics

The empirical literature on DTC in labor economics is comparatively smaller. A few papers show that labor market conditions affect labor-saving technology *adoption* in health care (Acemoglu and Finkelstein, 2008), agriculture (Hornbeck and Naidu, 2014, and Clemens, Lewis and Postel, 2018), and manufacturing (Lewis, 2011). Both Lordan and Neumark (2018) and Aaronson and Phelan (2019) look at the consequences of minimum wage hikes on routine jobs. Acemoglu and Restrepo (2019c) find that aging is associated with greater adoption of robots and other automation technologies in cross-country regressions. Further, this effect is stronger in industries relying more on middle-skill workers in industry x country-level regressions. More importantly, for the purpose of this review, they also find a positive correlation between aging and patenting in robotics. Alesina, Battisti and Zeira (2018) find that across countries, labor market regulations are positively correlated with innovation in low-skill intensive sectors.

A few recent papers use micro evidence. Dechezleprêtre, Hémous, Olsen and Zanella (2019) develop a new classification of patents in the machinery sector as automation or non-automation by combining information on patent texts and technological classes. They build on the empirical strategy of Aghion et al. (2016) taking advantage of the market structure for most innovation in automation technology: innovation is highly

concentrated in a few large companies which sell their technology to other (typically manufacturing) firms around the world. Consequently, the demand for automation, and with it the incentive to innovate, is determined by the wages faced by these potential customers. They compute a proxy for the low- and high- skill wages faced by these customers by taking a weighted average of country-level wages where the weights are calculated using the geographical dispersion of patents pre-sample. They find a large positive effect of low-skill wages on automation innovations with an elasticity between 2 and 4. In line with capital-skill complementarity, high-skill wages tend to reduce automation innovations. In contrast, wages do not have a significant effect on non-automation innovations in machinery. Moreover, they show that the Hartz reforms – which increased the flexibility of labor market – in Germany led to a relative decrease in automation innovations in non-German firms more exposed to Germany.Relatedly, Bena and Simintzi (2019) attempt to distinguish process from product innovations in patent data and find that firms with better access to the Chinese labor market decrease their share of process innovations after the 1999 U.S.-China trade agreement. Note that process and automation innovations may overlap but are distinct concepts.

Several papers use immigration to relate labor scarcity to innovation. San (2019) shows that following the exclusion of Mexican seasonal agricultural workers, patenting increased for crops that rely more on them. Danzer, Feurerbaum and Gaessler (2020) rely on the German regional allocation of ethnic German migrants from the collapsing Soviet Union. They also classify patents as automation or non-automation and find that regions receiving more immigrants developed fewer automation patents. Andersson, Karadja and Prawitz (2020) use an IV strategy to show that Swedish emigration to the US led to higher wages and innovation in the most affected municipalities. They do not, however, look at the direction of innovation. In contrast, Doran and Soon (2020) find that innovation decreased in the US cities most affected by the 1920 immigration quotas which reduced immigration from Southern and Eastern Europe. These, perhaps, contradictory results highlight that analyzing the effect of labor scarcity on innovation requires distinguishing between different forms of innovation.

6 Conclusion and Future Avenues

The literature has established that innovation responds strongly to market incentives and that its endogeneity matters for macroeconomic outcomes. The original DTC framework

of Acemoglu (1998) has been successfully applied in various contexts. A recent literature has developed new DTC models to analyze automation. A potential avenue for future research is to use these new models in other contexts, notably environmental economics.

Our review has identified two important issues. First, whether the economy is on a BGP? Should this not be the case, DTC can account for path dependence in energy technologies in environmental models and for growing income inequality in labor models. In contrast, on a BGP, an economy would revert to the same path after shocks. Testing for the existence of a BGP would be a complex but rewarding empirical endeavor.

Second, is the gap between the private and social returns of innovation the same for all technologies? The answer to this question determines whether industrial innovation policies are called for. In the environmental context, AABH and the literature that followed provide a strong case for a green innovation policy: climate policy should be designed with innovation at the forefront. The question is more open in the labor context: should automation be encouraged or hindered? Future research should delve deeper into this important issue, particularly, because the DTC labor literature has paradoxically sidelined the distributional aspects of innovation policy.

References

Aaronson, Daniel, and Brian Phelan. 2019. "Wage Shocks and the Technological Substitution of Low-wage Jobs." *The Economic Journal*, 219(617): 1–34.

Acemoglu, Daron. 1998. "Why do new technologies complement skills? Directed technical change and wage inequality." *Quarterly Journal of Economics*, 113(4): 1055–1089.

Acemoglu, Daron. 2002. "Directed Technical Change." *Review of Economic Studies*, 69(4): 871–809.

Acemoglu, Daron. 2003. "Labor- and Capital-Augmenting Technical Change." *Journal of the European Economic Association*, 1(1): 1–37.

Acemoglu, Daron. 2007. "Equilibrium Bias of Technology." *Econometrica*, 75(5).

Acemoglu, Daron. 2010. "When Does Labor Scarcity Encourage Innovation?" *Journal of Political Economy*, 118(6): 1037–1078.

Acemoglu, Daron, and Amy Finkelstein. 2008. "Input and Technology Choices in Regulated Industries: Evidence from the Health Care Sector." *Journal of Political Economy*, 116(5): 837–880.

Acemoglu, Daron, and David Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." *Handbook of Labor Economics*, 4(B).

Acemoglu, Daron, and Fabrizio Zilibotti. 2001. "Productivity Differences." *Quarterly Journal of Economics*, 116(2).

Acemoglu, Daron, and Pascual Restrepo. 2018. "The Race between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment." *American Economic Review*, 108(6): 1488–1542.

Acemoglu, Daron, and Pascual Restrepo. 2019*a*. "Artificial Intelligence, Automation, and Work." *Chapter in The Economics of Artificial Intelligence: An Agenda.*

Acemoglu, Daron, and Pascual Restrepo. 2019*b*. "Automation and New Tasks: How Technology Displaces and Reinstates Labor." *Journal of Economic Perspectives*, 33(2).

Acemoglu, Daron, and Pascual Restrepo. 2019*c*. "Demographics and Automation." *Working Paper.*

Acemoglu, Daron, and Pascual Restrepo. 2020*a*. "Robots and Jobs, Evidence from US labor markets." *Journal of Political Economy*, 128(6).

Acemoglu, Daron, and Pascual Restrepo. 2020*b*. "Unpacking Skill Bias: Automation and New Tasks." *American Economic Association, Papers and Proceedings.*

Acemoglu, Daron, and Pascual Restrepo. 2020*c*. "The wrong kind of AI? Artificial intelligence and the future of labour demand." *Cambridge Journal of Regions, Economy and Society*, 13(1).

Acemoglu, Daron, and Will Rafey. 2019. "Mirage on the horizon: Geoengineering and carbon taxation without commitment." NBER working paper 24411.

Acemoglu, Daron, Gino Gancia, and Fabrizio Zilibotti. 2012. "Competing engines of growth: Innovation and stardardization." *Journal of Economic Theory*, 147.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hémous. 2012. "The Environment and Directed Technical Change." *The American Economic Review*, 102 (1): 131–166.

Acemoglu, Daron, Philippe Aghion, Lint Barrage, and David Hémous. 2019. "Climate Change, Directed Innovation and Energy Transition: The Long-run Consequences of the Shale Gas Revolution." mimeo.

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2016. "The Transition to Clean Technology." *Journal of Political Economy*, 124(1): 52–104.

Aghion, Philippe, and Peter Howitt. 1992. "A Model of Growth Through Creative

Destruction." *Econometrica*, 60(2): 323–351.

Aghion, Philippe, and Peter Howitt. 1996. "Research and Development in the Growth Process." *Journal of Economic Growth*, 1(1): 49–73.

Aghion, Philippe, and Peter Howitt. 2009. *The Economics of Growth.* MIT Press.

Aghion, Philippe, Antoine Dechezleprêtre, David Hémous, Ralf Martin, and John Van Reenen. 2016. "Carbon Taxes, Path Dependency, and Directed Technical Change: Evidence from the Auto Industry." *Journal of Political Economy*, 214(1): 1–51.

Aghion, Philippe, Benjamin Jones, and Charles Jones. 2017. "Artificial Intelligence and Economic Growth." NBER Working Paper 23928.

Aghion, Philippe, Cameron Hepburn, Alexander Teytelboym, and Dimitri Zenghelis. 2019. "Handbook on Green Growth." , ed. Roger Fouquet, Chapter Path dependence, innovation and the economics of climate change, 67–83. Cheltenham, UK:Edward Elgar Publishing.

Alesina, Alberto, Michele Battisti, and Joseph Zeira. 2018. "Technology and labor regulations: theory and evidence." *Journal of Economic Growth*, 23(1): 41–78.

Andersson, David, Mounir Karadja, and Erik Prawitz. 2020. "Mass Migration and Technological Change." mimeo.

André, Francisco, and Sjak Smulders. 2014. "Fueling growth when oil peaks: Directed technological change and the limits to efficiency." *European Economic Review*, 69: 18–39.

Autor, David. 2015. "Polanyi's Paradox and the Shape of Employment Growth." *Reevaluating Labor Market Dynamics*.

Autor, David, Frank Levy, and Richard Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *The Quarterly Journal of Economics*, 118(4): 1279–1333.

Barrett, Scott. 2006. "Climate Treaties and "Breakthrough" Technologies." *American Economic Review*, 96(2): 22–25.

Bena, Jan, and Elena Simintzi. 2019. "Machines could not compete with Chinese labor: Evidence from U.S. firms' innovation." mimeo.

Bovenberg, A. Lans, and Sjak Smulders. 1995. "Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model." *Journal of Public Economics*, 57(3): 369–391.

Bovenberg, A. Lans, and Sjak Smulders. 1996. "Transitional Impacts of Envi-

ronmental Policy in an Endogenous Growth Model." *International Economic Review*, 37(4): 861–893.

Brunnermeier, Smita, and Mark Cohen. 2003. "Determinants of environmental innovation in US manufacturing industries." *Journal of Environmental Economics and Management*, 45(2): 278–293.

Calel, Raphael. 2020. "Adopt or Innovate: Understanding Technological Responses to Cap-and-Trade." *American Economic Journal: Economic Policy*, 12(3): 170–201.

Calel, Raphael, and Antoine Dechezleprêtre. 2016. "Environmental Policy and Directed Technical Change: Evidence from the European Carbon Market." *The Review of Economics and Statistics*, 98(1).

Casey, Gregory. 2018. "Growth, Unemployment, and Labor-Saving Technical Change." *Working Paper*.

Casey, Gregory. 2019. "Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation." mimeo.

Casey, Gregory, and Ryo Horri. 2019. "A Multi-factor Uzawa Growth Theorem and Endogenous Capital-Augmenting Technological Change." *Working Paper*.

Clemens, Michael, Ethan Lewis, and Hannah Postel. 2018. "Immigration Restrictions as Active Labor Market Policy: Evidence from the Mexican Bracero Exclusion." *American Economic Review*, 108(6): 1468–87.

Coelli, Federica, Andreas Moxnes, and Karen Helene Ulltveit-Moe. forthcoming. "Better, Faster, Stronger: Global Innovation and Trade Liberalization." *Review of Economics and Statistics*.

Danzer, Alexander, Carsten Feuerbaum, and Fabian Gaessler. 2020. "Labor Supply and Automation Innovation." IZA Discussion Paper No. 13429.

Dechezleprêtre, Antoine, and Mathieu Glachant. 2011. "Does Foreign Environmental Policy Influence Domestic Innovation? Evidence from the Wind Industry." *Environmetnal and Resource Econonomics*, 58: 391–413.

Dechezleprêtre, Antoine, David Hémous, Morten Olsen, and Carlo Zanella. 2019. "Automating Labor: Evidence from Firm-level Patent Data." CEPR Discussion Paper 14249.

Dietz, Simon, and Bruno Lanz. 2019. "Can a growing world be fed when the climate is changing?" IRENE Working Papers 19-09, IRENE Institute of Economic Research.

Di Maria, Corrado, and Simone Valente. 2008. "Hicks meets Hotelling: the direction of technical change in capital-resource economies." *Environment and Development*

Economics, 13: 691–717.

Doran, Kirk, and Chungeun Yoon. 2020. "Immigration and Invention: Evidence from the Quota Acts." mimeo.

Dugoua, Eugenie. 2020. "Induced Innovation and International Environmental Agreements: Evidence from the Ozone Regime."

Fischer, Carolyn, and Garth Heutel. 2013. "Environmental Macroeconomics: Environmental Policy, Business Cycles, and Directed Technical Change." *Annual Review of Resource Economics*, 5: 197–210.

Fried, Stephi. 2018. "Climate Policy and Innovation: A Quantitative Macroeconomic Analysis." *American Economic Journal: Macroeconomics*, 10(1): 9–118.

Gans, Joshua. 2012. "Innovation and Climate Change Policy." *American Economic Journal: Economic Policy*, 4(4): 125–145.

Gars, Johan, and Conny Olovsson. 2019. "Fuel for economic growth?" *Journal of Economic Theory*.

Gerglagh, Reyer, Snorre Kverndokk, and Knut Einar Rosendahl. 2009. "Optimal Timing of Climate Change Policy: Interaction Between Carbon Taxes and Innovation Externalities." *Environmental Resource Economics*, 43: 369–390.

Gerlagh, Reyer, and Wietze Lise. 2005. "Carbon taxes: A drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change." *Ecological Economics*, 54: 241–260.

Gerlagh, Reyer, Snorre Kverndokk, and Knut Einar Rosendahl. 2014. "The optimal time path of clean energy R&D policy when patents have finite lifetime." *Journal of Environmental Economics and Management*, 67: 2–19.

Goldin, Claudia, and Lawrence Katz. 2008. *The Race between Education and Technology.* Harvard University Press.

Goulder, Lawrence H., and Stephen H. Schneider. 1999. "Induced Technological Change and the Attractiveness of CO2 Abatement Policies." *Resource and Energy Economics*, 21(3-4): 211–253.

Greaker, Mads, Tom-Reiel Heggedal, and Knut Einar Rosendahl. 2018. "Environmental Policy and the Direction of Technical Change." *The Scandinavian Journal of Economics*, 120(4): 1100–1138.

Grimaud, André, and Luc Rouge. 2008. "Environment, Directed Technical Change and Economic Policy." *Environmental and Resource Economics*, 41(4): 439–463.

Habakkuk, John. 1962. *American and British Technology in the Nineteenth Century.*

Cambridge University Press.

Harstad, Bard, Francesco Lancia, and Alessia Russo. 2019. "Compliance Technology and Self-enforcing Agreements." *Journal of the European Economic Association*, 17(1): 1–29.

Hart, Rob. 2004. "Growth, environment and innovation - a model with production vintages and environmentally oriented research." *Journal of Environmental Economics and Management*, 48: 1078–1098.

Hart, Rob. 2008. "The timing of taxes on CO2 emissions when technological change is endogenous." *Journal of Environmental Economics and Management*, 55: 194–212.

Hart, Rob. 2019. "To Everything There Is a Season: Carbon Pricing, Research Subsidies, and the Transition to Fossil-Free Energy." *Journal of the Association of Environmental and Resource Economists*, 6(2): 349–389.

Hassler, John, Per Krusell, and Conny Olovsson. 2019. "Directed technical change as a response to natural-resource scarcity."

Hémous, David. 2016. "The dynamic impact of unilateral environmental policies." *Journal of International Economics*, 103: 80–95.

Hémous, David, and Morten Olsen. forthcoming. "The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality." *American Economic Journal: Macroeconomics*.

Heutel, Garth, Juan Moreno-Cruz, and Soheil Shayegh. 2018. "Solar geoengineering, uncertainty, and the price of carbon." *Journal of Environmental Economics and Management*, 87: 24–41.

Hicks, John. 1932. *The Theory of Wages.* Macmillan.

Hornbeck, Richard, and Suresh Naidu. 2014. "When the Levee Breaks: Black Migration and Economic Development in the American South." *American Economic Review*, 104(3).

Howell, Sabrina. 2017. "Financing Innovation, Evidence from RD Grants." *The American Economic Review*, 107(4).

Irmen, Andreas. 2017. "Capital- and Labor-saving Technical Change in an Aging Economy." *International Economic Review*, 58(1).

Irmen, Andreas, and Amer Tabakovic. 2017. "Endogenous capital-and labor-augmenting technical change in the neoclassical growth model." *Journal of Economic Theory*, 170.

Jonstone, Nick, Ivan Hascic, and David Popp. 2010. "Renewable Energy Policies

and Technological Innovation: Evidence Based on Patent Counts." *Environmental and Resource Economics*, 45(1): 133–55.

Katz, Lawrence, and Kevin Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *Quarterly Journal of Economics*, 107(1): 35–78.

Kennedy, Charles. 1964. "Induced Bias in Innovation and the Theory of Distribution." *The Economic Journal*, 74(295).

Knittel, Christopher. 2011. "Automobiles on Steroids: Product Attribute Trade-Offs and Technological Progress in the Automobile Sector." *American Economic Review*, 101: 3368–3399.

Kruse-Andersen, Peter. 2020. "Directed Technical Change, EnvironmentalSustainability, and Population Growth." mimeo.

Lazkano, Itziar, Linda Nøstbakken, and Martino Pelli. 2017. "From fossil fuels to renewables: The role of electricity storage." *European Economic Review*, 99: 113–129.

Lewis, Ethan. 2011. "Immigration, Skill Mix, and Capital Skill Complementarity." *The Quarterly Journal of Economics*, 126(2).

Loebbing, Jonas. 2020. "An Elementary Theory of Directed Technical Change and Wage Inequality." *Working Paper*.

Lööf, Hans, Christopher Baum, and Luis Perez. 2018. "Directed Technical Change in Clean Energy: Evidence from the Solar Industry,." Working Paper Series in Economics and Institutions of Innovation 470, Royal Institute of Technology, CESIS - Centre of Excellence for Science and Innovation Studies.

Lordan, Grace, and David Neumark. 2018. "People versus machines: The impact of minimum wages on automatable jobs." *Labour Economics*, 52(40-53).

Maria, Corrado Di, and Sjak A. Smulders. 2004. "Trade Pessimists vs Technology Optimists: Induced Technical Change and Pollution Havens." *The B.E. Journal of Economic Analysis and Policy*, 4(2): Article 7.

Martinez, Joseba. 2019. "Automation, Growth and Factor Shares." mimeo.

Massetti, Emanuele, Carlo Carraro, and Lea Nicita. 2009. "How Does Climate Policy Affect Technical Change? An Analysis of the Direction and Pace of Technical Progress in a Climate-Economy Model." *The Energy Journal*, 30(2): 7–38.

Mookherjee, Dilip, and Debraj Ray. 2020. "Growth, Automation and the Long Run Share of Labor." *Working Paper*.

Nakamura, Hideki, and Joseph Zeira. 2018. "Automation and Unemployment: Help is on the Way." *Working Paper*.

Newell, Richard G., Adam B. Jaffe, and Robert N. Stavins. 1999. "The Induced Innovation Hypothesis and Energy-Saving Technological Change." *The Quarterly Journal of Economics*, 114(3): 941–975.

Noailly, Joëlle, and Roger Smeets. 2015. "Directing technical change from fossil-fuel to renewable energy innovation: An application using firm-level patent data." *Journal of Environmental Economics and Management*, 72: 15–37.

Nordhaus, William D. 2002. "Modeling induced innovation in climate-change policy." In *Technological Change and the Environment.* , ed. Arnulf Grübler, Nebojsa Nakicenovic and William D. Nordhaus, Chapter 8, 182–209. Resources for the Future-Washington, DC, USA.

Papageorgiou, Chris, Marianne Saam, and Patrick Schulte. 2017. "Substitution between clean and dirty energy inputs, a macroeconomic perspective." *The Review of Economics and Statistics*, 99(2): 281–290.

Peretto, Pietro, and John Seater. 2013. "Factor-eliminating technical change." *Journal of Monetary Economics*, 60.

Popp, David. 2002. "Induced Innovation and Energy Prices." *The American Economic Review*, 92(1): 160–180.

Popp, David. 2004. "ENTICE: Endogenouse Technological Change in the DICE Model of Global Warming." *Journal of Environmental Economics and Management*, 24(1): 742–768.

Popp, David. 2006. "ENTICE-BR: The effects of backstop technology R&D on climate policy models." *Energy Economics*, 28: 188–222.

Popp, David. 2019. "Environmental Policy and Innovation: A Decade of Research." *International Review of Environmental and Resource Economics*, 13: 265–337.

Popp, David, Richard Newell, and Adam Jaffe. 2010. "Energy, the Environment and Technological Change." In *Handbook of the Economics of Innovation.* , ed. Bronwyn Hall and Nathan Rosenberg, Chapter 21, 873–937. Elsevier.

Prettner, Klaus, and Prettner Strulik. 2019. "Innovation, automation and inequality: Policy challenges in the race against the machine." *Journal of Monetary Economics*, 10.

Ricci, Francesco. 2007. "Environmental policy and growth when inputs are differentiated in pollution intensity." *Environmental and Resource Economics*, 38(285-310).

Romer, Paul. 1990. "Endogenous Technological Change." *Journal of Political Economy*, 98(5): 71–S102.

San, Shmuel. 2019. "Labor Supply and Directed Technical Change: Evidence from the Abrogation of the Bracero Program in 1964." mimeo.

Shanker, Akshay, and David Stern. 2018. "Energy intensity , growth and technical change." CAMA Working Paper 46/2018.

Smulders, Sjak, and Michiel de Nooij. 2003. "The impact of energy conservation on technology and economic growth." *Resource and Energy Economics*, 25: 59–79.

Stern, David, John Pezzey, and Yingying Lu. 2020. "Directed Technical Change and the British Industrial Revolution." mimeo.

Sue Wing, Ian. 2003. "Induced Technical Change and the Cost of Climate Policy." MIT Joint Programon the Science and Policy of Global Change.

van den Bijgaart, Inge. 2017. "The unilateral implementation of a sustainable growth path with directed technical change." *European Economic Review*, 91: 305–327.

van der Meijden, Gerard, and Sjak Smulders. 2017. "Carbon Lock-In: The Role of Expectations." *International Economic Review*, 58(4): 1371–1415.

Verdolini, Elena, and Marzio Galeotti. 2011. "At home and abroad: An empirical analysis of innovation and diffusion in energy technologies." *Journal of Environmental Economics and Management*, 61(2): 119–134.

Witajewski-Baltvilks, Jan, and Fischer Carolyn. 2019. "Green Innovation and Economic Growth in a North South Model." Resources for the Future Working Paper 19-04.

Zeira, Joseph. 1998. "Workers, Machines, and Economic Growth." *Quarterly Journal of Economics*, 113(4): 1091–1117.

7 Appendix

We consider the model presented at the beginning of Section 4.3 and establish that low-skill and high-skill wages grow asymptotically at the same rate if and only if $\varsigma = 0$.

Denote by $g(i, t)$ the share of products indexed by i which are automated by time and by η the Poisson rate of automation. One then gets:

$$g(i, t) = 1 - \exp\left(-\eta\left(t - \frac{i}{n}\right)\right) \text{ for } t \geq \frac{i}{n}, \tag{25}$$

since products are born non-automated and a product indexed by i was born at time $\frac{i}{n}$.

The unit cost function of product i can be written as:

$$c_i(w_L, w_H, \alpha(i)) = \left(\min\left(\frac{w_L}{b(i)^{1-\varsigma}}, \frac{1}{\alpha(i)}\right)\right)^\beta \frac{w_H^{1-\beta}}{b(i)^{1-\beta(1-\varsigma)}\beta^\beta(1-\beta)^{1-\beta}},$$

where $\alpha(i) \in \{0, 1\}$ denotes whether a product is non-automated ($\alpha(i) = 0$) or automated ($\alpha(i) = 1$). Therefore an automated firm uses machines instead of low-skill labor if $w_L > b(i)^{1-\alpha}$. This implies that two cases must be considered, if $w_{Lt} > \exp(B(1 - \varsigma)nt)$, then all automated firms use machines. Otherwise, there exists a $I_t \in [0, N_t)$ such that $w_{Lt} = \exp(B(1 - \varsigma)I_t)$ and automated firms with an index $i < I_t$ use machines while those with an index $i > I_t$ use low-skill workers instead. We fix $I_t = N_t$ if $w_{Lt} \geq \exp(B(1 - \varsigma)nt$.

Firms charge a mark-up $\sigma/(\sigma - 1)$ and revenues of firm i are given by

$$R_i(w_L, w_H, \alpha(i)) = ((\sigma - 1)/\sigma)^{\sigma-1} c_i(w_L, w_H, \alpha(i))^{1-\sigma} Y.$$

A share $(1 - \beta)(\sigma - 1)/\sigma$ of firms' revenues accrue to high-skill workers, while low-skill workers obtain a share $\beta(\sigma - 1)/\sigma$ if the firm is non-automated or has an index $i > I_t$ and 0 otherwise. Therefore

$$w_{Ht}H = (1 - \beta)\frac{\sigma - 1}{\sigma}\int_0^{N_t}(1 - g(i, t))R_i(w_{Lt}, w_{Ht}, 0) + g(i, t)R_i(w_{Lt}, w_{Ht}, 1)\,di,$$

$$w_{Lt}L = \beta\frac{\sigma - 1}{\sigma}\int_0^{N_t}(1 - g(i, t))R_i(w_L, w_H, 0) + 1_{i>I_t}g(i, t)R_i(w_{Lt}, w_{Ht}, 1)\,di,$$

where $1_{i>I_t}$ denotes the index function for $i > I_t$. Taking the ratio between these two

expressions, we obtain:

$$\frac{w_{Ht}H}{w_{Lt}L} = \frac{1-\beta}{\beta}\left[1 + w_{Lt}^{\beta(\sigma-1)}\frac{\int_0^{N_t} 1_{i<I_t}g\left(i,t\right)b\left(i\right)^{(1-\beta(1-\varsigma))(\sigma-1)}di}{\int_0^{N_t}\left(1-1_{i<I_t}g\left(i,t\right)\right)b\left(i\right)^{(\sigma-1)}di}\right], \qquad (26)$$

which traces the relative demand curve. Similarly, using the price normalization, we obtain:

$$\frac{\sigma}{\sigma-1}\frac{w_{Ht}^{1-\beta}}{\beta^\beta\left(1-\beta\right)^{1-\beta}}\left(\int_0^{N_t}\left(\begin{array}{c}\left(1-1_{i<I}g\left(i,t\right)\right)w_{Lt}^{\beta(1-\sigma)}b\left(i\right)^{(\sigma-1)}\\ +1_{i<I}g\left(i,t\right)b\left(i\right)^{(1-\beta(1-\varsigma))(\sigma-1)}\end{array}\right)di\right)^{\frac{1}{1-\sigma}} = 1, \qquad (27)$$

which traces an isocost curve. (26) and (27) determine wages for given technologies $(N_t, g(i,t))$. We consider the two cases in turn: $I_t = N_t$ and $I_t < N_t$.

Case 1: $I_t = N_t$. Using (25) and the expression for $b\left(i\right)$ we can compute the integral in (26) and obtain:

$$\frac{w_{Ht}H}{w_{Lt}L} = \frac{1-\beta}{\beta}\left[1 + w_{Lt}^{\beta(\sigma-1)}\frac{\frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)nt)-1}{B(1-\beta(1-\varsigma))(\sigma-1)} - \frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)nt)-\exp(-\eta t)}{\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)}}{\frac{\exp(B(\sigma-1)nt)-\exp(-\eta t)}{\left(B(\sigma-1)+\frac{\eta}{n}\right)}}\right].$$

For t large enough (and since $(1 - \beta(1-\varsigma))(\sigma-1) > 0$), we get:

$$\frac{w_{Ht}H}{w_{Lt}L} \underset{t\to\infty}{\sim} \frac{1-\beta}{\beta}\left[1 + \frac{\left(B\left(\sigma-1\right)+\frac{\eta}{n}\right)\frac{\eta}{n}\left(w_L\exp\left(-B\left(1-\varsigma\right)nt\right)\right)^{\beta(\sigma-1)}}{\left(B\left(1-\beta\left(1-\varsigma\right)\right)\left(\sigma-1\right)\right)\left(\left(B\left(1-\beta\left(1-\varsigma\right)\right)\left(\sigma-1\right)+\frac{\eta}{n}\right)\right)}\right]. \tag{28}$$

Similarly (27) gives

$$\frac{\sigma}{\sigma-1}\frac{w_{Ht}^{1-\beta}}{\beta^\beta\left(1-\beta\right)^{1-\beta}}$$

$$= \left(\begin{array}{c}w_{Lt}^{\beta(1-\sigma)}\frac{\exp(B(\sigma-1)nt)-\exp(-\eta t)}{B(\sigma-1)+\frac{\eta}{n}} + \frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)nt)-1}{B(1-\beta(1-\varsigma))(\sigma-1)}\\ -\frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)nt)-\exp(-\eta t)}{\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)}\end{array}\right)^{\frac{1}{\sigma-1}}.$$

From this, we obtain:

$$w_{Ht} \underset{t\to\infty}{\sim} \exp\left(\frac{B\left(1-\beta\left(1-\varsigma\right)\right)nt}{1-\beta}\right)(1-\beta)\left(\frac{\sigma-1}{\sigma}\beta^\beta\right)^{\frac{1}{1-\beta}} \tag{29}$$

$$\left(\frac{\left(\frac{\exp(B(1-\varsigma)nt)}{w_{Lt}}\right)^{\beta(\sigma-1)}}{B\left(\sigma-1\right)+\frac{\eta}{n}}+\frac{\frac{\eta}{n}}{B\left(1-\beta\left(1-\varsigma\right)\right)\left(\sigma-1\right)\left(B\left(1-\beta\left(1-\varsigma\right)\right)\left(\sigma-1\right)+\frac{\eta}{n}\right)}\right)^\psi.$$

with $\psi \equiv 1/\left[\left(\sigma-1\right)\left(1-\beta\right)\right]$.

Since $I_t = N_t$, then we must have $w_{Lt} \geq \exp\left(B\left(1-\varsigma\right)nt\right)$. Therefore (29) implies that

$$w_{Ht} = O\left(\exp\left(\frac{B\left(1-\beta\left(1-\varsigma\right)\right)n}{1-\beta}t\right)\right).$$

Further (28) implies that

$$\frac{w_{Ht}}{w_{Lt}} = O\left(\left(w_L\exp\left(-B\left(1-\varsigma\right)nt\right)\right)^{\beta(\sigma-1)}\right),$$

from which we get that

$$w_{Lt} = O\left(\exp\left(\left(\frac{1-\beta\left(1-\varsigma\right)+\left(1-\beta\right)\beta\left(\sigma-1\right)\left(1-\varsigma\right)}{\left(1+\beta\left(\sigma-1\right)\right)\left(1-\beta\right)}\right)Bnt\right)\right).$$

We need to verify that $w_{Lt} \geq \exp\left(B\left(1-\varsigma\right)nt\right)$. Note that

$$\left(\frac{1-\beta\left(1-\varsigma\right)+\left(1-\beta\right)\beta\left(\sigma-1\right)\left(1-\varsigma\right)}{\left(1+\beta\left(\sigma-1\right)\right)\left(1-\beta\right)}\right)Bn \geq B\left(1-\varsigma\right)n \Longleftrightarrow$$
$$\varsigma \geq 0.$$

Therefore, if $\varsigma > 0$, then $w_{Lt} > \exp\left(B\left(1-\varsigma\right)nt\right)$ is verified for sure for large t, and we get that $g_\infty^{w_H}$ and $g_\infty^{w_L}$ exist with $g_\infty^{w_H} = \frac{(1-\beta(1-\varsigma))Bn}{1-\beta}$ and $g_\infty^{w_L} = \frac{1-\beta(1-\varsigma)+(1-\beta)\beta(\sigma-1)(1-\varsigma)}{(1+\beta(\sigma-1))(1-\beta)}Bn$ and we can verify that $g_\infty^{w_H} > g_\infty^{w_L}$.

In contrast if $\varsigma = 0$, then we have $g_\infty^{w_H} = g_\infty^{w_L} = Bn$ but this case only applies if there exist constant $\overline{w}_H$ and $\overline{w}_L$ such that

$$\overline{w}_H = (1-\beta)\left(\frac{\sigma-1}{\sigma}\beta^\beta\right)^{\frac{1}{1-\beta}}\left(\frac{\overline{w}_L^{\beta(1-\sigma)}}{B\left(\sigma-1\right)+\frac{\eta}{n}}+\frac{\frac{\eta}{n}}{B\left(1-\beta\right)\left(\sigma-1\right)\left(B\left(1-\beta\right)\left(\sigma-1\right)+\frac{\eta}{n}\right)}\right)^\psi, \tag{30}$$

$$\frac{\overline{w}_H H}{\overline{w}_L L} = \frac{1-\beta}{\beta} \left[1 + \overline{w}_L^{\beta(\sigma-1)} \frac{\left(B(\sigma-1)+\frac{\eta}{n}\right)\frac{\eta}{n}}{B(1-\beta)(\sigma-1)\left(\left(B(1-\beta)(\sigma-1)+\frac{\eta}{n}\right)\right)} \right], \qquad (31)$$

with $\overline{w}_L \geq 1$, in which case $w_{Ht} \underset{t\to\infty}{\sim} \overline{w}_H \exp(Bnt)$ and $w_{Lt} \underset{t\to\infty}{\sim} \overline{w}_L \exp(Bnt)$ and $I_t = N_t$ is a possibility.

Case 2: $I_t < N_t$. Then (26) implies

$$\frac{w_{Ht} H}{w_{Lt} L} = \frac{1-\beta}{\beta} \left[1 + w_{Lt}^{\beta(\sigma-1)} \frac{\frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)I_t)-1}{B(1-\beta(1-\varsigma))(\sigma-1)} - \exp(-\eta t)\frac{\exp\left(\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)I_t\right)-1}{\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)}}{\frac{\exp(B(\sigma-1)nt)-\exp(B(\sigma-1)I_t)}{B(\sigma-1)} + \exp(-\eta t)\frac{\exp\left(\left(B(\sigma-1)+\frac{\eta}{n}\right)I_t\right)-1}{\left(B(\sigma-1)+\frac{\eta}{n}\right)}} \right].$$

Note that we must have $\lim I_t = \infty$, otherwise, there would be periods where w_{Lt} remain bounded even though an arbitrarily large number of products use low-skill workers. Therefore, using $w_{Lt} = \exp((1-\varsigma)BI_t)$, the previous equation leads to:

$$\frac{w_{Ht} H}{w_{Lt} L} \underset{t\to\infty}{\sim} \frac{1-\beta}{\beta} \left[1 + \frac{\frac{1}{B(1-\beta(1-\varsigma))(\sigma-1)} - \exp\left(-\frac{\eta}{n}(nt-I_t)\right)\frac{1}{B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}}}{\frac{\exp(B(\sigma-1)(nt-I_t))-1}{B(\sigma-1)} + \exp\left(-\frac{\eta}{n}(nt-I_t)\right)\frac{1}{B(\sigma-1)+\frac{\eta}{n}}} \right],$$

Since $nt \geq I_t$, then we must have that $w_{Ht} = O(w_{Lt}) = O(\exp((1-\varsigma)BI_t))$. Similarly (27) now implies:

$$\frac{\sigma w_{Ht}^{1-\beta}}{(\sigma-1)\beta^\beta(1-\beta)^{1-\beta}} = \left(\begin{array}{c} w_{Lt}^{\beta(1-\sigma)}\frac{\exp(B(\sigma-1)nt)-\exp(B(\sigma-1)I_t)}{B(\sigma-1)} + w_{Lt}^{\beta(1-\sigma)}\exp(-\eta t)\frac{\exp\left(\left(B(\sigma-1)+\frac{\eta}{n}\right)I_t\right)-1}{B(\sigma-1)+\frac{\eta}{n}} \\ + \frac{\exp(B(1-\beta(1-\varsigma))(\sigma-1)I_t)-1}{B(1-\beta(1-\varsigma))(\sigma-1)} - \exp(-\eta t)\frac{\exp\left(\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)I_t\right)-1}{\left(B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}\right)} \end{array} \right)^{\frac{1}{\sigma-1}}$$

Using that $I_t \to \infty$ and that $w_{Lt} = \exp((1-\varsigma)BI_t)$, we then get

$$\frac{\sigma w_{Ht}^{1-\beta}}{(\sigma-1)\beta^\beta(1-\beta)^{1-\beta}} \underset{t\to\infty}{\sim} \exp((1-(1-\varsigma)\beta)BI_t) \left(\begin{array}{c} \frac{\exp(B(\sigma-1)(nt-I_t))-1}{B(\sigma-1)} + \frac{\exp\left(-\frac{\eta}{n}(nt-I_t)\right)}{B(\sigma-1)+\frac{\eta}{n}} \\ + \frac{1}{B(1-\beta(1-\varsigma))(\sigma-1)} - \frac{\exp\left(-\frac{\eta}{n}(nt-I_t)\right)}{B(1-\beta(1-\varsigma))(\sigma-1)+\frac{\eta}{n}} \end{array} \right)^{\frac{1}{\sigma-1}}.$$

The left-hand side is of order $\exp((1-\varsigma-\beta(1-\varsigma))BI_t)$ while the the right-hand side is of order at least $\exp((1-(1-\varsigma)\beta)BI_t)$, therefore this situation is only possible if $\varsigma = 0$. Moreover, if $nt - I_t$ is unbounded then the ratio of the right-hand side to the left-hand side is also unbounded, which is a contradiction. Therefore it must be that $nt - I_t$ remains bounded. In that case, we must then have that w_{Lt} and w_{Ht} are of the same order as $\exp(BI_t)$.

We then obtain an equilibrium with $I_t < N_t$ if there exist $\overline{w}_H, \overline{w}_L < 1$ (which implies $\lim nt - I_t = -\frac{\log \overline{w}_L}{B}$) such that:

$$\frac{\overline{w}_H H}{\overline{w}_L L} = \frac{1-\beta}{\beta}\left[1 + \frac{\frac{1}{B(1-\beta)(\sigma-1)} - \overline{w}_L^{\frac{\eta}{Bn}}\frac{1}{B(1-\beta)(\sigma-1)+\frac{\eta}{n}}}{\frac{\overline{w}_L^{(1-\sigma)}-1}{B(\sigma-1)} + \overline{w}_L^{\frac{\eta}{Bn}}\frac{1}{B(\sigma-1)+\frac{\eta}{n}}}\right], \tag{32}$$

$$\overline{w}_H = (1-\beta)\left(\frac{\sigma-1}{\sigma}\beta^\beta\right)^{\frac{1}{1-\beta}}\overline{w}_L\left(\frac{\frac{\overline{w}_L^{(1-\sigma)}-1}{B(\sigma-1)} - \frac{\overline{w}_L^{\frac{\eta}{Bn}}B\beta(\sigma-1)}{\left(B(\sigma-1)+\frac{\eta}{n}\right)\left(B(1-\beta)(\sigma-1)+\frac{\eta}{n}\right)}}{+\frac{1}{B(1-\beta)(\sigma-1)}}\right)^\psi. \tag{33}$$

(32) can be rewritten as $\overline{w}_H = \frac{1-\beta}{\beta}\frac{L}{H}s_1(\overline{w}_L)$ with

$$s_1(\overline{w}_L) \equiv \overline{w}_L\left[1 + \frac{\frac{1}{B(1-\beta)(\sigma-1)} - \overline{w}_L^{\frac{\eta}{Bn}}\frac{1}{B(1-\beta)(\sigma-1)+\frac{\eta}{n}}}{\frac{\overline{w}_L^{(1-\sigma)}-1}{B(\sigma-1)} + \overline{w}_L^{\frac{\eta}{Bn}}\frac{1}{B(\sigma-1)+\frac{\eta}{n}}}\right].$$

We obtain:

$$s_1'(\overline{w}_L) = \frac{\left\{\begin{array}{c}\left(\frac{\frac{\eta}{n}\left(\frac{\eta}{Bn}+1\right)}{\left(B(\sigma-1)+\frac{\eta}{n}\right)\left(B(1-\beta)(\sigma-1)+\frac{\eta}{n}\right)} + \frac{1}{B(1-\beta)(\sigma-1)} + \frac{\frac{\beta(\sigma-1)}{1-\beta}+1+\frac{\beta}{B(1-\beta)}\frac{\eta}{n}}{B(\sigma-1)+\frac{\eta}{n}}\right)\overline{w}_L^{\frac{\eta}{Bn}}\frac{\overline{w}_L^{1-\sigma}-1}{B(\sigma-1)} \\ +\left(\frac{\overline{w}_L^{1-\sigma}-1}{B(\sigma-1)} + \frac{1-\overline{w}_L^{\frac{\eta}{Bn}}}{B(1-\beta)(\sigma-1)}\right)\frac{\overline{w}_L^{1-\sigma}-1}{B(\sigma-1)} + \frac{1-\overline{w}_L^{\frac{\eta}{Bn}}}{B(1-\beta)(\sigma-1)}\frac{\overline{w}_L^{1-\sigma}}{B} + \frac{1+\beta(\sigma-1)}{B(1-\beta)(\sigma-1)}\frac{\overline{w}_L^{\frac{\eta}{Bn}}}{B(\sigma-1)+\frac{\eta}{n}} \\ +\frac{\frac{\eta}{n}\overline{w}_L^{\frac{\eta}{Bn}}}{\left(B(\sigma-1)+\frac{\eta}{n}\right)\left(B(1-\beta)(\sigma-1)+\frac{\eta}{n}\right)}\left(\frac{\overline{w}_L^{1-\sigma}}{B} + \frac{\overline{w}_L^{\frac{\eta}{Bn}}}{B(\sigma-1)+\frac{\eta}{n}}\right)\end{array}\right\}}{\left(\frac{\overline{w}_L^{1-\sigma}-1}{B(\sigma-1)} + \frac{\overline{w}_L^{\frac{\eta}{Bn}}}{B(\sigma-1)+\frac{\eta}{n}}\right)^2},$$

which is positive when $\overline{w}_L \leq 1$ (as then $\overline{w}_L^{1-\sigma} - 1 \geq 0$ and $1 - \overline{w}_L^{\frac{\eta}{Bn}} \geq 0$), so that s_1 is increasing in $\overline{w}_L$ for $\overline{w}_L \leq 1$. Similarly (33) can be rewritten as $\overline{w}_H = (1-\beta)\left(\frac{\sigma-1}{\sigma}\beta^\beta\right)^{\frac{1}{1-\beta}}s_2(\overline{w}_L)^\psi$, with

$$s_2(\overline{w}_L) \equiv \overline{w}_L^{(\sigma-1)(1-\beta)}\left(\frac{\overline{w}_L^{(1-\sigma)}-1}{B(\sigma-1)} - \frac{\overline{w}_L^{\frac{\eta}{Bn}}B\beta(\sigma-1)}{\left(B(\sigma-1)+\frac{\eta}{n}\right)\left(B(1-\beta)(\sigma-1)+\frac{\eta}{n}\right)} + \frac{1}{B(1-\beta)(\sigma-1)}\right).$$

We get

$$s_2'(\overline{w}_L) = -\overline{w}_L^{(\sigma-1)(1-\beta)-1}\left[\left(\frac{B(\sigma-1)\beta}{B(\sigma-1)+\frac{\eta}{n}}\right)\frac{\overline{w}_L^{\frac{\eta}{Bn}}}{B} + \beta\frac{\overline{w}_L^{1-\sigma}-1}{B}\right],$$

which is negative for $\overline{w}_L \leq 1$. Therefore (31) and (32) together trace a positive relationship between $\overline{w}_H$ and $\overline{w}_L$ (it is easy to show continuity between the two expressions) and similarly (30) and (33) trace a positive relationship (continuity is also easily verified), checking the limits when $\overline{w}_L \to 0, \infty$, we obtain that there exist a single solution. This ensures that asymptotically when $\varsigma = 0$, the asymptotic equilibrium described above exists, and depending on other parameters it is in case 1 or case 2.

Taking stock. Overall, we find that if $\varsigma > 0$, then for t large enough, $I_t = N_t$ and $g^{w_H} > g^{w_L}$; while if $\varsigma = 0$, $g^{w_H} = g^{w_L}$ with $I_t = N_t$ or $I_t < N_t$ depending on parameters.