## DISCUSSION PAPER SERIES



# Targeted product design 

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#### Abstract

We present a model of product design. In our framework, there is an inherent trade-off associated with choosing between broad or niche designs. More-targeted designs are able to excite specific types of consumers, but at the cost of alienating others. We adapt the familiar Salop (1979) circle by allowing firms to locate on the interior. In contrast to previous research that exogenously assumed extreme designs, we provide conditions that ensure extreme or intermediate designs in monopoly, monopolistic competition, and duopoly. We use the framework to qualify earlier findings in the literature and support the robustness of others.


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# Targeted product design ${ }^{1}$ 

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#### Abstract

We present a model of product design. In our framework, there is an inherent trade-off associated with choosing between broad or niche designs. More-targeted designs are able to excite specific types of consumers, but at the cost of alienating others. We adapt the familiar Salop (1979) circle by allowing firms to locate on the interior. We provide conditions that ensure extreme or intermediate designs in monopoly, monopolistic competition, and duopoly and conduct comparative statics exercises that provide further characterization and intuition.


[^0]
## 1 Introduction

Firms constantly make decisions not only about prices and quantities, but also regarding the kind of goods that they produce. In this paper, we analyze product design as a form of consumer targeting: a choice of the degree to which a product is attractive to a broad versus a narrow group of consumers. Even though some choices of product characteristics may be costless to the firm, they are non-trivial decisions. A broader product design has the advantage of appealing to a wider consumer base, but it often comes at a cost; the same design features that aim to alienate few consumers are also unlikely to excite the passions of any. For example, a fashion or product designer might choose a neutral colour that is unlikely to offend any consumers, but that may not thrill any of them either. Software designers might choose designs that can handle many uses; but these may be slower and more cumbersome than very slick clean programs designed to address specific needs.

We model product design choices as a trade-off between conventional representations of horizontal and vertical differentiation. ${ }^{2}$ This naturally leads different designs to create demand rotations, as introduced by Johnson and Myatt (2006). That is, moving towards a relatively more generic design involves a rotation of a firm's demand curve: the consumers who enjoy the good the most gain less utility, but the consumers who enjoy it the least gain more utility. ${ }^{3}$ As an example, consider the dining experience in a new Persian restaurant that can offer only a limited number of dishes and must choose between menu items that are designed with broader audiences

[^1]in mind (for example, offering burgers), or items that might appeal only to more refined palettes (such as kalleh pacheh, a traditional broth prepared with lamb's head and trotters). This kind of design choice creates an interesting trade-off. A blander, more conventional menu might appeal to a broader audience; at the same time, no individual diner is likely to be enamoured, and, thus, prices would have to be much lower. In seeking to attract a wider range of horizontal preferences through design, there is a sense in which there is a vertical quality drop through the loss of authenticity.

We introduce a framework that can allow for the terms of this trade-off to vary smoothly and, so, lead to interior design choices that change marginally with parameters of the models (such as marginal costs). The model also allows for parametrizations where extreme outcomes - most broad or most niche designs - necessarily arise, so that marginal changes in costs lead to drastic changes in design. Extreme design choices can be a convenient assumption, as it effectively restricts the attention to two kinds of designs. This has been the focus of recent literature (including Johnson and Myatt; 2006; Kuksov, 2004; Larson, 2011; and Bar-Isaac, Caruana, Cuñat, 2012, from now on BCC). ${ }^{4}$ We provide a more general approach, and a simple characterization of when we should expect extreme or interior design choices.

Another contribution of our framework is to present an intuitive representation of demand rotations; one that encompasses all linear demand rotations. In our model, consumers are located on the circumference of a circle (as in Salop (1979)), but firms can locate on any point on the interior of the circle. Consumer preferences are reflected in linear horizontal costs associated with moving around the circle and vertical costs associated with moving to the interior. ${ }^{5}$ Locations closer to the center of the circle correspond to broader, more generic offerings with higher vertical costs and lower horizontal costs.

[^2]Whether firms choose extreme or interior designs depends on vertical transport costs and, more specifically, their convexity. Intuitively, following our previous example, if a customer cares a great deal for authenticity but is relatively insensitive once moving away from a genuinely authentic cuisine, then the restaurateur will do best by choosing an extreme offering: either as bland and generic as possible to cater to a wide audience, or as authentic as possible to target extreme tastes. Instead, if the aficionado intensely dislikes bland generic offerings but is relatively insensitive across offerings that are somewhat authentic, then the restaurateur optimizes with an intermediate menu that balances between the aficionado's tastes and those of the broader population. Both possibilities seem plausible and motivate our model, as it can accommodate both interior and extreme designs. ${ }^{6}$

Indeed, recent empirical work has considered endogenous product design and highlights that interior designs are relevant for application and worth modelling. For example, Ershov (2020) examines a change in categories on the Android app store and its effects on the introduction of more-niche and more-broad apps. More recently, Gong (2021) and Bonelli, Buyalskaya, and Yao (2021) use natural language processing techniques to provide continuous measures of design for social media influencers and mutual fund prospectuses. These highlight not only that design can be measured and quantified, but that varied types of design (rather than solely extremal ones) do indeed arise. These works also empirically confirm some of our findings; in particular, as we discuss below, that higher quality goods choose broader designs and that more intense competition leads to nichier designs.

Of course, these empirical settings involve competition, rather than a single firm. Embedding our model of product design into a monopolistic competition model, we establish several further results. First, we find sufficient conditions to ensure extreme product offerings (either maximally

[^3]generic or niche) or interior product offerings. As suggested above, these key conditions are related to the convexity of the vertical transport cost that regulates the relative speed by which horizontal and vertical cost vary. Different and rich market configurations in terms of product design arise through the interplay of firms' marginal cost heterogeneity and the tradeoff between vertical and horizontal quality. In particular, a continuum of different designs, rather than only two extreme designs, can arise in equilibrium. Second, we show that the higher the marginal cost of production, the nichier the offering. ${ }^{7}$ The intuition is a familiar one: a firm with a very high marginal cost must charge a relatively high price and, thus, values variance in consumer valuations in the hope of finding some consumers willing to buy. Third, we show that when search costs fall, firms offer nichier designs. In this context, we provide examples of how long-tail and superstar effects arise. ${ }^{8}$ Finally, while this demonstrates that many results in BCC are robust in this new setting in which intermediate designs arise, we qualify other results; notably, we show that profits may be non-monotonic in search costs even when all firms are ex-ante identical, and that a small change in search costs can lead many firms to change their designs.

In the last section of the paper, we briefly introduce a Bertrand duopoly model to further highlight the strategic interactions of design choices. We show that the firm with the higher marginal cost always chooses an extremal niche design. Meanwhile, the other firm chooses a design that is broader the lower its marginal cost. These results are in line with those from the monopolistic competition model. This Bertrand framework has been adopted and further developed to consider symmetric firms and the welfare effects of entry or exit of firms (González-Maestre and Granero, 2018, 2020).

Throughout we also compare the decentralized solutions to the planner's problem and highlight a force that leads the planner to prefer nichier

[^4]designs. Specifically, firms choose their strategies to attract marginal consumers whereas the planner cares about the welfare of all consumers. This force, which applies in all our models, is analogous to the familiar Spence (1975) quality distortion. There are other forces at play, that depend on the particular model, as we discuss in each case.

## 2 Baseline Model of Product Design

We adapt the well-known Salop (1979) circular model of horizontal differentiation to consider product design. As in Salop's model, we assume that consumers are uniformly distributed on the circumference of a circle of radius 1 . It is convenient and without loss of generality to suppose that there is a mass $2 \pi$ of consumers. However, we break with the standard model in supposing that firms can locate not only on the circumference of this circle, but also on the interior of a ring. The outer edge of the ring is a circle of radius 1 , corresponding to consumer locations, and the inner edge is a circle with inner radius $B$, where $1>B>0$. Locations anywhere in this ring correspond to different possible designs. ${ }^{9}$

In this section, we start by considering a single monopolist firm facing a single consumer. This allows us to highlight economic forces and serves as a building block to analyze the models of competition in later sections. The firm has a constant per-unit marginal cost $m$ and can locate anywhere within the ring. Thus, a firm's location is determined by the angle and the distance to the center. In our example of the restaurant, a location consists of the type of cuisine (Italian, Persian, etc.) corresponding to an angle of the circle, in addition to a choice of how authentic (further out towards the outer edge of the ring) or bland/generic (towards the inner edge of the ring) the restaurant's food is.

If the firm locates exactly at the location of a consumer, this consumer's value for the product is $V$. Otherwise, the consumer must incur travel costs

[^5]to reach the firm. She first travels along a radius towards the center of the ring and, only then, travels along the arc. ${ }^{10}$ If she travels a distance $y$ along the radius and $d$ along the arc, the travel costs are assumed to be $c(y)+d$ with $c(\cdot)$ twice continuously differentiable and $c^{\prime}(\cdot)>0$. That is, we assume linear unit travel costs along the arc but allow any increasing shape for the cost associated with travelling along a radius. ${ }^{11}$ By construction, the cost of travelling along a radius is common to all consumers and can be interpreted as vertical differentiation. Meanwhile, the cost of travelling along the arc varies across consumers, depending on their locations. Thus, a change in this cost can be interpreted as a change in horizontal differentiation. Throughout the paper, we refer to the cost of travelling along the arc as a horizontal cost and the cost of traveling along a radius as a vertical cost. A central element of the model is that firm strategies always involve a trade-off between these two costs. This framework is illustrated in Figure 1.

[^6]

Figure 1: Design and consumer travel costs.

Without loss of generality, the monopolist is located at angle 0 . Thus, the location decision consists of choosing how far inside the ring it wants to be, which we capture by $s \in[B, 1]$. Locating at $s=1$ corresponds to a fully tailored design in which the firm aims for a niche consumer base. Such a design maximizes the valuation of the consumer located at angle 0 , but it also maximizes the heterogeneity in consumer valuations. Locating closer to the centre reduces this and has a similar effect to reducing horizontal transport costs in a standard circular setting. However, moving towards the center also reduces the vertical quality of the good by imposing a common additional cost on all consumers.

If a monopolist chooses a price $p$ and a design $s$, the marginal consumers who are indifferent between purchasing or not are located at angles $x$ and $-x$, where $x \in[0, \pi]$ satisfies

$$
\begin{equation*}
V-c(1-s)-s x-p=0 . \tag{1}
\end{equation*}
$$

The right hand side, of course, simply represents the payoff, 0 , associated with choosing not to purchase. The right hand side is the net utility for
consumer $x$, who values the good at $V$ but must incur vertical transport costs, $c(1-s)$, horizontal transport costs $s x$ given the design $s$ and their angle relative to the good $x$, and the price of the good, $p$. It is clear that when the value of $x$ that is the solution to (1) is interior then overall demand is $2 x$; if the solution is negative then the firm makes no sales, and if $x>\pi$ then the firm sells to all consumers (that is the quantity is $2 \pi$ ). Solving for $x$ in (1), therefore, allows us to write the demand for a firm that chooses price $p$ and design $s$ as

$$
\begin{equation*}
q(p, s)=\max \left(0, \min \left(2 \pi, \frac{2}{s}(V-c(1-s)-p)\right)\right) . \tag{2}
\end{equation*}
$$

### 2.1 Optimal Design

We focus on the case in which optimal choices lead to a demand that is in the interval $(0,2 \pi))^{12}$ This is guaranteed to be the case if intermediate values of $V$ are considered. In this case, the demand function simplifies to

$$
\begin{equation*}
q(p, s)=\frac{2}{s}(V-c(1-s)-p) . \tag{3}
\end{equation*}
$$

Note that the demand function $q(p, s)$ is linear in $p$ and that the higher is $s$-i.e. the nichier the design - the steeper is the slope of the demand curve. This steeper slope reflects more-diverse valuations by different consumers (if there were no diversity, the demand curve would simply be a single step reflecting the common valuation). A higher $s$ also involves a higher intercept with the price axis, representing a higher valuation of the consumer who likes the good the most. Since a higher $s$ leads to a higher intercept and a steeper slope, it follows that any two designs result in demands that cross only once, and so different design choices induce demand rotations as in Johnson and Myatt (2006).

[^7]The monopolist's problem is to choose $s$ and $p$ in order to maximize:

$$
\begin{equation*}
\Pi(p, s)=\frac{2}{s}[V-c(1-s)-p](p-m) . \tag{4}
\end{equation*}
$$

Our assumption on the differentiability of the vertical transport cost implies the differentiability of the profit function, and allow us to use firstand second-order conditions to characterize the optimal design if this is intermediate. Here in the text, we present the results, while proofs are deferred to the Appendix.

Proposition 1 When the optimal design $s^{*}$ is intermediate, it satisfies the following condition:

$$
\begin{equation*}
c^{\prime}\left(1-s^{*}\right)=\frac{1}{2} q\left(p^{*}, s^{*}\right) . \tag{5}
\end{equation*}
$$

This condition has an intuitive economic interpretation. If the monopolist decides to set a price that leads to sales $q$, the marginal consumer is located at horizontal distance $\frac{1}{2} q$ from the best-matched consumer. The monopolist optimizes by choosing a design that minimizes the overall combined (horizontal and vertical) transport for this marginal consumer. Thus, the marginal vertical quality loss from standardizing the product on the lefthand side of Equation (5) is equated to the marginal horizontal gain on the right-hand side. In other words, the optimal design ensures that the marginal rates of substitution of vertical and horizontal quality for the indifferent consumer are equalized. The next result provides further characterization of the optimal design.

Proposition $2 A$ necessary condition for an intermediate design solution is that a consumer's vertical transport cost is locally convex. Meanwhile, if vertical transport costs are concave-that is, $c^{\prime \prime}(x)<0$ for all $x$-then a monopolist optimally chooses an extremal design $s^{*} \in\{B, 1\}$.

We provide some intuition for this result. Note that Proposition 1 shows that the marginal cost of standardizing the product is key in determining the optimal design. Thus, it is no surprise that how this cost changes explains whether an intermediate or an extreme design arises. Consider the following
experiment: For a given fixed price, let the firm change the design towards standardization (reducing $s$ ). If a marginal change attracts more consumers, would a further move still do so? That is the case when $c^{\prime \prime}(\cdot)<0$ because, while the horizontal costs are reduced linearly, vertical costs increase only at a slower speed, which makes the good more attractive to inframarginal consumers. A similar argument would lead to an optimal fully niche design if the initial marginal change induced a reduction of the customer base. As a result, and as Proposition 2 states, when the vertical transport costs are concave, the optimal design must be extreme.

One can reinterpret this idea in the context of whether or not the demand rotations induced by design changes are ordered, in the sense of Johnson and Myatt (2006). ${ }^{13}$ Consider the different demand curves that are traced out as the firm chooses different designs in Figure 2. Indeed, our model provides a simple way to visualize all possible demand rotations in a linear demand model and intuitively decomposes them into simple horizontal and a vertical differentiation attributes. A concave travel cost $c(\cdot)$ ensures that, as the firm moves from niche designs that induce steep demand functions to flatter broad designs, the drop-off in the price intercept is not too severe, as in the top right panel of Figure 2. This implies that the family of rotations is ordered. In particular, the upper envelope of sales/price combinations (traced out in grey) that can be achieved by the family of demand rotations is composed of the most niche and the most broad designs. ${ }^{14}$ Thus, the firm chooses one of these two designs.

[^8]

Figure 2: Summary of rotation orderings as a function of transport costs.
Meanwhile, when $c(\cdot)$ is convex, one cannot immediately conclude that the optimal design is going to be an intermediate one. Note that the case of linear transport costs still entails an extreme optimal design. As illustrated in the top left panel of Figure 2, in this case, all demand curves cross through the same point of rotation. Thus, it is still the case that the upper envelope of the demand curves is composed by only the most niche and the most broad designs. But once $c(\cdot)$ is convex, the family of rotation is no longer ordered, and all designs contribute to the upper envelope of the demand curves (see the bottom left panel of Figure 2). Of course, since the firm also chooses prices only one point on this outer envelope is relevant. We show that if the degree of convexity is sufficiently high, the potential gains from choosing an intermediate design become strong enough to make such a choice optimal.

Note that Propositions 1 and 2 are necessary conditions for an optimal
intermediate design. We can, however, establish elementary conditions that are sufficient to guarantee it.

Proposition 3 An intermediate optimal design arises if the vertical cost function $c(\cdot)$ satisfies the following two inequalities:

$$
2 B c^{\prime}(1-B)+c(1-B)>V-m>2 c^{\prime}(0)
$$

Essentially, a sufficient condition for a solution to be interior is that the cost function $c(y)$ is sufficiently flat at $y=0$ and steep enough at $y=1-B$. While these two conditions may not be always satisfied, they are interesting for two reasons. First, they are simple to check and interpret, and second, they do not impose any particular functional behavior in the interior of the domain-in particular, whether the function needs to be globally concave or convex. In the context of the restaurant example, these conditions correspond to checking the extent to which an aficionado suffers from moving from full authenticity and gains from departing from the most bland cuisine. More broadly, a rich literature in marketing and psychology analyzes the contexts and product categories in which consumers value uniqueness and those in which uniqueness concerns are less marked (Lynn and Synder, 2002; Han et al, 2010; Chan et al. 2012); thus, illustrating where such convexity/concavity in preferences is likely to be found.

Next, we turn to the comparative statics of the optimal design, and show that a firm with higher marginal costs would choose a nichier design. As discussed in the introduction, this result has a simple intuition-a firm with a high marginal cost would need to charge a relatively high price, so for the firm to make sales, it needs to find some consumers who fall in love with the product, leading to a niche design. Instead, firms with very low marginal costs hope to sell to many consumers and avoid choices that put off any potential consumers. Therefore, they choose more-generic designs.

Proposition 4 A monopolist with a higher marginal cost of production, $m$, chooses a nichier design.

### 2.2 Welfare Analysis

Now we turn our attention to the optimal product choice from a welfare perspective. Our goal is to understand the distortions that the monopolist creates with its decisions. The planner's problem is to choose a price $p$ and design $s$ that maximizes

$$
\begin{equation*}
W(p, s)=q(p, s)[V-c(1-s)-s E(x \mid x \text { buys })-m] . \tag{6}
\end{equation*}
$$

This expression is simply the number of units sold multiplied by the average surplus created per sale. As may be readily anticipated (and shown in the appendix), the optimal price is set equal the marginal cost $m$. The monopolist, by charging a higher price, creates a quantity distortion; that is, fewer customers than optimal are served.

Meanwhile, the condition for an optimal design is

$$
\begin{equation*}
c^{\prime}(1-s)=\frac{1}{4} q(m, s) . \tag{7}
\end{equation*}
$$

The left hand side is the average vertical cost, and the right hand side is the average horizontal cost. These are efficiently equated at the optimum. Note that this condition differs from

$$
c^{\prime}(1-s)=\frac{1}{2} q(p, s),
$$

the one obtained in Equation (29) characterizing an optimal design for the monopolist for a given price $p$. Here, the monopolist equates the horizontal and vertical costs for the marginal consumer, as opposed to the average consumer. This is the well known quality distortion described in Spence (1975).

Now, with the use of (3) one can rewrite (7) as

$$
c^{\prime}(1-s)=\frac{V-c(1-s)-m}{2 s},
$$

which is the same expression determining $s^{*}$ in (30). Thus, in this model, the planner and the monopolist eventually choose the same design. The
reason is that, due to the linear demand structure of the model, the design adjustments to correct for the monopolist's quantity and quality distortions fully offset each other. The planner prefers a nichier design to cater to the average consumer. But, as mentioned above, the planner also has an incentive to reduce the price so that more consumers are served. But then the average consumer now is further away horizontally, creating a force towards a broader design. The linearity in the model leads these two effects to perfectly cancel each other out when $p=m$.

### 2.3 Reinterpretation of the cost structure

We introduce two reinterpretations of our model that allow for a broader scope and additional insights. The first one assigns the vertical transport cost to the firm and the second one redefines firm heterogeneity in terms of a vertical quality component.

In our model, vertical transport costs are borne by consumers. As a first re-interpretation of the model, we argue that they can easily be interpreted as borne by the firm instead. That is, instead of having a utility-based tradeoff in which broader goods come at the cost of alienating core consumers, we can model the trade-off between appealing to more consumers at the cost of an increased marginal cost of production that gets passed-through to consumers via prices. This allows for additional applications of the model; for example, a firm could produce a good with more and more features that would be appreciated by all consumers, and make the product more general purpose. However, making the product suit a wider range of consumers would come at a higher production cost. This alternative model has identical predictions as our utility-based trade-off.

To see this, note that, as obtained in expression (28) in the proof of Proposition 1, the optimal price for a given a design $s$ is:

$$
p=\frac{V-c(1-s)+m}{2} .
$$

Further, one can substitute this in (4). and express profits as

$$
\Pi=\frac{(V-c(1-s)-m)^{2}}{2 s}
$$

Rather than interpreting the vertical transport cost $c(1-s)$ as one that is borne by the consumer, it could instead be understood as part of the marginal cost incurred by the firm. As one can see, the consumer valuation, $V$, the marginal cost of production, $m$, and the vertical transport cost $c(1-s)$ appear additively in the determination of the optimal price, and in the calculation of profits.

This observation immediately allows for a reinterpretation of costs in which a lower $s$ (that moves towards the centre) raises the marginal costs of the firm in order to reduce the horizontal transport costs of the consumer. This interpretation can also provide further intuition for Proposition 2: when these costs are concave, the firm will chooses $s$ to be as low or as high as possible. Moreover, the fact that one often thinks of such costs as convex lends additional support to our focus on intermediate designs.

Second, looking at the profit expression in Equation (4), one can immediately see that the effect of reducing the marginal cost $m$ is identical to the effect of increasing the consumer valuation $V$ by the same quantity. This equilibrium property allows for our second re-interpretation of the model. The effect of a lower $m$ is identical to the effect of being able to produce a higher quality good (i.e., higher $V$ ). This implies that Proposition 4 can also be understood as a comparative static in the ability of the firm to produce high-quality goods; arguing that firms with goods of an inherently lower quality tend to choose nichier designs.

So far, we have characterized the design choices of a monopolist. This provides the intuitions and tools to analyze the competitive models. First, we embed the previous setting within a sequential search model of monopolistic competition. Then, we analyze the duopoly case, in which firms simultaneously compete in the design space. We show that the results obtained so far extend easily to these competitive environments. We then explore the market configurations that arise in such settings.

## 3 Monopolistic Competition

In this section, we maintain the form of consumer preferences and firm design choices from the baseline model, but we introduce competition from multiple firms. We do so tractably by supposing that consumers must incur search costs to observe product offerings.

Formally, we adapt the model of Wolinsky (1986) or Anderson and Renault (1999) in which consumers incur a search cost $a$ to learn both the price and utility they would obtain from a new firm that they encounter at random. This modelling approach has been widely used to consider the impact of changes in search costs on market outcomes (examples include, Bakos (1997), Cachon, Terwiesch and Xu (2008) and Goldmanis, Hortaçsu, Onsel and Syverson (2010)). As in BCC, we adapt the supply side to suppose that, in addition to choosing prices, firms can choose designs. While that paper considers a simpler design decision, in which only two optimal designs ever arise, here we consider the specific design choice outlined in Section 2. We highlight both some robustness of earlier findings and some qualifications.

We begin by describing the model more formally. First, we show that a necessary condition for interior designs is convexity of the vertical transport cost and that higher-cost firms choose nichier designs. These results build on the findings in Section 2. This setting, however, provides considerably more richness and allows us to consider both industry- and firm-level effects. In particular, many different design types coexist in a market. In order to explore these industry-level effects, we examine an environment in which all firms are assumed to be symmetric before discussing the consequences of firm heterogeneity.

### 3.1 Model and preliminary analysis

Consider a continuum of active firms indexed by $i \in I$ that is uniformly distributed around the circle. ${ }^{15}$ We allow for heterogeneity in firms' marginal

[^9]costs of production $m_{i}$ and assume that this attribute is independent of the horizontal location. All firms simultaneously decide their design $s_{i}$ and price $p_{i}$.

Consumers have to decide whether or not to search in a sequential way. Before visiting a store $i$, a consumer is ignorant of the actual price $p_{i}$, the design $s_{i}$, and the horizontal distance to the firm that depends on her angle of location, $x$. She can learn all of these if she visits the store at a cost $a>0$. Then, she decides whether to buy or to incur an additional cost $a$ to visit another store.

In equilibrium, consumers hold the right expectations on the joint distribution of these three attributes in the market. Just as in McCall (1970), if a consumer finds it worthwhile to search at all, then she optimizes by choosing a threshold rule. This rule establishes that a consumer buys if and only if she obtains a net utility from purchase greater than or equal to some threshold level, $U$; otherwise, the consumer continues to search. ${ }^{16}$

Firm $i$ 's location from the consumer's perspective is uniformly distributed on $(0, \pi)$. Consequently, it can be shown that $U$ is implicitly defined by

$$
\begin{equation*}
\int_{i \in I} \int_{0}^{X_{i}}\left(V-c\left(1-s_{i}\right)-s_{i} x-p_{i}-U\right) d x d i=a \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
X_{i}=\max \left(0, \min \left(\frac{1}{s_{i}}\left(V-c\left(1-s_{i}\right)-p_{i}-U, \pi\right)\right)\right. \tag{9}
\end{equation*}
$$

represents the distance that makes the consumer indifferent between buying from firm $i$ or continue searching.

In the appendix we provide more detail on how to derive expression (8). It has an intuitive interpretation. The left-hand side is the average gain over purchasing a product that delivers net utility $U$ if a new search is conducted.

[^10]The right-hand side is the cost of doing so. Thus, this formula determines $U$ as the net utility from a product that leaves the agent indifferent between searching again and buying it.

If a firm's demand per consumer visit is interior $\left(X_{i} \in(0, \pi)\right)$, it is determined by

$$
\begin{equation*}
q\left(p_{i}, s_{i}, U\right)=2 X_{i}=\frac{2}{s_{i}}\left(V-c\left(1-s_{i}\right)-p_{i}-U\right) \tag{10}
\end{equation*}
$$

and firm total profits can be written as:

$$
\begin{equation*}
\Pi=\frac{2 X_{i}}{\rho}\left(p_{i}-m_{i}\right) \tag{11}
\end{equation*}
$$

where $\rho$ denotes the probability of a consumer purchasing on a visit to a random firm, and so $\rho=\int_{j \in I} \frac{X_{j}}{\pi} d j .{ }^{17}$

We can define an equilibrium as a family of $\left(U,\left(p_{i}, s_{i}\right)_{i \in I}\right)$, where $U$ solves (8) and ( $p_{i}, s_{i}$ ) maximizes profits for each firm $i$.

The analysis of the model builds on that of Section 2. In effect, one can view the firm's problem as acting like a monopolist on residual demandthis is immediate on comparing (10) to (3) and noting that they differ only in the appearance of $U$ and that, from an individual firm's perspective, the firm can treat this as a constant. Consequently, analogues to Propositions $1,2,3$, and 4 apply:

Proposition 5 In an equilibrium in which the consumer search threshold is given by $U$ as defined in (8) and with each firm's design and price denoted $s_{i}^{*}$ and $p_{i}^{*}$ :

1. When firm $i^{\prime}$ 's equilibrium design, $s_{i}^{*}$, is intermediate, it satisfies the

[^11]following condition:
\[

$$
\begin{equation*}
c^{\prime}\left(1-s_{i}^{*}\right)=\frac{1}{2} q\left(p_{i}^{*}, s_{i}^{*}, U\right) . \tag{12}
\end{equation*}
$$

\]

2. A necessary condition for an intermediate design solution is that a consumer's vertical transport cost is locally convex. Meanwhile, if vertical transport costs are concave-that is, $c^{\prime \prime}(x)<0$ for all $x$-then firms optimally choose extremal designs $s_{i}^{*} \in\{B, 1\}$ for all $i$.
3. Firm $i$ will choose an intermediate optimal design if the vertical cost function $c(\cdot)$ satisfies the following two inequalities:

$$
\begin{equation*}
2 B c^{\prime}(1-B)+c(1-B)>V-U-m_{i}>2 c^{\prime}(0) . \tag{13}
\end{equation*}
$$

4. Consider two firms with different marginal costs $m_{1}>m_{2}$; then, in equilibrium, Firm 1 has a (weakly) nichier design; that is, $s_{1} \geq s_{2}$.

### 3.2 Symmetric Firms

The case where are all firms are symmetric is a natural benchmark that allows for further clear analytic results and intuition. In this section, we consider all firms to be identical - that is, having the same marginal cost $m$. Given a consumer threshold, $U$, implicitly defined in (8), and the probability of consumers purchasing at a random firm $\rho$, every firm faces the identical problem of choosing $p$ and $s$ in order to maximize

$$
\begin{equation*}
\Pi=\frac{2}{\rho s}(V-U-c(1-s)-p)(p-m) . \tag{14}
\end{equation*}
$$

By maximizing profit with respect to price, we derive the optimal price

$$
\begin{equation*}
p^{+}=\frac{m+V-U-c\left(1-s^{+}\right)}{2} . \tag{15}
\end{equation*}
$$

Following (12) in Proposition 5, the optimal design satisfies:

$$
\begin{equation*}
c^{\prime}\left(1-s^{+}\right)=\frac{q\left(p^{+}, s^{+}\right)}{2}=\frac{1}{s^{+}}\left(V-U-c\left(1-s^{+}\right)-p^{+}\right) . \tag{16}
\end{equation*}
$$

Consider a reduction in the cost of search-that is, a fall in $a$. By inspection of (8), this leads to more-intense competition, or a higher value of $U$ : Consumers can shop at lower cost and, thus, may be more demanding in terms of price and match with the products offered. Consequently, firms respond by seeking to offer more to customers, in part through opportunities of better matches that arise out of nichier designs. Of course, all firms change their behaviors simultaneously, and so this argument is incomplete in ignoring aggregate equilibrium effects. However, the following proposition establishes that the overall effect is consistent with this intuition.

Proposition 6 In the case of symmetric firms that choose an interior design, the design becomes nichier the lower the search costs.

These result suggests an ambiguous relationship between search costs and industry profitability. There is a direct effect: Lower search costs might increase competition; but as Proposition 6 shows, there will also be a counteracting effect through design choices. When other firms choose nichier designs, they are, in effect, more differentiated, and this can, therefore, moderate price competition; moreover, with nichier designs and lower search costs, better matches arise and so more surplus is created. These effects can (though need not) counteract and overwhelm the more direct effect of lower search costs on intensifying competition and reducing prices.

Proposition 7 In the case of symmetric firms that choose an interior design, profits increase in search costs if and only if $\frac{c^{\prime}\left(1-s^{+}\right)-s^{+} c^{\prime \prime}\left(1-s^{+}\right)}{c^{\prime}\left(1-s^{+}\right)-2 s^{+} c^{\prime \prime}\left(1-s^{+}\right)}>0$. Both cases-that is, profits increasing or decreasing in search costs-can arise.

It is worth contrasting this result with the parallel one in BCC-namely, Proposition 5(iv). There, in the case of homogeneous firms, profits necessar-
ily increase as search costs fall. ${ }^{18}$ The difference stems from the fact that in BCC, only extremal designs arise, but there can still be an "intermediate" outcome in which some firms choose the most broad design while others choose the most niche one. Here, instead, firms directly choose intermediate designs. Thus, the comparative static involves design changes of a different nature. In this paper, there are smooth changes towards nichier designs by all firms, while in BCC, there are abrupt changes from fully broad to fully niche by only a few firms.

### 3.2.1 Welfare Analysis

This section highlights that the monopolistic competitive equilibrium suffers from the same Spence (1975) quality distortion discussed in Section 2.2. Again, each firm - a local monopolist-optimizes by attracting the marginal consumer. Instead the planner is concerned for all consumers. However, in this monopolistic competition model, welfare also incorporates the consumers' search costs.

Total welfare is composed of the sum of all consumers' expected utilities and all firms' profits. Given that firms end up selling to all consumers, and that there is a mass 1 of consumers, total welfare can be expressed as

$$
W=U+\Pi,
$$

where profits are simply $\Pi=p-m$. Meanwhile, the consumer surplus, equal to the value of searching and denoted, $U$, as shown in the appendix in (39), can be written as

$$
U=V-c(1-s)-p-\sqrt{2 a s}
$$

We can use these two expressions to write welfare as:

$$
W=V-m-c(1-s)-\sqrt{2 a s} .
$$

[^12]Prices do not appear in this expression and are irrelevant for welfare given that all consumers end up buying one unit. Thus, total welfare is equal to $V$ minus the different costs: the production cost, $m$; the vertical transport cost, $c(1-s)$, which is common to all consumers; and the $\sqrt{2 a s}$ term, which subsumes the horizontal transport costs and the search costs.

An interior welfare maximizing design $s^{w}$ satisfies

$$
\begin{equation*}
c^{\prime}\left(1-s^{w}\right)=\sqrt{\frac{a}{2 s^{w}}} . \tag{17}
\end{equation*}
$$

This expression is "half" of the one obtained in (40) for the monopolistic competitive equilibrium design $s^{+}$:

$$
\begin{equation*}
c^{\prime}\left(1-s^{+}\right)=2 \sqrt{\frac{a}{2 s^{+}}}, \tag{18}
\end{equation*}
$$

where the monopolist equates the marginal horizontal cost with the combination of the vertical and search cost for the marginal consumer.

Thus, it is clear that, starting from the competitive design $s^{+}$, the social planner has an incentive to deliver nichier designs.

The argument above is a little imprecise in that conditions (17) and (18) could deliver multiple solutions, making a comparison between the multiple $s^{w}$ and $s^{+}$difficult. However, with sufficient assumptions on the functional form of $c(\cdot)$, that comparison is straight forward. For instance, if one assumes that $c^{\prime \prime \prime}<0$ one gets a unique local (and thus global) welfare optimum and a unique monopolistic competitive equilibrium. Then one can easily conclude that $s^{w}>s^{+}$. In this way, we can conclude that the Spence (1975) quality distortion is the leading force in this model; that is, the decentralized equilibrium involves designs that are too broad.

### 3.3 Asymmetric Firms

The contrast between Proposition 6 and the corresponding result in BCC might lead to a broader concern that other results in that paper also crucially rely on the assumption that only extreme (fully niche or fully generic)
designs arise. Most notably, BCC show that market restructuring following a fall in search costs (through the diffusion of the internet, for example) can simultaneously account for higher market shares of: (i) the most successful "superstar" firms; and (ii) the least successful ones, creating the "long tail" effect. We highlight below that these results can also arise when a range of intermediate designs are offered in the market.

Assume, now, that firms vary in their marginal costs of production, $m_{i} .{ }^{19}$ For concreteness, it is convenient to suppose that these marginal costs are uniformly distributed on the unit interval $[0,1]$. If the vertical transport $\operatorname{costs} c(\cdot)$ are concave, then, since Proposition 5.2 applies, a polarized distribution of product designs arises, with firms choosing either the most niche kind of design $s=0$ or the broadest one $s=B$. By Proposition 5.4, firms with low marginal costs prefer the broadest kind of design. Simultaneously, the least efficient firms opt for the most niche kind of design. Thus, a threshold determines which firms choose each of the two designs.

Similarly, when Condition (13) of Proposition 5 is satisfied, firms choose intermediate designs according to each firm's marginal cost-again, with more efficient firms preferring broader designs. These market configurations also have immediate implications for prices and quantities sold. It is straightforward to show that, keeping design fixed, higher marginal costs are associated with higher prices and lower sales. Moreover, the endogenous design choices reinforce these effects, as higher marginal costs induce firms to choose more-specific designs that, in turn, induce higher prices and lower sales. Therefore, regardless of the market configuration, prices are monotonically increasing and sales are monotonically decreasing as the design moves from being more generic to nichier.

To illustrate the above case with convex transport costs and intermediate designs, consider the example described in Figure 3. Panels 1 and 2 plot designs and sales against a firm's marginal costs. In this example, $B$ is set at 0 , so that a fully broad product is valued identically by all consumers-in particular, this implies that any firm that chooses a fully broad design and

[^13]makes sales would sell to all consumers who visit the firm. The blue line corresponds to a lower search cost and the green line to a higher search cost.


Figure 3: Designs and sales against marginal cost

$$
\text { with } c(y)=3 / 2 y^{2}, B=0 \text { and } V=3, \text { at } a_{1}=0.4 \text { and } a_{2}=0.5
$$

The first panel of Figure 3 shows that with a lower search cost, fewer firms choose a fully broad design, and that all firms choose nichier designs. Intuitively, with the more intense competition implied by lower search costs, firms compete, in part, by offering consumers products that are better targeted. This is consistent with Proposition 7 for the case of homogeneous firms. This example and the symmetric case are more broadly illustrative. Following a line of argument similar to that in BCC, a fall in search costs results in consumers searching more intensively (higher $U$ ). ${ }^{20}$ Following expressions (10) and (11) and reasoning similar to that in Proposition 4, it is immediate that as search costs fall, firm designs become nichier.

The second panel shows an interesting implication: sales of both the most and least efficient firms increase when consumer search costs falls. In other words, a superstar and a "long-tail" effect arise simultaneously. That such superstar and long-tail effects can simultaneously arise is consistent with the findings in BCC. But this example highlights that this can arise with convex as well as concave vertical transport costs and, consequently, in the

[^14]more realistic case that firms choose intermediate designs and there are a range of different design types offered in the market rather than just two.

The example also highlights that by relaxing the assumptions that lead to extremal designs, it is possible to encounter more-complex responses to a marginal reduction in search costs: While some firms react by only marginally changing their pricing, others marginally change both their pricing and their product design. Finally, as in BCC, yet another group of a few firms with extremal designs discretely change their whole design and pricing, switching to a strategy entailing much higher prices and lower sales and, thus, releasing consumers that fuel the superstar and long-tail effects.

## 4 Bertrand Duopoly

We now move on to consider a classic competitive model of a Bertrand duopoly where consumers can observe prices and designs of both firms at no cost.

This section highlights the robustness of some of our earlier insights. Notably, we show the necessity of convex vertical transport costs for interior designs, and that higher marginal costs are associated with niche designs in this setting too. The setting and, especially Proposition 9 below, also highlights that design choices can be strategic to affect price competition, as in more standard models of vertical differentiation.

Further, this Bertrand case points to further directions in which this framework can be explored. Indeed, building on an earlier version of this paper, González-Maestre and Granero $(2018,2020)$ adapt and extend this analysis to allow for $n$ identical firms and to consider comparative statics in $n$. They focus on equilibria with symmetric designs and highlight that higher competition can lead to higher prices (similar to the insight in Section 3.2, where the indirect effect of design choice may counteract the direct effect).

In this section, we analyze two firms $i=1,2$ with differing marginal costs of production, $m_{i} \geq 0$, competing in their choices of design and price. Specifically, we suppose that firms, first, choose their design simultaneously. These decisions then become public, and in a second stage, firms simulta-
neously choose prices. This timing is intended to capture that prices can adjust more easily than demand. ${ }^{21}$ Finally, consumers observe locations, prices and designs of both firms and choose one of the two products (if any).

We assume that $V$ is sufficiently high to guarantee full market coverage: that is, all consumers buy from one or other of the two firms. Further, we assume that, with respect to the horizontal dimension, firms are located opposite each other (Firm 1 at angle 0 and Firm 2 at angle $\pi$ ). ${ }^{22}$ We denote $x_{12}$ to be the consumer that is indifferent between buying from Firm 1 or Firm 2; this can be written explicitly as

$$
\begin{equation*}
x_{12}=\frac{c\left(1-s_{2}\right)-c\left(1-s_{1}\right)+p_{2}-p_{1}+\pi s_{2}}{s_{1}+s_{2}} . \tag{19}
\end{equation*}
$$

In the following analysis, we concentrate on the case in which both firms are active in the market - that is, when $x_{12} \in(0, \pi)$. Firms' profits can be written simply as:

$$
\begin{align*}
& \Pi_{1}=2 x_{12}\left(p_{1}-m_{1}\right), \text { and }  \tag{20}\\
& \Pi_{2}=2\left(\pi-x_{12}\right)\left(p_{2}-m_{2}\right) . \tag{21}
\end{align*}
$$

Given fixed designs, one can calculate optimal prices by solving for the Nash equilibrium of the last stage. Solving for these prices and substituting them into Equation (20), allows us to write Firm 1's profits as a function of $s_{1}$ and $s_{2}$ as follows:

$$
\begin{equation*}
\Pi_{1}(s)=\frac{2}{9} \frac{\left(c\left(1-s_{2}\right)-c\left(1-s_{1}\right)-m_{1}+m_{2}+\pi s_{1}+2 \pi s_{2}\right)^{2}}{s_{1}+s_{2}} . \tag{22}
\end{equation*}
$$

Simple calculus and algebraic manipulation imply that Firm 1's optimal

[^15]design (when interior) satisfies
\[

$$
\begin{equation*}
c^{\prime}\left(1-s_{1}^{d}\right)=\frac{3}{2} x_{12}-\pi . \tag{23}
\end{equation*}
$$

\]

This condition is the counterpart of Equation (5) in the monopoly model. The different expression stems from the fact that, now, the design choice has a strategic aspect (to influence the subsequent price competition) that was absent in the monopoly and monopolistic competition models.

We begin by establishing a familiar relationship between the shape of the vertical transport costs and the choice of product design that was present in the monopoly model:

Proposition 8 A necessary condition for an intermediate design in the duopoly setting is that the vertical transport costs are locally convex. Moreover, if these costs are concave, then both firms choose an extremal design.

The duopoly setting provides the following new result: At least one firm adopts an extreme most niche design in equilibrium.

Proposition 9 The firm with higher cost chooses an extreme most niche design; that is, if $m_{2} \geq m_{1}$, then without loss of generality, firm 2's equilibrium design, $s_{2}^{d}$, satisfies $s_{2}^{d}=1$.

The intuition for this result is a familiar one. When first choosing locations and then prices, firms (and particularly high-cost firms) have an incentive to differentiate in order to soften price competition. Depending on parameters, one might have the better firm choosing an intermediate or even a fully broad design. This firm may not want to fully soften competition and may prefer to exploit its comparative productive advantage.

This result partially replicates the intuition in the sections above, as it establishes that, in competition, it is the firm with lower marginal costs that would choose a broader design. This is further corroborated by the next result, which performs comparative statics on marginal costs.

Proposition 10 Holding constant the marginal cost of the rival, a higher marginal cost of production $m$ leads to a (weakly) nichier design.

The results in this section allow us to characterize the equilibrium configurations that arise in the duopoly setting with perfectly informed consumers. When vertical transport costs are concave, the firm with the higher marginal cost chooses a fully niche design. The firm with the lower marginal cost chooses either a fully broad or a niche design depending on its cost advantage. When vertical transport costs are convex, the same configurations can occur, but a third possibility may arise in which the low-marginal-cost firm chooses an intermediate design.

### 4.1 Welfare Analysis

Given that we assume that $V$ is sufficiently high for full market coverage, the fraction of consumers who buy from from Firm 1 is $2 x_{12}$ and from Firm 2 is $2\left(\pi-x_{12}\right)$. Then, analogous to expression (6) for the monopoly case, the planner's problem here is:

$$
\begin{gathered}
W=2 x_{12}\left[V-c\left(1-s_{1}\right)-s_{1} E[x \mid x \text { buys from Firm } 1]-m_{1}\right]+ \\
+2\left(\pi-x_{12}\right)\left[V-c\left(1-s_{2}\right)-s_{2} E[\pi-x \mid x \text { buys from Firm } 2]-m_{2}\right]
\end{gathered}
$$

As one can see, with full market coverage, the planner does not care about prices per se. Prices just determine the extent of transfers between firms and consumers. Prices are relevant; however, in as much as they determine the identity of the marginal consumer, $x_{12}$. It is convenient, therefore, to take the planner's problem as choosing the identity of the marginal consumer $x_{12}$, and of course, designs.

The first order analysis delivers the following conditions for the optimal designs $s_{1}^{w}$ and $s_{2}^{w}$ :

$$
\begin{align*}
c^{\prime}\left(1-s_{1}^{w}\right) & =\frac{x_{12}}{2}, \text { and }  \tag{24}\\
c^{\prime}\left(1-s_{2}^{w}\right) & =\frac{\pi-x_{12}}{2} \tag{25}
\end{align*}
$$

It is instructive to compare (24) with the corresponding solution for the decentralized design solutions duopoly, $s_{2}^{d}$, which satisfies (23). Note that
since we assume that Firms 1 and 2 both make sales, then $x_{12}<\pi$ and, so, $\frac{3}{2} x_{12}-\pi<\frac{x_{12}}{2}$. This suggests that starting from the duopoly outcome $x_{12}^{d}$, the planner would prefer firm 1 to choose a nichier design. This is consistent with the intuition from the monopolist, and monopolistic cases discussed above that highlight a force that leads the planner to prefer nichier designs - that firms optimize with respect to the marginal consumer but the planner optimizes with respect to all consumers.

Of course, there are other forces at play. In particular, the planner also cares about the trade-off between transport costs and the costs of production (which differ between Firms 1 and 2) so that, generically, the market shares in the planner's problem, captured by $x_{12}^{w}$ in the following first order condition:

$$
c\left(1-s_{1}\right)+s_{1} x_{12}+m_{1}=c\left(1-s_{2}\right)+s_{2}\left(\pi-x_{12}\right)+m_{2}
$$

differ from the corresponding ones in the decentralized solution, $x_{12}^{d}$. Moreover, as noted above, since in the decentralized case firms choose designs and quantities sequentially, there is an strategic vertical differentiation effect (with the inefficient firm choosing a most niche design) to soften price competition. This effect that is irrelevant to the planner.

## 5 Conclusions

We present a general framework to analyze product design decisions, focusing on the trade-off between having a strong appeal to a narrow consumer segment versus having some (though a more limited) appeal to a broader audience.

This framework allows us to characterize all linear rotations and to show, intuitively, when demand rotations would be ordered. This question is closely related to the question of when firms would opt for extreme designs (either fully niche or fully broad) and when they would opt for interior designs that balance the intensity and the breadth of their consumer appeal. We provide sufficient conditions for these two outcomes.

In addition, we show that better firms tend to choose broader designs, while worse firms tend to choose niche designs. When consumers face lower search costs, all firms weakly shift towards more-niche designs. We show these results are robust to different forms of competition and contrast the decentralized and planner solutions.

We embed this representation of design in a model of monopolistic competition. In contrast with the previous literature, interior equilibrium configurations with varied design can arise and demonstrate possibilities that are new to the literature. For example, we present market configurations in which, as consumers become more selective, fewer firms choose broad designs. Some firms drift towards slightly nichier designs, while others radically change to an extreme niche design. This is a useful representation of how firms adapt their product design and marketing strategies to environments in which consumers can search more efficiently for products.

The framework is general, flexible, and easy to interpret. It can also be used as a building block to study further questions on product design. As an illustration, González-Maestre and Granero (2018) and González-Maestre and Granero (2020) focus on the impact of changing the number of competing firms in an extension to our model of Section 4. In addition, the model allows for alternative interpretations within the same setting. As we discuss in Section 2.3, one alternative interpretation is one in which production costs take the role of vertical transportation costs (i.e., if it is more costly to produce a good that is more homogeneously liked by all consumers); and another reinterpretation of the model portrays firm heterogeneity as being due to intrinsic differences in the quality of the goods that each firm produces. Finally, in line with the literature on informative marketing, it is also possible to interpret design in the model as the degree of transparency and information that the firm provides about the product, with more-opaque strategies leading to less-dispersed valuations of the good. These alternative settings lead to different interpretations and a broader applicability of the model outcomes.

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## A Appendix

Proof of Propositions 1 and 2 If the solution is interior, the first-order conditions of the profit maximization of (4) are:

$$
\begin{align*}
\frac{2}{s}(V-c(1-s)-2 p+m) & =0  \tag{26}\\
\frac{2(p-m)}{s}\left(c^{\prime}(1-s)-\frac{1}{s}(V-c(1-s)-p)\right) & =0 \tag{27}
\end{align*}
$$

The first equation delivers the optimal price

$$
\begin{equation*}
p=\frac{V-c(1-s)+m}{2}, \tag{28}
\end{equation*}
$$

while the second equation combined with (3) delivers

$$
\begin{equation*}
c^{\prime}(1-s)=\frac{1}{2} q(p, s), \tag{29}
\end{equation*}
$$

Both of them combined prove Proposition 1.
Note also that if one substitutes (28) in (27) one gets

$$
\begin{equation*}
c^{\prime}(1-s)=\frac{V-c(1-s)-m}{2 s} . \tag{30}
\end{equation*}
$$

At an optimal interior design $s^{*}$, the second-order conditions must also be
satisfied. In particular, the one with respect to design delivers

$$
\begin{equation*}
\frac{2\left(p^{*}-m\right)}{s^{*}}\left(\frac{2}{s^{* 2}}\left(V-c\left(1-s^{*}\right)-p^{*}\right)-\frac{2}{s^{*}} c^{\prime}\left(1-s^{*}\right)-c^{\prime \prime}\left(1-s^{*}\right)\right)<0 \tag{31}
\end{equation*}
$$

Now, one can use Equations (3) and (29) and simplify this to

$$
\begin{equation*}
\left(\frac{q\left(p^{*}, s^{*}\right)}{s^{*}}-\frac{2}{s^{*}} c^{\prime}\left(1-s^{*}\right)-c^{\prime \prime}\left(1-s^{*}\right)\right)<0 \Leftrightarrow c^{\prime \prime}\left(1-s^{*}\right)>0 \tag{32}
\end{equation*}
$$

which proves Proposition 2.
Proof of Proposition 3 If one substitutes the expression (28) for the optimal price into (4), profits can then be expressed as a function of design:

$$
\begin{equation*}
\Pi(s)=\frac{(V-c(1-s)-m)^{2}}{2 s} \tag{33}
\end{equation*}
$$

The firm necessarily prefers an interior solution if $\Pi^{\prime}(1)<0$ and $\Pi^{\prime}(B)>0$. Given that

$$
\begin{equation*}
\Pi^{\prime}(s)=-\frac{(V-c(1-s)-m)^{2}}{2 s^{2}}+\frac{c^{\prime}(1-s)}{s}(V-c(1-s)-m) \tag{34}
\end{equation*}
$$

we can write

$$
\begin{gather*}
\Pi^{\prime}(1)<0 \Leftrightarrow V-m>2 c^{\prime}(0)  \tag{35}\\
\Pi^{\prime}(B)>0 \Leftrightarrow 2 B c^{\prime}(1-B)+c(1-B)>V-m
\end{gather*}
$$

which concludes the proof.
Proof of Proposition 4 To prove this, it is sufficient to show that

$$
\begin{equation*}
\forall m_{1}>m_{2}, \forall s_{1}>s_{2} \Pi\left(s_{1}, m_{2}\right)>\Pi\left(s_{2}, m_{2}\right) \Rightarrow \Pi\left(s_{1}, m_{1}\right)>\Pi\left(s_{2}, m_{1}\right) \tag{36}
\end{equation*}
$$

Note that $\frac{2}{s_{1}}\left(\frac{V-c\left(1-s_{1}\right)-m_{2}}{2}\right)^{2}=\Pi\left(s_{1}, m_{2}\right)>\Pi\left(s_{2}, m_{2}\right)=\frac{2}{s_{2}}\left(\frac{V-c\left(1-s_{2}\right)-m_{2}}{2}\right)^{2}$ implies that $\left(\sqrt{s_{1}}-\sqrt{s_{2}}\right) m_{2}>\sqrt{s_{1}}\left(V-c\left(1-s_{2}\right)\right)-\sqrt{s_{2}}\left(V-c\left(1-s_{1}\right)\right)$. Given that $\left(\sqrt{s_{1}}-\sqrt{s_{2}}\right) m_{1}>\left(\sqrt{s_{1}}-\sqrt{s_{2}}\right) m_{2}$, we can write $\left(\sqrt{s_{1}}-\sqrt{s_{2}}\right) m_{1}>\sqrt{s_{1}}(V-$ $\left.c\left(1-s_{2}\right)\right)-\sqrt{s_{2}}\left(V-c\left(1-s_{1}\right)\right)$, which implies that $\Pi\left(s_{1}, m_{1}\right)>\Pi\left(s_{2}, m_{1}\right)$.

Given that $\Pi$ is continuous in $(s, m)$, the condition (36) above implies that $\Pi$ satisfies the single crossing property in $(s, m)$ as defined in Milgrom and Shannon (1994). Thus, this proposition is just a particular case of Theorem 4 in Milgrom and Shannon (1994), which establishes monotone comparative statics.

Algebra details on Section 2.2 The welfare function is

$$
W(p, s)=q(p, s)[V-c(1-s)-s E(x \mid x \text { buys })-m]
$$

Noting that $E(x \mid x$ buys $)=E\left(x \left\lvert\, x \leq \frac{q(p, s)}{2}\right.\right)$ and substituting for $q(p, s)$ as defined in (3), we obtain:

$$
W(p, s)=\frac{1}{s}(V-c(1-s)-p)(V-c(1-s)+p-2 m)
$$

The first order condition with respect price $p$ delivers:

$$
-\frac{2}{s}(p-m)=0
$$

And, thus, it is optimal to set price equal to marginal cost. The welfare function can then be simplified to obtain

$$
W(m, s)=\frac{1}{s}(V-c(1-s)-m)^{2}
$$

which delivers the following optimal condition with respect to quality:

$$
c^{\prime}(1-s)=\frac{V-c(1-s)-m}{2 s}=\frac{q(m, s)}{4}
$$

Algebra details on Section 3.1 As it is standard in sequential search models a la McCall (1970), given the stationary environment, it is optimal to purchase a product if and only if it delivers a utility larger than $U$. Conditional on the strategies ( $p_{i}, s_{i}$ ) by all firms, we define

$$
P(U):=\operatorname{Pr}\left(V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \leq U\right)
$$

as the probability that, if a new store is visited, the utility obtained from buying its product is not higher than $U$. Thus, for $U$ to be the equilibrium threshold value, it has to be the case that a customer holding a product that delivers exactly a utility $U$ has to be indifferent between buying it, and conducting a new search. In other words, the following condition needs to be met:
$U=P(U) U+(1-P(U)) E\left[V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \mid V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \geq U\right]-a$.
After some immediate algebra this can be written as

$$
(1-P(U))\left(E\left[V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \mid V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \geq U\right]-U\right)=a
$$

Finally, it is sufficient to realize that

$$
\begin{gathered}
E\left[V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \mid V-c\left(1-s_{i}\right)-s_{i} x-p_{i} \geq U\right]= \\
=\int_{i \in I} \int_{0}^{X_{i}}\left(V-c\left(1-s_{i}\right)-s_{i} x-p_{i}\right) \frac{d x d i}{(1-P(U))}
\end{gathered}
$$

where

$$
\begin{equation*}
X_{i}=\max \left(0, \min \left(\frac{1}{s_{i}}\left(V-c\left(1-s_{i}\right)-p_{i}-U, \pi\right)\right)\right. \tag{37}
\end{equation*}
$$

to obtain expression (8) in the text.
Proof of Proposition 5 The proof is immediate given the arguments presented in the text right above the proposition.

Proof of Proposition 6 Using (16), we can rewrite (15) as

$$
\begin{equation*}
p^{+}=m+s^{+} c^{\prime}\left(1-s^{+}\right) \tag{38}
\end{equation*}
$$

Further, the expression for the consumer stopping rule (8) can be simplified (by first integrating and then replacing for the optimal price) to:

$$
\begin{equation*}
a=\frac{\left(V-c\left(1-s^{+}\right)-p^{+}-U\right)^{2}}{2 s^{+}}=\frac{\left(p^{+}-m\right)^{2}}{2 s^{+}} \tag{39}
\end{equation*}
$$

Now, we can substitute out $p^{+}$in Equation (39) using (38) to obtain an implicit formula for the optimal interior design.

$$
\begin{equation*}
c^{\prime}\left(1-s^{+}\right)=\sqrt{\frac{2 a}{s^{+}}} \tag{40}
\end{equation*}
$$

Using this expression, we immediately obtain that

$$
\begin{equation*}
\frac{d s^{+}}{d a}=\frac{2}{\left(c^{\prime}\left(1-s^{+}\right)\right)^{2}-2 s^{+} c^{\prime}\left(1-s^{+}\right) c^{\prime \prime}\left(1-s^{+}\right)}=\frac{2}{c^{\prime}\left(c^{\prime}-2 s^{+} c^{\prime \prime}\right)} \tag{41}
\end{equation*}
$$

(from now on, and with some abuse of notation, we write $c, c^{\prime}$, and $c^{\prime \prime}$ to refer to $c\left(1-s^{+}\right), c^{\prime}\left(1-s^{+}\right)$, and $c^{\prime \prime}\left(1-s^{+}\right)$respectively $)$

Thus,

$$
\begin{equation*}
\frac{d s^{+}}{d a}<0 \Leftrightarrow c^{\prime}-2 s^{+} c^{\prime \prime}<0 \tag{42}
\end{equation*}
$$

Next, we check that this is a necessary condition for the design implicitly defined by (40) to be optimal. That is, we check that the SOC condition of the maximization problem implies the previous inequality. Take the profit function; substitute its optimal price (15); and denote $D:=V-U-m$. Then, we have that profits are given by

$$
\begin{equation*}
\Pi=\frac{2}{\rho s}\left(\frac{D-c}{2}\right)^{2} \tag{43}
\end{equation*}
$$

The first-order condition with respect to design is given by

$$
\begin{align*}
\frac{\partial \Pi}{\partial s} & =-\frac{2}{s^{2}}\left(\frac{D-c}{2}\right)^{2}+\frac{1}{s}(D-c) c^{\prime}=0 \Leftrightarrow  \tag{44}\\
& \Leftrightarrow 2 s c^{\prime}=D-c \tag{45}
\end{align*}
$$

and the corresponding second-order condition that ensures that the solution does indeed maximize profits is given by

$$
\begin{equation*}
\frac{\partial^{2} \Pi}{\partial s^{2}}=\frac{4}{s^{3}}\left(\frac{D-c}{2}\right)^{2}-\frac{1}{s^{2}}(D-c) c^{\prime}-\frac{1}{s^{2}}(D-c) c^{\prime}+\frac{1}{s}\left(c^{\prime}\right)^{2}-\frac{1}{s}(D-c) c^{\prime \prime}<0 . \tag{46}
\end{equation*}
$$

Substituting for $(D-c)$ from expression (45), we get:

$$
\begin{equation*}
\frac{4}{\left(s^{+}\right)^{3}}\left(s^{+} c^{\prime}\right)^{2}-\frac{4}{\left(s^{+}\right)^{2}} s^{+}\left(c^{\prime}\right)^{2}+\frac{1}{s^{+}}\left(c^{\prime}\right)^{2}-\frac{1}{s^{+}} 2 s^{+} c^{\prime} c^{\prime \prime}<0 \Leftrightarrow c^{\prime}-2 s^{+} c^{\prime \prime}<0, \tag{47}
\end{equation*}
$$

which, as above, establishes that $\frac{d s^{+}}{d a}<0$, or, equivalently, designs become nichier as search costs fall.

Proof of Proposition 7 Note that, by symmetry, all firms enjoy the same number of customers and, by assumption, there is full market coverage, and, so, total industry profits are equal to $\Pi=2 \pi\left(p^{+}-m\right)$. Differentiating this expression, we obtain that

$$
\begin{equation*}
\frac{d \Pi}{d a}=2 \pi \frac{d p^{+}}{d a} \tag{48}
\end{equation*}
$$

Thus, it suffices to establish the sign of $\frac{d p^{+}}{d a}$. Now, using (38) and (40), we can express the equilibrium price as

$$
\begin{equation*}
p^{+}=m+s^{+} c^{\prime}=m+\frac{2 a}{c^{\prime}} \tag{49}
\end{equation*}
$$

and differentiating, we get
$\frac{d p^{+}}{d a}=\frac{2 c^{\prime}+2 a c^{\prime \prime} \frac{d s^{+}}{d a}}{c^{\prime 2}}=2 \frac{c^{\prime}+\frac{a 2 c^{\prime \prime}}{c^{\prime}\left(c^{\prime}-2 s^{+} c^{\prime \prime}\right)}}{c^{\prime 2}}=2 \frac{c^{2}\left(c^{\prime}-2 s^{+} c^{\prime \prime}\right)+2 a c^{\prime \prime}}{c^{2} c^{\prime}\left(c^{\prime}-2 s^{+} c^{\prime \prime}\right)} \stackrel{(40)}{=} 2 \frac{c^{\prime}-s^{+} c^{\prime \prime}}{c^{\prime}\left(c^{\prime}-2 s^{+*} c^{\prime \prime}\right)}$.
The result follows immediately.
We conclude the proof by highlighting that there are indeed parameters where symmetric equilibria exist and where either case arises. It can be verified that with $c(y)=2 y^{3}$ and $m=0$, at $a=1.35$ profits decrease in $a$, whereas at $a=0.15$, they increase in $a$.

Proof of Proposition 8 In order to have an interior design decision, one needs $\frac{d^{2} \Pi_{1}(s)}{d s_{1}^{2}} \leq 0$.

$$
\begin{gather*}
\frac{d^{2} \Pi_{1}(s)}{d s_{1}^{2}}=2 \frac{\partial x_{12}}{\partial s_{1}}\left(\frac{2}{3}\left(c^{\prime}\left(1-s_{1}\right)+\pi\right)-x_{12}\right)+2 x_{12}\left(-\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right)-\frac{\partial x_{12}}{\partial s_{1}}\right)  \tag{51}\\
= \\
\stackrel{4}{3}\left(\frac{\partial x_{12}}{\partial s_{1}}\left(c^{\prime}\left(1-s_{1}\right)+\pi\right)-x_{12} c^{\prime \prime}\left(1-s_{1}\right)\right)-4 x_{12} \frac{\partial x_{12}}{\partial s_{1}} \\
\stackrel{F O C}{=} 2 x_{12} \frac{\partial x_{12}}{\partial s_{1}}+2 x_{12}\left(-\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right)\right)-4 x_{12} \frac{\partial x_{12}}{\partial s_{1}}=2 x_{12}\left(-\frac{\partial x_{12}}{\partial s_{1}}-\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right)\right) \leq 0 .
\end{gather*}
$$

This is equivalent to

$$
\begin{align*}
& \frac{\partial x_{12}}{\partial s_{1}}+\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right) \geq 0 \Leftrightarrow \frac{1}{3} \frac{c^{\prime}\left(1-s_{1}\right)+\pi}{s_{1}+s_{2}}-\frac{x_{12}}{s_{1}+s_{2}}+\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right) \geq 0  \tag{52}\\
& \stackrel{F O C}{\Leftrightarrow} \frac{\frac{1}{2} x_{12}}{s_{1}+s_{2}}-\frac{x_{12}}{s_{1}+s_{2}}+\frac{2}{3} c^{\prime \prime}\left(1-s_{1}\right) \geq 0 \Leftrightarrow c^{\prime \prime}\left(1-s_{1}\right) \geq \frac{3}{4} \frac{x_{12}}{\left(s_{1}+s_{2}\right)},
\end{align*}
$$

which shows that $c^{\prime \prime}\left(1-s_{1}\right)>0$ is a necessary but not a sufficient condition for an interior decision.

Proof of Proposition 9 We proceed in two stages. First, we argue that at least one firm chooses the most niche design, and then we show that this must be the one with higher marginal cost.

Suppose, for contradiction, that both firms choose designs $s_{1}^{d}, s_{2}^{d}$ that are either interior or broad. Then, (23) states that $c^{\prime}\left(1-s_{1}^{d}\right)+\pi \leq \frac{3}{2} x_{12}^{d}$. Similarly, the first-order condition for Firm 2 requires that $c^{\prime}\left(1-s_{2}^{d}\right)+\pi \leq \frac{3}{2}\left(\pi-x_{12}^{d}\right)$. Summing these, we obtain $c^{\prime}\left(1-s_{1}^{d}\right)+c^{\prime}\left(1-s_{2}^{d}\right)+2 \pi \leq \frac{3}{2} \pi$ or $c^{\prime}\left(1-s_{1}^{d}\right)+c^{\prime}\left(1-s_{2}^{d}\right) \leq-\frac{\pi}{2}$. Since $c^{\prime}>0$, this provides a contradiction.

Next, suppose that $s_{1}^{d} \in[0,1)$, then, $c^{\prime}\left(1-s_{1}^{d}\right)+\pi \leq \frac{3}{2} x_{12}^{d}$. Since $c^{\prime}\left(1-s_{1}^{d}\right)>0$, it follows that $\frac{3}{2} x_{12}^{d}>\pi$. Substituting for optimal prices in (19) and rearranging this last inequality, we obtain: $m_{2}-m_{1}>\pi s_{1}^{d}+c\left(1-s_{1}^{d}\right)-c\left(1-s_{2}^{d}\right)$. Now, from the first half of the proof, if $s_{1}^{d} \in[0,1)$, then, necessarily, $s_{2}^{d}=1$, and since $c$ is an increasing function $c\left(1-s_{1}^{d}\right)>c\left(1-s_{2}^{d}\right)=c(0)$, this, in turn, implies that $m_{2}-m_{1}>\pi s_{1}^{d}>0$, which completes the proof.

Proof of Proposition 10 Following Proposition 9, the firm with a higher marginal cost necessarily chooses the most extreme niche design. Thus without loss of generality, suppose that $m_{1}>m_{2}$ and that $s_{1}^{d}=1$, and consider $\frac{d s_{2}^{d}}{d m_{2}}$ for an interior design. Substituting for optimal prices and setting $s_{1}^{d}=1$ allows us to write Firm 2's profit function as

$$
\begin{equation*}
\Pi_{2}=\frac{2}{9} \frac{\left(c(0)-c\left(1-s_{2}^{d}\right)-m_{2}+m_{1}+\pi s_{2}^{d}+2 \pi\right)^{2}}{1+s_{2}^{d}} \tag{53}
\end{equation*}
$$

The second-order condition can be shown to be equivalent to

$$
\begin{equation*}
\left(\pi+c^{\prime}\left(1-s_{2}^{d}\right)\right)^{2}<c^{\prime \prime}\left(1-s_{2}^{d}\right) \tag{54}
\end{equation*}
$$

The first-order condition is equivalent to $c(0)-c\left(1-s_{2}^{d}\right)-m_{2}+m_{1}-\pi s_{2}^{d}=$ $2\left(1+s_{2}^{d}\right) c^{\prime}\left(1-s_{2}^{d}\right)$. Taking the total derivative of this expression with respect to $m_{2}$, we obtain that $\frac{d s_{2}^{d}}{d m_{2}}\left(2\left(1+s_{2}^{d}\right) c^{\prime \prime}\left(1-s_{2}^{d}\right)-c^{\prime}\left(1-s_{2}^{d}\right)-\pi\right)=1$. It follows that $\frac{d s_{2}^{d}}{d m_{2}}>0$ as long as

$$
\begin{equation*}
2\left(1+s_{2}^{d}\right) c^{\prime \prime}\left(1-s_{2}^{d}\right)>c^{\prime}\left(1-s_{2}^{d}\right)+\pi \tag{55}
\end{equation*}
$$

Following (54), $2\left(1+s_{2}^{d}\right) c^{\prime \prime}\left(1-s_{2}^{d}\right)>2\left(1+s_{2}^{d}\right)\left(\pi+c^{\prime}\left(1-s_{2}^{d}\right)\right)^{2}$, so that the above expression is implied by $2\left(1+s_{2}^{d}\right)\left(\pi+c^{\prime}\left(1-s_{2}^{d}\right)\right)^{2}>c^{\prime}\left(1-s_{2}^{d}\right)+\pi$, which is necessarily true since $c^{\prime}>0$ and $s_{2}^{d} \geq 0$.


[^0]:    ${ }^{1}$ A previous version of this paper circulated as "Targeted product design: Locating inside the Salop circle." We thank Jessica Burshell, Haritz Garro and Mariano Irace for superb research assistance, and Jan Bouckaert, Justin Johnson, Gerard Llobet, Matthew Mitchell, David Myatt, Michael Ryall, Kostas Serfes, and audiences at several conferences and seminars for helpful comments and discussions.

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[^1]:    ${ }^{2}$ There is a literature that considers firms that choose both vertical and horizontal features-notably, Economides (1989) and Neven and Thisse (1990). However, in their models, firms can separately choose horizontal and vertical characteristics (see, also, Irmen and Thisse (1998) and Vandenbosch and Weinberg (1995)). Instead, in this paper, horizontal and vertical aspects are inherently linked so that it is impossible to affect a consumer's horizontal transport cost without also influencing the vertical distaste. This also provides a contrast with the analysis in Dos Santos Ferreira and Thisse (1996) where there is no vertical cost (in our language) associated with choosing different horizontal transport costs.
    ${ }^{3}$ This design decision, thus, contrasts with standard models of horizontal differentiation, in which the concern is which consumers to satisfy rather than how much to satisfy segments of the population; and models of vertical differentiation in which designs are commonly ranked in consumers' preferences.

[^2]:    ${ }^{4}$ Von Ungern-Sternberg (1988), instead, imposes conditions that ensure an interior solution and analyses a symmetric equilibrium.
    ${ }^{5}$ Alternatively, as we describe in Section 2.3, the vertical costs can be understood as higher marginal costs that the firm incurs.

[^3]:    ${ }^{6}$ This discussion is reminiscent of aspects of the strategy literature and, specifically, Porter's (1998) notion of firms "stuck in the middle" in their strategic orientation, which suggests that "extremal" approaches may be optimal. The empirical validity of Porter's conjecture has found mixed support (see Campbell-Hunt (2000) for a meta-analysis of empirical studies). Our analysis highlights when extremal product designs are optimal and when, instead, intermediate designs may be preferred.

[^4]:    ${ }^{7}$ The paper and its results are stated in terms of differences in marginal costs of production. As is standard, and as we discuss below, these can be rewritten in terms of differences in vertical quality. Under this interpretation, firms with better products will choose broader designs.
    ${ }^{8}$ The term "long tail" was introduced in an article in Wired (Anderson (2004)). See, also, Brynjolfsson, Hu and Smith (2006).

[^5]:    ${ }^{9}$ We present the paper with $B>0$ to reduce algebra and ease exposition. The results for $B=0$ are similar, and they coincide with those obtained for the limit case where $B$ tends to 0 .

[^6]:    ${ }^{10}$ The consumer travels towards the center and along a ring independently, with different associated costs. Hence, these dimensions are better suited for two different characteristics of a good (such as the brightness and hue of its colour), rather than dimensions in a physical space.
    ${ }^{11}$ Linear costs along the arc deliver linear demand functions, while unit costs are without loss of generality.

[^7]:    ${ }^{12}$ Similar qualitative results arise when the constraint that demand can be no higher than $2 \pi$ binds-that is, all consumers are served. In particular, this case naturally arises when $s$ is close enough to 0 .

[^8]:    ${ }^{13}$ Demand rotations are ordered if the intersection point between two (inverse) demand functions moves upwards as the demand becomes flatter. See Johnson and Myatt (2006) for a formal definition of such ordered demand rotations.
    ${ }^{14}$ This visualisation of the outer envelope also provides an intuition for Lemma 1 of Johnson and Myatt (2007) which provides sufficient conditions for extremal designs based on properties of the induced demand curves.

[^9]:    ${ }^{15}$ In particular we take the mass of firms as exogenously given, and abstract from firms' entry decisions for ease of exposition.

[^10]:    ${ }^{16}$ Note that since there is a continuum of firms, the continuation value of continuing searching does not depend on the history of search. Therefore, the consumer prefers to visit a new random store rather than to go back to one that she has discarded before. In particular, this implies that it is irrelevant whether or not consumers can costlessly visit stores that they have visited in the past.

[^11]:    ${ }^{17}$ We can write profits as $\Pi=T 2 X_{i}\left(p_{i}-m_{i}\right)$, where $T$ is the total number of customers that visit the store in all periods. Given, the definition of $\rho, T=\sum_{t=0}^{\infty}(1-\rho)^{t}=\frac{1}{\rho}$. The expression in (11) follows directly.

[^12]:    ${ }^{18}$ Of course, both in that paper and in this one, in a region where designs are unaffected (for example, if all firms are choosing the nichiest design before any fall in search costs), profits rise monotonically with search costs.

[^13]:    ${ }^{19}$ Recall that this is equivalent to having all firms homogeneous in their marginal costs, but heterogeneous in their vertical quality, $V_{i}$. This is the case, for instance, in BCC.

[^14]:    ${ }^{20}$ More specifically, they show this for all stable equilibria in a simplified dynamic system that captures the interplay between $V$ and $U$ (see BCC for formal definitions and proofs.)

[^15]:    ${ }^{21}$ This contrasts with González-Maestre and Granero (2018), who consider the case in which the design and pricing decision are simultaneous.
    ${ }^{22}$ For simplicity, we abstract from the firms' choices of angle of location. Such analysis has been shown to be involved or intractable even in the simpler Hotelling framework (Osborne and Pitchik, 1987; and Vogel, 2008).

