DISCUSSION PAPER SERIES

DP15697

A Second-best Argument for Low Optimal Tariffs

Lorenzo Caliendo, Robert Feenstra, John Romalis and Alan M. Taylor

INTERNATIONAL TRADE AND REGIONAL ECONOMICS

ISSN 0265-8003

A Second-best Argument for Low Optimal Tariffs

Lorenzo Caliendo, Robert Feenstra, John Romalis and Alan M. Taylor

Discussion Paper DP15697
Published 22 January 2021
Submitted 17 January 2021

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

A Second-best Argument for Low Optimal Tariffs

Abstract

We derive a new formula for the optimal uniform tariff in a small-country, heterogeneous-firm model with roundabout production and a nontraded good. Tariffs are applied on imported intermediate inputs. First-best policy requires that markups on domestic intermediate inputs are offset by subsidies. In a second-best setting where such subsidies are not used, the double-marginalization of domestic markups creates a strong incentive to lower the optimal tariff on imported inputs. In a 186-country quantitative model, the median optimal tariff is 10%, and negative for five countries, as compared to 27% in manufacturing from the one-sector, optimal tariff formula without roundabout production.

JEL Classification: F12, F13, F17, F61

Keywords: trade policy, monopolistic competition, Gains from trade, input-output linkages

Lorenzo Caliendo - lorenzo.caliendo@yale.edu
Yale University

Robert Feenstra - rcfeenstra@ucdavis.edu
University of California, Davis

John Romalis - john.romalis@sydney.edu.au
The University of Sydney

Alan M. Taylor - amtaylor@ucdavis.edu
University of California, Davis and CEPR

Acknowledgements
We thank Pol Antràs, Kyle Bagwell, John Fernald, Andrés Rodríguez-Clare, and participants at an NBER seminar for helpful comments, along with Anna Ignatenko and Ninghui Li for providing excellent research assistance.

A Second-best Argument for Low Optimal Tariffs *

Lorenzo Caliendo
Yale University and NBER

Robert C. Feenstra
UC Davis and NBER

John Romalis
University of Sydney and ABFER

Alan M. Taylor
UC Davis, NBER, and CEPR

January 14, 2021

Abstract

We derive a new formula for the optimal uniform tariff in a small-country, heterogeneous-firm model with roundabout production and a nontraded good. Tariffs are applied on imported intermediate inputs. First-best policy requires that markups on domestic intermediate inputs are offset by subsidies. In a second-best setting where such subsidies are not used, the double-marginalization of domestic markups creates a strong incentive to lower the optimal tariff on imported inputs. In a 186-country quantitative model, the median optimal tariff is 10%, and negative for five countries, as compared to 27% in manufacturing from the one-sector, optimal tariff formula without roundabout production.

Keywords: trade policy, monopolistic competition, gains from trade, input-output linkages

JEL Codes: F12, F13, F17, F61

*Contact information: Caliendo: lorenzo.caliendo@yale.edu; Feenstra, rcfeenstra@ucdavis.edu; Romalis, john.romalis@sydney.edu.au; Taylor, amtaylor@ucdavis.edu.

We thank Pol Antràs, Kyle Bagwell, John Fernald, Andrés Rodríguez-Clare, and participants at an NBER seminar for helpful comments, along with Anna Ignatenko and Ninghui Li for providing excellent research assistance.

1 Introduction

The use of tariffs to protect traded goods such as manufactures has a long history. In his famous *Report on Manufactures*, Alexander Hamilton argues for moderate tariffs combined with direct subsidies to promote manufacturing. Opposition to the proposed subsidies came from Thomas Jefferson and James Madison, who favored even higher tariffs, and Madison's administration produced the first protectionist tariff in the United States (Irwin, 2004). The administration of President Donald Trump enacted tariffs, often at 25%, to protect several manufacturing industries and against a broad range of products from China. Significantly, the Chinese products were initially selected to minimize the direct impact on consumer prices, leaving American businesses facing the brunt of tariffs on their imported inputs (Fajgelbaum, Goldberg, Kennedy and Khandelwal, 2020).

Does modern trade theory offer any new answer to this old question of whether to protect the traded sector? To answer this, we investigate a small-country model with two sectors – one traded and the other nontraded – and with heterogeneous firms, monopolistic competition and CES preferences (as in Melitz, 2003). We adopt a Pareto distribution for productivity (as in Chaney, 2008) and also roundabout production (as in Caliendo and Parro, 2015). The differentiated intermediate inputs in each sector are bundled into a finished good that is sold to home consumers and firms in that sector, but not traded, while the differentiated inputs are traded in one sector. A tariff is applied to imports of these differentiated intermediate inputs.

Demidova and Rodríguez-Clare (2009) obtain a formula for the optimal uniform tariff in a small country with one sector and no roundabout production, which we denote by t^{opt}. Because there is no roundabout production, we can think of this tariff as applying to imported final differentiated goods. They argue that this single tariff instrument obtains the first-best by offsetting two distortions: the need to correct for the markup on domestic final goods (by applying a tariff equal to that markup) and the externality present because imported varieties bring surplus that is not taken into account in domestic spending (by slightly lowering the tariff). When there is roundabout production, t^{opt} does not correct for the double-marginalization that occurs when the markup on domestic differentiated inputs is passed-through to the price of the bundled finished good, which is further used as an input to the production of other differentiated inputs. In a closed economy, we show that this double-marginalization is corrected by applying a subsidy on the bundled finished good. When this subsidy is not used, however, then a second-best policy available in an open economy is to lower the import tariff below t^{opt}, thereby lowering the price of the bundled finished good. *Our main result is to show that the optimal second-best tariff on intermediate inputs is below* t^{opt}*, provided that a certain (small) amount of roundabout production is present.*

We obtain the optimal uniform, second-best tariff as a fixed-point of a formula that has two new terms: a M term that reflects the relative monopoly distortion between the traded and non-traded sectors; and a R term that reflects roundabout production in the traded sector, which amplifies the monopoly distortion there. In a 186-country, 15-sector quantitative version of the model, the optimal uniform tariff has a median value of only 10% (or 7.5% for countries with above-median shares of manufacturing production), as compared to t^{opt} of 27.3% in a one-sector model with manufacturing parameters, and is negative for five countries: Bhutan, Myanmar, New Caledonia, Hong Kong, and Spain.

Costinot, Rodríguez-Clare and Werning (2020) analyze optimal tariffs on final differentiated goods with very general tastes and technologies, and they show that optimal tariffs can be lowered (and even made negative) by having multiple sectors, a non-Pareto distribution for productivity, or linear foreign preferences. They are the first to extend the analysis to *nonuniform* tariffs, and they find that the importing country should use an import subsidy on the least efficient foreign exporters. Haaland and Venables (2016) demonstrated a potential second-best role for reduced trade taxes to offset a monopoly distortion, building on earlier work by Flam and Helpman (1987). Lashkaripour and Lugovsky (2020) analyze optimal uniform first-best tariffs with multiple sectors and input-output linkages, but when considering second-best tariffs, they do not incorporate these linkages. As far as we are aware, then, the literature has not addressed the realistic case that we examine here: second-best tariffs in the presence of roundabout production and a nontraded sector. That is the critical gap we aim to fill, by providing a formula for the second-best tariff in this setting and by characterizing the *generality* of low optimal tariffs on intermediate inputs.

2 Two-Sector Economy with Roundabout Production

We analyze a two-sector Melitz (2003)-Chaney (2008) model with roundabout production, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012, section IV) and Costinot and Rodríguez-Clare (2014). We summarize key equations here and Appendix A contains the full model. There are two countries, $k = i, j$, and two sectors $s = 1, 2$, where sector 1 is traded and sector 2 is nontraded. County i is a small open economy, and the foreign country $j \neq i$ is the rest of the world. In the foreign country, for simplicity we assume a single traded sector, $s = 1$.

In both sectors, firms produce differentiated inputs under monopolistic competition, which are costlessly bundled into a *finished good* in CES fashion, with elasticity $\sigma_s > 1$. The finished good is non-traded, and it is sold to domestic consumers as final goods and also to domestic firms as intermediate inputs, used to produce differentiated inputs (e.g. firms produce machinery parts using machines). In sector 1, the traded differentiated inputs are subject to iceberg costs and the imported varieties are subject to a tariff, where one plus the *ad valorem* tariff for country i imports from j is denoted by t_{ji1}; for simplicity, there is no foreign tariff.

The finished output in each sector has quantity Q_{is}, price index P_{is}, and value $Y_{is} \equiv P_{is} Q_{is}$. With roundabout production, the marginal cost of producing a differentiated input for a firm with productivity $\varphi_s = 1$ in sector s is

$$x_{is} \equiv w_i^{\gamma_{is}} P_{is}^{1-\gamma_{is}}, \tag{1}$$

where $0 < \gamma_{is} \leq 1$ is the labor share. We refer to (1) as the *input cost index*.

A mass of firms N_{is} incur fixed labor costs of entry f_{is}^e to receive a productivity draw from a Pareto distribution, $G_s(\varphi_s) = 1 - \varphi_s^{-\theta_s}$, with $\varphi_s \geq 1$ and $\theta_s > \sigma_s - 1$. As is familiar from the Melitz-Chaney model, firms choose to produce the differentiated input for the domestic market or to export if their productivities exceed some *cutoff* levels, and in each case, the firms then incur additional fixed labor costs.

Consumers have Cobb-Douglas preferences over final goods in the two sectors:

$$U_i = C_{i1}^{\alpha_i} C_{i2}^{1-\alpha_i}, \tag{2}$$

where $\alpha_i > 0$ is the expenditure share on the traded sector 1. Consumer income I_i includes labor income (the only factor of production) $w_i L_i$, plus rebated tariff and tax revenue B_i, while free entry ensures that expected firm profits equal zero.

Domestic consumer demand for finished goods equals $\alpha_i I_i$ in sector 1 and $(1-\alpha_i)I_i$ in sector 2. Let λ_{ijs} denote the expenditure share of differentiated inputs that country j purchases from country i, so $\lambda_{iis} = 1 - \lambda_{jis}$ is the domestic expenditure share with $\lambda_{ji2} \equiv 0$ and $\lambda_{ii2} \equiv 1$ in the nontraded sector 2. Then the total production cost of all differentiated inputs in sector 1 in country i is $\frac{\sigma_1 - 1}{\sigma_1} \sum_{k=i,j} \lambda_{ik1} Y_{k1}$; namely, the domestic sales and exports of differentiated inputs in sector 1, adjusted by markups. Given the share $(1-\gamma_{is})$ of costs in (1) going to the finished good, the demand for that good in sector 1 comes from domestic consumers and from domestic firms producing those differentiated inputs for sale in both countries:

$$Y_{i1} = \alpha_i(w_i L_i + B_i) + \tilde{\gamma}_{i1}\left(\lambda_{iis} Y_{i1} + \lambda_{ij1} Y_{j1}\right), \text{with } \tilde{\gamma}_{is} \equiv (1-\gamma_{is})\left(\frac{\sigma_s - 1}{\sigma_s}\right) < 1, \tag{3}$$

while in the nontraded sector 2 this equation is simply $Y_{i2} = (1-\alpha_i)(w_i L_i + B_i) + \tilde{\gamma}_{i2} Y_{i2}$. The parameter $\tilde{\gamma}_{is}$ eliminates markups from the value of intermediate inputs before computing the cost share, $(1-\gamma_{is})$, devoted to the finished good as an input.

The expenditure shares and the cutoff productivities are determined in equilibrium (see Appendix A), and we normalize the foreign wage at unity. The term $\lambda_{ij1} Y_{j1}$ appearing in (3) is the value of country i exports of the differentiated inputs. Under balanced trade, this must equal the net-of-tariff value of imports. Letting t_{ji1} denote one plus the *ad valorem* import tariff used by country i, then $\lambda_{ij1} Y_{j1} = \frac{\lambda_{ji1}}{t_{ji1}} Y_{i1}$. Entry is proportional to the demand for those inputs for home sale, $\lambda_{ii1} Y_{i1}$, plus the demand for exports, $\lambda_{ij1} Y_{j1} = \frac{\lambda_{ji1}}{t_{ji1}} Y_{i1}$. Solving for Y_{i1} from (3) we find that entry is

$$N_{i1} = \frac{\alpha_i(\sigma_1 - 1)}{f_{i1}^e \theta_1 \sigma_1}\left[\frac{L_i}{\frac{1-\alpha_i}{\Lambda_{i1}} + (\alpha_i - \tilde{\gamma}_{i1})}\right], \text{ with } \Lambda_{i1} \equiv \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}}\right). \tag{4}$$

Since $\lambda_{iis} + \lambda_{jis} = 1$, then $\Lambda_{i1} = 1$ in free trade (with $t_{ji1} = 1$) and autarky ($t_{ji1} \to +\infty$ so $\lambda_{ii1} = 1$ and $\lambda_{ji1} = 0$). It follows that N_{i1} is equal at these two points. But for $1 < t_{ji1} < +\infty$ then $\Lambda_{i1} < 1$, so that Λ_{i1} is a $\cup$-shaped function of the tariff. We show (see Appendix A.2) that Λ_{i1} achieves its minimum at the same tariff at which tariff revenue B_i/w_i is maximized. It follows from (4) that entry is a $\cup$-shaped function of the tariff, just like Λ_{i1}, unless there is no nontraded sector and $\alpha_i = 1$, in which case entry is constant. The intuition for this result is Lerner symmetry (Costinot and Werning, 2019), whereby the import tariff acts like an export tax, and starting from free trade the tariff depresses entry into the traded sector. Entry into the nontraded sector is

$$N_{i2} = \frac{(1-\alpha_i)(\sigma_2 - 1)}{f_{i2}^e \theta_2 \, \sigma_2 (1 - \tilde{\gamma}_{i2})} \left(L_i + \frac{B_i}{w_i} \right), \tag{5}$$

which is a $\cap$-shaped function of the tariff because revenue B_i/w_i has that pattern.

3 Optimal Consumer and Producer Taxes in a Closed Economy

We first discuss the distortions arising in a closed economy from having monopolistic production of the differentiated inputs, where both sectors $s = 1, 2$ are nontraded. The markup on the differentiated inputs is fully passed-through to the price of the bundled, finished good. That distortion then operates on two margins: consumer purchases of finished goods; and firm purchases of finished goods as inputs, where the higher price on the finished good is *further* passed-through to raise the price of intermediate inputs, creating a double-marginalization of the markup on intermediate inputs. Rather than correcting the monopoly distortion at its source (i.e. in the price of differentiated inputs), it will be instructive to correct it by using tax/subsidies on purchases of the finished goods on these two margins. So we consider both consumer and producer tax/subsidies on purchases of the finished goods, where one plus the *ad valorem* rates are denoted by t_{is}^c and t_{is}^p, respectively.

We consider two solutions to the closed-economy problem (see Appendix B): first, choosing both the consumer and producer tax/subsidies optimally; and second, using only the consumer tax/subsidy while setting $t_{is}^p \equiv 1$. When both instruments are used, we obtain the solution

$$t_{is}^p = \left(\frac{\sigma_s - 1}{\sigma_s} \right) < 1 \quad \text{and} \quad \frac{t_{i1}^c}{t_{i2}^c} = \left(\frac{\sigma_1 - 1}{\sigma_1} \right) \bigg/ \left(\frac{\sigma_2 - 1}{\sigma_2} \right). \tag{6}$$

The optimal producer subsidies $t_{is}^p < 1$ exactly counteract the markups on differentiated inputs which would otherwise be fully passed-through to finished goods prices.[1] With these subsidies, firms pay prices for finished goods that reflect their marginal costs. In addition, optimal consumption tax/subsidies are needed so that, in relative terms, these prices offset the markups implicit in finished goods' prices faced by consumers.

In contrast to this first-best case, consider the second-best policy that involves consumption tax/subsidies only. Because of double-marginalization of the markups charged on differentiated outputs, the sector s elasticity σ_s effectively becomes $\tilde{\sigma}_{is} \equiv 1 + \gamma_{is}(\sigma_s - 1)$, and the markup is $\frac{\tilde{\sigma}_{is}}{(\tilde{\sigma}_{is}-1)}$. The solution for the optimal consumption tax/subsidies is

$$\frac{t^c_{i1}}{t^c_{i2}} = \left(\frac{\tilde{\sigma}_{i1} - 1}{\tilde{\sigma}_{i1}}\right) \Big/ \left(\frac{\tilde{\sigma}_{i2} - 1}{\tilde{\sigma}_{i2}}\right) \quad \text{for} \quad \tilde{\sigma}_{is} \equiv 1 + \gamma_{is}(\sigma_s - 1). \tag{7}$$

To interpret (7), the sector with the lowest effective elasticity must have the lowest tax (i.e. greatest subsidy) to offset the *effective* monopoly distortion, which is inversely measured by $\gamma_{is}(\sigma_s - 1)$. Even if the elasticities $\sigma_s \equiv \sigma > 1$ are identical then the sector with the strongest roundabout production (lowest γ_{is}) must be subsidized in consumption, because it has the highest effective markup due to double-marginalization. The intuition from the second-best case will be useful as we examine tariffs on trade, as we turn to next.

4 Optimal Uniform Tariffs in a Small Open Economy

4.1 First-best tariff

Demidova and Rodríguez-Clare (2009) analyze a small, open economy with one sector, $s = 1$, and no roundabout production, so we should think of the imports as final differentiated goods. They identify two distortions arising from monopolistic competition. The first is the markup charged on the domestic differentiated varieties which can be corrected by subsidizing domestic buyers of those inputs, where one minus the *ad valorem* subsidy is the inverse of the markup:

$$t^*_{ii1} = \rho_1 \quad \text{with} \quad \rho_i \equiv \frac{\sigma_i - 1}{\sigma_i} < 1. \tag{8}$$

[1] The need for such subsidies in a dynamic monopolistic competition model was noted by Judd (1997, 2002).

Alternatively, the markup on domestic varieties can be offset by using a tariff on imported varieties equal to the markup, $t_1 \equiv 1/\rho_1$, which is the optimal tariff with homogeneous firms (Gros, 1987).

With heterogeneous firms, however, Demidova and Rodríguez-Clare (2009) find that there is a second distortion: each new foreign variety brings surplus, which domestic buyers do not take account of in their spending. One way to correct this externality is to use an import subsidy, and they find that one minus the optimal *ad valorem* subsidy is

$$t^*_{ji1} = \frac{\theta_1 \rho_1}{(\theta_1 - \rho_1)} < 1, \tag{9}$$

where the inequality follows from $\theta_1 > \sigma_1 - 1$. Furthermore, they argue that that an *equivalent* policy to using t^*_{ii1}, t^*_{ji1} is to multiply the tariff $t_1 = 1/\rho_1$ by the import subsidy in (9), and then both distortions are corrected by a single instrument, which is the optimal tariff:[2]

$$t^{opt} \equiv t_1 \times t^*_{ji1} = \frac{\theta_1}{(\theta_1 - \rho_1)} > 1. \tag{10}$$

If we add a second sector or roundabout production, however, then the equivalence of using the policy $t^*_{ii1}, t^*_{ji1} < 1$ and the optimal tariff $t^{opt} > 1$ no longer holds. To see this, suppose that we "scale-up" t^*_{ii1}, t^*_{ji1} by dividing by ρ_1, thereby obtaining $t_{ii1} = 1$ and t^{opt}, and then use a subsidy of ρ_1 on the finished good to offset this scaling-up. With a single sector and no roundabout production, this subsidy does not make any difference because consumers cannot substitute away from the finished good and firms do not purchase it. But once we add multiple sectors and/or roundabout production, then substitution by consumers and firms means that the subsidy of ρ_i is needed to avoid double-marginalization, as we found in the closed economy. Analogously, for an open economy with multiple sectors and input-output linkages, Lashkaripour and Lugovsky (2020) argue that such subsidies must be applied in the first-best; in that case, the first-best tariffs for a *small* country are the same with and without input-output linkages.[3] Our interest is in the

[2]The same small-country formula for the optimal tariff as (10) is obtained by Felbermayr, Jung and Larch (2013), who show that the optimal tariff in a large country is higher.

[3]See their section 4(ii) and especially footnote 23, which explains that for a small open economy the equations for the first-best taxes and tariffs are identical with and without input-output linkages. These authors consider a wide class of models introduced by Kucheryavyy, Lyn and Rodríguez-Clare (2016), which includes the Melitz-Pareto case. Depending on the model being considered, the first-best tariff formula is not necessarily the same as t^{opt}, but in all cases the first-best tariff does not depend on input-output linkages for a small country. When considering the second-best

second-best tariff obtained in the absence of such subsidies, as we turn to next.

4.2 Second-best Tariff

We now add the nontraded sector 2, and we suppose that the *only* policy instrument available is a uniform import tariff (or subsidy). Because we are no longer using the instruments t_{ii1}, t_{i1}^{p} or t_{i1}^{c}, for convenience we drop subscripts from the import tariff t_{ji1} and simply denote it by t_i with an optimal second-best value t_i^*. The fact that a subsidy on the finished good is not used creates a robust reason for lowering the optimal tariff below t^{opt}. A slight reduction of the tariff below its first-best value ordinarily causes only a second-order loss in welfare, but it now brings a first-order gain in welfare because it lowers the price of the finished good purchased by firms.

Entry provides a second possible reason to have $t_i^* < t^{opt}$. As we showed in section 2, starting from free trade a tariff in sector 1 leads firms to *exit* that sector and move into sector 2. That will lead to a welfare loss if the monopoly distortion is greater in the traded sector. Let $D(t_i)$ denote the marginal welfare impact of firms entering the traded sector – holding the cutoff productivities constant – relative to the share of spending on that sector (α_i).[4] We find that

$$D(t_i) = \left[\frac{\tilde{\sigma}_{i1}}{(\tilde{\sigma}_{i1} - 1)} - \frac{\tilde{\sigma}_{i2}}{(\tilde{\sigma}_{i2} - 1)} \frac{\Lambda_{i1}(1 - \tilde{\gamma}_{i1})}{1 - \tilde{\gamma}_{i1}\Lambda_{i1}} - \mathcal{E}_d \right] \tag{11}$$

where $\mathcal{E}_d > 0$ and all such script-variables depend on sector 1 parameters and λ_{ii1} (and therefore depend on the tariff). The first term appearing in (11), $\frac{\tilde{\sigma}_{i1}}{(\tilde{\sigma}_{i1}-1)}$, is the effective markup in sector 1, and the second term is the effective markup in sector 2 multiplied by $\frac{\Lambda_{i1}(1-\tilde{\gamma}_{i1})}{1-\tilde{\gamma}_{i1}\Lambda_{i1}}$ (which is ≤ 1 for $t_i \geq 1$) that reflects tariff revenue. The third term $-\mathcal{E}_d < 0$ appears because the tariff is an inefficient instrument to influence entry, so it has a deadweight loss.

We see from (11) that $D(t_i) > 0$ so that entry into the traded sector leads to a welfare gain – and exit leads to a welfare loss –when that effective markup there is *sufficiently above* the effective markup in the nontraded sector. For the 186 country quantitative model used in the next section, we find that $\tilde{\sigma}_{i1}$ in manufacturing (one of the industries in the traded sector) and $\tilde{\sigma}_{i2}$ in the non-traded sector (services) both have median values of about two. It follows that $D(t_i) < 0$ at the

tariffs without such subsidies, however, they do not incorporate input-output linkages.

[4] See Appendix C.1 for the total change in utility from selection and entry (i.e. the term $D(t_i)$).

median , so it is inefficient to lower the tariff to promote entry into manufacturing. But for about 10% of countries we find that $D(t_i) > 0$ when comparing manufacturing with services, which creates an argument for encouraging entry into manufacturing by lowering the tariff.

In our theoretical work, we want to allow the effective distortion in the traded sector to be greater or less than that in the nontraded sector. We will impose an upper-bound on the *inverse* distortion of the traded sector as compared to the nontraded sector:

$$\frac{(\tilde{\sigma}_{i1}-1)}{\tilde{\sigma}_{i1}} < \kappa_i \frac{(\tilde{\sigma}_{i2}-1)}{\tilde{\sigma}_{i2}}, \tag{12}$$

where the parameter $\kappa_i \geq 1$ will be specified in Theorem 1 below. Our aim is to choose κ_i high enough to include a wide range of effective distortions in (12).

We can now state a general formula for the optimal second-best tariff t_i^*, as compared to t^{opt} (see Appendix C). Specifically, t_i^* is obtained as a fixed point of the equation

$$t_i^* = t^{opt}\, F(t_i^*), \text{ with } F(t_i) \equiv \left[\frac{1-(1-\gamma_{i1})R(t_i)}{1+(1-\alpha_i)M(t_i)}\right], \tag{13}$$

where $M(t_i)$ captures the impact of the higher monopoly distortion in the traded versus the non-traded sectors, and is defined by

$$M(t_i) \equiv \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1\right)\frac{D(t_i)}{A(t_i)} \quad \text{with} \quad \mathcal{M} > 0,\ \mathcal{E}_m > 0, \tag{14}$$

where $A(t_i)$ is defined by

$$A(t_i) \equiv \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\mathcal{E}_a \quad \text{with} \quad \mathcal{E}_a > 0, \tag{15}$$

while $R(t_i)$ reflects the impact of roundabout production and is defined by

$$R(t_i) = \mathcal{R} \times \left[\frac{\theta_1 - \rho_1\,(1-\lambda_{ii1})}{\Lambda_{i1}} - \theta_1\rho_1\right] \quad \text{with} \quad \mathcal{R} > 0. \tag{16}$$

To explain these terms more carefully, recall that the distortion term $D(t_i)$ measures the marginal welfare impact of firms moving from the nontraded to the traded sector, and notice that it enters $(1-\alpha_i)M(t_i)$, which appears in the denominator of (13), reflecting the impact of the *relative* monopoly distortion on the optimal tariff. When $\alpha_i = 1$ so there is only the traded sector, then this term vanishes, because there is no impact of the *relative* distortion between traded and non-traded goods. But there is still roundabout production in traded goods alone, and the impact of that roundabout production on the optimal tariff is captured by the term $R(t_i)$, appearing in the numerator of (13).

More specifically, when $\alpha_i = 1$ and $\gamma_{i1} = 1$ in (13), then we are back in the one-sector, no-roundabout model and that formula immediately gives $t_i^* = t^{opt}$. Outside of that special case, there will be a lower optimal tariff, $t_i^* < t^{opt}$, whenever $(1-\alpha_i)M(t_i^*) \geq 0$ and $(1-\gamma_{i1})R(t_i^*) \geq 0$ with one of these inequalities holding strictly. For example, suppose that $\alpha_i = 1$ so there is only a traded sector, but $\gamma_{i1} < 1$ so there is some roundabout production. Then we can show that $R(t_i^*) > 0$ at the fixed point of (13), so that roundabout production lowers the optimal tariff.

Next, suppose we add the nontraded sector so that $\alpha_i < 1$, in which case the denominator of $F(t_i^*)$ equaling $[1 + (1-\alpha_i)M(t_i^*)]$ comes into play. If the relative distortion in the traded sector is positive, $D(t_i^*) > 0$, then provided that the other terms in (14) are positive we will have $M(t_i^*) > 0$, so the denominator *further reduces* the optimal tariff. One of those other terms is $A(t_i)$. Recall that we initially defined $D(t_i)$ as the marginal impact of entry into sector 1 relative to the size of that sector (α_i), and we loosely interpret $A(t_i)$ as the effective size of sector 1. As a regularity condition we need to impose $A(t_i) > 0$, which is guaranteed by the sufficient conditions specified in the following result (proved in Appendix D).

Theorem 1.

(a) ***Pure roundabout:*** *If $\alpha_i = 1$ and $\gamma_{i1} < 1$, then $R(t_i^*) > 0$ and the optimal tariff is $t_i^* < t^{opt}$.*

(b) ***No roundabout:*** *If $\gamma_{i1} = \gamma_{i2} = 1$ then (i) $D(t_i^*) > 0$ and the optimal tariff is $t_i^* < t^{opt}$ when*

$$\sigma_1 < \sigma_2 \left[\frac{\sigma_1(\theta_1 - \rho_1)}{\sigma_1\theta_1 - \rho_1} \right] < \sigma_2, \tag{17}$$

(ii) if $\sigma_1 \geq \sigma_2$ then $D(t_i^*) < 0$ and the optimal tariff is $t_i^* > t^{opt}$.

(c) ***Two sectors with roundabout:*** *Assume that $\alpha_i < 1$ and the following two conditions hold:*

$$\gamma_{i1} \geq \frac{1}{1 + \frac{\sigma_1}{\rho_1}\left(\theta_1 - \rho_1\right)\left(1 - \rho_1\right)}, \tag{18}$$

$$\alpha_i \geq \min\left\{\tilde{\gamma}_{i1}, \frac{-\gamma_{i1}\theta_1 + \rho_1\left(1 + \frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}{\frac{\theta_1(1-\rho_1)}{\rho_1} + \rho_1\left(1 + \frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}\right)}\right\}. \tag{19}$$

Then $A(t_i) > 0$ for $t_i > t_i'$, where $t_i' < 1$ is an import subsidy. Furthermore, if there is enough roundabout production so that

$$\gamma_{i1} \leq 1 - \frac{\rho_1}{\left[\theta_1(1-\rho_1) + \rho_1^2\right]\left(\theta_1 - \rho_1\right)} < 1, \tag{20}$$

and the bounds in (12) hold where we specify κ_i as

$$\kappa_i = \left[\delta_i + \frac{\tilde{\gamma}_{i1}\theta_1\left(\alpha_i(1-\rho_1) + \gamma_{i1}\rho_1\right)\left(\theta_1\left(1-\rho_1\right) + \left(t^{opt} - \tilde{\gamma}_{i1}\right)\rho_1\right)}{\left(1-\alpha_i\right)\left(1-\tilde{\gamma}_{i1}\right)^2}\right]\frac{\left(t^{opt} - \tilde{\gamma}_{i1}\right)}{\left(1-\tilde{\gamma}_{i1}\right)}, \tag{21}$$

for $\delta_i \equiv \frac{1-\rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1-\frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt}+\frac{1}{\sigma_1}}$, then the optimal tariff is $t_i^* < t^{opt}$ with $R(t_i^*) > 0$.

Part (a) has already been discussed, and shows that roundabout production in a one-sector model always lowers the optimal tariff. This result is the simplest demonstration that the tariff t_i^* on intermediate inputs is less than the tariff t^{opt} on final goods.

Part (b) deals with the opposite case where there is no roundabout production, so the imports are differentiated final goods. In that case, $A(t_i) > 0$ is guaranteed. Condition (17) used in part (b)(i) ensures that the relative distortion in the traded sector sufficiently exceeds that in the non-traded sector so that $D(t_i) > 0$ for $t_i \in [1, t^{opt}]$. In that case, the relative monopoly distortion is the only factor operating to reduce the optimal tariff and we find that $t_i^* < t^{opt}$ because $D(t_i^*) > 0$ and the denominator of $F(t_i^*)$ exceeds unity. On the other hand, if the traded sector is less distorted than the nontraded sector, with $\sigma_1 \geq \sigma_2$, then we have the reverse outcome with $D(t_i^*) < 0$ and $t_i^* > t^{opt}$. So the tariff on final goods can be greater or less than that found in a one-sector model, depending on the relative monopoly distortion across sectors.

In part (c) we allow for two sectors and roundabout production, and now we need $A(t_i) > 0$.

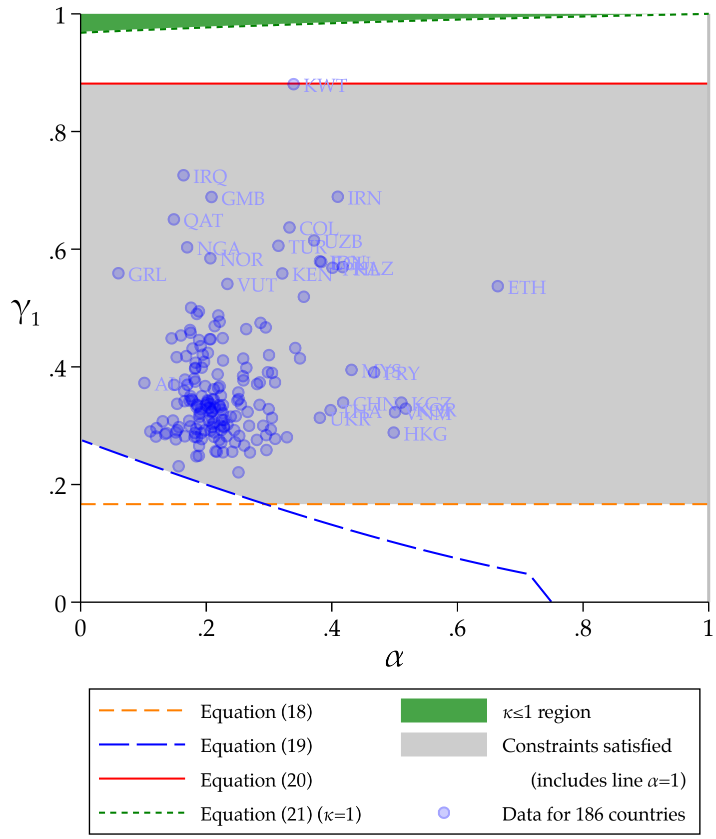
To ensure $A(t_i) > 0$ for $t_i > t_i'$, where $t_i' < 1$ is an import subsidy specified in the proof, we require the sufficient conditions (18) and (19): the former is a lower-bound on γ_{i1} and the latter is a lower-bound on α_i (but also depending on γ_{i1}). To illustrate these two lower-bound constraints, we rely on parameter values that we adopt in our quantitative model of the next section. There we use the EORA dataset that has input-output matrices for 186 countries in 2010 and 15 sectors (Lenzen, Moran, Kanemoto and Geschke, 2013).[5] The traded sectors include Manufacturing, Agriculture and Mining (including petroleum extraction), while all Service industries are treated as nontraded. We adopt parameters values for each of these sectors, and these are $\sigma_1 = 4.4$ and $\theta_1 = 5.1$ for Manufacturing which are used for illustrative purposes in Figure 1. The dots in Figure 1 are the values of α_i and γ_{i1} when aggregating within all three traded sectors. We see that the two lower-bounds constraints (18) and (19) are satisfied for all countries so that the regularity condition $A(t_i) > 0$ holds.

Now we check whether $t_i^* < t^{opt}$ holds in part (c), which allows for the nontraded sector and roundabout production in both sectors. We already know from part (b) – where we excluded roundabout production – that it is possible to find the reverse outcome $t_i^* > t^{opt}$ if the traded sector is less distorted than the nontraded sector ($\sigma_1 \geq \sigma_2$). We would like to know, however, if a small amount of roundabout production is enough to overwhelm that relative distortion, so that $t_i^* < t^{opt}$ due to $R(t_i^*) > 0$ regardless of the sign of $D(t_i^*)$. Part (c) answers that question in the affirmative. The needed amount of roundabout production is shown by the constraint (20), which is an upper-bound on γ_{i1} as graphed in Figure 1 and is very weak: all countries in our sample satisfy this constraint, with Kuwait (KWT) near the borderline of (20) due to high γ_{i1} (little roundabout production) in petroleum extraction and thus in overall traded production.

To ensure that $t_i^* < t^{opt}$ in part (c), we also need to put a constraint on the relative distortion across sectors, as was indicated by (12) with $\kappa_i \geq 1$. The needed value of κ_i is indicated by (21), which has a large median value of 9.1 in our sample of 186 countries. The line for which $\kappa_i = 1$ is shown in Figure 1 with the thin solid region $\kappa_i \leq 1$ illustrated by that line and the region above it: for parameters in this region, we have $\kappa_i < 1$ and we can only find $t_i^* < t^{opt}$ if the traded

[5] EORA has 190 countries including the Rest of the World, which we omit, along with Belarus, Moldova and the Former Soviet Union because their input-output tables are nonsensical.

Figure 1: Parameter Restrictions



1

sector is more distorted than the nontraded sector ($\tilde{\sigma}_{i1} < \tilde{\sigma}_{i2}$ in (12)). Conversely, if the traded sector is less distorted in effective terms ($\tilde{\sigma}_1 \geq \tilde{\sigma}_{i2}$), then we find that $t_i^* > t^{opt}$. So just as (b)(ii) of Theorem 1 illustrates that a high optimal tariff can arise in the case of a final good not used in production, the thin solid region $\kappa_i \leq 1$ shows that a high optimal tariff can arise even when there is a small amount of roundabout production in the tradable sector, provided that this sector is less distorted. As just noted, Kuwait (KWT) has little roundabout production and it is near to this thin solid region at the top of Figure 1. Even though this country meets the sufficient conditions to have $t_i^* < t^{opt}$ in our two-sector model – since it lies just within the lightly shaded region of Figure 1[6] – in the multisector quantitative model analyzed in the next section we will find that Kuwait and other OPEC countries have high optimal tariffs, $t_i^* > t^{opt}$. So the result that a certain

[6] In the region of Figure 1 shown in white, we are unsure whether the second-best tariff is greater or less than t^{opt} because Theorem 1 only provides sufficient conditions for each case.

(small) amount of roundabout production is needed to ensure a low optimal tariff, as established by Theorem 1(c), will carry over to the quantitative model.

We conclude this section by noting that the optimal tariff can be negative. In our working paper (CFRT, 2020), we examine the conditions to ensure that the optimal tariff is negative, and we find that it occurs for two types of countries: a *Highly Linked Economy* that has high roundabout production (low γ_{i1}) and is very open (low λ_{ii1}); and a *Remote Economy*, with a small traded sector and with $\lambda_{ii1} \to 1$, so that the economy is nearly closed to trade due to high iceberg costs, as may occur for very distant countries. We find examples of both types in our quantitative analysis that is discussed next.

5 Second-best Uniform Tariffs in a General, Calibrated Model

The quantitative model from our working paper (CFRT, 2020) uses the EORA dataset. Table 1 contains the model elasticities and summary statistics, after grouping the 15 sectors into four broader sectors. The estimates of σ_s and θ_s for goods are from Caliendo and Parro (2015), and satisfy the relationship $\theta_s / (\sigma_s - 1) = 1.5$. Gervais and Jensen (2019) find that services have elasticities of substitution about one-quarter *smaller* than for manufacturing. We follow them, by setting $\sigma_s = 2.8$ for services and, given $\theta_s / (\sigma_s - 1) = 1.5$, setting $\theta_s = 2.7$. We therefore have σ_s for traded goods exceeding σ_s for services, generating higher markups in the nontraded sector.[7]

We slightly generalize the Melitz-Chaney model by allowing for nested CES with the upper-level elasticity of substitution ω_s, between the aggregates of home and foreign varieties, differing from the lower-level elasticity σ_s, across different foreign (or home) varieties. Setting $\omega_s = \sigma_s / 1.25$ best reproduces global trade growth between 1990 and 2010. With this structure we obtain the one-sector, no-roundabout, small-country formula for the optimal tariff from Costinot, Rodríguez-Clare and Werning (2020), which for nested CES is:[8]

$$t^{opt} = \frac{\omega_1}{\left[\omega_1 - \frac{(\sigma_1 - 1)}{\theta_1}\right]}. \tag{22}$$

[7]EORA allows for trade in service sectors, but we excluded that trade from our quantitative model.

[8]See their footnotes 21 and 20 and set $x^*_{FF} = 1$.

Table 1: Elasticities and Linkages by Broad Sector

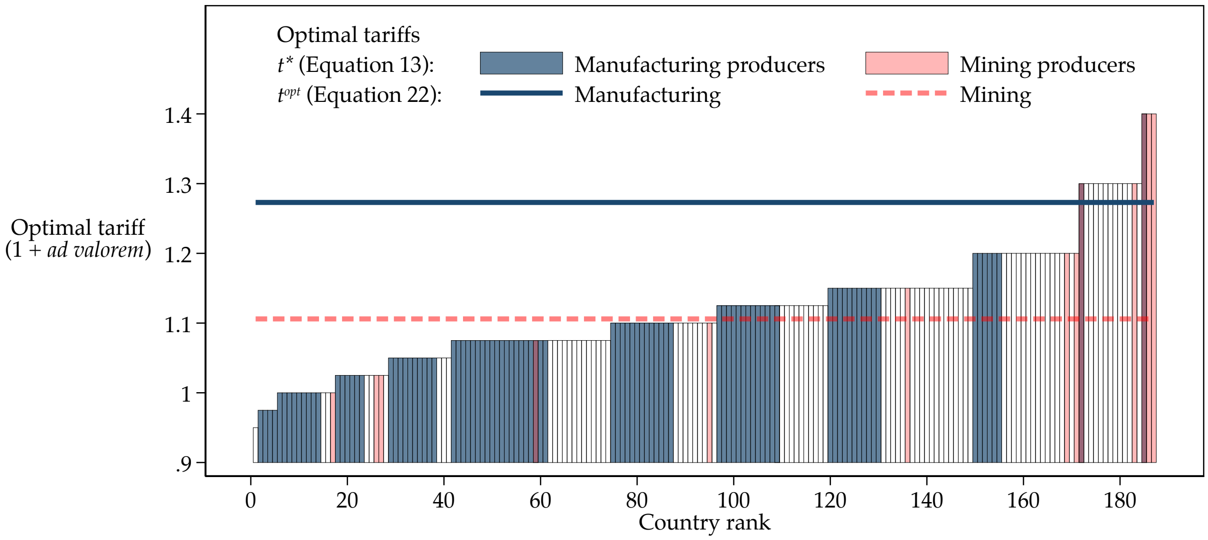
Statistic	Agriculture	Mining	Manufacturing	Services
θ_s	8.61	13.03	5.05	2.70
σ_s	6.74	9.69	4.36	2.80
$\alpha_{is} \quad (p10)$	0.00	0.00	0.15	0.66
$\alpha_{is} \quad (median)$	0.01	0.00	0.20	0.79
$\alpha_{is} \quad (p90)$	0.05	0.01	0.28	0.84
$\gamma_{is} \quad (p10)$	0.31	0.29	0.24	0.46
$\gamma_{is} \quad (median)$	0.51	0.46	0.28	0.56
$\gamma_{is} \quad (p90)$	0.76	0.74	0.38	0.69
$\tilde{\sigma}_{is} = 1 + \gamma_{is}(\sigma_s - 1) \quad (p10)$	2.77	3.56	1.81	1.83
$\tilde{\sigma}_{is} = 1 + \gamma_{is}(\sigma_s - 1) \quad (median)$	3.93	4.98	1.96	2.01
$\tilde{\sigma}_{is} = 1 + \gamma_{is}(\sigma_s - 1) \quad (p90)$	5.37	7.42	2.28	2.24

We measure γ_{is} from our theoretical model by treating "labor" in that model as an aggregate factor that includes both labor and capital services. Accordingly, Table 1 reports the shares of industry revenue from EORA that go to value-added, γ_{is}, which for Manufacturing varies across countries from 24% at the 10th percentile to 38% at the 90th. Also reported is the effective elasticity $\tilde{\sigma}_{is} \equiv 1 + \gamma_{is}(\sigma_s - 1)$ in each sector. We find that the median effective elasticity in Manufacturing (1.96) is only slightly lower than the median effective elasticity in Services (2.01), with much heterogeneity across countries, and both of these broad sectors are more distorted than the two primary sectors, Agriculture and Mining.

The optimal tariffs in the quantitative model are computed numerically. Specifically, we start at a world free trade equilibrium, calculated using 2010 input-output tables, and use a grid search over positive and negative tariffs for each country. We evaluate the welfare effects from imposing unilateral uniform tariffs across sectors of between -20% to +40% in increments mostly of 2.5% one country at a time and summarize results in Figure 2.

Five countries have negative optimal tariffs in 2010: Bhutan; Myanmar; New Caledonia; Hong Kong; and Spain. Bhutan, Myanmar and New Caledonia seem to fit our description of remote economies, while Hong Kong and Spain are highly-linked economies. Our median 2010 optimal tariff is 10%. For comparison we also plot optimal uniform tariffs using the one-sector formula from (22). Using our parameters for each sector yields optimal tariffs of 16.0% for Agriculture; 10.6% for Mining; and 27.3% for Manufacturing. We plot two horizontal lines for the optimal tariffs t^{opt} in Manufacturing and in Mining (which includes petroleum extraction).

Figure 2: Optimal One-sector Tariffs and from the Quantitative Model



Our median optimal tariff t_i^* of 10% is less than two-fifths of t^{opt} in (22) which is 27.3% for Manufacturing parameters (the dominant sector in trade). There is much variation across countries. Economies with at least the median proportion of Manufacturing production tend to have lower optimal tariffs, with a median of 7.5%, or just over one-quarter of t^{opt} from equation (22) for that sector. In contrast, the 13 countries where Mining (including petroleum extraction) accounts for at least 10% of production – including many OPEC countries – have a median optimal tariff of 20%, which greatly exceeds t^{opt} in (22) of 10.6% for that sector. Even though a resource sector like Mining was not introduced into our two-sector theoretical model, the possibility of high optimal tariffs in oil-rich countries was suggested by our discussion of Kuwait in the previous section, which has a low amount of roundabout production. Our finding of high optimal tariffs for oil exporters is also supported by the high elasticity σ_s for Mining in Table 2, implying low markups so that resources should be shifted to other sectors, and by large oil exporters exploiting the terms of trade. But these countries are the exceptions that prove the rule: for countries that specialize in Manufacturing, the numerical optimal tariffs are considerably lower than t^{opt} in nearly all cases.

6 Conclusions

We began by asking whether modern trade theory has anything new to say about arguments for protecting the traded sector. We did not mention a line of recent literature that to some extent argues in favor of such protection. Specifically, this is the *firm-delocation* literature that combines a monopolistically competitive traded sector with a competitive *traded* outside good (see e.g. Melitz and Ottaviano, 2008, section 4; Bagwell and Lee, 2020). The traded numeraire good pins down relative wages between countries, so the country applying tariffs is "small" in the sense that its wages do not respond to its tariff. In this literature, encouraging entry into traded goods requires positive import tariffs. Essentially, the ability to attract firms into the home country takes the place of a conventional terms-of-trade motive for tariffs, so that the optimal tariff is positive even though wages are fixed. Of course, with multiple countries pursuing this motive for protection, there is ample scope for trade agreements to reduce the deadweight losses due to the tariffs (Ossa, 2011; Bagwell and Staiger, 2015).

The major differences between this class of models and our own are: (i) roundabout production, so that tariffs are applied on imported intermediate inputs rather than final goods; and (ii) the *nontraded* service sector, which does not fix relative wages between countries. Lerner symmetry holds in the traded sector, so that import tariffs are equivalent to export taxes and inhibit entry into that sector. That logic does not apply when the numeraire good is traded, which gives firm-delocation models a very different flavor: they act like partial equilibrium models because wages are fixed, and perhaps are most appropriate to narrowly targeted tariffs, whereas our results depend on Lerner symmetry, which is a general equilibrium result and depends on having broad tariffs applied to the traded sector. Determining the most appropriate range of applications for each class of models is one important area for further research.

References

[1] Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. New Trade Models, Same Old Gains? *American Economic Review* 102(1): 94–130.

[2] Bagwell, Kyle, and Robert W. Staiger. 2015. Delocation and Trade Agreements in Imperfectly Competitive Markets. *Research in Economics* 69(2): 139–156.

[3] Bagwell, Kyle, and Seung Hoon Lee. 2020. Trade Policy under Monopolistic Competition with Firm Selection. *Journal of International Economics* 127, November, https://doi.org/10.1016/j.jinteco.2020.103379.

[4] Caliendo, Lorenzo, Robert C. Feenstra, John Romalis, and Alan M. Taylor. 2020 (revised version). Tariff Reductions, Entry and Welfare: Theory and Evidence for the last Two Decades. NBER Working Paper 21768.

[5] Caliendo, Lorenzo, and Fernando Parro. 2015. Estimates of the Trade and Welfare Effects of NAFTA. *Review of Economic Studies* 82 (1): 1–44.

[6] Chaney, Thomas. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98(4): 1707–21.

[7] Costinot, Arnaud, and Andrés Rodríguez-Clare. 2014. Trade Theory with Numbers: Quantifying the Consequences of Globalization. In *Handbook of International Economics*, volume 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. New York: Elsevier, pp. 197–262.

[8] Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning. 2020. Micro to Macro: Optimal Trade Policy with Firm Heterogeneity. *Econometrica*, forthcoming, https://www.econometricsociety.org/system/files/14763-4.pdf .

[9] Costinot, Arnaud and Iván Werning. 2019. Lerner Symmetry: A Modern Treatment, *American Economic Review: Insights*, 1(1), June, 13-26.

[10] Demidova, Svetlana and Andrés Rodríguez-Clare. 2009. Trade Policy under Firm-Level Heterogeneity in a Small Economy. *Journal of International Economics* 78(1), June: 100–112.

[11] Demidova, Svetlana and Andrés Rodríguez-Clare. 2013. The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics* 90(2), July: 266–272.

[12] Fajgelbaum, Pablo, Pinelopi Goldberg, Patrick Kennedy and Amit Khandelwal. 2020. The Return to Protectionism. *Quarterly Journal of Economics* 135(1): 1–55.

[13] Felbermayr, Gabriel, Benjamin Jung, and Mario Larch. 2013. Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model. *Journal of International Economics* 89: 13-25.

[14] Flam, Harry and Elhanan Helpman. 1987. Industrial Policy under Monopolistic Competition. *Journal of International Economics* 22: 79–102.

[15] Gervais, Antoine and J. Bradford Jensen. 2019. The Tradability of Services: Geographic Concentration and Trade Costs. *Journal of International Economics* 118(May): 331–350

[16] Gros, Daniel. 1987. A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade. *Journal of International Economics* 23: 357–367.

[17] Haaland, Jan I. and Anthony J. Venables. 2016. Optimal Trade Policy with Monopolistic Competition and Heterogeneous Firms. *Journal of International Economics.* 102: 85-95.

[18] Irwin, Douglas A. (2004). The Aftermath of Hamilton's 'Report on Manufactures'. *The Journal of Economic History*. 64 (3): 800–821.

[19] Judd, Kenneth L. 1997. The Optimal Tax on Capital Income is Negative. NBER Working Paper 6004.

[20] Judd, Kenneth L. 2002. Capital Income Taxation with Imperfect Competition. *American Economic Review* 92(2), May: 417–421.

[21] Kucheryavyy, Konstantin, Gary Lyn, and Andrés Rodríguez-Clare. 2016. Grounded by Gravity: A Well-Behaved Trade Model with Industry-Level Economies of Scale. NBER Working Paper 22484.

[22] Lashkaripour, Ahmad and and Volodymyr Lugovsky. 2020. Profits, Scale Economies, and the Gains from Trade and Industrial Policy. Indiana University, Working Paper, April, https://alashkar.pages.iu.edu/Lashkaripour_Lugovskyy_2020.pdf.

[23] Lenzen, Manfred, Daniel Moran, Keiichiro Kanemoto, and Arne Geschke. 2013. Building Eora: A Global Multi-Regional Input-Output Database at High Country and Sector Resolution. *Economic Systems Research* 25(1): 20–49.

[24] Melitz, Marc J. 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71(6):1695–1725.

[25] Melitz, Marc J. and Gianmarco I. P. Ottaviano. 2008. Market Size, Trade, and Productivity. *Review of Economic Studies* 75: 295–316.

[26] Melitz, Marc J., and Stephen J. Redding. 2014. Heterogeneous Firms and Trade. In Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, eds. *Handbook of International Trade*, Volume 4. Amsterdam: Elsevier, 1–54.

[27] Ossa, Ralph. 2011. A New Trade Theory of GATT/WTO Negotiations. *Journal of Political Economy* 119(1): February, 122–152

APPENDIX

A Two-Sector Small Open Economy Model

We focus on a small open economy model, with two sectors $s = 1, 2$ and roundabout production in both sectors. County i is the small home economy and the foreign country $j \neq i$ is the rest of the world. In the foreign country, for simplicity we assume a single traded sector denoted by $s = 1$ with no roundabout production and no import tariff.

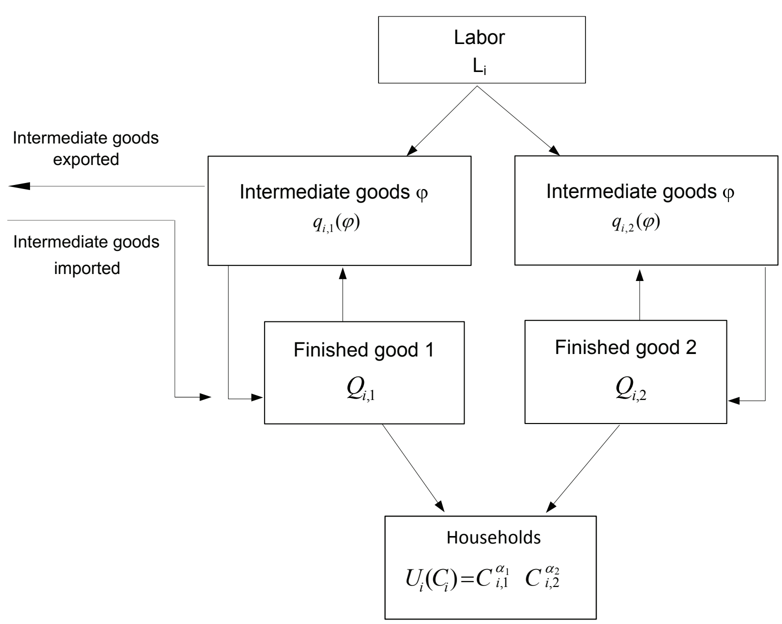
A.1 Description of Economy

The structure of the country i economy is illustrated in Figure A1. Firms in sector 1 of country i can source differentiated inputs from countries $k = i, j$ for $i \neq j$, and the CES production functions over the differentiated inputs purchased from each country k and in total are

$$Q_{i1} \equiv \left(\sum_{k=i,j} Q_{ki1}^{\frac{\sigma_1 - 1}{\sigma_1}} \right)^{\frac{\sigma_S}{\sigma_S - 1}} \text{ with } Q_{ki1} \equiv \left(N_{k1} \int_{\varphi_{ki1}^*}^{\infty} q_{ki1}(\varphi)^{\frac{\sigma_1 - 1}{\sigma_1}} g_1(\varphi) d\varphi \right)^{\frac{\sigma_1}{\sigma_1 - 1}}, \tag{23}$$

where N_{k1} are the mass of entrants in each of countries k who sell at the prices $p_{ki1}(\varphi)$, depending on their productivities φ with Pareto distribution $g_1(\varphi)$. The cutoff productivity $\varphi \geq \varphi_{ki1}^*$ needed to sell from country k to i will be derived below. The CES price indexes over the differentiated inputs purchased from

Figure A1: Schematic production structure



each country $k = i, j$ and in total are

$$P_{i1} \equiv \left(\sum_{k=i,j} P_{ki1}^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}} \text{ and } P_{ki1} \equiv \left(N_{k1} \int_{\varphi^*_{ki1}}^{\infty} p_{ki1}(\varphi)^{1-\sigma_1} g_1(\varphi) \, d\varphi \right)^{\frac{1}{1-\sigma_1}}. \tag{24}$$

The mass of input varieties sold from country $k = i, j$ to i for firms with productivity $\varphi \geq \varphi^*_{ki1}$ is

$$N_{ki1} \equiv N_{k1}[1 - G_1(\varphi^*_{ki1})] = N_{k1} \varphi^{*-\theta_1}_{ki1}, \tag{25}$$

using the Pareto distribution $G_1(\varphi) = 1 - \varphi^{-\theta_1}$ with $\varphi \geq 1$. Notice that the entry of firms N_{k1} appearing in (23) and (24) can be converted into the mass of varieties by multiplying and dividing by $[1 - G_1(\varphi^*_{ki1})]$, in which case the unconditional densities $g_1(\varphi)$ become conditional densities $g_1(\varphi)/[1 - G_1(\varphi^*_{ki1})]$. In sector 2 we use the analogous definitions of Q_{ii2}, P_{ii2}, and N_{ii2}.

The total value of production of the finished good in country i and sector s is $Y_{is} = P_{is} Q_{is}$, and the CES demand for intermediates of variety φ sold from country $k = i, j$ to i is given by

$$q_{kis}(\varphi) = \left(\frac{p_{kis}(\varphi)}{P_{is}} \right)^{-\sigma_s} \frac{Y_{is}}{P_{is}}. \tag{26}$$

In sector 2, however, the intermediates used in country i are purchased only from country i. A firm in i supplying differentiated inputs has the marginal costs x_{is}/φ, with the costs of input bundle supply x_{is} given by (1). We assume that fixed costs of the firm require only labor and are denoted by f_{iks} and f^e_{is}. In country j, we ignore roundabout production and there is only sector 1, so that $x_{j1} \equiv w_j$ and the fixed labor costs are f_{jk1} and f^e_{j1}.

The profits in country $k' = i, j$ from supplying differentiated inputs to country $k = i, j$ are

$$\pi_{k'ks}(\varphi) = \max_{p_{k'ks}(\varphi) \geq 0} \left\{ \frac{p_{k'ks}(\varphi)}{t_{k'ks}} q_{k'ks}(\varphi) - \frac{x_{k's}}{\varphi} \tau_{k'ks} \, q_{k'ks}(\varphi) - w_{k'} \, f_{k'ks} \right\}, \tag{27}$$

where $\tau_{k'ks}$ are iceberg trade costs with $\tau_{iis} \equiv \tau_{jjs} \equiv 1$, and t_{ji1} is one plus the *ad valorem* tariff charged for country i imports from j in sector 1, with all other tariffs at unity: $t_{kjs} \equiv 1$, $k = i, j$.[9] The first-order conditions for profit maximization yield

$$\frac{p_{k'ks}(\varphi)}{t_{k'ks}} = \frac{\sigma_s}{\sigma_s - 1} \frac{x_{k's} \, \tau_{k'ks}}{\varphi}, \tag{28}$$

$$q_{k'ks}(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1} \frac{x_{k's} \, \tau_{k'ks} t_{k'ks}}{\varphi} \right)^{-\sigma_s} \frac{Y_{ks}}{P_{ks}^{1-\sigma_s}}. \tag{29}$$

Substituting these expressions back into profits, we can readily solve for the cutoff productivity $\varphi^*_{k'ks}$ at which profits are zero:

$$\pi_{k'ks}(\varphi^*_{k'ks}) = 0 \implies \varphi^*_{k'ks} = \left(\frac{\sigma_s}{\sigma_s - 1} \right) \left(\frac{\sigma_s w_{k'} \, f_{k'ks} t_{k'ks}}{Y_{ks} P_{ks}^{\sigma_s - 1}} \right)^{\frac{1}{(\sigma_s - 1)}} x_{k's} \, \tau_{k'ks} t_{k'ks}. \tag{30}$$

[9] We briefly allowed for a domestic tax/subsidy of $t_{iis} \neq 1$ in our discussion of first-best policies in the main text, but that instrument is not used otherwise.

We follow Melitz (2003) in defining the average productivity as

$$\bar{\varphi}_{k'ks} \equiv \left(\int_{\varphi^*_{k'ks}}^{\infty} \varphi^{\sigma_s - 1} \frac{g_s(\varphi)}{\left[1 - G_s(\varphi^*_{k'ks})\right]} d\varphi \right)^{\frac{1}{\sigma_s - 1}} = K_s \varphi^*_{k'ks}, \text{with } K_s \equiv \left(\frac{\theta_s}{\theta_s - \sigma_s + 1} \right)^{\frac{1}{\sigma_s - 1}}, \tag{31}$$

where the constant K_s is obtained by computing the integral using the Pareto distribution.

We can substitute (29) into (23) to obtain the output of the finished good:

$$Q_{is} = K_s^{\sigma_s} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{-\sigma_s} \left(\frac{Y_{is}}{P_{is}^{1-\sigma_s}} \right) \left[N_{is} \varphi^{*-\theta_s}_{iis} \left(\frac{x_{is}}{\varphi^*_{iis}} \right)^{1-\sigma_s} + \ N_{js} \varphi^{*-\theta_s}_{jis} \left(\frac{w_j \, \tau_{jis} \, t_{jis}}{\varphi^*_{jis}} \right)^{1-\sigma_s} \right]^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{32}$$

where in the nontraded sector 2 the second terms in brackets does not appear, and this should also be understood in the next two equations. Using (24) and (28) we obtain an expression for P_{is}:

$$P_{is} = \left({\varphi^*_{iis}}^{-\theta_s} N_{is} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{x_{is}}{\bar{\varphi}_{iis}} \right)^{1-\sigma_s} + {\varphi^*_{jis}}^{-\theta_s} N_{j_s} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{w_j t_{jis}}{\bar{\varphi}_{jis}} \right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}. \tag{33}$$

We can multiply this by (32) to obtain a preliminary expression for the value of production of the finished goods in country i and sector s, $Y_{is} \equiv P_{is} Q_{is}$:

$$Y_{is} = K_s^{\sigma_s - 1} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{1-\sigma_s} \left(\frac{Y_{is}}{P_{is}^{1-\sigma_s}} \right) \left[N_{is} \varphi^{*-\theta_s}_{iis} \left(\frac{x_{is}}{\varphi^*_{iis}} \right)^{1-\sigma_s} + \ N_{js} \varphi^{*-\theta_s}_{jis} \left(\frac{w_j \, \tau_{jis} \, t_{jis}}{\varphi^*_{jis}} \right)^{1-\sigma_s} \right].$$

To simplify this expression, we can use (30) twice to obtain

$$\frac{Y_{is}}{P_{is}^{1-\sigma_s}} = \sigma_s w_i f_{iis} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{x_{is}}{\varphi^*_{iis}} \right)^{\sigma_s - 1} = \sigma_s w_j f_{jis} t_{jis} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{w_j \, \tau_{jis} t_{jis}}{\varphi^*_{jis}} \right)^{\sigma_s - 1}.$$

and substituting above we obtain

$$Y_{is} = K_s^{\sigma_s - 1} \sigma_s \left(N_{is} \varphi^{*-\theta_s}_{iis} \, w_i \, f_{iis} + \ N_{js} \varphi^{*-\theta_s}_{jis} \, w_j \, f_{jis} \, t_{jis} \right). \tag{34}$$

The value of finished output in each sector, Y_{ks}, is sold to consumers and also back to domestic firms. That finished output is costlessly bundled from home and (for sector 1) imported differentiated inputs. Let $\lambda_{k'k1}$ denote the share of country k total expenditure in sector 1 on intermediate goods from country k'. Using conditions (28)–(33) we can obtain the following expressions for the expenditure shares for inputs sold by country $k' = i$ to country $k = i, j$:

$$\lambda_{ik1} = {\varphi^*_{ik1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{\tau_{ik1} x_{i1}}{\bar{\varphi}_{ik1} \, P_{k1}} \right)^{1-\sigma_1} \tag{35}$$

$$= {\varphi^*_{ik1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1 \, w_i \, f_{ik1}}{Y_{k1}} \right) \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right), \tag{36}$$

and for country i imported inputs:

$$\lambda_{ji1} = {\varphi^*_{ji1}}^{-\theta_1} N_{j1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{\tau_{ji1}\, w_j\, t_{ji1}}{\bar{\varphi}_{ji1}\, P_{i1}} \right)^{1-\sigma_1} \tag{37}$$

$$= {\varphi^*_{ji1}}^{-\theta_1} N_{j1} \left(\frac{\sigma_1 w_j f_{ji1} t_{ji1}}{Y_{i1}} \right) \left(\frac{\theta_1}{\theta_1 + 1 - \sigma_1} \right). \tag{38}$$

The model is closed by making use of the market clearing condition described in the main text in (3), which in sector 2 is simply $Y_{i2} = (1 - \alpha_i)(w_i L_i + B_i) + \tilde{\gamma}_{i2} Y_{i2}$, together with trade balances. Duty-free imports in sector 1 of country i are $E_{ji1} = (\lambda_{ji1} Y_{i1}) / t_{ji1}$ while exports are $E_{ij1} = \lambda_{ij1} Y_{j1}$, so that trade balance requires

$$\frac{\lambda_{ji1} Y_{i1}}{t_{ji1}} = \lambda_{ij1} Y_{j1}. \tag{39}$$

Note that using (36) and (38), then trade balance (39) implies

$${\varphi^*_{ij1}}^{-\theta_1} N_{i1}\, w_i\, f_{ij1} = {\varphi^*_{ji1}}^{-\theta_1} N_{j1}\, w_j\, f_{ji1}. \tag{40}$$

Again using (36) and (38) with home sales $E_{ii1} = \lambda_{ii1} Y_{i1}$ and exports $E_{ij1} = \lambda_{ij1} Y_{j1}$, we obtain an expression for total sales of intermediate inputs in sector 1 by country i:

$$E_{ii1} + E_{ij1} = \sum_{k=i,j} {\varphi^*_{ik1}}^{-\theta_1} N_{i1} \left(\frac{\theta_1 \sigma_1\, w_i\, f_{ik1}}{\theta_1 + 1 - \sigma_1} \right).$$

This equation is simplified by making use of free entry in country i. Expected profits must equal the fixed costs of entry, so that for a country i firm:

$$\sum_{k=i,j} \int_{\varphi^*_{ik1}}^{\infty} \pi_{ik1}(\varphi) g_1(\varphi) d\varphi = w_i f^e_{i1}. \tag{41}$$

To evaluate this integral we follow the approach of Melitz and Redding (2014), who note that CES demand implies that $\pi_{ik1}(\varphi) + w_i f_{ik1} = \left[\pi_{ik1}(\varphi^*_{ik1}) + w_i f_{ik1}\right] (\varphi / \varphi^*_{ik1})^{\sigma_1 - 1}$. It follows from (27) that $\pi_{ik1}(\varphi) = \left[(\varphi/\varphi^*_{ik1})^{\sigma_1 - 1} - 1\right] w_i f_{ik1}$, and so the above entry condition becomes:

$$\sum_{k=i,j} J_1(\varphi^*_{ik1}) f_{ik1} = f^e_{i1} \quad \text{with} \quad J_s(\varphi^*) \equiv \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma_s - 1} - 1 \right] g_s(\varphi) d\varphi.$$

Completing the integral above using the Pareto distribution, we arrive at

$$\left(\frac{\sigma_1 - 1}{\theta_1 - \sigma_1 + 1} \right) \sum_{k=i,j} \varphi^{*-\theta_1}_{ik1} f_{ik1} = f^e_{i1}, \tag{42}$$

from which we can obtain an equation governing the mass of entrants N_{is}, namely

$$N_{i1} = \left(E_{ii1} + E_{ij1}\right) \Big/ \left[w_i\, f^e_{i1} \left(\frac{\theta_1\, \sigma_1}{\sigma_1 - 1} \right) \right]. \tag{43}$$

In sector 2 the mass of entrants is governed by the same equation but without E_{ij2} appearing

$$N_{i2} = E_{ii2} \bigg/ \left[w_i\, f_{i2}^e \left(\frac{\theta_1\, \sigma_2}{\sigma_2 - 1} \right) \right]. \tag{44}$$

The free entry condition for sector 2 is defined analogously to (41) but summing over country $k = i$ only, obtaining a condition that determines φ_{ii2}:

$$J_2(\varphi_{ii2}^*) f_{ii2} = \left(\frac{\sigma_2 - 1}{\theta_2 - \sigma_2 + 1} \right) \varphi_{ii2}^{*-\theta_2}\, f_{ii2} = f_{i2}^e. \tag{45}$$

A.2 Output, Entry and Λ_{i1}

As explained in the main text, the term $\lambda_{ij1} Y_{j1}$ appearing in (3) is the value of country i exports of the differentiated inputs. Under balanced trade, this must equal the net-of-tariff value of imports. Letting t_{ji1} denote one plus the *ad valorem* import tariff used by country i, then $\lambda_{ij1} Y_{j1} = \frac{\lambda_{ji1}}{t_{ji1}} Y_{i1}$. Tariff revenue is $B_i = \frac{t_{ji1}-1}{t_{ji1}} \lambda_{ji1} Y_{i1}$. We re-express tariff revenue as

$$B_i = (1 - \Lambda_{i1})\, Y_{i1} \;\; \text{with} \;\; \Lambda_{i1} \equiv \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}} \right), \tag{46}$$

and using these terms in (3), we can solve for real output Y_{i1}/w_i as

$$\frac{Y_{i1}}{w_i} = \frac{\alpha_i I_i}{1 - \tilde{\gamma}_{i1} \Lambda_{i1}} \quad \Longrightarrow \quad \frac{Y_{i1}}{w_i} = \frac{\alpha_i L_i}{1 - \alpha_i + (\alpha_i - \tilde{\gamma}_{i1}) \Lambda_{i1}}. \tag{47}$$

Since $\lambda_{iis} + \lambda_{jis} = 1$, then $\Lambda_{i1} = 1$ in free trade (with $t_{ji1} = 1$) and autarky ($t_{ji1} \to +\infty$ so $\lambda_{ii1} = 1$ and $\lambda_{ji1} = 0$). It follows that Y_{i1}/w_i is equal at these two points. But for $1 < t_{ji1} < +\infty$ then $\Lambda_{i1} < 1$, so that Λ_{i1} is a ∪-shaped function of the tariff. We show below that Λ_{i1} achieves its minimum at the same tariff at which tariff revenue B_i/w_i is maximized. Then we see from the the denominator of the second expression in (47) that real output can be either a ∩-shaped or ∪-shaped function of the tariff depending on whether $\alpha_i > (<) \tilde{\gamma}_{i1}$. This ambiguity does not extend, however, to the entry of firms producing differentiated inputs in sector 1. Using entry from (43) and noting that home sales are $E_{ii1} = \lambda_{ii1} Y_{i1}$ and exports are $E_{ij1} = \lambda_{ij1} Y_{j1}$, we solve for entry into sector 1 as shown in (4), which is a ∪-shaped function of the tariff provided that $\alpha_i < 1$.

For sector 2, the market clearing condition is $Y_{i2} = (1 - \alpha_i)(w_i L_i + B_i) + \tilde{\gamma}_{i2} Y_{i2}$, which directly leads to

$$Y_{i2} = \frac{1 - \alpha_i}{1 - \tilde{\gamma}_{i2}} I_i. \tag{48}$$

Then making use of (44) with (48) we immediately obtain (5) in the main text.

It remains to be shown that tariff revenue B_i has its maximum at the same tariff at which Λ_{i1} has its minimum, as asserted in the main text. This follows directly from (46) and (47), from which we obtain

$$B_i = Y_{i1}\,(1 - \Lambda_{i1}) = \frac{\alpha_i w_i L_i\,(1 - \Lambda_{i1})}{1 - \tilde{\gamma}_{i1} - (\alpha_i - \tilde{\gamma}_{i1})(1 - \Lambda_{i1})} = \frac{\alpha_i w_i L_i}{\frac{1 - \tilde{\gamma}_{i1}}{(1 - \Lambda_{i1})} - (\alpha_i - \tilde{\gamma}_{i1})}. \tag{49}$$

It follows that B_i is monotonically decreasing in Λ_{i1}, so their critical points are at the same maximum-revenue tariff.

A.3 Domestic Production Share and $T(t_{ji1})$

We now introduce the share of *production* (value-added) devoted to differentiated intermediate inputs that are sold domestically in country i, which will be used many times in our derivations. The expenditure share on imported intermediate inputs is λ_{ji1} in (37), so $\lambda_{ji1} Y_{i1}$ measures the value of imports inclusive of tariffs (and iceberg costs). We can instead evaluate imports at the net-of-tariff prices by dividing by t_{ji1} obtaining $\lambda_{ji1} Y_{i1}/t_{ji1} = (1-\lambda_{ii1}) Y_{i1}/t_{ji1}$, which equals exports and can be summed with $\lambda_{ii1} Y_{i1}$ to obtain the total value of production. It follows that the *share* of production sold to domestic firms – or the domestic production share – is

$$\tilde{\lambda}_{ii1} \equiv \frac{\lambda_{ii1}}{\lambda_{ii1} + \frac{(1-\lambda_{ii1})}{t_{ji1}}} = \frac{t_{ji1}\lambda_{ii1}}{1 + \lambda_{ii1}(t_{ji1}-1)}. \tag{50}$$

We now claim that this share can be measured by

$$\tilde{\lambda}_{ii1} = \frac{\varphi_{ii1}^{*}{}^{-\theta_1} f_{ii1}}{\varphi_{ii1}^{*}{}^{-\theta_1} f_{ii1} + \varphi_{ij1}^{*}{}^{-\theta_1} f_{ij1}}. \tag{51}$$

To show this, we first rewrite the domestic expenditure share λ_{ii1} using (34), (36) for $k = i$, (38) and trade balance (40) as

$$\lambda_{ii1} = \frac{\varphi_{ii1}^{*}{}^{-\theta_1} f_{ii1}}{\varphi_{ii1}^{*}{}^{-\theta_1} f_{ii1} + \varphi_{ij1}^{*}{}^{-\theta_1} f_{ij1} t_{ji1}}. \tag{52}$$

For the above two equations we obtain the relationship

$$t_{ji1} = \frac{(1-\lambda_{ii1})}{\lambda_{ii1}} \frac{\tilde{\lambda}_{ii1}}{(1-\tilde{\lambda}_{ii1})}, \tag{53}$$

and as a result

$$1 - t_{ji1} = \frac{\lambda_{ii1} - \tilde{\lambda}_{ii1}}{\lambda_{ii1}\left(1-\tilde{\lambda}_{ii1}\right)}. \tag{54}$$

From these two equations we can readily confirm the second equality in (50), which shows that it is equivalent to (51), so that is a correct formula for the domestic production share.

We can use this production share to define a simple function of the tariff $T(t_{ji1})$ given by

$$T(t_{ji1}) \equiv 1 - \tilde{\gamma}_{i1} + \left(t_{ji1} - 1\right)\left(1 - \tilde{\lambda}_{ii1}\right). \tag{55}$$

Notice that $T(t_{ji1}) = 1 - \tilde{\gamma}_{i1}$ in free trade (with $t_{ji1} = 1$) and autarky ($t_{ji1} \to +\infty$ so $\lambda_{ii1} = 1$ and $\lambda_{ji1} = 0$), but $T(t_{ji1}) > 1 - \tilde{\gamma}_{i1}$ for $1 < t_{ji1} =< +\infty$. It follows that $T(t_{ji1})$ is a $\cap$-shaped function of the tariff between these two points, which is the same shape as tariff revenue B_i. In fact, $T(t_{ji1})$ and B_i have their critical points at the same tariff, as we show just below.

In the main text we use Λ_{i1} to characterize entry into sector 1, but throughout the rest of the Appendix we mainly find it convenient to instead use the function $T(t_{ji1})$. These two concepts are inversely related, which can be seen by using (53) and (54) to obtain

$$t_{ji1} = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \frac{(1-\lambda_{ii1})}{(1-\tilde{\lambda}_{ii1})} = [(t_{ji1}-1)(1-\tilde{\lambda}_{ii1}) + 1] \frac{(1-\lambda_{ii1})}{(1-\tilde{\lambda}_{ii1})}.$$

Using this expression and $T(t_{ji1})$ from (55), with $\lambda_{ji1} = 1 - \lambda_{ii1}$ we can solve for

$$\Lambda_{i1} \equiv \lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}} = 1 - \frac{(t_{ji1}-1)[(T(t_{ji1}) + \tilde{\gamma}_{i1}) - 1]}{(t_{ji1}-1)(T(t_{ji1}) + \tilde{\gamma}_{i1})} = \frac{1}{(T(t_{ji1}) + \tilde{\gamma}_{i1})}. \tag{56}$$

We see that Λ_{i1} and $T(t_{ji1})$ are inversely related, as asserted. Since $1-\Lambda_{i1}=\frac{T(t_{ji1})-(1-\tilde{\gamma}_{i1})}{T(t_{ji1})+\tilde{\gamma}_{i1}}$ from the above equation, we can substitute this into (49) to obtain

$$B_i=\frac{\alpha_i w_i L_i[T(t_{ji1})-(1-\tilde{\gamma}_{i1})]}{[T(t_{ji1})-(1-\tilde{\gamma}_{i1})](1-\alpha_i)+1-\tilde{\gamma}_{i1}}=\frac{\alpha_i w_i L_i}{1-\alpha_i+\frac{1-\tilde{\gamma}_{i1}}{[T(t_{ji1})-(1-\tilde{\gamma}_{i1})]}}. \tag{57}$$

We see that B_i is monotonically increasing in $T(t_{ji1})$, so they have their critical points at the same maximum-revenue tariff. Note that if we take $\alpha_i=1$ so we are in a one-sector model, then B_i and $T(t_{ji1})$ are especially simple affine transformations of each other, given by

$$B_i=w_iL_i\left(\frac{T(t_{ji1})}{1-\gamma_{i1}}-1\right).$$

A.4 Labor Allocation

We now derive expressions for labor market demand in sectors 1 and 2:

$$\begin{aligned}
L_{i1} &= N_{i1}f_{i1}^e+N_{i1}f_{ii1}\int_{\varphi_{ii1}^*}^{\infty} g(\varphi)\,d\varphi+N_{i1}f_{ij1}\int_{\varphi_{ij1}^*}^{\infty} g(\varphi)\,d\varphi \\
&\quad +\gamma_{i1}(\sigma_1-1)N_{i1}\sum_{k=i,j}\left[\int_{\varphi_{ik1}^*}^{\infty}\frac{\pi_{ik1}}{w_i}(\varphi)\,g(\varphi)\,d\varphi+f_{ik1}\int_{\varphi_{ik1}^*}^{\infty} g(\varphi)\,d\varphi\right], \\
L_{i2} &= N_{i2}f_{i2}^e+N_{i2}f_{ii2}\int_{\varphi_{ii2}^*}^{\infty} g(\varphi)\,d\varphi \\
&\quad +\gamma_{i2}(\sigma_2-1)N_{i2}\left[\int_{\varphi_{ii2}^*}^{\infty}\frac{\pi_{ii2}}{w_i}(\varphi)\,g(\varphi)\,d\varphi+f_{ii2}\int_{\varphi_{ii2}^*}^{\infty} g(\varphi)\,d\varphi\right].
\end{aligned}$$

Using the free entry condition (41), we obtain

$$\frac{L_{i1}}{N_{i1}}=(1+\gamma_{i1}(\sigma_1-1))\left(f_{i1}^e+f_{ii1}{\varphi_{ii1}^*}^{-\theta_1}+f_{ij1}{\varphi_{ij1}^*}^{-\theta_1}\right), \tag{58}$$

$$\frac{L_{i2}}{N_{i2}}=(1+\gamma_{i2}(\sigma_2-1))\left(f_{i2}^e+f_{ii2}{\varphi_{ii2}^*}^{-\theta_2}\right). \tag{59}$$

Also using (42) and (45), entry into sectors 1 and 2 becomes

$$N_{i1}=\frac{(\sigma_1-1)}{[1+\gamma_{i1}(\sigma_1-1)]\,\theta_1}\frac{L_{i1}}{f_{i1}^e}, \tag{60}$$

$$N_{i2}=\frac{(\sigma_2-1)}{[1+\gamma_{i2}(\sigma_2-1)]\theta_2}\frac{L_{i2}}{f_{i2}^e}. \tag{61}$$

Combining the expressions, we obtain

$$\frac{L_{i1}}{L_{i2}}=\frac{N_{i1}}{N_{i2}}\frac{\frac{[1+\gamma_{i1}(\sigma_1-1)]\theta_1 f_{i1}^e}{(\sigma_1-1)}}{\frac{[1+\gamma_{i2}(\sigma_2-1)]\theta_2 f_{i2}^e}{(\sigma_2-1)}}. \tag{62}$$

To characterize the labor allocation across sectors, we need to use entry. We have already solved for Y_{1i} and Y_{2i} in (47) and (48). Use these results in (43) and (44) and recall that home sales are $E_{ii1} = \lambda_{ii1} Y_{i1}$ and $E_{ii2} = Y_{i2}$ while exports are $E_{ij1} = \lambda_{ij1} Y_{j1} = \lambda_{ji1} Y_{i1}/t_{ji1}$. Substituting the resulting expressions into (62), labor allocation across sectors can be written as

$$\frac{L_{i1}}{L_{i2}} = \frac{\alpha_i}{(1-\alpha_i)} \frac{(1-\tilde{\gamma}_{i1}) \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}}\right)}{1-\tilde{\gamma}_{i1}\left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}}\right)}. \tag{63}$$

The tariff formula (53) derived earlier can be used to simplify this expression for labor allocation. Using (53) in (63), we obtain

$$\begin{aligned}
\frac{L_{i1}}{L_{i2}} &= \frac{\alpha_i}{(1-\alpha_i)} \frac{(1-\tilde{\gamma}_{i1}) \left(\frac{1+(t_{ji1}-1)\lambda_{ii1}}{t_{ji1}}\right)}{1-\tilde{\gamma}_{i1}\left(\frac{1+(t_{ji1}-1)\lambda_{ii1}}{t_{ji1}}\right)} \\
&= \frac{\alpha_i}{(1-\alpha_i)} \frac{(1-\tilde{\gamma}_{i1}) \left(\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1}\right)^{-1}}{1-\tilde{\gamma}_{i1}\left(\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1}\right)^{-1}}.
\end{aligned}$$

Then we can also express the labor allocation as a fraction of total labor supply:

$$\frac{L_{i2}}{L_{i1}+L_{i2}} = \left(\frac{L_{i1}}{L_{i2}}+1\right)^{-1} = \frac{\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1} + \frac{\alpha_i}{1-\alpha_i}\left(1-\tilde{\gamma}_{i1}\right)}, \tag{64}$$

$$\frac{L_{i1}}{L_{i1}+L_{i2}} = \frac{\frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right) + \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}. \tag{65}$$

A.5 Income and Intermediate Demand

The tariff formula (53) can also be used to derive an alternative expression for income I_i, which depends on tariff revenue given by $B_i = \left(t_{ji1}-1\right) \frac{\lambda_{ji1}}{t_{ji1}} Y_{i1}$. From trade balance we have $\frac{\lambda_{ji1} Y_{i1}}{t_{ji1}} = \lambda_{ij1} Y_{j1}$, and using (36) and (60) we obtain

$$\begin{aligned}
B_i = \left(t_{ji1}-1\right)\lambda_{ij1} Y_{j,1} &= \left(t_{ji1}-1\right) \varphi_{ij1}^{*\,-\theta_1} N_{i1} \left(\sigma_1 w_i f_{ij1}\right) \left(\frac{\theta_1}{\theta_1+1-\sigma_1}\right) \\
&= \frac{\left(t_{ji1}-1\right)\sigma_1\left(\sigma_1-1\right) f_{ij1}}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)\left(\theta_1+1-\sigma_1\right) f_{i1}^e} w_i L_{i1} \varphi_{ij1}^{*\,-\theta_1}.
\end{aligned}$$

Then income $I_i = w_i L_i + B_i$ equals

$$I_i = w_i L_i + \left(t_{ji1}-1\right) \frac{\sigma_1}{1+\gamma_{i1}\left(\sigma_1-1\right)} \frac{f_{ij1}\varphi_{ij1}^{*\,-\theta_1}}{\frac{(\theta_1+1-\sigma_1)}{(\sigma_1-1)} f_{i1}^e} w_i L_{i1} = w_i L_i + \left(t_{ji1}-1\right)\left(\frac{1-\tilde{\lambda}_{ii1}}{1-\tilde{\gamma}_{i1}}\right) w_i L_{i1}.$$

Combining with (65), we have

$$\begin{aligned}\frac{I_i}{w_i L_{i1}} &= \frac{\left(1-\tilde{\lambda}_{ii1}\right) t_{ji1} + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}{\frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right) + \tilde{\gamma}_{i1}} + \left(t_{ji1}-1\right)\left(\frac{1}{1-\tilde{\gamma}_{i1}}\right)\left(1-\tilde{\lambda}_{ii1}\right), \\ &= \frac{\left(1-\alpha_i\right)\left(t_{ji1}-1\right)\left(1-\tilde{\lambda}_{ii1}\right) + 1 - \tilde{\gamma}_{i1}}{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)} + \left(t_{ji1}-1\right)\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{1-\tilde{\gamma}_{i1}} \\ &= \frac{1}{\alpha_i} + \frac{\left(t_{ji1}-1\right)\left(1-\tilde{\lambda}_{ii1}\right)}{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)}.\end{aligned}$$

Using $T(t_{ji1}) \equiv 1 - \tilde{\gamma}_{i1} + \left(t_{ji1}-1\right)\left(1-\tilde{\lambda}_{ii1}\right)$ from (55), we then obtain

$$I_i = \frac{w_i L_{i1}}{\alpha_i}\frac{T(t_{ji1})}{\left(1-\tilde{\gamma}_{i1}\right)}. \tag{66}$$

Next, we derive the expression for the value of the finished goods used as an intermediate input in sector 1:

$$\text{Intermediate demand} = N_{i1}\left(1-\gamma_{i1}\right)\left(\int_{\varphi^*_{ii1}}^{\infty}\frac{x_{i1}q_{ii1}\left(\varphi\right)}{\varphi}g\left(\varphi\right)d\varphi + \int_{\varphi^*_{ij1}}^{\infty}\frac{x_{i1}\tau_{ij1}q_{ij1}\left(\varphi\right)}{\varphi}g\left(\varphi\right)d\varphi\right),$$

and using the expression for profits:

$$\frac{x_{i1}\tau_{ij1}q_{ij1}\left(\varphi\right)}{\varphi} = \left(\sigma_1-1\right)\pi_{ij1}\left(\varphi\right) + \left(\sigma_1-1\right)w_i f_{ij1},$$

we then obtain

$$\text{Intermediate demand} = N_{i1}\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)\sum\nolimits_{k=i,j}\int_{\varphi^*_{ik1}}^{\infty}\left(\pi_{ik1}\left(\varphi\right) + w_i f_{ik1}\right)g\left(\varphi\right)d\varphi.$$

Using the free entry condition (41), we have

$$\text{Intermediate demand} = N_{i1}\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)w_i\left(f^e_{i1} + f_{ii1}{\varphi^*_{ii1}}^{-\theta_1} + f_{ij1}{\varphi^*_{ij1}}^{-\theta_1}\right).$$

Using labor market clearing (58), the intermediate demand is then given by

$$\text{Intermediate demand} = w_i L_{i1}\frac{\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}.$$

It follows that the total demand for finished goods in sector 1 is

$$Y_{i1} = \alpha_i I_i + w_i L_{i1}\frac{\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}.$$

After combining these expressions with (34) and (40), I_i is given in terms of sector 1 variables by

$$I_i = \frac{w_i L_{i1}}{\alpha_i}\frac{\left(\sigma_1-1\right)}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}\left(K_1^{\sigma_1-1}\frac{\sigma_1}{\theta_1 f^e_{i1}}\left(\varphi^{*-\theta_1}_{ii1} f_{ii1} + {\varphi^*_{ij1}}^{-\theta_1} f_{ij1}\, t_{ji1}\right) - \left(1-\gamma_{i1}\right)\right). \tag{67}$$

For sector 2, there are no exports so that dividing the numerator and denominator of the ratio $\frac{(\sigma_2-1)}{1+\gamma_{i2}(\sigma_2-1)}$

by σ_2 to obtain $\frac{\rho_2}{1-\tilde{\gamma}_{i2}}$, then income can also be written in terms of sector 2 variables as

$$I_i = \frac{w_i L_{i2}}{1-\alpha_i} \frac{\rho_2}{1-\tilde{\gamma}_{i2}} \left(K_2^{\sigma_2 - 1} \frac{\sigma_2}{\theta_2 f_{i2}^e} \varphi_{ii2}^{* - \theta_2} f_{ii2} - (1 - \gamma_{i2}) \right) = \frac{w_i L_{i2}}{(1-\alpha_i)}, \tag{68}$$

because $K_2^{\sigma_2 - 1} \left(\frac{\sigma_2 - 1}{\theta_2 f_{i2}^e} \right) \varphi_{ii2}^{* - \theta_2} f_{ii2} = 1$ from (31) and (45).

A.6 Equilibrium Conditions

We use the definition of small open economy following Demidova and Rodríguez-Clare (2013), where we impose a fixed demand curve for country i exports to country j. In particular the wages, prices, entry, and expenditure at county j are not affected by changes in the trade policy of i. Formally, the equilibrium conditions of the small open economy are the following.

Definition 1. *An equilibrium of small open economy, two-sector roundabout model using domestic labor for fixed costs is characterized for a set of prices $(w_i, x_{i1}, x_{i2}, P_{i1}, P_{i2})$ productivity cutoffs $\left(\varphi_{ii1}^*, \varphi_{ii2}^*, \varphi_{ji1}^*, \varphi_{ij1}^* \right)$, finished outputs (Y_{i1}, Y_{i2}), mass of firms (N_{i1}, N_{i2}), and expenditure shares $(\lambda_{ii1}, \lambda_{ji1})$ that solve the following equilibrium conditions taking as given $\{P_{j1}, Y_{j1}, N_{j1}, w_j\}$:*
Zero cut-off productivity (ZCP) *from (30),*

$$\varphi_{ii1}^* = \left(\frac{\sigma_1}{\sigma_1 - 1} \right) \left(\frac{\sigma_1 w_i f_{ii1}}{Y_{i1}} \right)^{\frac{1}{\sigma_1 - 1}} \frac{x_{i1}}{P_{i1}},$$

$$\varphi_{ij1}^* = \left(\frac{\sigma_1}{\sigma_1 - 1} \right) \left(\frac{\sigma_1 w_i f_{ij1}}{Y_{j1}} \right)^{\frac{1}{\sigma_1 - 1}} \frac{x_{i1} \tau_{ij1}}{P_{j1}},$$

$$\varphi_{ji1}^* = \left(\frac{\sigma_1}{\sigma_1 - 1} \right) \left(\frac{\sigma_1 w_j f_{ji1}}{Y_{i1}} \right)^{\frac{1}{\sigma_1 - 1}} \frac{w_j \tau_{ji1} (t_{ji1})^{\frac{\sigma_1}{\sigma_1 - 1}}}{P_{i1}},$$

$$\varphi_{ii2}^* = \left(\frac{\sigma_2}{\sigma_2 - 1} \right) \left(\frac{\sigma_2 w_i f_{ii2}}{Y_{i2}} \right)^{\frac{1}{\sigma_2 - 1}} \frac{x_{i2}}{P_{i2}},$$

Input cost indexes *from (1),*

$$x_{i1} = (w_i)^{\gamma_{i1}} (P_{i1})^{1-\gamma_{i1}},$$
$$x_{i2} = (w_i)^{\gamma_{i2}} (P_{i2})^{1-\gamma_{i2}},$$

Value of finished output *from (47) and (48),*

$$\frac{Y_{i1}}{w_i} = \frac{\alpha_i I_i}{1 - \tilde{\gamma}_{i1} \Lambda_{i1}},$$

$$Y_{i2} = \frac{1-\alpha_i}{1-\tilde{\gamma}_{i2}} I_i,$$

with $I_i = w_i L_i + B_i = w_i L_i + (1 - \Lambda_{i1}) Y_{i1}$, $\Lambda_{i1} \equiv \left(\lambda_{ii1} + \frac{\lambda_{ji1}}{t_{ji1}} \right)$ and $\tilde{\gamma}_1 = (1 - \gamma_{i1}) \frac{\sigma_1 - 1}{\sigma_1}$,
Price indexes *from (33),*

$$P_{i1} = \left(\varphi_{ii1}^{* \; -\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{\bar{\varphi}_{ii1}} \right)^{1-\sigma_1} + \varphi_{ji1}^{* \; -\theta_1} N_{j_1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{w_j t_{ji1}}{\bar{\varphi}_{ji1}} \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}},$$

$$P_{i2} = \left({\varphi^*_{ii2}}^{-\theta_2} N_{i2} \left(\frac{\sigma_2}{\sigma_2 - 1} \frac{x_{i2}}{\bar{\varphi}_{ii2}} \right)^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}},$$

Entry *from (43) and (44),*

$$N_{i1} = \frac{\Lambda_{i1} Y_{i1}}{w_i f^e_{i1} \left(\frac{\theta_1 \sigma_1}{\sigma_1 - 1} \right)},$$

$$N_{i2} = \frac{Y_{i2}}{w_i f^e_{i2} \left(\frac{\theta_2 \sigma_2}{\sigma_2 - 1} \right)},$$

Expenditure share *from (35),*

$$\lambda_{ii1} = {\varphi^*_{ii1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{P_{i1} \bar{\varphi}_{ii1}} \right)^{1-\sigma_1} \quad \textit{and} \quad \lambda_{ji1} = 1 - \lambda_{ii1},$$

with $\bar{\varphi}_{iis} = K_s \varphi^*_{iis}$, $K_s \equiv \left(\frac{\theta_s}{\theta_s + 1 - \sigma_s} \right)^{\frac{1}{\sigma_s - 1}}$ *and* $\lambda_{ii2} \equiv 1$,
Trade balance *from (36), (38) and (39),*

$${\varphi^*_{ij1}}^{-\theta_1} N_{i1} w_i f_{ij} = {\varphi^*_{ji1}}^{-\theta_1} N_{j1} w_j f_{ji1}.$$

Several other expressions such as ***free entry*** *in (42) and (45),* ***labor allocation*** *in (64)–(65) and* ***income*** *in (67)–(68) continue to hold but are not needed for the definition of equilibrium; these expressions will be useful in our analysis below.*

B Closed Economy Model

In the closed-economy model we allow for multiple sectors $s = 1, ..., S$, where we use $\alpha_{is} > 0$ to denote the consumption share in each sector with with $\sum_{s=1}^{S} \alpha_{is} = 1$. We now introduce producer and consumer tax/subsidies t^c_{is} and t^p_{is} on purchases of the finished good. The producer tax/subsidy means that the input cost index is modified from (1) as

$$x_{is} = w_i^{\gamma_{is}} (t^p_{is} P_{is})^{1-\gamma_{is}}, \tag{69}$$

where P_{is} denotes the price of the finished good before the application of any tax/subsidies.

Without loss of generality, we assume that the government budget is balanced so that $B_i = 0$. In the market clearing condition (3), there is no trade so that $\lambda_{iis} = 1$ and $\lambda_{ijs} = 0$, and the consumer and firm purchases must be divided by t^c_{is} and t^p_{is}, respectively, to obtain the net-of-tax purchases. Further multiplying these purchases by the *ad valorem* tax rates $t^c_{is} - 1$ and $t^p_{is} - 1$, respectively, we obtain the balanced budget

$$0 = \sum_{s=1}^{S} (t^c_{is} - 1) \frac{\alpha_{is} w_i L_i}{t^c_{is}} + (t^p_{is} - 1) \frac{\tilde{\gamma}_s Y_{is}}{t^p_{is}}. \tag{70}$$

The term $\alpha_{is} w_i L_i / t^c_{is}$ on the right of (70) is the value of consumer purchases of the finished good. Dividing this by the duty-free price index of the finished good, P_{is}, we obtain consumption in each sector, and so the objective function for the government is

$$\max_{t^c_{is}, t^p_{is} > 0} \prod_{s=1}^{S} C_{is}^{\alpha_{is}} = \prod_{s=1}^{S} \left(\frac{\alpha_{is} w_i L_i}{t^c_{is} P_{is}} \right)^{\alpha_{is}}, \tag{71}$$

subject to the constraint (70).

To determine the optimal policies, we need an expression for the price index in each sector under autarky. Recall from (24) that P_{iis} is the CES price index for differentiated inputs purchased from domestic firms in each sector. Using the input price index in (69), we can substitute prices from (28) into (24) to obtain

$$P_{iis} = (N_{iis})^{\frac{1}{(1-\sigma_s)}} \left(\frac{\sigma_s}{\sigma_s - 1}\right) \frac{w_i^{\gamma_{is}} \left(t_{is}^p P_{is}\right)^{1-\gamma_{is}}}{\bar{\varphi}_{is}}. \tag{72}$$

In a closed economy we have $P_{iis} = P_{is}$, and so we can solve for the price index P_{is} from (72)as

$$P_{is} = w_i \left[\left(\frac{1}{N_{iis}}\right)^{\frac{1}{(\sigma_s-1)}} \left(\frac{\sigma_s}{\sigma_s - 1}\right) \frac{(t_{is}^p)^{1-\gamma_{is}}}{\bar{\varphi}_{iis}}\right]^{\frac{1}{\gamma_{is}}}. \tag{73}$$

This expression includes the average productivities, but these are not affected by the consumer or producer taxes because from (31) they are proportional to the cutoff productivities, which are determined by the free-entry condition like (45) but in each sector: $J_s(\varphi_{iis}^*) f_{iis} = f_{is}^E$.

Entry into each sector, N_{is}, is endogenous and is determined by (43), where the expenditure on the differentiated inputs in the closed economy, E_{iis}, equals the net-of-tax value of the final good that are bundled from them, Y_{is}, and we ignore the term E_{ijs}. In the market clearing condition (3), with no trade then $\lambda_{iis} = 1$ and $\lambda_{ijs} = 0$, and the consumer and firm purchases must be divided by t_{is}^c and t_{is}^p, respectively, to obtain the net-of-tax purchases

$$Y_{is} = \frac{\alpha_{is}}{t_{is}^c} w_i L_i + \frac{\tilde{\gamma}_{is}}{t_{is}^p} Y_{is}, \tag{74}$$

recalling that we have set $B_i = 0$ so that $w_i L_i$ is consumer income. We solve for $Y_{is} = \frac{\alpha_{is} w_i L_i}{t_{is}^c [1-(\tilde{\gamma}_{is}/t_{is}^p)]}$, and then entry from (43) is

$$N_{is} = (\alpha_{is} L_i) \Big/ \left[t_{is}^c \left(1 - \frac{\tilde{\gamma}_{is}}{t_{is}^p}\right) f_{is}^e \left(\frac{\theta_s \sigma_s}{\sigma_s - 1}\right)\right]. \tag{75}$$

Substituting (75) into (25), (73) and then (71) and ignoring constants, the objective function is

$$\max_{t_{is}^c, t_{is}^p > 0} \prod_{s=1}^{S} \left\{ t_{is}^c \left[t_{is}^c \left(1 - \frac{\tilde{\gamma}_{is}}{t_{is}^p}\right)\right]^{\frac{1}{\gamma_{is}(\sigma_s - 1)}} \left(t_{is}^p\right)^{\frac{(1-\gamma_{is})}{\gamma_{is}}} \right\}^{-\alpha_{is}}. \tag{76}$$

We solve the problem (76) subject to (70) twice: in the first-best by choosing the optimal consumer and producer tax/subsidies; and in the second-best by choosing t_{is}^c while setting $t_{is}^p \equiv 1$. The solutions are shown in (6) and (7), respectively, for the case of just two sectors.

C Fixed-point Formula for the Second-Best Tariff

We now assume that no consumer or producer tax/subsidies apply to purchases of the finished good in either sector, and the only policy instrument used is the tariff t_{ji1} on imports of the differentiated inputs in sector 1. For convenience we drop subscripts from the import tariff t_{ji1} and simply denote it by t_i. In this Appendix we perform the comparative statics with respect to a change in the tariff to obtain the fixed-point formula for the optimal tariff (13), and in Appendix D we develop the proof of Theorem 1.

We first derive an expression for the price index that is going to be used in order to express welfare as a function of productivity thresholds. From (33) the sector 1 price index is

$$P_{i1} = \left({\varphi^*_{ii1}}^{-\theta_1} N_{i1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{\bar{\varphi}_{ii1}} \right)^{1-\sigma_1} + {\varphi^*_{ji1}}^{-\theta_1} N_{j_1} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{w_j t_{ji1}}{\bar{\varphi}_{ji1}} \right)^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}}.$$

We combine the entry thresholds φ^*_{ji1}, and φ^*_{ii1}, to obtain

$$\left(\frac{\sigma_1}{\sigma_1 - 1} \frac{w_j \tau_{ji1} \, t_i}{\varphi^*_{ji1}} \right)^{1-\sigma_1} = \frac{\sigma_1 \, w_j \, f_{ji1} \, t_i}{\sigma_1 \, w_i \, f_{ii1}} \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{x_{i1}}{\varphi^*_{ii1}} \right)^{1-\sigma_1}. \tag{77}$$

Using this expression together with trade balance (40) and (1) we obtain

$$P_{i1} = \left(\frac{\sigma_1}{\sigma_1 - 1} \frac{1}{K_1} \left(\frac{N_{i1}}{f_{ii1}} \right)^{\frac{1}{(1-\sigma_1)}} \right)^{\frac{1}{\gamma_{i1}}} (\varphi^*_{ii1})^{-\frac{1}{\gamma_{i1}}} \, w_i \left(f_{ii1} \varphi^{*-\theta_1}_{ii1} + \; {\varphi^*_{ij1}}^{-\theta_1} \; f_{ij1} t_i \right)^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}. \tag{78}$$

Similarly, we obtain

$$P_{i2} = \left(\frac{\sigma_2}{\sigma_2 - 1} \frac{1}{K_2} \left(\frac{N_{i2}}{f_{ii2}} \right)^{\frac{1}{(1-\sigma_2)}} \right)^{\frac{1}{\gamma_{i2}}} (\, \varphi^*_{ii2})^{-\frac{1}{\gamma_{i2}}} \, w_i \left(f_{ii2} \varphi^{*-\theta_2}_{ii2} \right)^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}.$$

Using expressions (67) and (68) for income and the above expressions for the price indexes, we substitute these into indirect utility or welfare, which from (2) is given by

$$U_i = \left(\frac{\alpha_i I_i}{P_{i1}} \right)^{\alpha_i} \left(\frac{(1-\alpha_i) I_i}{P_{i2}} \right)^{1-\alpha_i}.$$

Define the term,

$$\Theta \equiv \left(\frac{\frac{(\sigma_1 - 1)}{(1+\gamma_{i1}(\sigma_1 - 1))}}{\left(\frac{\sigma_1}{\sigma_1 - 1} \frac{1}{K_1} \left(\frac{1}{f_{ii1}} \right)^{\frac{1}{(1-\sigma_1)}} \right)^{\frac{1-\sigma_1}{\gamma_{i1}(1-\sigma_1)}}} \right)^{\alpha_i} \left(\frac{1}{\left(\frac{\sigma_2}{\sigma_2 - 1} \frac{1}{K_2} \left(\frac{1}{f_{ii2}} \right)^{\frac{1}{(1-\sigma_2)}} \right)^{\frac{1}{\gamma_{i2}}} (\varphi^*_{ii2})^{-\frac{1}{\gamma_{i2}}} \left(f_{ii2} \, \varphi^{*-\theta_2}_{ii2} \right)^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}} \right)^{1-\alpha_i}$$

which is a constant because φ^*_{ii2} is constant from (45). We then obtain the welfare expression,

$$\begin{aligned} U_i \quad &= \quad \Theta \left(\frac{\frac{K_1^{\sigma_1 - 1} \sigma_1}{\theta_1 f^e_{i1}} \left(\varphi^{*-\theta_1}_{ii1} \; f_{ii1} + \; {\varphi^*_{ij1}}^{-\theta_1} \; f_{ij1} \, t_i \right) - (1 - \gamma_{i1})}{\left(f_{ii1} \varphi^{*-\theta_1}_{ii1} + {\varphi^*_{ij1}}^{-\theta_1} \; f_{ij1} t_i \right)^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}} (\varphi^*_{ii1})^{\frac{1}{\gamma_{i1}}} \right)^{\alpha_i} \left(\frac{L_{i1}}{(N_{i1})^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}} \right)^{\alpha_i} \\ &\quad \times \left(\frac{L_{i2}}{(N_{i2})^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}} \right)^{1-\alpha_i}. \end{aligned}$$

There is new term in this expression, given by

$$\left(\frac{L_{i1}}{(N_{i1})^{\frac{1}{\gamma_{i1}(1-\sigma_1)}}}\right)^{\alpha_i}\left(\frac{L_{i2}}{(N_{i2})^{\frac{1}{\gamma_{i2}(1-\sigma_2)}}}\right)^{1-\alpha_i} =$$
$$\left(\frac{(1+\gamma_{i1}(\sigma_1-1))\,\theta_1 f_{i1}^e}{(\sigma_1-1)}(N_{i1})^{\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)}}\right)^{\alpha_i}\left(\frac{(1+\gamma_2(\sigma_2-1))\,\theta_2 f_{i1}^e}{(\sigma_2-1)}(N_{i2})^{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}}\right)^{1-\alpha_i}$$

using (60), (61).

Totally differentiating welfare, $\hat{U}_i$ can be written as

$$\begin{aligned}
\hat{U}_i &= \alpha_i\left(1+\frac{1-\gamma_{i1}}{\frac{\alpha_i I_i}{w_i L_{i1}}\frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\left((1-\lambda_{ii1})\left(-\theta_1\hat{\varphi}_{ij1}^*+\hat{t}_i\right)-\theta_1\lambda_{ii1}\hat{\varphi}_{ii1}^*\right)\\
&\quad+\frac{\alpha_i}{\gamma_{i1}}\hat{\varphi}_{ii1}^*+\alpha_i\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)}\hat{N}_{i1}+(1-\alpha_i)\frac{1+\gamma_2(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\hat{N}_{i2}\\
&= \alpha_i\left(1+\frac{(1-\gamma_{i1})(\sigma_1-1)}{(1+\gamma_{i1}(\sigma_1-1))+\sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\\
&\quad\times\left((1-\lambda_{ii1})\left(-\theta_1\hat{\varphi}_{ij1}^*+\hat{t}_i\right)-\theta_1\lambda_{ii1}\hat{\varphi}_{ii1}^*\right)+\frac{1}{\gamma_{i1}}\hat{\varphi}_{ii1}^*\\
&\quad+\alpha_i\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)}\hat{N}_{i1}+(1-\alpha_i)\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\hat{N}_{i2},
\end{aligned}\tag{79}$$

where the equality is obtained by using the following expression

$$\begin{aligned}
1+\frac{1-\gamma_{i1}}{\frac{\alpha_i I_i}{w_i L_{i1}}\frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}} &+ \frac{1}{\gamma_{i1}(\sigma_1-1)} = 1+\frac{1-\gamma_{i1}}{\left(1+\frac{\sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}{(1+\gamma_{i1}(\sigma_1-1))}\right)\frac{(1+\gamma_{i1}(\sigma_1-1))}{(\sigma_1-1)}}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\\
&= 1+\frac{(1-\gamma_{i1})(\sigma_1-1)}{(1+\gamma_{i1}(\sigma_1-1))+\sigma_1(t_i-1)\left(1-\tilde{\lambda}_{ii1}\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)},
\end{aligned}$$

Now the strategy is to obtain expressions for $\hat{\varphi}_{ii1}^*$, and $\hat{\varphi}_{ij1}^*$. First, totally differentiate the free entry condition (43) and use (51) to obtain

$$\hat{\varphi}_{ii1}^* = -\left(\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right)\hat{\varphi}_{ij1}^*.\tag{80}$$

Then we totally differentiate the price index (78) to obtain

$$\begin{aligned}
\hat{P}_{i1} &= \frac{1}{\gamma_{i1}(1-\sigma_1)}\hat{N}_{i1}+\hat{w}_i+\frac{1}{\gamma_{i1}}\left(\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\left(\frac{\theta_1}{(1-\sigma_1)}\lambda_{ii1}+1\right)-\frac{\theta_1(1-\lambda_{ii1})}{(1-\sigma_1)}\right)\hat{\varphi}_{ij1}^*\\
&\quad+\frac{1}{\gamma_{i1}(1-\sigma_1)}(1-\lambda_{ii1})\,\hat{t}_i.
\end{aligned}\tag{81}$$

Next, totally differentiate the expression for φ_{ij1}^* in (30) and recall that country i is a small open economy so that the country j price index, value of output and input-cost index are fixed. It follows that $\hat{\varphi}_{ij1}^*$ is given by

$$\hat{\varphi}_{ij1}^* - \left(\frac{1}{\sigma_1-1}+\gamma_{i1}\right)\hat{w}_i = (1-\gamma_{i1})\,\hat{P}_{i1}.\tag{82}$$

Now combine (81) and (82) to obtain

$$\frac{\hat{\varphi}^*_{ij1}}{1-\gamma_{i1}} - \frac{1}{1-\gamma_{i1}}\left(\frac{1}{\sigma_1-1}+\gamma_{i1}\right)\hat{w}_i =$$
$$\frac{1}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{N}_{i1} + \hat{w}_i + \frac{1}{\gamma_{i1}}\left(\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\theta_1\lambda_{ii1}}{1-\sigma_1}+1\right) - \frac{\theta_1\left(1-\lambda_{ii1}\right)}{1-\sigma_1}\right)\hat{\varphi}^*_{ij1}$$
$$+\frac{1-\lambda_{ii1}}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{t}_i. \tag{83}$$

From trade balance (40), we have

$$\hat{\varphi}^*_{ji1} = \hat{\varphi}^*_{ij1} - \frac{1}{\theta_1}\hat{w}_i - \frac{1}{\theta_1}\hat{N}_{i1}. \tag{84}$$

From the equality for $\hat{\varphi}^*_{ji1}$ and $\hat{\varphi}^*_{ii1}$ (77), we can see that

$$\hat{\varphi}^*_{ji1} = \hat{\varphi}^*_{ii1} - \left(\frac{1}{\sigma_1-1}+\gamma_{i1}\right)\hat{w}_i - \left(1-\gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1-1}\hat{t}_i.$$

Combining (80) and (82), $\hat{\varphi}^*_{ji1}$ is given by

$$\hat{\varphi}^*_{ji1} = -\frac{1}{\tilde{\lambda}_{ii1}}\left(\frac{1}{\sigma_1-1}+\gamma_{i1}\right)\hat{w}_i - \frac{1}{\tilde{\lambda}_{ii1}}\left(1-\gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1-1}\hat{t}_i,$$

and after using (84), we obtain

$$\hat{\varphi}^*_{ij1} - \frac{1}{\theta_1}\hat{w}_i - \frac{1}{\theta_1}\hat{N}_{i1} = -\frac{1}{\tilde{\lambda}_{ii1}}\left(\frac{1}{\sigma_1-1}+\gamma_{i1}\right)\hat{w}_i - \frac{1}{\tilde{\lambda}_{ii1}}\left(1-\gamma_{i1}\right)\hat{P}_{i1} + \frac{\sigma_1}{\sigma_1-1}\hat{t}_i.$$

Then from (82) we have

$$\hat{\varphi}^*_{ij1} = \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(\hat{w}_i+\hat{N}_{i1}\right) + \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{\sigma_1}{\sigma_1-1}\hat{t}_i. \tag{85}$$

Using (83) and multiplying both sides by $\left(1-\gamma_{i1}\right)$ we have

$$\begin{aligned} -\left(\frac{\sigma_1}{\sigma_1-1}\right)\hat{w}_i &= \left(-1+\frac{1-\gamma_{i1}}{\gamma_{i1}}\left(\frac{\theta_1}{\left(1-\sigma_1\right)}\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right)+\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right)\right)\hat{\varphi}^*_{ij1} \\ &+ \frac{1-\gamma_{i1}}{\gamma_{i1}\left(1-\sigma_1\right)}\hat{N}_{i1} + \frac{\left(1-\gamma_{i1}\right)}{\gamma_{i1}\left(1-\sigma_1\right)}\left(1-\lambda_{ii1}\right)\hat{t}_i. \end{aligned}$$

Combining this expression with (85) and using $\rho_1 \equiv \frac{\sigma_1-1}{\sigma_1}$, we finally obtain

$$\hat{w}_i = \mathcal{E}_1\hat{t}_i + \mathcal{E}_2\hat{N}_{i1},$$

where

$$\mathcal{E}_1 = \frac{1-\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(1-\frac{1}{\sigma_1}\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(\frac{1}{\sigma_1}-\frac{\theta_1}{\sigma_1-1}\right)\left(1-t_i\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}-\frac{\rho_1}{\theta_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}\left(1-t_i\right)\lambda_{ii1}\right)}, \tag{86}$$

$$\mathcal{E}_2 = \frac{\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}+\frac{\rho_1}{\theta_1}-\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}\left(1-t_i\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}-\frac{\rho_1}{\theta_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}\left(1-t_i\right)\lambda_{ii1}\right)}. \tag{87}$$

Now substituting these expressions into (85), we obtain

$$\begin{aligned}
\hat{\varphi}^*_{ij1} &= \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right)\hat{N}_{i1} + \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{\mathcal{E}_1}{\theta_1}+\frac{1}{\rho_1}\right)\hat{t}_i \\
&= \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(t_i-1\right)\lambda_{ii1}\right)}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}\right)\hat{t}_i \\
&+ \frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right)\hat{N}_{i1}. \qquad (88)
\end{aligned}$$

Note that

$$\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\frac{1}{\theta_1}\left(1+\mathcal{E}_2\right) = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}-\frac{1}{\sigma_1}\left(1-t_i\right)\lambda_{ii1}\right)}.$$

Then from the welfare equation (79), using (80) we obtain

$$\begin{aligned}
\hat{U}_i &= \alpha_i\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\hat{t}_i + \left(\mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right)\hat{\varphi}^*_{ij1}\right] \\
&+\alpha_i\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_{i1}\left(\sigma_1-1\right)}\hat{N}_{i1} + \left(1-\alpha_i\right)\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\hat{N}_{i2}, \qquad (89)
\end{aligned}$$

where

$$\mathcal{E}_3 \equiv \left(1+\frac{\left(1-\gamma_{i1}\right)\left(\sigma_1-1\right)}{\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)+\sigma_1\left(t_i-1\right)\left(1-\tilde{\lambda}_{ii1}\right)}+\frac{1}{\gamma_{i1}\left(\sigma_1-1\right)}\right). \qquad (90)$$

Inverting (88), $\hat{t}_i$ is given by

$$\begin{aligned}
\hat{t}_i &= -\frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(t_i-1\right)\lambda_{ii1}\right)\right)}\hat{N}_{i1} \\
&+ \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(t_i-1\right)\lambda_{ii1}\right)\right)}\hat{\varphi}^*_{ij1}.
\end{aligned}$$

Write this expression for $\hat{t}_i$ as

$$\hat{t}_i = -\mathcal{E}_4\hat{N}_{i1} + \mathcal{E}_5\hat{\varphi}^*_{ij1}, \qquad (91)$$

where

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(t_i-1\right)\lambda_{ii1}\right)\right)},$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}{\frac{\tilde{\lambda}_{ii1}}{1+\tilde{\lambda}_{ii1}}\left(\frac{1}{\rho_1}\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{1-\lambda_{ii1}\tilde{\lambda}_{ii1}}{1-\tilde{\lambda}_{ii1}}+\left(t_i-1\right)\lambda_{ii1}\right)\right)}.$$

Using (54), $\mathcal{E}_4$ and $\mathcal{E}_5$ can be written as

$$\mathcal{E}_4 = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}, \qquad (92)$$

$$\mathcal{E}_5 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}\left(t_i - 1\right)\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}. \tag{93}$$

Now we simplify the welfare expression in (89). First, note that using (91), we obtain

$$\begin{aligned}
\hat{U}_i &= \alpha_i\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right]\hat{\varphi}^*_{ij1} \\
&\quad + \alpha_i\left[\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_1\left(\sigma_1-1\right)} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right]\hat{N}_{i1} + \left(1-\alpha_i\right)\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\hat{N}_{i2}.
\end{aligned} \tag{94}$$

We seek to express $\hat{N}_{i1}$, $\hat{N}_{i2}$ as a function of $\hat{\varphi}^*_{ij1}$. From the labor market clearing condition $L_{i1}+L_{i2}=L_i$ and using (60) and (61), we have

$$0 = \frac{L_{i2}}{L_i}\hat{N}_{i2} + \frac{L_{i1}}{L_i}\hat{N}_{i1},$$

which implies that

$$\hat{N}_{i2} = -\frac{L_{i1}}{L_{i2}}\hat{N}_{i1}. \tag{95}$$

In addition, recalling (60), (61) and (64), we obtain

$$N_{i2}\,w_i f^e_{i2}\left(\frac{\theta_2\sigma_2}{\sigma_2-1}\right) = \frac{1}{1-\tilde{\gamma}_{i2}}w_i L_{i2} = \frac{1}{1-\tilde{\gamma}_{i2}}w_i L_i\left(\frac{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}\right).$$

Here we define

$$l_{i2} \equiv \frac{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)} = \frac{L_{i2}}{L_i},$$

and then $\hat{N}_{i2}$ is given by

$$\hat{N}_{i2} = \hat{l}_{i2},$$

where

$$\hat{l}_{i2} = \left(1-l_{i2}\right)\frac{\left(1-t_i\right)\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right)t_i\hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}},$$

Combining this expression with (95), $\hat{N}_{i1}$ can be written as

$$\begin{aligned}
\hat{N}_{i1} &= -\frac{L_{i2}}{L_{i1}}\left(1-l_{i2}\right)\left(\frac{\left(1-t_i\right)\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right)t_i\hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}\right) \\
&= -\left(\frac{\left(1-t_i\right)\tilde{\lambda}_{ii1}\hat{\tilde{\lambda}}_{ii1} + \left(1-\tilde{\lambda}_{ii1}\right)t_i\hat{t}_i}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)}\right).
\end{aligned}$$

From (51) and (77), we can use

$$\hat{\tilde{\lambda}}_{ii1} = \theta_1\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\hat{\varphi}^*_{ij1}, \tag{96}$$

and combining with (91), we obtain

$$\begin{aligned}
&-\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} + \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)\hat{N}_{i1} \\
&= \left(1-t_i\right)\tilde{\lambda}_{ii1}\theta_1\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\,\hat{\varphi}^*_{ij1} + \left(1-\tilde{\lambda}_{ii1}\right)t_i(\mathcal{E}_5\hat{\varphi}^*_{ij1} - \mathcal{E}_4\hat{N}_{i1}).
\end{aligned}$$

Then we arrive at

$$\hat{N}_{i1} = \frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\left(1-t_i\right)\theta_1 + t_i\mathcal{E}_5\right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)}\hat{\varphi}^*_{ij1}. \tag{97}$$

Finally, from (64), (65) and (95), $\hat{N}_{i2}$ can be written as

$$\hat{N}_{i2} = -\left(\frac{\alpha_i}{1-\alpha_i}\right)\frac{\left(1-\tilde{\gamma}_1\right)}{\left(1-\tilde{\lambda}_{ii1}\right)t_i + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}\hat{N}_{i1}. \tag{98}$$

C.1 Total Change in Utility and Definition of $D(t_i)$

We can use the above equations to obtain the total change in utility. Substituting (97) and (98) into the second term of welfare in (94), we have

$$\alpha_i\left(\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_1\left(\sigma_1-1\right)} - \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right)\hat{N}_{i1} + \left(1-\alpha_i\right)\frac{\sigma_2}{\sigma_2-1}\hat{N}_{i2} = D(t_i)\alpha_i\hat{N}_{i1},$$

where

$$D(t_i) \equiv \frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\gamma_{i1}\left(\sigma_1-1\right)} - \frac{1-\tilde{\gamma}_{i2}}{\gamma_{i2}\rho_2}\frac{1+\gamma_{i1}\left(\sigma_1-1\right)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i-1\right) + \left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)} - \mathcal{E}_3\mathcal{E}_4\left(1-\lambda_{ii1}\right).$$

This initial definition $D(t_i)$ can be re-expressed using the function $T(t_i)$ in (55) to obtain the alternative definition

$$D(t_i) \equiv \left[\frac{1+\gamma_{i1}(\sigma_1-1)}{\gamma_{i1}(\sigma_1-1)} - \left(\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\right)\frac{(1-\tilde{\gamma}_{i1})}{T(t_i)} - \mathcal{E}_3\mathcal{E}_4(1-\lambda_{ii1})\right]. \tag{99}$$

Notice that the definition of $D(t_i)$ used in the main text, is obtained by further defining

$$\mathcal{E}_d \equiv \mathcal{E}_3\mathcal{E}_4(1-\lambda_{ii1}), \tag{100}$$

and using (56) to derive $\frac{(1-\tilde{\gamma}_{i1})}{T(t_i)} = \frac{\Lambda_{i1}(1-\tilde{\gamma}_{i1})}{1-\tilde{\gamma}_{i1}\Lambda_{i1}}$, and also using the effective markups $\frac{\tilde{\sigma}_{is}}{(\tilde{\sigma}_{is}-1)}$ defined in (7) so that expression (11) in the main text follows.

It follows that $\hat{U}_i$ can be written as

$$\hat{U}_i = \alpha_i\left[\mathcal{E}_\phi\hat{\varphi}^*_{ij1} + D(t_i)\hat{N}_{i1}\right], \tag{101}$$

where

$$\mathcal{E}_\phi \equiv \mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \left(\frac{1-\tilde{\lambda}_{ii1}}{\gamma_{i1}\tilde{\lambda}_{ii1}}\right). \tag{102}$$

We see that the total change in utility in (101) is written as the sum of two terms: the first given by $\alpha_i\mathcal{E}_\phi\hat{\varphi}^*_{ij1}$ reflects selection and includes all the changes in cutoff productivities; and second $\alpha_i D(t_i)\hat{N}_{i1}$ reflects entry. At the optimum, $\hat{U}_i/\left(\alpha_i\hat{\varphi}^*_{ij1}\right) = 0$, which implies from (97) that

$$\begin{aligned}&\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5 + \mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right) - \frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right] \\ &= -D(t_i)\frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\left(1-t_i\right)\theta_1 + t_i\mathcal{E}_5\right)}{\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(\mathcal{E}_4-1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i-\tilde{\gamma}_{i1}\right)}.\end{aligned} \tag{103}$$

Using the tariff formula (53) repeatedly, we define

$$\tilde{M}(t_i) = \frac{\gamma_{i1}\left(\mathcal{E}_5 - \frac{(t_i-1)}{t_i}\theta_1\right)}{(1-\alpha_i)\left(\left(1-\tilde{\lambda}_{ii1}\right)t_i\left(1-\mathcal{E}_4\right)+\tilde{\lambda}_{ii1}\right)+\alpha_i-\tilde{\gamma}_{i1}}\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}}D(t_i), \tag{104}$$

and then the first-order condition (103) becomes

$$\left[\mathcal{E}_3\left(1-\lambda_{ii1}\right)\mathcal{E}_5+\mathcal{E}_3\theta_1\left(\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\right)-\frac{1}{\gamma_{i1}}\frac{\left(1-\tilde{\lambda}_{ii1}\right)}{\tilde{\lambda}_{ii1}}\right]=\frac{\left(1-\lambda_{ii1}\right)\left(1-\alpha_i\right)}{\gamma_{i1}}\tilde{M}(t_i).$$

Using $\frac{1-t_i}{t_i}\left(1-\lambda_{ii1}\right)=\frac{\lambda_{ii1}-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}$ from (53), we get

$$\frac{\left(1-\lambda_{ii1}\right)}{\gamma_{i1}}\left[\gamma_{i1}\mathcal{E}_3\mathcal{E}_5+\gamma_{i1}\mathcal{E}_3\theta_1\left(\frac{1-t_i}{t_i}\right)-\frac{1}{\lambda_{ii1}t_i}\right]=\frac{\left(1-\lambda_{ii1}\right)\left(1-\alpha_i\right)}{\gamma_{i1}}\tilde{M}(t_i),$$

$$\mathcal{E}_5+\theta_1\left(\frac{1-t_i}{t_i}\right)-\frac{1}{\lambda_{ii1}t_i\gamma_{i1}\mathcal{E}_3}=\left(1-\alpha_i\right)\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}.$$

Using (93), we obtain

$$\begin{aligned}&\left(\frac{1-t_i}{t_i}\right)\frac{\theta_1}{\rho_1}+\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}-\frac{\rho_1}{\theta_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\frac{\rho_1}{\theta_1}-\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}t_i\gamma_{i1}\mathcal{E}_3}\\&=\left(1-\alpha_i\right)\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right).\end{aligned}$$

We multiply both sides by t_i and use (53) again to get

$$\begin{aligned}&\left(1-t_i\right)\frac{\theta_1}{\rho_1}+\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}t_i-\frac{\rho_1}{\theta_1}t_i+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}\gamma_{i1}\mathcal{E}_3}\\&=\left(1-\alpha_i\right)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right).\end{aligned}$$

Next we add and subtract $\frac{1}{\lambda_{ii1}}$ and use $\frac{t_i}{\tilde{\lambda}_{ii1}}=t_i-1+\frac{1}{\lambda_{ii1}}$, to obtain

$$\begin{aligned}&\left(1-t_i\right)\left(\frac{\theta_1-\rho_1}{\rho_1}\right)+t_i-\frac{\rho_1}{\theta_1}t_i+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-\left(\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}{\gamma_{i1}\mathcal{E}_3}-1\right)\frac{1}{\lambda_{ii1}}\\=\;&\left(1-\alpha_i\right)\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right).\end{aligned}$$

Note that

$$\frac{\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}{\gamma_{i1}\mathcal{E}_3}-1=\left(1-\gamma_{i1}\right)\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)+\frac{\rho_1+\left(1-t_i\right)\left(1-\tilde{\lambda}_{ii1}\right)-1}{\left(1-\gamma_{i1}\right)\rho_1+\left(1-t_i\right)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}.$$

Then the first-order condition becomes

$$\left(\frac{\theta_1}{\theta_1-\rho_1}-t_i\right)\frac{\left(\theta_1-\rho_1\right)^2}{\theta_1\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{\left(1-\lambda_{ii1}\right)}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-\left(1-\gamma_{i1}\right)\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)+\frac{\rho_1+\left(1-t_i\right)\left(1-\tilde{\lambda}_{ii1}\right)-1}{\left(1-\gamma_{i1}\right)\rho_1+\left(1-t_i\right)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\lambda_{ii1}}$$

$$= (1-\alpha_i)\,\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right).$$

Now we find the common denominator for the second terms on the left-hand side using (90):

$$\begin{aligned}
&\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\rho_1}{\theta_1}-(1-\gamma_{i1})\,\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\lambda_{ii1}}\\
&=\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\rho_1\left(\frac{(1-\lambda_{ii1})}{\lambda_{ii1}}\frac{\mathcal{E}_3}{\theta_1}-\frac{\frac{1}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)+\frac{\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}{(1-\gamma_{i1})\rho_1+(1-t_i)\left(1-\tilde{\lambda}_{ii1}\right)-1}}{\rho_1}\frac{1}{\lambda_{ii1}}\right)\\
&=\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\frac{1}{\theta_1}\frac{1}{\lambda_{ii1}}\frac{\theta_1-\theta_1\rho_1-(1-\lambda_{ii1})\,\rho_1+(\theta_1-\rho_1\left(1-\lambda_{ii1}\right))\,(t_i-1)\,\left(1-\tilde{\lambda}_{ii1}\right)}{(1-\gamma_{i1})\,\rho_1+(1-t_i)\,\left(1-\tilde{\lambda}_{ii1}\right)-1}.
\end{aligned}$$

Then the first-order condition becomes

$$\left(\frac{\theta_1}{\theta_1-\rho_1}-t_i\right)\frac{(\theta_1-\rho_1)^2}{\theta_1\rho_1}-\frac{(1-\gamma_{i1})}{\gamma_{i1}\mathcal{E}_3}\frac{\tilde{R}(t_i)}{\theta_1}=(1-\alpha_i)\,\frac{\tilde{M}(t_i)t_i}{\gamma_{i1}\mathcal{E}_3}\left(\frac{1}{\rho_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right),$$

where

$$\tilde{R}(t_i)\equiv\frac{1}{\lambda_{ii1}}\left(\frac{\theta_1-\theta_1\rho_1-\rho_1\left(1-\lambda_{ii1}\right)+(\theta_1-\rho_1\left(1-\lambda_{ii1}\right))\,(t_i-1)\,\left(1-\tilde{\lambda}_{ii1}\right)}{1-(1-\gamma_{i1})\,\rho_1+(t_i-1)\,\left(1-\tilde{\lambda}_{ii1}\right)}\right).\tag{105}$$

The first-order condition can then be written succinctly using $t^{opt}=\frac{\theta_1}{\theta_1-\rho_1}$ as

$$t^{opt}[1-(1-\gamma_{i1})R(t_i)]=t_i[1+(1-\alpha_i)\,M(t_i)],\tag{106}$$

where we define the terms $M(t_i)$ and $R(t_i)$ as in the following subsections. Dividing through by $[1+(1-\alpha_i)\,M(t_i)]$, we obtain the fixed-point formula (13).

C.2 Definition of $M(t_i)$ and $A(t_i)$

Use $\tilde{M}(t_i)$ from (104), and replace $\mathcal{E}_5$ with $\mathcal{E}_m\equiv\mathcal{E}_5$ that was defined in (93):

$$\mathcal{E}_m=\frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}-\frac{\rho_1}{\theta_1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1}+\frac{1}{\sigma_1}\left(t_i-1\right)\lambda_{ii1}\right)}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}>0.$$

Then we define $M(t_i)$ as

$$\begin{aligned}
M(t_i)\;&\equiv\;\frac{\theta_1}{(\theta_1-\rho_1)^2}\frac{\tilde{M}(t_i)}{\gamma_{i1}\mathcal{E}_3}\left(1+\frac{(1-\gamma_{i1})}{\gamma_{i1}}\frac{\rho_1}{\theta_1\sigma_1}\left(1-\lambda_{ii1}\right)\right)\\
&=\;\frac{\theta_1\left(\mathcal{E}_m-\frac{(t_i-1)}{t_i}\theta_1\right)}{(\theta_1-\rho_1)^2\,\mathcal{E}_3}\left(1+\frac{\rho_1(1-\gamma_{i1})}{\theta_1\sigma_1\gamma_{i1}}\left(1-\lambda_{ii1}\right)\right)\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}}\frac{D(t_i)}{A(t_i)}\\
&=\;\mathcal{M}\times\left(\mathcal{E}_m-\frac{(t_i-1)}{t_i}\theta_1\right)\frac{D(t_i)}{A(t_i)},
\end{aligned}\tag{107}$$

where $\mathcal{M}$ is defined by

$$\mathcal{M}\equiv\frac{\theta_1}{(\theta_1-\rho_1)^2\,\mathcal{E}_3}\left(1+\frac{\rho_1(1-\gamma_{i1})}{\theta_1\sigma_1\gamma_{i1}}\left(1-\lambda_{ii1}\right)\right)\frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}}>0,\tag{108}$$

and the term $A(t_i)$ is given by the denominator of $\tilde{M}(t_i)$ from (104):

$$A(t_i) \equiv \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{ii1}\right], \tag{109}$$

and we define $\mathcal{E}_a \equiv [(1-\tilde{\lambda}_{ii1})t_i\,(1-\mathcal{E}_4)+\tilde{\lambda}_{ii1}]$ to obtain expression (15) in the main text.

These expressions give us the definition of $M(t_i)$ used in (14) in the main text. To establish the sign of $\mathcal{E}_a$, we use $\mathcal{E}_4$ from (92) to note that

$$\mathcal{E}_4\,(1-\lambda_{ii1}) = \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}\,(1-\lambda_{ii1})}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\,(1-\lambda_{ii1})} < 1.$$

Therefore, $1-\mathcal{E}_4 > 1-\frac{1}{(1-\lambda_{ii1})} = -\frac{\lambda_{ii1}}{(1-\lambda_{ii1})}$, and it follows using (53) that

$$\mathcal{E}_a = (1-\tilde{\lambda}_{ii1})t_i\,(1-\mathcal{E}_4)+\tilde{\lambda}_{ii1} > -\frac{t_i(1-\tilde{\lambda}_{ii1})\lambda_{ii1}}{(1-\lambda_{ii1})}+\tilde{\lambda}_{ii1} = 0. \tag{110}$$

C.3 Definition of $R(t_i)$

The term $R(t_i)$ appearing in (106) is a transformation of $\tilde{R}(t_i)$ from (105)

$$R(t_i) \equiv \frac{\rho_1}{\theta_1\,(\theta_1-\rho_1)}\frac{\tilde{R}(t_i)}{\gamma_{i1}\mathcal{E}_3} = \frac{\rho_1}{\theta_1\,(\theta_1-\rho_1)}\frac{1}{\lambda_{ii1}}\frac{(\theta_1-\rho_1\,(1-\lambda_{ii1}))\,(T+\tilde{\gamma}_{i1})-\theta_1\rho_1}{(1-\tilde{\gamma}_{i1})\,\frac{T}{\rho_1}+\gamma_{i1}\tilde{\gamma}_{i1}},$$

where the equality follows using $T(t_i)$ from (55) in (105). We rewrite this as

$$R(t_i) = \mathcal{R}\times[(\theta_1-\rho_1\,(1-\lambda_{ii1}))\,(T(t_i)+\tilde{\gamma}_{i1})-\theta_1\rho_1]\,,\ \mathcal{R} \equiv \frac{\frac{\rho_1}{\theta_1(\theta_1-\rho_1)\lambda_{ii1}}}{(1-\tilde{\gamma}_{i1})\,\frac{T(t_i)}{\rho_1}+\gamma_{i1}\tilde{\gamma}_{i1}} > 0. \tag{111}$$

Then expression (16) in the main text follows by using use (56) to rewrite $T(t_i)+\tilde{\gamma}_{i1} = \frac{1}{\Lambda_{i1}}$.

D Proof of Theorem 1

While a fixed point to (13) exists under general conditions,[10] to establish the properties of this fixed point we rely on a slightly different form of the equation. Taking the difference between the numerator of $F(t_i)$ times t^{opt} and the denominator times t_i, we obtain

$$H(t_i) \equiv t^{opt}\,[1-(1-\gamma_{i1})R(t_i)] - t_i\,[1+(1-\alpha_i)\,M(t_i)]\,. \tag{112}$$

The function $H(t_i)$ is a continuous function of the tariff provided that $A(t_i) > 0$ in the interval of tariffs we are interested in, so that $M(t_i)$ does not have any discontinuities. Our general approach in the next result is to find high and low tariffs at which the sign of $H(t_i)$ switches, and then we apply the intermediate value theorem to obtain a point where $H(t_i^*) = 0$, which by construction is a fixed-point of (13).

In order to apply the intermediate value theorem, we need to consider values of t_i below unity, meaning an import subsidy, so the revenue cost of the subsidy needs to be deducted from labor income w_iL_i to obtain net income I_i. With enough roundabout production, it seems possible that at a very low tariff – meaning a very high import subsidy – the revenue-cost of the subsidy could exhaust the labor income of the economy, so that net income $I_i = w_iL_i - B_i$ is zero. In that case, the there is no consumption by consumers in country

[10] Let $W(t_i)$ denote utility U_i as a function of the tariff. Provided that $W(t_i)$ is continuous and differentiable, then it will reach some maximum over the (compact) range of all possible tariffs and subsidies, and t_i^* at that maximum will satisfy the first-order condition (13).

i (though the labor endowment L_i is still provided), which is run like an overseas factory solely for the benefit of foreign consumers. We need to check whether this extreme case can occur, and if it does, we denote that minimal tariff (maximum import subsidy) by $t_i^{\min} \geq 0$. We give a more formal definition with:

Definition 2. $t_i^{\min} \equiv \arg\max_{t_i \geq 0} \{T(t_i) | T(t_i) = 0\}$, *with* $\lambda_{ii1}^{\min}$ *denoting the value of* λ_{ii1} *at* $t_i^{\min}$.

To explain this definition, notice that using B_i from the main text in (57) that we can solve for income I_i as

$$I_i = w_i L_i + B_i = w_i L_i \left[\frac{T(t_i)}{T(t_i) + \alpha_i (T(t_i) - (1 - \tilde{\gamma}_{i1}))} \right]. \tag{113}$$

We see that $I_i = 0 \iff T(t_i) = 0$. We do not rule out in Definition 2 the possibility that there might be multiple tariffs at which $T(t_i) = 0$, in which case $t_i^{\min}$ is chosen as the maximum of these.[11] To solve for $t_i^{\min}$, we use the market clearing condition (3) together with trade balance (39) to get

$$Y_{i1} = \alpha_i I_i + \tilde{\gamma}_{i1} \left(\lambda_{ii1} Y_{i1} + \frac{\lambda_{ji1}}{t_i} Y_{i1} \right).$$

If we set $I_i = 0$ and use $\lambda_{ii1} + \lambda_{ij1} = 1$, then we solve for

$$1 = \tilde{\gamma}_{i1} \lambda_{ii1}^{\min} + \tilde{\gamma}_{i1} \frac{1 - \lambda_{ii1}^{\min}}{t_i^{\min}} \implies t_i^{\min} = \frac{\tilde{\gamma}_{i1}(1 - \lambda_{ii1}^{\min})}{(1 - \tilde{\gamma}_{i1} \lambda_{ii1}^{\min})}. \tag{114}$$

We see that $t_i^{\min} = 0$ for $\gamma_{i1} = 1$ because then $\tilde{\gamma}_{i1} = (1 - \gamma_{i1})\rho_1 = 0$. More generally for $0 \leq \gamma_{i1} \leq 1$ we have $0 \leq \tilde{\gamma}_{i1} \leq \rho_1$, and it follows from (114) that $0 \leq t_i^{\min} \leq \tilde{\gamma}_{i1}$. Because we solved for $t_i^{\min}$ from the market clearing condition when $I_i = 0$, it follows from (113) that $T(t_i^{min}) = 0$. Negative income is not possible, so at higher tariffs we have $I_i > 0$ and then $T(t_i) > 0$ from (113).

Remark 2. *We henceforth restrict our attention to tariffs in the range* $t_i \geq t_i^{\min}$, *where* $T^{\min} \equiv T(t_i^{\min}) = 0$ *and* $T(t_i) > 0 \iff t_i > t_i^{\min}$.

Before proceeding with the proof of Theorem 1, we make use of the $T(t_i)$ function to slightly transform the terms used within $D(t_i)$ and $M(t_i)$, as defined in Appendix C.1 and C.2. We first transform the elasticity $\mathcal{E}_3$ appearing in (90) using $T(t_i)$ in (55) to obtain

$$\mathcal{E}_3 = \left(\frac{T(t_i) + \tilde{\gamma}_{i1}}{T(t_i)} + \frac{1}{\gamma_{i1}(\sigma_1 - 1)} \right) > 0,$$

and so

$$\mathcal{E}_3 T(t_i) = \left(1 + \frac{1}{\gamma_{i1} (\sigma_1 - 1)} \right) T + \tilde{\gamma}_{i1} = (1 - \tilde{\gamma}_{i1}) \frac{T(t_i)}{\gamma_{i1} \rho_1} + \tilde{\gamma}_{i1}.$$

Using the above equation with (99), and noting that $\frac{1+\gamma_{i1}(\sigma_1 - 1)}{\gamma_{i1}(\sigma_1 - 1)} = \frac{1 - \tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}$, we obtain

$$D(t_i)T(t_i) = \frac{1 - \tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1} \left[T(t_i) - \frac{1 + \gamma_{i2}(\sigma_2 - 1)}{\gamma_{i2}(\sigma_2 - 1)} \gamma_{i1}\rho_1 - \left(T(t_i) + \gamma_{i1}\rho_1 \frac{\tilde{\gamma}_{i1}}{1 - \tilde{\gamma}_{i1}} \right) (1 - \lambda_{ii1}) \mathcal{E}_4 \right]. \tag{115}$$

Also, note that (109) can be rewritten using $T(t_i)$ from (55) as

$$A(t_i) = \alpha_i - \tilde{\gamma}_{i1} + (1 - \alpha_i) \left[\left(T(t_i) + \tilde{\gamma}_{i1} - \tilde{\lambda}_{ii1} \right) (1 - \mathcal{E}_4) + \tilde{\lambda}_{ii1} \right]. \tag{116}$$

We now prove Theorem 1 by a series of Definitions, Lemmas and Remarks.

[11] If $T(t_i)$ is increasing in t_i then $t_i^{\min}$ will be unique, and conditions to ensure that are provided in Lemma 7.

From (111) we have $R(t_i) = \mathcal{R} \times [(\theta_1 - \rho_1 (1 - \lambda_{ii1}))(T(t_i) + \tilde{\gamma}_{i1}) - \theta_1\rho_1]$, where $\mathcal{R} > 0$. It appears that the term $[(\theta_1 - \rho_1 (1 - \lambda_{ii1}))(T(t_i) + \tilde{\gamma}_{i1}) - \theta_1\rho_1]$ can be zero, particularly as $T(t_i)$ is low, so that $R(t_i) = 0$. For the proof of parts (a) and (c) in Theorem 1, we will make extensive use of this low tariff, which is defined more formally as follows:

Definition 3. *Define* $t_i^{R0} \equiv \arg\max_{t_i \geq t_i^{min}} \{R(t_i) | R(t_i) = 0\}$ *and denote* $T^{R0} \equiv T(t_i^{R0})$*, where it is understood that* T^{R0} *uses the shares* $\tilde{\lambda}_{ii1}^{R0}$ *and* λ_{ii1}^{R0} *which are evaluated at* t_i^{R0}*.*

This definition allows for the possibility that there could be multiple tariffs at which $R(t_i) = 0$, in which case t_i^{R0} is chosen as the maximum of these points.

Lemma 3. *The tariff* t_i^{R0} *is given by*

$$t_i^{R0} = 1 + \frac{\rho_1}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)} \frac{\rho_1 \left(1 - \lambda_{ii1}^{R0}\right)}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - \frac{(1 - \rho_1)}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)}, \tag{117}$$

with $t_i^{\min} < t_i^{R0} < 1$ *and* $R(t_i) > 0$ *for* $t_i > t_i^{R0}$*.*

Proof: Because $\mathcal{R} > 0$ in (16), then $R(t_i) = 0$ implies $[(\theta_1 - \rho_1 (1 - \lambda_{ii1}))(T(t_i) + \tilde{\gamma}_{i1}) - \theta_1\rho_1] = 0$. This condition is rewritten as

$$T^{R0} = \frac{\theta_1\rho_1}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1} = \left(\frac{\theta_1}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)} - (1 - \gamma_{i1})\right)\rho_1 > 0, \tag{118}$$

because the first ratio on the right is greater than 1 and so it exceeds $(1 - \gamma_{i1})$. It follows from Remark 2 that $t_i^{R0} > t_i^{\min}$, and from Definition 3 that $R(t_i) > 0$ for $t_i > t_i^{R0}$.

Using (55) we can solve for t_i^{R0} to obtain obtain (117), which can also be written as

$$t_i^{R0} = 1 + \frac{1}{\left(1 - \tilde{\lambda}_{ii1}^{R0}\right)} \left(\frac{-(1 - \rho_1)\theta_1 + \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)}{\theta_1 - \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)}\right) < 1$$

where the final inequality follows from $\theta_1 > (\sigma_1 - 1) \Rightarrow \theta_1 (1 - \rho_1) > \rho_1 \left(1 - \lambda_{ii1}^{R0}\right)$. QED

PROOF OF PART (a)

We assume that $\alpha_i = 1$, and then $H(t_i)$ from (112) becomes $H(t_i) = t^{opt} - t_i - t^{opt}(1 - \gamma_{i1})R(t_i)$. With $R(t_i^{R0}) = 0$ for $t_i^{R0} < 1$, we obtain $H(t_i^{R0}) = t^{opt} - t_i^{R0} > 0$. Checking the sign of $R(t^{opt})$, because $T(t^{opt}) > 1 - \tilde{\gamma}_{i1}$ it readily follows from (16) that $R(t^{opt}) > 0$. In that case we obtain $H(t_i^{opt}) = -t^{opt}(1 - \gamma_{i1})R(t^{opt}) < 0$ for $\gamma_{i1} < 1$. Using the continuity of $R(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff t_i^* with $t_i^{R0} < t_i^* < t^{opt}$ at which $H(t_i^*) = 0$. By construction, this tariff is a fixed point of (13). QED

PROOF OF PARTS (b) AND (c)

From (107) we have $M(t_i) = \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1\right) \frac{D(t_i)}{A(t_i)}$, where $\mathcal{M} > 0$ from (108). It appears to be possible that $M(t_i) = 0$ for two reasons: either $D(t_i) = 0$ at some tariff; or $\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1$ at some tariff. For the proof of parts (b) and (c) in Theorem 1, we will make extensive use of the first of these points, where $D(t_i) = 0$, which is defined more formally as follows:

Definition 4. *Define*

$$t_i^{D0} \equiv \begin{cases} \arg\min_{t_i \geq t_i^{\min}} \{D(t_i) = 0\} & \textit{if this value exists,} \\ +\infty & \textit{otherwise,} \end{cases}$$

and denote $T^{D0} \equiv T(t_i^{D0})$ *and likewise for the shares* $\tilde{\lambda}_{ii1}^{D0}$ *and* λ_{ii1}^{D0} *evaluated at* t_i^{D0}*.*

Once again, we allow for multiple solutions for the tariff where $D(t_i) = 0$, and in this case we choose t_i^{D0} as the minimum of them. Next, we establish a result for the term $\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1$ that also appears within $M(t_i) = \mathcal{M} \times \left(\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1\right)\frac{D(t_i)}{A(t_i)}$, and could possibly make this expression equal to zero.

Lemma 4. *$\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 > 0$ for all $t_i \in [t_i^{\min}, t_i^{opt}]$. In addition, if $\gamma_{i1} = 1$ then $\mathcal{E}_m - \frac{(t_i''-1)}{t_i''}\theta_1 = 0$ at a tariff $t_i'' > t_i^{opt}$.*

Proof: From (93) we see that

$$\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 = \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}} - \frac{(t_i-1)}{t_i}\theta_1. \tag{119}$$

Notice that the final term on the right is increasing in t_i, so it takes its highest value over $t_i \in [t_i^{\min}, t_i^{opt}]$ at $t_i = t^{opt}$, in which that term equals ρ_1. It follows that

$$\begin{aligned}
\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 \quad &\geq \quad \frac{\frac{1+\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}} - \frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1-\tilde{\lambda}_{ii1}}{\tilde{\lambda}_{ii1}}\left(\frac{\rho_1}{\theta_1} + \frac{1}{\sigma_1}(t_i-1)\lambda_{ii1}\right)}{\frac{1}{\rho_1} + \frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}} - \rho_1 \\
&= \quad \frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}\sigma_1}\left(\left(1-\tilde{\lambda}_{ii1}\right)(t_i-1)\lambda_{ii1} - \frac{\rho_1}{\theta_1}(1-\lambda_{ii1})\tilde{\lambda}_{ii1}\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right) \\
&= \quad \frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1 - \frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1} + \frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\frac{\rho_1}{\theta_1} + \frac{1-\gamma_{i1}}{\gamma_{i1}\sigma_1}\left(1-\tilde{\lambda}_{ii1}\right)\lambda_{ii1}\left(\frac{t_i}{t^{opt}}-1\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right)
\end{aligned}$$

where the final line follows using (53). The second two terms in the numerator are

$$\frac{1-\gamma_{i1}}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\left(\frac{\rho_1}{\theta_1} - \frac{\lambda_{ii1}}{\sigma_1}\left(1-\frac{t_i}{t^{opt}}\right)\right) \geq \frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\theta_1\sigma_1}\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i(\theta_1-\rho_1) - (\theta_1 - (\sigma_1-1))\right)$$

which is positive for $t_i \geq 1$ and proves that $\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 > 0$ for all $t_i \in [t_i^{\min}, t_i^{opt}]$.

To evaluate (119) at higher levels of the tariff, note that with $\gamma_{i1} = 1$ we have that

$$\lim_{t_i\to\infty}\left(\mathcal{E}_m|_{\gamma_{i1}=1} - \frac{(t_i-1)}{t_i}\theta_1\right) = \rho_1\left(2-\frac{\rho_1}{\theta_1}\right) - \theta_1 < 0$$

because $\left(2 - \frac{\rho_1}{\theta_1} - \frac{\theta_1}{\rho_1}\right) < 0$ for $\frac{\theta_1}{\rho_1} > 1$. It follows that for $\gamma_{i1} = 1$ then there exists a tariff $t_i'' > t_i^{opt}$ at which $\mathcal{E}_m - \frac{(t_i''-1)}{t_i''}\theta_1 = 0$. QED

PROOF OF PART (b)(i)

If $\gamma_{is} = 1$ for $s = 1, 2$ then from (92) and (93) we have we have $\mathcal{E}_3 = \frac{\sigma_1}{(\sigma_1-1)}$ and $\mathcal{E}_4 = \frac{\rho_1}{\theta_1\sigma_1}$. Substituting these into (99) we obtain

$$\begin{aligned}
D(t_i) \quad &= \quad \left[\frac{\sigma_1}{(\sigma_1-1)} - \left(\frac{\sigma_2}{(\sigma_2-1)}\right)\frac{1}{T(t_i)} - \frac{1}{\sigma_1\theta_1}(1-\lambda_{ii1})\right] \\
&> \quad \left[\frac{\sigma_1}{(\sigma_1-1)} - \frac{\sigma_2}{(\sigma_2-1)} - \frac{1}{\sigma_1\theta_1}\right],
\end{aligned} \tag{120}$$

where the inequality follows from $T(t_i) \geq 1$ for $t_i \in [1, t^{opt}]$ and $\lambda_{ii1} < 1$. It follows that $D(t_i) > 0$ when condition (17) holds.

Notice that when $\gamma_{i1} = 1$ and $\mathcal{E}_4 = \frac{\rho_1}{\theta_1 \sigma_1}$ then $A(t_i)$ in (109) becomes

$$A(t_i) \equiv \alpha_i + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(1 - \frac{\rho_1}{\theta_1 \sigma_1}\right) + \tilde{\lambda}_{ii1}\right] > \alpha_i > 0, \tag{121}$$

which is bounded away from zero so that $M(t_i)$ is continuous. Then because $\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 > 0$ for $t_i \in [1, t^{opt}]$ from Lemma 4, it follows that $M(t_i) > 0$ in that interval, and in particular $M(t_i^{opt}) > 0$. From (112) with $\gamma_{i1} = 1$ it follows that $H(t^{opt}) = -t^{opt}(1-\alpha_i) M(t^{opt}) < 0$.

Now we make use of t_i^{D0} which for $\gamma_{i1} = 1$ is solved by setting (120) equal to zero, giving

$$\frac{\sigma_1}{(\sigma_1 - 1)} - \left(\frac{\sigma_2}{(\sigma_2 - 1)}\right) \frac{1}{T(t_i^{D0})} - \frac{1}{\sigma_1 \theta_1}(1 - \lambda_{ii1}) = 0. \tag{122}$$

It follows that

$$T(t_i^{D0}) = \frac{\frac{\sigma_2}{\sigma_2 - 1}}{\frac{\sigma_1}{(\sigma_1-1)} - \frac{(1-\lambda_{ii1})}{\sigma_1\theta_1}} < \frac{\frac{\sigma_2}{\sigma_2 - 1}}{\frac{\sigma_1}{(\sigma_1-1)} - \frac{1}{\sigma_1\theta_1}} < 1,$$

since condition (17) implies $\frac{\sigma_1}{\sigma_1 - 1} - \frac{1}{\sigma_1\theta_1} > \frac{\sigma_2}{(\sigma_2-1)}$. Because $T(t_i^{D0}) = 1 + \left(t_i^{D0} - 1\right)\left(1 - \tilde{\lambda}_{ii1}\right)$ when $\gamma_{i1} = 1$, it follows immediately that $t_i^{D0} < 1$.

We have already shown $H(t^{opt}) = -t^{opt}(1-\alpha_i) M(t^{opt}) < 0$. Since $t_i^{D0} < 1$ then $M(t_i^{D0}) = 0$ and so $H(t_i^{D0}) = t^{opt} - t_i^{D0}\left[1 + (1-\alpha_i) M(t_i^{D0})\right] = t^{opt} - t_i^{D0} > 0$. Using the continuity of $M(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff t_i^* with $t_i^{D0} < t_i^* < t^{opt}$ at which $H(t_i^*) = 0$. By construction, this tariff is a fixed point of (13). QED

PROOF OF PART (b)(ii)

Under $\sigma_1 \geq \sigma_2$, we have $D(t_i^{opt}) < 0$ from (120). Using Lemma 4 it follows that $M(t_i^{opt}) < 0$, and therefore from $H(t_i)$ in (112), with $\gamma_{i1} = 1$, we have $H(t^{opt}) = -t^{opt}(1-\alpha_i) M(t^{opt}) > 0$.

Now we check a higher tariff $t_i'' > t^{opt}$ from Lemma 4 at which $\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1 = 0$ and therefore $M(t_i'') = 0$. From $H(t_i)$ in (112), with $\gamma_{i1} = 1$, we have $H(t_i'') = t^{opt} - t_i''[1 + (1-\alpha_i) M(t_i'')] = t^{opt} - t_i'' < 0$. Using from the continuity of $M(t_i)$ and therefore of $H(t_i)$, it follows from the intermediate value theorem that there exists a tariff exists a tariff t_i^* with $t_i^{opt} < t_i^* < t_i''$ at which $H(t_i^*) = 0$. By construction, this tariff is a fixed point of (13). QED

PROOF OF PART (c)

We first establish conditions to ensure that $A(t_i) > 0$, starting with the region $t_i \geq t^{opt}$.

Lemma 5. $(1-\mathcal{E}_4)t^{opt} \geq \rho_1$ *when condition (18) holds, where $\mathcal{E}_4$ can be evaluated at any tariff. It follows that* $A(t_i) > \alpha_i(1-\rho_1) + \gamma_{i1}\rho_1 > 0$ *for all* $t_i \geq t^{opt}$.

Proof: We want to ensure that $(1-\mathcal{E}_4) \geq \frac{\rho_1}{t^{opt}} = \frac{\rho_1(\theta_1 - \rho_1)}{\theta_1}$. Use (92) to obtain

$$1 - \frac{\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})} \geq \frac{\rho_1(\theta_1 - \rho_1)}{\theta_1}.$$

Then we take $\lambda_{ii1} = 1$ to get a sufficient condition

$$1 - \frac{\rho_1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1 - 1)}{\sigma_1\gamma_{i1}} \geq \frac{\rho_1(\theta_1-\rho_1)}{\theta_1} \iff \frac{\sigma_1}{\rho_1}(\theta_1 - \rho_1)(1-\rho_1) \geq \frac{1-\gamma_{i1}}{\gamma_{i1}}. \tag{123}$$

which is equivalent to (18) in the main text. Now the magnitude of $A(t_i)$ is established from

$$\begin{aligned}
A(t_i) &= \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{ii1}\right] \\
&\geq \alpha_i - \tilde{\gamma}_{i1} + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right) t^{opt} \left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{ii1}\right] \\
&\geq \alpha_i - (1-\gamma_{i1})\rho_1 + (1-\alpha_i)\left[\left(1-\tilde{\lambda}_{ii1}\right) \rho_1 + \tilde{\lambda}_{ii1}\right] \\
&> \alpha_i(1-\rho_1) + \gamma_{i1}\rho_1,
\end{aligned}$$

where the first inequality follows from $t_i \geq t^{opt}$, the second from $(1-\mathcal{E}_4)t^{opt} \geq \rho_1$, and the final inequality from $[(1-\tilde{\lambda}_{ii1})\rho_1 + \tilde{\lambda}_{ii1}] > \rho_1$. QED

Next, we define the tariff t_i^{A0} at which $A(t_i)$ becomes zero, if it exists:

Definition 5. *a) Define*

$$t_i^{A0} \equiv \begin{cases} \arg\max_{t_i \geq t_i^{\min}} \{A(t_i) = 0\} & \textit{if this value exists,} \\ t_i^{\min} & \textit{otherwise,} \end{cases}$$

and denote $T^{A0} \equiv T(t_i^{A0})$ and likewise for the shares $\tilde{\lambda}_{ii1}^{A0}$ and λ_{ii1}^{A0}, which are evaluated at t_i^{A0}.

In this definition we are looking for tariffs at which $A(t_i) = 0$, but there will be no such tariffs if $A(t_i) > 0$ for all $t_i \geq t_i^{\min}$. In that case, $t_i^{A0} = t_i^{\min} < 1$. On the other hand, if there are multiple tariffs at which $A(t_i) = 0$, then t_i^{A0} is the maximum of these. From Lemma 5 which relies on condition (18) we know that $t_i^{A0} < t^{opt}$. In Lemma 8 below, we will further show that condition (19) ensures that $t_i^{A0} < t_i^{R0}$, and we know that $t_i^{R0} < 1$ from Lemma 3, so $t_i^{A0} < 1$.

Remark 6. *The tariff t_i^{A0} is the import subsidy referred to as t_i' in the statement of Theorem 1(c).*

Lemma 7. *For $t_i \in (t_i^{\min}, 1)$, $T(t_i)$ is monotonically increasing in t_i provided that $A(t_i) > 0$.*

Proof: From (54) combined with (55), $T(t_i)$ is given by

$$T = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} - \tilde{\gamma}_{i1}, \tag{124}$$

which we differentiate to obtain,

$$dT = \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \left(\hat{\tilde{\lambda}}_{ii1} - \hat{\lambda}_{ii1}\right).$$

Totally differentiate (53), we can show that

$$\hat{\lambda}_{ii1} = \frac{(1-\lambda_{ii1})}{(1-\tilde{\lambda}_{ii1})} \left[\hat{\tilde{\lambda}}_{ii1} - \left(1-\tilde{\lambda}_{ii1}\right) \hat{t}_i\right]. \tag{125}$$

Then combining with (96), we obtain

$$\hat{\lambda}_{ii1} - \hat{\tilde{\lambda}}_{ii1} = \frac{\theta_1}{\tilde{\lambda}_{ii1}} \left(\tilde{\lambda}_{ii1} - \lambda_{ii1}\right) \hat{\varphi}_{ij1}^{*} - \left(1-\lambda_{ii1}\right) \hat{t}_i.$$

It follows that,

$$dT = -\frac{\theta_1}{\lambda_{ii1}} \left(\tilde{\lambda}_{ii1} - \lambda_{ii1}\right) \hat{\varphi}_{ij1}^{*} + \frac{\tilde{\lambda}_{ii1}}{\lambda_{ii1}} \left(1-\lambda_{ii1}\right)\hat{t}_i. \tag{126}$$

Notice that the coefficient of $\hat{t}_i$ in the final term is positive. We now show that $\hat{\varphi}_{ij1}/\hat{t}_i$ is positive, so then because $\tilde{\lambda}_{ii1} < \lambda_{ii1}$ for $t_i < 1$, we have established the monotonicity of $T(t_i)$.

Using (91) and (97), we have

$$\hat{t}_i = \left(\mathcal{E}_m - t_i \mathcal{E}_4 \frac{\left(1-\tilde{\lambda}_{ii1}\right)\left(\mathcal{E}_m - \frac{(t_i-1)}{t_i}\theta_1\right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)} \right) \hat{\varphi}^*_{ij1},$$

and so

$$\hat{\varphi}^*_{ij1} = \frac{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)}{\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i \left(\mathcal{E}_4 - 1\right) - \tilde{\lambda}_{ii1} - \frac{1}{1-\alpha_i}\left(\alpha_i - \tilde{\gamma}_{i1}\right)\right)\mathcal{E}_m - \mathcal{E}_4\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i\mathcal{E}_m - (t_i-1)\theta_1\right)} \hat{t}_i.$$

Multiply the numerator and denominator by $-(1-\alpha_i)$ and use (109) to obtain

$$\hat{\varphi}^*_{ij1} = \frac{A}{A\mathcal{E}_m + (1-\alpha_i)\mathcal{E}_4\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i\mathcal{E}_m - (t_i-1)\theta_1\right)} \hat{t}_i.$$

Because $\mathcal{E}_4 > 0$ and $\mathcal{E}_m > 0$, then for $t_i < 1$ we have $\hat{\varphi}_{ij1}/\hat{t}_i > 0$. QED

Lemma 8. *$A(t_i) > 0$ for $t_i \in [t_i^{R0}, t^{opt}]$ provided that (18) and (19) hold.*

Proof:

There are two cases to consider. The first case is where $t_i^{A0} = t_i^{min}$ so that $A(t_i) > 0$ for all $t_i > t_i^{min}$. In that case, the lemma holds trivially.

The second case is where $t_i^{A0} > t_i^{min}$. Then according to (109), $A(t_i^{A0}) = 0$ at the tariff

$$(1-\alpha_i)\left(\left(1-\tilde{\lambda}_{ii1}\right) t_i^{A0}\left(1-\mathcal{E}_4\right) + \tilde{\lambda}_{ii1}\right) + \alpha_i - \tilde{\gamma}_{i1} = 0,$$

so that,

$$\left(1-\tilde{\lambda}_{ii1}\right) t_i^{A0} = -\frac{\alpha_i\left(1-\tilde{\lambda}_{ii1}\right) + \tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}}{\left(1-\mathcal{E}_4\right)\left(1-\alpha_i\right)}. \tag{127}$$

Using the definition of $T(t_i)$ in (55), we can rewrite (109) as

$$(1-\alpha_i)\left(T^{A0} - \left(1-\tilde{\lambda}_{ii1}\right) t_i^{A0}\mathcal{E}_4\right) + \alpha_i\left(1-\tilde{\gamma}_{i1}\right) = 0, \quad \text{so that,}$$

$$T^{A0} = \left(1-\tilde{\lambda}_{ii1}\right) t_i^{A0}\mathcal{E}_4 - \frac{\alpha_i}{1-\alpha_i}\left(1-\tilde{\gamma}_{i1}\right).$$

Combining with (127), T^{A0} can be written as

$$\begin{aligned} T^{A0} &= -\frac{1}{1-\alpha_i}\left(\frac{\mathcal{E}_4\left(1-\alpha_i\right)\left(\tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}\right) + \alpha_i\left(1-\tilde{\gamma}_{i1}\right)}{\left(1-\mathcal{E}_4\right)}\right) \\ &= -\left(\tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}\right)\frac{\mathcal{E}_4}{\left(1-\mathcal{E}_4\right)} - \frac{\alpha_i\left(1-\tilde{\gamma}_{i1}\right)}{\left(1-\alpha_i\right)\left(1-\mathcal{E}_4\right)}. \end{aligned} \tag{128}$$

We know that the tariff t_i^{R0} at which $R(t_i^{R0}) = 0$ occurs at

$$T^{R0} = \frac{\theta_1\rho_1}{\theta_1 - \rho_1\left(1-\lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1}.$$

Our goal is to show that $T^{A0} \leq T^{R0}$, which will ensure that $T(t_i)$ is invertible in the range $[t_i^{A0}, 1]$ using

Lemma 7 with $A(t_i) > 0$ in that range. The condition $T^{A0} \le T^{R0}$ holds if

$$-\left(\tilde{\lambda}_{ii1} - \tilde{\gamma}_{i1}\right) \frac{\mathcal{E}_4}{(1-\mathcal{E}_4)} - \frac{\alpha_i \left(1-\tilde{\gamma}_{i1}\right)}{(1-\alpha_i)\left(1-\mathcal{E}_4\right)} \le \frac{\theta_1 \rho_1}{\theta_1 - \rho_1 \left(1-\lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1}, \quad \text{or,}$$

$$-\tilde{\lambda}_{ii1} \frac{\mathcal{E}_4}{(1-\mathcal{E}_4)} - \frac{\alpha_i \left(1-\tilde{\gamma}_{i1}\right)}{(1-\alpha_i)\left(1-\mathcal{E}_4\right)} \le \frac{\theta_1 \rho_1}{\theta_1 - \rho_1 \left(1-\lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1} \frac{1}{1-\mathcal{E}_4}.$$

Drop the share on the left and we get the sufficient condition

$$\frac{1}{(1-\mathcal{E}_4)} \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)} \le \frac{\theta_1 \rho_1}{\theta_1 - \rho_1 \left(1-\lambda_{ii1}^{R0}\right)}. \tag{129}$$

If $\tilde{\gamma}_{i1} < \alpha_i$, this restriction is satisfied. However, for $\tilde{\gamma}_{i1} > \alpha_i$, then we need $\mathcal{E}_4$ sufficiently small so that the above condition holds.

From Lemma 5 we know that $(1-\mathcal{E}_4)t^{opt} < 1$ and it follows that $\mathcal{E}_4 < 1$. Then the sufficient condition for (129) is

$$(1-\mathcal{E}_4) \ge \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)} \left[\frac{1}{\rho_1} - \frac{\left(1-\lambda_{ii1}^{R0}\right)}{\theta_1}\right].$$

If $\alpha_i \ge \tilde{\gamma}_{i1}$ then this condition is automatically satisfied, since then the right-hand side is less than or equal to zero, while the left-hand side is positive. For $\alpha_i < \tilde{\gamma}_{i1}$, we can take $\lambda_{ii1}^{R0} = 1$ to get the sufficient condition

$$1-\mathcal{E}_4 \ge \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)} \left[\frac{1}{\rho_1} - \frac{1}{\theta_1} + \frac{1}{\theta_1}\right] \ge \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)} \left[\frac{1}{\rho_1} - \frac{1}{\theta_1} + \frac{\lambda_{ii1}^{R0}}{\theta_1}\right].$$

We can substitute for $\mathcal{E}_4$ and the sufficient condition becomes

$$1 - \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\sigma_1 \gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\gamma_{i1}} \frac{1}{\sigma_1} \left(1-\lambda_{ii1}\right)} \ge \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)\rho_1}.$$

A sufficient condition for this inequality is obtained by taking $\lambda_{ii1} = 1$ on the left, so

$$1 - \frac{\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\sigma_1 \gamma_{i1}}}{\frac{1}{\rho_1}} \ge \frac{\left(\tilde{\gamma}_{i1} - \alpha_i\right)}{(1-\alpha_i)\rho_1} \implies \alpha_i \ge \frac{-\gamma_{i1} + \rho_1 \left(\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\sigma_1 \gamma_{i1}}\right)}{\frac{1}{\rho_1} - 1 + \rho_1 \left(\frac{1}{\theta_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\sigma_1 \gamma_{i1}}\right)}.$$

We therefore obtain (19) as the sufficient condition for $t_i^{A0} < t_i^{R0}$, which ensures the $A(t_i) > 0$ for $t_i \in [t_i^{R0}, t^{opt}]$. QED

Lemma 9. *$D(t_i^{R0}) < 0$. It follows by also using conditions (18) and (19) together with Lemma 4 that $M(t_i^{R0}) < 0$.*

Proof: We evaluate $D(t_i)$ from (99) at t_i^{R0} where we also evaluate the elasticities $\mathcal{E}_3$, and $\mathcal{E}_4$ at t_i^{R0}. Then $D(t_i^{R0}) < 0$ if the following expression is negative

$$1 - \frac{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)} \gamma_{i1}\left(\sigma_1 - 1\right)}{\sigma_1 \left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0} - 1\right) + \left(1 + \gamma_{i1}\left(\sigma_1 - 1\right)\right)} - \frac{\gamma_{i1}\left(\sigma_1 - 1\right)\left(1-\lambda_{ii1}\right)}{1 + \gamma_{i1}\left(\sigma_1 - 1\right)} \frac{\left(\frac{T(t_i) + \tilde{\gamma}_{i1}}{T(t_i)} + \frac{1}{\gamma_{i1}(\sigma_1 - 1)}\right) \frac{1}{\theta_1} \frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1 \gamma_{i1}}}{\frac{1}{\rho_1} + \frac{1}{\theta_1} \frac{1-\gamma_{i1}}{\gamma_{i1}} \frac{1}{\sigma_1} \left(1-\lambda_{ii1}\right)} < 0.$$

Using $T^{R0} = \frac{\theta_1\rho_1}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)} - \tilde{\gamma}_{i1}$, then we require

$$1 - \frac{\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)}\gamma_{i1}\left(\sigma_1-1\right)}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}$$
$$-\frac{\gamma_{i1}\left(\sigma_1-1\right)\left(1-\lambda_{ii1}\right)}{1+\gamma_{i1}\left(\sigma_1-1\right)}\frac{\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1+\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)} < 0.$$

Given that $\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_2(\sigma_2-1)} > 1$ then a sufficient condition is

$$\frac{1}{\sigma_1\left(1-\tilde{\lambda}_{ii1}\right)\left(t_i^{R0}-1\right)+\left(1+\gamma_{i1}\left(\sigma_1-1\right)\right)}$$
$$+\frac{\left(1-\lambda_{ii1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\frac{1}{\theta_1}\frac{1}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)} \geq \frac{1}{\gamma_{i1}\left(\sigma_1-1\right)}.$$

Using $t_i^{R0} = 1 + \frac{\rho_1}{\left(1-\tilde{\lambda}_{ii1}^{R0}\right)}\frac{\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)} - \frac{(1-\rho_1)}{\left(1-\tilde{\lambda}_{ii1}^{R0}\right)}$, we simply this inequality as

$$\frac{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}$$
$$+\frac{\left(1-\lambda_{ii1}\right)\left(\frac{\theta_1\rho_1}{\theta_1\rho_1-\tilde{\gamma}_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{1}{\gamma_{i1}(\sigma_1-1)}\right)\rho_1\frac{\gamma_{i1}(\sigma_1-1)}{\sigma_1\gamma_{i1}}}{\rho_1\frac{1}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\gamma_{i1}\left(\sigma_1-1\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)\right)} \geq \frac{1}{\gamma_{i1}\left(\sigma_1-1\right)}.$$

With simplifications, this inequality is expressed as

$$\frac{\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)}+\frac{\left(\frac{\theta_1\rho_1(1-\lambda_{ii1})(\sigma_1-1)}{\gamma_{i1}\theta_1+(1-\gamma_{i1})\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+-\sigma_1\theta_1+\rho_1\left(1-\lambda_{ii1}\right)\right)}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)} \geq 0,$$

or,

$$\frac{1}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)} \geq \frac{\frac{(\sigma_1-1)\theta_1\gamma_{i1}}{\gamma_{i1}\theta_1+(1-\gamma_{i1})\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+1}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)}.$$

Cross-multiplying terms we obtain

$$1+\frac{\gamma_{i1}\left(\sigma_1-1\right)\left(\sigma_1-1\right)\theta_1}{\sigma_1\rho_1^2\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\sigma_1-1\right)\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)} \geq \frac{\left(\sigma_1-1\right)\theta_1\gamma_{i1}}{\gamma_{i1}\theta_1+\left(1-\gamma_{i1}\right)\rho_1\left(1-\lambda_{ii1}^{R0}\right)}+1,$$

so that we finally obtain the inequality

$$\gamma_{i1}\theta_1+\left(1-\gamma_{i1}\right)\rho_1\left(1-\lambda_{ii1}^{R0}\right) \geq \rho_1\left(1-\lambda_{ii1}^{R0}\right)+\gamma_{i1}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}^{R0}\right)\right)$$

which is true because by canceling common terms it holds as an equality. QED

To prove Theorem 1(c), we rely on two, final Lemmas.

Lemma 10. *Provided that condition (20) holds, then*

$$D^{opt} > \delta_i \left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1} \right) - \frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)} \left(\frac{1-\tilde{\gamma}_{i1}}{t^{opt}-\tilde{\gamma}_{i1}} \right), \tag{130}$$

where

$$\delta_i \equiv \frac{1-\rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1-\frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt}+\frac{1}{\sigma_1}}. \tag{131}$$

Proof: We define $T^{opt} \equiv T(t^{opt})$ and $D^{opt} \equiv D(t^{opt})$. It follows from substituting the expenditure share (36) for $k = i$ into the production share (50) and then into $T(t_i)$ in (55) that

$$T^{opt} = 1 - \tilde{\gamma}_{i1} + \frac{\left(t^{opt}-1\right)(1-\lambda_{ii1})}{1+(t^{opt}-1)\lambda_{ii1}}. \tag{132}$$

It follows from (115) that

$$D^{opt}T^{opt} = \left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1} \right) \left[T^{opt}\left(1-(1-\lambda_{ii1})\mathcal{E}_4\right) - \frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1 - \gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}(1-\lambda_{ii1})\mathcal{E}_4 \right]. \tag{133}$$

It should be understood that the shares appearing in these equations are also evaluated at t^{opt}. Our strategy, however, is to treat these shares as parameters and differentiate $D^{opt}T^{opt}$ with respect to the share λ_{ii1} so as to obtain a lower-bound on $D^{opt}T^{opt}$. During this process, we are allowing the production share to adjust parametrically according to (50).

The value $D^{opt}T^{opt}$ changes with the share λ_{ii1} according to

$$\frac{\partial D^{opt}T^{opt}}{\partial \lambda_{ii1}} = \left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1} \right) \left[\frac{\partial T^{opt}}{\partial \lambda_{ii1}}\left(1-(1-\lambda_{ii1})\mathcal{E}_4\right) - \left(T^{opt} + \gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}} \right) \frac{\partial(1-\lambda_{ii1})\mathcal{E}_4}{\partial \lambda_{ii1}} \right]. \tag{134}$$

From (132) we have

$$\begin{aligned} \frac{\partial T^{opt}}{\partial \lambda_{ii1}} &= -\frac{\left(t^{opt}-1\right)}{1+(t^{opt}-1)\lambda_{ii1}} - \frac{\left(t^{opt}-1\right)^2(1-\lambda_{ii1})}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2} \\ &= -\frac{\left(t^{opt}-1\right)t^{opt}}{\left[1+(t^{opt}-1)\lambda_{ii1}\right]^2}. \end{aligned}$$

Also from (92) we see that $(1-\lambda_{ii1})\mathcal{E}_4 = \frac{\frac{(1-\lambda_{ii1})}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}(1-\lambda_{ii1})}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}(1-\lambda_{ii1})} = \frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}}$, and so

$$\begin{aligned} \frac{\partial(1-\lambda_{ii1})\mathcal{E}_4}{\partial \lambda_{ii1}} &= -\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]^2}\frac{1}{\rho_1(1-\lambda_{ii1})^2} \\ &= -\mathcal{E}_4\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]}. \end{aligned}$$

Substituting these expressions into (134), we obtain

$$
\begin{aligned}
\frac{\partial D^{opt} T^{opt}}{\partial \lambda_{ii1}} &= -\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right) \frac{\left(t^{opt}-1\right) t^{opt}}{\left[1+\left(t^{opt}-1\right)\lambda_{ii1}\right]^2} \left[1-\left(1-\lambda_{ii1}\right)\mathcal{E}_4\right] \\
&+ \left(T^{opt}+\gamma_{i1}\rho_1 \frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right) \mathcal{E}_4 \frac{\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right) \frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]} \\
&= -\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right) \frac{\left(t^{opt}-1\right) t^{opt}}{\left[1+\left(t^{opt}-1\right)\lambda_{ii1}\right]^2} \left[\frac{\frac{1}{\rho_1(1-\lambda_{ii1})}-\frac{1}{\theta_1}}{\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}}\right] \\
&+ \left(T^{opt}+\gamma_{i1}\rho_1 \frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right) \mathcal{E}_4 \frac{\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right) \frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]} \\
&> \left\{-\frac{\left(t^{opt}-1\right) t^{opt}}{\left[1+\left(t^{opt}-1\right)\lambda_{ii1}\right]^2}+\left(T^{opt}+\gamma_{i1}\rho_1 \frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\mathcal{E}_4\right\} \frac{\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right) \frac{1}{\rho_1(1-\lambda_{ii1})}}{\left[\frac{1}{\rho_1(1-\lambda_{ii1})}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\right]}.
\end{aligned}
$$

Using (132) it follows that $\frac{\partial D^{opt} T^{opt}}{\partial \lambda_{ii1}} > 0$ if

$$
\mathcal{E}_4 \frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}} > \frac{\left(t^{opt}-1\right)}{1+\left(t^{opt}-1\right)\lambda_{ii1}} \left(\frac{t^{opt}}{\left[1+\left(t^{opt}-1\right)\lambda_{ii1}\right]}-\mathcal{E}_4\left(1-\lambda_{ii1}\right)\right). \tag{135}
$$

Substituting $t^{opt} = \frac{\theta_1}{\theta_1-\rho_1}$ so that $t^{opt}-1 = \frac{\rho_1}{\theta_1-\rho_1}$, we simplify this expression to obtain

$$
\left(\mathcal{E}_4 - \frac{\rho_1}{\theta_1-\rho_1(1-\lambda_{ii1})}\right)\left(\frac{\rho_1(1-\lambda_{ii1})}{\theta_1-\rho_1(1-\lambda_{ii1})}+1\right) > \mathcal{E}_4\left(1-\frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right).
$$

Then we substitute (92) on the left and we use the bound from Lemma 5 on the right, which implies that $\mathcal{E}_4 \leq \frac{\theta_1-\rho_1(\theta_1-\rho_1)}{\theta_1}$, to obtain the sufficient condition

$$
\begin{aligned}
&\left(\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}\left(1-\lambda_{ii1}\right)}-\frac{\rho_1}{\theta_1-\rho_1(1-\lambda_{ii1})}\right)\left(\frac{\rho_1(1-\lambda_{ii1})}{\theta_1-\rho_1(1-\lambda_{ii1})}+1\right) \\
&\qquad > \frac{\theta_1-\rho_1\left(\theta_1-\rho_1\right)}{\theta_1}\left(1-\frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right).
\end{aligned}
$$

We set $\lambda_{ii1} = 0$ on the left to obtain a further sufficient condition

$$
\frac{\frac{1}{\theta_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\sigma_1\gamma_{i1}}}{\frac{1}{\rho_1}+\frac{1}{\theta_1}\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{1}{\sigma_1}}-\frac{\rho_1}{\theta_1-\rho_1} > \frac{\theta_1-\rho_1\left(\theta_1-\rho_1\right)}{\theta_1}\left(1-\frac{1+\gamma_{i1}\rho_1\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)
$$

After extensive simplification, this inequality is written as

$$
\begin{aligned}
&\left(1-\gamma_{i1}\right)^2 \rho_1\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+\gamma_{i1}\rho_1\right)\frac{\theta_1-\rho_1}{\theta_1}+\left(1-\gamma_{i1}\right)\left(\theta_1-2\rho_1\right)\left(1-\left(1-\gamma_{i1}\right).\rho_1\right) \\
&\quad > \sigma_1\gamma_{i1}\rho_1\left(1-\left(1-\gamma_{i1}\right)\rho_1\right)-\sigma_1\gamma_{i1}\left(1-\gamma_{i1}\right)\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+\gamma_{i1}\rho_1\right)\left(\theta_1-\rho_1\right)
\end{aligned}
$$

This inequality fails to hold at $\gamma_{i1} = 1$, so lower values of γ_{i1} are needed. The first set of terms on the left will involve a cubic in γ_{i1}, so to avoid that a sufficient condition is obtained by ignoring those (positive) terms, resulting in

$$\left(1-\gamma_{i1}\right)\left(\theta_1-2\rho_1\right)\left(1-\left(1-\gamma_{i1}\right)\rho_1\right)-\sigma_1\gamma_{i1}\rho_1\left(1-\left(1-\gamma_{i1}\right)\rho_1\right)$$
$$\geq-\sigma_1\gamma_{i1}\left(1-\gamma_{i1}\right)\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(1+\gamma_{i1}\rho_1\right)\left(\theta_1-\rho_1\right).$$

A further simplification is obtained by observing that $(1+\gamma_{i1}\rho_1)$ on the right is made smaller by replacing it with $(1-(1-\gamma_{i1})\rho_1)$, and dividing out that common term to obtain

$$\left(1-\gamma_{i1}\right)\left(\theta_1-2\rho_1\right)\geq\sigma_1\gamma_{i1}\left[\rho_1-\left(1-\gamma_{i1}\right)\left(\theta_1-\rho_1\left(\theta_1-\rho_1\right)\right)\left(\theta_1-\rho_1\right)\right].$$

A sufficient condition for this inequality to hold is provided by (20).

It follows that we can take $\lambda_{ii1}=0$ to obtain a lower-bound for $D^{opt}T^{opt}$, and also $\tilde{\lambda}_{ii1}=0$ from (50). So we set both these shares at zero in (132) and (133) to obtain $T^{opt}|_{\lambda_{ii1}=0}=t^{opt}-\tilde{\gamma}_{i1}$ and using this in (133), we obtain

$$\begin{aligned}&D^{opt}|_{\lambda_{ii1}=0}T^{opt}|_{\lambda_{ii1}=0}\\&=\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right)\left[\left(t^{opt}-\tilde{\gamma}_{i1}\right)\left(1-\mathcal{E}_4|_{\lambda_{ii1}=0}\right)+\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}\right].\end{aligned}$$

We use these expressions to obtain a bound on D^{opt} of

$$\begin{aligned}D^{opt}&=\frac{D^{opt}T^{opt}}{T^{opt}}\geq\frac{D^{opt}|_{\lambda_{ii1}=0}T^{opt}|_{\lambda_{ii1}=0}}{T^{opt}}\\&=\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right)\frac{\left(t^{opt}-\tilde{\gamma}_{i1}\right)\left(1-\mathcal{E}_4|_{\lambda_{ii1}=0}\right)+\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}}{1-\tilde{\gamma}_{i1}+\left(t^{opt}-1\right)\left(1-\tilde{\lambda}_{ii1}\right)}.\end{aligned}$$

We further set $\tilde{\lambda}_{ii1}$ in the denominator at zero to obtain another lower bound on D^{opt} of

$$\begin{aligned}D^{opt}&>\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right)\frac{\left(t^{opt}-\tilde{\gamma}_{i1}\right)\left(1-\mathcal{E}_4|_{\lambda_{ii1}=0}\right)-\frac{1+\gamma_{i2}(\sigma_2-1)}{\gamma_{i2}(\sigma_2-1)}\gamma_{i1}\rho_1-\gamma_{i1}\rho_1\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\mathcal{E}_4|_{\lambda_{ii1}=0}}{t^{opt}-\tilde{\gamma}_{i1}}\\&=\left(\frac{1-\tilde{\gamma}_{i1}}{\gamma_{i1}\rho_1}\right)\left[1-\mathcal{E}_4|_{\lambda_{ii1}=0}\left(1+\frac{\gamma_{i1}\rho_1}{\left(t^{opt}-\tilde{\gamma}_{i1}\right)}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)-\frac{1+\gamma_{i2}\left(\sigma_2-1\right)}{\gamma_{i2}\left(\sigma_2-1\right)}\frac{\gamma_{i1}\rho_1}{t^{opt}-\tilde{\gamma}_{i1}}\right].\end{aligned}\tag{136}$$

Evaluating the second term in this expression, we apply (18) which is equivalent to (123) to obtain

$$\mathcal{E}_4|_{\lambda_{ii1}=0}\leq\frac{\rho_1+\left(\theta_1-\rho_1\right)\left(1-\rho_1\right)}{\theta_1+\left(\theta_1-\rho_1\right)\left(1-\rho_1\right)}=\frac{t^{opt}-\rho_1}{t^{opt}+\frac{1}{\sigma_1}}.$$

In addition, the denominator of the term $1+\frac{\gamma_{i1}\rho_1}{(t^{opt}-\tilde{\gamma}_{i1})}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}$ is bounded below by using (18) again to obtain $\tilde{\gamma}_{i1}=(1-\gamma_{i1})\rho_1\leq\rho_1/t^{opt}$ and so $\left(t^{opt}-\tilde{\gamma}_{i1}\right)\left(1-\tilde{\gamma}_{i1}\right)\geq\left(t^{opt}-\frac{\rho_1}{t^{opt}}\right)\left(1-\frac{\rho_1}{t^{opt}}\right)$. It follows that

$$1-\mathcal{E}_4|_{\lambda_{ii1}=0}\left(1+\frac{\gamma_{i1}\rho_1}{\left(t^{opt}-\tilde{\gamma}_{i1}\right)}\frac{\tilde{\gamma}_{i1}}{1-\tilde{\gamma}_{i1}}\right)\geq\frac{1-\rho_1^2\gamma_{i1}(1-\gamma_{i1})\left(1-\frac{\rho_1}{(t^{opt})^2}\right)^{-1}}{t^{opt}+\frac{1}{\sigma_1}}\equiv\delta_i.$$

Substituting these results into (136) we have shown (130), with δ_i defined as in (131). QED

Lemma 11. *When conditions (18) and (20) hold, then $H^{opt}<0$ for all parameters satisfying (12) when κ_i is chosen as stated in part (c) of Theorem 1.*

Proof: Using (112), the needed condition is that

$$-\left(1-\alpha_i\right)\mathcal{M}^{opt}\left(\mathcal{E}_m^{opt}-\frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1\right)\frac{D^{opt}}{A^{opt}}<(1-\gamma_{i1})R^{opt},$$

where $\mathcal{M}^{opt}$, $\mathcal{E}_m^{opt}$, D^{opt}, A^{opt} and R^{opt} are all evaluated at t^{opt}. Using Lemma 10, we can rewrite this expression as

$$\frac{\gamma_{i1}(\sigma_1-1)}{1+\gamma_{i1}(\sigma_1-1)}<\left[\delta_i+\frac{(1-\gamma_{i1})R^{opt}A^{opt}}{\left(1-\alpha_i\right)\mathcal{M}^{opt}\left(\mathcal{E}_m^{opt}-\frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})\gamma_{i2}\left(\sigma_2-1\right)}{(1-\tilde{\gamma}_{i1})[1+\gamma_{i2}\left(\sigma_2-1\right)]}.$$

Therefore, we satisfy condition (12), $\frac{\gamma_{i1}(\sigma_1-1)}{1+\gamma_{i1}(\sigma_1-1)}<\kappa_i\frac{\gamma_{i2}(\sigma_2-1)}{1+\gamma_{i2}(\sigma_2-1)}$, by choosing κ_i as

$$\kappa_i=\left[\delta_i+\frac{(1-\gamma_{i1})R^{opt}A^{opt}}{\left(1-\alpha_i\right)\mathcal{M}^{opt}\left(\mathcal{E}_m^{opt}-\frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

Because many of the variables on the right-hand side of this equation depend on expenditure or production shares, we now develop a lower-bound for κ_i that is independent of these shares.

Using the method in the proof of Lemma 4, we first obtain

$$\mathcal{E}_m^{opt}-\frac{\left(t^{opt}-1\right)}{t^{opt}}\theta_1=\frac{\rho_1}{\tilde{\lambda}_{ii1}}\left(\frac{1-\frac{\rho_1}{\theta_1}\tilde{\lambda}_{ii1}+\frac{1-\gamma_{i1}}{\gamma_{i1}}\frac{\rho_1}{\theta_1}\left(1-\tilde{\lambda}_{ii1}\right)}{1+\rho_1\frac{(1-\gamma_{i1})(1-\lambda_{ii1})}{\gamma_{i1}\sigma_1\theta_1}}\right).$$

We substitute this along with the lower-bound for $A(t^{opt})$ from Lemma 5, which we rewrite as $A(t^{opt})>A_i\equiv\alpha_i(1-\rho_1)+\gamma_{i1}\rho_1$, together with the expressions for $\mathcal{M}^{opt}$, D^{opt} and R^{opt}, to obtain

$$\kappa_i>\left[\delta_i+\frac{(1-\gamma_{i1})\theta_1\left(\theta_1-\rho_1\right)^2\rho_1 A_i\left(\begin{array}{c}\left(\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)\frac{1}{\theta_1-\rho_1}\\+\frac{-\theta_1\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(1-\gamma_{i1})\rho_1}{\left(\theta_1-\rho_1+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)-(\theta_1-\rho_1)(1-\gamma_{i1})\rho_1\right)}\end{array}\right)}{\left(1-\alpha_i\right)\left(1-\tilde{\gamma}_{i1}\right)\left(\theta_1-\rho_1+\frac{\rho_1}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}.$$

Now we use $\frac{\theta_1-\rho_1+\rho_1\lambda_{ii1}}{\theta_1-\rho_1}>1$ and $\left(\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)\frac{1}{\theta_1-\rho_1}<\frac{\theta_1}{\theta_1-\rho_1}$ to obtain the further lower-bound

$$\kappa_i>\left[\delta_i+\frac{(1-\gamma_{i1})\theta_1\left(\theta_1-\rho_1\right)^2\rho_1 A_i\left(\frac{\theta_1}{\theta_1-\rho_1}+\frac{-\theta_1\rho_1+(\theta_1-\rho_1(1-\lambda_{ii1}))(1-\gamma_{i1})\rho_1}{\left(\theta_1-\rho_1+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)-(\theta_1-\rho_1)(1-\gamma_{i1})\rho_1\right)}\right)}{\left(1-\alpha_i\right)\left(1-\tilde{\gamma}_{i1}\right)\left(\theta_1-\rho_1+\frac{\rho_1}{\gamma_{i1}}\left(1-\tilde{\lambda}_{ii1}\right)\right)}\right]\frac{(t^{opt}-\tilde{\gamma}_{i1})}{(1-\tilde{\gamma}_{i1})}. \tag{137}$$

Notice that

$$\frac{\theta_1}{\theta_1-\rho_1}+\frac{-\theta_1\rho_1+\left(\theta_1-\rho_1\left(1-\lambda_{ii1}\right)\right)\left(1-\gamma_{i1}\right)\rho_1}{\left(\theta_1-\rho_1+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)-\left(\theta_1-\rho_1\right)\left(1-\gamma_{i1}\right)\rho_1\right)}=\frac{\theta_1\left(1-\rho_1\right)-\rho_1\tilde{\gamma}_{i1}\left(1-\lambda_{ii1}\right)+\frac{\theta_1}{(\theta_1-\rho_1)}\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}{\left(\theta_1-\rho_1\right)\left(1-\tilde{\gamma}_{i1}\right)+\rho_1\left(1-\tilde{\lambda}_{ii1}\right)}$$

and then since $\lambda_{ii1} < \tilde{\lambda}_{ii1}$ at t^{opt} we have

$$\begin{aligned}
\frac{\theta_1 (1-\rho_1) - \rho_1 \tilde{\gamma}_{i1} (1-\lambda_{ii1}) + \frac{\theta_1}{(\theta_1-\rho_1)} \rho_1 \left(1-\tilde{\lambda}_{ii1}\right)}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1}) + \rho_1 \left(1-\tilde{\lambda}_{ii1}\right)} &> \frac{\theta_1 (1-\rho_1) + \left(\frac{\theta_1}{\theta_1-\rho_1} - \tilde{\gamma}_{i1}\right) \rho_1 \left(1-\tilde{\lambda}_{ii1}\right)}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1}) + \rho_1 \left(1-\tilde{\lambda}_{ii1}\right)} \\
&> \frac{\theta_1 (1-\rho_1) + \left(\frac{\theta_1}{\theta_1-\rho_1} - \tilde{\gamma}_{i1}\right) \rho_1}{(\theta_1-\rho_1)(1-\tilde{\gamma}_{i1})},
\end{aligned}$$

where the second line is obtained by using $\tilde{\lambda}_{ii1} = 0$ in numerator and $\tilde{\lambda}_{ii1} = 1$ in the denominator. Substituting these results into (137) and again using $\tilde{\lambda}_{ii1} = 1$ in the denominator, we obtain

$$\kappa_i > \left[\delta_i + \frac{\tilde{\gamma}_{i1} \theta_1 A_i \left(\theta_1 (1-\rho_1) + \left(t^{opt} - \tilde{\gamma}_{i1}\right) \rho_1\right)}{(1-\alpha_i)(1-\tilde{\gamma}_{i1})^2}\right] \frac{\left(t^{opt} - \tilde{\gamma}_{i1}\right)}{(1-\tilde{\gamma}_{i1})}.$$

In the statement of Theorem 1(c), we use the lower-bound on the right with $A_i \equiv \alpha_i(1-\rho_1) + \gamma_{i1}\rho_1$ to specify κ_i, which gives a smaller (and therefore sufficient) range of effective markups in (12) to ensure that $H(t_i^{opt}) < 0$. QED

To complete the proof of part (c) we need to establish the tariff $t_i^* \in (t_i^{R0}, t_i^{opt})$ with $H(t_i^*) = 0$. Using $R(t_i^{R0}) = 0$, it follows from (112) that $H(t_i^{R0}) = \left(t^{opt} - t_i^{R0}\right) - t_i^{R0}(1-\alpha_i)M(t_i^{R0}) > 0$, because $M(t_i^{R0}) < 0$ from Lemma 9 since $D\left(t_i^{R0}\right) < 0$. From Lemma 11 we have $H(t_i^{opt}) < 0$. It follows from the continuity of $H(t_i)$ that there exists a tariff $t_i^* < t^{opt}$ at which $H(t_i^*) = 0$. QED