DISCUSSION PAPER SERIES

DP15692

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MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper DP15692
Published 21 January 2021
Submitted 19 January 2021

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JEL Classification: C32, E32

Keywords: Uncertainty shocks, VAR models, OLS estimation, stochastic volatility

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Acknowledgements
Forni, Gambetti and Sala gratefully acknowledge the financial support from the Italian Ministry of Research and University, PRIN 2017 grant J44I20000180001. Luca Gambetti acknowledges the financial support from the Spanish Ministry of Science and Innovation, through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S), the financial support of the Spanish Ministry of Science, Innovation and Universities through grant PGC2018-094364-B-I00, and the Barcelona Graduate School Research Network.

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Abstract

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1 Introduction

Uncertainty shocks have been in recent years at the heart of the business cycle debate. Since Bloom (2009), a vast literature studying the link between uncertainty and economic fluctuations has grown up.[1] From a theoretical point of view, uncertainty might induce agents to postpone private expenditures and investment, thus producing a potentially important temporary downturn in economic activity.

A huge effort has been devoted to construct measures of uncertainty. Several papers propose proxies of uncertainty which are not model-based but exploit different sources of information, such as stock market volatility (Bloom, 2009, Bekaert et al., 2013, Caldara et al., 2016), forecast disagreement in survey data (Bachmann et al., 2013), the frequency of selected keywords in journal articles (Baker et al., 2016) or the unconditional distribution of forecast errors (Jo and Sekkel, 2019). On the contrary, other papers (e.g. Jurado et al. 2015, JLN henceforth, Ludvigson et al. 2019, LMN henceforth) start from a rigorous statistical definition of uncertainty as the conditional volatility of a forecast error and specify and estimate a stochastic volatility model by using sophisticated time series techniques.

A common feature of these studies is that, once the external measure is available, the effects of uncertainty on the economy are estimated by including such measure into a SVAR model and then identifying the shock by means of a set of restrictions. Results so far are mixed. Stock market volatility measures (VIX and VXO) and the index developed by Rossi and Sekhposyan (RS henceforth) have small and barely significant effects on output, whereas other measures, such as the Economic Policy Uncertainty index (EPU henceforth) of Baker et al. (2016), have large and significant effects. A few papers, among others, Jurado et al. (2015), find big and persistent effects. Almost all papers use VAR models to estimate the effects of uncertainty, whereas no one uses them to estimate uncertainty itself. This opens the door to a potential problem of inconsistency between the estimates of uncertainty and its effects.

Our research is motivated by two questions. First, is it possible to estimate uncertainty

[1] A few prominent contributions are Fernandez-Villaverde et al. (2011), Bachmann et al. (2013), Bekaert et al. (2013), Caggiano et al. (2014), Rossi and Sekhposyan (2015), Jurado et al. (2015), Scotti (2016), Baker et al. (2016), Caldara et al. (2016), Leduc and Liu (2016), Basu and Bundik (2017), Fajgelbaum et al. (2017), Piffer and Podstawsky (2017), Nakamura et al. (2017), Bloom et al. (2018), Carriero et al. (2018a, 2018b), Shin and Zhong (2018), Jo and Sekkel (2019), Ludvigson et al. (2019), Angelini and Fanelli (2019).

and its effects within a single model without relying on any external measures or proxies? Second, can standard VAR models deliver trustworthy uncertainty estimates? The answers are yes and yes. In this paper we propose a simple approach to estimate uncertainty and its effect within a single framework based on standard VAR models.

Throughout the paper, we focus on the definition of uncertainty used in JLN: uncertainty is the forecast error variance, conditional to agents' information, or, equivalently, the conditional expectation of the square of the forecast error. Our procedure is extremely simple and unfolds in four steps: (i) estimate a VAR; (ii) compute the implied squared forecast error for the combination variable-forecast horizon of interest; (iii) regress the squared forecast error onto the current and past values of the VAR variables; (iv) use the coefficients of this regression to compute the impulse response functions of the uncertainty shock and the related variance decomposition. The fitted values of the regression in step (iii) provide an estimate of uncertainty. The innovation of this uncertainty estimate is the uncertainty shock. This procedure ensures consistency between the estimate of uncertainty and the estimated effects of uncertainty shocks.

Under suitable conditions, steps (iii-iv) are equivalent to using the squared forecast error as the instrument within a proxy SVAR. Hence, our method can be thought of as a proxy SVAR, where the proxy, instead of being an external variable, is a function of the estimated forecast error. The relevance condition of the instrument is clearly satisfied: the squared forecast error *is* correlated with the uncertainty shock by the very definition of uncertainty. However, in order for the exogeneity condition to hold, we need the additional assumption that uncertainty (or, more precisely, the squared prediction error) is not affected on impact by other structural shocks. This assumption is questionable.[2] To relax it, we impose orthogonality constraints with respect to other structural shocks within the standard proxy SVAR procedure. This represents a methodological innovation in the literature on Proxy SVAR where the effects are typically estimated without relying on any other additional restriction.

Our method has a few noticeable advantages. First, we have a clear and rigorous definition of uncertainty for each variable and horizon in the VAR. Second, we avoid the problematic choice of an external uncertainty measure. Third, we avoid the inconsistency implied by the use of two different models to estimate uncertainty and assess its business-cycle effects, since we use the same model for both purposes. Fourth, we avoid

[2] Notice however that most papers in the uncertainty literature make precisely the same assumption, by adopting a Cholesky identification scheme with the external uncertainty measure ordered first.

the restrictive assumptions on the form of the conditional distribution of the shocks, typical of fully specified time-varying volatility models. Finally, estimation is quite simple, in that we use just ordinary least squares.

We apply our procedure to a US macroeconomic data set and find that (a) our estimates of uncertainty are reliable, in that (a.1) the squared prediction errors are significantly predicted by a linear combination of the VAR variables, with sizable explained variances; (a.2) uncertainty estimates obtained with our linear approximation are strongly correlated with comparable estimates in the literature (notably, JLN and LMN measures); (a.3) price uncertainty and interest-rate uncertainty are related to recognizable economic events. As for the impulse response functions and variance decomposition, we find that (b) a substantial fraction of uncertainty is exogenous, that is, generated by exogenous shocks to uncertainty; (c) exogenous uncertainty shocks explain a large fraction of business-cycle fluctuations; (d) results are robust with respect to the choice of the uncertainty horizon and variable, the number of lags and the choice of the variables included in the VAR.

The remainder of the paper is organized as follows. Section 2 discusses the econometric approach. Section 3 presents the results. Section 4 concludes.

2 Econometric approach

This section discusses the econometric approach to estimate uncertainty and identify the effects of the uncertainty shock in a simple VAR model.

2.1 The VAR model

Our starting point is the assumption that the macroeconomic variables in the n-dimensional vector y_t follow the VAR model

$$A(L)y_t = \mu + \varepsilon_t, \tag{1}$$

where ε_t is orthogonal to y_{t-k}, $k > 0$, and $A(L) = I - \sum_{k=1}^{p} A_k L^k$ is a matrix of degree-p polynomials in the lag operator L. By inverting the VAR, we get the VMA representation

$$y_t = \delta + B(L)\varepsilon_t, \tag{2}$$

where $B(L) = \sum_{k=0}^{\infty} B_k L^k = A(L)^{-1}$, with $B_0 = I_n$, is the matrix of reduced form impulse response functions and $\delta = B(1)\mu$. The implied h-step ahead prediction error is

$$e_{t+h} = \sum_{k=0}^{h-1} B_k \varepsilon_{t+h-k}. \tag{3}$$

2.2 VAR-based uncertainty

Following JLN, we define uncertainty as the conditional volatility of the prediction error. For variable i and horizon h we have:

$$U^i_{ht} = E_t e^2_{i,t+h}. \tag{4}$$

The expected value cannot be computed without introducing additional assumptions about the conditional distribution of the VAR residuals (for example, a stochastic volatility model). However, we can approximate it by linear projections. Precisely, we approximate the log of uncertainty by taking the orthogonal projection of the log of the squared prediction error onto the linear space spanned by the constant and the present and past values of the y's:[3]

$$\begin{aligned} \log(U^i_{ht}) &\approx Proj\left(\log(e^2_{i,t+h})|y_{i,t-k}, i=1,\dots,n; k=0,\dots,q\right) \\ &= \theta + c(L)'y_t \\ &= \theta + c_0'y_t + \cdots + c_q'y_{t-q}. \end{aligned} \tag{5}$$

where c_j is an n-dimensional column vector of coefficients. Notice that, if the VAR residuals were serially independent (and therefore independent of lagged y's), then $\log(e^2_{i,t+h})$ would be orthogonal to the predictors, implying $c(L) = 0$. Hence our procedure requires that the VAR residuals, while being serially uncorrelated, are not serially independent.

Using the estimated (in-sample) forecast errors, the parameters of the projection above can be estimated from the regression

$$\log(e^2_{i,t+h}) = \theta + c(L)'y_t + \nu_t = \theta + c_0'y_t + \cdots + c_q'y_{t-q} + \nu_t, \tag{6}$$

where the error ν_t is orthogonal to y_t and its past history. In the empirical section we document that the estimated coefficients are significantly different from zero thus rejecting serial independence. Uncertainty can then be estimated as the exponential of the fitted values $\hat\theta + \hat c(L)'y_t$.

2.3 Identifying uncertainty shocks

Here we discuss how to identify the uncertainty shock and estimate its effects. First of all, notice that log uncertainty is a linear combination of the VAR variables, see equation

[3] We approximate the log uncertainty rather than uncertainty itself to avoid negative estimates of uncertainty. However by approximating directly uncertainty very similar results are obtained.

(5), and therefore a combination of VAR residuals. Precisely

$$\begin{aligned}\log(U^i_{ht}) &\approx \theta + c(L)'y_t \\ &= \theta + c(L)'B(L)\varepsilon_t \\ &= \theta + g(L)'\varepsilon_t. \end{aligned} \tag{7}$$

where $g(L) = \sum_{j=0}^{\infty} g_j L^j$.

To begin, we consider the simple case in which the uncertainty shock is simply the innovation of log uncertainty, normalized to have unit variance. Although quite common, this is a strong assumption and will be relaxed later on. From equation (7) the innovation is

$$g_0'\varepsilon_t = c_0'B_0\varepsilon_t = c_0'\varepsilon_t$$

(recall that $B_0 = I_n$). Then the normalized innovation $u*_t$ is

$$u_t^* = \frac{c_0'}{\sqrt{c_0'\Sigma_\varepsilon c_0}}\varepsilon_t = v'\varepsilon_t, \tag{8}$$

where Σ_ε is the variance-covariance matrix of ε_t. The corresponding vector of impulse response functions for the variables included in the VAR is

$$d^*(L) = B(L)\Sigma_\varepsilon v, \tag{9}$$

with contemporaneous effects equal to $\Sigma_\varepsilon v$, being $B(0) = I_n$ (see Appendix A for details).

2.4 Adding orthogonality constraints

The normalized innovation u_t^* is the uncertainty shock under the assumption that no other shock has non-zero contemporaneous effects on uncertainty. It is the same assumption made when identifying the uncertainty shock as the first Cholesky shock in a VAR with the external measure of uncertainty ordered first. While common in the literature, this assumption is questionable, see for instance Bachmann et al. (2013).

We can relax this assumption by imposing orthogonality restrictions with respect to other identified shocks. This can be done by projecting the uncertainty innovation onto these shocks and taking the residual. More formally, the non-normalized uncertainty shock orthogonal to the structural shock $D_1\varepsilon_t$, where D_1 is an identifying vector, is $u_t = [c_0' - c_0'\Sigma_\varepsilon D_1' D_1]\varepsilon_t$. As an example, we could impose orthogonality with respect to a long-run shock, identified as the only one shock affecting GDP in the long run. Under this identification scheme, the uncertainty shock has transitory effects on output.

Similarly, one can restrict to zero the impact coefficient of the uncertainty shock on a given variable by imposing orthogonality with respect to the VAR residual of that variable. For instance, to impose a zero impact effect on GDP, GDP being ordered first in y_t, it suffices to impose orthogonality with respect to $\varepsilon_{1t} = D_2\varepsilon_t$, where $D_2 = [1\ 0\ \cdots\ 0]$.

More generally, let D be the $m \times n$ matrix having on the rows the vectors D_1, D_2,. . ., D_m, with $m < n$. If we want to impose orthogonality with respect to the corresponding m shocks $D_1\varepsilon_t, D_2\varepsilon_t \ldots, D_m\varepsilon_t$, we have to take the residual of the orthogonal projection of the uncertainty innovation u_t^* onto $D\varepsilon_t$, normalized to have unit variance. The corresponding uncertainty shock, call it u_t, can be computed from the VAR coefficients by applying the formulas

$$\begin{aligned} u_t &= \gamma\varepsilon_t \\ \gamma &= \frac{\beta}{\sqrt{\beta'\Sigma_\varepsilon\beta}} \\ \beta &= c_0' - c_0'\Sigma_\varepsilon D'(D\Sigma_\varepsilon D')^{-1}D. \end{aligned} \tag{10}$$

The impulse-response functions for the variables included in the VAR corresponding to the shock $u_t = \gamma\varepsilon_t$ are

$$d(L) = B(L)\Sigma_\varepsilon\gamma. \tag{11}$$

Notice that the impulse response functions derived in equations (9) and (11) do not include the effects of u_t^* (or u_t) on uncertainty itself. It can be seen from equation (6) that such responses are

$$d_u^*(L) = c(L)d^*(L)' \tag{12}$$

$$d_u(L) = c(L)d(L)'. \tag{13}$$

for u_t^* and u_t, respectively. The last equation identifies the effect of the uncertainty shock on uncertainty as $d_u(L)u_t = d_u(L)\gamma\varepsilon_t$, let us call it the *exogenous component*. The part of uncertainty not driven by the uncertainty shock, i.e. the *endogenous component*, is therefore $c(L)y_t - d_u(L)u_t = [c(L)B(L) - d_u(L)\gamma]\varepsilon_t$. Since the two components are orthogonal, we can compute a variance decomposition both for the total variance and for the prediction errors at all horizons.

2.5 Equivalence with proxy SVAR

Our procedure is equivalent in population to estimating a proxy SVAR using $z_t = \log(e_{i,t+h}^2)$ as the external instrument for the uncertainty shock.[4] When the number of lags in equa-

[4] On the proxy SVAR approach see Mertens and Ravn (2013) and Stock and Watson (2018).

tion (5) is the same as the number of lags in the VAR, the results of the two procedures are identical even in sample.

For the instrument to be valid, the standard assumptions of relevance and exogeneity have to hold. The intuition of why the squared forecast error is a good candidate is the following. Consider the orthogonal decomposition

$$e_{i,t+h}^2 = E_t e_{i,t+h}^2 + v_{it} = U_{ht}^i + v_{it}.$$

Since v_{it} is independent of uncertainty, $e_{i,t+h}^2$ must be correlated with the uncertainty shock and so will be the log, which is the instrument we use. Hence relevance is ensured by the very definition of uncertainty. If the other shocks have zero impact effect on uncertainty, as assumed in Section 2.3, then the exogeneity assumption is also fulfilled, so that $\log(e_{i,t+h}^2)$ is a valid proxy to identify the uncertainty shock.

Let us come now to the equivalence. The proxy SVAR approach consists in projecting the VAR residuals ε_t onto the proxy z_t. The population parameters are $\phi = E z_t \varepsilon_t / E z_t^2$ (see Mertens and Ravn, 2013). The impact effects ϕ are therefore proportional to $E z_t \varepsilon_t$. It is easily seen that our population impact effects are also proportional to $E z_t \varepsilon_t$, so that they are equal to those of the proxy SVAR when the same normalization is imposed. If the proxy z_t is $\log(e_{i,t+h}^2)$, from equations (6) and (2) we get

$$z_t = \omega + c(L)' B(L) \varepsilon_t + \nu_t, \tag{14}$$

where $\omega = \theta + c_0' \delta$ and ν_t is orthogonal to y_{t-k}, $k \geq 0$ and therefore to ε_{t-k}, $k \geq 0$. Post-multiplying by ε_t' and taking expected values we get $E z_t \varepsilon_t' = c_0' \Sigma_\varepsilon$, since $B(0) = I$. But we have already seen that our impact effects are $\Sigma_\varepsilon v = \Sigma_\varepsilon c_0 / \alpha$ with $\alpha = \sqrt{c_0' \Sigma_\varepsilon c_0}$ (see equations (8) and (9)). Hence our impact effects are $E z_t \varepsilon_t / \alpha$.

In Appedix B we also show that the OLS estimates are equal to those of Mertens and Ravn (2013) if $q = p$, i.e. when the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR. Hence, as far as the estimation of the effects of uncertainty are concerned, our approach and the standard proxy SVAR approach produce the same results.

Te advantage of our method is that it allows us to get an estimate of uncertainty itself, besides the uncertainty shock and its impulse-response functions. On the other hand, the above discussion clarifies that, for the identification of the uncertainty shock, the linear approximation of uncertainty in equation (5) is not needed: we just need the standard assumptions of relevance and exogeneity.

2.6 Summary of the procedure

Summing up, our procedure is the following.

1. Estimate by OLS the VAR in equation (1) to get $\hat{B}(L) = \hat{A}(L)^{-1}$, the vector of residuals $\hat{\varepsilon}_t$ and its sample variance-covariance matrix $\hat{\Sigma}_\varepsilon$. Compute $\hat{e}_{t+h}$ according to equation (3).

2. Compute $\hat{z}_t = \log(\hat{e}^2_{i,t+h})$. Estimate by OLS equation (6) to get $\hat{\theta}$ and $\hat{c}(L)$ and compute $\hat{U}^i_{ht}$ according to equation (6) as $\hat{U}^i_{ht} = \exp(\hat{\theta} + \hat{c}(L)'y_t)$.

3. Compute $\hat{u}^*_t$ and $\hat{d}^*(L)$ according to equations (8) and (9) by replacing c_0 and Σ_ε with the corresponding estimates. Alternatively:

$3'$. Specify the relevant orthogonality restrictions by choosing the matrix D. Compute the estimates $\hat{u}_t$ and $\hat{d}(L)$ according to equations (10) and (11) by replacing c_0 and Σ_ε with the corresponding estimates.

4. Get the estimate of the IRFs of uncertainty, either $d^*_u(L)$ or $d_u(L)$, according to (12).

In Appendix C we describe in detail our bootstrap procedure to construct confidence bands.

If the goal is to exclusively estimate the effects of uncertainty shocks, an alternative and equivalent procedure is the following.

a. Estimate by OLS the VAR in equation (1) to get $\hat{B}(L) = \hat{A}(L)^{-1}$, the vector of residuals $\hat{\varepsilon}_t$ and its sample variance-covariance matrix $\hat{\Sigma}_\varepsilon$. Compute $\hat{e}_{t+h}$ according to equation (3).

b. Compute $\hat{z}_t = \log(\hat{e}^2_{i,t+h})$ and use it as the external instrument in a proxy SVAR to obtain the effects of the uncertainty shock.

3 Empirics

In this section, we present the main results of our empirical analysis.

3.1 Specification

We collect data for the US economy. The data span is 1960:Q1-2019:Q3. Our benchmark VAR includes seven variables: the log of real per-capita GDP, the unemployment rate, CPI inflation, the federal funds rate, the log of the S&P500 stock price index, a component of the Michigan Consumer Confidence Index, i.e. expected business conditions for the next

12 months (E1Y), and the spread between BAA corporate bond yield and GS10 (BAA-GS10).[5] The last four variables are included essentially because they are supposed to quickly react to shocks and therefore are hopefully able to better capture the information necessary to reveal uncertainty. In the robustness section, we replace stock prices and the spread BAA-GS10 with a different set of forward-looking variables. Note that we do not include uncertainty measures in the model, since we want to verify whether the VAR is able to produce reliable estimates of uncertainty without specific external information.

We include just one lag in the VAR, as suggested by the BIC criterion. In the robustness section we show results for 2 and 4 lags.

We estimate equation (6) for all variables and forecast horizons equal to 1, 4 and 8. In all cases, following the BIC criterion, we include y_t without further lags on the right-hand side (i.e. $q = 0$ and $c(L) = c_0$). In the robustness section we include also y_{t-1}, so that $p = q$ and our method produce exactly the same result as the proxy-SVAR method discussed above.

We perform a number of robustness checks, which will be discussed below.

3.2 Estimated uncertainty

Our procedure requires that the log of future squared forecast errors are predictable by means of current (and possibly lagged) y's. It is therefore important to document the overall significance of the regressors in equation (6).

Table 1 shows the R^2 statistic along with the F-test for the overall significance of the regression, for all variables and horizons, when using just the contemporaneous VAR variables as regressors ($q = 0$). All regressions but the one for stock price uncertainty at horizon 8 are significant at the 5% level, and 16 regressions out of 21 are significant at the 1% level. The VAR variables predict the squared prediction errors implied by the VAR itself. This result, to our knowledge, was not found before and, as already observed, implies that the VAR residuals are not serially independent. This preliminary step lends support to the validity of our approximation procedure.[6]

Table 2 shows the correlation coefficients of three of our uncertainty indexes, computed according to equation (5), namely the GDP uncertainty index, 4 quarters ahead ($\hat{U}^{GDP}_{4,t}$), the unemployment rate uncertainty index, 4 quarters ahead ($\hat{U}^{UN}_{4,t}$) and the stock price

[5] GDP and stock prices are taken in log levels to take into account potential cointegration relations.

[6] The R^2 might appear small for several equations; notice however that $R^2 = 0.15$, corresponding to unemployment uncertainty at the one-year horizon (which is the uncertainty used in our baseline VAR below) roughly corresponds to the R^2 of a univariate AR(1) model with the sizable coefficient 0.4.

uncertainty index, 1-quarter ahead ($\hat{U}^{S\&P}_{1,t}$), with (a) the VXO index, extended as in Bloom (2009), (b) the LMN (2020) financial uncertainty index 3-months (LMN fin), (c) the JLN (2015) macroeconomic uncertainty index 12 months (JLN), (d) the LMN (2020) real uncertainty index 12-months (LMN real), (e) the Becker et al. (2016) US Economic Policy Uncertainty index (EPU) and (f) the Rossi and Sekhposyan (2015) 4-quarters ahead uncertainty index (RS).

Our indexes are highly positively correlated with each other and with JLN and LMN indexes, which are consistent with ours as for the definition of uncertainty. In particular, our GDP uncertainty 4-quarters and unemployment rate uncertainty 4-quarters exhibit correlation coefficients with JLN uncertainty 12-months as high as 0.71 and 0.79, respectively.

Figure 1 shows the graphs of the above uncertainty indexes, along with gray areas indicating US recessions according to the NBER dating. It is seen that in most cases the indexes anticipate recessions; they start increasing before the beginning of the recessions, and start reducing before the end of the recessions.

Figure 2 shows two additional uncertainty indexes: inflation uncertainty, 4-quarters, and federal funds rate uncertainty, 1-quarter. These uncertainties are considerably different from the previous ones, particularly because they do not exhibit a peak corresponding to the Great Recession. Inflation uncertainty is large during periods of high inflation, with peaks corresponding roughly with oil shocks. Federal funds rate uncertainty is high when the federal funds rate is high, i.e. during the so-called "stop and go" monetary policy period and during the Volcker era; it is very low at the end of the sample, when interest rates are close to zero.

3.3 Impulse response functions

To begin, we have to choose the relevant uncertainty. Two quite natural choices for macroeconomic uncertainty (see LMN, 2020) are GDP uncertainty and unemployment uncertainty. As a benchmark, we choose unemployment uncertainty, mainly because the R^2 reported in Table 1 are larger and more significant than those for GDP. In the robustness section we show results for GDP uncertainty. As for the horizon, we choose 4 quarters. In the robustness section, we show results for $h = 1$ and $h = 8$.

The literature does not provide a widespread consensus about a set of identification restrictions for the exogenous uncertainty shock. Here we present results for three identification schemes.

With Identification I, the uncertainty shock is simply the VAR innovation of uncertainty u_t^*, see Section (2.3). Therefore, the only shock affecting uncertainty on impact is the uncertainty shock. As already observed, this scheme is questionable. On the other hand, it is quite common in the literature, hence results may be useful for comparison.

With Identification II, we just impose that the uncertainty shock is orthogonal to a long-run shock, identified as the only shock having effects on the level of GDP after 40 quarters. Hence, we include just one row in the matrix D appearing in equation (10). This amounts to assuming that (i) the uncertainty shock has transitory effects on output, and (ii) only the long run shock and the uncertainty shock itself affect uncertainty on impact.

With Identification III, we impose that the uncertainty shock is orthogonal to the long-run shock above and, in addition, to the VAR innovations of GDP, unemployment, CPI and the federal funds rate (hence, we add four rows to the matrix D). In this way we impose that (i) the uncertainty shock has transitory effects on output; (ii) the slow-moving variables (output, unemployment and prices) do not react to uncertainty on impact, as is assumed for the monetary policy shock *à la* Christiano et al. (1999); in addition, (iii) the federal funds rate does not react to uncertainty on impact. The last constraint is imposed because, given (ii), (iii) entails that the uncertainty shock is orthogonal to a monetary policy shock which moves on impact the federal funds rate and therefore cannot be confused with it. On the other hand, the monetary policy shock, as well as the long-run shock and, possibly, other unidentified transitory shocks, may affect uncertainty on impact.

Figure 4 shows results for Identification I. As expected, the uncertainty shock reduces output and increases unemployment. The effects are very large, as in JLN (2015), but not that much persistent, since they vanish after about 4 years. This result is different from those in JLN and Carriero et al. (2018b). Inflation is not affected significantly. The federal funds rate reduces, reacting to the slowdown of real activity and prices. Stock prices reduce on impact. The confidence index goes down on impact, reflecting consumers' expectations. The BAA-GS10 spread increases, reflecting the increased risk premium of Baa Corporate bonds.

Table 3 shows variance decomposition. The uncertainty shock accounts for a very high fraction of GDP and especially the unemployment rate. The shock explains more than one half of unemployment fluctuations at the one-year horizon. The effect on the risk premium is also big: according to this identification, the uncertainty shock explains about three quarters of the spread variance at the one-year horizon.

Figure 5 shows results for Identification II. Results are very similar to those of Identification I. Again, inflation is not significantly affected.

The variance explained by the uncertainty shock (see Table 3) is slightly reduced but still very high. As for the stock market, the effects are smaller, consistently with Carriero et al. (2018b): the uncertainty shock explains about 20% of volatility at the one year horizon. Finally, the shock explains more than 90% of uncertainty itself on impact, leaving a very limited role for the long-term shock.

Figure 6 shows results for Identification III. Results are qualitatively similar to those of Identification I. The effects on output and unemployment are now smaller, but still significant for both GDP and unemployment.

Overall the variance explained by the uncertainty shock (see Table 3) is now much smaller. Still, at the one-year and the 4-year horizons, uncertainty shocks explains about 10% of output volatility and about 30% of unemployment volatility. Exogenous uncertainty considerably reduces at all horizons; however, it is still close to 80% on impact and about 50% at medium- and long-term horizons.

3.4 Robustness checks

For all robustness exercises we use Identification I as our benchmark. In the first exercise, we change the uncertainty horizon, by using $h = 1$ and $h = 8$ in place of $h = 4$. Results are reported in Figure 6. The black solid lines correspond to the benchmark $h = 4$, the blue dotted lines correspond to horizon $h = 1$ and the magenta dotted-dashed lines correspond to horizon $h = 8$. Results are very similar. We conclude that changing the horizon does not change the results.

In the second exercise, reported in Figure 7, we change the uncertainty variable and use (i) GDP uncertainty at the 4 quarter horizon (blue dotted lines) and (ii) stock prices uncertainty at the 1 quarter horizon (magenta dotted-dashed lines), in place of the benchmark unemployment uncertainty (black solid lines). For GDP uncertainty, results are very similar to the benchmark. As for stock market uncertainty, the effects are much smaller, suggesting that financial uncertainty does not affect systematically real activity.

In the third exercise, reported in Figure 8, we change the number of lags and use 2 lags (blue dotted lines) and 4 lags (magenta dotted-dashed lines) instead of 1 lag (benchmark case, black solid lines). Results are somewhat different from those obtained in the baseline model, particularly because the effects on GDP and stock prices are more persistent. However, both the sign and the size of the responses are similar to those of the baseline specification.

In the next exercise we change the VAR specification, by removing stock prices and the spread BAA-GS10, and including two different forward-looking variables: the ISM New Order Index and another component of the Michigan Consumer Confidence Index, the expected business conditions for the next five years (E5Y).[7] We remove the spread mainly to avoid a possible contamination of uncertainty shocks with credit market shocks (Gilchrist and Zakrajsek, 2012, Caldara et al., 2016). Results are reported in Figure 9. The effects of uncertainty shocks on the variables which are included in both specifications are similar.

In the last two exercises we retain the baseline specification for the VAR, but change the way we estimate uncertainty. First, we use the squares of the prediction error in place of their logs, i.e. we do not use equation (5), but simply replace the conditional expectation appearing in equation (4) with the linear projection. The effects of the implied uncertainty shock are very similar to those of the baseline model (Figure 10). Second, we specify $q = 1$ instead of $q = 0$ in equation (5), so that we have $q = p$ and the results are identical to those obtained with the proxy SVAR approach. The results are reported in Figure 11. The effects on GDP and stock prices are larger and more persistent than in the benchmark model, whereas those on unemployment are smaller. However, the main results are confirmed: a positive uncertainty shock has large negative effects on economic activity.

All in all the results appear to be robust to changes in several features of the model specification.

4 Conclusions

We have shown that it is possible to produce reliable uncertainty estimates with a standard VAR model, without modeling time-varying volatility and using only OLS. The basic idea is to compute the squares of the prediction errors implied by the VAR model and replace expected values with linear projections.

Our estimate of uncertainty is a linear combination of the VAR variables. Therefore, the uncertainty shock is a linear combination of the VAR residuals and its effects can be computed by applying simple formulas to the reduced form impulse response functions. In this way, the same VAR model is used to estimate both uncertainty and its effects on

[7] The latter variable is studied in depth in Barsky and Sims, 2012.

the macro economy.

We have also provided simple formulas that can be used to impose suitable orthogonality constraints on the uncertainty shock.

The advantage of our procedure is twofold: on the one hand, we avoid the problematic choice of an external uncertainty measure; on the other hand, we avoid imposing restrictive assumption about the structure of conditional volatility.

Our procedure can be regarded as a variant of a proxy SVAR with the log of the squared prediction error taken as the relevant proxy. Under suitable conditions, the two methods yield the same results.

The procedure described here can easily be adapted to a factor model or a factor-augmented VAR. Moreover, it can be applied to survey-based forecast errors associated with local projection impulse-response functions estimation.

We have applied our procedure to a US macroeconomic quarterly data set. Our main conclusion is that a substantial fraction of macroeconomic uncertainty is exogenous and uncertainty shocks explain a large part of business cycle fluctuations.

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Appendix A: A useful formula

If the unit-variance structural shock is $v'\varepsilon_t$, its impact effects are $d = \Sigma_\varepsilon v$. To see this, consider first the Cholesky representation with orthonormal shocks: $y_t = B(L)CC^{-1}\varepsilon_t$, where C is such that $CC' = \Sigma_\varepsilon$. Any other fundamental representation with orthogonal, unit-variance shocks will be given by

$$y_t = B(L)CUU'C^{-1}\varepsilon_t,$$

where U is a unitary matrix (i.e. $UU' = I$). Assuming, without loss of generality, that the structural shock of interest is the first one, the impact effects are $d = CU_1$, where U_1 is the first column of U, and the vector identifying the structural shock is $v' = U_1'C^{-1}$. Hence $U_1 = C'v$ and $d = CC'v = \Sigma_\varepsilon v$.

Appendix B: The relation with standard proxy SVAR

In the main text we have shown that in population our procedure is equivalent to the proxy-SVAR methodology.

Here we show that the OLS estimates are identical to those of Mertens and Ravn (2013) if the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR for y_t (see equation (14)).

Let us begin with OLS estimation of the VAR in equation (1), which we report here for convenience:

$$y_t = \mu - A_1 y_{t-1} - \cdots - A_p y_{t-p} + \varepsilon_t. \tag{15}$$

We need some additional notation. Let

$$Y_k = \begin{pmatrix} y'_{p+1-k} \\ y'_{p+2-k} \\ \vdots \\ y'_{T-k} \end{pmatrix}, \quad \imath = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad X = \begin{pmatrix} \imath & Y_1 & \cdots & Y_p \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} \varepsilon'_{p+1-k} \\ \varepsilon'_{p+2-k} \\ \vdots \\ \varepsilon'_{T-k} \end{pmatrix}.$$

Moreover, let $Y = Y_0$. Hence the VAR equation can be written as

$$Y = XA + \mathcal{E},$$

where $A = \begin{pmatrix} \mu & -A_1 & \cdots & -A_p \end{pmatrix}'$. The OLS estimates of A and $\mathcal{E}$ are

$$\hat{A} = (X'X)^{-1}X'Y, \qquad \hat{\mathcal{E}} = Y - X(X'X)^{-1}X'Y.$$

Of course we have $X'\hat{\mathcal{E}} = 0$.

Mertens and Ravn (2013) focuses on the effects of the structural shock. Such effects are estimated by performing the OLS regression of $\hat{\varepsilon}_t$ onto the proxy z_t, which for ease of exposition and without loss of generality we assume to be zero-mean. Precisely, let $z = \begin{pmatrix} z'_{p+1} & z'_{p+2} & \cdots & z'_T \end{pmatrix}'$, and consider the regression equation

$$\hat{\mathcal{E}} = z\phi' + V.$$

The vector of the impact effects is obtained as the OLS estimator of ϕ, suitably normalized (for instance to get unit variance for the corresponding structural shock). The OLS estimator of ϕ is

$$\hat{\phi} = \hat{\mathcal{E}}'z/z'z. \tag{16}$$

The vector of the impact effects is then obtained by normalizing the above vector in the desired way.

Our proposed procedure focuses on the estimation of the structural shock, rather than the estimation of the corresponding impulse-response functions. We compute the OLS regression of z onto the columns of Y and X:

$$z = Yc_0 + Xb + \nu,$$

where $b = (\theta' \ c_1' \ \cdots \ c_p')'$ (see equation 5). Letting $W = \begin{pmatrix} Y & X \end{pmatrix}$, the fitted value of z (which in our case is the estimate of uncertainty) is $W(W'W)^{-1}W'z$ and the residual is $\hat{\nu} = z - W(W'W)^{-1}W'z$. Clearly, $W'\hat{\nu} = 0$, so that $Y'\hat{\nu} = 0$ and $X'\hat{\nu} = 0$. Hence $\hat{\mathcal{E}}'\hat{\nu} = 0$. Pre-multiplying the above equation by $\hat{\mathcal{E}}'$ we get

$$\hat{c}_0 = (\hat{\mathcal{E}}'Y)^{-1}\hat{\mathcal{E}}'z = (\hat{\mathcal{E}}'\hat{\mathcal{E}})^{-1}\hat{\mathcal{E}}'z,$$

where the last equality is obtained by observing that $\hat{\mathcal{E}}'Y = \hat{\mathcal{E}}' \left(X(X'X)^{-1}X'Y + \hat{\mathcal{E}} \right) = \hat{\mathcal{E}}'\hat{\mathcal{E}}$. Hence $\hat{c}_0$ could be obtained equivalently by OLS regression of z_t onto ε_t. This makes sense: the estimated structural shock is nothing else than the OLS projection of the proxy z_t onto the VAR residuals. The reason why we do not follow this way is that it would not enable us to get an estimate of uncertainty.

We have shown above that the impact effects of $c_0'\varepsilon_t$ are proportional to $\Sigma_\varepsilon c_0$. Hence we estimate such impact effects as $\hat{\mathcal{E}}'\hat{\mathcal{E}}\hat{c}_0 = \hat{\mathcal{E}}'z$, up to a multiplicative constant which is fixed by the unit variance normalization. These effects are proportional to the ones in equation (16) and are equal once we impose the same normalization.

Appendix C: The bootstrap procedure

To construct confidence bands we draw randomly $T - p$ times (with replacement) from the uniform discrete distribution with possible values $p + 1, \dots, T$, to get the sequence $t(\tau)$, $\tau = p + 1, \dots, T$ and the corresponding sequences $\varepsilon_\tau = \hat{\varepsilon}_{t(\tau)}$, $r_\tau = \hat{r}_{t(\tau)}$, $\tau = p + 1, \dots, T$. Then we set $y_\tau = y_t$ for $\tau = 1, \dots, p$. Moreover, according to (15), we set $y_\tau = \hat{\mu} - \hat{A}_1 y_{\tau-1} - \cdots, -A_p y_{\tau-p} + \varepsilon_\tau$, and, according to (6), $z_\tau = \hat{\theta} + \hat{c}_0' y_\tau + \cdots + \hat{c}_p' y_{\tau-p} + r_\tau$, for $\tau = p + 1, \dots, T$. Having the artificial series y_τ, $\tau = 1, \dots, T$, and z_τ, $\tau = p + 1, \dots, T$, we re-estimate the relevant impulse-response functions. We repeat the procedure N times to get a distribution of IRFs and take the desired point-wise percentiles to form the confidence bands.

The above procedure takes into account the parameter estimate uncertainty of both the VAR and the proxy equation (6). On the other hand, we treat z_t as an observed variable, whereas in our case it is estimated. This cannot be avoided since we do not have a fully specified stochastic volatility model enabling us to reproduce the correct covariances between the squared prediction errors and the lagged variables.

Tables

	R^2			p-value (F-test)		
	$h=1$	$h=4$	$h=8$	$h=1$	$h=4$	$h=8$
Per Capita GDP	0.15	0.08	0.06	0.00	0.01	0.04
Unemployment rate	0.19	0.15	0.13	0.00	0.00	0.00
CPI inflation	0.09	0.08	0.07	0.00	0.01	0.02
Federal Funds Rate	0.43	0.25	0.27	0.00	0.00	0.00
S&P500	0.10	0.09	0.05	0.00	0.00	0.08
E1Y	0.08	0.08	0.06	0.01	0.01	0.04
spread BAA-GS10	0.21	0.09	0.07	0.00	0.00	0.02

Table 1: R^2 of regression (5) and p-values of the F-test of the significance of the regression.

	$\hat{U}^{GDP}_{4,t}$	$\hat{U}^{UN}_{4,t}$	$\hat{U}^{S\&P}_{1,t}$	VXO	LMN F12m	JLN 12m	LMN R12m	EPU	RS 4q
$\hat{U}^{GDP}_{4,t}$	1.00	-	-	-	-	-	-	-	-
$\hat{U}^{UN}_{4,t}$	0.76	1.00	-	-	-	-	-	-	-
$\hat{U}^{S\&P}_{1,t}$	0.45	0.69	1.00	-	-	-	-	-	-
VXO	0.29	0.56	0.43	1.00	-	-	-	-	-
LMN F12m	0.45	0.60	0.50	0.78	1.00	-	-	-	-
JLN 12m	0.71	0.79	0.48	0.47	0.52	1.00	-	-	-
LMN R12m	0.68	0.76	0.49	0.28	0.44	0.82	1.00	-	-
EPU	0.45	0.48	0.33	0.35	0.38	0.29	0.25	1.00	-
RS 4q	0.13	0.13	0.16	0.28	0.31	0.14	0.12	-0.14	1.00

Table 2: Correlation of our estimated uncertainty measures, GDP uncertainty 4-quarter ahead ($\hat{U}^{GDP}_{4,t}$), unemployment rate uncertainty 4-quarter ahead ($\hat{U}^{UN}_{4,t}$) and S&P uncertainty 1-quarter ahead ($\hat{U}^{S\&P}_{1,t}$) with existing measures: VXO, LMN financial 12-month ahead (LMN F12m), JLN 12-month ahead (JLN 12m), LMN real 12-month ahead (LMN R12m), economic policy uncertainty (EPU), and Rossi and Sekhposyan 4-quarter ahead (RS 4q).

Identification I				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	12.7	38.6	27.4	14.7
Unemployment rate	9.9	54.3	55.4	42.2
CPI inflation	1.1	1.3	6.4	6.4
Federal Funds Rate	1.2	8.9	22.6	19.2
S&P500	21.4	24.4	12.2	6.5
E1Y	62.2	50.6	38.8	36.4
spread BAA-GS10	61.3	75.9	68.2	67.6
Uncertainty	100.0	89.4	68.7	67.8

Identification II				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	10.8	35.0	23.8	13.3
Unemployment rate	11.4	56.0	54.9	42.2
CPI inflation	0.7	0.9	6.8	6.7
Federal Funds Rate	1.4	9.8	24.4	20.7
S&P500	18.5	21.0	10.0	5.7
E1Y	60.7	48.5	37.4	35.1
spread BAA-GS10	63.3	77.7	69.5	68.8
Uncertainty	99.6	88.2	68.2	67.5

Identification III				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	0.0	11.0	9.8	5.7
Unemployment rate	0.0	27.4	35.9	26.7
CPI inflation	0.0	0.4	1.9	2.4
Federal Funds Rate	0.0	3.2	9.4	8.5
S&P500	27.9	29.9	17.2	9.6
E1Y	47.7	38.6	29.4	27.6
spread BAA-GS10	54.0	65.2	58.1	55.9
Uncertainty	80.1	72.9	53.8	52.1

Table 3: Variance decomposition. Identification I: uncertainty innovation. Identification II: orthogonal to long run shock. Identification III: zero contemporaneous effects on GDO, unemployment rate, CPI and federal funds rate.

Figures

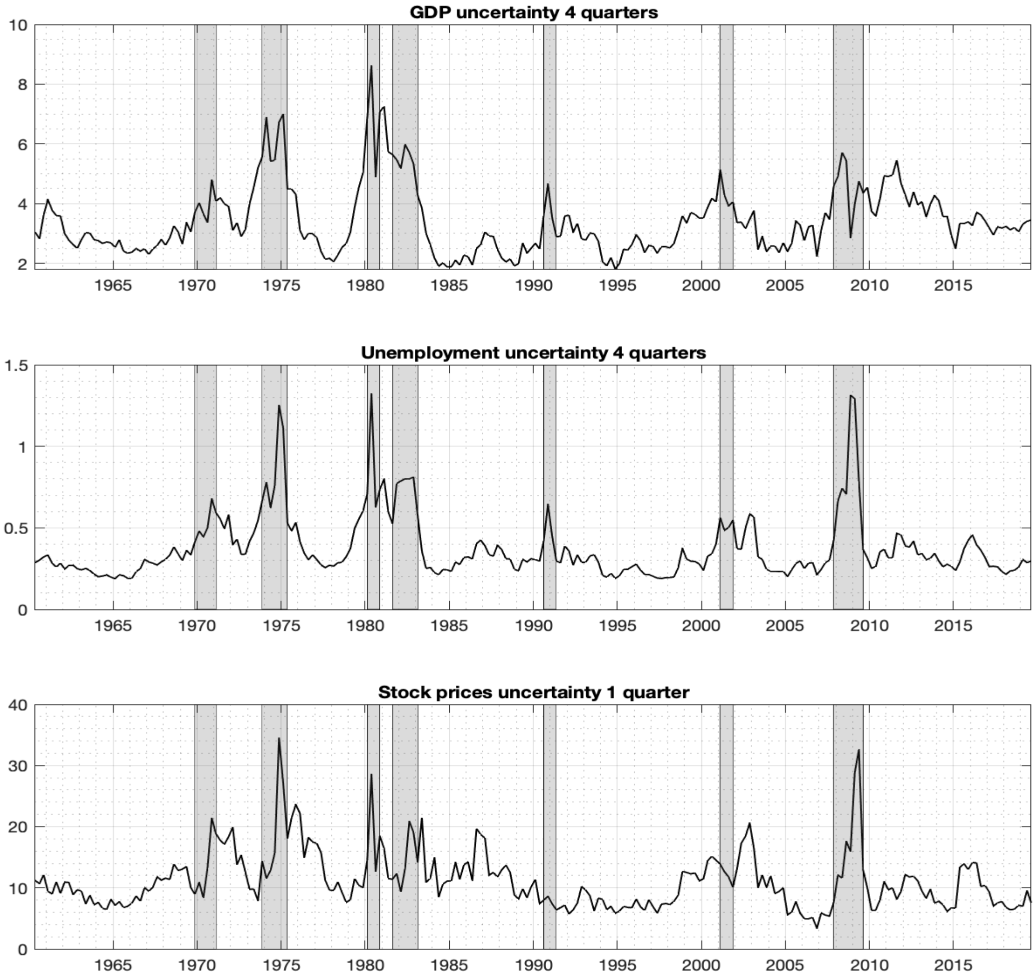

Figure 1: US estimated uncertainties. Top: GDP uncertainty. Middle: unemployment uncertainty. Bottom: stock prices uncertainty. Gray vertical bands are US recessions.

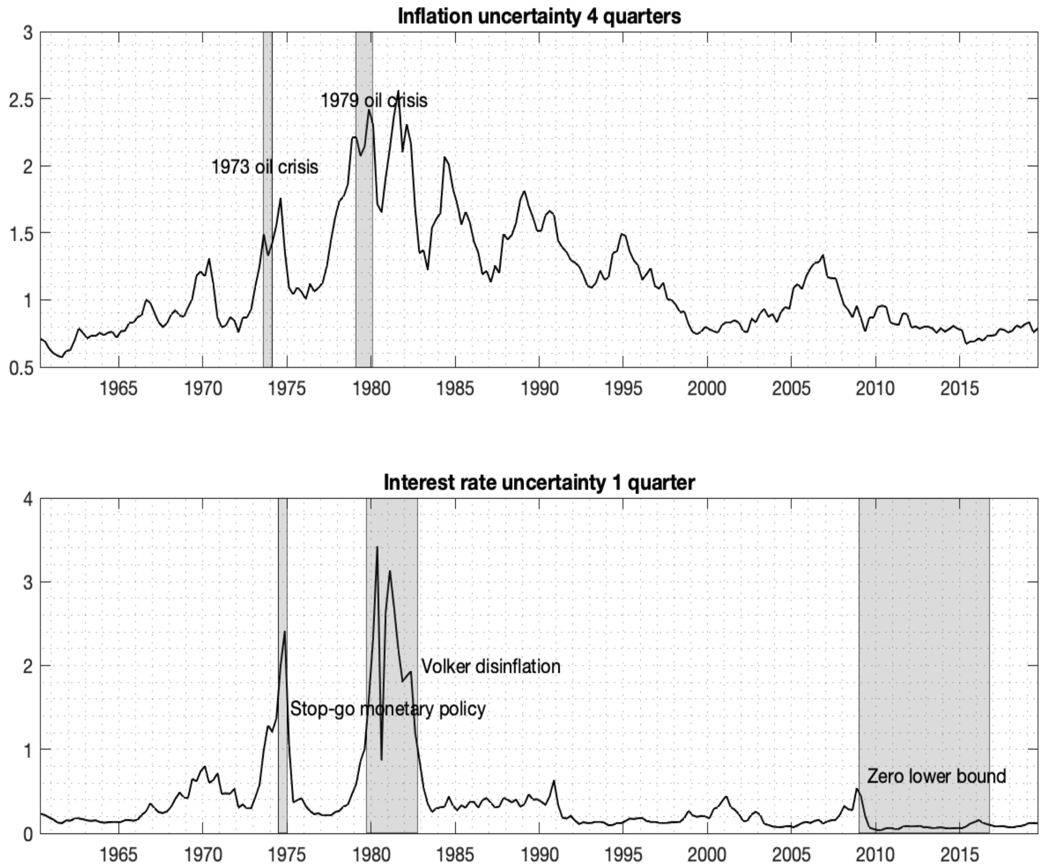

Figure 2: US estimated uncertainties. Top: inflation uncertainty. Bottom: interest rate uncertainty. Gray vertical bands are oil crisis periods (upper graph) and monetary policy periods (lower graph).

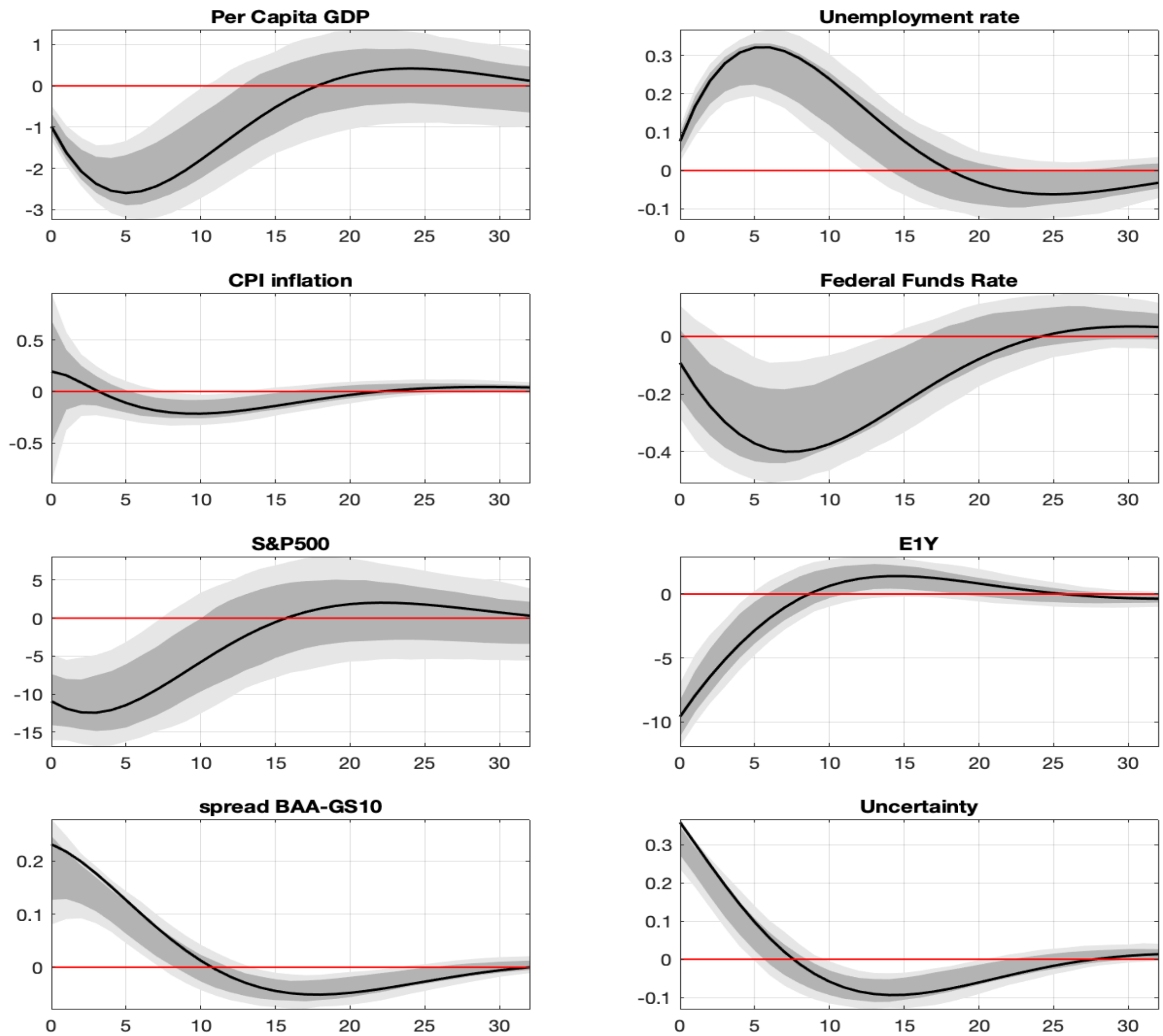

Figure 3: Impulse response functions of the unemployment rate uncertainty shock, 4 quarters. The shock is identified as the residual of the projection of the uncertainty innovation onto a long-run shock (Identification I). The latter shock is identified as the only one shock having effect on GDP at the 40 quarter horizon. Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

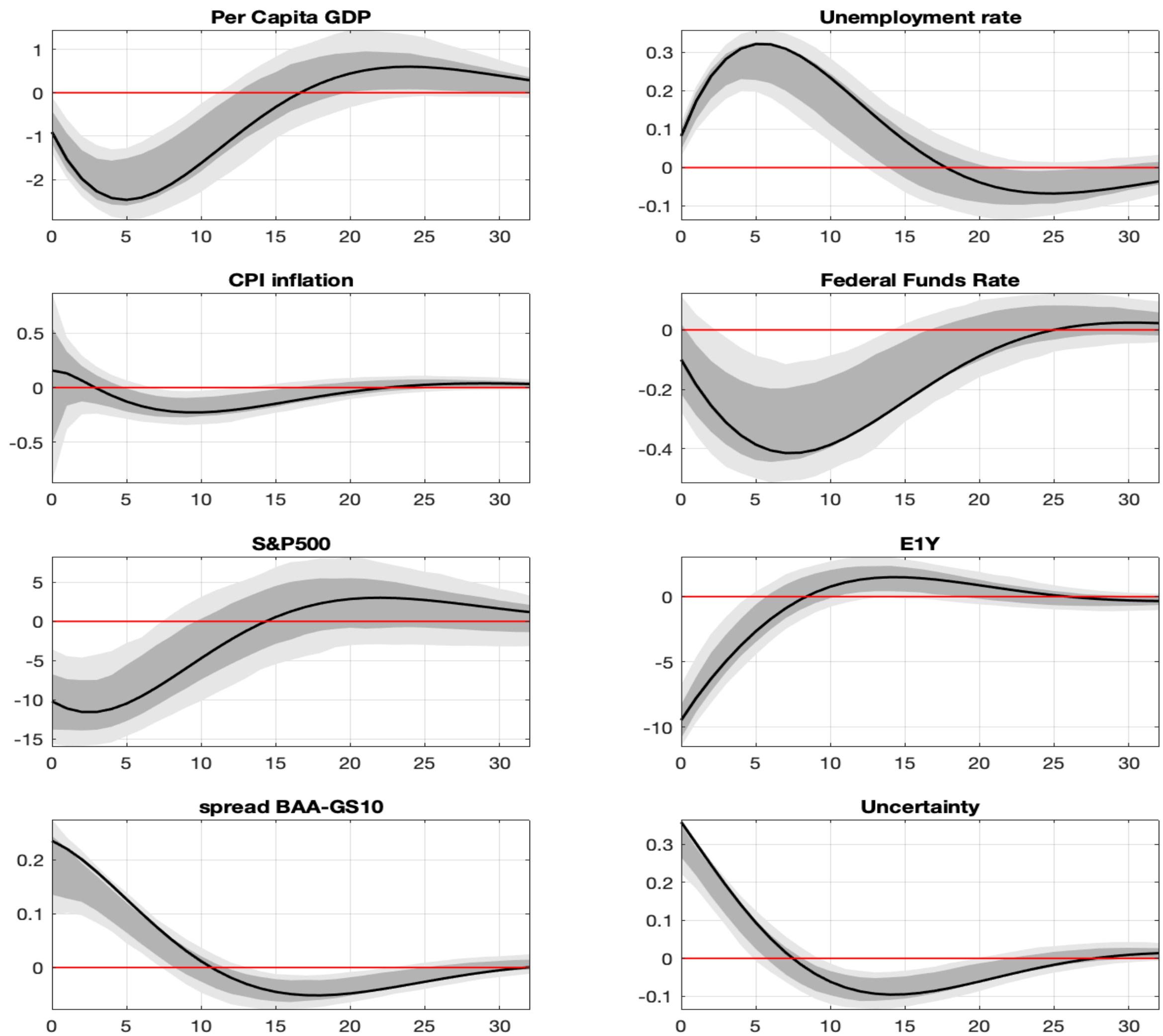

Figure 4: Impulse response functions of the unemployment rate uncertainty shock, 4 quarters. The shock is identified as the residual of the projection of the uncertainty innovation onto a long-run shock (Identification II). The latter shock is identified as the only one shock having effect on GDP at the 40 quarter horizon. Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

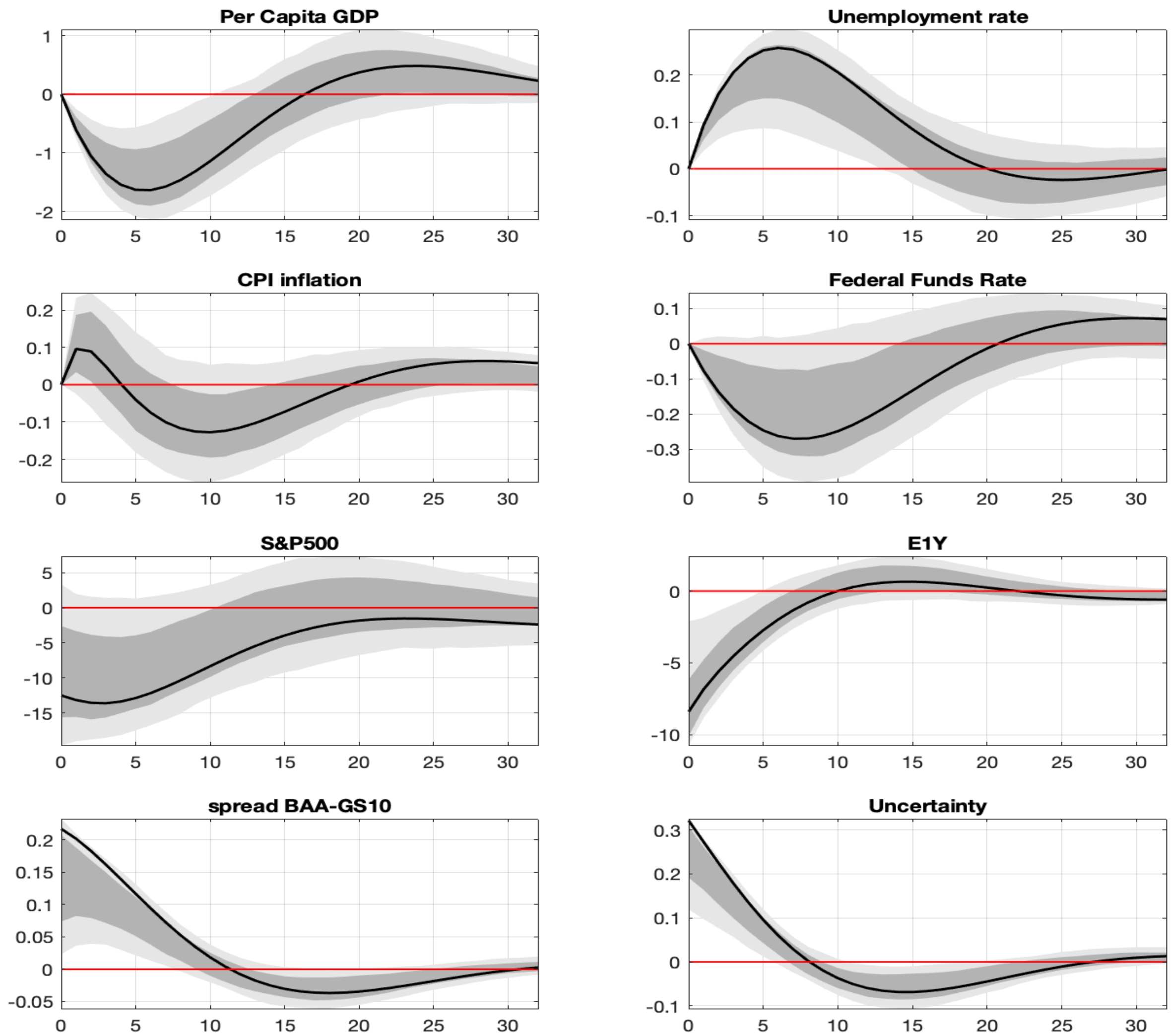

Figure 5: Impulse response functions of the unemployment rate uncertainty shock, 4 quarters. The shock is identified as the residual of the projection of the uncertainty innovation onto the long-run shock, the GDP innovation, the unemployment rate innovation, the CPI innovation and the federal funds rate innovation (Identification III). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

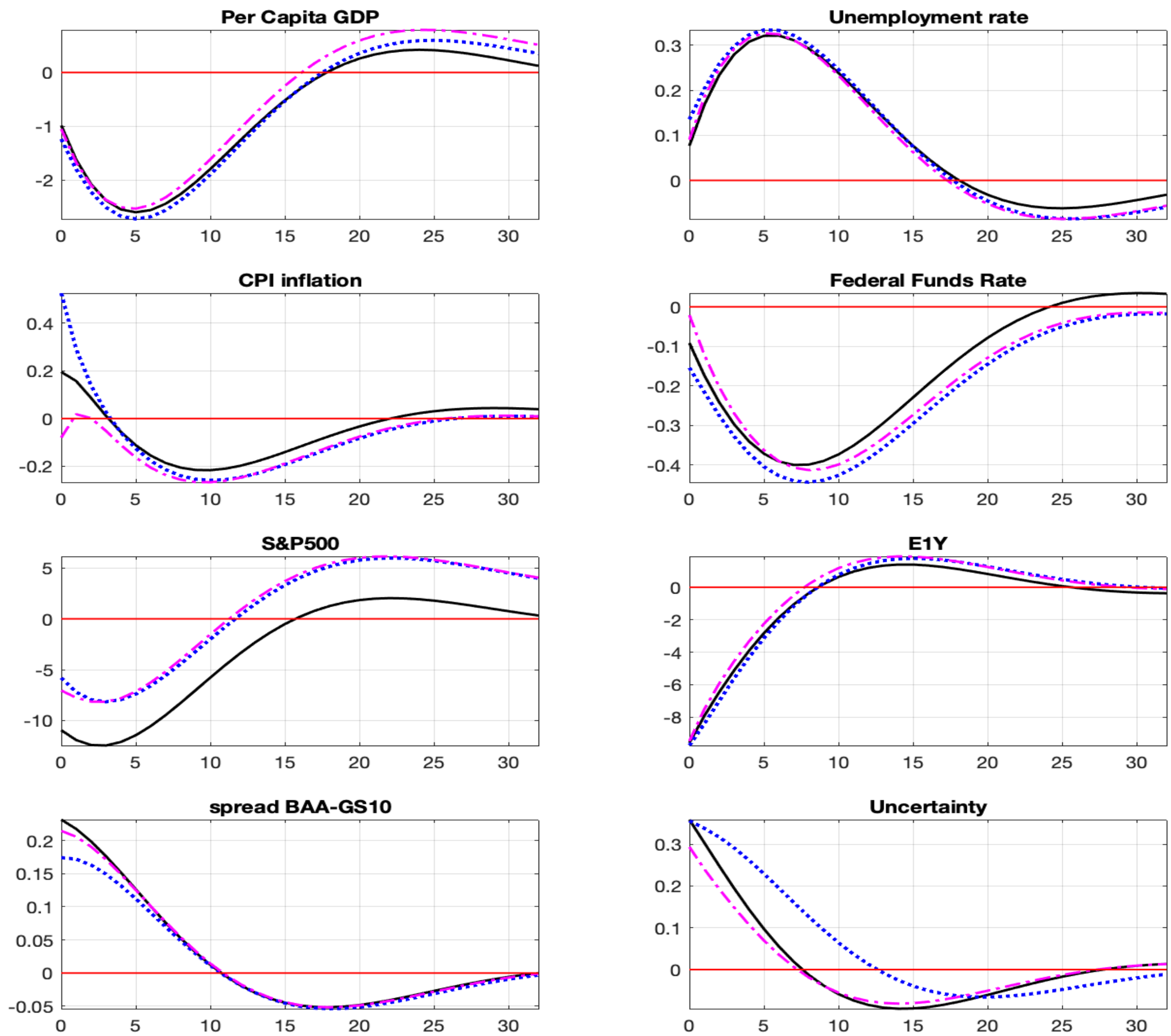

Figure 6: Comparison between the benchmark impulse response functions of Identification I (solid black lines), obtained with unemployment rate uncertainty 4 quarters, and the corresponding impulse response functions for unemployment rate uncertainty 1 quarter (dotted blue lines) and unemployment rate uncertainty 8 quarter (dashed-dotted magenta lines).

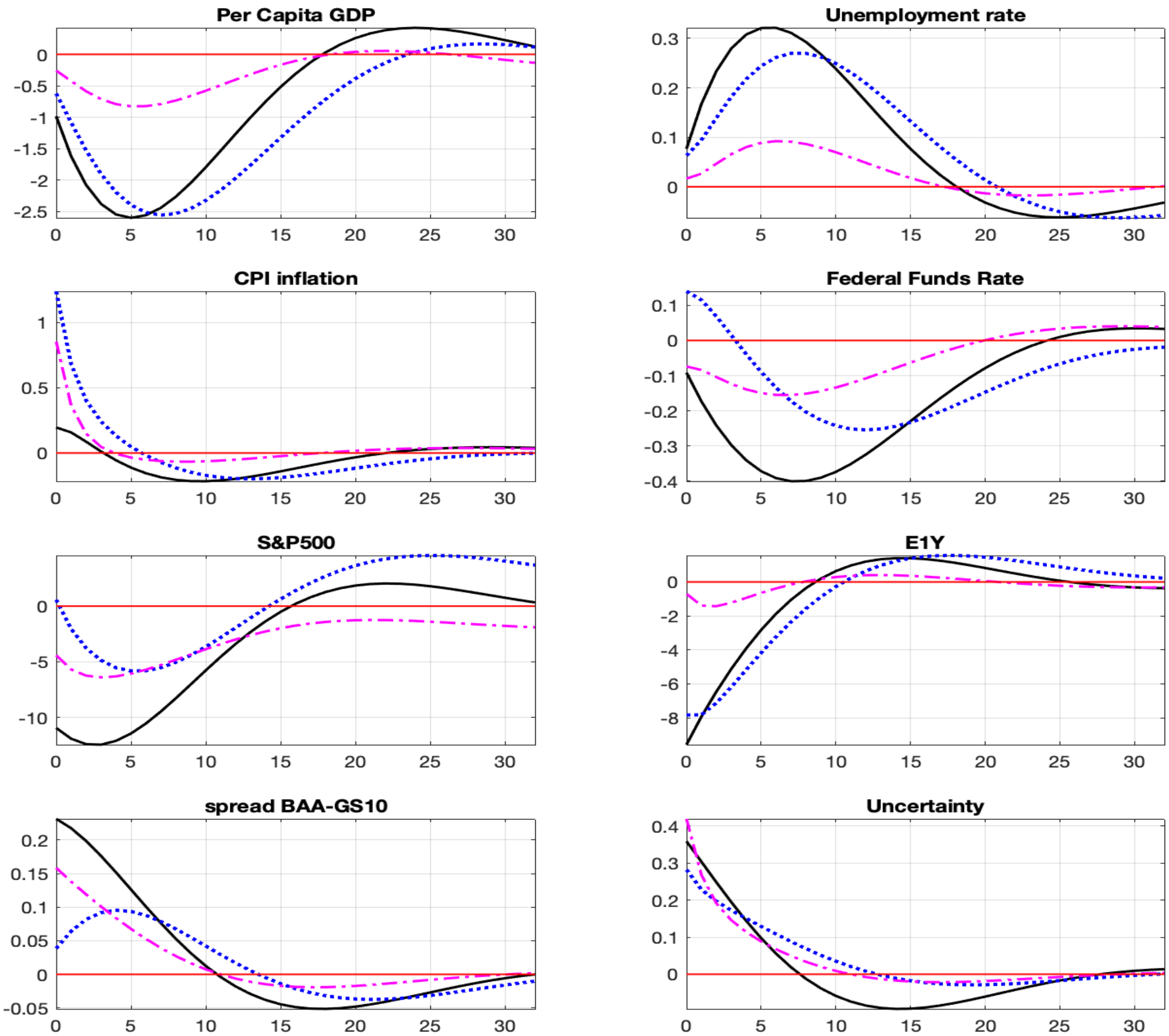

Figure 7: Comparison between the benchmark impulse response functions of Identification I (solid black lines) and the corresponding impulse response functions for GDP uncertainty 4 quarters (dotted blue lines) and S&P500 uncertainty 1 quarter (dashed-dotted magenta lines).

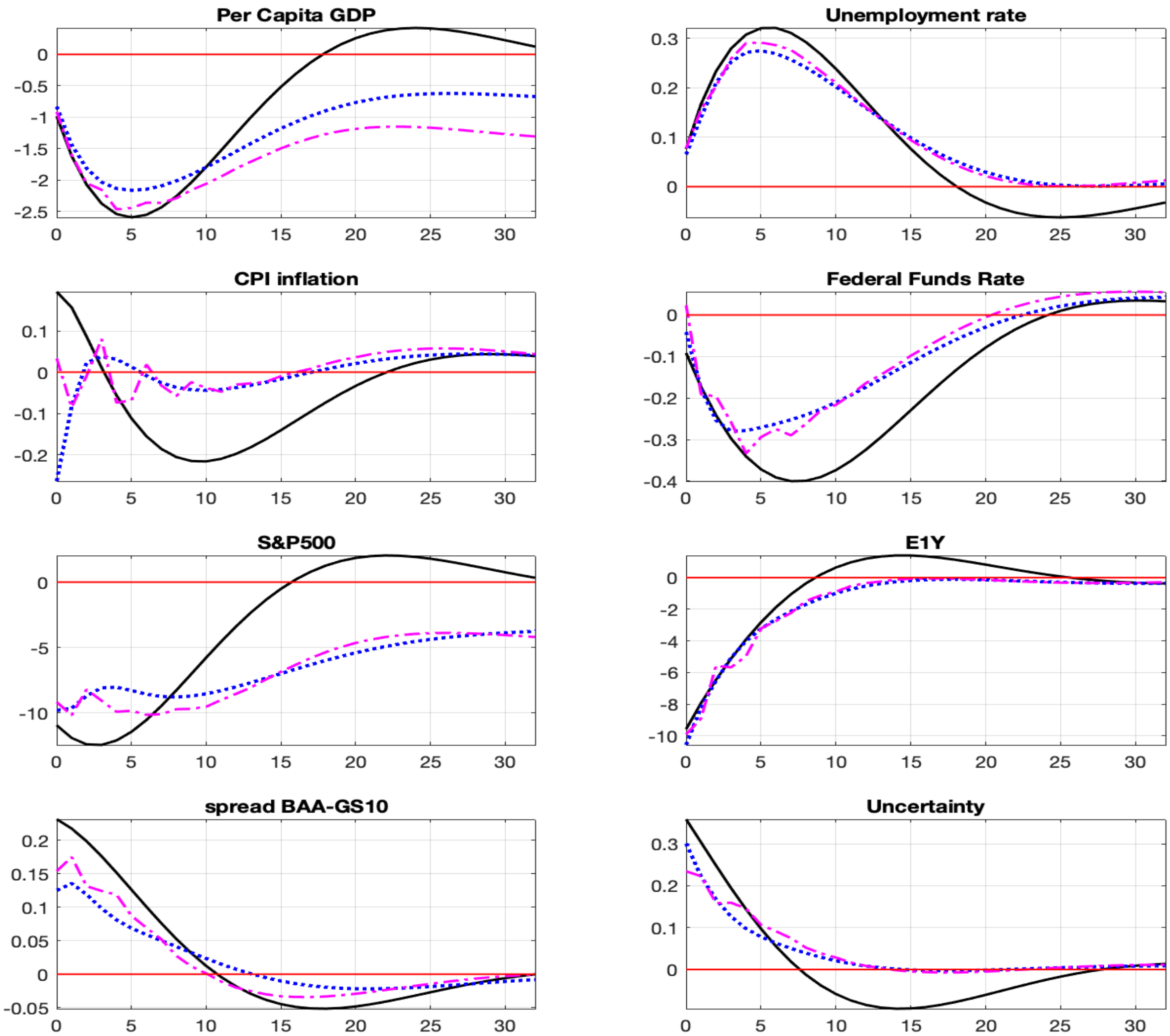

Figure 8: Comparison between the benchmark impulse response functions of Identification I (solid black lines), obtained with 1 lag in the VAR and the corresponding impulse response functions obtained with 2 lags (dotted blue lines) and 4 lags (dashed-dotted magenta lines).

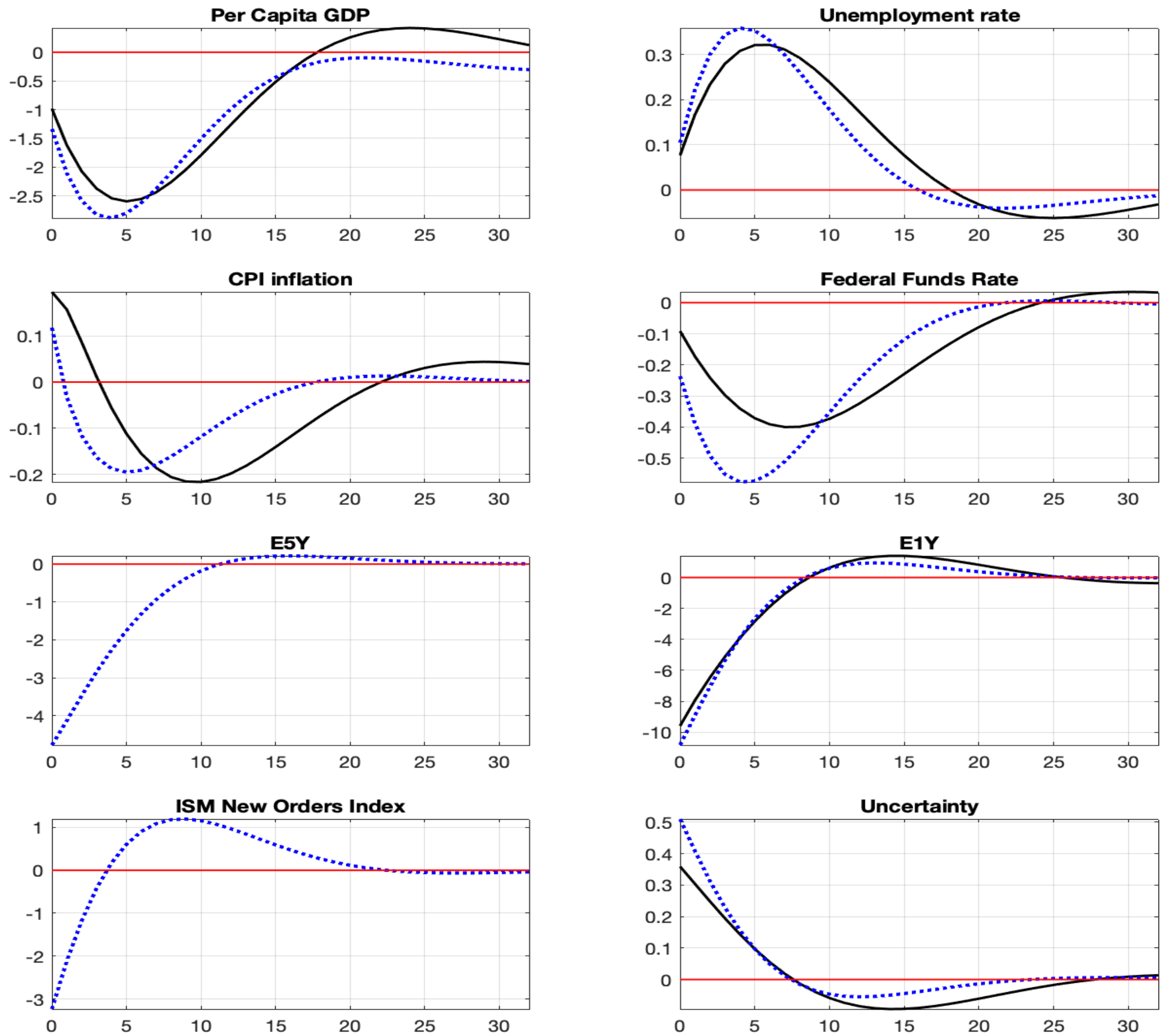

Figure 9: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained with a different VAR specification, including E5Y (a component of the Michigan University Consumer Confidence Index) and the ISM New Order Index in place of S&P500 and the spread BAA-GS10 (dotted blue lines).

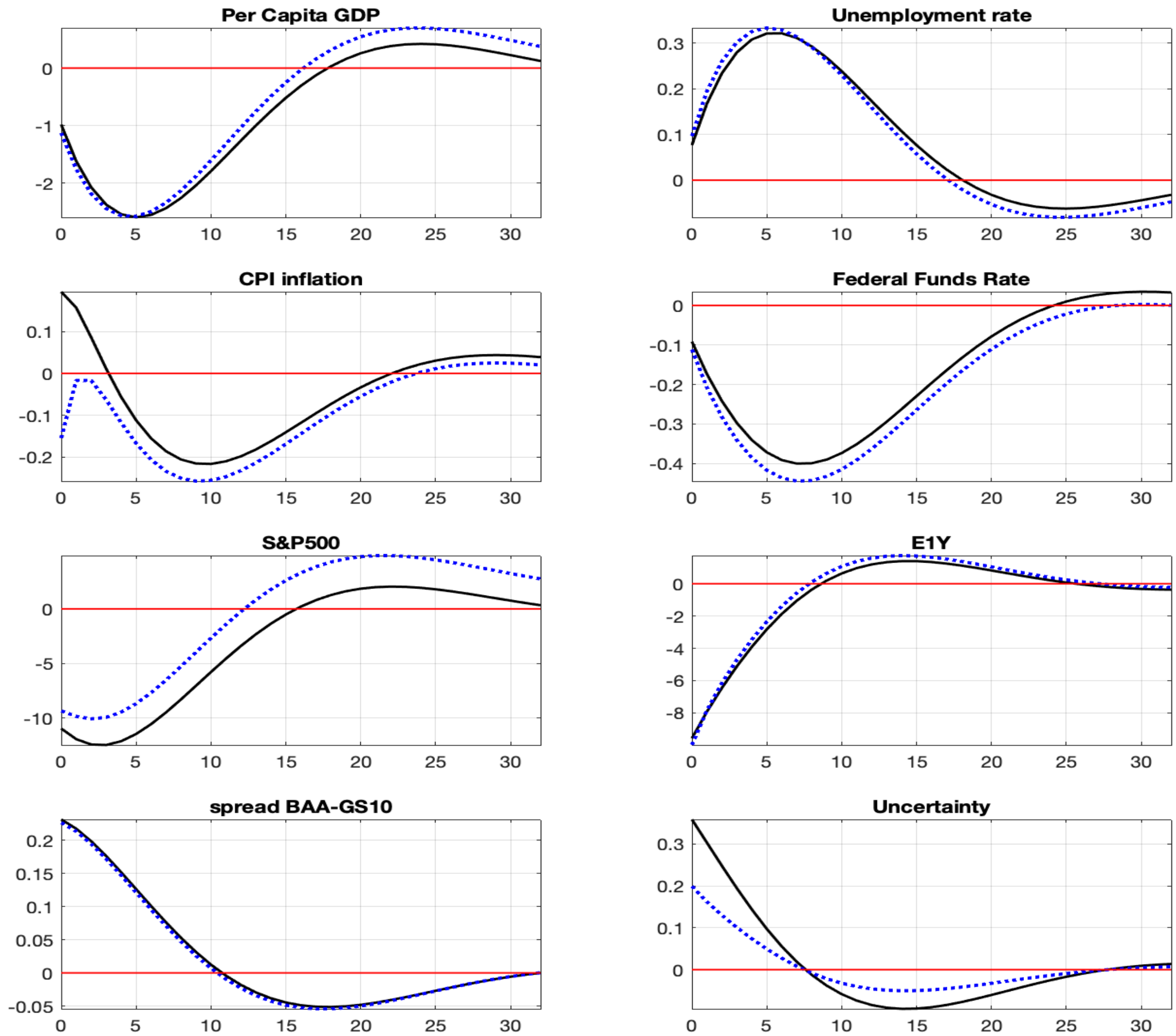

Figure 10: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using the squared predictions error in place of the log of the squared prediction error to compute uncertainty (dotted blue lines).

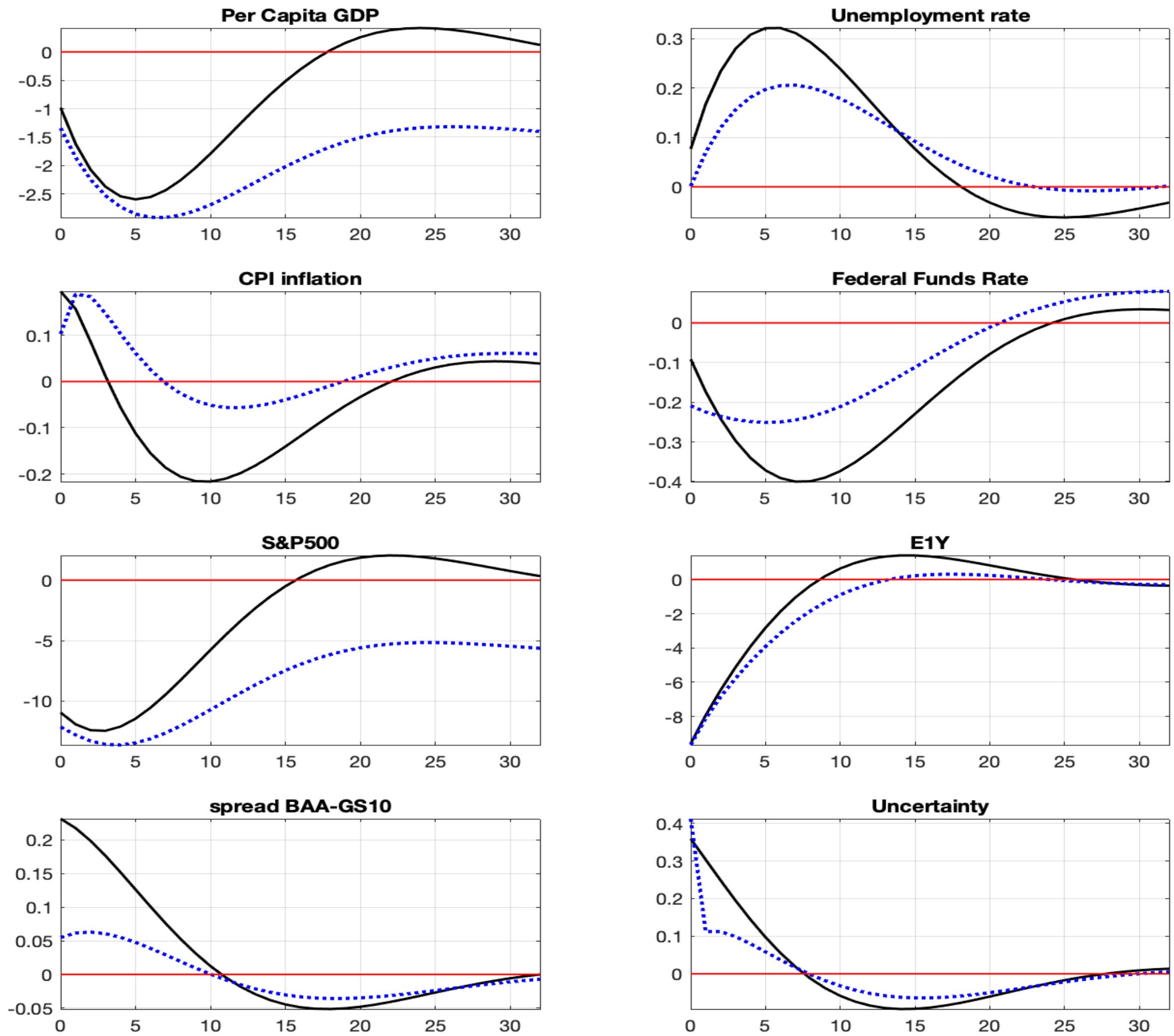

Figure 11: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using 1 lag of the variables, in addition to the current values, to compute uncertainty (dotted blue lines).