DISCUSSION PAPER SERIES

DP15607

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PUBLIC ECONOMICS

ISSN 0265-8003

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Discussion Paper DP15607
Published 24 December 2020
Submitted 22 December 2020

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The current polarization of elites in the U.S., particularly in Congress, is frequently ascribed to the emergence of cohorts of ideologically extreme legislators replacing moderate ones. Politicians, however, do not operate as isolated agents, driven solely by their preferences. They act within organized parties, whose leaders exert control over the rank-and-file, directing support for and against policies. This paper shows that the omission of party discipline as a driver of political polarization is consequential for our understanding of this phenomenon. We present a multi-dimensional voting model and identification strategy designed to decouple the ideological preferences of lawmakers from the control exerted by their party leadership. Applying this structural framework to the U.S. Congress between 1927- 2018, we find that the influence of leaders over their rank-and-file has been a growing driver of polarization in voting, particularly since the 1970s. In 2018, party discipline accounts for around 65% of the polarization in roll call voting. Our findings qualify the interpretation of – and in two important cases subvert – a number of empirical claims in the literature that measures polarization with models that lack a formal role for parties.

JEL Classification: P48, D72

Keywords: Political Polarization, Parties, discipline, Ideology, Spatial voting

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Acknowledgements
We thank Matilde Bombardini, Ernesto Dal Bo, Fred Finan, Tasos Kalandrakis, Vadim Marmer, as well as seminar participants at various institutions for comments. Yihang Zhang provided excellent research assistance. We are grateful for funding from CIFAR and from the Bank of Canada.

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The current polarization of elites in the U.S., particularly in Congress, is frequently ascribed to the emergence of cohorts of ideologically extreme legislators replacing moderate ones. Politicians, however, do not operate as isolated agents, driven solely by their preferences. They act within organized parties, whose leaders exert control over the rank-and-file, directing support for and against policies. This paper shows that the omission of party discipline as a driver of political polarization is consequential for our understanding of this phenomenon. We present a multi-dimensional voting model and identification strategy designed to decouple the ideological preferences of lawmakers from the control exerted by their party leadership. Applying this structural framework to the U.S. Congress between 1927-2018, we find that the influence of leaders over their rank-and-file has been a growing driver of polarization in voting, particularly since the 1970s. In 2018, party discipline accounts for around 65% of the polarization in roll call voting. Our findings qualify the interpretation of – and in two important cases subvert – a number of empirical claims in the literature that measures polarization with models that lack a formal role for parties.

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We thank Matilde Bombardini, Ernesto Dal Bo, Fred Finan, Tasos Kalandrakis, Vadim Marmer, as well as seminar participants at various institutions for comments. Yihang Zhang provided excellent research assistance. We are grateful for funding from CIFAR and from the Bank of Canada.

1 Introduction

The sharp increase in political polarization over the last forty years in the United States is an uncontroversial phenomenon. In terms of political elite polarization, evidence stems from congressional voting records (McCarty, 2016), candidate survey responses (Moskowitz et al., 2017), congressional speech scores (Gentzkow et al., 2019), and campaign donation measures (Bonica, 2014). In the electorate at large, the picture appears less sharp in terms of the polarization of the policy preferences of voters (Fiorina et al., 2005), but stark evidence of partisan sorting emerges more consistently in other dimensions – particularly in the affective polarization of citizens (Iyengar and Westwood, 2015; Iyengar et al., 2019; Boxell et al., 2020) and other indicators of culture (Bertrand and Kamenica, 2018) and beliefs (Alesina et al., 2020). Currently, both the political economy and political science literature characterize a context of growing mutual antagonism across political caucuses, and of increasing animus among voters identifying with different political parties (Gentzkow, 2016). Growing evidence of the adverse economic consequences of polarization also exists, arising through delay in fiscal stabilizations, uncompromising obstructionism, political gridlock, and policy uncertainty due to partisan cycles and electoral shocks (Pastor and Veronesi, 2012; Baker et al., 2014; Mian et al., 2014; Davis, 2019; Binder, 2003).

To contribute to our understanding of this phenomenon, we study the role of the two main political parties and their leadership in driving polarization over the last ninety years in the U.S. Specifically, we attempt to assess the extent of the influence that party leaders exert on the behavior of rank-and-file members as they drive the passage of laws and create wedges across lawmakers belonging to different parties.

Within liberal democracies, political parties are more than just the sum of their individual members (Aldrich, 1995), having time horizons and strategies that span those of individual politicians. The party leadership devises, coordinates, and enacts the policy agenda (Caillaud and Tirole, 1999, 2002). In representative bodies, the relative strength, internal cohesion, and mechanisms of discipline utilized by political organizations are determinants of effective (if not efficient) policy making (Cox and McCubbins, 1993). Tight control exerted by political organizations on their members, however, may also act as an instrument of division and separation (Evans, 2018) and such divisions may be tactically valuable.[1]

In this context, we ask whether the sharp increase in polarization in congressional voting over the last forty years is the sole result of more ideologically extreme politicians replacing moderates (Poole and Rosenthal, 1997; McCarty et al., 2006; Moskowitz et al., 2017), or whether strategic

[1]E.g. Newt Gingrich, the architect of the 1994 Republican Revolution and former Party Whip, notably stated in 1984: "*The No. 1 fact about the news media is they love fights ... When you give them confrontations, you get attention; when you get attention, you can educate.*"

party discipline also plays a role in the progressive separation between partisan camps (Sinclair, 2014; Stonecash, 2018; Canen et al., 2020). How much pressure do the leaders of the U.S. parties of today exercise on their rank-and-file, by influencing member behavior and pulling them away from the middle ground (Snyder and Groseclose, 2000; Forgette, 2004)? How has the role of parties evolved over time or around structural breaks in political strategies?[2]

Because the decisions of politicians are function of both their unobserved individual policy preferences (their "ideologies") and the (often unobserved) influence exerted by the political organization to which they belong, quantifying the role of these different drivers of behavior is nontrivial on grounds of identification (Krehbiel, 1993, 1999, 2000).

In previous work, Canen et al. (2020) leverage detailed internal party records for identification, showing that party discipline is an important component of political polarization in the decade between 1977 and 1986.[3] Because these detailed internal records are only available for the House of Representatives for that specific decade, however, this identification strategy does not generalize. That is, it cannot be used to systematically study how party discipline has evolved over the long term, one of the main goals of this work.

In this paper, we develop a novel, more general identification strategy that requires information on congressional vote choices ("roll call" votes in the terminology of the U.S. legislative branch) and on the party leadership positions on each vote.[4] We are able to address questions of how party control drives polarization over the last century.[5] Furthermore, because we study party discipline over periods in which a second dimension of policy preferences (in addition to the standard liberal-conservative ideological dimension) is relevant (i.e. the Civil Rights era), our approach incorporates multiple policy dimensions. This extension turns out to be non-trivial from the perspective of identification relative to the one-dimensional approach of Canen et al. (2020).

Focusing on congressional roll calls, we show how information about the direction of pressure implied by leaders' votes can be combined with an economic model of legislative choice to recover parameters related to the disciplining technology of each party. This technology (occasionally referred to as "whipping" and here meant to encompass both persuasion and horse trading with

[2]See Jenkins (2011).

[3]The use of internal party records (i.e. whip counts by the leadership) in Canen et al. (2020) also allowed us to identify a rich model of agenda setting to determine which bills are pursued by the party and which are dropped, and to produce counterfactuals demonstrating how this selection process interacts with the technology of party discipline. Absent whip counts, we do not have sufficient information to study agenda setting over the last century. Thus, while we allow for a general form of agenda setting in our empirical model, a quantitative assessment of policy counterfactuals over the 1927-2019 period is beyond the scope of this paper.

[4]As such, the method is applicable to any institution for which voting data is available and the direction of potential influence (via party leadership, special interests, etc.) is known.

[5]Reassuringly, in the subsample overlapping with Canen et al. (2020), we find very similar measures of party discipline, validating our identification strategy.

the rank-and-file[6]) can be parameterized by how far the party organization is able reach within the set of dissident members, to persuade them to vote with the leadership on occasions when they would not do so otherwise.

To provide an intuition, suppose that we observe the vote decisions of each member of Congress and know the direction in the policy space towards which each party leadership is whipping for each roll call. In standard spatial models of legislative behavior (Poole and Rosenthal, 1997; Heckman and Snyder, 1997; Clinton et al., 2004), a multidimensonal random utility framework is applied to individual vote choices, obtaining preference parameters and cutlines that indicate indifference between support and opposition to each specific bill. Absent party discipline, for each bill, this cutline is unique, separating Yes and No votes. Empirically, however, one observes two cutlines for each roll call, one for each party. In our framework, these party-specific cutlines are determined by how far into the subset of dissidents each party is willing to reach in order to have some members change their votes to follow party cues. We also observe in which direction the party leadership votes and from this we infer information (which we probe) on in which direction they exert pressure. The direction in which party discipline is applied allows us to pin down whether the observed distance between the party cutlines in each congressional vote is either the sum or the difference of the party discipline parameters applied to members on the fence. Figures 1 and 2 illustrate an example for a two-dimensional policy space. As the party leadership applies its pressure selectively on each bill, our spatial model identifies which members are subject to pressure by the party – those nearest to being on the fence on that vote. Although Cox and McCubbins (1993) discuss leadership votes in their analysis of party organizations and McCarty et al. (2001) allow for party-specific cutlines in assessing their model's fit[7], the intuition of jointly using these insights is the key to identifying the model.

We formally prove that our approach resolves the identification problem of separating politicians' multidimensional preferences from the pressure exercised on them by their parties, and we then pursue estimation using a likelihood-based estimator. Our approach spells out the identification requirements of our method and clarifies the role of agenda setting for inference in this setting.

We further note that formal identification results in multiple policy dimensions (even absent a role for parties) are unavailable for what is arguably one of the most influential methods in the literature, DW-Nominate (Poole and Rosenthal, 1984, 1997), a statistical approach designed to recover policy preferences of legislators from a random utility framework within a spatial context similar to ours.[8] Because of Nominate's relevance to the literature, in Appendix B we prove the

[6]For a comprehensive discussion, see Evans (2018).

[7]The use of party-specific cutlines is in itself insufficient for identification of ideology and discipline parameters, as demonstrated in our proof of identification.

[8]The prominent exception is the proof in Rivers (2003) for the special case of a random utility model with

lack of identification of the DW-Nominate two-dimensional case, and clarify the features of our methodology that allow us to improve upon this established approach.

Our principal finding is that political party influence bears a substantial weight in driving observed polarization in congressional voting behavior. The leaderships of both parties have played a similar role in driving an increasing wedge between groups of politicians that appear substantially less ideologically extreme than that inferred from extant methodologies which omit a role for parties. A misspecified model estimated without a role for parties is statistically rejected at high confidence levels in every congressional cycle in our sample, and we show that the misspecification is large from a quantitative perspective. Misattributing these effects solely to individual ideology misses salient features of the data, and clouds the debate on how to address the effects of polarization.

In a second finding, we show that the commonly assumed U-shaped trajectory of ideological polarization traced over the 20th century by McCarty et al. (2006) disappears when the true ideological positions of members are decoupled from party discipline. Instead, our estimates of ideological polarization increase monotonically. Reconciling this discrepancy, we find that the ability of parties to push the leadership's line and forge internal rules has varied quantitatively (and non-monotonically) over time both in the House and Senate. The low point of party discipline appears around the second half of the 1960s, during the Civil Rights Era, and early 1970s. In the early part of the 1980s an increment in party discipline starts to appear and a sharp increment is detected after the mid-1990s, the time of Newt Gingrich's speakership and the Republican Revolution.[9] As a result, we do not find support for the theory that the present levels of ideological polarization have been previously observed. Our results suggest, instead, that the U.S. Congress is currently in a period of unprecedented ideological polarization and of strong party discipline. By comparison, in the post-war period, while party discipline was high, ideological polarization was relatively low.

Overall, we find party leaders have been responsible for a significant share of polarization in congressional voting – approximately 65% in recent decades in both the Senate and in the House – and the phenomenon appears fairly symmetric between the parties. These findings are present in both the one-dimensional and in the two-dimensional versions of our model.

We next address the question of how party leaders were able to increase discipline, whipping members further out in the ideological distribution over time. Having estimates of party discipline

quadratic two-dimensional preferences. This identification result does not apply to the standard DW-Nominate method, which employs non-convex preferences within a random utility choice framework and multiple policy dimensions. Rivers (2003) is related to, but also does not apply to, the IDEAL estimator of Clinton et al. (2004), which employs quadratic preferences, but within a Bayesian, not classical, statistical environment. We expand on this discussion in Section 2.

[9] This finding appears in line with extant quantitative, but less systematic evidence, e.g. (Sinclair, 2014).

over time allows us to investigate the technology of internal party organization around known structural breaks (Theriault, 2013) and how it is affected by majority size and divided government. We then discuss which theories of party influence are consistent with our estimates (Smith, 2007), particularly with respect to the correlation of party discipline and time varying within-party heterogeneity.[10] We observe that increases in party discipline appear positively correlated with within party ideological homogeneity (the variance of ideologies within a party). This result holds for both parties and it is consistent with the Conditional Party Government theory of Aldrich (1995) and Rohde (1991).[11]

Discipline parameters are highly correlated between parties and over time. Further, they appear symmetrical, contrary to extant research focused on asymmetric polarization in ideology (i.e. emerging solely from the conservative end of the spectrum). A conjecture is that technological innovations may be an important piece of the explanation: when one party favorably innovates in its internal organization, the other party can follow closely by imitation. This is not inconsistent with qualitative and quantitative evidence on the spread of technological political innovation, both within the U.S. system and abroad.[12]

This paper relates to several strands of literature. Mayhew (2004) presents U.S. parties as exerting weak control and the members of Congress as having limited party loyalty. The debate on decoupling the drivers of political polarization is active (Moskowitz et al., 2017), and explicitly linked to economically consequential phenomena, such as changes in income inequality over time (e.g. McCarty et al., 2006, but also Rajan, 2011), the policy response to financial crises (Mian et al., 2014), and legislative inaction more generally (Binder, 2003).

As a result of our identification method, we differ in many respects from extant empirical approaches on the study of parties and political polarization. These approaches include, to cite just a few prominent examples, the use of historical natural experiments during the American Civil War (Jenkins, 2000), functional form identification of voting models with heterogeneous

[10]It is beyond the scope of the paper to explore the motivations behind the actions taken by party leaders and why they result in party polarization. Polborn and Snyder Jr (2017) offer an example of what mechanisms may be at play.

[11]The latter states that as parties become more homogeneous, party members are willing to delegate more (agenda setting and control) power to party leaders - they will be more likely to get bills approved that are in the interest of a majority of the party. Our evidence supports this explanation over the past ninety years.

[12]Examples include the use of coordinated partisan vocabularies by the 1994 Revolution Republicans (e.g. Gentzkow et al., 2019), a practice also followed by Democrats, and by the simultaneous adoption of focus-group-tested language and messaging. This may also explain the diffusion of political strategies and tactics across political systems due to the international visibility of the U.S. system. For example, in 2001 Prime Minister Silvio Berlusconi in Italy hired strategist Frank Luntz, who inspired the 1994 Contract with America, and transposed the Republican public relations approach to the Italian context (see Luntz, 2007, p.138). President Emmanuel Macron of France notoriously adopted campaigning techniques form the 2008 Obama campaign. Another example appears to be the diffusion of certain strategies adopted by the Trump campaign to other populist movements in Europe and Latin America. These examples suggest a potential mechanism through which U.S. party-driven political polarization may spread internationally, via imitation of internal organization and branding tactics.

legislators (Levitt, 1996; Poole and Rosenthal, 1997; Heckman and Snyder, 1997; McCarty et al., 2001; Clinton et al., 2004), the exclusion of lopsided legislative bills from party discipline (Snyder and Groseclose, 2000), and the use of detailed internal party records (Evans, 2018; Canen et al., 2020). We provide more detailed comparisons to extant methodologies in Section 2.5.

This paper also relates to works on the study of political organizations. Parties play a crucial role in agenda setting and in drafting statutes (Cox and McCubbins, 1993; Aldrich, 1995; Cox and McCubbins, 2005). Their leadership also systematically organizes and coordinates members' political behavior (Smith, 2007), from setting policy platforms (Caillaud and Tirole, 2002) to coordinating internal communication and the whipping of votes (Meinke, 2008; Evans, 2018). Making explicit the empirical role of these dimensions, which are latent and unobserved relative to the formal operations of government, has been an open question in political economy and political science for decades and has resulted in a rich, but far from complete line of inquiry.[13] We contribute with an economic model and a structural estimation approach designed to consistently infer the extent of party influence over the last century in the U.S., but also likely applicable to other contexts.

Providing a measure for the degree of control exercised by one party against the other is important because it offers evidence of elite organizations driving partisan separation though political action that is strategic and deliberate (Smith, 2007; Evans, 2011). These actions may take additional forms that we do not explore here, but our time series evidence in recent times is consistent with a contemporaneous role for elites in driving systematic wedges in public opinion (Robison and Mullinix, 2016; Alesina et al., 2020) and using divisive speech (Gentzkow et al., 2019), which may ultimately manifest in affective polarization of voters.

The paper is organized as follows. Section 2 presents the econometric model used in the structural estimation, including an analysis of the issues of selection, agenda setting, and the derivation of the likelihood function. Section 3 presents the data. For the most part the data is standard within political economy, but a few details, such as the absence of selective pruning of roll call votes, are important. Section 4 presents our main estimates of party discipline over time and our analysis of different mechanisms behind its rise. Section 5 concludes.

[13] Most prominently, see Snyder and Groseclose (2000), but also see McCarty et al. (2001) for a critique of this approach. For a detailed discussion of the complexity and identification issues of party influence in the context of the U.S. Congress see Krehbiel (1993, 1999) and Cox and McCubbins (1993). For related work on the decomposition of polarization trends, see the analyses in Theriault (2008); Moskowitz et al. (2017).

2 Empirical Model

2.1 Setup

Legislators $i = 1, ..., N$, where N is large, belong to one of two parties $p \in \{D, R\}$.[14] Each legislator is characterized by her constant policy preferences: a $d \geq 1$ dimensional characteristic of i, which we refer to as her ideology.[15] Specifically, each i has a fixed ideology denoted by her ideal point, $\bar{\theta}^i = (\theta_1^i, ..., \theta_d^i)$. In what follows, an upper bar (e.g. $\bar{x}$) denotes a vector.

Each congressional cycle defines a set $\boldsymbol{\Theta} = \left\{\bar{\theta}^1, \bar{\theta}^2, ..., \bar{\theta}^i, ..., \bar{\theta}^N\right\}$ where $\boldsymbol{\Theta}$ may change from one congressional cycle to the next due to the potential replacement of some members of the legislature. Within each congressional cycle (a two year period), let $t = 1, 2, ..., T$ indicate the discrete times at which a single bill may be introduced and voted on. We assume T is large for each congressional cycle. For exposition, we consider the case of a single congressional cycle, but discuss in Section 4 how our estimation procedure handles multiple cycles.

Individual i's preferences over policies are represented within a random utility framework. For any policy $\bar{k}_t \in \mathbb{R}^d$, we assume that i's preferences are given by:

$$u\left(\bar{k}_t, \bar{\theta}^i\right) = u\left(\left\|\bar{\omega}_t^i - \bar{k}_t + \bar{y}_t^i\right\|\right), \tag{1}$$

with $u'(\cdot) < 0$. $\|.\|$ indicates the weighted Euclidean norm with weights $w_1, w_2, ..., w_d$. We indicate by $\bar{\omega}_t^i = \bar{\theta}^i + \bar{\varepsilon}_t^i$ i's realized ideal point at t. $\bar{\omega}_t^i$ includes i's ideology plus a random shock, $\bar{\varepsilon}_t^i$, that is independently and identically distributed across individuals i and votes t according to a continuous CDF, $G(\bar{\varepsilon})$.[16]

Utility is also a function of $\bar{y}_t^i$, the extent of party influence exerted on politician i on roll call t. We refer to $\bar{y}_t^i$ as 'party influence', 'party discipline', or 'whipping', and specify it in detail in Section 2.2.2. Party discipline may be exerted in favor of or against the status quo, depending the preference of the politician's party. Each party can only discipline its own members.

Absent whipping, a member i votes for a policy $\bar{x}_t \in \mathbb{R}^d$ and against the status quo $\bar{q}_t \in \mathbb{R}^d$ if and only if $u\left(\| \bar{\omega}_t^i - \bar{q}_t \|\right) \leq u\left(\| \bar{\omega}_t^i - \bar{x}_t \|\right)$. Given that $u'(\cdot) < 0$, this inequality is equivalent to $\| \bar{\omega}_t^i - \bar{q}_t \| \geq \| \bar{\omega}_t^i - \bar{x}_t \|$.

The case of $d = 2$ is central to our empirical analysis, so we focus on it here. Additional dimensions could be included analogously, at a cost of higher identification requirements. For the case of $d = 2$, the set of members that vote for $\bar{x}_t = (x_{1,t}, x_{2,t})$, X_t, is the set:

[14] $N = 435$ for the House and $N = 100$ for the Senate.

[15] We focus on the case $d = 2$ in this section, but we also study and estimate models for the $d = 1$ case, which is considered appropriate especially for the period between 1975 and 2018 (McCarty, 2016).

[16] Assuming ideology shocks instead of utility shocks (as in Canen et al. (2020)) allows us to avoid making an assumption about the exact shape of the utility function (i.e. quadratic), as shown below.

$$X_t = \left\{ \bar{\omega}_t^i | \omega_{2,t}^i \geq \omega_{1,t}^i \frac{w_1 \left(q_{1,t} - x_{1,t}\right)}{w_2 \left(x_{2,t} - q_{2,t}\right)} + \frac{w_1(x_{1,t}^2 - q_{1,t}^2) + w_2(x_{2,t}^2 - q_{2,t}^2)}{2w_2 \left(x_{2,t} - q_{2,t}\right)} \right\}, \tag{2}$$

when $x_{2,t} > q_{2,t}$ (otherwise, the inequality is reversed).[17]

The formulation in (2) is useful because it makes explicit that the set of members that votes for $\bar{x}_t$ is the set of those who lie above a cutline in the two-dimensional space given by

$$\omega_{2,t}^i = m_t \omega_{1,t}^i + b_t \tag{3}$$

where

$$m_t \equiv \frac{w_1 \left(q_{1,t} - x_{1,t}\right)}{w_2 \left(x_{2,t} - q_{2,t}\right)},$$

$$b_t \equiv \frac{w_1(x_{1,t}^2 - q_{1,t}^2) + w_2(x_{2,t}^2 - q_{2,t}^2)}{2w_2 \left(x_{2,t} - q_{2,t}\right)}.$$

We make immediate use of (3) to simplify the structure of the shocks. Recall that $\bar{\varepsilon}_t^i = \bar{\omega}_t^i - \bar{\theta}^i$. We assume that $G(\bar{\varepsilon})$ has the following structure: (i) the vector $\bar{\varepsilon}_t^i$ has magnitude $e_t^i = \|\bar{\varepsilon}_t^i\|$, a scalar which is assumed to be distributed i.i.d. with distribution, $e_t^i \sim N(0,1)$; (ii) shocks are assumed to shift a member's ideal point along the direction orthogonal to the cutline (3) with a positive shock increasing $\omega_{1,t}^i$.

This structure ensures that $\bar{\varepsilon}_t^i$ moves a politician in the direction most likely to change her vote, a feature which greatly simplifies the construction of the likelihood function and its computation. Notice further that an unrestricted $\bar{\varepsilon}_t^i$ vector shock could move politicians from $\bar{\theta}^i$ in any direction in $\mathbb{R}^2$, but this vector can be always represented in terms of its projection onto the line orthogonal to (3), obtaining an identical vote choice in our context.

Similarly, we assume that party discipline $\bar{y}_t^i$ also acts along the direction orthogonal to the cutline (i.e. in the direction most likely to make politician i change her vote). We discuss further benefits of the structure induced by these assumptions in Section 2.5 below.

2.2 Timing and Structure

The timing of the legislative process is as follows:

(I) Each period t, one of two parties is recognized to set the agenda.[18]

[17] In the special case in which $x_{2,t} = q_{2,t}$, we have $X_t = \left\{ \omega_{it} | \omega_{1,it} \geq \frac{x_{1,t}+q_{1,t}}{2} \right\}$ for $x_{1,t} > q_{1,t}$ (and otherwise the inequality is reversed).

[18] For now, we allow for an arbitrary rule that picks the proposing party in each period. For example, we can let

(II) The agenda setting party, p_t, draws (with replacement) a status quo, $\bar{q}_t$, from the distribution of possible policy status quo's $W(\bar{q})$ with support $Q \subseteq \mathbb{R}^2$. For each status quo, $\bar{q}_t$, the agenda setter can decide whether or not to propose an alternative, $\bar{x}_t = x(\bar{q}_t)$, or not pursue any alternative.

(III) If an alternative is proposed, preference shocks realize and then each party whips a subset of their members.

(IV) Politicians vote for $\bar{x}_t$ or $\bar{q}_t$, payoffs realize, and the chamber moves to $t + 1$.

2.2.1 Parts (I) and (II): Agenda Setting

A congressional cycle includes a series of recognition draws $\{p_1, p_2, ..., p_T\}$ and status quo draws $\{\bar{q}_1, \bar{q}_2, ..., \bar{q}_T\}$. Notice that, due to selection, only a subset of $\{\bar{q}_1, \bar{q}_2, ..., \bar{q}_T\}$ is considered, producing the actual vote data observable to the econometrician. We use $Q_p^1 \subseteq Q$ and $Q_p^0 \subseteq Q$ to denote the sets of status quo's that are considered and not considered for a vote by p_t, respectively, such that $Q_p^1 \cap Q_p^0 = \emptyset$ and $Q_p^1 \cup Q_p^0 = \{\bar{q}_1, \bar{q}_2, ..., \bar{q}_T\}$.

Agenda selection defines an optimal partition $Q_p^0(\boldsymbol{\Theta}, \bar{y}^{max})$ and $Q_p^1(\boldsymbol{\Theta}, \bar{y}^{max})$, which is a function of the vector of members' ideologies, $\boldsymbol{\Theta}$, and the party discipline technologies represented by the vector $\bar{y}^{max} = \{y_D^{max}, y_R^{max}\}$ where $\|\bar{y}_t^i\| \leq y_p^{max}$ for all i in both parties.

We assume that the random shocks $\bar{\varepsilon}$ are drawn after the partition $\left\{Q_p^1, Q_p^0\right\}$ is designed. We do not need to restrict the game that induces the partition $\left\{Q_p^0, Q_p^1\right\}$ in any way, as long the game includes: i) large N, ii) a random component for the politicians' votes as above, and iii) the shocks are realized after the agenda is set. The first two conditions are used for the statistical identification of the model, as we show below, while the third guarantees that the party has uncertainty about whether a bill gets passed or not. This last condition is empirically relevant, as not all bills that are brought to the floor pass a vote.

The optimal choice of dropping an issue $\bar{q}_t$ from the agenda or challenging it with an optimal alternative $\bar{x}_t$, as well as how the optimal alternative $\bar{x}_t$ itself may be designed, depend upon the specific characteristics of the legislative game considered.

We assume that party p's ideal point coincides with the party median $\bar{\theta}^p$ and that preferences over policy are represented deterministically[19] for a policy $\bar{k}_t \in \mathbb{R}^2$ as:

$$u\left(\bar{k}_t, \bar{\theta}^p\right) = u\left(\left\|\bar{k}_t - \bar{\theta}^p\right\|\right).$$

party D be recognized with probability γ and party R with $1 - \gamma$, where γ can be allowed to vary by Congress or to depend upon party characteristics.

[19] The preferences of the party are better represented as deterministic, as opposed to also having a random component as in the case of politicians, because the former rules out the possibility of an agenda setting party choosing policy x_t ex ante and then preferring q_t to it ex post (and thus whipping against its own bill).

2.2.2 Part (III): Whipping

Party discipline is enforced by each party's whips. Whips are a subset of members of each party that are responsible for the votes of a subset of legislators within the same party. Whips are rewarded $r_p > 0$ for each member under their oversight who votes with the party at t. The party is deep-pocketed, in the sense that the rewards r_p are not scarce, so that no budget constraint (either within or across bills) limits the extent of whipping. The cost of whipping is borne by the whip herself. Each whip bears a private cost, $c\left(\| \bar{\omega}_i - \bar{\omega}_i' \|\right)$ from moving member i from point $\bar{\omega}_i$ to $\bar{\omega}_i'$, where $\| \,.\, \|$ is the same Euclidean norm that enters the utility function (i.e. if members weight the first dimension more heavily, it costs more to move them along this dimension). We assume $c'(\cdot) > 0$ and $c(0) < r_p$. These assumptions ensure that any member that already prefers to vote for the party's preferred policy is not whipped and that a member that prefers to vote against the party's preferred policy will be whipped only if the distance she must be moved to get her to change her position is less than $c^{-1}(r_p)$. Whips have full information about all members preferences and shocks.

Consider the case in which a party prefers the alternative $\bar{x}_t$ to $\bar{q}_t$ (i.e. the party "whips" for $\bar{x}_t$). In the case $d = 2$, the set of members that are whipped are those outside of X_t (the set that prefers $\bar{x}_t$ in the absence of whipping) and such that the distance between the member's ideology to a point within X_t is less than $y_p^{max} \equiv c^{-1}(r_p)$. Because the boundary of X_t is a line, the set of whipped members is the set of members that lie within a distance y_p^{max} of the bounding line. Specifically, using equation (2), if a party p whips for policy $\bar{x}_t$ against $\bar{q}_t$ and $x_{2,t} > q_{2,t}$, we have that the set of members which vote for $\bar{x}_t$ is given by

$$X_{p,t}^{whipped} = \left\{\bar{\omega}_t^i | \omega_{2,t}^i \geq m_t \omega_{1,t}^i + b_t - y_{p,t}\right\} \tag{4}$$

where

$$y_{p,t} \equiv y_p^{max} \sqrt{\frac{w_1 + m_t^2 w_2}{w_1 w_2}}.$$

Let us indicate that a party p whips 'up' (for the policy with the largest second dimension) with the expression $W_{p,t} = 1$; $W_{p,t} = -1$, otherwise. Further define $\mathcal{I}_t \equiv I\left(x_{2,t} > q_{2,t}\right)$, where $I(.)$ is the indicator function. Then we have:

$$X_{p,t}^{whipped} = \begin{cases} \left\{\bar{\omega}_t^i | \omega_{2,t}^i \geq m_t \omega_{1,t}^i + b_t - W_{p,t} \times y_{p,t}\right\} & if\, \mathcal{I}_t = 1 \\ \left\{\bar{\omega}_t^i | \omega_{2,t}^i \leq m_t \omega_{1,t}^i + b_t - W_{p,t} \times y_{p,t}\right\} & if\, \mathcal{I}_t = 0. \end{cases}$$

2.2.3 Part (IV): Voting

Let Y_{it} be a random variable taking value 1 if politician i votes Yes in favor of $\bar{x}_t$, conditional on $\bar{q}_t$ having been selected for consideration (i.e. $\bar{q}_t \in Q_p^1$) by party p, and 0 otherwise.

The probability that i from party p supports alternative $\bar{x}_t$ over the status quo $\bar{q}_t$ is then

$$\Pr\left(Y_{it} = 1 | \bar{q}_t \in Q_p^1, \bar{x}_t; \mathbf{\Theta}, y_p^{max}\right) \quad = \quad \Pr\left(\bar{\omega}_t^i \in X_{p,t}^{whipped} | \bar{q}_t \in Q_p^1, \bar{x}_t; \mathbf{\Theta}, y_p^{max}\right).$$

To calculate this probability, consider that the (signed) minimum distance of a member at $\bar{\theta}^i$ from the boundary line with slope m_t and intercept b_t, is given by

$$\sqrt{\frac{w_1 w_2}{w_1 + m_t^2 w_2}} \left(\theta_2^i - m_t \theta_1^i - b_t + W_{p,t} \times y_{p,t}\right).$$

Given that positive shocks increase $\omega_{1,t}^i$, a positive shock implies $\theta_2^i > m_t \theta_1^i + b_t - W_{p,t} \times y_{p,t}$. Since e_{it} is distributed as a standard normal,[20] we have that the probability a member votes for $\bar{x}_t$ is given by:

$$\Pr\left(Y_{it} = 1 | \bar{q}_t \in Q_p^1, \bar{x}_t; \mathbf{\Theta}, y_p^{max}\right) = \tag{5}$$
$$\begin{cases} \Phi\left(\sqrt{\frac{w_1 w_2}{w_1 + m_t^2 w_2}} \left(\theta_2^i - m_t \theta_1^i - b_t\right) + W_{p,t} \times y_p^{max}\right) \text{ if } \mathcal{I}_t = 1 \\ 1 - \Phi\left(\sqrt{\frac{w_1 w_2}{w_1 + m_t^2 w_2}} \left(\theta_2^i - m_t \theta_1^i - b_t\right) + W_{p,t} \times y_p^{max}\right) \text{ if } \mathcal{I}_t = 0, \end{cases}$$

where Φ indicates the standard normal CDF.[21]

2.3 Identification

This section discusses the identification proof for the two-dimensional case of our model. A formal derivation is provided in Appendix A. Identification of a more complex version of the model in the one-dimensional case is proven in Canen et al. (2020). The analysis can be extended to three or more dimensions, but the set of identifying assumptions needs to increase with the higher parametric requirements.

[20] The use of a standardized distribution is necessary for statistical identification and is a common feature of any discrete choice model. If we used a different normal distribution, we could simply rescale all parameters by the distribution's standard deviation and de-mean the model to obtain the same probability of voting Yes, implying a failure of identification.

[21] With the same expressions, but the sign of y_p^{max} reversed when the party whips for the status quo $\bar{q}_t$, we can construct a likelihood function, provided the direction, $x_{2,t} \lesseqgtr q_{2,t}$, is known at each t. We address this issue in the construction of the full likelihood below.

2.3.1 Preliminaries

The Euclidean norm weights are imposed to be $w_1 = w_2 = 1$. This is an identifying condition, as even the case $w_1 = 1, w_2$ cannot be identified. We emphasize that these weights cannot be identified in the DW-Nominate model either. In fact, even under $w_1 = 1$, $0 < w_2 < 1$ or $w_1 = w_2 = 1$, DW-Nominate is not identified as we show in Appendix B.

Notice also that members' vote probabilities depend on $\mathcal{I}_t$,which is unobserved and must be identified from the data in conjunction with the other parameters. Once $\mathcal{I}_t$ is identified, we know each party's whipping direction, $W_{p,t}$, based on the direction of the leadership votes, as discussed in Section 3. We address the estimation of $\mathcal{I}_t$ in Subsection 2.4.

2.3.2 Main Identifying Assumptions

To identify the parameters $\left\{\mathbf{\Theta}, \{m_t, b_t, \mathcal{I}_t\}_{t=1}^{T}, \{y_p^{max}\}_{p\in\{D,R\}}\right\}$, we assume the following:

Assumptions ID:

1. The ideal points, $\{(\theta_1^i, \theta_2^i)\}$, are not perfectly collinear within at least one party.

2. (i) There exists a politician 0 such that $\bar{\theta}^0 = (0,0)$. (ii) There exists a politician k whose first dimension ideology, θ_1^k, is known.

3. (i) There exists a bill 0 such that $m_0 = 0$. (ii) There exists a bill s such that the slope of the cutline, $m_s \neq 0$, and is such that if parties whip in the same (opposite) direction on bill 0, they whip in the same (opposite) direction on s.

4. Parties D and R whip in the same direction on at least one bill, and in opposite directions on at least one other bill.

In addition, we trivially require that the data include at least two roll calls with cutlines different from $t = 0$ (this restriction is satisfied, as the data includes thousands of bills), and at least one politician with ideology different from $i = 0, k$ (the data include hundreds of politicians). This set of assumptions does not need to be imposed repeatedly for every congressional cycle, only once for any set of congressional cycles that are jointly analyzed.

In terms of intuition, Assumption ID1 is the requirement that two dimensions are in fact necessary. If ideal points are collinear, then the problem is one-dimensional. ID2(i) is a natural location choice, equivalent to the normalization of a single individual fixed effect to zero in standard panel data models. Assumptions ID2(ii) and ID3(i) pin down the rotation of the estimates in the two-dimensional space. In addition, Assumption ID3(i) facilitates identification of the second dimension of ideology, as for bill 0 only the second dimension is relevant. Assumption ID3(ii), together with ID1, ensures sufficient variation in the data to identify more than one dimension. Assumption ID4 is necessary to identify the party discipline parameters from changes in whipping directions. It is possible to show, in fact, that party-specific cutlines can be recovered and that by comparing the relative positions of these cutlines in cases where parties exert their discipline in opposition to each other versus cases where they exert it in the same direction, party discipline can be point identified for both $p \in \{D, R\}$. As is standard in discrete choice models, the underlying normalization of the variance of the utility shock magnitude (implicit in equation 5) pins down the scale of the estimates.

Under these assumptions, Appendix A proves identification of our model in two dimensions. Note here that several innovations in our structure are crucial for identification in addition to Assumptions ID1-4. First, shocks to ideology allow us to forgo any complication due to nonlinearity in $u(.)$ when comparing vote choices, and to maintain general utility functions (e.g. we are not restricted to quadratic or Gaussian). Renouncing the additive separability between the deterministic and stochastic components of the utility function might appear to complicate the analysis, but, as we show, it greatly simplifies it in this instance. Second, the assumption of the orthogonality of the shocks to the cutlines allows us to focus on simple univariate probability functions in describing vote probabilities even when preferences are two-dimensional. Third, the use of the specific information coming from the inference on whipping directions of both parties allows us to separate the individual party discipline parameters.

2.4 Likelihood

We derive the likelihood function for the problem presented in Parts (I)-(IV) of Section 2.2.

Consider the sequences $\{p_1, p_2, ..., p_T\}$ and $\{\bar{q}_1, \bar{q}_2, ..., \bar{q}_T\}$, only partially observed by the econometrician. Without loss of generality, order periods so that all $\{\bar{q}_1, ..., \bar{q}_{\tau-1}\}$ belong to Q^0 and are therefore unobserved, while $\{\bar{q}_\tau, ..., \bar{q}_T\}$ belong to Q^1 and are potentially estimable by the econometrician, as actual votes occurred on these bills.

For the i-th legislator, we observe $T - \tau$ vote choices, $\mathbf{Y}_i = \{Y_{i\tau}, ..., Y_{iT}\}$. Let us now define a theoretical sample likelihood constructed assuming we have complete information. Let γ denote the generic probability that party D is recognized as the proposer. Under full knowledge of the sequence $\{\bar{q}_1, \bar{q}_2, ..., \bar{q}_T\}$, the density for the i-th observation can be theoretically expressed as:

$$
\begin{aligned}
\mathcal{L}^{*}\left(\mathbf{Y}_{i}\right) \;=\; & \prod_{t=1}^{\tau-1}\left[\gamma \Pr(\bar{q}_t \in Q_D^0)\right]^{I[p_t=D]} \times \left[(1-\gamma)\Pr(\bar{q}_t \in Q_R^0)\right]^{I[p_t=R]} \\
& \times \prod_{t=\tau}^{T}\Big[\gamma \Pr(\bar{q}_t \in Q_D^1)\left(\Pr\left(Y_{it}=1|\bar{q}_t \in Q_D^1, \bar{x}_t; \boldsymbol{\Theta}, \bar{y}^{max}\right)\right)^{Y_{it}} \\
& \times \left(\Pr\left(Y_{it}=0|\bar{q}_t \in Q_D^1, \bar{x}_t; \boldsymbol{\Theta}, \bar{y}^{max}\right)\right)^{1-Y_{it}}\Big]^{I[p_t=D]} \\
& \times \Big[(1-\gamma)\Pr(\bar{q}_t \in Q_R^1)\left(\Pr\left(Y_{it}=1|\bar{q}_t \in Q_R^1, \bar{x}_t; \boldsymbol{\Theta}, \bar{y}^{max}\right)\right)^{Y_{it}} \\
& \times \left(\Pr\left(Y_{it}=0|\bar{q}_t \in Q_R^1, \bar{x}_t; \boldsymbol{\Theta}, \bar{y}^{max}\right)\right)^{1-Y_{it}}\Big]^{I[p_t=R]}.
\end{aligned}
$$

Notice, that the terms $\Pr(q_t \in Q_p^0)$ which indicate the status quo policies not pursued by party p cannot be observed in reality. Notice further that, conditioning the vote probabilities on $\bar{x}_t$ implicitly conditions on $\mathcal{I}_t$, which, given data on leadership votes, determines $W_{p,t}$ for each party. In essence, both the parameters pertinent to the recognition and agenda selection components of the model (Parts (I) and (II) of the structure in Section 2.2) and the parameters pertinent to the party discipline and voting components (Parts (III) and (IV)) enter the estimation problem.

As the information concerning Parts (I) and (II) is unobserved, a consistent estimator of ideology, party discipline and the other voting parameters would seem infeasible. Consistent with this view, the literature has suggested that such omission may be consequential to the study of polarization. For instance, Clinton et al. (2014) and others[22] point out that agenda setting may play a key role in producing polarization: politicians may vote more similarly with their co-partisans not because of ideologies or party discipline, but simply because divisive bills are left out of the agenda or bills that clearly separate the two parties are brought forth.

To the contrary, we now show how one can obtain consistent estimates of the vote parameters independent of the policies that are voted upon.[23] As our argument holds independently of how the proposing party is chosen, for illustrative purposes, consider the simplified case of $\gamma = 1$ (i.e. all bills are proposed by the same party D). In this case, the infeasible log likelihood is:

[22] E.g. McCarty (2019) ch. 5, pp.83-84.

[23] If one is explicitly interested in the agenda setting parameters, one can explicitly model the agenda setting process as in Canen et al. (2020).

$$
\begin{aligned}
\log \mathcal{L}^* \quad (\mathbf{Y}_i) = \quad & \sum_{t=1}^{\tau-1} \log\left(\Pr(q_t \in Q_D^0)\right) + \sum_{t=\tau}^{T} \log\left(\Pr(q_t \in Q_D^1)\right) \\
& + \sum_{t=\tau}^{T}\sum_{i=1}^{N} \left[Y_{it} \log\left(\Pr\left(Y_{it}=1 | \bar{q}_t \in Q_D^1, \bar{x}_t; \mathbf{\Theta}, \bar{y}^{max}\right)\right)\right. \\
& \left. + (1-Y_{it}) \log\left(\Pr\left(Y_{it}=0 | \bar{q}_t \in Q_D^1, \bar{x}_t; \mathbf{\Theta}, \bar{y}^{max}\right)\right)\right].
\end{aligned} \tag{6}
$$

The log likelihood (6) is separable. The double summation corresponds to the conditional likelihood of roll call votes based on the selected status quo $\bar{q}_t$ that are brought to the floor for a vote, and the corresponding selected alternative $\bar{x}_t$. This likelihood component corresponds to Parts (III) and (IV) of the structure in Section 2.2.

Define $\Xi = \{m_t, b_t, \mathcal{I}_t\}_{t=1}^{T}$.[24] Consider maximizing the (feasible) conditional likelihood $\mathcal{L}$ of individual vote decisions:

$$
\begin{aligned}
\log \mathcal{L}(\mathbf{Y}_i) = & \sum_{t=\tau}^{T}\sum_{i=1}^{N} \left[Y_{it} \log\left(\Pr\left(Y_{it}=1 | \mathbf{\Theta}, \Xi, \bar{y}^{max}\right)\right)\right. \\
& \left. + (1-Y_{it}) \log\left(1 - \Pr\left(Y_{it}=1 | \mathbf{\Theta}, \Xi, \bar{y}^{max}\right)\right)\right],
\end{aligned} \tag{7}
$$

where $\{\mathbf{\Theta}, \Xi, \bar{y}^{max}\}$ is the set of parameters to estimate. Equation (7) can be used to consistently estimate $\{\mathbf{\Theta}, \Xi, \bar{y}^{max}\}$ based on vote data alone even if (i) the range of party discipline y_p^{max} influences the selection decisions of status quos (i.e. the sets $\{Q_D^0, Q_D^1\}$), and (ii) the policy alternatives $\bar{x}_t$ are endogenously set.

The key reason for this result is that each m_t, b_t, and $\mathcal{I}_t$ can be consistently estimated from the vote data alone so that it does not matter how they arise through agenda selection. Each of these parameters can be estimated because (i) preference shocks realize independently after the selection of the status quo, $\bar{q}_t$, and of the alternative, $\bar{x}_t$, have occurred, and (ii) the support of the preference shocks is unbounded – so that no matter the choices of $\bar{q}_t, \bar{x}_t$ the probability that each politician votes for either alternative is non-zero.

To see the intuition for this result, consider a one-dimensional environment and two politicians i and j, with $\theta^i < \theta^j$. Take a Congress where only one bill is voted upon repeatedly T times so that we observe only one cutline m. No matter how extreme the cutline, nor how it is selected by the agenda setter, if one shocks the politicians with full-support shocks, each politician, i, will cross the cutline with a certain frequency given by the distribution of the shocks and her ideal

[24] Notice here that for each bill we can characterize vote choices as functions of the three parameters m_t, b_t, and $\mathcal{I}_t$, rather than the four parameters in $\bar{q}_t, \bar{x}_t$. We therefore have one less parameter per bill, which facilitates identification and estimation.

point location relative to m. The politician with θ^j immediately to the right of θ^i will cross the cutline as well, but with a slightly different frequency. If in the next Congress, the agenda setter changes the cutline m, then the frequencies will change, but θ^i and θ^j cannot change given the structure and the nature of the shocks: the vote probabilities will adjust for the different cutline accordingly. Given unbounded shocks and large T, no two politicians with different ideologies can have identical voting records, no matter which bills are proposed: the ideal points will be separated asymptotically.[25]

Finally, notice that using the definition (5), $\mathcal{I}_t$ can be simply estimated as selecting for every bill t, $I(x_{2,t} < q_{2,t}) = 1$ if

$$\begin{aligned}\sum_{i=1}^{N} &\left[Y_{it} \log\left(\Pr\left(Y_{it}=1|\boldsymbol{\Theta}, m_t, b_t, 1, \bar{y}^{max}\right)\right)\right.\\ &\left.+\left(1-Y_{it}\right)\log\left(1-\Pr\left(Y_{it}=1|\boldsymbol{\Theta}, m_t, b_t, 1, \bar{y}^{max}\right)\right)\right] \quad >\\ \sum_{i=1}^{N} &\left[Y_{it} \log\left(\Pr\left(Y_{it}=1|\boldsymbol{\Theta}, m_t, b_t, 0, \bar{y}^{max}\right)\right)\right.\\ &\left.+\left(1-Y_{it}\right)\log\left(1-\Pr\left(Y_{it}=1|\boldsymbol{\Theta}, m_t, b_t, 0, \bar{y}^{max}\right)\right)\right]\end{aligned}$$

and $I(x_{2,t} < q_{2,t}) = 0$ otherwise. By calculating the likelihood for each $\mathcal{I}_t$, we avoid estimation of a binary parameter.

Consistency of the estimator for $\left\{\boldsymbol{\Theta}, \{m_t, b_t\}_{t=1}^{T}, \bar{y}^{max}\right\}$ is guaranteed for large $T-\tau$ and N. The requirement for a large number of bills, which holds in our application, is to estimate each $\bar{\theta}^i$ consistently by MLE without nuisance parameter problems (Fernández-Val and Weidner, 2016). Further, as N is also large, one can also consistently estimate all elements of $\{m_t, b_t\}_{t=1}^{T}$ and $\bar{y}^{max}$.

2.5 Comparison to Other Established Methodologies

Here we discuss how our methodology contrasts with established methodologies in the literature, focusing on three main approaches. As a first point of departure, note that none of the approaches below incorporates a role for party discipline in our current form.

The first method for comparison is the Bayesian approach of Clinton et al. (2004). This approach posits quadratic preferences for the deterministic component of utility and normal idiosyncratic shocks. We share the use of the latter, but do not need to impose a quadratic utility function. The authors' use of Markov Chain Monte Carlo methods to estimate posterior densities, typical of Bayesian methods, is also in sharp contrast to our setup in terms of identification. The

[25] It follows then that agenda-setting can only potentially affect estimates in finite samples (as demonstrated in the simulations of Clinton et al. (2014)). Given our very large T, finite sample effects are likely negligible, as confirmed by our Monte Carlo simulations.

Bayesian approach allows the authors to sidestep classical identification issues, but also requires the reader to trust the assumed priors. When the authors extend their approach to allow for parties to discipline votes, they assume (as in Snyder and Groseclose (2000)) that lopsided votes are not whipped in order to be able to identify (only) the net effect (Republican-Democrat) of party discipline. By incorporating the leadership positions to identify whipping directions, we do not need to assume some votes are not whipped and can individually identify the discipline exerted by each party.

Heckman and Snyder (1997) share our classical approach: their structurally-derived linear probability model is close in spirit to this paper. Yet their assumptions of quadratic preferences and additive separable uniform shocks are differ from ours. We introduce non-separable additive shocks in the argument of the utility functions, an innovation that helps in terms of identification and estimation of the explicit effects of discipline. The usefulness of our approach comes in two forms. First, we do not impose restrictive utility functions. Second, it allows for a simple characterization of the cutline in equation (4), becoming a function of an intercept, slope, and direction, rather than a function of $\bar{q}_t$ and $\bar{x}_t$. With two dimensions, this simplification reduces the number of parameters by one for each bill. Finally, for their analysis with an unobservable number of policy dimensions, the authors implement their linear model as a factor model under an orthornormality assumption.[26]

The most influential and cited approach in the analysis of congressional behavior and political polarization is arguably DW-Nominate (Dynamic Weighted NOMINAL Three-step Estimation), a method that has gone through multiple incarnations (Poole and Rosenthal, 1997, 2001; McCarty et al., 2006) and is at the core of the path-breaking VoteView.com repository. This well-established methodology relies on somewhat unique assumptions, however. Politicians' preferences are given by a Gaussian function (which are not globally concave). The model is also often written as if multiple policy dimensions could be estimated from the vote data without increasing the identification requirements.

An unappreciated consequence of the former assumption is that strong nonlinearity in the preference parameters immensely complicates identification when one tries to map choice data into the model structure, even absent the weighting of different policy dimensions (the W, for Weighted in the name) or linear trends in legislator preferences (the D, for Dynamic in the name).[27] In fact, to the best of our knowledge, no formal proof of identification for the Nominate method exists in

[26] The authors estimate six latent policy factors using χ^2 and AIC methods. These tests however are known to produce over-estimates of the number of factors in small and medium samples. More conservative modern tests for the number of latent factors could be implemented to re-assess their PCA analysis (for instance, the eigenvalue ratio method of Ahn and Horenstein (2013)).

[27] Heckman and Snyder (1997) discuss the problem arising from the nonlinearity of the estimator explicitly in their analysis and point to its consequences for consistency of the MLE estimator.

two dimensions or higher. Indeed, we prove in Appendix B that DW-Nominate in two dimensions is not identified. We show that a specific nonlinear transformations of the parameters can in fact change the DW-Nominate ranking of legislators along any dimension. Notice further that this difficulty is not resolved by imposing additional identifying restrictions, such as that legislators' ideal points need to be constrained to lie within a unit circle. In fact, this often-emphasized "unit circle" identification constraint operates as an additional source of distortion: legislators are not allowed to simultaneously be extreme on both policy dimensions, as they would fall outside the circle. A substantial share of politicians are located at the artificial boundary of the circle (7% of our sample from the House, and approximately 8% of our sample from the Senate lie on the boundary) and all estimates are affected by this restriction through comparisons to the subset of politicians located on the boundary. We provide further details and discussions in Appendix B.[28]

After experimenting with replications of the DW-Nominate approach on our part, we can only surmise that see the lack of identification of the preference and (therefore bill) parameters is being disciplined by the addition of external information about the locations of a number of (initial) politicians. According to Boche et al. (2018) "*It has been said that Poole himself was the 'outer loop' of this estimation process: his judgment and expertise were required in the estimation of the original values*" (p.24). The additional identifying information of this outer loop continues to be important in estimates for new bills and legislators today. In the current VoteView.com structure, Boche et al. (2018) avoid any adjustment in ideal point estimates for past members when new voting information is added (no "*back propagating*", p.24).

3 Data

Our data on roll call votes for both the House of Representatives and the Senate comes from VoteView.com. This standard dataset was originally created by Keith Poole and Howard Rosenthal (Poole et al., 1997), who collected the roll call votes for each member of Congress over time and made them widely available.[29] We map these votes to the binary variable $Y_{i,t}$ (politician i voting Yes or No on roll call t) in the model and employ all roll call votes available.

Figure 3 shows the number of roll call votes over time in each chamber. The number of roll calls in the Senate increases from just under 200 in Congress 70 to a peak of almost 1,500 by Congress 94, before settling to around 500 in more recent Congresses. For the House, the average number of roll calls increases from around 200 in Congress 70 to around 1,200 in recent times. Regarding

[28]For instance, see Figure 22 in Appendix B for an illustration of this problem. In summary, the unit circle limits the correlation of ideologies across both dimensions as no legislator can be set at $(1, 1)$, for example. The most extreme legislator in both dimensions would be located at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, implying that they would seem less extreme in some dimension than legislators $(0, 0.8)$ and $(0.8, 0)$, for example, even though that may not be the case.

[29]See Boche et al. (2018) for a recent overview.

agenda setting, we present summary statistics for bills in Table 2 in Appendix D, including the number of bills introduced, approval rates, and the number of bills passed in a congressional cycle, for both the House and the Senate, from the mid 1940's until the early 2010's. This data is drawn from the Vital Statistics on Congress by the Brooking Institute. In both chambers, the approval rate of bills has dropped sharply: for the Senate, from over 50% in Congress 80 to around 10-20% more recently, and from around 20% to under 6% in the same time period for the House.

We restrict our sample to the post-WWI period from 1927 (Congress 70) to January 2019 (the end of Congress 115). We impose this restriction because our identification strategy requires clear party leadership positions for every roll call (necessary to obtain whipping directions, as described below). Formal leadership positions were not fully consolidated until the 1920's (Evans, 2018, ch.1). In the Senate (the focus of our main quantitative exercises in two dimensions), the first Republican leader was only officially nominated in 1925 (the beginning of Congress 69), while the first Democrat party leader was elected in 1920 (see Senate, 2020). Since the first Republican leader (Charles Curtis) was elected months into Congress 69, we begin our sample in Congress 70. For similar reasons, we maintain the same restrictions when using data from the House of Representatives.[30]

To determine the whipping directions, $W_{p,t}$, we make use of leadership votes. For each roll call vote, we code whether the party leadership voted Yes or No using the decisions by the Majority and the Minority Leader. When such votes are unavailable, we use the Majority or Minority Whip's vote instead, and when that is also missing, the direction of the vote of the majority of the party. For the Senate, out of 25,824 roll calls in our time period, only 2,181 votes do not have the Democratic Leader's vote, 1,388 do not have the Republican Leader's vote, 161 do not have the vote of either the Democratic Leader or the Democratic Whip, and 355 do not have the vote of either the Republican Leader or Republican Whip. Out of 32,763 roll calls in the House, only 2,808 do not have the vote of the Democratic Leader and 285 have neither that of the Democratic Leader or Whip. For the Republicans in the House, 2,502 roll calls do not have the Republican Leader and 429 do not have either the Republican Leader or Whip. [31] Whipping directions are

[30] While party leadership in the House of Representatives was already formally established by both parties by the late 1890's, the scope, powers and election of those leaders changed significantly between 1900 and 1920. Most notably, the Democratic Party instituted elections for Majority Leader in 1911 (Congress 62) to limit the power of the Speaker (initially, the Majority Leader was appointed by the Speaker). Meanwhile, the Republicans only began electing Majority Leaders in the House in 1923 (United States House of Representatives History and Archives (2020b)). There were also changes in the committee membership and selection of Majority Leaders: between 1899-1919, the Majority Leader was also the chairman of the Ways and Means Committee regardless of party, although from 1919 onward it became commonplace that such leaders would not serve in committees. Finally, we face data limitations when using data earlier than 1921: no official records for the Democratic Whip between 1909-1921 exist due to missing documentation (see United States House of Representatives History and Archives (2020a)).

[31] The choice of using the Majority and Minority Leaders as the main information source for leadership behavior follows such seminal work as Cox and McCubbins (1993). We show in Section 4 that using only votes where both the Leaders and Whips agree yields qualitatively and quantitatively similar results. Another potential alternative

then based simply on how the leader votes and the direction of the vote (which is estimated by the maximum likelihood estimator in equation (7)). If the leader says Yes, the whipping direction is in the direction of Yes. If the leader says No, it is in the opposite direction (i.e. towards the direction of the No vote). This coding characterizes the random variable $W_{p,t}$ and allows us to generate subsets of bills where leaders from both parties whip in the same/opposite directions. \

In Figure 3, we provide summary information on the variation in whipping directions in our sample. We present the number of roll call votes available in each Congress and then decompose this number into votes for which the two party leaders voted identically and differently. This decomposition is informative about the amount of variation available in the data, which is important because identification of the party discipline parameters requires both types of votes. We see that we have a large sample of each type of vote. Although it varies over time, approximately 40% of roll calls have both leaders whipping in the same direction. Figure 3(b) shows the same information for the House of Representatives, again indicating many roll call votes in each group. The amount of data for the House is much larger than that for the Senate, with many more roll calls per Congress, and 435 member voters per roll call versus 100.

We are use all roll call votes in the sample to estimate both the two-dimensional model for the Senate and the one-dimensional model for both the House and the Senate. The computational cost of estimating our model increases sharply when moving to the two-dimensional case. Both the number of ideology parameters and the number of bill specific parameters double which precludes estimation of our two-dimensional model for the House for the time being. However, as computational power is constantly improving, our approach should soon be feasible for two dimensions in the House as well.

To give a better sense of the dimensionality of our problem, in Table 1 we include the total number of parameters estimated in our roll call analyses. It reports all classes of parameters for the Senate (two-dimension and one-dimension model) for the period 1927-2018 (i.e. up to Congress 115th) and for the House of Representatives (one dimension model) for the period 1927-2018.

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would be to use the median party member's vote. However, this approach is problematic because we can only identify the median member after performing the estimation. Finally, one could simultaneously use the votes of the Majority/Minority Leader, Majority/Minority Whip and other ranking members of the party together to jointly determine the whipping direction. Unfortunately, it becomes unclear how to treat aggregate that data when there are missing votes of one or more members. We clarify that most of the missing values for Majority/Minority Leadership votes is due to unclear or missing data on leadership, particularly due to leadership transitions in the middle of a Congress, where the timing of a particular roll call is hard to assess (i.e. before or after the transition). For instance, in the middle of Congress 87, Majority Leader John McCormack became the Speaker of the House. As Speaker, he did not vote on roll calls. However, the previous Majority Whip (Carl Albert) became the Majority Leader, so using his votes when McCormack's are unavailable is still appropriate.

4 Results

Our main application for the empirical analysis is the U.S. Senate model in two dimensions, but we also include results for the one-dimensional House and Senate models. We refer to the two-dimensional model as 2D and the one-dimensional as 1D.

We estimate the likelihood presented in Section 2.4 jointly for the 70th-115th Congresses. Given the number of parameters to be estimated, ensuring global convergence for every set of starting parameters is not guaranteed. Therefore, we evaluate the estimation results for many sets of starting parameters, finding similar estimates across many runs. We also performed extensive Monte Carlo simulations of the model to prove that all parameters of the data generating process can be recovered, providing additional assurance that the model is identified. We provide more details on the implementation of our estimator in Appendix C.

4.1 Party Discipline and Polarization

The large number of parameters (see Table 1) requires us to focus on the parameters of most interest. We begin with the party specific discipline parameters, $\bar{y}^{max} = \{y_D^{max}, y_R^{max}\}$. We estimate a different vector $\bar{y}^{max}$ for each congressional cycle, therefore allowing discipline to vary across parties and time. Figure 4 reports the point estimates for party discipline in the Senate 2D model for the time period 1927-2018 together with a smooth fit line to show the trends in party discipline for each party.

Figure 4 illustrates fairly persistent, but evolving, levels of party discipline for the two main American political parties. For both parties, we observe a U-shaped profile over our time period. Neither party appears to lead or lag the other, with substantial contemporary correlation (0.515), but typically higher party discipline for the Republican Party in the Post-War period. Party discipline appears to be declining until the early 1970s, increasing until the end of the 1990s, and then takes on an even steeper increase more recently. Interestingly, this time series evidence accurately fits more descriptive analyses, like the one in Sinclair (2014). The inflection points in the time series match the qualitative discussions of Congress experts, with a sharp separation between the Committee ascendancy period of 1933-1960 to the period of stronger leadership and realignment of 1960-1994 to the modern 1994-2018 Congress (Deering and Smith, 1997; Jenkins, 2011; Sinclair, 2014; Evans, 2018).[32]

[32] For example, Jenkins (2011) specifically mentions rule changes that affect the organization of the House and Senate over the 1960-1994 period ("*To control proceedings, the leadership began relying on special (restrictive) rules to structure debate and floor voting*") and in explaining the uptick in polarization for the post-1994 period ("*... as Senate parties have become more effective in recent years at steering the legislative agenda toward party cleavage issues—those on which there is internal party unity and wide divergence between the two parties—a strengthening of formal leadership structures in the Senate has also occurred, with party caucuses meeting more frequently and*

All the point estimates of party discipline are statistically significant (p-values < 0.001)[33], implying that the data strongly rejects, for every single congressional cycle in our sample, the null hypothesis of the absence of party discipline. This fact remains true even at discipline's historical lows of around $0.3 - 0.4$ units in the 92nd-95th Congresses (1971-1976). More recent estimates are historical high points, between 1.5 and 2. The 2018 level of y_R^{max}, for example, is 2.04, indicating a substantial ability of the Republican leadership to reach far into the set of (potentially) dissenting members. Intuitively then, even ideologically moderate Republican members of the 115th Senate, such as Sen. Susan Collins or Sen. Mitt Romney, may appear more conservative in terms of their vote profile along the first dimension than they truly are because of the powerful reach of the Senate leadership.

Our second main result is the time series of ideological polarization reported in Figure 6 for the Senate 2D model over the 1927-2018 period. As with DW-Nominate and other methods, our approach requires to specify location, scale, and rotation through normalizations (Assumptions ID of Section 2.3). Although our assumptions pin down a rotation, such rotation is arbitrary, as it depends on the particular normalizing bill chosen to have $m_0 = 0$. Thus, to make our results more comparable to DW-Nominate and to others in the literature (a comparison we return to in the next section) – which is required for the correct interpretation of the correlations between approaches[34] – we rotate our estimates using the Procrustes rotation of our ideology estimates onto those of DW-Nominate. Procrustes analysis is a popular and theoretically founded approach to comparing these two multidimensional scaling methods (Goodall 1991; Kendall 1989). A Procrustes rotation minimizes the sum of the squared differences between points in our matrix of estimates and the DW-Nominate matrix, which constitutes the reference space.

We derive estimates of ideological polarization from our estimates of politicians' ideologies, noting that we assume that these ideologies are constant across Congresses (and serve to create intertemporal linkages across Congressional cycles). We focus here on polarization in the first dimension, but also report results for polarization along the second dimension (in Figure 7). Following the standard in the literature, we define ideological polarization as the difference between the ideological positions of the median Republican and the median Democrat in each dimension.

The most salient fact in Figure 6 is the steady growth of ideological polarization over the

enhanced resources (both funds and staff levels) being devoted to party leadership offices.") (p.13). Also see Canen et al. (2020) and the references therein for a discussion of rule changes in Congress that strengthened party leadership over the 1970s. Such rule changes, which occurred both in the House and Senate over the 1970s, include megabills, omnibus legislation, and time-limitation agreements, allowing leaders more control over the party rank-and-file and the agenda.

[33] We estimate the variance of the parameters using the empirical counterpart to the asymptotic variance of the MLE, as is standard.

[34] As linear correlation is dependent on the rotation of the data, calculating the naive correlation of our first dimension estimates and DW-Nominate's first dimension would be uninformative. Appropriate transposition of our estimates into the DW-Nominate space is necessary.

sample period. Ideological polarization appears to double approximately every forty years. This result is inconsistent with the the U-shaped dynamic reported by McCarty et al. (2006) which was obtained through DW-Nominate estimation. Our results imply that the standard intuition that more moderate members are increasingly replaced with more extreme ones appears correct (although with lower absolute levels due to the presence of party discipline). However, theories of ideological polarization that revolve around non-monotonic dynamics are less robust. Rather, our analysis suggests that the U-shaped profile observed in estimates where party discipline is ignored is in fact due to party discipline itself changing non-monotonically.

The U-shaped profile in party discipline is confirmed with both of the 1D Senate and 1D House models, and is in fact more marked in these instances (see Figure 5(a) for the Senate 1D model and Figure 5(b) for the House 1D model). Qualitative studies for the House, like Sinclair (1992), match the timing and the sign of the time derivatives of our estimates. Figures 24 and 25 in Appendix D report the time series for ideological polarization for the Senate and House 1D models, showing remarkably similar profiles to the results of our Senate 2D model.

To put the magnitudes of the party discipline parameters into perspective, we plot the share of polarization attributable to party discipline (i.e. total party discipline divided by party discipline plus ideological polarization) in Figure 8. As first demonstrated in Canen et al. (2020), the denominator of this measure is the ideological polarization one would obtain with a model that ignores the role of party discipline (a "misspecified" model that we turn to the next section), a consequence of the fact ignoring whipping results in a misattribution of vote differences to difference in ideologies across parties.[35] The share of polarization attributable to discipline has highs of over 80% in the 1930's, falls to around 60% in the Civil Rights Era, and is between 65-75% in recent decades. Results for the Senate and House 1D models are quantitatively similar (Figure 26 in Appendix D).

In Figure 9, we report the ideology of the median member in each party and further split the Democratic Party into the Southern Democrats and Northern Democrats, to emphasize this important component of historical heterogeneity within that organization. The well known ideological convergence between Southern Democrats and the Republican party along the first ideological dimension is evident in Figure 9.[36]

To provide a more complete presentation of the distributions of ideological preferences along the two policy dimensions, we report the kernel density estimates for the two parties over time. The first dimension marginal distributions are reported in Figure 10, and the second dimension distributions in Figure 11. We report only the 2D Senate model for brevity. Not only have the

[35]Neglecting discipline shifts the ideologies of all members of a party by the same amount because of unbounded ideology shocks: each member will, with some probability, be subject to discipline on every bill.

[36]We report analogous figures for the second dimension in Appendix D.

first moments of the Democratic and Republican Parties been diverging over time, most visibly from the 95th Congress (started in 1977) in Figure 10, but the variances in the first dimension of each party have also fallen over time. Our model is consistent with the extant literature for these well-established facts.

In terms of symmetry, the pattern of ideological polarization does not appear to be driven by one party relative to the other. Instead, both Republican and Democratic parties contribute to the ideological divergence highlighted in Figure 9. The extant literature has discussed asymmetries in voting polarization based on DW-Nominate (Grossmann and Hopkins, 2016), but they appear driven by a marginally higher party discipline parameter for the Republican Party in the last part of the sample and not by asymmetric ideological divergence.

In summary, our first group of results shows that party discipline has played a significant role over time, particularly in recent Congresses. The data clearly rejects models that omit party whipping. While we confirm standard findings in terms of a recent increase in ideological polarization, existing results of non-monotonic and asymmetric dynamics appear unsupported by the data once the role of parties is included in the analysis.

4.2 Comparison to DW-Nominate

We compare our results to those of the DW-Nominate method. Recall that a comparison of our 2D estimates to those of DW-Nominate is appropriate because we analyze our estimates after a Procrustes rotation on to DW-Nominate's space. Nevertheless, we must emphasize that this basis for comparison is not unique – using other rotations would likely produce similar, but not identical results.

Figure 12 reports the time series of polarization in the first and second dimensions according to DW-Nominate and a third model, which we call the "misspecified" model. The misspecified model implements our main model with a constraint of no party discipline. It is therefore an identified version of our two-dimensional model that is directly comparable to DW-Nominate in that it lacks a role for parties. However, as shown in this figure, the misspecified model does not replicate the early sharp decline in liberal-conservative polarization that so typically defines the time series for DW-Nominate over the 20th century. This confirms our findings from Figure 6 for our benchmark model, and also suggests that the lack of a U-shape in ideological polarization could also in part be due to our model being identified (further details in Appendix B).

Figures 13 and 14 provide scatter plots of our first dimension estimates versus those of DW-Nominate, both for our baseline and misspecified models. Figures 15 and 16 present the same scatter plots for the second dimension estimates. The first dimension estimates of the misspecified model align reasonably to those of DW-Nominate, but in our model with party discipline, a sizable gap opens up between members of the two parties located at the same first-dimensional ideological level. This gap is driven by the fact that our model recognizes that individuals who have the same preferences, but belong to different parties, are often whipped in opposite directions, appearing less moderate. Ignoring party discipline, DW-Nominate misattributes the difference in voting behavior exclusively to differences in preferences, as does our misspecified model. This shift is ultimately responsible for the mismeasurement of ideological polarization in DW-Nominate, and leads to a different interpretation of the data.

Pairwise rank correlations between model estimates in the first and second dimension are also informative. Notice, however, that these correlations paint a different picture than the location of the marginal densities or consistency of the estimated ideology parameters. Rank correlations simply capture the similarity in rankings of politicians between methodologies. The rank correlation of the first dimension of ideological positions of our baseline model (after imposing the rotation) and DW-Nominate is 0.857. This high correlation means that that our ordering and that of DW-Nominate are quite similar along the first dimension. As the ordering of legislators along the first dimension is probably the most widely-accepted feature of DW-Nominate, we find this correlation reassuring. On the other hand, the rank correlation of second dimension ideological positions across models is much lower, 0.435. This low correlation is most likely because the second dimension of ideologies and the cutline parameters appear the most sensitive to the lack of identification in DW-Nominate. One plausible explanation is the short time period over which this second dimension makes up an important feature of the legislative voting data (the 1960s and 1970s), while the first dimension appears relevant for the entire sample period. Lack of identification unsurprisingly translates into less predictable estimates along the second dimension. Comparing DW-Nominate to our misspecified model produces similar results. The rank correlation along the first dimension is higher at 0.910, but the correlation along the second dimension is slightly lower at 0.365.

4.3 Fit and Robustness

We assess the in-sample fit of our empirical model congressional cycle by congressional cycle, further quantitatively validating our approach. We begin by reporting the time series for the in-sample fraction of correctly predicted roll call votes in each Congressional cycle in Figure 17. The share of correctly predicted votes increases over time, with at least 80 percent of all individual choices being correctly predicted in any cycle. The share of correctly predicted votes in 2018 reaches about 95 percent of all votes cast, which is extremely high.

However, it is important to remark that the ability to predict votes to a high degree may not necessarily be fully indicative of model quality, especially with respect to bias along certain dimensions and the distributions of congressmembers. The increase in ideological polarization over the past forty years may allow for high levels of correct predictions of a binary decision even with biased estimates. DW-Nominate also has excellent predictive power, yet we have shown that its estimates of preference parameters are biased by the omission of party discipline, an important feature of the data. The misattribution by omission can be substantial - as discussed in Section 4.1, party discipline makes up on average, $65 - 70$ percent of voting polarization from the misspecified model (Figure 8) over the entire period (with the remaining $30 - 35$ percent being correctly attributed to ideological polarization).

Apart from the standard identification assumptions discussed in Section 2.3, our results may depend on the way in which we construct the whipping direction variable, $W_{p,t}$. To probe the reliance of our estimates on this variable's exact definition, we consider alternative whipping directions based on suggestions within the extant literature. We re-estimate our model under three alternative scenarios: (i) no whipping (i.e. $W_{p,t} = 0$) on lopsided votes (where lopsided is defined as either a 65/35 percent vote or a 70/30 percent depending on the size of the majority party),[37] (ii) dropping votes where a party's Leader and Whip voted in different directions and (iii) dropping votes where the leaderships of both parties vote in the same way (in this case we can only identify the aggregate amount of party discipline, $y_D^{max} + y_R^{max}$). Each of these specifications probes a particular assumption about either our modeling of whipping or $W_{p,t}$. The first specification tests whether our results rely on the assumption of whipping on every vote. It does so by incorporating an idea that has received extensive attention in the literature following Snyder and Groseclose (2000), but still maintaining identification of the party discipline and ideology parameters. The second specification tests the robustness of the empirical construction of $W_{p,t}$ itself. The econometrician does not observe the exact direction of party discipline. Instead, we currently proxy it by leadership votes. This proxy might seem less appropriate when leaders within the same party disagree (e.g. the Majority Whip's decision differs from the Majority Leader's). One particular reason for this difference in voting could be the use of a motion to reconsider in the Senate, whereby a senator on the prevailing side or who did not vote can motion for a revote. This may incentivize a leader to vote against his/her preferred policy in order to preserve the possibility of a future revote.[38] Finally, the last specification tests whether our results

[37] This is a a specification inspired, but different, than the one presented in Snyder and Groseclose (2000). In contrast to their work, identification of this specification does not rely on comparing voting behavior of the same legislators in lopsided and non-lopsided votes, a source of weak identification due to the lack of variation in voting behavior in lopsided votes (McCarty et al. (2001)). Instead, our parameters for party discipline (y_p^{max}) are identified by information on the leadership voting/whipping directions within non-lopsided votes. As a result, individual ideologies are recovered from average voting behavior conditional on discipline, using information on both lopsided and non-lopsided votes.

[38] Only a senator on the prevailing side or who did not vote can motion to reconsider. In most cases, this motion is pro-forma: after it gets proposed, another senator who voted alike immediately motions to table it. This dual procedure guarantees that the first vote is final (i.e. it will not be revoted). See Schneider and Koempel (2012) for

are driven by the roll calls where opposing leaders vote the same way. Such votes may be less confrontational or salient, requiring different types of party discipline.

We present the results for total discipline $y_D^{max} + y_R^{max}$ across models in Figure 18, and the results for individual party disciplines for (i) and (ii) in Figure 19 (i.e. the specifications where the individual parameters are identified). It is clear that our quantitative and qualitative results are remarkably similar across specifications, assuaging concerns that the results described in Section 4.1 are due to a particular definition or construction of $W_{p,t}$, or how we model whipping behavior.

Finally, we compare our benchmark estimates of party discipline to those from Canen et al. (2020), which derives identification from information contained in detailed internal party records before floor votes (whip counts, as cataloged by Evans, 2018). This comparison is possible only for the short subsample in which both sets of results are available: for both parties in the House of Representatives between 1977-1986 (i.e. Congresses 95-99). Figure 27 in Appendix D shows that the estimates of party discipline are remarkably correlated across identification strategies, with a linear correlation of 0.878,[39] and that the different identification strategies produce quantitatively similar estimates of the role of parties. This result is reassuring in that it demonstrates the robustness of our approach, and that our more parsimonious method is picking up informative variation on the direction of party pressure from the data.

4.4 Implications for Theories of Party Organization

Our results also allow us to speak to different theories of political party organization. Such theories for the most part have remained either theoretical or have been guided by less formal quantitative approaches (Sinclair, 2014). We do not aim here for a complete analysis of the historical determinants of party discipline, as this would be beyond the scope of the paper, but include this discussion to demonstrate the potential value of having estimates of party power.

Figure 20 reports evidence of a inverse U-shape time series in the variance of the first dimension of ideologies within each party, in contrast to the U-shape in party discipline of Figure 4. This negative correlation between the time series of party discipline and within party variance along the liberal-conservative dimension is strong and statistically significant for both Republicans and Democrats.[40] This result would seem to be in line with predictions from the Conditional Party

details. Nevertheless, a Majority Leader may sometimes deviate from his/her preferred vote in order to file a motion to reconsider. For example, when (s)he is about to lose a vote in the Senate, (s)he might prefer to switch sides and vote with the opposition, preserving the possibility of a future revote due to a motion to reconsider. A recent example was Mitch McConnell's vote with the Democrats in the failure to confirm Judy Shelton's nomination to the Federal Reserve Bank (see: https://www.washingtonpost.com/business/2020/11/17/shelton-fed-mcconnell/). This motion is pro-forma in the House of Representatives (Schneider and Koempel, 2012).

[39] To make the results comparable, we scale up the estimates from Canen et al. (2020) by a factor of $\sqrt{2}$ because of differences in the way in which the ideologies and party discipline parameters were scaled in the two models.

[40] The estimates from separate regressions of y_p^{max} on the variance of ideology estimates for party p are -9.218 for

Government theory of Aldrich (1995) and Rohde (1991). The theory states that legislators delegate more agenda setting power to leaders when the party is more ideologically homogeneous – exactly the pattern that our results seem to indicate. The intuition is that, as party members become more aligned, it is more beneficial to yield power to leaders who are more likely to advance commonly desired policies. To explain the trends in the data, one could hypothesize a dynamic version of this argument: increases in party discipline due to more homogeneous parties may induce the exit of moderate members, increasing ideological homogeneity even further. Increasing homogeneity could then lead to a further increase in party discipline, and so on, in a self-reinforcing mechanism.

We find a high degree of correlation between party discipline across parties (0.515) in our sample. This correlation is high even though there is extensive evidence of technological innovations during this period, including the introduction of focus-group tested languages and coordinated vocabularies by the 1994 Revolution Republicans (see Gentzkow et al., 2019). Because of these innovations, one might have thought that increases in discipline would have come first for the innovating party, followed by the other (as seen by the adoption of these tactics by Democrats). Although still possible, the high correlation in discipline across parties suggests that such technological innovations disseminate quickly across the political spectrum.[41]

Finally, we expect that our approach could prove fruitful to testing other existing theories of party behavior. Our model recovers consistent estimates for y_p^{max} without imposing structure on its explanatory sources (e.g. majority status or divided government). As a result, we can use it as a dependent variable in a regression framework to test such sources. Table 3 in Appendix reports the estimates of such an exercise. To highlight one result, we find suggestive evidence that unified and divided governments have similar party behavior. This finding is consistent with Krehbiel (1998) and Mayhew (2004), but in contrast to work as Sundquist (1988), who argue that there is something institutionally different about party behavior when the president's party does not coincide with the majority in Congress.

Democrats and -3.529 for Republicans. Robust standard errors are 2.881 and 1.305, respectively.

[41] In fact, qualitative evidence suggests that this spread may not be constrained to the U.S. alone – other countries often adopt the same American legislative tactics and electoral innovations in their own campaigns and legislative proceedings. For example, in the early 2000s, Silvio Berlusconi in Italy applied similar public relations techniques to the U.S. Republican Revolution, in 2017 Emmanuel Macron in France employed some of the campaigning techniques experimented with in the Democratic presidential campaigns of 2008 and 2012, and in 2018 Jair Bolsonaro in Brazil explicitly mirrored Republican tactics (see https://apnews.com/article/e6d1ef0d496545dd86d21584253b2312). This international spread of U.S.-born parliamentary innovations could possibly drive similar patterns of political polarization across different political systems.

5 Conclusion

Political polarization is currently at an all-time high in the United States many other Western Democracies. This phenomenon is attributed by many to the election of representatives who express radically more extreme views than their predecessors Under this reading, without compromising the integrity of the electoral process, there would seem to be little remedy to the current adversarial state of liberal democracies. Voters are purposefully electing extreme types over moderates.

Elected legislators, however, do not act as independent decision-makers. They belong to structured political organizations. These organizations operate with formal systems of leadership and pursue specific party goals by incentivizing their members. Perhaps more encouragingly, party strategies and the technology of whipping appear more amenable to transformation and policy change than slow-moving secular trends in voters' attitudes.

We show that U.S. parties have been critical in driving elite polarization, essentially carving out, through stronger control and discipline, the moderate middle ground between the two parties. Employing a structural model and a new methodology for the analysis of legislative voting in the U.S., we show that the Democratic and Republican Party leaderships have played a substantial role in driving political polarization over the last century. We estimate that about 65-70 percent of current polarization in congressional voting is due to the ability of U.S. parties to discipline and control the votes of their rank and file. Increasing ideological polarization accounts for the remaining portion of the variation.

Virtually all extant methods for the analysis of elite polarization currently attribute no role to party discipline, instead ascribing the entirety of the variation to ideological polarization. Based upon our tests, this assumption is statistically rejected by the data. In addition, within extant models, legislators appear substantially farther from each other than they are in reality, misattributing influence from the party leadership as extreme preferences.

Because our methodology requires only vote data and leadership positions, we are also able to document how the role of party discipline has changed over time. The well known U-shaped profile of political polarization over the last century appears to be the combination of a monotonic increase in ideological separation between median party members' policy preferences and a U-shaped profile of party discipline over time (with a low point in discipline in the 1960's-early 1970's). Strategies of "slash and burn", in which parties describe other members disparagingly, are now commonplace, and the timing of their emergence aligns with the inflection points in party discipline estimated in the data.[42]

At the moment, U.S. political parties appear to be at a high point of party control, with the

[42] https://www.nytimes.com/1990/09/20/opinion/the-politics-of-slash-and-burn.html

technological tools and strategic abilities that allow them to direct their members (and to offer incentives to toe the party line) more readily than ever before. We do not study these specific tools and tactics here, but the ability to measure and analyze party control that we offer will hopefully open the path to new research in this area.

References

Ahn, S. C. and A. R. Horenstein (2013). Eigenvalue ratio test for the number of factors. *Econometrica 81*(3), 1203–1227.

Aldrich, J. H. (1995). *Why parties?: The origin and transformation of political parties in America.* University of Chicago Press.

Alesina, A., A. Miano, and S. Stantcheva (2020). The polarization of reality. In *AEA Papers and Proceedings*, Volume 110, pp. 324–28.

Armstrong, D. A., R. Bakker, R. Carroll, C. Hare, K. T. Poole, H. Rosenthal, et al. (2014). *Analyzing spatial models of choice and judgment with R.* CRC Press.

Baker, S. R., N. Bloom, B. Canes-Wrone, S. J. Davis, and J. Rodden (2014, May). Why has us policy uncertainty risen since 1960? *American Economic Review 104*(5), 56–60.

Bertrand, M. and E. Kamenica (2018). Coming apart? cultural distances in the united states over time. Technical report, National Bureau of Economic Research.

Binder, S. (2003). *Stalemate: Causes and consequences of legislative gridlock.* Brookings DC.

Boche, A., J. B. Lewis, A. Rudkin, and L. Sonnet (2018). The new voteview.com: preserving and continuing keith poole's infrastructure for scholars, students and observers of congress. *Public Choice 176*(1-2), 17–32.

Bonica, A. (2014). Mapping the ideological market place. *American Journal of Political Science 58*(2), 367–386.

Boxell, L., M. Gentzkow, and J. M. Shapiro (2020). Cross-country trends in affective polarization. Technical report, National Bureau of Economic Research.

Caillaud, B. and J. Tirole (1999). Party governance and ideological bias. *European Economic Review 43*(4-6), 779–789.

Caillaud, B. and J. Tirole (2002). Parties as political intermediaries. *The Quarterly Journal of Economics 117*(4), 1453–1489.

Canen, N., C. Kendall, and F. Trebbi (2020). Unbundling polarization. *Econometrica 88*(3), 1197–1233.

Carroll, R., J. B. Lewis, J. Lo, K. T. Poole, and H. Rosenthal (2009). Measuring bias and uncertainty in dw-nominate ideal point estimates via the parametric bootstrap. *Political Analysis 17*(3), 261–275.

Clinton, J., S. Jackman, and D. Rivers (2004). The statistical analysis of roll call data. *American Political Science Review 98*(2), 355–370.

Clinton, J., I. Katznelson, and J. Lapinski (2014). Where measures meet history: Party polarization during the new deal and fair deal. *Governing in a Polarized Age: Elections, Parties, and Representation in America*.

Cox, G. W. and M. D. McCubbins (1993). *Legislative Leviathan: Party Government in the House*, Volume 23. Univ of California Press.

Cox, G. W. and M. D. McCubbins (2005). *Setting the agenda: Responsible party government in the US House of Representatives*. Cambridge University Press.

Davis, S. J. (2019). Rising policy uncertainty. Technical report, National Bureau of Economic Research.

Deering, C. J. and S. S. Smith (1997). *Committees in congress*. Sage.

Evans, C. L. (2011). Growing the vote: Majority party whipping in the us house, 1955-2002. In *10th Annual Congress and History Conference, Brown University, June*, pp. 9–10.

Evans, C. L. (2018). *The Whips: Building Party Coalitions in Congress*. University of Michigan Press.

Fernández-Val, I. and M. Weidner (2016). Individual and time effects in nonlinear panel models with large n, t. *Journal of Econometrics 192*(1), 291–312.

Fiorina, M. P., S. J. Abrams, and J. C. Pope (2005). Culture war. *The myth of a polarized America*.

Forgette, R. (2004). Party caucuses and coordination: Assessing caucus activity and party effects. *Legislative Studies Quarterly 29*(3), 407–430.

Gentzkow, M. (2016). Polarization in 2016. *Toulouse Network of Information Technology white paper*.

Gentzkow, M., J. M. Shapiro, and M. Taddy (2019). Measuring group differences in high-dimensional choices: method and application to congressional speech. *Econometrica 87*(4), 1307–1340.

Goodall, C. (1991). Procrustes methods in the statistical analysis of shape. *Journal of the Royal Statistical Society: Series B (Methodological) 53*(2), 285–321.

Grossmann, M. and D. A. Hopkins (2016). *Asymmetric politics: Ideological Republicans and group interest Democrats*. Oxford University Press.

Heckman, J. J. and J. M. Snyder (1997). Linear probability models of the demand for attributes with an empirical application to estimating the preferences of legislators. *The RAND Journal of Economics 28*.

Iyengar, S., Y. Lelkes, M. Levendusky, N. Malhotra, and S. J. Westwood (2019). The origins and consequences of affective polarization in the united states. *Annual Review of Political Science 22*, 129–146.

Iyengar, S. and S. J. Westwood (2015). Fear and loathing across party lines: New evidence on group polarization. *American Journal of Political Science 59*(3), 690–707.

Jenkins, J. A. (2000). Examining the robustness of ideological voting: evidence from the confederate house of representatives. *American Journal of Political Science*, 811–822.

Jenkins, J. A. (2011). The evolution of party leadership. In *The Oxford Handbook of the American Congress*, pp. 684–711. Citeseer.

Kendall, D. G. (1989). A survey of the statistical theory of shape. *Statistical Science*, 87–99.

Kingma, D. P. and J. Ba (2014). Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*.

Krehbiel, K. (1993). Where's the party? *British Journal of Political Science 23*(2), 235–266.

Krehbiel, K. (1998). *Pivotal politics: A theory of US lawmaking*. University of Chicago Press.

Krehbiel, K. (1999). Paradoxes of parties in congress. *Legislative Studies Quarterly*, 31–64.

Krehbiel, K. (2000). Party discipline and measures of partisanship. *American Journal of Political Science*, 212–227.

Levitt, S. D. (1996). How do senators vote? disentangling the role of voter preferences, party affiliation, and senator ideology. *The American Economic Review 86*(3), 425–441.

Luntz, F. (2007). *Words that work: It's not what you say, it's what people hear.* Hachette UK.

Mayhew, D. R. (2004). *Congress: The electoral connection.* Yale university press.

McCarty, N. (2016). Polarization, congressional dysfunction, and constitutional change symposium. *Indiana Law Review 50*, 223.

McCarty, N. (2019). *Polarization: What Everyone Needs to Know.* Oxford University Press.

McCarty, N., K. T. Poole, and H. Rosenthal (2001). The hunt for party discipline in congress. *American Political Science Review 95*(3), 673–687.

McCarty, N., K. T. Poole, and H. Rosenthal (2006). *Polarized America: The Dance of Ideology and Unequal Riches.* Cambridge: MIT Press.

Meinke, S. R. (2008). Who whips? party government and the house extended whip networks. *American Politics Research 36*(5), 639–668.

Mian, A., A. Sufi, and F. Trebbi (2014, 04). Resolving debt overhang: Political constraints in the aftermath of financial crises. *American Economic Journal: Macroeconomics 6*(2), 1–28.

Moskowitz, D. J., J. Rogowski, and J. M. S. Jr. (2017). Parsing party polarization. mimeo.

Pastor, L. and P. Veronesi (2012). Uncertainty about government policy and stock prices. *The Journal of Finance 67*(4), 1219–1264.

Polborn, M. K. and J. M. Snyder Jr (2017). Party polarization in legislatures with office-motivated candidates. *The Quarterly Journal of Economics 132*(3), 1509–1550.

Poole, K. T. and H. Rosenthal (1984). The polarization of american politics. *Journal of Politics 46*(4), 1061–1079.

Poole, K. T. and H. Rosenthal (1997). *Congress: A Political-Economic History of Roll Call Voting.* New York: Oxford University Press.

Poole, K. T. and H. Rosenthal (2001). D-nominate after 10 years: A comparative update to congress: A political-economic history of roll-call voting. *Legislative Studies Quarterly*, 5–29.

Poole, K. T., H. Rosenthal, et al. (1997). *Congress: A Political-economic History of Roll Call Voting.* Oxford University Press on Demand.

Potthoff, R. F. (2018). Estimating ideal points from roll-call data: explore principal components analysis, especially for more than one dimension? *Social Sciences 7*(1), 12.

Rajan, R. G. (2011). *Fault lines: How hidden fractures still threaten the world economy*. princeton University press.

Rivers, D. (2003). Identification of multidimensional spatial voting models. *Typescript. Stanford University*.

Robison, J. and K. J. Mullinix (2016). Elite polarization and public opinion: How polarization is communicated and its effects. *Political Communication 33*(2), 261–282.

Rohde, D. W. (1991). *Parties and Leaders in the Postreform House*. University of Chicago Press.

Schneider, J. and M. L. Koempel (2012). *Congressional Deskbook: The Practical and Comprehensive Guide to Congress*. The Capitol Net Inc.

Senate, U. S. (2020). Majority and minority leaders. *Online, Retrieved on Senate.gov on June 25, 2020*.

Sinclair, B. (1992). The emergence of strong leadership in the 1980s house of representatives. *The Journal of Politics 54*(3), 657–684.

Sinclair, B. (2014). *Party wars: Polarization and the politics of national policy making*, Volume 10. University of Oklahoma Press.

Smith, S. S. (2007). *Party influence in Congress*. Cambridge University Press.

Snyder, J. M. and T. Groseclose (2000). Estimating party influence in congressional roll-call voting. *American Journal of Political Science*, 193–211.

Stonecash, J. (2018). *Diverging parties: Social change, realignment, and party polarization*. Routledge.

Sundquist, J. L. (1988). Needed: A political theory for the new era of coalition government in the united states. *Political Science Quarterly 103*(4), 613–635.

Theriault, S. M. (2008). *Party Polarization in Congress.* New York: Cambridge University Press.

Theriault, S. M. (2013). *The Gingrich senators: The roots of partisan warfare in Congress*. Oxford University Press.

United States House of Representatives History, A. and Archives (2020a). Democratic whips (1899 to present). *Online, Retrieved on House.gov on June 25, 2020*.

United States House of Representatives History, A. and Archives (2020b). Majority leaders of the house (1899 to present). *Online, Retrieved on House.gov on June 25, 2020*.

6 Tables and Figures

Figure 1: Party Leaders Whipping in Opposite Directions in 2 Dimensions

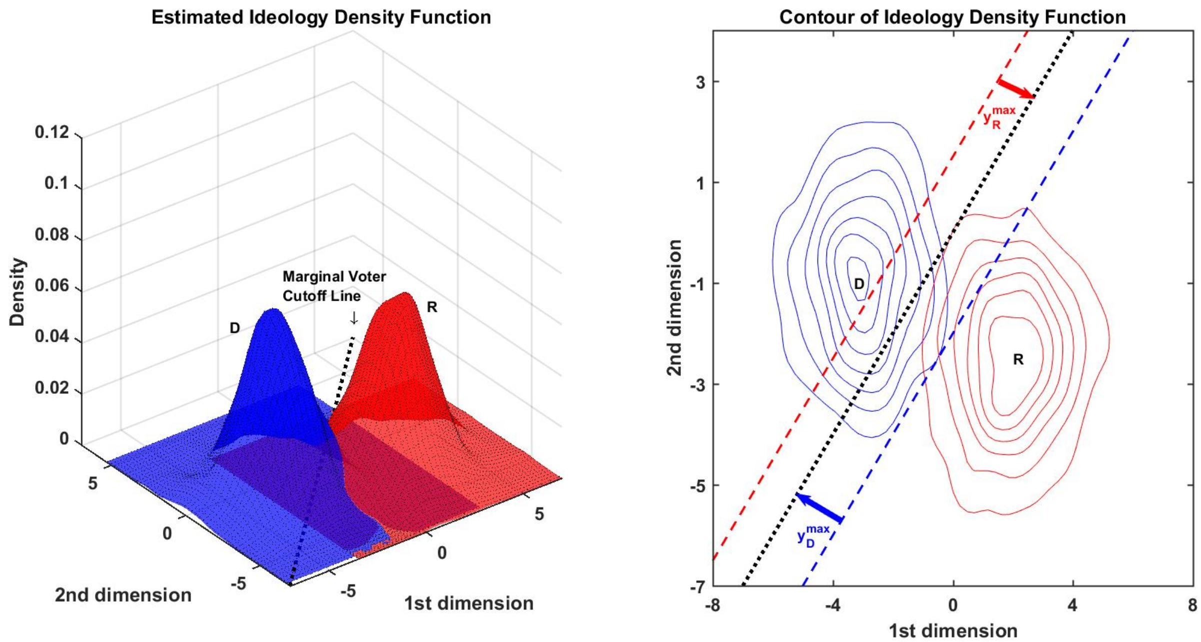


Figure 2: Party Leaders Whipping in the Same Direction in 2 Dimensions

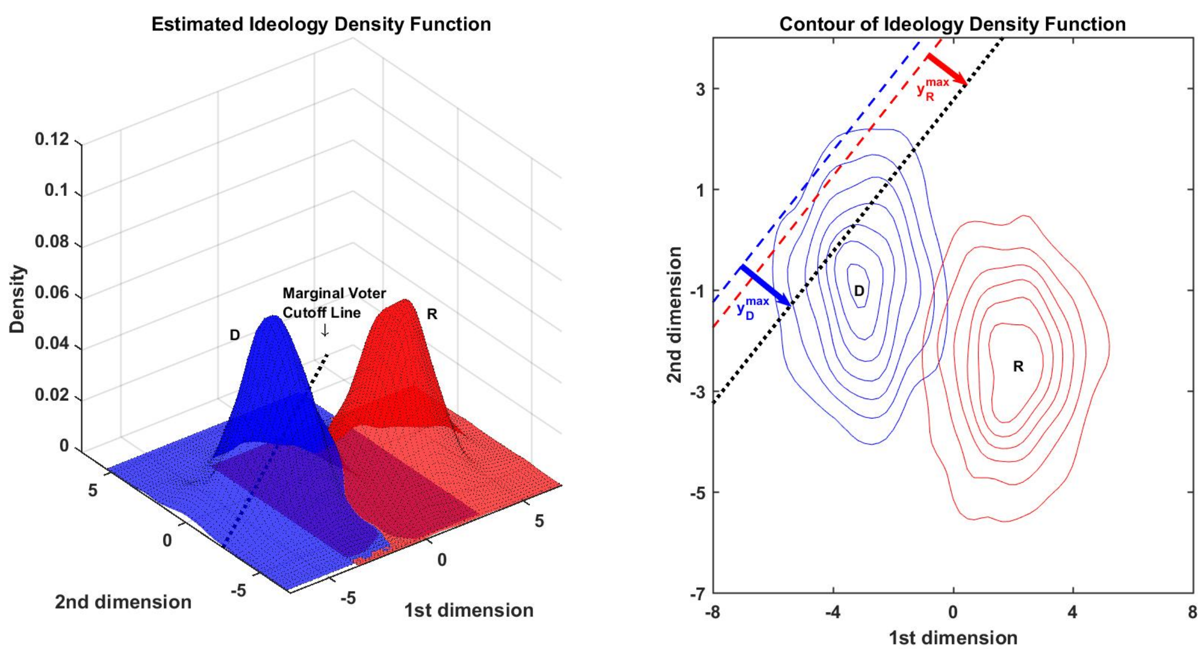
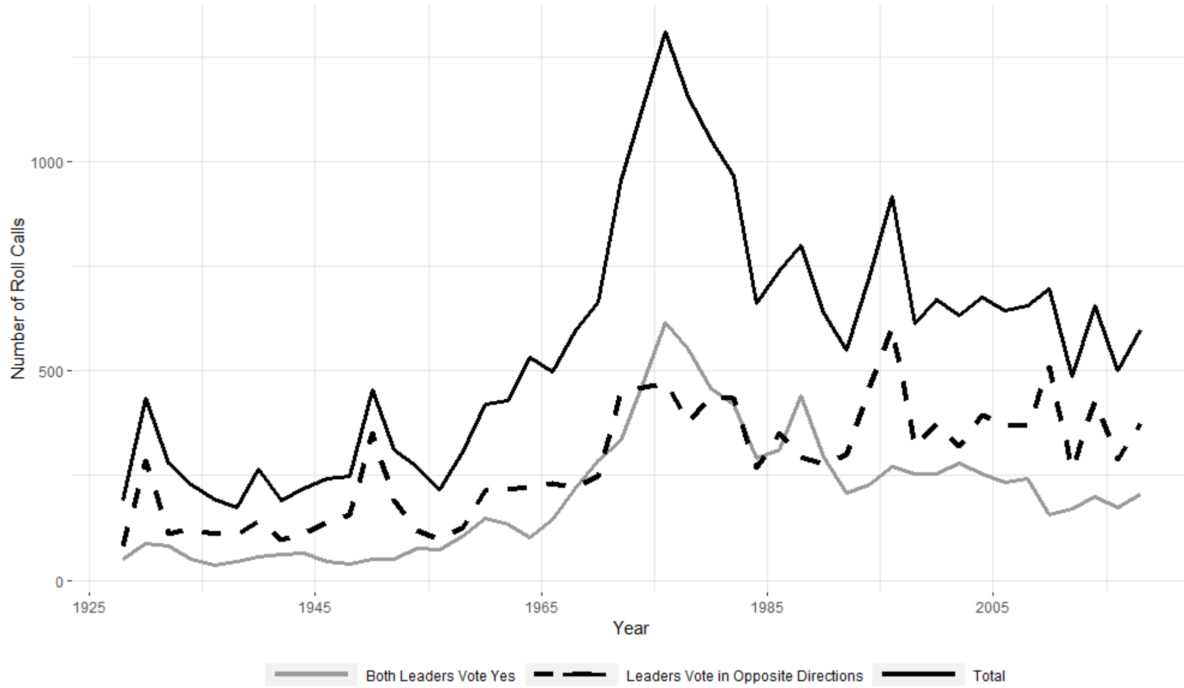
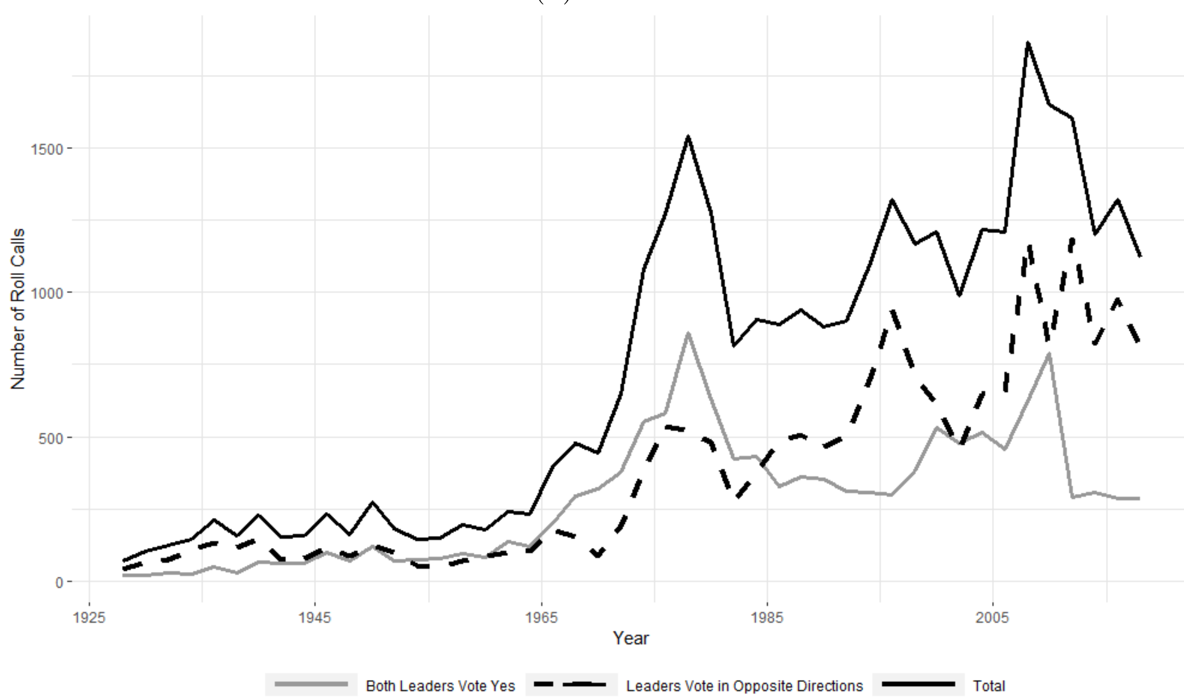


Figure 3: Roll Call Votes Across the Sample

(a) Senate

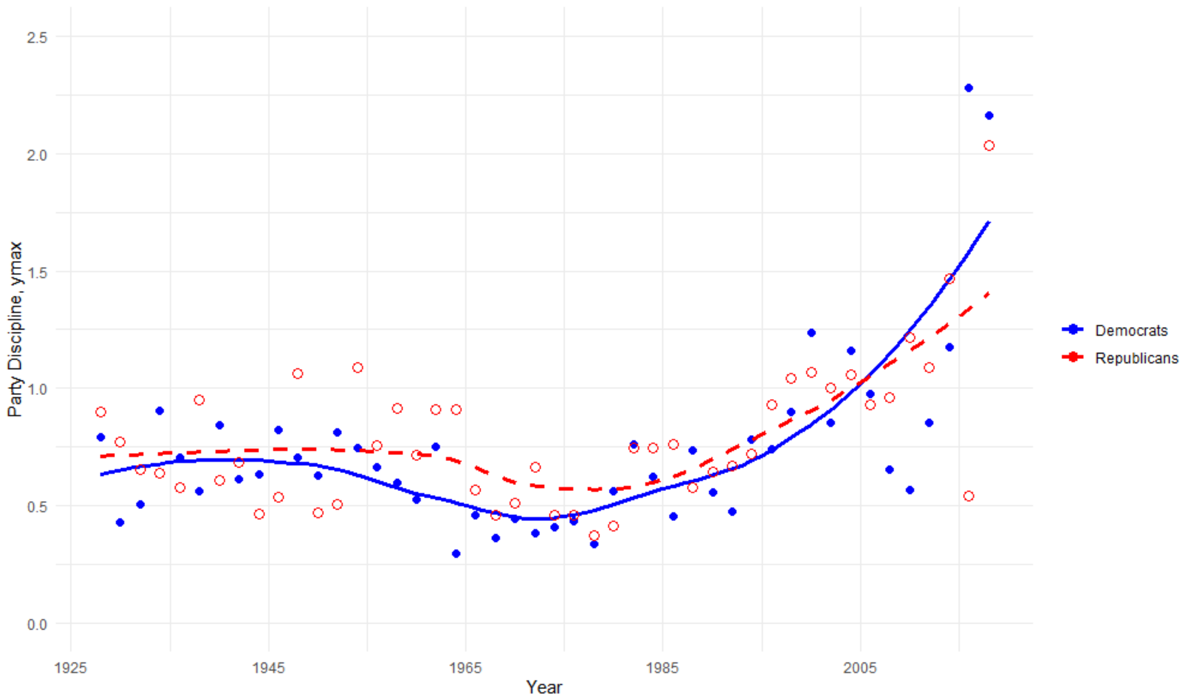


(b) House



Notes: The figures present summary statistics on the key variation necessary for the identification of party discipline. They show, for each Congress, how many total roll calls there are, and how those votes are split between roll calls in which both party leaders vote in favor of the new policy, and those in which they vote in opposite directions.

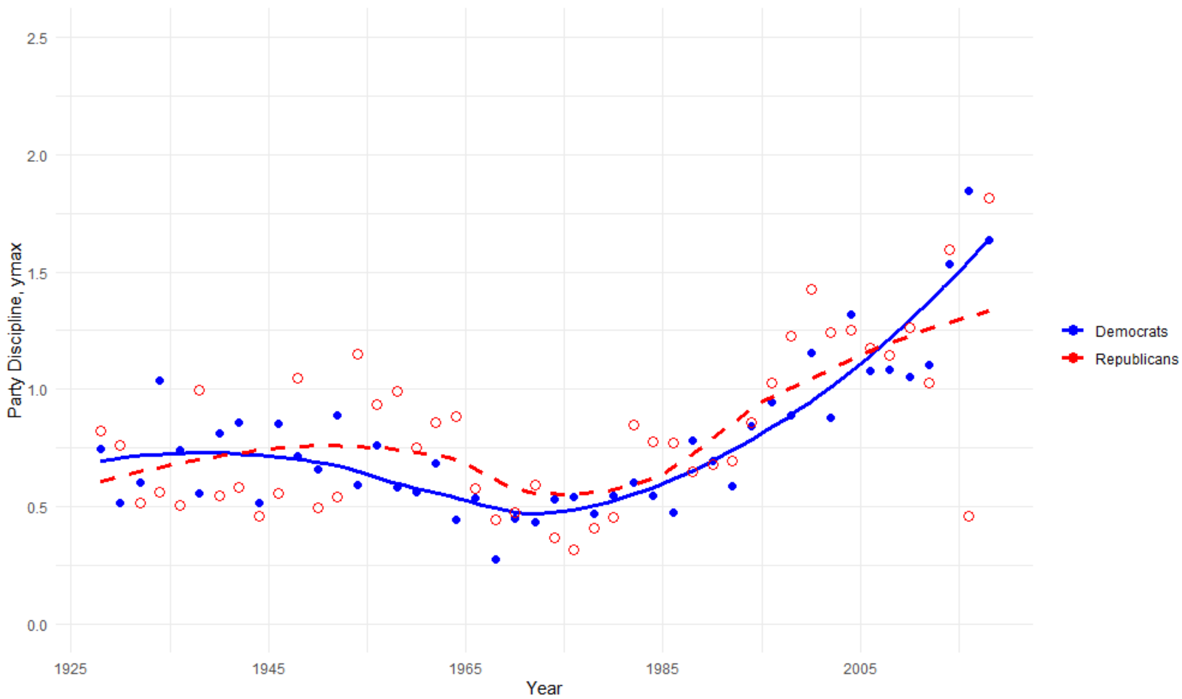
Figure 4: Party Discipline Over Time, 1927-2019 - Senate 2D Model



Notes: Estimates of y_p^{max} shown for each party, Democrats in filled blue, Republicans in unfilled red. Party-specific smoothed fit (Loess) curves with span 0.7 are also shown.

Figure 5: Party Discipline in the 1D Model

(a) Senate



(b) House

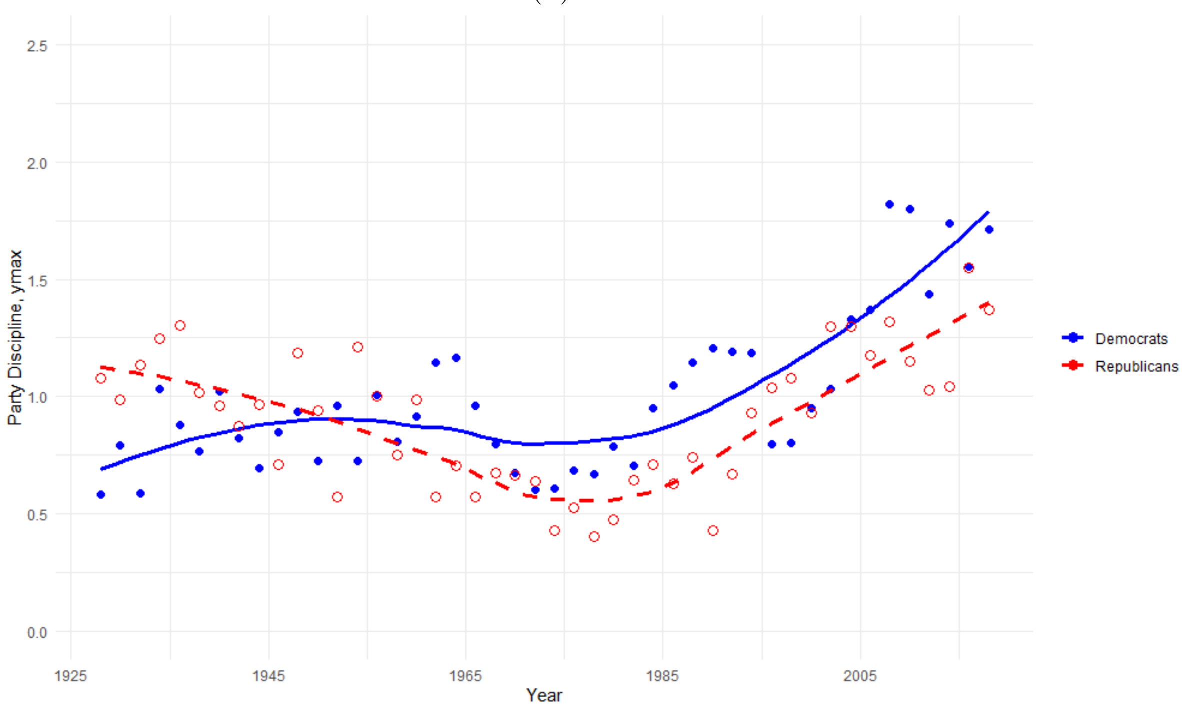
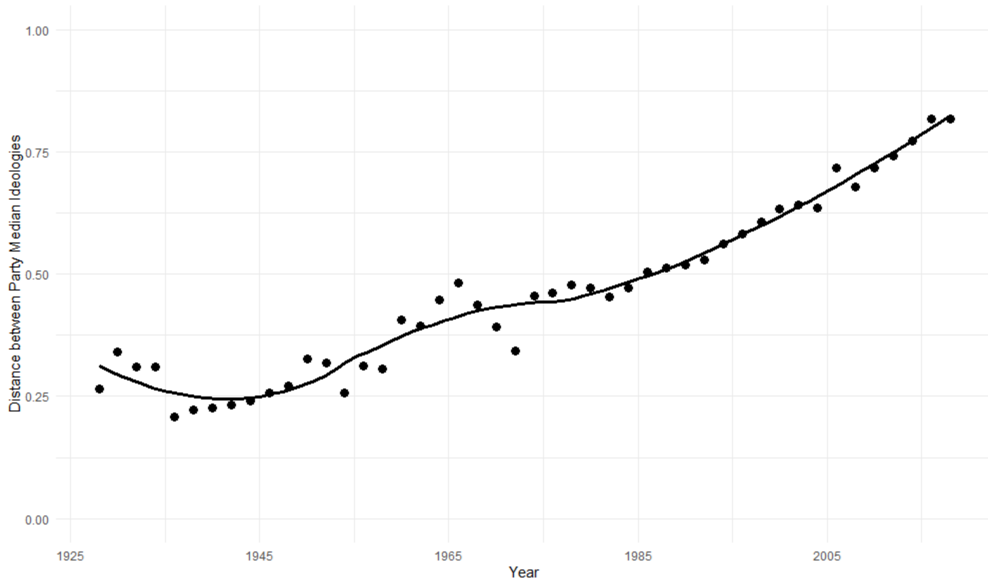
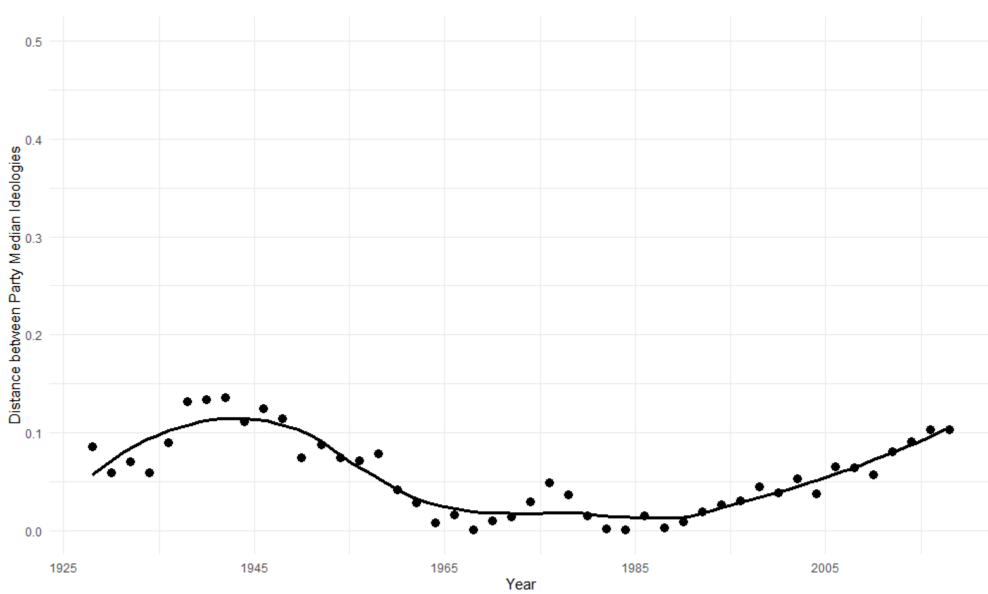


Figure 6: Ideological Polarization Between Senate Members, 1927-2019 (1st Dimension) - Senate 2D Model



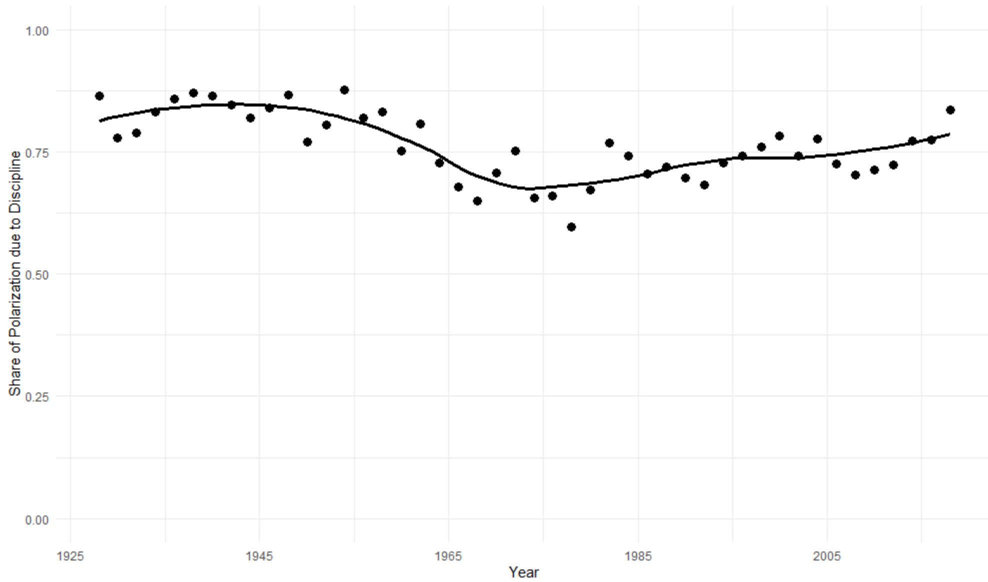
Notes: Estimates of the distance between party medians in the 1st dimension for the Senate 2D Model are shown, together with a smoothed fit (Loess) curve with span 0.5.

Figure 7: Ideological Polarization Between Senate Members, 1927-2019 (2nd Dimension) - Senate 2D Model



Notes: Estimates of the distance between party medians in the 2nd dimension for the Senate 2D Model are shown, together with a smoothed fit (Loess) curve with span 0.5.

Figure 8: Share of Polarization Attributable to Party Discipline (Relative to 1st Dimension Ideologies) - Senate 2D Model



Notes: The estimated share of polarization attributed to party discipline is shown for each Congress, computed by the total amount of party discipline divided by that amount plus the distance between party medians in the first dimension (i.e. $\frac{y_D^{max}+y_R^{max}}{y_D^{max}+y_R^{max}+(\theta_{m,R}-\theta_{m,D})}$), together with a smoothed fit (Loess) curve with span 0.5.

Figure 9: Ideological Polarization Over Time (1st dimension), 1927-2019 - Senate 2D Model

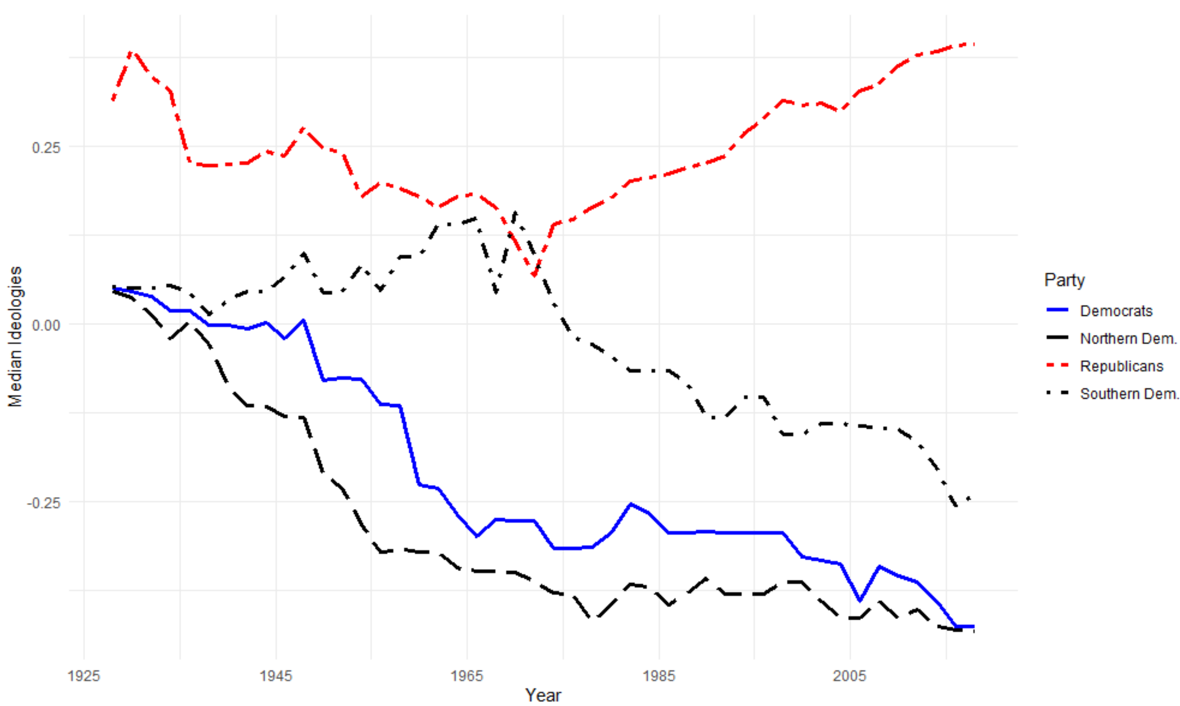
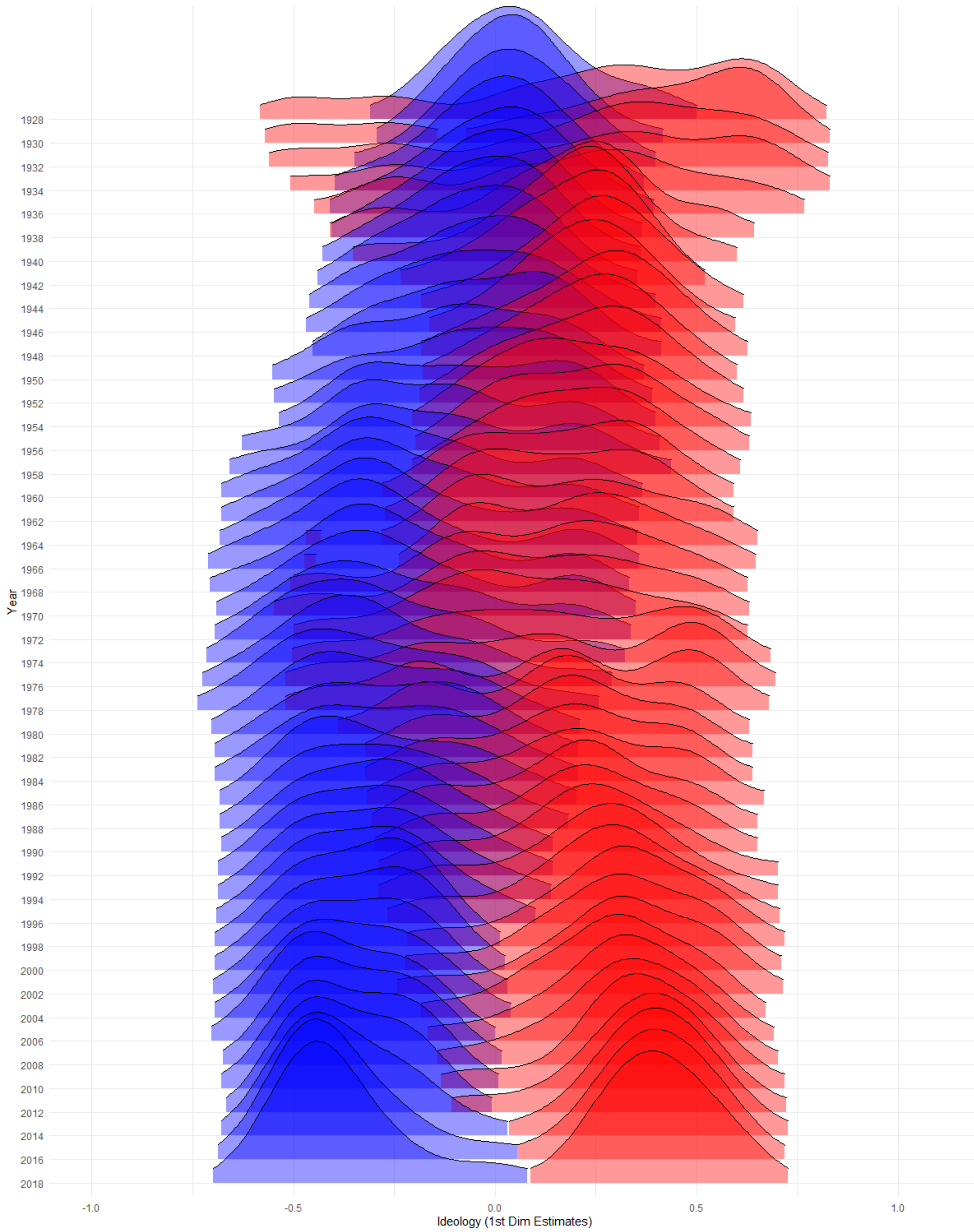
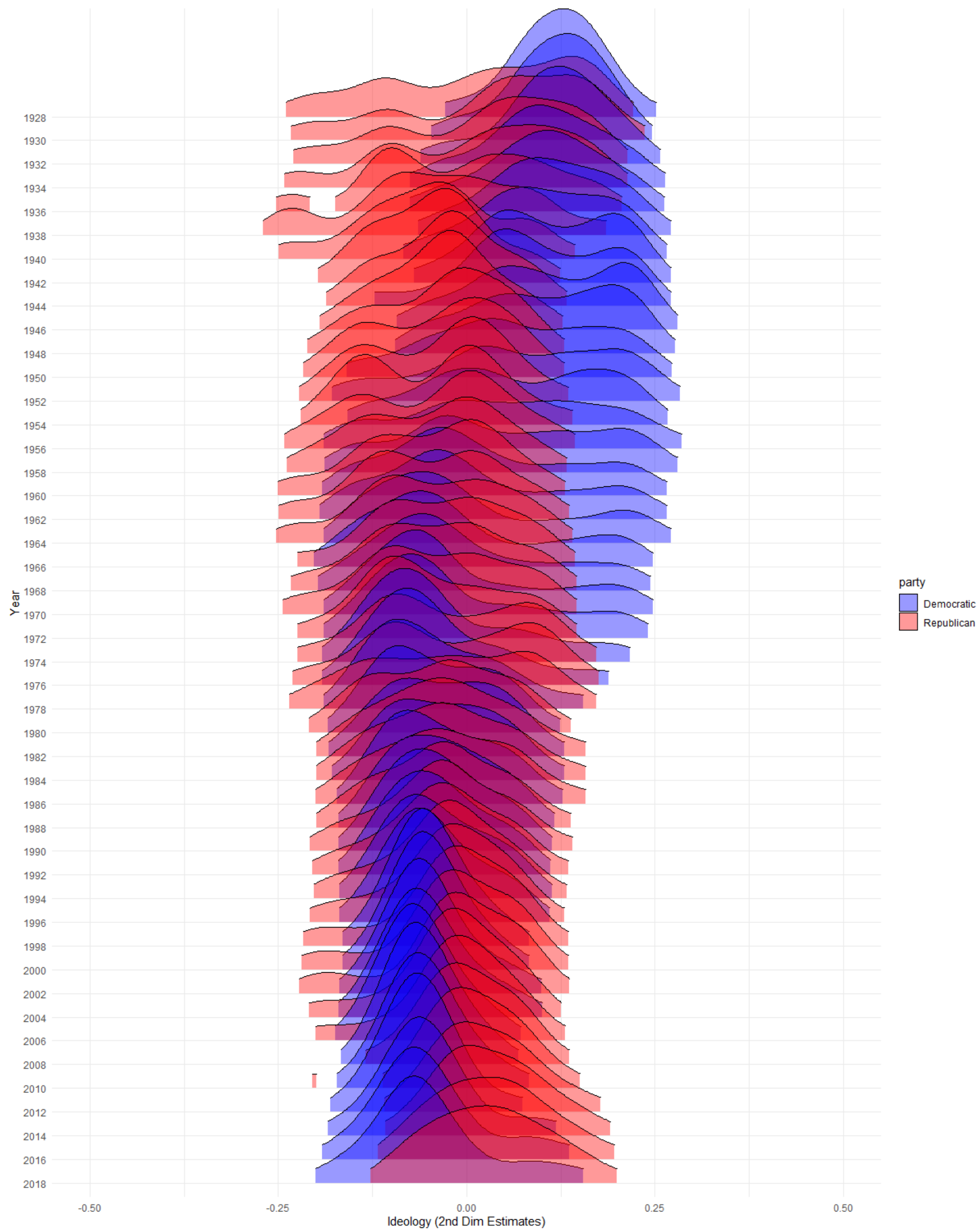


Figure 10: Ideological Polarization Between Senate Members, 1927-2019 - Senate 2D Model



Notes: Kernel density estimates of the ideological parameters for the first dimension from the Senate 2D Model across Congresses.

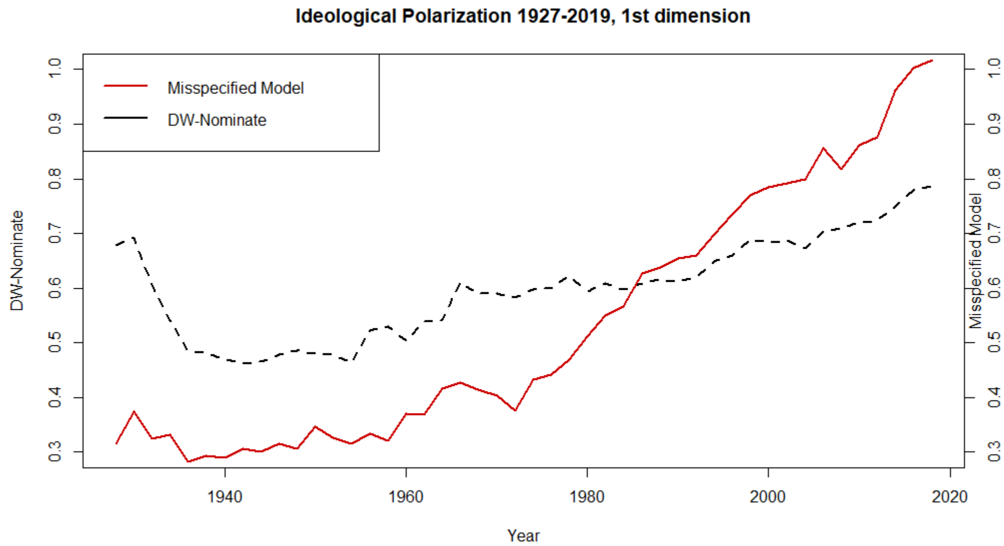
Figure 11: Ideological Polarization Between Senate Members, 1927-2019 - Senate 2D Model



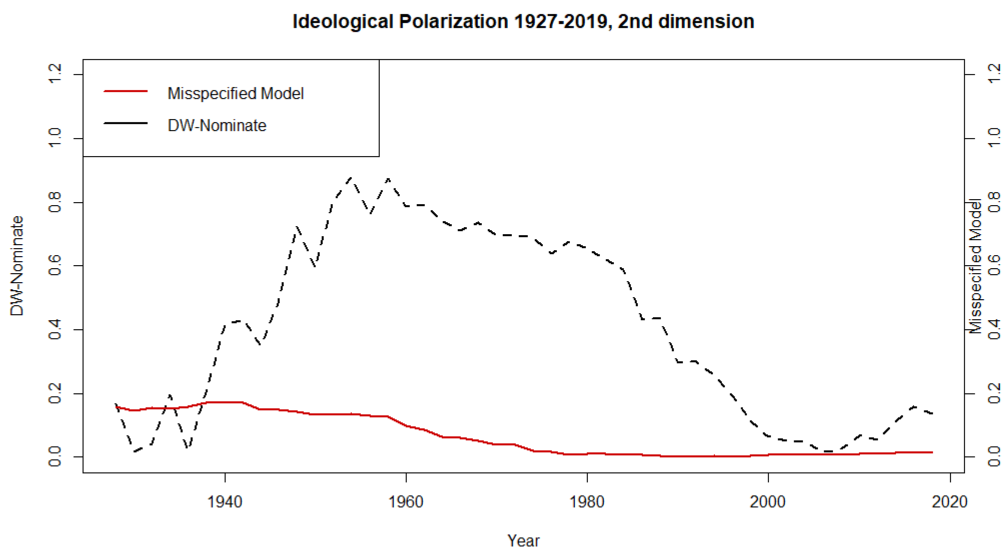
Notes: Kernel density estimates of the ideological parameters for the second dimension from the Senate 2D Model across Congresses.

Figure 12: Trends in Ideological Polarization: Misspecified Model vs. DW-Nominate - Senate 2D Model

(a) First Dimension

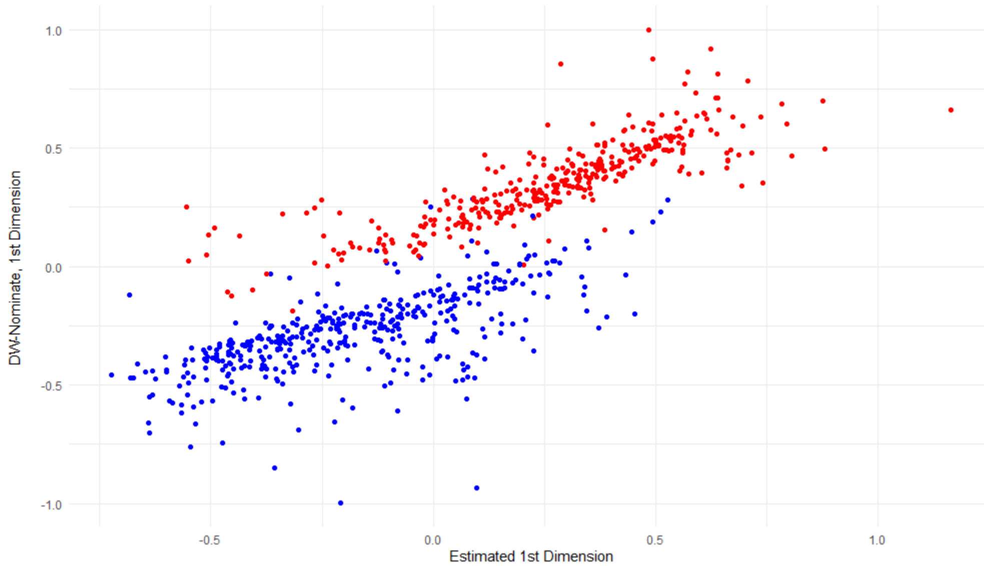


(b) Second Dimension



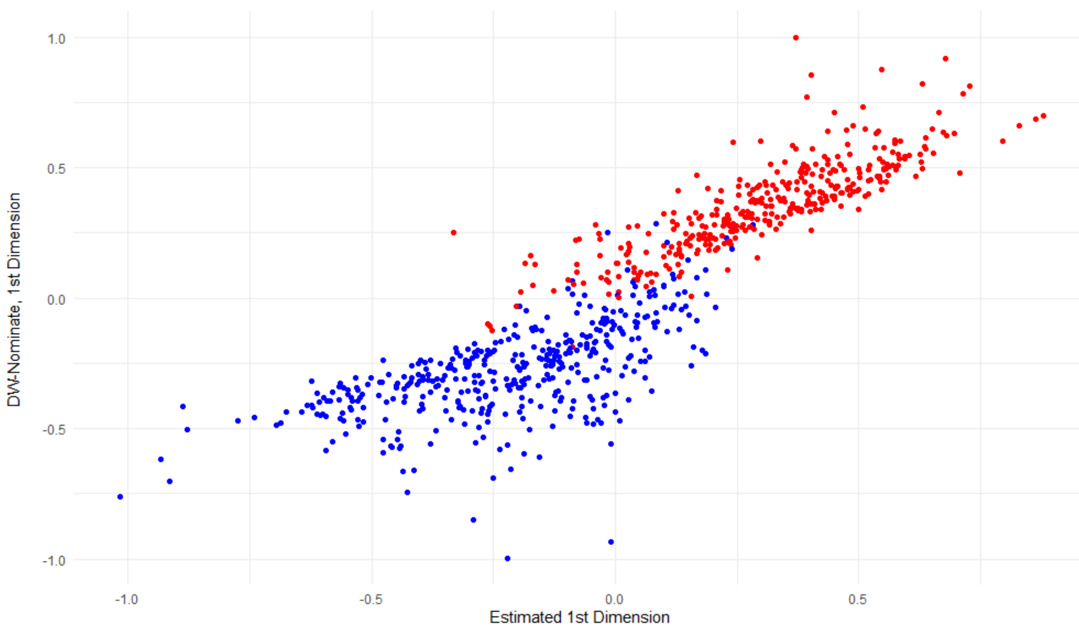
Notes: The two graphs compare the ideological polarization (difference between estimated party medians) across time for the misspecified model (no whipping) and DW-Nominate.

Figure 13: Estimated (Senate 2D) Model vs. DW-Nominate, 1st Dimension



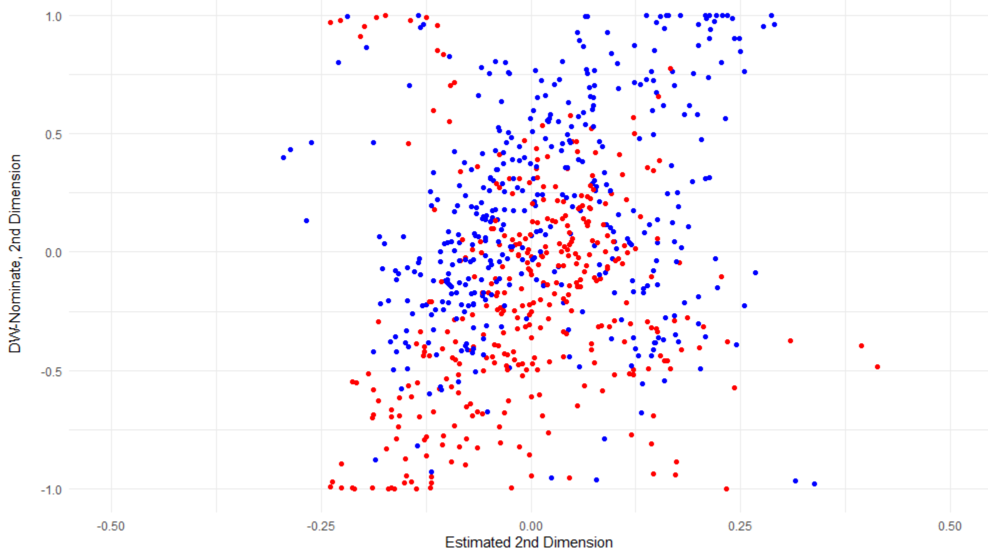
Notes: Scatter plot of first dimension estimated ideologies versus those from DW-Nominate, pooled across all Congresses. Democrats are shown in blue, Republicans are shown in red. The correlation is 0.857.

Figure 14: Misspecified (Senate 2D) Model vs. DW-Nominate, 1st Dimension



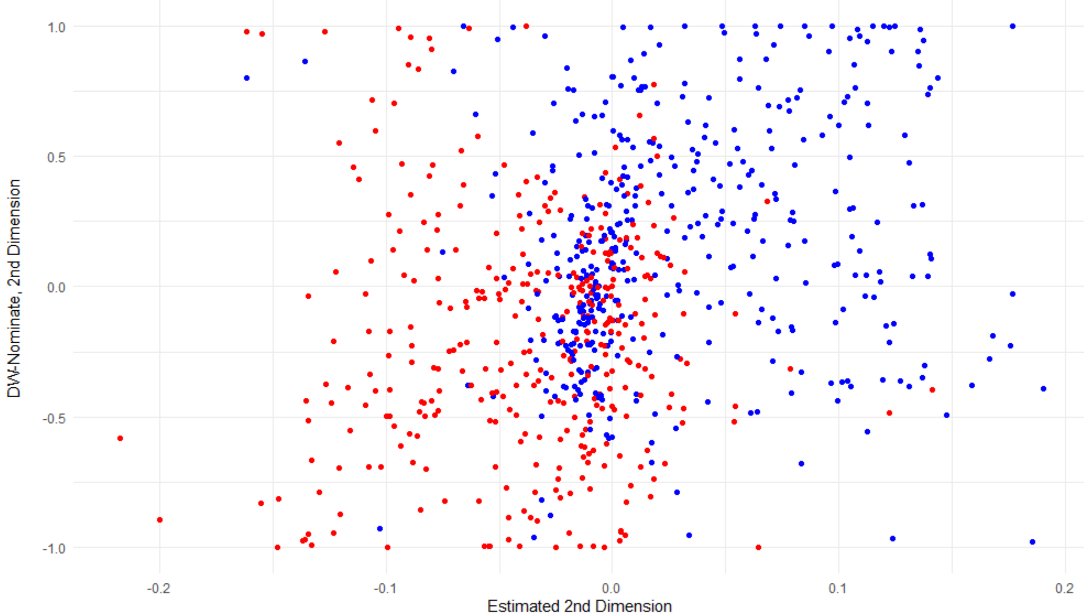
Notes: Scatter plot of the first dimension estimated ideologies of the misspecified model (no whipping) versus those from DW-Nominate, pooled across all Congresses. Democrats are shown in blue, Republicans are shown in red. The correlation is 0.910.

Figure 15: Estimated (Senate 2D) Model vs. DW-Nominate, 2nd Dimension



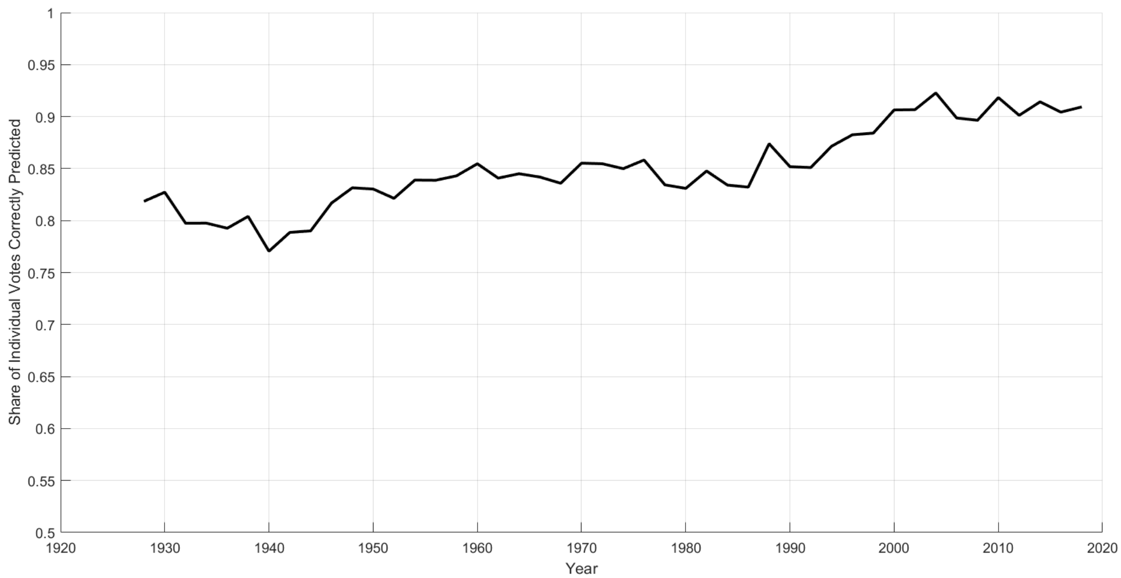
Notes: Scatter plot of the second dimension estimated ideologies versus those from DW-Nominate, pooled across all Congresses. Democrats are shown in blue, Republicans are shown in red. The correlation is 0.435.

Figure 16: Misspecified (Senate 2D) Model vs. DW-Nominate, 2nd Dimension



Notes: Scatter plot of the second dimension estimated ideologies of the misspecified model (no whipping) versus those from DW-Nominate, pooled across all Congresses. Democrats are shown in blue, Republicans are shown in red. The correlation is 0.365.

Figure 17: Model Fit: Share of Votes Correctly Predicted in the Senate (2D Model)



Notes: Average share of votes that are correctly predicted in each Congress. A vote is considered to be correctly predicted if, under our estimated parameters, the probability of a congressmember voting as observed in the data is larger than 0.5.

Figure 18: Robustness of Total Party Discipline ($y_D^{max} + y_R^{max}$) Across Whipping Assumptions - Senate 2D Model

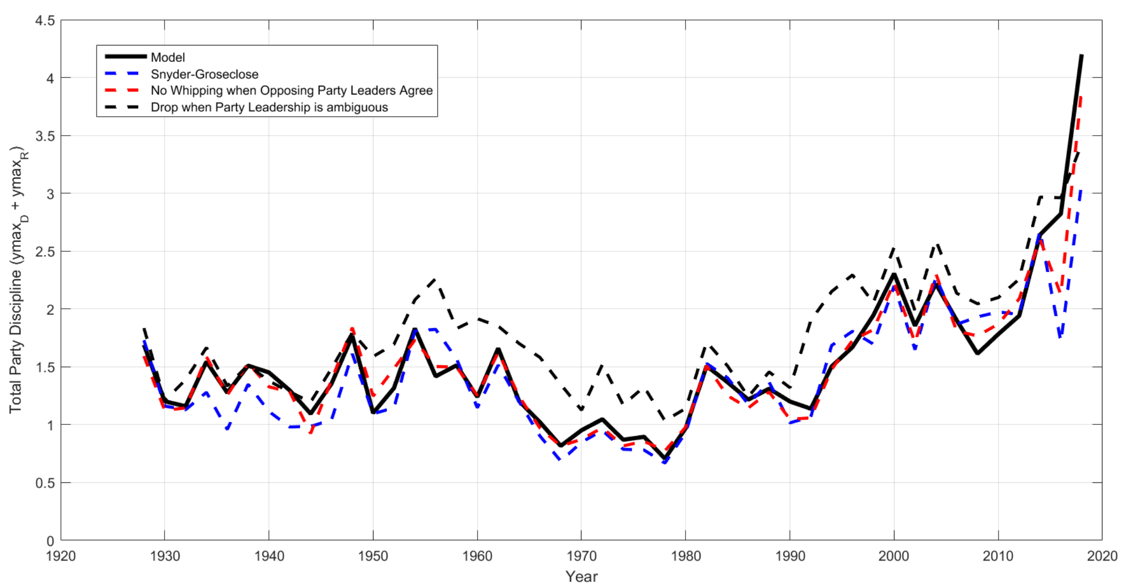


Figure 19: Robustness of the Estimates of Party Discipline Across Whipping Assumptions - Senate 2D Model

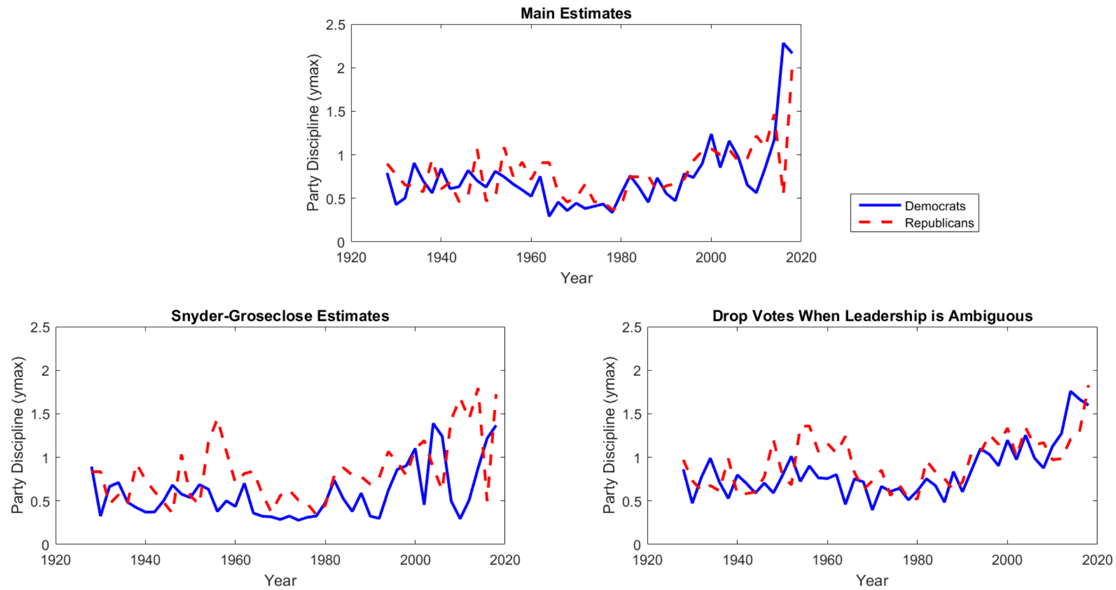
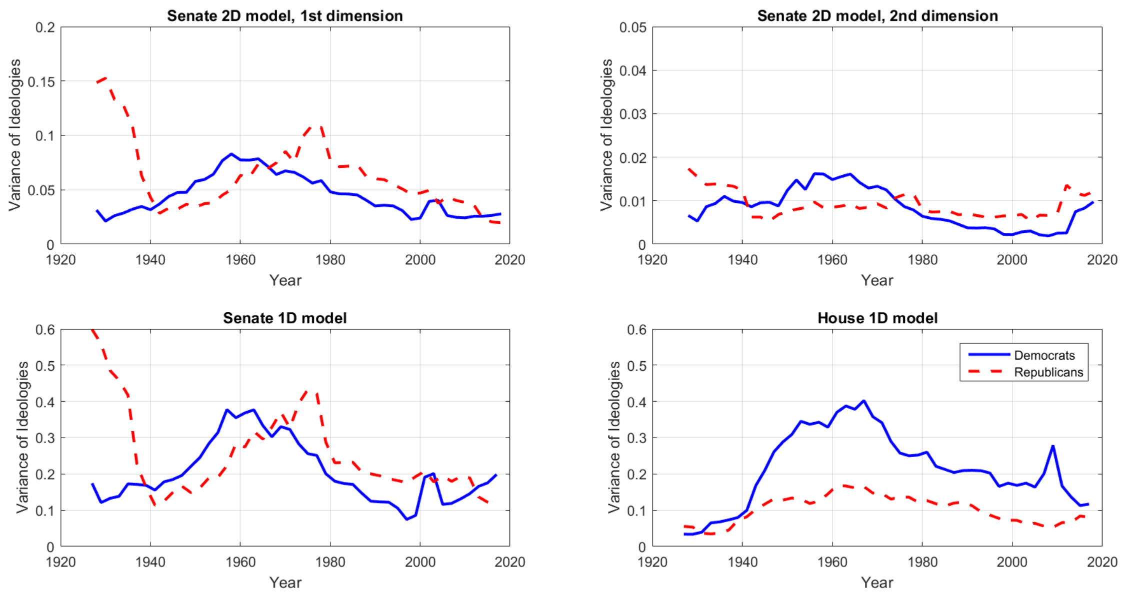


Figure 20: Variance of Estimated Ideologies over Time



Notes: Each panel shows the variance of estimated ideologies within party over time.

Table 1: Number of Parameters Across Specifications

Model	Ideology	Party Discipline	Roll Call	Total
Senate - 1 Dimensional	789	92	25824	26705
House - 1 Dimensional	3938	92	32763	36793
Senate - 2 Dimensional	1568	92	22314	23974

Appendix A: Identification

This Appendix proves the Identification of our model in two dimensions.

For $\mathcal{I}_t = 1$, we can rewrite (5) as:

$$\Pr\left(Y_{it} = 1 | \bar{q}_t \in Q_p^1, \bar{x}_t; \theta^i, y_p^{max}, m_t\right) = \Phi\left(\sqrt{\tfrac{1}{1+m_t^2}}\left(\theta_2^i - m_t\theta_1^i - b_t\right) + W_{p,t} \times y_p^{max}\right)$$

Let us use the simplified notation, $\Pr\left(Y_{it} = 1\right) = \Pr\left(Y_{it} = 1 | \bar{q}_t \in Q_p^1, \bar{x}_t; \theta^i, y_p^{max}, m_t\right)$. This term is the likelihood component of politician i voting Yes on a bill t if $\mathcal{I}_t = 1$. It is more convenient for us to work with the standardized likelihood:

$$\Phi^{-1}\left(\Pr\left(Y_{it} = 1\right)\right) = \sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^i - m_t\theta_1^i - b_t\right) + W_{p,t} \times y_p^{max}, \tag{8}$$

which makes explicit the unique correspondence between data (on the left hand side) and model parameters (on the right hand side).

Using Assumption ID3(i), we begin by comparing the probability of voting Yes on the normalizing bill 0 between any two politicians, i and j, belonging to the same party:

$$\Phi^{-1}\left(\Pr\left(Y_{i,0} = 1\right)\right) - \Phi^{-1}\left(\Pr\left(Y_{j,0} = 1\right)\right) = \theta_2^i - \theta_2^j$$

It is immediate that with $j = 0$ (the normalizing agent in Assumption ID2(i)), we obtain identification of θ_2^i for all i. In the absence of this normalization, we would only identify the differences of second dimension ideologies across legislators. Intuitively, identification of $\{\theta_2^i\}_{i=1}^N$ relies on the normalizing bill $m_0 = 0$. This bill is such that differences in voting behavior only come from the second preference dimension.

For $\mathcal{I}_t = 0$, we have instead

$$\Phi^{-1}\left(1 - \Pr\left(Y_{it} = 1\right)\right) = \sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^i - m_t\theta_1^i - b_t\right) + W_{p,t} \times y_p^{max}. \tag{9}$$

One can see immediately that the difference in standardized likelihoods using (9) again identifies the second dimension ideologies, $\{\theta_2^i\}_{i=1}^N$.

Given the second dimension ideologies, we can identify the direction, $\mathcal{I}_0$, of the normalized bill by comparing the standardized likelihoods of two members of the same party with different second dimension ideologies (which must exist for at least one party by Assumption ID1). Without loss, consider two members, i and j, with $\theta_2^i > \theta_2^j$. If $\mathcal{I}_t = 1$ then (8) implies $Pr\left(Y_{it} = 1\right) > \Pr\left(Y_{jt} = 1\right)$. On the other hand, if $\mathcal{I}_0 = 0$, then (9) implies the opposite. Thus, knowledge of the relative size of the voting probabilities identifies $\mathcal{I}_0$, which also implies knowledge of $W_{p,0}$.

From the normalizing bill, for two politicians i and h that belong to different parties, who whip in opposite directions, we obtain:

$$\begin{aligned} & \Phi^{-1}\left(\Pr\left(Y_{i0}=1\right)\right)-\Phi^{-1}\left(\Pr\left(Y_{h0}=1\right)\right)= \\ = \; & \theta_2^i-\theta_2^h+W_{D,0}\times y_D^{max}-W_{R,0}\times y_R^{max} \\ = \; & \theta_2^i-\theta_2^h\pm\left(y_D^{max}+y_R^{max}\right) \end{aligned} \tag{10}$$

which implies $y_D^{max}+y_R^{max}$ is known and uniquely identified in this case.

If, instead, in the normalizing bill, the two parties whip in the same direction we obtain:

$$\begin{aligned} & \Phi^{-1}\left(\Pr\left(Y_{i,0}=1\right)\right)-\Phi^{-1}\left(\Pr\left(Y_{h,0}=1\right)\right)= \\ = \; & \theta_2^i-\theta_2^h+W_{D,0}\times y_D^{max}+W_{R,0}\times y_R^{max} \\ = \; & \theta_2^i-\theta_2^h\pm\left(y_D^{max}-y_R^{max}\right) \end{aligned} \tag{11}$$

which implies $y_D^{max}-y_R^{max}$ is known and uniquely identified in this case.

Let $\boldsymbol{W_0}$ denote the subset of bills in which the two parties whip in the same (opposing) direction if they whip in the same (opposing) direction on bill 0. Within the set, $\boldsymbol{W_0}$, knowledge of $\mathcal{I}_t$ implies knowledge of $\Delta y_t \equiv W_{D,t}\times y_D^{max}\pm W_{R,t}\times y_R^{max}$, either from (10) or (11).

We now identify the cutlines and directions for the subset of bills, $\boldsymbol{W_0}$. Consider politicians 0 and k's vote decisions on an arbitrary bill, t in $\boldsymbol{W_0}$. The ideal points, (θ_1^0,θ_2^0) and (θ_1^k,θ_2^k) are known for these members, with θ_2^k having been identified. In our estimation procedure, we assume that the two politicians are members of opposing parties (D and R respectively). The standardized likelihoods are then given by:

$$\begin{aligned} \Phi^{-1}\left(\Pr\left(Y_{0t}=1\right)\right) &= \pm\sqrt{\tfrac{1}{1+m_t^2}}\left(\theta_2^0-m_t\theta_1^0-b_t\right)\pm W_{D,t}\times y_D^{max} \\ \Phi^{-1}\left(\Pr\left(Y_{kt}=1\right)\right) &= \pm\sqrt{\tfrac{1}{1+m_t^2}}\left(\theta_2^k-m_t\theta_1^k-b_t\right)\pm W_{R,t}\times y_R^{max}, \end{aligned} \tag{12}$$

where the sign of the RHS depends upon $\mathcal{I}_t$.

It is convenient to create a hypothetical member of party D, k', with identical voting probability to member k of party R. This (artificial) member has known ideal points $\theta_1^{k'}=\theta_1^k$ and $\theta_2^{k'}=\theta_2^k-\Delta y_t$ because Δy_t has been identified for bills in $\boldsymbol{W_0}$. The standardized likelihood for k' satisfies

$$
\begin{aligned}
\Phi^{-1}\left(\Pr\left(Y_{kt}=1\right)\right) &= \pm\sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^k - m_t\theta_1^k - b_t\right) \pm W_{R,t}\times y_R^{max} \\
&= \pm\sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^k - m_t\theta_1^k - b_t\right) \pm W_{R,t}\times y_R^{max} \pm W_{D,t}\times y_D^{max} \mp W_{D,t}\times y_D^{max} \\
&= \pm\sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^{k'} - m_t\theta_1^{k'} - b_t\right) \pm W_{D,t}\times y_D^{max} \qquad (13)
\end{aligned}
$$

The set of points in the (θ_1, θ_2) space that are at distance $\Phi^{-1}\left(\Pr\left(Y_{it}=1\right)\right)$ from i's ideal point define a circle centered at $\bar{\theta}^i$. Allowing for both $\mathcal{I}_t = 0$ and $\mathcal{I}_t = 1$, the equations for member 0 in (12) and member k' in (13) define the tangents to each of the two circles for members 0 and k'. At most four $(m_t, \hat{b}_t)$ pairs define cutlines that are tangent to both circles: at most two outer tangents that place members 0 and k' on the same side of a cutline, and at most two inner tangents that place the members 0 and k' on opposite sides of a cutline. Figure 21 presents a visualization of the possible tangent cutlines.

For an outer tangent such that both members lie on the same side of it, we have $\theta_2^i < m_t\theta_1^i + \hat{b}_t$ for $i = 0, k'$, or $\theta_2^i > m_t\theta_1^i + \hat{b}_t$ for $i = 0, k'$. These inequalities imply $\Pr\left(Y_{it}=1\right) < \frac{1}{2}$ for both members or $\Pr\left(Y_{it}=1\right) > \frac{1}{2}$ for both members.

For an inner tangent such that one member lies on each side, we instead must have $\Pr\left(Y_{0t}=1\right) < \frac{1}{2}$ and $\Pr\left(Y_{k't}=1\right) > \frac{1}{2}$, or $\Pr\left(Y_{0t}=1\right) > \frac{1}{2}$ and $\Pr\left(Y_{k't}=1\right) < \frac{1}{2}$.

Therefore, given knowledge of the voting probabilities, at most two of the four possible cutlines (with an appropriate $\mathcal{I}_t$ associated with that cutline) can simultaneously satisfy the equations for the standardized likelihood of 0 and k' : either two cutlines that form outer tangents, or two cutlines that form inner tangents.[43]

Assumption ID1 allows us to pin down $m_t, b_t,$ and$\mathcal{I}_t$ uniquely from the two remaining possibilities by means of contradiction. Suppose, to the contrary, that two pairs of solutions, $(m_t^*, b_t^*, \mathcal{I}_t^*)$ and $(m_t^{**}, b_t^{**}, \mathcal{I}_t^{**})$, satisfy the two standardized likelihood equations for 0 and k' (and therefore k). Recall that each associated cutline must be tangent to both of the circles centered on each member's ideal point.

Now consider the possible locations of the other members of party D. To ensure $(m_t^*, b_t^*, \mathcal{I}_t^*)$ is indistinguishable from $(m_t^{**}, b_t^{**}, \mathcal{I}_t^{**})$, the circle centered on $\bar{\theta}^i$ with radius $\Phi^{-1}\left(\Pr\left(Y_{it}=1\right)\right)$ for each member, i, must also be tangent to both potential cutlines. Following the Locus theorem,

[43]In the two limiting cases in which one party's cutline passes exactly through its respective member's ideal point, the two possible cutlines are such that they pass on opposite sides of the other member's ideal point. The appropriate cutline is then immediately identified by knowing whether the other member's voting probability is greater or less than one-half.

a generic D member i must then lie on the line, A, passing through $\bar{\theta}^0$ and the intersection of the two potential cutlines or on the line orthogonal to A, A'. Points on these two lines are the only points that ensure i is equidistant from both cutlines, so that the circle associated with i is tangent to both.

We can rule out points on the line A'. If the two potential cutlines are outer tangents to the circles of 0 and k', then if a member i is located on A', he lies on the same side as 0 and k' for one cutline and on the opposite side for the other. But, we know whether each of the three probabilities, $\Pr(Y_{0t}=1), \Pr(Y_{k't}=1), \Pr(Y_{it}=1)$, is greater or less than one-half.[44] If all are on the same side, all must be greater than one-half or all must be less. If i is on the opposite side, then his probability must be greater than one-half if the other two are less than one-half, or vice versa. Thus, if i lies on A', we can distinguish between the two pairs of solutions, a contradiction. Similarly, if the two potential cutlines are inner tangents to the circles of 0 and k' then for one of the cutlines, i is on the same side as 0 (and opposite to k') and for the other i is on the same side as k' (and opposite to 0). Knowing which voting probabilities are greater or less than one-half again allows us to tell the solutions apart.

We have then shown that if we have two potential solutions, all members of party D must lie on the line A. One can construct an identical argument for party R: if there are two potential cutlines, all members of party R must be collinear. But, all members of either party being collinear violates Assumption ID1. Thus, the triplet $(m_t, \hat{b}_t, \mathcal{I}_t)$ is uniquely identified for all bills in the set $\boldsymbol{W_0}$.

Given uniqueness of the solution, $\Pr(Y_{0t}=1)$ greater or less than one-half provides $\mathcal{I}_t$.

m_t can then be recovered from the difference of the normalized likelihoods of members 0 and k:

$$\begin{aligned} & \Phi^{-1}\left(\Pr\left(Y_{0t}=1\right)\right) - \Phi^{-1}\left(\Pr\left(Y_{kt}=1\right)\right) \\ = \quad & \pm\sqrt{\frac{1}{1+m_t^2}}\left(\theta_2^0 - \theta_2^k - m_t(\theta_1^0 - \theta_1^k)\right) + \Delta y_t \end{aligned}$$

because the ideological parameters and Δy_t are known. Finally, $\hat{b}_t \equiv \pm\sqrt{\frac{1}{1+m_t^2}} b_t \mp W_{D,t} \times y_D^{max}$ is identified through the normalized likelihood of member 0 in (12), where its sign depends on the known $\mathcal{I}_t$.

Now consider bill s which, by assumption ID3(ii) is assumed to satisfy $m_s \neq 0$ and is contained in the set $\boldsymbol{W_0}$. The difference in the normalized likelihoods of member 0 and an arbitrary member

[44] Notice that $\Pr(Y_{k't}=1) = \Pr(Y_{kt}=1)$ by construction, so the hypothetical vote probabilities of k' are known because k's vote probabilities are known.

i of party R on this bill is given by

$$\Phi^{-1}\left(\Pr\left(Y_{0s}=1\right)\right)-\Phi^{-1}\left(\Pr\left(Y_{is}=1\right)\right)=\sqrt{\frac{1}{1+m_s^2}}\left(-\theta_2^i+m_s\theta_1^i\right)+\Delta y_s$$

so that θ_1^i is identified for all members of party R, because all of the other parameters have been identified. For members of party D, the argument is the same except that Δy_s does not enter the equation.

Having identified all of the Θ parameters, turn to the set of bills not in $\boldsymbol{W_0}$. Consider the normalized likelihoods of member 0 and another member from party D, j,:

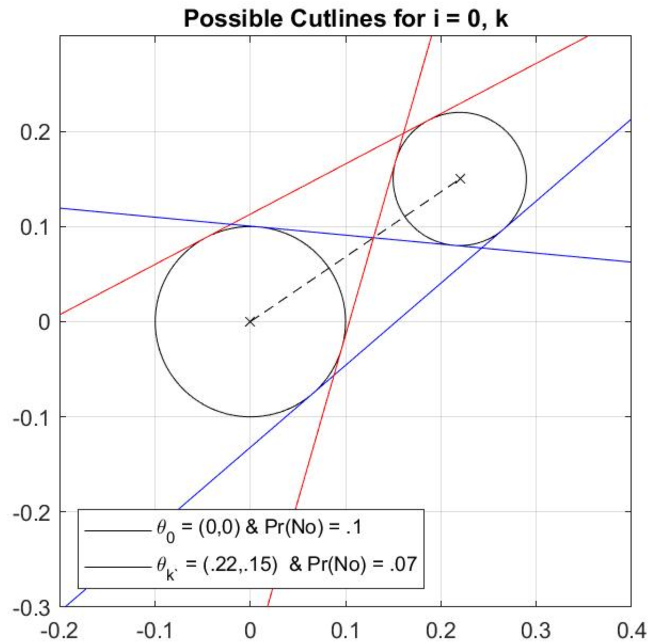
$$\begin{aligned}\Phi^{-1}\left(\Pr\left(Y_{0s}=1\right)\right)&=\pm\sqrt{\tfrac{1}{1+m_t^2}}\left(\theta_2^0-m_t\theta_1^0-b_t\right)\pm W_{D,t}\times y_D^{max}\\ \Phi^{-1}\left(\Pr\left(Y_{js}=1\right)\right)&=\pm\sqrt{\tfrac{1}{1+m_t^2}}\left(\theta_2^j-m_t\theta_1^j-b_t\right)\pm W_{D,t}\times y_D^{max}\end{aligned}$$

We can repeat the above arguments that utilized 0 and k's normalized likelihoods to identify $m_t, \hat{b}_t$, and $\mathcal{I}_t$ now that j's ideal point, $\bar{\theta}^j$, has been identified. Thus, $\left\{m_t, \hat{b}_t, \mathcal{I}_t\right\}_{t=1}^T$ is identified.

Finally, we can identify b_t, y_D^{max}, and y_R^{max}. Recall that, from the normalized bill, we have either (10) if the parties whip in opposite directions or (11) if the parties whip in the same direction. From Assumption ID4, we can perform the same exercise on another bill in which the parties whip differently (in opposite directions if they whip in the same direction on bill 0, or vice versa), so that we have both the sum and the difference of the $\bar{y}^{max}$ parameters, thus identifying both individually. Given y_D^{max}, m_t, and $\mathcal{I}_t$, we can then recover b_t from $\hat{b}_t \equiv \pm\sqrt{\frac{1}{1+m_t^2}}b_t \mp W_{D,t}\times y_D^{max}$ for all t.[45] □

[45] In the version of the model in which parties only whip (in opposite directions) when the party leaderships disagree, we can't separately identify the party discipline parameters, but can recover $y_D^{max}+y_R^{max}$ and each b_t. From member 0's normalized likelihood on bills that are not whipped, we obtain b_t for these bills. On bills that are whipped, the difference between normalized likelihoods of members of opposing parties gives $y_R^{max}+y_D^{max}$. Finally, assuming $y_R^{max}=y_D^{max}$ (without loss because only the sum is identifiable), we can recover b_t for bills that are whipped.

Figure 21: Identification Assumptions in a Numerical Example



Appendix B: DW-Nominate's Lack of Identification in Two Dimensions (or more)

In this section, we provide new insights as to the lack of identification of DW-Nominate (Dynamically Weighted Nominal Three-Step Estimation) in two dimensions. In Section B.1, we formally prove (building on, but correcting the proof in Potthoff 2018), that W-Nominate is not identified. This result immediately extends to DW-Nominate, as it is a generalization of W-Nominate with dynamically changing ideal points (i.e. preferences linearly changing in time). In Section B.2, we show that, even if the utility weight in W-Nominate were constrained to 1, the Gaussian utility function assumed in Nominate makes it very difficult to determine the number of normalizations necessary for it to be identified. This section builds on the work of Rivers (2003), which is, to date, the best formal discussion of identification of multidimensional spatial models. Finally, in Section B.3, we consider the effect of normalizing members' ideologies to lie within a unit circle: the only clearly specified normalization that Nominate imposes.

As background, the current version of DW-Nominate, updates active members' ideologies and estimates the cutline parameters for new bills as they become available (Boche et al., 2018). To do so, it holds constant inactive members' ideologies and the cutlines of previous bills (no "back-propagation"). New ideology and cutline estimates all rely on previous runs of DW-Nominate for identification. To quote Boche et al. (2018), p.24, "*...By effectively locking in place the locations that Poole last estimated for past members, we guarantee that our scores maintain compatibility with the widely used DW-Nominate scores with which scholars are familiar.*" Thus, unfortunately, beyond the unit circle normalization that DW-Nominate imposes, we do not know what other normalizations were initially imposed. As we show, however, no matter what these normalizations were, DW-Nominate is not identified.

B.1: Lack of Identification of W-Nominate

In W-Nominate, the 'W' stands for 'weighted'. It normalizes the utility weight in the first dimension to be one and allows the weight in second dimension, w_2, to be estimated. Here, we prove that this model is not identified by providing a transformation that can change the rank ordering of members in either dimension. Importantly, the transformation we provide is not a combination of a rotation, scale, and translation and thus poses a problem even if the rotation, scale, and location of the estimates are constrained via suitable normalization (as in our work).

Consider the likelihood argument in Carroll et al. (2009):

$$\begin{aligned} Pr(Y_{i,t}=1) &= \Phi\left[u(\theta_i,\mathbf{x}_t)-u(\theta_i,\mathbf{q}_t)\right]= \\ & \Phi\left[\beta e^{-\frac{1}{2}\left(\theta_i^1-x_t^1\right)^2-\frac{w_2}{2}\left(\theta_i^2-x_t^2\right)^2}-\beta e^{-\frac{1}{2}\left(\theta_i^1-q_t^1\right)^2-\frac{w_2}{2}\left(\theta_i^2-q_t^2\right)^2}\right] \end{aligned}$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. The vector of parameters of interest is $\Theta=\{\theta_i^1,x_t^1,q_t^1,\theta_i^2,x_t^2,q_t^2,w_2\}$.

Consider $s>0$ and $0<r<1$ and define the following candidate (nonlinear) transformation of the parameter vector, which can be proven to not be a rotation (other than in the special case $w_2=s=1$):

$$\begin{aligned} \tilde{\theta}_i^1 &= \theta_i^1\sqrt{r}-\theta_i^2\sqrt{w_2\left(1-r\right)} \\ \tilde{x}_t^1 &= x_t^1\sqrt{r}-x_t^2\sqrt{w_2\left(1-r\right)} \\ \tilde{q}_t^1 &= q_t^1\sqrt{r}-q_t^2\sqrt{w_2\left(1-r\right)} \\ \tilde{\theta}_i^2 &= s\times\left(\theta_i^1\sqrt{(1-r)}+\theta_i^2\sqrt{w_2 r}\right) \\ \tilde{x}_t^2 &= s\times\left(x_t^1\sqrt{(1-r)}+x_t^2\sqrt{w_2 r}\right) \\ \tilde{q}_t^2 &= s\times\left(q_t^1\sqrt{(1-r)}+q_t^2\sqrt{w_2 r}\right) \\ \tilde{w_2} &= \frac{1}{s^2} \end{aligned}$$

To check that within this class of transformations one obtains the same likelihood of the vote data:

$$\Phi\left[\beta e^{-\frac{1}{2}\left(\tilde{\theta}_i^1-\tilde{x}_t^1\right)^2-\frac{\tilde{w}_2}{2}\left(\tilde{\theta}_i^2-\tilde{x}_t^2\right)^2}-\beta e^{-\frac{1}{2}\left(\tilde{\theta}_i^1-\tilde{q}_t^1\right)^2-\frac{\tilde{w}_2}{2}\left(\tilde{\theta}_i^2-\tilde{q}_t^2\right)^2}\right]=$$
$$\Phi\left[\beta e^{-\frac{1}{2}\left(\theta_i^1-x_t^1\right)^2-\frac{w_2}{2}\left(\theta_i^2-x_t^2\right)^2}-\beta e^{-\frac{1}{2}\left(\theta_i^1-q_t^1\right)^2-\frac{w_2}{2}\left(\theta_i^2-q_t^2\right)^2}\right]$$

it suffices to show that:

$$\left(\tilde{\theta}_i^1-\tilde{x}_t^1\right)^2+\tilde{w}_2\left(\tilde{\theta}_i^2-\tilde{x}_t^2\right)^2=$$

$$\left(\theta_i^1\sqrt{r}-\theta_i^2\sqrt{w_2\left(1-r\right)}-x_t^1\sqrt{r}+x_t^2\sqrt{w_2\left(1-r\right)}\right)^2$$
$$+\frac{1}{s^2}\left(s\times\left(\theta_i^1\sqrt{(1-r)}+\theta_i^2\sqrt{w_2r}\right)-s\times\left(x_t^1\sqrt{(1-r)}+x_t^2\sqrt{w_2r}\right)\right)^2=$$

$$\left(\left(\theta_i^1-x_t^1\right)\sqrt{r}-\left(\theta_i^2-x_t^2\right)\sqrt{w_2\left(1-r\right)}\right)^2+\left(\left(\theta_i^1-x_t^1\right)\sqrt{(1-r)}+\left(\theta_i^2-x_t^2\right)\sqrt{w_2r}\right)^2=$$

$$\left(\theta_i^1-x_t^1\right)^2r+\left(\theta_i^2-x_t^2\right)^2w_2\left(1-r\right)-2\left(\theta_i^1-x_t^1\right)\sqrt{r}\left(\theta_i^2-x_t^2\right)\sqrt{w_2\left(1-r\right)}$$
$$+\left(\theta_i^1-x_t^1\right)^2(1-r)+\left(\theta_i^2-x_t^2\right)^2w_2r+2\left(\theta_i^1-x_t^1\right)\sqrt{(1-r)}\left(\theta_i^2-x_t^2\right)\sqrt{w_2r}=$$

$$\left(\theta_i^1-x_t^1\right)^2+w_2\left(\theta_i^2-x_t^2\right)^2$$

This proves that W-Nominate in two dimensions is not identified up to this class of transformations, which is broader than than the class of transformation that only rotate, scale, and/or change the location of the ideal points.

To show how this class of transformations is particularly damaging, consider three individuals $i=a,b,c$, each more conservative than the other with respect to the first dimension (i.e. $0<\theta_a^1<\theta_b^1<\theta_c^1$). Suppose the ideal points of a and c are known. We can show that, for an infinite set of values of r, either we can achieve the incorrect ranking, $\tilde{\theta}_b^1<\theta_a^1<\theta_c^1$, or the incorrect ranking, $\theta_a^1<\theta_c^1<\tilde{\theta}_b^1$, along the first dimension.

Consider the proposed transformation:

$$\tilde{\theta}_b^1=\theta_b^1\sqrt{r}-\theta_b^2\sqrt{w_2\left(1-r\right)}.$$

For $\theta_b^2\geq 0$, the inequality

$$\theta_b^1\sqrt{r}-\theta_b^2\sqrt{w_2\left(1-r\right)}<\theta_a^1$$

is always satisfied for $0 < r < 1$ sufficiently small and the inequality

$$\theta_b^1 \sqrt{r} - \theta_b^2 \sqrt{w_2 (1-r)} < \theta_c^1$$

is always satisfied for any r. Then, $\tilde{\theta}_b^1 < \theta_a^1 < \theta_c^1$, which is the incorrect ranking.

For $\theta_b^2 < -\frac{\theta_c^1}{\sqrt{w_2}} < 0$, the inequality

$$\theta_c^1 < \theta_b^1 \sqrt{r} - \theta_b^2 \sqrt{w_2 (1-r)}$$

is always satisfied for r sufficiently small. Then, $\theta_a^1 < \theta_c^1 < \tilde{\theta}_b^1$, which is the incorrect ranking.

For $-\frac{\theta_c^1}{\sqrt{w_2}} < \theta_b^2 < -\frac{\theta_a^1}{\sqrt{w_2}} < 0$, the inequality

$$\theta_a^1 > \theta_b^1 \sqrt{r} - \theta_b^2 \sqrt{w_2 (1-r)}$$

is always satisfied for r sufficiently small. Then, $\tilde{\theta}_b^1 < \theta_a^1 < \theta_c^1$, which is the incorrect ranking.

For $-\frac{\theta_a^1}{\sqrt{w_2}} < \theta_b^2 < 0$, both inequalities

$$\theta_a^1 < \theta_b^1 \sqrt{r} - \theta_b^2 \sqrt{w_2 (1-r)}$$
$$\theta_c^1 > \theta_b^1 \sqrt{r} - \theta_b^2 \sqrt{w_2 (1-r)}$$

are always satisfied. Thus, only in this case do we obtain the correct ranking, $\theta_a^1 < \tilde{\theta}_b^1 < \theta_c^1$.

Notice that we can apply a similar argument along the second preference dimension as well, further restricting the set of individuals who would be correctly ranked along both dimensions.

Because the specific case under examination is not known, this result implies full indeterminacy of the ranking of the ideology parameter estimates under this class of transformations. Specifically, the ranking along one of the two dimensions can be wrong for an infinite number of transformations.

B.2: Identification of Nominate

The previous section proves lack of identification for nonlinear transformations when, as in W-Nominate and DW-Nominate, the utility weight in the second dimension is estimated. Here, we discuss the identification of Nominate, which constrains all utility weights to be equal to one.[46]

In Section B.2.1, we consider the problem of identifying members' ideologies under the assumption that some of the cutline parameters, $\bar{x}_t$ and $\bar{q}_t$, are known. In Section B.2.2, we discuss the reverse problem: identifying the cutline parameters assuming some of the ideology parameters are known. Sections B.2.1 and B.2.2 are illustrative of the interim steps of the Nominate method

[46] We discuss the difficulties a Gaussian utility function creates even when $\beta = 1$ is assumed (Nominate estimates the parameter β as well, creating a further burden for identification on top of the ones discussed here).

(Nominal Three-Step Estimation), where either the cutlines or the ideal points are taken as given and the remaining set of parameters are estimated, iterating until convergence.

B.3.1: Known Bill Parameters

Making use of the Gaussian preferences employed in Nominate, let us start by highlighting that, for known roll call "0"

$$\Phi^{-1}\left[Pr(Y_{i,0}=1)\right] = u(\theta_i, \bar{x}_0) - u(\theta_i, \bar{q}_0)$$
$$e^{-\frac{1}{2}\left[\left(\theta_i^1 - x_0^1\right)^2 + \left(\theta_i^2 - x_0^2\right)^2\right]} - e^{-\frac{1}{2}\left[\left(\theta_i^1 - q_0^1\right)^2 + \left(\theta_i^2 - q_0^2\right)^2\right]}$$

is a highly-nonlinear equation in two unknowns (θ_i^1, θ_i^2). A generalized cubic equation in (θ_i^1, θ_i^2) follows from a second-order Taylor expansion of the difference in the deterministic utilities on the RHS for each vote:

$$\Phi^{-1}\left[Pr(Y_{i,0}=1)\right] =$$
$$e^{-\frac{1}{2}\left[\left(\theta_i^1 - x_0^1\right)^2 + \left(\theta_i^2 - x_0^2\right)^2\right]} - e^{-\frac{1}{2}\left[\left(\theta_i^1 - q_0^1\right)^2 + \left(\theta_i^2 - q_0^2\right)^2\right]} =$$
$$\sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^n}{n!} \left[\left[(\theta_i^1 - x_0^1)^2 + (\theta_i^2 - x_0^2)^2\right]^n - \left[(\theta_i^1 - q_0^1)^2 - (\theta_i^2 - q_0^2)^2\right]^n\right] \approx$$
$$-\frac{1}{2}\left[\sum_{j=1}^2 \left(\theta_i^j - x_0^j\right)^2 - \sum_{j=1}^2 \left(\theta_i^j - q_0^j\right)^2\right] + \frac{1}{8}\left[\left[\sum_{j=1}^2 \left(\theta_i^j - x_0^j\right)^2\right]^2 - \left[\sum_{j=1}^2 \left(\theta_i^j - q_0^j\right)^2\right]^2\right] =$$
$$-\frac{1}{2}\left[\sum_{j=1}^2 \left(x_0^j - q_0^j\right)\left(x_0^j + q_0^j - 2\theta_i^j\right)\right] \times \left[1 - \frac{1}{4}\sum_{j=1}^2 \left[\left(x_0^j\right)^2 + \left(q_0^j\right)^2 - 2\theta_i^j\left(x_0^j + q_0^j - \theta_i^j\right)\right]\right]$$

It is therefore possible to see that, even using approximations, a single normalization on a "0" bill is insufficient to uniquely pin down the (θ_i^1, θ_i^2) unknowns from the data $\Phi^{-1}\left[Pr(Y_{i,0}=1)\right]$.

Notice further that even for a quadratic loss function, instead of a Gaussian utility function, a single roll call normalization would still be insufficient for an unique mapping:

$$\Phi^{-1}\left[Pr(Y_{i,0}=1)\right] =$$
$$-\frac{1}{2}\left(\left(\theta_i^1 - x_0^1\right)^2 + \left(\theta_i^2 - x_0^2\right)^2\right) + \frac{1}{2}\left(\left(\theta_i^1 - q_0^1\right)^2 + \left(\theta_i^2 - q_0^2\right)^2\right) =$$
$$-\frac{1}{2}\left[\sum_{j=1}^2 \left(\theta_i^j - x_0^j\right)^2 - \sum_{j=1}^2 \left(\theta_i^j - q_0^j\right)^2\right] =$$
$$-\frac{1}{2}\sum_{j=1}^2 \left(x_0^j - q_0^j\right)\left(x_0^j + q_0^j - 2\theta_i^j\right)$$

To see the extent of the normalizations needed for different classes of individual utility functions, consider full knowledge of all policy issues $\bar{x}_t, \bar{q}_t$ for the set of T bill upon which a politician i votes, which can be treated as data. Then we can write the system of polynomials in for the unknown ideology, (θ_i^1, θ_i^2):

$$
\begin{cases}
\Phi^{-1}\left[Pr(Y_{i,0}=1)\right] = \delta_0^0 + \delta_1^0\theta_i^1 + \delta_2^0\theta_i^2 + \delta_3^0\left(\theta_i^1\right)^2 + \delta_4^0\left(\theta_i^2\right)^2 + \delta_5^0\theta_i^1\theta_i^2 + ... \\
... \\
\Phi^{-1}\left[Pr(Y_{i,t}=1)\right] = \delta_0^t + \delta_1^t\theta_i^1 + \delta_2^t\theta_i^2 + \delta_3^t\left(\theta_i^1\right)^2 + \delta_4^t\left(\theta_i^2\right)^2 + \delta_5^t\theta_i^1\theta_i^2 + ... \\
... \\
\Phi^{-1}\left[Pr(Y_{i,T}=1)\right] = \delta_0^T + \delta_1^T\theta_i^1 + \delta_2^T\theta_i^2 + \delta_3^T\left(\theta_i^1\right)^2 + \delta_4^T\left(\theta_i^2\right)^2 + \delta_5^T\theta_i^1\theta_i^2 + ...
\end{cases} \tag{14}
$$

Here, full knowledge of all $\bar{x}_t = \left(x_t^1, x_t^2\right), \bar{q}_t = \left(q_t^1, q_t^2\right)$ delivers what essentially amounts to bill-specific data $\{\delta_0^t, \delta_1^t, \delta_2^t, \delta_3^t, \delta_4^t, \delta_5^t, ...\}$, and (14) remains a system of T (typically nonlinear) equations in the two original unknowns (θ_i^1, θ_i^2). Generally, there cannot be any theoretical assurance of a unique exact mapping from the data on the LHS of the equations in the system to a unique $(\theta_i^1, \theta_i^2)^*$ for every i beyond the linear system case. However, operating under the hypothesis that the model is correctly specified the (14) will admit a unique solution for T large enough. In fact, (θ_i^1, θ_i^2) may be identifiable given knowledge of only the bill parameters for $\tau < T$ bills. We illustrate a few cases here, but emphasize that a general proof is not available (to the best of our knowledge).

For the quadratic utility case, the number of necessary normalizations is $\tau = 2$ bills (i.e. 8 parameter restrictions for $\bar{x}_0, \bar{x}_1, \bar{q}_0, \bar{q}_1$), given that the polynomials in (14) are of the first order. This implies that two roll calls can uniquely identify a solution (θ_i^1, θ_i^2) to (14), i.e. there is no observationally equivalent $\left(\tilde{\theta}_i^1, \tilde{\theta}_i^2\right) \neq (\theta_i^1, \theta_i^2)$ delivering the same set of values $\Phi^{-1}\left[Pr(Y_{i,t}=1)\right]$.

This result for quadratic utility is conceptually identical to the result in Rivers (2003), which proves that, for $d = 2$, the number of required restrictions is $d(d+1) = 6$. The difference here is that here we are considering as parameters the policy points, and not simply the policy cutlines (the 6 parameter restrictions on $\{\delta_0^0, \delta_1^0, \delta_2^0, \delta_0^1, \delta_1^1, \delta_2^1\}$). This difference does not affect the identification of the set of ideal points, but makes identification of the bill parameters more burdensome.

For utility functions that deliver conic functions in the system (14), the number of required normalizations $\tau = 5$ (i.e. 20 parameter restrictions). To see why, consider first that any system of two conic equations admits at most four solutions. Define these solutions as $\left\{\theta^A, \theta^B, \theta^C, \theta^D\right\}$. All of these solution are observationally equivalent in the sense of exactly satisfying both equations. This system defines the first two roll calls $\{\bar{x}_t, \bar{q}_t\}_{t=0,1}$ that are required for normalization. Let us now add an additional third bill $\bar{x}_2, \bar{q}_2$ introducing another conic equation and under the assumption that such conic equation is non-redundant in the sense of the direction of axes of the associated ellipse are not the same as those of any of the previously normalized conic equations. At most, three of the elements of the set $\left\{\theta^A, \theta^B, \theta^C, \theta^D\right\}$ will satisfy this third equation (if all the elements of $\left\{\theta^A, \theta^B, \theta^C, \theta^D\right\}$ satisfied this third restriction, than that would imply that the third conic equation is, in fact, redundant). Without loss, define the remaining set of candidate solutions as

$\{\theta^A, \theta^B, \theta^C\}$. Adding a fourth bill to the normalization (again assuming non-redundancy), delivers a set of candidate solutions satisfying this fourth constraint of (at most) two elements $\{\theta^A, \theta^B\}$, and a fifth bill, pins down the ideology vector uniquely to, say, $\{\theta^A\}$. In summary, normalization of five bills is needed for theoretical identification of the ideology parameters (θ_i^1, θ_i^2) under the assumption that the model is correctly specified.

For utility functions that deliver cubic functions in (14), as in the case of a second-order approximation of the difference in Gaussian utilities used in Nominate, the number of normalizations is higher than $\tau = 5$ bills, as the number of conditions grows. This exercise illustrates that the number of normalizations required for Gaussian utility functions in Nominate is likely much higher than that required for quadratic utility functions, and that it is difficult to determine how many bills must be normalized to uniquely identify the ideal points for N members.

The discussion in this subsection illustrates the inherent difficulty in proving identification within each of Nominate's interim steps (i.e. the algorithm's iteration step where all of the cutline parameters are assumed given and the ideal points are estimated). It is not immediate that each iteration is guaranteed to deliver a unique vector of ideal point estimates.

B.3.2: Known Ideal Points

Concerning the policy choice parameters $\bar{x}_t, \bar{q}_t$, let us focus on the expression

$$Pr(Y_{i,t} = 1) =$$
$$\Phi\left[e^{-\frac{1}{2}\left(\theta_i^1 - x_t^1\right)^2 - \frac{1}{2}\left(\theta_i^2 - x_t^2\right)^2} - e^{-\frac{1}{2}\left(\theta_i^1 - q_t^1\right)^2 - \frac{1}{2}\left(\theta_i^2 - q_t^2\right)^2}\right]$$

for known ideology parameters. Specifically, under a normalization for $\theta_0 = (\theta_0^1, \theta_0^2)$, we can write:

$$\Phi^{-1}\left[Pr(Y_{0,t} = 1)\right] =$$
$$e^{-\frac{1}{2}\left[\left(\theta_0^1 - x_t^1\right)^2 + \left(\theta_0^2 - x_t^2\right)^2\right]} - e^{-\frac{1}{2}\left[\left(\theta_0^1 - q_t^1\right)^2 + \left(\theta_0^2 - q_t^2\right)^2\right]} =$$
$$\textstyle\sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^n}{n!}\left[\left[(\theta_0^1 - x_t^1)^2 + (\theta_0^2 - x_t^2)^2\right]^n - \left[(\theta_0^1 - q_t^1)^2 + (\theta_0^2 - q_t^2)^2\right]^n\right] \approx$$
$$-\frac{1}{2}\left[\textstyle\sum_{j=1}^{2}\left(x_t^j - q_t^j\right)\left(x_t^j + q_t^j - 2\theta_0^j\right)\right] \times \left[1 - \frac{1}{4}\textstyle\sum_{j=1}^{2}\left[\left(x_t^j\right)^2 + \left(q_t^j\right)^2 - 2\theta_0^j\left(x_t^j + q_t^j - \theta_0^j\right)\right]\right]$$

which, even in second-order approximate form, does not lend to an immediate analysis of the mapping from data to policy points and generally admits multiple solutions.

With a further normalization for $\theta_1 = (\theta_1^1, \theta_1^2)$ one can make more progress focusing on quadratic losses or first-order approximation of the (difference in) Gaussian utilities. In particular, note that with quadratic losses:

$$
\begin{aligned}
\Phi^{-1}\left[Pr(Y_{0,t}=1)\right]-\Phi^{-1}\left[Pr(Y_{1,t}=1)\right] &= \\
-\frac{1}{2}\sum_{j=1}^{2}\left(x_t^j-q_t^j\right)\left(x_t^j+q_t^j-2\theta_0^j\right)+\frac{1}{2}\sum_{j=1}^{2}\left(x_t^j-q_t^j\right)\left(x_t^j+q_t^j-2\theta_1^j\right) &= \\
\sum_{j=1}^{2}\left(x_t^j-q_t^j\right)\left(\theta_0^j-\theta_1^j\right).
\end{aligned}
\tag{15}
$$

Following a similar approach to that laid out in the preceding section, we can observe that for every roll call t, four equations of the type (15) are necessary for the four unknown bill parameters. We require therefore four politicians to be normalized (i.e. 8 parameters) to uniquely identify all parameters $\bar{x}_t, \bar{q}_t$ from the data.

For the case of Gaussian preferences such as those used in Nominate, however, the situation appears more complex. For the case of the second order Taylor expansion, we see that the system of equations of conditions for identification will be composed of generalized quartic equations and so that we know that we need at least 20 restrictions. Again, this fact illustrates that Nominate with Gaussian preferences requires a substantially higher number of identification restrictions than for the quadratic utility case of Rivers (2003). Mirroring the problem with estimating the ideal points holding the cutlines fixed, it is not immediate that the alternative iteration steps in which the ideal points are held fixed and the cutlines estimated will deliver unique cutline estimates.

B.3: A discussion of further normalizations in DW-Nominate

The only normalization that DW-Nominate imposes that is consistently specified (see p.268 of Armstrong et al. 2014) is that all of the ideologies must lie within a unit circle. This normalization may at first appear intuitive, but we point out two difficulties that it creates. Both of the difficulties arise because DW-Nominate does not re-estimate all ideologies and cutline parameters when new roll call data arrives (i.e. no back-propagation). If one were to estimate everything *without* restricting ideologies to the unit circle and then simply rescale them to lie within the unit circle, the normalization would pose no problem. For example, one could take our estimates and simply rescale them all to lie within the unit circle given that the scaling is arbitrary. But, because DW-Nominate imposes the restriction in the estimation process, two complications arise.

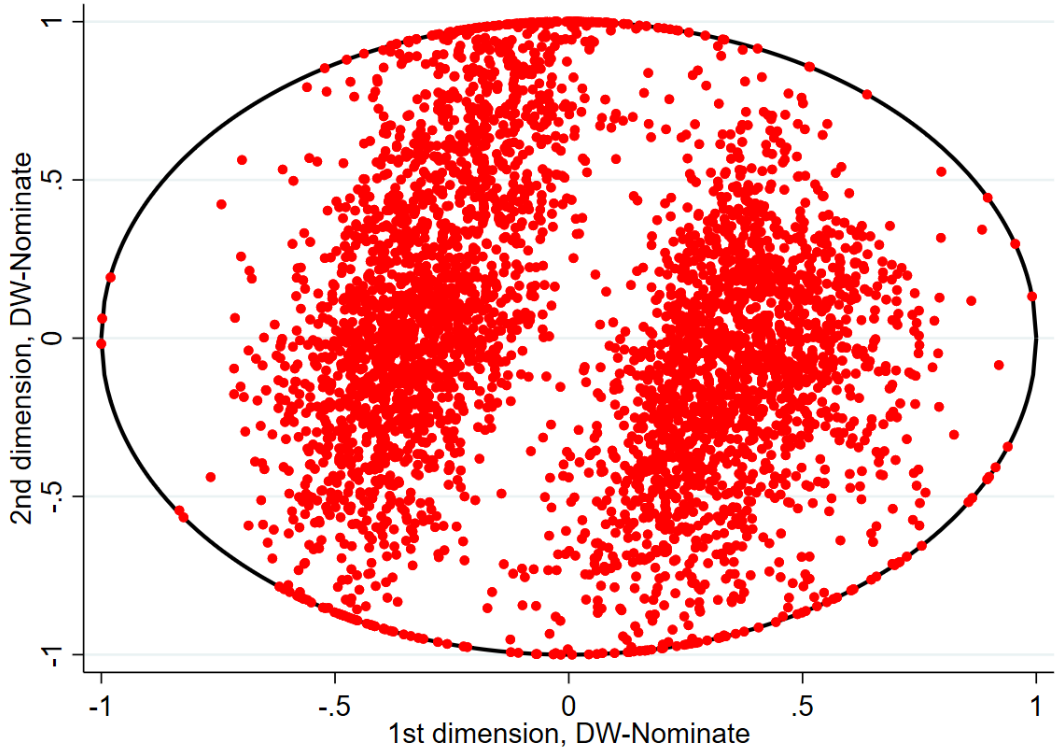
The first difficulty is that a unit circle restriction creates an artificial negative correlation between the two dimensions of members' ideological positions. To see this problem most clearly, consider a new member of Congress, i, that is very liberal in the first dimension. Locating this member at $\theta_1^i = -1$ forces him or her to be perfectly moderate in the second dimension (θ_2^i

must be 0). In reality, the estimation procedure will be forced to make a compromise: to place a member at an extreme position along the first dimension, it must mechanically moderate the member in the second dimension (and similarly, for placing a member at an extreme position along the second dimension). We do not believe there is any ex ante reason to think that politicians cannot simultaneously hold extreme positions in both dimensions, but DW-Nominate rules out this possibility through the unit circle normalization.

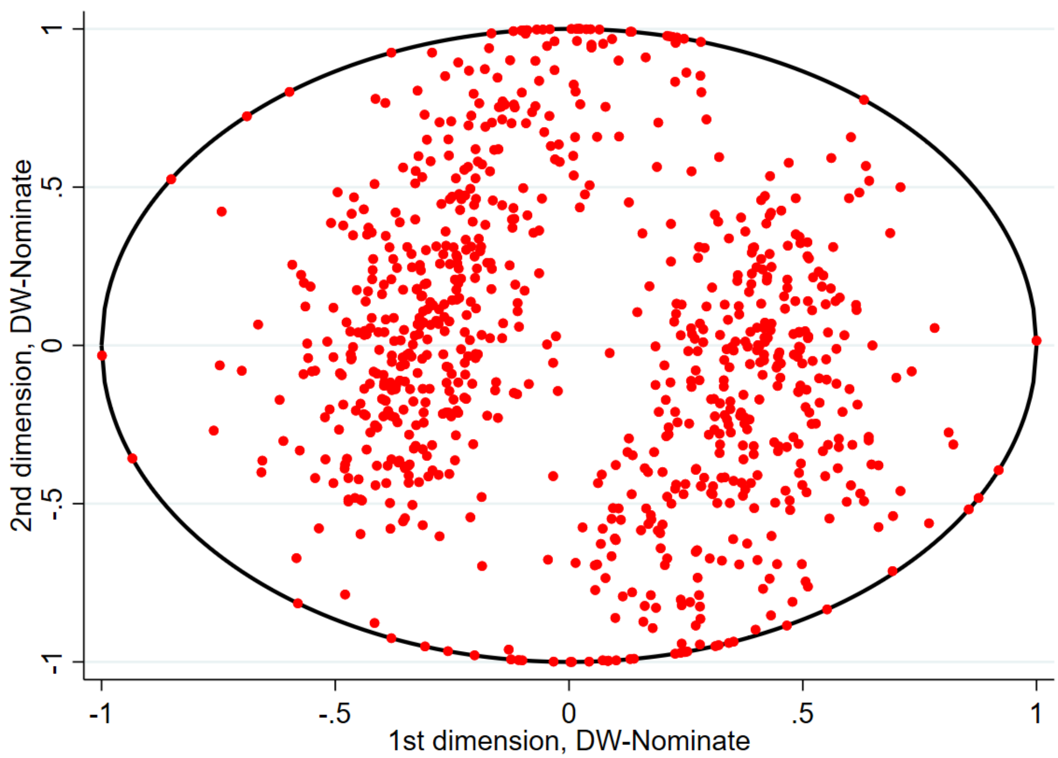
The second difficulty directly stems from the lack of back-propagation. At one point in time, prior to knowing all future members' ideological points, DW-Nominate was scaled such that all members at that time lied within the unit circle. But, unless the constraint was originally 'slack' (no members were located on the unit circle), this scaling implies that any future member that is more extreme than any of those in this initial set will lie on the unit circle boundary artificially. If progressively more extreme politicians are in fact replacing more moderate ones, this normalization starts to progressively become more problematic. To provide suggestive evidence that this artificial constraint is binding, in Figure 22, we plot the unit circle together with all DW-Nominate estimates for each ideology from Congress 70 to Congress 115, both for the House and for the Senate. Since Congress 70, approximately 7% of estimates in the House sit on the boundary of the unit circle, with 8% being on the boundary for the Senate. This evidence suggests that the unit circle boundary is directly and artificially constraining the estimated ideologies for a non-trivial number of legislators. Furthermore, note that this constraint also affects estimates of members away from the boundary, because their ideologies are estimated by incorporating information from those who sit on the boundary.

Figure 22: The Role of the Unit Circle Restriction in DW-Nominate

(a) House of Representatives



(b) Senate



Appendix C: Computational Details of the Estimation Procedure

We maximize the likelihood in (7) via an unconstrained optimization procedure, providing the analytic gradient to the algorithm to greatly improve estimation speed. Rather than using an off-the-shelf quasi-newton algorithm (such as Matlab's fminunc), which proved to perform very poorly given the non-convexity of our likelihood function, we instead use Adam, a version of the steepest descent algorithm. Adaptive Moment Estimation (Adam) is a stochastic optimization algorithm which is also ideal for problems with a large number of parameters like ours (Kingma and Ba, 2014).

As is standard, we run the estimation procedure until either the stepsize or the gradient is small (for the 2D model, typically the estimation procedure terminated due to the stepsize being small, on the order of 1e-4).

Because for non-convex optimization problems, convergence to a global maximum cannot be guaranteed, we ran the estimation procedure for our main model (Senate 2D) with 60 starting points, with each batch of 12 taking roughly one day on a 64G RAM machine. This is an extensive search for a problem of the size as the one that we study (90 years of Congressional voting, including all available roll calls). For the Senate 2D model, we use the first dimension ideological positions from the Senate 1D model as starting points. For the misspecified Senate 2D model (without discipline), we use ideology estimates from the full Senate 2D model. Starting points were otherwise randomly chosen (i.e. for the cutlines, party discipline parameters, and ideologies for the 1D models).

We report the estimates for the estimation run that produced the largest likelihood across runs. But, we emphasize that the estimates of the main parameters of interest (namely, the party discipline parameters) were quantitatively very similar (although not identical) across runs.

Appendix D: Additional Tables and Figures

Figure 23: Ideological Polarization Over Time (2nd dimension), 1927-2019 - Senate 2D Model

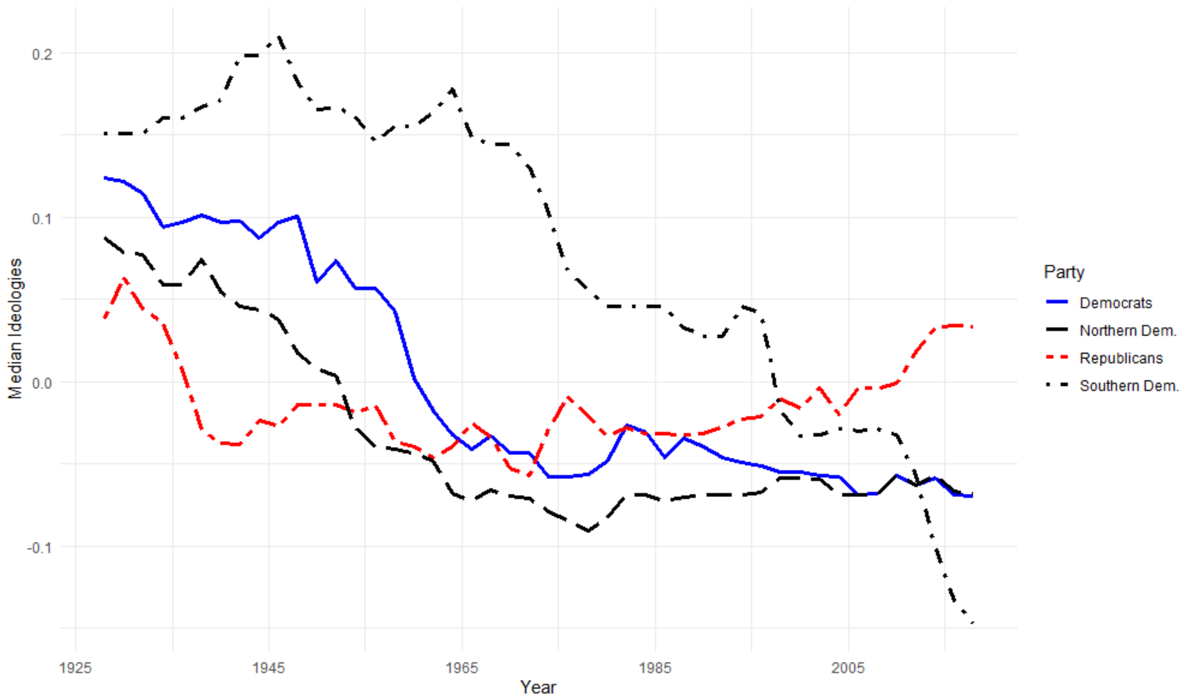
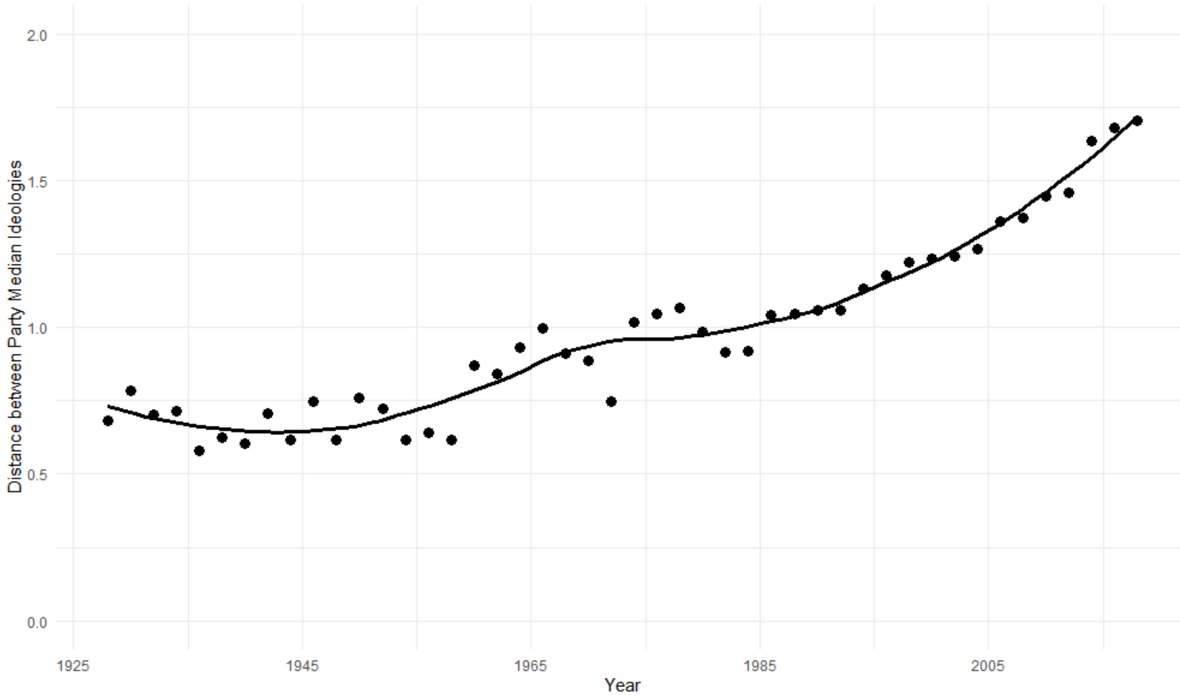


Figure 24: Ideological Polarization in the 1D Model

(a) Senate



(b) House

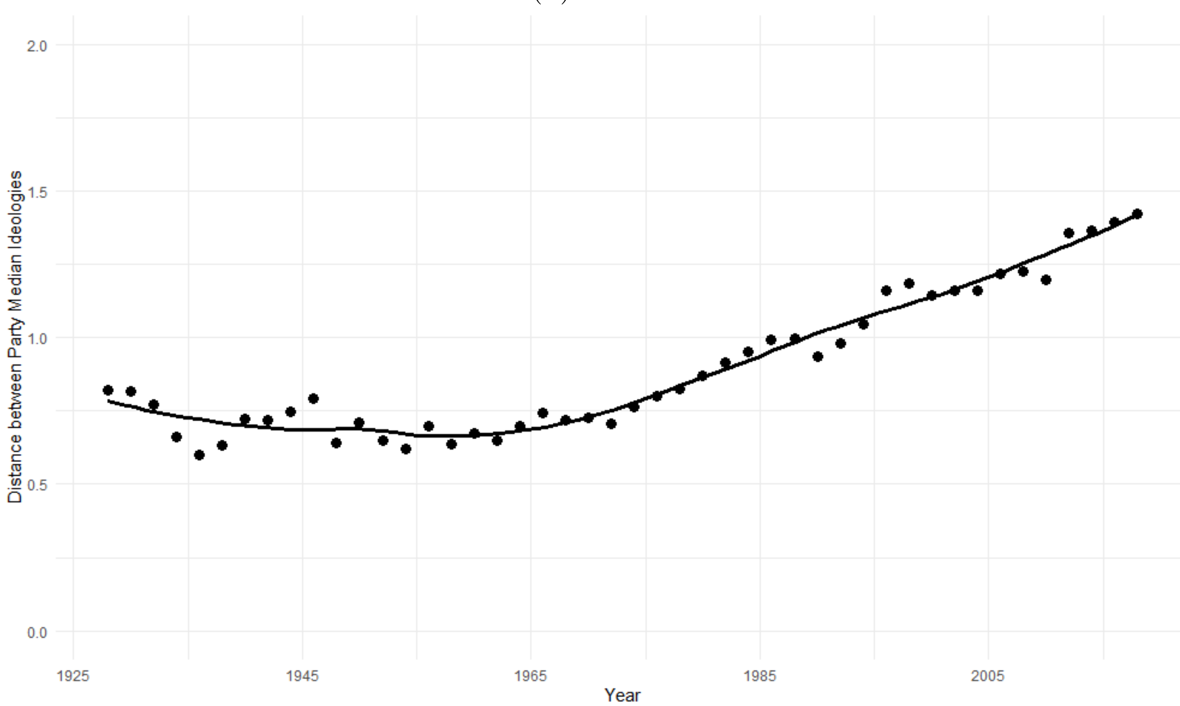
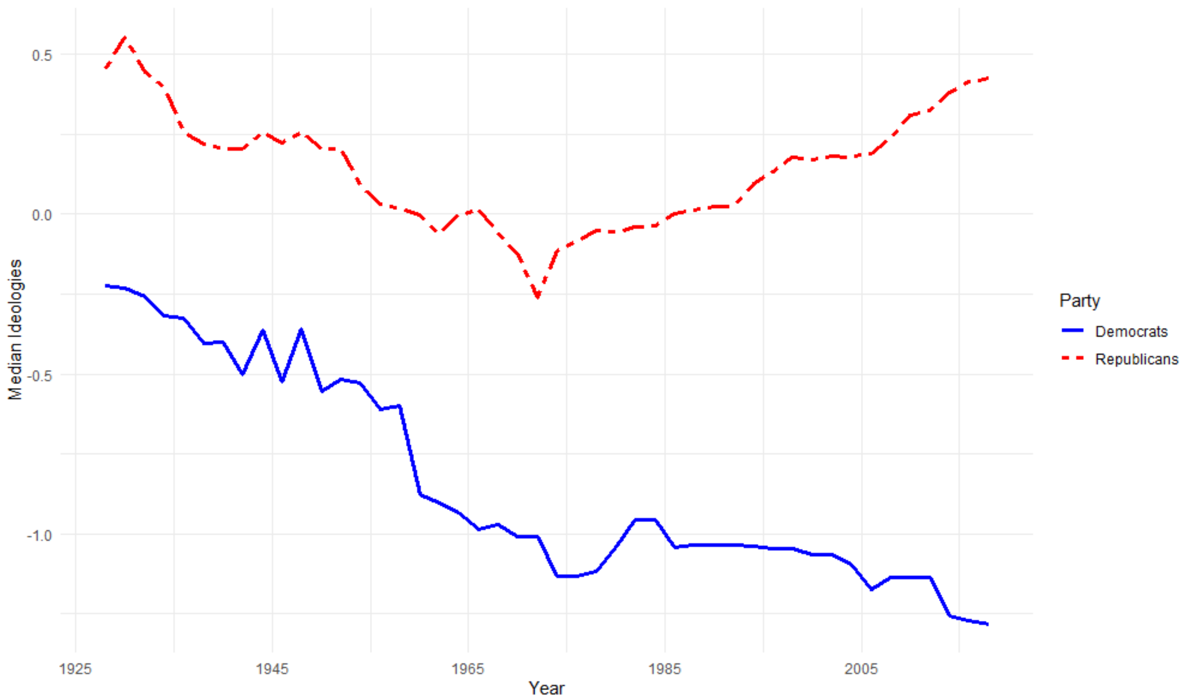


Figure 25: Ideological Polarization over Time, 1927-2019 - 1D Model

(a) Senate



(b) House

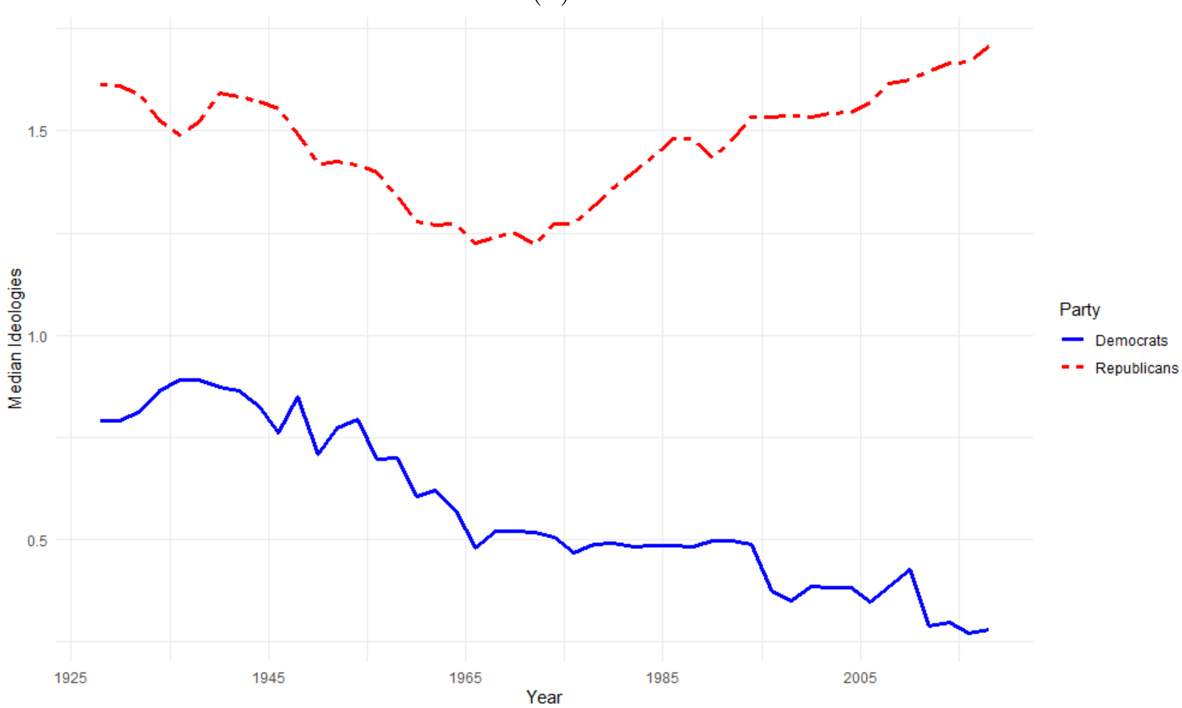
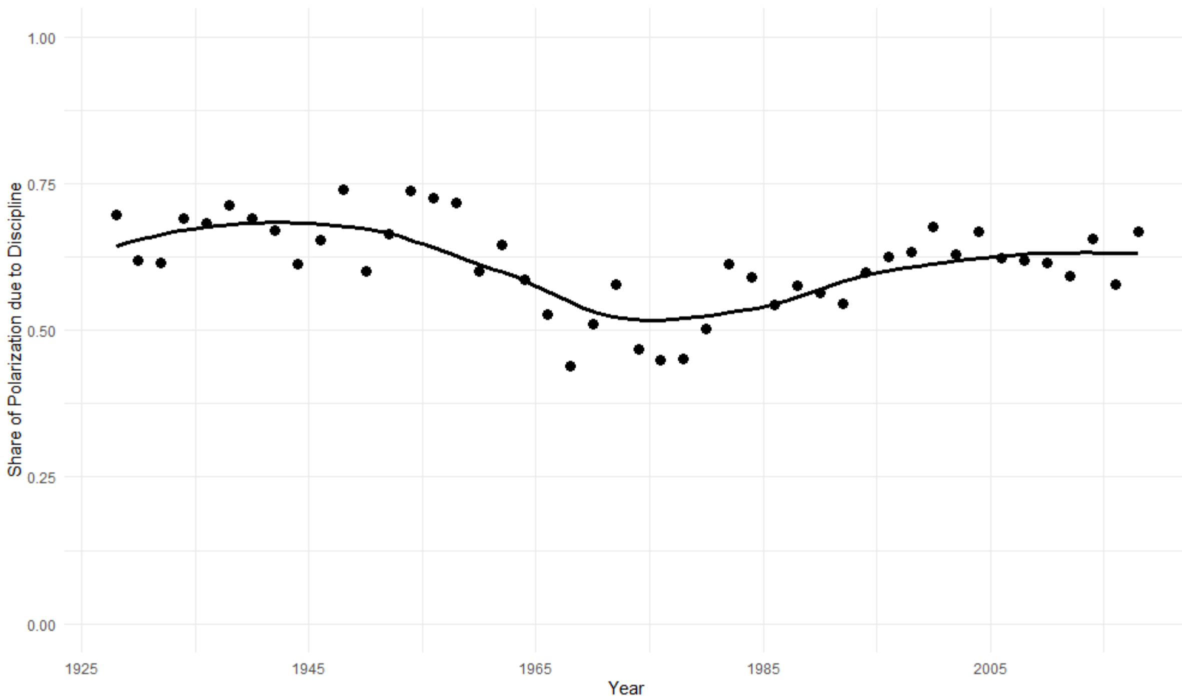


Figure 26: Share of Ideological Polarization Attributable to Party Discipline - 1D Model

(a) Senate



(b) House

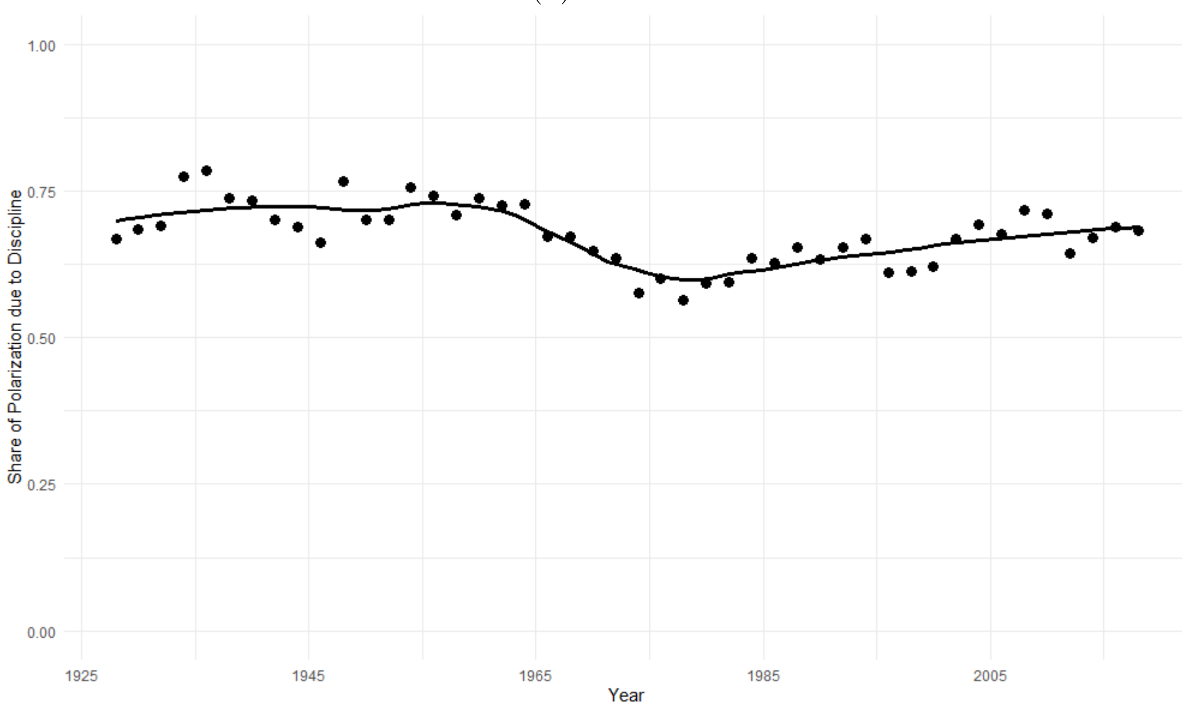
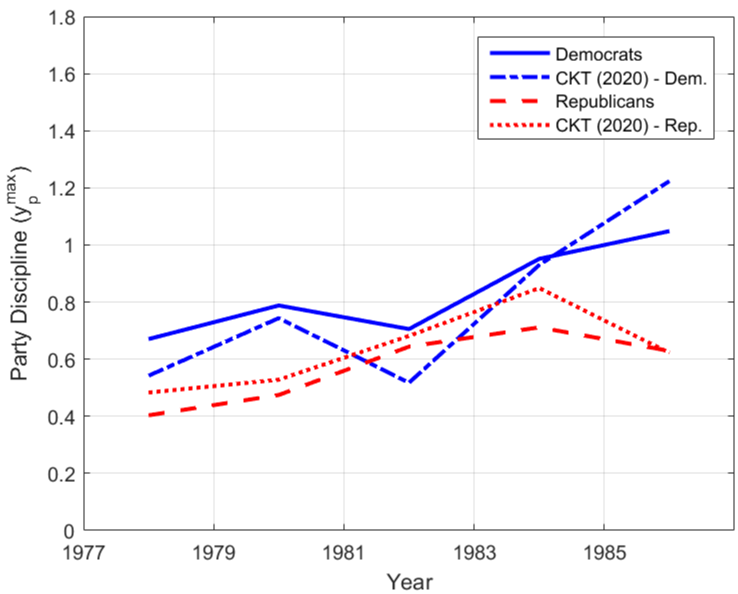


Figure 27: Comparison of Party Discipline Estimates with and without agenda setting



Notes: Estimates of y_p^{max} compared to those from Canen et al. (2020) for 1977-1986 (i.e. Congresses 95-99). Canen et al. (2020) assumed utility shocks have a variance equal to two (instead of one), so the prior estimates are rescaled by $\sqrt{2}$.

Table 2: Summary Statistics

Congress	Senate				House			
	Bills introduced	Avg. bills per member	Bills passed	Fraction that pass	Bills introduced	Avg. bills per member	Bills passed	Fraction that pass
80th (1947-1948)	3,186	33.2	1,670	0.524	7,611	17.5	1,739	0.228
81st (1949-1950)	4,486	46.7	2,362	0.527	10,502	24.1	2,482	0.236
82nd (1951-1952)	3,665	38.2	1,849	0.505	9,065	20.8	2,008	0.222
83rd (1953-1954)	4,077	42.5	2,231	0.547	10,875	25.0	2,129	0.196
84th (1955-1956)	4,518	47.1	2,550	0.564	13,169	30.3	2,360	0.179
85th (1957-1958)	4,532	47.2	2,202	0.486	14,580	33.5	2,064	0.142
86th (1959-1960)	4,149	41.5	1,680	0.405	14,112	32.3	1,636	0.116
87th (1961-1962)	4,048	40.5	1,953	0.482	14,328	32.8	1,927	0.134
88th (1963-1964)	3,457	34.6	1,341	0.388	14,022	32.2	1,267	0.090
89th (1965-1966)	4,129	41.3	1,636	0.396	19,874	45.7	1,565	0.079
90th (1967-1968)	4,400	44.0	1,376	0.313	22,060	50.7	1,213	0.055
91st (1969-1971)	4,867	48.7	1,271	0.261	21,436	49.3	1,130	0.053
92nd (1971-1972)	4,408	44.1	1,035	0.235	18,561	42.7	970	0.052
93rd (1973-1974)	4,524	45.2	1,115	0.246	18,872	43.4	923	0.049
94th (1975-1976)	4,115	41.2	1,038	0.252	16,982	39.0	968	0.057
95th (1977-1978)	3,800	38.0	1,070	0.282	15,587	35.8	1,027	0.066
96th (1979-1980)	3,480	34.8	976	0.280	9,103	20.9	929	0.102
97th (1981-1982)	3,396	34.0	786	0.231	8,094	18.6	704	0.087
98th (1983-1984)	3,454	34.5	936	0.271	7,105	16.3	978	0.138
99th (1985-1986)	3,386	33.9	940	0.278	6,499	14.9	973	0.150
100th (1987-1988)	3,325	33.3	1,002	0.301	6,263	14.4	1,061	0.169
101st (1989-1990)	3,669	36.7	980	0.267	6,664	15.3	968	0.145
102nd (1991-1992)	3,738	37.4	947	0.253	6,775	15.6	932	0.138
103rd (1993-1994)	2,805	28.1	682	0.243	5,739	13.2	749	0.131
104th (1995-1996)	2,266	22.7	518	0.229	4,542	10.4	611	0.135
105th (1997-1998)	2,718	27.2	586	0.216	5,014	11.5	710	0.142
106th (1999-2000)	3,343	33.4	819	0.245	5,815	13.4	957	0.165
107th (2001-2002)	3,242	32.4	554	0.171	5,892	13.5	677	0.115
108th (2003-2004)	3,078	30.8	759	0.247	5,547	12.8	801	0.144
109th (2005-2006)	4,163	41.6	684	0.164	6,540	15.0	770	0.118
110th (2007-2008)	3,738	37.4	556	0.149	7,441	17.1	1101	0.148
111th (2009-2010)	4,101	41.0	176	0.043	6,677	15.3	861	0.129
112th (2011-2012)	3,767	37.7	364	0.097	6,845	15.7	561	0.082

Table 3: Regression Results - Sources of Party Discipline

	Estimates of y_p^{max}				
Party (Republican)	0.051 (0.073)	0.036 (0.097)	0.051 (0.073)	0.036 (0.097)	0.036 (0.060)
Majority Status		-0.045 (0.097)		-0.045 (0.097)	-0.045 (0.060)
Divided Government (1 if Divided)			0.032 (0.073)	0.032 (0.074)	0.087 (0.051)
Observations	92	92	92	92	92
Decade Fixed Effect					Yes
R^2	0.005	0.009	0.008	0.011	0.635

Notes: Regressions of the time series of estimates of $\{y_p^{max}\}_{p\in\{D,R\}}$ for the Senate 2D model on a Party level dummy variable (equal to 1 if p is Republican), dummy variable for Majority Status (which equals 1 if party p held the majority of seats in the Senate, and 0 otherwise) and dummy variable for divided government (which is equal to 0 if the president's party is the same as the majority party in the House and in the Senate and 1 otherwise). Robust standard errors in parentheses.