DISCUSSION PAPER SERIES

DP15572

THE STORY OF THE REAL EXCHANGE RATE

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INTERNATIONAL MACROECONOMICS AND FINANCE

MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper DP15572
Published 17 December 2020
Submitted 15 December 2020

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Abstract

The real exchange rate (RER) measures relative price levels across countries, capturing deviations from purchasing power parity (PPP). RER is a key variable in international macroeconomic models as it is central to equilibrium conditions in both goods and asset markets. It is also one of the most starkly-behaved variables empirically, tightly co-moving with the nominal exchange rate and virtually uncorrelated with most other macroeconomic variables, nominal or real. This survey lays out an equilibrium framework of RER determination, focusing separately on each building block and discussing corresponding empirical evidence. We emphasize home bias and incomplete pass-through into prices with expenditure switching and goods market clearing, imperfect international risk sharing, country budget constraint and monetary policy regime. We show that RER is inherently a general-equilibrium variable, which depends on the full model structure and policy regime, and therefore partial theories like PPP are insufficient to explain it. We also discuss issues of stationarity and predictability of exchange rates.

JEL Classification: F31, F41, E30

Keywords: real exchange rate, PPP puzzle, Backus-Smith puzzle

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Acknowledgements
When citing this paper, please use the following: Itskhoki O. 2021. The Story of the Real Exchange Rate. Annu. Rev. Econ. 13: DOI: 10.1146/annurev-economics-080218-025532. Based on my graduate lectures that I taught at Princeton in 2009-2019. I am grateful to Ken Rogoff for introducing me to this topic in his Ph.D. class at Harvard in 2005, as well as to my coauthors Gita Gopinath, Roberto Rigobon, Emmanuel Farhi, Brent Neiman, Mary Amiti, Joep Konings and Dmitry Mukhin, with whom I worked in this area and who shaped my understanding of the topic. I also benefited from conversations with Mark Aguiar, Andy Atkeson, Ariel Burstein, Charles Engel, Pierre-Olivier Gourinchas, Jon Steinsson and many others. All errors are, of course, my own.

The Story of the Real Exchange Rate*

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December 8, 2020

Abstract

The real exchange rate (RER) measures relative price levels across countries, capturing deviations from purchasing power parity (PPP). RER is a key variable in international macroeconomic models as it is central to equilibrium conditions in both goods and asset markets. It is also one of the most starkly-behaved variables empirically, tightly co-moving with the nominal exchange rate and virtually uncorrelated with most other macroeconomic variables, nominal or real. This survey lays out an equilibrium framework of RER determination, focusing separately on each building block and discussing corresponding empirical evidence. We emphasize home bias and incomplete pass-through into prices with expenditure switching and goods market clearing, imperfect international risk sharing, country budget constraint and monetary policy regime. We show that RER is inherently a general-equilibrium variable, which depends on the full model structure and policy regime, and therefore partial theories like PPP are insufficient to explain it. We also discuss issues of stationarity and predictability of exchange rates.

*Prepared for the *Annual Review of Economics*, https://doi.org/10.1146/annurev-economics-080218-025532. Based on my graduate lectures that I taught at Princeton in 2009–2019. I am grateful to Ken Rogoff for introducing me to this topic in his Ph.D. class at Harvard in 2005, as well as to my coauthors Gita Gopinath, Roberto Rigobon, Emmanuel Farhi, Brent Neiman, Mary Amiti, Joep Konings and Dmitry Mukhin, with whom I worked in this area and who shaped my understanding of the topic. I also benefited from conversations with Mark Aguiar, Andy Atkeson, Ariel Burstein, Charles Engel, Pierre-Olivier Gourinchas, Jón Steinsson and many others. All errors are, of course, my own.

Contents

1 Introduction

The real exchange rate (RER) measures relative price levels across countries. In other words, RER captures deviations from purchasing power parity (PPP), according to which price levels should be equalized across space (countries). A country with a higher consumer price level is said to have an appreciated real exchange rate. Movements in RER reflect gaps between relative inflation rates across countries and nominal exchange rate depreciation: when nominal depreciation is not met with more price inflation at home, a real depreciation occurs.

Why should economists care about RER, and in particular about its short-to-medium run dynamics? Indeed, it is an artificial construct: while it looks like a relative price, in fact there is no market in which it acts like one, since we cannot exchange consumer baskets across countries. This contrasts with the terms of trade (ToT), a closely related concept, which is in fact a relative price in the export-import market. Nonetheless, RER often plays a more central role than ToT in academic and policy discussions. In part, it is perhaps because popular discussion sometimes conflates RER and ToT. Still, the focus on RER is justified from a theoretical perspective: RER deserves its central place in international macroeconomics as a crucial diagnostic variable for our modeling frameworks — both in the goods market and in the asset market — as we explore in this manuscript.

Furthermore, RER is one of the most starkly-behaved variables. It co-moves tightly with the nominal exchange rate (NER) and is virtually uncorrelated with most other macroeconomic fundamentals, real or nominal. These facts provide sharp testable implications for models, which often have a hard time matching the empirical properties of exchange rates, resulting in numerous puzzles. Historically, the behavior of RER was viewed as prime evidence of nominal non-neutralities (the Mussa puzzle) and nominal rigidities (the PPP Puzzle). We argue here that the focus on nominal rigidities, and more specifically models in which monetary shocks are the driving force of RER, sidetracked the literature from focusing on alternative mechanisms, which are more successful at explaining the behavior of exchange rates.

We further argue that RER is inherently a general-equilibrium object, which depends on the full model structure and policy regime. As a result, partial theories like PPP or non-tradables, which were often the focus of the literature, are insufficient to explain RER. As we will show, PPP deviations, while necessary for RER to move, shed little light on the nature of RER volatility and persistence, which are shaped by general-equilibrium forces — both in the asset and in the goods markets.

In Section 2, we start with basic definitions and empirical facts about exchange rates that we address in the remainder of the manuscript. Section 3 focuses on PPP and the nature of PPP deviations: non-tradables and home bias in tradables, and various forms of the law of one price (LOP) violations — pricing to market and foreign-currency price stickiness. We review both theory and empirical evidence supporting or falsifying it. While some form of PPP deviations is necessary for a theory of RER, no partial theory of PPP deviations is sufficient to explain its equilibrium properties. A combination of aggregate home bias with incomplete exchange rate pass-through into prices is essential for a successful quantitative theory of RER. Yet, this offers only an entry point into the full equilibrium analysis of RER.

The following three sections study various general equilibrium aspects of RER determination. In Section 4, we explore the second step of international transmission — expenditure switching from movements in international relative prices into import quantities and net exports. This section also discusses goods market clearing in an open economy and the implied equilibrium relationship between RER and aggregate consumption. For the goods market to clear, a decline in home aggregate consumption requires a RER depreciation to shift global expenditure towards domestically produced goods in order to balance out their excess supply. We show that expenditure switching in the goods market is at the core of general equilibrium comovement between RER and macroeconomic quantities, or lack thereof, as emphasized empirically by the Backus-Smith correlation (puzzle).

Section 5 switches focus to the asset market, exploring the role of exchange rates in international risk sharing. We begin with two limiting cases — complete asset markets and financial autarky. Surprisingly, these two opposite extremes result in qualitatively equivalent implications of strong counterfactual comovement between aggregate consumption and RER, as they both turn a generally dynamic risk sharing condition into a static equilibrium relationship — the Backus-Smith risk sharing condition in the former case and the balanced trade condition in the latter case. We then consider the general case of incomplete asset markets, both without additional frictions and under financial constraints or market segmentation. This relaxes the tight state-by-state relationship between RER and macroeconomic aggregates, and instead results in a *martingale* condition for RER with possible sources of departure from a pure random walk. We show that the asset market equilibrium shapes the expected future changes in RER without pinning down the equilibrium level of RER or its unexpected level jumps.

Finally, in Section 6, we explore the remaining general-equilibrium determinants of RER — the country budget constraint and monetary policy regime. While equilibrium in the financial market shapes the future expected changes in exchange rates, the intertemporal budget constraint provides an integral condition on the equilibrium level of RER. This provides the remaining condition required to determine the unexpected level jumps in RER which are necessary to balance a country's intertemporal budget constraint. Together with the martingale risk sharing property, this establishes equilibrium discipline on the long-run behavior of RER. In general, RER follows an integrated non-stationary process that exhibits partial mean reversion, implying very long measured half lives consistent with the empirical evidence on PPP.

Therefore, it is the general equilibrium determinants of RER — in the goods and asset markets — that shape the dynamic properties of RER, independently of the specific nature of PPP deviations at the micro level. Inflation-stabilizing monetary policy then ensures the tight relationship between nominal and real exchange rates observed in the data.

To summarize, this survey emphasizes a general equilibrium framework that combinines home bias and incomplete pass-through into prices with expenditure switching and goods market clearing, imperfect international risk sharing, intertemporal budget constraint and monetary policy. These elements jointly determine the equilibrium behavior of exchange rates and their comovement with macroeconomic variables that is consistent with empirical evidence. This analysis builds on excellent previous surveys, including Rogoff (1996), Goldberg and Knetter (1997), Burstein and Gopinath (2012), Engel (2014), with a focus on the general equilibrium nature of exchange rates and the recent theoretical and empirical work in this area.

Overview of the equilibrium system Sections 3–6 analyze sequentially the various equilibrium blocks of RER determination. We identify here the key equilibrium equations to make it easier for the reader to navigate the resulting equilibrium system. Section 3 starts with a general RER decomposition (4) to outline all possible sources of PPP deviations in Lemma 1. It then proceeds to characterize relationships between RER and international relative prices and wages in (10), (16) and (17). The outcome of the expenditure switching mechanism in Section 4 is the goods market clearing condition (20). The key result of Section 5 is the international risk sharing condition (28), with special case solutions given by (23) for complete markets and (25) for autarky. The country budget constraint (30) is the main result of Section 6. The dynamic equilibrium system consists of (20), (28) and (30), resulting in the equilibrium characterization of RER in (31).

2 Definitions and Facts

The nominal exchange rate (NER, or simply exchange rate) is the relative price of currencies. We denote it with $\mathcal{E}_t$, defined as the price of foreign currency in units of the home currency. Therefore, an increase in $\mathcal{E}_t$ corresponds to a *nominal depreciation* of the home currency, as more units of home currency need to be paid for one unit of foreign currency.[1] We use small letters to denote logs, e.g. $e_t \equiv \log \mathcal{E}_t$ is the log exchange rate and $\Delta e_t \equiv e_t - e_{t-1}$ corresponds to an exchange rate depreciation (appreciation, if negative) in log points.

[1]This definition where an increases in the exchange rate corresponds to a depreciation is conventional in the international macro literature, in particular in monetary models where all nominal quantities are cointegrated and $\mathcal{E}_t$ increases together with the quantity of money M_t and the nominal price level P_t, at least in the long run. NER is typically defined as a bilateral exchange rate between a pair of currencies; occasionally, it is defined as a trade-weighted exchange rate of the home country's currency against the rest of the world.

In the data, under floating exchange rate regimes, bilateral log exchange rates follow a process nearly indistinguishable from a *random walk*. In other words, exchange rate depreciations are nearly unpredictable, $\mathbb{E}_t\Delta e_{t+1} \approx 0$, and the current level of the exchange rate offers *nearly* the best forecast for the future exchange rates, $\mathbb{E}_t e_{t+h} \approx e_t$ for any $h > 0$. Furthermore, exchange rate changes Δe_{t+1} exhibit *no* robust contemporaneous correlation with almost any macro variable. As a result, it is very difficult to improve the out-of-sample forecasts of the exchange rates over a random-walk forecast, even using macro information from future periods. This property of exchange rates is often called the Meese and Rogoff puzzle, after their seminal 1983 paper, and is sometimes referred to as the *exchange rate disconnect* in a narrow sense, emphasizing the lack of contemporaneous correlation between exchange rates and macroeconomic fundamentals.[2]

Macroeconomists view exchange rates as "excessively" volatile. Indeed, floating exchange rate changes have annualized standard deviation of around 10-12 percentage points — an order of magnitude larger than that for inflation, consumption and output growth. In contrast, finance economists view exchange rates as "insufficiently" volatile, as they are about a third less volatile than the stock market (Brandt, Cochrane, and Santa-Clara 2006, Lustig and Verdelhan 2019). Indeed, exchange rates can be viewed as assets, and hence should inherit the properties of the same stochastic discount factor that prices stocks and bonds (see Section 5).

The real exchange rate (RER) is the relative price of consumption baskets across countries. We denote it with $\mathcal{Q}_t$, which is defined as the ratio of price levels denominated in a common currency:

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}, \qquad \text{or in logs} \quad q_t \equiv e_t + p_t^* - p_t, \tag{1}$$

where P_t and P_t^* are consumer price levels at home and abroad, respectively ($*$ denotes foreign variables and/or variables denominated in foreign currency, where it causes no confusion). Therefore, $\mathcal{Q}_t$ measures the number of home consumption baskets one needs to sell at home (each valued at P_t) in order to buy one foreign consumption basket abroad (valued at P_t^* and converted to home currency using $\mathcal{E}_t$). This is, of course, an imaginary transaction as consumptions baskets are generally non-tradable and RER is *not* an actual price in any existing market.

An increase in $\mathcal{Q}_t$ corresponds to a *real depreciation* — a decline in the *relative* purchasing

[2]There are a number of documented departures from a purely unpredictable random walk process for exchange rates (see surveys by Rossi 2013, Rogoff and Stavrakeva 2008). For example, there exists predictability at high frequencies using order flows (Evans and Lyons 2002), for "commodity currencies" (Chen and Rogoff 2003), with past trade deficits (Gourinchas and Rey 2007), RER deviations (Eichenbaum, Johannsen, and Rebelo 2020), interest rates (Stavrakeva and Tang 2015), portfolio flows (Lilley, Maggiori, Neiman, and Schreger 2020) and risk premia (Jiang, Krishnamurthy, and Lustig 2018). Engel, Mark, and West (2008) study the reverse predictability of future macroeconomic fundamentals by current exchange rate movements. In many case, predictability exists only for specific currencies (e.g., the US dollar) or during specific time periods (e.g., amid and after the global financial crisis).

power of one unit of currency (e.g., dollar) abroad. International macroeconomics focuses on the dynamics of RER, rather than on its level, namely:

$$\Delta q_t = \Delta e_t + \pi_t^* - \pi_t, \tag{2}$$

where $\Delta q_t = q_t - q_{t-1}$ is the real depreciation, $\pi_t = \log P_t - \log P_{t-1}$ is the home CPI inflation rate (analogously, π_t^* is the CPI inflation abroad), with the length of the period typically taken as a month, a quarter, or a year.[3]

Since there are different goods baskets and different corresponding price levels, we can define different concepts of RER. In addition to CPI-based RER, we often consider three alternative measures. One corresponds to the RER for tradable goods, denoted $\mathcal{Q}_t^T \equiv \mathcal{E}_t P_{Tt}^*/P_{Tt}$, where P_{Tt} is the home price of the tradable basket. The second is the RER for the locally produced goods, or PPI-based RER, $\mathcal{Q}_t^P \equiv \mathcal{E}_t P_{Pt}^*/P_{Pt}$, where P_{Pt} is the producer price index, i.e. the price level of goods produced and distributed domestically. Lastly, we measure RER for the basket of labor inputs, or the wage-based RER, $\mathcal{Q}_t^W \equiv \mathcal{E}_t W_t^*/W_t$, where W_t is the wage rate (index) at home; $\mathcal{Q}_t^W$ measures the relative cost of foreign labor in units of home labor.

We outline the main empirical facts about RER, focusing on unconditional moments at business cycle frequencies broadly defined. First, RER is nearly indistinguishable from NER at most horizons, from days to years, and thus also follows a volatile near-random-walk process. The half life of RER's mean reversion is estimated to be very long, on the order of 3-to-5 years, resulting in the famous PPP Puzzle (Rogoff 1996). The literature conventionally assumes RER to be long-run stationary, however, there is *no* conclusive empirical evidence to support this (see e.g. Burstein and Gopinath 2012, Müller and Watson 2018, 2019).

Second, all types of RERs (CPI, PPI, tradable, and wage-based) comove closely, with similar volatility and persistence, and all nearly perfectly correlate with NER (see Itskhoki and Mukhin 2017). RER is almost an order of magnitude more volatile than various inflation rates (which have annualized standard deviations of 1–2%). RER is also nearly uncorrelated with inflation rates. Furthermore, while RER in log changes (Δq_t) is close to *iid*, inflation rates are persistent and sometimes considered integrated (see Stock and Watson 2007).

Third, RER is an order of magnitude more volatile and nearly uncorrelated with real macro variables, such as output, employment, consumption, or productivity. There is a mild negative (yet unstable sample-to-sample) correlation with the relative consumption growth across countries — a pattern often referred to as the Backus and Smith (1993) puzzle.

Finally, RER comoves closely with NER not only under a floating exchange rate, but also changes its equilibrium properties when the policy regime is switched to an exchange rate peg,

[3] The level of RER, or the relative PPP violation, is important for cross-country income comparisons (see e.g. Deaton and Aten 2017, and the Penn World Tables).

in which case the volatility of RER discontinuously drops by an order of magnitude together with that of NER — the Mussa (1986) puzzle.

The terms of trade (ToT) measure the relative price of imports and exports of a country:

$$\mathcal{S}_t = \frac{P_{Ft}}{P^*_{Ht}\mathcal{E}_t}, \tag{3}$$

where P_{Ft} and P^*_{Ht} are home and foreign import price indexes in local currency. Therefore, ToT measures the relative price of the import basket in units of export baskets. An increase in $\mathcal{S}_t$ corresponds to a ToT *deterioration*, when a country needs to export more in order to afford the same quantity of imports.

ToT and RER — and in particular ToT deterioration and real depreciation — are often confused. Indeed, in many models the two variables are closely linked. This is not, however, the case in practice, and large RER depreciations are not always accompanied by significant ToT deteriorations. On average, ToT is about 2-3 times less volatile than RER and the two are only weakly positively correlated over short-to-medium horizons (see Atkeson and Burstein 2008, Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller 2020).

3 Real Exchange Rate and International Relative Prices

We start our analysis in the goods market and first study the relationship between exchange rates and international relative prices — the first stage of the transmission mechanism to macroeconomic quantities — which gives rise to the most popular partial theories of RER. For simplicity, throughout the analysis we focus on the case of two large symmetric countries — home and foreign. The results generalize to the more involved case of asymmetric home and the rest of the world or to the case of home-biased small open economies (as in Galí and Monacelli 2005).

3.1 The PPP hypothesis

A foundational concept in international macroeconomics is the *Purchasing Power Parity (PPP) hypothesis* (see Rogoff 1996). Narrowly interpreted, the PPP hypothesis states that one dollar should buy the same amount of goods in all countries, implying convergence of price levels in space. More generally, there are three forms of PPP:

1. *Absolute PPP* is the strongest form and requires the equality of the price levels, $P_t = P^*_t\mathcal{E}_t$, which in turn implies that RER in (1) always equals 1, $\mathcal{Q}_t \equiv 1$, or in logs $q_t \equiv 0$. In other words, RER equals PPP deviations.

2. *Relative PPP* is a weaker form that allows price levels to differ, but the relative prices must be stable over time, implying that the nominal depreciation equals the relative inflation rate, $\Delta e_t = \pi_t - \pi_t^*$. This links together the key domestic and international monetary quantities, requiring that inflation and nominal depreciation move in lockstep. RER is then constant over time, $\Delta q_t \equiv 0$.
3. *Weak relative PPP* only requires mean reversion in the relative price levels, or equivalently (mean) stationarity of the real exchange, q_t. In other words, $\Delta e_t = \pi_t - \pi_t^*$ may be violated period-by-period, but it must hold over long time intervals.

Clearly, both absolute and relative PPP are strongly violated in the data, as RER is highly volatile at all horizons and its deviations are persistent if not permanent.

We now dig deeper into what the PPP hypothesis implies. Consider the (empirical) definition of the CPI inflation rate, $\pi_t = \Delta p_t = \sum_{i\in\Omega_t} \omega_{it}\Delta p_{it}$, which we sometimes write in log-deviation terms as $p_t = \sum_{i\in\Omega_t} \omega_{it} p_{it}$. We denote with Ω_t the set of products available for consumption and ω_{it} are expenditure weights. For now we do not specify the granularity of the set Ω_t, which may be as detailed as individual barcodes or as crude as $\{T, N\}$ for tradables and non-tradables respectively.[4] With this definition of CPI inflation, the change in RER (2) can be expanded as:

$$\Delta q_t = \Delta e_t + \sum\nolimits_{i\in\Omega_t^*} \omega_{it}^* \Delta p_{it}^* - \sum\nolimits_{i\in\Omega_t} \omega_{it}\Delta p_{it}. \tag{4}$$

This expression implies the following conditions for the relative PPP, $\Delta q_t = 0$:

Lemma 1 *The relative PPP holds if the following three conditions are simultaneously satisfied:*

(i) all goods are traded, $\Omega_t = \Omega_t^*$*;*

(ii) there is no home bias in the expenditure shares, $\omega_{it} = \omega_{it}^*$ *for all* $i \in \Omega_t = \Omega_t^*$*;*

(iii) the law of one price holds, at least in changes, $\Delta p_{it} = \Delta p_{it}^* + \Delta e_t$ *for all* $i \in \Omega_t = \Omega_t^*$.

Three remarks are in order. First, Lemma 1 offers only a set of sufficient requirements, and $\Delta q_t = 0$ could be the case even when none of the conditions in the lemma hold. However, this would be a knife-edge coincidence, and one should not expect the relative PPP to hold when either of the conditions in Lemma 1 is systematically violated. Therefore, Lemma 1 should be rather viewed as describing alternative necessary conditions for PPP violations — at least one of the conditions must fail for $\Delta q_t \neq 0$. Viewed this way, Lemma 1 provides guidance for both theory and empirics. Specifically, in order to have a model with non-constant RER, as

[4] The finer is the granularity of Ω_t, the more important become questions of churning, extensive margin and quality adjustment (see Feenstra 1994, Broda and Weinstein 2006, 2008, Nakamura and Steinsson 2012).

is the case in the data, it needs to feature either non-tradables/home bias or law of one price deviations for traded goods, or some combination of both, as we explore in Sections 3.2–3.4.

Second, the macroeconomic concept of PPP is often motivated with a microeconomic concept of the *law of one price* (LOP). Indeed, LOP is one of the conditions in Lemma 1. However, it is easy to see that LOP does not ensure that PPP holds. In fact, if the main reason for PPP violations are non-tradables and/or home bias in tradables, LOP violations are not necessary to explain the behavior of RER. What is less obvious, but perhaps more important, is the converse statement: even vast LOP violations in the micro data are *not* sufficient for PPP violations at the macro level. If $\Delta p_{it} = \Delta p_{it}^* + \Delta e_t$ holds *on average* across goods, neither permanent nor idiosyncratic violations in LOP would cause deviations from PPP.[5]

Third, Lemma 1 clearly illustrates why PPP hypothesis is a very tall order, and should not be expected to hold in general. Even if LOP held in the data, the world is characterized by abundance of non-tradables and strong home bias in tradable expenditure (Obstfeld and Rogoff 2001, Anderson and van Wincoop 2004). Given this, one may wonder why PPP hypothesis plays such a prominent role in international macroeconomics, and furthermore why the idea of a stationary RER is so profoundly rooted in the literature. My conjecture is that this is not merely an intellectual path-dependence, but rather a deeply-rooted implicit assumption that the main drivers of NER are monetary shocks, which should not have lasting real effects including on RER (see Section 3.4).

3.2 Non-tradables and home bias

The most popular theoretical approach to modeling RER is using non-tradable goods and assuming that LOP holds for tradables. This is why RER is often viewed as the international relative price of non-tradables (see Obstfeld and Rogoff 1996, Ch.4). Using the general expansion of the real exchange rate in (4), consider the case with $\Omega = \{T, N\}$, and denote with $\omega = \omega_N = 1 - \omega_T$ the expenditure share on non-tradables in both countries. Following Engel (1999), we decompose RER in log-deviation terms (thus dropping Δ) as:

$$q_t = \overbrace{(p_{Tt}^* + e_t - p_{Tt})}^{\equiv q_t^T} \; + \; \omega[\overbrace{(p_{Nt}^* - p_{Tt}^*) - (p_{Nt} - p_{Tt})}^{\equiv v_t^N}], \tag{5}$$

where q_t^T is the tradable RER and v_t^N is the international relative price of non-tradables.

[5]For example, trade costs τ_i result in LOP violations at the micro level, yet may lead to no relative PPP violations at the aggregate, with $\Delta q_t = 0$ (and even no absolute PPP violations, $\mathcal{Q}_t = 1$, in a symmetric equilibrium). Similarly, Balassa-Samuelson forces (discussed in Section 3.2), may lead to LOP violations in levels for certain goods and services, yet no dynamic PPP violations. Even dynamic micro-level LOP violations, if idiosyncratic, can wash out in the aggregate and cause no PPP violations (along the lines of Caplin and Spulber 1987).

Balassa-Samuelson hypothesis We begin with a neoclassical Balassa-Samuelson model in which countries produce a *homogenous* tradable good with its prices equalized internationally, so that $q_t^T = 0$ in (5) and RER is proportional to the relative price of non-tradables, $q_t = \omega v_t^N$. Assuming a linear production technology, perfect competition implies unit-labor-cost pricing:

$$p_{it} = w_t - a_{it} \qquad \text{and} \qquad p_{jt}^* = w_t^* - a_{jt}^* \tag{6}$$

for $i \in \Omega = \{T, N\}$ and $j \in \Omega^* = \{T, N^*\}$, where w_t denotes the log nominal wage rate at home and (a_{Tt}, a_{Nt}) are log domestic productivities in tradables and non-tradables respectively, and similarly in the foreign country (denoted with a $*$).[6]

Combining competitive pricing (6) with LOP for tradables, $p_{Tt} = p_{Tt}^* + e_t$, relative wages must equal relative tradable productivity across countries and thus the wage-based RER, $q_t^W \equiv w_t^* + e_t - w_t = a_{Tt}^* - a_{Tt}$. In turn, the relative price of non-tradables equals the relative tradable productivity within a country, e.g. $p_{Nt} - p_{Tt} = a_{Tt} - a_{Nt}$. Countries that are relatively more productive in the tradable sector have higher wage rates and, as a result, higher prices of non-tradables. From (5), this results in an appreciated real exchange rate (i.e., lower q_t when $a_{Tt} - a_{Nt}$ is higher at home):

$$q_t = \omega \nu_t^N \qquad \text{with} \qquad \nu_t^N \equiv (a_{Tt}^* - a_{Nt}^*) - (a_{Tt} - a_{Nt}), \tag{7}$$

thus $v_t^N = \nu_t^N$ equals the relative tradable productivity abroad. According to the Balassa-Samuelson hypothesis, richer countries have higher average productivity in particular in the tradable sector, resulting in higher wages and higher non-tradable prices and thus an appreciated RER. Hence, the main driver of RER in this case is the relative tradable productivity.

Empirical evidence The Balassa-Samuelson hypothesis accurately captures the persistent differences in price levels (and RER) between very rich and very poor countries, as well as over time for countries experiencing growth miracles, such as post-war Japan. At the same time, its explanatory power in the general cross-section of countries and in particular over short and medium horizons is low, as discussed in Rogoff (1996). Neither relative tradable productivity ν_t^N in (7), nor more generally the relative price of non-tradables v_t^N in (5), appears to be the principal driver of RER. Engel (1999) develops a sharp empirical test and shows that the non-tradable component v_t^N in (5) contributes very little to the overall volatility of RER q_t, at all practical horizons (i.e., up to 10 years).[7]

[6] Obstfeld and Rogoff (1996, Ch.4.2) provide a comprehensive treatment with multiple factors of production and decreasing returns in the labor input.

[7] In the original paper, Engel (1999) considered the bilateral price data between the US and other rich countries, and only 19 categories of goods, split into tradable and non-tradable categories. The follow-up studies have improved on both the set of country pairs and the granularity of good categories and found consistent results, as discussed in Burstein and Gopinath (2012). Betts and Kehoe (2008) show that the contribution of the tradable

In other words, it is the tradable RER, q_t^T, that accounts for the bulk of RER volatility. Heuristically, q_t^T is the term in (5) that contains NER e_t, and thus is the "wild" one in comparison with the much tamer non-tradable term v_t^N containing relative local prices. Indeed, under floating exchange rate regimes, changes in NER are an order of magnitude more volatile than changes in productivity, wages and prices.[8] It may be tempting to conclude that this offers strong evidence against the role of non-tradables in the dynamics of RER, which is largely driven by the tradable component q_t^T. This is not quite the case, as we explore next.

Home bias in tradables We extend now decomposition (5) to allow the tradable good to be an aggregator of home and foreign value added, which features imperfect substitutability and home bias. Specifically, we assume that the log of the home tradable price index is given by:

$$p_T = (1-\tilde{\gamma})p_H + \tilde{\gamma}p_F, \tag{8}$$

where p_H and p_F are prices of the home- and foreign-produced value added (whether final goods or intermediate inputs) and $\tilde{\gamma} \in [0,1)$ is the measure of openness. Correspondingly, $1-\tilde{\gamma}$ is a measure of home bias, provided it exceeds the country's share in the world GDP.

For simplicity, we assume a world of two symmetric countries, so that the foreign tradable price index is given analogously by $p_T^* = (1-\tilde{\gamma})p_F^* + \tilde{\gamma}p_H^*$, and it is also biased towards the locally-produced value added. With two symmetric countries, home bias occurs when $\tilde{\gamma} < 1/2$, which we assume is the case. In terms of the general expansion (4), we now have $\Omega = \{H, F, N\}$ and $\Omega^* = \{H, F, N^*\}$, so that $\Omega \neq \Omega^*$ if the non-tradable share is positive, $\omega_N = \omega > 0$. Furthermore, individual tradable shares are different across countries due to home bias, e.g. $\omega_H = (1-\tilde{\gamma})(1-\omega) \neq \tilde{\gamma}(1-\omega) = \omega_H^*$. Therefore, both conditions (i) and (ii) of Lemma 1 are generally violated in this case.

Still assuming LOP holds for individual tradable goods, that is $p_{it} = p_{it}^* + e_t$ for $i \in \{H, F\}$, we use the tradable price index (8) and its foreign counterpart to rewrite the tradable RER as:

$$q_t^T = (1-2\tilde{\gamma})q_t^P, \qquad \text{where} \qquad q_t^P \equiv p_{Ft}^* + e_t - p_{Ht} = q_t^W - (a_{Tt}^* - a_{Tt}). \tag{9}$$

Due to the marginal cost pricing (6), producer-price RER q_t^P reflects wage-based RER q_t^W adjusted for relative tradable productivity. Using (9), we characterize the overall RER in:[9]

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component to RER volatility is somewhat smaller for country pairs that extensively trade with each other, and also if import price indexes are used to construct the tradable RER component (Burstein and Gopinath 2012).

[8] In contrast, under a fixed exchange rate, in the European monetary union, Berka, Devereux, and Engel (2012, 2018) find a considerably larger contribution of productivity shocks to the dynamics of RER, which is in turn a lot less volatile under the peg.

[9] To complete the proof, note that the non-tradable component in (5) is now given by $v_t^N = \nu_t^N + 2\tilde{\gamma}q_t^P$, and thus it partially comoves with the tradable component $q_t^T = (1-2\tilde{\gamma})q_t^P$ due to home bias ($\tilde{\gamma} < 1/2$).

Proposition 1 *Under marginal cost pricing* (6)*, the general expression for RER in the presence of home bias* $\tilde{\gamma} \le 1/2$ *and non-tradables* $\omega \ge 0$ *is given by:*

$$q_t = (1 - 2\gamma)\big[q_t^W - (a_{Tt}^* - a_{Tt})\big] + \omega\nu_t^N, \tag{10}$$

where $\gamma \equiv \tilde{\gamma}(1-\omega)$ *is aggregate foreign share and* ν_t^N *is the relative non-tradable productivity defined in* (7).

We emphasize two key implications of this result. First, the model with home bias links together domestic consumer and producer prices, and hence CPI and PPI-based RER, q_t and q_t^P, which both comove with the wage-based RER q_t^W. In contrast, *without* non-tradables and home bias ($\omega = 0, \tilde{\gamma} = 1/2$), the volatility in relative producer prices q_t^P does not transmit to relative consumer prices q_t. Furthermore, home bias in tradables (small $\tilde{\gamma}$) and the presence of non-tradables ($\omega > 0$) act in the exact same way from the point of transmission from q_t^W to q_t: what matters is the overall share of expenditure on foreign value added, $\gamma \equiv \tilde{\gamma}(1-\omega)$.

Second, the tradable component q_t^T can exhibit considerable volatility together with NER e_t and closely comove with the overall RER q_t, even when LOP holds at the micro level. In other words, the model can reproduce the empirical patterns documented by Engel (1999) without relying on micro-level LOP deviations.[10] This, however, requires two conditions: (a) significant home bias in tradables, $\tilde{\gamma} \ll 1/2$, and (b) volatile wage-based RER q_t^W relative to productivity shocks. In this sense, it is a partial equilibrium theory of RER, which leaves out the explanation of the equilibrium properties of the wage-based RER. As we will see, a variant of equation (10) is a focal point for a variety of partial theories of RER, whether they rely on home bias or LOP deviations, and further progress can only be made in general equilibrium (see Sections 4–6).

Trade costs, distribution margin and intermediate inputs What is the economic nature of home bias in tradables? It can emerge directly from home bias in preferences, which always acts as a residual explanation. A largely equivalent mechanism that gives rise to home bias in expenditure is due to trade costs.[11] Furthermore, there are two additional mechanisms that result in effective home bias — local distribution margin and imported intermediate inputs.

In order to reach the domestic consumer, a foreign-produced good needs to incur a considerable local distribution cost, which contributes to the overall cost of the good and constitutes domestic value added. As a result, there are few purely tradable goods with the entirety of

[10] To be sure, this is not to say that micro-level LOP deviations are unimportant in the data, but rather to emphasize that they are not conceptually necessary to rationalize the evidence in Engel (1999).

[11] See Obstfeld and Rogoff (2001) for a discussion of the role of trade costs and home bias in international macroeconomics (see also recent quantitative work by Eaton, Kortum, and Neiman 2016, Reyes-Heroles 2016). See Anderson and van Wincoop (2004) for a survey of trade costs in international trade literature. A recent work by Coçar, Grieco, Li, and Tintelnot (2018) attempts to identify empirically the differential role played by trade costs and home bias in preferences in explaining home bias in expenditure on cars.

value added accrued abroad.[12] Empirically, the local distribution share is sizable even for typical tradable goods: Burstein, Neves, and Rebelo (2003) measure it to be over 40% in the US market and even higher in smaller developing countries. The distribution margin mutes the difference between tradable and non-tradable goods, contributing to the stark patterns documented by Engel (1999). Nonetheless, from the aggregate perspective, what matters is the overall share of foreign value added in domestic absorption, γ. It can be quantified directly from the aggregate trade-to-GDP ratio without delving into details of its particular decomposition into non-tradables, home bias and distribution costs — an implication of Hulten's theorem in an open economy (see Burstein and Cravino 2015, Baqaee and Farhi 2019).

Finally, we consider briefly imported intermediate inputs. If they are used uniformly by all firms (e.g., in a round-about fashion, as in Basu 1995, Eaton and Kortum 2002), they reduce the foreign value-added content of imports, which increases effective home bias. In the data, however, the use of imported intermediate inputs is highly heterogenous across firms, with large exporting firms relying a lot more intensively on imported inputs (see Amiti, Itskhoki, and Konings 2014). Combined with the Melitz (2003) selection force in international trade, the average good shipped abroad has a considerably lower domestic value added component relative to the average good shipped domestically. While this does not lead to LOP violations at the individual product level, it has consequences for the tradable RER, as in (9). More importantly, such input-ouput structure biases the inference on home bias from the aggregate trade-to-GDP ratio downwards, and particularly so for small and open economies.[13]

3.3 Variable markups and pricing to market

Relaxing the assumption of competitive marginal cost pricing, we consider two fundamental reasons for LOP deviations at the micro-level — short-run LOP deviations due to sticky prices in the following section and long-run LOP deviations due to markup pricing in this section.[14]

[12] Local distribution costs include transportation, insurance, storage, wholesale, retail, advertising and marketing, warranty and service (for a prominent application see Burstein, Eichenbaum, and Rebelo 2005). In addition, the home distribution sector, if not perfectly competitive, may add on a considerable markup for its services, which is also part of domestic value added (see also Corsetti and Dedola 2005). The distribution cost can be captured by an analog of (8), which applies at the individual product level, $p_t(i) = (1 - \tilde{\gamma}_i)p_{Ht} + \tilde{\gamma}_i p_{Ft}$, where p_{Ht} and p_{Ft} are prices of one unit of home and foreign value added respectively and $\tilde{\gamma}_i$ is the share of foreign value added in the cost structure of a given product i delivered to the home consumer. Crucini and Landry (2019) argue that accounting for the distribution margin at the product-level reconciles the data with LOP that holds for the foreign value-added component of the costs, even as LOP fails for individual tradable goods.

[13] Consider an extreme case of a 'maquiladora' assembly plant, which adds marginally to the overall value of a product, yet in the trade data the *full* value of the product is reflected *twice* — first in imports and then in exports.

[14] In both cases we omit considerations of entry and exit of products, which may play an important role in the aggregate adjustment to exchange rate shocks over the long run; the empirical literature, however, has yet to establish such patterns if they are present (e.g. see discussion in Fitzgerald and Haller 2018).

Markup identity Prices of a home product i in the home and foreign markets (expressed in the destination currency) can be decomposed respectively as:

$$p_{Ht}(i) = \mu_{it} + mc_{it} \qquad \text{and} \qquad p^*_{Ht}(i) = \mu^*_{it} + mc_{it} - e_t + \tau_i, \tag{11}$$

where μ_{it} and μ^*_{it} are log markups in the home and foreign markets, mc_{it} is the marginal cost of production, and τ_i is the iceberg trade cost assumed to be constant over time. The expressions in (11) define the realized markups. In this subsection we assume that prices are set flexibly, and thus μ_{it} and μ^*_{it} are also the optimal (*desired*) markups.

Taking the difference between the foreign and home market prices for the same product, we arrive at the LOP deviation, $q_{Ht}(i) \equiv p^*_{Ht}(i) + e_t - p_{Ht}(i) = \mu^*_{it} - \mu_{it} + \tau_i$. Thus, as long as a firm finds it optimal to set different markups in the two markets, that is *price to market*, one should not expect LOP to hold even at the very micro product level. Our analysis focuses on the dynamic response of markups, rather than their levels (see Simonovska 2015, Cavallo, Neiman, and Rigobon 2014), and hence we take the first difference, $\Delta q_{Ht}(i) = \Delta\mu^*_{it} - \Delta\mu_{it}$. While direct assumption-free measurement of markups is not possible (see the discussion in De Loecker and Goldberg 2014), this cross-market transformation of the data allows one to obtain a direct measure of the relative markup. This insight has been leveraged in empirical work, for example in Fitzgerald and Haller (2013), who use it to provide a direct model-free test of *pricing to market* (PTM) by projecting $\Delta q_{Ht}(i)$ on Δe_t.[15]

A model of markups To further study the implications of PTM, we adopt a structural model of markups. Under monopolistic competition with CES demand, the optimal markup is constant, and therefore $\Delta\mu_{it} \equiv 0$ for all firms in every market, resulting in an empirically-counterfactual prediction of no pricing to market and complete pass-through of cost shocks.

In contrast, models that relax either the CES assumption (following Dornbusch 1987) or the monopolistic competition assumption (that is, study oligopolistic competition, following Krugman 1987), generally predict variable markups, PTM and incomplete exchange rate pass-through (ERPT) into prices.[16] Both classes of models share the same general predictions for the dynamics of the destination-specific markups. In particular, the optimal markup of the firm decreases with its price relative to prices of its competitors in a given destination market,

[15] The null hypothesis of no PTM implies that $\Delta q_{Ht}(i)$ should not move with Δe_t, while in the data $\Delta q_{Ht}(i)$ closely tracks Δe_t over time. An additional complication in testing this hypothesis is that firms have sticky prices, and hence the test needs to be carried out conditional on a price adjustment (as in Gopinath, Itskhoki, and Rigobon 2010, Fitzgerald and Haller 2013).

[16] Alternatively, CES monopolistic competition models with flexible prices also result in variable markups and incomplete pass-through under dynamic pricing due, for example, to consumption habits (Ravn, Schmitt-Grohé, and Uribe 2006), inventory management (Alessandria, Kaboski, and Midrigan 2010) or durable/storable goods (Fabinger, Gopinath, and Itskhoki 2011).

that is $\mu_{it} = \mathcal{M}(p_{Ht}(i) - p_t)$ with $\mathcal{M}'(\cdot) < 0$, where p_t is the local competitor price index. Amiti, Itskhoki, and Konings (2019) show that this characterization applies across various classes of models, including models of oligopolistic competition as in Atkeson and Burstein (2008) and models with non-CES demand as in Gopinath and Itskhoki (2010).

Using this model of markups with (11), the first order approximation to the firm's prices in the two markets is given in log deviations by:

$$p_{Ht}(i) = (1-\alpha)mc_{it} + \alpha p_t \quad \text{and} \quad p^*_{Ht}(i) = (1-\alpha)(mc_{it} - e_t) + \alpha p^*_t, \tag{12}$$

where $\alpha \equiv \frac{-\mathcal{M}'(p_{Ht}(i)-p_t)}{1-\mathcal{M}'(p_{Ht}(i)-p_t)} \in [0,1)$, and α depends in general on the characteristics of the firm and the market it serves. Elasticity α measures the extent of *strategic complementarities* in price setting. In other words, firms find it optimal to increase their prices not only when their costs increase, but also when their competitors' prices go up. The complementary quantity $(1-\alpha)$ is the cost pass-through elasticity, which is in general incomplete. Amiti, Itskhoki, and Konings (2019) estimate α to be around 0.4–0.5 on average across manufacturing firms: that is, firms put roughly equal weight on their marginal costs and the prices of their competitors in determining their own optimal price.

Combining price setting in the two markets in (12), we evaluate the LOP deviation:

$$q_{Ht}(i) = p^*_{Ht}(i) + e_t - p_{Ht}(i) = \alpha q_t, \tag{13}$$

with $q_{Ht}(i) \neq 0$ whenever $q_t \neq 0$ and $\alpha > 0$. As firms are responsive to the prices of their competitors, they optimally choose different markups in markets with different competitive pressures, as well as adjust markups differentially in response to shocks. What is essential for this latter prediction is that firms face a different mix of competitors in different markets with local bias due to fixed and/or variable trade costs (see Atkeson and Burstein 2008).

When α is common across the firms, LOP deviations are the same for all home firms, $q_{Ht} = \alpha q_t$. Similarly, for foreign firms, $q_{Ft} = p^*_{Ft}(i) + e_t - p_{Ft}(i) = \alpha q_t$. Therefore all micro-level LOP deviations move in concert with RER q_t, as is the case in the data.[17]

Real exchange rate and terms of trade We now combine this model of price setting with the definitions of the aggregate price index, $p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}$, where γ represents the overall home bias whether due to non-tradables or home bias in tradables. In addition to CPI-

[17] Micro-level LOP deviations are hugely dispersed in the cross-section: Broda and Weinstein (2008) document, at the barcode level, a standard deviation from LOP of 50 log points across US cities, which is 5 times larger than the annual standard deviation of the exchange rate. Yet, this dispersion is largely idiosyncratic and washes out in the aggregate. In contrast, in the time series, LOP deviations tend to move in concert with NER for all products (see Crucini and Telmer 2012, Gopinath, Gourinchas, Hsieh, and Li 2011, Burstein and Jaimovich 2012).

based RER q_t, we also consider PPI-based RER, $q_t^P \equiv p_{Ft}^* + e_t - p_{Ht}$, and ToT, $s_t = p_{Ft} - p_{Ht}^* - e_t$. Note how, in the presence of LOP deviations, the two differ from each other, as PPI-based RER reflects producer prices set for the local market, while ToT reflects export prices.

Following Itskhoki and Mukhin (2017), we use the definitions of RER and ToT to write two relationships that tie them together. First, PPI-RER and ToT differ only by LOP deviation terms:

$$q_t^P = s_t + (q_{Ht} + q_{Ft}) = s_t + 2\alpha q_t, \tag{14}$$

where the first equality is general accounting, while the second equality substitutes the expressions for LOP deviations (13). Second, CPI-RER equals PPI-RER adjusted for prices of imported and exported goods, namely ToT:

$$q_t = (1-\gamma) q_t^P - \gamma s_t, \tag{15}$$

which is a general accounting identity. Combining (14) and (15), we have:

Proposition 2 *In the PTM model, the equilibrium relationships between ToT and RERs are:*

$$s_t = \frac{1 - 2\alpha(1-\gamma)}{1-2\gamma} q_t \qquad \textit{and} \qquad q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t. \tag{16}$$

Note that this relationship depends only on two parameters, α and γ. In the absence of PTM ($\alpha = 0$), it reduces to $s_t = q_t^P = q_t/(1-2\gamma)$, implying that ToT are more volatile than RER.[18] This is, of course, counterfactual, as ToT are considerably more stable than RER. PTM with $\alpha > 0$ improves the fit of the data by reducing the volatility of ToT relative to RER and increasing the volatility of CPI-RER towards that of PPI-RER. Atkeson and Burstein (2008), using a quantitative model of oligopolistic competition under CES demand, show how the PTM mechanism, when combined with variable and fixed trade costs, can be simultaneously consistent with the empirical behavior of the main international relative prices.[19]

Beyond Proposition 2, can PTM explain the volatility and persistence of the aggregate RER? For concreteness, assume that marginal cost in (12) is $mc_t = w_t - a_t$. Together with the foreign counterpart of (12), we can solve for the domestic price level using its definition, $p_t = (1-\gamma)(w_t - a_t) + \gamma(w_t^* - a_t^* + e_t)$. Therefore, despite markup pricing, p_t still reflects the expenditure-weighted average marginal cost of the products served in the market (note that

[18]Intuitively, without LOP deviations, relative consumer prices are necessarily more stable than relative producer prices and ToT, as a mixed consumption basket offers a "diversification" benefit, which in the limit of no home bias ($\gamma = 1/2$) results in a perfectly stable RER ($q_t \equiv 0$) independently of the producer-price RER.

[19]While PTM mechanism can reproduce the empirical patterns of relative volatility of ToT and RER, it fails on their correlation, which in the data is positive but small and far from perfect. Only LOP deviation shocks in (14) can break the tight comovement between ToT and RER in the model. A natural source of such shocks is foreign-currency stickiness of border prices, as we discuss in the following subsection.

the mean markup wedge drops out from the expressions in log deviations). Using a parallel expression for the foreign price level p_t^*, we can solve for RER, $q_t = (1-2\gamma)\big[q_t^W - (a_t^* - a_t)\big]$, which is a special case of the expression in Proposition 1. Therefore, PTM with a common $\alpha > 0$ does not affect the equilibrium behavior of RER relative to the competitive no-markup case (or $\alpha = 0$), as was noted by Atkeson and Burstein (2007). This is despite incomplete ERPT and possibly large LOP deviations at the micro level, which end up not mattering for PPP deviations at the macro level.

How can this be? It turns out that strategic complementarities work both ways — reducing exchange rate pass-through for foreign firms, yet resulting in markup adjustment by domestic firms, with the two exactly offsetting each other when α is common across all firms. In other words, the average markup in a market stays unchanged, even though importers reduce markups and local firms increase markups in response to a home currency depreciation. Put differently, in the language of international trade, RER movements are neither pro- nor anti-competitive (see Amiti, Itskhoki, and Konings 2019, Arkolakis, Costinot, Donaldson, and Rodríguez-Clare 2019). While PTM is useful for explaining LOP deviations, incomplete ERPT, and low volatility of ToT, its implications are limited for aggregate price levels and RER. This is an example of how micro-level LOP deviations may have little consequence for aggregate RER.

Heterogeneity The data suggest substantial heterogeneity in the extent of strategic complementarities and ERPT across firms as first pointed out by Berman, Martin, and Mayer (2012). Amiti, Itskhoki, and Konings (2019) document that small firms exhibit complete pass-through and no strategic complementarities ($\alpha = 0$), just like constant-markup monopolistic competitors under CES demand. In contrast, large firms exhibit very strong strategic complementarities with $\alpha \geq 0.5$, putting at least as much weight on prices of their competitors as on their own marginal cost. Given that it is large firms that account for the majority of exports and imports, the average strategic complementarities among local and foreign firms are not symmetric. As a result, we expect a larger reduction in markups of foreign firms than an increase in markups of domestic firms in response to a home currency depreciation. This leads to incomplete pass-through into the *aggregate* price level, and hence results in PTM having aggregate RER consequences. In particular, the price levels p_t and p_t^* respond less to exchange rate fluctuations, and thus RER q_t tracks more closely NER e_t.[20]

Finally, following Amiti, Itskhoki, and Konings (2014, 2019), we briefly consider the marginal cost channel of international transmission, and in particular heterogeneous exposure to im-

[20]Formally, consider a generalization of (12) to feature firm-specific α_i increasing in firm size (see Amiti, Itskhoki, and Konings 2019, for details). Selection into exporting suggests that $\bar{\alpha}_X > \bar{\alpha}$, which are sales-weighted strategic complementarity elasticities among exporters only and all firms serving the market respectively. One can then show that $q_t = \big(1 - 2\gamma\frac{1-\bar{\alpha}_X}{1-\bar{\alpha}}\big)\big[q_t^W - (a_t^* - a_t)\big]$, with heterogeneity in pass-through ($1-\bar{\alpha}_X < 1-\bar{\alpha}$) reinforcing the effect of home bias (small γ).

ported intermediate inputs captured by $mc_{it} = (1-\phi_i)w_t + \phi_i(w_t^* + e_t) - a_{it}$. Such a cost structure is both empirically relevant, with small local firms having $\phi_i = 0$ and large exporting firms significantly exposed to foreign intermediates $\phi_i > 0$, and has a number of aggregate implications.[21] Large exporting firms exhibit simultaneously high import intensity ϕ_i and high strategic complementarities α_i. Due to home bias, these firms mostly compete with other domestic firms at home and with foreign firms abroad. A home currency depreciation, thus, makes them simultaneously lose competitiveness at home (due to exposure to foreign inputs) and gain competitiveness abroad (as they still rely on domestic inputs). Due to their high α_i, such firms significantly reduce their markups at home and raise them abroad, limiting ERPT in both markets and further muting the response of local price levels p_t and p_t^* to exchange rates. Finally, from the point of view of a country's trade balance, this limits the competitive effects of a devaluation, making it uncompetitive in the short run. Indeed, the largest exporters that are expected to expand their foreign sales in response to a home currency devaluation are simultaneously adversely affected by the increasing costs of their foreign inputs, making it harder for them to expand production (see Rodnyansky 2018, Blaum 2018).

3.4 Foreign-currency price stickiness

The other key source of LOP deviations is price stickiness in local currency, which drives a short-run wedge between home and foreign prices of a product good, even if desired prices are the same in both locations. Before prices adjust, there is an induced markup wedge equal in size to the accumulated exchange rate change, making such models promising in explaining large LOP and PPP deviations observed in the data. We consider here a baseline sticky-price model of RER, as analyzed in Kehoe and Midrigan (2008), Carvalho and Nechio (2011) and Blanco and Cravino (2020). Despite its simplicity, this model delivers the main insights and showcases the limitations of the sticky price mechanism more generally, and in particular its shortcoming in explaining RER persistence — the celebrated PPP Puzzle (Rogoff 1996, Chari, Kehoe, and McGrattan 2002, Itskhoki and Mukhin 2017). In contrast with the earlier PTM analysis, this model provides a full general-equilibrium framework, in particular endogenizing q_t^W.

The model features a combination of the following three assumptions, which make it immediately tractable: (i) cash-in-advance, $P_tC_t = M_t$, instead of dynamic money demand; (ii) log-linear utility, $u_t = \log C_t - L_t$, which implies perfectly elastic labor supply at a wage rate $W_t/P_t = C_t$ (*real neutrality*, in the terminology of Ball and Romer 1990); and (iii) complete asset markets resulting in the Backus-Smith condition, $C_t/C_t^* = \mathcal{Q}_t$ (see Section 5).

[21] Halpern, Koren, and Szeidl (2015) provide a microfoundation for such a cost structure with endogenous expenditure share ϕ_i based on the Melitz (2003) selection mechanism applied to sourcing of intermediate inputs. Dhyne, Kikkawa, Mogstad, and Tintelnot (2020) study the full exposure of domestic firms to foreign value added using the detailed input-output structure of the Belgian firms.

These assumptions together result in a simple equilibrium solution for wages and NER:[22]

$$w_t = m_t, \qquad w_t^* = m_t^*, \qquad \text{and} \qquad e_t = m_t - m_t^*,$$

which in particular implies $q_t^W = w_t^* + e_t - w_t = 0$. For concreteness, the only source of shocks is to money supply, m_t and m_t^*, which follow random walks (in logs).

Lastly, we assume Calvo sticky prices in local currency, so that the home-market price of product i (home or foreign) remains unchanged, $p_{it} = p_{it-1}$, with probability λ and is adjusted to $\bar{p}_t$ with probability $1-\lambda$. The desired prices (in log deviations) are given by $\tilde{p}_{Ht} = w_t = m_t$ and $\tilde{p}_{Ft}^* = w_t^* = m_t^*$ in producer currency, tracing the costs. In the absence of PTM, desired prices satisfy LOP, and thus desired export prices are $\tilde{p}_{Ht}^* = w_t - e_t = m_t^*$ and $p_{Ft} = w_t^* + e_t = m_t$. An interesting property of this monetary model is that local currency desired prices are the same for both home and foreign firms, both fluctuating with the local money supply. With money supply following random walk, the optimal reset price is $\bar{p}_t = m_t$ for all firms (home and foreign) serving the home market and $\bar{p}_t^* = m_t^*$ for all firms serving the foreign market.[23] Calvo pricing implies that the aggregate price level evolves as $p_t = \lambda p_{t-1} + (1-\lambda)\bar{p}_t$. Combined with a similar equation abroad, the dynamics of RER are given by $q_t = \lambda q_{t-1} + \lambda \Delta e_t + (1-\lambda)\bar{q}_t$, where $\bar{q}_t \equiv \bar{p}_t^* + e_t - \bar{p}_t$ is the *reset-price RER* (see Bils, Klenow, and Malin 2012, Gopinath and Itskhoki 2011, Blanco and Cravino 2020). Using the solutions for reset prices and NER, the reset-price RER is zero, $\bar{q}_t \equiv 0$; in other words, LOP (or PPP) holds for reset prices. PPP does *not* hold, however, for the regular consumer prices, as not all prices adjust.

Proposition 3 *Under local currency price stickiness, RER follows an AR(1) process:*

$$q_t = \lambda q_{t-1} + \lambda \Delta e_t, \tag{17}$$

with an iid innovation $\lambda \Delta e_t$ and persistence λ, a parameter governing the duration of sticky prices.

Note that NER, $e_t = m_t - m_t^*$, indeed follows a random walk. Crucially, the standard deviation of RER (relative to NER) and the persistence of RER are *both* given by the Calvo parameter λ. Intuitively, a fraction λ of firms do not adjust on impact, resulting in a proportional PPP violation. This fraction decreases as λ^t over time, and thus λ also represents the autoregressive coefficient. The stickier are the prices, the more persistent and volatile is RER, and the closer it tracks the random walk in NER.

[22] Using cash-in-advance and risk-sharing $\frac{M_t}{M_t^*} = \frac{P_t C_t}{P_t^* C_t^*} = \frac{\mathcal{Q}_t P_t}{P_t^*} = \mathcal{E}_t$ and using labor supply $W_t = P_t C_t = M_t$.

[23] Consider, for example, a domestic firm setting prices for the foreign market with a desired price $\tilde{p}_{Ht}^* = m_t^*$. The log-linearized reset price is then $\bar{p}_{Ht} = (1-\beta\lambda)\sum_{j=0}^{\infty}(\beta\lambda)^j \mathbb{E}_t \tilde{p}_{H,t+j} = m_t^*$, a weighted-average of future desired prices, where β is the discount factor. Similar logic establishes reset prices for other firms in different markets. For further details and derivations, see Galí (2008) and Farhi, Gopinath, and Itskhoki (2014).

Proposition 3 suggests a qualitative success for the sticky-price model, as it reproduces a random walk in NER and a closely comoving (yet mean-reverting) RER, at least when $\lambda \approx 1$. The difficulty, however, is quantitative, as in the data λ is not nearly large enough. Micro data on price stickiness suggests that consumer and producer prices adjust at least once a year on average, implying $\lambda \approx 0.75$ quarterly (see Bils and Klenow 2004, Nakamura and Steinsson 2008, Gopinath and Rigobon 2008). This is insufficient to generate a persistent enough RER, as its half life in such models is around $\log(0.5)/\log\lambda \approx 2.4$ quarters, while in the data it is over 3 years (or 12 quarters), as forcefully argued in Rogoff (1996), giving rise to the *PPP Puzzle*. Chari, Kehoe, and McGrattan (2002) further show the robustness of this failure in a more general class of sticky-price models, and Itskhoki and Mukhin (2017, 2019) argue that such models necessarily result in a range of additional exchange rate puzzles (including the Meese and Rogoff disconnect puzzle, the Backus and Smith risk-sharing puzzle and the Mussa puzzle).[24]

Further empirical falsification Kehoe and Midrigan (2008) apply this sticky price model in the cross section of sectors, generalizing (17) to sectoral RER, $q_{zt} = p^*_{zt} + e_t - p_{zt}$, where p_{zt} is sectoral price index. Consequently, $q_t = \sum_{z \in Z} \omega_z q_{zt}$ is the overall RER, where ω_z are sectoral expenditure weights. Instead of studying the properties of q_t upon aggregation, as originally proposed by Imbs, Mumtaz, Ravn, and Rey (2005), they focus on the comparative behavior of sectoral RERs q_{zt}, both their volatility and persistence, which are starkly captured in a simple model by λ_z — the sectoral Calvo price stickiness parameter. In the data, however, both volatility and persistence of q_{zt} are uniformly large across sectors and almost unrelated to sectoral price durations λ_z, in sharp violation of the theory.[25]

Blanco and Cravino (2020) adopt instead a time-series approach, focusing on the reset-price RER $\bar{q}_t$ defined above. The simple model here predicts sharply that $\bar{q}_t \equiv 0$, that is PPP is satisfied for reset prices. Intuitively, reset RER uses prices that are filtered of nominal stickiness, and if nominal rigidities are the main source of PPP violations, the reset RER would be immune to them, and thus should exhibit starkly different properties. However, in the data, this is not the case, and the reset RER tracks closely the conventional RER, and we observe nearly as large and persistent PPP violations for reset prices as for regular price levels. This empha-

[24]The core counterfactual prediction of the monetary model is that $\mathcal{E} = (PC)/(P^*C^*)$, and thus NER must be cointegrated with a combination of relative price and relative consumption levels, which in the data are both an order of magnitude less volatile and virtually uncorrelated with the nominal exchange rate. This argument extends beyond a simple complete-markets cash-in-advance model presented here.

[25]Carvalho and Nechio (2011) generalize Kehoe and Midrigan (2008) to study aggregate RER, clarifying the earlier results of Imbs, Mumtaz, Ravn, and Rey (2005). If there are N sectors, and sectoral RER q_{zt} follow AR(1) with heterogeneous persistence λ_z, the overall RER q_t follows an $ARMA(N, N-1)$, which generally has a greater persistence than the average persistence of sectoral RERs. Persistent shocks to money growth Δm_t can increase arbitrarily the persistence of q_t, yet at the cost of making Δe_t counterfactually positively autocorrelated (see Gopinath and Itskhoki 2011). Steinsson (2008) considers alternative macroeconomic shocks which give rise to a higher-order AR(2) process for RER with a considerably longer half life.

sizes that it is not the lack of price adjustment that cause the PPP puzzle. These two pieces of evidence reinforce considerably the original PPP puzzle argument about lacking persistence, and suggest that sticky prices alone cannot succeed in explaining the behavior of RER.

Generalizations The model studied above is extremely simple and makes a number of strong assumptions: (i) Calvo price stickiness (as opposed to menu costs) allows for no selection in price adjustment, which tends to increase the persistence of RER; (ii) money supply rule instead of interest rate rule, which tends to also increase the persistence (see Engel 2019); (iii) no strategic complementarities or input-output linkages across products/sectors, which reduces the persistence; (iv) flexible wages, which also reduces the persistence. Many of these assumptions are relaxed in the literature, and the main results carry through quantitatively.

One obvious issues with the baseline model is that wage-based RER under flexible wages is $q_t^W = 0$, and this is why reset RER is also zero, since more generally $\bar{q}_t = (1-2\gamma)\big[q_t^W - (a_t^* - a_t)\big]$. When wages are slow to adjust, q_t^W itself tracks NER, introducing additional persistence in all subsequent measures of relative prices, including $\bar{q}_t$ and q_t.[26] In this case, however, the properties of RER, including sectoral RER and reset RER, rely mostly on wage stickiness λ_w and home bias γ, and could be reconciled assuming an extreme extent of wage stickiness ($\lambda_w \to 1$). This does not, however, relieve the model from producing a variety of other major exchange rate puzzles (see Itskhoki and Mukhin 2019, and the discussion above).

Why does the sticky price model fail? The issue is not in the structure of the model or in the nominal rigidities *per se*; instead, it is the premise that monetary shocks are the main drivers of exchange rates. The appropriate conclusion is not that sticky prices are unimportant, or absent in the data — we observe their presence and, arguably, importance for understanding exchange rates. Rather the conclusion is that sticky prices with monetary shocks are not sufficient to explain the behavior of exchange rates, thus refuting a long-standing workhorse model for thinking about exchange rates in general equilibrium (see Itskhoki and Mukhin 2017).

Currency of pricing We briefly consider here the implications of currency of pricing for the equilibrium behavior of RER and ToT, and refer the reader for further details to the forthcoming handbook chapter by Gopinath and Itskhoki (2021). Historically, the conventional assumption has been producer currency pricing (PCP), whereby the exporter simply converts the producer price into the destination currency using the spot exchange rate, with LOP thus maintained (see Obstfeld and Rogoff 1995a). More recently, evidence of incomplete pass-through and LOP deviations has shifted the literature towards the assumption of local currency pricing (LCP; see Betts and Devereux 2000), as we adopted in the model above. While these assumptions have

[26] Specifically, with Calvo wage stickiness with parameter λ_w, we have $q_t^W = \lambda_w q_{t-1}^W + \lambda_w \Delta e_t$. Then, RER q_t follows an ARMA(2,1) given by $q_t = \lambda q_{t-1} + \lambda \Delta e_t + (1-\lambda)\frac{1-\beta\lambda}{1-\beta\lambda\lambda_w}(1-2\gamma)q_t^W$.

qualitatively different, and often opposite, implications for the patterns of international transmission and cross-country spillovers (see Lane 2001), quantitatively they result in very similar implications for macroeconomic aggregates and RER, provided sufficiently strong home bias.[27] Conditional on the behavior of the wage-based RER q_t^W, the currency of export price stickiness plays only a limited role for the behavior of the aggregate RER q_t.

Nonetheless, the currency of price stickiness is central for the comovement of exchange rates with ToT, and therefore the transmission into import quantities and net exports. Since ToT is the ratio of import and export prices, $s_t = p_{Ft} - p^*_{Ht} - e_t$, LCP implies a short-run negative correlation between exchange rate and ToT, as it is the export prices that increase in the home currency when it depreciates. In contrast, PCP implies a positive correlation, as in this situation it is the import prices that increase (due to LOP, which holds under PCP and not under LCP, $s_t = p^*_{Ft} + e_t - p_{Ht}$ in this case). This sharp theoretical distinction led to a lively debate in the literature (see Obstfeld and Rogoff 2000, Engel 2003).

What both LCP and PCP fail to capture, though, is not as much the sign of the correlation, but the fact that in the data this correlation is very weak (if positive), and ToT does not move in concert with RER in the short run. The novel dominant currency paradigm captures this empirical pattern by postulating that both export and import prices are sticky in the same dominant currency (DCP), currently the US dollar. This allows the model to match the muted movements in ToT in response to large exchange rate fluctuations (see Gopinath 2016, Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller 2020). Muted ToT response, however, does not imply muted response of net exports, as we discuss in the next section.

Crucially, currency choice is an active firm-level decision with substantial variation in currency use across exporters even within destination countries and narrowly defined industries. At the same time, currency choice is persistent over time, driven in part by strategic complementarities in price setting across firms and other macroeconomic complementarities (e.g., currency of financing, monetary anchors; see Gourinchas 2019). The key theoretical insight, developed in Engel (2006) and Gopinath, Itskhoki, and Rigobon (2010), is that currency choice is shaped by desired ERPT prior to price adjustment, which in turn reflects properties of the firm's marginal cost and desired markup, as we discussed in Section 3.3. In particular, constant-markup firms with marginal costs stable in producer currency ($\alpha_i = \phi_i = 0$) favor PCP, while firms with strong strategic complementarities with their local competitors ($\alpha_i > 0$) favor LCP, and finally firms that source intermediate inputs in dollars ($\phi_i > 0$) favor DCP (see Mukhin 2017, Amiti, Itskhoki, and Konings 2020). The more firms adopt local currency price stability, the lower is the desired ERPT into local prices due to strategic complementarities and the smaller is the incentive to adjust prices for any individual firm (see Gopinath and Itskhoki

[27]Under PCP, equilibrium RER is $q_t = \lambda q_{t-1} + \lambda(1-2\gamma)\Delta e_t$, instead of (17), a difference by a factor of $(1-2\gamma)$.

2010). This is the mechanism by which endogenous foreign-currency price stickiness, just like PTM and imported inputs, propagates incomplete ERPT into destination prices, tying closer together the dynamics of e_t, q_t^W and q_t.

Summary The mechanisms of LOP deviations discussed in Sections 3.3–3.4, while altering in important ways the dynamics of individual prices, do not change the qualitative relationship between nominal and real exchange rates. Over a one-to-five-year horizon, the flexible-price relationship in Proposition 1 offers a useful benchmark for thinking about RER q_t, with home bias γ and general equilibrium behavior of the wage-based RER q_t^W being the two crucial determinants. At the same time, variable markups, imported intermediate inputs, firm heterogeneity and foreign-currency price stickiness are important additional forces which further mute pass-through of NER, reinforcing the effect of home bias γ, and are necessary for a complete quantitative model of RER and international transmission of shocks. The rest of the manuscript focuses on the general equilibrium determination of q_t (and q_t^W), for concreteness making use of Propositions 1 and 2; it illustrates the overall limited role of a specific mechanism of transmission via prices beyond some form of a PPP violation identified in Lemma 1.

4 Real Exchange Rate and Expenditure Switching

This section studies the next step of international transmission, going from border prices to import quantities and trade balance, emphasizing empirical implications which pose challenges for current international models. We also study goods market clearing in an open economy, and its implications for the relationship between RER and aggregate consumption, and the associated Backus-Smith puzzle. This provides an important segue into the general equilibrium analysis in the remaining two sections.

4.1 Import demand and net exports

Net exports, or exports minus imports, are given by $NX_t = \mathcal{E}_t P_{Ht}^* C_{Ht}^* - P_{Ft} C_{Ft}$, where C_{Ft} and C_{Ht}^* are aggregate import quantities at home and abroad, and P_{Ft} and P_{Ht}^* are local-currency import price indexes, defining ToT in (3). For simplicity, we log-linearize net exports around a symmetric equilibrium with $\bar{NX} = 0$, which results in $nx_t = \gamma[c_{Ht}^* - c_{Ft} - s_t]$, where s_t is ToT, $nx_t \equiv NX_t/\bar{Y}$, and γ is the steady-state trade share (imports or exports) in GDP.

Import demand can be written in log deviations as $c_{Ft} = -\theta(p_{Ft} - p_t) + c_t$, where θ is the elasticity of the import demand schedule, which in addition shifts out with the overall consumption level, c_t at home and c_t^* abroad. A parallel expression characterizes foreign import demand c_{Ht}^*. These expressions are exact under CES and emerge more generally as a first-order approximation to an aggregate demand system. Conventional models of trade rely on $\theta > 1$, and often considerably so (see cross-sectional estimates in e.g. Broda and Weinstein 2006). In the time series, however, the response of import quantities to the exchange rate is muted and often characterized by $\theta \approx 1$ (see e.g. Feenstra, Luck, Obstfeld, and Russ 2018, Amiti, Itskhoki, and Konings 2020), which is sometimes referred to as the international elasticity puzzle (Ruhl 2008, Fontagné, Martin, and Orefice 2018). Furthermore, import quantities, while varying a lot in the cross-section of products, are relatively stable in the time series, and in particular are considerably less volatile than the exchange rate.

The relative price of imports can be expressed as $p_{Ft} - p_t = (1-\gamma)(p_{Ft} - p_{Ht})$ and $p_{Ft} - p_{Ht} = (s_t + q_{Ht})$, thus reflecting simultaneously ToT and LOP deviation for domestic tradables. Therefore, the response of import quantities is muted if either the responses of both ToT and LOP deviations are small, or alternatively if the two move in offsetting directions. Models that mute ToT fluctuations typically do so by creating large LOP deviations, resulting in an inconvenient tradeoff for import quantities,[28] which also persists for net exports.

Using import demand schedules, we obtain a rather general expression for net exports which holds independently of the nature of price setting and price stickiness:

$$nx_t = \gamma[\theta q_t + (\theta - 1)s_t - (c_t - c_t^*)]. \tag{18}$$

Both RER and ToT are important for net exports, as ToT characterizes the relative import-export prices, while RER additionally captures their movement relative to local prices, which shapes import demand. A consumption boom at home leads to an increase in imports and hence a deterioration of trade balance. The *expenditure switching* mechanism characterizes the properties of comovement between RER and net exports. A classical question in international economics is under which circumstances does trade balance improve in response to an exchange rate devaluation. The seminal Marshall-Lerner condition requires that the sum of the export and import elasticities is greater than one, or in our case $2\theta > 1$, which is equivalent to $\theta + (\theta - 1) > 0$. From (18), this could be the case if q_t and s_t moved proportionally with the exchange rate, which is not typically the case when prices are determined in equilibrium.

[28]Competitive pricing models and PCP sticky-prices feature no LOP deviations, yet imply ToT that are more volatile than RER, thus resulting in highly volatile import quantities. PTM models limit the volatility of ToT by means of LOP deviations (recall (13) and (16)), yet still in this model $s_t + q_{Ht} = \frac{1-\alpha}{1-2\gamma} q_t$ decreases with incomplete pass-through $(1-\alpha)$, which thus also mutes the quantity response. The DCP sticky-price model mutes ToT volatility, consistent with the data, yet implies very large LOP deviations for imports and volatile import quantities (as relative import prices move with the dominant exchange rate). LCP, while implying counterfactual negative correlation of ToT and RER, is in contrast successful at stabilizing import quantities (as import prices are stable relative to domestic prices). For further discussion of tradeoffs in matching the volatility of international prices and quantities across models in general equilibrium see Itskhoki and Mukhin (2017).

Using Proposition 2, we simplify (18) as:

$$nx_t = \gamma\big[\vartheta q_t - (c_t - c_t^*)\big], \qquad \text{where} \qquad \vartheta \equiv 2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \frac{1-2\alpha(1-\gamma)}{1-2\gamma}. \tag{19}$$

The required condition for trade balance to improve with a real devaluation is then $\vartheta > 0$.[29] The two terms of ϑ summarize the effects of RER on trade quantities and trade prices respectively, and under competitive or constant-markup pricing ($\alpha = 0$) the expression simplifies to $\vartheta = \frac{2\theta(1-\gamma)-1}{1-2\gamma}$. The requirement $2\theta > 1$ is sufficient for $\vartheta > 0$ in the closed economy limit ($\gamma \to 0$), and a general sufficient condition is $\theta \geq 1$, which works even in the absence of home bias ($\gamma \to 1/2$). The reason is that exchange rate shocks affect not only export and import prices, but also domestic price levels, and the more so in more open economies. For a given γ, strategic complementarities ($\alpha > 0$) relax the necessary requirement.[30]

In the data, net exports are both weakly correlated with q_t in the short and medium run, and also significantly less volatile than what is suggested by (18), even for conservative values of θ. Over long horizons (10 years), net exports track RER with a lag, as Alessandria and Choi (2019) show for the United States. A successful model of both trade prices and trade quantities must, therefore, include multiple ingredients: for example, a combination of DCP pricing at the border with local distribution margin and LCP pricing to consumers (see e.g. Auer, Burstein, and Lein 2020) and/or predetermined import quantities along with sticky import prices (see e.g. Alessandria, Kaboski, and Midrigan 2010, Fitzgerald, Yedid-Levi, and Haller 2019, Amiti, Itskhoki, and Konings 2020), resulting in a J-curve pattern of net export response.

4.2 Market clearing and aggregate consumption

Domestically-produced output Y_t is used for domestic consumption C_{Ht} and exports C_{Ht}^*, with the log-linearized market clearing for the domestic good given by $y_t = (1-\gamma)c_{Ht} + \gamma c_{Ht}^*$, reflecting home bias. Using the import demand schedule for c_{Ht}^* and a corresponding expression for domestic demand c_{Ht}, and taking the difference with their foreign counterparts, we arrive at the equilibrium relationship between relative consumption and RER:

$$c_t - c_t^* = \frac{1}{1-2\gamma}\big[(y_t - y_t^*) - 2\gamma\varkappa q_t\big], \qquad \text{where} \qquad \varkappa \equiv \frac{2\theta(1-\alpha)(1-\gamma)}{1-2\gamma} > 0, \tag{20}$$

[29] This condition is still partial equilibrium as it holds aggregate consumption constant, while it generally also comoves with RER (see (20)), and the full GE condition must account for this (see Itskhoki and Mukhin 2017).

[30] Matters are further involved with sticky prices: the patterns of short-run expenditure switching are different under PCP, LCP and DCP, as discussed in detail in Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller (2020) and Barbiero, Farhi, Gopinath, and Itskhoki (2019).

where for concreteness we adopt the flexible-price case of Proposition 2.[31] Equation (20) is the result of goods market clearing in an open economy: it summarizes how globally produced output is allocated to its final consumption use, mediated by the expenditure switching mechanism. Output y_t and y_t^* can be both exogenous endowment shaped by productivity shocks alone or endogenous aggregate supply of goods shaped in part by the interaction of equilibrium labor supply, intermediate inputs, markups and nominal rigidities.

Equilibrium relationship (20) has a number of implications. In a closed economy ($\gamma \to 0$), it reduces to $c_t = y_t$, as all output must be consumed locally. In an open economy with home bias, $0 < \gamma < 1/2$, an increase in relative output $y_t - y_t^*$ must be accommodated by a more than proportional increase in relative consumption $c_t - c_t^*$, if international relative prices q_t remain unchanged. In other words, when $\gamma > 0$, a 10% increase in $c_t - c_t^*$ requires a less than 10% increase in $y_t - y_t^*$, as home agents want to increase consumption of both home- and foreign-produced goods. Of course, an increase in the relative supply of home goods, $y_t - y_t^*$, tends to also reduce their relative prices, that is depreciate RER.[32]

Finally, RER depreciation (an increase in q_t), with a constant supply $y_t - y_t^*$, requires a reduction in relative consumption $c_t - c_t^*$. This effect is easier to see in reverse: a reduction in consumption $c_t - c_t^*$ for a given level of output $y_t - y_t^*$ must be accommodated by a reduction in relative prices in order for expenditure switching towards domestic goods to clear the market, a *Keynes' transfer effect*. The more home-biased is the economy (smaller γ), the larger is the required change in RER to accommodate a given movement in consumption, or equivalently the smaller is the change in consumption (nil in the limit $\gamma \to 0$) for a given movement in RER.

These implications of conventional goods market clearing in open economies are crucial for understanding the equilibrium patterns of comovement, or the lack thereof, between RER and macroeconomic aggregates, such as consumption and output. In particular, it illustrates the challenges for the standard International RBC and New Keynesian Open Economy models in explaining the empirical negative correlation between q_t and $c_t - c_t^*$, or the celebrated *Backus-Smith puzzle*. Independently of asset market completeness, the driving force in such models are product-market shocks — namely, an increase in y_t either driven by productivity or reduction in markups (in response to expansionary monetary shocks under nominal rigidities) — which tend to simultaneously raise $c_t - c_t^*$ and q_t (depreciate RER). In contrast, shocks

[31]The more general relationship is $c_t - c_t^* = \frac{1}{1-2\gamma}(y_t - y_t^*) - 2\theta(1-\gamma)\frac{2\gamma}{1-2\gamma}\big[s_t + \frac{q_{Ht}+q_{Ft}}{2}\big]$, and (20) derives from it after we use $q_{Ht} = q_{Ft} = \alpha q_t$ and $s_t + \alpha q_t = \frac{1-\alpha}{1-2\gamma} q_t$ implied by Proposition 2. Under sticky prices, this equation acts as an error correction target, with departures from it generalizing the closed-economy concept of an *output gap*, which enters the international Phillips curve that governs the dynamic adjustment of RER towards its long-run equilibrium (see Itskhoki and Mukhin 2019).

[32]This is particularly transparent in the case of no home bias, $\gamma \to 1/2$, when goods market clearing (20) implies $c_t - c_t^* = 0$ and $q_t^P = \frac{(1-\alpha)q_t}{1-2\gamma} = \frac{1}{\theta}(y_t - y_t^*)$. That is, an increase in relative supply results in a proportional reduction in relative producer prices, q_t^P defined in (16), leaving relative consumption unchanged.

to RER arising outside the goods market, produce a negative comovement between $c_t - c_t^*$ and q_t, with a low relative consumption volatility when home bias is strong (small γ) and pass-through $(1-\alpha)$ is incomplete (see Itskhoki and Mukhin 2017).[33]

5 Real Exchange Rate and International Risk Sharing

We now shift our focus to the role of RER in financial markets, and in particular in international risk sharing. The analysis of international risk sharing builds on the fundamental asset pricing equation, $\mathbb{E}_t\{\mathcal{M}^h_{t+1}\mathcal{R}^j_{t+1}\} = 1$, where $\mathcal{M}^h_{t+1}$ is the stochastic discount factor (SDF) of agent/household h and $\mathcal{R}^j_{t+1} \equiv (\mathcal{P}^j_{t+1} + \mathcal{D}^j_{t+1})/\mathcal{P}^j_t$ is the rate of return on asset j with price $\mathcal{P}^j_t$ and dividend $\mathcal{D}^j_{t+1}$ (see e.g. Cochrane 2001). The asset pricing equation holds for every asset j and every household h that can freely purchase this asset.[34] We focus on SDFs that arise from separable CRRA utility in consumption with relative risk aversion σ, $\mathcal{M}_{t=1} = \beta\big(\frac{C_{t+1}}{C_t}\big)^{-\sigma}\frac{P_t}{P_{t+1}}$, for a representative domestic household facing consumption price level P_t. A foreign representative household has a symmetric SDF, $\mathcal{M}^*_{t=1}$. We denote with J_t and J^*_t the sets of assets j available at time t to home and foreign households, respectively. An asset j with return $\mathcal{R}^{j*}_{t+1}$ in foreign currency has return $\mathcal{R}^j_{t+1} = \mathcal{R}^{j*}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$ when converted to home currency, adjusting for its nominal depreciation.

All assets traded by both foreign and home households yield a foreign-currency return $\mathcal{R}^{j*}_{t+1}$ that simultaneously satisfies $\mathbb{E}_t\big\{\mathcal{M}^*_{t+1}\mathcal{R}^{j*}_{t+1}\big\} = 1$ and $\mathbb{E}_t\Big\{\mathcal{M}_{t+1}\mathcal{R}^{j*}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\Big\} = 1$. Subtracting one from the other and expressing out SDFs, the *international risk sharing condition* is:

$$\mathbb{E}_t\left\{\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} - \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\right]\cdot\frac{\mathcal{R}^{j*}_{t+1}}{P^*_{t+1}/P^*_t}\right\} = 0 \qquad \text{for all } \ j \in J_t \cap J^*_t. \tag{21}$$

Note that $\frac{\mathcal{R}^{j*}_{t+1}}{P^*_{t+1}/P^*_t}$ is the realized real return in terms of foreign consumption basket, that is nominal return adjusted for foreign inflation. The terms in the square brackets in (21) are home and foreign household's *real* SDFs, with home SDF adjusted for real depreciation. Each asset available to both home and foreign households brings the two SDFs closer together.[35]

[33] Corsetti, Dedola, and Leduc (2008) and Colacito and Croce (2013) provide related explanations to the Backus-Smith puzzle based on long-lasting shocks to y_{t+j} for $j \geq 0$, which have a relatively small effect on contemporaneous goods supply y_t, yet trigger a large forward-looking response in q_t that dominates the overall shifts on the right-hand side of (20). Alternatively, the Backus-Smith puzzle can be resolved if RER *appreciates* with a positive goods supply shock, either due to Balassa-Samuelson forces (Benigno and Thoenissen 2008) or a low elasticity of substitution between home and foreign goods, $\theta < 1$ (the second case in Corsetti, Dedola, and Leduc 2008).

[34] The asset pricing equation derives from the Euler equation of an investor, an asset demand condition taking asset prices $\mathcal{P}^j_t$ and returns $\mathcal{R}^j_{t+1}$ as given. When an investor faces binding financial or borrowing constraints, similar conditions still hold, but with asset returns adjusted by the Lagrange multiplier on the constraint.

[35] According to (21), the gap between home and foreign SDFs (adjusted for real depreciation) must be orthogonal with real returns on every asset j they can trade. Note how (21) is akin to an IV GMM condition minimizing the variance of the projection residuals (state-by-state SDF gaps) using asset returns as instruments.

The richer is the set (span) of available assets, the more perfect is the extent of international risk sharing; in the limit, this makes the two SDFs equal up to a real depreciation term.

5.1 Complete markets and financial autarky

We start by considering the two opposite limiting cases — complete international asset markets and financial autarky — which surprisingly yield qualitatively similar implications for RER. The intuition is that in both of these cases, an inherently dynamic international risk sharing condition turns into a static relationship between aggregate consumption and RER.

Complete markets Markets are complete when the set of assets $j \in J_t \cap J_t^*$ allows agents to span every state of the world; that is, assets replicate a full set of *Arrow securities* which pay one unit in a given state at $t+1$ conditional on state at t. In this case, the set of conditions (21) holds not just in expectation, but also state-by-state. That is, (21) becomes $\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\left(\frac{C^*_{t+1}}{C^*_t}\right)^{\sigma}$, or equivalently $\left(\frac{C_t}{C_t^*}\right)^{\sigma} = \chi \mathcal{Q}_t$ across all periods and states for some constant factor $\chi \in (0, \infty)$, which is determined by the intertemporal budget constraint of the countries in the initial state at $t = 0$.[36] In log deviation terms, this further simplifies to the following static condition:

$$\sigma(c_t - c_t^*) = q_t, \tag{22}$$

and thus the constant factor χ and the budget constraint are irrelevant for the *dynamics* of the variables, as state-contingent transfers provide full insurance between the two countries. Backus and Smith (1993) and Kollmann (1995) first pointed out this stark state-by-state implication of complete international asset markets for the relationship between relative consumption and RER. In particular, relative consumption growth must track real depreciations, or in other words home consumption must be relatively high when home price level is relatively low.

What is the logic behind (22)? If PPP holds ($q_t \equiv 0$), then perfect risk sharing implies perfect comovement between home and foreign consumption, $\Delta c_{t+1} = \Delta c^*_{t+1}$. However, when relative prices fluctuate, the relative cost of an extra unit of marginal utility is not constant, and thus keeping relative consumption constant is no longer optimal. What is optimal is to equalize the utility gains that can be obtained from an extra "dollar", that is $\frac{u'(C_t)}{P_t} = \frac{u'(C_t^*)}{\mathcal{E}_t P_t^*}$, which results in (22) under CRRA utility. Simply put, consumption must be temporarily higher where the cost of delivering consumption is temporarily lower. This is the opposite of what is observed in the data, giving rise to the Backus-Smith puzzle.

The risk sharing condition (22) emphasizes the central role played by RER in complete international asset markets. Any movement in RER has immediate and direct implications for

[36] Another implication of (21) under complete markets is that nominal depreciations must satisfy $\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{\mathcal{M}_{t+1}}{\mathcal{M}^*_{t+1}}$, making NER volatility equal to that of the ratio of SDFs (see e.g. Brandt, Cochrane, and Santa-Clara 2006).

the key macroeconomic aggregates, independently of openness to trade. This is different from the goods market clearing (20), where the effect of RER on macro aggregates is indirect, via expenditure switching, and is muted by home bias γ and incomplete pass-through $(1-\alpha)$. Combining (22) with (20) allows us to immediately solve for both equilibrium RER and relative consumption as a function of equilibrium output (given by the supply side of the economy):

$$q_t = \xi \cdot (y_t - y_t^*), \qquad \text{where} \qquad \xi \equiv \frac{\sigma}{(1-2\gamma) + 2\gamma\sigma\varkappa}. \tag{23}$$

As a result, under complete financial markets, the only source of volatility in RER emerges from the supply of goods and results in strong counterfactual comovement between RER and macroeconomics aggregates — consumption and output — with the relative volatility of RER roughly proportional to the relative risk aversion σ.[37]

Finally, note from (23) the limited role played by the transmission via international relative prices (the focus of Section 3) for equilibrium RER determination in this case, and in particular if home bias is substantial (γ premultiplies $\varkappa$ which summarizes the effects via prices).

Financial autarky At the other extreme is the case of financial autarky, whereby home and foreign households cannot share risk at all ($J_t \cap J_t^* = \emptyset$), which implies balanced trade period-by-period and state-by-state, $NX_t \equiv 0$. Gross exports and imports, at the same time, are in general non-zero, and countries exchange goods within each period-state. Surprisingly, this case results in a similarly tight relationship between relative consumption and RER:

$$c_t - c_t^* = \vartheta q_t, \tag{24}$$

which obtains directly from $nx_t = 0$ in (19), where $\vartheta = 1 + 2(\theta - 1)(1-\alpha)\frac{1-\gamma}{1-2\gamma} > 0$ corresponds to the Marshall-Lerner condition discussed above. The logic in this case is, however, very different from the risk-sharing logic behind the Backus-Smith condition (22). Balanced trade requires price movements to offset shifts in import demand (aggregate consumption): when demand is high, imports are high given prices, and in order to balance trade, home prices must fall to encourage exports and discourage imports.

Despite different logic, the implications for a positive consumption-RER comovement are qualitatively the same, just with the goods-market elasticity θ replacing risk aversion σ. In fact, autarky and complete-market allocations coincide under the celebrated Cole and Obstfeld (1991) case with $\sigma = \theta = 1$, which implies $\vartheta = 1$ independently of the values of α and γ.[38]

[37] The only way to break this tight relationship is by changing the utility function — namely, by introducing marginal utility shocks as in Stockman and Tesar (1995), non-separable leisure with home production as in Karabarbounis (2014), or non-time-separable Epstein-Zin preferences as in Colacito and Croce (2011).

[38] Outside the exact Cole-Obstfeld case, this result applies as a first-order approximation when $\sigma\vartheta = 1$.

In this case, movements in ToT provide perfect insurance without any need to trade financial assets, ensuring the complete-market allocation independently of the asset market structure.

Regardless of the exact equivalence, the autarky case (24) is qualitatively similar with the complete markets case (22), as they both imply direct and strong comovement of relative consumption with RER, and in combination with the goods market clearing (20), pin both of them down as a function of aggregate supply in the goods market:

$$q_t = \zeta \cdot (y_t - y_t^*), \qquad \text{where} \qquad \zeta \equiv \frac{1}{(1-2\gamma)\vartheta + 2\gamma\varkappa}, \tag{25}$$

which parallels (23) with σ replaced by $1/\vartheta$ (note that $\zeta = \xi$ when $\sigma\vartheta = 1$), and with the relative volatility of RER now decreasing in the product-market elasticity θ. This comovement is again empirically counterfactual, and thus both extreme benchmarks — autarky and complete markets — offer a poor approximation to the observed empirical patterns of RER.[39]

5.2 General incomplete markets

In between the two extremes of autarky and complete markets, the case of incomplete markets is starkly different, implying a much weaker link between consumption and RER in expected changes instead of state-by-state comovement. Thus, incomplete markets limit the centrality of international risk sharing in shaping the equilibrium behavior of RER. We briefly explore two cases: conventional incomplete markets and financial frictions in risk sharing.

Conventional incomplete markets We consider first the case when $J_t \cap J_t^*$ contains at least one bond that home and foreign households can trade without constraints. In this case, a quantitatively accurate first-order approximation to the risk-sharing condition (21) is given by:[40]

$$\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\} = 0. \tag{26}$$

Thus, instead of a static relationship as in (22), incomplete-market risk sharing results in a martingale condition for a combination of consumption and RER, $\sigma(c_t - c_t^*) - q_t$. In other words, it ties together expected depreciation and relative consumption growth without constraining the comovement of these variables across states of the world.

[39] Just like preference/utility shocks under complete markets, product-market taste shocks which shift import demand (e.g. as in Pavlova and Rigobon 2008) can break this tight relationship between consumption and RER. Even outside autarky, such shocks are useful to reduce counterfactually strong correlation between nx_t and q_t.

[40] Consider a foreign bond with deterministic real return $R_t^{r*} = \mathcal{R}_{t+1}^{r*}/(P_{t+1}^*/P_t^*)$, so that (21) becomes $\mathbb{E}_t \exp\{-\sigma\Delta c_{t+1} + \Delta q_{t+1}\} = \mathbb{E}_t \exp\{-\sigma\Delta c_{t+1}^*\}$, which is equivalent to (26) up to higher-order terms that tend to be both small and non-time-varying. These higher-order terms could be amplified with alternative preferences (e.g. habits as in Verdelhan 2010), heterogeneous agents (e.g. Kocherlakota and Pistaferri 2007) or time-varying disaster risk (e.g. Farhi and Gabaix 2016).

As with complete markets, we can substitute market clearing (20) into the risk-sharing condition (26), which results in:

$$\mathbb{E}_t\Delta q_{t+1} = \xi \cdot \mathbb{E}_t\{\Delta y_{t+1} - \Delta y^*_{t+1}\}, \tag{27}$$

shaping the future expected path of RER instead of its equilibrium value as in (23). If aggregate supply in the economy follows a martingale process, then RER is also a martingale unpredictable in changes, $\mathbb{E}_t\Delta q_{t+1} = 0$. RER predictability (and, in particular, mean reversion) in this case must come from predictable future changes in relative GDP growth.

There are two further notable implications of (27). First, international risk sharing and goods market clearing are, in general, insufficient to determine equilibrium RER. More specifically, they characterize its expected path (in changes), but not its current equilibrium level q_t.[41] Second, (27) suggests that RER process contains a unit root, and even transitory shocks can permanently shift its entire equilibrium path including its long-run expectation, $\lim_{j\to\infty}\mathbb{E}_t q_{t+j}$.[42]

Risk sharing under financial frictions With limited participation in the financial market, borrowing constraints, and other types of financial frictions, condition (26) generalizes to additionally feature a risk-sharing wedge ψ_t:[43]

$$\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\} = \psi_t. \tag{28}$$

Risk-sharing wedge ψ_t reflects both exogenous shocks in the financial market, as well as endogenous feedback from conventional productivity and monetary shocks, and in general also depends on state variables (e.g., net foreign assets) which may ensure long-run stationarity of RER (see Itskhoki and Mukhin 2019). Time-series variation in ψ_t generates departures from the martingale property of RER. Itskhoki and Mukhin (2017) argue that such shocks are essential to breaking the tight link between RER and macroeconomic aggregates implied by the

[41] By consequence, fully *unexpected* level jumps in RER may leave the equilibrium allocation unchanged, at least from the point of the financial market (21). For example, if goods market pass-through from RER is fully muted by a border tax, the same macroeconomic allocation can be consistent with a level shift in RER, as is the case under a *fiscal devaluation* policy studied in Farhi, Gopinath, and Itskhoki (2014).

[42] To see both implications, roll forward (27) to solve for $q_t = \lim_{j\to\infty}\mathbb{E}_t q_{t+j} + \sum_{j=0}^{\infty}\mathbb{E}_t\{\Delta y_{t+j} - \Delta y^*_{t+j}\}$. The long-run expectation is finite (thus cannot be ruled out by a 'no-bubble condition') and undetermined without an additional condition on the path of $\{q_{t+j}\}_{j\geq 0}$, namely the intertemporal budget constraint (see Section 6). Shocks to the short-run path of q_{t+j} generally affect $\lim_{j\to\infty}\mathbb{E}_t q_{t+j}$. This last property is similar to the general non-stationarity of (log-linearized) incomplete market open economies emphasized by Obstfeld and Rogoff (1995a).

[43] For example, ψ_t in (28) can arise in models with borrowing constraints (e.g. Mendoza, Quadrini, and Ríos-Rull 2009), limited market participation (e.g. Alvarez, Atkeson, and Kehoe 2009), or imperfect intermediation under limits-to-arbitrage (e.g. Jeanne and Rose 2002, Gabaix and Maggiori 2015, Itskhoki and Mukhin 2019), but also can capture time-varying risk-premia terms in models without financial frictions (see footnote 40). Note the relationship between (28) and the *uncovered interest parity* (UIP; see Engel 2014) for RER, $r^f_t - r^{f*}_t - \mathbb{E}_t\Delta q_{t+1} = \psi_t$, where $r^f_t = \sigma\mathbb{E}_t\Delta c_{t+1}$ is the real rate; thus, ψ_t is often referred to as a UIP deviation (shock).

goods-market shocks (as in (23) and (27)), in order to reproduce the empirical exchange rate disconnect properties.[44] An outstanding question is the fundamental nature of such "financial" shocks, as well as their likely endogeneity to policy and other macro-fundamental shocks.

To summarize, outside of the two limiting cases of complete markets and autarky, equilibrium conditions in the financial and goods markets, (28) and (20), are generally insufficient to characterize equilibrium exchange rates, as we explore in the final section.

6 Real Exchange Rate in General Equilibrium

6.1 Country budget constraint

The final key determinant of the equilibrium RER is the intertemporal budget constraint of a country. Following Farhi, Gopinath, and Itskhoki (2014) and Itskhoki and Mukhin (2019), we can write the flow budget constraint of a country without loss of generality as:

$$\mathcal{B}_t = \mathcal{R}_t \mathcal{B}_{t-1} + NX_t, \tag{29}$$

where $\mathcal{B}_t$ is net foreign assets (NFA) and $\mathcal{R}_t$ is gross return on the NFA position.[45] Flow budget constraint holds state-by-state, and it can be rolled forward along any path of future states (imposing no bubble condition) to arrive at the intertemporal budget constraint, $\mathcal{R}_t\mathcal{B}_{t-1} + \sum_{j=0}^{\infty} \frac{NX_{t+j}}{\mathcal{R}_{t,t+\ell}} = 0$, where $\mathcal{R}_{t,t+j} \equiv \prod_{\ell=1}^{j} \mathcal{R}_{t+\ell}$. A negative net export surprise today needs to be compensated either by future trade surpluses or by favorable returns on NFA, along every path of the future (see Obstfeld and Rogoff 1995b, Gourinchas and Rey 2007).

An open economy equilibrium is characterized by the interplay of two *dynamic* forces — international risk sharing (21) and intertemporal budget constraint (29). The relative role of these two forces depends on asset market completeness. Under complete markets, the role of (29) is reduced to determining the constant factor χ in the Backus-Smith condition, with the dynamics of allocations fully determined by risk sharing (21). Under financial autarky, $\mathcal{B}_t = NX_t = 0$ in (29) state-by-state, which fully determined the allocation as the set of risk-sharing conditions (21) is empty. Under general incomplete markets, risk sharing (21) (or equivalently (28)) shapes the path of the future expected RER changes, while the budget

[44] Furthermore, recent work in international finance finds an important empirical role for such shocks: see e.g. Du, Tepper, and Verdelhan (2018), Lustig and Verdelhan (2019), Jiang, Krishnamurthy, and Lustig (2018).

[45] If a country's portfolio positions (private and public) at t consist of $\{B_t^j\}_{j\in J_t}$, then $\mathcal{B}_t = \sum_{j\in J_t} \mathcal{P}_t^j B_t^j$ and $\mathcal{R}_{t+1}\mathcal{B}_t = \sum_{j\in J_t} (\mathcal{P}_{t+1}^j + \mathcal{D}_{t+1}^j) B_t^j$, which together define $\mathcal{R}_{t+1}$. Gross return $\mathcal{R}_t$ is stochastic: for example, under complete markets, it generates a state-contingent wealth transfer required to support (22) as the outcome of international risk sharing. Farhi, Gopinath, and Itskhoki (2014) show how various asset classes map into this definition. In practice, capital gains on NFA positions (especially FDI) are hard to capture, which leads to mismeasurement of $\mathcal{P}_{t+1}^j$, $\mathcal{R}_{t+1}$ and $\mathcal{B}_{t+1}$ (see e.g. Lane and Milesi-Ferretti 2007). This, however, is not consequential for the present analysis, as it focuses on NX_t rather than the NFA position $\mathcal{B}_t$.

constraint (29) determines its current value (unexpected level jump), as we now discuss.

We log-linearize the flow budget constraint (29) around a symmetric steady state:

$$\beta b_{t+1} - b_t = nx_t = \gamma[\vartheta q_t - (c_t - c_t^*)], \tag{30}$$

where $b_t \equiv \bar{\mathcal{R}}\mathcal{B}_{t-1}/\bar{Y}$ is the NFA-to-GDP ratio, $\beta = 1/\bar{\mathcal{R}} < 1$ is the discount factor, and nx_t is as defined in (19). The dynamic equilibrium system combines (28) and (30) with a static goods market clearing (20). This allows us to solve out aggregate consumption from the dynamic equilibrium system, leaving RER q_t and NFA b_t as the two endogenous dynamic variables — a forward-looking jump variable and a predetermined state variable. The equilibrium path of these variables is characterized given dynamic paths of relative output shocks $\tilde{y} \equiv y_t - y_t^*$ and financial shocks ψ_t, the initial condition b_0 and the no-bubble condition $\lim_{j\to\infty}\beta^j b_{t+j} = 0$.

For concreteness, assuming that both $\tilde{y}_t$ and ψ_t follow an AR(1) process with a common autoregressive coefficient $\rho \le 1$, we obtain the following cointegration relationship for RER:

$$q_t = -\frac{1}{\vartheta + \frac{2\gamma\varkappa}{1-2\gamma}}\frac{1-\beta}{\gamma}b_t + \frac{1}{1+\frac{2\gamma\sigma\varkappa}{1-2\gamma}}\frac{\beta}{1-\beta\rho}\psi_t + \left[\frac{\beta(1-\rho)}{1-\beta\rho}\xi + \frac{1-\beta}{1-\beta\rho}\zeta\right]\tilde{y}_t, \tag{31}$$

where ξ and ζ are defined in (23) and (25) respectively. Thus, equilibrium RER depreciates ($q_t\uparrow$) with lower NFA b_t, an increase in the relative supply of domestic goods $\tilde{y}_t$, and a financial shock ψ_t (which leads home to delay current consumption and buy foreign assets).

Without financial shocks ψ_t, we can further show that:

$$\Delta q_t = \zeta\Delta\tilde{y}_t + \frac{\beta(1-\rho)}{1-\beta\rho}(\xi-\zeta)\Big(\tilde{y}_t - \frac{1}{\beta}\tilde{y}_{t-1}\Big), \tag{32}$$

generalizing both the complete-market and the autarky solutions, (23) and (25), which are nested as special cases.[46] In the absence of financial shocks, equilibrium RER inherits the properties from these two special cases more generally — namely, it is insufficiently volatile relative to macroeconomic aggregates and it is strongly positively correlated with consumption and output, violating disconnect properties and resulting in the Backus-Smith puzzle.

These deficiencies are overcome in the presence of financial risk-sharing shocks ψ_t, as shown in Itskhoki and Mukhin (2017). Combining the equilibrium relationship (31) with the budget constraint (30), one can solve for equilibrium RER q_t, which follows an ARIMA(1,1,1) process, that is Δq_t follows an ARMA(1,1), with an AR root ρ and MA root $1/\beta$. Furthermore, this process is indistinguishable from a random walk when β and ρ are close to 1, which also ensures excessive RER volatility relative to consumption and output, as well as a weak negative

[46] Note that $\beta \to 1$ turns (32) into (23), while $\rho \to 1$ turns it into (25); all three are equivalent in the Cole-Obstfeld case which implies $\xi = \zeta$ independently of the value of β and ρ.

correlation between RER and relative consumption driven by expenditure switching in the goods market (20), in line with the empirical properties of RER.[47] It is, thus, essential that the bulk of equilibrium RER volatility emerges from the financial, rather than the goods, market.

Is RER stationary? It follows from our discussion that RER is, in general, non-stationary, and is in fact integrated even when underlying shocks are transitory, and thus the assumption of long-run mean reversion in RER is not generally justified. Intertemporal budget constraint (30) provides the theoretically-coherent discipline on the future path of q_t – via the no-bubble condition, $\lim_{j\to\infty}\beta^j b_{t+j} = 0$, instead of an *ad hoc* assumption of mean reversion, $\lim_{j\to\infty} q_{t+j} = \bar{q}$.[48] At the same time, the integrated nature of RER does not exclude notable departures from a pure random-walk behavior and a tendency for an imperfect mean reversion at different horizons. In particular, the finite-sample autocorrelation is shaped by $\mathrm{corr}(\Delta q_t, \Delta q_{t-1}) < 0$, and thus when observing q_t in finite samples, one may confuse it for a persistent AR(1) with a finite half life. The more persistent are the underlying shocks, the closer is RER to a pure random walk. Lastly, note that equilibrium persistence of RER is a result of general equilibrium forces, and does *not* directly depend on the specific nature of PPP deviations outlined in Lemma 1 (as long as $\xi, \zeta > 0$, which is ensured e.g. by home bias, $\gamma < 1/2$).

6.2 Monetary policy and exchange rate regimes

The final step in our general equilibrium analysis is to bring back together nominal and real exchange rates, defined in (1). In partial equilibrium, it is conventional to take NER shocks as exogenous and study their transmission into prices, thus shaping the RER response, as we discussed in Section 3. From the general equilibrium perspective, however, a reverse approach is, arguably, more fruitful. Specifically, we combine the equilibrium behavior of RER, described in the previous Section 6.1, with the assumption that monetary policy stabilizes consumer price inflation, and does so independently of the equilibrium exchange rate volatility. To the extent that π_t and π_t^* are stable, NER e_t inherits the volatility and persistent properties of RER q_t, $\Delta e_t = \Delta q_t + \pi_t - \pi_t^*$. The two exchange rates track each other closely at most horizons, until the gap in inflation rates $\pi_t - \pi_t^*$ gradually accumulates to magnitudes comparable with exchange rate volatility.

[47] Small persistent ψ_t shocks generate small persistent departures from $\mathbb{E}_t\Delta q_{t+j} = 0$, which must be met with a large surprise jump in q_t in the opposite direction to satisfy the intertemporal budget constraint (30); then, home bias combined with incomplete ERPT ensures a muted transmission into aggregate prices and quantities, which are affected only indirectly via the expenditure switching mechanism in (20).

[48] Formally, it is the interplay between the martingale risk sharing condition and the budget constraint that results in an integrated ARIMA process for RER, with the MA root shaped by the discount factor β in the budget constraint (see Itskhoki and Mukhin 2017 and *cf.* Engel and West 2005 where characterization uses the financial market equilibrium alone without the country budget constraint). Departure from the martingale risk-sharing property, e.g. due to risk premia or financial frictions, may render RER mean reverting, yet with virtually unbounded half lives.

Despite being unconventional, this appears to be an empirically relevant description, as inflation rates in OECD countries under floating regimes are an order of magnitude less volatile than both exchange rates, as well as virtually uncorrelated with them (see Itskhoki and Mukhin 2019). Furthermore, any departures from a pure random walk in RER can be used to predict changes in NER as well, provided the long-run adjustment towards equilibrium RER is not achieved exclusively via accumulated inflation differentials. This is indeed the case in the data (see Eichenbaum, Johannsen, and Rebelo 2020).

A remaining issue to address is how inflation-stabilizing monetary policy can be consistent with a volatile and persistent NER, violating the monetarist view that Δe_t must track $\pi_t - \pi_t^*$, at least over the long run. There is, of course, no inconsistency from a neoclassical perspective, where each monetary authority chooses a nominal anchor (consumer price level), and NER freely floats tracing RER which is determined in general equilibrium, as we discussed above.[49] This requires that monetary shocks are *not* the key drivers of RER, as we discussed in Section 3.4. The same logic applies in a monetary model with nominal rigidities, provided that home bias and incomplete ERPT limit exchange rate fluctuations from being the key contributors to domestic CPI inflation targeted by monetary policy (see Itskhoki and Mukhin 2017).

Exchange rate regimes and the Mussa puzzle Matters are different when monetary authorities adopt a fixed exchange rate regime (peg), which is a common occurrence in practice (see Ilzetzki, Reinhart, and Rogoff 2019). Under a peg, $\Delta e_t = 0$, and therefore $\Delta q_t = -(\pi_t - \pi_t^*)$, which holds by definition of RER. As a result, RER changes dramatically its equilibrium properties across policy regimes — it closely tracks a volatile NER under a float and becomes an order of magnitude less volatile, tracing smooth inflation differentials under a peg. This offers an important source of identification and inference in macroeconomics, constituting prime evidence of monetary non-neutrality, as RER is a real variable affected by a change in monetary policy (see Mussa 1986, Baxter and Stockman 1989, Nakamura and Steinsson 2018). However, the properties of most other macro aggregates — including CPI inflation, consumption and output — remain largely unchanged across monetary regimes, in a stark challenge to conventional monetary models, which predict that non-neutrality must persist across all these variables. Itskhoki and Mukhin (2019) present a resolution to this broader puzzle in a model of segmented financial market, which results in a risk-sharing wedge ψ_t in (28) that is endogenous to the exchange rate regime, while nominal rigidities play only a secondary role if present.[50]

[49] While monetary policy ensures $p_t = p_t^* = 0$, relative prices are determined from equilibrium conditions: e.g. real wages $w_t - p_t = a_t - \frac{\gamma}{1-2\gamma} q_t$ and wage-based RER $q_t^W = \frac{1}{1-2\gamma} q_t - (a_t - a_t^*)$, given equilibrium RER q_t.

[50] A peg switches off NER volatility, a source of risk in international interest rate arbitrage, reducing the extent of the equilibrium risk-sharing wedge ψ_t and hence its contribution to equilibrium RER volatility, independently of nominal rigidities in price setting.

7 Conclusion

The real exchange rate plays a central role in international macroeconomic models, in both goods and asset markets. Some form of PPP violation is necessary for a theory of RER (Lemma 1). Home bias in tradables and due to non-tradables acts as the primary source of PPP violations, amplified by pricing to market, the distribution margin, imported intermediate inputs, and foreign-currency export price stickiness.[51] This is, however, insufficient to explain the equilibrium behavior of RER, which is inherently a general equilibrium variable. In particular, its equilibrium dynamics are shaped by the interplay of international risk sharing, goods market clearing and the intertemporal budget constraint. These forces determine simultaneously the expected future changes in RER, which equilibrate the financial market, and the surprise level jumps in RER, which balance the country's budget constraint. In turn, goods market clearing shapes the comovement properties between RER and macroeconomic aggregates.

Our analysis distinguishes between two types of exogenous shocks that account for RER volatility — namely, shocks to the relative equilibrium supply of goods (whether due to productivity shocks or due to markup shifts induced by monetary shocks) and shocks to relative asset demand/savings supply (or international risk sharing wedges). While both types of shocks, if sufficiently persistent, can produce near-random-walk behavior of RER, the product-market shocks fail to reproduce the disconnect property (including a sufficient RER volatility) and the Backus-Smith comovement observed in the data. Asset-market shocks, in contrast, ensure the empirically relevant comovement properties and a large gap in volatility between RER and macroeconomic aggregates. An important outstanding question concerns the specific nature of risk-sharing shocks and the extent to which they may be induced by macroeconomic fundamental shocks (e.g., monetary policy shocks), which is essential for normative analyses (e.g., weighing the relative benefits of RER stabilization and expenditure switching).

We conclude that RER should be taken as a generally non-stationary variable: its long-run behavior is shaped by the intertemporal budget constraint rather than by PPP, which is generally violated at all horizons. Nonetheless, the budget constraint together with international risk sharing typically imply imperfect mean reversion in RER, which may be confused for stationarity even in large finite samples. Partial mean reversion in RER with monetary policy effectively stabilizing consumer price inflation results in long-run predictability of the nominal exchange rate, which enables the long-run adjustment in RER. Our analysis focused on broad unconditional exchange rate moments. Outstanding empirical questions concern exchange rate behavior conditional on various well-identified shocks (including monetary and productivity shocks), which would further shed light on international transmission mechanism.

[51] The interplay of these forces (e.g., input-output networks with dominant currency pricing), as well as the role of very long-lasting nominal wage rigidities and import quantity adjustment frictions remain as some of the open questions for future literature.

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