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JEL Classification: L13, L15, D82, D43

Keywords: asymmetric information, product quality, delegation, intermediary, signaling

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Asymmetric Information and Delegated Selling*

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Abstract

Asymmetric information about product quality can create incentives for a privately informed manufacturer to sell to uninformed consumers through a retailer and to maintain secrecy of upstream pricing. Delegating retail price setting to an intermediary generates pooling equilibria that avoid signaling distortions associated with direct selling even under reasonable restrictions on beliefs; these beliefs can also prevent double marginalization by the retailer. Expected profit, consumer surplus and social welfare can all be higher with intermediated selling. However, if secrecy of upstream pricing cannot be maintained, selling through a retailer can only lower the expected profit of the manufacturer.

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1 Introduction

Manufacturers often sell their products through retailers instead of selling directly to consumers; they delegate to retailers the task of setting prices faced by consumers. Consumers rarely observe the upstream pricing scheme used by manufacturers (when selling to retailers). In this paper we argue this kind of delegation may be an optimal response to asymmetric information about product quality, a pervasive feature in many markets. Here, product quality attributes may include health hazards as well as ethical dimensions of the production process that consumers care about such as environmental footprint, the working conditions of employees (including use of child labour) and prices paid to suppliers (fair trade). Manufacturers acquire such information through their intimate knowledge of the supply chain, the production process, results of product testing and quality control. When the manufacturer has private information about such quality attributes, selling through a retailer while maintaining secrecy of the vertical pricing scheme can help avoid the signaling distortions that arise when the manufacturer sells directly. This not only increases the expected profit of the manufacturer but can also lead to a more efficient market outcome. This result does not depend on whether the retailer has information about product quality and it is robust to having some consumers that are *a priori* informed about product quality.

The key idea behind our result is simple and based on the existence of asymmetric information about product quality between producers and consumers. In a world where upstream and downstream activities are integrated, the manufacturer controls the (retail) price that consumers pay. This price can signal private information the firm has about product quality. Invoking the seminal insight due to Bagwell and Riordan (1991), to deter imitation by low quality, the high quality price should be sufficiently distorted upward (relative to the full information outcome). Signaling introduces significant distortions and the resulting equilibrium may actually be quite inefficient, resulting in low profits for the firm and low consumer welfare.

Contrast this with a situation where the manufacturer sells through a retailer. When the price charged by the manufacturer to the retailer is not observed by consumers, the manufacturer can *hide* information about quality by selling to the retailer at a price that is independent of quality and as a result, the retail price does not convey any information about quality to the buyers. Note that the retailer's own information about product quality does not matter here.[1] We show that the resulting pooling equilibrium not only eliminates the signaling distortion that arises when the manufacturer directly sells to buyers, but may also avoid the well-known double marginalization problem associated with linear wholesale pricing. Note that delegation of selling to a retailer involves long-run commitment on the part of the manufacturer and for this reason we focus on the manufacturer's ex ante incentives prior to the actual realization of quality. The delegation outcome may yield higher ex ante expected industry profit and also higher consumer welfare, i.e., consumers may be better off being in the dark about product quality and *not being able to infer* quality from prices.

Secrecy of the manufacturer's pricing plays an important role in generating this kind of attractive pooling outcome under delegation to a retailer. If the price set by the manufacturer is publicly observable, final buyers may infer product quality directly from the wholesale price and this leads to a signaling outcome that is qualitatively similar to that in the vertically integrated industry except that in addition to the signaling distortion in wholesale price there is an added distortion due to double marginalization by the retailer. We show that even if the manufacturer uses a two-part tariff pricing scheme and extracts all the surplus earned by the retailer, his expected profit with observable vertical contracts can never exceed that under direct selling. Thus, we provide a new economic explanation for secrecy of vertical contracts.[2]

When the manufacturer sells through a retailer, the market interaction is not anymore that of a standard signaling game. The sender (manufacturer) sends a signal to an intermediary (retailer) who sends a signal to the receiver (consumer). Communication through intermediated interactions are natural in vertical supply chains but have been mostly ignored in the game theoretic signaling literature.[3] Depending on whether or not the price of the manufacturer is observed, some interesting game theoretic issues arise in applying standard refinement concepts such as the widely used Intuitive Criterion (Cho and Kreps

[1] The retailer's cost of supplying the product is independent of product quality and his payoff does not depend on product quality (it only depends on buyers' beliefs about product quality after observing a retail price).

[2] As in our settings two-part tariffs do not increase firms' profits, our paper may also be interpreted as providing an explanation for why the lump-sum component in actual wholesale contracts is small relative to the overall payment between firms (see, e.g., Blair and Lafontaine 2015 and Kaufmann and Lafontaine 1994).

[3] There is a small literature on communication through intermediaries in cheap talk games (see, e.g., Ambrus et al. 2013), but the signaling aspect of price that is important in vertical supply chains is absent in this literature.

1987) and the D1 criterion (Cho and Sobel 1990).

When the manufacturer's pricing is unobserved, buyers can also not observe any deviation by the manufacturer. They only observe deviations from the equilibrium retail price and when this can be fully accounted for by a unilateral deviation by the retailer, buyers may not attribute the deviation to the manufacturer. Thus, the pooling equilibria discussed above can be sustained even when buyers are fairly sophisticated in forming their beliefs. In addition, buyers' beliefs can discipline the mark-up charged by the retailer.

If the price set by the manufacturer is publicly observable, consumers can update their beliefs about quality based on the manufacturer's price in a way that is similar to the case where the manufacturer sells directly. In thinking about the manufacturer's gain from any deviation for certain beliefs of the consumers, one needs to think about the retailer's reaction to such a deviation which will depend on the latter's "second order" belief about the beliefs of the consumers. We outline modified restrictions on out-of-equilibrium beliefs for our specific game that are in the spirit of the standard refinements mentioned above to show that reasonable beliefs create incentives for the manufacturer to deviate from lucrative pooling outcomes.

Our paper contributes to the literature on the role of intermediaries in markets with asymmetric information about quality that has largely focused on information or certification intermediaries that use their own information, skill or reputation to provide information to buyers (Biglaiser 1993, Lizzeri 1999, Albano and Lizzeri 2001 and Glode and Opp 2016); in our framework, the intermediary retailer has no skill or market reputation and in fact, may have no more information about product quality than the uninformed consumer. In contrast to this literature, our key result is based on the beneficiary role of using a retailer to *hide* information from final consumers.

A number of papers have analyzed the role of leasing of new durable goods in reducing the extent of the lemons problem in the used goods markets. The leasing firm's opportunity cost of selling the used good (at the end of the lease) is determined prior to the realization of actual quality or performance of the used good and therefore independent of it; see, among others, Lizzeri and Hendel (2002) and Johnson and Waldman (2003). One may view leasing as delegation of reselling of the used good to the leasing firm. Further, the timing of actions rules out the possibility of signaling. Unlike this literature, our paper focuses on private information about producer's quality. The manufacturer is informed about quality before he sets the terms under which the retailer acquires the good and he chooses whether or not the retailer's cost of acquiring the good varies with quality.

Signaling by the manufacturer is potentially possible, but the manufacturer abstain from doing so. Further, in this setting observability of the terms of the vertical contract by final consumers affects the market outcome significantly, whereas this does not play a role in the leasing literature.

Our paper also contributes to a large literature on informational factors behind vertical integration and separation. In particular, beginning with Arrow (1975), a significant body of theoretical literature has argued that information frictions create private and social incentives for vertical integration by facilitating exchange or monitoring of information between the integrating firms (see, among others, Crocker 1983 and Riordan and Sappington 1987) or by concealing information from rival firms (see, Choi 1998). In contrast, our paper provides an argument why information frictions can create incentives for vertical separation. We focus on the information revealed or concealed *to consumers* rather than on discovery or revelation of information among firms.

Finally, we contribute to the literature initiated by Bonanno and Vickers (1988), Katz (1989) and Hart and Tirole (1990) on the strategic use of vertical contracts. That literature showed, among other things, that observable contracts with downstream firms create a strategic advantage in the presence of market competition. In contrast, we highlight the strategic advantage of keeping vertical contracts secret within a supply chain irrespective of the interaction with competitors.[4]

The rest of the paper is organized as follows. Section 2 outlines the basic framework and the market outcome when the manufacturer sells directly to consumers. Section 3 contains our main result and in particular, the pooling outcomes that result when the manufacturer sells through a retailer with secret vertical pricing. Section 4 analyzes the outcome when vertical contracts are observable. Section 5 discusses an extension where we allow a fraction of consumers to be informed about quality. Section 6 concludes. Appendix A contains a precise definition of an extended belief refinement criterion. Appendix B contains proofs of all results.

2 Basic Framework

The basic framework is adopted from Bagwell and Riordan (1991) who analyzed price signaling of product quality by a monopolist. The monopolist, who we shall henceforth

[4]Fershtman and Kalai (1997) and Ok and Kockesen (2004), among others, study the effect of strategic delegation with unobservable contracts in games of perfect information.

refer to as the manufacturer, produces a good whose quality can be either high (H) or low (L). The unit cost of production is constant and depends only on the quality of the good; in particular, high quality has a unit cost of $c > 0$ while the cost of low quality is normalized to zero. There is a unit mass of consumers. All consumers have unit demand. They have identical valuation $v_L > 0$ for low quality, while their valuation of high quality is uniformly distributed on $[v_L, 1 + v_L]$. Thus, if the consumers face a price p and assign probability μ to high quality, then the quantity demanded $d(p, \mu)$ is given by:

$$\begin{aligned} d(p,\mu) &= 0, \text{ if } p \geq \mu + v_L \\ &= 1 - \frac{p - v_L}{\mu}, \text{ if } p \in [v_L, \mu + v_L] \\ &= 1, \text{ if } p \leq v_L \end{aligned} \tag{1}$$

The prior probability that quality is high is common knowledge and denoted by $\alpha \in (0, 1)$. The realized quality of the good is observed only by the manufacturer. The manufacturer maximizes expected profit and each consumer maximizes her expected net surplus.

Our focus is on markets where in a signaling equilibrium a high quality firm has to distort its price relative to the full information outcome. This is the case if

$$v_L + c < 1. \tag{2}$$

Bagwell and Riordan (1991) fully characterize the equilibria when the manufacturer sells directly to buyers. In particular, the manufacturer sets a price p after observing the true (realized) product quality; buyers use this price to update their belief and make their purchase decision. There is a unique perfect Bayesian equilibrium (hereafter, PBE) outcome that can be supported by beliefs that satisfy the Intuitive Criterion (hereafter, IC). It is the least distortionary of all separating PBE outcomes.[5] In particular, under restriction (2), the high quality manufacturer charges a price $p_H^I = 1$ that exceeds his full information optimal price and earns profit equal to $v_L(1 - c)$, while the low quality manufacturer charges his full information optimal price $p_L^I = v_L$ (which is also his profit) and is indifferent between charging this price and imitating the high quality price. Thus,

[5] Note that while there are pooling outcomes that can be sustained as PBE, they are eliminated once beliefs are restricted to satisfy the IC. Pooling equilibria that satisfy the IC may exist if a significant proportion of buyers observe the realized product quality. In our framework, all buyers are ex ante uninformed.

the ex ante expected equilibrium profit of the manufacturer is given by

$$\pi^I = \upsilon_L(1 - \alpha c).$$

We shall refer to this as the *direct selling* outcome. Note that from the manufacturer's perspective, the signaling distortion is relatively large if c is small, α is large and υ_L is small. Pooling equilibria cannot be sustained with beliefs satisfying IC as after observing a deviation to a sufficiently high price p, buyers would infer that only the high quality type (with higher marginal cost) could possibly gain from this deviation and IC then suggests that the out-of-equilibrium belief $\mu(p)$ should equal 1 which would in turn make it gainful for the high quality type to deviate.

In subsequent sections, we analyze the consequences of the manufacturer selling exclusively through an intermediary retailer. The retailer has no specific expertise. For our analysis, it is irrelevant whether the retailer knows the quality of the good provided by the manufacturer; we make no assumption in this respect. The only cost incurred by the retailer is what he pays the manufacturer for the good; his payoff is his expected profit net of this payment. We assume that the retailer's outside option is zero. The manufacturer sets a linear wholesale price w at which it sells to the retailer. It is observed by the retailer before setting the retail price p at which it sells to consumers.[6]

As it takes time to set up a retail distribution channel, we view the decision whether or not to delegate to a retailer as a long-term commitment; a manufacturer that delegates to a retailer no longer has a distribution network to sell directly to consumers at a later stage. The decision whether or not to delegate is evaluated by comparing the ex ante pay-offs to the manufacturer. We will consider two variations of the market where the manufacturer sells through a retailer: one in which the wholesale price (or, the upstream contract) is secret, i.e., observed only by the retailer and not by the consumers, and the other where it is also observed by consumers.

3 Selling through a Retailer: Secret Wholesale Pricing

Consider first the situation where the manufacturer sells his product to the retailer at a wholesale price w that is not observable by the consumers. Consumers observe only the

[6] We indicate later that our results continue to hold if the manufacturer uses a two-part tariff pricing scheme.

retail price p and update their beliefs about product quality (i.e., the manufacturer's type) on that basis; we use $\mu(p)$ to denote the updated probability of high quality when consumers observe retail price p.The strategy of the manufacturer is specified by a wholesale price for each quality (type), while the strategy of the retailer is a function $p(w)$ that specifies a retail price for every possible wholesale price.

We begin by characterizing a class of *pooling* perfect Bayesian equilibria (PBE) that can yield higher expected profit for the manufacturer compared to the vertically integrated outcome. In these pooling outcomes, the manufacturer sets a wholesale price w^* regardless of product quality and the retailer follows up by selling at a retail price $p^* = w^*$, i.e., the retailer is fully squeezed and there is no double marginalization; after observing the retail price p^*, buyer's updated belief is identical to their prior belief, i.e., $\mu(p^*) = \alpha$, while the manufacturer's sells a quantity $d(p^*, \alpha)$. We confine attention to outcomes where $p^* = w^* < \alpha + \upsilon_L$ so that the manufacturer sells a strictly positive quantity. Further,

$$p^* = w^* \geq \max\{\upsilon_L, c\} \tag{3}$$

as $d(p, \mu) = 1$ for all $p < \upsilon_L$ and $\mu \in [0, 1]$ so that the retailer will always want to deviate if p^* were smaller than υ_L,[7] while the high quality manufacturer would not agree to set set $w^* < c$. We will focus on the set of such pooling outcomes that can be sustained by pessimistic out-of-equilibrium beliefs:

$$\mu(p) = 0 \text{ for all } p \neq p^*. \tag{4}$$

Consider $w^* = p^* \in [\upsilon_L, \alpha + \upsilon_L]$. Given beliefs (4), the retailer cannot sell at any retail price $p > w \geq w^*$ so that it is optimal for the retailer to set $p = w$ for all such w. If $\upsilon_L < w < w^*$, then given buyers' beliefs, the only retail price $p > w$ at which the retailer can sell a strictly positive quantity is $p = p^*$ and so that is the unique optimal retail price for such w. If $w \leq \upsilon_L$, the retailer has effectively two choices: p^* and υ_L. It is easy to check that the former is more profitable if, and only if, $w \geq p^* - \alpha$ and $p^* \geq \alpha$. Thus, we

[7] Note that (3) ensures that $p^* = w^* > c$ so that both types of the manufacturer earn strictly positive profit.

can write the optimal strategy for the retailer as

$$\begin{aligned} p(w) &= w, \text{ if } w \geq p^* \\ &= p^*, \text{ if } p^* \geq w > \max\{p^* - \alpha, 0\} \\ &= \upsilon_L, \text{ if } 0 \leq w \leq p^* - \alpha. \end{aligned} \tag{5}$$

Given beliefs (4) and the retailer's strategy (5), consider the manufacturer's incentive to deviate from setting w^*. If he sets $w > w^*$, he sells nothing. Reducing the wholesale price to $w \in (\max\{p^* - \alpha, 0\}, p^*)$ leads to the same retail price as $w = w^*$ and therefore is not gainful. The manufacturer may consider deviating to $0 \leq w \leq \max\{p^* - \alpha, 0\}$ as it reduces the retail price to υ_L and increases the quantity sold to 1. Obviously, the best deviation here is to set $p^* - \alpha$. If such a deviation is not profitable for the low quality manufacturer, it is certainly not profitable for the high quality manufacturer. Further, this deviation is not profitable for the low quality manufacturer if, and only if,

$$p^* \leq \frac{\upsilon_L + \sqrt{(\upsilon_L)^2 + \alpha^2}}{2}.$$

To sum up, neither type has an incentive to deviate from the proposed equilibrium if, and only if,

$$p^* \leq \max\left\{\alpha, \frac{\upsilon_L + \sqrt{(\upsilon_L)^2 + \alpha^2}}{2}\right\} = \overline{p}. \tag{6}$$

It is easy to see that the pooling PBE outcome described above can also be sustained as a PBE in a market where the manufacturer sells directly. In that setting, however, the beliefs needed to sustain pooling do not satisfy IC. We will now argue that similar considerations do not hold in the current setting and, in fact, the equilibrium construction above satisfies a reasonable extension of the IC to this setting.

When wholesale prices are secret, consumers only observe the retail price and can only infer there is a deviation from this price. Thus, in the pooling PBE described above, if buyers observe a price $p > p^*$, they cannot rule out the possibility that this is simply a unilateral opportunistic deviation by the retailer hoping some buyers with optimistic beliefs would buy. It is irrelevant whether the retailer is informed or uninformed about product quality, as having a cost that is independent of product quality means that he cannot signal quality even if he knows. If buyers attribute a deviation to a unilateral deviation by the retailer, there is no way to invoke a criterion like IC or any similar belief refinement to impose restrictions on out-of-equilibrium beliefs.

What about out-of-equilibrium beliefs at a retail price $p < p^*$? This cannot be attributed to a unilateral deviation by the retailer and therefore the buyers must consider the incentive of the manufacturer to deviate to some w that motivates the retailer to deviate to p. Obviously, only $w \leq p$ could motivate such a deviation by the retailer. It is easy to check that if the high quality manufacturer with higher marginal cost weakly gains by reducing his wholesale price from w^* to $w \leq p < p^*$ for some belief of buyers μ' (after observing retail price p), *i.e.*, if

$$(w - c)d(p, \mu') \geq (p^* - c)d(p^*, \alpha),$$

then the low quality manufacturer must strictly gain from this deviation, *i.e.*,

$$wd(p, \mu') > p^* d(p^*, \alpha)$$

and so an Intuitive Criterion like reasoning (or even other stronger criteria like D1) would suggest that it is perfectly reasonable for buyers to hold the belief $\mu(p) = 0$ at $p < p^*$. We conclude that unlike the case of direct selling, the pessimistic beliefs underlying the pooling PBE outcomes described in this section are robust to the reasoning underlying refinement notions like IC.

The vertically separated structure with secret wholesale prices is not a standard signaling game as the price (or signal) chosen by the manufacturer is not observed by the final buyers (the receivers), while buyers only observe the action of the retailer. Therefore, criteria like IC cannot be directly applied here without careful modification. In Appendix A we outline a modified criterion that extends the reasoning behind IC to the specific game analyzed in this section; we refer to this as the extended IC. Using arguments outlined above, it is easy to check that the pooling PBE described satisfy the extended IC.

Proposition 1 *Suppose the manufacturer sells through a retailer with secret wholesale pricing. For every $p^* \in [\upsilon_L, \overline{p}]$ there exists a pooling PBE with $w^* = p^*$ that satisfies the extended IC. The ex ante expected profit of the manufacturer in such a pooling outcome is given by:*

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - \alpha c). \tag{7}$$

Thus, there exists a continuum of pooling equilibria where the retailer is fully squeezed. As the manufacturer is already able to fully extract the retailer's rent, it is easy to show that these pooling outcomes can also be sustained as equilibria satisfying an appropriately extended version of IC if the manufacturer uses a nonlinear pricing scheme such as two-part tariff.

We now outline sufficient conditions under which the manufacturer makes more profit when selling through a retailer with secret wholesale pricing and whether consumer surplus could also be higher. In particular, we identify a pooling price

$$p_S^* = \frac{\upsilon_L + \alpha(1+c)}{2}$$

that maximizes (7). The sufficient conditions that are stated in the proposition below guarantee that *(i)* $p_S^* \geq c$, *(ii)* $p_S^* \in [\upsilon_L, \overline{p}]$, the range of poling prices identified in Proposition 1, and *(iii)* the ex ante expected profit in the pooling outcome with $p^* = p_S^*$ is larger than that under direct selling.

Proposition 2 *Suppose the manufacturer sells through a retailer with secret wholesale pricing. If*

$$c \leq \max\left\{\frac{\upsilon_L + \alpha}{2-\alpha}, \upsilon_L\right\} \tag{8}$$

and

$$\frac{\upsilon_L}{\alpha} < [(1 + c(1-2\alpha)) - 2\sqrt{(1-\alpha)c(1-\alpha c)}], \tag{9}$$

then there are pooling equilibria satisfying the extended IC that generate higher ex ante expected profit for the manufacturer and higher expected consumer surplus (and therefore, higher social surplus) than in the direct selling outcome.

Observe that (8) always holds if α is large or c is small. Further, (9) is likely to hold if υ_L or c is small, or if α is large. In particular, as α approaches 1 both (8) and (9) hold (using assumption (2)). When c approaches 0, both (8) and (9) hold if $\upsilon_L < \alpha$. As indicated in the previous section, low values of υ_L and c imply that the signaling distortion is high when the manufacturer sells directly; further, when α is large, the ex ante surplus puts a higher weight on the high quality state which is where the price is distorted under direct selling. The pooling outcome generated when the manufacturer sells through a retailer with secret upstream pricing can avoid much of this signaling distortion in the high quality state. As a result, even though the high quality manufacturer faces lower demand by pooling, it can be more profitable than selling directly provided the distortion due to double marginalization can be kept to a minimal level. Keeping product quality hidden for consumers by preventing the retailer from signaling quality by setting a wholesale price that is independent of quality helps prevent double marginalization. Surprisingly, the pooling equilibria that generate higher ex ante expected profit for the manufacturer also benefit consumers through lower prices on average, though they remain uninformed about product quality before purchase.

4 Selling through a Retailer: Observable Wholesale Pricing

A key feature underlying the analysis of the previous section is that wholesale pricing is secret. In this section, we show how the consequence of selling through a retailer is affected if consumers are able to observe wholesale pricing in addition to the retail price. We show that with observable wholesale pricing, any equilibrium that satisfies reasonable restrictions on consumers' out-of-equilibrium beliefs yields less profit for the manufacturer than direct selling. Further, this holds even if the manufacturer can use two-part tariffs to extract rent from the retailer.

In particular, consider the situation where the manufacturer sells through a retailer and sets a two-part tariff that is directly observed by both the retailer and the consumers; let w_τ be the unit wholesale price and F_τ the fixed fee charged by manufacturer of type $\tau, \tau \in \{H, L\}$. Our first result shows that if the manufacturer uses upstream pricing to signal its product quality to consumers, his expected profit is lower than that earned when he sells directly to consumers. As consumers can infer quality from wholesale prices, the retailer can mark-up the retail price without affecting consumers' beliefs about quality. This leads to a distortion due to double marginalization and an excessively high retail price for the high quality good. This argument is independent of out-of-equilibrium beliefs.

Proposition 3 *When the manufacturer sells through a retailer with observable two-part tariff wholesale contracts, his ex ante expected profit in any separating perfect Bayesian equilibrium is lower than in the direct selling outcome.*

Next, we investigate whether pooling equilibria can increase profits. In such equilibria the manufacturer sets a per unit wholesale price w^* and a fixed fee F^* regardless of his product quality, and the retailer's equilibrium strategy is $p_R^*(w)$ where $p^* = p_R^*(w^*)$. Now, consider the incentive of the manufacturer to deviate to some $(\widehat{w}, \widehat{F}) \neq (w^*, F^*)$, which is now observed by consumers. Importantly, the retailer's belief about the quality provided by the manufacturer is not pay-off relevant for either the manufacturer or the retailer.

What matters is consumer demand $d(p, \mu(\widehat{w}, \widehat{F}))$, which depends on consumer beliefs and on the price set by the retailer, which in turn depends on $(\widehat{w}, \widehat{F})$ and on the *second-order belief* of the retailer what consumers believe about product quality. Note that while in any PBE, (both on and off-the-equilibrium path) the second-order beliefs of the retailer must necessarily coincide with consumers' first-order beliefs as specified in the equilibrium, the difficulty arises when we want to determine the reasonableness of out-of-equilibrium beliefs by looking at the relative incentive of different types of the manufacturer to choose an out-of-equilibrium wholesale price.

Whether or not the deviation is profitable depends on (i) the criteria restricting consumer beliefs and (ii) on the relation between consumer beliefs and the retailer's *second-order belief* about consumer beliefs. Obviously, the more optimistic consumers are about product quality and the less optimistic the retailer believes the consumer is, the more incentives the manufacturer has to deviate. Below, we will outline two different approaches to refinement of beliefs; under both approaches, delegation with observable contracts creates lower profits than direct selling. The first approach extends the weaker Intuitive Criterion, but imposes the requirement that first- and second-order beliefs are coordinated, i.e., the retailer holds a correct belief about the beliefs of consumers. The second approach extends the strict D1 criterion to the current setting where there is an intermediary between Sender (producer) and Receiver (consumer) without imposing restrictions on the relation between consumer beliefs and the retailer's *second-order belief.*

We first consider the extended Intuitive Criterion, which we call the IC+ criterion, where first- and second-order beliefs coincide. To this end, suppose the manufacturer deviate from the equilibrium contract and sets $(\widehat{w}, \widehat{F}) \neq (w^*, F^*)$. Coordinated beliefs imply that if, after observing this deviation, consumers believe with probability μ that the manufacturer sells high quality, then the retailer also believes that consumers have belief μ. Therefore, the retailer sets his retail price assuming that the demand is $d(p, \mu)$ at any retail price p; the optimal response of the retailer, denoted by $p(\widehat{w}, \mu)$, is then given by:

$$p(\widehat{w}, \mu) = \arg\max_{p \geq \widehat{w}} [(p - \widehat{w}) d(p, \mu)]$$

Following the standard approach set by the Intuitive Criterion, IC+ requires that if for some $\tau \in \{H, L\}$

$$(w^* - c_\tau) d(p^*, \alpha) + F^* \geq (\widehat{w} - c_\tau) d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for all } \mu \in [0, 1],$$

while for $\tau' \in \{H, L\}, \tau' \neq \tau$

$$(w^* - c_{\tau'})d(p^*, \alpha) + F^* < (\widehat{w} - c_{\tau'})d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for some } \mu \in [0, 1],$$

then the out of equilibrium belief $\mu(\widehat{w}, \widehat{F})$ should be such that consumers believe that the manufacturer of type τ' has deviated with probability one. If, given such a belief, at least one type of manufacturer has an incentive to deviate, then we will say that the pooling equilibrium does not satisfy IC+.

From Proposition 3, we know that there is no separating perfect Bayesian equilibrium that generates higher ex ante expected profit for the manufacturer as under direct selling. The next proposition argues that this is in fact true for all equilibria satisfying IC+. The proof argues that for any pooling equilibrium where the pooling pooling two-part tariff (w^*, F^*) is such that $w^* > \upsilon_L$ one can find deviations $(\widehat{w}, \widehat{F})$ such that, using IC+, consumers have to believe that they come from high quality manufacturers, making these deviations profitable. This part of the argument is similar to the argument used by Bagwell and Riordan (1991) to eliminate pooling equilibria under direct selling; in our setting the manufacturer also has to take into account double marginalization by the retailer. If $\widehat{w}$ is sufficiently large, low quality manufacturers would never have an incentive to deviate, while due to higher production cost one can still find wholesale prices in this range (and appropriately chosen fixed fees) that may be profitable for the high quality manufacturer (if beliefs of consumers and the retailer are coordinated). For $\upsilon_L - \alpha < w^* < \upsilon_L$, there are deviations to $\widehat{w} < w^*$ (and appropriately chosen levels of fixed fees) that have to be attributed to low quality manufacturers, again making these deviations profitable for the low type. Thus, a pooling equilibrium satisfying IC+ with $w^* > \upsilon_L - \alpha$ does not exist. If pooling equilibria exist for $w^* \leq \upsilon_L - \alpha$, the manufacturer's profit cannot be larger than under direct selling.

Proposition 4 *When the manufacturer sells through a retailer with observable two-part tariff upstream pricing, his ex ante expected profit in any equilibrium satisfying IC+ is lower than in the direct selling outcome.*

Next, we show how one can apply D1 reasoning to the current setting without imposing any restriction on the relation between consumer beliefs and the retailer's *second-order belief* to demonstrate that with observable wholesale contracts, delegating retail pricing decisions to an independent third party cannot create higher profits than direct selling in reasonable pooling outcomes.

In a standard signaling game, the D1 criterion considers the set of possible responses of the receiver for which a particular deviation by any type of sender is profitable. Here, we look at the set of possible demands for the manufacturer's product for which the deviation may be profitable, where, as mentioned above, the demand depends on the action of an intermediary receiver (retailer) and the final receiver (consumer beliefs). Thus, for any pooling two-part tariff (w^*, F^*) where F^* represents the fixed fee that the manufacturer charges in equilibrium, let $Q(\widehat{w}, \widehat{F})$ be the set of quantities that can be sold through undominated actions of the retailer and the consumers for all possible consumer beliefs:

$$Q(\widehat{w}, \widehat{F}) = \{q \geq 0 : q \leq d(p, 1) \text{ for some } p \geq \widehat{w} \text{ where } (p - \widehat{w})q \geq \widehat{F}\}$$

and let $Q^H(\widehat{w}, \widehat{F})$ and $Q^L(\widehat{w}, \widehat{F})$ be the subsets of quantities in $Q(\widehat{w}, \widehat{F})$ for which the high and the low type manufacturer, respectively, find it strictly gainful to deviate to $(\widehat{w}, \widehat{F})$. We interpret the D1 criterion in our context as requiring that $\mu(\widehat{w}, \widehat{F}) = 1$ (respectively, 0) if $Q^H(\widehat{w}, \widehat{F}) \supset Q^L(\widehat{w}, \widehat{F})$ (respectively, $Q^H(\widehat{w}, \widehat{F}) \subset Q^L(\widehat{w}, \widehat{F})$) and $Q^H(\widehat{w}, \widehat{F}) \neq Q^L(\widehat{w}, \widehat{F})$.

In the proof of the proposition below we show that if $w^* > \upsilon_L$ we can choose $\widehat{w} = w^* + \epsilon$ for $\epsilon > 0$ small enough such that $Q^H(\widehat{w}, F^*) \supset Q^L(\widehat{w}, F^*)$ and $Q^H(\widehat{w}, F^*) \neq Q^L(\widehat{w}, F^*)$ so that D1 implies $\mu(\widehat{w}, F^*) = 1$. This, in turn, implies that the equilibrium strategy of the retailer must specify that $p_R^*(\widehat{w}, F^*) = \frac{1+\upsilon_L+\widehat{w}}{2}$ so that the deviation is strictly gainful for the high type manufacturer. By considering a deviation by the manufacturer to $\widehat{w} = w^* - \epsilon$ for $\epsilon > 0$ small enough, a similar argument shows that there cannot be a pooling equilibrium satisfying D1 with $w^* \in (\upsilon_L - \alpha, \upsilon_L]$. Finally, a pooling outcome with $w^* \leq \upsilon_L - \alpha$ can only yield lower expected profit than direct selling. Combining all this with Proposition 3, we have the following

Proposition 5 *When the manufacturer sells through a retailer with observable two-part tariff wholesale contracts, his ex ante expected profit in any D1 equilibrium outcome is lower than in the direct selling outcome.*

5 Informed Consumers

In this section, we extend the basic model with unobservable wholesale pricing discussed in Section 3 to allow for some informed consumers. We show that our qualitative result that delegation yields more profit to the manufacturer and increases consumer surplus continues to hold. With a positive fraction of informed consumers, pooling outcomes that generate higher expected profit than direct selling are associated with strictly positive retail margin and retailer's profit.

In their analysis of the market where the seller with private information sells directly to consumers, Bagwell and Riordan (1991) allow for a positive fraction of informed consumers. Their analysis shows that the larger the fraction of informed consumers, the less the high quality seller has to distort its prices to have a fully revealing equilibrium. If the fraction of informed consumers is large enough, the high quality seller does not need to distort its price at all as low quality sellers cannot deceive many buyers by imitating that price.

If some consumers are informed about product quality, then it would be natural to assume that the retailer is also informed. We analyze whether a pooling equilibrium continues to exist if a fraction of consumers and the retailer are informed about product quality. We denote the fraction of informed consumers by $\lambda \in (0, 1)$ and consider two cases: one where λ is small and one where it is large.

If λ is small, we will construct pooling equilibria that are close to the ones we constructed in Section 3 for markets where all consumers are uninformed. With a positive fraction of consumers being informed, the retailer should make some profit in any equilibrium. Otherwise, a high quality retailer will have an incentive to deviate knowing that some consumers will still be willing to buy knowing that it sells high quality. Thus, unlike the pooling outcomes in Section 3, the retailer has a strictly positive markup.

This has the following implications for a pooling equilibrium. First, if consumers would believe that quality is low for all prices off-the-equilibrium path, then the low quality manufacturer would have an incentive to squeeze the retailer and increase the wholesale price. Thus, in order for the manufacturer not to have an incentive to do so, retailers should react to wholesale price deviations and there should be at least one price off-the-equilibrium path where consumers think quality may be high. Second, the extended intuitive criterion described in Appendix A imposes more restrictions when some consumers are informed. As retailers will make positive profits in equilibrium, they do not have an incentive to deviate to very high retail prices even if consumers believe quality is high after such deviation. Therefore, consumers cannot attribute such deviation to a unilateral deviation by the retailer. The question then is whether the high quality manufacturer will have an incentive to deviate to high wholesale prices that can induce the retailer to deviate to high retail prices and whether uninformed consumers should then infer that a deviation by a high quality manufacturer must have occurred.

The next proposition shows that if λ is small enough, we can construct pooling equilibria satisfying the extended intuitive criterion (as defined in Appendix A) and that these equilibria may yield higher ex ante expected profit for the manufacturer as well as higher expected consumer surplus.

Proposition 6 *If the fraction of informed consumers λ is small, then (i) there exists a continuum of pooling equilibria satisfying the extended intuitive criterion, and (ii) under similar parametric restrictions as in Proposition 2 ,some of these equilibria generate higher ex ante expected profit for the manufacturer and higher expected consumer surplus (and therefore, higher social surplus) than in the direct selling outcome.*

Finally, we argue that if almost all consumers are informed, pooling equilibria do not exist. Thus, asymmetric information is essential for the argument to hold. The argument is relatively simple. First, in a candidate equilibrium a retailer knowing quality is high will charge a price close to the retail monopoly price for that quality, while the low quality retailer will not want to charge such a price as he will not sell much (given that most consumers believe quality is low). But given that double marginalization anyway arises and the high quality manufacturer does not have to distort its price much as price anyway does not signal quality to most consumers, it is optimal for the manufacturer to choose quality-dependent prices.

Proposition 7 *If the fraction of informed consumers λ is large, then a pooling equilibrium does not exist.*

This last proposition points at an interesting externality that informed consumers impose on the other consumers. Individually, consumers are better off being informed as this may prevent purchasing low quality at a price above their true valuation. However as indicated above, if too many consumers become informed, a pooling equilibrium does not exist and the market is necessarily characterized by a separating outcome where consumer surplus is fully extracted by the manufacturer in the low quality state and in the high quality state, double marginalization results in much higher prices than in a pooling equilibrium. It follows that as a group, consumers may be better off if they are uninformed.

6 Discussion and Conclusion

In this paper we have argued that when consumers are uninformed about product quality, a manufacturer with private information can increase his expected profit and at the same time, increase consumer and social welfare, by delegating the task of setting the price faced by consumers to an intermediary retailer. By delegating and not imposing vertical control, while withholding information about the wholesale pricing contract between manufacturer and retailer, the manufacturer can prevent signaling distortions. We have also shown that the argument extends if a fraction of consumers is informed about product quality and that in that case, pooling outcomes that generate higher expected profit than direct selling are associated with strictly positive retail margin and retailer's profit. Interestingly, by increasing the retail margin, an increase in the fraction of informed consumers may leave all consumers worse off.

Our analysis points to a class of intermediated signaling games that is of clear economic interest but has not been studied extensively, namely games where the sender chooses an action that is not directly pay-off relevant to the final receiver but that potentially influences the behavior of the intermediate receiver. Our analysis indicates the different implications that arise depending on whether or not the sender's action is observed by the final receiver. Future research directed to understanding the general nature of this type of three-player interaction and their economic implications will be useful.

Appendix

Appendix A: Extended Intuitive Criterion

In this appendix we present a modified version of Intuitive Criterion for the game where manufacturer sells through a retailer and manufacturer's pricing is unobserved by consumers.

Let $c_H = c, c_L = 0$.

Consider any perfect Bayesian equilibrium where the manufacturer of type τ sets wholesale price $w^*_\tau, \tau = H, L$, the retailer's equilibrium strategy is $p(w)$ with $p(w^*_\tau) = p^*_\tau$ and let $\mu_\tau(p)$ be the (updated) belief that type is τ after observing retail price p.

Define

$$\underline{\pi}^*_R = \min_{\tau \in \{L,H\}} [(p^*_\tau - w^*_\tau) D(p^*_\tau, \mu(p^*_\tau))]$$

and

$$D_R = \{p : [p - \min\{w^*_L, w^*_H\}] D(p, 1) \leq \underline{\pi}^*_R\} .$$

Intuitively, D_R is the set of retail prices such that a retailer will not have an incentive to choose them. In the pooling equilibria we focus on in Section 3 of the main text D_R is the set of retail prices that is smaller than the equilibrium retail price. Fix any $\widehat{p} \in D_R$. Let

$$D_M(\widehat{p}) = \{w : [\widehat{p} - w] D(\widehat{p}, 1) \geq \underline{\pi}^*_R\} .$$

Thus, $D_M(\widehat{p})$ is the set of wholesale prices such that if one of them is set by the manufacturer, then the retailer may have an incentive to react by setting $\widehat{p}$.

We then require that if one type of manufacturer will not have an incentive to deviate to any wholesale price in $D_M(\widehat{p})$, i.e., for some $\tau \in \{H, L\}$

$$(\widetilde{w} - c_\tau) D(\widetilde{w}, 1) \leq (w^*_\tau - c_\tau) D(p^*_\tau, \mu(p^*_\tau)) \text{ for all } \widetilde{w} \in D_M(\widehat{p}),$$

while the other type may have an incentive to deviate to at least some prices in $D_M(\widehat{p})$, i.e., for $\tau' \in \{H, L\}, \tau' \neq \tau$

$$(w' - c_{\tau'}) D(w', 1) > (w^*_{\tau'} - c_{\tau'}) D(p^*_{\tau'}, \mu(p^*_{\tau'})) \text{ for some } w' \in D_M(\widehat{p})$$

then consumers should believe that with probability one it is the latter type of manufacturer that has deviated, i.e.,

$$\mu_\tau(\widehat{p}) = 0 = 1 - \mu_{\tau'}(\widehat{p}).$$

Appendix B: Proofs

Proof of Proposition 2. The expected profit the manufacturer may in a pooling equilibrium with retail price p^* as described in Proposition 1 exceeds that when selling directly if .

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - \alpha c) > \upsilon_L(1 - \alpha c) \tag{10}$$

The maximal expected profit across all such pooling outcome is when the pooling price $p^* = p_S^* = \frac{\upsilon_L + \alpha(1+c)}{2}$ provided $p_S^* \in [\upsilon_L, \overline{p}]$ and $p_S^* \geq c$. (10) holds if and only if

$$(\alpha(1 + c) - \upsilon_L)^2 > 4\alpha^2 c(1 - \upsilon_L),$$

which can be rewritten as

$$(\upsilon_L)^2 - 2\alpha(1 + c(1 - 2\alpha))\upsilon_L + \alpha^2(1 - c)^2 > 0, \tag{11}$$

which holds if

$$\upsilon_L < \alpha[(1 + c(1 - 2\alpha)) - 2\sqrt{(1 - \alpha)c(1 - \alpha c)}] \tag{12}$$

Observe that

$$1 + c(1 - 2\alpha)) - 2\sqrt{(1 - \alpha)c(1 - \alpha c)} < 1 - c$$

Thus, if (12) holds, $\upsilon_L < \alpha(1-c)$ so that $p_S^* = \frac{\upsilon_L+\alpha(1+c)}{2} < \alpha \leq \overline{p}$ and further, $\upsilon_L < \alpha(1-c)$ $< \alpha(1 + c)$ ensures $p_S^* > \upsilon_L$. If $c \leq \upsilon_L, p_S^* > \upsilon_L$ implies $p_S^* > c$. On the other hand, if $c > \upsilon_L$, (8) implies $c < \frac{\upsilon_L+\alpha}{2-\alpha}$ which in turn implies that $p_S^* = \frac{\upsilon_L+\alpha(1+c)}{2} \geq c$. Note that (12) is identical to (9). Thus, (8) and (9) ensure that (10) holds for $p^* = p_S^*$, $p_S^* \in (\upsilon_L, \overline{p}]$ and $p_S^* \geq c$.

When the manufacturer sells directly, the ex ante expected consumer surplus is given by $\frac{\alpha}{2}(\upsilon_L)^2$. When selling through a retailer with secret pricing, in the pooling equilibrium where $p^* = p_S^* = \frac{\upsilon_L + \alpha(1+c)}{2}$, the ex ante expected consumer surplus is given by

$$\begin{aligned}\frac{1}{2\alpha}(\alpha + \upsilon_L - p_S^*)^2 &= \frac{1}{8\alpha}(\alpha(1-c) + \upsilon_L)^2 \\ &> \frac{\alpha}{2}(\upsilon_L)^2\end{aligned}$$

if $2\alpha\upsilon_L < \alpha(1-c) + \upsilon_L$ i.e., $\upsilon_L(2 - \frac{1}{\alpha}) + c < 1$ which always holds.

Proof of Proposition 3. Consider a separating perfect Bayesian equilibrium where the low and high type manufacturers set distinct two part tariffs (w_L, F_L) and (w_H, F_H) where $w_L \leq \upsilon_L$ and $w_H > w_L$. Given that $d(p, 0) = 1$ for all $p \leq \upsilon_L$, it is clear that the retailer's optimal strategy must be such that $p_R(w_L, F_L) = \upsilon_L$ and $p_R(w_H, F_H) = (1 + \upsilon_L + w_H)/2$. The condition that the low quality manufacturer should not have an incentive to imitate the high quality type is:

$$\frac{1}{2}w_H\left(1 - w_H + \upsilon_L\right) + F_H \leq w_L + F_L. \tag{13}$$

Suppose that the ex ante expected profit of the manufacturer in a separating equilibrium is at least as high as $\pi^I = \upsilon_L(1 - \alpha c)$. Then,

$$\begin{aligned}\upsilon_L(1-\alpha c) &\leq (1-\alpha)(w_L + F_L) + \alpha\left(\frac{1}{2}(w_H - c)\left(1 - w_H + \upsilon_L\right) + F_H\right) \\ &\leq w_L + F_L - \alpha c\frac{1}{2}\left(1 - w_H + \upsilon_L\right) \;\; \text{(using (13))} \\ &= \upsilon_L(1-\alpha c) + \alpha c\upsilon_L - \alpha c\frac{1}{2}\left(1 - w_H + \upsilon_L\right) \\ &= \upsilon_L(1-\alpha c) + \frac{\alpha c}{2}(w_H + \upsilon_L - 1),\end{aligned}$$

which can only hold if $w_H \geq 1 - \upsilon_L$. The equilibrium retail price if the manufacturer is of high type is then $p_H = p_R^*(w_H, F_H) = (1 + \upsilon_L + w_H)/2 > 1$; as $(p-c)(1 + \upsilon_L - p)$ is strictly decreasing in p for $p \geq 1$ (by assumption (2)), the total *industry* profit in this state of the world is $(p_H - c)(1 + \upsilon_L - p_H) \leq \upsilon_L(1-c)$. The ex ante industry profit can then not be larger than $\upsilon_L(1 - \alpha c) = \pi^I$, a contradiction.

Proof of Proposition 4. Given Proposition 3, it is sufficient to show that if there is a pooling equilibrium satisfying IC+, then it yields lower ex ante expected profit for the manufacturer than in the direct selling outcome. We begin with some useful facts. Suppose that the (coordinated) belief is that quality is high with probability μ. Then for any unit wholesale price $w \leq \mu + \upsilon_L$ the optimal price set by the retailer (if he accepts the contract) is

$$\begin{aligned} p(w,\mu) &= \frac{\mu + \upsilon_L + w}{2}, \text{ if } w \geq \upsilon_L - \mu \\ &= \upsilon_L, \text{ if } w < \upsilon_L - \mu. \end{aligned}$$

For $w > \mu + \upsilon_L$, the retailer sells zero at any $p \geq w$ and so $p(w,\mu)$ is any price at least as large as w. The quantity sold by the retailer is then

$$\begin{aligned} d(p(w,\mu),\mu) &= \frac{\mu + \upsilon_L - w}{2\mu}, \text{if } w \in [\upsilon_L - \mu, \upsilon_L + \mu] \\ &= 1, \text{if } w \leq \upsilon_L - \mu \\ &= 0, \text{ if } w \geq \upsilon_L + \mu. \end{aligned}$$

Note that given $w \geq \upsilon_L$, $d(p(w,\mu),\mu)$ is non-decreasing in μ and for $w < \upsilon_L$, $d(p(w,\mu),\mu)$ is non-increasing in μ.

Proof. Consider a pooling equilibrium where the manufacturer sets two part tariff (w^*, F^*). Then,

$$\begin{aligned} c &\leq w^* \leq \alpha + \upsilon_L \\ F^* &\leq (p^* - w^*)d(p^*, \alpha) \end{aligned}$$

The retailer's equilibrium strategy $p_R(w,F)$ in such an outcome must be such that $p^* = p_R(w^*, F^*)$ is given by

$$\begin{aligned} p^* &= \frac{\alpha + \upsilon_L + w^*}{2}, \text{ if } w^* \in [\upsilon_L - \alpha, \upsilon_L + \alpha] \\ &= \upsilon_L, \text{ if } w^* < \upsilon_L - \alpha. \end{aligned}$$

Note that the equilibrium profits of the high and low type manufacturers would then be

$$\begin{aligned} \pi^H &= (w^* - c)d(p^*, \alpha) + F^* \\ &= \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha \\ &= (w^* - c) + F^*, \text{ if } w^* < \upsilon_L - \alpha, \end{aligned}$$

$$
\begin{aligned}
\pi^L &= w^* d(p^*, \alpha) + F^* \\
&= \frac{1}{2\alpha} w^*(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha \\
&= w^* + F^*, \text{if } w^* < \upsilon_L - \alpha.
\end{aligned}
$$

First, suppose that $w^* \geq \upsilon_L$. Consider any unit wholesale price $w \in (w^*, 1 + \upsilon_L)$ and an associated fixed fee $F(w)$

$$
F(w) = [p(w, 1) - w] d(p(w, 1), 1).
$$

The profit earned by the low type manufacturer by deviating to such $(w, F(w))$ when buyers' belief is $\mu = 1$ equals

$$
\begin{aligned}
g(w) &= p(w, 1) d(p(w, 1), 1) \\
&= \frac{1}{4}(1 + \upsilon_L + w)(1 + \upsilon_L - w).
\end{aligned}
$$

Note that $g(w)$ is continuous (and strictly decreasing) in w on $[\upsilon_L, 1 + \upsilon_L]$. Note that as $w \downarrow w^*$,

$$
\begin{aligned}
g(w) &\to p(w^*, 1) d(p(w^*, 1), 1) = \frac{1}{4}(1 + \upsilon_L + w^*) \left(1 - (w^* - \upsilon_L)\right) \\
&> \frac{1}{4}(\alpha + \upsilon_L + w^*) \left(1 - \frac{w^* - \upsilon_L}{\alpha}\right) \\
&= p^* d(p^*, \alpha) = w^* d(p^*, \alpha) + (p^* - w^*) d(p^*, \alpha) \\
&\geq \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^* = \pi^L
\end{aligned}
$$

On the other hand, as $w \uparrow (1 + \upsilon_L), g(w) \to 0$. Thus, there exists a unique $w_0 \in (w^*, 1 + \upsilon_L)$ such that

$$
g(w_0) = \pi^L. \tag{14}
$$

We now claim that the low type manufacturer can never strictly gain by deviating to the contract $(w_0, F(w_0))$ for any belief $\mu \in [0, 1]$. As noted above, $w_0 > \upsilon_L$ implies $d(p(w_0, \mu), \mu)$ is non-decreasing in μ. So, if the contract $(w_0, F(w_0))$ is feasible for belief μ

(i.e., the retailer makes non-negative profit) the low type manufacturer's deviation profit:

$$\begin{aligned}
& w_0 d(p(w_0,\mu),\mu) + F(w_0) \\
= \;& g(w_0) - w_0[d(p(w_0,1),1) - d(p(w_0,\mu),\mu)] \\
\leq \;& g(w_0) = \pi^L.
\end{aligned}$$

If the contract $(w_0, F(w_0))$ is not feasible for belief μ, the low type manufacturer makes zero profit. Thus, regardless of the beliefs of buyers, the low type manufacturer can never gain by deviating to a contract $(w_0, F(w_0))$. Note that (14) implies

$$p(w_0,1)d(p(w_0,1),1) = w^* d(p^*,\alpha) + F^* \leq p^* d(p^*,\alpha) = \frac{\alpha + \upsilon_L + w^*}{2} d(p^*,\alpha).$$

As $\alpha < 1, w_0 > w^*$

$$p(w_0,1) = \frac{1 + \upsilon_L + w_0}{2} > \frac{\alpha + \upsilon_L + w^*}{2}$$

it follows that

$$d(p(w_0,1),1) < d(p^*,\alpha). \tag{15}$$

If a high type manufacturer deviates to a contract $(w_0, F(w_0))$ and belief is $\mu = 1$ his deviation profit is:

$$\begin{aligned}
& (p(w_0,1) - c)d(p(w_0,1),1) \\
= \;& g(w_0) - cd(p(w_0,1),1) = \pi^L - c\, d(p(w_0,1),1) \\
= \;& w^* d(p^*,\alpha) + F^* - c\, d(p(w_0,1),1), \text{ using (14)} \\
= \;& (w^* - c)d(p^*,\alpha) + F^* + c[d(p^*,\alpha) - d(p(w_0,1),1)] \\
= \;& \pi^H + c[d(p^*,\alpha) - d(p(w_0,1),1)] \\
> \;& \pi^H, \text{ using (15).}
\end{aligned}$$

IC+ therefore requires that the out-of-equilibrium belief satisfies

$$\mu(w_0, F(w_0)) = 1,$$

which immediately implies that the high quality manufacturer has an incentive to deviate to $(w_0, F(w_0))$. Thus, there is no pooling equilibrium satisfying IC+ where the marginal wholesale price $w^* \geq \upsilon_L$

Next, suppose $w^* \in (\upsilon_L - \alpha, \upsilon_L)$. Note that when the manufacturer is of low type, the total industry profit in the pooling equilibrium (w^*, F^*) is given by

$$\begin{aligned} & \frac{1}{2\alpha}(p^* - w^*)(\alpha + \upsilon_L - w^*) + \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*) = \frac{1}{4\alpha}[(\alpha + \upsilon_L)^2 - (w^*)^2] \\ < & \frac{1}{4\alpha}[(\alpha + \upsilon_L)^2 - (\upsilon_L - \alpha)^2], \text{ as } w^* > \upsilon_L - \alpha \\ = & \upsilon_L \end{aligned}$$

so that

$$\frac{1}{2\alpha}(p^* - w^*)(\alpha + \upsilon_L - w^*) < \upsilon_L - \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*)$$

and as

$$F^* \leq \frac{1}{2\alpha}(p^* - w^*)(\alpha + \upsilon_L - w^*)$$

we have

$$F^* < \upsilon_L - \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*)$$

so that there exists $h > 0$ such that

$$F^* < \upsilon_L - \left(\frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*) + \epsilon\right) \text{ for all } \epsilon \in (0, h). \tag{16}$$

Now, consider a deviation by the manufacturer to a two-part tariff $(\widehat{w}, F^*)$ where $\widehat{w} < w^*$and in particular:

$$(w^* - c)[\frac{w^* - (\upsilon_L - \alpha)}{2\alpha}] \leq w^* - \widehat{w} < w^*[\frac{w^* - (\upsilon_L - \alpha)}{2\alpha}] - \epsilon_0 \tag{17}$$

for some $\epsilon_0 \in (0, h)$. Note that as $c > 0$, the left most expression in (17) must be strictly less than the right most expression for ϵ_0 small enough.

We claim that a high type manufacturer can never gain from such a deviation regardless of belief μ. Suppose to the contrary there exists belief $\mu' \in [0, 1]$ such that the deviation is strictly gainful for the high type manufacturer. Then, for such μ', the retailer makes non-negative profit and

$$\begin{aligned} & (\widehat{w} - c)d(p(\widehat{w}, \mu'), \mu') + F^* \\ > & \pi^H = \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^* \end{aligned}$$

i.e.,

$$(\widehat{w}-c)d(p(\widehat{w},\mu'),\mu') > \frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*)$$

and recalling that $\upsilon_L > w$ implies $d(p(w,\mu),\mu)$ is non-increasing in μ we have

$$(\widehat{w}-c)d(p(\widehat{w},0),0) > \frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*).$$

As $d(p(\widehat{w},0),0)=1$, we must have

$$\widehat{w} > \frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*)+c$$

which yields,

$$w^*-\widehat{w} < (w^*-c)\frac{w^*-(\upsilon_L-\alpha)}{2\alpha}$$

This contradicts the left inequality in (17). Thus, the high type manufacturer can never gain from the deviation regardless of belief.

On the other hand, a low type manufacturer strictly gains from this deviation if belief $\mu=0$ as the second inequality in (17) implies

$$\widehat{w} > \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*) \tag{18}$$

so that

$$\widehat{w}+F^* > \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)+F^* = \pi^L.$$

To verify that the contract $(\widehat{w},F^*)$ yields non-negative profit for the retailer when belief $\mu=0$ i.e., $F^* \leq \upsilon_L-\widehat{w}$, note that (16) and $\epsilon_0 \in (0,h)$ imply

$$\begin{aligned} F^* &< \upsilon_L - \left(\frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)+\epsilon_0\right) \\ &< \upsilon_L-\widehat{w}, \end{aligned}$$

using the second inequality in (17). Thus, there is no pooling equilibrium satisfying IC+ where $w^* \in (\upsilon_L-\alpha,\upsilon_L)$.

This only leaves possibility of pooling equilibria where $w^* \leq \upsilon_L-\alpha$. On the equilibrium path in such an equilibrium, the retailer sets price equal to υ_L (sells quantity equal to 1) and the manufacturer's ex ante expected profit is bounded above by the expected industry profit, $\upsilon_L-\alpha c$. The latter is (strictly) smaller than $\upsilon_L(1-\alpha c)$, the ex ante expected profit of the manufacturer under direct selling. This completes the proof. ■

Proof of Proposition 5. Given Proposition 3, it is sufficient to show that if there is a pooling equilibrium satisfying the D1 criterion, then it yields lower ex ante expected profit for the manufacturer than in the direct selling outcome.

We begin by showing that there is no pooling D1 equilibrium where the manufacturer sets a two part tariff (w^*, F^*) and the unit wholesale price $w^* > \upsilon_L - \alpha$. Suppose to the contrary there is such an equilibrium. Let $p_R(w, F)$ be the retailer's equilibrium strategy. Clearly,

$$w^* \leq p^* = p_R(w^*, F^*) = \frac{\alpha + \upsilon_L + w^*}{2} \leq \alpha + \upsilon_L$$

and

$$F^* \leq (p^* - w^*) d(p^*, \alpha)$$

where

$$d(p^*, \alpha) = \frac{1}{2\alpha}(\alpha + \upsilon_L - w^*)$$

Given any two part tariff $(\widehat{w}, \widehat{F})$ to which the manufacturer deviates, we have defined $Q(\widehat{w}, \widehat{F})$ to be the set of quantities that can be sold by the manufacturer for all possible undominated actions of the retailer and the consumers

$$Q(\widehat{w}, \widehat{F}) = \{q \geq 0 : q \leq d(p, 1) \text{ and } (p - \widehat{w})q \geq \widehat{F} \text{ for some } p \geq \widehat{w} \ \}$$

It can be checked that if $Q(\widehat{w}, \widehat{F})$ is non-empty, then it is an interval of the form $[\underline{q}(\widehat{w}, \widehat{F}), \overline{q}(\widehat{w}, \widehat{F})]$ where $0 \leq \underline{q}(\widehat{w}, \widehat{F}) \leq \overline{q}(\widehat{w}, \widehat{F}) \leq d(\widehat{w}, 1)$. We also defined $Q^\tau(\widehat{w}, \widehat{F}), \tau \in \{H, L\}$ to be the subset of $Q(\widehat{w}, \widehat{F})$ such that a type τ manufacturer finds it strictly gainful to deviate to $(\widehat{w}, \widehat{F})$ if, and only if, it can sell a quantity $q \in Q^\tau(\widehat{w}, \widehat{F})$. If $Q^\tau(\widehat{w}, \widehat{F})$ is non-empty then it is also an interval of the form $(q^\tau(\widehat{w}, \widehat{F}), \overline{q}(\widehat{w}, \widehat{F})]$ where $q^\tau(\widehat{w}, \widehat{F})$ makes the type τ manufacturer indifferent between deviating to $(\widehat{w}, \widehat{F})$ and sticking to his equilibrium strategy (w^*, F^*). The D1 criterion can then applied by simply comparing $q^\tau(\widehat{w}, \widehat{F})$ for $\tau = H$ and $\tau = L$.

First, consider $w^* > \upsilon_L$. Note that as $\alpha < 1$,

$$F^* \leq (p^* - w^*) d(p^*, \alpha) < (p^* - w^*) d(p^*, 1). \tag{19}$$

Choose $\epsilon > 0$ is small enough

$$F^* < ((p^* + \epsilon) - (w^* + \epsilon))d(p^* + \epsilon, 1) = (p^* - w^*)d(p^* + \epsilon, 1) \tag{20}$$

and

$$d(p^*, \alpha) < d(p^* + \epsilon, 1). \tag{21}$$

Suppose the manufacturer deviates to $(w^* + \epsilon, F^*)$ for $\epsilon > 0$ sufficiently small. Clearly, $Q(w^* + \epsilon, F^*)$ is non-empty as (20) implies $d(p^* + \epsilon, 1) \in Q(w^* + \epsilon, F^*)$. Using (21) and the fact that (19) implies

$$F^* \leq ((p^* + \epsilon) - (w^* + \epsilon))d(p^*, \alpha) \tag{22}$$

we have $d(p^*, \alpha) \in Q(w^* + \epsilon, F^*)$. Observe that the strict inequality in (20) implies there exists $p' > p^* + \epsilon$ such that

$$(p' - (w^* + \epsilon))d(p', 1) = F^* \tag{23}$$

so that $d(p', 1) \in Q(w^* + \epsilon, F^*)$. Comparing (23) with (22), we have $d(p', 1) < d(p^*, \alpha)$. Thus,

$$\underline{q}(\widehat{w}, \widehat{F}) \leq d(p', 1) < d(p^*, \alpha) < d(p^* + \epsilon, 1) \leq \overline{q}(\widehat{w}, \widehat{F}) \tag{24}$$

i.e., $d(p^*, \alpha)$ is in the interior of $Q(w^* + \epsilon, F^*)$.

The high type manufacturer strictly gains from the deviation to $(w^* + \epsilon, F^*)$ if, and only if,

$$q > \frac{1}{2\alpha} \frac{(w^* - c)(\alpha + \upsilon_L - w^*)}{(w^* + \epsilon - c)} = q^H(w^* + \epsilon, F^*), \tag{25}$$

while the low type manufacturer strictly gains if, and only if,

$$q > \frac{1}{2\alpha} \frac{w^*(\alpha + \upsilon_L - w^*)}{(w^* + \epsilon)} = q^L(w^* + \epsilon, F^*). \tag{26}$$

Note that for ϵ small enough, both $q^H(w^* + \epsilon, F^*)$ and $q^L(w^* + \epsilon, F^*)$ are sufficiently close to $d(p^*, \alpha)$ and using (24) we then have that $q^\tau(w^* + \epsilon, F^*) \in Q(w^* + \epsilon, F^*)$. Thus, $Q^\tau(w^* + \epsilon, F^*)$ is a non-empty subset of $Q(w^* + \epsilon, F^*)$ for $\tau \in \{H, L\}$. As $\frac{w^* - c}{w^* + \epsilon - c} < \frac{w^*}{w^* + \epsilon}$, we have $q^H(w^* + \epsilon, F^*) < q^L(w^* + \epsilon, F^*)$, i.e., $Q^L(w^* + \epsilon, F^*)$ is a strict subset of $Q^H(w^* + \epsilon, F^*)$. The D1 criterion then requires that $\mu(w^* + \epsilon, F^*) = 1$ which in turn implies that after deviation to $(w^*+\epsilon, F^*)$, the retailer's optimal price is $\frac{1+\upsilon_L+w^*+\epsilon}{2}$ so that the realized deviation profit of the high type manufacturer is

$$\begin{aligned} & \frac{1}{2}(w^*+\epsilon-c)(1+\upsilon_L-(w^*+\epsilon))+F^* \\ \rightarrow \quad & \frac{1}{2}(w^*-c)(1+\upsilon_L-w^*)+F^*, \end{aligned}$$

as $\epsilon \rightarrow 0$. As $w^* > \upsilon_L$,

$$\frac{1}{2}(w^*-c)(1+\upsilon_L-w^*)+F^* > \frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*)+F^*$$

so that the deviation by the high type manufacturer to $(w^*+\epsilon, F^*)$ is strictly gainful for ϵ small enough.

Next, consider $w^* \in (\upsilon_L-\alpha, \upsilon_L]$. Suppose the manufacturer deviates to $(w^*-\epsilon, F^*)$ where $\epsilon > 0$ is small enough such that $F^* < (p^*-w^*)d(p^*-\epsilon, 1)$. A symmetric argument to that outlined above can be used to show that the D1 criterion requires $\mu(w^*-\epsilon, F^*) = 0$ so that after a deviation by the manufacturer, the retailer sets the optimal retail price at υ_L. Note that as $w^* \in (\upsilon_L-\alpha, \upsilon_L]$,

$$w^*d(p^*,\alpha)+(p^*-w^*)d(p^*,\alpha) = p^*d(p^*,\alpha) = \frac{1}{2\alpha}[(\alpha+\upsilon_L)^2-(w^*)^2] < \upsilon_L$$

so that

$$F^* \leq (p^*-w^*)d(p^*,\alpha) < \upsilon_L - w^*d(p^*,\alpha) = \upsilon_L - \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*) \leq \upsilon_L - w^*,$$

which implies that for $\epsilon > 0$ sufficiently small

$$F^* < \upsilon_L - (w^*-\epsilon),$$

i.e., given belief $\mu(w^*-\epsilon, F^*) = 0$, the retailer earns nonnegative profit under the deviation tariff $(w^*-\epsilon, F^*)$ if ϵ is small enough. Using the fact that $w^* > \upsilon_L-\alpha$, the realized deviation profit of the low type manufacturer is $(w^*-\epsilon)+F^* > \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)+F^*$, the equilibrium profit of the low type manufacturer. Thus, there is no pooling equilibrium satisfying the D1 criterion where the manufacturer sets a two part tariff (w^*, F^*) and the unit wholesale price $w^* > \upsilon_L-\alpha$.

We are only left with the possibility of a pooling equilibrium (w^*, F^*) is one where

where $w^* \leq \upsilon_L - \alpha$. This implies that the equilibrium retail price $p_R(w^*, F^*) = \upsilon_L$ and the quantity sold is 1; the industry profits when the manufacturer is of high and low types are $\upsilon_L - c$ and υ_L, respectively. The ex ante expected industry profit (and therefore the expected profit of the manufacturer) in any pooling outcome cannot exceed $\upsilon_L - \alpha c < \pi^I = \upsilon_L(1 - \alpha c)$.

Proof of Proposition 6. We construct a pooling equilibrium where retailers make some profits, i.e., $p^*(w^*) > w^*$. In such an equilibrium the low quality retailer makes a profit of

$$(1-\lambda)\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - w^*)$$

and in an equilibrium it should be the case that this is larger than or equal to $(1 - \lambda)\left(1 - \frac{p - \upsilon_L}{\mu(p)}\right)(p - w^*)$ for all $p > w^*$. A high quality retailer makes a profit of

$$\left[\lambda\left(1 - p^* + \upsilon_L\right) + (1-\lambda)\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)\right](p^* - w^*)$$

and in an equilibrium this should be larger than or equal to

$$\left[\lambda\left(1 - p + \upsilon_L\right) + (1-\lambda)\left(1 - \frac{p - \upsilon_L}{\mu(p)}\right)\right](p - w^*)$$

for all $p > w^*$. Note that for any w and independent of $\mu(p)$ a high cost retailer can always guarantee himself a profit of $\lambda\left(1 + \upsilon_L - w\right)^2/4$ by setting a price equal to $\left(1 + \upsilon_L + w\right)/2$ so that p^* should be such that

$$\left[\lambda\left(1 - p^* + \upsilon_L\right) + (1-\lambda)\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)\right](p^* - w^*) \geq \lambda\frac{\left(1 + \upsilon_L - w^*\right)^2}{4}. \tag{27}$$

We will first argue that there is a range of prices p larger than the equilibrium retail price such that both types of retailers may have an incentive to deviate so that the extended IC implies that we can choose any out-of-equilibrium belief $\mu(p)$. The argument runs as follows. No matter what the belief $\mu(p)$ is, a low quality retailer will not have an incentive to deviate to prices p if

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - w^*) > \left(1 - p + \upsilon_L\right)(p - w^*). \tag{28}$$

Similarly, a high quality retailer will not have an incentive to set a price p if

$$\begin{aligned}
& \left[\lambda\left(1-p^{*}+\upsilon_{L}\right)+(1-\lambda)\left(1-\frac{p^{*}-\upsilon_{L}}{\alpha}\right)\right]\left(p^{*}-w^{*}\right) \\
> \; & \left[\lambda\left(1-p+\upsilon_{L}\right)+(1-\lambda)\left(1-p+\upsilon_{L}\right)\right](p-w^{*}),
\end{aligned} \tag{29}$$

which, taking (28) into account, will be the case if

$$\left(1-p^{*}+\upsilon_{L}\right)\left(p^{*}-w^{*}\right)>\left(1-p+\upsilon_{L}\right)\left(p-w^{*}\right).$$

As $(1-p^{*}+\upsilon_{L})>\left(1-\frac{p^{*}-\upsilon_{L}}{\alpha}\right)$ this inequality follows from (28).

(28) implicitly identifies two prices, $\underline{p}_L$ and $\overline{p}_L$, such that for all $\underline{p}_L<p<\overline{p}_L$ the low quality retailer may have an incentive to deviate. Similarly, (29) implicitly identifies two prices, $\underline{p}_H$ and $\overline{p}_H$, such that the high quality retailer may have an incentive to deviate for all $\underline{p}_H<p<\overline{p}_H$. From the discussion above, it is clear that $\underline{p}_L<\underline{p}_H<p^{*}<\overline{p}_H<\overline{p}_L$. (Interestingly, if the low quality retailer may have an incentive to deviate, the high quality retailer certainly has an incentive to deviate. This is of importance for our extended Intuitive Criterion as it implies that there are no retail prices that can only be accounted for by deviations of the high quality retailer.[8] Thus, for all p such that $p^{*}<p<\overline{p}_H$ both low and high cost retailers may have an incentive to deviate and for these prices our extended IC does not impose any restriction on the out-of-equilibrium beliefs, i.e., we can choose any $0\leq\mu(p)\leq 1$.

We now construct a price $\widehat{p}$ in this interval and a corresponding $0<\mu(\widehat{p})<1$ such that if the manufacturer sets the equilibrium wholesale price w^{*} both types of retailers are indifferent between setting this price and the equilibrium price p^{*}. This guarantees that if the manufacturer does deviate locally the retailers will react by setting $\widehat{p}$ making the local deviation by the manufacturer unprofitable. Thus, the price $\widehat{p}$ and the belief $\mu(\widehat{p})$ have to satisfy:

$$\left(1-\frac{p^{*}-\upsilon_{L}}{\alpha}\right)\left(p^{*}-w^{*}\right)=\left(1-\frac{\widehat{p}-\upsilon_{L}}{\mu(\widehat{p})}\right)\left(\widehat{p}-w^{*}\right)$$

[8] Intuitively, the reason is the following. First, in a pooling equilibrium, both types of retailers have the same cost. Cost reasons therefore do not distinguish the two types. Second, price has to be relatively high, and demand therefore relatively low, for a low cost retailer not to set that price even if consumers believe they can buy high quality at that price. High quality retailers suffer even more from such a high price deviation as they will sell in addition to a fraction of informed consumers, but these consumers are also price sensitive.

and

$$\left[\lambda\left(1-p^*+\upsilon_L\right)+(1-\lambda)\left(1-\frac{p^*-\upsilon_L}{\alpha}\right)\right](p^*-w^*)$$
$$=\quad \left[\lambda\left(1-\widehat{p}+\upsilon_L\right)+(1-\lambda)\left(1-\frac{\widehat{p}-\upsilon_L}{\mu(\widehat{p})}\right)\right](\widehat{p}-w^*).$$

Writing $\widehat{p}=p^*+\delta$, the first equality can be written as

$$\left(\frac{p^*-\upsilon_L}{\mu(\widehat{p})}-\frac{p^*-\upsilon_L}{\alpha}\right)(p^*-w^*)=-\frac{\delta}{\mu(\widehat{p})}(2p^*+\delta-w^*-\mu(\widehat{p})-\upsilon_L),$$

while the second equality can be written as

$$\begin{aligned}(1-\lambda)\left(\frac{p^*-\upsilon_L}{\mu(\widehat{p})}-\frac{p^*-\upsilon_L}{\alpha}\right)(p^*-w^*) \quad &= \quad -\frac{\delta(1-\lambda)}{\mu(\widehat{p})}(2p^*+\delta-w^*-\mu(\widehat{p})-\upsilon_L)\\ &\quad -\delta\lambda(2p^*+\delta-w^*-1-\upsilon_L).\end{aligned}$$

It is clear that if these two equalities have to hold together, it must be the case that $2p^*+\delta-w^*-1-\upsilon_L=0$, or

$$\widehat{p}=p^*+\delta=1+\upsilon_L-(p^*-w^*)$$

so that $\mu(\widehat{p})$ has to solve $\left(\frac{p^*-\upsilon_L}{\mu(\widehat{p})}-\frac{p^*-\upsilon_L}{\alpha}\right)(p^*-w^*)=\delta-\frac{\delta}{\mu(\widehat{p})}$ or

$$\mu(\widehat{p})=\frac{(p^*-w^*)(p^*-\upsilon_L)+\delta}{\frac{1}{\alpha}(p^*-w^*)(p^*-\upsilon_L)+\delta}<1.$$

As $\mu(\widehat{p})<1$ it follows that $\widehat{p}$ is indeed smaller than $\overline{p}_H$ so that we can indeed choose any $0<\mu(\widehat{p})<1$ and be consistent with the extended intuitive criterion. Importantly, if retail margins are small, $\widehat{p}$ is close to $1+\upsilon_L$ so that demand at this price is very small even if consumers believe it is likely that quality is high at this price.

The equilibrium profit of the low and high quality manufacturer are, respectively,

$$\left(1-\frac{p^*-\upsilon_L}{\alpha}\right)w^*$$

and

$$\left[\lambda\left(1-p^*+\upsilon_L\right)+(1-\lambda)\left(1-\frac{p^*-\upsilon_L}{\alpha}\right)\right](w^*-c).$$

If either type of the manufacturer would locally deviate upwards, its profit would be

$$\left(1-\frac{\widehat{p}-\upsilon_L}{\mu(\widehat{p})}\right)(w^*+\varepsilon)<(1+\upsilon_L-\widehat{p})(w^*+\varepsilon)=(p^*-w^*)(w^*+\varepsilon)$$

when type is low, and

$$\begin{aligned}&\left[\lambda(1-\widehat{p}+\upsilon_L)+(1-\lambda)\left(1-\frac{\widehat{p}-\upsilon_L}{\mu(\widehat{p})}\right)\right](w^*-c+\varepsilon)\\ <\quad&(p^*-w^*)(w^*-c+\varepsilon).\end{aligned}$$

when type is high. Importantly, (27) implies that for λ small enough, we can choose p^* close to w^* implying that the deviation profit of both types of manufacturers is small and certainly smaller than the equilibrium pay-off.

To consider other possible deviations for both retailers and manufacturers, it is most convenient to choose $\mu(p)=0$ for all $p\neq p^*,\widehat{p}$ and $\upsilon_L<p<\overline{p}_L$. From the above discussion it follows that these beliefs are consistent with the extended intuitive criterion. We can then specify the equilibrium strategies of the retailer as follows. For both types of the retailer, as in the case of the pooling equilibrium without informed consumers, they choose $p=p^*$ for all w such that $\underline{w}_i'<w\leq w^*$ and they choose $p=\upsilon_L$ for all w such that $w\leq\underline{w}_i'$, $i=L,H$, where $\underline{w}_L'=\underline{w}$ is the wholesale price such that the low quality retailer is indifferent between setting υ_L and p^* (defined similarly as in Section 3 of the main paper) and $\underline{w}_H'$ is defined accordingly. For all $w>w^*$, the retailers choose a price $p\geq\widehat{p}$. Depending on the beliefs $\mu(p)$ for $p\geq\overline{p}_L$ and the wholesale price w this optimal retail price is either $\widehat{p}$ or a price $p\geq\overline{p}_L$.

It is clear that given the beliefs, the retail strategies are optimal. We do not need to specify the retail strategies in more detail to argue that given these retail strategies, the manufacturer cannot do better than charging w^*. Any deviation upwards results in a price reaction $p\geq\widehat{p}=1+\upsilon_L-(p^*-w^*)$. Even if consumers believe after observing such a high retail price that quality is high, demand at these prices is smaller than $1+\upsilon_L-\widehat{p}=(p^*-w^*)$. As one can make p^*-w^* close to 0 for λ small enough, the deviation profits can therefore be made arbitrarily close to 0, making upward deviations unprofitable. The argument that downward deviations are not profitable is similar to the case where all consumers are uninformed and therefore omitted.

As the transition from all consumers being uninformed to some being informed is a continuous transition, it is also clear that the profit and welfare analysis in Proposition

2 continues to hold under similar parametric restrictions. Specifically, if condition (8) holds as strict inequality and condition (9) holds, then for λ small enough and following the arguments in the proof of Proposition 2, there are pooling equilibria under delegated selling with unobservable upstream pricing that yield more ex ante expected profit to the manufacturer and more expected social surplus than direct selling.

Proof of Proposition 7. Suppose a pooling equilibrium does exist and that both types of manufacturers set a wholesale price equal to w^*. If λ is close enough to 1, in any such equilibrium the high quality retailer will set a retail price close to $(1+v_L+w^*)/2$, while the high quality manufacturer will therefore find it optimal to set a wholesale price close to $(1+v_L+c)/4$ resulting in a retail price larger than v_L. If the low quality manufacturer also sets this price, it will (together with the the low quality retailer) only make profit over the fraction $(1-\lambda)$ of uninformed consumers. As this fraction is close to 0 for λ values close to 1, its profit is also close to 0. By deviating downward to a wholesale price close to, but strictly smaller than v_L, the low quality manufacturer can guarantee that the low quality retailer reacts by setting v_L so that it will sell to all consumers. The total profit for the low quality manufacturer and retailer together is then equal to v_L. As the manufacturer has to leave only a very small fraction of this total profit to the retailer, it will strictly benefit from this deviation.

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