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ABSTRACT

On Strategic Community Development*

This paper examines strategic behaviour of developers who, through offering different public good packages and revenue/fiscal schemes, compete for residents who are differentiated by income. There is an endogenous determination of numbers and sizes of communities. Developers have an incentive to strongly differentiate their public good offerings. In terms of pricing strategies, developers exhibit sharply contrasting behaviours. In low-income communities housing consumption is subsidized once lots are priced. In high-income communities housing consumption is generally taxed.

JEL Classification: H7, L13, R5
Keywords: land development, Tiebout model, strategic pricing, tax competition

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NON-TECHNICAL SUMMARY

It is widely recognized that many public goods are local in nature and are, in many countries, provided by communities competing to attract residents. The existing literature models the community decision-making process in two different ways. In the first, residents are voters who, according to the median-voter rule, choose the quantity of public goods and the taxation scheme to be established within the community. In the second, communities are designed by land development corporations which set public goods and revenue/fiscal schemes in order to attract residents and maximize profits. The first group of papers seems to better model the behaviour of a given number of established communities, while the second seems more appropriate for describing new urban developments such as retirement communities and, more generally, suburban communities as well as edge-cities.

In the first set of papers based on the median-voter principle, consumers have the desirable feature of being heterogeneous in terms of income and/or tastes. This leads to a segmentation of the population in which communities do not behave strategically in their attempt to attract residents. Within the second set of papers, there are recent contributions focusing on the strategic choice of local instruments, but assuming that consumers are either homogeneous or immobile.

It would be desirable to develop models which encapsulate both heterogeneous and mobile consumers as well as communities adopting strategic behaviour. In the case of consumers differentiated by income, communities compete for high- versus low-income segments of the market, similar to firms selling durables of different qualities to different income groups of consumers. Unlike what we observe in the tax competition literature focusing on simple fiscal externalities, communities competing strategically to attract or retain residents may be induced to adopt sharply contrasting pricing policies.

It is also appealing to construct models where the number and size of communities are not fixed *a priori,* but result from the interplay among potential developers. In other words, such models should permit us to determine when entry of new developers is feasible, thus making the structure of the urban landscape endogenous.

This paper can be viewed as a first attempt to capture some of the desirable features discussed above. We present a model of imperfect competition

between land developers when consumers are differentiated by income. Developers either offer different public good packages or are endowed with land characterized by different natural amenity bundles desired by potential residents. In competing for residents, developers may use different pricing strategies and we consider two relevant cases. In the former, which we call *the sites and services model*, profit-maximizing developers select lump-sum fees to segment the population between different income groups, as well as the sites allowing residents' entry to communities. Consumers choose their amount of housing at a given market price. In the latter, which we call *the housing development model*, developers also choose the unit price of housing facing consumers. This could correspond to market situations where either developers set their own housing price for their residents or set a tax/subsidy on housing purchases by their residents in competitive markets.

Our main results include the endogenous determination of the number of communities that can be occupied in equilibrium. This number increases with income disparities. When developers choose public good levels, they face an incentive to strongly differentiate their provision. In terms of pricing, higher-income communities always charge higher lump-sum fees and unit prices as might be expected. It turns out that, in order to retain its relatively high-income residents, the low-income community will be induced to subsidize housing consumption for its residents, however. In the two-community case, the lower-income community will always subsidize housing consumption, while, generally, the higher-income community will tax housing consumption, thus offering a startling contrast in pricing policies.

1. Introduction

Ever since Tiebout (1956), it has been widely recognized that many public goods are local in nature and are provided by communities competing to attract residents. The existing literature models the community decision making process in two different ways. In the first one, residents are voters who, according to the median voter rule, choose the quantity of public goods and the taxation scheme to be established within the community. Papers following this approach include Ellickson (1971), Westhoff (1977), Starrett (1980), Epple and Romer (1991). In the second way, communities are designed by land development corporations which set public goods and revenue/fiscal schemes in order to attract residents and maximize profits. Among papers using this approach are Wildasin (1979), Henderson (1980), Scotchmer (1986), Pines (1991). The first group of papers seems to better model the behavior of a given number of established communities (Epple and Romer (1989)). The second seems more appropriate to describe new urban developments such as retirement communities and, more generally, suburban communities as well as edge-cities (Reichman (1976), Garreau (1991)).

In the first set of papers based on the median voter principle, consumers have the desirable feature of being heterogeneous in terms of income and/or tastes. This leads to a segmentation of the population as envisioned by Tiebout, but one in which communities do not behave strategically in their attempt to attract residents. Within the second set of papers, there are recent contributions focusing on the strategic choice of local instruments; but assuming that consumers are either homogeneous or immobile (Scotchmer (1986), Hoyt (1991), Henderson (1994)). In these recent papers, communities competing for identical potential residents offer strategically chosen tax public service packages. One result is that all developers will finance public services with effective head taxes and price housing at opportunity cost, profiting from head taxes set in excessive of service costs.

It would be desirable to develop models which encapsulate both heterogeneous and mobile consumers, as well as communities adopting strategic behavior. In the case of consumers differentiated by income, communities compete for high versus low income segments of the market, similar to firms selling durables of different qualities to different income groups of consumers. Unlike what we just noted in the recent tax competition literature, with heterogeneous potential residents, communities competing strategically to attract or retain residents may be induced to adopt sharply contrasting housing pricing policies. It is also appealing to construct models where the number and size of communities are not fixed a priori but result from the interplay among potential developers. In other words, such models should permit us to determine when entry of new developers is feasible, thus making endogenous the structure of the urban landscape.

This paper can be viewed as a first attempt to capture some of the desirable features discussed above. In what follows, we present a model of imperfect competition between land developers when consumers are differentiated by income. Developers either offer different public good packages or are endowed with land characterized by different natural amenity bundles desired by potential residents. In competing for residents, developers may use different pricing strategies and we consider here two relevant cases. In the former, that we call *the sites and services model,* profit-maximizing developers select lump-sum fees to segment the population between different income groups. These fees can be viewed as pricing both the services provided by public goods, as well as the sites allowing residents' entry to communities. Consumers choose their amount of housing at a given market price. In the latter, that we call *the housing development model,* developers also choose the unit price of housing facing consumers. This could correspond to market situations where either developers set their own housing price for their residents or set a tax/subsidy on housing purchased by their residents in competitive markets.

Our main results include the determination of the endogenous number of communities that can be occupied in equilibrium. This number increases with income disparities. When developers choose public good levels, they face an incentive to strongly differentiate their provision. In terms of pricing, higher income communities always charge higher lump-sum fees and unit prices as might be expected. However, it turns out that, in order to retain its relatively high income residents, the low income community will be induced to subsidize housing consumption for its residents. In the two-community case, the lower income community will always subsidize housing consumption, while, generally, the higher income community will tax housing consumption, offering a startling contrast in pricing policies.

Before proceeding, we wish to substantiate our assertion that the development of new communities with a key role for developers is an important aspect of the urban landscape. In Table 1 we show that the between-decade growth rate in number of urban places typically exceeds the population growth rate in the USA. Since 1950 urban places include unincorporated places of 2500 or more, as well as incorporated cities, villages, and boroughs, where unincorporated places would include land developments not incorporated into existing urban places. Viewing the issue from another perspective between 1952 and 1992 the number of municipalities, towns, townships, and special districts grew by 49% (Statistical Abstract 1995, Bureau of Census, US Government). Most of this consisted of growth in special districts — bodies which have organizational existence and substantial autonomy — that often serve newly developing private communities with utilities.

Table 1

Growth in Community Numbers: Urban Places

	Percentage Increase in National Population	Percentage Increase in No. of Urban Places*
1910-20	15	20
1920-30	16	16
1930-40	8	9
1940-50	15	17
1950-60	19	27
1960-70	13	18
1970-80	9	23

Source: D. J. Bogue, *Population of the US: Historical Trends and Future Projections,* Free Press, New York, 1985.

*The urban place definition changes in 1950. The 1940-50 growth rate is based on the old definition, and 1950-60 on the new.

With respect to the specific role of developers, Henderson and Mitra (1996) telephone surveyed a sample of Garreau's (1991) listed set of 123 edge cities — large new communities outside existing cities that have formed since 1970. Garreau also identifies 77 more emerging edge cities. The telephone survey substantiated Garreau's portrait of edge cities as being planned controlled entites started by land development companies and often administered by "shadow governments." Henderson and Mitra found no examples of edge cities not started by a single developer and provide a table of examples of large edge cities still tightly controlled by a single developer.

2. The Sites and Services Model

In this model developers provide local public goods to the resident population. Developers charge their residents fixed fees which can be viewed as the cost of purchasing a lot and the right of benefiting from the local public goods available in the corresponding community. We assume that these goods are not subject to congestion.

There are two developers, each offering a land development to a continuum of consumers ranked by income who each choose to live in one of the communities. Below we will develop conditions under which two communities may form and the generalization to n communities. Consumers are free to choose their level of housing available at a given unit price.

There are three commodities, namely housing, h, a local public good, q, and a composite good, x, which serves as the numeraire. Consumers have identical preferences which are log-linear in the three commodities:

$$U(x, h, q) = x^{\alpha} h^{\beta} q \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are constant; without loss of generality, the exponent of q is normalized to one. However, they have different incomes. For simplicity, it is assumed that consumers are uniformly distributed over the interval $[\underline{y}, \overline{y}]$ with unit density and with $0 < \underline{y} < \overline{y}$. This assumption is made only to ease exposition; our main results (e.g., the finiteness property) hold true for any continuous income density. The budget constraint of consumer y in this interval is

$$y = f + ph + x \tag{2}$$

where f denotes the lump-sum fee paid to reside within a community and p is the given unit price of housing. It is readily verified that housing demand of this consumer is

$$h = \frac{\beta}{\alpha+\beta}\,\frac{y-f}{p}. \tag{3}$$

The corresponding indirect utility is

$$V = A(y-f)^{\alpha+\beta}p^{-\beta}q \tag{4}$$

where $A \equiv \left(\frac{\beta}{\alpha+\beta}\right)^{\beta}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}$.

Developers play a two-stage game. First, they choose the quantity q_i of local public goods and, then, the corresponding fee f_i, where $i = 1,2$ denotes the developer and indexes their strategy. The division into two stages is motivated by the fact that, before competing for residents, developers commit to roads, parks, water mains, sewer lines and other facilities, which are all subsumed in a public good index q_i. Alternatively q_1 and q_2 could be predetermined by nature, as amenity characteristics of the land owned by each developer. Once q_1 and q_2 have been chosen, developers compete in stage two of the game for residents through the choice of fees f_i. Consumers choose in which community to reside, as well as their housing level. We assume the supply of housing is perfectly elastic to each community at a given market price, p. This also implies developers have flexible geographic community boundaries (in initially designing their communities), with contiguous land available at a given price. Thus there is no need to give the specifics of housing production technology.

The market outcome is given by a subgame perfect Nash equilibrium. As usual, we solve the game by backward induction; and assume, without loss of generality, that $q_1 < q_2$. (We will see below that land developers never want to choose the same level of public good.) Typically, the consumer population will be segmented into a high-quality community where each resident enjoys q_2 and a low-quality community associated with q_1. Below we will present a necessary and sufficient condition on the income spread in order for the two communities to be occupied in the second-stage equilibrium. Since the two communities span the whole interval of consumers, there is a marginal consumer $\tilde{y}$ who is indifferent between residing in either community. From (4), it must be that $\tilde{y}$ solves $V_1(\tilde{y}) = V_2(\tilde{y})$, which has a unique solution given by

$$\tilde{y} = \frac{f_2 - Qf_1}{1 - Q} \tag{5}$$

where $Q \equiv (q_1/q_2)^{1/(\alpha+\beta)} < 1$ since $q_1 < q_2$.

We now state a necessary condition on fees for the consumers to distribute themselves into the two communities. This condition is that the fee paid in the higher income community, indexed by f_2, is larger than that in the lower income community, indexed by f_1. Furthermore, the richer segment chooses community 2 with q_2 and the poorer segment community 1 with q_1. Why? Maintaining a Nash equilibrium segmentation of the market requires that those in the higher income community don't want to switch to the lower income community. In the neighborhood of $\tilde{y}$, this means

$$\frac{dV_2}{dy}|_{y=\tilde{y}} > \frac{dV_1}{dy}|_{y=\tilde{y}}.$$

This condition requires the fee in the higher income community to be larger, or that $f_2 > f_1$. $f_2 > f_1$ also means the higher income community consumes q_2, where $q_2 > q_1$ (otherwise, no one would live in a high price, low quality community). The condition $f_2 > f_1$ implies that $V_2(y)$ grows faster than $V_1(y)$ for $y > \tilde{y}$. This result is known as the single crossing property.

In summary, in the second-stage equilibrium, developers select fees f_1^* and f_2^* such that the rich consumers pay more to live in the high-quality community, while the poor consumers live in the low-quality community because they do not want to pay the higher fee for the greater q. Hence, once the f_i's are chosen by developers, consumers are allocated to one community or the other; and none has an incentive to move to the other community. Such consumer equilibria are free mobility ones.

Consider now the developers' profits. Since each consumer must reside in one community, the profits of developers are given by

$$\Pi_1 = (\tilde{y} - \overline{y})f_1 = \left(\frac{f_2 - Qf_1}{1-Q} - \overline{y}\right) f_1 \tag{6}$$

with $f_1 \in [0, \overline{y}]$ to guarantee that the poorest consumer can live in community 1. Similarly, the profits of developer 2 are such that

$$\Pi_2 = (\overline{\overline{y}} - \tilde{y})f_2 = \left(\overline{\overline{y}} - \frac{f_2 - Qf_1}{1-Q}\right) f_2 \tag{7}$$

with $f_2 \in [0, \overline{\overline{y}}]$. Both profit functions, (6) and (7), are continuous in (f_1, f_2) and concave in their own fee. Therefore, there exists a Nash equilibrium (f_1^*, f_2^*) in pure strategies. Furthermore, since the first-order conditions can be seen to be linear in f_1 and f_2, the equilibrium is unique. Solving $\partial\Pi_1/\partial f_1 = 0$ and $\partial\Pi/\partial f_2 = 0$ with respect to f_1 and f_2 yields the equilibrium fees as given by

$$f_1^* = (\overline{\overline{y}} - 2\overline{y})\frac{1-Q}{3Q} \tag{8}$$

and

$$f_2^* = (2\overline{\overline{y}} - \overline{y})\frac{1-Q}{3}. \tag{9}$$

Clearly, $f_2^* > 0$ regardless of $\overline{\overline{y}}$ and $\overline{y}$. We assume for the moment that

$$\overline{\overline{y}} > 2\overline{y} \tag{10}$$

so that f_1^* is positive too, thus implying that equilibrium profits of both developers are positive. It is readily verified that both fees decrease (increase) when $q_1 (q_2)$ increases. This occurs because the quality gap between communities narrows (expands), thus making competition between developers tougher (softer).

In equilibrium, from (5) the marginal consumer is given by

$$\tilde{y}^* = (\overline{\overline{y}} + \overline{y})/3 \tag{11}$$

which means that the population border is independent of q_1 and q_2. Comparing the populations in each community, $\tilde{y} - \overline{y}$ and $\overline{\overline{y}} - \tilde{y}$, we can see that community 1 has less than a third of the total population. In addition, as the income spread, $\overline{\overline{y}}/\overline{y}$, shrinks, developer 1's market share falls.

Proposition 1. In the sites and services model, there is only one occupied community when $\overline{\overline{y}} < 2\overline{y}$.

Proof: For the two communities to be occupied, it must be that $\tilde{y}^* > \overline{y}$, which holds if and only $\overline{\overline{y}} > 2\overline{y}$. Q.E.D.

The condition identified in Proposition 1 is identical to (10) which ensures f_1^* is positive.

If (10) does not hold, then *all consumers will be bunched in the high-quality community.* This occurs because *the income range is narrow enough for developer 2 to find it profitable to accommodate the whole population* by charging a fee low enough for all consumers to choose to reside there, even when developer 1 selects a zero fee. On the other hand, when (10) is satisfied, the income range is wide enough to induce developer 2 to specialize on the segment of richer consumers, thus leaving room for developer 1 to supply poorer consumers.

These results are reminiscent of the "finiteness property" derived in a different context of an oligoplistic model with vertical product differentiation (Gabszewicz and Thisse (1979), Shaked and Sutton (1983)). The major difference here is that individuals consume different amounts of housing within and across communities, making the present model richer. As in the cited papers, our model generalizes to the case of n developers. So, for example, with three communities, by examining the equations for the two border consumers, the first-order conditions for profit maximization, and the expressions for market size for each community, one can show that the existence of three communities requires $\overline{\overline{y}} > 4\overline{y}$. With n communities the conditon is $\overline{\overline{y}} > 2^{n-1}\overline{y}$, so that the upper bound on the number of communities that can survive in equilibrium rises as the income gap expands. It is worth noting that this bound does not rest upon any cost considerations. It is only driven by the fact that all consumers like more public goods than less, but have different willingness-to-pay for more. Only if the population income span is large enough, can a second (and then a third, and so on) developer find a market niche by offering low income consumers relatively low fees, for less public goods.

Furthermore, we also require for consistency that the following inequalities be satisfied: (i) $f_1^* < f_2^*$, (ii) $f_1^* \leq \underline{y}$, and (iii) $f_2^* \leq \tilde{y}^*$. Condition (i) was derived above and states that developer 1 charges a lower fee in order to have customers. Condition (ii) means developer 1 must select a fee less than the income of the poorest consumer in community 1 and condition (iii) is the corresponding restriction on developer 2. Given (10), some simple calculations show that condition (ii) is the most stringent one, that is,

$$Q \geq \frac{\overline{\overline{y}} - 2\underline{y}}{\overline{\overline{y}} + \underline{y}}. \tag{12}$$

We will use this condition below.

We now turn to the first stage of the game. For that, we substitute into the profit functions (6) and (7) for the equilibrium fees (8) and (9) to get

$$\Pi_1(q_1, q_2) = \frac{1}{9}(\overline{\overline{y}} - 2\underline{y})^2(1/Q - 1) \tag{13}$$

and

$$\Pi_2(q_1, q_2) = \frac{1}{9}(2\overline{\overline{y}} - \underline{y})^2(1 - Q). \tag{14}$$

Then, we have

$$\frac{\partial \Pi_1}{\partial q_1} = -\frac{1}{9}(\overline{\overline{y}} - 2\underline{y})^2(1/Q^2)\partial Q/\partial q_1 < 0$$

and

$$\frac{\partial \Pi_2}{\partial q_2} = -\frac{1}{9}(2\overline{\overline{y}} - \underline{y})^2 \partial Q/\partial q_2 > 0$$

since $\partial Q/\partial q_1 > 0$ and $\partial Q/\partial q_2 < 0$. This suggests, based upon revenue considerations, that *both developers have an incentive to differentiate as much as possible their public good* *provision.* However, condition (12) which ensures that $f_1^* \leq \bar{y}$ places a lower bound on Q and hence on the quality gap expressed by q_1/q_2.

A common way to proceed is to assume that q_1 and q_2 are chosen costlessly in a given interval $[\overline{q}, \overline{\overline{q}}]$, where $\overline{q}$ is a minimum quality standard imposed by the government while $\overline{\overline{q}}$ stands for the maximum public good level. Alternatively, q_1 and q_2 could be predetermined quantities of natural amenities with which each developer's lands are endowed. If the ratio $\overline{q}/\overline{\overline{q}}$ satisfies (12), then the equilibrium configuration is $q_1^* = \overline{q}$ and $q_2^* = \overline{\overline{q}}$ and maximum differentiation between the two communities arises. If the ratio $\overline{q}/\overline{\overline{q}}$ is so large that (12) is violated, then developer 1 may not want to serve the poorest consumers, a possibility further considered below.

Another obvious consideration is the cost of provision of public goods. Let us assume a cost function $c(q)$ which is increasing in q and the same for each developer. Sufficiently increasing costs place an upper bound on the choice of q_2. On the other hand, cost considerations reinforce the desire of developer 1 to reduce q_1 so that there is still a strong tendency to differentiate communities in terms of public good provision.

Apart from imposing either a minimum quality standard or exogenous amenity level, we must recognize that as developer 1 reduces q_1 and, therefore, increases f_1, he will cease to serve the lowest income consumers. A natural way to model that is to assume that consumers obtain a reservation utility outside the two communities, given, for example, by $y^{\alpha+\beta}$. Then, if y_m is the consumer marginal to the market, y_m, must satisfy the equality

$$A(y_m - f_1)^{\alpha+\beta} p^{-\beta} q_1 = y_m^{\alpha+\beta}.$$

Developer 1's profit now becomes

$$\Pi_1 = (\tilde{y} - y_m) f_1$$

where $\tilde{y}$ is given as before by (5) and $y_m = f_1[1 - (A^{-1} p^{\beta} q_1^{-1})^{\frac{1}{\alpha+\beta}}]^{-1}$. If one proceeds with this maximization problem through the second and first stages of the game, one sees that when choosing q_1 developer 1 faces a trade-off between losing consumers as he lowers q_1 versus raising f_1 paid by his remaining consumers. In this way the equilibrium value of q_1 may be determined.

3. The Housing Development Model

In addition to lump-sum fees as in the previous section, in the housing development model, developers also choose the unit price to be charged for housing, p_i, in community $i = 1, 2$. One interpretation is that developers allow residents to choose their level of housing at the prevailing price, p_i, set by the developer. An alternative interpretation is that consumers provide their own housing at a given unit price p but the developer adds a tax/subsidy per unit of housing.

In stage two of the game, developers now choose simultaneously the lump-sum fee f_i and unit price p_i. While this model bears some resemblance to a two-part tariff model, the main difference is that, here, the pair (f_i, p_i) is pricing two different goods, that is, q_i and h_i. The first is an indivisible public good and the second a divisible private good. The developers choose the value of q_i while consumers choose the community in which they reside and their housing consumption. Roughly speaking, the fee is pricing the choice of community i where q_i prevails, and p_i is pricing housing consumption in i.

The primitives of the model are the same as in the previous section. Hence the population will be segmented into two different communities as before. However, because the unit price of housing differs across communities, in solving for the marginal consumer $\tilde{y}$ by equating $V_1(\tilde{y}) = V_2(\tilde{y})$ from (4), we now get

$$\tilde{y} = \frac{f_2 - Q_p f_1}{1 - Q_p} \tag{15}$$

where

$$Q_p \equiv (p_2/p_1)^{\beta/(\alpha+\beta)}(q_1/q_2)^{1/(\alpha+\beta)}$$

which is identical to Q when $p_1 = p_2$.

Observe that for a marginal consumer to exist such that the high income community is community 2 requires that

$$\frac{dV_2}{dy}|_{y=\tilde{y}} > \frac{dV_1}{dy}|_{y=\tilde{y}}.$$

It is readily verified that this inequality holds if and only if

$$f_2 > f_1.$$

Given $f_2 > f_1$, it can be shown that $V_2(y)$ increases faster than $V_1(y)$ if and only if $y > \tilde{y}$. Consequently, there is a unique marginal consumer who is given by (15) and the whole population of consumers is segmented as desired. Further, it is shown in the appendix that $f_2 > f_1$ together with $\tilde{y} > \underline{y} > 0$ implies that $Q_p < 1$. As will be seen below in Proposition 4, p_1 must be smaller than p_2, so that $Q_p < 1$ can hold only if $q_2 > q_1$. In other words, *an equilibrium where both communities are occupied requires the high-income community to be that with the higher level of public goods.*

Since developers now collect revenue from selling housing, we need to define the aggregate demand for housing in each community, assuming that the two communities span the whole population. For that, we integrate the individual demands for housing given by (3) over the two intervals $[\underline{y}, \tilde{y}]$ and $[\tilde{y}, \overline{y}]$. This yields the total demands for housing in communities 1 and 2, H_1 and H_2, such that

$$H_1 = \frac{1}{2}\frac{\beta}{\alpha+\beta}p_1^{-1}(\tilde{y}-\underline{y})(\tilde{y}+\underline{y}-2f_1) \tag{16}$$

and

$$H_2 = \frac{1}{2}\frac{\beta}{\alpha+\beta}p_2^{-1}(\overline{y}-\tilde{y})(\overline{y}+\tilde{y}-2f_2). \tag{17}$$

The developers' profits are made up of two parts: fees collected from residents and the difference between sales revenue and housing opportunity cost, where the unit opportunity cost is p as in Section 2. Thus,

$$\begin{aligned} \Pi_1 &= (\tilde{y}-\underline{y})f_1 + H_1(p_1 - p) \\ &= (\tilde{y}-\underline{y})[\tfrac{1}{2}b_1(\tilde{y}+\underline{y}) + (1-b_1)f_1] \end{aligned} \tag{18}$$

and

$$\begin{aligned}\Pi_2 &= (\bar{\bar{y}} - \tilde{y})f_2 + H_2(p_2 - p) \\ &= (\bar{\bar{y}} - \tilde{y})[\tfrac{1}{2}b_2(\bar{\bar{y}} + \tilde{y}) + (1 - b_2)f_2]\end{aligned} \tag{19}$$

where

$$b_i \equiv \frac{\beta}{\alpha + \beta}(1 - p/p_i).$$

In what follows, we focus on market equilibria where both communities are occupied, although it is not the only outcome. Since each developer has two decision variables in stage 2, we must recognize that, a priori, *fees need not be positive and/or unit housing prices need not be above opportunity cost.* For example, developer 1 could either select a negative fee or price housing below opportunity cost and still earn positive profits. He might want to do so in order to attract more consumers from community 2. This is a distinctive feature of the housing development model, compared to the sites and services model. The sign of the fees and the relative values of the unit prices are to be determined at the equilibrium. This will be the focus of the analysis that follows.

Applying the first-order conditions to Π_1 and Π_2 with respect to f_1 and f_2 leads respectively to

$$f_1^* = \frac{1 - Q_p}{Q_p}(\tilde{y} - \bar{y}) - \frac{b_1}{1 - b_1}\tilde{y} \tag{20}$$

and

$$f_2^* = (1 - Q_p)(\bar{\bar{y}} - \tilde{y}) - \frac{b_2}{1 - b_2}\tilde{y}. \tag{21}$$

Substituting (20) and (21) into (15), we get

$$\tilde{y} = \frac{(1 - Q_p)(\bar{\bar{y}} + \bar{y})}{3(1 - Q_p) + b_2/(1 - b_2) - Q_p b_1/(1 - b_1)}. \tag{22}$$

As in Section 2, we can show the following result.

Proposition 2. In the housing development model, there is only one occupied community when $\bar{\bar{y}} \leq 2\bar{y}$.

Proof. For the two communities to be occupied it must be that $\tilde{y} > \bar{y}$. Using (22), this condition is equivalent to

$$\bar{\bar{y}} > 2\bar{y} + \left(\frac{1}{1-Q_p}\cdot\frac{b_2}{1-b_2} - \frac{Q_p}{1-Q_p}\cdot\frac{b_1}{1-b_1}\right)\bar{y} \tag{23}$$

For the proposition to hold, we must be able to show that

$$\frac{1}{1-Q_p}\cdot\frac{b_2}{1-b_2} - \frac{Q_p}{1-Q_p}\cdot\frac{b_1}{1-b_1} > 0$$

in equilibrium. Since we know from the appendix that $1 - Q_p > 0$, the inequality above is equivalent to

$$\frac{b_2}{1-b_2} - Q_p\frac{b_1}{1-b_1} > 0. \tag{24}$$

We will show below in Proposition 4 that $p_2^* > p_1^*$ in equilibrium so that $b_2 > b_1$. Therefore, we have $b_2/(1-b_2) > b_1/(1-b_1)$. Since $0 < Q_p < 1$, the LHS of (24) is strictly positive. Q.E.D.

Observe that (23) which must hold for two communities to be occupied is more stringent than equation (10) in Section 2. While it is true that equation (23) reduces to (10) when $p_1 = p_2 = p$ so that $b_1 = b_2 = 0$, here as we will show below $b_2 > b_1 \neq 0$. The intuition behind the finiteness property in Proposition 2 is similar to that discussed in Section 2. The fact that developers have more pricing instruments does not prevent developer 1 from being forced out of market when the income spread is small enough.

We now characterize equilibrium pricing.

Proposition 3. In equilibrium with two communities, we have $p_1^* < p$.

Proof: In equilibrium, consistency conditions require that $f_1^* < \tilde{y}$, where $\tilde{y}$ is evaluated at the Nash equilibrium of the second-stage subgame. Substituting for f_1^* from (20), we obtain

$$\tilde{y} - \frac{(1-b_1)(1-Q_p)}{Q_p}(\tilde{y} - \bar{y}) > 0. \tag{25}$$

We use the first-order condition for the first term $\tilde{y}$. Computing $\partial \Pi_1/\partial p_1 = 0$ and substituting f_1 from (20) leads to

$$-[\hat{y} - \frac{1-Q_p}{Q_p}(1-b_1)(\hat{y}-\tilde{y})] + \frac{p}{p_1}\left[\frac{\tilde{y}+\overline{y}}{2} - \frac{1-Q_p}{Q_p}(\tilde{y}-\overline{y}) + \frac{b_1}{1-b_1}\tilde{y}\right] = 0. \qquad (26)$$

Rearranging terms, we get

$$\tilde{y} = (\tilde{y}-\overline{y})\left[\frac{1-Q_p}{Q_p}(1-b_1) - \frac{(\alpha+\beta)^2}{2\alpha\beta}\frac{p}{p_1}\frac{1-b_1}{b_1}\right].$$

Substituting for the first term of (25), we get

$$-(\tilde{y}-\overline{y})\frac{(\alpha+\beta)^2}{2\alpha\beta}\frac{p}{p_1}\frac{1-b_1}{b_1} > 0$$

which holds if and only if $b_1 < 0$. That is, $p_1^* < p$. Q.E.D.

Hence, since profits in community 1 must be positive for community 1 to exist in equilibrium. Proposition 3 implies that developer 1 must select a positive fee. Consequently, we have

$$0 < f_1^* < f_2^*.$$

Proposition 3 tells us that *strategic competition leads the developer of the lower income community to subsidize housing.* This result was unanticipated. Our intuition was that, within a community, since everyone pays the same fee which magnitude is restricted by the income of the lowest income resident, developers would charge unit prices above opportunity cost as a way to extract more revenue from higher income consumers in this community. Such a result prevails in a monopoly version of this model; and appears to generally hold in the high-income community here. However, for the low income community, it is apparent that the developer is induced to subsidize housing in order to attract the higher income residents of his community from community 2. Roughly speaking, this occurs because these relatively big housing consumers within the community are more sensitive to price reductions than to a decrease in the corresponding fee, which is uniform within the community.

Moreover, we note that, in the n community case where communities span the income interval, we expect the lowest income community will always subsidize housing consumption.

Proposition 3 is proved with reference to only the adjacent community and utilizes expressions for f_1, $\tilde{y}$, and $\partial\pi_1/\partial p_1 = 0$, which are independent of the number of communities.

It remains to characterize $\overset{*}{p}_2$. This is done in the next proposition.

Proposition 4. In equilibrium with two communities, we have $p_1^* < p_2^*$.

Proof: Computing the first-order condition for $p_2, \partial\Pi_2/\partial p_2 = 0$, and substituting for f_1^* and f_2^* as given by (20) and (21), we obtain

$$-Q_p\frac{1-b_2}{1-b_1}[\tilde{y} - \frac{1-Q_p}{Q_p}(1-b_1)(\tilde{y}-\overline{y})]$$

$$+\frac{p}{p_2}\left[\frac{\overline{y}+\overline{\overline{y}}}{2} - (1-Q_p)(\overline{\overline{y}}-\tilde{y}) + \frac{b_2}{1-b_2}\tilde{y}\right] = 0.$$

Using (26) and multiplying by $(1-b_1)p_2/p$ yields

$$(\alpha p_1^* + \beta p)A - (\alpha p_2^* + \beta p)B = 0 \tag{27}$$

where

$$A \equiv \frac{\overline{y}+\overline{\overline{y}}}{2} - (1-Q_p)(\overline{\overline{y}}-\tilde{y}) + \frac{b_2}{1-b_2}\tilde{y}$$

and

$$B \equiv \frac{\overline{y}+\tilde{y}}{2}Q_p - (1-Q_p)(\tilde{y}-\overline{y}) + \frac{b_1}{1-b_1}Q_p\tilde{y}.$$

Solving (22) for $\overline{\overline{y}}$ and substituting in the second term of A, we find that

$$A - B = \frac{1}{2(1-Q_p)}[(\overline{\overline{y}}-\tilde{y}) + Q_p(\tilde{y}-\overline{y})]$$

which is strictly positive because the two communities are occupied and $1-Q_p > 0$ by the appendix. Hence $A > B$ in (27) implies that $p_2^* > p_1^*$. Q.E.D.

Hence, developer 2 always charges a higher price for housing than developer 1. This result was shown to be sufficient to establish Proposition 2 and the finiteness property. It also permitted us to show that the high income consumers always reside in the community offering the better package of public goods.

We have not been able to prove that $p_2^* > p$. Computing equilibria for a range of specific values of the model parameters, we have not succeeded in constructing an example of an equilibrium where $p_2^* < p$, thus suggesting that, in general, developer 2 sets housing price above opportunity cost.

Unlike in the sites and services model, we have not been able to determine conditions under which existence of a pricing equilibrium in pure strategies that spans the population holds true. Investigation of many numerical examples suggests the following problems. First, solutions to the first-order conditions may yield $f_1 > \overline{y}$, indicating that developer 1 does not wish to serve the lowest income segment of his community for some parameter constellations. Second, such solutions may lead to $\tilde{y} < \overline{y}$ in which case there would be only one community. Last, there are solutions to the first-order conditions, consistent with our requirements, for which the second-order conditions do not hold. In particular, we observe that one of the developers is at a saddle point. In this case, it is hard to conjecture what the equilibrium (if any) would be. However, for "non-extreme" values of the income spread and of the quality gap, we have obtained interior solutions satisfying first- and second-order conditions, as well as the consistency conditions for the two communities to span the whole population. This seems to accord with what we know from the study of simpler models of vertical product differentiation where equilibrium market configurations may be very sensitive to small changes in parameter values (Wauthy (1996)).

Not surprisingly, the housing development model does not really lend itself to a ready analysis of the first stage of the game, the choice of q_1 and q_2. The issues concerning the provision of public goods that arose in Section 2 would occur here too. In particular, absent cost considerations, we expect the same strong tendency for the developers to differentiate their provision of public goods.

4. Concluding Remarks

In one of the early formulations of the Tiebout model, Hamilton (1975) assumes a large, but finite, number of consumer groups with identical demands for housing and public goods within each group and different demands across groups. Each community is operated by a single developer who behaves competitively. In such a setting, there is no room for a nontrivial determination of the sizes and number of communities. To a large extent, a similar criticism can be applied to subsequent contributions.

This paper is an attempt to formulate the Tiebout problem, when there is a continuum of consumers with internally heterogeneous communities and there is strategic behavior on the part of developers as each community competes only with its neighbors for residents. Our approach allows for a nontrivial determination of the number and sizes of communities. In particular, the finiteness property places a limit on the number of active communities, depending on the income span of the population. The relative sizes of those communities are also related to the income span. Observe that in our model with pure local public goods, it is socially desirable to have only one community. Competition among developers typically involves formation of multiple communities, each corresponding to a specific income niche exploited by a developer.

Even though our model is static, the finiteness property allows us to suggest interesting dynamics in the process of creation and disappearance of communities. Assume indeed that the upper bound associated with the finiteness property is binding. Then, if innovation takes place in public goods, it is always possible for a new developer to enter the market and to capture a segment of residents, typically those who have high incomes. Other consumers then move to higher quality existing communities; and, at the very least, the lowest quality community finds itself with no residents. In other words, entry induces exit, suggesting that the finiteness property could pave the way for a much more detailed dynamic analysis of the land development industry. Of course, such an analysis should also allow for income growth as well as for changes in income disparities.

Note, finally, that we have restricted ourselves to two relevant pricing schemes – fees and fees plus pricing of housing (or, housing tax/subsidy). Our approach could be extended to cope with more sophisticated contracts, such as those considered by Spulber (1989). Clearly, more work is called for here.

Appendix

In equilibrium with two communities, we have $Q_p < 1$.

Proof: Assume that $Q_p \geq 1$. First, $Q_p = 1$ is impossible from (20) and (21) because we must have $f_1^* < f_2^*$. Then, consider $Q_p > 1$. Given (15) and $\tilde{y} > \overline{y} > 0, 1 - Q_p$ implies that $f_2^* - Q_p f_1^* < 0$. Substituting (20) and (21) into this inequality leads to

$$(1 - Q_p)(\overline{\overline{y}} + \overline{y} - 2\tilde{y}) - Q_p\tilde{y}\left(\frac{b_2}{1 - b_2}Q_p^{-1} - \frac{b_1}{1 - b_1}\right) < 0. \tag{A.1}$$

Similarly, we have $Q_p(f_2^* - f_1^*) > 0$. Hence after substitution we obtain

$$(1 - Q_p)[Q_p(\overline{\overline{y}} - \tilde{y}) + \overline{y} - \tilde{y}] - Q_p\tilde{y}\left(\frac{b_2}{1 - b_2} - \frac{b_1}{1 - b_1}\right) > 0. \tag{A.2}$$

Subtracting (A.2) from (A.1), we get

$$-(1 - Q_p)^2(\overline{\overline{y}} - \tilde{y}) - (Q_p - 1)\tilde{y}\frac{b_2}{1 - b_2} > 0.$$

Since $Q_p > 1$ by assumption, it must be that $b_2 < 0$. However, after having substituted f_2^* in Π_2, we obtain

$$\Pi_2 = (\overline{\overline{y}} - \tilde{y})[\frac{1}{2}b_2 + (1 - Q_p)(1 - b_2)] < 0$$

since $b_2 < 0$, $1 - b_2 > 0$ and $Q_p > 1$. This contradicts the fact that profits are positive in equilibrium. Q.E.D.

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