DISCUSSION PAPER SERIES

DP15475

CONSTANT PASS-THROUGH

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INDUSTRIAL ORGANIZATION

INTERNATIONAL MACROECONOMICS AND FINANCE

INTERNATIONAL TRADE AND REGIONAL ECONOMICS

MACROECONOMICS AND GROWTH

MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper DP15475
Published 20 November 2020
Submitted 16 November 2020

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JEL Classification: D21, D43, L13

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Constant Pass-Through*

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November 15, 2020

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*We thank Gadi Barlevy, Oleg Itskhoki and Ezra Oberfeld for discussion. The usual disclaimer applies.

1. Introduction

Monopolistic competition with the CES demand system, with or without entry, as well as with or without heterogeneous firms, is a workhorse model across many applied general equilibrium fields, including growth, macroeconomics, international trade, regional and urban economics, and economic geography. However, the CES demand system imposes the strong restrictions on the pricing behavior, such as the exogenously constant markup rate that is common across firms, and the complete pass-through. Of course, in a multi-sector setting with nested CES, the markup rate can vary across sectors but not across firms within each sector. Furthermore, the pass-through rate has to be equal to one across all firms and across all sectors. Another restrictive feature of CES is that various types of heterogeneity across firms are isomorphic to each other and hence cannot be distinguished.

In this paper, we propose and characterize parametric families of homothetic demand systems, which accommodate endogenous markup rates, incomplete pass-through, as well as various types of heterogeneity across firms and yet maintain much of tractability of CES, and hence can serve as a building block in a wide range of monopolistic competition models. More specifically, we consider parametric families that feature a constant pass-through rate as a parameter, which is common across firms. We shall call these parametric families, CoPaTh, since the constant and common pass-through rate is the key restriction, which buys us a lot of tractability.

To explain the properties of CoPaTh in more detail, let us first recall some terminology. The pass-through rate of firm ω is defined as $\rho_\omega \equiv \partial \ln p_\omega / \partial \ln \psi_\omega$, where p_ω is its profit-maximizing price and ψ_ω is its own marginal cost. The pass-through rate is closely related to the rate of change in its markup rate, $\mu_\omega \equiv p_\omega / \psi_\omega$, in response to a change in its own marginal cost, since

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1 + \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega}.$$

Under the CES demand system, used by Dixit and Stiglitz (1977; Section I) and most existing models of monopolistic competition, the markup rate is constant and common across firms, $\mu_\omega = \mu$. It is thus independent of the firm's marginal cost, which implies a unitary pass-through rate, $\rho_\omega = 1$.

Under CoPaTh, the markup rate is no longer restricted to be constant nor common across firms. Instead, the pass-through rate is restricted to be constant and common across firms with $0 < \rho_\omega = \rho \leq 1$. Or equivalently, the rate of change in the markup rate in response to a change in its marginal cost is restricted to be constant and common across firms, because of the identity, $0 < \partial \ln p_\omega / \partial \ln \psi_\omega = 1 + \partial \ln \mu_\omega / \partial \ln \psi_\omega = \rho \leq 1$. Firms can be heterogenous in many other dimensions.[1] Furthermore, under CoPaTh, the prices of competing firms affect the profit-maximizing price of each firm through a single common aggregator, $\mathcal{A}(\mathbf{p})$, which is linear homogeneous in $\mathbf{p}$, the vector of all prices. These restrictions jointly imply that the price of each firm p_ω can be expressed as:

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} = \left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega}\right)^\rho \tag{1}$$

with $\sigma_\omega > 1$ is the firm-specific markup shifter, and $\beta_\omega > 0$ is the firm-specific price shifter and $\mathcal{A} = \mathcal{A}(\mathbf{p})$ as the (inverse) measure of the "toughness" of competition, which is common across firms.

In the complete pass-through case ($\rho = 1$), this pricing formula, eq.(1), is simplified to:

$$p_\omega = \frac{\sigma_\omega}{\sigma_\omega - 1}\psi_\omega.$$

This is the case in which each firm faces the constant (but firm-specific) price elasticity. We shall call this case CPE for *Constant Price Elasticity*. CES is a special case of CPE, where the price elasticity is not only constant but also common across firms.[2] With the firm-specific constant price elasticity, the firm-specific markup rate is constant, and independent of its marginal cost ψ_ω, the price shifter, β_ω, and $\mathcal{A} = \mathcal{A}(\mathbf{p})$.

In the incomplete pass-through case ($0 < \rho < 1$), the pricing formula, eq.(1), becomes:

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[1]We allow firms to be heterogenous, not only in the marginal cost, as in Melitz (2004), and in the markup rate, but also in the quantity shifter and price shifter, which can be interpreted as differences in market size and in quality of their products. We do so for three reasons. First, we want to highlight the implications of the pass-through rate being common across firms. Second, various sources of heterogeneity are no longer isomorphic to each other under non-CES demand systems. Third, we hope to make CoPaTh useful as a building block in a wide range of monopolistic competition models.

[2]Uzawa (1962) and McFadden (1963) showed that the elasticity of substitution between each pair of products being constant implies that all pairs share the common elasticity of substitution, which in turn implies that the price elasticity of demand for each product has to be constant and common. However, the reverse is not true. The price elasticity of demand being constant for each product does *not* imply that all products share the common price elasticity. Furthermore, it does *not* imply that the elasticity of substitution between every pair of products is *not* constant, unless all products share the common price elasticity.

$$\ln p_\omega = (1-\rho)\ln \bar{p}_\omega + \rho \ln \psi_\omega,$$

where $\bar{p}_\omega \equiv \mathcal{A}(\mathbf{p})\beta_\omega(1-1/\sigma_\omega)^{\frac{\rho}{\rho-1}} < \infty$ is the choke price of firm ω. Thus, the price of each firm is log-linear in its marginal cost and its choke price with the common coefficients across firms. Furthermore, the markup rate is $\mu_\omega = (\bar{p}_\omega/\psi_\omega)^{1-\rho} > 1$, and hence decreasing in its own marginal cost. Tougher competition, a reduction in $\mathcal{A}$, causes a reduction in the price (strategic complementarity) through its effect on the choke price. Yet, it does not affect their relative prices, because this effect is uniform across firms. In spite of such independence of the relative price on $\mathcal{A}$, a change in $\mathcal{A}$ has nontrivial effects on the relative performance of firms. It will be shown that a reduction in $\mathcal{A}$ causes a disproportionately larger decline in the revenue and the profit among firms facing higher price elasticity of demand, and hence those setting lower markup rates. It should be pointed out that these firms, which suffer disproportionately more from tougher competition, are not necessarily smaller nor less productive, because firms can be heterogenous in many dimensions. If firms are heterogenous *only* in the marginal cost *and* if pass-through is incomplete, then it is less productive firms, which happen to be smaller in size, that set lower markup rates and suffer more from tougher competition.

These properties make the CoPaTh families of the demand system natural, flexible, and yet tractable extensions of CES. For example, think of any shock that changes the relative cost across firms, such as the exchange rate movement or tariffs, that change the relative cost of domestic and foreign firms. Or perhaps, if some firms are more dependent of energy than others, their relative cost varies more when energy prices go up. Under CoPaTh, the impact of such shocks on the markup rates and relative prices can be calculated without worrying about the general equilibrium feedback effect. This could be a great advantage when studying the general equilibrium effects of such shocks.[3] Furthermore, under CoPaTh, the pass-through rate is a parameter, which means that, in a multi-sector setting with nested CoPaTh, the pass-through rate can be sector-specific, and hence the average pass-through rate in the economy can be endogenized through a change in the sectoral composition. This would not be possible with nested CES.

[3]In their work on exchange rate pass-through, Gopinath and Itskhoki (2010) and Itskhoki and Muhkin (2017) justified their log-linear pricing formula under their non-CES homothetic demand systems as a linear approximation around a point of no heterogeneity. Under CoPaTh, the log-linearity holds exactly for any degree of heterogeneity.

It turns out that the CoPaTh families come in three different forms, as depicted by the three green petals (and three yellow petals of CPE, which are subfamilies of CoPaTh) in Figure. This is because there exist three different classes of homothetic demand systems, H.S.A., H.D.I.A., H.I.I.A., as depicted by the three blue petals in Figure, all of which share the property that the price elasticity of demand curve each firm faces is a function of $p_\omega/\mathcal{A}(\mathbf{p})$ only. These three classes differ in $\mathcal{A}(\mathbf{p})$, and they are disjoint with the sole exception of CES. In Section 2, we first discuss general properties that hold in all three CoPaTh families, assuming the existence of the single price aggregator, $\mathcal{A}(\mathbf{p})$. Then, we deal with each of these three classes and show how $\mathcal{A}(\mathbf{p})$ is constructed under H.S.A. in Section 3, under H.D.I.A. in Section 4, and under H.I.I.A. in Section 5.

There have been many attempts to depart from the CES demand system in monopolistic competition; see Parenti, Ushchev, and Thisse (2017) and Thisse and Ushchev (2019) for extensive reference. A vast majority of these studies, however, depart from CES by introducing nonhomotheticity in their demand systems[4], which make them less suitable as a building block for general equilibrium models with monopolistic competition. Homothetic non-CES demand systems are used in Feenstra (2003) and Kimball (1995). However, neither the translog specification by Feenstra nor the popular parametrization of the Kimball demand system by Klenow and Willis (2016) exhibit a constant pass-through rate. Furthermore, their functional forms are not well-suited for accommodating a variety of heterogeneity across firms. For these reasons, we believe that CoPaTh can serve as a useful building block for a wide range of monopolistic competition models.

In our earlier paper, Matsuyama and Ushchev (2017), we introduced the three non-parametric classes of homothetic demand systems, H.S.A., H.D.I.A., and H.I.I.A., and showed that they are pairwise disjoint with the sole exception of CES. That paper was just about demand systems and made no assumption about market structure. In a recent paper, Matsuyama and Ushchev (2020), we applied these three classes to a model of monopolistic competition among homogeneous firms with free entry, in order to investigate the relation between the optimal and equilibrium product variety. For that purpose, we assumed a continuum of homogeneous firms, symmetric demand systems, and gross substitutability of products, with additional restrictions

[4]See, e.g., Behrens and Murata (2007), Melitz and Ottaviano (2008), and Zhelobodko, Kokovin, Parenti, and Thisse (2012).

that ensure the uniqueness of the symmetric free entry equilibrium. However, the analysis there was completely nonparametric. In contrast, we pursue here parametric restrictions of these demand systems that offer natural, flexible, and yet tractable extensions to the CES, while allowing for various dimensions of heterogeneity across firms, except that they share the common constant pass-through rate, which is the restriction that buys us a lot of tractability. This is because the goal of this paper is to enrich our set of tools that is simple and yet flexible enough to be used as a building block in a wide range of monopolistic competition models, with or without heterogeneity, and with or without free entry.

2. **General Properties**

Consider a sector, which consists of a continuum of monopolistically competitive firms, $\omega \in \Omega$, each producing and supplying its own differentiated input variety to competitive producers, which assemble these input varieties by using the CRS assembly technology. This CRS technology is represented by its production function, $X = X(\mathbf{x})$, where $\mathbf{x} = \{x_\omega; \omega \in \Omega\}$ is a quantity vector of inputs. $X(\mathbf{x})$ is assumed to satisfy linear homogeneity, monotonicity, and quasi-concavity for a given set of Ω. The unit cost function dual to $X = X(\mathbf{x})$ can be obtained by:

$$P = P(\mathbf{p}) \equiv \min_{\mathbf{x}} \left\{ \mathbf{px} = \int_\Omega p_\omega x_\omega d\omega \,\middle|\, X(\mathbf{x}) \geq 1 \right\}, \tag{2}$$

where $\mathbf{p} = \{p_\omega; \omega \in \Omega\}$ is a price vector of inputs, and $P(\mathbf{p})$ also satisfies linear homogeneity, monotonicity, and quasi-concavity for a given set of Ω. Conversely, starting from any linear homogeneous, monotonic, and quasi-concave $P(\mathbf{p})$, one could recover the underlying linear homogenous, monotonic, and quasi-concave production function as follows:

$$X = X(\mathbf{x}) \equiv \min_{\mathbf{p}} \left\{ \mathbf{px} = \int_\Omega p_\omega x_\omega d\omega \,\middle|\, P(\mathbf{p}) \geq 1 \right\}. \tag{3}$$

Thus, either $X = X(\mathbf{x})$ or $P = P(\mathbf{p})$ can be used as a primitive of this CRS technology.

As is well-known from the duality theory, the cost minimization by competitive producers generates the demand curve and the inverse demand curve for each input,

$$x_\omega = X(\mathbf{x}) \frac{\partial P(\mathbf{p})}{\partial p_\omega}; \; p_\omega = P(\mathbf{p}) \frac{\partial X(\mathbf{x})}{\partial x_\omega},$$

from either of which one could show, using Euler's theorem on linear homogeneous functions,

$$\mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega = P(\mathbf{p})X(\mathbf{x}).$$

Furthermore, the market share of each input can be expressed as

$$\frac{p_\omega x_\omega}{\mathbf{px}} = \frac{p_\omega x_\omega}{P(\mathbf{p})X(\mathbf{x})} = \frac{\partial \log P(\mathbf{p})}{\partial \log p_\omega} = \frac{\partial \log X(\mathbf{x})}{\partial \log x_\omega}.$$

Each monopolistically competitive firm $\omega \in \Omega$ produces its own variety with the marginal cost, ψ_ω. We allow for firms (and varieties their produce) to be heterogenous in many dimensions. They may differ not only in their marginal costs, ψ_ω, but also in the way in which they enter in $X(\mathbf{x})$ or $P(\mathbf{p})$, so that they face different demand curves. (Note that we do not impose symmetry in the production technology.)

In this paper, we treat Ω as given. We also treat various sources of heterogeneity, including the marginal cost, as given. This is because the goal of this paper is *not* to develop a particular monopolistically competitive model to address a particular question, but rather to enrich our set of tools that can be applied to a wide range of monopolistic competition models. This is also the reason why we allow the firms to be heterogenous in many dimensions, except that they share the common, constant pass-through rate.[5]

In each of the three classes of homothetic demand systems discussed in the next three sections, the price elasticity of demand for each $\omega \in \Omega$ can be written as $\zeta_\omega(p_\omega/\mathcal{A}(\mathbf{p}))$, where $\mathcal{A}(\mathbf{p})$ is a price aggregator, which is linear homogeneous in $\mathbf{p}$, and common across all $\omega \in \Omega$. Thus, the familiar Lerner formula for the profit-maximizing price for ω, p_ω, can be written as

$$p_\omega \left[1 - \frac{1}{\zeta_\omega(p_\omega/\mathcal{A}(\mathbf{p}))}\right] = \psi_\omega.$$

Under the assumption that the LHS is increasing in p_ω, this can be further rewritten as:

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})} = \mathcal{G}_\omega\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right), \tag{4}$$

---

[5]Of course, when using one of the CoPaTh families as a building block of a particular model, one may want to suppress some, or even all, sources of heterogeneity, depending on the questions to be addressed. One may also want to endogenize Ω by introducing some entry/exit processes, as in most growth or trade models or to fix it exogenously, as in most macro models. One may also want to make some assumptions behind the marginal cost, whether it is an exogenously given characteristic of the firm, or it is a random variable the firm draws from some distribution, similar to Melitz (2004), or whether it may change through changes in the tariffs or exchange rates, as in the exchange rate pass-through literature, or through changes in factor prices, such as labor cost or capital cost, in multi-factor settings. Furthermore, one may also want to be specific about how many monopolistic competitive sectors exist and how they interact with one another as well as other (possibly competitive) sectors. And, of course, one needs to take into account the general equilibrium resource constraints to close the model.

where $\mathcal{G}_\omega(\cdot)$ is increasing. Here, $\mathcal{A} = \mathcal{A}(\mathbf{p})$ is "the average price," against which the relative price of each $\omega \in \Omega$ is measured. Thus, a *single* price aggregator, $\mathcal{A} = \mathcal{A}(\mathbf{p})$, serves as the sufficient statistic for an inverse measure of the "toughness" of competition for all $\omega \in \Omega$. (It should be pointed out that $\mathcal{A}(\mathbf{p})$ is not necessarily the same with the unit cost function, $P(\mathbf{p})$. The three classes differ in terms of what the relevant price aggregator $\mathcal{A} = \mathcal{A}(\mathbf{p})$ is.)

This also means that the markup rate of each firm, μ_ω, and the price elasticity of demand it faces, ζ_ω, can be written as functions of $\psi_\omega/\mathcal{A}(\mathbf{p})$:

$$\mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\mathcal{G}_\omega(\psi_\omega/\mathcal{A}(\mathbf{p}))}{\psi_\omega/\mathcal{A}(\mathbf{p})}; \quad \zeta_\omega\left(\frac{p_\omega}{\mathcal{A}(\mathbf{p})}\right) = \zeta_\omega\left(\mathcal{G}_\omega\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right)\right) = \frac{1}{1 - \frac{\psi_\omega/\mathcal{A}(\mathbf{p})}{\mathcal{G}_\omega(\psi_\omega/\mathcal{A}(\mathbf{p}))}}.$$

Furthermore, the firm-level pass-through rate, ρ_ω, is equal to the elasticity of $\mathcal{G}_\omega(\cdot)$, and hence also a function of $\psi_\omega/\mathcal{A}(\mathbf{p})$:

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = \frac{\partial \ln(p_\omega/\mathcal{A}(\mathbf{p}))}{\partial \ln(\psi_\omega/\mathcal{A}(\mathbf{p}))} = \frac{d \ln \mathcal{G}_\omega(\psi_\omega/\mathcal{A}(\mathbf{p}))}{d \ln(\psi_\omega/\mathcal{A}(\mathbf{p}))} \equiv \mathcal{E}_{\mathcal{G}_\omega}\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right) > 0,$$

where the notation, $\mathcal{E}_f(\xi) \equiv \frac{d \ln f(\xi)}{d \ln \xi} = \frac{\xi f'(\xi)}{f(\xi)}$, denotes the elasticity of function, $f: \mathbb{R}_{++} \to \mathbb{R}_{++}$, with respect to its argument. Likewise, the rate of change in the markup rate in response to the rate of change in its own marginal cost,

$$\frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \frac{\partial \ln(p_\omega/\psi_\omega)}{\partial \ln \psi_\omega} = \rho_\omega - 1 = \mathcal{E}_{\mathcal{G}_\omega}\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right) - 1,$$

is also a function of $\psi_\omega/\mathcal{A}(\mathbf{p})$.

In the next three sections, we will characterize the parametric families of demand systems within each of the three classes, which generate eq.(4) of the following form,

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})} = \mathcal{G}_\omega\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right) = (\beta_\omega)^{1-\rho}\left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right)^\rho,$$

with $0 < \rho \leq 1$, where $\sigma_\omega > 1$ and $\beta_\omega > 0$ can be interpreted as the firm-specific markup shifter, and price shifter, respectively. This expression can be further simplified to eq.(1), which is reproduced here:

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} = \left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega}\right)^\rho. \tag{5}$$

The pass-through rate is thus constant and uniform across varieties:

$$0 < \rho_\omega = \mathcal{E}_{\mathcal{G}_\omega}\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right) = \rho \leq 1.$$

We shall call this class of homothetic demand systems, CoPaTh. Clearly, the CES demand system is its special case with $\rho = 1$ and $\sigma_\omega = \sigma$.

For the case of the unitary (or complete) pass-through rate, $\rho = 1$, eq.(5) is simplified to:

$$p_\omega = \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\psi_\omega$$

with the constant (but not necessarily common across firms) markup rates and price elasticities:

$$\mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma_\omega}{\sigma_\omega - 1} \Leftrightarrow \zeta_\omega = \sigma_\omega.$$

We shall call this subfamily of CoPaTh, CPE for Constant Price Elasticity. Clearly, CES is a special case of CPE with $\sigma_\omega = \sigma$.

For the incomplete pass-through case, $0 < \rho < 1$, it is convenient to define

$$\bar{\beta}_\omega \equiv \beta_\omega\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{\rho}{1-\rho}} < \infty,$$

so that one could express eq.(5) as:

$$\bar{\beta}_\omega > \frac{p_\omega}{\mathcal{A}(\mathbf{p})} = \mathcal{G}_\omega\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right) = (\bar{\beta}_\omega)^{1-\rho}\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right)^\rho > \frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}$$

and the markup rate as:

$$\mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \left(\frac{\mathcal{A}(\mathbf{p})\bar{\beta}_\omega}{\psi_\omega}\right)^{1-\rho} > 1$$

for $\psi_\omega < \mathcal{A}(\mathbf{p})\bar{\beta}_\omega < \infty$. Thus, the markup rate is decreasing in ψ_ω at the rate equal to $\partial \ln \mu_\omega / \partial \ln \psi_\omega = \rho - 1 < 0$ and it converges to one, as $\psi_\omega \to \mathcal{A}(\mathbf{p})\bar{\beta}_\omega < \infty$. Thus, the firm can earn positive profit only for $\psi_\omega < \mathcal{A}(\mathbf{p})\bar{\beta}_\omega < \infty$, which suggests that $\bar{p}_\omega \equiv \mathcal{A}(\mathbf{p})\bar{\beta}_\omega$ is the choke price, which is linear homogeneous in **p**. With this notation for the choke price, eq.(5) can also be written as:

$$\bar{p}_\omega > p_\omega = (\bar{p}_\omega)^{1-\rho}(\psi_\omega)^\rho > \psi_\omega,$$

so that the price of each ω, p_ω, is the weighted geometric average of its own marginal cost, ψ_ω, and its own choke price, $\bar{p}_\omega$. Hence, it is log-linear in ψ_ω and $\bar{p}_\omega$:

$$\ln p_\omega = (1-\rho)\ln \bar{p}_\omega + \rho \ln \psi_\omega.$$

For (the inverse of) the price elasticity of demand,

$$\frac{1}{\zeta_\omega} = 1 - \left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\bar{\beta}_\omega}\right)^{1-\rho} = 1 - \left(\frac{\psi_\omega}{\bar{p}_\omega}\right)^{1-\rho} = 1 - \left(\frac{p_\omega}{\bar{p}_\omega}\right)^{\frac{1-\rho}{\rho}} > 0$$

for $\psi_\omega < p_\omega < \bar{p}_\omega < \infty$. Thus, the price elasticity is monotonically increasing in ψ_ω and converges to infinity as $\psi_\omega \to \bar{p}_\omega < \infty$, and hence $p_\omega \to \bar{p}_\omega < \infty$.

These CoPaTh demand systems offer natural, flexible, and tractable extensions of the CES demand system for the following reasons.

First, under eq. (5), the relative price of two varieties, $\omega_1, \omega_2 \in \Omega$, is given by

$$\frac{p_{\omega_1}}{p_{\omega_2}} = \left(\frac{\beta_{\omega_1}}{\beta_{\omega_2}}\right)^{1-\rho} \left(\frac{\sigma_{\omega_1}/(\sigma_{\omega_1}-1)}{\sigma_{\omega_2}/(\sigma_{\omega_2}-1)}\frac{\psi_{\omega_1}}{\psi_{\omega_2}}\right)^{\rho}, \tag{6}$$

while their relative markup rate is

$$\frac{\mu_{\omega_1}}{\mu_{\omega_2}} = \left(\frac{\sigma_{\omega_1}/(\sigma_{\omega_1}-1)}{\sigma_{\omega_2}/(\sigma_{\omega_2}-1)}\right)^{\rho} \left(\frac{\beta_{\omega_1}/\psi_{\omega_1}}{\beta_{\omega_2}/\psi_{\omega_2}}\right)^{1-\rho}. \tag{7}$$

Note that they are both independent of $\mathcal{A} = \mathcal{A}(\mathbf{p})$.

Second, even though $\mathcal{A} = \mathcal{A}(\mathbf{p})$ does not affect the relative prices, it has nontrivial implications on the relative performance of the firms. Indeed, when we derive the expressions for the relative revenue and the relative profit of $\omega_1, \omega_2 \in \Omega$, in the next three sections, it turns out that they take the form, for $\rho = 1$,

$$\frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}} = \frac{\gamma_{\omega_1}\beta_{\omega_1}\left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1}-1}\frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p})\beta_{\omega_1}}\right)^{1-\sigma_{\omega_1}}}{\gamma_{\omega_2}\beta_{\omega_2}\left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2}-1}\frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p})\beta_{\omega_2}}\right)^{1-\sigma_{\omega_2}}} \propto [\mathcal{A}(\mathbf{p})]^{\sigma_{\omega_1}-\sigma_{\omega_2}} \tag{8}$$

$$\frac{\pi_{\omega_1}}{\pi_{\omega_2}} = \frac{\frac{\gamma_{\omega_1}\beta_{\omega_1}}{\sigma_{\omega_1}}\left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1}-1}\frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p})\beta_{\omega_1}}\right)^{1-\sigma_{\omega_1}}}{\frac{\gamma_{\omega_2}\beta_{\omega_2}}{\sigma_{\omega_2}}\left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2}-1}\frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p})\beta_{\omega_2}}\right)^{1-\sigma_{\omega_2}}} \propto [\mathcal{A}(\mathbf{p})]^{\sigma_{\omega_1}-\sigma_{\omega_2}}. \tag{9}$$

where $\gamma_\omega > 0$ is the quantity shifter, i.e., the parameter that affects the demand curve for $\omega \in \Omega$ only multiplicatively, and hence does not affect the price elasticity of demand. Eqs.(8)- (9) clearly show that a decline in $\mathcal{A}$ reduces the relative revenue and the relative profit of ω_1, if and only if $\zeta_{\omega_1} = \sigma_{\omega_1} > \zeta_{\omega_2} = \sigma_{\omega_2}$. For $0 < \rho < 1$, the expressions for the relative revenue and the relative profit are:

$$\frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}}\frac{\bar{\beta}_{\omega_1}}{\bar{\beta}_{\omega_2}}\left(\frac{\sigma_{\omega_1}-1}{\sigma_{\omega_2}-1}\right)^{\frac{\rho}{1-\rho}}\left[\frac{1-\left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}{1-\left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}\right]^{\frac{\rho}{1-\rho}} \tag{10}$$

$$= \frac{\gamma_{\omega_1}\beta_{\omega_1}}{\gamma_{\omega_2}\beta_{\omega_2}}\left[\frac{\sigma_{\omega_1}-(\sigma_{\omega_1}-1)\left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1}-1}\frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p})\beta_{\omega_1}}\right)^{1-\rho}}{\sigma_{\omega_2}-(\sigma_{\omega_2}-1)\left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2}-1}\frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p})\beta_{\omega_2}}\right)^{1-\rho}}\right]^{\frac{\rho}{1-\rho}}$$

$$\frac{\pi_{\omega_1}}{\pi_{\omega_2}} = \frac{p_{\omega_1}x_{\omega_1}/\zeta_{\omega_1}}{p_{\omega_2}x_{\omega_2}/\zeta_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}}\frac{\bar{\beta}_{\omega_1}}{\bar{\beta}_{\omega_2}}\left(\frac{\sigma_{\omega_1}-1}{\sigma_{\omega_2}-1}\right)^{\frac{\rho}{1-\rho}}\left[\frac{1-\left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}{1-\left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}} \tag{11}$$

$$= \frac{\frac{\gamma_{\omega_1}\beta_{\omega_1}}{\sigma_{\omega_1}}}{\frac{\gamma_{\omega_2}\beta_{\omega_2}}{\sigma_{\omega_2}}}\left[\frac{\sigma_{\omega_1}-(\sigma_{\omega_1}-1)\left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1}-1}\frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p})\beta_{\omega_1}}\right)^{1-\rho}}{\sigma_{\omega_2}-(\sigma_{\omega_2}-1)\left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2}-1}\frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p})\beta_{\omega_2}}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}.$$

One could easily verify that, as $\rho \to 1$, eq.(10) converges to eq.(8), and eq.(11) converges to eq.(9).[6] Furthermore, eqs.(10)-(11) imply [7]

$$sgn\left\{\frac{\partial}{\partial\mathcal{A}}\left(\frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}}\right)\right\} = sgn\left\{\frac{\partial}{\partial\mathcal{A}}\left(\frac{\pi_{\omega_1}}{\pi_{\omega_2}}\right)\right\} = sgn\{\zeta_{\omega_1}-\zeta_{\omega_2}\},$$

hence tougher competition, a reduction in $\mathcal{A} = \mathcal{A}(\mathbf{p})$, causes a disproportionately larger decline in the revenue and the profit among firms that face higher price elasticity and hence those firms that set lower markup rates. It should be pointed that these firms with lower markup rates are not necessarily smaller or less productive, because firms can be heterogenous in many dimensions. If firms are heterogenous only in the marginal cost, ψ_ω, and identical in the quantity shifter, γ_ω, the price shifter, β_ω, and the markup shifter, σ_ω, then less productive firms are smaller, and face higher price elasticities, and hence suffer disproportionately more from tougher competition under the incomplete pass-through ($0 < \rho < 1$).

3. Constant Pass-Through under H.S.A.

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[6] This follows from $\lim_{\rho\to 1}[\sigma-(\sigma-1)\xi^{1-\rho}]^{\frac{1}{1-\rho}} = \xi^{1-\sigma}$, which in turn follows from applying L'Hospital's rule to $\lim_{\rho\to 1}\frac{\ln[\sigma-(\sigma-1)\xi^{1-\rho}]}{1-\rho} = \lim_{\rho\to 1}\frac{-(\sigma-1)\xi^{1-\rho}\ln(\xi)}{\sigma-(\sigma-1)\xi^{1-\rho}} = (1-\sigma)\ln\xi$.

[7] This is because $1/\zeta_\omega = 1-\left(\psi_\omega/\mathcal{A}\bar{\beta}_\omega\right)^{1-\rho}$ for $0 < \rho < 1$ and $\psi_\omega/\mathcal{A}\bar{\beta}_\omega < 1$ is log-supermodular in $\psi_\omega/\bar{\beta}_\omega$ and $\mathcal{A}$, and decreasing in $\psi_\omega/\mathcal{A}\bar{\beta}_\omega$.

3.1 H.S.A. Demand System

A CRS technology, $X = X(\mathbf{x})$, or its unit cost function, $P = P(\mathbf{p})$, is called *homothetic with a single aggregator* (H.S.A.) if the market share of ω can be written as:

$$\frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right). \tag{12}$$

Here, $s_\omega(\cdot): \mathbb{R}_{++} \to \mathbb{R}_+$ is the *market share function of* $\omega \in \Omega$, which is twice continuously differentiable, and *strictly decreasing*, as long as $s_\omega(z) > 0$, with $\lim_{z\to 0} s_\omega(z) = \infty$ and $\lim_{z\to\bar{\beta}_\omega} s_\omega(z) = 0$, where $\bar{\beta}_\omega \equiv \inf\{z > 0 | s_\omega(z) = 0\}$, which can be finite or infinite, and $A(\mathbf{p})$ is linear homogenous in $\mathbf{p}$, defined implicitly and uniquely by

$$\int_\Omega s_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega = 1, \tag{13}$$

which ensures, by construction, that the market shares of all inputs are added up to one. By integrating eq.(12), one could verify that the unit cost function, $P(\mathbf{p})$, is related to $A(\mathbf{p})$, as:

$$\ln\left(\frac{P(\mathbf{p})}{A(\mathbf{p})}\right) = const. - \int_\Omega \left[\int_{p_\omega/A(\mathbf{p})}^{\bar{\beta}_\omega} \frac{s_\omega(\xi)}{\xi} d\xi\right] d\omega. \tag{14}$$

Note that the RHS of eq.(14) generally depends on $\mathbf{p}$, and hence, $P(\mathbf{p}) \neq cA(\mathbf{p})$ for any constant $c > 0$. In fact, CES is the only case where $P(\mathbf{p})/A(\mathbf{p})$ is a constant.[8]

Eqs.(12)-(13) state that the market share of ω is decreasing in its *relative price*, $p_\omega/A(\mathbf{p})$, which is defined as its own price, p_ω, divided by the *common price aggregator,* $A(\mathbf{p})$. Notice that $A(\mathbf{p})$ is independent of ω. Thus, it is a *common* measure of the "toughness" of competition for all varieties, as it captures "the average price" against which the relative prices of *all* inputs are measured. In other words, one could keep track of all the cross-price effects in the demand system by looking at a single aggregator, $A(\mathbf{p})$, which is the key feature of H.S.A. The monotonicity of $s_\omega(\cdot)$, combined with the assumptions, $\lim_{z\to 0} s_\omega(z) = \infty$ and $\lim_{z\to\bar{\beta}_\omega} s_\omega(z) = 0$ ensures that $A(\mathbf{p})$ is well-defined by eq.(13) for any arbitrarily small positive measure of Ω.

Note also that we allow for the possibility of $\bar{\beta}_\omega < \infty$, which means the existence of the choke (relative) price for ω. That is, for $p_\omega > \bar{p}_\omega \equiv \bar{\beta}_\omega A(\mathbf{p})$, demand for ω is zero. If $\bar{\beta}_\omega = \infty$, the choke price does not exist and demand for ω always remains positive for any $A = A(\mathbf{p}) > 0$.

[8] See Matsuyama and Ushchev (2017; Proposition 1-iii))

3.2 Monopolistically Competitive Firms under H.S.A.

Each $\omega \in \Omega$ is produced by a single monopolistically competitive firm, also indexed by $\omega \in \Omega$, with the marginal cost, ψ_ω. Its profit is

$$\pi_\omega = (p_\omega - \psi_\omega)x_\omega = \left(1 - \frac{\psi_\omega}{p_\omega}\right)p_\omega x_\omega = \left(1 - \frac{\psi_\omega/A(\mathbf{p})}{z_\omega}\right)s_\omega(z_\omega)E$$

where $z_\omega = p_\omega/A(\mathbf{p})$ is the relative price and $E = \mathbf{px} = P(\mathbf{p})X(\mathbf{x})$ is the total spending. Firm ω chooses its price, p_ω or equivalently $z_\omega = p_\omega/A(\mathbf{p})$, to maximize π_ω, taking the aggregate variables, $A = A(\mathbf{p})$ and E as given. The FOC is

$$z_\omega\left(1 - \frac{1}{\zeta_\omega(z_\omega)}\right) = \frac{\psi_\omega}{A(\mathbf{p})} \tag{15}$$

where $\zeta_\omega\colon (0, \bar{\beta}_\omega) \to (1, \infty)$ is defined by:

$$\zeta_\omega(z) \equiv 1 - \frac{z s'_\omega(z)}{s_\omega(z)} > 1.$$

Note that $\zeta_\omega(\cdot)$ is continuously differentiable for $z \in \left(0, \bar{\beta}_\omega\right)$, and $\lim_{z\to\bar{\beta}_\omega} \zeta_\omega(z) = \infty$ if $\bar{\beta}_\omega < \infty$. Conversely, from any continuously differentiable $\zeta_\omega\colon (0, \bar{\beta}_\omega) \to (1, \infty)$, satisfying $\lim_{z\to\bar{\beta}_\omega} \zeta_\omega(z) = \infty$ if $\bar{\beta}_\omega < \infty$, one could recover the market share function as follows:

$$s_\omega(z) = \exp\left[\int_{c_\omega}^{z} \frac{1 - \zeta_\omega(\xi)}{\xi} d\xi\right],$$

where $c_\omega \in (0, \bar{\beta}_\omega)$ is a constant, which means that $\zeta_\omega(\cdot)$ determines the market share function, $s_\omega(\cdot)$, up to a positive scalar multiplier.

Generally, the FOC is a necessary condition for the global optimum. The following assumption ensures that the FOC is also sufficient for the global optimum.

Assumption S1: *For all $\omega \in \Omega$ and all $z \in \left(0, \bar{\beta}_\omega\right)$,*

$$\frac{d}{dz}\left(z\left[1 - \frac{1}{\zeta_\omega(z)}\right]\right) = \frac{1}{\zeta_\omega(z)}\left[\zeta_\omega(z) - 1 + \frac{z\zeta'_\omega(z)}{\zeta_\omega(z)}\right] > 0.$$

Under **S1**, the LHS of eq.(15) is strictly increasing in z_ω. Hence, eq.(15) determines the unique profit-maximizing relative price for each firm, as an increasing function of $\psi_\omega/A(\mathbf{p})$

$$\frac{p_\omega}{A(\mathbf{p})} = z_\omega = Z_\omega\left(\frac{\psi_\omega}{A(\mathbf{p})}\right) \in \left(0, \bar{\beta}_\omega\right)$$

with

$$Z'_\omega\left(\frac{\psi_\omega}{A}\right) = \left.\frac{\zeta_\omega(z)}{\zeta_\omega(z) - 1 + \frac{z\zeta'_\omega(z)}{\zeta_\omega(z)}}\right|_{z=Z_\omega(\psi_\omega/A)} > 0,$$

which shows that, with **S1,** the pricing rule under H.S.A. takes the form of eq.(4) with $\mathcal{A}(\mathbf{p}) = A(\mathbf{p})$ and $\mathcal{G}_\omega(\cdot) = Z_\omega(\cdot)$. Recall that the pass-through rate is the elasticity of this function, so that

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = \mathcal{E}_{Z_\omega}\left(\frac{\psi_\omega}{A}\right) = \frac{(\psi_\omega/A)Z'_\omega(\psi_\omega/A)}{Z_\omega(\psi_\omega/A)} = Z'_\omega(\psi_\omega/A)\left.\left(1 - \frac{1}{\zeta_\omega(z)}\right)\right|_{z=Z_\omega(\psi_\omega/A)}$$

$$= \left.\frac{\zeta_\omega(z) - 1}{\zeta_\omega(z) - 1 + \frac{z\zeta'_\omega(z)}{\zeta_\omega(z)}}\right|_{z=Z_\omega(\psi_\omega/A)} > 0,$$

which implies

$$\rho_\omega = \mathcal{E}_{Z_\omega}\left(\frac{\psi_\omega}{A}\right) \lesseqqgtr 1 \Leftrightarrow \zeta'_\omega\left(Z_\omega\left(\frac{\psi_\omega}{A}\right)\right) \gtreqqless 0.$$

and hence

$$\frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \frac{\partial \ln(p_\omega/\psi_\omega)}{\partial \ln \psi_\omega} = \rho_\omega - 1 \lesseqqgtr 0 \Leftrightarrow \zeta'_\omega\left(Z_\omega\left(\frac{\psi_\omega}{A}\right)\right) \gtreqqless 0.$$

Note also that, using eq.(15), the maximized profit is now written as:

$$\pi_\omega = (p_\omega - \psi_\omega)x_\omega = \frac{p_\omega x_\omega}{\zeta_\omega(z_\omega)} = \frac{s_\omega(z_\omega)}{\zeta_\omega(z_\omega)}P(\mathbf{p})X(\mathbf{x}). \tag{16}$$

3.3 Constant Pass-Through Families of H.S.A.

We now turn to the cases where the pass-through rate is constant and common across varieties, $\rho_\omega = \rho$. In the first and second cases, $\rho = 1$, hence the pass-through is complete. In the third case, $0 < \rho < 1$, hence the pass-through is incomplete.

Constant Elasticity of Substitution (CES): For $\gamma_\omega > 0$, $\beta_\omega > 0$, and $\sigma > 1$,

$$s_\omega(z) = \gamma_\omega \beta_\omega \left(\frac{z}{\beta_\omega}\right)^{1-\sigma} \Longrightarrow \zeta_\omega(z) = \sigma$$

In this case, the choke price does not exist, i.e., $\bar{\beta}_\omega = \infty$, and

$$A(\mathbf{p}) = \left[\int_{\Omega} \gamma_\omega {\beta_\omega}^\sigma (p_\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

and

$$p_\omega\left(1-\frac{1}{\sigma}\right) = \psi_\omega;\ \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma}{\sigma-1}.$$

Hence, this corresponds to the case of $\sigma_\omega = \sigma$ and $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the (common) markup rate is constant. The relative price, the relative revenue, and the relative profits of two firms, $\omega_1, \omega_2 \in \Omega$, are

$$\frac{p_{\omega_1}}{p_{\omega_2}} = \frac{\psi_{\omega_1}}{\psi_{\omega_2}};\ \frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}}\frac{\beta_{\omega_1}}{\beta_{\omega_2}}\left(\frac{\psi_{\omega_1}/\beta_{\omega_1}}{\psi_{\omega_2}/\beta_{\omega_2}}\right)^{1-\sigma} = \frac{\pi_{\omega_1}}{\pi_{\omega_2}},$$

which correspond to the case of $\sigma_\omega = \sigma$ in eqs.(8)-(9), and all of them are independent of $A(\mathbf{p})$.

Constant (but Differential) Price Elasticity (CPE): For $\gamma_\omega > 0$, $\beta_\omega > 0$, and $\sigma_\omega > 1$,

$$s_\omega(z) = \gamma_\omega\beta_\omega\left(\frac{z}{\beta_\omega}\right)^{1-\sigma_\omega} \Longrightarrow \zeta_\omega(z) = \sigma_\omega$$

In this case, the choke price does not exist, i.e., $\bar{\beta}_\omega = \infty$ and, $A = A(\mathbf{p})$ is the uniquely solution to:

$$\int_{\Omega} \gamma_\omega(\beta_\omega)^{\sigma_\omega}\left(\frac{p_\omega}{A}\right)^{1-\sigma_\omega} d\omega = 1,$$

but it does not have a closed-form solution. From the pricing formula,

$$p_\omega\left(1-\frac{1}{\sigma_\omega}\right) = \psi_\omega, \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma_\omega}{\sigma_\omega - 1},$$

Hence, this corresponds to the case of $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the markup rates are constant, though no longer uniform across the firms:

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1;\ \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \rho_\omega - 1 = 0.$$

Then, using eq.(16), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(8)-(9) with $\mathcal{A}(\mathbf{p}) = A(\mathbf{p})$. In particular, this means that a reduction in $A = A(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\sigma_{\omega_1} > \sigma_{\omega_2}$.

Incomplete Constant (and Common) Pass-Through (CoPaTh): $(0 < \rho < 1)$

For $\Delta > 0$, $\sigma > 1$, $\varepsilon > 0$, $\beta_\omega > 0$, and $\gamma_\omega > 0$, let

$$\bar{\beta}_\omega \equiv \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{1/\Delta}$$

and

$$s_\omega(z) = \begin{cases} \gamma_\omega \beta_\omega \left[\sigma_\omega - (\sigma_\omega - 1)\left(\frac{z}{\beta_\omega}\right)^\Delta\right]^{1/\Delta} = \gamma_\omega \bar{\beta}_\omega (\sigma_\omega - 1)^{\frac{1}{\Delta}} \left[1 - \left(\frac{z}{\bar{\beta}_\omega}\right)^\Delta\right]^{1/\Delta} & \text{for } \varepsilon < z < \bar{\beta}_\omega \\ 0 & \text{for } z \geq \bar{\beta}_\omega \end{cases} \tag{17}$$

so that

$$\zeta_\omega(z) = \frac{1}{1 - \left(1 - \frac{1}{\sigma_\omega}\right)\left(\frac{z}{\beta_\omega}\right)^\Delta} = \frac{1}{1 - \left(\frac{z}{\bar{\beta}_\omega}\right)^\Delta}, \qquad \text{for } \varepsilon < z < \bar{\beta}_\omega.$$

which is increasing for $\varepsilon < z < \bar{\beta}_\omega$. Clearly, $s_\omega(z)$ can be extended to $0 < z \leq \varepsilon$ such that $\zeta_\omega(z)$ is increasing and continuously differentiable, $s_\omega(0) = \infty$, and **S1** holds.[9]

Then, by inserting this expression for $\zeta_\omega(z)$ for eq.(15), we obtain the pricing formula,

$$z_\omega = \frac{p_\omega}{A(\mathbf{p})} = (\beta_\omega)^{\frac{\Delta}{1+\Delta}} \left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{A(\mathbf{p})}\right)^{\frac{1}{1+\Delta}} = (\bar{\beta}_\omega)^{\frac{\Delta}{1+\Delta}} \left(\frac{\psi_\omega}{A(\mathbf{p})}\right)^{\frac{1}{1+\Delta}} \quad \text{for } \varepsilon < \frac{p_\omega}{A(\mathbf{p})} < \bar{\beta}_\omega,$$

which is indeed eq.(5), by setting $0 < \rho = 1/(1+\Delta) < 1$, with $\mathcal{A}(\mathbf{p}) = A(\mathbf{p})$, which can be determined by applying eq.(17) to eq.(13).

Then, using eq.(16), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(10)-(11) with $\mathcal{A}(\mathbf{p}) = A(\mathbf{p})$. In particular, this means that a reduction in $A = A(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\zeta_{\omega_1} > \zeta_{\omega_2}$. Thus, as in the CPE case, the firms facing higher price elasticity of demand suffer more from an increase in competition. Unlike in the CPE case, however, the price elasticity is endogenous, because, even after controlling for σ_ω, the firms with lower productivity face higher price elasticity. Note also that the above condition,

[9]We need to define $s_\omega(z)$ separately for $0 < z \leq \varepsilon$ with an arbitrarily small $\varepsilon > 0$, to ensure $s_\omega(0) = \infty$, so that $A(\mathbf{p})$ is well-defined for any arbitrarily small positive measure of Ω. If the support of ψ_ω is bounded away from zero, we could choose ε sufficiently small to ensure $\psi_\omega/A(\mathbf{p}) > \varepsilon$ for all $\omega \in \Omega$. Even if the support of ψ_ω is not bounded away from zero, we could make the fraction of firms with $\psi_\omega/A(\mathbf{p}) \leq \varepsilon$ arbitrarily small making $\varepsilon \searrow 0$. Note also that CPE (and hence CES) is the limit case, as $\Delta \searrow 0$ and $\varepsilon \searrow 0$, while holding $\beta_\omega > 0$ and $\sigma_\omega > 1$ fixed, because $\bar{\beta}_\omega \equiv \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{1/\Delta} \to \infty$; $\zeta_\omega(z) = \frac{\sigma_\omega}{\sigma_\omega - (\sigma_\omega - 1)(z/\beta_\omega)^\Delta} \to \sigma_\omega$, and $s_\omega(z) = \gamma_\omega \beta_\omega [\sigma_\omega - (\sigma_\omega - 1)(z/\beta_\omega)^\Delta]^{1/\Delta} \to \gamma_\omega \beta_\omega (z/\beta_\omega)^{1-\sigma_\omega}$. The last can be shown by applying L'Hospital's rule, $\lim_{\Delta \to 0} \frac{\ln[\sigma_\omega - (\sigma_\omega - 1)(z/\beta_\omega)^\Delta]}{\Delta} = \lim_{\Delta \to 0} \frac{(1-\sigma_\omega)(z/\beta_\omega)^\Delta \log(z/\beta_\omega)}{\sigma_\omega - (\sigma_\omega - 1)(z/\beta_\omega)^\Delta} = \ln\ (z/\beta_\omega)^{1-\sigma_\omega}$.

$\zeta_{\omega_1} > \zeta_{\omega_2}$, implies neither $p_{\omega_1}x_{\omega_1} < p_{\omega_2}x_{\omega_2}$ nor $\pi_{\omega_1} < \pi_{\omega_2}$, unless γ_ω is independent of ω. Hence, smaller firms, measured in the revenue or the profit, do not always suffer more from increased competition.

Before proceeding to H.D.I.A. and H.I.I.A., we point out that there exists an alternative (but equivalent) definition of H.S.A.. That is, $X = X(\mathbf{x})$ or $P = P(\mathbf{p})$ is called *homothetic with a single aggregator* (H.S.A.) if the market share of input ω, as a function of $\mathbf{x}$, can be written as:

$$\frac{p_\omega x_\omega}{\mathbf{px}} = \frac{p_\omega x_\omega}{P(\mathbf{p})X(\mathbf{x})} = \frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega} = s_\omega^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right).$$

Here, $s_\omega^*: \mathbb{R}_{++} \to \mathbb{R}_+$ is the *market share function of* ω and it is assumed to be twice continuously differentiable with $0 < ys_\omega^{*\prime}(y)/s_\omega^*(y) < 1$, $s_\omega^*(0) = 0$ and $s_\omega^*(\infty) = \infty$, and $A^*(\mathbf{x})$ is linear homogenous in $\mathbf{x}$, defined implicitly and uniquely by

$$\int_\Omega s_\omega^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right) d\omega \; = 1,$$

which ensures that the market shares of all inputs are added up to one. Thus, the market share of input ω is a function of its *relative quantity*, defined as its own quantity x_ω divided by the *common quantity aggregator* $A^*(\mathbf{x})$, which is strictly increasing with the elasticity less than one. This common quantity aggregator, $A^*(\mathbf{x})$, is related to the production function, $X(\mathbf{x})$, as follows:

$$\log\left(\frac{X(\mathbf{x})}{A^*(\mathbf{x})}\right) = const. + \int\limits_\Omega \left[\int\limits_0^{x_\omega/A^*(\mathbf{x})} \frac{s_\omega^*(\xi)}{\xi} d\xi\right] d\omega,$$

where the RHS depends on $\mathbf{x}$ and hence $X(\mathbf{x}) \neq cA^*(\mathbf{x})$ with any constant $c > 0$, unless it is CES.

These two alternative definitions of H.S.A. are isomorphic to each other via the one-to-one mapping between $s_\omega(z) \leftrightarrow s_\omega^*(y)$, defined by:

$$s_\omega^*(y) = s_\omega\left(\frac{s_\omega^*(y)}{y}\right); \; s_\omega(z) = s_\omega^*\left(\frac{s_\omega(z)}{z}\right). \tag{18}$$

With this mapping, the relative quantity, $y_\omega \equiv x_\omega/A^*(\mathbf{x})$, and the relative price, $z_\omega \equiv p_\omega/A(\mathbf{p})$, are negatively related as $z_\omega = s_\omega^*(y_\omega)/y_\omega$ and $y_\omega = s_\omega(z_\omega)/z_\omega$, with $\lim_{y\to 0} s_\omega^*(y)/y = s_\omega^{*\prime}(0) \equiv \bar{\beta}_\omega$. Moreover, differentiating either of the two equalities in eq.(18) yields the identity,

$$\zeta_\omega^*(y) \equiv \left[1 - \frac{ys_\omega^{*\prime}(y)}{s_\omega^*(y)}\right]^{-1} = \zeta_\omega(z) \equiv 1 - \frac{zs_\omega'(z)}{s_\omega(z)} > 1,$$

where $\zeta_\omega^*(y)$ is the price elasticity as a function of $y \equiv x/A^*(\mathbf{x})$, which shows that the condition, $0 < ys_\omega^{*\prime}(y)/s_\omega^*(y) < 1$, is equivalent to $s_\omega'(z) < 0$.[10]

Using the mapping eq.(18), one could easily verify that, under this alternative definition of H.S.A., stated in quantity, the complete pass-through family of CPE ($\rho = 1$) can be obtained with

$$s_\omega^*(y) = \gamma_\omega \beta_\omega \left(\frac{y}{\gamma_\omega}\right)^{1-\frac{1}{\sigma_\omega}}$$

for $\gamma_\omega > 0, \beta_\omega > 0$, and $\sigma_\omega > 1$, which contains CES as a special case with $\sigma_\omega = \sigma$. In this case, the choke price does not exist, because $\bar{\beta}_\omega \equiv s_\omega^{*\prime}(0) = \infty$. The constant incomplete pass-through family ($0 < \rho < 1$) can also be represented by, for $\Delta = (1-\rho)/\rho > 0, \sigma > 1, \varepsilon > 0$, $\beta_\omega > 0$, and $\gamma_\omega > 0$, defining

$$\bar{\beta}_\omega \equiv \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{1/\Delta} < \infty$$

and

$$s_\omega^*(y) = \gamma_\omega \beta_\omega \left[\frac{1}{\sigma_\omega} + \left(1 - \frac{1}{\sigma_\omega}\right)\left(\frac{y}{\gamma_\omega}\right)^{-\Delta}\right]^{-1/\Delta} = \gamma_\omega \bar{\beta}_\omega \left[\frac{1}{\sigma_\omega - 1} + \left(\frac{y}{\gamma_\omega}\right)^{-\Delta}\right]^{-1/\Delta},$$

$$\Longrightarrow \zeta_\omega^*(y) \equiv \left[1 - \frac{ys_\omega^{*\prime}(y)}{s_\omega^*(y)}\right]^{-1} = 1 + (\sigma_\omega - 1)\left(\frac{y}{\gamma_\omega}\right)^{-\Delta} > 1$$

for $y < 1/\varepsilon$, for an arbitrarily small $\varepsilon > 0$, with $s_\omega^*(\cdot)$ extended for $y \geq 1/\varepsilon$ such that $s_\omega^*(\infty) = \infty$.[11]

4. Constant Pass-Through under H.D.I.A.

4.1.H.D.I.A. Demand System

We call a CRS technology, $X = X(\mathbf{x})$ or $P = P(\mathbf{p})$, *homothetic with direct implicit additivity* (H.D.I.A.)[12] if $X = X(\mathbf{x})$ can be defined implicitly by:

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[10]This isomorphism has been shown for the broader class of H.S.A., including the case of gross complements, where $s_\omega^{*\prime}(y) < 0 \Leftrightarrow 0 < zs_\omega'(z)/s_\omega(z) < 1$ holds: see Matsuyama and Ushchev (2017, Section 3, Remark 3).

[11]We need to define $s_\omega^*(\cdot)$ separately for $y \geq 1/\varepsilon$ with an arbitrarily small $\varepsilon > 0$, to ensure $s_\omega^*(\infty) = \infty$, and hence $A^*(\mathbf{x})$ is well-defined for any arbitrarily small positive measure of Ω. As in the price representation of H.S.A., the fraction of firms operating in the range $y_\omega > 1/\varepsilon$ becomes negligible for a sufficiently small $\varepsilon > 0$.

[12]More generally, a function, $X = X(\mathbf{x})$ satisfies *direct implicit additivity* (D.I.A.) if it is defined implicitly by $\int_\Omega \tilde{\phi}_\omega(x_\omega, X)d\omega = 1$. See Hanoch (1975; Section 2). H.D.I.A. is the homothetic restriction of D.I.A., with $\tilde{\phi}_\omega(x_\omega, X) = \phi_\omega(x_\omega/X)$. In contrast, a function, $X = X(\mathbf{x})$ satisfies *direct explicit additivity* (D.E.A.) if it can be

$$\int_{\Omega} \phi_\omega\left(\frac{x_\omega}{X}\right) d\omega = 1, \tag{19}$$

where $\phi_\omega(\cdot): \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, strictly concave, and at least thrice continuously differentiable with $\phi_\omega(0) = 0$ and $\phi_\omega(\infty) = \infty$. The monotonicity of $\phi_\omega(\cdot)$, combined with $\phi_\omega(0) = 0$ and $\phi_\omega(\infty) = \infty$, ensures that $X = X(\mathbf{x})$ is well-defined by eq.(19) for any positive measure of Ω.

The cost minimization problem, eq.(2) subject to eq.(19) implies that the inverse demand curve for each $\omega \in \Omega$ can be written as:

$$\frac{p_\omega}{B(\mathbf{p})} = \phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right), \tag{20}$$

where $B(\mathbf{p})$ is the Lagrange multiplier associated with eq. (19), and linear homogenous in $\mathbf{p}$, given by

$$\int_{\Omega} \phi_\omega\left((\phi'_\omega)^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)\right) d\omega \equiv 1. \tag{21}$$

From eq.(20), the market share of each $\omega \in \Omega$ can be expressed either as a function of $\mathbf{p}$, or as a function of $\mathbf{x}$, as follows:

$$\frac{p_\omega}{P(\mathbf{p})}\frac{x_\omega}{X(\mathbf{x})} = \frac{p_\omega}{P(\mathbf{p})}(\phi'_\omega)^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right) = \frac{x_\omega}{C^*(\mathbf{x})}\phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right),$$

where the unit cost function is given by:

$$P(\mathbf{p}) = \int_{\Omega} p_\omega(\phi'_\omega)^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right) d\omega,$$

and $C^*(\mathbf{x})$ is a linear homogenous function of $\mathbf{x}$, given by

$$C^*(\mathbf{x}) \equiv \int_{\Omega} x_\omega\phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) d\omega,$$

and it satisfies

$$\frac{P(\mathbf{p})}{B(\mathbf{p})} = \int_{\Omega} \frac{p_\omega}{B(\mathbf{p})}(\phi'_\omega)^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right) d\omega = \int_{\Omega} \phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right)\frac{x_\omega}{X(\mathbf{x})} d\omega = \frac{C^*(\mathbf{x})}{X(\mathbf{x})}.$$

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written as $X = \mathcal{M}\left(\int_{\Omega} \phi_\omega(x_\omega)d\omega\right)$, where $\mathcal{M}(\cdot)$ is a monotone transformation. Even though both D.E.A. and H.D.I.A. are both subclasses of D.I.A., they are disjoint with the sole exception of CES.

These two expressions for the market share under H.D.I.A. show that it is a function of the two relative prices, $p_\omega/P(\mathbf{p})$ and $p_\omega/B(\mathbf{p})$, or a function of the two relative quantities, $x_\omega/X(\mathbf{x})$ and $x_\omega/C^*(\mathbf{x})$, unless $P(\mathbf{p})/B(\mathbf{p}) = C^*(\mathbf{x})/X(\mathbf{x}) = c > 0$ for a constant c, which occurs if and only if it is CES. Thus, H.D.I.A. and H.S.A. do not overlap with the sole exception of CES. [13]

4.2 Monopolistically Competitive Firms under H.D.I.A.

From the inverse demand curve, eq.(20), the profit of firm $\omega \in \Omega$ is given by:

$$\pi_\omega = (p_\omega - \psi_\omega)x_\omega = \left(B(\mathbf{p})\phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) - \psi_\omega\right)x_\omega,$$

which firm ω chooses its output, x_ω, to maximize, taking the aggregate variables, $B = B(\mathbf{p})$ and $X = X(\mathbf{x})$ as given. Or equivalently, it chooses $y_\omega \equiv x_\omega/X(\mathbf{x})$ to maximize

$$\left(\phi'_\omega(y_\omega) - \frac{\psi_\omega}{B(\mathbf{p})}\right)y_\omega.$$

The FOC is:

$$\phi'_\omega(y_\omega) + y_\omega\phi''_\omega(y_\omega) = \phi'_\omega(y_\omega)\left[1 - \frac{1}{\zeta^D_\omega(y_\omega)}\right] = \frac{\psi_\omega}{B(\mathbf{p})} \tag{22}$$

where

$$\zeta^D_\omega(y) \equiv -\frac{\phi'_\omega(y)}{y\phi''_\omega(y)} > 1, \tag{23}$$

the inverse of the elasticity of $\phi'_\omega(\cdot)$ in its absolute value, is the price elasticity of demand, expressed as a (continuously differentiable) function of y. The monotonicity and concavity of $\phi_\omega(\cdot)$ jointly ensure $\zeta^D_\omega(\cdot) > 0$. In addition, it is necessary to assume that $\zeta^D_\omega(\cdot) > 1$ to ensure that the FOC is well-defined. Conversely, from any continuously differentiable $\zeta^D_\omega(\cdot) > 1$, one could recover $\phi'_\omega(\cdot)$ and hence $\phi_\omega(\cdot)$ up to a positive scalar multiplier as:

$$\phi'_\omega(y) = \exp\left[-\int_{c^D_\omega}^{y}\frac{d\xi'}{\zeta^D_\omega(\xi')\xi'}\right] \Rightarrow \phi_\omega(y) = \int_0^{y}\exp\left[-\int_{c^D_\omega}^{\xi}\frac{d\xi'}{\zeta^D_\omega(\xi')\xi'}\right]d\xi,$$

where $c^D_\omega > 0$ is a constant. It should be also clear from eq.(20) that the choke price exists if and only if

$$\bar{\beta}_\omega \equiv \phi'_\omega(0) = \lim_{y\to 0}\exp\left[\int_{y}^{c^D_\omega}\frac{d\xi}{\zeta^D_\omega(\xi)\xi}\right] < \infty,$$

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[13] See Proposition 2-(ii) in Matsuyama and Ushchev (2017).

which implies $\lim_{y \to 0} \zeta_\omega^D(y) = \infty$.

Generally, the FOC is a necessary condition for the global optimum. The following assumption ensures that the FOC is also sufficient for the global optimum.

Assumption D1: *For all* $\omega \in \Omega$ *and all* $y > 0$,

$$\frac{d}{dy}\left\{\phi_\omega'(y)\left[1 - \frac{1}{\zeta_\omega^D(y)}\right]\right\} = \phi_\omega''(y)\left[1 - \frac{1}{\zeta_\omega^D(y)} - \frac{y\zeta_\omega^{D'}(y)}{\zeta_\omega^D(y)}\right] < 0.$$

Under **D1**, the LHS of eq.(22) is strictly decreasing in y_ω. Hence, eq.(22) determines the profit-maximizing relative output for each firm, y_ω, uniquely as a decreasing function of $\psi_\omega/B(\mathbf{p})$

$$y_\omega = \mathcal{Y}_\omega\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)$$

and hence its relative price, $p_\omega/B(\mathbf{p})$, as an increasing function of $\psi_\omega/B(\mathbf{p})$, as follows:

$$\frac{p_\omega}{B(\mathbf{p})} = \phi_\omega'\left(\mathcal{Y}_\omega\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)\right)$$

with

$$\frac{d(p_\omega/B)}{d(\psi_\omega/B)} = \phi_\omega''\left(\mathcal{Y}_\omega\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)\right)\mathcal{Y}_\omega'\left(\frac{\psi_\omega}{B(\mathbf{p})}\right) = \left.\frac{1}{1 - \frac{1}{\zeta_\omega^D(y)} - \frac{y\zeta_\omega^{D'}(y)}{\zeta_\omega^D(y)}}\right|_{y=\mathcal{Y}_\omega(\psi_\omega/B)} > 0,$$

which shows that, with **D1,** the pricing rule under H.D.I.A. takes the form of eq.(4) with $\mathcal{A}(\mathbf{p}) = B(\mathbf{p})$ and $\mathcal{G}_\omega(\cdot) = \phi_\omega'(\mathcal{Y}_\omega(\cdot))$. Recall that the pass-through rate is the elasticity of this function, so that

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = \mathcal{E}_{\phi_\omega'(\mathcal{Y}_\omega)}\left(\frac{\psi_\omega}{B(\mathbf{p})}\right) = \left.\frac{1 - \frac{1}{\zeta_\omega^D(y)}}{1 - \frac{1}{\zeta_\omega^D(y)} - \frac{y\zeta_\omega^{D'}(y)}{\zeta_\omega^D(y)}}\right|_{y=\mathcal{Y}_\omega(\psi_\omega/B(\mathbf{p}))} > 0$$

which implies

$$\rho_\omega = \mathcal{E}_{\phi_\omega'(\mathcal{Y}_\omega)}\left(\frac{\psi_\omega}{B(\mathbf{p})}\right) \lesseqgtr 1 \;\Leftrightarrow \zeta_\omega^{D'}\left(\mathcal{Y}_\omega\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)\right) \lesseqgtr 0.$$

and hence

$$\frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \frac{\partial \ln(p_\omega/\psi_\omega)}{\partial \ln \psi_\omega} = \rho_\omega - 1 \lesseqgtr 0 \Leftrightarrow \zeta_\omega^{D'}\left(\mathcal{Y}_\omega\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)\right) \lesseqgtr 0.$$

Note also that, using eq.(22), the maximized profit is now written as:

$$\pi_\omega = (p_\omega - \psi_\omega)x_\omega = \frac{p_\omega x_\omega}{\zeta_\omega^D(y_\omega)} = \frac{\phi'_\omega(y_\omega)y_\omega}{\zeta_\omega^D(y_\omega)} B(\mathbf{p})X(\mathbf{x}). \tag{24}$$

4.3 Constant Pass-Through Families of H.D.I.A.

We now turn to the cases where the pass-through rate is constant and common across varieties, $\rho_\omega = \rho$. In the first and second classes, $\rho = 1$, hence the pass-through is complete. In the third and fourth class, $0 < \rho < 1$, hence the pass-through is incomplete.

Constant Elasticity of Substitution (CES): For $\beta_\omega > 0,\ \gamma_\omega > 0$, and $\sigma > 1$,

$$\phi_\omega(y) = \left(\frac{\sigma}{\sigma - 1}\right)\gamma_\omega\beta_\omega\left(\frac{y}{\gamma_\omega}\right)^{1-\frac{1}{\sigma}} \Rightarrow \phi'_\omega(y) = \beta_\omega\left(\frac{y}{\gamma_\omega}\right)^{-\frac{1}{\sigma}} \Rightarrow \zeta_\omega^D(y) = \sigma > 1.$$

In this case, the choke price does not exist, i.e., $\bar{\beta}_\omega \equiv \phi'_\omega(0) = \infty$, and

$$X(\mathbf{x}) = \left[\int_\Omega \left(\frac{\sigma}{\sigma - 1}\right)\beta_\omega\gamma_\omega^{\frac{1}{\sigma}}(x_\omega)^{1-\frac{1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

And

$$p_\omega\left(1 - \frac{1}{\sigma}\right) = \psi_\omega;\ \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma}{\sigma - 1},$$

hence, the pass-through rate is unitary and the (common) markup rate is constant.

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1;\ \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \rho_\omega - 1 = 0.$$

Hence, this corresponds to the case of $\sigma_\omega = \sigma$ and $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the (common) markup rate is constant. The relative price, the relative revenue, and the relative profits of two firms, $\omega_1, \omega_2 \in \Omega$, are

$$\frac{p_{\omega_1}}{p_{\omega_2}} = \frac{\psi_{\omega_1}}{\psi_{\omega_2}};\ \frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}}\frac{\beta_{\omega_1}}{\beta_{\omega_2}}\left(\frac{\psi_{\omega_1}/\beta_{\omega_1}}{\psi_{\omega_2}/\beta_{\omega_2}}\right)^{1-\sigma} = \frac{\pi_{\omega_1}}{\pi_{\omega_2}},$$

which correspond to the case of $\sigma_\omega = \sigma$ in eqs.(8)-(9), and all of them are independent of $B(\mathbf{p})$.

Constant (but Differential) Price Elasticity (CPE): For $\beta_\omega > 0, \gamma_\omega > 0$, and $\sigma_\omega > 1$,

$$\phi_\omega(y) = \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\gamma_\omega\beta_\omega\left(\frac{y}{\gamma_\omega}\right)^{1-\frac{1}{\sigma_\omega}} \Rightarrow \phi'_\omega(y) = \beta_\omega\left(\frac{y}{\gamma_\omega}\right)^{-\frac{1}{\sigma_\omega}} \Rightarrow \zeta_\omega^D(y) = \sigma_\omega > 1.$$

In this case, the choked price does not exit, i.e., $\bar{\beta}_\omega \equiv \phi'_\omega(0) = \infty$, and $X = X(\mathbf{x})$ is the unique solution to:

$$\int_\Omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right) \beta_\omega \gamma_\omega^{\frac{1}{\sigma_\omega}} \left(\frac{x_\omega}{X}\right)^{1-\frac{1}{\sigma_\omega}} d\omega = 1,$$

but it does not have a closed-form solution. From the pricing rule,

$$p_\omega\left(1 - \frac{1}{\sigma_\omega}\right) = \psi_\omega;\ \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma_\omega}{\sigma_\omega - 1},$$

Hence, this corresponds to the case of $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the markup rates are constant, though no longer uniform across the firms:

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1;\ \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \rho_\omega - 1 = 0.$$

Then, using eq.(16), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(8)-(9) with $\mathcal{A}(\mathbf{p}) = B(\mathbf{p})$. In particular, this means that a reduction in $B = B(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\sigma_{\omega_1} > \sigma_{\omega_2}$.

Incomplete Constant (and Common) Pass-Through (CoPaTh): ($0 < \rho < 1$)

For $\Delta > 0, \sigma_\omega > 1, \beta_\omega > 0,$ and $\gamma_\omega > 0,$ define

$$\phi'_\omega(0) = \bar{\beta}_\omega \equiv \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{1}{\Delta}} < \infty$$

and

$$\phi_\omega(y) = \bar{\beta}_\omega \int_0^y \left(1 + \frac{1}{\sigma_\omega - 1}\left(\frac{\xi}{\gamma_\omega}\right)^\Delta\right)^{-\frac{1}{\Delta}} d\xi \tag{25}$$

so that

$$\phi'_\omega(y) = \beta_\omega \left(\frac{1}{\sigma_\omega}\left(\frac{y}{\gamma_\omega}\right)^\Delta + \left(1 - \frac{1}{\sigma_\omega}\right)\right)^{-\frac{1}{\Delta}} = \bar{\beta}_\omega \left(1 + \frac{1}{\sigma_\omega - 1}\left(\frac{y}{\gamma_\omega}\right)^\Delta\right)^{-\frac{1}{\Delta}},$$

$$\zeta^D_\omega(y) \equiv -\frac{\phi'_\omega(y)}{y\phi''_\omega(y)} = 1 + (\sigma_\omega - 1)\left(\frac{y}{\gamma_\omega}\right)^{-\Delta} > 1.$$

Clearly, $\phi_\omega(\cdot)$ is increasing, concave, and $\phi_\omega(0) = 0$. With $\zeta_\omega^D(\cdot)$ decreasing, **D1** holds.[14] Then, by inserting this expression for $\zeta_\omega^D(\mathcal{y})$ to eq.(22), we obtain,

$$\frac{p_\omega}{B(\mathbf{p})} = \phi'_\omega(\mathcal{y}_\omega) = (\beta_\omega)^{\frac{\Delta}{1+\Delta}}\left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{B(\mathbf{p})}\right)^{\frac{1}{1+\Delta}} = (\bar{\beta}_\omega)^{\frac{\Delta}{1+\Delta}}\left(\frac{\psi_\omega}{B(\mathbf{p})}\right)^{\frac{1}{1+\Delta}} \quad \text{for } 0 < \frac{\psi_\omega}{B(\mathbf{p})} < \bar{\beta}_\omega,$$

which is indeed eq.(5), by setting $0 < \rho = 1/(1+\Delta) < 1$ and $\mathcal{A}(\mathbf{p}) = B(\mathbf{p})$, which can be determined by applying eq.(25) to eq.(21).

Then, using eq.(24), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(10)-(11) with $\mathcal{A}(\mathbf{p}) = B(\mathbf{p})$. In particular, this means that a reduction in $B = B(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\zeta_{\omega_1} > \zeta_{\omega_2}$. Thus, as in the CPE case, the firms facing higher price elasticity of demand suffer more from an increase in competition. Unlike in the CPE case, however, the price elasticity is partially endogenous, because, even after controlling for σ_ω, the firms with lower productivity face higher price elasticity. Note also that the above condition, $\zeta_{\omega_1} > \zeta_{\omega_2}$, implies neither $p_{\omega_1}x_{\omega_1} < p_{\omega_2}x_{\omega_2}$ nor $\pi_{\omega_1} < \pi_{\omega_2}$, unless γ_ω is independent of ω. Hence, smaller firms, measured in the revenue or the profit, do not always suffer more from increased competition.

5. Constant Pass-Through under H.I.I.A.

5.1 HIIA Demand System

We call CRS technology, $X = X(\mathbf{x})$ or $P = P(\mathbf{p})$, *homothetic with indirect implicit additivity* (H.I.I.A.)[15] if $P = P(\mathbf{p})$ can be defined implicitly by:

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[14] CPE (and hence CES) is the limit case with $\Delta \searrow 0$, while holding $\beta_\omega > 0$ and $\sigma_\omega > 1$ fixed, since $\phi'_\omega(0) = \bar{\beta}_\omega \equiv \beta_\omega\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{1}{\Delta}} \to \infty$; $\zeta_\omega^D(\mathcal{y}) \to \sigma_\omega > 1$; and, since $\lim_{\Delta\to 0}\frac{\ln\left[\frac{1}{\sigma_\omega}\left(\frac{\mathcal{y}}{\gamma_\omega}\right)^\Delta + \left(1 - \frac{1}{\sigma_\omega}\right)\right]}{-\Delta} = \lim_{\Delta\to 0}\frac{-\frac{1}{\sigma_\omega}\left(\frac{\mathcal{y}}{\gamma_\omega}\right)^\Delta \ln(\mathcal{y}/\gamma_\omega)}{\frac{1}{\sigma_\omega}\left(\frac{\mathcal{y}}{\gamma_\omega}\right)^\Delta + \left(1 - \frac{1}{\sigma_\omega}\right)} = -\frac{1}{\sigma_\omega}\ln(\mathcal{y}/\gamma_\omega)$ by applying L'Hospital's rule, $\phi'_\omega(\mathcal{y}) \to \beta_\omega\left(\frac{\mathcal{y}}{\gamma_\omega}\right)^{-\frac{1}{\sigma_\omega}}$, and hence, $\phi_\omega(\mathcal{y}) \to \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\beta_\omega\gamma_\omega^{\frac{1}{\sigma_\omega}}\mathcal{y}^{1-\frac{1}{\sigma_\omega}}$.

[15] More generally, a function, $P = P(\mathbf{p})$, satisfies *indirect implicit additivity* (I.I.A.) if it is defined implicitly by $\int_\Omega \tilde{\theta}_\omega(p_\omega, P)d\omega = 1$. See Hanoch (1975; Section 3). H.I.I.A. is the homothetic restriction of I.I.A., with $\tilde{\theta}_\omega(p_\omega, P) = \theta_\omega(p_\omega/P)$. In contrast, a function, $P = P(\mathbf{p})$, satisfies *indirect explicit additivity* (I.E.A.) if it can be written as $P = \mathcal{M}\left(\int_\Omega \theta_\omega(p_\omega)d\omega\right)$ where $\mathcal{M}(\cdot)$ is a monotone transformation. Even though both I.E.A. and H.I.I.A. are both subclasses of I.I.A., they are disjoint with the sole exception of CES.

$$\int_{\Omega} \theta_{\omega}\left(\frac{p_{\omega}}{P(\mathbf{p})}\right) d\omega = 1, \tag{26}$$

where $\theta_{\omega}(\cdot): \mathbb{R}_{+} \to \mathbb{R}_{+}$ is thrice continuously differentiable, strictly decreasing, and strictly convex, as long as $\theta_{\omega}(z) > 0$ with $\lim_{z\to 0} \theta_{\omega}(z) = \infty$ and $\lim_{z\to \bar{\beta}_{\omega}} \theta_{\omega}(z) = \lim\limits_{z\to \bar{\beta}_{\omega}} \theta'_{\omega}(z) = 0$, where $\bar{\beta}_{\omega} \equiv \inf\{z > 0 | \theta_{\omega}(z) = 0\}$. The monotonicity of $\theta_{\omega}(\cdot)$, combined with $\lim_{z\to 0} \theta_{\omega}(z) = \infty$ and $\lim_{z\to \bar{\beta}_{\omega}} \theta_{\omega}(z) = 0$, ensures that $P = P(\mathbf{x})$ is well-defined by eq.(26) for any positive measure of **Ω.**

The cost minimization problem, eq.(3) subject to eq. (26) implies that the demand curve for each $\omega \in \Omega$ can be written as:

$$\frac{x_{\omega}}{B^{*}(\mathbf{x})} = -\theta'_{\omega}\left(\frac{p_{\omega}}{P(\mathbf{p})}\right) > 0, \tag{27}$$

where $B^{*}(\mathbf{x}) > 0$ is the Lagrange multiplier associated with eq.(26), and linear homogenous in **x**, given by

$$\int_{\Omega} \theta'_{\omega}\left((-\theta'_{\omega})^{-1}\left(\frac{x_{\omega}}{B^{*}(\mathbf{x})}\right)\right) d\omega \equiv 1.$$

From eq.(27), the market share of each input can be written either as a function of **p**, or as a function of **x**, as follows:

$$\frac{p_{\omega}}{P(\mathbf{p})}\frac{x_{\omega}}{X(\mathbf{x})} = -\theta'_{\omega}\left(\frac{p_{\omega}}{P(\mathbf{p})}\right)\frac{p_{\omega}}{C(\mathbf{p})} = (-\theta'_{\omega})^{-1}\left(\frac{x_{\omega}}{B^{*}(\mathbf{x})}\right)\frac{x_{\omega}}{X(\mathbf{x})},$$

where the production function is given by:

$$X = X(\mathbf{x}) = \int_{\Omega} (-\theta'_{\omega})^{-1}\left(\frac{x_{\omega}}{B^{*}(\mathbf{x})}\right) x_{\omega} d\omega \tag{28}$$

and $C(\mathbf{p})$ is a linear homogenous function of **p**, given by

$$C(\mathbf{p}) \equiv -\int_{\Omega} \theta'_{\omega}\left(\frac{p_{\omega}}{P(\mathbf{p})}\right) p_{\omega} d\omega > 0,$$

and it satisfies

$$\frac{C(\mathbf{p})}{P(\mathbf{p})} = -\int_{\Omega} \theta'_{\omega}\left(\frac{p_{\omega}}{P(\mathbf{p})}\right)\frac{p_{\omega}}{P(\mathbf{p})} d\omega = \int_{\Omega} (-\theta'_{\omega})^{-1}\left(\frac{x_{\omega}}{B^{*}(\mathbf{x})}\right)\frac{x_{\omega}}{B^{*}(\mathbf{x})} d\omega = \frac{X(\mathbf{x})}{B^{*}(\mathbf{x})}.$$

These two expressions for the market share under H.I.I.A. show that it is either a function of the two relative prices, $p_{\omega}/P(\mathbf{p})$ and $p_{\omega}/C(\mathbf{p})$, or a function of the two relative quantities,

$x_\omega/X(\mathbf{x})$ and $x_\omega/B^*(\mathbf{x})$, unless $P(\mathbf{p})/C(\mathbf{p}) = B^*(\mathbf{x})/X(\mathbf{x}) = c > 0$ for a constant c, which occurs if and only if it is CES. Thus, H.I.I.A. and H.S.A. do not overlap with the sole exception of CES. [16]

Since $\lim_{z\to\bar{\beta}_\omega} \theta'_\omega(z) = 0$, the choke price exists if $\bar{\beta}_\omega < \infty$. If $\bar{\beta}_\omega = \infty$, the choke price does not exist and demand for each input always remains positive for any positive price vector.

5.2 Monopolistically Competitive Firms under H.I.I.A.

From the demand curve, eq.(27), the profit of firm $\omega \in \Omega$ is given by:

$$\pi_\omega = -(p_\omega - \psi_\omega)B^*(\mathbf{x})\theta'_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right).$$

Firm ω chooses its price, p_ω, to maximize its profit π_ω, taking the aggregate variables, $P = P(\mathbf{p})$ and $B^*(\mathbf{x})$ as given. Or equivalently, it chooses $z_\omega \equiv p_\omega/P$ to *minimize*

$$\left(z_\omega - \frac{\psi_\omega}{P(\mathbf{p})}\right)\theta'_\omega(z_\omega).$$

The FOC is:

$$\theta'_\omega(z_\omega) + \left(z_\omega - \frac{\psi_\omega}{P(\mathbf{p})}\right)\theta''_\omega(z(\omega)) = \theta''_\omega(z_\omega)\left\{z_\omega\left(1 - \frac{1}{\zeta^I_\omega(z_\omega)}\right) - \frac{\psi_\omega}{P(\mathbf{p})}\right\} = 0. \tag{29}$$

where

$$\zeta^I_\omega(z) \equiv -\frac{z\theta''_\omega(z)}{\theta'_\omega(z)} > 1, \tag{30}$$

defined over $(0, \bar{\beta}_\omega)$, is the elasticity of $\theta'_\omega(\cdot)$ in its absolute value, and equal to the price elasticity of demand. That $\theta_\omega(z)$ is strictly decreasing and strictly convex in $(0, \bar{\beta}_\omega)$ ensures $\zeta^I_\omega(z) > 0$. In addition, it is necessary to assume $\zeta^I_\omega(z) > 1$ to ensure that inputs are gross substitutes, as will be seen below. Note that $\zeta^I_\omega(z) > 1$ is continuously differentiable in $(0, \bar{\beta}_\omega)$ and satisfies $\lim_{z\to\bar{z}} \zeta^I_\omega(z) = \infty$ if $\bar{\beta}_\omega < \infty$. Conversely, from any continuously differentiable

[16] See Proposition 3-(ii) in Matsuyama and Ushchev (2017). Its Proposition 4-(iii) also shows that H.D.I.A. and H.I.I.A. do not overlap with the sole exception of CES.

$\zeta_\omega^I(z) > 1$, defined over $\left(0, \bar{\beta}_\omega\right)$, satisfying $\lim_{z \to \bar{z}} \zeta_\omega^I(z) = \infty$ if $\bar{\beta}_\omega < \infty$, one could recover $\theta_\omega'(z)$ and hence $\theta_\omega(z)$ up to a positive scalar multiplier as follows: $\theta_\omega'(z)$

$$\theta_\omega'(z) = -\exp\left[-\int_{c_\omega^I}^{z} \zeta_\omega^I(\xi')\frac{d\xi'}{\xi'}\right] \Rightarrow \theta_\omega(z) = \int_z^\infty \exp\left[-\int_{c_\omega^I}^{\xi} \zeta_\omega^I(\xi')\frac{d\xi'}{\xi'}\right] d\xi.$$

where $c_\omega^I > 0$ is a constant.

Generally, the FOC is a necessary condition for the global optimum. The following assumption ensures that the FOC is also sufficient for the global optimum.

Assumption I1: *For all* $\omega \in \Omega$ *and all* $z \in (0, \bar{\beta}_\omega)$,

$$\frac{d}{dz}\left[z\left(1 - \frac{1}{\zeta_\omega^I(z)}\right)\right] = \frac{1}{\zeta_\omega^I(z)}\left[\frac{z\zeta_\omega^{I\prime}(z)}{\zeta_\omega^I(z)} + \zeta_\omega^I(z) - 1\right] > 0.$$

I1 is equivalent to the strict concavity of $1/\theta_\omega'(\cdot)$. Under **I1**, the LHS of eq.(29) is increasing in z_ω in the neighborhood of every solution to eq.(29). Hence, eq.(29) determines the profit-maximizing relative price for each firm, z_ω, uniquely as an increasing function of ψ_ω/P

$$\frac{p_\omega}{P(\mathbf{p})} = z_\omega = \mathcal{Z}_\omega\left(\frac{\psi_\omega}{P(\mathbf{p})}\right)$$

with

$$\mathcal{Z}_\omega'\left(\frac{\psi_\omega}{P(\mathbf{p})}\right) = \left.\frac{\zeta_\omega^I(z)}{\frac{z\zeta_\omega^{I\prime}(z)}{\zeta_\omega^I(z)} + \zeta_\omega^I(z) - 1}\right|_{z=\mathcal{Z}_\omega(\psi_\omega/P)} > 0,$$

which shows that, with **I1,** the pricing rule under H.I.I.A. takes the form of eq.(4) with $\mathcal{A}(\mathbf{p}) = P(\mathbf{p})$ and $\mathcal{G}_\omega(\cdot) = \mathcal{Z}_\omega(\cdot)$. Recall that the pass-through rate is the elasticity of this function, so that

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = \mathcal{E}_{\mathcal{Z}_\omega}\left(\frac{\psi_\omega}{P(\mathbf{p})}\right) = \frac{(\psi_\omega/P)\mathcal{Z}_\omega'(\psi_\omega/P)}{\mathcal{Z}_\omega(\psi_\omega/P)} = \mathcal{Z}_\omega'\left(\frac{\psi_\omega}{P(\mathbf{p})}\right)\left.\left(1 - \frac{1}{\zeta_\omega^I(z)}\right)\right|_{z=\mathcal{Z}_\omega(\psi_\omega/P)}$$

$$= \left.\frac{\zeta_\omega^I(z) - 1}{\zeta_\omega^I(z) - 1 + \frac{z\zeta_\omega^{I\prime}(z)}{\zeta_\omega^I(z)}}\right|_{z=\mathcal{Z}_\omega(\psi_\omega/P)} > 0,$$

which implies

$$\rho_\omega = \mathcal{E}_{Z_\omega}\left(\frac{\psi_\omega}{P(\mathbf{p})}\right) \lesseqgtr 1 \Leftrightarrow \zeta_\omega^{I\prime}\left(Z_\omega\left(\frac{\psi_\omega}{P(\mathbf{p})}\right)\right) \gtreqless 0.$$

and hence

$$\frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \frac{\partial \ln(p_\omega/\psi_\omega)}{\partial \ln \psi_\omega} = \rho_\omega - 1 \lesseqgtr 0 \Leftrightarrow \zeta_\omega^{I\prime}\left(Z_\omega\left(\frac{\psi_\omega}{P(\mathbf{p})}\right)\right) \gtreqless 0.$$

Note also that, using eq.(29), the maximized profit is now written as:

$$\pi_\omega = (p_\omega - \psi_\omega)x_\omega = \frac{p_\omega x_\omega}{\zeta_\omega^I(z_\omega)} = -\frac{z_\omega \theta_\omega'(z_\omega)}{\zeta_\omega^I(z_\omega)} P(\mathbf{p})B^*(\mathbf{x}). \tag{31}$$

5.3. Constant Pass-Through Families of H.I.I.A.

We now turn to the cases where the pass-through rate is constant and common across varieties, $\rho_\omega = \rho$. In the first and second classes, $\rho = 1$, hence the pass-through is complete. In the third and fourth classes, $0 < \rho < 1$, hence the pass-through is incomplete.

Constant Elasticity of Substitution (CES): For $\beta_\omega > 0$, $\gamma_\omega > 0$, and $\sigma > 1$,

$$\theta_\omega(z) = \frac{\gamma_\omega \beta_\omega}{\sigma - 1}\left(\frac{z}{\beta_\omega}\right)^{1-\sigma} \Rightarrow \theta_\omega'(z) = -\gamma_\omega \left(\frac{z}{\beta_\omega}\right)^{-\sigma} \Rightarrow \zeta_\omega^I(z) = \sigma > 1.$$

In this case, the choke price does not exist, i.e., $\bar{\beta}_\omega = \infty$, and

$$P(\mathbf{p}) = \left[\int_\Omega \frac{\gamma_\omega}{\sigma - 1}(\beta_\omega)^\sigma (p_\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

From the pricing rule,

$$p_\omega\left(1 - \frac{1}{\sigma}\right) = \psi_\omega;\ \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma}{\sigma - 1},$$

hence, the pass-through rate is unitary and the (common) markup rate is constant.

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1;\ \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \rho_\omega - 1 = 0.$$

Hence, this corresponds to the case of $\sigma_\omega = \sigma$ and $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the (common) markup rate is constant. The relative price, the relative revenue, and the relative profits of two firms, $\omega_1, \omega_2 \in \Omega$, are

$$\frac{p_{\omega_1}}{p_{\omega_2}} = \frac{\psi_{\omega_1}}{\psi_{\omega_2}};\ \frac{p_{\omega_1}x_{\omega_1}}{p_{\omega_2}x_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}}\frac{\beta_{\omega_1}}{\beta_{\omega_2}}\left(\frac{\psi_{\omega_1}/\beta_{\omega_1}}{\psi_{\omega_2}/\beta_{\omega_2}}\right)^{1-\sigma} = \frac{\pi_{\omega_1}}{\pi_{\omega_2}},$$

which correspond to the case of $\sigma_\omega = \sigma$ in eqs.(8)-(9), and all of them are independent of $P(\mathbf{p})$.

Constant (but Differential) Price Elasticity (CPE): For $\beta_\omega > 0, \gamma_\omega > 0$, and $\sigma_\omega > 1$,

$$\theta_\omega(z) = \frac{\gamma_\omega \beta_\omega}{\sigma_\omega - 1}\left(\frac{z}{\beta_\omega}\right)^{1-\sigma_\omega} \Rightarrow \theta'_\omega(z) = -\gamma_\omega \left(\frac{z}{\beta_\omega}\right)^{-\sigma_\omega} \Rightarrow \zeta^I_\omega(z) = \sigma_\omega > 1$$

In this case, the choke price does not exist, i.e., $\bar{\beta}_\omega = \infty$ and, $P = P(\mathbf{p})$ is the uniquely solution to:

$$\int_\Omega \frac{\gamma_\omega}{\sigma_\omega - 1}\left(\frac{p_\omega}{P\beta_\omega}\right)^{1-\sigma_\omega} d\omega = 1,$$

but it does not have a closed-form solution. From the pricing formula,

$$p_\omega \left(1 - \frac{1}{\sigma_\omega}\right) = \psi_\omega, \mu_\omega \equiv \frac{p_\omega}{\psi_\omega} = \frac{\sigma_\omega}{\sigma_\omega - 1},$$

Hence, this corresponds to the case of $\rho = 1$ in eqs.(5)-(7), so that the pass-through rate is unitary and the markup rates are constant, though no longer uniform across the firms:

$$\rho_\omega \equiv \frac{\partial \ln p_\omega}{\partial \ln \psi_\omega} = 1; \frac{\partial \ln \mu_\omega}{\partial \ln \psi_\omega} = \rho_\omega - 1 = 0.$$

Then, using eq.(16), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(8)-(9) with $\mathcal{A}(\mathbf{p}) = P(\mathbf{p})$. In particular, this means that a reduction in $P = P(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\sigma_{\omega_1} > \sigma_{\omega_2}$.

Incomplete Constant (and Common) Pass-Through (CoPaTh) ($0 < \rho < 1$):

For $\Delta > 0, \sigma > 1, \beta_\omega > 0$, and $\gamma_\omega > 0$, define

$$\bar{\beta}_\omega \equiv \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{1}{\Delta}}$$

and

$$\theta_\omega(z) = \begin{cases} \gamma_\omega(\sigma_\omega - 1)^{\frac{1}{\Delta}} \int_z^{\bar{\beta}_\omega} \left(\left(\frac{\xi}{\bar{\beta}_\omega}\right)^{-\Delta} - 1\right)^{\frac{1}{\Delta}} d\xi & \text{for } z < \bar{\beta}_\omega \\ 0 & \text{for } z \geq \bar{\beta}_\omega \end{cases} \tag{32}$$

so that

$$\theta'_\omega(z) = -\gamma_\omega \left(\sigma_\omega \left(\frac{z}{\beta_\omega}\right)^{-\Delta} - (\sigma_\omega - 1)\right)^{\frac{1}{\Delta}} = -\gamma_\omega(\sigma_\omega - 1)^{\frac{1}{\Delta}}\left(\left(\frac{z}{\bar{\beta}_\omega}\right)^{-\Delta} - 1\right)^{\frac{1}{\Delta}} < 0 \text{ for } z < \bar{\beta}_\omega$$

$$\zeta_\omega^I(z) \equiv -\frac{z\theta_\omega''(z)}{\theta_\omega'(z)} = \frac{1}{1-\left(1-\frac{1}{\sigma_\omega}\right)\left(\frac{z}{\beta_\omega}\right)^\Delta} = \frac{1}{1-\left(\frac{z}{\bar{\beta}_\omega}\right)^\Delta} > 1, \qquad \text{for } z < \bar{\beta}_\omega$$

Clearly, $\theta_\omega(\cdot)$ is decreasing, convex, with $\theta_\omega(0) = \infty$ and $\theta_\omega(\bar{\beta}_\omega) = 0$. With $\zeta_\omega^I(\cdot)$ increasing, I1 holds.[17]

Then, by inserting this expression for $\zeta_\omega^I(z)$ into eq.(29), we obtain

$$\frac{p_\omega}{P(\mathbf{p})} = z_\omega = (\beta_\omega)^{\frac{\Delta}{1+\Delta}}\left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{P(\mathbf{p})}\right)^{\frac{1}{1+\Delta}} = (\bar{\beta}_\omega)^{\frac{\Delta}{1+\Delta}}\left(\frac{\psi_\omega}{P(\mathbf{p})}\right)^{\frac{1}{1+\Delta}}, \quad \text{for } 0 < \frac{p_\omega}{P(\mathbf{p})} < \bar{\beta}_\omega.$$

which is indeed eq.(5), by setting $0 < \rho = 1/(1+\Delta) < 1$, and $\mathcal{A}(\mathbf{p}) = P(\mathbf{p})$, which can be determined by applying eq.(32) to eq.(26).

Then, using eq.(31), one could easily verify that the relative price, the relative revenue, and the relative profit of two firms, ω_1 and ω_2, can take the form given in eqs.(10)-(11) with $\mathcal{A}(\mathbf{p}) = P(\mathbf{p})$. In particular, this means that a reduction in $P = P(\mathbf{p})$ reduces the revenue and the profit of ω_1 relative to ω_2 if and only if $\zeta_{\omega_1} > \zeta_{\omega_2}$. Thus, as in the CPE case, the firms facing higher price elasticity of demand suffer more from an increase in competition. Unlike in the CPE case, however, the price elasticity is partially endogenous, because, even after controlling for σ_ω, the firms with lower productivity face higher price elasticity. Note also that the above condition, $\zeta_{\omega_1} > \zeta_{\omega_2}$, implies neither $p_{\omega_1}x_{\omega_1} < p_{\omega_2}x_{\omega_2}$ nor $\pi_{\omega_1} < \pi_{\omega_2}$, unless γ_ω is independent of ω. Hence, smaller firms, measured in the revenue or the profit, do not always suffer more from increased competition.

6. Concluding Remarks

In this paper, we proposed and characterized what we call CoPaTh, the three parametric families of homothetic demand systems that feature a constant pass-through rate as a parameter, $0 < \rho \leq 1$, which is common across otherwise heterogenous monopolistically competitive firms. In the case of complete pass-through ($\rho = 1$), the markup rate is constant, as in CES, yet it can

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[17]CPE(and hence CES) is the limit case, as $\Delta \searrow 0$, while holding $\beta_\omega > 0$ and $\sigma_\omega > 1$ fixed: $\bar{\beta}_\omega \equiv \beta_\omega\left(\frac{\sigma_\omega}{\sigma_\omega-1}\right)^{1/\Delta} \to \infty$; $\zeta_\omega^I(z) \equiv -\frac{z\theta_\omega''(z)}{\theta_\omega'(z)} = \frac{1}{1-\left(1-\frac{1}{\sigma_\omega}\right)(z/\beta_\omega)^\Delta} \to \sigma_\omega$. Since $\lim_{\Delta\to 0}\frac{\ln\left[\left(\frac{\sigma_\omega}{\sigma_\omega-1}\right)(z/\beta_\omega)^{-\Delta}-1\right]}{\Delta} = \lim_{\Delta\to 0}\frac{-(z/\beta_\omega)^{-\Delta}\ln(z/\beta_\omega)}{(z/\beta_\omega)^{-\Delta}-\left(1-\frac{1}{\sigma_\omega}\right)} = -\sigma_\omega\ln(z/\beta_\omega)$ from L'Hospital's rule, $\theta_\omega'(z) \to -\gamma_\omega(z/\beta_\omega)^{-\sigma_\omega}$, and hence $\theta_\omega(z) \to \frac{\gamma_\omega}{\sigma_\omega-1}(\beta_\omega)^{\sigma_\omega}(z)^{1-\sigma_\omega}$.

be heterogenous across firms, unlike in CES. In the case of incomplete pass-thorough ($0 < \rho < 1$), the price of each firm is log-linear in its marginal cost and its choke price with the common coefficients across firms. Tougher competition, captured by a lower "average price," reduces the prices of all firms at a uniform rate, so that their relative prices are not affected. Yet, it causes a disproportionately larger decline in the revenue and the profit among firms with lower markup rates. Furthermore, because the pass-through rate is a parameter, use of nested CoPaTh allows us to introduce the sector-specific pass-through rates, so that the average pass-through rate in the economy could vary through a change in the sectoral composition. We believe that these features make CoPaTh flexible, and yet tractable extensions of CES.

It should be pointed out that the goal of this paper is not to generalize the demand systems for the sake of generalization. If that were the case, we would have proposed a more general version of CoPaTh, where the pass-through rate is constant and firm-specific, $0 < \rho_\omega < 1$. Constructing such a demand system itself is straightforward; all we have to do would be to make $\Delta = 1/\rho - 1 > 0$ in eq.(17), eq.(25), and eq.(32) firm-specific.[18] However, with this type of heterogeneity, the resulting demand system would be not only intractable, but also of little predictive content; almost "anything can happen" in term of cross-sectional implications. We have chosen to impose the restriction that the firm has the common pass-through rate, because that buys us a lot of tractability, and at the same time allows us to keep other dimensions of heterogeneity across firms in a meaningful way.

It should also be pointed out that we are not arguing that CES should be abandoned. We believe that CES remains useful for many purposes. In many ways, the role of CoPaTh relative to CES is similar to that of CES relative to Cobb-Douglas. We have not abandoned the use of Cobb-Douglass simply because CES became available. CES is useful as a more flexible and yet tractable extension of Cobb-Douglas, when the key implication of Cobb-Douglas, the unitary elasticity of substitution, is too restrictive. Likewise, we should not abandon the use of CES. CoPaTh is useful as a more flexible and yet tractable extension of CES, when the key implications of CES, the constant and common markup rate and the unitary pass-through elasticity of substitution, are too restrictive.

[18]If we dropped the requirement that the pass-through rate is globally constant, then we could also have the case of excessive pass-through ($\rho > 1$), by allowing $-1 < \Delta = 1/\rho - 1 < 0$ in eq.(16), eq.(24), and eq.(31) over certain ranges. However, this means that we would have to impose some upper bounds on the heterogeneity of firms to ensure that all firms would operate within the range.

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Figure: The Three Families of CoPaTh Demand Systems