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JEL Classification: D43, L11, L42, L81

Keywords: asymmetric information, distribution network, opportunism, retail market structure, Vertical contracting

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Vertical Contracting with Endogenous Market Structure*

Marco Pagnozzi[†] Salvatore Piccolo[‡] Markus Reisinger[§]

November 18, 2020

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1 Introduction

The retail market structure chosen by a manufacturer and the contractual arrangements with retailers are crucial determinants of firms' profits and social welfare. As a consequence, distribution systems and vertical mergers are often under the scrutiny of antitrust authorities, worried that these practices may reduce downstream competition. In fact, several antitrust cases consider whether manufacturers restrict intrabrand competition and harm consumers through the design of their distribution networks.[1]

The seminal papers analyzing vertical contracting—Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994), and Segal and Whinston (2003)—show that such worries are indeed well grounded.[2] In fact, when secretly contracting with multiple retailers, an *opportunism problem* arises because a manufacturer has an incentive to raise the quantity traded in each bilateral negotiation, which reduces industry profits. This induces the manufacturer to restrict the number of retailers that distribute its product in order to soften downstream competition, thereby harming final consumers. Moreover, a vertical merger allows the manufacturer to reduce (and, under some conditions, fully solve) the opportunism problem by engaging in foreclosure practices, which again harms consumers.

Most of this literature, however, does not take into account information asymmetries between manufacturers and retailers, even though this is a prevalent feature of distribution networks. In fact, retailers are typically better informed than manufacturers about demand and/or cost characteristics. For example, they are likely to obtain better information about demand by interacting directly with final consumers and observing their idiosyncratic tastes. Similarly, retailers often have superior information about their production technology because downstream costs may depend on price shocks to local inputs that are not directly observable by manufacturers.

In this paper, we analyze the interplay between asymmetric information and the opportunism problem, and provide two main contributions. First, we characterize the manufacturer's choice of the optimal retail market structure, thereby showing how industry characteristics affect the number of retailers, and determine the resulting effects for competition policy. Second, we advance the theoretical literature by analyzing secret contracting with externalities and asymmetric information.

[1]For example, in two recent cases, the distribution systems of the cosmetics manufacturer Pierre Fabre and of the sport shoe producer Asics were ruled to violate competition law because they prohibited sales on third-party websites. See European Court of Justice, judgment of 13 October 2011, Case C - 439/09, *Pierre Fabre Dermo-Cosmétique*, and German Federal Cartel Office (Bundeskartellamt), 13 January 2016, Case Summary, "Unlawful Restrictions of Online Sales of ASICS Running Shoes", B2-98/11.

[2]See Rey and Tirole (2007) for a comprehensive summary of the literature.

In our model, a manufacturer chooses the retail market structure—i.e., the number of retailers distributing its product—and retailers have private information about a common cost (or demand) shock.[3] Subsequently, the manufacturer bilaterally and secretly contracts with each retailer by offering non-linear tariffs consisting of a quantity sold by the manufacturer and a transfer paid by the retailer. Retailers choose how much to buy from the manufacturer and compete in the final-consumer market.

Our framework therefore builds on two theoretical pillars. First, the manufacturer commits to the size of the distribution network before contracting with retailers. In fact, a change in the distribution network is a long-term decision that requires costly investments and negotiations, and many brand manufacturers choose few selected outlets to distribute their products (e.g., Schmidt-Kessen, 2018). For example, car and clothing manufacturers usually use a selective distribution system, which includes only retailers that comply with specific quality criteria (e.g., Bhasin, 2018). Second, we focus on bilateral and secret contracts that cannot be contingent on elements external to the bilateral relationship between the manufacturer and each retailer (like the quantity sold by other retailers).[4] This typically occurs due to institutional constraints. In fact, it is usually too costly for a manufacturer to write a complete multilateral contract with all retailers in the network since this requires to foresee and verify many contingencies, and the costs of handling and processing the required information raises considerably if more parties are involved (McAfee and Schwartz, 1994; Dequiedt and Martimort, 2015). In addition, antitrust laws often preclude multilateral agreements in which the quantity sold to a retailer depends on trades made with competitors.[5]

We show that, in contrast to an environment with complete information (e.g., Rey and Tirole, 2007), with asymmetric information the manufacturer may benefit from a sizable distribution network. Hence, monopolization through exclusive distribution is less likely in industries with strong uncertainty about, for example, retail costs or downstream demand. This result arises because of a novel trade-off between the opportunism problem and information asymmetries in vertical contracting. On the one hand, as is well-known, with a monopolistic retailer the manufacturer solves the opportunism problem. On the other hand, competition between retailers has a disciplining effect on each retailer's incentive to mimic types with higher cost

[3]The assumption of a common shock allows us to derive our results in the clearest way. However, the effects we highlight also arise with imperfect correlation (see Section 6).

[4]This assumption prevents the manufacturer from obtaining monopoly profits. First, the manufacturer cannot use contracts based on the aggregate quantity to eliminate the opportunism problem when distributing through multiple retailers. Second, the manufacturer cannot exploit yardstick competition in the spirit of Crémer and McLean (1985) to eliminate the retailers' information rent and cannot select retailers by auctioning the right to distribute its product.

[5]We provide a more detailed motivation for this pillar in the next section.

or lower demand, and hence reduces the information rent—a *competing-contracts effect* in the spirit of Martimort (1996).

To see this, consider the incentive of a low-cost retailer to unilaterally deviate from a truthful equilibrium.[6] In order to pay a lower transfer, the retailer has an incentive to pretend to have a high cost and acquire a lower quantity. Because the market price depends on the aggregate quantity and competing retailers sell a larger quantity than the deviating retailer, the market price and the deviating retailer's profit are lower than without competition (or if all retailers deviate). However, due to the common shock, the manufacturer requests a transfer based on the presumption that all retailers sell a lower quantity and, hence, obtain a relatively high profit. Therefore, competition in the downstream market reduces a retailer's incentive to deviate and thereby lowers the information rent that the manufacturer pays to elicit truthful information.[7] Although the extent of information asymmetry is independent of the number of retailers, more competition in the retail market allows the manufacturer to reduce retailers' information rent.

In their key contribution on vertical relationships, Rey and Tirole (1986) also show that a manufacturer benefits from retail competition in a framework with public offers and risk averse retailers, because competition provides insurance to retailers and therefore allows the manufacturer to extract more surplus. We find that competition benefits the manufacturer even with risk-neutral retailers, when they possess private information.

We examine the trade-off between the opportunism problem and the competing-contracts effect with a general demand and cost function and show that, starting from a monopolistic retail market structure, the manufacturer always benefits (at the margin) from competition between retailers.[8] Nonetheless, the optimal number of retailers is always finite.

To explicitly determine the optimal size of the distribution network, we consider a specification with linear demand, quadratic costs of the manufacturer, and a beta distribution of the retailers' cost. We show that the optimal number of retailers is larger if the manufacturer's cost function is convex or if the elasticity of demand is large. The intuition is that a convex cost function and a high demand elasticity reduce the manufacturer's incentive to increase a retailer's quantity, thus limiting the opportunism problem. In this case, the disciplining effect of competition on information rents dominates.

[6]That is, an equilibrium in which all retailers acquire the quantity intended by the manufacturer for a low-cost realization.

[7]This disciplining effect of competition on agency costs is also obtained through yardstick competition (see, e.g., Shleifer, 1985), which requires principals to design sophisticated contractual rules that play agents against each other. Our analysis shows that a disciplining effect of competition emerges even with simple quantity discounts as long as the retailers' private information is correlated.

[8]More precisely, the manufacturer's profit increases with a marginal increase in the number of retailers if there is only one retailer.

On the normative side, the welfare effects arising in a manufacturer-retailer relationship with secret offers and asymmetric information are considerably different from the ones with complete information. With complete information, the number of retailers chosen by the manufacturer is always lower than the socially optimal one (due to the opportunism problem that occurs with multiple retailers), and vertical integration reduces consumer surplus and welfare because it eliminates downstream competition. By contrast, with asymmetric information, larger distribution networks reduce retailers' information rents, which may induce the manufacturer to choose more retailers than socially optimal.[9] Moreover, vertical integration may increase consumer surplus and welfare because it eliminates asymmetric information and, hence, downward distortion of quantities for rent extraction reasons. This happens, for example, when the manufacturer's cost function features sufficiently high diseconomies of scale because in this case asymmetric information leads to severe quantity distortions.

In sum, our analysis unveils a novel trade-off arising from the presence of information asymmetries in a canonical vertical-contracting framework. Asymmetric information tends to increase the size of optimal distribution networks and mitigate concerns of low intrabrand competition.[10] Our results are consistent with anecdotal evidence from some observed distribution structures. For example, in the automobile industry, in which demand and cost conditions are relatively stable over time and, hence, asymmetric information is less relevant, manufacturers often choose a single retailer in a region. By contrast, in the market for electronic products in which demand and cost conditions usually fluctuate over time,[11] thereby enhancing problems of asymmetric information, manufacturers typically sell through multiple retailers.[12]

In the remainder of this section, we discuss the related literature. In Section 2, we set out the main model. In Section 3, we consider a simple example with two types. In Section 4, we analyze the optimal retail market structure, first in the complete information benchmark and then with asymmetric information. In Section 5, we discuss the welfare effects of our analysis.

[9] Information rents do not affect social welfare because they are transfers between the manufacturer and the retailers.

[10] Naturally, there are many other factors that influence the size of a manufacturer's distribution network. For example, a manufacturer may prefer a larger number of retailers: when consumers perceive retailers' products as differentiated (see, e.g., Motta, 2004, Ch. 6); to sell in geographically differentiated areas that cannot be served by a single retailer (e.g., Rey and Stiglitz, 1995); or when the hold-up problem distorts investments (Bolton and Whinston, 1993; and Hart and Tirole, 1990). These factors are complementary to the effects of asymmetric information that we highlight.

[11] Dyer *et al.* (2014) classify uncertainty in different industries and find that the automobile and truck industry is exposed to significantly less uncertainty than the electronic and electrical equipment industry.

[12] We acknowledge, however, that there may be many other reasons why electronic products are sold in larger retail networks than cars—e.g., consumers are less willing to travel long distances for electronic products as their value is on average much smaller than the one of a car.

We consider imperfectly correlated costs in Section 6, and discuss alternative mechanisms in Section 7. Section 8 concludes. The proofs of all results are in Appendix A.

Related Literature. Hart and Tirole (1990) were the first to highlight the opportunism problem of a manufacturer dealing with multiple retailers. Building on their framework, many subsequent papers further analyzed this issue. O'Brien and Shaffer (1992) show that exclusive territories, or appropriate forms of resale price control, such as a market-wide price floor, can solve the opportunism problem in a contract equilibrium.[13] McAfee and Schwartz (1994) and Rey and Vergé (2004) explore how the problem depends on different types of off-equilibrium beliefs by retailers. Recently, Nocke and Rey (2018) consider the opportunism problem in a framework with multiple differentiated manufacturers and show that exclusive dealing or vertical integration increase profits but lower consumer welfare. Taking a different approach, Segal and Whinston (2003) consider menus of contracts in the bilateral negotiation between manufacturer and retailers and provide bounds for their equilibrium payoffs, which do not depend on the choice of off-equilibrium beliefs.[14] In addition to this literature, we consider asymmetric information between retailers and the manufacturer and determine its consequences for the size of the distribution network and the resulting competitive effects.

Our paper also contributes to the literature on contracting with externalities. Segal (1999) provides a very general treatment and shows, among many other results, that private contracts usually fail to achieve an efficient outcome for the principal. This literature has been extended in many directions—e.g., allowing for pre-play side payments (Jackson and Wilkie, 2005) or sequential offers by the principal (Genicot and Ray, 2006; Möller, 2007). However, this literature focuses on complete information between contracting parties and takes the number of agents as fixed.

Our work is also related to the strand of literature analyzing asymmetric information in manufacturer-retailer relationships. These papers usually examine common agency games (e.g., Calzolari and Denicolò, 2013, 2015; Martimort, 1996; Martimort and Stole, 2009a, 2009b) or games played by competing organizations (e.g., Caillaud *et al.*, 1995; Gal-Or, 1999; and Kastl *et al.*, 2011). None of these papers, however, jointly considers the opportunism problem and asymmetric information in vertical contracting and endogenizes the retail market structure.

[13] In a contract equilibrium, the manufacturer behaves optimally in each bilateral relationship, given the contracts with other retailers. This equilibrium concept, however, does not consider multi-lateral deviations. Recently, Montez (2015) finds that commitment of the manufacturer to buy back units of unsold stock from retailers may restore the monopoly outcome in a contract equilibrium.

[14] In our model, on top of solving the opportunism problem, the manufacturer also needs to elicit truthful information. While achieving both objectives with a belief-free approach is feasible, this would require a complex contract space and is outside the scope of our paper.

As mentioned above, Rey and Tirole (1986) obtain a similar result as we do—i.e., a manufacturer benefits from a larger retail network. However, their effect is very different from ours, as it relies on risk aversion of retailers: inducing competition between retailers exposes each one to a lower level of uncertainty, which allows the manufacturer to charge a larger fixed fee. Hansen and Motta (2019) also analyze a model with privately-informed risk averse retailers. In contrast to Rey and Tirole (1986), they consider the case in which retailers' shocks are not perfectly correlated, and show that competition then exposes retailers to additional risk, as each one is uncertain about the shock realization of its competitors. Because the manufacturer must compensate retailers for their risk, using a single retailer maximizes profits if retailers are sufficiently risk averse. In contrast to these papers, which analyze public contracts and risk-averse retailers, we consider secret contracts and risk-neutral retailers.

To the best of our knowledge, only Dequiedt and Martimort (2015) examine the link between opportunism and asymmetric information. They consider a framework with public contracting in which the manufacturer can condition contracts on the information obtained from other retailers. Dequiedt and Martimort (2015) show that this creates a new form of informational opportunism—even when retailers do not impose production externalities on each other—which prevents the manufacturer from achieving the monopoly outcome. By contrast, we consider secret contracting and focus on how the optimal retail market structure is shaped by asymmetric information and classical opportunism.

2 The Model

Players and Environment. We consider a vertical contracting model with externalities *à la* Segal and Whinston (2003), where a (female) manufacturer M chooses (publicly) a fixed number of (male) retailers $N \geq 1$ to which to sell her product. A retailer R_i, $i = 1, .., N$, sells in the downstream market by converting each unit of M's product into one unit of the final good. We denote by x_i the quantity sold by R_i and let $X \triangleq \sum_{i=1}^{N} x_i$. The downstream demand function is $P(X)$, with $P'(\cdot) < 0$—i.e., retailers sell undifferentiated products.[15]

The assumption that the manufacturer commits to the size of her distribution network is consistent with industries in which choosing the number of retailers is a long-term decision that cannot be secretly modified in the short term, or in which distributing the manufacturer's

[15] This assumption simplifies the analysis and makes our results directly comparable to Hart and Tirole (1990). Our results, however, do not hinge on it: in Appendix B, we analyze price competition and consider differentiated products.

product requires specific investments that can be observed by competitors.[16] We assume without loss of insights that increasing the number of retailers is costless for the manufacturer.[17]

We assume that M's cost function $c(X)$ is increasing and weakly convex, with $c(0) = 0$.[18] In order to make our model equivalent to Hart and Tirole (1990) in the case of complete information, we assume that retailers are symmetric and have a constant common marginal cost θ—i.e., retail costs are the same for all retailers (e.g., because they depend on a common input price shock, which affects all retailers symmetrically).[19] This assumption, however, is not crucial for the results—in Section 6, we show that the effects we identify also arise with imperfect correlation among costs.

Retailers are privately informed about θ, which is drawn from a common knowledge, non-negative, twice continuously differentiable, bounded, and atomless density function $f(\theta)$ on the compact support $\Theta \triangleq [\underline{\theta}, \overline{\theta}]$. The associated distribution function $F(\theta)$ satisfies the (inverse) Monotone Hazard Rate Property—i.e., $h(\theta) \triangleq F(\theta)/f(\theta)$ is increasing. We would obtain equivalent results if retailers were privately informed about demand rather than costs.

Contracts. The manufacturer contracts with all retailers simultaneously (see, e.g., Segal, 1999). Contracts are secret—i.e., a retailer does not observe the contracts that M offers to other retailers. (However, he knows N—i.e., the number of competitors he is facing.)

Following the literature on screening in vertical contracting—e.g., Caillaud et al. (1992), Martimort (1996), Martimort and Stole (2009a, 2009b)—we assume that M offers a direct quantity-forcing mechanism (contract) to R_i, which consists of a menu

$$C_i \triangleq \{T_i(m_i), x_i(m_i)\}_{m_i \in \Theta},$$

specifying the quantity $x_i(m_i)$ that M supplies to R_i and that R_i sells in the downstream market, and the tariff $T_i(m_i)$ that R_i pays to M, contingent on R_i's report $m_i \in \Theta$ about the cost θ. In Section 4.2, we explain that there is no loss of generality in considering quantity-forcing contracts because, in equilibrium, a retailer has no incentive to sell a quantity lower

[16] The assumption is also consistent with the literature on the opportunism problem in which a retailer knows the number of competitors. If the manufacturer could instantaneously increase the number of retailers, there would be a unique equilibrium with price equal to marginal cost. The assumption is also implicit in the seminal work by Hart and Tirole (1990) on exclusive dealing, which is a commitment not to supply additional retailers.

[17] When we discuss social welfare and consumer surplus in Section 5, we introduce a fixed cost that the manufacturer has to pay for each retailer she uses.

[18] Our main results do not hinge on the convexity of costs and also hold with constant marginal costs. However, the assumption allows us to provide comparative-static results on the curvature of the manufacturer's cost function (see Section 4.3).

[19] Therefore, if the manufacturer chooses multiple retailers, this is not due to asymmetry among them.

than the one acquired from the manufacturer. As in most of the literature, we restrict to equilibria in which the functions $x_i(m_i)$ and $T_i(m_i)$ are continuously differentiable.[20]

A retailer's outside option is normalized to zero. If contracts are accepted by retailers, M's total profit is

$$\sum_{i=1}^{N} T_i(m_i) - c\left(\sum_{i=1}^{N} x_i(m_i)\right),$$

while R_i's profit is

$$\left[P\left(\sum_{j=1}^{N} x_j(m_j)\right) - \theta\right] x_i(m_i) - T_i(m_i).$$

The simple bilateral contracts we consider are fully determined by a retailer's report about the (common) cost and depend neither on the reports of other retailers nor on their offered quantities. This assumption rules out mechanisms *à la* Crémer and McLean, which allow the manufacturer to costlessly extract correlated information from retailers.[21] There are several motivations behind this assumption.

First, contracting and communication is typically secret, and it is usually too costly for the manufacturer to credibly disclose to a retailer the reports of all other retailers. In fact, to implement a mechanism *à la* Crémer and Mclean, a retailer's contract needs to be a comprehensive agreement that conditions on the whole array of verifiable messages of other retailers. Writing and enforcing such a multi-lateral agreement requires substantial auditing costs, which might be too large to render the mechanism profitable (see e.g., Dequiedt and Martimort, 2015).

Second, such multi-lateral contracts usually violate antitrust law. Indeed, contracts that condition the quantity and payment of a retailer on the vector of trades that the manufacturer executes with his competitors are often forbidden by competition law on the premise that they facilitate collusion and harm consumers (see, e.g., Rey and Tirole, 2007). Therefore, the vertical contracting literature has focused on bilateral contracts (McAfee and Schwartz, 1994; Rey and Vergé, 2004).

Third, when costs are not perfectly correlated, a mechanism along the lines of Crémer and McLean may impose large penalties to retailers, which are impossible to enforce if retailers are protected by limited liability (see, e.g., Robert, 1991, and Herweg and Schmidt, 2020). By contrast, our bilateral contracts do not require unlimited liability.

[20]By the Taxation Principle, these direct mechanisms are equivalent to M using indirect, non-linear tariffs $T_i(x_i)$—which are usually observed in practice—specifying a payment $T_i(\cdot)$ to M for every quantity x_i ordered by R_i (see, e.g., Laffont and Martimort, 2002). However, compared to indirect mechanisms, direct mechanisms make it easier to pin down and understand the structure of the retailers' information rents, which are the key drivers of our results (as we show below).

[21]See Crémer and McLean (1985, 1988) and McAfee and Reny (1992), among others.

We finally note that our restriction in the contract space also prevents the manufacturer from selecting retailers through auctions, where the probability of a retailer winning depends on the bids by other retailers (see also Section 7).

Timing and Equilibrium Concept. The timing of the game is as follows:

1. *Retail Market Structure.* M chooses the number of retailers N.

2. *Contracting.* Retailers observe N and θ. M (simultaneously and secretly) offers contracts. If R_i accepts his contract, he reports m_i, obtains the quantity $x_i(m_i)$ and pays the tariff $T_i(m_i)$ accordingly.[22]

3. *Downstream Competition.* Retailers sell their quantities in the downstream market and profits are realized.

Retailers play a Cournot game in the downstream market. In Appendix B, we show that our results also hold with price competition. Moreover, the equilibrium that we characterize is equivalent to the one of a game in which retailers set prices but are capacity constrained in the downstream market because the manufacturer produces to order before prices are set and final demand is realized (Rey and Tirole, 2007).[23] Essentially, price competition with capacity constraints as in Kreps and Scheinkman (1983) leads to a Cournot outcome.

We consider a Perfect Bayesian Nash Equilibrium in direct revelation mechanisms such that retailers truthfully report their cost—i.e., $m_i = \theta$ for every $i = 1, .., N$. In our main analysis, with impose the refinement of 'passive beliefs', which is the one most widely used in the literature (Hart and Tirole, 1990; McAfee and Schwartz, 1994; Rey and Tirole, 2007). With passive beliefs and multiple retailers, a retailer's conjecture about the contracts offered to other retailers is not influenced by an out-of-equilibrium offer he receives. In Appendix C, we analyze wary beliefs and show that our qualitative results are robust to the use of this alternative refinement.

[22]The assumption that contracting occurs in the second stage implies that no payment is made in the first stage. This is not restrictive: even if the manufacturer could pay retailers a fixed (perhaps negative) sum in the first stage, each retailer still chooses in the second stage his individually rational contract (or refuse to trade at all), given the manufacturer's offered menu. Hence, the analysis of the second stage is unaffected and the manufacturer cannot benefit from possible payments in the first stage. In other words, as long as the manufacturer cannot force the chosen retailers to trade with her, it is immaterial whether payments are made in the first or the second stage.

[23]Alternatively, one could imagine that the transformation activity is sufficiently time consuming that a downstream firm cannot instantaneously re-order the manufacturer's product and satisfy customers when their demand is larger than expected, or reduce it when demand is unexpectedly low.

Assumptions. We first treat N as a continuous variable and ignore the integer constraint on the number of retailers (see, e.g., Mankiw and Whinston, 1986). In Section 4.3, we analyze a closed-form example of our model and explicitly consider the effects of the integer constraint.

We also impose the following technical assumptions.

Assumption 1. $P(0) - c'(0) > \overline{\theta} + h\left(\overline{\theta}\right)$.

This assumption guarantees that production is always positive.

Assumption 2. The inverse demand function satisfies the following conditions:

(i) $P'(X) + P''(X)X < 0$;

(ii) $\lim_{X\to+\infty} P(X) = 0$ and $|P'(X)|$ is bounded;

(iii) $P'''(X)$ is not too large—i.e., $P'''(X) \leq -P''(X)/x_i$, $\forall x_i \geq 0$.

Part (i) of Assumption 2 implies that all profit functions are strictly concave and that quantities are strategic substitutes.[24] Part (ii) guarantees that the equilibrium market price converges to zero as the quantity produced becomes arbitrarily large, and that the equilibrium quantity is positive. Part (iii) ensures that the single-crossing property holds and also implies global optimality of the truth-telling strategy for each retailer.

3 A Binary Example

In order to gain intuition on the main trade-off and insights of the paper, we first analyze a stylized model with a binary type space in which M can only choose between one and two retailers. Suppose that $\Theta \triangleq \left\{0, \overline{\theta}\right\}$, with $\Pr[\theta = 0] = \Pr\left[\theta = \overline{\theta}\right] = 1/2$ and $\overline{\theta} < 1/2$ (to guarantee positive quantities in equilibrium). Let the manufacturer's cost function be $c(X) \triangleq \beta X^2/2$ and the (inverse) demand function be $P(X) \triangleq \max\{1 - X, 0\}$.

With a monopolistic retailer, the manufacturer faces a standard monopoly screening problem. We use "underline" (resp. "overline") to denote variables referring to a low-cost (resp. high-cost) retailer.[25] Let $\overline{u} \triangleq \left(P(\overline{x}) - \overline{\theta}\right)\overline{x} - \overline{T}$ and $\underline{u} \triangleq P(\underline{x})\,\underline{x} - \underline{T}$ define the high-cost and low-cost retailer's utility, respectively, when he truthfully reports his type. The participation constraint for the high-cost retailer and the incentive compatibility constraint for the

[24] This is a standard assumption in games with quantity competition (see e.g., Vives, 2001).

[25] For example, $\overline{x}$ is the equilibrium quantity sold to a retailer with cost $\theta = \overline{\theta}$ and $\underline{u}$ is the equilibrium utility of a retailer with cost $\theta = 0$.

low-cost retailer are binding, which implies $\overline{u} = 0$ and $\underline{u} = \overline{u} + \overline{\theta}\overline{x}$. Hence, the manufacturer's maximization problem is

$$\max_{(\overline{x},\underline{x})\geq 0} \tfrac{1}{2}\left\{\left[P\left(\underline{x}\right)\underline{x} - \overline{\theta}\overline{x} - c\left(\underline{x}\right)\right] + \left[\left(P\left(\overline{x}\right) - \overline{\theta}\right)\overline{x} - c\left(\overline{x}\right)\right]\right\},$$

yielding solutions[26]

$$\underline{x}^*\left(1\right) \triangleq \frac{1}{2+\beta} > \overline{x}^*\left(1\right) \triangleq \frac{1-2\overline{\theta}}{2+\beta}.$$

The quantity of the low-cost retailer is the optimal quantity with complete information, while the quantity of the high-cost retailer is distorted downward.[27]

Suppose now that M uses two retailers (who hold passive beliefs) and consider a symmetric equilibrium in which retailers with the same cost sell the same quantity $\overline{x}^*\left(2\right)$ when the cost is high and $\underline{x}^*\left(2\right)$ when it is low. Let

$$\overline{u}_i \triangleq \left(P\left(\overline{x}_i + \overline{x}^*\left(2\right)\right) - \overline{\theta}\right)\overline{x}_i - \overline{T}_i,$$

and

$$\underline{u}_i \triangleq P\left(\underline{x}_i + \underline{x}^*\left(2\right)\right)\underline{x}_i - \underline{T}_i,$$

define the high-cost and low-cost retailer's utility, respectively, when he truthfully reports his type and the rival produces the equilibrium quantity. It can be shown that the incentive compatibility constraint of a low-cost retailer is

$$\begin{aligned}\underline{u}_i &\geq \overline{u}_i + \overline{\theta}\overline{x}_i - \left[P\left(\overline{x}_i + \overline{x}^*\left(2\right)\right) - P\left(\overline{x}_i + \underline{x}^*\left(2\right)\right)\right]\overline{x}_i \\ &= \overline{u}_i + \underbrace{\overline{\theta}\overline{x}_i}_{\text{Standard rent}} - \underbrace{\Delta x^*\overline{x}_i}_{\text{Competing contracts}},\end{aligned} \tag{1}$$

where $\Delta x^* \triangleq \underline{x}^*\left(2\right) - \overline{x}^*\left(2\right) > 0$ represents the difference between the equilibrium quantities of a low-cost and a high-cost retailer. Expression (1) embeds two contrasting effects. First, a low-cost retailer R_i has an incentive to over-report the cost in order to pay a lower tariff, which allows him to obtain a standard monopoly information rent—see, e.g., Baron and Myerson (1982), Maskin and Riley (1985) and Mussa and Rosen (1978).

Second, there is a *competing-contracts effect*. When the low-cost retailer R_i over-reports his cost, he knows that the other retailer sells a larger quantity than he does because the other retailer has the same low cost and truthfully reports it in equilibrium. Hence, the price in

[26]The argument of $\underline{x}^*\left(\cdot\right)$ and $\overline{x}^*\left(\cdot\right)$ indicates the number of retailers.

[27]With complete information about θ, the profit maximizing quantity is $\left(1-\theta\right)/\left(2+\beta\right)$.

the downstream market is relatively low. However, the tariff requested by the manufacturer is based on the assumption that both retailers have high cost, according to R_i's report, so that the downstream price is relatively high. This implies that the manufacturer demands a tariff based on a higher retail price than the one which occurs in the market. As a consequence, R_i's utility is lower and, other things being equal, R_i's incentive to overstate his cost is weaker than without competition in the downstream market.[28] Therefore, compared to the monopoly case, a duopolistic retail market structure reduces each retailer's information rent.

Notice that a duopoly in the retail market does not reduce information asymmetry between the manufacturer and the retailers compared to a monopoly, but it does reduce the information rent that the manufacturer has to pay to a low-cost retailer to induce him to truthfully reveal his type (as indicated by the last term in expression (1)).

With two retailers, M's maximization problem is

$$\max_{(\underline{x}_1,\overline{x}_1);(\underline{x}_2,\overline{x}_2)} \tfrac{1}{2}\sum_{i=1}^{2}\left[P\left(\underline{x}_i+\underline{x}^*(2)\right)\underline{x}_i-\left(\overline{\theta}-\Delta x^*\right)\overline{x}_i+\left(P\left(\overline{x}_i+\overline{x}^*(2)\right)-\overline{\theta}\right)\overline{x}_i\right]$$
$$-\tfrac{1}{2}\left\{c(\Sigma_{i=1,2}\underline{x}_i)+c\left(\Sigma_{i=1,2}\overline{x}_i\right)\right\}.$$

Differentiating with respect to $\underline{x}_i$ and $\overline{x}_i$, $i=1,2$, yields the symmetric equilibrium quantities

$$\underline{x}^*(2)\triangleq\frac{1}{3+2\beta}>\overline{x}^*(2)\triangleq\underbrace{\frac{1-\overline{\theta}}{3+2\beta}}_{\text{First-best}}-\underbrace{\frac{(1+\beta)\,\overline{\theta}}{(2+\beta)\,(3+2\beta)}}_{\text{Distortion}}. \tag{2}$$

Hence, low-cost retailers obtain the same quantity as with complete information, while there is a downward distortion for high-cost retailers, which is represented by the second term of $\overline{x}^*(2)$ in expression (2). The aggregate quantity in both states of the world, however, is larger than with a monopolistic retailer due to the opportunism problem (i.e., $2\overline{x}^*(2)>\overline{x}^*(1)$ and $2\underline{x}^*(2)>\underline{x}^*(1)$). As β increases, the opportunism problem becomes weaker and the difference between the aggregate quantity produced with two retailers and the quantity produced with one retailer decreases.

Comparing the manufacturer's expected profit with one retailer $\pi^*(1)$ and her expected profit with two retailers $\pi^*(2)$, we obtain the following result.

[28]In other words, a 'coordinated' deviation by both retailers to over-report the cost is more attractive than a 'partial' deviation by a single retailer, when the other retailer truthfully reports a low cost and sells a high quantity.

Proposition 1 *If $\overline{\theta} = 0$, then $\pi^*(1) > \pi^*(2)$. For any $\overline{\theta} \in (0, \frac{1}{2})$, there exists a threshold $\hat{\beta} \geq 0$ such that $\pi^*(2) > \pi^*(1)$ if and only if $\beta > \hat{\beta}$, with $\hat{\beta} \to \infty$ if $\overline{\theta} \to 0$.*

When there is no asymmetry of information—i.e., $\overline{\theta} = 0$—the model converges to the standard Hart and Tirole (1990) framework. In this case, a market structure with two retailers can only harm the manufacturer (compared to the monopoly case) due to the opportunism problem. However, with asymmetric information—i.e., $\overline{\theta} > 0$—a market structure with two retailers reduces their information rents because of the competing-contracts effect and, other things being equal, increases the manufacturer's profit. When increasing production is sufficiently costly for the manufacturer—i.e., β is large—this effect dominates the opportunism problem and induces the manufacturer to distribute via two retailers.[29] The threshold $\hat{\beta}$, however, tends to infinity as the uncertainty disappears (i.e., $\overline{\theta} \to 0$).

4 Optimal Retail Market Structure

In this section, we analyze our more general model with a continuum of types, a generic demand and cost function, and $N \geq 1$.

4.1 Benchmark with Complete Information

As a benchmark, first assume that the manufacturer knows retailers' cost. The manufacturer optimally offers to each of the N retailers a single contract and fully extracts their surplus.

Let $x_i(\theta)$ be the quantity offered by M to R_i and denote by $x_i^e(\theta)$ the belief of all retailers $j \neq i$ about $x_i(\theta)$. With secret contracts and passive beliefs, a tariff accepted by R_i must satisfy

$$T_i(\theta) \leq \left[P\left(x_i(\theta) + \sum_{j=1,\, j\neq i}^{N} x_j^e(\theta) \right) - \theta \right] x_i(\theta).$$

Since this constraint is binding at the optimal contract, M's maximization problem is

$$\max_{(x_i(\theta)\geq 0)_{i=1}^{N}} \sum_{i=1}^{N} \left[P\left(x_i(\theta) + \sum_{j=1,\, j\neq i}^{N} x_j^e(\theta) \right) - \theta \right] x_i(\theta) - c\left(\sum_{i=1}^{N} x_i(\theta) \right). \tag{3}$$

Differentiating with respect to $x_i(\theta)$ and using the fact that retailers' expectations are correct in equilibrium, the quantity of each retailer in a symmetric equilibrium, $x_N^{CI}(\theta)$, is characterized

[29] As shown in the proof of Proposition 1, $\hat{\beta} = 0$ when $\overline{\theta} = 1/3$, which implies that using two retailers can be more profitable than using one retailer even if the manufacturer's cost is close to zero.

by the first-order condition

$$P(X_N^{CI}(\theta)) + P'(X_N^{CI}(\theta))x_N^{CI}(\theta) = \theta + c'(X_N^{CI}(\theta)), \tag{4}$$

where $X_N^{CI}(\theta) \triangleq N x_N^{CI}(\theta)$.[30]

Condition (4) shows that, in equilibrium, each retailer sells a quantity such that the retailer's marginal revenue equals the bilateral marginal cost, which is the sum of the manufacturer's and the retailer's cost. Because the manufacturer contracts bilaterally with each retailer, she does not internalize the effect of selling an additional unit to a retailer on the profit of the other $N-1$ retailers. Hence, retailers only accept contracts with the Cournot quantity (since, for any lower quantity, each retailer would expect the manufacturer to secretly sell a larger quantity to his rivals). This prevents the manufacturer from achieving the monopoly profit—the *opportunism problem.*

Although the maximization problem (3) is a multilateral one (i.e., M chooses the quantity offered to each retailer), the equilibrium quantities are equivalent to those obtained when splitting M's maximization problem into N bilateral contracting problems. While this result is well known for additive-separable cost functions (i.e., when $x_i(\theta)$ and $x_j(\theta)$, $j \neq i$, do not interact in M's cost function), it also holds for cost functions which are not additive separable. The reason is that, due to unobservability of offers, the tariff that M obtains from retailer $j \neq i$ is independent of $x_i(\theta)$, which implies that the marginal revenue is the same when considering a multilateral and a bilateral maximization problem. As a consequence, the aggregate quantity of R_i's rivals is $(N-1)x_N^{CI}$ in both maximization problems, and, hence, the total marginal costs are also the same.

The manufacturer chooses the number of retailers to maximize her aggregate expected profit

$$\pi^{CI}(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(P\left(X_N^{CI}(\theta)\right) - \theta\right) X_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right)\right] dF(\theta).$$

Maximizing this profit with respect to N yields the following result.

Proposition 2 *With complete information, M distributes through a single monopolistic retailer.*

With complete information, the manufacturer's optimal choice is to use a single retailer in

[30] M's problem cannot feature corner solutions. First, the quantity sold to retailers cannot be too large as this would reduce the market price to zero. Second, $x_i = 0$ cannot be optimal because of Assumption 1.

order to avoid the opportunism problem. This exclusive retailer monopolizes the downstream market, and the manufacturer obtains the monopoly profit.[31]

4.2 Asymmetric Information

Assume now that retailers have private information about their costs. We first characterize the optimal contract offered by M for a given a number of retailers, and then analyze the optimal retail market structure.

Consider a (differentiable) symmetric equilibrium in which each retailer sells the same quantity $x_N^*(\theta)$, given the cost θ. Because expectations are correct in equilibrium, this implies that each retailer's belief about his rivals' quantity is $x_N^*(\theta)$. Let

$$u_i(m_i, \theta) \triangleq \left(P\left(x_i(m_i) + (N-1)\, x_N^*(\theta)\right) - \theta\right) x_i(m_i) - T_i(m_i)$$

be R_i's utility when M offers the contract $\{T_i(m_i), x_i(m_i)\}$, he reports m_i and the cost is θ; and let $u_i(\theta) \triangleq u_i(m_i = \theta, \theta)$ be R_i's information rent. Following standard techniques, the necessary (local) first-order condition for R_i to truthfully report his type is[32]

$$\dot{x}_i(\theta)\, P'\left(x_i(\theta) + (N-1)\, x_N^*(\theta)\right) x_i(\theta) + \left(P\left(x_i(\theta) + (N-1)\, x_N^*(\theta)\right) - \theta\right) \dot{x}_i(\theta) - \dot{T}_i(\theta) = 0,$$

which yields the derivative of R_i's information rent

$$\dot{u}_i(\theta) = -x_i(\theta) + (N-1)\, P'\left(x_i(\theta) + (N-1)\, x_N^*(\theta)\right) \dot{x}_N^*(\theta)\, x_i(\theta).$$

Hence, R_i's information rent is

$$u_i(\theta) \triangleq u_i\left(\overline{\theta}\right) + \int_\theta^{\overline{\theta}} x_i(z)\, dz - \underbrace{(N-1) \int_\theta^{\overline{\theta}} P'\left(x_i(z) + (N-1)\, x_N^*(z)\right) \dot{x}_N^*(z)\, x_i(z)\, dz}_{\text{Competing-contracts effect}}. \quad (5)$$

This expression generalizes equation (1)—the information rent in the two-types example—and reflects the *competing-contracts effect*. When R_i over-reports his cost, he knows that his rivals acquire a larger quantity because they report a lower cost to the manufacturer, which reduces the downstream price. The tariff requested by the manufacturer, however, assumes that all retailers have a cost equal to R_i's report, which implies that the tariff is based on a higher

[31]This extreme result arises because retailers sell undifferentiated products. By contrast, a manufacturer may prefer multiple retailers if, for example, final consumers perceive retailers' products as differentiated.

[32]In Appendix A, we show that the global incentive compatibility constraint is satisfied.

retail price than the one that forms in the market. As a consequence, R_i's utility from over-reporting (i.e., the information rent he receives) is lower.

Other things being equal, R_i's incentive to overstate his cost decreases in the number of competing retailers in the downstream market. The reason is that the competing-contracts effect strengthens as the downstream market becomes more competitive, while it vanishes for $N \to 1$. In fact, as N increases, each retailer knows that he faces an even lower price when he over-reports his cost because the aggregate quantity produced by other retailers is larger. Therefore, a larger a number of retailers reduces the information rent that the manufacturer must pay to each of them (whereas the level of information asymmetry is independent of N).

To solve the manufacturer's problem, we split it into N bilateral maximization problems. In Appendix A, following the same logic as in case with complete information, we show that the resulting equilibrium is equivalent to the one of the multilateral maximization problem. Substituting for $u_i(\theta)$ into M's objective function and integrating by parts, in the bilateral (relaxed) contracting problem with R_i, M solves[33]

$$\max_{x_i(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[P(\cdot) - \theta - h(\cdot)\left(1 - (N-1) P'(\cdot)\, \dot{x}_N^*(\cdot)\right) \right] x_i(\cdot) - c\left(x_i(\cdot) + (N-1)\, x_N^*(\cdot)\right) \right\} dF(\theta).$$

Let $X_N^*(\theta) \triangleq N x_N^*(\theta)$. Differentiating pointwisely with respect to $x_i(\cdot)$ and rearranging, the symmetric equilibrium of the game is characterized by the differential equation

$$\dot{x}_N^*(\theta) = \frac{\theta + h(\theta) + c'\left(X_N^*(\theta)\right) - \left(P\left(X_N^*(\theta)\right) + P'\left(X_N^*(\theta)\right) x_N^*(\theta)\right)}{h(\theta)\,(N-1)\left(P'\left(X_N^*(\theta)\right) + P''\left(X_N^*(\theta)\right) x_N^*(\theta)\right)}, \tag{6}$$

with boundary condition $x_N^*(\underline{\theta}) = x_N^{CI}(\underline{\theta})$.

The unique solution of this differential equation has the following properties:

Lemma 1 *With asymmetric information, $\dot{x}_N^*(\theta) < 0$ and $x_N^*(\theta) \leq x_N^{CI}(\theta)$ for every θ, with equality only at $\theta = \underline{\theta}$.*

Therefore, in the presence of asymmetric information, the manufacturer sells to each retailer a lower quantity than with complete information, in order to limit information rents. As expected, the equilibrium output is decreasing in the marginal cost, there is no distortion at the top (i.e., for type $\underline{\theta}$) and a downward distortion for all types $\theta > \underline{\theta}$.

[33] In Appendix A, we show that M's maximization problem is additive separable across types and concave; hence, first-order conditions are necessary and sufficient for global optimality.

Moving backward to determine the equilibrium market structure, the manufacturer chooses the optimal number of retailers N^* to maximize her aggregate expected profit

$$\pi^*(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[P\left(X_N^*(\cdot)\right) - \theta - h(\cdot)\left(1 - (N-1) P'\left(X_N^*(\cdot)\right) \dot{x}_N^*(\cdot)\right) \right] X_N^*(\cdot) - c\left(X_N^*(\cdot)\right) \right\} dF(\theta). \tag{7}$$

The effect of a change in the number of retailers can be decomposed in two terms:

$$\frac{\partial \pi^*(N)}{\partial N} = \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} \left[P\left(X_N^*(\cdot)\right) + P'\left(X_N^*(\cdot)\right) X_N^*(\cdot) - \theta - h(\cdot) - c'(\cdot) \right] \frac{\partial X_N^*(\cdot)}{\partial N} dF(\theta) +}_{\text{Strategic effect}}$$

$$+ \underbrace{\frac{\partial}{\partial N} \int_{\underline{\theta}}^{\overline{\theta}} h(\cdot)(N-1) P'\left(X_N^*(\cdot)\right) \dot{x}_N^*(\cdot) X_N^*(\cdot) \, dF(\theta)}_{\text{Rent-extraction effect}}. \tag{8}$$

The first term of the right-hand side of (8) reflects the strategic effect of a change in N on aggregate profit, excluding the competing-contracts effect. In fact, the term in square parenthesis is the difference between marginal revenue and total marginal cost, minus the retailers' monopoly rent (i.e., the information rent of a retailer who has no competition in the downstream market). The second term, by contrast, only reflects the effect of a change in N on the competing-contracts effect.

The interaction between these two effects determines the optimal retail market structure. When $N \to 1$, the first effect vanishes because the aggregate quantity converges to the second-best monopoly one,[34] while the second effect is positive, as we show in the following result.

Proposition 3 *With asymmetric information, distributing through a single monopolistic retailer may not be optimal because*

$$\lim_{N \to 1^+} \frac{\partial \pi^*(N)}{\partial N} > 0.$$

Moreover, the optimal number of retailers N^* is finite because $\pi^*(N) < \pi^*(1)$ for N sufficiently large.

The intuition of this result is as follows. With a single retailer, there is no competition in the downstream market. A marginal increase in competition has a second-order effect on the manufacturer's profit through the opportunism problem because this problem is relatively weak.

[34]In fact, when $N \to 1$, equation (6) implies that $P\left(X_N^*(\cdot)\right) + P'\left(X_N^*(\cdot)\right) X_N^*(\cdot) = c'(\cdot) + \theta + h(\cdot)$.

By contrast, the competing-contracts effect is of first-order magnitude because the retailers' cost is distributed over a non-negligible support. As a consequence, a slight increase number of retailers increases the manufacturer's profit. It is important to note, however, that this result does not imply that a monopolistic retail market structure is never optimal, because the result only refers to a marginal increase in the number of retailers. As retailers can only be added in discrete numbers, distributing through one retailer can still be more profitable than distributing through two or more retailers (see Section 4.3). Proposition 3 also shows that the manufacturer never chooses a retail market structure that approaches the perfectly competitive level. In fact, as downstream competition becomes more intense, the opportunism problem strengthens and offsets the competing-contracts effect (in the perfectly competitive limit, when $N \to +\infty$, the manufacturer makes zero profit).

Remark. Our focus on quantity-forcing contracts is without loss of generality. In fact, even if the manufacturer does not control the quantity sold by retailers in the downstream market, retailers have no incentive to sell a quantity that is lower than the one acquired from the manufacturer. The reason is that, as we have shown, each retailer acquires a quantity that is weakly lower than the Cournot quantity. Therefore, no retailer has an incentive to individually reduce the quantity sold in the downstream market because their marginal revenue is higher than the marginal cost at the quantity acquired from the manufacturer.[35]

4.2.1 Unique versus Nash Implementation

We focused on truthful equilibria in the retailers' reporting game (i.e., in the continuation game of the second stage after the contract offers of the manufacturer), and showed that condition (6) characterizes a Nash equilibrium. One may wonder whether this is the unique symmetric equilibrium of the reporting game between retailers, or whether there are other symmetric but non-truthful reporting equilibria—i.e., whether, given the contracts characterized above, the unique equilibrium of the reporting game is the one in which retailers report truthfully. Specifically, a different equilibrium of the reporting game may exist in which all retailers report a higher cost (say $\theta' > \theta$) in order to weaken the competing-contracts effect and obtain a higher rent. This behavior could be interpreted as 'implicit collusion' between retailers, allowing them to coordinate on an equilibrium with a higher (expected) profit.

To show that the truthful-reporting equilibrium is the unique symmetric equilibrium in the second stage, given the equilibrium offer of the manufacturer, we show that every retailer is better off when reporting his cost truthfully even if all rivals mis-report. For any cost realization

[35] For a formal proof of this point, see Martimort and Piccolo (2007).

θ, this is equivalent to

$$\left[P\left(x_N^*\left(\theta\right)+\left(N-1\right)x_N^*\left(\theta'\right)\right)-\theta\right]x_N^*\left(\theta\right)-T_N^*\left(\theta\right)\geq\left[P\left(Nx_N^*\left(\theta'\right)\right)-\theta\right]x_N^*\left(\theta'\right)-T_N^*\left(\theta'\right). \quad (9)$$

Recall that, for every θ,

$$T_N^*\left(\theta\right)\triangleq\left[P\left(X_N^*\left(\theta\right)\right)-\theta\right]x_N^*\left(\theta\right)-\underbrace{\int_{\theta}^{\overline{\theta}}\left[1-\left(N-1\right)P'\left(X_N^*\left(z\right)\right)\dot{x}_N^*\left(z\right)\right]x_N^*\left(z\right)dx}_{\triangleq u_N^*\left(\theta\right)}.$$

Using (6), we can show that (9) is always fulfilled (see Appendix A). We therefore obtain the following result.

Proposition 4 *Suppose that M offers the contract $\left\{x_N^*\left(\cdot\right),T_N^*\left(\cdot\right)\right\}$ to every retailer. Then, in the unique symmetric equilibrium of the message game, retailers truthfully report the cost.*

The intuition behind this result is as follows. If all rivals over-report their cost, they sell a lower quantity than with truthful reporting, which increases the marginal revenue. A retailer then benefits more from each unit he sells compared to the case in which his rivals report truthfully. Since he obtains a lower quantity when over-reporting than when reporting truthfully, and a truth-telling strategy is optimal when the rivals also follow this strategy, by a revealed-preference argument, each retailer is better off with a truthful report than with over-reporting.

4.3 Linear-Quadratic Framework

In order to explicitly characterize the optimal number of retailers and to analyze how the retail market structure depends on demand and cost conditions, in this section we consider a more specialized model. Specifically, we assume that the manufacturer's cost function is quadratic—i.e.,

$$c\left(X\right)\triangleq\beta\frac{X^2}{2},$$

and that the demand function is linear—i.e., the (inverse) demand function is

$$P\left(X\right)\triangleq\max\left\{a-bX,0\right\}.$$

The parameter a (i.e., the highest willingness to pay) determines the price elasticity of demand, which is equal to $-P/(a-P)$; hence, a larger a implies less elastic demand. Instead, the parameter b is a measure of how the market price reacts to changes in the quantity sold by retailers.

We also assume that the random variable θ is distributed on $[0,1]$ according to the beta distribution—i.e., $\theta \sim \text{Beta}\left[1, \lambda^{-1}\right]$ such that $F(\theta) = \theta^{\frac{1}{\lambda}}$ and $h(\theta) = \lambda\theta$, with $\lambda \geq 0$ (see, e.g., Miravete, 2002). Since $F(\theta)$ is increasing in λ,[36] beta distributions parametrized by a lower value of λ first-order stochastically dominate those parametrized by higher values of λ. This implies that, as λ increases, retailers' marginal costs are more likely to be low, and therefore distortions are lower too (*ceteris paribus*). When $\lambda = 1$, the beta distribution is equal to the uniform distribution. All our assumptions are satisfied if $a \geq 1 + \lambda$.[37]

Condition (6) yields the following linear differential equation

$$\dot{x}_N^*(\theta) = \frac{a - \theta - h(\theta) - (b(N+1) + \beta N)\, x_N^*(\theta)}{h(\theta)\, b\,(N-1)},$$

with boundary condition

$$x_N^*(0) = \frac{a}{b(N+1) + \beta N}.$$

In Appendix A, we show that this differential equation has a unique linear solution

$$x_N^*(\theta) = \frac{a}{b(N+1) + \beta N} - \frac{\theta(1+\lambda)}{b(N+1) + \beta N + \lambda b\,(N-1)},$$

and that M's expected profit is

$$\pi^*(N) \triangleq \frac{2Nb + \beta N^2}{2} \int_0^1 x_N^*(\theta)^2\, d\theta^{\frac{1}{\lambda}}.$$

The next proposition compares the manufacturer's expected profit with 1 and 2 retailers.

Proposition 5 *There exist two thresholds $\hat{a} > 1 + \lambda$ and $\left(\frac{\hat{\beta}}{b}\right) \geq 0$ such that: (i) when $a \in (1+\lambda, \hat{a}]$, $\pi^*(2) \geq \pi^*(1)$; (ii) when $a > \hat{a}$, $\pi^*(2) \geq \pi^*(1)$ if and only if $\frac{\beta}{b} > \left(\frac{\hat{\beta}}{b}\right)$.*

Therefore, the manufacturer prefers a duopolistic retail market structure rather than a monopolistic one when either (*i*) demand is relatively elastic or (*ii*) her cost function is sufficiently convex and/or the market price is not too responsive to aggregate quantity.

The intuition for these results is as follows. First, when a is low—i.e., the elasticity of demand is large—the manufacturer optimally sells a relatively small quantity. Therefore, her

[36] In fact, $\frac{\partial F(\theta)}{\partial \lambda} = -\frac{\theta^{\frac{1}{\lambda}}}{\lambda^2} \ln\theta > 0$ for $\theta \in [0,1]$.

[37] Assuming that $a \geq 1 + \lambda$ also guarantees that $x_N^*(1) > 0$ at $N = 1$. Hence, it is a sufficient condition to rule out shut down.

incentives to expand the quantity to each retailer is weak. As a consequence, the opportunism problem is weak as well, so that the manufacturer prefers multiple retailers. Second, as the manufacturer's cost becomes more convex, increasing production becomes (relatively) more costly for her. This also implies that the opportunism problem gets weaker because expanding the quantity of one retailer is less profitable. In this case, the importance of the disciplining effect of competition on information rents is magnified. Third, if b increases, the market price becomes more responsive to changes in quantity. As a consequence, the opportunism problem gets worse because each retailer suffers more from an expansion in the quantity of his rivals; hence, the manufacturer tends to prefer a monopolistic retailer.

As Proposition 5 shows, when a is small, a market structure with two retailers is more profitable than with a single retailer even if the manufacturer's cost function is linear—i.e., $\beta = 0$. In general, for any combination of a and λ, if β is sufficiently high or b sufficiently low, the duopolistic retail structure dominates the monopolistic one. Of course, market structures with a much larger number of retailers can be optimal for the manufacturer as well. For example, if $a = 10$, $b = 1$, and $\lambda = 3$, then the optimal number of retailers is $N^* = 4$ for $\beta = 4$, and $N^* = 7$ for $\beta = 5$.

To provide a full analysis of the comparative statics of the parameters of the model, we consider a uniform distribution of the retailers' cost—i.e., we assume that $\lambda = 1$. In this case, expression (8), which characterizes the effects of a change in N on the manufacturer's profit, is

$$\frac{\partial \pi^*(N)}{\partial N} = \int_0^1 \left[a - 2bX_N^*(\cdot) - \beta X_N^*(\cdot) - 2\theta\right] \frac{\partial X_N^*(\cdot)}{\partial N} d\theta + \\ + \frac{\partial}{\partial N} \int_0^1 \theta (N-1) b |\dot{x}_N^*(\cdot)| X_N^*(\cdot) \, d\theta.$$

Again, the first term is the strategic effect, which captures the opportunism problem, and is strictly negative, while the second term is the rent-extraction effect, which reflects the effect of competing contracts, and is strictly positive.

Proposition 6 *When $\lambda = 1$, the optimal number of retailers is increasing in β and decreasing in a and b.*

Hence, with a uniform distribution, the effects shaping the comparison between a duopolistic and a monopolistic market structure in Proposition 5 apply more generally.

Turning to the comparative statics of λ, Figure 1 shows by numerical simulations that the optimal number of retailers is increasing in λ (for the chosen parameters).[38] The reason is that

[38] Specifically, $N^* = 2$ if $\lambda = 0.5$, $N^* = 3$ if $\lambda = 1$, and $N^* = 6$ if $\lambda = 2$. We cannot obtain an analytical solution for the comparative statics with respect to λ.

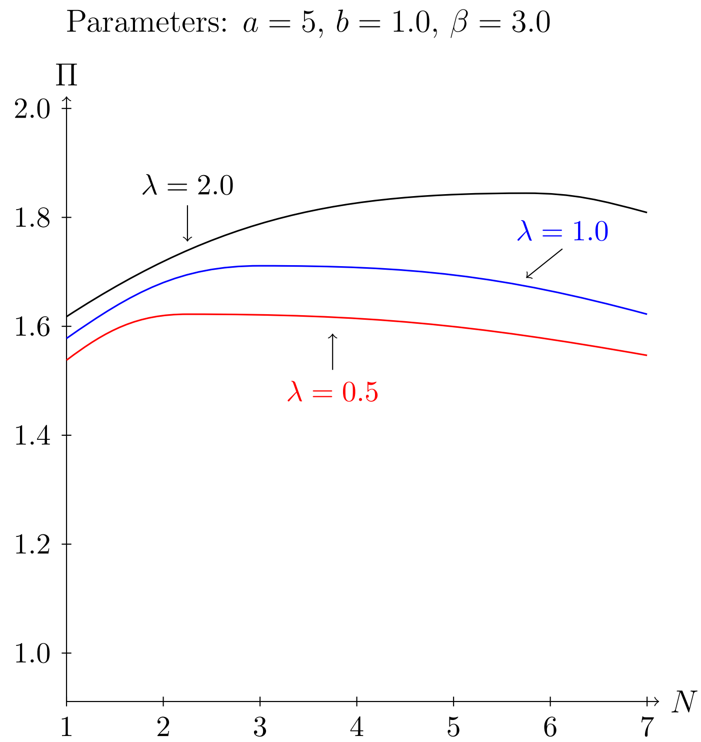

Figure 1: $N^\star$ for different values of λ

an increase in λ increases the mass of types distributed on the lower tail of the support, so that the retailers' cost is likely to be low. This implies that a retailer has a weaker incentive to overstate his type by reporting a higher marginal cost, as the manufacturer expects retailers to have low cost with high probability, leading to a relatively high aggregate quantity and a low price. The competing-contracts effect then becomes relatively more important, implying that the manufacturer benefits from using more retailers. By contrast, as λ goes to zero, the distribution converges to a mass point on the highest cost, which eliminates asymmetric information and yields $N^* = 1$.[39]

[39] This comparative-static result holds as long as λ is sufficiently low (i.e., it satisfies our assumption $a \geq 1+\lambda$). As mentioned in footnote 37, if this constraint is not satisfied, the optimal contract specifies a quantity of zero if retailers' costs are close to 1, which implies a shutdown of the market. The optimal number of retailers is then, in addition to our effects, determined by the boundary of θ above which a market shutdown occurs. If λ grows large, this effect dominates. For example, if $\lambda \to \infty$, the distribution converges to a mass point on 0, which implies that asymmetric information vanishes and $N^* = 1$.

5 Welfare Consequences

In this section, we draw the normative consequences of our analysis and show that our results have novel implications for competition policy with respect to the effects resulting from the size of the distribution network and from a vertical merger. We discuss these two points in turn, starting with a comparison of the privately and socially optimal network size (Section 5.1) and then turning to the effects of vertical integration (Section 5.2).[40]

In our main analysis, in order to focus on the novel effects, we did not consider costs of establishing a distribution network. Without such costs, the socially optimal number of retailers is infinity, as this leads to a price equal to marginal costs. In this section, to provide a meaningful welfare analysis, we assume that there is a fixed cost f of installing each retailer. This is a realistic assumption as distributing through an additional retailer requires investments and monitoring by the manufacturer to ensure that the retailer meets the required quality standards. We note that introducing such costs in our previous analysis does not affect the qualitative insights as these costs are orthogonal to the effects shown there.

5.1 Socially Optimal Number of Retailers

Consider a social planner who chooses the number of retailers but cannot control the contracts between the manufacturer and the retailers. In order to maximize the sum of the expected consumer surplus and the manufacturer's and retailers' profits, net of the costs of installing retailers, the social planner solves

$$\max_N \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \int_0^{X_N^*(\theta)} \left[P(x) - P(X_N^*(\theta))\right] dx + \left(P(X_N^*(\theta)) - \theta\right) X_N^*(\theta) - c(X_N^*(\theta)) \right\} dF(\theta) - fN.$$

Instead, M's objective function is given by (7) net of the cost fN. In contrast to the problem of the social planner, the manufacturer's problem neglects consumer surplus. However, it includes the information rent that the manufacturer must leave to retailers. As the information rent is just a transfer between the manufacturer and the retailers, it does not affect social welfare. Since consumer surplus is increasing in N, and the information rent is decreasing in N but enters the manufacturer's problem negatively, the comparison between the socially and the privately optimal number of retailers depends on the how these two terms react to a change in

[40] Following previous literature (e.g., Mankiw and Whinston, 1986, or Rey and Tirole, 2019), the regulator cannot directly choose prices, which are determined by firms. This is arguably the most relevant scenario for competition policy, as authorities usually face cases in which manufacturers restrict the number of distribution channels or propose vertical mergers.

N. Considering the linear-quadratic framework of Section 4.3 with $\lambda = 1$, the next proposition shows that the information rent might be more responsive to N than consumer surplus.

Proposition 7 *If β/b is sufficiently large, M chooses a number of retailers that is larger than the socially optimal one.*

The intuition behind the results rests again on the importance of the competing-contracts effect. If the inverse demand has a relatively flat slope (i.e., b is small), the manufacturer can reduce a retailer's information rent by a sizable amount when establishing more retailers, while an increase in the number of retailers has a limited effect on the market price, and hence on consumer surplus. Similarly, if the manufacturer's cost function is sufficiently convex, an increase in N raises the aggregate quantity only by a small amount but considerably lowers the information rent. The competing-contracts effect then dominates the opportunism problem, leading to a relatively large number of retailers in the private optimum.[41]

The result is in stark contract to the one obtained in models with complete information, in which the manufacturer always chooses a number of retailers that is below the socially optimal one, thus justifying the concern of antitrust authorities that distribution networks limit intra-brand competition and may reduce consumer surplus. Our analysis shows that this result can be reversed by asymmetric information.

5.2 Vertical Integration

Second, we consider the competitive effects of vertical integration. When the manufacturer vertically merges with a retailer, she distributes exclusively through this retailer and monopolizes the retail market. Indeed, with homogeneous products and symmetric retailers, the manufacturer has no incentive to distribute through other retailers after a vertical merger.[42] We assume that, in order to maximize industry profits, an integrated retailer reveals his cost to the manufacturer, so that the manufacturer does not distort quantities to reduce the information rent.[43]

[41] This result is reminiscent of excessive entry in a classic oligopoly model with free entry (e.g., Mankiw and Whinston, 1986). However, while excessive entry in oligopoly arises because additional firms reduce price but also steal business from rivals, our result hinges on asymmetric information in vertical contracting and on the competing-contracts effect that benefits the manufacturer but harms retailers.

[42] Reisinger and Tarantino (2015) also analyze the effects of vertical mergers in the presence of opportunism, but do not consider asymmetric information. They show that if the manufacturer is integrated with an inefficient retailer (i.e., one that has a higher marginal cost than a competing retailer), the manufacturer always distributes through both retailers and may even favor the non-integrated retailer via the wholesale contract to discipline its integrated unit.

[43] The assumption that all information problems are solved when the manufacturer and a retailer merge is admittedly strong. However, it follows the literature on vertical integration, which assumes that negotiations inside a vertically integrated company are efficient (e.g., Hart and Tirole, 1990). We also note that our results

Therefore, the downstream quantity $X^{VI}(\theta)$ is equal to the monopoly solution and implicitly given by

$$P(X^{VI}(\cdot)) + P'(X^{VI}(\cdot))X^{VI}(\cdot) = \theta + c'(X^{VI}(\cdot)).$$

Without vertical integration, given a distribution network N, the aggregate quantity $X_N^*(\theta)$ is distorted downward for rent extraction reasons. The optimal distribution network solves a modified version of the first-order condition (8), which takes into account the installment cost f.

Considering again the linear-quadratic framework of Section 4.3 with $\lambda = 1$, we can show the following result:

Proposition 8 *For any $f \geq 0$, if β/b is sufficiently large, consumer surplus and total welfare are higher with vertical integration than at the optimal distribution network with privately-informed retailers.*

The intuition behind the result is as follows. The quantity distortion imposed by the manufacturer to induce truthful reporting is particularly severe when the manufacturer's cost function is convex or the inverse demand is flat. In these cases, a retailer has a relatively strong incentive to report a higher-than-actual cost: with convex costs, the quantities sold by rival retailers who report truthfully are relatively low; with flat demand, the downstream price does not react much to changes in the quantities of rivals. The expected total production with asymmetric information is then relatively low, so that vertical integration raises consumer surplus and total welfare.

We note that this result occurs even if $f = 0$, which implies that the optimal distribution network without vertical integration can be large.[44] This shows the strength of the effect resulting from the elimination of quantity distortions as compared to reduced competition.

Several papers (e.g., Hart and Tirole, 1990; Rey and Tirole, 2007, Nocke and Rey, 2018) show that vertical integration reduces both consumer surplus and total welfare with complete information (because it eliminates downstream competition and leads to monopoly). By contrast, our analysis reveals that this result is not necessarily true with asymmetric information, when vertical integration avoids distortions resulting from information asymmetry.

arise as long as distortions within an integrated company are smaller than with separated firms, even if the distortions are not fully eliminated.

[44] If the optimal distribution network consists only of a single retailer, vertical integration trivially increases welfare because it eliminates the distortion generated by asymmetric information and does not affect competition in the downstream market.

6 Imperfect Correlation

In this section, we show that our qualitative results also hold when retailers' costs are not perfectly correlated. To keep this analysis tractable, we follow previous literature—e.g., Armstrong and Vickers (2010) and Dequiedt and Martimort (2015)—and suppose that retailers' costs are either identical or perfectly independent. Specifically, given a realization of a retailer's cost, we assume that with probability ν other retailers have the same cost, while with probability $1-\nu$ each rival retailer's cost is independently distributed according to $F(\theta)$.

Consider a symmetric equilibrium in which each retailer sells $x_N^*(\theta_i)$ when his cost is θ_i and let

$$\begin{aligned} u_i(\theta_i, m_i) \triangleq \; & [\nu P(x_i(m_i) + (N-1)x_N^*(\theta_i)) \\ & + (1-\nu)\mathbb{E}_{\theta_{-i}}\left[P\left(x_i(m_i) + X_{-i}^*(\theta_{-i})\right)\right] - \theta_i] \, x_i(m_i) - T_i(m_i) \end{aligned}$$

be R_i's rent when he reports m_i and his type is θ_i. By the Envelope Theorem,

$$\dot{u}_i(\theta_i) = -x_i(\theta_i) + \nu(N-1)P'(x_i(\theta_i) + (N-1)x_N^*(\theta_i))\,\dot{x}_N^*(\theta_i)\,x_i(\theta_i).$$

Integrating and assuming no rent at the bottom—i.e., $u_i(\overline{\theta}) = 0$—$R_i$'s information rent is

$$u_i(\theta_i) = \underbrace{\int_{\theta_i}^{\overline{\theta}} x_i(z)\,dz}_{\text{Standard rent}} \underbrace{- \nu(N-1)\int_{\theta_i}^{\overline{\theta}} P'(x_i(z) + (N-1)x_N^*(z))\,\dot{x}_N^*(z)\,x_i(z)\,dz.}_{\text{Competing-contracts effect}}$$

If $\dot{x}_N^*(\cdot) \leq 0$, that is, if in equilibrium each retailer produces a lower quantity when his marginal cost increases, this expression shows that the competing-contracts effect continues to be present even with imperfect cost correlation but becomes weaker as ν decreases. As intuition suggests, the expression converges to the standard Baron-Myerson rent for $\nu \to 0$ (i.e., when costs are uncorrelated) and to expression (5) for $\nu \to 1$ (i.e., when costs are perfectly correlated). In Appendix D, we consider an example with linear demand and quadratic cost and show that M prefers to use two retailers rather than one when β and ν are sufficiently large.

With imperfectly correlated types there is an additional 'sampling' reason that may induce M to implement a market structure with a relatively large number of retailers. In fact, by increasing the number of retailers, M increases the variance of aggregate output, and hence the price variability, which increases the fixed fees collected by M due to the fact that the (indirect) profit functions in the downstream market are convex in prices.

7 Sequential Contracting and Alternative Mechanisms

While the assumption that the manufacturer and the retailers contract simultaneously is commonly used in the literature and realistic in many environments, one may wonder whether with incomplete information the manufacturer may obtain a higher profit with sequential contracting due to better information extraction. In this section, we explain that the manufacturer can in fact improve her profit with a sequential-contracting mechanism but that such a mechanism, in contrast to our simultaneous mechanism, is fragile to small changes in the environment. Moreover, we also discuss other perturbations of the game—i.e., pre-contracting communication, the use of a bidding rather than an offer game, and public rather than private contracts—and show that, under natural assumptions, our main results survive when these aspects are taken into account.

Sequential Contracting. In our baseline model with perfectly correlated costs, the manufacturer's can increase her profit through a sequential mechanism with the following characteristics. The manufacturer uses two retailers and offers a contract to the first retailer in which, regardless of his report, the retailer makes no payment and receives no quantity. The retailer then has no incentive to mis-report, so that the manufacturer learns retailers' cost and can extract the entire monopoly surplus in the contracting with the second retailer. However, this mechanism, and more generally sequential contracting in the manufacturer-retailer relationship, is not robust to small and realistic changes to the environment.

First, with sequential contracting the manufacturer always has an incentive to go back to the first retailer and contract with him again, after contracting with the second retailer. This has two consequences: (i) it again creates the opportunism problem, and (ii) it provides the first retailer with an incentive to mis-report his cost, in order to obtain an information rent in future contracting.[45] A related problem arises if the manufacturer has to pay an arbitrarily small contracting cost to interact with an additional retailer because, in this case, the manufacturer would choose not to contract with the second retailer at all, after obtaining all information from the first retailer.

Second, if retailers' costs are imperfectly correlated, sequential contracting does not eliminate information rents. In this case, the report of the first retailer is only an imperfect signal for the second retailer's costs, so that the manufacturer still has to pay an information rent to the second retailer. In Appendix E, we consider an example with two types and imperfectly

[45]The more general point is that, although a sequential mechanism allows the manufacturer to use information obtained in early stages to improve later contracting, it also introduces an incentive for retailers who contract early to influence contracts offered later on, which tends to increase their information rents. Therefore, the manufacturer does not necessarily benefit from sequential contracting.

correlated costs, and compare the sequential mechanism that we have described with simultaneous contracting (where the competing-contracts effect reduces information rents as shown in Section 6). We show that the manufacturer's profit is higher with simultaneous contracting if the coefficient of correlation between retailers' costs is not too large.

Third, the sequential mechanism that we have described can only be implemented through a direct mechanism, in which retailers report their types, but not through indirect non-linear tariffs that are usually observed in practice. These tariffs specify a payment $T_i(x_i)$ for every quantity x_i and do not require communication between the manufacturer and the retailer. Therefore, with non-linear tariffs, if the manufacturer only offers to the retailer a contract with no payment and quantity, she learns no information from the retailer. By contrast, our contracts can be implemented through an indirect mechanism.

Finally, the sequential mechanism is discriminatory since it treats identical retailers differently. In some markets (e.g., licensing), this is problematic as antitrust laws require non-discriminatory contracts between upstream and downstream firms.[46] Instead, the equilibrium contracts with simultaneous contracting are indeed non-discriminatory.

In general, although mechanisms that eliminate all retailers' information rents exist in our environment, we chose to focus on simultaneous and secret contracting because this is a simple and realistic mechanism that captures the main elements of many types of real-world contracting between a manufacturer and the retailers. Moreover, a substantial part of the literature on vertical contracting and on the opportunism problem has focused on this mechanism.

Pre-Contracting Communication. In the spirit of sequential contracting, a manufacturer may try to use 'pre-play' communication with all retailers to exploit types' correlation and learn their cost before offering contracts. To fix ideas, suppose that there is an additional stage in our game between stages 1 and 2—i.e., after M has selected the retailers but before offering simulatneous contracts—in which all selected retailers report their cost realization. If all retailers report truthfully, the manufacturer learns θ before offering contracts and therefore does no leave an information rent to retailers.

However, retailers do not have an incentive to reveal their types in this pre-contracting communication. This is obvious in case of only one retailer as the retailer would then forego the information rent. With two or more retailers, the argument is more intricate. In fact, an equilibrium may exist in which, given that the retailers' reports about θ do not coincide, the

[46]For example, according to the Robinson-Patman Act in the US and Article 102 TFEU in the EU, a manufacturer who charges discriminatory contracts may be found liable of an infringement of the law. For a recent US case on golf equipment, see *Games People Play, Inc. v. Nike, Inc.; case number 1:14-CV-321.*

manufacturer's out-of-equilibrium belief is $\theta = \underline{\theta}$. Then, no retailer has a incentive to mis-report and the manufacturer, consequently, does not choose more than two retailers.

There are, however, two problems with this equilibrium. First, if retailer's costs are not perfectly correlated, the manufacturer's out-of-equilibrium belief is no longer justified, and the proposed mechanism to elicit the retailers' information does not work. Second, even with perfect correlation of costs, the equilibrium does not survive reasonable refinements (such as allowing for small trembles of retailers in their pre-contracting reports). The manufacturer's out-of equilibrium belief of $\theta = \underline{\theta}$ can then not be supported. As retailers are harmed by revealing their true types in this pre-contracting communication stage, in both cases, they will not transmit information to the manufacturer.

Bidding Game. In our analysis, we have considered an offer game in which, after deciding how many retailers to use, M contracts with them simultaneously. Could M benefit from committing to implement a monopolistic market structure and allowing potential retailers to bid for the right to be the monopolist in the final market? Consider, for example, a (first-price) auction in which retailers bid to acquire an exclusive license to distribute the manufacturer's product and the winner pays his bid to the manufacturer.[47] The manufacturer then simply chooses the quantity that the winning monopolistic retailer distributes in the downstream market. In this environment, competition erodes retailers' rents and allows the manufacturer to maximize profit: the auction price paid by the winning retailer is equal to the monopoly profit in the downstream market, and the manufacturer sells the monopoly quantity.[48]

This result, however, hinges on the absence of bidding costs. If there is an arbitrarily small bidding cost and retailers sequentially choose whether to enter the auction, then only one retailer participates and pays a price equal to zero to the manufacturer.[49] The reason is that, if there are two or more retailers, their profit from contracting with the manufacturer is equal to zero and hence they have no incentive to participate in the auction organized by the manufacturer. By contrast, in our game retailers obtain a strictly positive profit in expectation due to the information rent. Therefore, with an arbitrarily small bidding cost, using an auction does not allow the manufacturer to obtain the monopoly profit.

Similarly, even without bidding costs, at most one retailer would have a *strict* incentive to participate in a bidding game that selects the monopolistic distributor to which the man-

[47] Analogous results would be obtained if retailers bid a unit price at which to acquire the manufacturer's product, rather than a fixed price.

[48] The logic of this bidding mechanism is closely related to the result by Crémer and McLean (1988) that, with correlated types, a principal can achieve full surplus extraction by conditioning the offer made to one agent on the reports of his rivals, which is in fact what an auction does.

[49] With simultaneous entry by retailers, there are multiple pure-strategy equilibria, but in all of them only one retailer participates in the auction.

ufacturer offers a contract. The reason is that, if two or more retailers participate, (in a pure-strategy equilibrium) their bids reveal the common cost and, hence, the winner obtains no rent from the contract offered by the manufacturer.

Public Contracts. The assumption of secret contracting is arguably the most realistic one, as contracts between a manufacturer and her distributors are usually not public (see e.g., Rey and Tirole, 2007). Our main insights, however, also hold with public contracts.

To see this, suppose that in our baseline model all contracts offered by the manufacturer are observable to all retailers. With complete information, the manufacturer can obtain the monopoly profit with any size of the retail network by choosing contracts appropriately. If there is an arbitrarily small cost to select multiple retailers, the manufacturer optimally chooses a single retailer. By contrast, with asymmetric information, the competing-contracts effect is still at work, which implies that the manufacturer optimally chooses a larger number of retailers.

8 Conclusion

Vertical foreclosure is a major concern for antitrust authorities. This concern is often driven by the logic of the opportunism problem, which induces a manufacturer to limit intra-brand competition by distributing through few selected retailers. Our results, however, suggest that this concern is less relevant when retailers have private information about cost or demand.

In order to highlight this issue, we have examined a vertical contracting environment *à la* Segal and Whinston (2003) in which a manufacturer chooses the number of retailers who distribute her product and secretly contracts with each of them. The interplay between vertical opportunism and asymmetric information may induce a manufacturer to implement a distribution network with multiple retailers—even if retailers are undifferentiated—in order to exploit the disciplining effect of downstream competition on their information rents. This effect is sizable, and is stronger when the manufacturer's marginal costs are increasing and when the inverse demand function is relatively inelastic. Our analysis contributes to a better understanding of the forces that shape the retail market structure and advances the theoretical literature by introducing asymmetric information in a framework with secret contracting and externalities.

We conclude with two policy implications of our results that may provide guidance for policy makers with respect to the competitive effects in vertical relationships.

The first is on selective distribution systems (SDS), in which a brand supplier authorizes few distributors to sell to final consumers in each region. In the European Union, several recent antitrust cases analyze whether the SDS established by a supplier violates antitrust laws.[50] Whereas many SDSs are legal as they are primarily in place to ensure that distributors meet certain quality criteria, for others antitrust authorities fear the restriction of intra-brand competition. Our theory contributes to this discussion by showing that a manufacturer's incentive to design an anti-competitive SDS depends on informational asymmetries with respect to its retailers. Specifically, impediments of competition within an SDS are more likely in industries that are only moderately plagued by such asymmetries. By contrast, if information asymmetries are strong, brand suppliers may even foster competition within a SDS. This insight can provide guidance for antitrust authorities regarding which markets to scrutinize.

Our second policy implication is on the pro- and anti-competitive effects of vertical mergers, which are an ongoing topic of debate. For example, in January 2020, the FTC and the DOJ released a draft of the Vertical Merger Guidelines describing foreclosure or raising rivals' costs as important anti-competitive effects. These effects are (among other examples) explicitly mentioned in the context of retail distribution. Our analysis contributes to this discussion by taking into account information asymmetries and showing that a vertical merger that eliminates such asymmetries can have pro-competitive effects, despite foreclosure of rival retailers.

[50] In addition to the two examples given in the Introduction, there is, for instance, *Coty Germany GmbH v Parfumerie Akzente GmbH* in Germany in 2017, or *Ping Europe Limited v Competition and Markets Authority* in the U.K. in 2018.

Appendix

The Appendix consists of five parts. In Appendix A, we provide the proofs of all our results. In Appendix B, we analyze the case in which retailers compete in prices and offer differentiated products. In Appendix C, we consider wary beliefs by retailers. In Appendix D, we study the case of imperfect correlation in the linear framework. Finally, in Appendix E, we compare simultaneous and sequential contracting in the two-types case.

Appendix A: Proofs and Various Derivations

This Appendix provides proofs of all propositions, the proof of Lemma 1, as well as the proof that our equilibrium satisfies the incentive-compatibility constraints, the equivalence of the bilateral maximization problems with the multilateral one, and several calculations needed for the proofs of the linear-quadratic framework. We present the material in chronological order as it appears in the main text.

Proof of Proposition 1. With a single retailer, using the expressions for $\underline{x}^*(1)$ and $\overline{x}^*(1)$, the manufacturer's expected profit is

$$\pi^*(1) \triangleq \frac{1-2\overline{\theta}\left(1-\overline{\theta}\right)}{2(2+\beta)}.$$

Let now $N=2$ and

$$\underline{u}_i \triangleq \left(P\left(\underline{x}_i+\underline{x}^*(2)\right)\right)\underline{x}_i-\underline{T}_i,$$

$$\overline{u}_i \triangleq \left(P\left(\overline{x}_i+\overline{x}^*(2)\right)-\overline{\theta}\right)\overline{x}_i-\overline{T}_i.$$

Using standard techniques, the incentive compatibility constraints are

$$\underline{u}_i \geq P\left(\overline{x}_i+\underline{x}^*(2)\right)\overline{x}_i-\overline{T}_i \triangleq \overline{u}_i+\overline{\theta}\overline{x}_i-\Delta x^*\overline{x}_i \tag{10}$$

and

$$\overline{u}_i \geq \left(P\left(\underline{x}_i+\overline{x}^*(2)\right)-\overline{\theta}\right)\underline{x}_i-\underline{T}_i \triangleq \underline{u}_i-\overline{\theta}\underline{x}_i+\Delta x^*\underline{x}_i,$$

and the participation constraints require that $\underline{u}_i \geq 0$ and $\overline{u}_i \geq 0$.

Conjecturing that only the incentive compatibility constraint (10) matters, and that $\overline{u}_i=0$ at the optimal contract, it is easy to obtain expressions (2), which imply

$$\Delta x^*=\frac{\overline{\theta}}{2+\beta}.$$

Condition (10) yields the equilibrium rents

$$\underline{u}^*(2) = \frac{1+\beta}{2+\beta}\overline{x}^*(2)\,\overline{\theta} > 0$$

and

$$\overline{u}^*(2) = 0 > \frac{1+\beta}{2+\beta}\left(\overline{x}^*(2) - \underline{x}^*(2)\right)\overline{\theta}.$$

Hence, the starting conjecture is correct. Finally, letting $\underline{X}^* \triangleq 2\underline{x}^*(2)$, $\overline{X}^* \triangleq 2\underline{x}^*(2)$ and using (2), the manufacturer's expected profit is

$$\begin{aligned}
\pi^*(2) &= \tfrac{1}{2}\left[P\left(\underline{X}^*\right)\underline{X}^* - c\left(\overline{X}^*\right) + P\left(\overline{X}^*\right)\overline{X}^* - c\left(\underline{X}^*\right)\right] - \frac{(1+\beta)\,\overline{\theta}}{2\,(2+\beta)}\overline{X}^* \\
&= \frac{2\,(1+\beta)}{(3+2\beta)^2} - \frac{2\,(1+\beta)\,\overline{\theta}}{(3+2\beta)\,(2+\beta)} + \frac{(1+\beta)\,\overline{\theta}^2}{(2+\beta)^2}.
\end{aligned}$$

Let $\Delta\pi \triangleq \pi^*(2) - \pi^*(1)$. Using the expression for M's expected profit we have

$$\Delta\pi = \frac{4\overline{\theta}\left(1-2\overline{\theta}\right)\beta^2 + \beta\left(12\overline{\theta}-1\right)\left(1-2\overline{\theta}\right) - 2\left(1-3\overline{\theta}\right)^2}{2\,(2+\beta)^2\,(3+2\beta)^2},$$

with

$$\hat{\beta} \triangleq \frac{1+24\overline{\theta}^2 - 14\overline{\theta} + \sqrt{\left(6\overline{\theta}+1\right)\left(1-2\overline{\theta}\right)}}{8\overline{\theta}\left(1-2\overline{\theta}\right)}$$

being the unique positive root of $\Delta\pi = 0$ in the relevant region of parameters—i.e., $\overline{\theta} \in [0, 1/2)$. Because the denominator of $\Delta\pi$ is strictly positive, the sign of $\Delta\pi$ is equal to the sign of the numerator. Taking the derivative of $\Delta\pi$ with respect to β and evaluating the result at $\beta = \hat{\beta}$ yields $\sqrt{(1-2\overline{\theta})(6\overline{\theta}+1)}$, which is strictly positive for $\overline{\theta} \in [0, 1/2)$. Therefore, $\Delta\pi > 0$ if $\beta > \hat{\beta}$. If $\overline{\theta} \to 0$, then $\hat{\beta} \to \infty$. This implies that $\Delta\pi < 0$ for all β if $\overline{\theta} \to 0$. Instead, $\hat{\beta} > 0$ and finite for $0 < \overline{\theta} < 1/2$. Moreover, $\hat{\beta} = 0$ if $\overline{\theta} = 1/3$. ■

Proof of Proposition 2. The quantity produced by a monopolist in the downstream market is

$$X^M(\theta) \triangleq \arg\max_{X \geq 0}\,(P(X) - \theta)\,X - c(X),$$

which is unique since the function $(P(X) - \theta)\,X - c(X)$ is strictly concave. From condition (4), when $N = 1$, $X^M(\theta) = X_1^{CI}(\theta)$ for every θ.

We now show that $X_N^{CI}(\theta)$ is strictly increasing in N. Recall that $X_N^{CI}(\theta) \triangleq N x_N^{CI}(\theta)$.

From the first-order condition (4), the Implicit Function Theorem yields

$$\frac{\partial x_N^{CI}(\theta)}{\partial N} = -x_N^{CI}(\theta)\frac{P''\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right) - c''\left(X_N^{CI}(\theta)\right)}{P''\left(X_N^{CI}(\theta)\right)X_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right)(1+N) - c''\left(X_N^{CI}(\theta)\right)N},$$

which is negative since $c''(\cdot) \geq 0$ and $P'(X) + P''(X)X < 0$. Hence,

$$\begin{aligned}
\frac{\partial X_N^{CI}(\theta)}{\partial N} &= x_N^{CI}(\theta) + N\frac{\partial x_N^{CI}(\theta)}{\partial N} \\
&= x_N^{CI}(\theta)\left[1 - N\frac{P''\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right) - c''\left(X_N^{CI}(\theta)\right)}{P''\left(X_N^{CI}(\theta)\right)X_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right)(1+N) - c''\left(X_N^{CI}(\theta)\right)N}\right] \\
&= x_N^{CI}(\theta)\left[\frac{P'\left(X_N^{CI}(\theta)\right)}{P''\left(X_N^{CI}(\theta)\right)X_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right)(1+N) - c''\left(X_N^{CI}(\theta)\right)N}\right] > 0.
\end{aligned}$$

It follows that $X^M(\theta) < Nx_N^{CI}(\theta) = X_N^{CI}(\theta)$ for $N > 1$ and every θ.

M's aggregate (state-contingent) profit is

$$\pi^{CI}(N,\theta) \triangleq N(P\left(X_N^{CI}(\theta)\right) - \theta)x_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right) = (P\left(X_N^{CI}(\theta)\right) - \theta)X_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right).$$

Hence,

$$\pi^{CI}(N=1,\theta) = (P\left(X^M(\theta)\right) - \theta)X^M(\theta) - c\left(X^M(\theta)\right),$$

and $\pi^{CI}(N,\theta) < \pi^{CI}(N=1,\theta)$ for every θ and $N > 1$. This implies that the manufacturer chooses a single retailer to maximize her profit. ■

Multilateral Maximization Problem. In order to show that the equilibrium obtained when splitting M's problem into N bilateral maximization problems is equivalent to the full multilateral maximization problem, suppose that every retailer's belief about the quantity M offers to his rivals is $x_N^*(\theta)$. The manufacturer's maximization problem is

$$\begin{aligned}
\max_{\{x_i(\theta)\}_{i=1}^N} \sum_{i=1}^N \int_{\underline{\theta}}^{\overline{\theta}} & \left(P\left(x_i(\theta) + (N-1)x_N^*(\theta)\right) - \theta - h(\theta)\right)x_i(\theta)\,dF(\theta) + \\
&+ \int_{\underline{\theta}}^{\overline{\theta}} h(\theta)(N-1)P'\left(x_i(\theta) + (N-1)x_N^*(\theta)\right)\dot{x}_N^*(\theta)\,x_i(\theta)\,dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} c(X(\theta))\,dF(\theta),
\end{aligned}$$

where $X(\theta) \triangleq \sum_{i=1}^N x_i(\theta)$ for every $\theta \in \Theta$. This maximization problem is additively-separable across types, and we can therefore maximize it in a pointwise manner.

Differentiating with respect to $x_i(\theta)$ (for every $\theta \in \Theta$ and $i = 1,..,N$), an interior solution satisfies

$$\theta + h(\theta) + c'(X(\theta)) = P(x_i + (N-1)x_N^*(\theta)) + x_i(\theta) P'(x_i(\theta) + (N-1)x_N^*(\theta)) + \\ + h(\theta)(N-1)(P'(x_i(\theta) + (N-1)x_N^*(\theta)) + x_i(\theta) P''(x_i(\theta) + (N-1)x_N^*(\theta)))\dot{x}_N^*(\theta).$$

Hence, in an interior solution, $x_i(\theta) = x_j(\theta)$ for every i, j. Moreover, $x_N^*(\theta)$ solves

$$\theta + h(\theta) + c'(Nx_N^*(\theta)) = P(Nx_N^*(\theta)) + x_N^*(\theta) P'(Nx_N^*(\theta)) + \\ + h(\theta)(N-1)(P'(Nx_N^*(\theta)) + x_N^*(\theta) P''(Nx_N^*(\theta)))\dot{x}_N^*(\theta).$$

Solving this expression for $\dot{x}_N^*(\theta)$ then yields (6). It follows that $x_i(\theta) = x_N^*(\theta)$ for every θ (in an interior solution of M's maximization problem).

Finally, it is immediate that corner solutions cannot be optimal. Hence, the equilibrium characterized when splitting M's problem into N bilateral maximization problems is robust to multilateral deviations.

Proof of Lemma 1. First, by the Cauchy-Lipschitz theorem, the differential equation (6) with boundary condition $x_N^*(\underline{\theta}) = x_N^{CI}(\underline{\theta})$ has a unique solution.[51] We denote the equilibrium output by $x_N^*(\theta)$.

We now show that $x_N^*(\theta) \leq x_N^{CI}(\theta)\ \forall\theta$. To simplify notation, let $P' \triangleq P'(X_N^{CI}(\underline{\theta}))$, $P'' \triangleq P''(X_N^{CI}(\underline{\theta}))$, and $c'' \triangleq c''\left(X_N^{CI}(\underline{\theta})\right)$. Notice that $\lim_{\theta\to\underline{\theta}} \dot{x}_N^*(\theta) = 0/0$. Using L'Hôpital's rule,

$$\dot{x}_N^*(\underline{\theta}) = \frac{2}{2NP' + (2N-1)P''x_N^{CI}(\underline{\theta}) - Nc''},$$

which is strictly negative under our assumptions. Hence, in a neighborhood of $\underline{\theta}$,

$$x_N^*(\theta) \approx x_N^{CI}(\underline{\theta}) + \dot{x}_N^*(\underline{\theta})(\theta - \underline{\theta}).$$

Similarly,

$$\dot{x}_N^{CI}(\underline{\theta}) = \frac{1}{(N+1)P' + NP''x_N^{CI}(\underline{\theta}) - Nc''},$$

so that in a neighborhood of $\underline{\theta}$

$$x_N^{CI}(\theta) \approx x_N^{CI}(\underline{\theta}) + \dot{x}_N^{CI}(\underline{\theta})(\theta - \underline{\theta}).$$

[51] See Martimort (1996) for a detailed exposition.

Therefore,

$$\dot{x}_N^*(\underline{\theta}) - \dot{x}_N^{CI}(\underline{\theta}) = \frac{2P' + P''x_N^{CI}(\underline{\theta}) - Nc''}{((N+1)P' + NP''x_N^{CI}(\underline{\theta}) - Nc'')\,(2NP' + (2N-1)P''x_N^{CI}(\underline{\theta}) - Nc'')},$$

which is strictly negative under our assumptions on $P(\cdot)$ and $c(\cdot)$. Hence, $x_N^*(\theta) < x_N^{CI}(\theta)$ for $\theta \to \underline{\theta}$.

We now show by contradiction that this property holds globally. Suppose that $x_N^*(\theta) > x_N^{CI}(\theta)$ for some θ. Then, consider the lowest θ (say $\theta_1 > \underline{\theta}$) at which $x_N^*(\theta) = x_N^{CI}(\theta)$. By definition of $x_N^{CI}(\theta)$, equation (4) yields

$$\dot{x}_N^*(\theta_1) = \frac{1}{(N-1)\left(P'\left(X_N^{CI}(\theta_1)\right) + P''\left(X_N^{CI}(\theta_1)\right)x_N^{CI}(\theta_1)\right)},$$

which is negative because $P'(X) + P''(X)X < 0$. Note that

$$\text{sign}\left[\dot{x}_N^*(\theta_1) - \dot{x}_N^{CI}(\theta_1)\right] = \text{sign}\left[2P'\left(X_N^{CI}(\theta_1)\right) + P''\left(X_N^{CI}(\theta_1)\right)x_N^{CI}(\theta_1) - Nc''(X_N^{CI}(\theta_1))\right],$$

which is negative under our assumptions. Hence, for ε positive and small, a first-order Taylor approximation yields

$$\text{sign}\left[x_N^*(\theta_1 - \varepsilon) - x_N^{CI}(\theta_1 - \varepsilon)\right] = \text{sign}\left[\dot{x}_N^{CI}(\theta_1) - \dot{x}_N^*(\theta_1)\right] > 0,$$

which implies the desired contradiction $x_N^*(\theta_1 - \varepsilon) > x_N^{CI}(\theta_1 - \varepsilon)$. Hence, $x_N^*(\theta) < x_N^{CI}(\theta)$ for every θ and N.

We now show that $\dot{x}_N^*(\theta) \leq 0$ for every θ. Let $x_N^S(\theta)$ be the solution of

$$\theta + h(\theta) + c'(Nx) - \left[P'(Nx)\,x + P(Nx)\right] = 0, \tag{11}$$

that is, the value of x such that the numerator of (6) is zero. The left-hand side of (11) is increasing in x because of Assumption 2 and $c''(\cdot) \geq 0$. Notice that

$$\dot{x}_N^S(\theta) = \frac{1 + \dot{h}(\theta)}{P'\left(Nx_N^S(\theta)\right)(N+1) + NP''\left(Nx_N^S(\theta)\right)x_N^S(\theta) - c''\left(Nx_N^S(\theta)\right)N},$$

which is strictly negative by assumption. For $\theta \to \underline{\theta}$ it can be shown that

$$\dot{x}_N^S(\underline{\theta}) = \frac{2}{(N+1)\,P' + NP''x_N^S(\underline{\theta}) - Nc''} < \dot{x}_N^*(\underline{\theta}) = \frac{2}{2NP' + (2N-1)P''x_N^{CI}(\underline{\theta}) - Nc''}.$$

Therefore, $x_N^S(\theta) \leq x_N^*(\theta)$ for $\theta \to \underline{\theta}$. But this implies that the numerator of (6) is positive. As the denominator is strictly negative, $\dot{x}_N^*(\underline{\theta}) < 0$.

Finally, we show by contradiction that $x_N^S(\theta) \leq x_N^*(\theta)$ also holds globally. Suppose that $x_N^S(\theta) > x_N^*(\theta)$ for some θ. Let θ_2 be the lowest value for which $x_N^S(\theta) = x_N^*(\theta)$. By definition $\dot{x}_N^*(\theta_2) = 0 > \dot{x}_N^S(\theta_2)$. Now consider $\varepsilon > 0$ and small enough. By definition of θ_2, $x_N^S(\theta_2 - \varepsilon) < x_N^*(\theta_2 - \varepsilon)$ must hold. But, taking the limit $\varepsilon \to 0$, we have

$$x_N^S(\theta_2 - \varepsilon) - x_N^*(\theta_2 - \varepsilon) \approx -\dot{x}_N^S(\theta_2) > 0,$$

which again yields a contradiction. ■

Incentive-Compatibility Constraints. We now show that the equilibrium satisfies the constraints that we have neglected in the analysis. We start with the local second-order incentive compatibility constraint. From the derivative of R_i's information rent, given by

$$\dot{u}_i(\theta) = -x_i(\theta) + (N-1)P'(x_i(\theta) + (N-1)x_N^*(\theta))\dot{x}_N^*(\theta)x_i(\theta).$$

, the local second-order condition for R_i's maximization problem is

$$-\dot{x}_i(\theta)[1 - (N-1)\dot{x}_N^*(\theta)(P''(\cdot)x_i(\theta) + P'(\cdot))] \geq 0.$$

Using (6), in a symmetric equilibrium, this constraint requires that

$$\begin{aligned} 0 &\leq -\dot{x}_N^*(\theta)[1 - (N-1)\dot{x}_N^*(\theta)(P'(X_N^*(\theta)) + P''(X_N^*(\theta))x_N^*(\theta))] = \\ &= \frac{\dot{x}_N^*(\theta)}{h(\theta)}[\theta + c'(X_N^*(\theta)) - (P'(X_N^*(\theta))x_N^*(\theta) + P(X_N^*(\theta)))]. \end{aligned} \tag{12}$$

Since $x_N^*(\theta) \leq x_N^{CI}(\theta)$, the assumptions $P'(X) + P''(X)X < 0$ and $c''(\cdot) \geq 0$ yield

$$P'(X_N^*(\theta))x_N^*(\theta) + P(X_N^*(\theta)) - c'(X_N^*(\theta)) > P'\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P\left(X_N^{CI}(\theta)\right) - c'\left(X_N^{CI}(\theta)\right) = \theta.$$

Hence, the constraint is satisfied since $\dot{x}_N^*(\theta) \leq 0$.

Finally, we show that the global incentive compatibility constraint holds, too. R_i's global incentive compatibility constraint holds if and only if, in equilibrium, $u^*(\theta) \geq u^*(m_i, \theta)$ for every $m_i \neq \theta$. Let $m_i \geq \theta$ (without loss of generality), we have

$$u^*(\theta) - u^*(m_i, \theta) = \int_{m_i}^{\theta} \{(P(x_N^*(z) + (N-1)x_N^*(\theta)) - \theta)\dot{x}_N^*(z) + \\ +P'(x_N^*(z) + (N-1)x_N^*(\theta))\dot{x}_N^*(z)x_N^*(z) - \dot{T}_N^*\Big\}(z)\,dz.$$

By definition,

$$\dot{T}_N^*(z) = P'(x_N^*(z) + (N-1)x_N^*(z))\dot{x}_N^*(z)x_N^*(z) + (P(x_N^*(z) + (N-1)x_N^*(z)) - z)\dot{x}_N^*(z).$$

Substituting and using Assumption 2 on $P'''(\cdot)$, we have

$$\begin{aligned} u^*(\theta) - u^*(m_i, \theta) &= \int_{m_i}^{\theta} \dot{x}_N^*(z) \int_z^{\theta} \{-1 + (N-1)\dot{x}_N^*(y)[P'(x_N^*(z) + (N-1)x_N^*(y)) \\ &\quad +P''(x_N^*(z) + (N-1)x_N^*(y))x_N^*(z)]\}\,dydz \\ &\geq -\int_{m_i}^{\theta} \dot{x}_N^*(z) \int_z^{\theta} \{1 - (N-1)\dot{x}_N^*(y)[P'(X_N^*(y)) + P''(X_N^*(y))x_N^*(y)]\}\,dydz, \end{aligned}$$

which is positive by the second-order incentive-compatibility constraint—see condition (12).

Proof of Proposition 3. In a symmetric equilibrium M's expected profit is

$$\pi^*(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \pi_N^*(\theta)dF(\theta),$$

where $\pi_N^*(\theta) \triangleq N\{[P(X_N^*(\cdot)) - \theta - h(\cdot)(1 - (N-1)P'(X_N^*(\cdot))\dot{x}_N^*(\cdot))]x_N^*(\cdot)\} - c(X_N^*(\cdot))$.

Differentiating $\pi_N^*(\theta)$ with respect to N, by the Envelope Theorem we have

$$\frac{\partial \pi_N^*(\theta)}{\partial N} = (P(\cdot) - \theta - h(\cdot)(1 - (N-1)P'(\cdot)\dot{x}_N^*(\cdot)))x_N^*(\cdot) + \\ -c'(\cdot)\left(x_N^*(\cdot) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right) + N\left[P'(\cdot)\left(x_N^*(\theta) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right) + h(\cdot)P'(\cdot)\dot{x}_N^*(\cdot)\right]x_N^*(\cdot) + \\ +(N-1)Nh(\cdot)\left[P''(\cdot)\left(x_N^*(\theta) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right)\dot{x}_N^*(\cdot) + P'(\cdot)\tfrac{\partial \dot{x}_N^*(\cdot)}{\partial N}\right]x_N^*(\cdot).$$

Hence,

$$\lim_{N\to 1^+}\frac{\partial \pi_N^*(\theta)}{\partial N} = \left(P\left(x_1^*(\cdot)\right)-\theta-h(\cdot)\right)x_1^*(\cdot)-c'\left(x_1^*(\cdot)\right)x_1^*(\cdot)+$$
$$+\left(P'\left(x_1^*(\cdot)\right)x_1^*(\theta)+h(\cdot)P'\left(x_1^*(\cdot)\right)\dot{x}_1^*(\cdot)\right)x_1^*(\cdot)$$
$$=\left(P\left(x_1^*(\cdot)\right)+P'\left(x_1^*(\cdot)\right)x_1^*(\theta)-\theta-h(\cdot)-c'\left(x_1^*(\cdot)\right)\right)x_1^*(\cdot)+$$
$$+h(\cdot)P'\left(x_1^*(\cdot)\right)\dot{x}_1^*(\cdot)x_1^*(\cdot).$$

By definition of $x_1^*(\theta)$,

$$P\left(x_1^*(\cdot)\right)+P'\left(x_1^*(\cdot)\right)x_1^*(\theta)-\theta-h(\cdot)-c'\left(x_1^*(\cdot)\right)=0,$$

Therefore,

$$\lim_{N\to 1^+}\frac{\partial \pi^*(N)}{\partial N}=\int_{\underline{\theta}}^{\overline{\theta}} h(\cdot)P'\left(x_1^*(\cdot)\right)\dot{x}_1^*(\cdot)x_1^*(\cdot)\,dF(\theta),$$

which is strictly positive since $\dot{x}_1^*(\cdot)\le 0$ and $P'(\cdot)<0$.

Finally, we show that $\pi^*(N)<\pi^*(1)$ for N sufficiently large. Notice that

$$\pi^*(N)\le\pi^{CI}(N)=\int_{\underline{\theta}}^{\overline{\theta}}\left[N\left(P\left(X_N^{CI}(\cdot)\right)-\theta\right)x_N^{CI}(\cdot)-c\left(X_N^*(\cdot)\right)\right]dF(\theta).$$

To determine $\lim_{N\to+\infty}x_N^{CI}(\theta)$, we can use the derivative of M's objective function under complete information. Doing so yields

$$P(X^C(\theta))+P'(X^C(\theta))\lim_{N\to+\infty}x_N^{CI}(\theta)-\left(\theta+c'(X^C(\theta))\right)=P'(X^C(\theta))\lim_{N\to+\infty}x_N^{CI}(\theta).$$

If $\lim_{N\to+\infty}x_N^{CI}(\theta)>0$, this derivative is strictly negative, which yields a contradiction because M would choose a smaller quantity to increase profit. Hence, $\lim_{N\to+\infty}x_N^{CI}(\theta)=0$. Hence,

$$\lim_{N\to+\infty}\pi^*(N)\le\lim_{N\to+\infty}\int_{\underline{\theta}}^{\overline{\theta}}\left[N\left(P\left(X_N^{CI}(\cdot)\right)-\theta\right)x_N^{CI}(\cdot)-c\left(X_N^*(\cdot)\right)\right]dF(\theta)=0.$$

Since $\pi^*(1)>0$, the result follows. ■

Proof of Proposition 4. Let

$$\Phi(z)\triangleq\left[1-(N-1)P'\left(X_N^*(z)\right)\dot{x}_N^*(z)\right]x_N^*(z).$$

We first consider the case $\theta' > \theta$. Substituting $T_N^*(\theta)$ into (9), we obtain

$$\int_\theta^{\theta'} (N-1)\, P'\left(x_N^*(\theta) + (N-1)\, x_N^*(z)\right) \dot{x}_N^*(z)\, x_N^*(\theta)\, dz + \int_\theta^{\overline{\theta}} \Phi(z)\, dz \geq x_N^*(\theta')(\theta' - \theta) + \int_{\theta'}^{\overline{\theta}} \Phi(z)\, dz$$

$$\Leftrightarrow \quad \int_\theta^{\theta'} (N-1)\, P'\left(x_N^*(\theta) + (N-1)\, x_N^*(z)\right) \dot{x}_N^*(z)\, x_N^*(\theta)\, dz + \int_\theta^{\theta'} \Phi(z)\, dz - x_N^*(\theta')(\theta' - \theta) \geq 0.$$

Using the definition of $\Phi(z)$ we can rewrite this as

$$(N-1) \int_\theta^{\theta'} \dot{x}_N^*(z) \int_\theta^{z} \dot{x}_N^*(y) \left[P'\left(x_N^*(y) + (N-1)\, x_N^*(z)\right) + x_N^*(y)\, P''\left(x_N^*(y) + (N-1)\, x_N^*(z)\right)\right] dydz$$

$$\leq - \int_\theta^{\theta'} \int_z^{\theta'} \dot{x}_N^*(y)\, dydz.$$

This inequality is strictly satisfied for all $\theta' > \theta$, since $\dot{x}_N^*(\cdot) < 0$ and $P'(\cdot) + xP''(\cdot) < 0$ by Assumption 2.

We now turn to the case $\theta' < \theta$. Retailers want to coordinate on a symmetric equilibrium in which they report θ' lower than the true cost θ if and only if

$$\left[P\left(x_N^*(\theta') + (N-1)\, x_N^*(\theta')\right) - \theta\right] x_N^*(\theta') - T_N^*(\theta') > \left[P\left(Nx_N^*(\theta)\right) - \theta\right] x_N^*(\theta) - T_N^*(\theta). \quad (13)$$

Suppose that (13) holds for some $\theta' < \theta$. Substituting $T_N^*(\theta)$ on both sides and rearranging, we obtain

$$\int_\theta^{\theta'} \left(x_N^*(\theta') - x_N^*(z)\right) dz + (N-1) \int_\theta^{\theta'} P'\left(Nx_N^*(z)\right) \dot{x}_N^*(z)\, x_N^*(z)\, dz > 0,$$

which cannot hold for any $\theta' < \theta$ since $\dot{x}_N^*(\cdot) < 0$ implies that $x_N^*(\theta') > x_N^*(z)$ for every $z > \theta'$ and $P'(\cdot)\, \dot{x}_N^*(\cdot) > 0$. ■

Linear-Quadratic Framework. We solve the differential equation

$$\dot{x}_N^*(\theta) = \frac{a - \theta - h(\theta) - (b(N+1) + \beta N)\, x_N^*(\theta)}{h(\theta)\, b\, (N-1)},$$

with boundary condition

$$x_N^*(0) = \frac{a}{b(N+1) + \beta N}.$$

Rearranging terms, we obtain

$$\dot{x}_N^*(\theta) + \frac{b(N+1)+\beta N}{\lambda\theta b(N-1)} x_N^*(\theta) = \frac{a-\theta(1+\lambda)}{\lambda\theta b(N-1)}.$$

Letting $\Gamma \triangleq \frac{b(N+1)+\beta N}{\lambda b(N-1)}$, the solution to the previous equation is

$$x_N^*(\theta) = k e^{-\int_0^\theta \frac{\Gamma}{z_1} dz_1} + \int_0^\theta e^{-\int_{z_2}^\theta \frac{\Gamma}{z_1} dz_1} \frac{a - z_2(1+\lambda)}{z_2 \lambda b(N-1)} dz_2. \tag{14}$$

Notice that $e^{-\int_0^\theta \frac{\Gamma}{z_1} dz_1} = e^{-\Gamma \ln z_1|_0^\theta} = 0$. Therefore, the term multiplying the constant k is zero, which implies that the boundary condition is not needed. As a consequence, (14) can be simplified to

$$x_N^*(\theta) = \int_0^\theta e^{-\Gamma \ln z_1|_{z_2}^\theta} \frac{a - z_2(1+\lambda)}{z_2 \lambda b(N-1)} dz_2.$$

Since $e^{-\Gamma \ln z_1|_{z_2}^\theta} = \frac{z_2^\Gamma}{\theta^\Gamma}$, rearranging yields

$$\begin{aligned} x_N^*(\theta) &= \frac{1}{\lambda b(N-1)\theta^\Gamma} \int_0^\theta \left(a z_2^{\Gamma-1} - z_2^\Gamma (1+\lambda) \right) dz_2 \\ &= \frac{1}{\lambda b(N-1)\theta^\Gamma} \left(\frac{a\theta^\Gamma}{\Gamma} - \frac{\theta^{\Gamma+1}(1+\lambda)}{\Gamma+1} \right) \\ &= \frac{a}{b(N+1)+\beta N} - \frac{\theta(1+\lambda)}{b(N+1)+\beta N + \lambda b(N-1)}. \end{aligned}$$

From this, we obtain

$$\dot{x}_N^*(\theta) = -\frac{(1+\lambda)}{b(N+1)+\beta N + \lambda b(N-1)}.$$

The expected profit of the manufacturer (7) can be written as

$$\begin{aligned} \pi^*(N) &= \int_0^1 \left(N x_N^*(\theta)^2 (b+\beta N) - \frac{\beta}{2} (N x_N^*(\theta))^2 \right) dF(\theta) \\ &= \int_0^1 \left(N x_N^*(\theta)^2 b + \frac{\beta}{2} (N x_N^*(\theta))^2 \right) dF(\theta) \\ &= \frac{2Nb + \beta N^2}{2} \int_0^1 x_N^*(\theta)^2 dF(\theta). \end{aligned}$$

Since $\theta \sim \text{Beta}\left[1, \lambda^{-1}\right]$

$$\mathbb{E}\left[\theta\right] = \frac{1}{\lambda} \int_0^1 \theta^{\frac{1}{\lambda}} d\theta = \frac{1}{1+\lambda} \left. \theta^{\frac{1}{\lambda}+1} \right|_0^1 = \frac{1}{1+\lambda}$$

and

$$\mathbb{E}\left[\theta^2\right] = \frac{1}{\lambda} \int_0^1 \theta^{\frac{1}{\lambda}+1} d\theta = \frac{1}{1+2\lambda} \left. \theta^{\frac{1}{\lambda}+2} \right|_0^1 = \frac{1}{1+2\lambda}.$$

Substituting for $x_N^*(\theta)$ and integrating yields

$$\pi^*(N) = \frac{a^2(2Nb+\beta N^2)}{2\left(b(N+1)+\beta N\right)^2} + \frac{\left(2Nb+\beta N^2\right)(1+\lambda)^2}{2(1+2\lambda)\left(b(N+1)+\beta N+\lambda b(N-1)\right)^2} + \\ - \frac{a\left(2Nb+\beta N^2\right)}{\left(b(N+1)+\beta N\right)\left(b(N+1)+\beta N+\lambda b(N-1)\right)}. \quad (15)$$

For $\lambda = 1$, the two terms of $\partial \pi^*(N)/\partial N$ are

$$\int_0^1 \left[a - 2bX_N^*(\cdot) - \beta X_N^*(\cdot) - 2\theta\right] \frac{\partial X_N^*(\cdot)}{\partial N} d\theta = -\frac{a^2 b (N-1)}{(b+N\beta+Nb)^3} < 0,$$

and

$$\frac{\partial}{\partial N} \int_0^1 \theta \frac{2(N-1)\, bX_N^*(\cdot)}{b(N+1)+\beta N+b(N-1)} dF(\theta).$$

This second expression is strictly positive because $\partial X_N^*(\cdot)/\partial N = (ab)/(b+N\beta+Nb)^2 > 0$ and

$$\frac{\partial}{\partial N}\left(\frac{2(N-1)\, b}{b(N+1)+\beta N+b(N-1)}\right) = \frac{2b}{N^2(2b+\beta)} > 0.$$

Proof of Proposition 5. We first divide both the numerator and the denominator of the equilibrium profit (15) by b^2. Doing so and denoting $\beta/b \equiv \psi$ yields

$$\pi^*(N) = \frac{a^2(2N+\psi N^2)}{2b\left((N+1)+\psi N\right)^2} + \frac{\left(2N+\psi N^2\right)(1+\lambda)^2}{2b(1+2\lambda)\left((N+1)+\psi N+\lambda(N-1)\right)^2} + \\ - \frac{a\left(2N+\psi N^2\right)}{b\left((N+1)+\psi N\right)\left((N+1)+\psi N+\lambda(N-1)\right)}. \quad (16)$$

Hence,

$$\pi^*(1) = \frac{1}{b(2+\psi)}\left(\frac{a^2}{2} - a + \frac{(1+\lambda)^2}{2(2\lambda+1)}\right),$$

and

$$\pi^*(2) = \frac{2(1+\psi)}{b}\left(\frac{a^2}{(3+2\psi)^2} + \frac{(1+\lambda)^2}{(3+2\psi+\lambda)^2(2\lambda+1)} - \frac{2a}{(3+2\psi)(3+2\psi+\lambda)}\right).$$

The sign of the difference $\pi^*(2) - \pi^*(1)$ is a polynomial function of third order in ψ, with a leading term of

$$16\lambda\left(a + 2\lambda(a-1) - 1 - \lambda^2\right). \tag{17}$$

Since $a > 1+\lambda$ by assumption, and (17) is equal to $16\lambda^2(1+\lambda) > 0$ for $a = 1+\lambda$ and is increasing in a, the leading term is positive for all a in the admissible range. Moreover, there is no bound on ψ. It follows that for any combination of a and λ, the difference $\pi^*(2) - \pi^*(1)$ is positive if ψ large enough.

We will now show that the difference $\pi^*(2) - \pi^*(1)$ is either positive for all $\psi \geq 0$ or that there exists a unique threshold, denoted by $\hat{\psi}$, such that the difference is positive if and only if $\psi > \hat{\psi}$.

Let us first consider the case $\psi = 0$. The sign of the difference $\pi^*(2) - \pi^*(1)$ is then given by the sign of

$$a(3+\lambda)(2\lambda+1)(6+18\lambda-3a-a\lambda) - 9(1+\lambda)^2(1+6\lambda+\lambda^2), \tag{18}$$

which is strictly concave in a. Setting this term equal to zero and solving for a yields two solutions,

$$a_1 = \frac{3\left(1+5\lambda+6\lambda^2 - \sqrt{\lambda^2(1+2\lambda)(7+10\lambda-\lambda^2)}\right)}{(3+\lambda)(1+2\lambda)}$$

and

$$a_2 = \frac{3\left(1+5\lambda+6\lambda^2 + \sqrt{\lambda^2(1+2\lambda)(7+10\lambda-\lambda^2)}\right)}{(3+\lambda)(1+2\lambda)}.$$

Due to the concavity of (18), $\pi^*(2) \geq \pi^*(1)$ for all $a \in [a_1, a_2]$. Since $a_1 < 1+\lambda$ and $a_2 > 1+\lambda$ for all $\lambda > 0$, the profit with two retailers is larger than with one retailer if $a \in (1+\lambda, a_2)$. We denote $\hat{a} \triangleq a_2$.

We now show that, if $a \in (1+\lambda, \hat{a})$, the same result holds for any $\psi > 0$. The derivative of $\pi^*(2) - \pi^*(1)$ with respect to ψ, evaluated at $\psi = 0$, is

$$-4(3+\lambda)^2(1+2\lambda)a^2 + 8(1+2\lambda)(3+14\lambda+3\lambda^2)a - 12(1+\lambda)^2(1+9\lambda+\lambda^2). \tag{19}$$

It is easy to check that (19) is positive for all $a \in [1+\lambda, \hat{a})$. Moreover, setting the derivative of $\pi^*(2) - \pi^*(1)$ with respect to ψ equal to 0, we obtain that the lower one of the two solutions is

$$\frac{(1+2\lambda)a\left(a-2(1+9\lambda+\lambda^2)\right)+(1+\lambda)^2(1+18\lambda+\lambda^2)-\sqrt{\xi}}{12\left(a+2\lambda(a-1)-1-\lambda^2\right)}, \tag{20}$$

with

$$\xi \equiv (1+2\lambda)^2a^3\left[a+4(2\lambda^2-1)\right]+(1+2\lambda)^2(1+\lambda)^4$$
$$+2(1+2\lambda)a\left[a(3+6\lambda(1-\lambda)-14\lambda^3-3\lambda^4+4\lambda^5)-2(1-\lambda^2+\lambda^4)(1+\lambda)^2\right].$$

Tedious but routine manipulations yield that (20) is negative for all admissible values of a and λ. Hence, since $\pi^*(2) - \pi^*(1)$ is a polynomial of third order with a positive leading term, the unique local maximum of this difference occurs at a negative value of ψ. As the difference is positive at $\psi = 0$ for $a \in (1+\lambda, \hat{a})$ and the derivative of the difference at is also positive at $\psi = 0$, $\pi^*(2) - \pi^*(1)$ is positive for $\psi > 0$.

We now turn to the range $a \geq \hat{a}$, where the difference between $\pi^*(2)$ and $\pi^*(1)$ is negative at $\psi = 0$. As the leading term of the third-order polynomial is positive and there is no bound on ψ, the difference is positive for ψ large enough. Using again the fact that the difference has a unique local maximum at $\psi < 0$, it follows that there must be a unique solution in the region $\psi > 0$, denoted by $\hat{\psi}$, such that the difference is positive if and only if $\psi > \hat{\psi}$. ■

Proof of Proposition 6. Setting $\lambda = 1$ in the profit function (16) and taking the derivative with respect to N, we obtain the first-order condition

$$\frac{a}{b(N+1+\psi N)^2} - \frac{3a^2N^2(2+\psi)^2(N-1)+4(N+1+\psi N)^3}{3bN^2(2+\psi)^2(N+1+\psi N)^3} = 0. \tag{21}$$

This condition cannot be solved explicitly for N. However, solving (21) for a yields two roots and the only one consistent with Assumption 1 (which requires that $a \geq 2$) is

$$a = \frac{3N(2+\psi)+\sqrt{3}\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{6N(N-1)(2+\psi)}. \tag{22}$$

To determine whether the profit function is concave in N, we differentiate (21) with respect to N and substitute a from (22) to obtain

$$-\frac{3\psi^3N^3+2\psi^2N^2(4+5N)+4\psi N(12N-4-N^2)+8(4N+2N^2-2-N^3)}{6N^3(N-1)^2(2+\psi)^2(N+1+\psi N)^2} - \frac{N^2(2+\psi)^2\sqrt{3}\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{6N^3(N-1)^2(2+\psi)^2(N+1+\psi N)^2}. \quad (23)$$

Since the denominator of both terms is the same and strictly positive, the sign of (23) is determined by the sign of the numerator. As there is a minus sign in front of each fraction, the numerator of (23) is strictly decreasing in ψ. Inserting $\psi = 0$ into (23) yields

$$-\frac{2(2N-1)+N^2(2-N)+N^2\sqrt{3}\sqrt{(2+N)(2-N)}}{3N^3(N+1)^2(N-1)^2},$$

which is strictly negative due to the fact that, at $\psi = 0$, the optimal N must be lower than or equal to 2 (because otherwise a given by (22) is not a real number and the first-order condition cannot be fulfilled). It follows that the second derivative of $\pi^*(N)$ with respect to N is negative at any N satisfying the first-order condition. Since we know from Theorem 1 that $\pi^*(N)$ is increasing in N at $N = 1$ but decreasing as N gets large, $\pi^*(N)$ must be globally concave for all $N \geq 1$.

We can now apply the Implicit-Function Theorem to determine how N^* changes in ψ and a. Differentiating (21) with respect to ψ and using (22), we obtain

$$\frac{dN^*}{d\psi} = -\frac{\frac{\partial^2\pi^*}{\partial\psi\partial N}}{\frac{\partial^2\pi^*}{\partial N^2}} = \frac{N(N-1)}{2+\psi} > 0.$$

Since $\psi = \beta/b$, the optimal number of retailers is increasing in β and decreasing in b. Following the same procedure for the derivative with respect to a, we obtain

$$\text{sign}\left\{\frac{dN^*}{da}\right\} = \text{sign}\left\{-\frac{\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{\sqrt{3}N(2+\psi)(N+1+\psi N)^2}\right\} < 0.$$

Hence, the optimal number of retailers is decreasing in a. ■

Proof of Proposition 7. Using (21) and considering the installment cost f per retailer, M's optimal number of retailers N^* is implicitly defined by

$$\frac{a}{b(N^*+1+\psi N^*)^2} - \frac{3a^2\left(N^*\right)^2(2+\psi)^2(N^*-1)+4(N^*+1+\psi N^*)^3}{3bN^2(2+\psi)^2(N^*+1+\psi N^*)^3} - f = 0. \quad (24)$$

The problem of the social planner in the linear-quadratic framework with $\lambda = 1$ is

$$\max_N \int_0^1 \left[\frac{1}{2} b \left(X_N(\theta)\right)^2 + \left(a - bX_N(\theta) - \theta\right) X_N(\theta) - \frac{\beta}{2} \left(X_N(\theta)\right)^2\right] d\theta,$$

with

$$X_N(\theta) = \frac{aN}{b(N+1)+\beta N} - \frac{2N\theta}{b(N+1)+\beta N + b\,(N-1)}.$$

The first term of the integrand is the consumer surplus and the second term the firms' profits. Taking the first-order condition,[52] the socially optimal number of retailers, denoted by N^{SP}, is implicitly defined by

$$\frac{2a(2+\psi)+\psi(N^{SP}+1)+\psi^2 N^{SP}}{2(2+\psi)(N^{SP}+1+N^{SP}\psi)^3 b} - f = 0. \tag{25}$$

Evaluating (25) at N^*, and using that both objective functions are single-peaked, we know that $N^* > N^{SP}$ if and only if

$$\left.\frac{2a(2+\psi)+\psi(N+1)+\psi^2 N}{2(2+\psi)(N+1+N\psi)^3 b} - f\right|_{N=N^*} < 0.$$

Using (24), we obtain that the latter inequality holds if and only if

$$-\psi^3 N^3(3a-8) + 3\psi^2 N^2(8(N+1) - a(7N+1) + 2a^2 N) + 6\psi N(4(2N+1+N^2(1+a^2))$$

$$-aN(3+7N)) + 8\left(1 + N(3(1+N)+N^2) + 3aN(aN^2 - N(N+1))\right) < 0.$$

If ψ is large, the first term on the left-hand side dominates all other terms as it has the highest polynomial of ψ. The sign of the expression is then given by the sign of $-N^3(3a-8)$. In the linear-quadratic model, Assumption 1, which ensures that all types are active, is given by $a > 2 + \beta$. As an increase in β also leads to an increase in ψ, it follows that $3a > 8$ if β, and therefore ψ, is sufficiently large. As a consequence, the inequality is fulfilled for ψ sufficiently large, which implies that $N^* > N^{SP}$ in that case. ■

Proof of Proposition 8. In case of a vertical merger, M is informed about the type of her affiliated retailer. This implies that there is no quantity distortion, and a retailer of type θ sells a quantity of $x^{VM}(\theta) = (a-\theta)/(2b+\beta)$. The consumer surplus is then given by

$$CS^{VM} = \int_0^1 \left(\frac{1}{2} b \left(x^{VM}(\theta)\right)^2\right) d\theta = \frac{1+3a(a-1)}{6b(2+\psi)^2},$$

[52] It is easy to show that the problem is strictly concave.

employing again the notation $\psi = \beta/b$.

Instead, without vertical integration, the optimal size of the distribution network N^* leads to a consumer surplus of

$$CS^* = \int_0^1 \left(\frac{1}{2} b \left(X_N^*(\theta) \right)^2 \right) d\theta,$$

with

$$X_N^*(\theta) = \frac{aN^*}{b(N^*+1)+\beta N^*} - \frac{2N^*\theta}{b(N^*+1)+\beta N^* + b\left(N^*-1\right)}.$$

Inserting $X_N^*(\theta)$ in CS^* and simplifying yields

$$CS^* = \frac{4(N^*+1+\psi N^*)^2}{6b(N^*+1+\psi N^*)(2+\psi)^2} + \frac{\left(aN^*(2+\psi)\right)\left(aN^*(2+\psi)-2(1+N^*+\psi N^*)\right)}{2b(N^*+1+\psi N^*)(2+\psi)^2}.$$

Comparing the consumer surplus in both scenarios, we obtain

$$CS^{VM} - CS^* = \frac{(a-1)\left(N^*\right)^2\psi^2 - 2(a-1)N^*(aN^*-N^*-a-1)\psi}{2b(1+N^*+\psi N^*)(2+\psi)^2}$$
$$- \frac{\left(N^*\right)^2(3a(1-a)+1) - 2aN^*(1+a) + 2N^* + a(a-1)+1}{2b(1+N^*+\psi N^*)(2+\psi)^2}. \qquad (26)$$

The denominator of the right-hand side of (26) is strictly positive; hence, the sign of $CS^{VM} - CS^*$ depends on the sign of the numerator. If ψ is large, the first term is the dominating one because it has the highest polynomial in ψ. Since $a > 2+\beta$, which implies $a > 1$, this term is strictly positive for any N^*.[53] As a consequence, if $\psi = \beta/b$ is sufficiently large, $CS^{VM} > CS^*$.

Proceeding in the same way for the welfare comparison yields that the sign of $WF^{VM} - WF^*$ is equal to the sign of

$$\left(N^*\right)^2\psi^3 + N^*(N^*+3a+2)\psi^2 - (N^*+1)(N^*(3a(a-1)+1) - 3a(a+1)-1)\psi$$
$$- \left[\left(N^*\right)^2+2\right](3a(a-1)+1) + 3(3a+1) - 1 + f\left(N^*-1\right),$$

which is also strictly positive if ψ is sufficiently large. ■

[53]Notice that the result holds regardless of the exact value of f, which determines N^*.

Appendix B: Price Competition

In this appendix, we show that our main insights carry over to the case of price competition. To make the problem interesting, we assume that retailers sell differentiated products.[54] We denote R_i's demand function by $D(p_i, p_{-i})$, where p_i is R_i's price and $p_{-i} \triangleq \sum_{j=1, j\neq i}^{N} p_j$. For simplicity, we assume that the demand system is symmetric and that $|D_i(\cdot)| > |D_{-i}(\cdot)|$, where $D_i(\cdot) \triangleq \partial D(\cdot)/\partial p_i < 0$ and $D_{-i}(\cdot) \triangleq \partial D(\cdot)/\partial p_{-i} \geq 0$ (see, e.g., Vives, 2001). Therefore, R_i's demand is more reactive (in absolute terms) to a change in R_i's price than to a change in the rivals' prices, which implies that retailers' products are differentiated.

Following the literature (e.g., Rey and Tirole, 2007), the contract offered by M to R_i is a menu of two-part tariffs

$$\{T_i(m_i), w_i(m_i)\}_{m_i \in \Theta},$$

where $T_i(m_i)$ is the fixed fee and $w_i(m_i)$ is the wholesale price (for every unit distributed in the final market) that R_i pays M, contingent on the R_i's report m_i about the cost θ.

To properly reflect price competition between retailers, the timing of the game is as follows:

1. M chooses the number of retailers N.

2. Retailers observe their cost θ, and M offers contracts. If R_i accepts his contract, he reports m_i.

3. Retailers choose prices. Consumer observe all prices and demand is realized.

4. Retailers order the quantities required to satisfy demand, and pay M accordingly.

Therefore, in contrast to quantity competition, retailers order quantities and pay tariffs *after* competing in the downstream market. This implies that the manufacturer supplies to demand and not to order, as in case of quantity competition.

As shown by Rey and Vergé (2004), with price competition in the retail market a Perfect Bayesian Equilibrium with passive beliefs and two-part tariffs does not exist if products are sufficiently homogeneous because of multilateral wholesale price deviations by the manufacturer. Therefore, we use the alternative solution concept of *Contract Equilibrium*, in which the manufacturer maximizes the bilateral profit with the contract offered to each retailer (Crémer and Riordan, 1987; Horn and Wolinsky, 1988).

As noted by Rey and Vergé (2019), a Contract Equilibrium has some of the features of a Perfect Bayesian Equilibrium with passive beliefs: R_i decides whether to accept his contract

[54] With homogenous goods, price competition drives downstream profits to zero.

assuming that rivals obtain the equilibrium contract—a logic in line with the passive beliefs or pairwise-proofness assumption of Hart and Tirole (1990) and McAfee and Schwartz (1994). In addition, a Contract Equilibrium is often used with price competition between retailers (e.g., O'Brien and Shaffer, 1992, and Montez, 2015).

We first show that there is a competing contracts effect similar to the one with quantity competition. Assume that there exists a symmetric (separating) equilibrium such that $w_i(\theta) = w^*(\theta)$ and $p_i(\theta) = p_N^*(\theta)$ for every $\theta \in \Theta$ and $i = 1, .., N$. Letting

$$u_i(\theta, m_i) \triangleq \max_{p_i \geq 0} \{(p_i - \theta - w_i(m_i)) D(p_i, (N-1)p_N^*(\theta)) - T(m_i)\}$$

be R_i's rent when he reports m_i and his type is θ_i, by the Envelope Theorem

$$\dot{u}_i(\theta) = -D(p_i(\theta), (N-1)p_N^*(\theta)) + (N-1)(p_i(\theta) - w_i(\theta) - \theta) D_{-i}(p_i(\theta), (N-1)p_N^*(\theta)) \dot{p}_N^*(\theta),$$

where the function $p_i(\theta)$ solves R_i's first-order condition

$$D_i(p_i, (N-1)p_N^*(\theta))(p_i - \theta - w_i(\theta)) + D(p_i, (N-1)p_N^*(\theta)) = 0.$$

Integrating and assuming no rent for the type with the highest cost—i.e., $u_i(\overline{\theta}) = 0$—$R_i$'s information rent is

$$u_i(\theta) = \underbrace{u_i(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} D(p_i(z), (N-1)p_N^*(z))\, dz}_{\text{Standard rent}} + \\ \underbrace{-(N-1)\int_{\theta}^{\overline{\theta}} (p_i(z) - z - w_i(z)) D_{-i}(p_i(z), (N-1)p_N^*(z)) \dot{p}_N^*(z)\, dz}_{\text{Competing-contracts effect}}.$$

Hence, if $\dot{p}_N^*(\cdot) \geq 0$—i.e., the equilibrium price is increasing in the marginal cost—the competing-contracts effect arises also with price competition so that (other things being equal) retailers obtain lower rents in more competitive retail market structures.[55] The intuition is as follows. Suppose that R_i over-reports his cost in order to be charged a lower tariff. The manufacturer, however, incorrectly assumes that R_i's rivals also have the same high cost and hence that R_i's residual demand is relatively high. This increases the tariff charged by M. However, because R_i's rivals have a lower cost, his demand and profit are actually lower than what M expects.

[55] Moreover, we can explicitly determine the optimal size of the retail network in the linear-quadratic framework and obtain very similar comparative-statics results as in our main model.

In order to show that the main insights of our analysis hold with price competition, we compare M's incentive to use one or two retailers with complete and with asymmetric information. Assume θ is uniform on $[0,1]$ and let $\beta = 0$ without loss of insights. Moreover, following Vives (2001), assume that the direct demand function is

$$D(p_i, p_{-i}) = \frac{a(1-\gamma) - p_i + \gamma p_{-i}}{1-\gamma^2},$$

which yields the demand function of a monopolist for $\gamma = 0$ and approaches perfect Bertrand competition as $\gamma \to 1$. Moreover, we assume that $a \geq 2$ in order to guarantee that demand is positive in the equilibrium.

Proposition 9 *With asymmetric information, M prefers two retailers rather than one for a larger range of γ than with complete information.*

The result shows M has a greater incentive to use multiple retailers under asymmetric information than under complete information. In both cases, M prefers to distribute through two retailers if differentiation between retailers is sufficiently strong, that is, if γ is sufficiently low. However, with asymmetric information, M additionally benefits from more retailers due to the competing-contracts effect, which implies that the threshold value of γ is larger than with complete information. Notice that this result holds even if $\beta = 0$—i.e., if capacity constraints do not limit the opportunism problem.

Proof of Proposition 9. We start with the case of complete information (i.e., M knows θ). Consider first $N = 2$. Denoting the equilibrium retail price by $p^{CI}_{N=2}(\theta)$, in the second stage, each retailer i maximizes his profit $D_i(p_i, p^{CI}_{N=2}(\theta))(p_i - w_i - \theta)$, as he expects the rival retailer to set the equilibrium price. Solving for p_i, we obtain

$$p_i(w_i) = \frac{a(1-\gamma) + \theta + w_i + \gamma p^{CI}(\theta)}{2}, \quad i = 1, 2.$$

Turning to the first stage, the notion of contract equilibrium implies that M chooses w_i to solve

$$\max_{w_i \geq 0} \frac{a(1-\gamma) - p_i(w_i) + \gamma p^{CI}_{N=2}(\theta)}{1-\gamma^2} w_i + T_i + \frac{a(1-\gamma) - p^{CI}_{N=2}(\theta) + \gamma p_i(w_i)}{1-\gamma^2} w^{CI}_{N=2}(\theta),$$

where $w^{CI}_{N=2}(\cdot)$ denotes the equilibrium wholesale price. R_i's participation constraint implies that

$$T_i = \max_{p_i \geq 0} \frac{a(1-\gamma) - p_i(w_i) + \gamma p^{CI}_{N=2}(\theta)}{1-\gamma^2}(p_i - w_i - \theta),$$

Hence, M's maximization problem is

$$\max_{w_i \geq 0} \frac{a\left(1-\gamma\right)-p_i\left(w_i\right)+\gamma p_{N=2}^{CI}\left(\theta\right)}{1-\gamma^2}\left(p_i\left(w_i\right)-\theta\right)+\frac{a\left(1-\gamma\right)-p_{N=2}^{CI}\left(\theta\right)+\gamma p_i\left(w_i\right)}{1-\gamma^2}w_{N=2}^{CI}\left(\theta\right).$$

The first-order condition yields $w_{N=2}^{CI}\left(\theta\right)=0$ for every $\theta\in[0,1]$. It follows that the equilibrium retail price is

$$p_{N=2}^{CI}\left(\theta\right)=\frac{a\left(1-\gamma\right)+\theta}{2-\gamma}\geq 0,$$

and M's expected (aggregate) profit is

$$\pi_{N=2}^{CI}=\frac{2}{3}\frac{1-\gamma}{1+\gamma}\frac{1+3a\left(a-1\right)}{\left(2-\gamma\right)^2}\geq 0,$$

with equality at $\gamma=1$.

When $N=1$, the problem is straightforward: the wholesale price is equal to zero (to avoid double marginalization) and the retailer obtains the monopoly profit, which is extracted by M via the fixed fee—i.e., $w_{N=1}^{CI}\left(\theta\right)=0$ for every θ, $p_{N=1}^{CI}\left(\theta\right)=(a+\theta)/2$ and

$$\pi_{N=1}^{CI}=\frac{1+3a\left(a-1\right)}{12}\geq 0.$$

Comparing $\pi_{N=2}^{CI}$ and $\pi_{N=1}^{CI}$, we obtain that there exists a unique threshold $\gamma^{CI}\in(0,1)$ such that M uses two retailers if and only if $\gamma\leq\gamma^{CI}$. This threshold is given by

$$\gamma^{CI}=1-\frac{\left(27+6\sqrt{114}\right)^{\frac{1}{3}}}{3}+\frac{5}{\left(27+6\sqrt{114}\right)^{\frac{1}{3}}}\approx 0.612.$$

We now turn the incomplete information (i.e., M is uninformed about θ). Following the same procedure as with quantity competition, R_i's rent with linear demand is

$$u_i(\theta)=\underbrace{u_i\left(1\right)+\int_{\theta}^{1}\frac{a\left(1-\gamma\right)-p_i\left(w_i\left(z\right)\right)+\gamma p_{N=2}^{*}\left(z\right)}{1-\gamma^2}dz}_{\text{Standard rent}}+$$
$$\underbrace{-\frac{\gamma}{1-\gamma^2}\int_{\theta}^{1}\left(p_i\left(w_i\left(z\right)\right)-z-w_i\left(z\right)\right)\dot{p}_N^{*}\left(z\right)dz}_{\text{Competing-contracts effect}}.$$

In a contract equilibrium, M sets $w_i(\cdot)$ to solve

$$\max_{w_i(\cdot)\geq 0}\left\{\int_0^1\left(\frac{a(1-\gamma)-p_i(w_i(\cdot))+\gamma p^*_{N=2}(\theta)}{1-\gamma^2}(p_i(w_i(\cdot))-2\theta)\right)d\theta+\right.$$
$$+\int_0^1\frac{a(1-\gamma)-p^*_{N=2}(\theta)+\gamma p_i(w_i(\cdot))}{1-\gamma^2}w^*_{N=2}(\theta)\,d\theta+$$
$$\left.+\frac{\gamma}{1-\gamma^2}\int_0^1\theta\left(p_i(w_i(\cdot))-\theta-w_i(\cdot)\right)\dot{p}^*_{N=2}(\theta)\,d\theta\right\}.$$

From the analysis above, in the second stage, the equilibrium retail price is given by

$$p^*_{N=2}(\theta)=\frac{a(1-\gamma)+w^*_{N=2}(\theta)+\theta}{2-\gamma}, \tag{27}$$

which implies

$$\dot{p}^*_{N=2}(z)=\frac{\dot{w}^*_{N=2}(\theta)+1}{2-\gamma}. \tag{28}$$

Pointwisely differentiating M's objective function and using (27) and (28), we obtain the following differential equation:

$$\frac{4(1-\gamma)+\gamma^2}{\theta\gamma}w^*_{N=2}(\theta)+\dot{w}^*_{N=2}(\theta)=\frac{2(1-\gamma)}{\gamma}.$$

By standard techniques, the solution is

$$w^*_{N=2}(\theta)=ke^{-\frac{4(1-\gamma)+\gamma^2}{\gamma}\int_0^\theta\frac{1}{z}dz}+\int_0^\theta e^{-\frac{4(1-\gamma)+\gamma^2}{\gamma}\int_z^\theta\frac{1}{x}dx}\frac{2(1-\gamma)}{\gamma}dz. \tag{29}$$

Using the same techniques as in the analysis of the linear-quadratic framework above yields the equilibrium wholesale price

$$w^*_{N=2}(\theta)=\frac{2(1-\gamma)\theta}{4-3\gamma+\gamma^2}.$$

and the equilibrium retail price

$$p^*_{N=2}(\theta)=a\frac{1-\gamma}{2-\gamma}+\theta\frac{3-\gamma}{4-3\gamma+\gamma^2}\geq 0. \tag{30}$$

M's expected total profit is thus

$$\pi^*_{N=2}=\frac{2}{1+\gamma}\int_0^1\left((a-p^*_{N=2}(\theta))(p^*_{N=2}(\theta)-2\theta)+\frac{\gamma\theta}{1-\gamma}(p^*_{N=2}(\theta)-\theta-w^*_{N=2}(\theta))\dot{p}^*_{N=2}(\theta)\,d\theta\right)d\theta.$$

Substituting $w^*_{N=2}(\theta)$ and $p^*_{N=2}(\theta)$ and integrating yields

$$\pi^*_{N=2} = \frac{2(1-\gamma)\left(a\left(4-3\gamma+\gamma^2\right)\left(a\left(4-3\gamma+\gamma^2\right)-(2-\gamma)(3-2\gamma)\right)+(2-\gamma)^2(3-\gamma)(1-\gamma)\right)}{(1+\gamma)(2-\gamma)^2\left(4-3\gamma+\gamma^2\right)^2}.$$

This expression equals 0 at $\gamma = 1$, and is positive for all $\gamma > 0$. Moreover, it can be shown that it is decreasing in γ.

The equilibrium outcome for the case in which $N = 1$ is obtained directly by imposing $\gamma = 0$ in (29) and (30), yielding $w^*_{N=1} = \theta$ and $p^*_{N=1}(\theta) = (a + 2\theta)/2$. Hence, M's expected profit is

$$\pi^*_{N=1} = \frac{4 + 3a(a-2)}{12} > 0.$$

We finally compare $\pi^*_{N=2}$ with $\pi^*_{N=2}$. First, at $\gamma = 1$, the difference $\pi^{CI}_{N=2} - \pi^{CI}_{N=1}$ is negative as $\pi^*_{N=1}$ is strictly positive whereas $\pi^*_{N=2}$ is zero then. Second, at $\gamma = 0$, the difference is positive because $\pi^*_{N=2}$ is then twice the monopoly profit. In addition, as $\pi^*_{N=2}$ is strictly decreasing in γ, there exists a unique threshold γ^* such that $\pi^*_{N=2} \geq 0$ if and only if $\gamma \leq \gamma^*$. As a consequence, the statement of the proposition follows if $\pi^*_{N=2} > \pi^*_{N=1}$ at $\gamma = \gamma^{CI}$. In fact, inserting $\gamma = \gamma^{CI}$ in this difference yields

$$\pi^*_{N=2} - \pi^*_{N=1}\big|_{\gamma=\gamma^{CI}} =$$

$$= \frac{32}{84375}\left((27+6\sqrt{114})^{\frac{1}{3}}\left(5\sqrt{114} - 60 + (8+\sqrt{114})(27+6\sqrt{114})^{\frac{1}{3}}\right) - 300\right)$$
$$\times\left((27+6\sqrt{114})^{\frac{1}{3}}\left(564 + 5\sqrt{114} + (71 - 13\sqrt{114})(27+6\sqrt{114})^{\frac{1}{3}}\right) + 3375a - 4425\right),$$

which is strictly positive for every $a \geq 2$. ■

Appendix C: Wary Beliefs

In our main model, we assumed that retailers have passive beliefs regarding the contracts offered to their rivals. This is the most prominent assumption in the literature when the manufacturer produces to order and retailers pay the tariff before competing in the product market. The manufacturer then receives all payments before downstream competition takes place, which justifies that a retailer expects the manufacturer to make equilibrium offers to rivals, regardless of the offers he receives.

In this appendix, we consider the alternative assumption of *wary beliefs*: when a retailer receives an unexpected offer from the manufacturer, he conjectures that the manufacturer acts optimally with his rival(s), given the offer he receives. Our main aims of this analysis are threefold. First, we show that wary beliefs are equivalent to passive beliefs only in case the manufacturer's cost function is linear but the equivalence breaks down if costs are strictly convex. Second, we demonstrate how to solve for wary beliefs in an environment with incomplete information. Finally, we show that our main insights carry over to the case of wary beliefs.

Following McAfee and Schwartz (1994) and Rey and Vergé (2004), a retailer R_i who has wary beliefs and is offered a contract C_i believes that:

- M expects that R_i accepts this contract;
- M offers to R_{-i} the contract $C_{-i}(C_i)$ that maximizes M's profit, among all contracts that R_{-i} would accept;
- R_{-i} reasons the same way.

As shown by Rey and Vergé (2004), because of their recursive nature, wary beliefs are hard to characterize with a general demand function, even with complete information. Therefore, to simplify the analysis, we consider the linear-quadratic framework of Section 4.3 with $\lambda = 1$ and $b = 1$, and focus on the case of two retailers. Since a retailer's profit is only affected by the quantity sold by his competitor but not by the transfer that the competitor pays, without loss of generality we restrict attention to beliefs that depend on quantities but not transfers. Moreover, we focus on belief functions that are affine.[56]

8.1 Complete Information

First, consider the case of complete information. With a slight abuse of notation, we denote by $x_{-i}(\theta, x_i)$ the belief of R_i about his rival's quantity as a function of his own quantity x_i and of θ. In every state θ, M's maximization is

$$\max_{x_1,x_2} \sum_{i=1}^{2} (\underbrace{a - x_i - x_{-i}(\theta, x_i) - \theta}_{P(\cdot)-\theta}) x_i - \underbrace{\tfrac{\beta}{2}(x_1 + x_2)^2}_{c(\cdot)}.$$

The first-order conditions are

$$a - 2x_i - x_{-i}(\theta, x_i) - \frac{dx_{-i}(\theta, x_i)}{dx_i} x_i - \theta - \beta(x_1 + x_2) = 0, \quad i = 1, 2, \tag{31}$$

[56]This is in line with Rey and Vergé (2004) who consider polynomial beliefs and show that within this class, belief functions are indeed affine.

where $dx_{-i}(\theta, x_i)/dx_i$ reflects the change in R_i's belief about his rival's quantity as x_i varies. Therefore, the equilibrium must pin down both the retailers' quantities and the sensitivity of their beliefs.

Proposition 10 *In the symmetric equilibrium with complete information and wary beliefs:*

(*i*) *Beliefs are such that*

$$\frac{dx_{-i}(\theta, x_i)}{dx_i} = -\frac{\beta}{2}, \quad i = 1, 2.$$

(*ii*) *The quantity sold by each retailer is larger than with passive beliefs.*

(*iii*) *The manufacturer's profit is higher with one rather than two retailers.*

Hence, wary beliefs and passive beliefs are equivalent if and only if $\beta = 0$—i.e., the manufacturer's cost function is linear. Instead, if it is strictly convex and retailers have wary beliefs, a retailer who receives an offer with a larger quantity than expected assumes that the manufacturer sells a lower quantity to his rival. The intuition is rooted in the fact that the two quantities interact in the manufacturer's cost function. A larger x_i makes it more costly for the manufacturer to increase x_{-i}. As a consequence, M's optimal response to an increased quantity of R_i is to lower the quantity to R_{-i}, which is reflected by the beliefs.

A consequence of this result is that wary beliefs generate a more competitive outcome than passive beliefs (the second part of the proposition). The reason is that, with wary beliefs, a retailer is willing to pay a higher transfer if M increases his quantity because he expects that M lowers the quantity to the rival, which leads to a higher market price and, hence, a higher profit. Therefore, M has a stronger incentive to increase retailers' quantities, which leads to a higher aggregate output.[57] Since the market price with two retailers and wary beliefs is lower than with passive beliefs, the manufacturer always prefers to use a single retailer with complete information.

8.2 Asymmetric Information

We now turn to asymmetric information.[58] Since M offers a quantity schedule $x_i(\cdot)$ to each retailer, R_i's belief about his rival's quantity could depend on the entire quantity schedule that

[57] By contrast, with price competition and differentiated products, Rey and Vergé (2004) show that the market outcome is less competitive and prices are higher with wary than with passive beliefs. The reason for the difference between their result and ours is rooted in the strategic nature of the game: as prices are strategic complements, a retailer who receives an offer with an unexpectedly high wholesale price believes that the rival is also offered a higher wholesale price than the expected one. Therefore, the retailer is willing to pay a higher transfer, which implies that the manufacturer has a stronger incentive to increase wholesale prices with wary beliefs. Instead, in our framework, quantities are strategic substitutes, leading to the opposite result.

[58] As in case of complete information, we restrict attention to beliefs that do not depend on transfers.

M offers to R_i and the state θ. However, in a truthful-reporting equilibrium, without loss of generality, we can focus on R_i's beliefs about his rival's quantity that only depend on the actual state θ and on the quantity offered to R_i in state θ—i.e., $x_{-i}(\theta, x_i(\theta))$. First, since costs are perfectly correlated, R_i only cares about the quantity sold by his rival in state θ, and not on the entire schedule offered by M to the rival. Hence, the relevant belief in state θ for R_i is about the rival's quantity in that state. Second, such belief only depends on the quantity offered to R_i in state θ and not on quantities in other states. The reason is that the quantity that M offers to R_i's rival in state θ in order to best respond to the contract offered to R_i does only depend on R_i's quantity in state θ. In fact, from M's perspective, the only interaction between the retailers' quantities occurs in the cost function, and the argument of this cost function always contains quantities in the same state of the world but never in different ones.

Using the same techniques as in our main analysis, in an equilibrium in which both retailers truthfully report their cost, R_i's information rent is

$$u_i(\theta) \triangleq u_i(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} x_i(z)\, dz - \underbrace{\int_{\theta}^{\overline{\theta}} P'(x_i(z) + x_{-i}(\cdot)) \left[\frac{dx_{-i}(\cdot)}{dz} + \frac{dx_{-i}(\cdot)}{dx_i(z)}\dot{x}_i(z)\right] x_i(z)\, dz}_{\text{Competing-contracts effect}},$$

with $x_{-i}(\cdot) = x_{-i}(z, x_i(z))$. Hence, the competing-contracts effect arises also with wary beliefs. However, in contrast to passive beliefs, its intensity is affected by R_i's belief about R_{-i}'s offer. Specifically, as can be seen from the terms in squared brackets, R_i's incentive to over-report his cost not only depends on the the direct impact of θ on $x_{-i}(\cdot)$ but also on R_i's belief about M's best reply to $x_i(\theta)$, which is determined by $(dx_{-i}(\cdot)/dx_i(\theta))\,\dot{x}_i(\theta)$. Both effects have an impact on the market price when reporting a different state of the world to M, and therefore affect R_i's information rent.

Following the same steps as in the baseline model, M's maximization problem is

$$\max_{x_1(\cdot),x_2(\cdot)} \int_{\theta=0}^{1} \left\{ \sum_{i=1,2} \left[a - x_i(\theta) - x_{-i}(\theta, x_i(\theta)) - 2\theta - \right.\right.$$
$$\left.\left. - \theta\left(\frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} + \frac{dx_{-i}(\theta, x_i(\theta))}{dx_i(\theta)}\dot{x}_i(\theta)\right)\right] x_i(\theta) - \frac{\beta}{2}\left(x_1(\theta) + x_2(\theta)\right)^2 \right\} dF(\theta).$$

This problem must be solved with Calculus of Variations because M's objective function depends not only on θ and $x_i(\theta)$, but also on the derivative $\dot{x}_i(\theta)$. Focusing on an equilibrium in which $x_{-i}(\theta, x_i(\theta))$ is an affine function, the associated Euler equations (see, e.g., Kamien and

Schwartz, 2012, Ch. 3) are

$$a-2x_i(\theta)-x_{-i}(\theta,x_i(\theta))-2\theta-\theta\left(\frac{dx_{-i}(\theta,x_i(\theta))}{d\theta}+\frac{dx_{-i}(\theta,x_i(\theta))}{dx_i(\theta)}\dot{x}_i(\theta)\right)-\frac{dx_{-i}(\theta,x_i(\theta))}{dx_i(\theta)}x_i(\theta)-$$
$$-\theta x_i(\theta)\frac{d^2x_{-i}(\theta,x_i(\theta))}{d\theta dx_i(\theta)}-\beta\left(x_1(\theta)+x_2(\theta)\right)=-\frac{dx_{-i}(\theta,x_i(\theta))}{dx_i(\theta)}\left(x_i(\theta)+\theta\dot{x}_i(\theta)\right),\quad i=1,2. \tag{32}$$

The left-hand side of (32) captures the standard trade off between rent extraction and efficiency: by increasing R_i's quantity, M affects both R_i's revenue (and thus the transfer that R_i is willing to pay) and the information rent that she must grant to R_i in order to elicit a truthful report because a higher quantity increases R_i's incentive to over-report the cost. The right-hand side of (32) reflects the fact that, when M changes $x_i(\cdot)$ and thereby also $\dot{x}_i(\cdot)$, with wary beliefs this determines the quantity that R_i expects M to offer to his rival and, thus, the market price.

Proposition 11 *With incomplete information and wary beliefs, the symmetric equilibrium with two retailers has the following properties:*

(*i*) *Beliefs are*
$$0\geq\frac{dx_{-i}(\theta,x_i(\theta))}{dx_i(\theta)}=-\frac{\beta}{2}-1+\frac{1}{2}\sqrt{\beta^2+4}\geq-\frac{\beta}{2},$$

(*ii*) *For any θ (i.e., even for $\theta=0$), the quantity sold by retailers is lower with incomplete information than with complete information.*

(*iii*) *The quantity sold by each retailer with wary beliefs is larger than with passive beliefs.*

This first statement of the proposition implies retailers' beliefs are less responsive with incomplete information than with compete information for all $\beta>0$. The reason is that retailers know that the manufacturer distorts quantities downwards for rent extraction reasons. This limits the opportunism problem and implies that she optimally reacts less (in absolute terms) in the quantity to R_{-i} in case R_i receives an offer with a quantity higher or lower than expected.

The second statement shows that there is a downward distortion of the quantities of all retailer's types, even the most efficient one (i.e., $\theta=0$), with incomplete information. This implies that there is distortion at the top. The intuition is based on the effect that beliefs are less responsive with incomplete information than with complete one: in general, the opportunism problem leads to the result that the manufacturer cannot convince retailers to buy a smaller quantity than the equilibrium one. This problem is particularly severe with wary beliefs as R_i, when being offered a lower quantity, conjectures this R_{-i} gets an even higher quantity. However, if R_{-i}'s quantity is distorted downward anyway, as is the case with incomplete information, R_i's belief is less suspicious. As a consequence, M can "more easily" convince retailers to accept a contract with a lower quantity, which leads to a downward distortion of all types.

Our result therefore identifies a new reason why distortion at the top can occur. In contrast to previous literature, which focuses on countervailing incentives to explain distortion at the top (i.e., Jullien, 2000; Stole, 2007), in our framework the result is based on beliefs and arises because the presence of different agents and contracting externalities leads to different beliefs with symmetric than with asymmetric information.

The last statement implies that the market outcome is more competitive with wary beliefs. Starting from the equilibrium quantities with passive beliefs, when receiving an offer with a larger quantity, a retailer holding wary beliefs conjectures that the manufacturer optimally reduces the quantity to his rival. This implies that the retailer obtains a higher profit downstream; hence, he is willing to pay a higher transfer. As a consequence, the manufacturer optimally offers a higher quantity to both retailers.

We can now compare the M's profit with wary beliefs and two retailers, $\pi^w(2)$, to the profit with passive beliefs and two retailers, $\pi^p(2)$, and to the profit with a single retailer, $\pi^*(1)$.

Proposition 12 *(i)* $\pi^w(2) \leq \pi^p(2)$ *with equality at* $\beta = 0$ *and* $\beta \to +\infty$; *(ii)* M *prefers two retailers over one retailer for all* $\beta \geq 0$*, if* $a \in [2, 3+\sqrt{3})$*, or, if* $a \geq 3+\sqrt{3}$*, there exists a unique threshold value of* β*, such that* $\pi^w(2) > \pi^*(1)$ *if and only if* β *is above this threshold.*

The first part of the proposition states that the manufacturer is worse off when retailers have wary rather than passive beliefs. Given the above discussion of the difference between the two forms of beliefs, this result is intuitive. First, from Proposition 11, the market outcome is more competitive and industry profits are lower with wary beliefs. Second, the competing-contracts effect is weaker with wary beliefs, which implies that retailers' information rent is higher. The reason is as follows: with wary beliefs, when the manufacturer lowers the quantity sold to a retailer to reduce the information rent, the retailer conjectures that the manufacturer increases the quantity sold to the rival. Since this reduces the retailer's profit in the downstream market, the manufacturer has to compensate the retailer with a lower tariff. Both effects reduce the manufacturer's profit.

Finally, the second part of Proposition 12 shows that the qualitative results of Proposition 5 regarding the comparison between M's profit with one and two retailers and passive beliefs also hold with wary beliefs. If a is relatively small, the competing-contracts effects dominates the opportunism effect even if the cost function is linear. Instead, if a is relatively large, the opportunism problem is severe and M prefers to distribute through two retailers if and only if the cost function is sufficiently convex. Therefore, although the expected profit with two retailers is smaller in case of wary beliefs, the qualitative result still carries through.

Proof of Proposition 10. Consider the case in which (dropping arguments) dx_i/dx_{-i} and dx_{-i}/dx_i are affine functions, that is, $d^2x_i/dx_{-i}^2 = d^2x_{-i}/dx_i^2 = 0$. Total differentiation of the system (31) with respect to x_i and x_{-i} then yields

$$-2\frac{dx_i}{dx_{-i}} - 2\frac{dx_{-i}}{dx_i}\frac{dx_i}{dx_{-i}} - \beta\frac{dx_i}{dx_{-i}} - \beta = 0, \quad i = 1, 2.$$

Solving this for a symmetric equilibrium, we obtain

$$\frac{dx_i}{dx_{-i}} = \frac{dx_{-i}}{dx_i} = -\frac{\beta}{2}, \quad i = 1, 2.$$

Therefore, in equilibrium, dx_i/dx_{-i} and dx_{-i}/dx_i are indeed affine functions, which justifies our assumption to start with. As a consequence, a symmetric equilibrium in which beliefs are affine functions exists and is unique.

Substituting $dx_i/dx_{-i} = dx_{-i}/dx_i = -\beta/2$ into (31) and solving for the equilibrium yields that equilibrium quantities are

$$x_1^{w,ci}(\theta) = x_2^{w,ci}(\theta) = \frac{a - \theta}{2 + \frac{3}{2}\beta},$$

where the superscript w, ci indicates that these are the equilibrium quantities with *w*ary beliefs under *c*omplete *i*nformation. With passive beliefs, equilibrium quantities with complete information are equal to $(a - \theta)/(2 + 2\beta)$, which is strictly lower than the equilibrium quantities with wary beliefs for $\beta > 0$. Following the same steps as in the proof of Proposition 2, one can show that with complete information M prefers to distribute through one retailer only. ■

Proof of Proposition 11. We first determine the wary beliefs $dx_{-i}(\theta, x_i(\theta))/dx_i(\theta)$, $i = 1, 2$ by totally differentiating the Euler equations (32) with respect to x_i and x_{-i}. Simplifying (32), the Euler equations can be written as

$$\begin{aligned} a - 2x_i(\theta) - x_{-i}(\theta, x_i(\theta)) - 2\theta - \theta\frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} - \\ - \theta x_i(\theta)\frac{d^2x_{-i}(\theta, x_i(\theta))}{d\theta dx_i(\theta)} - \beta(x_1(\theta) + x_2(\theta)) = 0, \quad i = 1, 2. \quad (33)\end{aligned}$$

We conjecture that in an equilibrium in which beliefs are affine functions (i.e., $d^2x_{-i}(\cdot)/d\left(x_i(\cdot)\right)^2 = 0$), the expression $d^2x_{-i}(\theta, x_i(\theta))/d\theta dx_i(\theta)$ is also zero. Total differentiation of (33) with respect to x_i and x_{-i} then yields (omitting arguments)

$$-2\frac{dx_i}{dx_{-i}} - \frac{dx_{-i}}{dx_i}\frac{dx_i}{dx_{-i}} - \beta\frac{dx_i}{dx_{-i}} - \beta = 0, \quad i = 1, 2.$$

Hence, in a symmetric equilibrium,

$$\frac{dx_i}{dx_{-i}} = \frac{dx_{-i}}{dx_i} = -\frac{\beta}{2} - 1 + \frac{1}{2}\sqrt{\beta^2 + 4}, \quad i = 1, 2. \tag{34}$$

It is evident that the equilibrium beliefs are indeed affine functions, which justifies our assumption to start with. In addition, the functions (34) do not depend on θ, which verifies our conjecture that $\partial^2 x_{-i}(\theta, x_i(\theta))/\partial\theta\partial x_i(\theta) = 0$. As a consequence, an equilibrium with affine wary beliefs exists and is characterized by (34). It is easy to check that the responsiveness of the equilibrium beliefs is 0 if $\beta = 0$, strictly negative for all $\beta > 0$, and larger than $-\beta/2$, which is the responsiveness of the equilibrium passive beliefs. This proves statement (i).

We next determine the equilibrium quantities. To do so, we first use the Euler equations (33) to solve for $dx_{-i}(\theta, x_i(\theta))/d\theta$. Since $d^2x_{-i}(\theta, x_i(\theta))/d\theta dx_i(\theta) = 0$, (33) can be written as

$$a - 2x_i(\theta) - x_{-i}(\theta, x_i(\theta)) - 2\theta - \theta\frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} - \beta\left(x_1(\theta) + x_2(\theta)\right) = 0, \quad i = 1, 2. \tag{35}$$

Differentiating (35) with respect to θ, under the conjecture that $d^2x_{-i}(\theta, x_i(\theta))/d\left(\theta\right)^2 = 0$, yields

$$-2 - 2\dot{x}_i(\theta) - 2\frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} - \frac{dx_i}{dx_{-i}}\dot{x}_i(\theta) - \beta\left(\dot{x}_1(\theta) + \dot{x}_2(\theta)\right) = 0, \quad i = 1, 2.$$

Imposing symmetry, that is, in equilibrium each retailer sells $x^{w,ii}(\theta)$,[59] we obtain

$$\frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} = -1 - \frac{\dot{x}^{w,ii}(\theta)}{2}\left(2 + \frac{dx_i}{dx_{-i}} + 2\beta\right). \tag{36}$$

Substituting $dx_{-i}(\theta, x_i(\theta))/d\theta$ into (33) and rearranging yields the differential equation

$$\dot{x}^{w,ii}(\theta) - \frac{4(3 + 2\beta)}{\theta\left(2 + 3\beta + \sqrt{\beta^2 + 4}\right)}x^{w,ii}(\theta) = -\frac{4(a - \theta)}{\theta\left(2 + 3\beta + \sqrt{\beta^2 + 4}\right)}.$$

[59]The superscript w, ii indicates the equilibrium with $w$ary beliefs under $i$ncomplete $i$nformation.

Proceeding in the same way as in Section 4.3 to solve this differential equation, we obtain

$$x^{w,ii}(\theta) = \frac{a}{3+2\beta} - \frac{4\theta}{10+5\beta-\sqrt{\beta^2+4}}. \tag{37}$$

It follows that

$$\dot{x}^{w,ii}(\theta) = -\frac{4}{10+5\beta-\sqrt{\beta^2+4}},$$

which is independent of θ. Since both $\dot{x}^{w,ii}(\theta)$ and dx_i/dx_{-i} do not depend on θ, it is evident from (36), that also $dx_{-i}(\theta, x_i(\theta))/d\theta$ does not depend on θ, which implies that our conjecture $d^2x_{-i}(\theta, x_i(\theta))/d(\theta)^2 = 0$ is indeed correct. As a consequence, the equilibrium quantity with wary beliefs and incomplete information is characterized by (37).

Comparing this equilibrium quantity with the one in case of passive beliefs, which is given by

$$x^*(\theta) = \frac{a}{3+2\beta} - \frac{2\theta}{4+2\beta},$$

yields

$$x^{w,ii}(\theta) - x^*(\theta) = \theta\frac{2+\beta-\sqrt{\beta^2+4}}{(\beta+2)\left(10+5\beta-\sqrt{\beta^2+4}\right)},$$

which is strictly positive for $\beta > 0$ and 0 for $\beta = 0$. This proves statement (*ii*).

Finally, comparing $x^{w,ii}(\theta)$ with $x^{w,ci}(\theta)$, which is given by $2(a-\theta)/(4+3\beta)$, we obtain

$$x^{w,ci}(\theta) - x^{w,ii}(\theta) = \frac{2(2+\beta)\left(10+5\beta-\sqrt{4+\beta^2}\right) + 2\theta(3+2\beta)\left(\beta-2+\sqrt{4+\beta^2}\right)}{(4+3\beta)(3+2\beta)\left(10+5\beta-\sqrt{4+\beta^2}\right)}.$$

Both the numerator and the denominator of the last expression are strictly positive; hence, for any $\theta \in [0,1]$, the equilibrium quantity with complete information is higher than with incomplete information. This proves statement (*iii*). ■

Proof of Proposition 12. We first determine M's expected profit with two retailers and wary beliefs, denoted by π^w, and compare it with the profit in case the two retailers hold passive beliefs, denoted by π^p. From the Euler equation (32), we have

$$a - 2x^{w,ii}(\theta) - \theta - \theta\left(1 + \frac{dx_{-i}(\theta, x_i(\theta))}{d\theta} + \frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\right) = (1+2\beta)\,x^{w,ii}(\theta) - \theta\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta).$$

Multiplying both sides with $x^{w,ii}(\theta)$ yields

$$\begin{aligned}\left[a-2x^{w,ii}(\theta)-2\theta-\theta\left(\frac{dx_{-i}(\theta,x_i(\theta))}{d\theta}+\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\right)\right]x^{w,ii}(\theta)= \\ =(1+2\beta)\left(x^{w,ii}(\theta)\right)^2-\theta\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\,x^{w,ii}(\theta),\end{aligned}$$

with the left-hand side being M's revenue in state θ. Using this equality and subtracting M's costs, we obtain that the expected profit can be written as

$$\begin{aligned}\pi^w &= 2\int_0^1\left[\left(x^{w,ii}(\theta)\right)^2(1+2\beta)-\theta\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\,x^{w,ii}(\theta)-\beta\left(x^{w,ii}(\theta)\right)^2\right]d\theta \\ &= 2\int_0^1\left[\left(x^{w,ii}(\theta)\right)^2(1+\beta)-\theta\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\,x^{w,ii}(\theta)\right]d\theta \\ &= 2(1+\beta)\int_0^1\left(x^{w,ii}(\theta)\right)^2d\theta-2\frac{dx_i}{dx_{-i}}\dot{x}_i(\theta)\int_0^1\theta x^{w,ii}(\theta)\,d\theta.\end{aligned}$$

From the derivation of the equilibrium in the linear-quadratic framework, the profit with passive beliefs can be written as

$$\pi^p=2(1+\beta)\int_0^1\left(x^{p,ii}(\theta)\right)^2d\theta,$$

where $x^{p,ii}(\theta)$ denotes the equilibrium quantity with passive beliefs. The connection between the equilibrium quantity with wary beliefs and passive beliefs is

$$x^{w,ii}(\theta)=x^{p,ii}(\theta)+\theta\frac{2+\beta-\sqrt{\beta^2+4}}{(\beta+2)\left(10+5\beta-\sqrt{\beta^2+4}\right)},$$

whereby

$$x^{p,ii}(\theta)=\frac{a}{3+2\beta}-\frac{\theta}{2+\beta}.$$

Integrating yields

$$\pi^w = \frac{12a^2(1+\beta)\left(52+50\beta+13\beta^2-5\sqrt{4+\beta^2}(2+\beta)\right)}{3(3+2\beta)^2\left(10+5\beta-\sqrt{4+\beta^2}\right)^2}$$
$$+\frac{4\left[(3+2\beta)^2\left(4+3\beta-\sqrt{4+\beta^2}\right)-3a(3+2\beta)\left(32+40\beta+13\beta^2-\sqrt{4+\beta^2}(8+5\beta)\right)\right]}{3(3+2\beta)^2\left(10+5\beta-\sqrt{4+\beta^2}\right)^2}$$

and

$$\pi^p = 2(1+\beta)\left[\frac{a^2}{(3+2\beta)^2}+\frac{1}{3(1+2\beta)^2}-\frac{a}{(3+2\beta)(1+2\beta)}\right].$$

Taking the difference between the two profits, we obtain that $\pi^p - \pi^w$ equals

$$\frac{12a(2+\beta)\left(12+10\beta+3\beta^2-3\sqrt{4+\beta^2}(2+\beta)\right)}{3(2+\beta)^2(3+2\beta)\left(10+5\beta-\sqrt{4+\beta^2}\right)^2}$$
$$-\frac{4(3+2\beta)\left(12+10\beta+\beta^2-\beta^3-\sqrt{4+\beta^2}(6+\beta-\beta^2)\right)}{3(2+\beta)^2(3+2\beta)\left(10+5\beta-\sqrt{4+\beta^2}\right)^2}. \quad (38)$$

Inserting $\beta = 0$ yields that the numerator of (38) equals zero whereas the denominator is strictly positive; hence, $\pi^p - \pi^w = 0$ at $\beta = 0$. For $\beta \to +\infty$, (38) also approaches zero. The reason is that both the numerator and the denominator go to $+\infty$, but the denominator is a polynomial of fifth order in β, whereas the numerator is a polynomial only of fourth order in β; hence, the denominator goes faster to $+\infty$ than the numerator. Finally, to see that (38) is strictly positive for $\beta \in (0,\infty)$, note first that (38) is strictly increasing in a for $\beta > 0$ because $12+10\beta+3\beta^2-3\sqrt{4+\beta^2}(2+\beta) > 0$ for $\beta > 0$. Inserting the lower bound for a, which is 2 in (38) yields

$$(3+\beta)\left(36+34\beta+13\beta^2+2\beta^3-\sqrt{4+\beta^2}\left(18+13\beta+2\beta^2\right)\right),$$

which is strictly positive for all $\beta > 0$.

Finally, we compare the profit with two retailers and wary beliefs with the profit in case M distributes through one retailer only. From the derivations for the linear-quadratic framework above, the latter profit is $\pi^*(1) = (3a^2+4-6a)/(6(2+\beta))$. The sign of the difference $\pi^w - \pi^*(1)$

is a polynomial of fourth order in β. The term involving β to the fourth order is

$$16\beta^3\left(\sqrt{4+\beta^2}-\beta\right),$$

which is strictly positive. It follows that $\pi^w - \pi^*(1)$ is strictly positive for β large enough.

To show that there is either a unique threshold value for β or that the difference $\pi^w - \pi^*(1)$ is positive for all β, we can follow a very similar procedure as in the proof of Proposition 5. First, setting $\beta = 0$, we obtain that $\pi^w - \pi^*(1) > 0$ for all $a \in [2, 3+\sqrt{3})$. Moreover, the derivative of $\pi^w - \pi^*(1)$ with respect to β is also positive for all $a \in [2, 3+\sqrt{3})$. In addition, there is a unique local maximum of this difference which occurs at $\beta = -2/3$. Taking this together, it follows that the difference between the two profits is strictly positive for all values of $\beta \geq 0$ if $a \in [2, 3+\sqrt{3})$. For $a \geq 3+\sqrt{3}$, the difference $\pi^w - \pi^*(1)$ is negative at $\beta = 0$. As the difference is positive for β large enough and there is a local maximum at a negative value, there must be a unique solution in the region $\beta > 0$ such that the difference is positive. ■

Appendix D: Imperfect Correlation—the Linear Example

Consider the linear-quadratic framework developed in Section 4.3. Assume that $\lambda = 1$ and focus on a symmetric equilibrium in which every retailer i produces $x_N^*(\theta_i)$. Following the same techniques developed above, the bilateral contracting problem between M and R_i is

$$\begin{aligned}\max_{x_i(\cdot)}\Bigg\{\int_{\theta_i}&\left\{\nu P\left(x_i(\cdot)+(N-1)\,x_N^*(\cdot)\right)+(1-\nu)\,\mathbb{E}_{\theta_{-i}}\left[P\left(x_i(\cdot)+X_{N,-i}^*(\theta_{-i})\right)\right]-\theta_i\right\}x_i(\cdot)\,dF(\theta_i)\\ &-\int_{\theta_i}h(\theta_i)\left[1-\nu(N-1)P'\left(x_i(\cdot)+(N-1)x_N^*(\cdot)\right)\dot{x}_N^*(\cdot)\right]x_i(\cdot)\,dF(\theta_i)\\ &-\int_{\theta_i}\left[\nu c\left(x_i(\cdot)+(N-1)\,x_N^*(\cdot)\right)+(1-\nu)\,\mathbb{E}_{\theta_{-i}}\left[c\left(x_i(\cdot)+X_{N,-i}^*(\theta_{-i})\right)\right]\right]dF(\theta_i)\Bigg\}.\end{aligned}$$

Differentiating with respect to $x_i(\cdot)$, and substituting the linear specification, yields the following differential equation:

$$\dot{x}_N^*(\theta)=\frac{a-\theta-(1-\nu)(N-1)(b+\beta)\,\hat{x}_N-x_N^*(\theta)\left(b+(b+\beta)\left(1+\nu(N-1)\right)\right)-\theta}{\theta b\nu(N-1)}, \quad (39)$$

with $\hat{x}_N \triangleq \mathbb{E}\left[x_N^*(\theta)\right]$ and boundary condition

$$x_N^*(0) = \frac{a - (1-\nu)(N-1)(b+\beta)\hat{x}_N}{b + (b+\beta)(1+\nu(N-1))}.$$

Equation (39) can be rewritten as

$$\dot{x}_N^*(\theta) + x_N^*(\theta)\frac{\Phi}{\theta b\nu(N-1)} = \frac{a-\bar{a}}{\theta b\nu(N-1)} - \frac{2}{b\nu(N-1)},$$

where

$$\Phi \triangleq 2b + b\nu(N-1) + \nu\beta N + (1-\nu)\beta$$

and

$$\bar{a} \triangleq (1-\nu)(N-1)(b+\beta)\hat{x}_N.$$

The solution is

$$x_N^*(\theta) = ke^{-\int_0^\theta \frac{\Phi}{z_1 b\nu(N-1)}dz_1} + \int_0^\theta e^{-\int_{z_2}^\theta \frac{\Phi}{z_1 b\nu(N-1)}dz_1}\left(\frac{a-\bar{a}}{z_2\nu b(N-1)} - \frac{2}{b\nu(N-1)}\right)dz_2.$$

It can be easily seen that, as before, the constant k must be equal to zero because $\Phi > 0$. Hence,

$$\begin{aligned}
x_N^*(\theta) &= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)}\int_0^\theta \theta_2^{\frac{\Phi}{b\nu(N-1)}}\left(\frac{a-\bar{a}}{\theta_2} - 2\right)d\theta_2 \\
&= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)}\int_0^\theta \left(\theta_2^{\frac{\Phi}{b\nu(N-1)}-1}(a-\bar{a}) - 2\theta_2^{\frac{\Phi}{b\nu(N-1)}}\right)d\theta_2 \\
&= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)}\left|\frac{(a-\bar{a})b\nu(N-1)}{\Phi}\theta_2^{\frac{\Phi}{b\nu(N-1)}} - \frac{2}{\frac{\Phi}{b\nu(N-1)}+1}\theta_2^{\frac{\Phi}{b\nu(N-1)}+1}\right|_0^\theta d\theta_2 \\
&= \frac{a-\bar{a}}{\Phi} - \frac{2\theta}{\Phi + b\nu(N-1)}.
\end{aligned}$$

Integrating yields

$$\begin{aligned}
\int_0^1 x_N^*(\theta)d\theta \triangleq \hat{x}_N &= \frac{a - (1-\nu)(N-1)(b+\beta)\hat{x}_N}{\Phi} - \frac{1}{\Phi + b\nu(N-1)} \\
\Leftrightarrow \quad \hat{x}_N &= \frac{a}{b + N(b+\beta)} - \frac{b + (b+\beta)(1+\nu(N-1))}{(b+N(b+\beta))(2b+\beta)(1+\nu(N-1))}.
\end{aligned}$$

Hence,

$$x_N^*(\theta) = \frac{a}{b+(b+\beta)(1+\nu(N-1))} - \underbrace{\frac{(1-\nu)(N-1)(b+\beta)}{b+(b+\beta)(1+\nu(N-1))}\hat{x}_N}_{\text{Bayes-Nash component}} - \frac{2\theta}{(2b+\beta)(1+\nu(N-1))}.$$

It can be shown that for $\nu \to 1$ the solution converges to

$$x_N^*(\theta) = \frac{a}{b(N+1)+\beta N} - \frac{\theta(1+\lambda)}{b(N+1)+\beta N+\lambda b(N-1)}.$$

Notice that, compared to the baseline model, with imperfectly correlated types there is a new term in the expression of the equilibrium quantity. This term reduces retailers' individual quantity since it captures the uncertainty about the rivals' types—i.e., the extent to which costs are independently distributed.

The expected profit $\pi_\nu^*(N)$ can be obtained by rearranging M's first-order condition with respect to $x_i(\theta)$ and aggregating over the number of retailers:

$$\pi_\nu^*(N) \triangleq N\int_0^1 x_N^*(\theta)\left((b+\nu\beta N)x_N^*(\theta)+\beta(1-\nu)\left(x_N^*(\theta)+(N-1)\hat{x}_N\right)\right)d\theta+ \\ -\frac{\nu\beta}{2}N^2\int_0^1 x_N^*(\theta)^2\,d\theta - \frac{(1-\nu)\beta}{2}\mathbb{E}\left[\left(\sum\nolimits_{i=1}^N x_N^*(\theta_i)\right)^2\right].$$

Since with probability $(1-\nu)$ types are i.i.d., it follows that

$$\frac{(1-\nu)\beta}{2}\mathbb{E}\left[\left(\sum\nolimits_{i=1}^N x_N^*(\theta_i)\right)^2\right] = \frac{(1-\nu)\beta}{2}\mathbb{E}\left[\sum\nolimits_{i=1}^N\sum\nolimits_{j=1}^N x_N^*(\theta_i)\,x_N^*(\theta_j)\right] = \frac{(1-\nu)\beta}{2}N^2\hat{x}_N^2.$$

We compare $\pi_\nu^*(2)$ and $\pi_\nu^*(1)$ numerically. For example, if $a=5$ and $b=1$, then

$$\pi_\nu^*(2)-\pi_\nu^*(1) = \frac{4(\nu+2)(1-\nu)^2\beta^3+4(11\nu+22\nu^2+3\nu^3+8)\beta^2}{6(2+\beta)^2(3+2\beta)^2(1+\nu)^2}+ \\ +\frac{3(39\nu+76\nu^2+3\nu^3-2)\beta+18\nu(5\nu-2)-78}{6(2+\beta)^2(3+2\beta)^2(1+\nu)^2},$$

It is then easy to check that M prefers a duopolistic market structure compared to a monopolistic one when β and ν are sufficiently large. For example, $\pi_\nu^*(2) \geq \pi_\nu^*(1)$ if and only if $\beta = 0.5$ and $\nu \geq 0.49$, or $\beta = 1$ and $\nu \geq 0.21$. Moreover, if $\beta = 1.5$, then $\pi_\nu^*(2) \geq \pi_\nu^*(1)$ for all $\nu \geq 0$.

Appendix E: Sequential Contracting for Information Extraction versus Simultaneous Contracting

In this appendix, we consider the sequential-contracting mechanism for information extraction reasons discussed in Section 7 of the paper. In particular, we consider sequential contracting with two retailers. The manufacturer asks the first retailer only to report his type, with no quantity and payment. She then uses this information in the contract with the second retailer. We will compare this mechanism to simultaneous contracting as considered in the main part of the paper, considering imperfect correlation between retailers' costs.

To keep the analysis simple, we focus on the binary-types case described in Section 3. Following Armstrong and Vickers (2010) and Dequiedt and Martimort (2015), we model the correlation between the two types by assuming that θ_{R_1} and θ_{R_2} are perfectly correlated with probability ν and independent with probability $1-\nu$. Therefore, ν corresponds to the degree of correlation between retailers' costs. Denoting the ex ante probability that $\theta = \underline{\theta}$ by α, the vector of random variables $\boldsymbol{\theta} = (\theta_{R_1}, \theta_{R_2})$ is then drawn from the following joint cumulative distribution function:

- $\Pr(\underline{\theta}, \underline{\theta}) = \alpha\left(\nu + (1-\nu)\alpha\right)$;
- $\Pr(\underline{\theta}, \overline{\theta}) = \Pr(\overline{\theta}, \underline{\theta}) = \alpha\left(1-\alpha\right)\left(1-\nu\right)$;
- $\Pr(\overline{\theta}, \overline{\theta}) = \left(1-\alpha\right)\left(\nu + (1-\nu)(1-\alpha)\right)$.

By Bayes rule, conditional probabilities are: $\Pr(\underline{\theta}|\underline{\theta}) = \nu + (1-\nu)\alpha$, $\Pr(\underline{\theta}|\overline{\theta}) = \left(1-\alpha\right)\left(1-\nu\right)$, $\Pr(\overline{\theta}|\underline{\theta}) = \alpha\left(1-\nu\right)$, and $\Pr(\overline{\theta}|\overline{\theta}) = \nu + (1-\nu)(1-\alpha)$. Letting $\alpha = 1/2$ and $\underline{\theta} = 0$ to simplify the exposition (as in Section 3) yields

$$\begin{aligned}
\Pr(\underline{\theta}|\underline{\theta}) &= \Pr(\overline{\theta}|\overline{\theta}) = \frac{1}{2}\left(1+\nu\right); \\
\Pr(\underline{\theta}|\overline{\theta}) &= \Pr(\overline{\theta}|\underline{\theta}) = \frac{1}{2}\left(1-\nu\right).
\end{aligned}$$

Sequential Contracting for Information Extraction

The first retailer sells no quantity (and makes no payment), and truthfully reports his type. Hence, we can focus on the contracting with the second retailer. The manufacturer's maximization problem is a standard screening problem with the appropriate conditional probabilities. As shown in Section 3, the utility of a second retailer with high cost equals zero, whereas his utility with low cost is $\overline{\theta}\overline{x}(\theta_1)$, where $\theta_1 \in \left\{0, \overline{\theta}\right\}$ is the report of the first retailer, and $\overline{x}(\theta_1)$ expresses the dependence of the second retailer's quantity on the first retailer's report.

Therefore, M's maximization problem is

$$\max_{\underline{x}(\theta_1), \overline{x}(\theta_1)} \Pr\left(\underline{\theta}|\theta_1\right)\left[P\left(\underline{x}(\theta_1)\right)\underline{x}(\theta_1) - \overline{\theta}\overline{x}(\theta_1) - c\left(\underline{x}(\theta_1)\right)\right] + \Pr\left(\overline{\theta}|\theta_1\right)\left[\left(P\left(\overline{x}(\theta_1)\right) - \overline{\theta}\right)\overline{x}(\theta_1) - c\left(\overline{x}(\theta_1)\right)\right].$$

Substituting probabilities and the (inverse) demand and cost functions (i.e., $P(x) = 1 - x$ and $c(x) = \beta x^2/2$), yields the optimal quantities:[60]

$$\underline{x}\left(\theta_1 = 0\right) = \underline{x}\left(\theta_1 = \overline{\theta}\right) = \frac{1}{2+\beta},$$

$$\overline{x}\left(\theta_1 = 0\right) = \frac{1 - \nu - 2\overline{\theta}}{\left(1-\nu\right)\left(2+\beta\right)}$$

and

$$\overline{x}\left(\theta_1 = \overline{\theta}\right) = \frac{1 + \nu - 2\overline{\theta}}{\left(1+\nu\right)\left(2+\beta\right)}.$$

Hence, the low-cost retailer receives the efficient quantity regardless of the report of the first retailer, while the quantity of a high-cost retailer is distorted downwards and depends on the report of the first retailer.

Using these quantities, M's profit conditional on θ_1 is

$$\pi\left(\theta_1 = 0\right) = \frac{\left(1-\nu\right)\left(1 - 2\overline{\theta}\right) + 2\overline{\theta}^2}{2\left(1-\nu\right)\left(2+\beta\right)}$$

and

$$\pi\left(\theta_1 = \overline{\theta}\right) = \frac{\left(1+\nu\right)\left(1 - 2\overline{\theta}\right) + 2\overline{\theta}^2}{2\left(1+\nu\right)\left(2+\beta\right)}.$$

As both cost realizations are equally likely, M's expected profit under sequential contracting is

$$\pi^M_{seq} = \frac{\left(1-\nu^2\right)\left(1 - 2\overline{\theta}\right) + 2\overline{\theta}^2}{2\left(1+\nu\right)\left(1-\nu\right)\left(2+\beta\right)}.$$

Simultaneous Contracting

[60] We focus on the case in which $\overline{\theta}$ is small enough so that all quantities are positive. This does not affect our qualitative results.

We now analyze the case in which M contracts with both retailers simultaneously. Following the same steps as in Section 3, the utility of a low-cost retailer, denoted by $\underline{u}_i$, is

$$\underline{u}_i = [1 - \underline{x} - (1/2(1+\nu)\underline{x}^* + 1/2(1-\nu)\overline{x}^*)]\,\underline{x} - \underline{T}$$

and the utility of a high-cost retailer, denoted by $\overline{u}_i$, is

$$\overline{u}_i = \left[1 - \overline{x} - (1/2(1-\nu)\underline{x}^* + 1/2(1+\nu)\overline{x}^*) - \overline{\theta}\right]\overline{x} - \overline{T}.$$

From the participation and the incentive-compatibility constraints, we obtain that, at the optimal contract, $\overline{u}_i = 0$ and

$$\begin{aligned}
\underline{u}_i &= [1 - \overline{x} - 1/2\left((1+\nu)\underline{x}^* + (1-\nu)\overline{x}^*\right)]\,\overline{x} - [1 - \overline{x} - 1/2\left((1-\nu)\underline{x}^* + (1+\nu)\overline{x}^*\right)]\,\overline{x} + \overline{\theta}\overline{x} \\
&= \overline{\theta}\overline{x} - \nu\overline{x}\left(\underline{x}^* - \overline{x}^*\right).
\end{aligned}$$

As in Section 3, the first term of the right-hand side is the standard information rent whereas the second term is the reduction in the information rent because of the competing-contracts effect.

Using the respective probabilities for the different states, and the fact that the optimal contact is symmetric for both retailers, M's maximization problem can be written as

$$\begin{aligned}
\max_{\underline{x},\overline{x}} \quad & [1 - \underline{x} - (1/2(1+\nu)\underline{x}^* + 1/2(1-\nu)\overline{x}^*)]\,\underline{x} - \overline{\theta}\overline{x} + \nu\overline{x}\left(\underline{x}^* - \overline{x}^*\right) \\
& + \left[1 - \overline{x} - (1/2(1-\nu)\underline{x}^* + 1/2(1+\nu)\overline{x}^*) - \overline{\theta}\right]\overline{x} \\
& - (1+\nu)\frac{\beta\underline{x}^2}{2} - (1+\nu)\frac{\beta\overline{x}^2}{2} - (1-\nu)\frac{\beta\left(\underline{x}+\overline{x}\right)^2}{4}.
\end{aligned}$$

Solving for the optimal quantities yields

$$\underline{x}^* = \frac{2(1+\nu) + \overline{\theta}(1-\nu) + \beta\left(1+\nu+\overline{\theta}(1-\nu)\right)}{(2+\beta)(3+2\beta)(1+\nu)}$$

and

$$\overline{x}^* = \frac{2(1+\nu) - \overline{\theta}(5+\nu) + \beta\left(1+\nu-\overline{\theta}(3+\nu)\right)}{(2+\beta)(3+2\beta)(1+\nu)}.\text{[61]}$$

[61] For $\nu \to 1$, these quantities correspond to the ones of Section 3.

Inserting these quantities into M's expected profit, we obtain

$$\pi_{sim}^{M} = \frac{2\left(1+\beta\right)}{\left(3+2\beta\right)^{2}} - \frac{4\overline{\theta}\left(1+\beta\right)\left(2+\nu+\beta+\nu\beta\right)}{\left(2+\beta\right)\left(3+2\beta\right)^{2}\left(1+\nu\right)}$$
$$+\frac{\overline{\theta}^{2}\left[2(13+4\nu+\nu^{2})+\beta\left(49+29\nu+6\nu^{2}\right)+2\beta^{2}\left(15+14\nu+3\nu^{2}\right)+2\beta^{3}\left(3+4\nu+\nu^{2}\right)\right]}{\left(2+\beta\right)^{2}\left(3+2\beta\right)^{2}\left(1+\nu\right)^{2}}.$$

We can now compare the manufacturer's expected profit from the sequential-contracting mechanism with her profit from simultaneous contracting. This yields

$$\pi_{sim}^{M} - \pi_{seq}^{M} = \frac{1}{\left(1-\nu\right)\left(2+\beta\right)^{2}\left(3+2\beta\right)^{2}\left(1+\nu\right)^{2}} \times \Bigg\{ (2+\beta)(1+2\overline{\theta}(1+\beta))(2\overline{\theta}(2+\beta-1))$$
$$-\nu\left[2(1+2\overline{\theta}(18\overline{\theta}-5))+\beta(1+2\overline{\theta}(53\overline{\theta}-13))+4\beta^{2}\overline{\theta}(11\overline{\theta}-4)+4\beta^{3}\overline{\theta}^{2}\right]$$
$$-\nu^{2}(2+\beta)\left[2\overline{\theta}+2\overline{\theta}^{2}(3+10\beta+6\beta^{2})-1\right]$$
$$-\nu^{3}\left[4\overline{\theta}(5+\overline{\theta})-2)+\beta(2\overline{\theta}(13+6\overline{\theta})-1)+4\beta^{2}\overline{\theta}(2+3\overline{\theta})+4\beta^{3}\overline{\theta}^{2}\right]\Bigg\}.$$

It is easy to see that for ν sufficiently low, the first term in the curly bracket is the dominant term. This term is positive if $\beta \geq \left(1-4\overline{\theta}\right)/\left(2\overline{\theta}\right)$. Therefore, if the last inequality is satisfied, simultaneous contracting is preferred to sequential contracting. By continuity, there always exists threshold value $\hat{\nu}$ such that simultaneous contracting leads to a higher expected profit if ν is below this threshold value.[62] As an example, if $\overline{\theta} = 1/5$ and $\beta = 2$, the threshold value is $\hat{\nu} \approx 0.431$.

Finally, we note that we do not solve for the optimal sequential mechanism with imperfect correlation. (While this is possible in the two-types case, it is extremely challenging in the general case and goes beyond the scope of the paper.) However, both retailers will be active (i.e., receive a positive quantity) at the optimal mechanism, as their costs are different with positive probability. As a consequence, the competing-contracts effect is also present then and shapes the size of the distribution network.

[62]Notice that if simultaneous contracting with two retailers yields a higher profit than sequential contracting, it must also yield a higher profit than simultaneous contracting with one retailer. The reason is that the latter profit is lower than the one with sequential contracting because the manufacturer has superior information and the retail market structure is the same.

References

[1] Armstrong, M, and Vickers, J. (2010). "Competitive Non-Linear Pricing and Bundling." *Review of Economic Studies*, 77: 30-60.

[2] Baron, D., and Myerson, R. (1982). "Regulating a Monopolist with Unknown Costs." *Econometrica*, 50: 911-930.

[3] Bhasin, H. (2018). "Selective Distribution and its role in Channel Distribution." digital article, available at: *https://www.marketing91.com/selective-distribution/*.

[4] Bolton, P., and Whinston, M. (1993), "Incomplete Contracts, Vertical Integration, and Supply Assurance." *Review of Economic Studies*, 60: 1-21.

[5] Caillaud, B., Jullien, B., and Picard, P. (1995). "Competing Vertical Structures: Precommitment and Renegotiation." *Econometrica,* 63: 621-646.

[6] Calzolari, G., and Denicolò, V. (2013). "Competition with Exclusive Contracts and Market-Share Discounts." *American Economic Review*, 103: 2384-2411.

[7] Calzolari, G., and Denicolò, V. (2015). "Exclusive Contracts and Market Dominance." *American Economic Review*, 105: 3321-3351.

[8] Crémer, J., and McLean, R. P. (1985). "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent." *Econometrica*, 53: 345-361.

[9] Crémer, J., and McLean, R. P. (1988). "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions." *Econometrica*, 56: 1247-1257.

[10] Crémer, J., and Riordan, M. (1987). "On Governing Multilateral Transactions with Bilateral Contracts." *RAND Journal of Economics*, 18: 436-451.

[11] Dequiedt, V., and Martimort, D. (2015). "Vertical Contracting with Informational Opportunism." *American Economic Review*, 105: 2141-2182.

[12] Dyer, J., Furr, N., and Lefrandt, C. (2014). "The Industries Plagued by the Most Uncertainty." *Harvard Business Review*, digital article, available at: *https://hbr.org/2014/09/the-industries-plagued-by-the-most-uncertainty*.

[13] Gal-Or, E. (1999). "Vertical Integration or Separation of the Sales Function as Implied by Competitive Forces." *International Journal of Industrial Organization*, 17: 641-662.

[14] Genicot, G., and Ray, D., (2006). "Contracts and Externalities: How Things Fall Apart." *Journal of Economic Theory*, 131: 71-100.

[15] Hart, O., and Tirole, J. (1990). "Vertical Integration and Market Foreclosure." *Brookings Papers on Economic Activity. Microeconomics*, 205-286.

[16] Hansen, S., and Motta, M., (2018). "Vertical Exclusion with Downstream Risk Aversion or Limited Liability." *Journal of Industrial Economics*, 67: 409-447.

[17] Herweg, F., and Schmidt, K. M., (2020). "Bayesian Implementation and Rent Extraction in a Multi-Dimensional Procurement Problem." *International Journal of Industrial Organization*, 70: Art.-No. 102521.

[18] Horn, H., and Wolinsky, A. (1988). "Bilateral Monopolies and Incentives for Merger." *RAND Journal of Economics*, 19: 408-419.

[19] Jackson, M.O., and Wilkie, S., (2005). "Endogenous Games and Mechanisms: Side Payments Among Players." *Review of Economic Studies*, 72: 543-566.

[20] Jullien, B. (2000). "Participation Constraints in Adverse Selection Models." *Journal of Economic Theory*, 93: 1-47.

[21] Kamien, M. I., and Schwartz, N (2012). *Dynamic Optimization: the Calculus of Variations and Optimal Control in Economics and Management.* Courier Corporation.

[22] Kastl, J., Martimort, D., and Piccolo, S. (2011) "When should Manufacturers Want Fair Trade? New Insights from Asymmetric Information when Supply Chains Compete." *Journal of Economics & Management Strategy*, 3: 649-677.

[23] Kreps, M. D., and Scheinkman, J. (1983). "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes." *Bell Journal of Economics*, 14: 326-337.

[24] Laffont, J.J., and Martimort, D. (2002). *The Theory of Incentives: The Principal-Agent Model.* Princeton University Press, Princeton, NJ, U.S.A.

[25] Mankiw, N. G., and Whinston, M. (1986). "Free entry and Social Inefficiency." *RAND Journal of Economics*, 17: 48-58.

[26] Martimort, D., (1996). "Exclusive Dealing, Common Agency and Multiprincipal Incentive Theory." *RAND Journal of Economics,* 27: 1-31.

[27] Martimort, D., and Piccolo, S. (2007). "Resale Price Maintenance under Asymmetric Information." *International Journal of Industrial Organization*, 25: 315-339.

[28] Martimort, D., and Stole, L. (2009a). "Selecting Equilibria in Common Agency Games." *Journal of Economic Theory*, 144: 604-634.

[29] Martimort, D., and Stole, L. (2009b). "Market Participation in Delegated and Intrinsic Common-Agency Games." *RAND Journal of Economics*, 40: 78-102.

[30] Maskin, E., and Riley, J. (1984). "Monopoly with Incomplete Information." *RAND Journal of Economics*, 15: 171-196.

[31] McAfee, R. P., and Reny, P. (1992). "Correlated Information and Mechanism Design." *Econometrica*, 60: 395-421.

[32] McAfee, R. P., and Schwartz, M. (1994). "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *American Economic Review*, 84: 210-230.

[33] Miravete, E. J. (2002). "Estimating Demand for Local Telephone Service with Symmetric Information and Optional Calling Plans." *Review of Economic Studies*, 69: 943-971.

[34] Möller, M. (2007). "The Timing of Contracting with Externalities." *Journal of Economic Theory*, 133: 484-503.

[35] Montez, J. (2015). "Controlling Opportunism in Vertical Contracting when Production Precedes Sales." *RAND Journal of Economics*, 46: 650-670.

[36] Motta, M., (2004). *Competition Policy: Theory and Practice*. Cambridge University Press.

[37] Mussa, M., and Rosen, S. (1978). "Monopoly and Product Quality." *Journal of Economic Theory*, 18: 301-317.

[38] Nocke, V., and Rey, P. (2018). "Exclusive Dealing and Vertical Integration in Interlocking Relationships." *Journal of Economic Theory*, 177: 183-221.

[39] O'Brien, D. P., and Shaffer, G. (1992). "Vertical Control with Bilateral Contracts." *RAND Journal of Economics*, 23: 299-308.

[40] Reisinger, M., Tarantino, E. (2015). "Vertical Integration, Foreclosure, and Productive Efficiency." *RAND Journal of Economics*, 46: 461-479.

[41] Rey, P., and Stiglitz, J. (1995). "The Role of Exclusive Territories in Producers' Competition." *RAND Journal of Economics*, 26: 431-451.

[42] Rey, P., and Tirole, J. (1986). "The Logic of Vertical Restraints." *American Economic Review*, 76: 921-939.

[43] Rey, P., and Tirole, J. (2007). "A Primer on Foreclosure," in *Handbook of Industrial Organization*, M. Armstrong and R. Porter (eds.), North Holland, vol. III, pp. 2145-2220.

[44] Rey, P., and Tirole, J. (2019). "Price Caps as Welfare-Enhancing Coopetition." *Journal of Political Economy*, 127: 3018-3069.

[45] Rey, P., and Vergé, T. (2004). "Bilateral Control with Vertical Contracts." *RAND Journal of Economics*, 35: 728-746.

[46] Robert, J., (1991). "Continuity in Auction Design." *Journal of Economic Theory*, 55: 169-179.

[47] Schmidt-Kessen, M.J. (2018) "Selective Distribution Systems in EU Competition and EU Trademark Law: Resolving the Tension." *Journal of European Competition Law & Practice*, 9: 304-316. 114: 337-388.

[48] Segal, I. (1999). "Contracting with Externalities." *Quarterly Journal of Economics*, 114: 337-388.

[49] Segal, I., and Whinston, M. D. (2003). "Robust Predictions for Bilateral Contracting with Externalities." *Econometrica*, 71: 757-791.

[50] Shleifer, A. (1985). "A Theory of Yardstick Competition." *RAND Journal of Economics,* 16(3): 319-327.

[51] Stole, L.A. (2007). "Price Discrmination and Competition," in *Handbook of Industrial Organization*, M. Armstrong and R. Porter (eds.), North Holland, vol. III, pp. 2222-2298.

[52] Vives, X. (2001). *Oligopoly Pricing: Old Ideas and New Tools.* MIT Press: Cambridge, Massachusetts.