DISCUSSION PAPER SERIES

DP15448
(v. 4)

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INDUSTRIAL ORGANIZATION

ISSN 0265-8003

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Discussion Paper DP15448
First Published 12 November 2020
This Revision 05 May 2022

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JEL Classification: D83, L11, L14

Keywords: consumer search, Pricing, Retailer Variety, Showrooming

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Acknowledgements
We thank Andrew Rhodes, Anton Sobolev, Daniel Garcia, Edona Reshidi, Jane Gu, Jidong Zhou, Maarten Janssen, Matthew Mitchell, Xianwen Shi, Zheng Gong, and audiences at 3rd Workshop on Advances in Industrial Organization, Berlin Micro Theory Seminar, the Consumer Search Digital Seminar Series, Canadian Economic Association, European University Institute, HSE-Vienna workshop on information economics and industrial organisation, MACCI, Universitat Pompeu Fabra, and University of Toronto for useful comments that improved the paper. Shelegia acknowledges financial support from BBVA foundation the Spanish Ministry of Economy and Competitiveness grants RYC-2016-20307 and ECO2017- 89240-P.

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1 Introduction

Online shopping and discount stores have grown over the last decades, (Hortaçsu and Syverson (2015)). This has led more-established brick-and-mortar stores to worry about showrooming—that is, the prospect that consumers might visit them to learn which products they most like but buy elsewhere (Zimmerman (2012); Tuttle (2012)). This kind of showrooming has been argued to have led to the demise of retailers, such as RadioShack and CircuitCity (Team (2015)). More recently, similar stores, such as BestBuy, seem to flourish (Pearson (2017)) and the trade press suggests that retailers are less concerned about the threat of showrooming than in the past (Kalogeropoulos (2019)). At the same time, online stores are establishing a brick-and-mortar presence (Herrera, Fung and Kapner (2021)).

The source of concern for stores that act as showrooms is clear. The ability of consumers to buy elsewhere at a lower price leads the "showrooms" to lose sales or to reduce prices. (See, for example Mehra, Kumar and Raju (2017) who consider how price-matching and exclusive products can counter these effects). But, showrooming also affects prices at the stores from which the showroomers buy. Showroomers know what they want and go to stores with shallow selections of products looking for a better price. Ironically (and in the spirit of Diamond (1971)), they are insensitive to prices at these stores.

A literature on showrooming (as in Balakrishnan, Sundaresan and Zhang (2014), Jing (2018), Mehra, Kumar and Raju (2017) and Kuksov and Liao (2018), for example) supposes that consumers observe prices before visiting a store. So, it does not discuss this more novel force. But, this force suggests that making showrooming easier can lead to higher or lower prices throughout the retail sector. We present a model to highlight both forces. We highlight the role of the mix of consumer shopping patterns—that is, the ways that consumers search to learn which products suit them best and what prices are available. It is key for retail price determination. Our model allows us to consider characteristics of consumers and stores that affect this mix.

To do so we introduce heterogeneous kinds of retail outlets, and heterogeneous consumer types who may engage in different kinds of shopping behaviours to determine which good to purchase and where to buy it.[1] Specifically, in our model, there are two varieties of a good, and consumers are initially uncertain about which of the two is the best fit. A consumer can choose between visiting a *deep* retailer, where she can discover all varieties at once—saving on the cost of inspecting them and learning their suitability—and visiting *shallow* stores, which stock only a single variety of the good, one by one.[2],[3] Alternatively, a consumer may "showroom" — that

[1]While we do not know of work that explicitly documents heterogeneous consumers showrooming behaviour, Kaplan and Menzio (2015) for example highlight heterogeneous consumer search behaviour more broadly.

[2]The appeal of deep stores or stores that are perceived as deep (if prices are the same) has been widely noted in Baumol and Ide (1956), Kahn and Meyer (1991), and Hoch, Bradlow and Wansink (1999). As in our paper, consumers prefer such stores to the extent that they anticipate that they are more likely to find a better match at reduced search costs. This literature has focussed on store choice but does not account for the possibility of learning in one store and buying elsewhere.

[3]Note that depth refers to a specific product category. In this way, a speciality audio store would be understood as deep in audio equipment but a larger store, such as Target or Walmart, as shallow.

is, go to a deep store with no intention of buying there, but to figure out which is her favourite variety, and then to buy it at another store where it may be available at a lower price.

Showrooming arises as an equilibrium phenomenon when consumers are heterogeneous in two dimensions: first, their "choosiness" or preference for one version of the good over other versions; and, second, their willingness to showroom—that is to go to another store knowing their match value but only looking for a new price quote.[4] Consumers know which types of stores are which before visiting. Thus, we characterise a consumer's directed search problem. A number of consumer behaviours can arise: some consumers start by visiting deep retailers, while others visit shallow stores first; some consumers anticipate buying at the first store they visit, while others (showroomers) may anticipate never buying from the first store they visit.

Prices depend on the mix of search behaviours that consumers employ (which in equilibrium, of course, depend on anticipated and realised prices). However, not all kinds of search behaviours serve to discipline prices. Indeed, only one kind of consumer behaviour acts to discipline industry prices: that is, (not-so-choosy) consumers who start off by visiting a shallow store and expect that they will buy there unless they find a sufficiently poor match; in this case, they move on to another kind of shallow store and learn about another good. This group of consumers is the only one in the economy that compares prices, and the (endogenous) size of this group and its composition, therefore, play a key role in price determination. Unlike this group, showroomers arrive at shallow stores already knowing that they like the product. Thus, just as in Diamond (1971), they never leave over small price deviations. Thus, in an equilibrium with showrooming, shallow stores charge prices that are disciplined by only the relatively few consumers who might search more than one store to learn about their matches with different goods. To the extent that retail variety or changes in the cost of showrooming affect the relative size of this group, they can affect prices through the whole retail sector.

Since consumers who buy from deep stores do not shop around (for match), such stores effectively have hold-up monopoly power over consumers. Prices at deep stores, therefore, are constrained only by the possibility that a consumer becomes a showroomer and leaves to buy at a shallow store.[5] Since this involves a cost to the consumer, deep stores charge higher prices than shallow stores do. Of course, if shallow stores charge the monopoly price, then so will deep stores, and such an equilibrium always exists.

We show that showrooming can arise as an equilibrium phenomenon in our model, as it appears to do in practice, as well. We explore marginal effects of more showrooming by allowing for a small fraction of consumers who do not showroom (for example, one could consider a campaign that makes some consumers feel so guilty about showrooming that they never do so). The effect of introducing such a group can either raise or lower prices, depending on whether these 'never showrooming' consumers are drawn primarily from relatively more picky consumers (who instead would buy from deep stores), or from the less picky who would instead search

[4]One interpretation is that this is a cost of visiting stores. Another interpretation is the "guilt" associated with spending time at one store and buying elsewhere.

[5]This constraint is precisely the classic concern over the effect of showrooming—it leads to lower prices at the deep stores that act as showrooms.

through shallow stores and, so, bring down prices.

We further extend our baseline model and our discussion in a couple of ways. First, we discuss the role of retailer variety and what happens if some types of stores (either deep or shallow) no longer operate. This is relevant, for example, if each type of store must cover a fixed cost in order to operate, and viability might be affected by changes in demand or the introduction of new kinds of competitors (such as alternative channels). The analysis highlights that even if prices are unaffected, there may be negative welfare consequences. Prices that are affected by such a change necessarily go up. In our model, to ease tractability and exposition, we do not consider an extensive margin for consumers; however, these observations clearly imply that an upstream manufacturer may have an interest in maintaining retailer variety to encourage consumer participation.

Second, we introduce into the model a different kind of retail sector—one that we call the price-only sector. We assume that consumers cannot discover their matches at retail outlets in this sector. This is an appropriate assumption for online retail, since in many product categories, physical interaction with a product is important—for example, hearing the sound quality of a high-end speaker.[6] However, prices are readily available in this sector (again consistent with e-commerce). We suppose that some "savvy" consumers have access to this sector, while naive consumers do not. Depending on whether these savvy consumers drawn away by the price-only sector are disproportionately picky types (who would otherwise be showrooming and making demand at shallow stores more inelastic) or less-picky types (who would otherwise be searching and exerting downward pressure), prices in the more traditional stores may go up or down as a result of the introduction of price-only venues.

1.1 Related Literature

While there are many models of consumer search and its relation to pricing, to our knowledge, ours is amongst the first to consider competition between shallow and deep stores that stock overlapping selections of substitute goods, and to examine equilibrium showrooming. We study how retail variety endogenously determines search behaviour and equilibrium prices and demonstrate that equilibrium showrooming requires consumer heterogeneity and retailer variety. Moreover, we highlight that the ability to showroom can lead to higher prices.

A nascent literature has begun to explore consumer search for both match-value and prices with multiproduct retailers (Zhou (2014); Rhodes (2014)), with more-recent studies shifting focus toward the co-existence of multiproduct retailers with other stores offering narrower selections (see Rhodes, Watanabe and Zhou (2018); Rhodes and Zhou (2019)). However, we depart substantively from this literature since we consider multi-product stores selling goods that are substitutes rather than independent in consumers' consumption utility, as well as different stores that have overlapping selections (in contrast to Cachon, Terwiesch and Xu

[6]Online stores often offer generous return policies to combat this problem but these do involve costs to the consumers (such as hassle and delay) and in practice many consumers anticipate that they will never use this option.

(2008) and Watson (2009)). We highlight that this diversity in retail outlets affects prices and welfare.

There are other recent contributions on the showrooming phenomenon. For example, Wang and Wright (2020) examine fees that platforms (similar to our deep stores) charge and price-parity clauses. They focus on consumers who are ex-ante homogeneous in preferences, and, indeed, in their analysis, showrooming is a possibility that is never observed in equilibrium and is a force that leads to lower rather than to higher prices.

There are a number of papers in which showrooming does arise in equilibrium. Notably, Balakrishnan, Sundaresan and Zhang (2014), Jing (2018), and Parakhonyak (2018) consider models of a single good, sold through different channels, in which match can be learned only at one kind of retailer. Instead, in our model, a consumer may showroom to figure out which good to buy (rather than whether to buy at all) and can learn about her match realisations at either deep stores or shallow stores. Loginova (2009), Mehra, Kumar and Raju (2017) and Kuksov and Liao (2018) consider product variants. In all these papers, in addition to only a single venue for learning matches, consumers observe all prices before visiting stores, leading to a different analysis and different effects—specifically, the effect that showroomers raise prices at shallow stores does not arise. In particular, Mehra, Kumar and Raju (2017) focus on the role of price matching and exclusivity, while Kuksov and Liao (2018) focus on retail service provision and a monopolist manufacturer's endogenous contracts with retailers.

Shin (2007) and Janssen and Ke (2020) study service provision in search markets rather than showrooming; however, there is a connection to our work. In Janssen and Ke (2020), service received is transferable to other variants of the product, whereas in Shin (2007), both firms sell the same product, and the service informs some consumers about the match. In both models, some form of showrooming occurs, but Shin (2007) is closer to ours in that consumers learn about a match at a store that provides service, and consumers with low visit costs purchase at the other, cheaper store that provides no service. This is akin to our extension, in which we introduce an online sector in which there is no ability to help discover matches. The key difference in our model is that firms differ in their assortment and consumers differ in their pickiness, neither of which is true in Shin (2007) (who considers only a single good) or Janssen and Ke (2020) (where each store offers a single distinct good).

Moorthy, Chen and Tehrani (2018) focus on channel management, but do allow for comparison shopping. They focus on the vertical arrangements between manufacturers and their integrated and rivals' retailers and the effect of these arrangements on consumers' decisions to participate in the market. In particular, rivals may sell each other's goods to encourage demand discovery and boost the size of the market. To allow this focus, Moorthy et al. present a model in which all retail prices are set at the monopoly level (given input costs), whereas our model highlights the interaction amongst retail variety, consumer search behaviour and equilibrium prices.

Our work is also related to the literature on retail formats and assortment choice (Lal and Matutes (1989); Cachon, Terwiesch and Xu (2008); Dukes, Geylani and Srinivasan (2009); Zhu, Singh and Dukes (2011); Kuksov and Lin (2017); Liu and Shuai (2013)). As in some of these studies, we consider how partial overlap in product assortment affects consumer behaviour and firm pricing; for example, Dukes, Geylani and Srinivasan (2009) and Zhu, Singh and Dukes (2011) show that partial overlap may relax price competition. However, in many of these works, consumers face no uncertainty concerning a product's fit or price before deciding where to purchase. Kuksov and Lin (2017) show that a deeper assortment attracts consumers by signaling low prices, but consumers in their model face uncertainty only on prices (and do not need to visit stores to learn match utility). Cachon, Terwiesch and Xu (2008) and Liu and Shuai (2013) consider assortment where consumers search to learn fit but in a setting where assortments do not overlap, in the sense that all firms offer unique varieties.[7] As in this literature, demand patterns in our model depend on retail formats; however, a key difference is that, in our model, consumers are shopping for one good to learn match values as well as prices, and, since the same good may be available at different prices at different stores, our model features equilibrium showrooming behaviour.

Many of the above studies focus on the vertical aspects associated with showrooming. Although we do not explicitly model these arrangements, we highlight that sustaining (or killing off) different kinds of stores can affect equilibrium prices and consumer surplus. In turn, to the extent that this boosts industry profitability (through the effect on prices and discrimination, as in Parakhonyak (2018)) or encourages or depresses consumer participation (through anticipated consumer surplus), this will have implications for manufacturers' preferred strategies.

Finally, in recent work, Armstrong and Vickers (2020) examine how different exogenous patterns of consumer consideration affect prices and firm profits. Our work is related inasmuch as the consumer search behaviour provides an endogenous model of the nature of consumer consideration.

2 Model

There are three types of retailers that sell two differentiated goods, 1 and 2. There are "deep" retailers that sell both goods. There are two types of "shallow" retailers that sell only one of the goods (one for each type of good).[8] As will become clear, as long as both types of retailers operate, the actual number of each type will not be important for determining equilibrium, as long as there are at least two of each type.[9]

There is a unit mass of consumers who wish to purchase one of the goods. Consumer j's utility from consuming good i at price p_i is

[7] Liu and Shuai (2013) are closer to our work in that they distinguish within-firm and across-firm search costs.

[8] Note that these designations of shallow and deep relate to this particular product segment. For example, for a consumer buying speakers, a large store (such as Costco or Walmart) might be considered shallow if it stocks a small product range, and a small specialised store might more appropriately be considered deep.

[9] For this reason, in the propositions that follow, we state profits earned by retailer type and not by individual retailers of such type.

$$\mu^j \varepsilon_i^j + u(p_i),$$

where ε_i^j is consumer j's idiosyncratic match value for good i and is an iid draw (across consumers and goods) from a distribution on the support $[0, 1]$ with twice continuously differentiable CDF $G(\cdot)$, where $1 - G(\cdot)$ is assumed to be log-concave. A consumer's choosiness, which we discuss below, is represented by $\mu^j > 0$. The inherent utility of consumers for the goods at price p is $u(p)$, which is derived from a downward-sloping, and log-concave demand function $Q(p)$:

$$u(p) = \int_p^\infty Q(x)\ dx.$$

Thus, $u(p)$ is the consumer surplus (excluding the component that comes from the match value) derived from consuming the good at price p. It is assumed to be the same for all consumers whose utility differs only in their match values. Note that match values are additive to $u(p)$, which greatly simplifies the analysis. Anderson and Renault (2000) show that, although this formulation with downward-sloping demand and an additively separable match term is qualitatively similar to the more standard unit demand formulation in terms of the consumer and firm problems, it allows a role for prices to affect welfare. We discuss it at greater length, where relevant, below.

We normalise firms' marginal costs to be equal to zero. Thus,

$$\pi(p) = Q(p)p$$

denotes the per-consumer profits earned by a firm on a good. We use the standard notation

$$p^m = \arg\max_p \pi(p)$$

to denote its maximiser—the monopoly price—and

$$\pi^m = \max_p \pi(p)$$

to denote its maximand—the monopoly profits. p^m exists and is the unique maximiser by the log-concavity of $Q(p)$.

The outside option is assumed to give sufficiently low utility that all consumers purchase some positive quantity.[10]

Consumer j is initially uninformed about how well-matched she is with each of the two goods; that is, she does not know ε_1^j and ε_2^j. To find out her valuation for a good (that is, to learn ε_i^j), as well as the price, she needs to inspect the good. Doing so incurs an inspection cost, s. In particular, we rule out the possibility of buying the good without first inspecting

[10] This assumption is restrictive and is made for the purpose of tractability. As we go along, we will comment, whenever necessary, about its implications. Given this assumption, imposing that ε_i^j has a finite support [0,1] is without loss of generality.

it.[11] If the consumer already knows her match value with a good and is visiting a store only to obtain a price quote, this 'visit' cost, which can also be understood as guilt associated with engaging in showrooming behaviour, is denoted as b^j, with $b^j \leq s$. Thus, inspection (including visit) at a shallow retailer that sells good i costs s; if the consumer already knows ε_i^j, the visit cost is only b^j.

The inspection cost at deep stores is $s(1+\gamma)$, where $\gamma \in [0, 1]$. During a visit to a deep retailer consumer learns the match values and prices for both goods.[12] The parameter γ measures economies of scale in inspection costs allowed by deep retailers that stock both goods. When $\gamma = 0$, such scale economies are at their highest, whereas when $\gamma = 1$, they are non-existent. Since a visit to a deep store involves no inspection when the consumer already knows her match realisations, we assume that the visit cost to a deep store is b^j; this plays no role in our analysis as long as this visit cost is non-negative.

Consumers are free to make their visits in any order and know the retailer's type before visiting; that is, they know whether a retailer is deep or shallow, and, if shallow, which good is stocked.[13] That is, given anticipated prices, consumer j can decide to make her first visit to a deep or shallow retailer. If consumers are indifferent amongst different stores in equilibrium, we assume that they are equally likely to visit any of them. To avoid issues related to prominence (Armstrong, Vickers and Zhou (2009)), we will seek equilibria in which all shallow stores charge the same price, and we will not allow consumers to target a particular type of shallow retailer (e.g., those selling good 1) for first visits. Consumers can go back to all visited stores to make a purchase at no extra cost.[14]

Note that while we allow consumers to differ in their choosiness μ^j and visit costs b^j, we suppose that the economies of scale associated with inspecting goods at a deep store γ and the inspection cost s are common amongst all consumers. Of course, this is not substantive for the analysis of an individual consumer's behaviour, which we consider in Section 3. Instead, it simplifies the analysis of the equilibrium pricing decisions in Section 4, where we must take a stance on the distribution of consumers. As we discuss there, we make assumptions for tractability (including that consumers vary only in their pickiness and in their visit costs)

[11] This can be justified while maintaining our analysis by supposing, for example, that there is a small probability of a very large negative match.

[12] In particular, we abstract from the search process within the store that has been considered by Gu and Liu (2013), for example. Allowing for consumers to choose not to search both goods within a store would add further analytic complexity, but the key force for our model—that expected inspection costs would be lower (and the anticipated match higher) at a deep stores—would be robust.

[13] In some applications, it may be more reasonable to suppose that, on inspection, consumers do not know the type of the shallow store (though they may be able to find out for the purpose of visiting or buying from such a store online); or that they do not know the type of the shallow store either for inspection or for visiting. Analysis in such an environment may be more involved; for example, a consumer may start by searching a shallow store and then choose to visit a deep store—a possibility that does not arise in our setting. However, we would anticipate that the general qualitative results of our analysis would also obtain in these cases.

[14] This is a simplifying assumption that can be relaxed at a cost of tractability. If there are positive costs of going back, such as having to pay the visit cost twice, fewer consumers would visit shallow stores since, in equilibrium, the only consumers who revisit stores are ones who search through shallow stores and find that the second match is even worse than the relatively low first match. Note that this 'costless recall' assumption is consistent with an interpretation of the visit cost b as reflecting a consumer's embarrassment or guilt at enjoying service in one store and buying elsewhere.

that we believe allow us to illustrate some economic forces that would also apply in richer environments.

The timing of the game is as follows. First, retailers simultaneously set prices for all the goods that they carry. Second, consumers decide which type of retailer to visit first. Once the first visit reveals match value(s) and price(s), consumers may decide to visit more stores, and visit and inspection costs are incurred, as outlined above. Once consumers finish their inspections and visits, they decide which good to buy and where.

We characterise (perfect Bayesian) equilibria in which prices are symmetric for the two goods. Next, we consider the welfare implications of retail variety and the effect of price-only retailers. Throughout, we assume passive beliefs: if a consumer observes unexpected prices at a retail store, this does not affect her expectations of prices at other stores.

3 The consumer problem

We assume that consumers expect a symmetric equilibrium in which both types of shallow retailers charge p_S^* for the good they sell, and deep retailers charge a price vector (p_D^*, p_D^*) for goods 1 and 2. Further, for this section we suppose that $p_D^* \geq p_S^*$. This will turn out to be the case in the equilibrium of the full game where prices are endogenous.[15] This is intuitive—deep stores offer inspection economies of scale and so can charge higher prices. For now, we assume that this is the case. Abusing notation, we sometimes use scalar p_D to denote the vector. This is because, unless noted otherwise, deep stores find it optimal to charge the same price for both goods.

In this section, under the assumption that pricing is as described in the above paragraph, we characterise the optimal search and purchase behaviour of a consumer given the parameters μ, b, s and γ.[16] Such a consumer has to decide which type of retailer to visit first and then what to do next. We suppose that a consumer will purchase one of the goods rather than drop out of the market. If indifferent, we assume a consumer chooses to visit deep stores. The possible consumer behaviours and how they depend on parameters are summarised in Section 3.3. The intervening sections derive these behaviours and introduce further notation.

The advantage of visiting a deep retailer is that no further search for match values is necessary. This may potentially save on inspection costs if γ is small enough and if the consumer is sensitive to match quality. The disadvantage is that consumers will have to either pay a higher price or incur a further visit cost in their search for a lower price elsewhere. We use the notation $\Delta(x, y) \equiv u(x) - u(y)$ to denote the utility difference in purchasing the same good at a price x rather than at a price y. It is convenient to introduce the notation $\Delta^* \equiv \Delta(p_S^*, p_D^*)$ for the gain in utility from buying a good at the equilibrium price of a shallow store rather than at the equilibrium price of a deep store. With some abuse, we refer to this as the price premium associated with a deep retailer (which would be accurate in the case of unit demand

[15] See Section 4, Lemma 4.

[16] Since we consider only a single consumer in this section, we drop the ^{j} superscript here for notational convenience

but, here, corresponds to a utility-adjusted price difference). A consumer who visits a deep retailer expects to pay an extra utility cost of $\min(\Delta^*, b)$ compared to buying the same good at a shallow store. Thus, a consumer at a deep store will have to compare Δ^* and b, and, accordingly, will showroom or not.

3.1 Starting at a shallow store

If a consumer chooses to make her first inspection at a shallow retailer, depending on her match value drawn for the one product that this retailer sells, she may choose to inspect again at the other type of shallow retailer. Because she expects $\Delta^* > 0$, the second inspection would never be at a deep retailer. Thus, if the consumer starts by searching at a shallow store, deep retailers are, in effect, irrelevant, and the way that she searches through shallow retailers is similar to that in the canonical Wolinsky (1986) model with $n = 2$, adjusted for our setup with downward-sloping rather than unit demand.[17]

It is convenient to define

$$w(x) \equiv \int_x^1 (\varepsilon - x) g(\varepsilon) d\varepsilon$$

as the expected gain from drawing a match value above x for $\mu = 1$. As is well known from the literature, and corresponding to the analysis in Wolinsky (1986), for example, a consumer who inspects at a shallow retailer will purchase there if the match value is high enough, and, otherwise, will inspect at the other type of shallow retailer. That is, the consumer searching amongst shallow retailers employs a threshold rule. It is convenient to introduce notation for the threshold match value r^*. This is the solution to

$$\mu w(r^*) = s.$$

If there is no solution, then $r^* = 0$, and the consumer will buy from the current store, irrespective of the match value.

The expression above is a little different from that in the standard model, in that the left-hand side of the equation includes the factor μ to take into account that match values are equal to $\mu\varepsilon$.

Consider a consumer who visits a shallow store selling good 1 (similar analysis applies for good 2) and finds price p_S when she expected that a shallow retailer selling good 2 would charge p_S^*. The consumer is indifferent between buying good 1 and inspecting good 2 if

$$\varepsilon_1 = r(p) \equiv r^* + \frac{\Delta(p_S^*, p_S)}{\mu}.$$

[17] Small price deviations by a shallow retailer selling good 1 do not lead to consumers switching to other retailers selling the same good—this is simply an instance of the Diamond paradox. However, due to our assumption of directed search, they do potentially lead consumers who are close to indifferent to inspect good 2 at a shallow retailer of that type. Thus, Diamond-paradox effects amongst similar retailers and inspection across different retailer types lead to predictions that are equivalent to those of the model with just two retailers with unique varieties.

A consumer will, therefore, continue and inspect good 2, rather than buying good 1, if $\varepsilon_1 < r(p_S)$. It never pays off for a such a consumer to buy good 1 from another shallow retailer selling it at p_S^* instead of learning about good 2, as that would give utility $\varepsilon_1 + u(p_S^*) - b$, which, by definition, is worse than searching for and inspecting good 2.

In principle, this problem seems involved; however, Choi, Dai and Kim (2018) and Armstrong (2017) note that there is a convenient representation. In particular, a sequential search problem faced by consumers who search amongst shallow stores is equivalent to a complete information discrete choice problem where, for each firm, the draw is replaced with $\min(\varepsilon_i, r^*)$, and consumers buy from the firm with the highest draw. For $r^* > 0$, we can then write the expected utility prior to the first inspection as

$$U_S = \mu \int_0^1 \min(\varepsilon, r^*)\tilde{g}(\varepsilon)\, d\varepsilon + u(p_S^*),$$

where $\tilde{g}(\varepsilon) \equiv 2g(\varepsilon)G(\varepsilon)$ is the density of $\max(\varepsilon_1, \varepsilon_2)$.[18] The beauty of this formulation is that the utility is as if the consumer does not pay any search costs (all, including the first one, are already accounted for through the definition of r^*). Instead, the utility simply involves the consumer drawing the maximum of two draws but only if the maximum is below r^*, or else she gets r^*.

3.2 Starting at a deep or a shallow store?

The expected utility when starting by inspecting at a deep retailer is given by

$$U_D = \mu \int_0^1 \varepsilon\tilde{g}(\varepsilon)\, d\varepsilon + u(p_S^*) - \min(b, \Delta^*) - (1+\gamma)s.$$

The first term reflects that the consumer always buys whichever of the two goods is a better match; the second two terms reflect that either the consumer pays b to showroom and enjoy the consumer surplus associated with a price of p_S^*, or else purchases at the deep store and enjoys consumer surplus $u(p_D^*) = \Delta^* - u(p_S^*)$, where the equality is immediate by the definition of Δ^*. The final term simply reflects the search costs at a deep store.

It is also useful to define the inspection efficiency benefit associated with visiting a deep store. Naturally, this depends on the consumer's sensitivity to the match value, μ. We write

$$\beta(\mu) \equiv \mu \int_{r^*}^1 (\varepsilon - r^*)\tilde{g}(\varepsilon) d\varepsilon - (1+\gamma)s$$

as the benefit of initially visiting a deep store versus a shallow store, absent any price difference.[19] The equality above holds for the case that $r^* > 0$.[20] This benefit reflects that the consumer who visits a deep store enjoys $\max(\varepsilon_1, \varepsilon_2)$ instead of ε_i for cases in which $\varepsilon_i > r^*$ but incurs a higher search cost. A consumer at a deep retailer necessarily obtains the maximum

[18]Instead, when $r^* = 0$, the consumer will never go beyond the first store, and so $U_S = \mu E(\varepsilon) + u(p_S^*) - s$.

[19]Note that at the same prices, $\Delta = 0$, and so $min(b, \Delta) = \Delta$ and b does not affect $\beta(\mu)$.

[20]For the case that $r^* = 0$, trivially, $U_S = \mu E(\varepsilon) + u(p_S^*) - s$ and so $\beta(\mu) = \mu[E(max(\varepsilon_1, \varepsilon_2)) - E(\varepsilon)] - \min(b, \Delta^*) - \gamma s$.

match value but has to pay an inspection cost of $(1+\gamma)s$. This inspection benefit naturally depends on (and increases with) a consumer's pickiness, μ.

With this notation, we can compare U_S and U_D at prevailing equilibrium prices:

$$U_D - U_S = \beta(\mu) - \min(b, \Delta^*).$$

We can see that consumers get an inspection benefit $\beta(\mu)$ from visiting deep stores, but this comes at the cost of incurring either the price premium at the deep store or an additional visit cost associated with showrooming: that is, $\min(b, \Delta^*)$. Following Lemma 1, $U_D - U_S$ is non-decreasing in μ, so that consumers with high μ are the ones who choose to visit deep retailers.[21] Of these, it is immediate, on inspecting $U_D - U_S$, that those with low b ($b < \Delta^*$) showroom, and those with $b \geq \Delta^*$ buy at deep stores.

We can now compare the expected utility from starting the search process at a deep store or at a shallow store. This is simply a comparison of U_D and U_S.

Lemma 1. *Suppose that $r^* > 0$; the choosier (higher μ) consumers are more likely to start by searching at a deep retailer.*

Proof. See Appendix A for the proof of the lemma and all other proofs. □

Lemma 2. *Consumers prefer visiting deep stores when stores are equally priced (that, is $p_S^* = p_D^*$) if the economies of scale from searching at a deep store are sufficiently large; that is, γ is small enough.*

Intuitively, at equal prices, consumers prefer to visit deep stores unless γ is high. To see this clearly, note that if $\gamma = 0$ and $p_S^* = p_D^*$, consumers strictly prefer to visit deep stores since they are guaranteed their best match at no additional search cost. Clearly, if γ is high enough, no consumer will wish to visit deep stores. However, more generally, the comparison between U_S and U_D then depends on other parameters.

3.3 Summary of consumer behaviour

We can now summarise a consumer's first visit. A ('deep loyal') consumer who visits and buys from a deep retailer will satisfy

$$b \geq \Delta^* \text{ and } U_D \geq U_S.$$

Consumers showroom (go to deep stores but buy at shallow stores) when

$$b < \Delta^* \text{ and } U_D \geq U_S.$$

Consumers with

$$r^* > 0 \text{ and } U_S > U_D$$

[21] It is clear that this property also holds when $r^* = 0$.

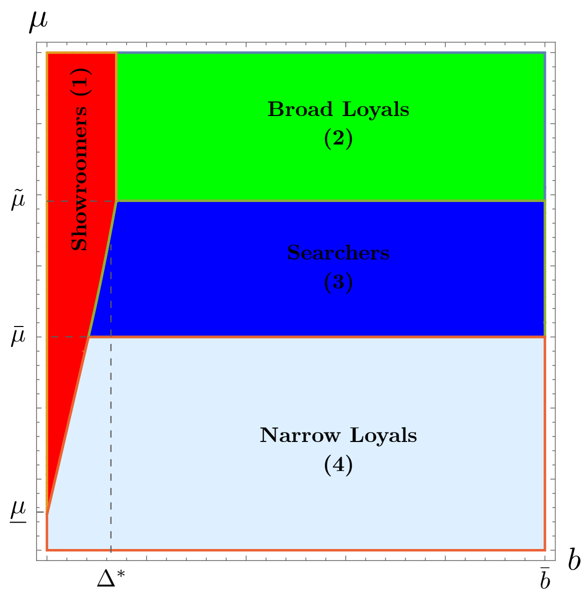

Figure 1: Consumer types for $G(\varepsilon) = \varepsilon$, $\Delta^* = 0.035$, $s = \bar{b} = 0.25$, $\gamma = 0.25$. In the red area (1), consumers showroom. In the green area (2), consumers visit a deep retailer and purchase there. In the two blue areas ((3) and (4)), consumers visit shallow retailers and buy from them. In the light-blue area (4), consumers do not search beyond the first retailer, whereas in the dark-blue area (3), they do.

visit shallow stores and may search if their first draw is below r^*. We call them 'searchers,' and, as highlighted, they play a crucial role in price determination. Finally, ('shallow loyal') consumers with

$$r^* \leq 0 \text{ and } U_S > U_D$$

make first visits to shallow stores but never search beyond the first one in equilibrium (but may search if the first firm deviates from p_S^*).

Fixing the other parameters, in (b, μ) space, the curve defined by $U_D = U_S$ for $b > \Delta^*$ is a flat line. For $b < \Delta^*$, because $U_D - U_S$ is increasing in μ, the curve is upward-sloping. Finally, some consumers never search when visiting shallow stores; for these, $r^* = 0$. Thus, the illustration in Figure 1, taken for particular parameter values, is more generally, representative. Throughout the analysis of the consumer problem, and in this figure in particular, Δ^* is exogenously given. However, the particular level of Δ^* shown here will be an equilibrium price difference, as illustrated in Section 4.3.

Naturally, choosy consumers who are very sensitive to match values—that is, those with high μ—visit deep retailers to discover $(\varepsilon_1, \varepsilon_2)$. Amongst these consumers, those with low visit cost b proceed to shallow retailers for actual purchases, effectively using deep retailers as showrooms. However, those with high b purchase from deep retailers because the price difference is not worth the extra cost associated with an additional visit.

This concludes the analysis of first visits. Second visits are simple to describe, though they depend on the actual encountered prices, which may differ from the anticipated equilibrium prices (when these differ, we denote such prices without a * to distinguish them from the equilibrium p_S^* and p_D^*). For those whose first visits are to deep stores, the decision to visit a shallow retailer depends on whether $\Delta(p_S^*, p_D)$ (using actual, not anticipated, p_D) is above or below b.[22] For example, if $\varepsilon_1 > \varepsilon_2$ and $b < \Delta(p_S^*, p_D)$, then this consumer will make the second trip to a shallow retailer selling good 1, and she will purchase there provided that p_S charged by that retailer satisfies $p_S \leq p_D$ and $\Delta(p_S^*, p_S) < b$.[23] Note that shallow retailers are able to hold up such showroomers because once they have arrived, only a drastic upward price deviation can result in losing them. Consumers who visit shallow stores never consider visiting deep stores for second inspections or visits ($p_S^* < p_D^*$). They stop at the first store if $\varepsilon_1 > r(p)$ and search otherwise.[24] They purchase the good they have inspected that gives the highest utility. We summarise this discussion as follows.

Lemma 3. *A consumer with (μ, s, γ, b) conducts the following optimal search:*

1. *If $U_D \geq U_S$, the consumer makes her first visit to a deep store. She discovers her best variety $i \in \arg\max_{k\in\{1,2\}} \varepsilon_k$. She purchases good $l \in \arg\max_{k\in\{1,2\}} \mu\varepsilon_k + u(p_D)$ from the deep store if $\mu\varepsilon_l + u(p_D) \geq \mu\varepsilon_i + u(p_S^*) - b$; otherwise, she visits a shallow store selling good i and buys there unless $\Delta(p_S, p_S^*) > b$, in which case she keeps going to shallow stores selling i until $\Delta(p_S, p_S^*) \leq b$, in which case she stops and buys, or she runs out of shallow stores selling good i and then recalls the best offer seen.*

2. *If $U_D < U_S$, then the consumer makes her first visit to a shallow store. She stops and buys if $\varepsilon_i \geq r(p_S)$, or else she searches shallow stores selling good $j \neq i$ and then buys the best offer seen.*

4 Retailer pricing and equilibrium

Retailers' equilibrium pricing and consumers' equilibrium choices of which kinds of stores to visit interact. The analysis in Section 3 is based on expectations of retailers' pricing decisions. We now consider the pricing decisions of different kinds of retailers who anticipate the consumer behaviour described above. As illustrated in Figure 1, for a given set of prices, different kinds of consumers typically engage in different kinds of search behaviour. Consequently, to address a firm's pricing problem, we must specify the number of each kind of consumer since they affect the elasticity of demand and, thus, the pricing decisions of each kind of retailer.

As described above, in addition to the endogenous prices, consumer behaviour depends on several exogenous parameters: a consumer's choosiness μ; visit cost b; inspection cost s; the inspection economy associated with a deep retailer γ; and the distribution of matches $G(\cdot)$. Clearly, allowing all of these to vary with no restrictions on their distributions would be

[22] Of course, which of the shallow stores to visit depends on whether ε_1 is more or less than ε_2.

[23] Visiting a shallow store is more attractive than visiting another deep store since $p_S^* \leq p_D^*$. Due to the free recall assumption, a consumer can go back to a deep retailer charging p_D at no cost; thus, the comparison is between p_S and p_D in that case. If p_S is too high, the consumer may opt to pay b and go to another shallow retailer selling good 1 at the equilibrium price p_S^*. This possibility explains the need for the second condition. We draw on our passive beliefs assumption throughout this discussion.

[24] If a consumer visits a store of type 1 and finds a match that she would buy at the anticipated price but the store charges an unexpectedly high price, she moves on to another store of type 1.

demanding. Instead, we impose some additional structure. Specifically, we assume that s, γ, and $G(\cdot)$ are common to all consumers. The remaining parameters are μ and b. We suppose that μ follows a binary distribution, where it takes a value μ_H with probability $1-\lambda$ and μ_L with probability λ, with $\mu_H > \mu_L > 0$; finally, $F_T(\cdot)$ is the CDF of $b \in [0, \bar{b}]$ for $T \in \{H, L\}$, and we assume that $1 - F_T(b)$ is log concave, and $f_T(0) = 0$.[25] It is convenient to assume that the high types' visit cost distribution has a weakly higher hazard rate than the distribution of the low types: $\frac{f_H(b)}{1-F_H(b)} \geq \frac{f_L(b)}{1-F_L(b)}$. This assumption is used in Section 5, where we introduce a price-only sector.

We consider consumer heterogeneity in pickiness in order to study equilibria in which showrooming and search occur, and all stores make sales. For this to be the case, we need some consumers to be deep loyals, some to be searchers and some to be showroomers. An inspection of Figure 1 makes it clear that in order to have all three types of consumers, there must be at least two levels of pickiness μ; thus, the assumption on μ is the minimum necessary for this type of equilibrium to emerge. Moreover, Figure 1 also highlights the need for some heterogeneity in b. Allowing one of these (in our case, b) to be smoothly distributed allows us to characterise prices through a first-order condition.

To simplify analysis, we will assume that the search economy associated with deep retailers is strong enough, or, equivalently, that γ is sufficiently low. Namely, define $\bar{\gamma} \equiv \frac{E[\max(\varepsilon_1, \varepsilon_2)]}{E[\varepsilon]} - 1 > 0$. From now on we will assume the following.

Assumption 1. *Search economies at deep stores are sufficiently strong; that is, $\gamma < \bar{\gamma}$.*

This assumption ensures that even consumers who are borderline searchers at shallow stores still strictly prefer to go to deep stores given equal prices; that is, consumers with $r^* = 0$ ($\mu = \frac{E[\varepsilon]}{s}$) have $\beta(\mu) > 0$. We discuss the importance of this below. In Appendix B, we consider the case in which this assumption fails.

We will focus on symmetric equilibria as described above. There are three possible types of such equilibria: (i) all directed first visits occur at shallow retailers; (ii) all such visits occur at deep retailers; and, finally, (iii) these visits are split between the two types of retailers. To reduce the number of equilibria to analyse and rule out unnatural cases, we assume that a store that anticipates no visits charges p^m for any products it sells. One can justify this assumption by trembling-hand-type arguments or by the presence of a small number of loyal consumers who purchase only from a given retailer. Even amongst these more-reasonable equilibria, the consumer model can lead to many possibilities.

To understand these possibilities, it is useful to define $\underline{\mu}$ as the solution to $\beta(\mu) = 0$—that is, the level of pickiness that would make a consumer indifferent between starting at a deep or a shallow store, absent price differences, or where the inspection efficiency is exactly 0. Given the

[25] Note that s is common to all consumers but b varies. Since the cost of visiting a store should enter both s and b, it is reasonable to expect that they are positively correlated. Explicitly accounting for such correlation would make this analysis less tractable. However, as long as this correlation is moderate, the results would not be significantly affected. This is consistent with much of the variation in b arising from psychological costs (guilt).=Furthermore, as noted earlier, for the analysis in Section 3 which considers the behaviour of individual consumers, no distributional assumptions are made about b and s.

monotonicity of the inspection efficiency, $\beta(\mu)$, in pickiness, μ, as described in Lemma 1, and the fact that at $\beta(0) \leq 0$, this solution exists and is unique. Further, define $\bar{\mu}$ as the solution to $r^* = 0$: the (lowest) level of pickiness that ensures that if the consumer searches through shallow stores, she will never move on to a second store, regardless of the match. By the monotonicity of r^* in μ and the fact that at $\mu = 0$ gains from search are zero, this solution also exists and is unique. By Assumption 1, we can rank $\bar{\mu} > \underline{\mu}$. Indeed, this is the simplification that the assumption affords.[26]

For what follows, we note that for $\mu < \underline{\mu}$, we have $\beta, r^* < 0$; if $\mu \in [\underline{\mu}, \bar{\mu})$, then $r^* < 0 \leq \beta$; and if $\mu \geq \bar{\mu}$, then $\beta, r^* \geq 0$. When μ is low, a consumer is not willing to search amongst shallow stores but would buy from a shallow store, regardless of the match (reflecting that $r^* < 0$), and would rather patronise shallow stores at equal prices (that is, $\beta(\mu) < 0$). When μ is intermediate, the consumer is unwilling to search amongst shallow stores ($r^* < 0$) but would rather patronise deep stores at equal prices ($\beta(\mu) > 0$). Finally, when μ is high, she would search through shallow stores ($r^* > 0$) but would still prefer deep stores to shallow ones ($\beta(\mu) > 0$).

Before analysing these cases, it is useful to introduce some additional notation. First, corresponding to the definition of r^* in Section 3 above, we define r^*_L as the reservation utility for consumers with μ_L, and r^*_H for those with μ_H. Similarly, we denote $\beta_H \equiv \beta(\mu_H)$ and $\beta_L \equiv \beta(\mu_L)$.

Lemma 4. *In any symmetric equilibrium, $p^*_D \geq p^*_S$. Furthermore, if $p^*_D = p^*_S$, then $p^*_D = p^*_S = p^m$.*

Lemma 4 claims that in all symmetric equilibria, deep stores are at least as expensive as shallow stores, and if stores charge equal prices, then all stores charge monopoly prices. The reason for this lies in Diamond-like reasoning, whereby consumers leave deep stores only for lower prices, and, thus, if no such prices are to be found, then deep stores will be charging monopoly prices.

Given the above lemma, we are left with two possible pricing configurations. First, all store types charge monopoly prices. Second, shallow stores charge lower prices than deep stores, in which case consumers ought to search and showroom. We will consider these types of equilibria in turn.

4.1 Equilibria in which all firms charge the monopoly price

In this section, we demonstrate that there always exists an equilibrium in which all firms charge the monopoly price. Such equilibria involve consumers anticipating that they will visit a single store and all retailers charging the monopoly price. Moreover, we outline that such an outcome may (but need not) arise as a unique symmetric equilibrium. However, depending on parameters—specifically how choosy the more- and less-choosy consumer-types are—this

[26] To see this, let us evaluate β at $\mu = \bar{\mu}$; substituting $r^* = 0$ yields $\beta = s\left[\frac{E[\max(\varepsilon_1, \varepsilon_2)]}{E[\varepsilon]} - 1 - \gamma\right]$, which given Assumption 1, implies that $\beta > 0$, and, thus, concludes that $\bar{\mu} > \underline{\mu}$.

outcome might depend on different consumer behaviours, with all visiting deep stores, all visiting a single shallow store, or the more-choosy visiting deep stores and the less-choosy visiting a single shallow store.

Proposition 1. *There always exists a symmetric equilibrium in which all stores charge p^m. If $\mu_L < \bar{\mu}$, such a symmetric equilibrium is unique. In such equilibria, consumer shopping behaviour is the following:*

1. *If $\mu_H < \underline{\mu}$, then all consumers visit shallow stores and buy without searching. Deep stores earn $\Pi_D^* = 0$, while shallow stores earn $\Pi_S^* = \frac{1}{2}\pi^m$.*
2. *If $\mu_H \geq \underline{\mu} > \mu_L$, then all high types visit and buy from deep stores, and all low types visit and buy from shallow stores. Deep stores earn $\Pi_D^* = (1-\lambda)\pi^m$, while shallow stores earn $\Pi_S^* = \frac{1}{2}\lambda\pi^m$.*
3. *If $\mu_L \geq \underline{\mu}$, then all consumers visit and buy from deep stores. Deep stores earn $\Pi_D^* = \pi^m$, while shallow stores earn $\Pi_S^* = 0$.*[27]

When all firms charge the monopoly price, if even the less choosy consumers are sufficiently choosy, $\mu_L \geq \underline{\mu}$, they value learning both match realisations before purchasing one of the goods; in this case, all consumers begin by visiting deep stores. Consequently, since shallow stores expect only showroomers who they can hold up, all stores end up charging monopoly prices.

When even picky consumers are so unfussy that they prefer to visit a single shallow store, regardless of the match value, $\mu_H < \underline{\mu}$, in equilibrium, all stores charge monopoly prices, but only shallow stores receive consumers.

Finally, when picky consumers are picky enough, and less-picky consumers are not, consumers with different choosiness visit different kinds of stores, but, again, in equilibrium, consumers will not visit more than one retailer (because the less-choosy are insufficiently choosy and visit a single shallow retailer, while the more-choosy are sufficiently choosy and visit a deep retailer, $\mu_H \geq \underline{\mu} > \mu_L$), and so all consumers end up paying the monopoly price.

Note, however, that Proposition 1 affords the possibility that when $\mu_L \geq \bar{\mu}$, there may exist other equilibria. We examine this possibility next.

4.2 Equilibrium with search and showrooming

Now we turn to the most interesting type of equilibrium, in which first visits are split between the retailer types and prices are below the monopoly level. This equilibrium does not always exist, and we characterise when it does.

In any such equilibrium, it must be the case that deep stores retain some consumers, or else they would choose to deviate to a lower p_D. Consequently, in equilibrium, it must be that

[27] Here, and elsewhere in the paper, equilibrium profits stated are for all firms of a given type; for example, in case 2, the total profits of all deep stores are qual to $(1-\lambda)\pi^m$, and the total profits of all shallow stores of each of the two types are equal to $\frac{1}{2}\lambda\pi^m$.

the inspection benefit associated with deep stores for those who value them most (the picky consumers) must be higher than the utility costs associated with anticipated price differences; that is,

$$\Delta^* \leq \beta_H.$$

Similarly, it must be that some consumers (the less-picky) prefer to start at shallow rather than at deep retailers; that is, equilibrium requires that

$$\Delta^* > \beta_L,$$

or else no consumers will make first visits to shallow stores. Note that this is a necessary condition but does not imply that all of the less-picky start by visiting shallow stores. Those with low visit costs prefer to visit a deep retailer and to showroom rather than to inspect at shallow stores.

Finally, for a non-monopoly price equilibrium to arise, low types must be sufficiently picky that they search through shallow stores,

$$\mu_L \geq \bar{\mu}.$$

We assume that these three conditions hold and check the parameter configurations that deliver them once prices are characterised. In such cases, all high types and low types with $b < \beta_L$ first visit deep stores. Around the equilibrium price (that is, for local deviations) p_D^*, only high types react to p_D (recall that $\Delta^* > \beta_L$) since low types strictly prefer to showroom. Thus, deep stores set p_D in order to maximise profits from high types, with high types staying at deep stores when $b \geq \Delta(p_S^*, p_D)$.[28]

4.2.1 Pricing for deep stores

In order to characterise deep store pricing, we introduce the notation $h(p) \equiv -\frac{\pi(p)u'(p)}{\pi'(p)}$. For a price p_D in the neighbourhood of p_D^* deep retailer's profit is

$$\Pi_D(p_D) = (1-\lambda)(1 - F_H(\Delta(p_S^*, p_D)))\pi(p_D).$$

This allows us to write the following, whose form should be familiar, as in a standard monopoly pricing problem.[29]

Lemma 5. *The first-order condition for a deep retailer's pricing problem is given by*

$$h(p_D^*) = \frac{1 - F_H(\Delta^*)}{f_H(\Delta^*)}. \tag{1}$$

Consumers leave deep stores only in order to claim a lower price at a shallow store. Because consumers find it costly to leave, deep stores are able to charge a positive price premium Δ^*. The

[28] See details on the deep store maximisation problem in the proof of Lemma 5.

[29] In the Appendix A.9 we show that $\Pi_D(p_D)$ is quasi-concave in p_D and so the FOC in Lemma 5 gives the unique maximiser.

higher this premium, the lower is the fraction of consumers that stay, and the trade-off between higher price and lower demand is resolved in the first-order condition shown above. Note that only high types may buy from a deep store, and only high types are marginal consumers, which is why (1) does not depend on F_L. To see this clearly, recall that amongst low types, only those with $b < \beta_L$ showroom, and, by construction, $\Delta^* > \beta_L$, even the low type consumer with the highest visit cost strictly prefers to leave rather than to stay.[30]

4.2.2 Pricing for shallow stores

Pricing for shallow stores is similar to that in the standard search model analysis, with the exception that, in addition to the standard demand from consumers who search through shallow stores to find a suitable match in the manner of Wolinksy (1986) or Anderson and Renault (1999), such stores also receive demand from two different kinds of showroomers. First, there are choosy (μ_H) consumers with low visit costs (below the price differential, $b < \Delta^*$). Second, there are less-choosy (μ_L) consumers with low visit costs (below the benefit associated with visiting a deep store at equal prices $b < \beta_L$). These two groups arrive at a shallow store for the good with which they found themselves to be better matched. Then, even if observing an off-equilibrium price, they continue to purchase the good unless the price deviation is more than b; given our assumption that $f_H(0) = f_L(0) = 0$, this means that local price deviations do not affect the demand of either kind of showroomer.

Thus, the only kind of consumers who are price-sensitive are the less-choosy L types with high enough visit costs that they prefer not to showroom, which is the case when $b > \beta_L$. The total mass of such consumers is $1 - F_L(\beta_L)$. Amongst the consumers arriving at a shallow retailer selling good i at price p_S, those with $\varepsilon_i < r_L^* + \frac{\Delta(p_S^*, p_S)}{\mu_L}$ will search another shallow retailer selling the other good, and the rest will buy from the first shallow retailer. Of the consumers who search, those discovering a low enough match with good j (specifically, $\varepsilon_j < \varepsilon_i + \frac{\Delta(p_S^*, p_S)}{\mu_L}$) will come back and buy. Furthermore, there will be consumers who arrive at other shallow retailers, discover match values below r_L^* for the other good, and visit this shallow retailer. Of these consumers, those with $\varepsilon_j < \varepsilon_i + \frac{\Delta(p_S^*, p_S)}{\mu_L}$ will also buy.[31]

The fraction of less-choosy (type L) consumers who start at shallow stores and end up purchasing at a shallow store that charges p_S is, therefore,[32]

[30] While F_L plays no role for p_D^*, because some low-type consumers showroom, it is important to check whether deep stores want to make a large deviation with the hope of retaining such consumers. This is analysed formally in the next section and is captured in Condition 1.

[31] In principle, if $p_S > p_S^*$, some consumers (of those who come back or do not search) may go to another shallow retailer selling i if their showrooming cost satisfies $b < \Delta(p_S^*, p_S)$, but given that only consumers with $b > \beta_L$ have arrived, for small price deviations, there will be no such consumers.

[32] This is different from total demand because each such consumer purchases $q(p_S)$ units; moreover, there are showroomers who do not start at shallow stores.

$$\sigma_L(p_S, p_S^*) = \frac{1}{2}\left[1 - G\left(r_L^* + \frac{\Delta(p_S^*, p_S)}{\mu_L}\right)\right](1 + G(r_L^*))$$
$$+ \int_0^{r_L^* + \frac{\Delta(p_S^*, p_S)}{\mu_L}} g(\varepsilon_i) G\left(\varepsilon_i - \frac{\Delta(p_S^*, p_S)}{\mu_L}\right) d\varepsilon_i.$$

Note that we normalised in such a way that $\sigma_L(p_S^*, p_S) = \frac{1}{2}$ (that is, in equilibrium, a store attracts half of those who start searching at shallow stores); hence, in equilibrium, the resulting profit of the shallow store will be equal to the profit of all shallow stores of its kind.

The total profit of a shallow retailer in the neighbourhood of p_S^* is given by:[33]

$$\Pi_S(p_S) = \left[\frac{(1-\lambda)}{2} F_H(\Delta^*) + \frac{\lambda}{2} F_L(\beta_L) + \lambda(1 - F_L(\beta_L))\sigma_L(p_S, p_S^*)\right] \pi(p_S),$$

where, inside the square brackets, we account for high-type showroomers with the first term, low-type showroomers with the second, and the share of searching low types that the shallow retailer retains with the third. For each of these consumers who purchases, the store earns a profit of $\pi(p_S)$.

In order to proceed, define

$$z_L \equiv \frac{\frac{\partial \sigma_L(p_S, p_S^*)}{\partial p_S}|_{p_S = p_S^*}}{u'(p_S^*)} = \frac{1}{\mu_L}\left(\frac{1}{2} g(r_L^*)(1 - G(r_L^*)) + \int_0^{r_L^*} g^2(\varepsilon) d\varepsilon\right)$$

as the derivative of $\sigma_L(p_S, p_S^*)$ with respect to p_S evaluated at the equilibrium price divided by $u'(p_S^*)$.[34] In the standard search model of Anderson and Renault (1990), with unit demand, z_L is the (negative of the) derivative of demand, so that the equilibrium price is $\frac{1}{2z_L}$.

We will make the following assumption for the rest of the paper. It ensures that the first order condition for shallow stores, as in Lemma 6, gives the maximiser which is unique.[35]

Assumption 2. *Assume that $F_H(\cdot)$, $F_L(\cdot)$, $G(\cdot)$ and $Q(\cdot)$ are such that $(A + B\sigma_L(p_S, p_S^*))\pi(p_S)$ is quasi-concave in p_S for any $A, B, p_S^* \in (0, 1]$ and z_L is decreasing in μ_L.*

It is not trivial to find direct conditions on $F_H(\cdot)$, $F_L(\cdot)$, $G(\cdot)$ and $Q(\cdot)$, but a simple numerical verification suffices to show that it holds for all the examples we consider below.

We are now ready to write the first-order condition for a shallow retailer.

[33] In this expression, we ignore consumers with b close to 0 who, if $p_S > p_S^*$, would leave and buy at another shallow retailer of the same type because at $p_S = p_S^*$, the mass of such consumers is zero. Accounting for these consumers only reduces the shallow store's incentive to deviate upward.

[34] We will occasionally need the notations z_H and $\sigma_H(p_S, p_S^*)$, which are the corresponding expressions when choosy consumers search through shallow retailers.

[35] It is used in the proof of Proposition 2. In particular, in inspecting $\Pi_S(p_S)$ note that $\Delta^*$and β_L are constant in p_S.

Lemma 6. *The first-order condition for a shallow retailer's pricing problem is given by*

$$h(p_S^*) = \left[\frac{1 + \frac{1-\lambda}{\lambda} F_H(\Delta^*)}{1 - F_L(\beta_L)}\right] \frac{1}{2z_L}. \tag{2}$$

It is immediate that the equilibrium price charged by shallow stores is higher than the price that would have been charged in the absence of showroomers. This is because of the choosy, H-type showroomers ($\frac{1-\lambda}{\lambda} F_H(\Delta^*)$ term in the numerator) and the less-choosy L-type showroomers ($(1 - F_L(\beta_L))$ term in the denominator).

4.2.3 Equilibrium

For an equilibrium of this type to exist, p_S^* and p_D^* have to simultaneously solve (1) and (2), while, at the same time, $\beta_H \geq \Delta^* > \beta_L$ and $r_L^* > 0$ should hold since if these fail, then consumer behaviour assumed in deriving (1) and (2) will not be correct.

Lemma 7. *There exists a unique solution to the system (1) and (2).*

Let (p_S^*, Δ^*) denote the solution to (1) and (2), which Lemma 7 establishes as well-defined. Note that Δ^* is a function of μ_L. By Assumption 2, we have that Δ^* is decreasing in μ_L because Equation (1) does not depend on μ_L; and, by Assumption 2 and the monotonicity of β_L in μ_L, equation (2) shifts upward in (Δ, p_S) space.

Further, let $\tilde{\mu}$ be the μ_L that solves $\beta(\mu_L) = \Delta^*(\mu_L)$.[36] Note that $\beta(\mu_L)$ is increasing in μ_L and goes from weakly negative to infinity, whereas, by Assumption 2, $\Delta^*(\mu_L)$ is decreasing in μ_L and starts at $\bar{b}$; therefore, a unique solution must exist. The role of $\tilde{\mu}$ turns out to be the following. If $\tilde{\mu} < \mu_L$, then there can be no showrooming equilibrium because, for such μ_L,we have $\Delta^* < \beta(\mu_L)$, and so there would be no searchers in equilibrium who would visit deep stores instead. Furthermore, we need μ_L to be above $\bar{\mu}$, or else $r_L^* \leq 0$, and so no L type consumer would search through shallow stores. In general, $\tilde{\mu}$ and $\bar{\mu}$ are not readily ordered; thus, the existence of the showrooming equilibrium depends on the primitives of the model beyond μ_L.

We are almost ready to characterise the search and showrooming equilibrium. In order to do so, we need to revisit the deep store pricing problem. In the case of a sufficiently large (rather than local) downward price deviation, such that $\Delta(p_S^*, p_D) < \beta_L$, deep stores are able to retain low-type—if any–showroomers. These are not taken into account in the first-order condition. The following condition rules out the optimality of such a strategy, using notation $\hat{p}^*$ as the implicit solution to $\Delta(p_S^*, \hat{p}^*) = \beta_L$.

Condition 1.

$$\max_{p_D \in [0, \hat{p}^*]} \left[(1-\lambda)(1 - F_H(\Delta(p_S^*, p_D))) + \lambda(F_L(\beta_L) - F_L(\Delta(p_S^*, p_D)))\right] \pi(p_D)$$
$$\leq (1-\lambda)(1 - F_H(\Delta^*))\pi(p_D^*).$$

[36] In more detail: $\mu_L = \tilde{\mu}$ alongside $\tilde{p}$ solve a system consisting of $h(\tilde{p}) = \left[\frac{1+\frac{1-\lambda}{\lambda}F_H(\beta_L)}{1-F_L(\beta_L)}\right]\frac{1}{2z_L}$ and $h(u^{-1}(u(\tilde{p}) - \beta_L)) = \frac{1-F_H(\beta_L)}{f_H(\beta_L)}$. See also the proof to Proposition 2.

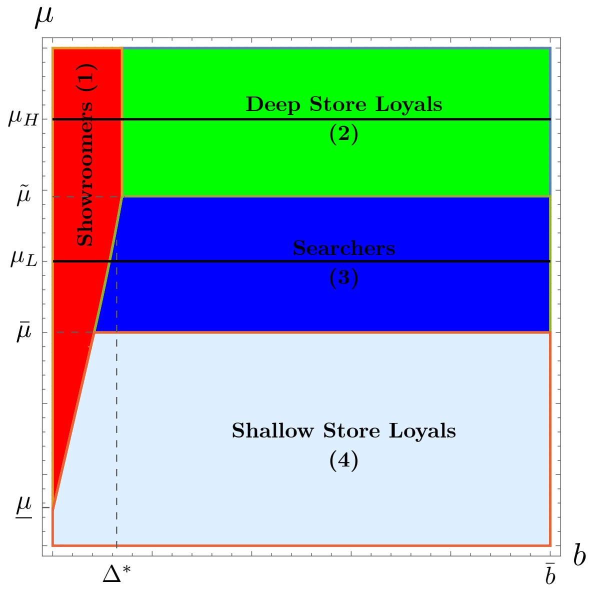

Figure 2: Consumer types for $G(\varepsilon) = \varepsilon$, $\mu_L = 0.55$, $\mu_H = 0.65$, $F_L(b) = (b/s)^6$, $F_H(b) = (b/s)^2$, $\Delta^* = 0.035$, $s = \bar{b} = 0.25$, $\gamma = 0.25$ and $\lambda = 0.5$.

This condition is trivially satisfied when $F_L(\cdot)$ has full mass above β_L or if the lower bound for the low-type visit costs is sufficiently high that there are no such low-type showroomers. However, the condition can also fail. For example, it necessarily fails when $F_L(\beta_L) = 1$ and $\lambda \to 1$, so that almost all consumers are low-type showroomers. In that case, deep stores receive negligible demand in equilibrium, and by deviating to a lower price so that $\Delta(p_S^*, p_D) < \beta_L$, they increase their demand infinitely.

We obtain the following result.

Proposition 2. *If $\tilde{\mu} > \mu_L > \bar{\mu}$, $\mu_H > \beta^{-1}(\Delta^*(\mu_L))$ and Condition 1 hold, then there is an equilibrium in which p_S^* and p_D^* jointly solve (1) and (2). All type L consumers with $b < \beta_L$ make first visits to deep stores and then showroom; the remaining L consumers visit shallow stores and conduct optimal sequential search, as described in Lemma 3. All type H consumers make first visits to deep stores, of these consumers, those with $b < \Delta(p_S^*, p_D^*)$ showroom and the rest buy from deep stores. Deep stores earn $\Pi_D^* = (1 - \lambda)(1 - F_H(\Delta^*))\pi(p_D^*)$, and shallow stores earn $\Pi_S^* = \frac{1}{2}(\lambda + (1 - \lambda)F_H(\Delta^*))\,\pi(p_S^*)$.*

As noted above, in general, $\tilde{\mu}$ and $\bar{\mu}$ cannot be ranked, and so there is no guarantee that an equilibrium of this form exists. However, it is a simple matter to verify that there are parameters that allow such an outcome. Figure 2 illustrates an equilibrium of this sort. The H types visit deep stores first, and some then showroom. Some of the L types also visit deep stores first, but all of them showroom. The rest of the L types visit shallow stores for the first visit— e.g., ones that sell good i—and search a store that sells j if $\varepsilon_i < r_L^*$.

It is worth noting that both Condition 1 and $\tilde{\mu} > \mu_L$ are more likely to hold when F_L shifts to the right (in the FOSD sense). That is, a search and showrooming equilibrium is more likely to hold when L types are less prone to showrooming. There are two reasons for this. First, Condition 1 is more likely to hold when there are fewer low-type showroomers who may tempt deep stores to deviate downwards to retain them. Second, the fewer low-type showroomers there are, the lower are shallow prices, which, in turn, means that higher levels of μ_L are still consistent with low types preferring to search shallow stores over buying at deep stores.

The equilibrium in Proposition 2 has several interesting properties that can be immediately derived.

Corollary 1. *Assume that the conditions in Proposition 2 hold. Then, within an equilibrium of that form:*

(i) Deep stores charge higher prices than shallow stores but lower than monopoly prices, $p^m > p_D^* > p_S^*$.

(ii) An increase in μ_H *has no effect on prices; an increase in* μ_L *increases prices.*

(iii) An increase in s *increases all prices.*

(iv) An increase in λ *reduces all prices.*

(v) An increase in γ *leads to a reduction in all prices.*

Note that these properties are mostly intuitive. It is, perhaps, worth highlighting, as in (ii), that increasing the choosiness of the more-choosy has no effect on prices; and that in (v), making the search process more efficient by increasing the search efficiency of deep stores (reducing γ) actually increases prices. This follows, as reducing γ makes it more attractive for the less-picky to visit deep stores and showroom. In turn, this increases the fraction of shallow stores visited by consumers who are showroomers rather than searchers, and so leads to higher prices.

Consequently, improving the efficiency of search at deep retailers (decreasing γ) has a potentially ambiguous impact on welfare: there is a direct saving in inspection costs for those who visit such stores, but Corollary 1 highlights that prices increase for all consumers. However, around the parameter ranges we use for illustration and in other parameterisations that we have explored, the direct effect dominates.

4.3 Equilibrium configurations: An illustration

Figure 4.3 illustrates the ranges for μ_H and μ_L for which various equilibrium configurations arise when $G \sim U(0,1)$, $q(p) = 1-p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$, and $s = \bar{b} = \gamma = 0.25$. In the red triangle, neither type is picky enough ($\mu_H < \underline{\mu}$), and so all consumers visit shallow stores that charge monopoly prices (Proposition 1). In the green region, high types are picky and low types are not ($\mu_H \geq \underline{\mu} > \mu_L$), so high types visit deep stores, and low types visit shallow stores, but since no one searches ($\bar{\mu} > \mu_L$), all prices are at the monopoly level. In the blue region, low types are picky but not too picky ($\mu_L > \underline{\mu}$) there exists an equilibrium in which all visits are to deep stores and prices are at monopoly levels. The orange region is where the previous type of equilibrium co-exists with the hybrid equilibrium of Proposition 2. Here, high types are picky, $\mu_H \geq \beta^{-1}(\Delta^*(\mu_L))$, and low types are picky enough, $\mu_L > \bar{\mu}$, but not too picky. So, $\tilde{\mu} > \mu_L$, and equilibrium prices are below monopoly levels, and there is active search and showrooming. Note that $\beta^{-1}(\Delta^*(\mu_L))$ (the bottom of orange area) is decreasing in μ_L because

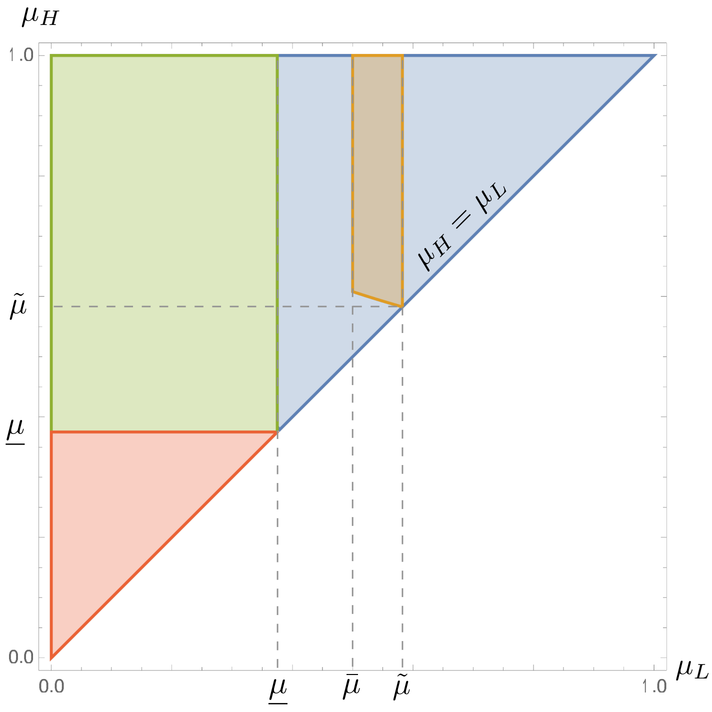

Figure 3: Equilibrium regimes depending on μ_L and μ_H, where $G \sim U(0,1)$, $Q(p) = 1 - p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$, $\lambda = 0.5$, and $s = \bar{b} = \gamma = 0.25$. In the red region, all visits occur at shallow stores. In the green region, high types visit deep and low types visit shallow stores. In the blue region, there is an equilibrium in which all visits are at deep stores. In all of these, all store types charge monopoly prices. In the orange region, there is an additional equilibrium of the type described in Proposition 2.

$\beta(\cdot)$ is monotone, and $\Delta^*(\mu_L)$ is decreasing. Further, for $\mu_L = \tilde{\mu}$, by the definition of $\tilde{\mu}$, we have $\Delta^*(\mu_L) = \beta_L$, so the condition $\mu_H \geq \beta^{-1}(\Delta^*(\mu_L))$ becomes $\mu_H \geq \tilde{\mu}$.

Inspecting Propositions 1 and 2, it is clear that the configuration above—that is, the order in which different equilibria arise as μ_H and μ_L vary, and the thresholds for these change—is quite general, with the important proviso that if $\bar{\mu} > \tilde{\mu}$, there is no equilibrium of the form characterised in Proposition 2 (the orange region in the graph).

4.4 The effect of showrooming

In this section, we explore the role of showrooming and its effect on prices. We do so by supposing that some fraction of consumers will never showroom and the remaining "regular" consumers are as in the model outlined above. We do this primarily, to illustrate the effect of showrooming in the model (and what precluding the possibility of showrooming for some consumers will do to prices); this could also have an interpretation around increasing the costs of showrooming—for example a public campaign that "shamed" some consumers from engaging in this behaviour.

In order to highlight effects, it is useful to allow for this fraction to be different between picky and less picky consumers. Specifically, say that a fraction ν_H of the picky are never showroomers, and a fraction ν_L of the less picky are never showroomers. Never showroomers are consumers for whom b, the cost of showrooming, is extremely high.

It is immediate, that for the regular consumers, their behaviour is as characterised in Lemma 1; instead for the never-showroomers while it is still the case that the expected value from visiting a regular store is given by $U_S = \mu \int_0^1 \min(\varepsilon, r^*)\tilde{g}(\varepsilon)\, d\varepsilon + u(p_S)$; the value of visiting a broad store is $U_D = \mu \int_0^1 \varepsilon\tilde{g}(\varepsilon)\, d\varepsilon + u(p_S) - \Delta^* - (1+\gamma)s$. Consequently, a never-showroomer shops from a broad store if and only $\beta(\mu) \geq \Delta^*$.

We can consider a departure from an equilibrium where showrooming exists, as characterised in Proposition 2 by supposing that a small fraction of never-showroomers are introduced.[37] Trivially, for a small enough ν_H and ν_L, a similar equilibrium would exist and Proposition 2 captures the behaviour of regular consumers. Further, given that a marginal change has a small impact on Δ^* then all picky never-showroomers would visit deep stores and buy there, and all less-picky never-showroomers would be searchers among shallow stores.

The pricing decisions, as captured in Lemma 5 and Lemma 6 (which have a unique solution as in Lemma 7) would be different from above. Specifically, we can now write the total profit of a shallow retailer (for small enough ν_H and ν_H) as

$$\Pi_S(p_S) = \Bigg[(1-\nu_H)\frac{(1-\lambda)}{2}F_H(\Delta^*) + (1-\nu_L)\frac{\lambda}{2}F_L(\beta_L) + (1-\nu_L)\lambda(1-F_L(\beta_L))\sigma_L(p_S,p_S^*) \\ + \nu_L\lambda\sigma_L(p_S,p_S^*)\Bigg]\pi(p_S). \tag{3}$$

The profits are as above for the regular consumers, adapting for the relevant consumer groups, but given that $\Delta^* > \beta_L$ (which must remain true for small enough ν_H and ν_H) all of the less-picky never-showroomers will search among shallow stores. Consider the case that $\nu_H = 0$ and $\nu_L > 0$, then it is clear that by increasing the fraction of (less picky) never-showroomers, there are relatively more shoppers among the consumers of shallow stores, and consequently the prices at these shallow stores would be lower. This is a force that suggests that having more showroomers can lead to higher rather than lower prices.

However, eliminating showroomers also has an effect on the prices of deep stores who are less threatened by the prospect that some of their consumers will go on to showroom. As in the proof of Lemma 5, the profit of deep retailers can be found by maximising

$$\Pi_D(p_D) = (1-\nu_H)(1-\lambda)F_H(\Delta(p_S^*, p_D))\pi(p_D) + \nu_H\lambda\pi(p_D). \tag{4}$$

It is immediate that holding constant p_S, a deep firm's price is rising in the fraction of picky never-showroomers ν_H and is entirely unaffected by (local) changes to ν_L.

Together these observations establish the following.

Proposition 3. *Reducing showrooming can lead to lower or higher prices. In particular, marginal departures from an equilibrium characterised in Proposition 2 will have lower prices when there are fewer potential showroomers among the less picky, that is, when ν_L is low and $\nu_H = 0$.*

[37] That is we perform a comparative statics exercise where we modify the probability that the showrooming cost, b, is prohibitively large

The intuition for the result is that in a showrooming equilibrium characterised in Proposition 2, the less picky never purchase from deep stores. If they showroom, this creates more-inelastic demand at shallow stores, and so higher prices at both shallow stores and deep stores (since a change in ν_L has no effect on Δ^*—the difference between buying at shallow and at deep stores). Instead, preventing them from showrooming leads to them shopping at shallow stores and, consequently, their more elastic demand leads to lower prices everywhere.

Of course, eliminating showroomers can also lead to higher prices. For example, consider the case that $\nu_L = 0$, then inspecting (4) it follows immediately that decreasing ν_H means that fewer picky consumers are captive at deep stores and may showroom. As a result deep stores charge lower prices in order to capture more of such consumers. The relative price at deep stores (Δ^*) would go down, through this effect. This is perhaps the concern that popular commentators have in mind regarding showrooming; deep stores threatened by the prospect of showrooming have to cut prices.[38] Note, however that such a change (fixing $\nu_L = 0$, and reducing ν_H) also has an effect on pricing of shallow stores. It decreases the mass of showroomers leading to lower prices at these shallow stores. In principle, either effect could dominate.

Of course, more broadly, varying both ν_H and ν_L simultaneously, e.g. by setting them equal, would allow all three effects (the effect through ν_L and the two effects of ν_H) to operate leading to ambiguous outcomes.

4.5 Retailer variety

In this section, we consider the implications of the changes in search behaviour or market structure.

4.5.1 No shallow stores

First, if shallow stores vanish, so that all stores are deep, then for any level of μ_L and μ_H, there is a unique equilibrium with monopoly prices in the style of Diamond (1971). This is because no consumer has an incentive to search to learn about varieties because all stores have identical offerings; moreover, no consumer is initially attracted by a lower price (since they are unaware of it) and has no reason to search elsewhere for lower prices.

Returning to our general characterisation, prices are the same in all regions other than the case analysed in Proposition 2. However, even if we ignore this possibility, the presence of shallow stores alongside deep ones alters welfare. In the region where case (1) of Proposition 1 applies (the red region in Figure 2), all consumers are so insensitive to matches (μ is so low) that they prefer to visit shallow stores. Consequently, if there are no such stores, consumers are worse off (since we require consumers to inspect all varieties at a deep store). In the (green) region corresponding to case (2), the less-choosy types visit shallow stores, so, again, their absence is detrimental to consumer surplus and, thus, welfare. In the (blue) region corresponding to

[38]Throughout the model we abstract from the costs associated with providing service to consumers who ultimately do not buy, though, of course, this may be another legitimate concern.

case (3), where all consumers are choosy and visit deep stores, welfare is the same regardless of shallow stores' presence.

In the case of Proposition 2, in which both types of retailers are active and there is search and showrooming (the orange region that overlaps with the blue), the disappearance of shallow stores causes welfare losses due to both higher prices and higher inspection costs for those consumers wishing to search shallow stores.

Proposition 4. *Removing shallow stores (weakly) reduces consumer surplus and welfare and (weakly) increases prices and total profits.*

4.5.2 No deep stores

Next, we consider what happens if deep stores vanish.[39] Nothing changes if consumers are so insensitive that they shop at a single shallow store (Proposition 1, case (1), corresponding to the red in Figure 2). In the region where Proposition 1, case (2), applies (the green region in Figure 2), with or without deep stores, there is no search and, therefore, monopoly prices, but high-type consumers are worse off because they would rather visit deep stores and obtain a better match. In the region corresponding to Proposition 1, case (3), where all consumers prefer to search deep stores, an exodus of deep stores leads to lower prices (equal to $h^{-1}\left(\frac{1}{2(\lambda z_L+(1-\lambda)z_H)}\right)$ when $\mu_L > \bar{\mu}$ or $h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$ when $\mu_L < \bar{\mu}$), but consumers prefer to search at deep stores, so their utility may go down since it is more costly to inspect both goods. When μ_H is sufficiently high, no price reduction can compensate H types for their increase in inspection costs, and so consumer surplus and welfare must fall as deep stores exit. Instead, the price reduction resulting from deep stores' disappearance must dominate any inspection cost efficiency losses when μ_H and μ_L are close to $\underline{\mu}$, because, in this case, the inspection cost efficiencies for both types are small. Thus, both situations exist: the disappearance of deep stores can be good or bad for consumers and for welfare: which case prevails depends on parameters and, as indicated above, crucially on how picky consumers are.

Finally, in the region corresponding to Proposition 2 (the orange region), if deep stores disappear, then there is a force for lower prices at shallow stores due to the disappearance of showrooming and the inelastic consumers that showrooming brings, but a force for higher prices due to shallow stores facing more high-type consumers with lower demand elasticity entering the pool of searchers. In particular, prices at shallow retailers would shift to $h^{-1}\left(\frac{1}{2(\lambda z_L+(1-\lambda)z_H)}\right)$, which can be higher or lower than $p_S^* = h^{-1}\left(\left[\frac{1+\frac{1-\lambda}{\lambda}F_H(\Delta^*)}{1-F_L(\beta_L)}\right]\frac{1}{2z_L}\right)$ depending on a variety of parameters of the model. In particular, when μ_H and μ_L are sufficiently close to each other and to $\tilde{\mu}$ (note that μ_H and μ_L can be close to each other only in the vicinity of $\tilde{\mu}$), the price must fall with the disappearance of deep stores, because, after the exit, the price is close to $h^{-1}\left(\frac{1}{2z_L}\right) < p_S^*$. Instead, when high-type consumers are sufficiently picky (when μ_H is sufficiently high, z_H is low), the disappearance of deep stores, which turns them into searchers

[39]In this case, the environment is similar to that in Anderson and Renault (1999), except for the downward-sloping demand assumption.

through shallow stores (rather than having them buy at deep stores, say), means that the shallow stores now face a more inelastic demand.

In addition to the ambiguous impact on prices that consumers pay, consumer surplus may rise or fall. Even if consumers face lower prices, they may be worse off due to higher inspection costs associated with the disappearance of deep retailers.

Proposition 5. *Removing deep stores can raise or lower prices, consumer surplus, welfare, and total profits; even if prices fall, consumer surplus may fall.*

5 Price-only sector

We extend the model by allowing for an alternative competitive retail channel. Specifically, in this retail sector, there is no opportunity to learn match quality; consumers must first learn this at either deep or shallow stores to purchase in this sector. Moreover, consumers can observe prices within this sector at no cost. We assume that the sector is competitive. Since consumers can observe prices, it is immediate that competition in prices between different stores in this sector leads them all to price at cost, which we have normalised to 0.[40]

Of course, if all consumers had access to this sector, then all would learn their match elsewhere but buy in this sector. Instead, we assume that not all consumers can access the price-only venues, or, equivalently, they may not be aware of this possibility. In particular, assume that a fraction θ_T of type $T \in \{L, H\}$ consumers do not consult prices in the price-only sector, whereas the rest have access and are able to purchase there. We will call consumers who have access 'savvy' and other consumers 'naive.' Savvy consumers can purchase in the price-only sector, and naive consumers cannot. Since savviness and pickiness are not assumed to be orthogonal, θ_L may be above, below or, indeed, equal to θ_H.

Trivially, since prices in the price-only sector are 0, all savvy consumers will buy there (and, trivially, picky consumers will showroom at deep stores, whereas sufficiently unpicky consumers might prefer to learn their match for only one good and showroom from a shallow store). Naive consumers cannot buy from the price-only sector, and their behaviour is characterised as in Section 3, given their expectations of prices at deep and shallow stores.

Deep and shallow stores cannot earn profits by matching or undercutting the price-only sector, so their behaviour will be similar to that characterised in Section 4, with the following proviso. In the overall population, there is a fraction λ of less-picky (μ_L) consumers out of the mass 1 of consumers; since only naive consumers are relevant in the presence of the price-only sector, out of this population that has mass $\lambda\theta_L + (1-\lambda)\theta_H$, a fraction $\hat{\lambda} \equiv \frac{\lambda\theta_L}{\lambda\theta_L+(1-\lambda)\theta_H}$ are less-picky. This can be higher or lower than the fraction in the overall population, depending on whether or not $\theta_H < \theta_L$. In the former case, the price-only sector attracts relatively more picky consumers, leaving relatively more of the less-picky to the deep and niche stores; and, in the

[40]In the working paper version of the paper, we consider the role of market power in the competitive sector by considering the case of a monopoly price-only retailer in which effects similar to those described in this section arise.

latter case, when $\theta_H > \theta_L$, it will be relatively picky consumers who buy from deep and niche stores. In short, when $\theta_H \neq \theta_L$, the price-only sector disproportionately attracts either picky or less-picky consumers who might otherwise be showrooming at shallow stores or searching and buying from shallow stores. In this way, the sector affects the average elasticity of consumers at shallow stores and, therefore, prices.

Specifically, we can apply the characterisation of prices in Section 4 directly. Introducing a price-only sector where none previously existed, as well as reducing the overall number of consumers who buy from deep and niche stores, has an effect similar to a change in λ and replacing λ by $\hat{\lambda}$. Applying our earlier results and, more specifically, Corollary 1, allows us to establish the following result immediately.

Proposition 6. *The introduction of a price-only sector, in which both before and after its introduction, the equilibrium is as described in Proposition 2, leads to higher prices at both deep and niche stores if $\theta_L < \theta_H$ and, instead, leads to lower prices at both deep and niche stores if $\theta_H < \theta_L$.*

5.1 Welfare

While Proposition 6 characterises prices, the welfare associated with the introduction of a price-only sector must also incorporate the benefit that savvy consumers enjoy from the opportunity to purchase at a price of 0 from the price-only sector. Clearly, the introduction of this sector makes savvy consumers better off. If prices at both deep and niche stores fall, then it is immediate that naive consumers are also better off (and by more than the fall in the profits of shallow and deep stores as their prices come closer to costs); however, if prices at deep and shallow stores rise, then the overall impact is ambiguous, as the gains to savvy consumers must be traded off against the decline in surplus associated with naive purchases.

Of course, this analysis assumes that the introduction of the price-only sector has no impact on the existence of deep and shallow stores. The introduction of a price-only sector necessarily implies that a fraction of consumers (the savvy) will no longer purchase from deep and shallow stores; moreover if the price-only sector disproportionally attracts less-picky consumers (that is, $\theta_H > \theta_L$), prices will be lower at deep and shallow stores. This may further endanger their viability, and lead to consequences similar to those described in Section 4.5.

6 Summary and Conclusions

Overall, this paper illustrates some familiar themes. In particular, the elasticity of demand at given stores depends on the pattern of consumer search: an equilibrium phenomenon. A literature on multiproduct search highlights that this pattern depends on the product mix across all stores. We extend this insight to observe that, perhaps unsurprisingly, it also applies to the case of stores that offer substitute goods and have overlapping offerings. As a consequence, seemingly beneficial changes (such as improving the search efficiency at a deep retailer, or introducing a relatively low-cost alternative venue) can lead to higher prices by affecting consumers' search patterns. Showrooming, by affecting search patterns, impacts prices both at deep stores (where consumers visit to learn about goods) but also at the shallow stores where showroomers purchase.

More specifically, we highlight that key to price determination in our environment are consumers who might pass through more than one (shallow) store to learn their match with the products on offer. These consumers are necessarily somewhat picky (or else there would be no need to visit more than one store) but not too picky (or else visiting a deep retailer would be more attractive). Marginal changes to the viability of showrooming can therefore raise or lower retail prices throughout a sector depending on how such changes affect the mix of such consumers among the customers of shallow stores.

The endogenous determination of search patterns suggests that welfare effects can be subtle, and the impact of the introduction of a price-only sector depends on the way in which savviness and pickiness are correlated.

From an antitrust perspective, we further highlight that even if the disappearance of stores has no impact on prices, it may impact consumer welfare. It is worth highlighting our assumption that consumers always participate; relaxing this and taking it together this observation about consumer welfare presents a rationale for manufacturers to seek to maintain retailer variety as a means of encouraging consumer participation (even with no impact on prices). Of course, such retailer variety can lead to lower prices and thereby encourage further consumer participation.

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Appendix A: Proofs

A1: Proof of Lemma 1.

Proof. It is sufficient to show that $\frac{\partial\beta(\mu,b)}{\partial\mu} > 0$.

$$\begin{aligned}
\frac{\partial\beta(\mu,b)}{\partial\mu} &= \int_{r^*}^{1}(\varepsilon - r^*)\tilde{g}(\varepsilon)\,d\varepsilon - \mu(1-G(r^*)^2)\frac{\partial r^*}{\partial\mu} \\
&= \int_{r^*}^{1}(\varepsilon - r^*)\tilde{g}(\varepsilon)\,d\varepsilon - \mu(1-G(r^*)^2)\frac{\int_{r^*}^{1}(\varepsilon - r^*)g(\varepsilon)\,d\varepsilon}{\mu(1-G(r^*))} \\
&= \int_{r^*}^{1}(\varepsilon - r^*)\tilde{g}(\varepsilon)\,d\varepsilon - (1+G(r^*))\int_{r^*}^{1}(\varepsilon - r^*)g(\varepsilon)\,d\varepsilon \\
&= \int_{r^*}^{1}\varepsilon(2G(\varepsilon) - (1+G(r^*)))g(\varepsilon)\,d\varepsilon \\
&= (1-G(r^*)^2)\left[\int_{r^*}^{1}\varepsilon\frac{\tilde{g}(\varepsilon)}{1-G(r^*)^2}\,d\varepsilon - \int_{r^*}^{1}\varepsilon\frac{g(\varepsilon)}{1-G(r^*)}\,d\varepsilon\right] > 0,
\end{aligned}$$

where we used the definition of $\tilde{g}(\varepsilon)$ and $\frac{\partial r^*}{\partial\mu} = \frac{\int_{r^*}^{1}(\varepsilon - r^*)g(\varepsilon)\,d\varepsilon}{\mu(1-G(r^*))}$, which follows from the definition of r^*; that $\int_{r^*}^{1}(2G(\varepsilon) - (1+G(r^*)))g(\varepsilon)\,d\varepsilon = 0$; and that $\int_{r^*}^{1}2G(\varepsilon)g(\varepsilon)\,d\varepsilon = 1 - G(r^*)^2$. The last line above can be seen to be positive by noting that the first term in square brackets is the conditional mean of $\max(\varepsilon_1, \varepsilon_2)$ above r^*, and the second term is the conditional mean of ε_1 above r^*. □

A2: Proof of Lemma 2.

Proof. This is immediate on noting that, for $\gamma = 0$, we have $U_D > U_S$, and for $\gamma = 1$, we have $U_D < U_S$, and that U_D is monotonically decreasing in γ. □

A3: Proof of Lemma 3.

Proof. Immediate from the discussion in the text. □

A4: Proof of Lemma 4.

Proof. Assume the contrary, so that $p_D^* < p_S^*$. Any consumer who visits deep stores will not leave for shallow stores. Given that $f_T(0) = 0$ for $T = H, L$, a zero mass of consumers will leave a deep store that deviates to a higher price; thus, we must have $p_D^* = p^m$. This leads to a contradiction because $p_S^* > p^m$ cannot hold in any equilibrium because shallow stores would profitably deviate to a lower price. □

A5: Proof of Proposition 1.

Proof. If all stores charge (p^m, p^m), then no consumer who visits deep stores has any incentive to search, and no other firm can attract these consumers by lowering its prices. Firms never wish to increase prices above p^m. By Assumption 1, $\underline{\mu} < \bar{\mu}$, so that if a consumer type prefers to patronise shallow stores given equal prices, this type is not willing to search. This means that if consumers visit shallow stores expecting $p_S^* = p^m$, then shallow stores have no incentive to deviate. In any such equilibrium, since prices are equal, type $T \in \{H, L\}$ visits deep stores and buys there if $\mu_T \geq \underline{\mu}$. This proves the existence of a symmetric equilibrium with monopoly prices, as well as the taxonomy of consumer behaviour stated.

For uniqueness, assume that $\mu_L < \bar{\mu}$ indeed holds. This condition is equivalent to $r_L^* < 0$. For contradiction, assume that there exists a symmetric equilibrium in which some firms charge sub-monopoly prices, since supra-monopoly prices can never be profitable. First, it must be that shallow stores charge sub-monopoly prices in such an equilibrium, because deep stores cannot be the only ones that do, due to Diamond-like reasoning.[41] For shallow stores to charge below monopoly prices, it must be that high types visit them, since, if only low types do, then $r_L^* < 0$ implies that no consumers would search, and prices at shallow stores will be at the monopoly level. Thus, it must that $\Delta^* \geq \beta_H$, so that at least some high types visit shallow stores. There are, then, two cases depending on μ_H. If $\mu_H > \underline{\mu}$ ($\beta_H > 0$), then some high types will showroom, and, in turn, $\Delta^* \geq \beta_H$ cannot be an equilibrium because deep stores will reduce p_D^* to retain some high types, a contradiction. If $\mu_H \leq \underline{\mu} < \bar{\mu}$, then no high type is willing to search, and so, in equilibrium, there can only be monopoly prices, a contradiction. □

A6: Proof of Lemma 5.

Proof. All types with μ_H visit deep stores on their first visits (since $\Delta^* \leq \beta_H$). In addition, less-choosy, μ_L types with low enough visit costs—specifically, with $b < \beta_L$—also inspect at deep retailers, but they do so with no intention to buy there but only to showroom. Deep stores have no incentive to charge different prices for the two goods so will charge the same price p_D^* given the same prices for the two goods at shallow stores p_S^*.

Define $\hat{p}^*$ as the solution to $\Delta(p_S^*, p_D) = \beta_L$. This solution exists because, by assumption, $\Delta^* = \Delta(p_S^*, p_D^*) > \beta_L$ and $\Delta(p_S^*, p_S^*) = 0$. Moreover, the previous sentence implies that and lies in $\hat{p}^* \in (p_S^*, p_D^*)$.

Then, deep store profit can be written as:

$$\Pi_B = \begin{cases} (1-\lambda)(1 - F_H(\Delta(p_S^*, p_D)))\pi(p_D) & \text{if } p_D \geq \hat{p}^* \\ \left[(1-\lambda)(1 - F_H(\Delta(p_S^*, p_D))) + \lambda(F_L(\beta_L) - F_L(\Delta(p_S^*, p_D)))\right]\pi(p_D) & \text{if } p_D < \hat{p}^* \end{cases}. \quad (5)$$

In this expression, the $(1-\lambda)$ choosy H consumers always visit a deep retailer (since they anticipate $\Delta^* \leq \beta_H$) and react to the actual price p_D, with some purchasing (if they have $b \leq \Delta(p_S^*, p_D)$) and the rest showrooming—that is, leaving to buy at a shallow retailer, anticipating the lower price p_S^*. Of the fraction λ comprised of less-choosy consumers, those with low visit costs ($b < \beta_L$) will find it worthwhile to visit a deep retailer and respond to the price posted: either to buy directly if their visit costs are moderately high (that is, if $b > \Delta(p_S^*, p_D)$,which does not happen in equilibrium for $p_D = p_D^*$ because, by assumption,

[41] Once at deep stores, consumers will never leave for small price deviations (by assumption $f_T(0) = 0$, the number of those who do leave is negligible).

$b < \beta_L < \Delta^*$), or else to showroom (if $b < \Delta(p_S^*, p_D)$). Therefore, for an equilibrium in which some of the less-choosy consumers visit deep stores and showroom, it must be that a deep store's profit is maximised where $\Delta(p_S^*, p_D) > \beta_L$, so the second part of demand is only to be checked against possible deviations there.

The first-order condition for deep stores is derived directly from the first line in (5). □

A7: Proof of Lemma 6.

Proof. Immediate from the discussion in the text and the results in the literature. □

A8: Proof of Lemma 7.

Proof. Log-concavity of $Q(p)$ ensures that $h(p)$ is increasing in p for $p < p^m$ and, moreover, $\lim_{p \to p^m} h(p) = \infty$. In addition, $\frac{1-F_H(\Delta^*)}{f_H(\Delta^*)}$ is decreasing in Δ^* because of the log-concavity of $1 - F_H(\cdot)$. Furthermore, $\lim_{\Delta^* \to 0} \frac{1-F_H(\Delta^*)}{f_H(\Delta^*)} = \infty$. We rewrite (1) using $p_D^* = u^{-1}(u(p_S^*) - \Delta^*)$ as

$$h(u^{-1}(u(p_S^*) - \Delta^*)) = \frac{1 - F_H(\Delta^*)}{f_H(\Delta^*)}.$$

Note that (1) implies an inverse relationship between p_S^* and Δ^* because of the discussion above. Define $\bar{\Delta}$ as the solution to $h(u^{-1}(u(0) - \bar{\Delta})) = \frac{1-F_H(\bar{\Delta})}{f_H(\bar{\Delta})}$. The left hand side is increasing in $\bar{\Delta}$ and goes from 0 at $\bar{\Delta} = 0$ to infinity at $\bar{\Delta} = u(0) - u(p^m)$. The right hand side is decreasing in $\bar{\Delta}$ and goes from p^m at $\bar{\Delta} = 0$ to 0 at $\bar{\Delta} = \bar{b}$. Thus the solution exists and is unique, and, furthermore, it satisfies $\bar{\Delta} < \bar{b}$. As Δ^* goes from 0 to $\bar{\Delta}$, p_S^* goes from p^m to 0. In contrast to (1), (2) establishes an increasing relationship between p_S^* and Δ^* (The right-hand side is clearly increasing in Δ^*, while the left-hand side is increasing in p_S^*). As Δ^* goes from 0 to $\bar{b}$, the associated p_S^* satisfies $0 < p_S^* \leq p^m$. Therefore, the system of these two equations in two unknowns has a unique solution. □

A9: Proof of Proposition 2.

Proof. First, we show that under Condition 1, the solution to (1) is the maximiser for deep store profits. The solution to (1) maximises $\Pi_B(p_D)$ for $p_D \in \{p : \ \Delta(p_S^*, p) \geq \beta_L\}$ because $\Pi_B(p_D)$ log-concave in that range. Log-concavity obtains because the first derivative is decreasing in p_D, as shown in the proof of Lemma 7. If, in addition, Condition 1 holds, then p_D^* is the optimal price for deep stores. Assumption 2 ensures that p_S^* defined uniquely by (2) is the optimal price for a shallow store, given assumed consumer behaviour and deep store and other shallow store pricing.

We have also shown that the solution to (1) and (2) is unique. Conditions on μ_L and μ_H were derived from assumed consumer behaviour, whereby all H types and some L types visit deep stores, and some L types (with $b > \beta_L$ and $\varepsilon_i < r_L^*$) search amongst shallow stores. For low types, $\Delta^* > \beta_L$ and $r_L^* > 0$ are equivalnet to $\tilde{\mu} > \mu_L > \bar{\mu}$, while $\beta_H > \Delta^*$ is the same as $\mu_H > \beta^{-1}(\Delta^*(\mu_L))$.

It remains to be shown that $\tilde{\mu}$ is well defined and unique. This is true by arguments similar to those in the proof to Lemma 7. Namely, consider $h(\tilde{p}) = \left[\frac{1+\frac{1-\lambda}{\lambda}F_H(\beta_L)}{1-F_L(\beta_L)}\right]\frac{1}{2z_L}$ and $h(u^{-1}(u(\tilde{p}) - \beta_L)) = \frac{1-F_H(\beta_L)}{f_H(\beta_L)}$. The latter implies an inverse relationship between $\tilde{p}$ and μ_L where $\tilde{p}$ goes from p^m to 0 as μ_L increases from $\underline{\mu}$ to $\bar{\bar{\mu}}$, where the latter is uniquely defined as the solution to $h(u^{-1}(u(0) - \beta(\bar{\bar{\mu}}))) = \frac{1-F_H(\beta(\bar{\bar{\mu}}))}{f_H(\beta(\bar{\bar{\mu}}))}$. The former implies increasing relationship between $\tilde{p}$ and μ_L where $\tilde{p}$ goes from strictly below p^m to p^m as μ_L goes from $\underline{\mu}$ to $\beta^{-1}(\bar{b})$. Therefore a unique solution exists. □

A10: Proof of Corollary 1.

Proof. Part (i) is immediate from the construction of the equilibrium. The remaining comparative statics follow from the following observations. Δ^* and p_S^* solve (1) and (2). In (Δ, p_S)-space, (1) is downward-sloping and (2) is upward-sloping. (1) does not depend on any of the parameters listed in the corollary. It is immediate to see that (2) shifts upwards with s and μ_L, and downwards with λ and γ. This implies that p_S^* goes up and Δ^* goes down. From the deep store's second-order condition, we also have that p_D^* goes up when p_S^* goes up. □

A11: Proof of Proposition 3.

Proof. Immediate from the discussion in the text. □

A12: Proof of Proposition 4.

Proof. Immediate from the discussion in the text. □

A13: Proof of Proposition 5.

Proof. Immediate from the discussion in the text. □

A14: Proof of Proposition 6.

Proof. Following arguments in the text. □

Appendix B: Relaxing Assumption 1

The characterisation of the search and showrooming equilibrium of Proposition 2 is not affected by Assumption 1, and, therefore, it applies when the assumption does not hold.

The results of Proposition 1, however, are affected. The reason is as follows. If Assumption 1 fails, then $\bar{\mu} \leq \underline{\mu}$. This means that consumers who prefer to patronise shallow stores at equal prices may now search through shallow stores, which then precludes monopoly prices in equilibrium.

We next provide a proposition that, like Proposition 1, characterises all non-showrooming equilibria, although, as the result shows, these are not necessarily characterised by monopoly prices.

Proposition 7. *The following non-showrooming equilibria obtain when Assumption 1 is violated. In all these cases, deep stores charge* (p^m, p^m)*. Further:*

1. *If* $\mu_H \leq \bar{\mu}$*, then all visits occur at shallow stores that charge* $p_S^* = p^m$*. Deep stores earn* $\Pi_B^* = 0$*, while shallow stores earn* $\Pi_D^* = \pi^m$*.*

2. *If* $\mu_L \leq \bar{\mu} < \mu_H < \underline{\mu}$*, then all visits occur at shallow stores that charge* $p_S^* = h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$*. Deep stores earn* $\Pi_D^* = 0$*, while shallow stores earn* $\Pi_S^* = \frac{1}{2}\pi(p_S^*)$*.*

3. *If* $\mu_H, \mu_L \in (\bar{\mu}, \underline{\mu})$*, then all visits occur to shallow stores that charge* $p_S^* = h^{-1}\left(\frac{1}{2(\lambda z_L + (1-\lambda) z_H)}\right)$*. Deep stores earn* $\Pi_D^* = 0$*, while shallow stores earn* $\Pi_S^* = \frac{1}{2}\pi(p_S^*)$*.*

4. *If* $\mu_L \leq \bar{\mu}$ *and* $\mu_H > \underline{\mu}$*, then all high types visit deep stores, and all low types visit shallow stores that charge* p^m*. Deep stores earn* $\Pi_D^* = (1-\lambda)\pi^m$*, while shallow stores earn* $\Pi_S^* = \frac{\lambda}{2}\pi^m$*.*

5. *If* $\mu_L \in (\bar{\mu}, \underline{\mu})$ *and* $\mu_H > \underline{\mu}$*, then non-showrooming equilibria in pure strategies do not exist.*

6. *If* $\mu_L > \underline{\mu}$*, then all consumers visit deep stores, and shallow stores charge* p^m*. Deep stores earn* $\Pi_D^* = \pi^m$*, while shallow stores earn* $\Pi^* = 0$*.*

Proof. Assume that $\mu_H \leq \bar{\mu}$. Given that $r_H^* < 0$ and $\beta_H \leq 0$ (by $\gamma < \bar{\gamma}$), there are no consumers who wish to showroom at deep stores, even with $b = 0$ and at equal prices. Given the holdup problem at deep stores, consumers cannot expect lower prices at deep stores than at shallow stores; thus, no consumer will visit deep stores. Given $r_L^* < 0$, no consumer will search through shallow stores. This implies that $p_S^* = p^m$ has to hold, and all consumers visit shallow stores.

Now assume that $\mu_L \leq \bar{\mu} < \mu_H < \underline{\mu}$. Since $\bar{\mu} < \mu_H < \underline{\mu}$, high types will not visit deep stores even at equal prices, but they will search through shallow stores. By $\mu_L \leq \bar{\mu}$, the low types are not willing to visit deep stores, and are unwilling to search through shallow stores; therefore, in equilibrium, all visits have to occur at shallow stores, which charge $p_S^* = h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$ because only high types search. The pricing details trivially follow from the results in the literature, when price-inelastic low types are taken into account.

Assume, now, that $\mu_H, \mu_L \in (\bar{\mu}, \underline{\mu})$. Here, both types are willing to search through shallow stores and will not visit deep stores at equal prices; thus, all consumers visit shallow stores and potentially search, with $p_S^* = h^{-1}\left(\frac{1}{2(\lambda z_L + (1-\lambda) z_H)}\right)$ as the equilibrium price.

Now consider $\mu_L \leq \bar{\mu}$ and $\mu_H > \underline{\mu}$. High types are willing to visit deep stores at equal prices, and low types are unwilling to search through shallow stores or visit deep ones, so all stores charging p^m is an equilibrium, in which high types visit deep stores and low types visit shallow stores. No other equilibrium exists because high types visiting shallow stores and searching is precluded for the same reason as was given in the proof of Proposition 1.

Assume that $\mu_L \in (\bar{\mu}, \underline{\mu})$ and $\mu_H > \underline{\mu}$ and the proof of Proposition 1. By $\mu_L > \bar{\mu}$, low types are willing to search amongst shallow stores, but, by $\mu_L < \underline{\mu}$, prefer shallow stores at equal prices. Thus, low types will visit shallow stores and some will search. If, as per statement of the proposition, no high types showroom, then it has to be that $\Delta^* = 0$ by $\mu_H > \underline{\mu}$, or else, with $\Delta^* > 0$, some high types with b sufficiently close to 0 will showroom. But $\Delta^* = 0$ cannot occur because deep stores will charge (p^m, p^m) to high types that visit them, whereas shallow stores will have to charge $p_S^* = h^{-1}\left(\frac{1}{2(1-\lambda) z_H}\right) < p^m$. Thus, in this range, no no-showrooming equilibrium exists.

Finally, assume that$\mu_L > \underline{\mu}$. By $\mu_H > \mu_L$, we have that both types prefer deep stores at equal prices. For showrooming not to occur, we need $\Delta^* = 0$, which then implies that deep stores will attract all consumers, and will charge (p^m, p^m) by Diamond-like reasoning. Since shallow stores do no attract consumers, they have to charge p^m, which is consistent with $\Delta^* = 0$. □

One interesting implication of the violation of Assumption 1 is that, now, for some parameters, no (pure strategy) equilibrium exists. This happens when μ_L is intermediate, so that low types are willing to search through shallow stores but are not willing to visit deep stores; yet μ_H is also intermediate, so that the showrooming equilibrium cannot be sustained (μ_H is not high enough), and all consumers going to shallow stores and searching amongst them is not an equilibrium either (μ_H is not low enough). Low types visit shallow stores and put downward pressure on their prices to ensure that $p_S^* < p^m$; this then implies that $\Delta^* > 0$, which, in turn, can occur only with showrooming. Just as in Proposition 2, such an equilibrium requires a high μ_H enough (generally higher than $\tilde{\mu}$) that there exists an interval $(\underline{\mu}, \mu')$ such that when $\mu_H \in (\underline{\mu}, \mu')$, no equilibrium exists.

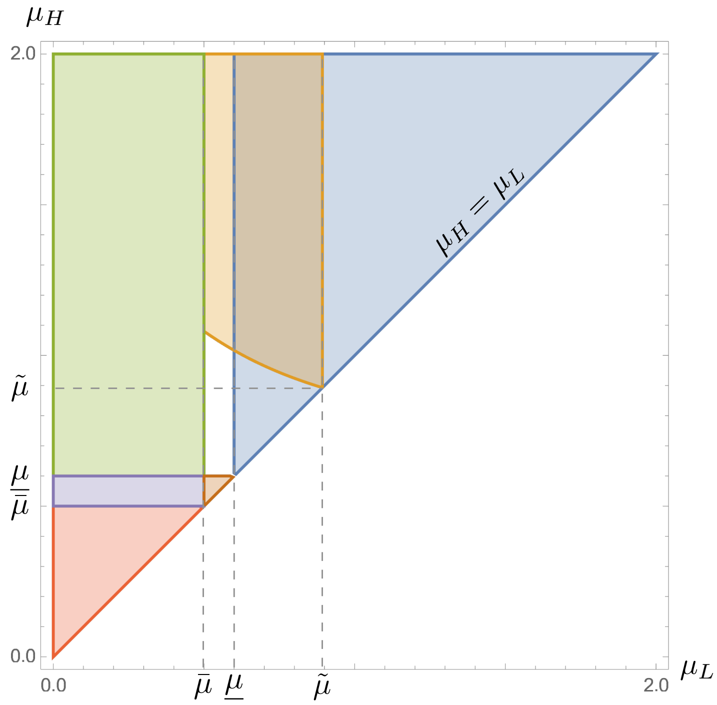

Figure 4: Equilibrium regimes depending on μ_L and μ_H, where $G \sim U(0,1)$, $q(p) = 1 - p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$,$\lambda = 0,5$, $s = \bar{b} = 0.25$, and $\gamma = 0.4$. In the red region, all visits occur at shallow stores. In the green region, high types visit deep and low types visit shallow stores. In the blue region, there is an equilibrium in which all visits are to deep stores. In all these, all store types charge monopoly prices. In the orange region, there is an additional equilibrium of the type described in Proposition 2. In the purple region, high types search and low types do not, but both types visit shallow stores. Finally, in the dark orange triangle, both types search but visit shallow stores.