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## **THE SCARS OF SUPPLY SHOCKS**

Luca Fornaro and Martin Wolf

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Centre for Economic Policy Research  
33 Great Sutton Street, London EC1V 0DX, UK  
Tel: +44 (0)20 7183 8801  
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## Abstract

We study the effects of supply disruptions - for instance due to energy price shocks or the emergence of a pandemic - in an economy with Keynesian unemployment and endogenous productivity growth. By temporarily disrupting investment, negative supply shocks generate permanent output losses - or scarring effects. By inducing a negative wealth effect, scarring effects depress aggregate demand, which may even fall below the exogenous fall in supply. However, that scarring effects depress aggregate demand does not necessarily translate into low rates of inflation. On the contrary, scarring effects may reinforce and prolong the inflationary impact of supply disruptions. A contractionary monetary policy response may end up deepening scarring effects and increasing inflation in the medium run. A successful disinflation may require a policy mix of monetary tightening and fiscal interventions aiming at supporting business investment and the economy's productive capacity.

JEL Classification: E22, E31, E32, E52, E62, O42

Keywords: Supply shocks, Covid-19, Hysteresis, investment, Endogenous growth, monetary policy, Fiscal policy, Zero lower bound, Keynesian growth

Luca Fornaro - [lfornaro@crei.cat](mailto:lfornaro@crei.cat)

*CREI and Universitat Pompeu Fabra, Barcelona and CEPR*

Martin Wolf - [martin.wolf@unisg.ch](mailto:martin.wolf@unisg.ch)

*Department of Economics, University of Bergen and CEPR*

# The Scars of Supply Shocks: Implications for Monetary Policy

Luca Fornaro and Martin Wolf\*

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## Abstract

We study the effects of supply disruptions - for instance due to energy price shocks or the emergence of a pandemic - in an economy with Keynesian unemployment and endogenous productivity growth. By temporarily disrupting investment, negative supply shocks generate permanent output losses - or scarring effects. By inducing a negative wealth effect, scarring effects depress aggregate demand, which may even fall below the exogenous fall in supply. However, that scarring effects depress aggregate demand does not necessarily translate into low rates of inflation. On the contrary, scarring effects may reinforce and prolong the inflationary impact of supply disruptions. A contractionary monetary policy response may end up deepening scarring effects and increasing inflation in the medium run. A successful disinflation may require a policy mix of monetary tightening and fiscal interventions aiming at supporting business investment and the economy's productive capacity.

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# 1 Introduction

In the last few years the global economy has been plagued by supply disruptions. First, the Covid-19 pandemic forced factories to shut down and disrupted global supply chains. Second, Russia's invasion of Ukraine has caused a sharp spike in energy and food prices. In addition to the short-run damage, many observers and policy institutions expect both shocks to leave deep scars, by inducing a very persistent drop in potential output below its pre-crisis trend (e.g., [IMF, 2022](#)).<sup>1</sup> In response to both shocks, moreover, inflation has surged in the global economy to levels not seen since the 1970s, and its persistence has been a surprise to many. These facts have renewed interest in the economic effects of supply disruptions. Can supply disruptions induce long-lasting damage to the economy? Can transitory negative supply shocks cause persistent rises of inflation? Which trade-offs are implied for monetary and fiscal policy? These questions are at the forefront of the current debate.

Much of the conventional thinking about supply shocks builds on the New Keynesian paradigm ([Galí, 2009](#)). In the New Keynesian model, following a negative supply shock demand contracts less than supply, and so the natural interest rate rises. Inflation is elevated during the period of the shock, but quickly falls back to trend once the shock abates. Monetary policy can single-handedly dampen the inflationary impact of supply disruptions, by slowing the economy and inducing a negative output gap. The New Keynesian framework, however, assumes that after the shock dissipates the economy quickly bounces back to its pre-shock trend, and so does not allow for the possibility that supply disruptions might have scarring effects.

This paper provides a theory in which negative supply shocks may leave persistent scars on the economy, and shows that this effect might radically change the macroeconomic implications of supply disruptions relative to the traditional view. Our idea is that negative supply shocks - even if purely transitory - induce firms to reduce investment, and thus destroy the future productive capacity of the economy. The associated drop in wealth depresses consumers' demand, in fact so much that the natural rate may fall in response to a supply disruption. Moreover, scarring effects may amplify and prolong the rise in inflation triggered by negative supply shocks, as they entail a long-lasting drop in firms' productivity. Monetary tightenings may backfire by inducing a drop in productivity and a rise in inflation in the medium run. A successful disinflation may thus require a policy mix of monetary tightening and fiscal interventions aiming at supporting business investment and the economy's productive capacity.

To formalize these insights, we provide a *Keynesian growth* framework with two key features. First, as in standard models of vertical innovation ([Aghion and Howitt, 1992](#)), firms invest in inno-

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<sup>1</sup>For instance, in its Spring 2022 World Economic Outlook, the International Monetary Fund (IMF) writes (see [IMF 2022](#)): “Beyond short-term output losses, the pandemic and geopolitical conflict are likely to leave longer-lasting footprints. [...] Sanctions can induce permanent dismantling of trade and supply chain linkages, entailing productivity and efficiency losses along the way. [...] And scarring effects from the pandemic are likely to materialize through several other channels - including corporate bankruptcies, productivity losses, lower capital accumulation due to a drag on investment, slower labor force growth, and human capital losses from school closures.” In response, the IMF has revised substantially downward its forecast for the growth rate of global real GDP per capita for the 2021-2027 period, relative to its pre-crisis projections.

vation in order to appropriate future monopoly rents. Second, as in the New Keynesian tradition, the presence of nominal wage rigidities implies that output may deviate from potential and that monetary policy has real effects. Our theory thus combines the Keynesian insight that unemployment may arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that sustained productivity growth is the result of investment in innovation by profit-maximizing agents.

We study the response of the economy to supply disruptions, modeled as standard temporary negative productivity shocks.<sup>2</sup> In our framework, supply shocks drive down the return to investment, by reducing firms' market size and their profits. The result is lower investment and slower productivity growth. Once the shock dissipates, investment recovers and productivity growth returns to its pre-shock level, but output falls permanently below its pre-shock trend. Our model thus captures the notion that deep recessions can have hysteresis effects on productivity and potential output.<sup>3</sup>

These scars of supply shocks matter critically for the response of aggregate demand. When productivity growth is exogenous, households' permanent income falls by little in response to transitory supply disruptions. In our endogenous growth model, instead, the drop in wealth caused by negative supply shocks is amplified by the associated decline in the trend component of productivity. In fact, we show that the fall in demand following a negative supply shock can even be larger than the exogenous fall in supply. In contrast with conventional wisdom, the natural interest rate may thus not rise sharply - and may even decline - during supply disruptions.

Against this background, a monetary tightening may trigger a "supply-demand doom loop", which amplifies the direct impact of the supply shock on employment and productivity. To see why, start by considering that a monetary contraction leads to lower aggregate demand and employment. In turn, lower aggregate demand reduces firms' profits and thus their incentives to invest. As firms cut back on investment, expected productivity growth declines. This causes a drop in households' wealth and another round of fall in aggregate demand, inducing a further decline in investment, and so forth. By triggering this vicious spiral, a tight monetary stance may thus depress both employment and productivity. This feature of the model is consistent with recent empirical evidence by [Garga and Singh \(2020\)](#), [Moran and Queraltó \(2018\)](#), [Jordà et al. \(2020\)](#) and [Grimm et al. \(2022\)](#), suggesting that monetary policy tightenings have a negative impact on investment in innovation and productivity growth.

We next turn to inflation. We first show that scarring effects may reinforce and prolong the inflationary impact of supply disruptions. The reason is that the endogenous drop in investment and trend productivity associated with supply disruptions raises firms' marginal costs, and so inflation. Moreover, since it takes time for investment to affect productivity, this effect arises with a delay. As a result, in our framework a temporary supply disruption causes a persistent rise in

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<sup>2</sup>In [Appendix B.1](#) we show that, under certain conditions, increases in energy prices have exactly the same effects as the productivity shocks considered in the paper.

<sup>3</sup>This result echoes the empirical findings of [Cerra and Saxena \(2008\)](#) and [Aikman et al. \(2022\)](#), who show that deep recessions are followed by permanent drops in output.

inflation. Our model thus helps understand why large supply disruptions - such as the oil shocks of the 1970s or the Covid-19 pandemic - are accompanied by highly persistent bursts of inflation.

What if the central bank hikes its policy rate in an attempt to reduce inflation? We show that such policy can be partially self-defeating. The reason is that, as argued above, a monetary tightening depresses firms' investment and future productivity. In turn, lower productivity sustains firms' marginal costs and inflation. Because the inflation triggered by these scarring effects arises with a delay, moreover, a tight monetary stance may be successful at reducing inflation in the short run, but at the cost of higher inflation in the medium run. Hence, monetary tightenings may end up exacerbating the inflationary consequences of supply disruptions over the medium run.

Central banks thus face a dilemma, as they may not be able to disinflate the economy without deepening the scarring effects. This suggests a potential role for fiscal interventions aiming at supporting business investment and the economy's productive capacity during disinflations. We show that a mix of monetary tightening and subsidies to investment lowers inflation both in the short and in the medium run, and improves the sacrifice ratio, that is the reduction in inflation associated with a given rise in unemployment. A successful disinflation can thus be seen as the outcome of both active supply and demand side management: while monetary policy slows inflation by reducing aggregate demand, fiscal policy slows inflation by supporting aggregate supply.<sup>4</sup> Interestingly, a similar policy mix characterized the 1980s disinflation in the United States, during which a sharp monetary tightening was accompanied by subsidies to business investment, especially in R&D (Blanchard, 1987; Modigliani, 1988).

This paper is related mainly to two strands of the literature. First, it is connected to the literature on supply disruptions and monetary policy. Compared to the standard New Keynesian approach (Blanchard and Galí, 2007a,b; Galí, 2009), our paper emphasizes the endogenous response of investment and productivity to supply disruptions and policy interventions. While in the New Keynesian model the focus is on overall aggregate demand, in our framework its composition between consumption and business investment plays a crucial role. There is also recent literature, motivated by the Covid-19 epidemic, revisiting the macroeconomic implications of supply disruptions. Guerrieri et al. (2022) study an economy with multiple consumption goods. In their model, a shock reducing the supply of some goods may induce consumers to cut spending also on those goods not directly affected by the shock. If this effect is strong enough, aggregate demand falls by more than supply. They dub supply shocks with this property *Keynesian supply shocks*. Baqaee and Farhi (2022) derive a similar result in an economy with production networks and multiple intermediate goods. Caballero and Simsek (2021) show that supply shocks can be Keynesian due to spillovers between asset prices and aggregate demand. In Bilbiie and Melitz (2020) supply disruptions depress demand by inducing firms' exit, while in L'Huillier et al. (2021) Keynesian supply shocks emerge due to the presence of diagnostic expectations.<sup>5</sup> Our paper studies a different - and complementary - channel through which supply shocks can become Keynesian,

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<sup>4</sup>Akcigit et al. (2018) and Cloyne et al. (2022) provide empirical evidence in favor of a positive impact of subsidies to R&D on firms' future productivity.

<sup>5</sup>See Bilbiie (2008) for an early model in which supply shocks have Keynesian features.

based on the endogenous response of investment and productivity growth.<sup>6</sup>

Second, this paper is related to the literature unifying the study of business cycles and endogenous growth. A central theme of this literature is the notion that deep recessions trigger hysteresis effects on productivity and potential output. Some examples of this literature are [Fatas \(2000\)](#), [Comin and Gertler \(2006\)](#), [Reifschneider et al. \(2015\)](#), [Benigno and Fornaro \(2018\)](#), [Moran and Queraltó \(2018\)](#), [Anzoategui et al. \(2019\)](#), [Bianchi et al. \(2019\)](#), [Queraltó \(2019\)](#), [Garga and Singh \(2020\)](#), [Cozzi et al. \(2021\)](#) and [Queraltó \(2022\)](#).<sup>7</sup> Our paper builds on the framework introduced by [Benigno and Fornaro \(2018\)](#), who study an endogenous growth model with vertical innovation and nominal wage rigidities. They show that in this Keynesian growth framework fluctuations can be driven by animal spirits, and derive the optimal monetary and fiscal policy. [Garga and Singh \(2020\)](#) and [Queraltó \(2022\)](#) derive, in a similar Keynesian growth model, the optimal monetary policy response to fundamental demand and cost-push shocks. Our paper, instead, employs a Keynesian growth model to study supply shocks. To the best of our knowledge, we are the first to show that the scars of supply shocks may change dramatically the macroeconomic implications of supply disruptions.

The rest of the paper is composed of five sections. Section 2 describes the baseline model. Section 3 studies the implications of scarring for aggregate demand and describes the supply-demand doom loop. Section 4 discusses the implications of scarring for inflation. Section 5 concludes. Appendix A provides the proofs of all propositions, while Appendix B contains additional derivations, as well as model extensions.

## 2 Baseline model

This section lays down our baseline *Keynesian growth* model. The economy has two key elements. First, the rate of productivity growth is endogenous, and it is the outcome of firms' investment. Second, the presence of nominal wage rigidities implies that output and employment can deviate from their potential levels. In order to illustrate transparently our key results, the framework in this section is kept voluntarily simple. Throughout the paper, however, we will extend this baseline model in several directions.

Consider an infinite-horizon closed economy. Time is discrete and indexed by  $t \in \{0, 1, 2, \dots\}$ . The economy is inhabited by households, firms, and by a central bank that sets monetary policy. For simplicity, we focus on a perfect foresight economy.

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<sup>6</sup>In our own earlier work ([Fornaro and Wolf, 2020](#)), we argued that permanent negative supply shocks can be Keynesian. In this paper, we move beyond the analysis in [Fornaro and Wolf \(2020\)](#) by providing a Keynesian growth model in which productivity growth is the result of firms' investment. We show that temporary negative supply shocks can be Keynesian by triggering endogenous drops in investment and productivity growth.

<sup>7</sup>See [Cerra et al. \(2020\)](#) for a recent survey of this literature.



## 2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous “final” good. The lifetime utility of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \log C_t, \quad (1)$$

where  $C_t$  denotes consumption and  $0 < \beta < 1$  is the subjective discount factor.

Each household is endowed with  $\bar{L}$  units of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only  $L_t < \bar{L}$  units of labor on the market. Moreover, households can trade in one-period, non-state contingent bonds  $B_t$ . Bonds are denominated in units of currency and pay the nominal interest rate  $i_t$ . Finally, households own all the firms and each period they receive dividends  $D_t$  from them.

The problem of the representative household consists in choosing  $C_t$  and  $B_{t+1}$  to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1+i_t} = W_t L_t + B_t + D_t,$$

where  $P_t$  is the nominal price of the final good,  $B_{t+1}$  is the stock of bonds purchased by the household in period  $t$ , and  $B_t$  is the payment received from its past investment in bonds.  $W_t$  denotes the nominal wage, so that  $W_t L_t$  is the household’s labor income. The optimality conditions are the Euler equation

$$C_t = \frac{C_{t+1}}{\beta(1+r_t)}, \quad (2)$$

where we have defined the real interest rate as  $1+r_t \equiv (1+i_t)P_t/P_{t+1}$ , and the standard transversality condition.

## 2.2 Final good production

The final good is produced by competitive firms using labor and a continuum of measure one of intermediate inputs  $x_{j,t}$ , indexed by  $j \in [0, 1]$ . Denoting by  $Y_t$  the output of the final good, the production function is

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^\alpha dj, \quad (3)$$

where  $0 < \alpha < 1$ , and  $A_{j,t}$  is the productivity, or quality, of input  $j$ .<sup>8</sup>  $Z_t$ , instead, is an exogenous productivity shock, which we refer to as the “supply shock” in our model. This term captures all the transitory factors affecting labor productivity which are not directly linked to firms’ investment. For instance, a reduction in  $Z_t$  could capture the fact that during a pandemic some firms cannot

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<sup>8</sup>More precisely, for every good  $j$ ,  $A_{j,t}$  represents the highest quality available. In principle, firms could produce using a lower quality of good  $j$ . However, the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

operate in order to preserve public health. Or, as we show in Appendix B.1, a fall in  $Z_t$  has a similar impact on the economy as an exogenous rise in the price of energy.

Profit maximization implies the demand functions

$$P_t(1 - \alpha)Z_t^{1-\alpha}L_t^{-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^\alpha dj = W_t \quad (4)$$

$$P_t \alpha (Z_t L_t)^{1-\alpha} A_{j,t}^{1-\alpha} x_{j,t}^{\alpha-1} = P_{j,t}, \quad (5)$$

where  $P_{j,t}$  is the nominal price of intermediate input  $j$ . Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

### 2.3 Intermediate goods production and profits

Every intermediate good is produced by a single monopolist. One unit of final output is needed to manufacture one unit of the intermediate good, regardless of quality. In order to maximize profits, each monopolist sets the price of its good according to

$$P_{j,t} = \frac{P_t}{\alpha}. \quad (6)$$

This expression implies that each monopolist charges a constant markup  $1/\alpha > 1$  over its marginal cost.

Equations (5) and (6) imply that the quantity produced of a generic intermediate good  $j$  is

$$x_{j,t} = \alpha^{\frac{2}{1-\alpha}} A_{j,t} Z_t L_t. \quad (7)$$

Combining equations (3) and (7) gives

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t Z_t L_t, \quad (8)$$

where  $A_t \equiv \int_0^1 A_{j,t} dj$  is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods, in the exogenous component of labor productivity, and in the amount of labor employed in production.

The profits earned by the monopolist in sector  $j$  are given by

$$(P_{j,t} - P_t)x_{j,t} = P_t \varpi A_{j,t} Z_t L_t,$$

where  $\varpi \equiv (1/\alpha - 1)\alpha^{2/(1-\alpha)}$ . According to this expression, the producer of an intermediate input of higher quality earns higher profits. Moreover, profits are increasing in  $Z_t L_t$  due to the presence of a market size effect. Intuitively, high production of the final good is associated with high demand for intermediate inputs, leading to high profits in the intermediate sector.

## 2.4 Investment and productivity growth

Firms operating in the intermediate sector can invest in innovation in order to improve the quality of their products. In particular, a firm that invests  $I_{j,t}$  units of the final good sees its productivity evolve according to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}, \quad (9)$$

where  $\chi > 0$  determines the productivity of investment.

Innovation-based endogenous growth models typically assume that knowledge is only partly excludable. For instance, this happens if inventors cannot prevent others from drawing on their ideas to innovate. For this reason, in most endogenous growth frameworks, the social return from investing in innovation is higher than the private one.<sup>9</sup> A simple way to introduce this effect in the model is to assume that every period, after production takes place, there is a constant probability  $1 - \eta$  that the incumbent firm dies, and is replaced by another firm that inherits its technology. This assumption encapsulates all the factors that might lead to the termination of the rents from innovation, including patent expiration and imitation by competitors.

Firms producing intermediate goods choose investment in innovation to maximize their discounted stream of profits net of investment costs

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}), \quad (10)$$

subject to (9) and given the initial condition  $A_{j,0} > 0$ . Since firms are fully owned by domestic households, they discount profits using the households' discount factor  $\beta^t / (P_t C_t)$ , adjusted for the survival probability  $\eta$ .

From now on, we assume that firms are symmetric and so  $A_{j,t} = A_t$ . Moreover, we focus on equilibria in which investment in innovation is always positive. Optimal investment in research then requires<sup>10</sup>

$$\frac{1}{\chi} = \frac{\beta C_t}{C_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \frac{\eta}{\chi} \right). \quad (11)$$

Intuitively, firms equalize the marginal cost from performing research  $1/\chi$ , to its marginal benefit discounted using the households' discount factor. The marginal benefit is given by the increase in next period profits ( $\varpi Z_{t+1} L_{t+1}$ ) plus the savings on future research costs  $1/\chi$ , adjusted for the firm survival probability  $\eta$ .

<sup>9</sup>See for instance Aghion and Howitt (1992).

<sup>10</sup>See Appendix B.2 for the derivation of equation (11).

## 2.5 Nominal rigidities and monetary policy

We consider an economy with frictions in the adjustment of nominal wages.<sup>11</sup> The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment, by ensuring that nominal wages remain positive even when employment falls short of households' labor supply. Second, it implies that monetary interventions have real effects. Since prices inherit part of wage stickiness, in fact, by setting the nominal interest rate the central bank can affect the real interest rate.

To streamline the analysis, in what follows we frame monetary policy directly in terms of a path for the real interest rate, as in [Werning \(2015\)](#) and [Mian et al. \(2021\)](#). This allows us to study the real side of the economy without specifying how nominal wages are set. We will turn to the nominal side of the economy in [Section 4](#), where we will make the nominal wage setting explicit and discuss the implications of our model for inflation and nominal interest rates.

## 2.6 Aggregation and market clearing

Market clearing for the final good implies<sup>12</sup>

$$Y_t - \int_0^1 x_{j,t} dj = C_t + I_t, \quad (12)$$

where  $I_t \equiv \int_0^1 I_{j,t} dj$ . The left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that all the GDP has to be either consumed or invested. Using [equations \(7\) and \(8\)](#) we can write GDP as

$$Y_t - \int_0^1 x_{j,t} dj = \Psi A_t Z_t L_t, \quad (13)$$

where  $\Psi \equiv \alpha^{2\alpha/(1-\alpha)}(1 - \alpha^2)$ .

Turning to labor market clearing, the assumption about labor endowment implies that  $L_t \leq \bar{L}$ . Since labor is supplied inelastically by the households,  $\bar{L} - L_t$  can be interpreted as the unemployment rate. For future reference, when  $L_t = \bar{L}$  we say that the economy is operating at full employment, while when  $L_t < \bar{L}$  the economy operates below capacity and there is a negative output gap.

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<sup>11</sup>A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by [Olivei and Tenreyro \(2007\)](#), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by [Fehr and Goette \(2005\)](#), [Gottschalk \(2005\)](#), [Barattieri et al. \(2014\)](#) and [Gertler et al. \(2020\)](#).

<sup>12</sup>The goods market clearing condition can be derived by combining the households' budget constraint with the expression for firms' profits

$$D_t = \underbrace{P_t Y_t - W_t L_t - P_t \frac{1}{\alpha} \int_0^1 x_{j,t} dj}_{\text{profits from final goods sector}} + \underbrace{P_t \int_0^1 \left( \frac{1}{\alpha} - 1 \right) x_{j,t} dj - P_t \int_0^1 I_{j,t} dj}_{\text{profits from intermediate goods sector}}.$$

We also use the equilibrium condition  $B_{t+1} = 0$ , which is implied by the assumption of identical households.

Long-run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index  $A_t$ . We can thus think of  $g_t \equiv A_t/A_{t-1}$  as the trend component of productivity. Using this definition, we can write equation (9) as

$$g_{t+1} = 1 + \chi \frac{I_t}{A_t}. \quad (14)$$

This expression implies that higher investment in research in period  $t$  is associated with faster productivity growth between periods  $t$  and  $t + 1$ . More precisely, the rate of productivity growth is determined by the ratio of investment in innovation  $I_t$  over the existing stock of knowledge  $A_t$ . In turn, the stock of knowledge depends on all past investment in innovation, that is on the R&D stock. Hence, there is a positive link between R&D intensity, captured by the ratio  $I_t/A_t$ , and future productivity growth.

## 2.7 Summary of equilibrium conditions

The equilibrium of the economy can be described by three equations. The first one captures consumers' behavior. It is obtained by rewriting the Euler equation (2) as

$$c_t = \frac{g_{t+1}c_{t+1}}{\beta(1+r_t)}, \quad (\text{Eq1})$$

where we have defined  $c_t \equiv C_t/A_t$  as consumption normalized by the trend component of productivity. As it is standard, this equation implies that current demand for consumption is increasing in future (normalized) consumption and decreasing in the real interest rate. It also implies a positive relationship between trend productivity growth and present demand for consumption. The reason is that faster productivity growth is associated with higher future wealth, and so stronger consumption demand.

The second key relationship in our model is the *growth* equation, which is obtained by combining equation (2) with the optimality condition for investment in research (11)

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} (\chi \varpi Z_{t+1} L_{t+1} + \eta). \quad (\text{Eq2})$$

This equation implies a positive relationship between growth and future market size. Intuitively, a rise in  $Z_{t+1}L_{t+1}$  is associated with higher future monopoly profits. In turn, higher profits induce entrepreneurs to invest more, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature.<sup>13</sup> At the same time, growth depends inversely on the growth rate of normalized consumption  $c_{t+1}/c_t$ . This is a cost of funds effect: when today's consumption is low relative to consumption in the future, firms pay out dividends to households rather than invest.

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<sup>13</sup>To be clear, what matters for our results is that productivity growth is increasing in employment relative to potential. This means that our key results would also apply to a setting in which scale effects related to population size were not present. For instance, in the spirit of Young (1998) and Howitt (1999), these scale effects could be removed by assuming that the number of intermediate inputs is proportional to population size.

The third equation combines the goods market clearing condition (12), the GDP equation (13) and the fact that  $I_t/A_t = (g_{t+1} - 1)/\chi$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}. \quad (\text{Eq3})$$

Keeping GDP constant, this equation implies a negative relationship between consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output to investment, reducing the resources available for consumption.

We are now ready to define an equilibrium as a set of sequences  $\{g_{t+1}, L_t, c_t\}_{t=0}^{+\infty}$  satisfying the three equations (Eq1), (Eq2) and (Eq3), as well as  $0 < L_t \leq \bar{L}$ ,  $g_{t+1} > 1$  and  $c_t > 0$  for all  $t \geq 0$ , given paths for monetary policy  $\{r_t\}_{t=0}^{+\infty}$  and the supply shock  $\{Z_t\}_{t=0}^{+\infty}$ .

## 2.8 The balanced growth path

Before studying the implications of the model, it is useful to spend a few words on the balanced growth path - or steady state - of the economy. A steady state is characterized by constant values for  $g_{t+1}$ ,  $L_t$ ,  $c_t$ ,  $r_t$  and  $Z_t$  that satisfy the three equilibrium conditions (Eq1), (Eq2) and (Eq3). For most of the paper, we will be studying economies that fluctuate around a full employment steady state. We denote the value of a variable in this steady state with an upper bar, and normalize steady state productivity to  $\bar{Z} = 1$ . We now make some assumptions to ensure that a full employment steady state exists.

**Proposition 1** *Suppose that the parameters satisfy*

$$\beta(\chi\varpi\bar{L} + \eta) > 1, \quad (15)$$

*and that monetary policy is such that*

$$1 + \bar{r} = \chi\varpi\bar{L} + \eta. \quad (16)$$

*Then there exists a unique full employment steady state. Moreover, this steady state is characterized by  $\bar{g} > 1$ .*

Intuitively, condition (15) guarantees that in the full employment steady state the return to investment is sufficiently high so that growth is positive. Condition (16), instead, ensures that the central bank policy is consistent with the existence of a full employment steady state.

## 3 Supply disruptions, scarring effects and aggregate demand

We start by studying how supply disruptions affect aggregate demand, and highlight the central role played by scarring effects. First, we will show that the persistent scars left by supply disruptions depress aggregate demand. Second, we will see how weak aggregate demand reinforces the scarring effects, by triggering a vicious cycle which we dub the supply-demand doom loop.

### 3.1 An exogenous growth benchmark

Let us first study a counterfactual economy in which there is no investment and trend productivity growth is exogenous. As is well known from the New Keynesian literature (Galí, 2009), in this case the natural interest rate - that is the value of the interest rate consistent with full employment - rises after a negative supply shock, indicating that the supply disruption depresses aggregate supply by more than demand.<sup>14</sup> Intuitively, this happens because households' permanent income falls by little in response to a temporary supply disruption.

More precisely, assume that firms don't undertake any investment, that trend growth is exogenous and equal to  $g_t = \bar{g}$ , and that  $Z_t$  follows the process

$$\log Z_t = \rho \log Z_{t-1}, \quad (17)$$

where  $0 \leq \rho < 1$  determines the persistence of the productivity shock.

Now suppose that the economy is hit by a previously unexpected negative supply shock, which corresponds to the initial condition  $Z_0 < 1$ . Rearranging the aggregate demand equation (Eq1) for the real interest rate, and imposing the full employment condition ( $L_t = \bar{L}$ ), the natural rate is then given by<sup>15</sup>

$$1 + \bar{r}_t = \frac{\bar{g}Z_{t+1}}{\beta Z_t} = \frac{\bar{g}Z_0^{\rho^t(\rho-1)}}{\beta}, \quad (18)$$

where, in this and the following equations, we use bars to denote variables in the natural allocation. Since  $Z_0 < 1$ , this expression implies that the natural interest rate increases in response to a temporary supply disruption.

This result forms part of the conventional wisdom on the macroeconomic implications of supply disruptions. As we will see, however, this conventional wisdom might fail once the impact of supply shocks on investment and productivity growth is taken into account.

### 3.2 Back to the Keynesian growth framework

We now present our first result: in an economy in which productivity growth is driven by firms' investment, a supply disruption triggers a negative wealth effect which depresses aggregate demand. Indeed, this effect can be so strong so that the shock might cause a demand shortage that is larger than the supply disruption itself. When this happens, the natural rate falls rather than rises following a supply disruption.

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<sup>14</sup>More formally, the natural interest rate is the equilibrium (real) interest rate when all wages and prices are flexible. In our simple model, this corresponds to the interest rate that prevails under full employment ( $L_t = \bar{L}$ ). The natural interest rate can be understood as a summary statistic of the balance between aggregate demand and supply in an economy. For instance, when the natural rate is high, this indicates an economy where aggregate demand is strong relative to supply. As we do, Guerrieri et al. (2022) use this insight to study the response of aggregate demand to supply disruptions.

<sup>15</sup>To derive this expression, notice that, since firms don't invest, all the output is consumed and so  $c_t = \Psi Z_t L_t$  for every  $t$ .

**Proposition 2** *Assume that  $Z_t$  is governed by the process (17), and that  $Z_0 < 1$ . If  $\rho > 0$ , the natural interest rate is characterized by  $\bar{r}_t < \bar{r}$  for all  $t \geq 0$ . If  $\rho = 0$ , the natural rate is unchanged in response to the shock,  $\bar{r}_t = \bar{r}$  for all  $t \geq 0$ .*

To understand Proposition 2, let us start by studying the behavior of investment and productivity growth. By equation (Eq2), in the full-employment allocation productivity growth evolves according to

$$\bar{g}_{t+1} = \beta \frac{\bar{c}_t}{\bar{c}_{t+1}} (\chi \varpi Z_{t+1} \bar{L} + \eta). \quad (19)$$

There are two channels through which the supply shock reduces growth. First, a transitory drop in  $Z_t$  leads to a drop in  $\bar{c}_t/\bar{c}_{t+1}$ . This corresponds to an increase in the rate at which households discount future profits, reducing firms' incentives to invest. Second, if the shock is persistent, the fall in  $Z_{t+1}$  lowers the profits that firms appropriate by investing in innovation. Both effects point toward a negative impact of supply disruptions on investment and productivity growth.

What are the implications of the fall in investment for aggregate demand? Because investment is a component of aggregate demand, the fall in investment constitutes a drag on aggregate demand in itself. Quantitatively, this effect is stronger, the higher the share of investment in GDP. Following standard practice in the endogenous growth literature, we may interpret firms' investment in innovation as their expenditure in R&D. Given that spending in R&D represents a small fraction of GDP, the direct impact of fluctuations in investment on aggregate demand in our model will be small.<sup>16</sup>

There is, however, a second channel through which a fall in investment depresses aggregate demand. Lower investment drives down productivity growth and so agents' future incomes. This negative wealth effect causes a drop in consumption demand. This second effect, on its own, might be strong enough to reverse the response of the natural rate to a supply shock relative to the case in which productivity growth is exogenous. To see this point most clearly, consider the limit case where the investment share of GDP goes to zero, so that  $c_t \approx \Psi Z_t L_t$ . Again we solve (Eq1) for the real interest rate, and impose the natural allocation, to obtain

$$1 + \bar{r}_t = \frac{\bar{g}_{t+1} Z_0^{\rho^t (\rho-1)}}{\beta}. \quad (20)$$

This expression shows how the endogenous drop in  $\bar{g}_{t+1}$  puts downward pressure on the natural rate. As we show in Proposition 2, as long as the supply disturbance has at least some persistence ( $\rho > 0$ ), this effect is strong enough to induce a drop in the natural rate following a supply disruption. Hence, in our baseline model, supply disruptions trigger demand shortages that are larger than the supply disruption itself.

To further illustrate this point, we resort to a simple numerical simulation.<sup>17</sup> We choose the

<sup>16</sup>That said, Howitt and Aghion (1998) highlight how investment in innovation may be complementary to other forms of investment, including in physical capital. Hence, a drop in investment in innovation may depress aggregate demand by triggering a fall in other types of investment. We leave the study of this transmission channel to future research.

<sup>17</sup>To be clear, our objective is not to provide a careful quantitative evaluation of the framework or to replicate



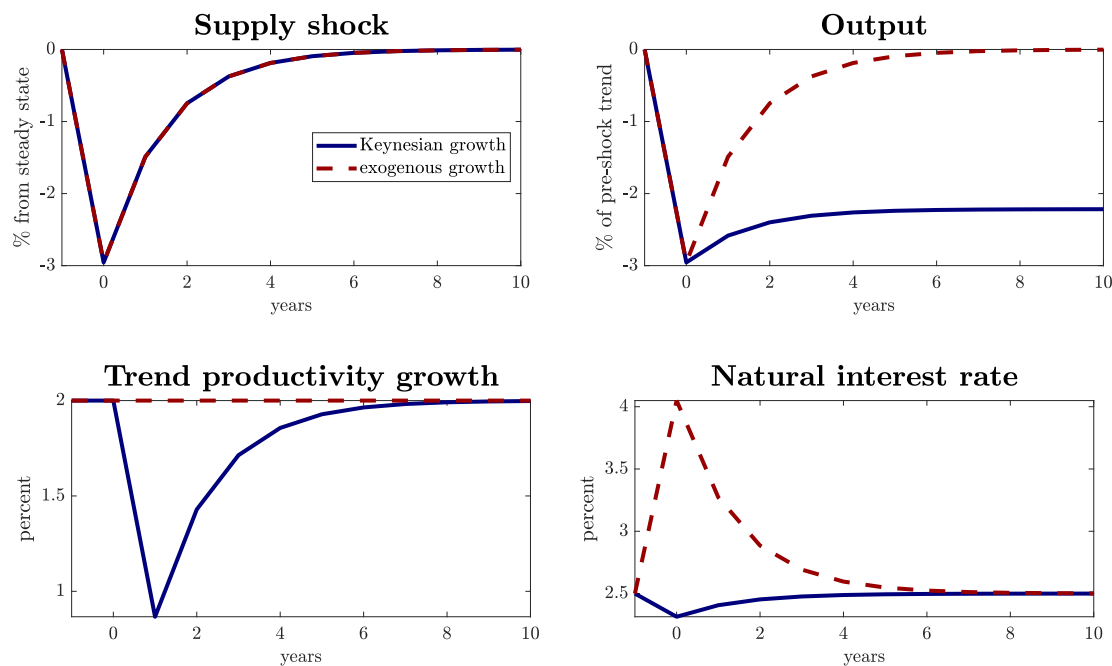


Figure 1: Scarring effects and the natural interest rate.

length of a period to correspond to one year. We set  $\chi$ ,  $\beta$  and  $\alpha$  by targeting three moments of the full employment steady state.  $\chi$  is set to 1.95 so that steady state productivity growth is equal to 2%, while we choose  $\beta = 0.995$  so that the real interest rate in steady state is equal to 2.5%. We set the labor share in gross output to  $1 - \alpha = 0.86$ , to match a ratio of spending in investment on innovation to GDP of 2%, close to the GDP share of business spending in R&D observed in the United States. This calibration choice implies that investment in innovation is a small component of aggregate demand. Following Benigno and Fornaro (2018), the firms' survival probability is set to  $\eta = 0.9$ . Finally, the shock is parametrized so that on impact potential output drops by 3%, and its persistence is set to  $\rho = 0.5$ .

Figure 1 illustrates the macroeconomic impact of a supply disruption, assuming that monetary policy replicates the full employment allocation. When productivity growth is exogenous, supply drops more than demand, and the natural rate rises. In our Keynesian growth model, instead, the temporary supply disruption causes a drop in investment, and so a permanent decline in long-run output.<sup>18</sup> The associated decline in households' wealth triggers a fall in consumers' demand, which explains why the natural interest rate declines.

We conclude this section with an observation. In our baseline model, there is a linear relationship between investment in innovation and productivity growth (see equation (9)). This is a common assumption in the theoretical literature on endogenous growth, since it is consistent with free entry in the research sector (Aghion and Howitt, 1992). Quantitative analyses, however, often

any particular historical event. In fact, both of these tasks would require a much richer model. Rather, our aim is to show how the model behaves for some reasonable parameter values.

<sup>18</sup>Cerra and Saxena (2008) and Aikman et al. (2022) provide empirical evidence supporting the notion that deep contractions are followed by permanent output declines.

assume that investment in research is subject to diminishing returns - to capture the existence of adjustment costs in investment in innovation (Acemoglu and Akcigit, 2012). In Appendix B.3 we show that, in this alternative case, it is no longer true that the natural rate unambiguously declines following a supply disruption. In fact, with diminishing returns to investment, a supply disruption triggers a drop in the natural rate only if it is sufficiently persistent. The intuition is that concavity in the investment function dampens the scarring effects of supply disruptions, implying a smaller drop in aggregate demand. However, even in the case of short-lived supply disruptions, it is still true that the natural interest rate rises by less compared to an economy with exogenous productivity growth.<sup>19</sup>

Summing up, by negatively affecting investment and productivity growth, even purely transitory supply shocks generate permanent output losses. The associated negative wealth effect induces consumers to cut on their demand, which depresses the natural interest rate. In our baseline model, this effect is strong enough to overturn the conventional wisdom according to which the natural rate rises following a supply disruption.

### 3.3 The supply-demand doom loop

To this point we have assumed the central bank closes the output gap at all times, by setting the real interest rate equal its natural level. But what if the central bank follows a tighter monetary stance, implying that the output gap turns negative?<sup>20</sup> We now show that, in this case, scarring effects are reinforced by the negative output gap, setting in motion a supply-demand doom loop.

To illustrate our point, we start by assuming that monetary policy follows the simple rule

$$1 + r_t = (1 + \bar{r}) \left( \frac{L_t}{\bar{L}} \right)^\phi. \quad (21)$$

Under this rule the central bank seeks to stabilize output around its potential level, by cutting the real interest rate in response to falls in employment. In addition to condition (16), we assume that

$$\phi > \frac{\chi \varpi \bar{L}}{\chi \varpi \bar{L} + \eta}, \quad (22)$$

which implies that the steady state under the rule (21) is locally determinate.<sup>21</sup>

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<sup>19</sup>Even if one maintains the assumption of a linear investment function, there is a case in which the natural rate might rise after a negative supply disruption. This might happen if the shock is large enough to drive investment in innovation, and the endogenous component of productivity growth, to zero. The reason is simple. Once firms stop investing in innovation, further drops in  $Z_t$  no longer depress the endogenous component of productivity growth - and the associated drag on consumers' demand becomes muted. This means that the impact of supply disruptions on the natural rate might become non-linear. In particular, the natural rate might rise in response to a negative supply shock severe enough to drive investment in innovation to zero.

<sup>20</sup>As we discuss in Section 4, the central bank may seek to produce a negative output gap in an attempt to reduce inflation.

<sup>21</sup>See Appendix B.4 for a proof.

### 3.3.1 An insightful case: a permanent supply disruption

While our focus is on temporary supply disruptions, it is useful to first study a case in which  $Z_t$  drops permanently to a lower level. The advantage is that, by focusing on a permanent shock, we can illustrate the key forces at the heart of the model using a simple graphical analysis.

Figure 2 shows how  $L$  and  $g$  are determined in steady state. The (GG) schedule corresponds to the growth equation (Eq2) evaluated in steady state, given by

$$g = \beta(\chi\varpi ZL + \eta). \quad (\text{GG})$$

The (GG) schedule implies a positive relationship between  $g$  and  $L$ . Intuitively, an increase in employment - and so in market size - is associated with a rise in the return from investing in innovation. Firms respond by increasing investment and productivity growth accelerates.

The (AD) curve, instead, summarizes the aggregate demand side of the model. It is obtained by combining equations (Eq1) and (21), evaluated in steady state

$$g = \beta(1 + \bar{r}) \left( \frac{L}{\bar{L}} \right)^\phi. \quad (\text{AD})$$

This equation implies a positive relationship between  $g$  and  $L$ . To understand the intuition behind this equation, consider what happens after a rise in productivity growth. Due to the associated positive wealth effect, households respond by increasing their demand for borrowing and consumption. Higher consumption, in turn, puts upward pressure on employment. The central bank reacts to the rise in employment by increasing the interest rate, which cools down households' demand for borrowing and restores equilibrium on the credit market.

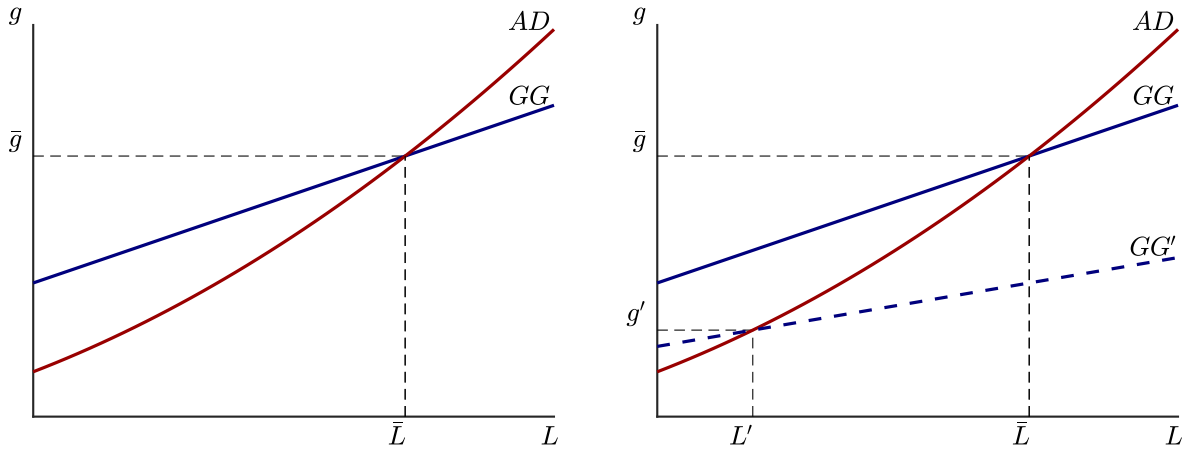
A steady state equilibrium corresponds to an intersection of the (AD) and (GG) curves. Under our assumption about  $\phi$ , there is only one intersection between the two curves satisfying  $L \leq \bar{L}$ , meaning that the steady state exists and is unique.<sup>22</sup> The steady state shown in the left panel of Figure 2 corresponds to the full employment steady state.

Now imagine that we start from the full employment steady state, and a previously unexpected permanent fall in  $Z$  occurs. As shown in the right panel of Figure 2, the decline in  $Z$  induces a downward shift of the (GG) curve. As already explained, the exogenous fall in labor productivity depresses firms' profits and their incentives to invest. Firms react by reducing investment and so, holding constant  $L$ , productivity growth  $g$  drops. The fall in productivity growth, through its negative wealth effect, translates into lower aggregate demand. By our assumptions, the central bank does not impart enough stimulus to prevent unemployment from arising. The result is a drop in employment below the full employment level ( $L < \bar{L}$ ). Therefore, the negative supply shock gives rise to a drop in aggregate demand and involuntary unemployment.<sup>23</sup>

This is not, however, the end of the story. Lower demand further reduces firms' profits and their

<sup>22</sup>Moreover, under our assumption about  $\phi$ , the (AD) curve is necessarily steeper than the (GG) curve at their intersection.

<sup>23</sup>This effect is well known from the literature on news shocks (e.g., Lorenzoni, 2009).



(a) The  $L - g$  diagram.

(b) A permanent supply disruption.

**Figure 2: The  $L - g$  diagram and permanent supply disruptions.**

incentives to invest. This effect generates another round of drops in investment and productivity growth. Lower productivity growth, in turn, induces a further cut in demand, which again lowers investment and growth. This vicious spiral, or supply-demand doom loop, amplifies the impact of the initial supply shock on employment and labor productivity growth.

It is possible to derive an expression for the elasticity of the endogenous component of productivity growth with respect to the supply shock. Combining (GG) and (AD) and differentiating gives

$$\left(\frac{\partial g}{\partial Z}\right) \frac{Z}{g} \Big|_{Z=1} = \frac{1 - \eta\beta/\bar{g}}{1 - \frac{1-\eta\beta/\bar{g}}{\phi}}. \quad (23)$$

In this expression, the numerator captures the direct impact of a change in  $Z$  on  $g$ . In this simple model, this direct effect is large when the externalities associated with innovation activities are substantial (i.e. when  $\eta$  is small). The denominator, instead, captures the multiplier effect associated with the supply-demand doom loop. This multiplier effect is decreasing in  $\phi$ .<sup>24</sup> As it is intuitive, a smaller response of monetary policy to changes in employment amplifies the impact of supply shocks on productivity growth.

Therefore, in our framework monetary policy has an impact on the endogenous component of productivity growth. In particular, a tighter monetary stance leads to a slowdown in investment in innovation and productivity growth. This feature of the model is consistent with a growing body of empirical evidence (Garga and Singh, 2020; Moran and Queraltó, 2018; Jordà et al., 2020; Grimm et al., 2022), suggesting that monetary policy tightenings induce firms to cut investment

<sup>24</sup>Notice that  $\bar{g} = \beta(\chi\varpi\bar{L} + \eta)$ . The denominator can thus be written as

$$1 - \frac{\chi\varpi\bar{L}}{\chi\varpi\bar{L} + \eta} / \phi.$$

By assumption (22), the denominator is therefore positive for any permissible level of  $\phi$ .

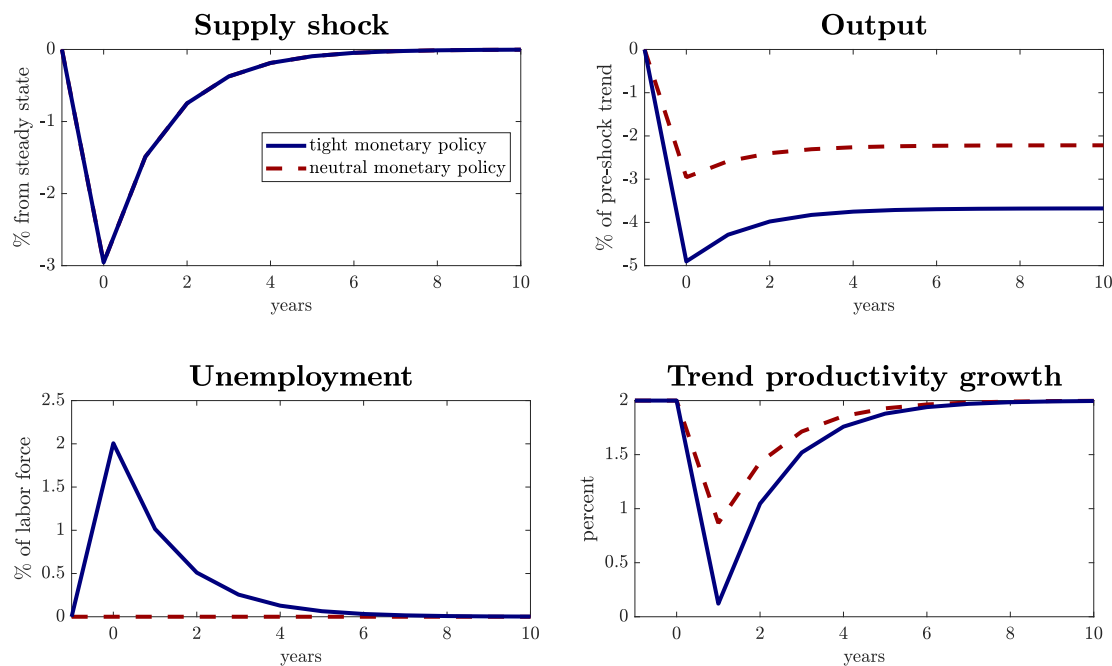


Figure 3: Tight monetary policy deepens scarring effects.

in innovation, such as R&D, and have a long lasting negative impact on productivity and potential output.

### 3.3.2 A temporary supply disruption

We turn back to the case of temporary shocks, by again assuming that  $Z_t$  evolves according to the process (17). In the case of temporary shocks, deriving analytic results is more challenging. However, some insights can be obtained by combining (Eq1), (Eq2) and (21) as follows

$$(1 + \bar{r}) \left( \frac{L_t}{\bar{L}} \right)^\phi = \chi \varpi Z_{t+1} L_{t+1} + \eta. \quad (24)$$

Since  $L_{t+1} \leq \bar{L}$ , this equation directly implies that a future supply disruption ( $Z_{t+1} < 1$ ) causes involuntary unemployment in the present ( $L_t < \bar{L}$ ). Of course, a lower  $L_t$  leads to a reduction in the incentives to invest in period  $t - 1$ , which lowers aggregate demand and employment in period  $t - 1$ , and so on. Thus in the case of temporary shocks, an intertemporal supply-demand doom loop emerges.

Figure 3 shows the response of the economy to a negative supply shock, contrasting the neutral monetary policy - which replicates the natural allocation - to a tighter monetary response which follows the rule (21). We choose the strength of the monetary response to unemployment to  $\phi = 0.15$ , so that in the impact period of the shock employment falls by 2%. Given our calibration of the size of the shock, this implies that on impact the supply disruption causes a 5% decline in output. All other model parameters are kept as in Section 3.2.

As we can see in Figure 3, a tighter monetary response induces an additional decline of produc-

tivity growth, which deepens the permanent output loss. Because of the logic of the supply-demand doom loop, the effect of a temporary negative output gap on the permanent component of productivity can be quite large. Hence a tight monetary stance may greatly amplify the direct impact of negative supply shocks on employment and productivity growth.

## 4 Implications for inflation

We now turn to inflation. Since scarring effects depress aggregate demand, one might be tempted to conclude that they dampen the rise in inflation conventionally associated with supply disruptions. In contrast, we will now show that scarring effects may amplify and prolong the burst of inflation caused by a negative supply shock. We will also show that monetary policy tightenings - by deepening scarring effects - can backfire and fail to reduce inflation. Finally, we will discuss the role of fiscal policy in complementing monetary policy during periods of disinflation.

Let us start by specifying a wage setting process. Inspired by the empirical literature on wage Phillips curves (Galí and Gambetti, 2020), we assume that nominal wages evolve according to

$$\frac{W_t}{W_{t-1}} = \bar{g} \left( \frac{L_t}{\bar{L}} \right)^\xi \pi_{t-1}^\lambda, \quad (25)$$

where  $\xi > 0$ ,  $0 \leq \lambda < 1$ , and  $\pi_{t-1}$  denotes (lagged) gross price inflation. According to this equation, as in standard Phillips curves, an increase in involuntary unemployment puts downward pressure on wage growth. Moreover, when  $\lambda > 0$  wages are partially indexed to past price inflation. As we will explain in a second, this feature is helpful to obtain reasonable inflation dynamics in response to supply shocks. Finally, we normalize wage inflation in the full employment steady state to be equal to productivity growth.<sup>25</sup>

Using the wage-setting rule (25), as well as (4) and (7), we get an expression for price inflation

$$\pi_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} \left( \frac{L_t}{\bar{L}} \right)^\xi \pi_{t-1}^\lambda. \quad (26)$$

Intuitively, firms set prices equal to their marginal cost. Higher wage inflation puts upward pressure on marginal costs and leads to higher price inflation, while faster productivity growth reduces marginal costs and lowers price inflation. This explains why price inflation is increasing in employment, and decreasing in productivity growth. The term  $\pi_{t-1}^\lambda$  captures the inflation persistence arising from the indexation of wages. Absent this term, and given that our model abstracts from price rigidities, inflation would be excessively volatile in response to supply shocks. Given our assumptions about wage inflation, steady state inflation - which can be interpreted as the medium run central bank's target - is normalized to zero ( $\bar{\pi} = 1$ ).

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<sup>25</sup>In Appendix B.5, we show that the main insights of this section are preserved under a conventional New Keynesian wage Phillips curve, derived from the presence of wage adjustment costs à la Rotemberg (1982).

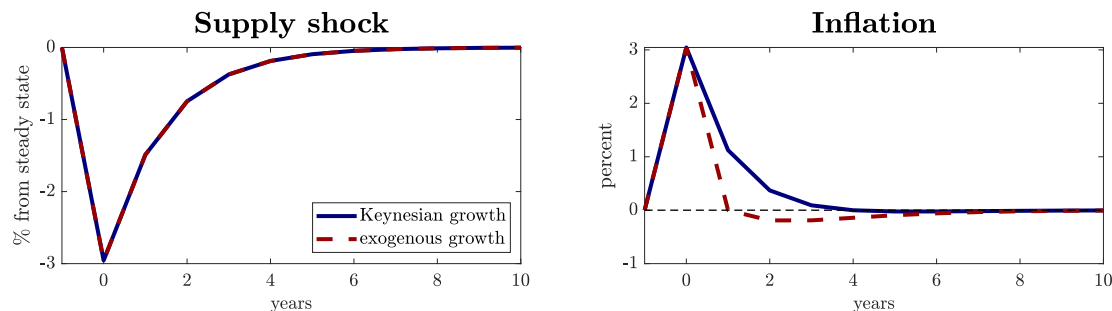


Figure 4: Inflationary impact of scarring effects.

#### 4.1 Can supply disruptions cause persistent rises in inflation?

Large supply disruptions tend to be associated with prolonged periods of high inflation. For instance, the 1970s oil shocks were accompanied by a full decade of high inflation. Similarly, the ongoing burst of inflation that is characterizing the recovery from the pandemic has been far more persistent than what many expected. We now show that scarring effects help explain why supply disruptions might trigger sustained rises in inflation.

Figure 4 illustrates the inflation response to the same transitory negative supply shock considered in Section 3.2, assuming that monetary policy maintains the economy at full employment ( $L_t = \bar{L}$ ).<sup>26</sup> What stands out is that the Keynesian growth model exhibits much more inflation persistence than the exogenous growth one. Let’s see why.

Start by considering the exogenous growth model ( $g_t = \bar{g}$ ). To gain analytic insights, abstract for a second from wage indexation ( $\lambda = 0$ ). Then equation (26) implies

$$\bar{\pi}_t = \frac{Z_{t-1}}{Z_t} = \begin{cases} \frac{1}{Z_0} & \text{for } t = 0 \\ Z_0^{(1-\rho)\rho^{t-1}} & \text{for } t > 0. \end{cases} \quad (27)$$

Since  $Z_0 < 1$ , on impact the negative shock triggers a rise in inflation. Intuitively, lower productivity drives down the equilibrium real wages. Since nominal wages are rigid, the drop in real wages takes place through a burst of inflation. These dynamics capture the standard view that negative supply shocks are inflationary (Blanchard and Galí, 2007a,b). The rise in inflation is, however, extremely transitory. As productivity recovers, the reason is, firms’ marginal costs decline over time, which results in a period of inflation below target.<sup>27</sup>

Now turn to the Keynesian growth model. To derive intuition, again assume that  $\lambda = 0$  and that the ratio of innovation spending to GDP is close to zero, so that  $c_t \approx \Psi Z_t L_t$ . Equation (26)

<sup>26</sup>We set the parameter capturing inflation persistence to  $\lambda = 0.5$ , in line with the estimates provided by Barnichon and Mesters (2020). All the other parameters are kept as in Section 3.2.

<sup>27</sup>Inflation falls below its steady state value for periods  $t > 0$ , as long as  $\lambda$  is not too large. For instance, this is the case in the example shown in Figure 4. With sufficient wage indexation to past inflation, the rise in inflation triggered by a transitory negative productivity shock could be persistent.

then implies

$$\bar{\pi}_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} = \begin{cases} \frac{1}{Z_0} & \text{for } t = 0 \\ \frac{\bar{g}/\beta}{\chi\varpi Z_0^{\rho^t} + \eta} & \text{for } t > 0. \end{cases} \quad (28)$$

Therefore, on impact the rise in inflation is exactly the same as in the exogenous growth case. Subsequently, however, the economy experiences a prolonged period of inflation above target.

What drives the inflation response? As argued above, the supply disruption triggers a drop in investment and in the endogenous component of productivity growth  $g_t$ . In turn, lower productivity growth sustains firms' marginal costs, leading to higher inflation. Thus, scarring effects reinforce the rise in inflation caused by the supply disruption. Moreover, the additional inflation due to scarring arises with a delay. This happens because current investment decisions affect productivity with a one-period lag. Hence if a supply shock disrupts investment in period  $t$ , the inflation triggered by the decline in productivity will only be felt in period  $t + 1$ .<sup>28</sup> This delay explains why, with scarring effects, a temporary supply disruption gives rise to a protracted period of stubbornly high inflation.<sup>29</sup>

## 4.2 Disinflation attempts by the central bank

What if the central bank seeks to contain inflation by implementing a tight monetary stance? The conventional view is that a monetary contraction leads to a rise in unemployment, which - due to the Phillips curve logic - lowers inflation. This force is present in our framework, but there is more. As we argued in Section 3.3, a monetary tightening causes a drop in firms' investment. The additional decline in productivity growth, in turn, tends to push inflation up. This effect happens with a delay, since it takes time for investment to have an impact on productivity. A monetary tightening may thus be partially self-defeating, since it may reduce inflation initially, but at the cost of higher inflation in the future.

To illustrate this effect, we start by considering a purely transitory monetary tightening. That is, assume that the central bank raises the nominal interest rate in period 0, but brings it back to its steady state from  $t = 1$  on. It is convenient to abstract from wage indexation ( $\lambda = 0$ ), and to exploit once again the approximation  $c_t \approx \Psi Z_t L_t$ . Combining (Eq1)-(Eq2) and (26), and using

<sup>28</sup>Indeed, the endogenous component of productivity does not react to the shock in period 0, since it is determined by investment decisions taken before the shock was foreseen. This is the reason why the Keynesian growth economy experiences the same rise in inflation in  $t = 0$  as the exogenous growth one.

<sup>29</sup>In our model, we have followed standard practice in the literature and assumed a one-period lag between the time of investment and the time when the higher productivity materializes. However, it is easy to imagine a scenario in which investment projects only pay off several periods in the future. For instance, Aghion et al. (Forthcoming) show that, following shocks to demand conditions faced by firms, it takes 2-5 years before a patent response of firms materializes, highlighting the time required to innovate. Comin and Gertler (2006) argue that, due to long diffusion lags, it takes on average about 10 years before new technologies are adopted by firms. By taking such additional lags into account, our model would predict even more endogenous persistence of the inflation spell triggered by supply disruptions.



the definition of the nominal interest rate  $1 + i_t = (1 + r_t)\pi_{t+1}$ , gives that

$$L_0 = \bar{L} \frac{1 + \bar{i}}{1 + i_0}$$

$$g_1 = \bar{g} \frac{1 + \bar{i}}{1 + i_0}$$

where  $1 + \bar{i} = \bar{g}/\beta$  is the nominal interest rate in steady state. The monetary tightening, as in the conventional view, lowers employment and the output gap on impact. However, it also depresses investment, and so future productivity growth. Using (26), we can then back out the response of inflation

$$\pi_0 = \left( \frac{1 + \bar{i}}{1 + i_0} \right)^\xi$$

$$\pi_1 = \frac{1 + i_0}{1 + \bar{i}}.$$

Thus, the monetary tightening lower inflation on impact, because it depresses employment and nominal wage growth. It does, however, increase inflation with a one-period delay, since lower productivity growth translates into higher future production costs.

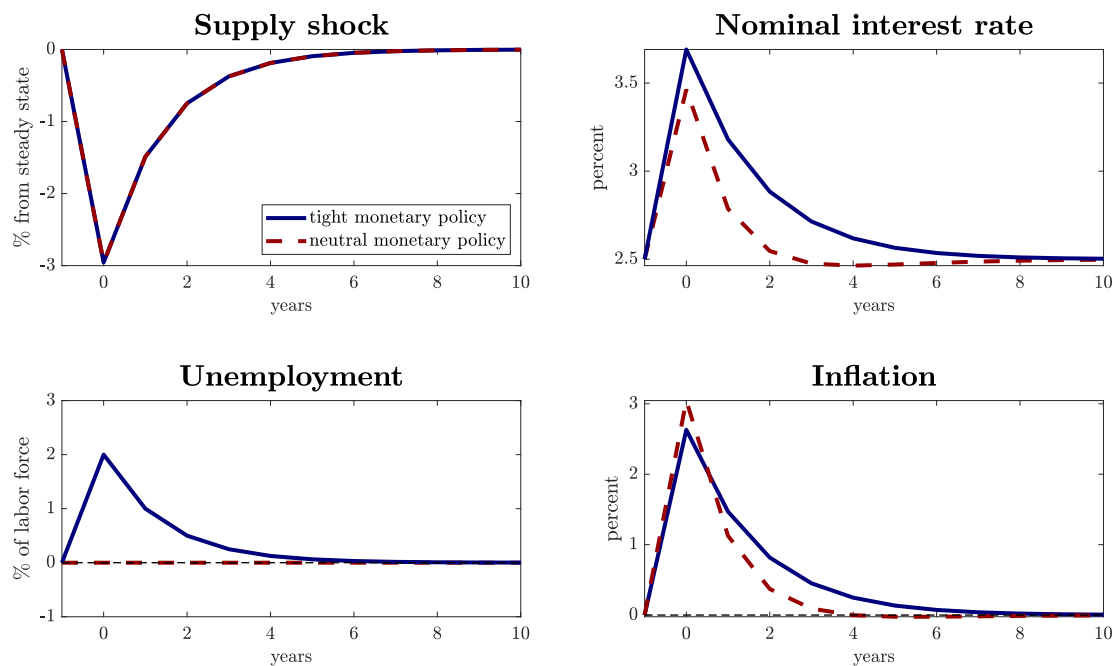
These insights are more general, and extend to an arbitrary process for the nominal interest rate, as we summarize in the following proposition.

**Proposition 3** *Assume that  $\lambda = 0$  and that  $c_t \approx \Psi Z_t L_t$ . Consider the effects of a monetary policy shock,  $i_t \geq \bar{i}$  for all  $t \geq 0$ . We have three results. First,  $L_t < \bar{L}$  whenever  $i_t > \bar{i}$ . Second,  $i_0 > \bar{i}$  implies that  $\pi_0 < 1$ . Third,  $\pi_{t+1} > 1$  whenever  $i_t > \bar{i}$ .*

Proposition 3 makes two points. First, tight monetary policy depresses employment (and thus wage inflation). Second, despite this fact, a monetary tightening reduces price inflation only on impact - that is in the period in which the monetary shock comes as a surprise. Thereafter, inflation rises above its steady state value. Intuitively, this happens because in our baseline model the unemployment effect - driving inflation down - is always dominated by the scarring effect - driving inflation up. This is true except in the impact period, when firms' productivity is predetermined with respect to the monetary shock.

At this point, it is useful to mention two caveats. First, Proposition 3 abstracts from wages indexation to past inflation. With indexation, a monetary tightening could lead to a persistent drop in inflation. Second, throughout the paper we are focusing on a linear investment function. With decreasing returns to investment in innovation the scarring effects are milder, as we discussed in Section 3.2. The implication is that, with decreasing returns to investment, one can think of scenarios in which the unemployment effect dominates the scarring one, and so a monetary tightening persistently lowers inflation. That said, even in this case the impact of a monetary tightening on inflation would be smaller than in an economy with exogenous growth.

To close this section, we consider the impact of a monetary tightening during a supply disruption by going back to our numerical example. In particular, we consider a scenario in which, in response



**Figure 5: Tight monetary policy partially backfires.**

to our usual negative supply shock, the central bank tightens the nominal interest rate to target the same unemployment path as in Figure 3.<sup>30</sup> Figure 5 shows the results, by contrasting the tight monetary policy, inducing some unemployment, with a neutral monetary policy that maintains full employment at all times.

As expected, the monetary tightening causes a reduction in inflation on impact. In the medium run, however, the monetary tightening becomes self-defeating. In fact, the productivity scars associated with tight money eventually push inflation above its value under the neutral policy. These results sound a note of caution on the macroeconomic impact of blunt monetary tightenings in response to supply disruptions. Not only a monetary tightening is likely to lead to lower investment and productivity growth, but it may also fail to mitigate significantly inflation in the medium run.

### 4.3 Protecting productive capacity during a disinflation

Scarring effects appear to put central banks in front of a dilemma: reducing inflation in the present comes at the cost of lower productivity growth and higher inflation in the future. This dynamic trade-off arises because monetary tightenings damage the future productive capacity of the economy through their negative effect on investment. But this line of reasoning also suggests a possible way

<sup>30</sup>We set the slope of the Phillips curve to  $\xi = 0.2$ . As we show in Appendix B.5, this slope is consistent with a model in which wage adjustment costs are calibrated to match a one-year duration of wage contracts. Moreover, a value of  $\xi = 0.2$  falls into the range of the empirical Phillips curves estimates for the United States. For instance, Barnichon and Mesters (2020) estimate a slope of 0.4, while Hazell et al. (2022) estimate a flatter Phillips curve with a slope close to 0.1. All other parameters are kept as in the earlier figures. In particular, as in Figure 4, we set the wage indexation parameter to  $\lambda = 0.5$ .

out of this dilemma. A successful disinflation can be the outcome of a monetary tightening that is coupled with fiscal interventions aiming at protecting business investment, especially in innovation.

Imagine that the government subsidizes investment in innovation at rate  $s_t$ , financing the subsidy with lump-sum taxes.<sup>31</sup> With this subsidy in place, firms' investment maximizes

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - (1 - s_t) \eta P_t I_{j,t}), \quad (29)$$

subject to the law of motion (9). Optimal investment in innovation now implies that

$$\frac{1 - s_t}{\chi} = \beta \frac{c_t}{c_{t+1} g_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1 - s_{t+1}}{\chi} \right) \right). \quad (30)$$

Naturally, a rise in the subsidy to innovation (i.e. an increase in  $s_t$ ) leads firms to invest more which generates faster productivity growth.

To build up intuition, consider a case in which the government grants a subsidy in period 0 ( $s_0 > 0$ ), but not in any subsequent period. Assume that the central bank sets monetary policy so that employment is not affected by this fiscal intervention, so that  $L_t = \bar{L}$  throughout, and that the exogenous component of productivity is constant and equal to its steady state value ( $Z_t = 1$ ). Using a bit of algebra, one can show that

$$g_1 = \frac{\bar{g} (\Psi \chi \bar{L} + 1)}{\chi (\Psi \bar{L} - s_0 \bar{c}) + 1}.$$

Unsurprisingly, investment in innovation and productivity growth are both increasing in the subsidy. This happens because, given our assumptions about monetary policy, the subsidy increases the fraction of output devoted to investment. Indeed, to prevent the economy from overheating, the central bank has to hike the nominal rate in period 0 according to

$$1 + i_0 = \frac{1 + \bar{i}}{1 - \frac{g_1 - \bar{g}}{c\chi}}.$$

A higher subsidy is thus associated with a higher policy rate, to ensure that consumption declines so as to offset the impact of higher investment on aggregate demand.

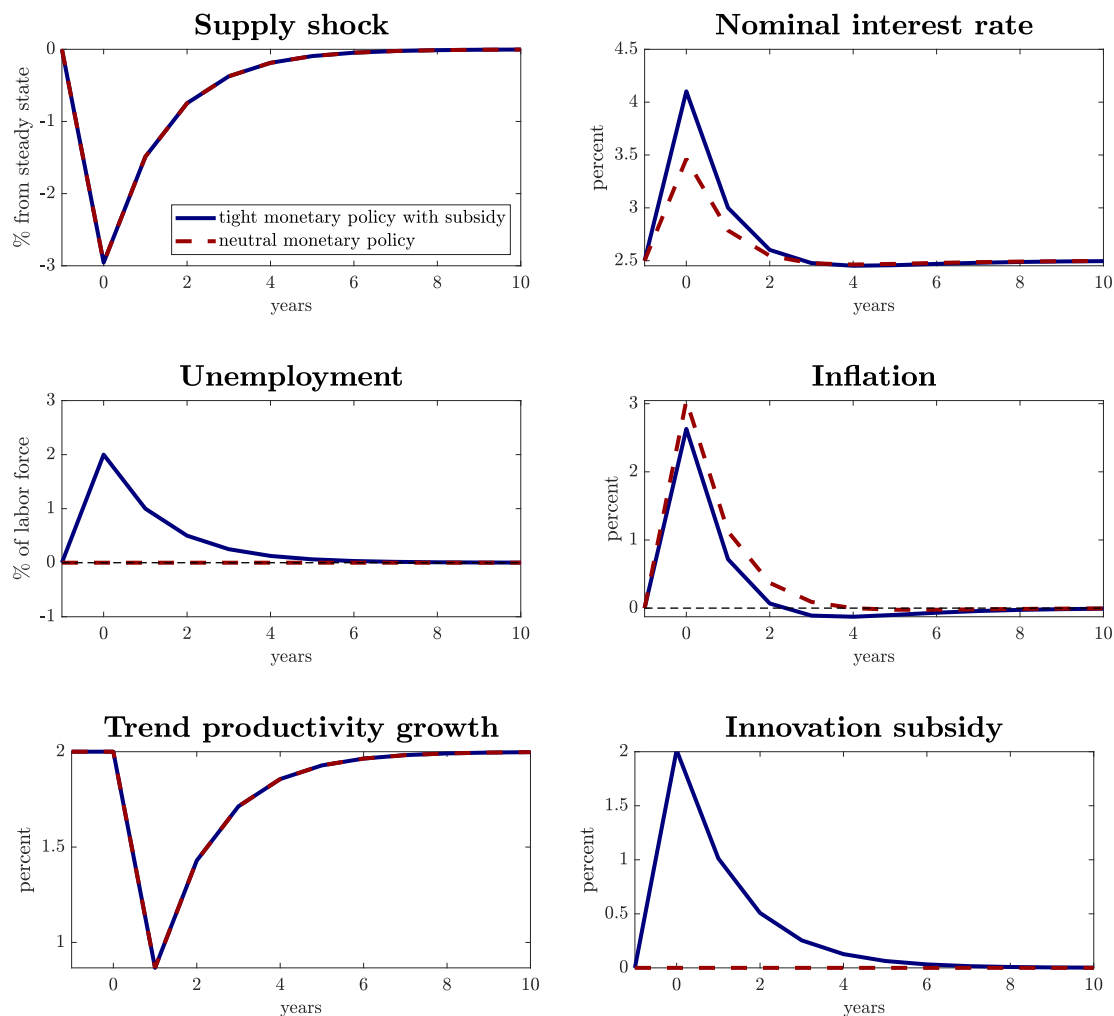
What about inflation? Since monetary policy counteracts the impact of the subsidy on aggregate demand in period 0, on impact inflation is not affected. Higher productivity growth, however, translates into lower inflation in period 1. More precisely, period 1 inflation is equal to

$$\pi_1 = \frac{\bar{g}}{g_1} = \frac{\chi (\Psi \bar{L} - s_0 \bar{c}) + 1}{(\Psi \chi \bar{L} + 1)}.$$

and it is therefore decreasing in the subsidy  $s_0$ . The message is that subsidizing investment dampens inflation in the medium run.

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<sup>31</sup>Since there are no financial frictions, it doesn't matter whether lump-sum taxes are levied on firms or households.



**Figure 6: Monetary tightening with subsidies to investment.**

We now illustrate how supporting firms' investment during a supply disruption can help a tight monetary policy in combating inflation. Let's use our, by now familiar, numerical example. As in the previous section, we consider a monetary tightening that produces the same unemployment path considered in Figure 4. But now we assume that the government shields firms' investment from the disinflation, by adjusting the subsidy so as to maintain the same growth rate of productivity as in the natural allocation ( $g_{t+1} = \bar{g}_{t+1}$ ). This policy mix ensures that the monetary tightening does not have any negative effect on the economy's productive capacity.

Figure 6 shows the economy's response to this policy mix, by contrasting it with an economy in which monetary policy is neutral ( $L_t = \bar{L}$ ) and in which there is no subsidy. The key result is that the monetary tightening, coupled with investment subsidies, successfully reduces inflation throughout the whole duration of the supply disruption. This contrasts with the solitary monetary tightening considered before, which lead to higher inflation in the medium run. Hence, subsidizing business investment during a disinflation improves the sacrifice ratio, that is the amount of inflation reduction brought about by a given rise in unemployment.

Taking stock, during a disinflation not only overall aggregate demand matters, but also its composition between consumption and business investment. A monetary tightening can have undesirable consequences for inflation, if it damages heavily the economy’s productive capacity by depressing firms’ investment. As we just showed, this problem can be mitigated using appropriately designed fiscal interventions. Interestingly, there are historical antecedents to our proposed policy mix. In fact, fiscal incentives for firms to invest, especially in R&D, were part of the Volcker-Reagan 1980s disinflation package (Blanchard, 1987; Modigliani, 1988). Recent evidence suggests that these fiscal incentives boosted innovation activities and productivity growth in the United States (Akcigit et al., 2018).<sup>32</sup> It would be interesting to evaluate their impact on inflation in future research.

## 5 Conclusion

In this paper, we have revisited the macroeconomic implications of supply disruptions through the lens of a Keynesian growth framework. In our model, negative supply shocks generate very persistent - or even permanent - drops in GDP below its pre-shock trend. These scars of supply shocks depress aggregate demand, and therefore the natural interest rate. Scarring effects also tend to amplify the inflationary impact of supply disruptions and make it more persistent. Monetary tightenings may backfire by depressing productivity growth and increasing inflation in the medium run. A successful disinflation may require a policy mix of monetary tightening coupled with subsidies to business investment.

We conclude by pointing out an insight from the Keynesian growth framework, on which future research can leverage. Traditional New Keynesian analyses focus on the relationship between monetary policy, overall aggregate demand and inflation. In the Keynesian growth framework, in contrast, the composition of aggregate demand between consumption and investment takes a central role. For instance, we have just seen that to predict the impact of a monetary tightening on inflation, it is important to understand how consumption and business investment will react to it. We believe that studying monetary policy in frameworks in which the composition of demand matters can be an exciting area for future research. In recent work, Fornaro and Wolf (2021), we take another step in this direction, but much more is yet to be done.

# Appendix

## A Proofs of all propositions

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<sup>32</sup>More broadly, Cloyne et al. (2022) show empirically that tax cuts on firms tend to boost investment in R&D and future productivity.

## A.1 Proof of Proposition 1

**Proposition 1** *Suppose that the parameters satisfy*

$$\beta(\chi\varpi\bar{L} + \eta) > 1,$$

*and that monetary policy is such that*

$$1 + \bar{r} = \chi\varpi\bar{L} + \eta.$$

*Then there exists a unique full employment steady state. Moreover, this steady state is characterized by  $\bar{g} > 1$ .*

**Proof.** The fact that growth is positive follows directly from the first condition in the proposition, because growth in the full employment steady state is given by  $\bar{g} = \beta(\chi\varpi\bar{L} + \eta)$ . Moreover, the assumption  $1 + \bar{r} = \bar{g}/\beta$  ensures that (Eq1) is consistent with the full employment steady state. Note next that, by using (Eq3),

$$\bar{c} = \Psi\bar{L} - \frac{\bar{g} - 1}{\chi} = \Psi\bar{L} - \frac{\beta(\chi\varpi\bar{L} + \eta) - 1}{\chi} = \bar{L}(\Psi - \beta\varpi) + \frac{1 - \beta\eta}{\chi}.$$

Due to  $\beta < 1$ ,  $\eta \leq 1$  and  $\varpi < \Psi$ , this expression implies that steady state consumption is positive. To see that  $\varpi < \Psi$ , note that  $\varpi/\Psi = \alpha/(1 + \alpha) < 1$ . These arguments also directly imply that the full employment steady state is unique. ■

## A.2 Proof of Proposition 2

**Proposition 2** *Assume that  $Z_t$  is governed by the process (17), and that  $Z_0 < 1$ . If  $\rho > 0$ , the natural interest rate is characterized by  $\bar{r}_t < \bar{r}$  for all  $t \geq 0$ . If  $\rho = 0$ , the natural rate is unchanged in response to the shock,  $\bar{r}_t = \bar{r}$  for all  $t \geq 0$ .*

**Proof.** In the natural allocation it holds that  $L_t = \bar{L}$  at all times. The system of equations (Eq1)-(Eq3) thus collapses to

$$\begin{aligned} \bar{c}_t &= \frac{\bar{g}_{t+1}\bar{c}_{t+1}}{\beta(1 + \bar{r}_t)}, \\ \bar{g}_{t+1} &= \beta \frac{\bar{c}_t}{\bar{c}_{t+1}} (\chi\varpi Z_{t+1}\bar{L} + \eta), \\ \Psi Z_t \bar{L} &= \bar{c}_t + \frac{\bar{g}_{t+1} - 1}{\chi}, \end{aligned}$$

where, as in the main text, we use bars to denote variables in the natural allocation.

Combining the first two equations reveals that

$$1 + \bar{r}_t = \chi\varpi Z_{t+1}\bar{L} + \eta.$$

Using the process (17) and  $Z_0 < 1$ , we can write

$$1 + \bar{r}_t = \chi \varpi Z_0^{\rho^{t+1}} \bar{L} + \eta. \quad (\text{A.1})$$

For  $\rho > 0$  ( $\rho = 0$ ), this expression implies that  $\bar{r}_t < \bar{r}$  ( $\bar{r}_t = \bar{r}$ ) for all  $t \geq 0$ . ■

### A.3 Proof of Proposition 3

**Proposition 3** *Assume that  $\lambda = 0$  and that  $c_t \approx \Psi Z_t L_t$ . Consider the effects of a monetary policy shock,  $i_t \geq \bar{i}$  for all  $t \geq 0$ . We have three results. First,  $L_t < \bar{L}$  whenever  $i_t > \bar{i}$ . Second,  $i_0 > \bar{i}$  implies that  $\pi_0 < 1$ . Third,  $\pi_{t+1} > 1$  whenever  $i_t > \bar{i}$ .*

**Proof.** Since study a monetary policy shock, we abstract from the supply shock and set  $Z_t = 1$  for all  $t$ . Hence the model reduces to

$$\begin{aligned} c_t &= \beta \frac{c_{t+1} g_{t+1} \pi_{t+1}}{1 + i_t} \\ g_{t+1} &= \beta \frac{c_t}{c_{t+1}} (\chi \varpi L_{t+1} + \eta) \\ \Psi L_t &= c_t \\ \pi_t &= \frac{\bar{g}}{g_t} \left( \frac{L_t}{\bar{L}} \right)^\xi. \end{aligned}$$

These are the Euler equation, the growth equation, the resource constraint and the inflation equation, respectively. In the resource constraint, we used our approximation  $(g_{t+1} - 1)/\chi \approx 0$ . Notice that  $g_0$  is predetermined from the point of view of period 0.

In steady state, inflation is equal to zero hence the nominal rate is given by  $1 + \bar{i} = \bar{g}/\beta$  (see the Euler equation), which from the growth equation can also be written as  $1 + \bar{i} = \chi \varpi \bar{L} + \eta$ .

Combining the Euler equation and the growth equation reveals that

$$\frac{1 + i_t}{\pi_{t+1}} = \chi \varpi L_{t+1} + \eta.$$

Comparing this with the steady state expression for  $1 + \bar{i}$ , and recognizing that  $L_{t+1} \leq \bar{L}$ , we can see that  $i_t > \bar{i}$  implies  $\pi_{t+1} > 1$ . This proves the third part of the proposition.

Next, we replace  $\pi_{t+1}$  in the previous expression by the inflation equation

$$\begin{aligned} 1 + i_t &= \pi_{t+1} (\chi \varpi L_{t+1} + \eta) \\ &= \frac{\bar{g}}{g_{t+1}} \left( \frac{L_{t+1}}{\bar{L}} \right)^\xi (\chi \varpi L_{t+1} + \eta) \\ &= (1 + \bar{i}) \frac{L_{t+1}}{L_t} \left( \frac{L_{t+1}}{\bar{L}} \right)^\xi, \end{aligned} \quad (\text{A.2})$$

where in the third line we replace  $g_{t+1}$  with the growth equation to cancel the term  $\chi \varpi L_{t+1} + \eta$ , we replace the consumption ratio  $c_{t+1}/c_t$  by  $L_{t+1}/L_t$  using the resource constraint, and we use

$$\bar{g}/\beta = 1 + \bar{i}.$$

Because  $L_{t+1} \leq \bar{L}$ , equation (A.2) shows that  $i_t > \bar{i}$  entails  $L_t < \bar{L}$ . This proves the first part of the proposition.

Finally, to prove the second part of the proposition, note that  $i_0 > \bar{i}$  implies that  $L_0 < \bar{L}$ , from previous arguments. But in the impact period of the shock,  $g_0$  is predetermined and inflation is hence given by

$$\pi_0 = \left( \frac{L_0}{\bar{L}} \right)^\xi.$$

This shows that  $\pi_0 < 1$  whenever  $i_0 > \bar{i}$ . ■

## B Additional derivations and model extensions

### B.1 Energy price shocks as productivity shocks

Let's consider a country that uses energy in production. For concreteness, all the energy is produced with oil, which is fully imported from the rest of the world. The production function is now given by

$$Y_t = o_t^\gamma \left( (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^\alpha dj \right)^{1-\gamma},$$

where  $o_t$  denotes the quantity of oil used in production and  $0 \leq \gamma < 1$ . We denote the price of oil in real terms (i.e. normalized by  $P_t$ ) as  $p_t^o$ . The price of oil is set exogenously on the global oil markets. The optimal demand for oil by firms implies

$$p_t^o o_t = \gamma Y_t. \tag{B.1}$$

We can then rewrite the production function as

$$Y_t = \left( \frac{\gamma}{p_t^o} \right)^{\frac{\gamma}{1-\gamma}} (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^\alpha dj. \tag{B.2}$$

Following the derivations in the main text, we can further write GDP as

$$Y_t - p_t^o o_t - \int_0^1 x_{j,t} dj = \left( \frac{\gamma}{p_t^o} \right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t, \tag{B.3}$$

where we define  $\tilde{\Psi} \equiv (1 - \alpha^2 - \gamma) \alpha^{\frac{2\alpha}{1-\alpha}}$ .

To close the model, we assume that the country is in financial autarky, so that trade is balanced every period. In this case, the market clearing condition for the final good is

$$\left( \frac{\gamma}{p_t^o} \right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t = C_t + I_t. \tag{B.4}$$

With these results, one can see that the response of the economy to an increase in the price of oil is qualitatively isomorphic to its response to a negative productivity shock.



## B.2 Optimal investment by firms

In this Appendix we derive the optimal investment strategy for firms (11). Firms producing intermediate goods choose investment in innovation to maximize

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}),$$

subject to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}$$

$$I_{j,t} \geq 0,$$

given the initial condition  $A_{j,0}$ . The last constraint takes into account the fact that investment cannot be negative.

We define the following Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t} (1 - \chi v_{j,t})) + \gamma_t (A_{j,t+1} - A_{j,t} - \chi I_{j,t}).$$

Optimal investment satisfies

$$\frac{1}{\chi} - v_{j,t} = \beta \frac{C_t}{C_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{1}{\chi} - v_{j,t+1} \right) \right)$$

$$v_{j,t} I_{j,t} = 0,$$

where  $v_{j,t} \geq 0$ .

Notice that if investment is always positive ( $v_{j,t} = 0$  for all  $t \geq 0$ ) the optimality condition reduces to equation (11). However, investing might not be profitable for firms. This happens if the marginal increase in profits obtained from investing is lower than its marginal cost. For instance, this might happen following very large negative supply shocks, as we mention in footnote 19.

## B.3 Model with diminishing returns to investment

In this Appendix we study the case where firms face diminishing returns to investment. As discussed in Section 3.2, we stress two key results. First, the natural interest rate falls following the negative supply shock when the persistence of the shock is high enough. Second, even when the shock is short-lived implying the natural rate rises, it still rises by less than when growth is exogenous.

Firms' investment technology is now

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}^\xi A_t^{1-\xi}, \tag{B.5}$$

where  $0 < \xi \leq 1$  and where aggregate technology  $A_t = \int_0^1 A_{j,t} dj$  is taken as given by the individual

firm. Under this formulation, firms combine investment and the aggregate stock of knowledge to increase their future productivity.<sup>33</sup> This is a simple way to introduce diminishing returns from investment in innovation.

Firms choose investment in innovation to maximize their expected profits

$$\sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}),$$

subject to (B.5). The non-negativity constraint on investment  $I_{j,t} \geq 0$  never binds in equilibrium, since the return to investment becomes infinity as  $I_{j,t}$  approaches zero. The Lagrangian becomes

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{(\beta\eta)^t}{P_t C_t} (P_t \varpi A_{j,t} Z_t L_t - \eta P_t I_{j,t}) + \gamma_t (A_{j,t+1} - A_{j,t} - \chi I_{j,t}^\xi A_t^{1-\xi}).$$

The optimality condition for investment is

$$\frac{1}{\chi\xi} \left( \frac{I_{j,t}}{A_t} \right)^{1-\xi} = \beta \frac{C_t}{C_{t+1}} \left( \varpi Z_{t+1} L_{t+1} + \frac{\eta}{\chi\xi} \left( \frac{I_{j,t+1}}{A_{t+1}} \right)^{1-\xi} \right).$$

As in the baseline model, the equilibrium can be summarized by three equations. The first one is the households' Euler equation (Eq1). The growth equation (Eq2) now becomes

$$\left( \frac{g_{t+1} - 1}{\chi} \right)^{\frac{1-\xi}{\xi}} g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left( \chi\xi \varpi Z_{t+1} L_{t+1} + \eta \left( \frac{g_{t+2} - 1}{\chi} \right)^{\frac{1-\xi}{\xi}} \right). \quad (\text{B.6})$$

Moreover, the market clearing condition (Eq3) is replaced by

$$\Psi Z_t L_t = c_t + \left( \frac{g_{t+1} - 1}{\chi} \right)^{\frac{1}{\xi}}. \quad (\text{B.7})$$

These expressions generalize the ones from the baseline model to the case  $0 < \xi < 1$ .

We are interested in deriving the path of the natural interest rate following a negative supply shock. To make progress, let us take a log-linear approximation of (B.6)-(B.7) around the full employment steady state to obtain

$$\bar{g} \left( \left[ \frac{1-\xi}{\xi} \frac{\bar{g}}{\bar{g}-1} + 1 \right] \hat{g}_{t+1} - \hat{c}_t + \hat{c}_{t+1} \right) = (\bar{g} - \beta\eta) \hat{Z}_{t+1} + \beta\eta \frac{1-\xi}{\xi} \frac{\bar{g}}{\bar{g}-1} \hat{g}_{t+2}.$$

$$\hat{Z}_t = s_c \hat{c}_t + (1 - s_c) \frac{1}{\xi} \frac{\bar{g}}{\bar{g}-1} \hat{g}_{t+1},$$

where  $s_c \equiv \bar{c}/\Psi\bar{L}$  is the consumption share of GDP in steady state. Here,  $\hat{x}_t \equiv \log(\bar{x}_t) - \log(\bar{x})$  for every variable  $\bar{x}_t$  (again, the bar denotes a variable in the natural allocation).

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<sup>33</sup> Assuming that firms combine investment with their individual stock of knowledge would not change any of the results that follow.

Using  $\hat{Z}_{t+1} = \rho \hat{Z}_t$ , we can guess and verify that the solution to the model can be written as

$$\hat{c}_t = \gamma_c \hat{Z}_t$$

$$\hat{g}_{t+1} = \gamma_g \hat{Z}_t,$$

where the two parameters  $\gamma_c$  and  $\gamma_g$  are given by

$$\gamma_c = \frac{1}{s_c} \frac{\bar{g} + \frac{\bar{g}}{\bar{g}-1} \left( \frac{1-\xi}{\xi} (\bar{g} - \beta\eta\rho) - \frac{1-s_c}{\xi} (\bar{g} - \beta\eta)\rho \right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1} \left( \frac{1-\xi}{\xi} (\bar{g} - \beta\eta\rho) + \frac{1}{\xi} \bar{g} (1-\rho) \frac{1-s_c}{s_c} \right)}$$

$$\gamma_g = \frac{(\bar{g} - \beta\eta)\rho + \frac{\bar{g}(1-\rho)}{s_c}}{\bar{g} + \frac{\bar{g}}{\bar{g}-1} \left[ \frac{1-\xi}{\xi} (\bar{g} - \beta\eta\rho) + \frac{1}{\xi} \bar{g} (1-\rho) \frac{1-s_c}{s_c} \right]}.$$

Log-linearizing (Eq1) yields an equation for the natural interest rate

$$\hat{c}_t = \hat{g}_{t+1} - \hat{r}_t + \hat{c}_{t+1}.$$

Inserting the solution for  $\hat{c}_t$  and  $\hat{g}_{t+1}$ , and rearranging we then obtain

$$\begin{aligned} \hat{r}_t &= (\gamma_g - (1-\rho)\gamma_c) \hat{Z}_t \\ &= \frac{(\bar{g} - \beta\eta)\rho + \frac{\bar{g}(1-\rho)}{s_c} - (1-\rho) \frac{1}{s_c} \left( \bar{g} + \frac{\bar{g}}{\bar{g}-1} \left[ \frac{1-\xi}{\xi} (\bar{g} - \beta\eta\rho) - \frac{1-s_c}{\xi} (\bar{g} - \beta\eta)\rho \right] \right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1} \left[ \frac{1-\xi}{\xi} (\bar{g} - \beta\eta\rho) + \frac{1}{\xi} \bar{g} (1-\rho) \frac{1-s_c}{s_c} \right]} \hat{Z}_t. \end{aligned}$$

Manipulating the expression above shows that the natural interest rate falls following a negative supply shock if and only if

$$\xi > \frac{\rho(\bar{g} - \beta\eta) + (1-\rho) \frac{1}{s_c} \bar{g}}{\frac{1}{s_c} (\bar{g} - \beta\eta\rho) + \frac{\rho}{1-\rho} \frac{\bar{g}-1}{\bar{g}} (\bar{g} - \beta\eta)} \equiv \bar{\xi}. \quad (\text{B.8})$$

Moreover, differentiating the expression above with respect to  $\rho$  gives

$$\frac{\partial \bar{\xi}}{\partial \rho} = - \frac{(\bar{g} - \beta\eta) \left( \frac{1-s_c}{s_c^2} \bar{g} + \frac{1}{1-\rho} \frac{\bar{g}-1}{\bar{g}} \left[ \frac{\rho^2}{1-\rho} (\bar{g} - \beta\eta) + \frac{1}{s_c} \bar{g} (1+\rho) \right] \right)}{\left( \frac{1}{s_c} (\bar{g} - \beta\eta\rho) + \frac{\rho}{1-\rho} \frac{\bar{g}-1}{\bar{g}} (\bar{g} - \beta\eta) \right)^2} < 0.$$

This means that, the stronger the diminishing returns to investment (i.e. the lower  $\xi$ ), the larger the shock persistence must be (i.e. the higher  $\rho$ ) for the interest rate to fall following a negative supply shock.

It is instructive to look at two limiting cases. First, assume that the shock is purely transitory ( $\rho = 0$ ). In this case, (B.8) reduces to  $\bar{\xi} = 1$ . From (B.5), this implies that when the shock is purely transitory, the natural rate unambiguously rises for any degree of curvature  $\xi < 1$ . The second limit case is the one of permanent shocks, in which case  $\rho = 1$ . In this case, (B.8) implies that

$\bar{\xi} = 0$ . Therefore, for permanent shocks the natural rate unambiguously declines for any degree of curvature  $\xi < 1$ .

In sum, when firms face diminishing returns to investment, the negative supply shock must be persistent enough to cause a decline in the natural rate.

Let us now consider a version in which growth is exogenous and there is no investment. Since productivity growth is exogenous, then  $\gamma_g = 0$ , while since all the output is consumed  $\gamma_c = 1$ . Now consider that the natural rate evolves according to  $\hat{r}_t = (\gamma_g - (1 - \rho)\gamma_c)\hat{Z}_t$ , and that under endogenous growth  $\gamma_c < 1$  and  $\gamma_g > 0$ . The implication is that, after a negative supply shock, the natural rate rises by less when trend productivity growth is endogenous, compared to an economy with fully exogenous productivity growth.

#### B.4 Determinacy under the interest rate rule

Here we establish that in the baseline model with the Taylor rule (21), the full employment steady state is locally determinate under condition (22), complementing the analysis from Section 3.3.

The baseline model with interest rate rule can be summarized by the following set of equations

$$c_t = \frac{g_{t+1}}{\beta c_{t+1}^{-1} (1 + \bar{r})} \left(\frac{L_t}{L}\right)^\phi.$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} (\chi \varpi Z_{t+1} L_{t+1} + \eta).$$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}.$$

where the first equation is the combination of (Eq1) and (21).

To show that the steady state is locally determinate, we take a log-linear approximation<sup>34</sup>

$$\hat{g}_{t+1} = \phi \hat{L}_t + \hat{c}_t - \hat{c}_{t+1}$$

$$\Psi \bar{L} \hat{L}_t = \bar{c} \hat{c}_t + \frac{\bar{g}}{\chi} \hat{g}_{t+1}$$

$$\hat{g}_{t+1} = \hat{c}_t - \hat{c}_{t+1} + \frac{\bar{g} - \beta \eta}{\bar{g}} \hat{L}_{t+1}$$

where  $\hat{x}_t \equiv \log(x_t) - \log(\bar{x})$  for every variable  $x_t$ . This system can be written as:

$$\hat{L}_t = \xi_1 \hat{L}_{t+1} + \xi_2 \hat{g}_{t+2}$$

$$\hat{g}_{t+1} = \xi_3 \hat{L}_{t+1} + \xi_4 \hat{g}_{t+2},$$

where

$$\xi_1 \equiv \frac{1}{\phi} \frac{\bar{g} - \beta \eta}{\bar{g}}$$

<sup>34</sup>In the approximation, we keep  $Z_t$  fixed at its steady state value  $Z_t = 1$  as variation in this variable is irrelevant for the determinacy properties of the dynamic system.

$$\begin{aligned}\xi_2 &\equiv 0 \\ \xi_3 &\equiv \frac{\bar{g} - 1}{\bar{g}\frac{\bar{\Psi}\bar{L}}{\bar{c}} - 1} \left( \phi\xi_1 + \frac{\bar{\Psi}\bar{L}}{\bar{c}} (\xi_1 - 1) \right) \\ \xi_4 &\equiv \frac{\bar{g} \left( 1 - \frac{\bar{c}}{\bar{\Psi}\bar{L}} \right)}{\bar{g} - \frac{\bar{c}}{\bar{\Psi}\bar{L}}}.\end{aligned}$$

The system is determinate if and only if:<sup>35</sup>

$$|\xi_1\xi_4 - \xi_2\xi_3| < 1 \tag{B.9}$$

$$|\xi_1 + \xi_4| < 1 + \xi_1\xi_4 - \xi_2\xi_3. \tag{B.10}$$

Condition (B.9) holds if

$$\phi > \frac{(\bar{g} - \beta\eta) \left( 1 - \frac{\bar{c}}{\bar{\Psi}\bar{L}} \right)}{\bar{g} - \frac{\bar{c}}{\bar{\Psi}\bar{L}}},$$

while condition (B.10) holds if

$$\phi > \frac{\bar{g} - \beta\eta}{\bar{g}}. \tag{B.11}$$

Because  $\bar{g} > 1$ , inserting  $\bar{g} = \beta(\chi\varpi\bar{L} + \eta)$  in (B.11) shows that the steady state is locally determinate if and only if condition (22) holds.

## B.5 Model with New Keynesian wage Phillips curve

In this appendix we replace the ad-hoc wage Phillips curve (25) by a microfounded New Keynesian wage Phillips curve. Specifically, we assume that households are monopolistically competitive suppliers of labor. Moreover, we assume that wage changes entail Rotemberg (1982)-type adjustment costs.

### B.5.1 Model

We continue to assume that the economy is populated by a unit mass of households, however, we now make the household index specific:  $k \in [0, 1]$ . Each household  $k$  has utility

$$\sum_{t=0}^{\infty} \beta^t (\log(C_t(k)) - G(L_t(k))),$$

where  $G(L_t(k))$  is positive, increasing and convex in labor supplied  $L_t(k)$ . The budget constraint is given by

$$P_t C_t(k) + \frac{B_{t+1}(k)}{1 + i_t} = W_t(k) L_t(k) + B_t(k) + D_t.$$

Households' Euler equation is as in the baseline model. We discuss households' labor supply choice below.

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<sup>35</sup>See Bullard and Mitra (2002).

Final goods firms' production technology is still given by

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^\alpha dj.$$

However, labor demand  $L_t$  is now a composite of the labor supplied by the different households  $k \in [0, 1]$

$$L_t = \left( \int_0^1 L_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (\text{B.12})$$

where  $\varepsilon > 1$  denotes the elasticity of substitution among different labor varieties. Firms' optimal labor demand solves

$$\min_{\{L_t(k)\}_{k \in [0,1]}} \int_0^1 W_t(k) L_t(k) dk \quad \text{subject to (B.12),}$$

by taking as given  $\{W_t(k)\}_{k \in [0,1]}$ . The first order condition is

$$L_t(k) = \left( \frac{W_t(k)}{W_t} \right)^{-\varepsilon} L_t. \quad (\text{B.13})$$

We now turn to labor supply. Each household maximizes utility subject to household-specific labor demand (B.13), subject to the budget constraint and subject to a wage adjustment cost. The optimization problem is to maximize the following objective function:

$$\sum_{t=0}^{\infty} \beta^t \left( \frac{1}{P_t C_t(k)} \left( W_t(k) L_t(k) - \frac{\theta}{2} A_t P_t \left( \frac{W_t(k)}{W_{t-1}(k)} - \bar{g} \right)^2 \right) - G(L_t(k)) \right)$$

subject to (B.13), where  $\theta \geq 0$  is the adjustment cost parameter. The adjustment cost is multiplied with  $A_t P_t$  to ensure the existence of a balanced growth path. Moreover, we assume that wage inflation is indexed to the growth rate of productivity in the full employment steady state,  $\bar{g}$  (we also make this assumption in the baseline model, see (25)). This ensures that the wage rigidities do not bind in steady state. Note that, to keep it simple, in the present model we do not assume indexation of wages to past inflation, an assumption that we made in the main text.

The first order condition is

$$\begin{aligned} \frac{1}{P_t C_t(k)} \left( (1 - \varepsilon) L_t(k) - \theta A_t P_t \left( \frac{W_t(k)}{W_{t-1}(k)} - \bar{g} \right) \frac{1}{W_{t-1}(k)} \right) - G'(L_t(k)) (-\varepsilon) \frac{L_t(k)}{W_t(k)} \\ + \beta \frac{1}{P_{t+1} C_{t+1}(k)} \theta A_{t+1} P_{t+1} \left( \frac{W_{t+1}(k)}{W_t(k)} - \bar{g} \right) \frac{W_{t+1}(k)}{W_t(k)^2} = 0. \end{aligned}$$

We now assume that  $W_{-1}(k) = W_{-1}$ , that is, all households face identical initial conditions. Because households face identical problems, this implies that in equilibrium, households make identical decisions. From now on, we thus omit the household index  $k$ .

Denoting nominal wage inflation  $\pi_t^W$  we can write

$$\frac{A_t}{C_t} \theta (\pi_t^W - \bar{g}) \pi_t^W - \beta \frac{A_{t+1}}{C_{t+1}} \theta (\pi_{t+1}^W - \bar{g}) \pi_{t+1}^W = \varepsilon L_t \left( G'(L_t) - \frac{\varepsilon - 1}{\varepsilon} \frac{W_t}{P_t} \frac{1}{C_t} \right). \quad (\text{B.14})$$

This wage Phillips curve replaces the reduced-form rule (25) in the main text.

The rest of the model is unchanged from the baseline model. In particular, we assume that the wage adjustment cost is rebated in a lump-sum manner to households in equilibrium, ensuring that the resource constraint of the economy is the same as in the baseline model. This implies the only difference to the baseline model is equation (B.14), which replaces equation (25).

### B.5.2 Equilibrium

Given a path for the supply shock  $\{Z_t\}_{t=0}^{+\infty}$  and a path for monetary policy  $\{i_t\}_{t=0}^{+\infty}$ , an equilibrium is a set of processes  $\{c_t, L_t, g_{t+1}, \pi_t, \pi_t^W\}_{t=0}^{+\infty}$  satisfying  $g_{t+1} > 1$ ,  $L_t > 0$ ,  $c_t > 0$  as well as the following equations for all  $t \geq 0$ . Households' Euler equation

$$\frac{g_{t+1}}{c_t} = \beta (1 + i_t) \pi_{t+1}^{-1} \frac{1}{c_{t+1}}, \quad (\text{B.15})$$

an equation linking price inflation to firms' marginal costs

$$\pi_t^W = \frac{Z_t}{Z_{t-1}} g_t \pi_t, \quad (\text{B.16})$$

the evolution of wage inflation, obtained by combining (B.14) and (4),

$$(\pi_t^W - \bar{g}) \pi_t^W - \beta \frac{c_t}{c_{t+1}} (\pi_{t+1}^W - \bar{g}) \pi_{t+1}^W = \frac{\varepsilon}{\theta} L_t \left( \frac{G'(L_t)}{c_t^{-1}} - \frac{\varepsilon - 1}{\varepsilon} (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}} Z_t \right), \quad (\text{B.17})$$

the growth equation

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} (\chi \varpi Z_{t+1} L_{t+1} + \eta), \quad (\text{B.18})$$

and the market clearing condition

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}. \quad (\text{B.19})$$

The flexible-wage allocation (the natural allocation) is nested for  $\theta \rightarrow 0$ . Denoting natural variables with a bar, note that in contrast to the baseline model, the natural level of employment  $\bar{L}_t$  is now time-varying.

### B.5.3 Steady state

We focus on the full employment steady state with zero inflation. In steady state,  $\bar{Z} = 1$ ,  $L_t = \bar{L}$  and  $\pi_t = 1$ , implying  $\pi_t^W = \bar{g}$ . The system of equations collapses to

$$\begin{aligned}\bar{g} &= \beta(1 + \bar{i}) \\ \frac{G'(\bar{L})}{\bar{c}^{-1}} &= \frac{\varepsilon - 1}{\varepsilon}(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \\ \bar{g} &= \beta(\chi\varpi\bar{L} + \eta), \\ \Psi\bar{L} &= \bar{c} + \frac{\bar{g} - 1}{\chi}.\end{aligned}$$

Relative to the baseline model,  $\bar{L}$  is now an endogenous variable, determined in the previous system of equations. More precisely, the previous system determines  $\{\bar{c}, \bar{L}, \bar{g}, \bar{i}\}$ , for given structural parameters.

### B.5.4 Calibration

To illustrate the properties of this version of the model, we resort to numerical simulations. First we make sure that the baseline model and the model presented in this section share the same steady state. To do so, we assume that  $G(\cdot)$  is given by the conventional constant-Frisch elasticity function, that is

$$G(L_t) = \zeta \frac{L_t^{1+\varphi}}{1+\varphi},$$

where  $\varphi > 0$  is the inverse Frisch elasticity of labor supply, and where  $\zeta > 0$  is a parameter. For given other parameters, we choose  $\zeta$  such that the full employment steady state is characterized by the same  $\bar{L}$  as in the baseline model (given by  $\bar{L} = 1$ ). Because the real side of this model and the baseline model is the same, this implies that the steady state values for all real variables are also identical. We thus set the same parameters  $\alpha, \beta, \chi$  and  $\eta$  as in the baseline model, to hit the same targets in steady state. This implies  $\chi = 1.95$ ,  $\beta = 0.995$ ,  $1 - \alpha = 0.86$  and  $\eta = 0.9$  (see Section 3). The implied value for  $\zeta$  is 0.8036.

Next, we choose 3 additional parameters related with the wage Phillips curve. We set the values of two of the three parameters in the range of those commonly used by the literature (e.g., Galí, 2011). First we assume that  $\varphi = 3$  for the inverse Frisch elasticity. Second, we set the elasticity of substitution between different labor types to  $\varepsilon = 10$ .

To calibrate the wage stickiness parameter, we start by drawing a parallel between the wage Phillips curve implied by our model and the one that would emerge under Calvo frictions in wage adjustment. Up to a first-order approximation, the wage Phillips curve (B.17) is given by

$$\hat{\pi}_t^w = \beta\hat{\pi}_{t+1}^w + \frac{\varepsilon - 1}{\theta}(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(\varphi\hat{L}_t + \hat{c}_t - \hat{Z}_t), \quad (\text{B.20})$$

where a hat above a variable denotes log-deviation from steady state,  $\hat{x}_t \equiv \log(x_t) - \log(\bar{x})$ . In



turn, had we set up our model using a Calvo framework, the Phillips curve would be (see [Born and Pfeifer \(2020\)](#))

$$\hat{\pi}_t^w = \beta \hat{\pi}_{t+1}^w + \frac{(1 - \theta_c)(1 - \beta\theta_c)}{\theta_c(1 + \varepsilon\varphi)}(\varphi \hat{L}_t + \hat{c}_t - \hat{Z}_t),$$

where  $\theta_c$  is the Calvo-type wage stickiness parameter.

Our model is calibrated at yearly frequency. The level of wage rigidities we use is given by  $\theta_c = 0.3164$ . This implies a per-quarter stickiness probability of  $0.3164^{1/4} = 0.75$ , and thus an average duration of a wage contract of  $1/(1 - 0.75) = 4$  quarters - or one year. This level of wage duration is in line with the empirical evidence (e.g., [Olivei and Tenreyro, 2007](#)). We then solve for the implied  $\theta$  in order to match the slopes of the two Phillips curves. This procedure implies  $\theta = 84.45$ .

How does the slope of the Phillips curve derived above compare to the slope of the Phillips curve that we used in [Section \(4.2\)](#) in the main text? In the main text, we considered a slope of  $\xi = 0.2$ . This number refers to the slope with respect to unemployment, whereas what appears in [equation \(B.20\)](#) is the slope with respect to the marginal-substitution-wage gap. To make the two numbers comparable, we write [\(B.20\)](#) as

$$\hat{\pi}_t^w = \beta \hat{\pi}_{t+1}^w + \frac{\varepsilon - 1}{\theta}(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(1 + \varphi)\hat{L}_t,$$

where we use  $\hat{c}_t \approx \hat{Z}_t + \hat{L}_t$  from the resource constraint [\(B.19\)](#), using again that the share of GDP going to investment in innovation is small. Because  $\hat{L}_t$  is (minus) the log of unemployment, the slope coefficient multiplying  $\hat{L}_t$  is exactly the elasticity of wage inflation with respect to unemployment. Our calibration implies  $slope = 0.191$ , which is therefore close to the value of 0.2 used in the main text.

### B.5.5 Can supply disruptions cause persistent rises in inflation?

We next go through the numerical simulations from [Section 4](#) to show that the model with micro-founded wage Phillips curve produces similar predictions for inflation as our baseline model.<sup>36</sup> We start by establishing that scarring effects make the inflation spell larger and more persistent.

The result is shown in [Figure 7](#), which is structured in the same way as [Figure 4](#) from the main text. That is, the figure shows the inflation response following a negative supply shock conditional on the central bank targeting full employment at all times. Again we contrast two cases, our baseline model and an exogenous-growth case where firms' investment and hence productivity growth are independent from the shock.

As we can see, the model with Rotemberg-type wage rigidity reproduces the main result from the main text, namely that scarring effects reinforce the inflationary impact of supply disruptions and make it more persistent. Compared to [Figure 4](#), we also see that the inflation spell is overall

<sup>36</sup>We don't need to repeat the analysis involving real variables from [Section 3](#), as the responses of all real variables are identical across the two models. This follows because the real side of the model is unchanged relative to the baseline model.

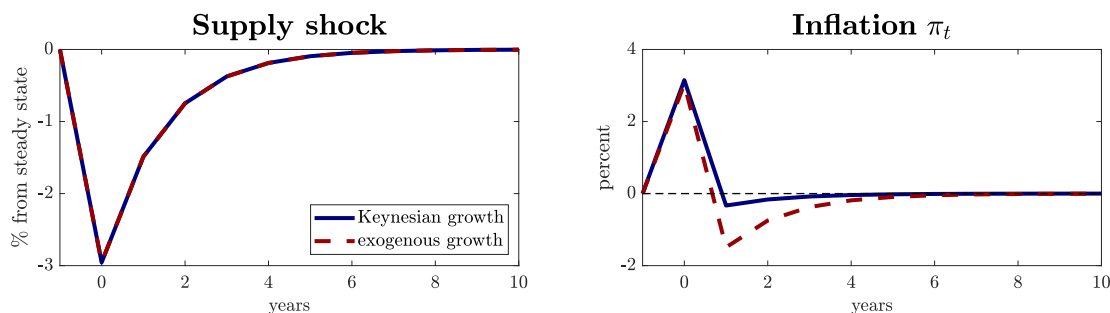


Figure 7: Inflationary impact of scarring effects. Model with Rotemberg-type wage rigidity.

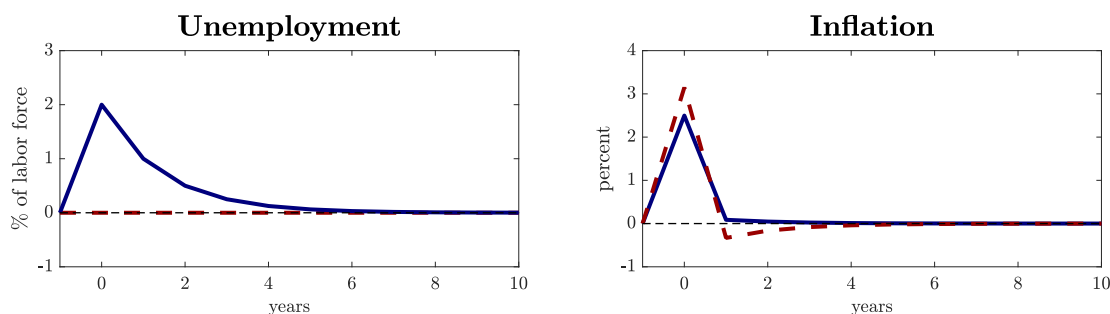


Figure 8: Disinflation attempts by the central bank. Model with Rotemberg-type wage rigidity.

less persistent, and that the period of inflation in the exogenous growth case is followed by a period of sharp deflation. This happens because, for simplicity, we have abstracted from indexation of wages to past inflation in the present environment.

### B.5.6 Disinflation attempts by the central bank

We next show that the model with Rotemberg-type wage rigidity also preserves the main result from Section 4.2, namely that attempts by the central bank to lower inflation can backfire.

The result is in Figure 8, which is structured in the same way as Figure 5 from the main text. That is, we assume the central bank sets its nominal interest rate to target the unemployment path from Figure 3, in an attempt to reduce inflation. We find that the central bank is successful initially at reducing inflation. However, due to the additional scarring the central bank induces, inflation is actually increased in the medium run, despite the fact that unemployment is still elevated.

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