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EXPECTATIONS-DRIVEN LIQUIDITY TRAPS: IMPLICATIONS FOR MONETARY AND FISCAL POLICY

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Expectations-Driven Liquidity Traps:
Implications for Monetary and Fiscal Policy[*]

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Abstract

We study optimal time-consistent monetary and fiscal policy in a New Keynesian model where occasional declines in agents' confidence give rise to persistent liquidity trap episodes. Insights from widely-studied fundamental-driven liquidity traps are not a useful guide for enhancing welfare in this model. Raising the inflation target, appointing an inflation-conservative central banker, or allowing for the use of government spending as an additional stabilization tool can exacerbate deflationary pressures and demand deficiencies during the liquidity trap episodes. However, appointing a policymaker who is sufficiently less concerned with government spending stabilization than society eliminates expectations-driven liquidity traps.

Keywords: Effective Lower Bound, Sunspot Equilibria, Monetary Policy, Fiscal Policy, Discretion, Policy Delegation

JEL-Codes: E52, E61, E62

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1   Introduction

The recent decade of low nominal interest rates and anemic inflation poses new challenges for monetary and fiscal policy. Current policy frameworks were predominantly designed at a time when the lower bound on nominal interest rates was not a major concern for central banks, and discretionary fiscal policy was not widely considered as an essential part of stabilization policies.[1]

This paper studies the implications of the lower bound on nominal interest rates for optimal monetary and fiscal policy in a standard New Keynesian model. The key difference to the existing literature on optimal policy with a lower bound is that we consider an equilibrium where liquidity trap episodes—i.e. periods where the lower bound constraint is binding—result from a self-fulfilling decline in people's confidence rather than from a deterioration of economic fundamentals. When confidence declines, people become more pessimistic about the economic outlook and lower their inflation expectations. The central bank cuts the policy rate in an attempt to stabilize inflation, inadvertently reinforcing the decline in inflation expectations. The resulting feedback loop culminates in a binding interest-rate lower bound, inflation below target and subdued economic activity. The concept of such expectations-driven liquidity traps is often used to characterize Japan's prolonged period of close-to-zero nominal interest rates and very low inflation. More recently, policymakers have raised concerns that other jurisdictions, too, are in danger of getting caught in a Japanese-style liquidity trap.[2]

We focus on two questions. First, is there a foolproof way to improve stabilization outcomes and welfare, taking as given the occasional occurrence of expectations-driven liquidity traps? Second, is it possible to prevent the economy from falling into an expectations-driven liquidity trap? Following the policy delegation literature (e.g. Rogoff, 1985; Walsh, 1995; Svensson, 1997), we address these questions by assuming that society designs the policy framework and a discretionary policymaker sets the policy instruments in accordance with the assigned objective function.

We first study monetary policy in the absence of fiscal stabilization policy. In this case, the policymaker has only one instrument, the short-term nominal interest rate, to stabilize economic activity and inflation. In our model, agents' confidence about the economic outlook is governed by 'sunspots' (Cass and Shell, 1983)—non-fundamental random variables that affect the economy if people believe it will. The sunspot shocks make macroeconomic stabilization non-trivial. In the sunspot equilibrium under optimal discretionary policy, the economy occasionally falls into a potentially long-lasting liquidity trap driven by a self-fulfilling decline in agents' confidence. These

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[1]Recently, the U.S. Federal Reserve, the European Central Bank, and the Bank of Canada have been officially reviewing their monetary policy frameworks. These central banks explicitly refer to the challenges for monetary policy associated with the lower bound (Wilkins, 2018; Clarida, 2019; Lagarde, 2020). See Eichenbaum (1997) for a summary of the pre-crisis consensus view on discretionary fiscal stabilization policy.

[2]In the words of Fed Chairman Jerome H. Powell: "You don't want to get behind the curve and let inflation drop well below 2%, because what happens is, you get into this unhealthy dynamic, potentially, where lower expected inflation gets baked into interest rates, which means lower interest rates, which means less room for the central bank to react. And that becomes a self-reinforcing thing. We've seen it in Japan and are now seeing it in Europe." (Semiannual Monetary Policy Report to the Congress, 11 July, 2019, Before the Committee on Financial Services, U.S. House of Representatives). See also de Guindos (2019).

expectations-driven liquidity traps are characterized by subdued economic activity and deflation. While liquidity traps are rare events in the model, agents' anticipation of possible future liquidity trap events gives rise to a systematic inflation shortfall even when confidence is high and the economy is not in a liquidity trap.

We consider two modifications of the monetary policy objective function that are known to improve stabilization outcomes and welfare in models with fundamental-driven liquidity traps. The first modification imposes a positive inflation target, and the second makes inflation stabilization the primary policy objective.[3] Both approaches reduce the inflation shortfall away from the lower bound. In models with fundamental-driven liquidity traps, higher inflation away from the lower bound mitigates the drop in output and inflation at the lower bound. However, in our model with expectations-driven liquidity traps, we find that increasing the inflation target or increasing the relative weight of the inflation objective reduces output and inflation in the state where confidence is low and the lower bound is binding. All else equal, both policy changes lead to an increase in households' desired consumption. But market clearing in the low-confidence state cannot be achieved by an increase in income, for any increase in income would, in turn, result in a disproportionate increase in desired consumption. The only way for markets to clear is that the relative price of consumption today, i.e. the real interest rate, increases. This required increase in the real rate is brought about by a drop in low-state inflation. In equilibrium, then, the increase in the real interest rate further depresses aggregate demand.

As a result of these opposite effects on inflation at and away from the lower bound, the optimal inflation target in the sunspot equilibrium can be negative or positive. Likewise, the optimal weight on inflation relative to output stabilization in the policymaker's objective function can be smaller or larger than the one in society's objective function.

Next, we turn to fiscal stabilization policy, and allow the policymaker to use government spending as an additional policy tool.[4] As in models with fundamental-driven liquidity traps, the optimizing policymaker raises government spending whenever the lower bound constraint binds. However, unlike in models with fundamental-driven liquidity traps—where the government spending stimulus improves allocations at and away from the lower bound—the persistent increase in government spending is deflationary and worsens stabilization outcomes both at and away from the lower bound. At the lower bound, more deflation raises the real interest rate and aggravates the drop in the output gap. More deflation at the lower bound also reduces conditional inflation expectations away from the lower bound and worsens the trade-off between output gap and inflation stabilization in the state where confidence is high and the lower bound constraint is slack.

Acting under discretion, the policymaker fails to internalize the adverse effect of the persistent government stimulus on expectations. Taking as given the occasional occurrence of expectations-driven liquidity traps, it is therefore best for society to disincentivize the use of government spending as a stabilization tool. To do so, society has to assign a sufficiently high relative weight to govern-

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[3]The latter approach is usually referred to as inflation conservatism and goes back to Rogoff (1985).

[4]Like most of the related literature, we assume that the provision of public goods generates some household utility.

ment spending stabilization in the policymaker's objective function.[5]

Thus, the answer to our first question—is there a foolproof way to improve welfare of an economy that is subject to occasional expectations-driven liquidity traps?—is rather disappointing. None of the reviewed policy delegation schemes—a higher inflation target, inflation conservatism, and fiscal activism—unambiguously improves stabilization outcomes and welfare in our model. However, the answer to our second question—is it possible to prevent the economy from falling into an expectations-driven liquidity trap?—turns out to be more promising if government spending is part of the policymaker's toolkit. Specifically, we find that when society assigns a sufficiently low relative weight on government spending stabilization to the policymaker's objective function the sunspot equilibrium ceases to exist.[6]

The mechanism behind this finding is as follows. Changes in government spending are a substitute for the short-term nominal interest rate in standard New Keynesian models.[7] However, variations in government spending—unlike variations in the interest rate—are costly from a welfare perspective because they lead to deviations from the efficient level of government spending. When the policymaker has the same objective function as society, the policymaker internalizes these welfare costs in her decision-making and refrains from using government spending as a strong substitute for the policy rate when the latter is constrained by the lower bound. Instead, with a sufficiently low relative weight on government spending stabilization, the policymaker stands ready to adjust government spending sufficiently elastically to deviations of inflation and the output gap from target that pessimistic expectations fail to be validated. In such a case, variations in the sunspot do not affect private sector decisions and government spending stays constant.[8]

For our baseline setup—a two-state model with linearized private sector behavioral constraints—we derive all results in closed form. In numerical extensions, we use the fully non-linear version of the model and explore expectations-driven liquidity trap equilibria with richer transition dynamics.

Our paper is related to a small but growing literature on equilibrium multiplicity and the lower bound on nominal interest rates. Benhabib et al. (2001) were the first to show that the lower bound constraint gives rise to two steady states in a model where monetary policy is governed by an interest-rate feedback rule. In one steady state the policy rate is strictly positive and inflation is at target, and in the other steady state the lower bound constraint is binding and inflation is below target.[9] Armenter (2018) and Nakata and Schmidt (2019a) show that the lower bound constraint can also give rise to multiple equilibria under optimal discretionary monetary policy.

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[5]In contrast, in models with fundamental-driven liquidity traps, society can further improve stabilization outcomes and welfare by appointing a policymaker who is less concerned with government spending stabilization than society as a whole. See Schmidt (2017).

[6]From an institutional perspective, this could be operationalized by the appointment of a decision-making fiscal council. Alternatively, one could think of society electing a policymaker with a certain type of preferences.

[7]In fact, if the policymaker commands a sufficiently rich set of fiscal instruments, she can implement the first best. See Correia et al. (2013).

[8]We find that the magnitude of the government spending expansion that would be implemented by a policymaker of this type if a decline in inflation and the output gap occurred and the lower bound became binding—maybe because of a fundamental shock—is plausible.

[9]Benhabib et al. (2001) also show that there usually exist an infinite number of perfect-foresight equilibria where the economy can originate arbitrarily close to the first steady state and converge to the second steady state.

This equilibrium multiplicity naturally opens the door for sunspot equilibria. Mertens and Ravn (2014) construct a sunspot equilibrium in a New Keynesian model and assess the effects of an exogenous increase in government spending when confidence is low and the lower bound is binding.[10] They find that a positive government spending shock is deflationary.[11] Bilbiie (2018) considers several exogenous policy interventions in an analytical model setup that allows for expectations-driven liquidity traps. He also studies optimal monetary and fiscal policies, assuming that the policymaker can commit to an interest rate or government spending level for each confidence state. Instead, we assume that the policymaker is unable to commit. This difference has an important bearing on the choice of policies and stabilization outcomes.[12,13]

A few papers have assessed the plausibility of expectations-driven liquidity traps empirically or used the concept for positive analysis of recent economic events. Aruoba et al. (2018) conduct an empirical assessment of a New Keynesian model with a sunspot shock and find that Japan transitioned in the late 1990s to an expectations-driven liquidity trap state and that the U.S. had been in a fundamental-driven liquidity trap equilibrium.[14] Schmitt-Grohé and Uribe (2017) show that a model with downward nominal wage rigidities and a sunspot shock can mimic the economic dynamics of a recessionary lower bound episode that is followed by a jobless recovery. Lansing (2017) develops a model in which agents' beliefs about the steady state to which the economy converges depends on aggregate outcomes, and applies it to the U.S. economy. Jarociński and Maćkowiak (2018) use a model with a sunspot shock to conduct counterfactual simulations of the euro area economic downturn in 2008-2015.

Our paper also makes contact with some existing studies on how to avoid expectations-driven liquidity traps. Benhabib et al. (2002) and Woodford (2003) show how non-Ricardian fiscal policies that entail an off-equilibrium violation of the transversality condition can rule out perfect-foresight equilibria in which the economy converges to the steady state where the lower bound constraint is binding. Schmidt (2016) and Tamanyu (2019) show that it is possible to design Ricardian fiscal policy rules that insulate the economy from expectations-driven liquidity traps. Sugo and Ueda (2008) and Schmitt-Grohé and Uribe (2014) consider alternative interest-rate feedback rules. Armenter (2018) shows that augmenting the objective function of a discretionary central bank with an objective for stabilizing the level of a long-run nominal interest rate can ensure the existence of

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[10]See also Boneva et al. (2016).

[11]Wieland (2018) shows that if one relaxes the assumption of Mertens and Ravn (2014) that the government spending shock is perfectly correlated with the sunspot shock, a sufficiently short-lived increase in government spending can be inflationary.

[12]For instance, in Bilbiie (2018)'s setup the policymaker finds it optimal to lower government spending in the low-confidence state whereas in our setup, the policymaker—unable to internalize the effect of her actions on past private sector expectations—finds it optimal to increase government spending.

[13]Coyle and Nakata (2018) numerically solve a fully non-linear New Keynesian model with an interest-rate rule allowing for both, fundamental-driven and expectations-driven liquidity traps, and find that the optimal inflation target in the policy rule is lower than in the model with fundamental-driven liquidity traps only.

[14]Hirose (2018) estimates a DSGE model log-linearized around the deflationary steady state on Japanese data. Cuba-Borda and Singh (2020) find that it is difficult to determine empirically whether Japan has been in an expectations-driven liquidity trap or in a secular-stagnation-driven liquidity trap characterized by a persistently low natural real rate of interest.

a unique Markov-perfect equilibrium.[15]

Finally, there is a rich literature on optimal monetary and fiscal policy in models with fundamental-driven liquidity traps. Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006, 2007), Nakov (2008) and Nakata and Schmidt (2019a,b), among others, study optimal monetary policy. Eggertsson and Woodford (2006), Eggertsson (2006), Schmidt (2013, 2017), Nakata (2016, 2017a), Bilbiie et al. (2019) and Bouakez et al. (2020), among others, study optimal fiscal policy.

The remainder of the paper is organized as follows. Section 2 presents the model and defines the equilibria of interest. Section 3 presents results on equilibrium existence and stabilization outcomes. Section 4 assesses the desirability of a positive inflation target and inflation conservatism in the sunspot equilibrium. Section 5 extends the analysis to fiscal policy. Section 6 considers sunspot equilibria with richer dynamics in a fully non-linear model. Section 7 concludes. An Online Appendix provides the proofs for the propositions, additional numerical examples, and further analyses based on extensions of the baseline model setup.

2 Model

We use a standard infinite-horizon New Keynesian model formulated in discrete time. The economy is inhabited by identical households who consume and work, goods-producing firms that act under monopolistic competition and are subject to price rigidities, and a government. For now, we assume that the one-period nominal interest rate is the only policy instrument. In Section 5, we extend the analysis to include government spending as an additional stabilization tool. Throughout the paper, fiscal policy is assumed to be Ricardian in the sense that policy ensures that the present discounted value of total government liabilities converges to zero under all possible equilibrium or off-equilibrium paths of the endogenous model variables. More detailed descriptions of the model can be found in Woodford (2003) and Galí (2015). We mainly work with a semi-loglinear version of the model that can be solved in closed form and allows us to derive analytical results.[16]

2.1 Private sector behavior and welfare

The aggregate private sector behavior is described by a Phillips curve and a consumption Euler equation

$$\pi_t = \kappa y_t + \beta \mathrm{E}_t \pi_{t+1} \tag{1}$$

$$y_t = \mathrm{E}_t y_{t+1} - \sigma \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right) \tag{2}$$

The private sector behavioral constraints have been log-linearized around the intended zero-inflation steady state. π_t is the inflation rate between periods $t-1$ and t, y_t denotes the output

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[15]Armenter (2018) also shows that price-level targeting does not ensure existence of a unique equilibrium.

[16]Section 6 and Section H in the Online Appendix presents results of a numerical analysis of the fully non-linear version of the model.

gap, i_t is the *level* of the riskless nominal interest rate between periods t and $t + 1$, r_t^n is the exogenous natural real rate of interest, and E_t is the rational expectations operator conditional on information available in period t. The parameters are defined as follows: $\beta \in (0, 1)$ is the households' subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, and κ represents the slope of the Phillips curve.[17]

Households' welfare at time t is given by the expected discounted sum of current and future utility flows. A second-order approximation to household preferences leads to

$$V_t = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \bar{\lambda} y_{t+j}^2 \right], \tag{3}$$

where $\bar{\lambda} = \kappa/\theta$.[18]

2.2 Central bank

At the beginning of time, society delegates monetary policy to a central bank. The central bank does not have a commitment technology; that is, it acts under discretion. The monetary policy objective is given by

$$V_t^{CB} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[(\pi_{t+j} - \pi^*)^2 + \lambda y_{t+j}^2 \right], \tag{4}$$

where $\lambda \geq 0$ and π^* are policy parameters to be set by society when designing the central bank's objective function. When $\lambda = \bar{\lambda}$ and $\pi^* = 0$, the central bank's objective function coincides with society's objective function (3).

The policy problem of a generic central bank is as follows. Each period t, it chooses the inflation rate, the output gap, and the nominal interest rate to maximize its objective function (4) subject to the behavioral constraints of the private sector (1)–(2), and the lower bound constraint $i_t \geq 0$, with the value and policy functions at time $t + 1$ taken as given.

Consolidating the first-order necessary conditions to this problem, one obtains

$$[\kappa(\pi_t - \pi^*) + \lambda y_t] \, i_t = 0, \tag{5}$$

where $\kappa(\pi_t - \pi^*) + \lambda y_t = 0$ whenever $i_t > 0$ and $\kappa(\pi_t - \pi^*) + \lambda y_t < 0$ when the lower bound constraint is binding, $i_t = 0$. In words, each period the central bank aims to stabilize a weighted sum of current period's inflation rate (in deviation from target) and the output gap.

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[17]κ is itself a function of several structural parameters of the economy: $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1} + \eta)$, where $\alpha \in (0, 1)$ denotes the share of firms that cannot reoptimize their price in a given period, $\eta > 0$ is the inverse of the labor-supply elasticity, and $\theta > 1$ denotes the price elasticity of demand for differentiated goods.

[18]See Woodford (2003). We assume that the steady state distortions arising from monopolistic competition are offset by a production subsidy.

2.3 Sunspot shock

For the benchmark setup, we assume that there is no uncertainty regarding the economy's fundamentals. Specifically, $r_t^n = r^n = 1/\beta - 1$ for all t. However, agents expectations may be affected by a non-fundamental sunspot or 'confidence' shock ξ_t.[19] The sunspot shock follows a two-state Markov process, $\xi_t \in (\xi_L, \xi_H)$. We refer to state ξ_L as the low-confidence state and to state ξ_H as the high-confidence state. The transition probabilities are given by

$$\text{Prob}\left(\xi_{t+1} = \xi_H | \xi_t = \xi_H\right) = p_H \tag{6}$$

$$\text{Prob}\left(\xi_{t+1} = \xi_L | \xi_t = \xi_L\right) = p_L \tag{7}$$

In words, $p_H \in (0, 1]$ is the probability of being in the high-confidence state in period $t+1$ conditional on being in the high-confidence state in period t, and can be interpreted as the persistence of high confidence. Note that while we allow the high-confidence state to be an absorbing state we do not restrict our analysis to this special case. $p_L \in (0, 1)$ is the probability of being in the low-confidence state in period $t + 1$ when the economy is in the low-confidence state in period t, and can be interpreted as the persistence of low confidence.[20]

Let x_s, $s \in \{L, H\}$ be the equilibrium value of some variable x in state ξ_s. *Sunspots matter* if there is an equilibrium in which $\{\pi_L, y_L, i_L, V_L\} \neq \{\pi_H, y_H, i_H, V_H\}$. We are interested in a sunspot equilibrium where the economy is subject to recurring liquidity trap episodes. We associate the occurrence of these liquidity trap events with the low-confidence state.

Definition 1 *The **sunspot equilibrium** with occasional liquidity traps is defined as a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations*

$$y_H = [p_H y_H + (1 - p_H)y_L] + \sigma [p_H \pi_H + (1 - p_H)\pi_L - i_H + r^n] \tag{8}$$

$$\pi_H = \kappa y_H + \beta [p_H \pi_H + (1 - p_H)\pi_L] \tag{9}$$

$$0 = \kappa(\pi_H - \pi^*) + \lambda y_H \tag{10}$$

$$y_L = [(1 - p_L)y_H + p_L y_L] + \sigma [(1 - p_L)\pi_H + p_L \pi_L - i_L + r^n] \tag{11}$$

$$\pi_L = \kappa y_L + \beta [(1 - p_L)\pi_H + p_L \pi_L] \tag{12}$$

$$i_L = 0, \tag{13}$$

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[19]In Section F of the Online Appendix we analyze a variant of the model where the economy is buffeted by both, sunspot and fundamental shocks. Arifovic et al. (2018) consider a model with both types of shocks under social learning.

[20]Mertens and Ravn (2014), Schmidt (2016), Aruoba et al. (2018) and Bilbiie (2018) also consider a sunspot shock that follows a two-state Markov process. Mertens and Ravn (2014), Schmidt (2016) and Bilbiie (2018) assume that the high-confidence state is an absorbing state, that is, $p_H = 1$. Aruoba et al. (2018) allow for recurring declines in confidence and assume that $p_H = 0.99$.

and satisfies the following two inequality constraints

$$i_H > 0 \tag{14}$$

$$\kappa(\pi_L - \pi^*) + \lambda y_L < 0. \tag{15}$$

As we will see shortly, the sunspot equilibrium may or may not exist, depending on the parameterization of the model and the sunspot shock. Even when the sunspot equilibrium does not exist, there still exists a solution to the problem of the central bank, albeit one in which the sunspot shock does not matter, so that $\{y_H, \pi_H, i_H\} = \{y_L, \pi_L, i_L\}$. We refer to such an equilibrium as a *no-sunspot equilibrium.* No-sunspot equilibria—while not of primary interest in our analysis—will be important to assess whether in those cases where society is able to design a policy framework that rules out the sunspot equilibrium, society also has the welfare incentive to do so.[21]

2.4 Alternative setup: Fundamental shock

Throughout the paper, we contrast results for the benchmark model—an economy that is subject to a sunspot shock—with those for an economy that is subject to a fundamental shock. In the model with a fundamental shock, the natural real rate is assumed to be stochastic.[22]

To keep the model setup as close as possible to the one with the sunspot shock, r_t^n is assumed to follow a two-state Markov process. In the high-fundamental state, the natural real rate is strictly positive $r_H^n > 0$, and in the low-fundamental state it is strictly negative $r_L^n < 0$. The transition probabilities for the natural real rate shock are given by

$$\text{Prob}\left(r_{t+1}^n = r_H^n | r_t^n = r_H^n\right) = p_H^f \tag{16}$$

$$\text{Prob}\left(r_{t+1}^n = r_L^n | r_t^n = r_L^n\right) = p_L^f, \tag{17}$$

and are distinguished from the transition probabilities of the sunspot shock via the superscript f. The fundamental equilibrium in the model with the natural real rate shock is defined as follows.

Definition 2 *The* **fundamental equilibrium** *with occasional liquidity traps is defined as a vector*

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[21]Whether or not the lower bound constraint is binding in the no-sunspot equilibrium depends on the policy framework and model parameters.

[22]We use a natural real rate shock rather than a cost-push shock in this model because liquidity traps caused by a natural real rate shock feature below-target inflation and a negative output gap. Instead, liquidity traps caused by a cost-push shock feature below-target inflation and a positive output gap, which is inconsistent with the data. See Nakata (2017b).

$\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ *that solves*

$$y_H = \left[p_H^f y_H + (1 - p_H^f)y_L\right] + \sigma \left[p_H^f \pi_H + (1 - p_H^f)\pi_L - i_H + r_H^n\right] \tag{18}$$

$$\pi_H = \kappa y_H + \beta \left[p_H^f \pi_H + (1 - p_H^f)\pi_L\right] \tag{19}$$

$$y_L = \left[(1 - p_L^f)y_H + p_L^f y_L\right] + \sigma \left[(1 - p_L^f)\pi_H + p_L^f \pi_L - i_L + r_L^n\right] \tag{20}$$

$$\pi_L = \kappa y_L + \beta \left[(1 - p_L^f)\pi_H + p_L^f \pi_L\right] \tag{21}$$

as well as (10) and (13), and satisfies inequality constraints (14) and (15).

This equilibrium has been analyzed in detail in Nakata and Schmidt (2019a).[23] To keep the exposition parsimonious, we will refer to this paper for the proofs related to the fundamental equilibrium whenever applicable.[24]

3 Basic properties of the sunspot equilibrium

This section presents conditions for existence of the sunspot equilibrium as well as equilibrium allocations and prices, and discusses how they compare to those of the fundamental equilibrium.

3.1 Equilibrium existence

The following proposition establishes the necessary and sufficient conditions for existence of the sunspot equilibrium.

Proposition 1 *The sunspot equilibrium exists if and only if*

$$p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa \sigma}\left(1 - \beta p_L + \beta(1 - p_H)\right) > 0, \tag{22}$$

and

$$\pi^* > -\frac{\kappa^2 + \lambda(1 - \beta)}{\kappa^2}r^n. \tag{23}$$

Proof: See Online Appendix, Section A.

Three observations are in order. First, for the sunspot equilibrium to exist, the two confidence states have to be sufficiently persistent, i.e. p_L and p_H have to be sufficiently close to one. In equilibrium, agents' expectations in one state are a weighted average of outcomes in both states.

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[23]Nakata and Schmidt (2019a) analytically show that in this model with a two-state fundamental shock there exists another equilibrium in which the lower bound constraint binds in both fundamental states. Here, we do not consider this equilibrium.

[24]The notation used in Nakata and Schmidt (2019a) is slightly different from the one used here. They use p_H to denote the probability that the economy is in the *low* state in the next period conditional on being in the high state today.

Hence, when the economy is in the low-confidence state and agents attach only a small probability to the possibility of low future inflation and economic activity, then the decline in conditional output and inflation expectations in the low-confidence state is too small to become self-fulfilling. Likewise, if the economy is in the high-confidence state and agents attach a high probability to the possibility of a future decline in inflation and economic activity, then the decline in conditional expectations is sufficiently large for agents' pessimism to become self-fulfilling, and the economy ends up being in a liquidity trap in both states. A high persistence of both confidence states implies that in the sunspot equilibrium liquidity traps are rare but long-lasting events.

Second, for the sunspot equilibrium to exist, prices have to be sufficiently flexible, i.e. κ has to be sufficiently large. If prices are too sticky, then inflation is not responsive enough to declines in confidence for the latter to become self-fulfilling. Third, for the sunspot equilibrium to exist, the central bank's inflation target π^* must be higher than some strictly negative lower bound, which depends on the central bank's relative weight on output gap stabilization λ. While the sunspot equilibrium fails to exist when condition (23) is violated, the remaining no-sunspot equilibrium is one where the lower bound is binding, and inflation and the output gap are both negative. See Section B in the Online Appendix for a formal characterization of this no-sunspot equilibrium.

The conditions for existence of the sunspot equilibrium qualitatively differ from the conditions for existence of the fundamental equilibrium in the model with the natural real rate shock. In particular, for the fundamental equilibrium to exist, the low-fundamental state must not be too persistent, and prices must not be too flexible (see Nakata and Schmidt, 2019a). If the low-fundamental state is too persistent or if prices are too flexible, the vicious cycle of higher desired saving, lower demand and deflationary pressures unfolding in the low-fundamental state fails to come to a halt, and no equilibrium exists.[25]

3.2 Allocations and prices

The allocations and prices in the sunspot equilibrium can be solved for in closed form. For the remainder of this section, we assume that the central bank has the same objective function as society. The signs of inflation and the output gap in the two confidence states are then unambiguously determined.

Proposition 2 *Suppose $\lambda = \bar{\lambda}$ and $\pi^* = 0$. In the sunspot equilibrium, $\pi_L < 0$, $y_L < 0$, $\pi_H \leq 0$, $y_H \geq 0$. When $p_H < 1$, then $\pi_H < 0$, $y_H > 0$.*

Proof: See Online Appendix, Section A

When confidence is low, agents expect persistently low future inflation. The ex-ante real interest rate increases, thereby putting downward pressure on economic activity and inflation. The central bank responds by lowering the policy rate, but if agents are sufficiently pessimistic, the lower

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[25]Section A in the Online Appendix provides a numerical illustration of the existence conditions for the sunspot equilibrium and for the fundamental equilibrium.

bound on the policy rate becomes binding. At the lower bound, agents' pessimistic expectations are validated, and inflation and the output gap both settle below target.

When confidence is high, the policy rate is strictly positive but if $p_H < 1$ the risk of a future decline in confidence creates a trade-off between inflation and output gap stabilization. Specifically, the possibility that confidence might fall in the future while the price set by a firm reoptimizing today is still in place provides an incentive for forward-looking firms to set a lower price than they would in the absence of any risk of a future drop in confidence. To counteract these deflationary forces, the central bank allows for a positive output gap, that is, it sets the policy rate in the high-confidence state such that the ex-ante real interest rate is below the constant natural real rate. In equilibrium, the high-confidence output gap is thus positive and inflation is negative. Negative inflation in both states implies that agents' long-run inflation expectations—represented by the unconditional average $\frac{1-p_L}{1-p_L+1-p_H}\pi_H + \frac{1-p_H}{1-p_L+1-p_H}\pi_L$—are negative as well.

The signs of output and inflation in the fundamental equilibrium are identical to those in the sunspot equilibrium. Output and inflation are negative in the low-fundamental state, and output (inflation) is positive (negative) in the high-fundamental state (see Nakata and Schmidt, 2019a). As in the sunspot equilibrium, long-run inflation expectations are negative.

3.3   Low-state aggregate demand and aggregate supply schedules

In order to better understand equilibrium outcomes in the model with the sunspot shock and in the model with the fundamental shock, and how they are affected by the policy framework, we recast the models in terms of aggregate demand (AD) and aggregate supply (AS) curves. The AD curve is the set of pairs of inflation rates and output gaps consistent with Euler equation (2) where the policy rate is set in line with target criterion (5), and the AS curve is the set of pairs of inflation rates and output gaps consistent with Phillips curve (1). Specifically, we focus on the AD and AS schedules conditional on the economy being in the low state of the respective model. In order to construct these schedules, one has to assume that the high state is an absorbing state, so that high-state output gap and inflation are unaffected by variations in low-state output gap and inflation. In spite of this more restrictive assumption, the AD-AS framework is a useful tool to shed light on the analytical results.

For the model with the sunspot shock the two curves in the low-confidence state are given by

$$\textbf{AD-sunspot:} \quad y_L = \min\left[\left(y_H + \sigma\pi_H + \frac{\sigma}{1-p_L}r^n\right) + \frac{\sigma p_L}{1-p_L}\pi_L, \frac{\kappa}{\lambda}(\pi^* - \pi_L)\right] \quad (24)$$

$$\textbf{AS-sunspot:} \quad y_L = -\frac{\beta(1-p_L)}{\kappa}\pi_H + \frac{1-\beta p_L}{\kappa}\pi_L, \quad (25)$$

where in each equation we distinguish between terms that are multiplied by π_L—the slope coefficient—and the other terms—the intercept. For the model with the fundamental shock, the two curves are

given by

$$\textbf{AD-fundamental:} \quad y_L = \min\left[\left(y_H + \sigma\pi_H + \frac{\sigma}{1-p_L^f}r_L^n\right) + \frac{\sigma p_L^f}{1-p_L^f}\pi_L, \frac{\kappa}{\lambda}(\pi^* - \pi_L)\right] \quad (26)$$

$$\textbf{AS-fundamental:} \quad y_L = -\frac{\beta(1-p_L^f)}{\kappa}\pi_H + \frac{1-\beta p_L^f}{\kappa}\pi_L. \quad (27)$$

Figure 1 plots these AD-AS curves for the model with the sunspot shock (left panel) and for the model with the fundamental shock (right panel). One period corresponds to one quarter. We set $p_L = 0.9375$ in the model with the sunspot shock, implying an average duration of lower bound episodes of 4 years in the sunspot equilibrium. In the model with the fundamental shock, we set $p_L^f = 0.85$, implying an average duration of lower bound episodes of 1 1/2 years. The other parameter values are summarized in Table 1.[26] Given $\pi^* = 0$, $\pi_H, y_H = 0$ (and $i_H = 1/\beta - 1$) is a

Table 1: Parameter values for numerical example

Parameter	Value	Economic interpretation
β	0.9975	Subjective discount factor
σ	0.5	Intertemporal elasticity of substitution in consumption
η	0.47	Inverse labor supply elasticity
θ	10	Price elasticity of demand
α	0.8106	Share of firms per period keeping prices unchanged
λ	$\bar{\lambda}$	Policy parameter: Relative weight on output term
π^*	0	Policy parameter: Inflation target
r_H^n	r^n	High-state natural real rate in model with fundamental shock
r_L^n	-0.005	Low-state natural real rate in model with fundamental shock

Note: This parameterization implies $r^n = 0.0025$, $\kappa = 0.0194$, $\bar{\lambda} = 0.0019$.

solution to the high-state equilibrium conditions in both models. Hence, the intercept terms in the low-state AS curves are zero, whereas the intercept terms in the low-state AD curves are positive (model with sunspot shock) and negative (model with fundamental shock), respectively.

The low-state AD-AS curves in the two models have several common features. First, due to the lower bound constraint, the AD curve has a kink. To the left of the kink, the lower bound constraint is binding and to the right of the kink the lower bound constraint is slack. Second, the AD curve is upward-sloping to the left of the kink—aggregate demand is increasing in inflation when the lower bound is binding because an increase in inflation lowers the ex-ante real interest rate—and downward-sloping to the right of the kink—aggregate demand is decreasing in inflation when the lower bound constraint is slack because the central bank raises the policy rate more than one-for-one with inflation. Third, the AS curve is monotonically upward-sloping—an increase in demand leads to an increase in inflation—and goes through the origin.

In the model with the sunspot shock, the AD curve is steeper than the AS curve. This is a

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[26]Parameter p_L^f has to be smaller than p_L for the fundamental equilibrium to exist, given our assumption that all other parameter values are the same across the two models.

Figure 1: Aggregate demand and aggregate supply in the low state

(a) Model with sunspot shock   (b) Model with fundamental shock
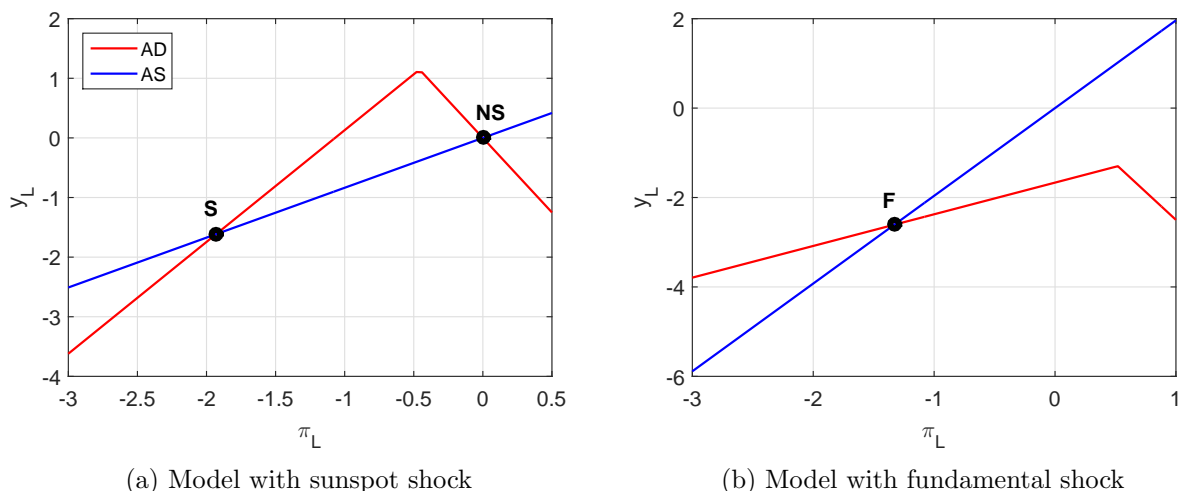
Note: In the left panel, S marks low-state output gap and inflation in the sunspot equilibrium and NS marks low-state output gap and inflation the no-sunspot equilibrium. In the right panel, F marks low-state output gap and inflation in the fundamental equilibrium. Inflation is expressed in annualized terms.

necessary—and, in case of $\pi^* = 0$, sufficient—condition for existence of the sunspot equilibrium.[27] Intuitively, since the low-confidence state is highly persistent, households' desired consumption is very sensitive to changes in low-state inflation, i.e. the AD curve is relatively steep. At the same time, the high persistence of the low-confidence state makes firms' price setting very sensitive to changes in aggregate demand, i.e. the AS curve is relatively flat. There are two intersections of the AD and AS curves. The intersection point on the left—denoted S—marks the low-state outcomes for the output gap and inflation in the sunspot equilibrium $\{y_H = 0, \pi_H = 0, i_H > 0, y_L < 0, \pi_L < 0, i_L = 0\}$. Consistent with Proposition 2, when confidence is low, output and inflation are strictly negative in the sunspot equilibrium. The intersection point on the right—denoted NS—marks the low-state outcomes for the output gap and inflation in the no-sunspot equilibrium $\{y_H = 0, \pi_H = 0, i_H > 0, y_L = 0, \pi_L = 0, i_L > 0\}$. In the 'no-sunspot' equilibrium, the sunspot shock does not affect agents' behavior, and outcomes in the low state are identical to those in the high state, i.e. the output gap and inflation are at target and the policy rate is strictly positive.

In the model with the fundamental shock, the AD curve is flatter than the AS curve, which is a necessary condition for the fundamental equilibrium to exist and reflects the relatively low persistence of the low-fundamental state. In the fundamental equilibrium, low-state output and inflation are negative, as marked by intersection point F, again in line with analytical results.

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[27]See the condition for existence of the sunspot equilibrium (22) with $p_H = 1$.

4 Monetary policy frameworks

Having shown that the sunspot equilibrium is associated with rare but long-lasting spells at the lower bound and chronic deflation, we now explore whether stabilization outcomes and welfare can be improved by assigning an objective function to the policymaker that differs from society's objective function. This section focuses on two monetary policy frameworks that are known to be desirable in models with fundamental-driven liquidity traps: a non-zero inflation target and inflation conservatism. The subsequent section extends the analysis to fiscal stabilization policy.

4.1 A non-zero inflation target

This subsection explores the desirability of a non-zero inflation target in the sunspot equilibrium. Throughout this subsection, we assume $\lambda = \bar{\lambda}$.

While the signs of allocations and prices are sensitive to the value of the central bank's inflation target π^*, the effects of a change in π^* on allocations and prices are unambiguously determined.

Proposition 3 *In the sunspot equilibrium, $\frac{\partial \pi_L}{\partial \pi^*} < 0$, $\frac{\partial y_L}{\partial \pi^*} < 0$, $\frac{\partial \pi_H}{\partial \pi^*} > 0$, $\frac{\partial y_H}{\partial \pi^*} > 0$.*

**Proof:** See Online Appendix, Section A.

In the sunspot equilibrium, a marginal increase in π^* lowers output and inflation in the low-confidence state and raises output and inflation in the high-confidence state.[28] Consider first the high-confidence state. All else equal, if π^* increases, the gap between π^* and actual inflation widens, and hence the central bank becomes more willing to tolerate a positive output gap to bring inflation again closer to π^*. In equilibrium, an increase in π^* therefore raises the output gap and inflation in the high-confidence state.

To understand why low-state output and inflation are decreasing in π^*, we make use of the AD-AS framework. The left panel of Figure 2 depicts how the low-confidence state AD and AS curves (24)–(25) are shifted in response to an increase in π^*. An increase in π^* shifts the AD curve upwards, because, all else equal, agents increase their desired consumption given higher expected inflation. At the same time, the AS curve shifts downwards, as firms' desired price increases in light of higher expected inflation for given current demand. Hence, at the inflation rate consistent with the sunspot equilibrium in the baseline, marked by intersection point S, there is now excess demand. Households would like to increase their consumption out of their current income. In order to do so, they have to borrow from others. Of course, in equilibrium, household saving is zero. To restore equilibrium, either household income or the relative price of current consumption—the real interest rate—has to adjust. The key point to notice is that an increase in income cannot restore equilibrium, for any increase in low-state income leads—via its effect on low-state inflation—to a

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[28]It can also be shown that $\frac{\partial \pi_H}{\partial \pi^*} < 1$. Together with Proposition 2, this implies that for any positive inflation target actual inflation in the high-confidence state is below target.

Figure 2: The effect of increasing the central bank's inflation target

(a) Model with sunspot shock

(b) Model with fundamental shock
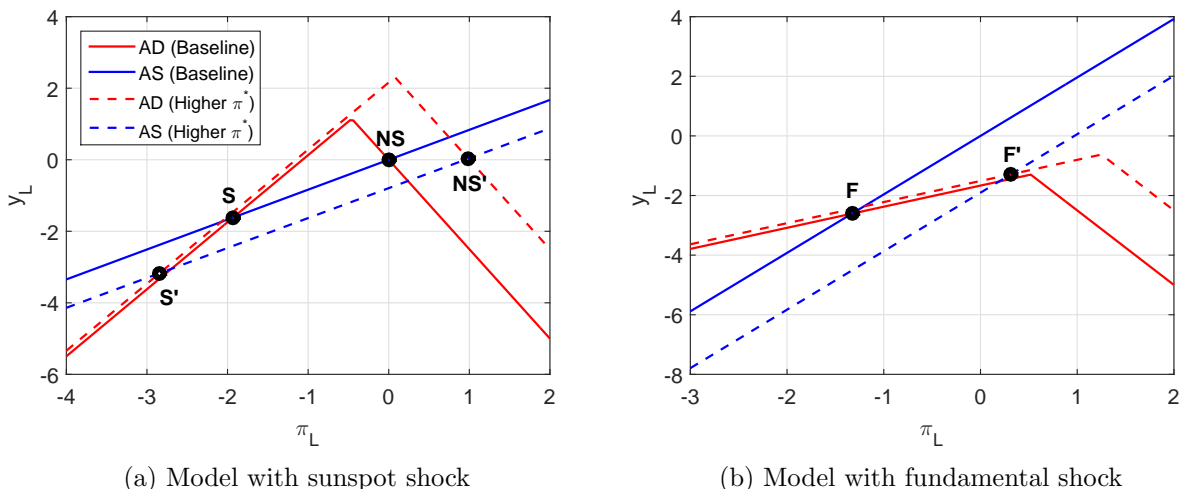
Note: Solid lines: $\pi^* = 0$; dashed lines: $\pi^* = 1/400$. In the left (right) panel, S (F) marks output gap and inflation in the sunspot (fundamental) equilibrium in the baseline and S' (F') marks outcomes in the sunspot (fundamental) equilibrium in the case of a higher π^*. NS marks output gap and inflation in the no-sunspot equilibrium in the baseline, and NS' marks outcomes in the no-sunspot equilibrium in the case of a higher π^*. Inflation is expressed in annualized terms.

more than one-for-one increase in desired consumption. That is why excess demand in the AD-AS figure is increasing in the inflation rate as long as the lower bound is binding.[29] Hence, for markets to clear, the real interest rate has to increase. With the nominal interest rate at the lower bound, inflation has to decline. The increase in the real interest rate lowers aggregate demand, and low-state output falls in equilibrium in response to the increase in π^*.

In the fundamental equilibrium, a marginal increase in π^* also raises high-state inflation. The effects on low-state outcomes, however, differ from those in the sunspot equilibrium. Higher inflation in the high-fundamental state lowers the ex-ante real interest rate in the low-fundamental state. This stimulates aggregate demand and leads to an increase in low-state output and inflation (Nakata and Schmidt, 2019a). The right panel of Figure 2 depicts how in the model with the fundamental shock the low-state AD and AS curves (26)–(27) are shifted in response to an increase in π^*.

For the characterization of the welfare-maximizing inflation target in the model with the sunspot shock, it is also useful to show that there exists an inflation target such that inflation in the high-confidence state is stabilized at zero.

Lemma 1 *There exists a $\pi^0 \geq 0$ such that in the sunspot equilibrium $\pi_H = 0$ if $\pi^* = \pi^0$. When $p_H = 1$, $\pi^0 = 0$, otherwise $\pi^0 > 0$.*

Proof: See Online Appendix, Section A.

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[29]This feature can also be seen from the (log-linearized) aggregated consumption function which relates consumption to income and the real interest rate. Using the Phillips curve to substitute out low-state inflation in the expression for the real interest rate, it can be shown that the coefficient on low-state income is larger than one in the model with the sunspot shock.

One can then establish the following result concerning the welfare-maximizing inflation target.

Proposition 4 *Let π^{**} denote the value of $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$ that maximizes households' unconditional welfare EV_t where V_t is defined in equation (3). In the sunspot equilibrium, $\pi^{**} < \pi^0$.*

Proof: See Online Appendix, Section A.

Together with Proposition 3 and Lemma 1, this proposition means that the optimal inflation target can be negative or positive.[30] This ambiguity can be understood from the fact that an increase in π^* has a negative effect on low-state inflation (moving low-state inflation further into negative territory), and a positive effect on high-state inflation (moving high-state inflation closer to zero as long as $\pi^* < \pi^0$).[31] Together with Lemma 1, the proposition also implies that when the optimal inflation target is positive, it will be below the level needed to engineer strictly positive inflation in the high-confidence state.

In contrast, in the fundamental equilibrium of the model with a natural real rate shock the optimal inflation target is unambiguously strictly positive, and it is high enough to engineer strictly positive inflation in the high-fundamental state (Nakata and Schmidt, 2019a).

In Section 3, we showed that for a sufficiently low value of π^* the sunspot equilibrium does not exist. Does society have an incentive to choose a π^* that is low enough such that the sunspot equilibrium does not exist? The answer is no.

Proposition 5 *Society's unconditional welfare EV_t is strictly larger in the sunspot equilibrium with an optimized inflation target $\pi^* = \pi^{**}$ than in the no-sunspot equilibrium with an inflation target π^* that violates condition (23) for the existence of the sunspot equilibrium.*

Proof: See Online Appendix, Section B.

The no-sunspot equilibrium with $\pi^* \leq -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$ is undesirable because in this equilibrium the economy is in a permanent—rather than in a temporary—liquidity trap, with permanent deflation and a permanently negative output gap.

Before moving to the next monetary policy framework, we numerically explore the role of the two parameters capturing the persistence of the two confidence states for the sign and the absolute size of the optimal inflation target in the sunspot equilibrium. The left panel of Figure 3 shows how the sign of π^{**} depends on p_H and p_L, the persistence of the high and the low-confidence state, respectively.[32] When the two confidence states are highly persistent, the optimal inflation target is positive, otherwise the optimal target is negative.

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[30]Only for the special case where the high-confidence state is an absorbing state, $p_H = 1$, the optimal inflation target is unambiguously negative. When $p_H = 1$ and $\pi^* = 0$, inflation in the high-confidence state is zero. Any increase in π^* above zero moves in inflation in both states further away from zero.

[31]Section A in the Online Appendix provides a numerical example of how π^* affects allocations and welfare in the sunspot equilibrium.

[32]The values for the other model parameters are reported in Table 1.

Figure 3: Optimal inflation target in model with sunspot shock
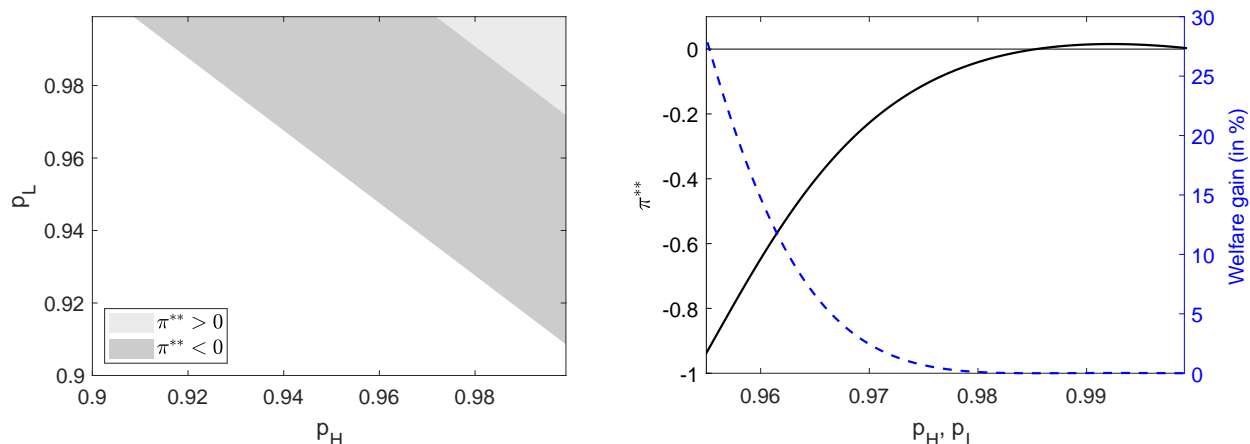
Note: In the left panel, the white-shaded area represents pairs of p_H and p_L for which the sunspot equilibrium does not exist. In the right panel, $p_H = p_L$. The optimal inflation target π^{**} is expressed in annualized terms.

The right panel of Figure 3 plots the optimal inflation target (left vertical axis, solid black line) and the welfare gain from assigning the optimal target to the central bank (right vertical axis, dashed blue line) as a function of the persistence of the two confidence states, assuming $p_H = p_L$. For sufficiently low values of p_H and p_L, the optimal inflation target is negative and is increasing in the persistence parameters. When p_H and p_L are high enough, the optimal inflation target is slightly positive. The welfare gain of assigning an optimized inflation target to the central bank is most elevated when the persistence parameters take on the lowest possible values for which the sunspot equilibrium exists.

Next, we assess the desirability of inflation conservatism.

4.2 Inflation conservatism

This subsection explores the desirability of inflation conservatism in the sunspot equilibrium. An inflation-conservative central banker is a policymaker who puts a higher relative weight on inflation stabilization than society. We first establish how a change in the central bank's relative weight on output stabilization λ affects allocations and prices in the sunspot equilibrium and then explore the welfare implications. To focus on the role of inflation conservatism, we assume $\pi^* = 0$.

Proposition 6 *Suppose $\pi^* = 0$ and $p_H < 1$. In the sunspot equilibrium, $\frac{\partial \pi_L}{\partial \lambda} > 0$, $\frac{\partial y_L}{\partial \lambda} > 0$, $\frac{\partial \pi_H}{\partial \lambda} < 0$, $\frac{\partial y_H}{\partial \lambda} < 0$.*

Proof: See Online Appendix, Section A.

In the sunspot equilibrium, a marginal increase in λ raises output and inflation in the low-confidence state and lowers output and inflation in the high-confidence state, provided that $p_H <$

1.[33] Qualitatively, the effects are thus the same as those of a marginal reduction in π^* (see Proposition 3).

The next proposition focuses on the welfare implications of inflation conservatism.

Proposition 7 *Suppose $\pi^* = 0$ and $p_H < 1$. Let λ^* denote the value of $\lambda \geq 0$ that maximizes households' unconditional welfare EV_t where V_t is defined in equation (3). In the sunspot equilibrium, $\lambda^* > 0$.*

Proof: See Online Appendix, Section A.

In words, strict inflation conservatism—the welfare-maximizing configuration in the fundamental equilibrium—is not desirable in the sunspot equilibrium. This stands in contrast to the optimality of strict inflation conservatism in the fundamental equilibrium of the model with the natural real rate shock (Nakata and Schmidt, 2019a).

In the Online Appendix, we show that in the sunspot equilibrium, the optimal relative weight, λ^*, can be either smaller or bigger than society's relative weight on output gap stabilization $\bar{\lambda}$ and provide the corresponding necessary and sufficient conditions. The reason for this ambiguity with regard to the desirability of inflation conservatism is similar to why the optimal inflation target can be negative or positive. In both cases, a change in the respective policy parameter moves inflation in the low-confidence and high-confidence states in opposite directions.

Finally, it is useful to point out that there is a close relationship between inflation conservatism and a non-zero inflation target.

Proposition 8 *Suppose $p_H < 1$. For any $\hat{\lambda} \geq 0$, there exists a $\hat{\pi}^*$ such that the sunspot equilibrium under optimal discretionary policy associated with the inflation conservatism regime satisfying ($\lambda = \hat{\lambda}, \pi^* = 0$) is replicated by the inflation target regime satisfying ($\lambda = \bar{\lambda}, \pi^* = \hat{\pi}^*$), where*

$$\hat{\pi}^* \equiv \frac{\beta(1 - p_H)r^n \left(\bar{\lambda} - \hat{\lambda}\right)}{\beta\hat{\lambda}(1 - p_H) - (\kappa^2 + \hat{\lambda}(1 - \beta))[(1 - p_L)(\sigma\kappa)^{-1}(1 - \beta p_L + \beta(1 - p_H)) - p_L]}, \quad (28)$$

Proof: See Online Appendix, Section A.

The reverse is not true, as a sufficiently negative inflation target results in a strictly negative high-state output gap, an allocation that is unattainable under inflation conservatism for any $\lambda \geq 0$.[34] An interesting implication of equation (28) is that if the allocation under the optimal inflation target is attainable under inflation conservatism, then the optimal inflation target π^{**} is positive if and only if the optimal relative output weight λ^* is smaller than society's weight $\bar{\lambda}$.[35]

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[33]If $p_H = 1$, a change in λ would not affect allocations.

[34]Likewise, a sufficiently positive inflation target results in a strictly positive high-state inflation rate, an allocation that is also unattainable under inflation conservatism for any $\lambda \geq 0$.

[35]To see this, note that $\beta\hat{\lambda}(1 - p_H) - (\kappa^2 + \hat{\lambda}(1 - \beta))[(1 - p_L)(\sigma\kappa)^{-1}(1 - \beta p_L + \beta(1 - p_H)) - p_L] > 0$ in the sunspot equilibrium.

In summary, there is no foolproof way to improve the sunspot equilibrium by means of a simple modification of the central bank's objective function. Depending on the parameterization the optimal inflation target may be negative or positive. Likewise, the optimal relative weight on output gap stabilization may be smaller or larger than the one implied by households' preferences.

5 Fiscal stabilization policy

This section extends the analysis to fiscal stabilization policy. To do so, we allow the discretionary policymaker to use government spending as an additional stabilization tool. We first show how the consideration of fiscal stabilization policy affects equilibrium existence and allocations. We then turn to the design of fiscal policy by asking how much relative weight should be put on government spending stabilization in the policymaker's objective function.[36]

5.1 The model with government spending

The aggregate private sector behavioral constraints in the model with government spending are

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{29}$$

$$x_t = (1 - \Gamma)g_t + E_t(x_{t+1} - (1 - \Gamma)g_{t+1}) - \sigma\left(i_t - E_t\pi_{t+1} - r_t^n\right), \tag{30}$$

where g_t denotes government spending as a share of steady-state output, expressed in deviation from the steady-state ratio, $x_t \equiv y_t - \Gamma g_t$, with $\Gamma = \frac{\sigma^{-1}}{\sigma^{-1}+\eta}$, will be referred to as the modified output gap, and, in a slight abuse of notation, σ now denotes the inverse of the elasticity of the marginal utility of private consumption with respect to *total output*.

We assume that the provision of public goods provides utility to households and that utility is separable in private and public consumption. A second-order approximation to household preferences leads to[37]

$$V_t = -\frac{1}{2}E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda}x_{t+j}^2 + \bar{\lambda}_g g_{t+j}^2\right). \tag{31}$$

The relative weight on government spending stabilization satisfies $\bar{\lambda}_g = \bar{\lambda}\Gamma\left(1 - \Gamma + \frac{\sigma}{\nu}\right) > 0$, where ν denotes the inverse of the elasticity of the marginal utility of public consumption with respect to total output. As before, $\bar{\lambda} = \kappa/\theta$.

At the beginning of time, society delegates monetary and fiscal policy to a discretionary poli-

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[36]Section I in the Online Appendix presents results of a numerical analysis of the fully non-linear version of the model with fiscal stabilization policy.

[37]See Schmidt (2013) for details.

cymaker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda}x_{t+j}^2 + \lambda_g g_{t+j}^2 \right), \tag{32}$$

where $\lambda_g \geq 0$ is a policy parameter to be set by society when designing the policymaker's objective function. When $\lambda_g = \bar{\lambda}_g$, the policymaker's objective function coincides with society's objective function. The policymaker's optimization problem and the first-order conditions are relegated to the Online Appendix (Section C).

As before, we focus on a sunspot equilibrium where the lower bound is binding in the low-confidence state and slack in the high-confidence state.

Definition 3 *The sunspot equilibrium with fiscal stabilization policy and occasional liquidity traps is defined as a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves the following system of linear equations*

$$x_H = p_H x_H + (1 - p_H)\left[x_L + (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[p_H \pi_H + (1 - p_H)\pi_L - i_H + r^n\right] \tag{33}$$

$$\pi_H = \kappa x_H + \beta\left[p_H \pi_H + (1 - p_H)\pi_L\right] \tag{34}$$

$$\lambda_g g_H = -(1 - \Gamma)\left(\kappa \pi_H + \bar{\lambda}x_H\right) \tag{35}$$

$$0 = \kappa \pi_H + \bar{\lambda}x_H \tag{36}$$

$$x_L = p_L x_L + (1 - p_L)\left[x_H - (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[(1 - p_L)\pi_H + p_L\pi_L - i_L + r^n\right] \tag{37}$$

$$\pi_L = \kappa x_L + \beta\left[(1 - p_L)\pi_H + p_L\pi_L\right] \tag{38}$$

$$\lambda_g g_L = -(1 - \Gamma)\left(\kappa \pi_L + \bar{\lambda}x_L\right) \tag{39}$$

$$i_L = 0, \tag{40}$$

and satisfies the following two inequality constraints

$$i_H > 0 \tag{41}$$

$$\kappa \pi_L + \bar{\lambda}x_L < 0. \tag{42}$$

Equations (35) and (39) represent the policymaker's first order condition for government spending. Under optimal discretionary policy, government spending responds countercyclically to deviations of inflation and the output gap from target.

The sunspot equilibrium is compared to a fundamental equilibrium in a setup where the two-state sunspot shock is replaced with a two-state natural real rate shock. As before, we consider an equilibrium where the lower bound constraint is slack in the high-fundamental state and binding in the low-fundamental state.

Definition 4 *The fundamental equilibrium with fiscal stabilization policy and occasional liquidity*

traps is defined as a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves the following system of linear equations

$$x_H = p_H^f x_H + (1 - p_H^f)\left[x_L + (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[p_H^f \pi_H + (1 - p_H^f)\pi_L - i_H + r_H^n\right] \qquad (43)$$

$$\pi_H = \kappa x_H + \beta\left[p_H^f \pi_H + (1 - p_H^f)\pi_L\right] \qquad (44)$$

$$x_L = p_L^f x_L + (1 - p_L^f)\left[x_H - (1 - \Gamma)(g_H - g_L)\right] + \sigma\left[(1 - p_L^f)\pi_H + p_L^f \pi_L - i_L + r_L^n\right] \qquad (45)$$

$$\pi_L = \kappa x_L + \beta\left[(1 - p_L^f)\pi_H + p_L^f \pi_L\right] \qquad (46)$$

as well as (35), (36), (39) and (40), and satisfies the inequality constraints (41) and (42).

5.2 Equilibrium existence and allocations

The following proposition establishes a necessary and sufficient condition for existence of the sunspot equilibrium in the model with fiscal stabilization policy. The condition for existence of the fundamental equilibrium in the model with the natural real rate shock is provided in the Online Appendix (Section E).

Proposition 9 *The sunspot equilibrium exists if and only if*

$$\lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1 - \Gamma)^2 \frac{1 - p_L + 1 - p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))\right] > 0, \qquad (47)$$

where $\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1 - p_H) - \frac{1 - p_L + 1 - p_H}{\kappa\sigma}\left(1 - \beta p_L + \beta(1 - p_H)\right).$

Proof: See Online Appendix, Section D.

From Proposition 1, we know that the sunspot equilibrium in the model without fiscal stabilization policy and a zero-inflation target exists if and only if $\Omega(\cdot) > 0$. In the model with fiscal stabilization policy, $\Omega(\cdot) > 0$ is a necessary but not a sufficient condition for existence of the sunspot equilibrium. Importantly, the condition for equilibrium existence depends on the policy parameter λ_g. Suppose $\Omega(\cdot) > 0$. Then, the sunspot equilibrium exists if and only if $\lambda_g > \frac{(1-\Gamma)^2}{\Omega(\cdot)}\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1 - \beta p_L + \beta(1 - p_H))\right] > 0.$

Next, we characterize allocations and prices in the sunspot equilibrium.

Proposition 10 *In the sunspot equilibrium, $\pi_L < 0$, $x_L < 0$, $g_L > 0$, $\pi_H \leq 0$, $x_H \geq 0$ and $g_H = 0$. When $p_H < 1$, then $\pi_H < 0$, $x_H > 0$.*

Proof: See Online Appendix, Section D.

The policymaker increases government spending when the lower bound is binding, and keeps government spending at its steady state level otherwise. The same holds true for the fundamental equilibrium. See Online Appendix (Section E).

5.3   Welfare implications of fiscal stabilization policy

The previous proposition tells us that the path of government spending in a liquidity trap under optimal discretionary policy is similar to the government spending path in Mertens and Ravn (2014)'s scenario of an exogenous government spending stimulus that prevails for as long as the economy is in the low state. Mertens and Ravn (2014) show that the government spending intervention lowers the inflation rate in an expectations-driven liquidity trap relative to an alternative scenario where government spending remains constant throughout the simulation. As we will see shortly, the same holds true under optimal discretionary policy when the economy is in an expectations-driven liquidity trap. This deflationary effect of the government spending stimulus will have a bearing on the optimal design of the policymaker's objective function. For now, whenever we consider the model with the sunspot shock, we assume that any change in policy parameters complies with the conditions for existence of the sunspot equilibrium.

Let us first establish how a marginal change in the policymaker's relative weight on government spending stabilization λ_g affects allocations and prices.

Proposition 11 *In the sunspot equilibrium,* $\frac{\partial \pi_L}{\partial \lambda_g} > 0$, $\frac{\partial x_L}{\partial \lambda_g} > 0$, $\frac{\partial g_L}{\partial \lambda_g} < 0$, $\frac{\partial \pi_H}{\partial \lambda_g} \geq 0$, $\frac{\partial x_H}{\partial \lambda_g} \leq 0$. *If* $p_H < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} > 0$, $\frac{\partial x_H}{\partial \lambda_g} < 0$.

Proof: See Online Appendix, Section D.

That is, the higher λ_g is, the smaller the fiscal stimulus in the low-confidence state. At the same time, an increase in λ_g raises the inflation rate and the modified output gap in the low-confidence state. Finally, an increase in λ_g raises the inflation rate and lowers the modified output gap in the high-confidence state. Thus, the higher λ_g the closer to target is the economy. As mentioned before, and consistent with the results in Mertens and Ravn (2014), a persistent increase in government spending in the low-confidence state is deflationary. With the policy rate stuck at the lower bound, the decline in inflation raises the real interest rate and aggravates the drop in the output gap. Higher deflation at the lower bound, in turn, reduces conditional inflation expectations away from the lower bound and worsens the trade-off between output gap and inflation stabilization in the high-confidence state. Since a higher λ_g is associated with a smaller government spending stimulus, an increase in λ_g improves stabilization outcomes.

In the fundamental equilibrium, an increase in λ_g mitigates the increase in government spending in the low state, as in the sunspot equilibrium. However, unlike in the sunspot equilibrium, the policymaker's government spending stimulus is inflationary in the fundamental-driven liquidity trap. A smaller government spending stimulus consequently leads to less inflation and, via real interest rates, to a larger drop in the output gap at the lower bound. The worsening of stabilization outcomes in the low-fundamental state fosters the trade-off between output gap and inflation stabilization in the high-fundamental state, leading to lower inflation and a bigger modified output gap. See Online Appendix, Section E.

Figure 4: The effect of reduction in λ_g on low-state aggregate demand and supply

(a) Model with sunspot shock   (b) Model with fundamental shock
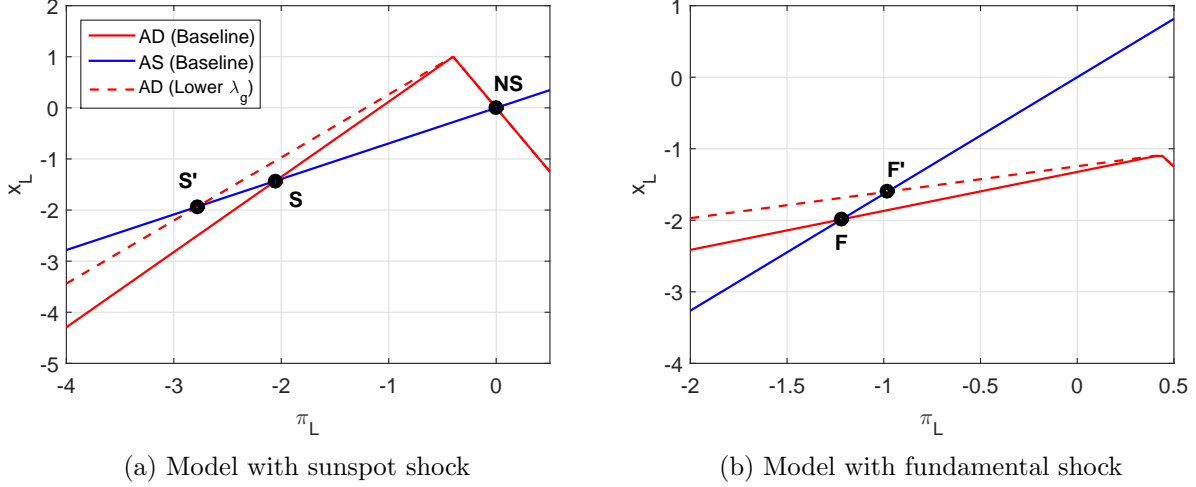
Note: Solid lines: $\lambda_g = \bar{\lambda}_g$; dashed lines: $\lambda_g = \bar{\lambda}_g/10$. In the left panel, S marks low-state output gap and inflation in the sunspot equilibrium in the baseline, S' marks outcomes in the sunspot equilibrium in case of a lower λ_g and NS marks outcomes in the no-sunspot equilibrium. In the right panel, F marks low-state output gap and inflation in the fundamental equilibrium in the baseline and F' marks outcomes in the fundamental equilibrium in case of a lower λ_g. Inflation is expressed in annualized terms.

It is instructive to show how a change in λ_g affects the low-state AD and AS curves in the two models.[38] The low-state AD and AS curves in the model with the sunspot shock are then given by

$$\textbf{AD-sunspot:} \quad x_L = \min\left[\frac{1}{\lambda_g + (1-\Gamma)^2\bar{\bar{\lambda}}}\left(\frac{\sigma\lambda_g}{1-p_L}r^n + \left(\frac{\sigma p_L \lambda_g}{1-p_L} - (1-\Gamma)^2\kappa\right)\pi_L\right), -\frac{\kappa}{\bar{\bar{\lambda}}}\pi_L\right]$$

$$\textbf{AS-sunspot:} \quad x_L = \frac{1-\beta p_L}{\kappa}\pi_L,$$

where π_H and x_H have been set equal to zero—reflecting the fact that the inflation target is zero. For the model with the fundamental shock, the low-state AD and AS curves are given by

$$\textbf{AD-fundamental:} \quad x_L = \min\left[\frac{1}{\lambda_g + (1-\Gamma)^2\bar{\bar{\lambda}}}\left(\frac{\sigma\lambda_g}{1-p_L^f}r_L^n + \left(\frac{\sigma p_L^f \lambda_g}{1-p_L^f} - (1-\Gamma)^2\kappa\right)\pi_L\right), -\frac{\kappa}{\bar{\bar{\lambda}}}\pi_L\right]$$

$$\textbf{AS-fundamental:} \quad x_L = \frac{1-\beta p_L^f}{\kappa}\pi_L,$$

where again π_H and x_H have been set equal to zero.

Figure 4 depicts how the AD-AS curves are affected by a reduction in λ_g.[39] The left panel considers the model with the sunspot shock. Intersection point S marks low-state output gap and inflation in the sunspot equilibrium, and intersection point NS marks low-state output gap and

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[38]As in the setup without fiscal stabilization policy, we have to assume that the high state is absorbing to construct the AD-AS schedules.

[39]The parameterization follows Table 1, except that we now account for a non-zero steady-state government spending to output ratio of 0.2. As before, $p_L = 0.9375$ and $p_L^f = 0.85$.

inflation in the no-sunspot equilibrium, i.e. the equilibrium in which the sunspot shock does not affect agents' decisions so that $\{x_L, \pi_L, i_L, g_L\} = \{x_H, \pi_H, i_H, g_H\}$. The right panel considers the model with the fundamental shock, and intersection point F marks the output gap and inflation in the low state of the fundamental equilibrium.

In both models, the AD curve becomes flatter to the left of the kink when λ_g is lowered. Intuitively, when the policymaker raises (lowers) government spending more aggressively in response to a decline (increase) in inflation, aggregate demand responds ceteris paribus less elastically to a change in inflation. In the model with the sunspot shock, the AD curve is steeper than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect when the lower bound is binding to the south-west. In contrast, in the model with the fundamental shock, the AD curve is flatter than the AS curve, and hence a flattening of the AD curve shifts the point at which the two curves intersect to the north-east. These results are consistent with those in Mertens and Ravn (2014) for an exogenous change in government spending in a liquidity trap, although an exogenous change in government spending results in a change of the level rather than the slope of the AD curve.[40]

Propositions 10 and 11 together have a straightforward implication for the optimal value of λ_g in the sunspot equilibrium.

Proposition 12 *Let λ_g^* denote the value of λ_g that maximizes households' unconditional welfare EV_t where V_t is defined in equation (31). In the sunspot equilibrium, $\lambda_g^* \to \infty$.*

It is easy to show that as $\lambda_g \to \infty$, $g_L \to 0$. Intuitively, if it becomes infinitely costly for the policymaker to adjust government spending, she will not use it as a stabilization tool. This turns out to be the optimal configuration in the sunspot equilibrium.[41]

In contrast, in the fundamental equilibrium of the model with the natural real rate shock, λ_g^* is finite and it is strictly smaller than $\bar{\lambda}_g$ (Schmidt, 2017).

5.4 Why is government spending raised in the low-confidence state?

If an expansionary fiscal policy in the low-confidence state moves the economy further away from target in both confidence states, why does the policymaker not refrain from raising government spending in the low-confidence state? To shed light on this question consider the following thought experiment. Suppose, $\lambda_g \to \infty$, i.e. there is no systematic use of government spending for stabilization purposes in the low-confidence state. Consider some period $T \geq 0$ where the economy is in the low-confidence state and the lower bound is binding. For ease of exposition, let $p_H = 1$. The

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[40]Section D in the Online Appendix provides a comparison of our setup where government spending is an endogenous variable set by an optimizing policymaker to the case where government spending is an exogenous variable.

[41]Section D in the Online Appendix provides a numerical example of how λ_g affects allocations and welfare in the sunspot equilibrium.

private sector behavioral constraints for period T can then be written as

$$x_L^T = (1 - \Gamma)g_L^T - p_L\frac{(1 - \beta p_L)\kappa^2 + (1 - \beta)(1 - \beta p_L + \beta(1 - p_H))\bar{\lambda} + \kappa\sigma\left(\kappa^2 + \bar{\lambda}(1 - \beta p_H)\right)}{\kappa E}r^n + \sigma r^n$$

$$\pi_L^T = \kappa x_L^T - \beta p_L\frac{\kappa^2 + \bar{\lambda}(1 - \beta p_H)}{E}r^n,$$

where π_L^T, x_L^T, g_L^T are the inflation rate, the modified output gap and government spending in period T. Now suppose that in period T there is an unexpected one-time increase in government spending. The marginal effect of this policy on the modified output gap and the inflation rate in period T is $(\partial x_L^T/\partial g_L^T) = 1 - \Gamma > 0$ and $(\partial \pi_L^T/\partial g_L^T) = \kappa(1 - \Gamma) > 0$. In words, the unexpected and temporary government spending stimulus raises the modified output gap and inflation in the low-confidence state.[42]

Hence, if expectations do not change, an increase in government spending is expansionary. However, a policy announcement of a purely temporary, one-time fiscal stimulus is not time-consistent. If the economy continues to be in the low-confidence state in the next period, any discretionary policymaker with an objective function satisfying $\lambda_g < \infty$ will have an incentive to implement a government spending stimulus in that period. In equilibrium, agents anticipate that the discretionary policymaker will raise government spending whenever the economy transitions from the high-confidence state to the low-confidence state and that she will keep government spending at a higher level for as long as the economy remains in the low-confidence state. Since the low-confidence state is highly persistent, expansionary government spending at the lower bound is contractionary, as in Mertens and Ravn (2014).

5.5 Avoiding the sunspot equilibrium

The results presented so far might appear disappointing from the perspective of policy design. Clearly, the sunspot equilibrium cannot be improved by allowing the discretionary policymaker to use government spending as an additional policy instrument. However, Proposition 9 implies that society can eliminate the sunspot equilibrium altogether. To do so it has to make the relative weight on government spending stabilization in the policymaker's objective function sufficiently small.

Having written the Phillips curve (29) and the Euler equation (30) in terms of the modified output gap x_t, it is apparent that the policy rate and government spending are substitutes—conditional on a given level of expected future government spending. Since deviations in government spending from the efficient steady state level are costly from a welfare perspective, an optimizing policymaker who faces the same objective function as society uses only the policy rate to stabilize inflation and the output gap when the lower bound constraint is not binding. When the lower bound is binding, the policymaker raises government spending to partially stabilize inflation and the output gap, but since doing so is costly, she refrains from using government spending as a

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[42]This echoes the result by Wieland (2018) that it is the persistence of the fiscal policy intervention at the lower bound rather than the type of the liquidity trap that matters for the sign of government spending multipliers.

Figure 5: Avoiding the sunspot equilibrium
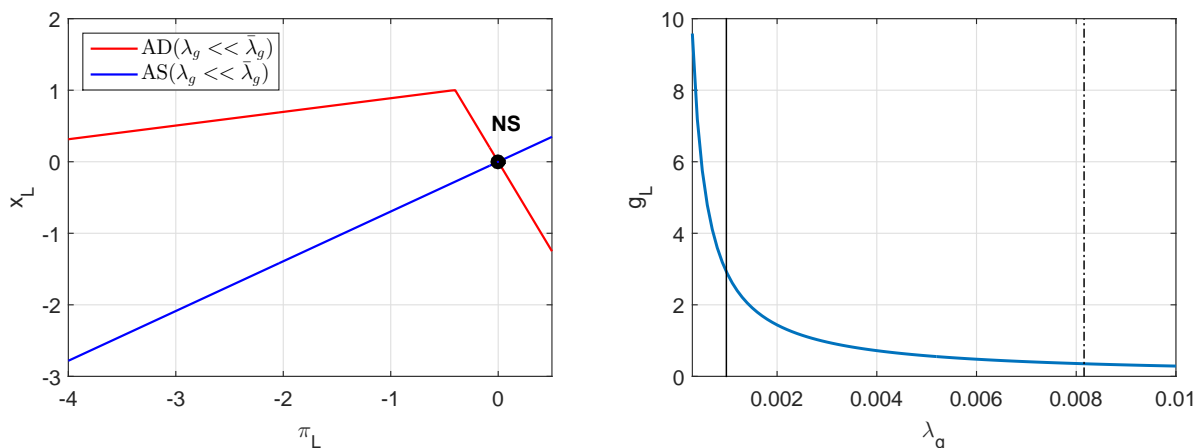
Note: The left panel shows the low-state AD-AS schedule for $\lambda_g = 0.00012 << \bar{\lambda}_g$. Inflation is expressed in annualized terms. The right panel shows how the counterfactual government spending response (expressed as a share of steady state output) to an inflation rate of -2.5% and an output gap of -1.6% depends on λ_g. The dash-dotted vertical line indicates the case where the policymaker has the same objective function as society as a whole, $\lambda_g = \bar{\lambda}_g$. For values of λ_g to the left of the solid vertical line, the sunspot equilibrium does not exist.

strong substitute for the policy rate.

By assigning a sufficiently small λ_g, society reduces the policymaker's perceived costs associated with deviations in government spending from the efficient level such that she is willing to use government spending aggressively as a substitute for the policy rate.[43] Since the lower bound is not binding when inflation and the modified output gap are stabilized, a sufficiently small λ_g rules out the sunspot equilibrium. In this case, the only equilibrium is the no-sunspot equilibrium where the sunspot shock does not affect agents' behavior. In the no-sunspot equilibrium, all variables are at target in both confidence states. The left panel in Figure 5 provides a graphical illustration. For a sufficiently low λ_g the AD curve to the left of the kink becomes flatter than the AS curve and there is only one intersection point left, which is the one associated with the no-sunspot equilibrium.

From a practical perspective, an important question is whether a policymaker with a sufficiently small λ_g to rule out the sunspot equilibrium would be consistent with quantitatively plausible variations in government spending in the face of actual fluctuations in output and inflation. To shed light on this question, we conduct the following counterfactual experiment. We first calculate the annualized inflation rate and the output gap in the low state of the sunspot equilibrium when the policymaker has the same objective function as society $(\lambda_g = \bar{\lambda}_g)$. Unlike for the numerical analysis based on the AD-AS curves, we do not have to assume that the high-confidence state is absorbing, and set $p_H = 0.98$. In this case, annualized inflation is -2.5% and the output gap is -1.6% in the low-confidence state. We then ask by how much a policymaker with a λ_g low enough to rule out the sunspot equilibrium would raise government spending *taking as given* the above outcomes for inflation and output.

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[43]Importantly, expected future government spending moves less than actual low-state government spending in response to a change in λ_g, for there is a positive probability to jump to the high state in the future.

The right panel of Figure 5 plots the counterfactual government spending response as a function of λ_g. A policymaker with a sufficiently small λ_g to rule out the sunspot equilibrium would raise government spending by at least 3% of total output, a quantitatively non-negligible but plausible number.

Finally, note that the mechanism underlying the elimination of the expectations-driven liquidity trap equilibrium under optimal discretionary policy with a sufficiently low relative weight on government spending stabilization differs from the result in Benhabib et al. (2002) and Woodford (2003) that an expectations-driven liquidity trap can be ruled out by the adoption of a non-Ricardian monetary-fiscal regime. While a non-Ricardian intervention to avoid a liquidity trap may also comprise a strong fiscal stimulus—for instance tax cuts or an increase in transfers—the key difference to the fiscal remedy studied in this paper is that under the non-Ricardian regime the policymaker does not adjust future budget surpluses to make sure that the intertemporal budget constraint of the government is satisfied. As a consequence, households' perceived after-tax net wealth increases in response to the fiscal intervention, which stimulates private demand and, thereby, inflation. Instead, under the Ricardian policy regime studied here, agents know that the policymaker adjusts budget surpluses in response to the government spending stimulus to make sure that the intertemporal government budget constraint is satisfied, and expectations-driven liquidity traps are ruled out by engineering sufficiently strong inter-temporal substitution towards present consumption.

6 Sunspot equilibria with richer dynamics

In our baseline setup, the economy jumped to the expectations-driven liquidity trap when the confidence state switched from high to low. We now consider sunspot equilibria with richer—and potentially more realistic—transition dynamics towards liquidity traps.

The economy is represented by a non-linear version of our New Keynesian baseline setup with government spending.[44] A detailed description of the household and firm problems, and the derivation of the aggregate private sector behavioral constraints is provided in Section G of the Online Appendix. It is, however, useful to explicitly state the representative household's welfare as represented by her expected discounted lifetime utility

$$
V_t = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \chi_Y \frac{Y_{t+j}^{1+\eta}}{1+\eta} + \chi_G \frac{G_{t+j}^{1-\frac{1}{\nu}}}{1 - \frac{1}{\nu}} \right),
\tag{48}
$$

where C_t denotes private consumption, Y_t is output (and has been substituted in for labor using the aggregated production technology), and G_t is utility-generating government spending.

Focusing on interior equilibria, the non-policy block of the model is described by the following

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[44]Using a fully non-linear model allows us to address potential concerns about the accuracy of the results in the baseline setup with log-linearized private sector behavioral constraints. See also Sections H and I in the Online Appendix.

equations[45]

$$C_t^{-\frac{1}{\sigma}} = \beta R_t \mathrm{E}_t C_{t+1}^{-\frac{1}{\sigma}} \Pi_{t+1}^{-1} \tag{49}$$

$$Y_t = (C_t + G_t)\left(1 + \frac{\phi}{2}(\Pi_t - 1)^2\right) \tag{50}$$

$$Y_t\left(\chi_Y Y_t^{\eta} C_t^{\frac{1}{\sigma}} - 1\right) = \frac{\phi}{\theta}\Big((\Pi_t - 1)\Pi_t(C_t + G_t)$$
$$- \beta \mathrm{E}_t \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}(\Pi_{t+1} - 1)\Pi_{t+1}(C_{t+1} + G_{t+1})\Big), \tag{51}$$

where R_t is the gross nominal interest rate, and Π_t is the gross inflation rate. Equation (49) is the consumption Euler equation, equation (50) is the aggregate resource constraint, and equation (51) is a non-linear Phillips curve. We assume that price stickiness arises in the form of quadratic price adjustment costs—with parameter $\phi > 0$ capturing their size—which facilitates the numerical solution of the model.[46] While there are no fundamental shocks, we allow for sunspot shocks. An interior private sector rational expectations equilibrium then consists of sequences of non-negative allocations $\{C_t, Y_t\}_{t=0}^{\infty}$, and prices $\{\Pi_t\}_{t=0}^{\infty}$, that solve equations (49)–(51) for a given sequence of policies $\{R_t \geq 1, G_t \geq 0\}_{t=0}^{\infty}$ and a process for the sunspot.

6.1 Gradual transitions to a liquidity trap

For starters, let us assume that the policymaker has the same objective function as society, i.e. the policymaker is benevolent. Each period, the benevolent policymaker maximizes (48) subject to (49)–(51) and the lower bound constraint $R_t \geq 1$. The formal optimization problem and the first order conditions are provided in the Online Appendix (Section G).

The model has two deterministic steady states. In the intended deterministic steady state, there is price stability ($\Pi = 1$) and the lower bound constraint is slack ($R = 1/\beta$). In the unintended deterministic steady state, there is deflation ($\Pi = \beta$) and the lower bound constraint is binding ($R = 1$). We exploit this steady state multiplicity to construct sunspot equilibria that originate away from the lower bound—potentially close to the intended steady state—and that converge to a point where the lower bound is binding and inflation is negative. In contrast to the perfect-foresight equilibria studied in Benhabib et al. (2001), and in order to comply with the focus of our baseline setup on temporary expectations-driven liquidity traps, we assume that each period, the economy may jump to the intended steady state with probability $1 - p_L$. Once the economy is in the intended steady state, all uncertainty is resolved. We refer to the point to which the economy is converging in these sunspot equilibria—before jumping to the intended steady state—as the liquidity trap state. Formally, after imposing a binding lower bound, the liquidity trap state is defined as a

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[45] As before, we assume that fiscal policy is Ricardian so that the household's transversality condition holds on- and off-equilibrium.

[46] A production subsidy ensures that the deterministic steady state with a slack lower bound constraint is efficient by offsetting the distortions arising from monopolistic competition.

vector $\{\Pi_L, Y_L, C_L, G_L, \lambda_L^{PC}, \lambda_L^{RC}\}$ that solves

$$1 = \beta \left(p_L \Pi_L^{-1} + (1 - p_L) \left(\frac{C_L}{C_{ISS}} \right)^{\frac{1}{\sigma}} \right) \tag{52}$$

$$0 = Y_L \left(\chi_Y Y_L^{\eta} C_L^{\frac{1}{\sigma}} - 1 \right) - \frac{\phi}{\theta}(1 - \beta p_L)(\Pi_L - 1)\Pi_L(C_L + G_L) \tag{53}$$

$$Y_L = \left(1 + \frac{\phi}{\theta}(\Pi_L - 1)^2 \right)(C_L + G_L) \tag{54}$$

$$\chi_Y Y_L^{\eta} = \lambda_L^{RC} + \left(\chi_Y(1 + \eta)Y_L^{\eta} C_L^{\frac{1}{\sigma}} - 1 \right)\lambda_L^{PC} \tag{55}$$

$$0 = \phi(C_L + G_L)(\Pi_L - 1)\lambda_L^{RC} + \frac{\phi}{\theta}(C_L + G_L)(2\Pi_L - 1)\lambda_L^{PC} \tag{56}$$

$$\chi_G G_L^{-\frac{1}{\nu}} = \left(1 + \frac{\phi}{2}(\Pi_L - 1)^2 \right)\lambda_L^{RC} + \frac{\phi}{\theta}(\Pi_L - 1)\Pi_L \lambda_L^{PC} \tag{57}$$

where λ_L^{RC} and λ_L^{PC} are the Lagrange multipliers associated with the resource constraint and the Phillips curve respectively, and C_{ISS} is the level of private consumption in the intended steady state. The sunspot equilibria are then constructed by perturbing the solution for the liquidity trap state and iterating backwards using the dynamic equilibrium conditions.

Figure 6 depicts global dynamics of private consumption, government spending and inflation. The parameterization is the same as in the baseline setup and is summarized in Table 1.[47] We set the probability of jumping to the intended steady state to 5 percent. The solid line in each of the two panels represents sunspot equilibria that converge to the liquidity trap state before jumping to the intended steady state at some point. Due to the lack of an initial condition, any path originating on this line is a sunspot equilibrium. The left panel depicts these equilibria in the inflation private-consumption space, and the right panel depicts them in the inflation government-spending space.

To give an example, suppose that the economy is initially somewhere on the solid line in the neighborhood of the intended steady state. Agents are pessimistic about the economic outlook, and expect inflation to decline. In the presence of sticky prices, firms respond to the expected decline in future prices by gradually lowering current prices. Eventually, the policy rate hits the lower bound, real interest rates start to increase and the policymaker begins to raise government spending. At this point, private consumption, which initially increases above the intended steady state, declines towards the liquidity trap state. At some point, potentially after having spent several years in the liquidity trap state, a positive sunspot shock occurs, agents' pessimism recedes, and the economy settles on the intended steady state.

Finally, for the sake of completeness, we also plot the unintended deterministic steady state under optimal time-consistent policy. Inflation and private consumption are lower, and government

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[47]We calibrate the price-adjustment-cost parameter ϕ such that the slope of the Phillips curve, when log-linearized around the intended steady state, is identical to the one in the baseline setup. Furthermore, we set χ_Y and χ_G such that total output equals one in the intended steady state, and the government spending to output ratio equals 0.2.

Figure 6: Global dynamics
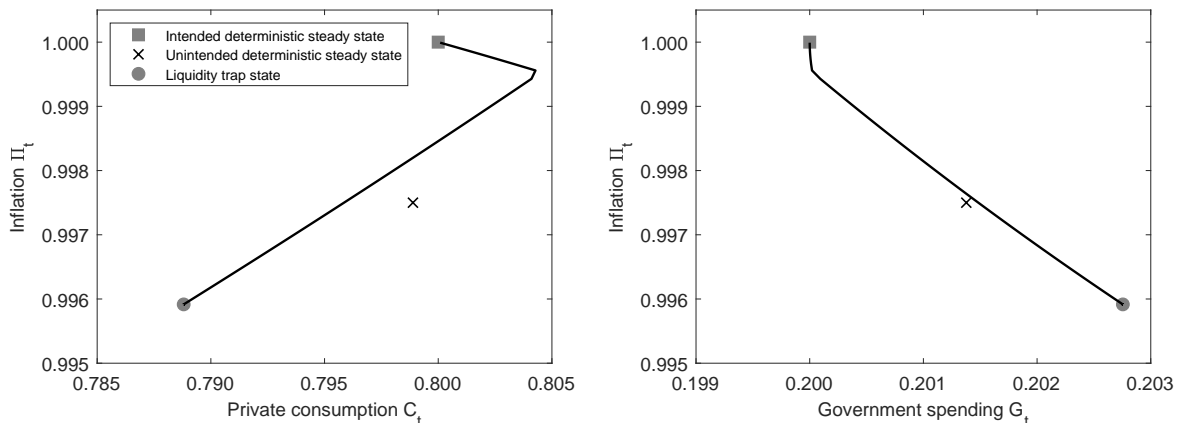
Note: The solid lines indicate equilibrium trajectories towards the liquidity trap state under optimal time-consistent monetary and fiscal policy.

spending is higher, in the liquidity trap state than in the unintended deterministic steady state. This is consistent with the property of the linearized baseline model that a marginal reduction in the persistence of the low-confidence state p_L lowers inflation and the output gap in the low-confidence state.

6.2 The perils of a higher inflation target

Next, we consider the case where society assigns a non-zero inflation target to the policymaker. In this case, the policymaker's objective function differs from society's objective function as follows

$$V_t^{MF} = V_t - \frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\Pi_{t+j} - \Pi^*\right)^2, \tag{58}$$

where Π^* is the gross inflation target.[48] We assume that firms' price adjustment costs are indexed to Π^*.[49] Figure 7 depicts global dynamics of private consumption, government spending and inflation for the case where the policymaker aims for an annualized inflation rate of 1% (dashed lines). We also plot dynamics for the benchmark case where the policymaker has the same objective function as society (solid lines). Due to price indexation, the intended steady-state levels of private and public consumption are unaffected by the assignment of a positive inflation target, and intended-steady-state inflation moves one-for-one with the target. At the same time, the liquidity trap state features more deflation, a lower level of private consumption and a higher level of government spending than

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[48] In the non-linear model, inflation does not show up directly in the welfare-relevant objective function (48) so that the inflation target is imposed by means of an additional term in the policymaker's objective function. This specification implies that the assignment of a non-zero inflation target goes hand-in-hand with some inflation conservatism.

[49] Hence, the resource constraint becomes $Y_t = (C_t + G_t)\left(1 + \frac{\phi}{2}(\Pi_t - \Pi^*)^2\right)$. Likewise, in the Phillips curve the terms $(\Pi_m - 1)$, $m = t, t+1$ are replaced with $(\Pi_m - \Pi^*)$.

Figure 7: Global dynamics with a positive inflation target
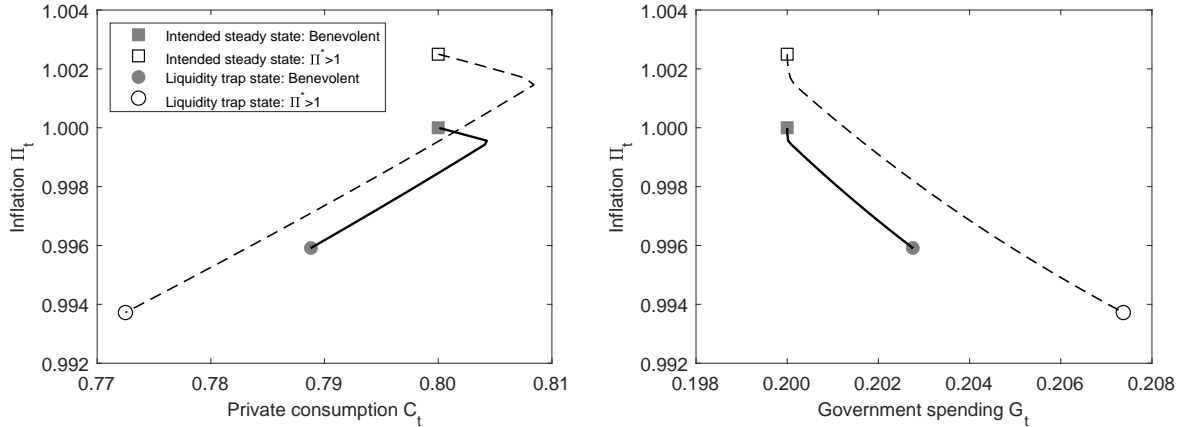
Note: The solid lines indicate equilibrium trajectories towards the liquidity trap state under optimal time-consistent monetary and fiscal policy when the policymaker has the same objective function as society. The dash-dotted lines indicate equilibrium trajectories when the policymaker pursues an inflation target of 1% (annualized).

in case of the benevolent policymaker.[50] This confirms the analytical result from the baseline setup that a higher inflation target goes hand-in-hand with lower inflation in expectations-driven liquidity traps.

6.3 Avoiding the sunspot equilibria

Finally, we turn to the question of how policy design can be used to prevent these sunspot equiliria. In our baseline setup, we were able to avoid expectations-driven liquidity traps based on the insight that fiscal policy can operate as a substitute for monetary policy when the policy rate is at its lower bound. We now demonstrate that this insight also holds true in the fully non-linear model in the context of sunspot equilibria that entail gradual transitions to a liquidity trap.

To do so, we have to show that we can design a policy objective function that eliminates the liquidity trap state. Suppose, the policymaker maximizes

$$V_t^{MF} = -\frac{1}{2}E_t \sum_{j=0}^{\infty} \beta^j \left(\Pi_{t+j} - 1\right)^2,$$ (59)

rather than society's expected lifetime utility (48). This objective function is not concerned with government spending stabilization and therefore belongs to the class of objective functions that was shown to avoid the sunspot equilibrium in the baseline model setup. With this policy objective function, the conditions for the liquidity trap state consist of (52)–(54), and

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[50]The increase in the rate of deflation in the liquidity trap state resulting from the increase in the inflation target is amplified by the endogenous government spending response.

Figure 8: Avoiding the sunspot equilibria
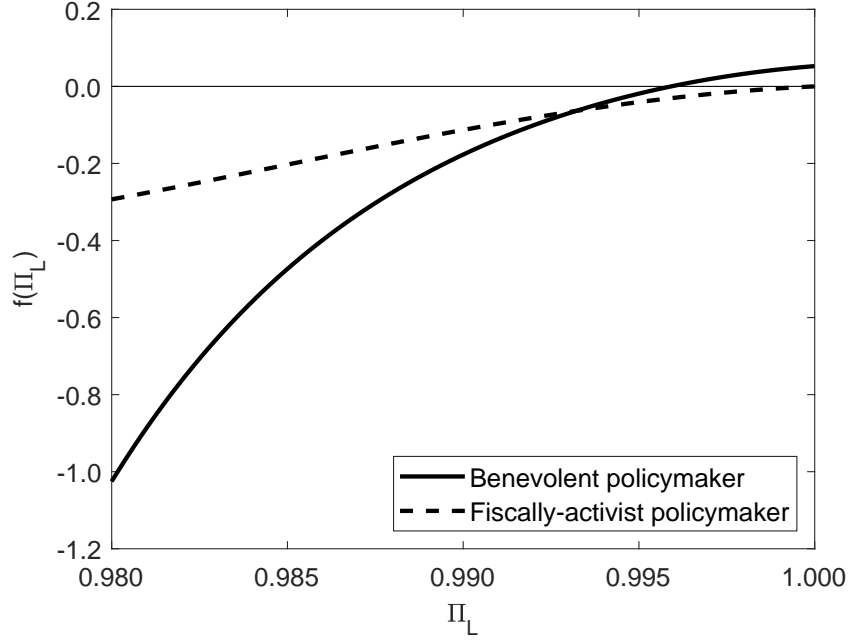
Note: In case of the fiscally-activist policymaker, the figure plots $10^3 \times f(\Pi_L)$.

$$0 = \lambda_L^{RC} + \left(\chi_Y (1+\eta) Y_L^\eta C_L^{\frac{1}{\sigma}} - 1 \right) \lambda_L^{PC} \tag{60}$$

$$0 = (\Pi_L - 1) + \phi(C_L + G_L)(\Pi_L - 1)\lambda_L^{RC} + \frac{\phi}{\theta}(C_L + G_L)(2\Pi_L - 1)\lambda_L^{PC} \tag{61}$$

$$0 = \left(1 + \frac{\phi}{2}(\Pi_L - 1)^2 \right) \lambda_L^{RC} + \frac{\phi}{\theta}(\Pi_L - 1)\Pi_L \lambda_L^{PC} \tag{62}$$

Starting with a guess for Π_L, we can solve (52)–(54) and (60)–(61) recursively. To be consistent with equilibrium, the solution associated with the guess for Π_L also has to satisfy (62). Similarly, when the policymaker has the same objective function as society, we can solve (52)–(56) recursively and check whether (57) holds for the particular guess. In both cases, the requirement that private consumption has to be positive in an interior equilibrium imposes a floor on permissible values for Π_L and the requirement that the lower bound constraint is binding in a liquidity trap imposes a ceiling. Thus, we only consider $\Pi_L \in (\beta p_L, 1)$.

Let $f(\pi_L)$ be the residual of either condition (57)—in the case of the benevolent policymaker— or condition (62)—in the case of the fiscally-activist policymaker. Figure 8 plots $f(\pi_L)$ for $\Pi_L \in [0.98, 1)$.[51] Under the benevolent policymaker, $f(\Pi_L)$ has one root at $\Pi_L = 0.996$. Under the fiscally-activist policymaker, $f(\Pi_L)$ does not have a root on the domain $\Pi_L \in (\beta p_L, 1)$. The fiscally-activist policymaker thus avoids the sunspot equilibria. At the same time, she supports the

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[51] For a better visualization, the figure considers only a subset of permissible values for Π_L. It has been verified that there exist no admissible roots outside of this subset.

intended steady state.

7 Conclusion

Expectations-driven liquidity traps differ from fundamental-driven liquidity traps in terms of their implications for the design of desirable monetary and fiscal stabilization policies.

In the model with fundamental-driven liquidity traps, it is desirable for society to assign a strictly positive inflation target or an inflation-conservative objective function to the central bank. No such clear-cut policy recommendations can be derived for the model with expectations-driven liquidity traps. The optimal inflation target may be negative or positive. Likewise, the optimal relative weight on inflation in the central bank's objective function may be smaller or larger than the weight that society puts on inflation stabilization, depending on parameter values.

Turning to fiscal policy, the use of government spending as an additional stabilization tool— welfare-improving in the case of fundamental-driven liquidity traps—is welfare-reducing in the case of expectations-driven liquidity traps. Nevertheless, it may be desirable to assign an explicit role to fiscal policy in an economy prone to the latter, for the appointment of a policymaker who puts a sufficiently small relative weight on government spending stabilization eliminates the sunspot equilibrium.

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