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JEL Classification: H32, H25, L12, L14

Keywords: Digital Platforms, multinational firms, corporate income taxation, Formula Apportionment, Separate accountin

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Profit-splitting Rules and the Taxation of Multinational Digital Platforms

Francis Bloch* Gabrielle Demange†

October 14, 2020

Abstract

This paper analyzes the strategy of a monopolistic digital platform serving users from two jurisdictions with different corporate tax rates. We consider two profit-splitting rules, Separate Accounting (SA) and Formula Apportionment (FA) based on the number of users in the two jurisdictions. We show that, even in the absence of transfer pricing, the platform shifts profit from the high-tax to the low-tax jurisdiction exploiting network externalities under SA and manipulating the apportionment key under FA. In order to shift profit, the platform distorts prices and quantities. Under SA, the direction of the distortions depends on the sign of the externalities. We use a numerical simulation to show that the ranking of fiscal revenues under the two régimes differ in the two jurisdictions: the high-tax jurisdiction prefers SA to FA whereas the low-tax jurisdiction prefers FA to SA.

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1 Introduction

The taxation of multinational digital platforms poses important conceptual and practical challenges. Currently, the profit of platforms is taxed mostly at the country of residence (oftentimes the United States) or in low-tax countries (such as Ireland or Luxembourg for business connected to the European Union). As platforms do not maintain physical presence in the countries in which they operate, the Permanent Establishment rules designed in the 1920's to allow countries to tax multinationals while avoiding double taxation do not apply. Furthermore, without physical presence, usual methods of formula apportionment cannot work as digital platforms do not employ personnel or own assets in the countries in which they operate. In order to address these difficulties, the OECD has launched in May 2019 a new work programme to replace the Permanent Establishment rules with a new nexus which is better adapted to the digital economy and to define new accounting rules for profit splitting.[1] At the same time, in order to put pressure on the platforms, some countries have decided to unilaterally implement a digital service tax.[2] Finally, some major digital platforms have entered voluntary agreements to let source countries tax part of their business income.[3] The rules for taxation of digital platforms are evolving rapidly, with source countries, where the value of the platforms are created, increasingly gaining the right to tax some of the corporate profits.

The objective of this paper is to shed light on the effect of the new tax régime which is likely to emerge from current policy discussions. We analyze the effect of taxes on a monopolistic digital platforms which generates externalities across users in different jurisdictions. All countries in which the firm operates have the right to tax corporate income and the value of the platform is shared across jurisdictions according to two profit-splitting methods. Under Separate Accounting (SA), the platform (truthfully) declares its profit in every country. Under Formula Apportionment (FA), in the absence of country by country profit reporting, fiscal authorities resort to another apportionment key, the number of users of the platform, to allocate profit

[1] See the proposal in OECD (2019)

[2] This is the case with France, which passed a law in July 2019 imposing a tax of 3% on the revenues based on intermediation and targeted advertising of 27 large digital platforms. Austria has passed a similar law in October 2019 imposing a tax of 5% on online advertising. Other countries, like Turkey, UK, Belgium, Spain and the UK, have also declared their intention to introduce taxes on digital services (ranging from 2% in the UK to 7.5% in Turkey) in 2020. See "Why digital taxes are the new trade war flashpoint", *The Washington Post*, December 10, 2019. However, to the best of our knowledge, none of the proposed taxes has actually been implemented yet.

[3] In January 2016, Google and HM Treasury reached an agreement on the tax liability of Google in the United Kingdom. See "Google agrees to pay British authorities £130 m in back taxes," *The Guardian*, January 23, 2016. In 2016, Facebook decided to stop declaring all its non-US profits in Ireland and agreed to submit profit declarations in all countries in which it operates a sales office.

across jurisdiction. Under both régimes, we study how the platform responds to differences in corporate income tax rates, shifting profits across countries and ultimately affecting the fiscal revenues of the jurisdictions in which it operates. We also study how differences in corporate tax rates result in distortions in the number of users and prices set by the platform in the different jurisdictions. Corporate tax rates – which apply to all firms in the countries not only digital platforms – are taken to be exogenous, and for a first cut we abstract away from tax competition across jurisdictions.

Our first result shows that, even in the absence of transfer pricing, the platform is able to shift profit away from the high-tax jurisdiction to the low-tax jurisdiction. The mechanism by which profit is shifted differs under SA and FA. Under SA, the platform exploits the network externalities created by the platform, raising or reducing the number of users in one country to affect the demand and the profit in the other. Under FA, the platform manipulates the apportionment key to reduce the tax base in one country and increase it in the other.[4] Profit-shifting is costly to the platform. An increase in the gap in corporate tax rates between the high- and low-tax jurisdictions increases distortions with respect to the platform's optimal choice and reduces overall pre-tax profit of the platform. Finally, as profit is shifted towards the low-tax jurisdiction, the fiscal revenues of the low-tax jurisdiction increase with an increase in the gap between the corporate tax rates of the two countries.[5]

In a second set of results, we delve deeper into the effect of differences in corporate tax rates on the quantities and prices chosen by the monopolistic platform. This effect can be decomposed into a direct effect (assuming that quantities in the other jurisdiction remain fixed) and an indirect effect (taking into account the sequence of adjustments due to externalities across jurisdictions). Under SA, the direction of the direct effect depends on the sign of network externalities. If externalities are positive, an increase in the corporate tax rate raises the number of users in the high-tax jurisdiction and reduces the number of users in the low-tax jurisdiction. If externalities are negative, the direct effect is reversed.[6] Under FA, the sign of the direct effect is independent of externalities, and always results in a decrease in the number of users in the

[4] We note that this manipulation under FA can happen even in the absence of network externalities across jurisdictions.

[5] For the high-tax jurisdiction, the effect is ambiguous as the tax base is reduced when the gap increases, and the total effect depends on the elasticity of the tax base with respect to the tax rate.

[6] The intuition underlying this result is easy to grasp: in order to shift profit assuming the number of users in the other jurisdiction fixed, when externalities are positive, the platform has an incentive to increase rather than decrease the number of users in the high-tax jurisdiction, as this increases demand in the low-tax jurisdiction. A similar reasoning shows that the number of users is reduced in the low-tax jurisdiction with positive externalities.

high-tax jurisdiction and an increase in the number of users in the low-tax jurisdiction in order to increase the fraction of profit taxed in the low-tax jurisdiction. Hence a platform with positive network externalities distorts the quantities in opposite ways under SA and FA, resulting in a decrease in the price in the high-tax jurisdiction under SA but an increase under FA.[7]

Finally, we use a numerical simulation to compare profits and tax revenues under SA and FA. We first show that the direct effect dominates the indirect effect for the entire range of tax rate values. Distortions in the number of users are stronger under FA than under SA, resulting in lower pre-tax profit. However, the tax bill of the platform is higher under SA than under FA, so that post-tax profits are comparable in the two régimes. Tax revenues in the high-tax jurisdiction are higher under SA than under FA, but tax revenues in the low-tax jurisdiction are higher under FA than under SA, suggesting that the two countries will disagree in the choice of the profit-splitting régime.

While our model is cast in terms of countries splitting corporate income taxes of a digital platform, the analysis applies more generally. The jurisdictions could be states or sub-national entities inside a federation rather than countries. The tax bill to be shared across jurisdictions could be direct taxes such as sales taxes or VAT rather than indirect taxes on profit. Our theoretical model of profit shifting by multinational digital platforms thus also sheds light on issues which have recently raised about platforms' liability to consumption and local taxes in the United States. As in the case of profit taxation at the OECD, one of the main question is whether local governments (at the state or city levels) have the right to tax online platforms with no physical presence. Recent decisions indicate that local governments are obtaining tax jurisdictions over digital platforms. The June 2018 Supreme Court in North Dakota vs. Wayfair establishes a nexus for online retailers without physical presence in the state, thereby opening the possibility for states to collect sales taxes from internet sales.[8] In the absence of a final court decision, AirBnB has reached voluntary agreements with state and city governments in the United States to collect occupancy taxes.[9] Our model helps understand how a platform reacts to differences in tax rates across cities and states.

The rest of the paper is organized as follows. We discuss the related literature in the next

[7] As we show in the paper, the direct and indirect effects may have opposite signs, making the total effect of an increase in the gap between the two corporate tax rates ambiguous. However, there are some simple cases - such as symmetric markets, or markets with one-sided externalities where the total effect can easily be signed.

[8] See "Supreme Court Widens Reach of Sales Taxes for online Retailers", *The New York Times*, June 21, 2018.

[9] See Bibler, Telster and Tremblay (2018) for a study of tax-compliance by AirBnB hosts in cities with and without tax collection agreements.

subsection. We present the model in Section 2. We analyze profit shifting in Section 3 and output and price distortions in Section 4. Section 5 contains a numerical comparison of SA and FA. Section 6 concludes.

Relation to the literature

This paper is related to two different strands of the theoretical literature: the literature on taxation of two-sided platforms and the literature on Formula Apportionment.

In a series of papers Kind, Koethenbuerger and Schjelderup (2008, 2009, 2010) and Kind, Schjelderup and Stähler (2013) study ad valorem taxes (like VAT) in two-sided platforms. They have generated two main results. First, they show that the classical result in public finance on the domination of ad valorem taxes over unit taxes no longer holds for two-sided markets. Second, the introduction of a value added tax for one side of the market can lead to a change in the entire business model of the platform. For example, the increase in VAT on the price of access for users could induce the platform to set a zero price for Internet access and switch all its revenues to the advertisers side. Bourreau, Caillaud and de Nijs (2018) supplement the model of a two-sided platform by considering data collection and letting consumers select the flow of data uploaded to the platform. Advertisers pay an ad valorem tax. Their main result shows that he introduction of a small tax rate on data collection a results in an increase in fiscal revenues and an increase in the prices and quantities of the platform. By contrast to our paper, they do not provide a general analysis of the comparative statics effect of the tax gap between the two sides of the market. Kotsogiannis and Serfes (2010) study competition between two jurisdictions and two platforms located in each of the jurisdiction. Platforms connect consumers and businesses in their jurisdictions. Jurisdictions compete á la Tiebout over the level of public good and the tax rate to attract mobile consumers and businesses. This model clearly differs from ours regarding mobility, competition, and taxation basis. Instead of taxing users, our focus is the corporate income tax paid by a monopolistic digital platform.

The literature on Formula Apportionment originates with a paper by Gordon and Wilson (1986) who show that the formula used in the United States, which puts positive weight on sales, wages and assets induces distortions in the optimal choice of inputs by the firms. Anand and Sansing (2000) analyze a model where two states bargain over the weights to place on different indicators and show that the weights placed on sales and inputs are typically inefficient in a decentralized equilibrium. In the United States, the weights placed on payroll, property and sales to apportion profits among states for corporate income taxation vary across states and have evolved over time, with an increasing weight placed on sales. This variation across time and space has been used to compute distortions in optimal imput choices (Goolsbee and Maydew (2000)) and the effect of corporate income taxes on economic activity and state tax revenues (Suarez Serrato and Zidar (2018)). Several papers compare SA and FA. Nielsen, Raimondos Moller and Schjelderup (2003) compare SA and FA in a model where transfer prices are used to manipulate the behavior of a subsidiary in an oligopolistic market. Kind, Midelfart and Schjelederup (2005) add a first stage of tax competition where two jurisdictions simultaneously select their corporate income tax rate to maximize fiscal revenues. Nielsen, Raimondos-Moller and Schjelderup (2010) analyze capital investment decisions of a multinational under the two régimes of SA and FA around symmetric tax rates. Finally, Gresik (2010) compares SA and FA when the production cost of the intermediate output is privately known by the multinational. None of the literature on Formula Apportionment has considered externalities in demand across jurisdictions as we do in this paper.

A small number of recent empirical papers analyze the effect of taxes on strategies of digital platforms. Bibler, Telster and Tremblay (2018) use data from AirBnB to study the pass-through of local occupancy taxes on the rental rates. They exploit the fact that AirBnB's tax collecting agreements with city and state authorities are staggered over time to estimate tax compliance and demand elasticity. Using the negative shock of tax collection agreements, the study finds that a 10% increase in the tax rate results in a decrease of nights booked by 3,6%. In a paper more closely related to our theoretical analysis, Lassmann, Liberini, Russo, Cuevas and Cuevas (2020) study the effect of taxes on Facebook advertising prices. They exploit the (voluntary) change in Facebook's tax declaration of non-US profit in 2016, switching from a declaration of all profit in the low-tax Ireland to a declaration of separate profit in all countries with sales offices. This switch, which can be interpreted as a switch to SA, has led Facebook to differentiate ad prices in different countries. They find that ad prices have significantly increased in countries with high corporate tax rates (from 10% to 32% depending on the countries). In order to explain this empirical finding, the authors propose a theoretical model where externalities between advertisers in different countries are induced by the adverse effect on consumers who dislike ads. Our theoretical model, though different, results in the same prediction in the special case where externalities are one-sided and negative: an increase in the corporate tax rate of one country results in an increase in price (and decrease in the number of users) in the high-tax country and a decrease in price (and increase in the number of users) in the low-tax country.

2 The model

We analyze the effect of corporate income taxes on the strategy of a monopolistic platform operating in two jurisdictions. We consider a general model where users of the platform experience network externalities. Depending on the platform, users can be consumers, peers, firms, advertisers, and externalities can be either be positive or negative. Examples of positive externalities abound, ranging from peer-to-peer platforms like Spotify or collaborative platforms connecting users of similar type to platforms connecting buyers and sellers like E-Bay, Airbnb or Amazon marketplace, search engines and digital social media platforms, like Google or Facebook, connecting advertisers to consumers. Advertisers benefit from the presence of more consumers on the platform as this improves the quality of matching consumers to targeted ads, inducing positive network externalities. Externalities can also be negative. For example, consumers who dislike ads are harmed by the presence of advertisers.

Because network externalities are experienced across borders, differences in corporate income tax rates will affect the behavior of the platform, distort its pricing and output strategy and affect profit and fiscal revenues in the two jurisdictions. The objective of our model is precisely to capture these effects. Throughout the analysis, we assume that users are immobile, either because their moving costs are too high or because they have already moved before the platform chooses its prices and the cost of relocation is high.

2.1 A monopolistic platform

A monopolistic platform connects users from two jurisdictions denoted A and B. In a 'general' model, users can be of different types, two to simplify, and present in both jurisdictions. This is the case for two-sided platforms when the two sides are present in the two jurisdictions, such as in most buyer-seller platforms, where buyers and sellers are both present in the two jurisdictions or, to some extent, platforms connecting Internet users (x) and advertisers (y).

We let x_A and y_A denote the total number of users of each type in jurisdiction A and x_B and y_B the total number of users in jurisdiction B. The platform's profit stems from fees paid by users on the two sides of the platform. We assume that the platform can discriminate according to the residence of users. Hence the platform charges four fees, denoted p_A, q_A, p_B and q_B to users of the two types in the two jurisdictions. The volume of use of the platform is supposed to be fixed and identical across users.

For a monopolistic platform, it is equivalent to study the optimal strategy in terms of prices or users, but turns out to be more convenient to let the number of users be the strategic variable chosen by the platform.[10] The interpretation is that the platform chooses the number of users in each jurisdiction, anticipating the maximum price that users are willing to pay to access the platform. Denote these prices by $P_A(z)$, $Q_A(z), P_B(z)$ and $Q_B(z)$ where $z = (x_A, y_A, x_B, y_B)$.[11]

Platform pre-tax profit We suppose, following empirical evidence, that the operating costs of the platform are negligible so that the pre-tax profit in each jurisdiction is given by

$$\begin{aligned} V_A(z) &= x_A P_A(z) + y_A Q_A(z), \\ V_B(z) &= x_B P_A(z) + y_B Q_B(z), \end{aligned}$$

and the total pre-tax profit as

$$V(z) = V_A(z) + V_B(z).$$

2.2 Separate Accounting and Formula Apportionment

The two jurisdictions charge corporate income tax rates t_A and t_B. We assume throughout that country A is the high-tax country so that $t_A \geq t_B$. We consider two regimes of profit-splitting depending on the availability of detailed data on the platform's profit. Such availability is part of the ongoing discussion and is changing fast, in part triggered by platforms themselves anticipating retaliation measures. For example, Facebook voluntarily agreed in 2016 to change its accounting rule to allow computation of profit for many non-US countries and allowing for taxation on these separate profits. Other companies, like Google, still do not provide a transparent account of their profit country by country. In that case, fiscal authorities willing to tax them need to resort to apportionment keys to allocate profit across jurisdictions. In our model, the only apportionment key that can be used is the number of users in each jurisdiction, often referred to as numerical presence.

Denote by $R_A(z)$ and $R_B(z)$ the taxes due to the platform in the two jurisdictions, equivalently the fiscal revenues of the two jurisdictions. We describe how they are computed in the two tax régimes.

[10]See Weyl (2010) for a general argument. showing equivalence between price and user strategies for monopolistic platforms. This equivalence does not hold for competitive platforms where competition in prices (Bertrand) leads to different results than competition in quantities (Cournot). See Belleflamme and Toulemonde (2018) for a study of two platforms competing in prices.

[11]In the online Appendix, we show how prices can be derived from a micro-founded model based on heterogeneous users in the two jurisdictions.

Separate Accounting (SA). The platform declares separately the profit it makes in the two jurisdictions and taxes are paid according to the profits declared in each jurisdiction. The taxes due by the platform to the two jurisdictions are computed as

$$R_A(z) = t_A V_A(z) \text{ and } R_B(z) = t_B V_B(z). \tag{1}$$

Formula Apportionment (FA). Data on profit made by the platform in each jurisdiction are not available and taxes are paid according to the *number of users in each jurisdiction* Under formula apportionment, a firm's tax payments to a given jurisdiction depend on its total profits times an average of the fractions of the users located in that jurisdiction. Letting ω denote the weight on the users of type x, the taxes due by the platform to the two jurisdictions are computed as

$$R_A(z) = t_A V(z)(\omega \frac{x_A}{x_A + x_B} + (1-\omega)\frac{y_A}{y_A + y_B}) \text{ and } R_B = t_B V(z)(\omega \frac{x_B}{x_A + x_B} + (1-\omega)\frac{y_B}{y_A + y_B}) \tag{2}$$

Whatever the tax régime, the post-tax profit of the platform is then given by

$$\Pi(z) = V(z) - R_A(z) - R_B(z) \tag{3}$$

3 Profit shifting

Our objective is to study how the platform reacts to differences in corporate tax rates by distorting its decisions, i.e. by deviating from maximization of the pre-tax profit V.

No externalities across jurisdictions We first observe that in both tax régimes, the platform chooses to maximize the pre-tax profit V whatever the common tax rate. Next we consider as a benchmark the optimal strategy of the platform when corporate tax rates differ, but there are no externalities across jurisdictions. We say that *there are no externalities between jurisdictions* if the demands hence the prices in a jurisdiction do not depend on the number of users in the other jurisdiction. In that case $V_A(z)$ only depend on (x_A, y_A) and $V_B(z)$ only depend on (x_B, y_B). It is easy to see that there will not be any distortion from an optimal strategy under SA, but that the platform will react to differences in corporate tax rates under FA, as stated in the following Proposition:

Proposition 1 *Let $t_A > t_B$ and suppose that there are no externalities across jurisdictions.*

Under SA, the platform maximizes the pre-tax profit V, hence chooses the optimal numbers of users in A and in B. An increase in t_A has no effect on the tax revenues of jurisdiction B.

Under FA, the platform does not maximize the pre-tax profit V. If V is strictly concave, maximized at z^*, the platform reduces the optimal numbers of users in the high-tax jurisdiction and raises them in the low-tax one: $x_A < x_A^*$ and $y_A < y_A^*$, $x_B > x_B^*$ and $y_B > y_B^*$.

Under FA, the distortions in the number of users is associated to distortions in fees, which are higher than at the optimum in the high-tax jurisdiction and lower than at the optimum in the low-tax one.

We now suppose that there are externalities across the two jurisdictions and analyze the platform's decisions under the two régimes.

Profit shifting under Separate Accounting We focus the analysis on the comparative statics effects of a change in the corporate tax rate t_A. This comparative statics analysis allows us to compare the platform's optimal decisions in high-tax and low-tax jurisdiction by starting from a situation where the tax rates are equal and gradually increasing the tax rate in one of the jurisdictions. Our first result shows that in response to an increase in t_A, the platform shifts profits from jurisdiction A to jurisdiction B.

Proposition 2 *Under SA, an increase in the corporate tax rate t_A results in a decrease in the overall pre-tax profit, a decrease in the pre-tax profit in jurisdiction A and an increase in the pre-tax profit in jurisdiction B:*

$V = V_A + V_B$ decreases, V_A decreases, and V_B increases.

Proposition 2 shows that even in the absence of transfer pricing, the platform shifts profit from the high-tax to the low-tax jurisdiction by exploiting externalities in consumption across the two jurisdictions. We first note that, keeping the corporate tax rate t_B constant, the optimal choice of the platform must involve a reduction in the pre-tax profit in jurisdiction A in response to an increase in the corporate tax rate t_A. Furthermore, an increase in the corporate tax rate t_A must have opposite effects on the pre-tax profits in the two jurisdictions. It cannot result in an increase or a decrease of the pre-tax profits in both jurisdictions, as this would contradict the optimality of the choice of the platform either before or after the tax change. Hence the platform increases pre-tax profit in the low-tax jurisdiction. In addition, Proposition 2 shows that an increase in the corporate tax rate of the high-tax jurisdiction exacerbates distortions in the choice of the platform, resulting in a decrease in pre-tax profit. Post-tax profit a fortiori decreases. We obtain an immediate corollary on the effect of an increase in t_A on the fiscal revenues of jurisdiction B.

Corollary 1 *Under SA, an increase in the corporate tax rate t_A results in an increase in the fiscal revenues of jurisdiction B, R_B.*

An increase in the corporate tax rate t_A results in the platform shifting profit to jurisdiction B so that the fiscal revenues in B increase. However, as expected, the effect of an increase in t_A on the fiscal revenues of jurisdiction A are not necessarily monotonic. An increase in t_A results in a reduction in the tax base V_A, and hence the effect of an increase in t_A on the fiscal revenues R_A depend on the elasticity of the tax base with respect to changes in the tax rate.

Profit shifting under Formula Apportionment Under Formula Apportionment, the platform has also an incentive to shift profit across jurisdictions. Profit shifting here is achieved by manipulating the apportionment key.[12] The next proposition shows that in response to an increase in the corporate tax rate t_A, the platform will reduce its tax base in jurisdiction A and increase its tax base in jurisdiction B by lowering the share of each type of users in jurisdiction A.

Proposition 3 *Under FA, an increase in the corporate tax rate t_A results in a decrease in the pre-tax profit V, in the shares of the number of users and the tax base in A and an increase in the share of the number of users and the tax base in B:*

V decreases

$\frac{x_A}{x_A+x_B}$ and $\frac{y_A}{y_A+y_B}$ decrease and the tax base in A, $(\omega\frac{x_A}{x_A+x_B}+(1-\omega)\frac{y_A}{y_A+y_B})V$, decreases

$\frac{x_B}{x_A+x_B}$ and $\frac{y_B}{y_A+y_B}V$ increase and the tax base in B, $V(\omega\frac{x_B}{x_A+x_B}+(1-\omega)\frac{y_B}{y_A+y_B})$, increases.

Under FA, an increase in the corporate tax rate t_A results in efficiency losses and in a decrease in the share of each type of users in jurisdiction A. Hence surely the tax base in A decreases. In jurisdiction B, the efficiency losses are outweighed by the increase in users' share and result in an increase in the tax base. We straightforwardly obtain the following corollary:

Corollary 2 *An increase in the corporate tax rate t_A results in an increase in the fiscal revenues of jurisdiction B, R_B.*

[12] Gordon and Wilson (1986) were the first to note that FA generates distortions in the choice of the platform, albeit in a different model where firms are competitive whereas we consider a monopolistic firm.

Propositions 2 and 3 show that the platform responds to an increase in t_A by shifting profit to the low-tax jurisdiction. The way to achieve this is clear under FA but not under SA; in particular Proposition 2 gives no indication on how a change in t_A affects the number of users. The next section examines these points in a simplified model.

4 Externalities and distortions

We focus on the comparative statics effects of a change in t_A on the number of users.[13] For that we consider a simplified model, with a single type of user in each jurisdiction. This simplified model captures two types of platforms. It first captures one-sided platforms connecting users of a single type. Examples of one-sided platforms are streaming platforms like Napster or Netflix, where users enjoy positive externalities from the presence of other users which improves the the accuracy of the recommendation system. Another example of one-sided platforms are exchanges connecting buyers and sellers where the same agents can be on the two sides of the market like e-Bay or AirBnB, even though these platforms also attract professional sellers and users often specialize as buyers or sellers. The model also captures two-sided platforms with a large imbalance in the number of users of the two types in the two jurisdictions. For example, Google books all advertising contracts in Ireland whereas consumers are located in all European markets. Hotel reservation platforms like Booking connect holiday-makers residing in one region (e.g. Canada or New York) with accommodation offers in another region (e.g. Florida or Arizona). Outsourcing platforms like Amazon Mechanical Turk connect entrepreneurs from high-income countries with workers in developing countries to perform simple tasks.

In the simplified model, cross-border network externalities are one-dimensional, allowing for a transparent analysis of the effect of corporate tax rates on the strategy of the platform. In this one-dimensional model, we let x_A and x_B denote the number of users in A and B. (There is a slight change in notation when the users in different countries are of different type, previously denoted by x_A and y_B).

How the number of users under SA vary depend on the sign of the externalities. Externalities from B to A are said to be *positive* if the inverse demand function $P_A(x_A, x_B)$ is increasing in x_B, $\frac{\partial P_B}{\partial x_A} > 0$. Externalities are *negative* if the inverse demand function $P_A(x_A, x_B)$ is decreasing in x_B. We also allow for situations where network externalities are only experienced on one side

[13] As shown in the online Appendix, this number is perfectly correlated with user surplus, so that the number of users is also a measure of user welfare in the two jurisdictions.

of the market. We say that *there are no externalities from B to A* if $P_A(x_A, x_B)$ only depends on x_A. The same definitions apply to network externalities from A to B, by considering the inverse demand function $P_B(x_A, x_B)$. Network externalities arising from users from the same jurisdiction can also either be positive or negative. We assume that these externalities are not too strong, to guarantee that the inverse demand function $P_A(x_A, x_B)$ is decreasing in x_A and the inverse demand function $P_B(x_A, x_B)$ is decreasing in x_B.

4.1 Externalities and distortions under SA

The post-tax profit is given by

$$\begin{aligned} \Pi(x_A, x_B) = &\quad (1 - t_A)V_A(x_A, x_B) + (1 - t_B)V_B(x_A, x_B) \text{ where} \\ V_A(x_A, x_B) = &\quad x_A P_A(x_A, x_B) \text{ and } V_B(x_A, x_B) = x_B P_B(x_A, x_B). \end{aligned}$$

We assume the strict concavity of V_A and V_B to ensure the uniqueness of the platform's optimal choice and its differentiability. The first-order conditions on the profit thus characterize the optimal choices of the platform:

$$\frac{\partial \Pi}{\partial x_A} = (1 - t_A)(x_A \frac{\partial P_A}{\partial x_A} + P_A) + (1 - t_B)x_B \frac{\partial P_B}{\partial x_A} = 0, \tag{4}$$

$$\frac{\partial \Pi}{\partial x_B} = (1 - t_A)x_A \frac{\partial P_A}{\partial x_B} + (1 - t_B)(x_B \frac{\partial P_B}{\partial x_B} + P_B) = 0 \tag{5}$$

The optimal choices are affected by externalities through the impact of the number of users in a jurisdiction on the price hence the profit in the other: $\frac{\partial V_B}{\partial x_A} = x_B \frac{\partial P_B}{\partial x_A}$ and $\frac{\partial V_A}{\partial x_B} = x_A \frac{\partial P_A}{\partial x_B}$. Proposition 4 is obtained by implicit differentiation of these conditions.

Proposition 4 *Suppose that the profit of the platform is strictly concave in x_A and x_B. At an interior optimal solution (X_A, X_B),*

$$X'_A(t_A) = \frac{\delta_A + s_A\delta_B}{1 - s_A s_B} \tag{6}$$

$$X'_B(t_A) = \frac{\delta_B + s_B\delta_A}{1 - s_A s_B} \tag{7}$$

where

$$\delta_A = -\frac{\partial^2\Pi}{\partial x_A \partial t_A} / \frac{\partial^2\Pi}{\partial x_A \partial x_A}, \qquad \delta_B = -\frac{\partial^2\Pi}{\partial x_B \partial t_A} / \frac{\partial^2\Pi}{\partial x_B \partial x_B} \tag{8}$$

$$s_A = -\frac{\partial^2\Pi}{\partial x_A \partial x_B} / \frac{\partial^2\Pi}{\partial x_A \partial x_A}, \qquad s_B = -\frac{\partial^2\Pi}{\partial x_A \partial x_B} / \frac{\partial^2\Pi}{\partial x_B \partial x_B} \tag{9}$$

with

$$\frac{\partial^2\Pi}{\partial x_A \partial t_A} = \frac{1-t_B}{t_A} x_B \frac{\partial P_B}{\partial x_A}, \tag{10}$$

$$\frac{\partial^2\Pi}{\partial x_B x_A} = -x_A \frac{\partial P_A}{\partial x_B}. \tag{11}$$

We can apply Proposition 4 to the limiting case where the two jurisdictions have symmetric inverse demand functions and the tax rates are initially equal. Using the decomposition into direct and indirect effects, we immediately assess the effect of an increase in the corporate tax rate t_A. By symmetry, $\delta_A = \delta_B = \delta$ and $s_A = s_B = s$. By concavity, $s^2 < 1$ so that $|s| < 1$.In response to an increase in t_A, the platform raises x_A and lowers x_B when externalities are positive and raises x_B and lowers x_A when externalities are negative. This result indicates that peer-to-peer platforms with positive externalities operating in similar jurisdictions, such as E-bay operating in France and Germany should charge a lower subscription fee in the jurisdiction with higher tax rate (France which has a corporate income tax rate of 34.4%) than in the jurisdiction with the lower tax rate (Germany with a corporate income tax rate of 29.8 %).

However, in general, the sign of the change in the number of users is not determined only by the direction of the externalities. Proposition 4 decomposes the effect of an increase in t_A on the number of users into direct and indirect effects. The direct effects δ_A and δ_B measure how the change in the corporate tax rate t_A affects the choice of the platform in jurisdiction i, X_i, assuming the number of users in the other jurisdiction, X_j, to be constant. These direct effects are followed by adjustments due to the presence of externalities: the direct effect in jurisdiction B, δ_B, induces a change in the number of users in jurisdiction A measured by $s_A\delta_B$ while direct effect δ_A induces a change in the number of users in jurisdiction B equal to $s_B\delta_A$. This in turn leads to further changes, resulting in an infinite series of adjustments, with limits given by (6) and (7). These expressions show the decomposition of the effect of a change in t_A on X_A into the direct effect $\frac{\delta_A}{1-s_As_B}$ and the indirect effect $\frac{s_A\delta_B}{1-s_As_B}$. Similarly, we decompose the effect on X_B into the direct effect $\frac{\delta_B}{1-s_As_B}$ and the indirect effect $\frac{s_B\delta_A}{1-s_As_B}$.

We now proceed to sign the direct and indirect effects. From (8), it immediately follows that

the direct effect in A has the same sign as the externalities from A to B, and the direct effect in B has the opposite sign of the externalities from B to A.

The intuition underlying this result is easy to grasp. Suppose for example that externalities from A to B are positive. In order to shift profit from jurisdiction A to jurisdiction B, the platform can either increase or decrease the number of users in jurisdiction A. So suppose that the platform initially served x_A users and consider two values x'_A and x''_A such that $x'_A < x_A < x''_A$ which result in the same profit V_A. For a fixed x_B, because of the positive network externalities from A to B, the platform generates a higher price in jurisdiction B by choosing x''_B rather than x'_B. Hence the profit V_B in jurisdiction B is higher under x''_A than under x'_A, showing that the platform has an incentive to increase the number of users in the jurisdiction with higher tax rate. Consider now δ_B. Since $\frac{\partial V_A}{\partial x_B} = x_A \frac{\partial P_A}{\partial x_B}$, the sign of δ_B is the opposite of the sign of $\frac{\partial P_A}{\partial x_B}$.

Consider now the sign of the indirect effects s_A and s_B. First observe that their signs are identical, given by the sign of the cross-derivative $\frac{\partial^2 \Pi}{\partial x_A \partial x_B}$. This sign reflects whether the marginal profit with respect to its number of users increases when the number of users in the other country increases. We compute:

$$\frac{\partial^2 \Pi}{\partial x_A \partial x_B} = (1 - t_A)[2\frac{\partial P_A}{\partial x_B} + x_A \frac{\partial^2 P_A}{\partial x_A \partial x_B}] + (1 - t_B)[2\frac{\partial P_B}{\partial x_A} + x_B \frac{\partial^2 P_B}{\partial x_A \partial x_B}]. \tag{12}$$

So the sign of $s_i, i = A, B$ depends on the elasticity of the externality $\frac{\partial P_A}{\partial x_B}$ with respect to the number of users x_A and of the elasticity of the externality $\frac{\partial P_B}{\partial x_A}$ with respect to the number of users x_B. Different inverse demand functions may generate positive or negative values of $\frac{\partial^2 \Pi}{\partial x_A \partial x_B}$. It follows that the signs of the direct and indirect effects may differ and an increase in the tax rate t_A can result in different combinations of effects depending on the magnitude of the direct and indirect effects. However, as shown in the next Proposition, one combination will never arise. It is impossible to sustain a situation where indirect effects dominate direct effects for both X_A and X_B.

Proposition 5 *Suppose that the direct and indirect effects have opposite signs. Then an increase in t_A cannot result in indirect effects dominating the direct effect for both X_A and X_B.*

Proposition 5 shows that, when externalities are positive, the only case which can be excluded is one where the number of users rises in the low-tax jurisdiction and decreases in the high-tax jurisdiction. When externalities are negative, the only case which can be excluded is one where the number of users decreases in the low-tax jurisdiction and increases in the high-tax jurisdiction.

To go further, we sign the indirect effects. Let us say that the markets in A and B are *complements* if $\frac{\partial^2 \Pi}{\partial x_A \partial x_B} > 0$ and *substitutes* if $\frac{\partial^2 \Pi}{\partial x_A \partial x_B} < 0$. In the former case s_A and s_B are both positive, and in the latter they are both negative. Table 1 highlights situations where the comparative statics effects of changes in t_A on X_A, X_B, P_A and P_B can be signed unambiguously.

$A \to B$	$B \to A$	complements	substitutes
+	+	?	$X_A \uparrow X_B \downarrow$ $P_A \downarrow P_B \uparrow$
+	−	$X_A \uparrow X_B \uparrow$ $P_A \downarrow P_B?$	?
−	+	$X_A \downarrow X_B \downarrow$ $P_A? P_B \uparrow$	?
−	−	?	$X_A \downarrow X_B \uparrow$?

Table 1: Comparative statics effects of t_A on X_A, X_B, P_A, P_B

Table 1 shows that the sign of the effect of t_A on X_A and X_B can be ascertained whenever the direct and indirect effects go in the same direction. This happens when markets are substitutes when the externalities have the same sign, and when markets are complements when externalities across jurisdictions have opposite signs. The effect on prices are even harder to sign. When externalities are positive and markets are substitute, an increase in t_A results in a decrease in the price of the service in the high-tax jurisdiction and an increase in the price of service in the low-tax jurisdiction. Hence, for peer-to-peer platforms such as E-Bay or AirBnB, commission rates should be lower in high-tax countries than in low-tax countries. When externalities are positive in one direction and negative in the other direction, one can only sign the price in the market generating positive externalities. For example, if advertisers in Ireland benefit from the presence of consumers in European markets, but consumers are harmed by the number of advertisers, the model predicts that prices charged to consumers in high-tax markets should be low - a finding which is consistent with the fact that prices charged by Google to users are actually equal to zero.

Table 2 computes the sign of the effects in the simpler case of one-sided externalities, where externalities only flow from one country to another. It shows that the comparative statics effects of t_A on the number of users X_A and X_B can be signed whenever externalities are one-sided.

$A \to B$	$B \to A$	complements	substitutes
$+$	0	$X_A \uparrow X_B \uparrow$ $P_A \downarrow P_B ?$	$X_A \uparrow X_B \downarrow$ $P_A \downarrow P_B \uparrow$
$-$	0	$X_A \downarrow X_B \downarrow$ $P_A \uparrow P_B \uparrow$	$X_A \downarrow X_B \uparrow$ $P_A \uparrow P_B ?$
0	$+$	$X_A \downarrow X_B \downarrow$ $P_A ? P_B \uparrow$	$X_A \uparrow X_B \downarrow$ $P_A \downarrow P_B \uparrow$
0	$-$	$X_A \uparrow X_B \uparrow$ $P_A \downarrow P_B \downarrow$	$X_A \downarrow X_B \uparrow$ $P_A ? P_B \downarrow$

Table 2: Comparative statics effects of t_A on X_A, X_B, P_A, P_B for one-sided externalities

This is due to the fact that one of the two direct effects δ_A or δ_B is equal to zero, so the sign of the effect is either the sign of the direct effect or of the indirect effect. The effect of an increase in t_A on prices P_A and P_B can only be signed when markets are substitutes when externalities are positive and complements when externalities are negative.

Finally, while our analysis of the sign of the change in the number of users is cast for one-dimensional externalities, the decomposition into direct and indirect effects also holds in the two-dimensional model. In that case, indirect effects come from three sources rather than one: from users on the same side of the platform in the other market and from users on the other side of the platform in the two markets. This multiplicity of indirect effects makes the model more complex and prevents an easy determination of the comparative statics effect of a change in t_A on the number of users and prices in the two jurisdictions.

4.2 Externalities and distortions under FA

We already know from Proposition 3 that an increase in t_A leads to a decrease in the share of users in A. We study here the effect on the absolute number of users. The decomposition of the comparative statics effects of an increase in t_A on the number of users X_A and X_B of Proposition 4 does not depend on the régime of profit-splitting and remains valid under FA. However, the formulas for δ_A, δ_B, s_A and s_B are different under FA. The next Proposition shows that, whatever the sign of externalities, the direct effect δ_A is negative whereas the direct effect δ_B is positive.

Proposition 6 *Suppose that $t_A \geq t_B$, then under FA, the direct effect of an increase in the corporate tax rate on X_A is negative and the direct effect on X_B is positive.*

Under Formula Apportionment, an increase in the corporate tax rate in jurisdiction A induces the platform to reduce its coverage in the high-tax jurisdiction and increase its coverage in the low-tax jurisdiction. This is easily explained: when the number of users in the low-tax jurisdiction is fixed, the platform has an incentive to lower the number of users in the high-tax jurisdiction in order to reduce the share of profit allocated to the high-tax jurisdiction. This first order effect dominates the second-order effect of a reduction in total profit due to the distortion in output. By a similar reasoning, when the number of users in the high-tax jurisdiction is fixed, the platform has an incentive to increase the number of users in the low-tax jurisdiction.

On the other hand, under FA, the signs of s_A and s_B are difficult to compute and, in fact, may not be constant for all values of the corporate tax rate t_A. This makes the indirect effects difficult to sign and prevents us from analyzing the comparative statics effects of a change in t_A on the number of users under FA even in the simple case of one-sided externalities. One model that can be handled is the limiting case of symmetric jurisdictions. As we saw above, direct effects always dominate indirect effects when jurisdictions are symmetric and tax rates equal. Hence the comparative statics effects of a change in t_A on the number of users follows the direct effect: a small increase of the corporate tax rate above the common rate results in a decrease in the number of users in the high-tax jurisdiction and an increase in the low-tax jurisdiction, the exact opposite to the effect under SA.

5 A Comparison between Separate Accounting and Formula Apportionment

We next use a numerical simulation to compare outcomes under Separate Accounting and Formula Apportionment. Suppose that demand is linear, so that the inverse demand functions are given by

$$\begin{aligned} P_A &= 1 - \sigma_A x_A + \beta x_B, \\ P_B &= 1 - \sigma_B x_B + \alpha x_A. \end{aligned}$$

The parameters σ_A (respectively σ_B) measures the sensitivity of the price in jurisdiction A (respectively B) to the number of users in the jurisdiction. These parameters are assumed to be positive, reflecting the fact that price in a jurisdiction is decreasing in the number of users in that jurisdiction. The parameters α and β measure externalities and can either be positive or negative.

In the benchmark case, we consider a peer-to-peer platform with symmetric externalities. The effect of the presence of users in jurisdiction B on users in jurisdiction A is the same as the effect of the presence of users in jurisdiction A on users in jurisdiction B, so that $\alpha = \beta$. We also assume that the two jurisdiction are of equal size, and that the elasticity of demand to price is the same in the two jurisdictions, so that $\sigma_A = \sigma_B$.[14] Finally, we fix the sensitivity of demand to $\sigma_A = \sigma_B = 0.9$, externalities to $\alpha = \beta = 0.1$ and the corporate tax rate of the low-tax jurisdiction to the tax rate in Ireland at $t_B = 0.125$.

The first two graphs illustrate the effect of differences in corporate tax rate on profit shifting. Figure 1 illustrates the effect of t_A on the tax base of the two jurisdictions under SA and FA. As expected from Propositions 2 and 3, a difference between the tax rates leads the platform to shift profit away from jurisdiction A to jurisdiction B. We note however that the effect of profit shifting is much stronger under FA than under SA. Because externalities are symmetric, the tax bases in the two countries are identical under FA and SA when the tax rates are identical. This is not necessarily the case when externalities are asymmetric. In that case, as shown in the online Appendix, even when tax rates are identical and there is no distortion in output, the optimal number of users in the two jurisdictions is different, affecting the apportionment key under FA, and resulting in different tax bases.

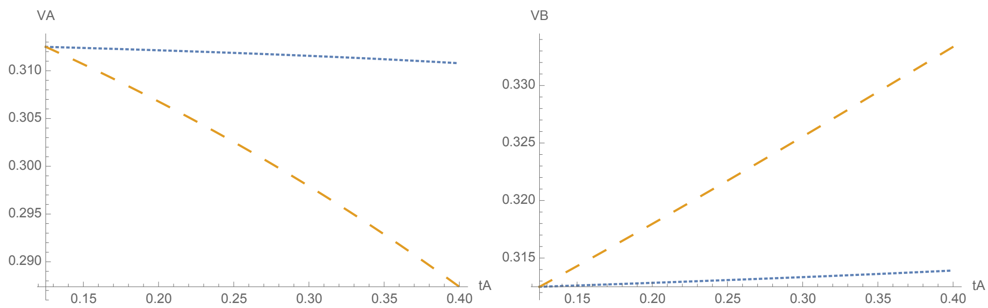

Figure 1: Tax base in the two jurisdictions

We next consider the effect of an increase in t_A on the tax revenues of the two countries. Figure 2 confirms the fact that the fiscal revenues of the low-tax country always increase with t_A. It also shows that the fiscal revenues of the high-tax country are also increasing with the

[14]In the Online Appendix, we present the detailed computations in the linear model and numerical simulations with asymmetric jurisdictions and asymmetric externalities.

corporate tax rate. The effect on the tax base is too small to compensate for the increase in the tax rate. This suggests that tax competition between the two jurisdictions will not result in a race to the bottom, but will stop at a positive tax rate for both jurisdictions.[15] Finally, we note that the two countries differ in their ranking between the two régimes of profit-splitting: the high-tax country obtains higher revenues under SA than under FA whereas the low-tax country obtains higher revenues under FA than under SA. Hence, in a negotiation over taxation of digital platforms, countries are likely to disagree over the optimal régime of profit sharing.[16]

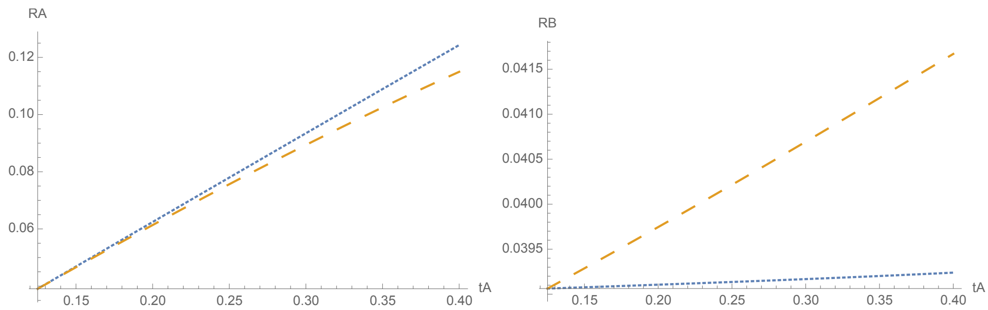

Figure 2: Tax revenues in the two jurisdictions

Figure 3 displays the pre-tax and post-tax profits of the platform as a function of the tax rate t_A. As shown in Propositions 2 and 3, in response to a difference in corporate tax rates, the platform distorts its price and quantity decisions, resulting in a decrease in both pre-tax and post-tax profits. The figure shows that distortions are higher, and the pre-tax profit lower, under SA than under FA. Interestingly, the post-tax profit is almost identical under the two régimes, indicating that the sum of tax revenues for both countries is higher under SA than under FA.

Turning to the optimal number of users, Figure 4 shows that under both régimes of profit-splitting, the direct effect dominates the indirect effect. Because externalities are positive, under SA, the direct effect results in an increase in the number of users in jurisdiction A and a decrease in the number of users in jurisdiction B. Under FA, the direct effect works in an opposite direction: an increase in t_A induces the platform to reduce the number of users in jurisdiction A

[15] In the Online Appendix, we compute the equilibrium tax rate in a symmetric model under SA. This tax rate is equal to $t^* = 1 - (\frac{\alpha}{\sigma - \alpha})^2$, a level which is very high when externalities are small.

[16] The finding that the low-tax country prefers FA and the high-tax country prefers SA holds true whenever externalities across jurisdictions are positive. If externalities from one jurisdiction are positive and from the other negative, as seen in the online Appendix, the ranking between the two régimes may be different. It remains true however that the two countries always disagree on the optimal profit-splitting régime: if one gets higher tax revenues under SA, the other one gets higher tax revenues under FA.

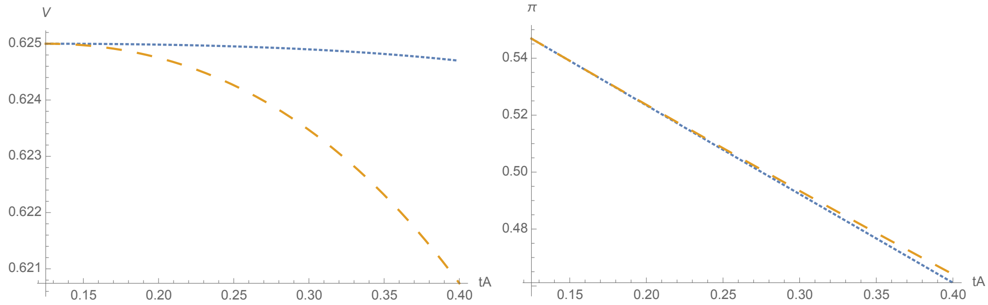

Figure 3: Pre-tax and post-tax profits of the platform

and increase the number of users in jurisdiction B. Because user surplus is perfectly correlated with the number of users, we deduce that when the corporate tax rate t_A increases, under SA, users in the high-tax jurisdiction gain whereas users in the low-tax jurisdiction lose. Under FA, we observe an opposite effect: users in the high-tax jurisdiction lose whereas users in the low-tax jurisdiction gain with an increase in the corporate tax rate t_A.

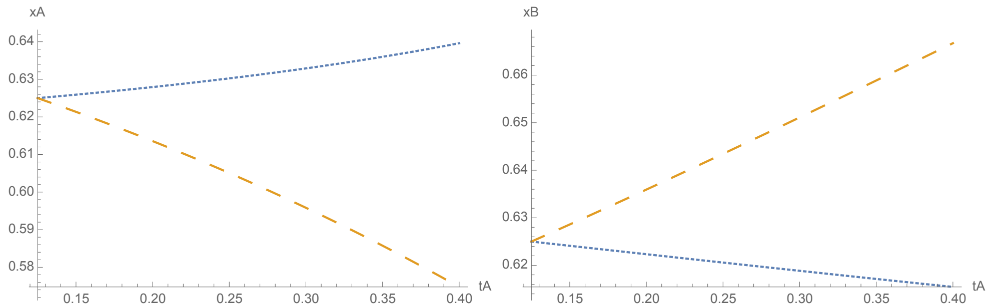

Figure 4: Number of users in the two jurisdictions

The distortion on the number of users also affects prices, as shown in Figure 5. Under SA, as the number of users in A increases and the number of users in B decreases, the price in jurisdiction A goes down while the price in jurisdiction B goes up. Hence an increase in the corporate tax rate t_A leads the platform to *reduce its price* p_A. Instead of passing through the tax increase to the users in jurisdiction A, the platform chooses to pass it through to users in the other jurisdiction, a result which is reminiscent of the analysis of tax incidence in two-sided platforms by Kind et al. (2008). By contrast, under FA, the increase in the corporate tax rate t_A is passed on to users in jurisdiction A rather than jurisdiction B: it results in an increase in the price p_A and a decrease in the price p_B.

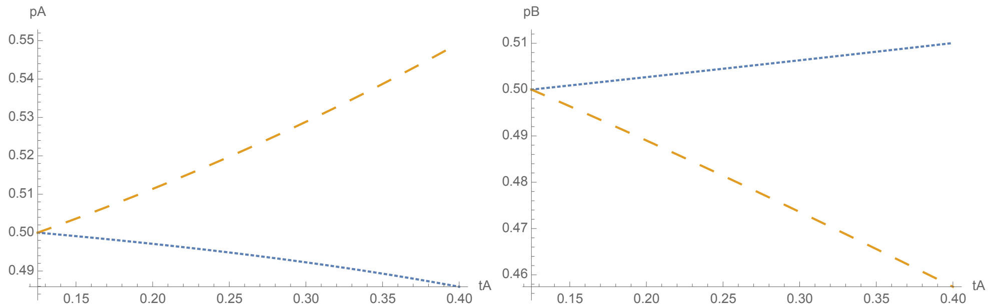

Figure 5: Prices in the two jurisdictions

6 Conclusion

This paper analyzes the strategy of a monopolistic digital platform serving users from two jurisdictions with different corporate tax rates. We show that even in the absence of transfer pricing, the platform shifts profit from the high-tax to the low-tax jurisdiction. Under Separate Accounting, the platform exploits network externalities to increase demand in the low-tax and reduce demand in the high-tax jurisdiction. Under Formula Apportionment, the platform manipulates the apportionment key to increase the tax base in the low-tax jurisdiction and reduce the tax base in the high-tax jurisdiction. In order to shift profit, the platform distorts prices and quantities in the two jurisdictions. These distortions can be decomposed into direct effects (assuming that quantities in the other jurisdiction remain fixed) and indirect effects (due to changes in the quantities in the other jurisdiction). When direct effects dominate, under Separate Accounting, the direction of the distortion depends on the sign of externalities. Under positive externalities, the platform has an incentive to reduce price in the high-tax jurisdiction and increase price in the low-tax jurisdiction ; under negative externalities, the direction of diversion is reversed. When direct effects dominate, under Formula Apportionment, the direction of the distortion is always the same: the platform increases prices in the high-tax jurisdiction and reduces prices in the low-tax jurisdiction. We use a numerical simulation to show that the ranking of fiscal revenues under the two régimes differ in the two jurisdictions: the high-tax jurisdiction prefers SA to FA whereas the low-tax jurisdiction prefers FA to SA.

We would like to conclude by pointing out important limitations of our analysis which need to be studied in further research. First, we study the effect of an exogenous change in the corporate tax rate and do not study tax competition between the jurisdictions. In our model, the platform's ability to shift profit is limited, and differences in corporate tax rates result in small changes in the tax base. As a consequence, it can be seen that in a symmetric model, the unique symmetric equilibrium of tax competition is for both jurisdictions to set a very high-tax rate. This suggests that a more thorough analysis of tax competition should incorporate the effect of transfer pricing, through the payment of royalties on intellectual property, which results in substantial levels of profit shifts. We finally note that our concept of Formula Apportionment is extremely coarse, relying on a single indicator – the number of users – to apportion profit across jurisdictions. A more realistic approach would use several factors, including for example sales or payroll as additional apportionment keys. However, in the absence of operating costs, sales and profits are perfectly correlated. Hence a formula using both the number of users and sales as apportionment factors would fall in between Separate Accounting and Formula Apportionment and the response of the platform to a difference in corporate tax rates would thus be a weighted average of the responses under SA and FA studied in this paper. We believe that more work, both empirical and theoretical, is needed to design optimal formulas for apportionment of the profit of multinational digital platforms.

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8 Appendix

Proof of Proposition 1: In the absence of externalities, under SA the platform chooses x_A and x_B to maximize the profits separately in the two jurisdictions,

$$\frac{\partial V_A}{\partial x_A} = \frac{\partial V_B}{\partial x_B} = 0, \tag{13}$$

resulting in the optimal number of users x_A^* and x_B^*.

Under FA, the platform chooses x_A and x_B to satisfy the first-order conditions:

$$\begin{aligned} \frac{\partial V}{\partial x_A}[1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}] &= \frac{V(t_A - t_B)x_B}{(x_A + x_B)^2}, & (14) \\ \frac{\partial V}{\partial x_B}[1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}] &= \frac{V(t_B - t_A)x_A}{(x_A + x_B)^2}, & (15) \\ & & (16) \end{aligned}$$

If $t_A > t_B$, $\frac{\partial V}{\partial x_A} > 0$. As the profit is concave in x_A, this implies that $x_A < x_A^*$. Similarly, as $\frac{\partial V}{\partial x_B} < 0$, $x_B > x_B^*$. ■

Proof of Proposiiton 2: Let $t'_A > t_A$. Let $z = (x_A, y_A, x_B, y_B)$ denote an optimal choice of the platform when the tax rate in A is t_A and $z' = (x'_A, y'_A, x'_B, y'_B)$ an optimal choice when the rate is t'_A. Optimality implies

$$\begin{aligned} (1 - t_A)V_A(z) + (1 - t_B)V_B(z) &\geq (1 - t_A)V_A(z') + (1 - t_B)V_B(z'), & (17) \\ (1 - t'_A)V_A(z') + (1 - t_B)V_B(z') &\geq (1 - t'_A)V_A(z) + (1 - t_B)V_B(z), & (18) \end{aligned}$$

which can be written as

$$(1 - t_A)(V_A(z) - V_A(z')) \geq (1 - t_B)(V_B(z') - V_B(z)) \geq (1 - t'_A)(V_A(z) - V_A(z')).$$

Since $t'_A \geq t_A$, the two extreme inequalities imply $V_A(z) \geq V_A(z')$, which in turn implies $V_B(z') - V_B(z) \geq 0$. When the profit is strictly concave, the optimal choices are unique and the above inequalities are strict.

Consider now the change in the pre-tax profit $\Delta = V_A(z') + V_B(z') - (V_A(z) + V_B(z))$. Writing the post tax profit $(1 - t_A)V_A(z) + (1 - t_B)V_B(z)$ as $(1 - t_B)V(z) + (t_A - t_B)V_A(z)$, we derive from (18) and (18) again,

$$-(t'_A - t_B)(V_A(z) - V_A(z')) \leq (1 - t_B)\Delta \leq -(t_A - t_B)(V_A(z) - V_A(z')).$$

Since $V_A(z) - V_A(z') \geq 0$, $\Delta \leq 0$ if $t_A \geq t_B$: starting with a tax in A at least equal to the level in B, increasing t_A further has a negative effect on the pre-tax profit. The opposite holds, i.e. $\Delta \geq 0$, when $t'_A \leq t_B$: in that case, $t_A < t_B$ hence the increase in the tax level diminishes the gap between the tax levels of the countries. ■

Proof of Proposition 3: It is convenient to operate a change in variables. Let us define the share of users of type x in A as $\lambda = \frac{x_A}{x_A + x_B}$, the share of users of type y in A as $\mu = \frac{y_A}{y_A + y_B}$, the total number of users of type x as $T = x_A + x_B$ and the total number of users of type y as $Z = y_A + y_B$. We rewrite the profit as

$$\Pi(\lambda, T) = [1 - t_B - (t_A - t_B)(\omega\lambda + (1-\omega)\mu)]V(\lambda T, \mu Z, (1-\lambda)T, (1-\mu)Z).$$

Let $t'_A > t_A$. Let (λ, μ, T, Z) denote an optimal choice of the platform when the tax rate in A is t_A and (λ', μ', T', Z') when the rate is t'_A, and $v = V(\lambda T, \mu Z, (1-\lambda)T, (1-\mu)Z)$ and $v' = V(\lambda' T', \mu' Z', (1-\lambda')T', (1-\mu')Z')$. Optimality implies that

$$\begin{array}{rl} (1 - t_B - (t_A - t_B)(\omega\lambda + (1-\omega)\mu))v & \geq \\ (1 - t_B - (t_A - t_B)(\omega\lambda' + (1-\omega)\mu'))v' & \\ (1 - t_B - (t'_A - t_B)(\omega\lambda' + (1-\omega)\mu'))v' & \geq \\ (1 - t_B - (t'_A - t_B)(\omega\lambda + (1-\omega)\mu))v & , \end{array}$$

which results in:

$$\frac{1 - t_B - (t'_A - t_B)(\omega\lambda' + (1-\omega)\mu')}{[1 - t_B - (t'_A - t_B)(\omega\lambda + (1-\omega)\mu)]} \geq \frac{v}{v'} \geq \frac{1 - t_B - (t_A - t_B)(\omega\lambda' + (1-\omega)\mu')}{1 - t_B - (t_A - t_B)(\omega\lambda + (1-\omega)\mu)}$$

or

$$1 + \frac{(t'_A - t_B)(\omega(\lambda - \lambda') + (1-\omega)(\mu - \mu'))}{1 - t_B - (t'_A - t_B)(\omega\lambda + (1-\omega)\mu)} \geq \frac{v}{v'} \geq 1 + \frac{(t_A - t_B)(\omega(\lambda - \lambda') + (1-\omega)(\mu - \mu'))}{1 - t_B - (t_A - t_B)(\omega\lambda + (1-\omega)\mu)}. \quad (19)$$

Consider the terms on the left and the right of the above equation. Since $t'_A > t_A$ the denominator in the left equation is larger than that on the right. Hence we must have $(t'_A - t_B)(\omega(\lambda - \lambda') + (1-\omega)(\mu - \mu')) \geq (t_A - t_B)(\omega(\lambda - \lambda') + (1-\omega)(\mu - \mu'))$, which in turn implies $(\omega(\lambda - \lambda') +$

$(1-\omega)(\mu-\mu')) > 0$ since $t'A > t_A \geq t_B$: the share of users in A goes down. This implies $\frac{V(\lambda T, \mu Z, (1-\lambda)T, (1-\mu)Z)}{V(\lambda' T', \mu' Z', (1-\lambda')T', (1-\mu')Z')} \geq 1$, so that the pre-tax profit V decreases.

To prove that the tax base in jurisdiction B increases, we need to show (with the change of variables) that $(1-(\omega\lambda+(1-\omega)\mu))V(\lambda T, \mu Z, (1-\lambda)T, (1-\mu)Z) \leq (1-(\omega\lambda'+(1-\omega)\mu'))V(\lambda' T', \mu' Z', (1-\lambda')T', (1-\mu')Z')$. Using (19), it suffices to show

$$\frac{1-(\omega\lambda'+(1-\omega)\mu')}{1-(\omega\lambda+(1-\omega)\mu)} \geq 1 + \frac{(t'_A - t_B)(\omega(\lambda-\lambda')+(1-\omega)(\mu-\mu'))}{1-t_B-(t'_A-t_B)(\omega\lambda+(1-\omega)\mu)}$$

which is equivalent to

$$\frac{(\omega(\lambda-\lambda')+(1-\omega)(\mu-\mu'))}{1-(\omega\lambda+(1-\omega)\mu)} \geq \frac{(t'_A - t_B)((\omega(\lambda-\lambda')+(1-\omega)(\mu-\mu'))}{1-t_B-(t'_A-t_B)(\omega\lambda+(1-\omega)\mu)}.$$

or, since $(\omega(\lambda-\lambda')+(1-\omega)(\mu-\mu') \geq 0$, to

$$[1-t_B-(t'_A-t_B)(\omega\lambda+(1-\omega)\mu)] \geq (t'_A-t_B)(1-(\omega\lambda+(1-\omega)\mu))$$

and finally to $1 \geq t'_A$, which is of course always satisfied. ■

Proof of Proposition 4: As noted in the text, if the profit is concave in (x_A, x_B), the first order conditions (4) and 5) hold at an interior solution. By the Implicit Function Theorem, X_A and X_B are differentiable in t_A and the derivatives satisfy:

$$\frac{\partial^2 \Pi}{\partial x_A \partial t_A} + \frac{\partial^2 \Pi}{\partial x_A \partial x_A} X'_A(t_A) + \frac{\partial^2 \Pi}{\partial x_A \partial x_B} X'_B(t_A) = 0 \tag{20}$$

$$\frac{\partial^2 \Pi}{\partial x_B \partial t_A} + \frac{\partial^2 \Pi}{\partial x_A \partial x_B} X'_A(t_A) + \frac{\partial^2 \Pi}{\partial x_B \partial x_B} X'_B(t_A) = 0 \tag{21}$$

Solving the system of linear equations, we obtain:

$$X'_A(t_A) = \frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}\frac{\partial^2 \Pi}{\partial x_B \partial t_A} - \frac{\partial^2 \Pi}{\partial x_B \partial x_B}\frac{\partial^2 \Pi}{\partial x_A \partial t_A}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}\frac{\partial^2 \Pi}{\partial x_B \partial x_B} - \frac{\partial^2 \Pi}{\partial x_A \partial x_B}^2}$$

$$X'_B(t_A) = \frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}\frac{\partial^2 \Pi}{\partial x_A \partial t_A} - \frac{\partial^2 \Pi}{\partial x_A \partial x_A}\frac{\partial^2 \Pi}{\partial x_B \partial t_A}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}\frac{\partial^2 \Pi}{\partial x_B \partial x_B} - \frac{\partial^2 \Pi}{\partial x_A \partial x_B}^2}$$

Factoring out the numerator and denominator by $\frac{\partial^2\Pi}{\partial x_A \partial x_A}\frac{\partial^2\Pi}{\partial x_B \partial x_B}$, we obtain expressions (8) and (9). Now, under SA,

$$\frac{\partial^2\Pi}{\partial x_A \partial t_A} = -\frac{\partial V_A}{\partial x_A} \text{ and } \frac{\partial^2\Pi}{\partial x_B \partial t_A} = -\frac{\partial V_A}{\partial x_B}.$$

Using the first order condition on profit maximization with respect to x_A (4), we have

$$-\frac{\partial V_A}{\partial x_A} = \frac{1-t_B}{1-t_A}\frac{\partial V_B}{\partial x_A} = \frac{1-t_B}{1-t_A}x_B\frac{\partial P_B}{\partial x_A}$$

which gives the expression for δ_A. Finally

$$-\frac{\partial V_A}{\partial x_B} = -x_B\frac{\partial P_B}{\partial x_A}$$

which gives the expression for δ_B. ∎

Proof of Proposition 5: Consider one of the situations where direct and indirect effects have opposite signs. Suppose that markets are complements and externalities are positive, $\delta_A > 0, \delta_B < 0$ and $s_A, s_B > 0$. All other situations are handled in a similar way. Suppose without loss of generality that the indirect effect dominates the direct effect for X_A so that

$$\delta_A + s_A\delta_B < 0.$$

yielding

$$\delta_B < -\frac{\delta_A}{s_A}.$$

Consider the effect of a change in t_A on X_B:

$$\delta_B + s_B\delta_A < -\frac{\delta_A}{s_A} + s_B\delta_A = \frac{\delta_A}{s_A}(-1 + s_A s_B) < 0.$$

where the last inequality is obtained because $\delta_A > 0$ and $s_A s_B < 1$ by strict concavity of the profit. This implies that the direct effect dominates the indirect effect for X_B. ∎

Proof of Proposition 6: The direct effects are given by (8) and (9) computed at an optimal solution. Necessarily, the second derivatives $\frac{\partial^2\Pi}{\partial x_A \partial x_A}$ and $\frac{\partial^2\Pi}{\partial x_B \partial x_B}$ are negative (even if the profit is not globally concave) so it suffices to compute the sign of $\frac{\partial^2\Pi}{\partial x_A \partial t_A}$ and $\frac{\partial^2\Pi}{\partial x_B \partial t_A}$.

$$\frac{\partial^2\Pi}{\partial x_A \partial t_A} = -\frac{\partial V \frac{x_A}{x_A+x_B}}{\partial x_A}$$
$$= -\frac{V x_B}{(x_A+x_B)^2} - \frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_A}.$$

By the first order conditions of profit maximization

$$\frac{\partial \Pi}{\partial x_A} = \frac{\partial V}{\partial x_A}[1 - t_A\frac{x_A}{x_A+x_B} - t_B\frac{x_B}{x_A+x_B}] - \frac{V x_B(t_A - t_B)}{(x_A+x_B)^2},$$
$$= 0$$

so that

$$\frac{\partial V}{\partial x_A} = \frac{1}{[1 - t_A\frac{x_A}{x_A+x_B} - t_B\frac{x_B}{x_A+x_B}]}\frac{V x_B(t_A - t_B)}{(x_A+x_B)^2} \geq 0,$$

showing that $\frac{\partial^2\Pi}{\partial x_A \partial t_A} < 0$. Similarly, we compute

$$\frac{\partial^2\Pi}{\partial x_B \partial t_A} = \frac{V x_A}{(x_A+x_B)^2} - \frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_B},$$
$$= \frac{V x_A}{(x_A+x_B)^2} + \frac{1}{[1 - t_A\frac{x_A}{x_A+x_B} - t_B\frac{x_B}{x_A+x_B}]}\frac{V x_A(t_A - t_B)}{(x_A+x_B)^2}$$
$$> 0,$$

showing that $\frac{\partial^2\Pi}{\partial x_B \partial t_A} > 0$. ∎

9 Online Appendix

9.1 Micro-foundation of the prices

In this Appendix, we describe a model of consumer behavior which gives rise to the inverse demand functions considered in the text. Every jurisdiction is populated by a continuum of heterogeneous users characterized by their willingness to pay for using the platform. In addition, users experience externalities from the presence of other users on the platform. Formally, letting U and W denote the utilities of users of the two types on the platform,

$$\begin{aligned} U_A &= \theta_A + u_A(x_A, y_A, x_B, y_B) - p_A, \\ W_A &= \eta_A + w_A(x_A, y_A, x_B, y_B) - q_A, \\ U_B &= \theta_B + u_B(x_A, y_A, x_B, y_B) - p_B \\ W_B &= \eta_B + w_B(x_A, y_A, x_B, y_B) - q_B, \end{aligned}$$

where θ_A and θ_B are distributed according to continuous distributions with full support F_A and F_B on $[\underline{\theta}, \overline{\theta}]$, and η_A and η_B are distributed according to continuous distributions with full support G_A and G_B on $[\underline{\eta}, \overline{\eta}]$.

We now derive the demand associated to the fees (p_A, q_A, p_B, q_B). Let x_A, y_A, x_B, y_B be the common expectation of every user over the number of users in the two jurisdictions. We compute the value of the user of type 1 in jurisdiction A who is indifferent between buying access to the platform or not. This value is given by

$$\widehat{\theta_A} = p_A - u_A(x_A, y_A, x_B, y_B).$$

provided that $p_A - u_A(x_A, y_A, x_B, y_B)$ belongs to the support of F_A; otherwise $\widehat{\theta_A}$ will be equal to one of the extreme values $\underline{\theta}$ (if the market is covered) or $\overline{\theta}$ (if no user accesses the platform). We can similarly compute the value of all other indifferent users. The two jurisdictions may have different sizes. We normalize the measure of users in jurisdiction B to 1, and let s denote the measure of users in jurisdiction A. Assuming that expectations are rational, the demand thus satisfies

$$x_A = s(1 - F_A(p_A - u_A(x_A, y_A, x_B, y_B)).$$

Similarly,

$$
\begin{aligned}
y_A &= s(1 - G_A(q_A - w_A(x_A, y_A, x_B, y_B)),\\
x_B &= 1 - F_B(p_B - u_B(x_A, y_A, x_B, y_B)),\\
y_B &= 1 - G_B(q_B - w_B(x_A, y_A, x_B, y_B)).
\end{aligned}
$$

From the computations above, the prices are given by[17]

$$
\begin{aligned}
P_A(x_A, y_A, x_B, y_B) &= u_A(x_A, y_A, x_B, y_B) + F_A^{-1}(1 - \frac{x_A}{s}), &(22)\\
Q_A(x_A, y_A, x_B, y_B) &= w_A(x_A, y_A, x_B, y_B) + G_A^{-1}(1 - \frac{y_A}{s}), &(23)\\
P_B(x_A, y_A, x_B, y_B) &= u_B(x_A, y_A, x_B, y_B) + F_B^{-1}(1 - x_B), &(24)\\
Q_B(x_A, y_A, x_B, y_B) &= w_B(x_A, y_A, x_B, y_B) + G_B^{-1}(1 - y_B). &(25)
\end{aligned}
$$

The sign of the derivatives $\partial P_A/\partial y_A$, $\partial P_A/\partial x_B$, $\partial P_A/\partial y_B$ depends on the sign of the externalities, $\partial u_A/\partial y_A$, $\partial u_A/\partial x_B$, $\partial u_A/\partial y_B$. For P_A to be decreasing in x_A, as required in the paper we need the externalities $\partial u_A/\partial x_A$ to be small relative to the direct effect measured by $\frac{\partial F_A^{-1}(1-\frac{x_A}{s})}{\partial x_A}$. The same computations hold for the inverse demands Q_A, P_B and Q_B.

Next, we compute the surplus of users and show that they only depend on the number of users of the same type in the same jurisdiction. Consider the surplus of users of type 1 in jurisdiction A. Taking into account their participation decision, the surplus of users of type 1 in jurisdiction A can be written as

$$
US_A = \int_{p_A - u(x_A, y_A, x_B, y_B)}^{\underline{\theta}} [\theta_A + u_A(x_A, y_A, x_B, y_B) - p_A] f_A(\theta_A) d\theta_A,
$$

which, using (22), writes

$$
US_A = \int_{F_A^{-1}(1-\frac{x_A}{s})}^{\underline{\theta}} [\theta_A - F_A^{-1}(1 - \frac{x_A}{s})] f_A(\theta_A) d\theta_A.
$$

The surplus of users of type 1 in jurisdiction A can thus be written as a function of x_A. Furthermore, it is easy to check that this function is non-decreasing: since $F_A^{-1}(1-\frac{x_A}{s})$ is non-increasing

[17] The price functions are akin to inverse demand functions, with one caveat: due to coordination issues, a given couple of prices (p_A, p_B) could lead to different demands. In that case we select the largest demands.

in x_A both the domain of integration and the integrand are non-decreasing in x_A. Hence, as intuition suggests, an increase in the number of users x_A results in an increase in the user surplus of type 1 users in jurisdiction A. The same reasoning holds for the surplus of all other type of users, showing that measures of number of users and of user surplus are perfectly correlated.

9.2 The linear model

In this Appendix we consider the linear model with two type of users. Let

$$P_A(x_A, x_B) = 1 - \sigma_A x_A + \beta x_B \tag{26}$$

$$P_B(x_A, x_B) = 1 - \sigma_B x_B + \alpha x_A \tag{27}$$

We first consider the optimal choice of the platform under Separate Accounting. The platform's profit is given by

$$\Pi = (1 - t_A)x_A(1 - \sigma_A x_A + \beta x_B) + (1 - t_B)x_B(1 - \sigma_B x_B + \alpha x_A).$$

The first order conditions are given by:

$$\begin{aligned} (1 - t_A)(1 - 2\sigma_A x_A + \beta x_B) + (1 - t_B)\alpha x_B &= 0, \\ (1 - t_A)\beta x_A + (1 - t_B)(1 - 2\sigma_B x_B + \alpha x_A) &= 0 \end{aligned}$$

Solving the system of linear equations, we obtain the interior solutions:

$$X_A = \frac{2\sigma_B(1 - t_A)(1 - t_B) + (1 - t_B)[\beta(1 - t_A) + \alpha(1 - t_B)]}{4\sigma_A\sigma_B(1 - t_A)(1 - t_B) - [\beta(1 - t_A) + \alpha(1 - t_B)]^2}, \tag{28}$$

$$X_B = \frac{2\sigma_A(1 - t_A)(1 - t_B) + (1 - t_A)[\beta(1 - t_A) + \alpha(1 - t_B)]}{4\sigma_A\sigma_B(1 - t_A)(1 - t_B) - [\beta(1 - t_A) + \alpha(1 - t_B)]^2}. \tag{29}$$

The second order conditions are satisfied if $4\sigma_A\sigma_B(1-t_A)(1-t_B) > [\beta(1-t_A)+\alpha(1-t_B)]^2$. We need to put additional restrictions (that we do not explicitly spell out) to guarantee that the prices P_A and P_B are positive.

Even in the linear model, the comparative statics effect of changes in t_A on X_A and X_B cannot be signed easily. The decomposition into direct and indirect effects gives

$$\delta_A = \frac{1-t_B}{1-t_A}\frac{\alpha x_B}{2(1-t_A)\sigma_A},\ \delta_B = -\frac{\beta x_A}{2(1-t_B)\sigma_B}$$

$$s_A = \frac{(1-t_A)\beta + (1-t_B)\alpha}{2(1-t_A)\sigma_A},\ s_B = \frac{(1-t_A)\beta + (1-t_B)\alpha}{2(1-t_B)\sigma_B}$$

Notice that s_A and s_B are both positive – the linear model captures a market with complements. The direct effects δ_A and δ_B depend on the optimal solutions x_A and x_B, making a comparison of the magnitude of the direct and indirect effects uneasy. Our next result provides a sufficient condition under which prices P_A and P_B are monotonic in the corporate tax rate t_A:

Proposition 7 *Suppose that* $\sigma_A\sigma_B \geq \max\{\alpha(\frac{1-t_B}{1-t_A}\alpha+\beta), \beta(\alpha+\frac{1-t_A}{1-t_B}\beta)\}$, *then at an interior solution, the equilibrium price* P_A *is decreasing in* t_A *while the equilibrium price* P_B *is increasing in* t_A.

Proof of Proposition 7: Note that

$$\begin{aligned} P_A'(t_A) &= -\sigma_A X_A'(t_A) + \beta X_B'(t_A), \\ P_B'(t_A) &= \alpha X_A'(t_A) - \sigma_B X_B'(t_A). \end{aligned}$$

We use the decomposition of $X_A'(t_A)$ and $X_B'(t_A)$ to write

$$\begin{aligned} P_A'(t_A) &= \frac{1}{1-s_As_B}[-\sigma_A + \beta s_B]\delta_A + [-\sigma_A s_A + \beta]\delta_B], \\ P_B'(t_A) &= \frac{1}{1-s_As_B}[\alpha - \sigma_B s_B]\delta_A + [\alpha s_A - \sigma_B]\delta_B. \end{aligned}$$

Using the formulas for $\delta_A, \delta_B, \sigma_A$ and σ_B, the sign of $P_A'(t_A)$ is the same as the sign of

$$\begin{aligned} R_A' &= (-2\sigma_A\sigma_B(1-t_B) + \beta[(1-t_A)\beta + (1-t_B)\alpha])\frac{1-t_B}{1-t_A}\alpha x_B \\ &+ (2\beta(1-t_A)\sigma_A - \sigma_A[(1-t_A)\beta + (1-t_B)\alpha])(-\beta x_A) \end{aligned}$$

Now, because the price P_B is positive

$$\sigma_B\frac{1-t_B}{1-t_A}x_B - \beta x_A > 0.$$

Hence

$$-\sigma_A\sigma_B(1-t_B)\frac{1-t_B}{1-t_A}\alpha x_B + \sigma_A(1-t_B)\alpha\beta x_A < 0.$$

so that

$$\begin{aligned} R'_A &< (-\sigma_A\sigma_B(1-t_B) + \beta[(1-t_A)\beta + (1-t_B)\alpha])\frac{1-t_B}{1-t_A}\alpha x_B \\ &- \beta\sigma_A(1-t_A)\beta x_A, \\ &< (-\sigma_A\sigma_B(1-t_B) + \beta[(1-t_A)\beta + (1-t_B)\alpha])\frac{1-t_B}{1-t_A}\alpha x_B. \end{aligned}$$

and using the condition $\beta(\alpha + \frac{1-t_A}{1-t_B}\beta)$, $R'_A < 0$.

Similarly, we see that $P'_B(t_A)$ has the same sign as

$$\begin{aligned} R'_B &= (2\alpha(1-t_B)\sigma_B - \sigma_B[(1-t_A)\beta + (1-t_B)\alpha])\frac{1-t_B}{1-t_A}\alpha x_B \\ &+ (-2(1-t_A)\sigma_A\sigma_B + \alpha[(1-t_A)\beta + (1-t_B)\alpha])(-\beta x_A) \end{aligned}$$

Because the price P_A is positive,

$$\sigma_A x_A > \alpha\frac{1-t_A}{1-t_B}x_B,$$

so that

$$(1-t_A)\sigma_A\sigma_B\beta x_A > (1-t_A)\sigma_B\beta\alpha\frac{1-t_A}{1-t_B}x_B.$$

and

$$\begin{aligned} R'_B &> \alpha(1-t_B)\sigma_B\frac{1-t_B}{1-t_A}\alpha x_B \\ &+ ((1-t_A)\sigma_A\sigma_B - \alpha[[(1-t_A)\beta + (1-t_B)\alpha])\beta x_A, \\ &> ((1-t_A)\sigma_A\sigma_B - \alpha[[(1-t_A)\beta + (1-t_B)\alpha])\beta x_A. \end{aligned}$$

Using the condition $\sigma_A\sigma_B > \alpha(\alpha + \frac{1-t_B}{1-t_A}\beta)$, the result follows. ■

Proposition 7 shows that, as in the analysis of Kind et al. (2005, 2008, 2010, 2013), an increase in the corporate tax rate t_A leads to a reduction in the price P_A and an increase in the price P_B in the linear model. Next, we analyze tax competition when platforms are symmetric.

Proposition 8 *Suppose that platforms are symmetric, $\sigma_A = \sigma_B = \sigma$, $\alpha = \beta$. In a model of tax competition, where both countries choose their corporate tax rate to maximize tax revenues, there exists a unique symmetric equilibrium where*

$$t^* = 1 - (\frac{\alpha}{\sigma - \alpha})^2..$$

Proof of Proposition 8: Consider the marginal effect of an increase in t_A on R_A at a point where $t_B = t_A = t$. We compute

$$\frac{\partial R_A}{\partial t_A} = V_A(t) + (1 - 2\sigma x + \alpha x)X'_A(t_A) + \alpha x X'_B(t_A)$$

Now, at $t_A = t_B = t$, $X'_A(t_A) = -X'_B(t_A)$. In addition, $V_A = x(1-\sigma x+\alpha x)$ and $1-2\sigma x+2\alpha x = 0$. Hence,

$$\frac{\partial R_A}{\partial t_A} = x(1 - \sigma x + \alpha x) - 2\alpha x X'_A(t_A).$$

Next, using the decomposition formula,

$$X'_A(t_A) = \frac{\delta}{1-s} = \frac{\frac{\alpha x}{2(1-t)\sigma}}{1 - \frac{\alpha}{\sigma}} = \frac{\alpha x}{2(\sigma - \alpha)(1-t)}.$$

Replacing,

$$\frac{\partial R_A}{\partial t_A} = x^2(\sigma - \alpha) - x^2 \frac{\alpha^2}{(\sigma - \alpha)(1-t)}.$$

Hence at a symmetric equilibrium where $\frac{\partial R_A}{\partial t_A} = 0$,

$$t^* = 1 - (\frac{\alpha}{\sigma - \alpha})^2.$$

■

We next consider the optimal choice of the platform under Formula Apportionment. As in the Proof of Proposition 3, let $\lambda = \frac{x_A}{x_A + x_B}$ and $T = x_A + x_B$. After this change of variable, the profit of the platform becomes:

$$\begin{aligned}\Pi &= (1-\lambda t_A-(1-\lambda)t_B)[\lambda T(1-\sigma_A\lambda T+\beta(1-\lambda T)\\&\quad+(1-\lambda)T(1-\sigma_B(1-\lambda)T+\alpha\lambda T)]\\&= (1-\lambda t_A-(1-\lambda)t_B)T[1+\lambda(1-\lambda)(\alpha+\beta)T-\sigma_A\lambda^2T-\sigma_B(1-\lambda)^2T]\end{aligned}$$

Notice in particular that the optimal choices of the platform under FA only depend on the sum of externalities $\alpha+\beta$. The first order conditions with respect to λ and T give rise to a system of quadratic equations:

$$\begin{aligned}0 &= (t_B-t_A)[1+\lambda(1-\lambda)(\alpha+\beta)T-\sigma_A\lambda^2T-\sigma_B(1-\lambda)^2T]\\&+ T[(\alpha+\beta)(1-2\lambda)-2\lambda\sigma_A-2(1-\lambda)\sigma_B](1-\lambda t_A-(1-\lambda)t_B),\\0 &= 1+2(\alpha+\beta)\lambda(1-\lambda)T-2\sigma_A\lambda^2T-2\sigma_B(1-\lambda)^2T\end{aligned}$$

Assuming that the parameters are chosen so that the second order conditions are satisfied, this system of equation gives rise to an interior solution in (λ, T) which allows us to compute the optimal number of users X_A and X_B. Unfortunately, under Formula Apportionment, even in the linear model, the decomposition of the effect of a change in t_A on the number of users into direct and indirect effects does not give rise to simple formulas that can easily be signed or interpreted.

9.3 Numerical simulations

9.3.1 Asymmetric countries

We suppose that jurisdiction A is three times larger than jurisdiction B, which results in $\sigma_A = \frac{1}{3}-0.1$, whereas $\sigma_B = 0.9$.

9.3.2 Asymmetric externalities I: positive from A to B, negative from B to A

We suppose that jurisdictions have the same size, with $\sigma_A = \sigma_B = 0.9$ but externalities are asymmetric. There are positive externalities from A to B, reflected in $\alpha = 0.3$, but negative externalities from B to A reflected in $\beta = -0.1$. These numbers capture the situation of a

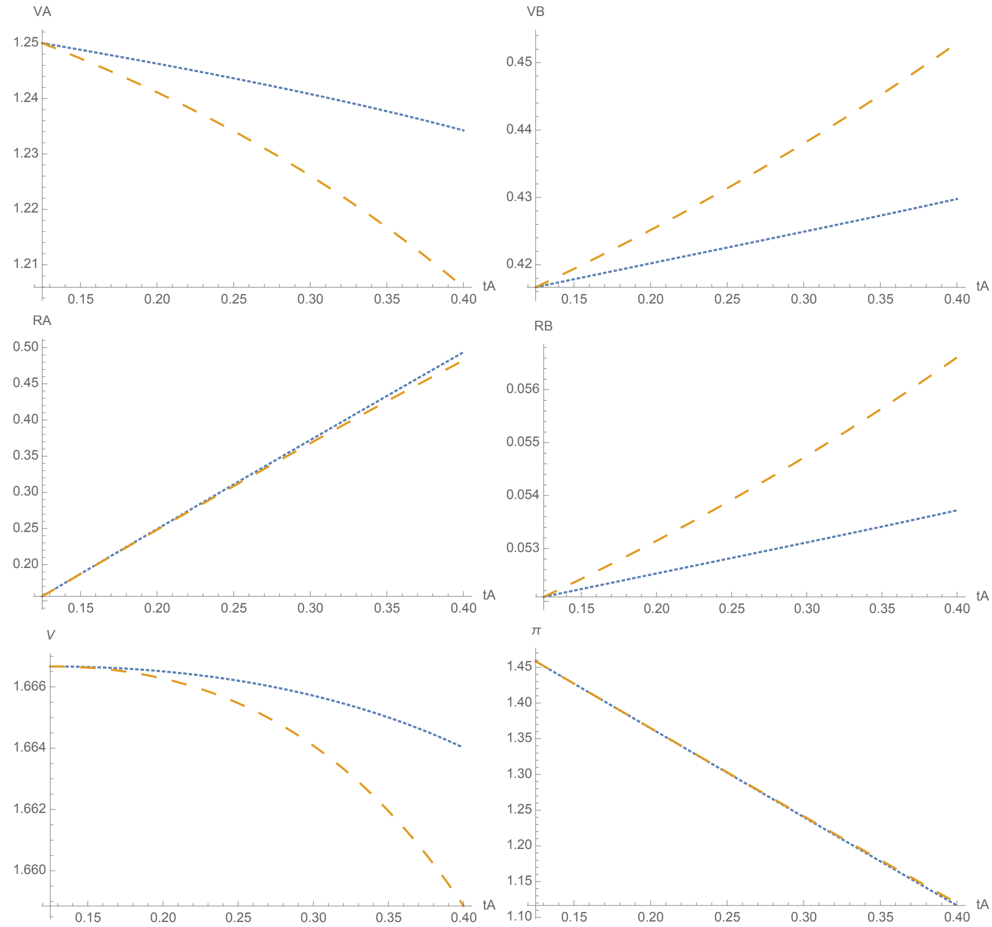

Figure 6: Asymmetric countries: tax base, tax revenues and profits

search engine where advertisers are located in the low-tax jurisdiction and users in the high-tax jurisdiction.

9.3.3 Asymmetric externalities II: negative from A to B, positive from B to A

We suppose that jurisdictions have the same size, with $\sigma_A = \sigma_B = 0.9$ but externalities are asymmetric. There are negative externalities from A to B, reflected in $\alpha = -0.1$, and positive externalities from B to A reflected in $\beta = 0.3$.

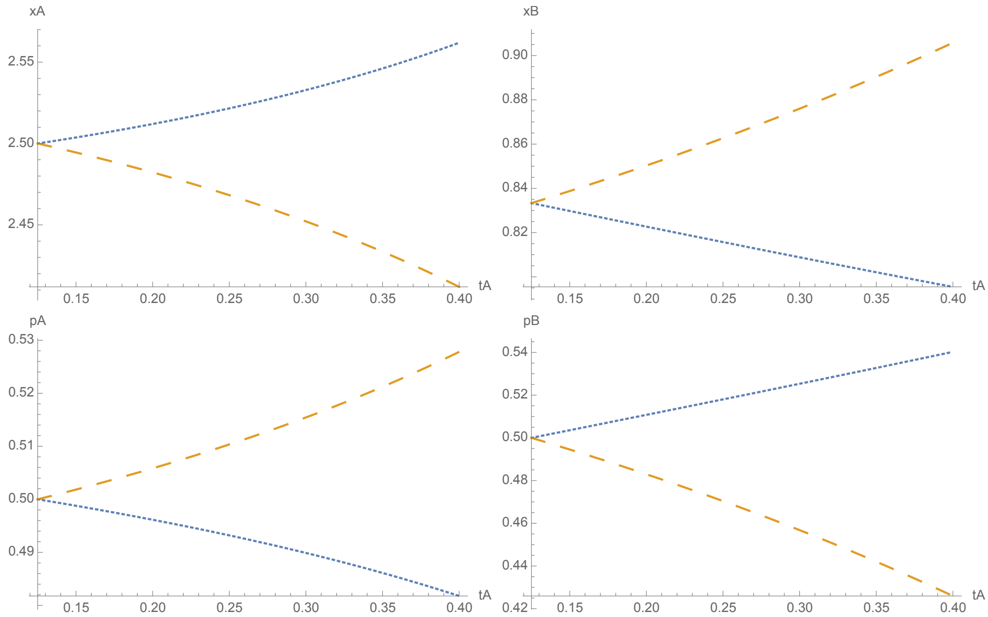

Figure 7: Asymmetric countries: users and prices

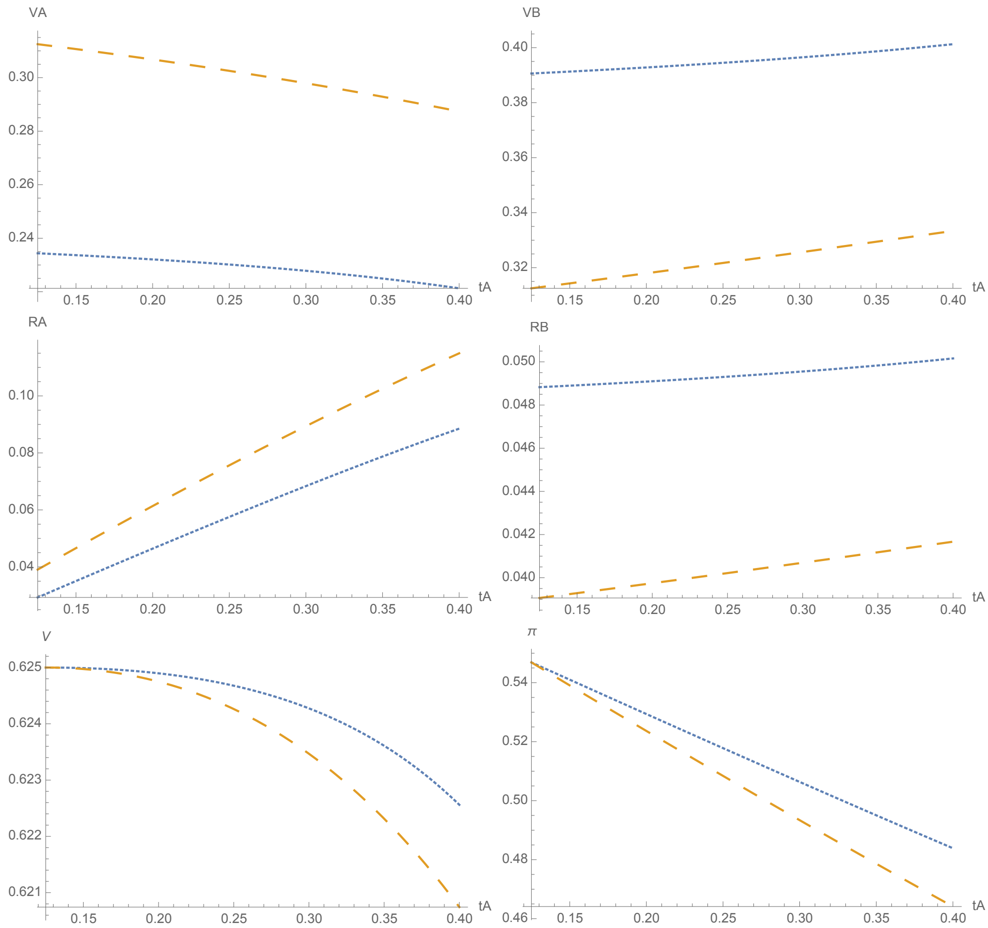

Figure 8: Asymmetric externalities I: tax base, tax revenues and profits

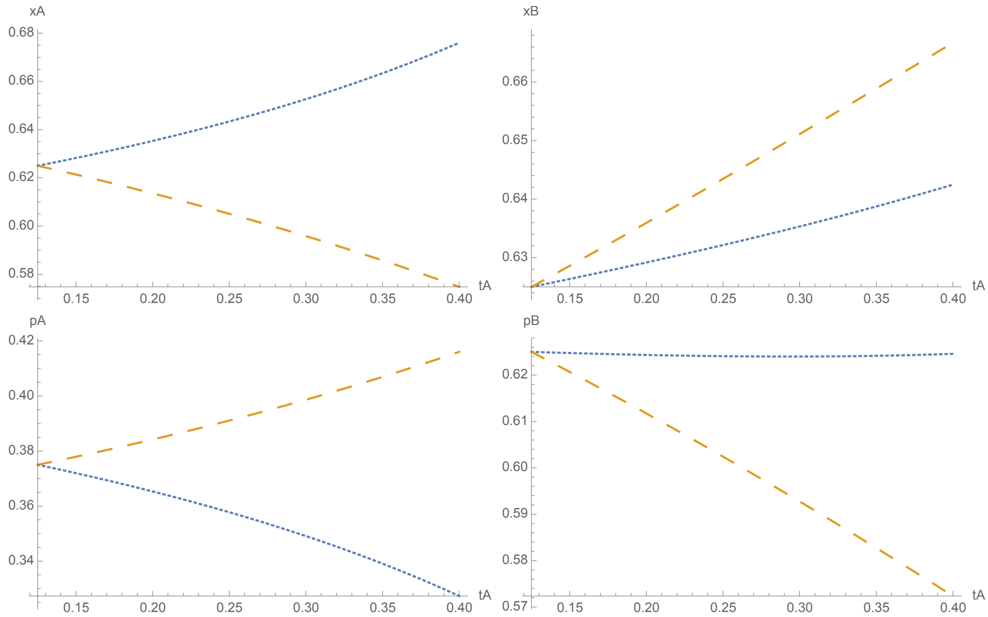

Figure 9: Asymmetric externalities I: users and prices

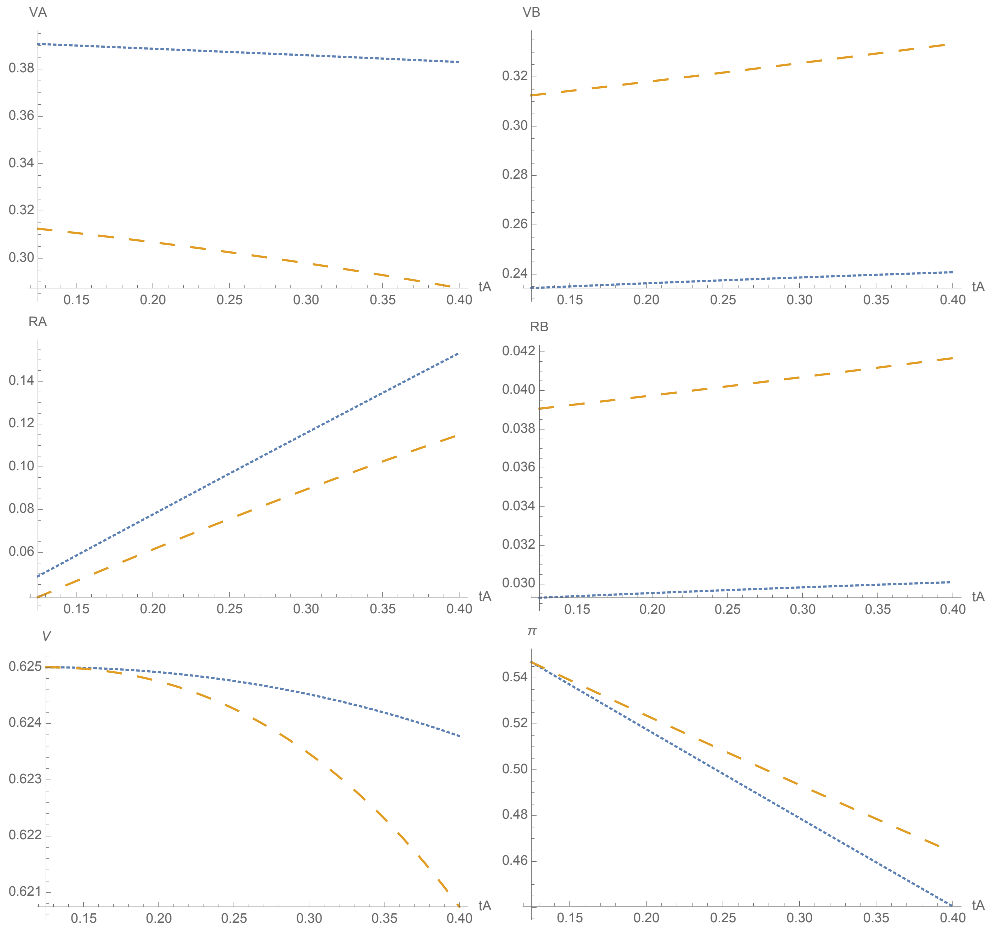

Figure 10: Asymmetric externalities II: tax base, tax revenues and profits

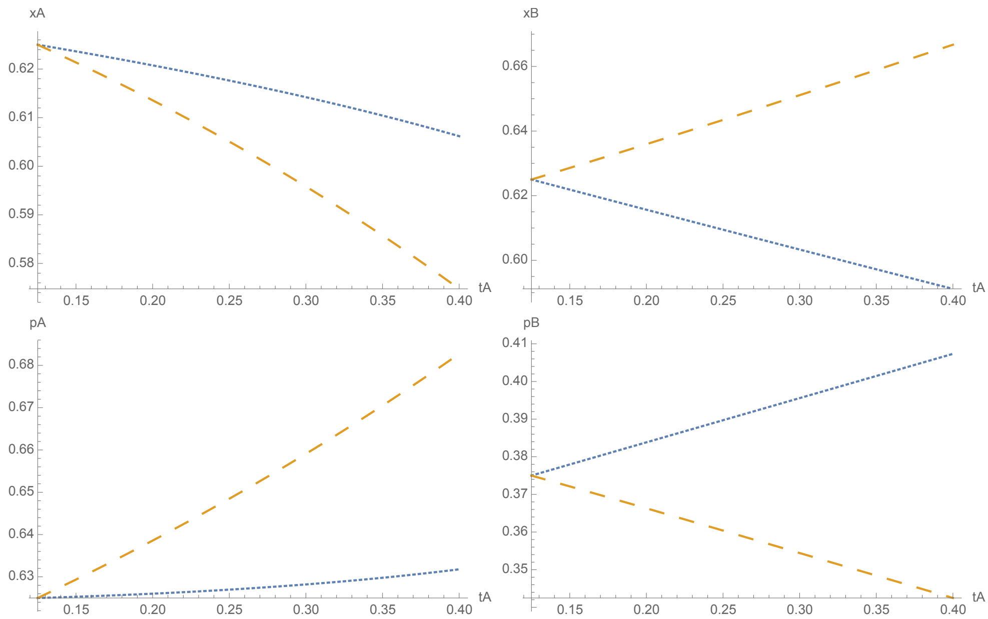

Figure 11: Asymmetric externalities II: users and prices