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MONETARY POLICY WITH A CENTRAL BANK DIGITAL CURRENCY: THE SHORT AND THE LONG TERM

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Abstract

We examine how the introduction of an interest-bearing central bank digital currency (CBDC) impacts bank activities and monetary policy. Depositors can switch from bank deposits to CBDC as a safe medium of exchange at any time. As banks face digital runs, either because depositors have a preference for CBDC or fear bank insolvency, monetary policy can use collateral requirements (and default penalties) to initially increase bankers' monitoring incentives. This leads to higher aggregate productivity. However, the mass of households holding CBDC will increase over time, causing additional liquidity risk for banks. After a certain period, monetary policy with tight collateral requirements generating liquidity risk for banks and exposing bankers to default penalties would render banking non-viable and prompt the central bank to abandon such policies. Under these circumstances, bankers' monitoring incentives will revert to low levels. Accordingly, a CBDC can at best yield short-term welfare gains.

JEL Classification: E42, E52, E58, G21, G28

Keywords: Central bank digital currency - Monetary policy - Banks - Deposits

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Monetary Policy with a Central Bank Digital Currency: The Short and the Long Term*

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1 Introduction

Whether and how governments should introduce a publicly available digital form of their national currency is a widely debated issue in academia and policymaking. Various countries are experiencing a decline in the use of cash and privately issued digital currencies are attracting increasing attention, thus making the issue doubly relevant. Figure 1 depicts the share of cash in the narrowest monetary aggregate M1 for a sample of developed countries over the last four decades until the recent outbreak of Covid-19, illustrating the diminishing importance of cash in some countries.[1]

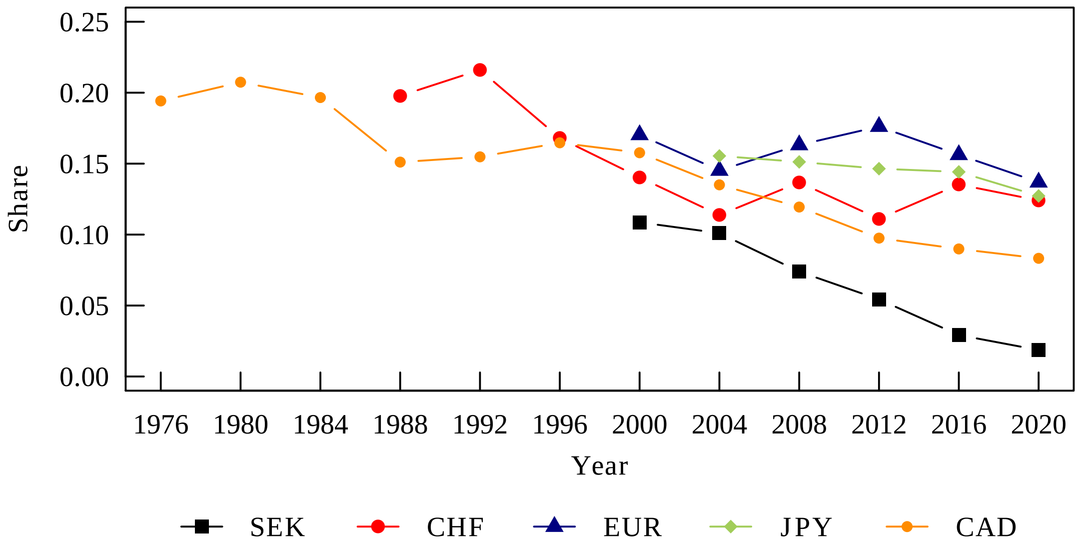

Figure 1: Share of cash in the monetary aggregate M1, average of monthly reported figures; *Source:* Bank of Canada, Bank of Japan, European Central Bank, Sveriges Riksbank, and Swiss National Bank; accessed 7 April, 2020.

Several central banks have announced plans to assess the implications of this new form of national currency, which, as it would be issued by the respective central bank, is often referred to as "central bank digital currency", or simply "CBDC".[2] Unresolved issues relate not only to functional and technical design, but also to economic consequences.

A central bank digital currency will naturally compete with the predominant types of money currently used by the public, i.e. deposits with private banks and cash. Substitutability will generally depend on how a CBDC compares to the existing monies regarding the three main functions of money: unit of account, medium of exchange, and store of value (Hicks, 1967). Differences are conceivable in the latter two dimensions. The low, and still declining, use of cash indicates that, compared to bank deposits, banknotes and coins are considered as of now to be the inferior medium of exchange. Exceptions are situations in which the infrastructure required for electronic transaction settlement is not available or anonymity is desired. Moreover,

[1] Covid-19 may have a strong impact on the use of cash and accelerate current trends initiated by technological innovation (Brown et al., 2020).

[2] For example, the Sveriges Riksbank (`https://www.riksbank.se/en-gb/payments--cash/e-krona/`, accessed 7 April, 2020) and the Bank of Canada (`https://www.bankofcanada.ca/research/digital-currencies-and-fintech/`, accessed 7 April, 2020).

outside crisis periods—when bank default is not considered an important issue—the fact that cash is does not bear interest also makes it an inferior store of value, in comparison to bank deposits. A central bank digital currency can therefore represent a near substitute for cash—non-interest-bearing but allowing for transactions of any kind, albeit with higher transaction costs than deposits with private banks—or as a near substitute for bank deposits—with interest payments and a similar ability to serve as a medium of exchange. In the latter case, a CBDC may even be considered superior to deposits as a store of value since it does not comprise default risk on the part of the issuer.

Arguably, with a central bank digital currency acting as a new substitute for bank deposits and with no restrictions to the right of converting deposits into CBDC using a digital infrastructure at negligible cost, a deposit insurance scheme of the kind we have in our current monetary system would be less needed. The aim of this paper is thus to analyze how the introduction of such a central bank digital currency would impact bank activities and monetary policy. We analyze the welfare implications of these institutional changes and provide a welfare comparison with today's monetary system, where bank deposits, as the principal form of money, are insured through governmental guarantees. We characterize the optimal monetary policy in the presence of a central bank digital currency and discuss its effectiveness in both the short and the long term. We also compare welfare in an economy with central bank digital currency and optimal monetary policy to welfare achieved by a (constrained) social planner.

The model developed in this paper aims to reproduce several stylized features of the current monetary system present in many developed countries. First, private banks supply money by financing loans and other investments with deposits. Accordingly, banks' investment and financing decisions, influenced by bank regulation, strongly affect the supply of money. Second, deposits represent a claim on the legal tender, which at present is cash. Hence, deposit withdrawals are effected by using banknotes and coins. Interbank liabilities such as those emerging from deposit transfers, are, in turn, settled by using interest-bearing reserves, a digital form of the national currency only available to banks. Third, the central bank only provides banks with cash and reserves, commonly referred to as liquidity. Hence, monetary policy as the organization and execution of liquidity provisions influences banks' investment and financing decisions and thus private money creation.[3]

In the presence of a deposit insurance scheme or an interest-bearing CBDC, cash is generally an inferior medium of exchange and store of value.[4] Accordingly, it will not concern us any further here. In our model for an alternative monetary system, the CBDC represents the only legal tender. Thus, any deposit withdrawal can be interpreted as a deposit transfer from a private bank to the central bank. Following today's institutional arrangement, deposit transfers are settled with reserves that can be borrowed from, and deposited with, the central bank. Liquidity provisions take the form of collateralized loans, such that monetary policy includes the choice of interest rates for reserves and the CBDC, and also the choice of the collateral

[3]Liquidity can be provided in different ways; see Bindseil (2004) on how monetary policy has evolved over time.

[4]We rule out situations in which transactions cannot be settled electronically or anonymity is desired. Moreover, in describing today's system we do not account for any legal tender. But in the presence of a deposit insurance scheme, banknotes and coins are not demanded, so we can do this without any loss of generality.

framework that determines the eligible assets and their valuation (Bindseil et al., 2017).

Our model accounts for standard banking elements. In particular, bankers act as delegated monitors able to alleviate moral hazard on the part of borrowers. However, bankers are also subject to moral hazard, leaving them with the alternative between costly monitoring or shirking. Bankers face penalties if they default on liabilities vis-à-vis the central bank. Finally, depositors face switching costs when transferring funds between private bankers or between a private banker and the central bank. These are motivated by the substantial effort involved in engineering a transfer. In our model, fiat money in the form of bank deposits has value due to three reasons: First, firms can only acquire investment goods if they obtain loans from banks and thus there are large gains from using money to buy these goods. Second, depositors can avoid default risk at banks by switching to CBCDs. Third, all money is destroyed at the end of the economy.[5]

Depositors can switch from bank deposits to CBDC as a safe medium of exchange at any time. We model digital bank runs arising either because depositors have a preference for CBDC or fear bank insolvency. We refer to the former type of bank runs as CBDC-induced, as they arise only because the central bank issues money to the public, which is equivalent to bank deposits as a medium of exchange and store of value. Deposits can be converted into CBDC without the consent of bankers or the central bank. Thus, following a bank run, bankers may face a liability vis-à-vis the central bank that exceeds their collateral capacity as determined by the central bank. In this case, the bank becomes illiquid and defaults, and the respective banker will face a default penalty scaling with the liability vis-à-vis the central bank that is not covered by available collateral. While bankers cannot influence the likelihood of CBDC-induced bank runs, they can engage in the costly monitoring of borrowers to alleviate moral hazard, which increases the probability of success for the financed project and ultimately reduces the likelihood of bank insolvency. Monitoring therefore also reduces the likelihood of a default penalty. Hence, a monetary policy with tight collateral requirements generating liquidity risk and exposing bankers to default penalties can increase bankers' monitoring incentives and lead to higher aggregate productivity. However, due to recurrent bank insolvencies and positive switching costs, the mass of households holding CBDC will increase over time and cause additional liquidity risk for banks. Thus, as the likelihood of CBDC-induced bank runs increases, the chances of bankers being exposed to default penalties will also increase, while the chances of earning returns from loan financing will decrease. After a certain period, monetary policy with tight collateral requirements would render banking non-viable and prompt the central bank to abandon such policies so that monitoring incentives will revert to low levels.

We provide necessary conditions for the optimality of tight collateral requirements and characterize the optimal monetary policy explicitly under specific assumptions on firm productivity and switching costs. We find that the default penalties that suffice to incentivize bankers to monitor decrease with banks' equity-to-deposit ratio and increase with the probability of success for the financed project without monitoring by the banker. The higher banks' equity financing, the larger the returns from monitoring skimmed by bankers and the lower the default penalties

[5] Note that imposing a deposit-in-advance constraint is not necessary since deposits are interest-bearing. For an alternative set of assumptions to ensure the value of bank deposits in a finite horizon model see Faure and Gersbach (2017).

necessary to incentivize bankers to monitor. Similarly, the higher the probability of success for the financed project without monitoring by bankers, the higher the expected returns without monitoring skimmed by bankers will be, so that there are correspondingly fewer incentives for bankers to monitor. Accordingly, the default penalty required to incentivize bankers to monitor will increase.

We also compare this alternative monetary system (a CBDC and no deposit insurance) with the current monetary system, where bank deposits are the principal form of money, which is often insured by such things as governmental guarantees. Most notably, if there are no switching costs and monetary policy is optimal, the alternative system will never entail welfare losses compared with today's monetary system. However, through the use of its collateral framework, the central bank can improve bankers' monitoring incentives and ultimately increase welfare. This effect exists at most for a finite period of time as in the presence of solvency risk, tight collateral requirements will at some point render banking non-viable. In this respect, introducing a central bank digital currency involves risks for both the individual bank and for the banking system as a whole. Since banks' liquidity demand is likely to rise with a CBDC, the rules for liquidity provisions by the central bank, including the collateral framework, come to the fore.

Welfare in a competitive equilibrium with optimal monetary is also compared with welfare achieved by a (constrained) social planner. The unconstrained social planner having complete information about agents' activities can achieve the first-best welfare by reallocating endowments between agents in order to rule out solvency risk for bankers, which guarantees a welfare-maximizing monitoring decision by bankers and avoids switching costs incurred by depositors in the case of bank insolvency. Accordingly, any competitive equilibrium without solvency risk and with loose collateral requirements representing the optimal monetary policy, i.e. no liquidity risk and default penalties for bankers, yields the first-best welfare. The constrained social planner having limited information about agents' activities and being restricted to taxes and transfers contingent on idiosyncratic states can only achieve the second-best welfare: Bankers' monitoring decision can be aligned with the objective of maximizing welfare but solvency risk for bankers and thus switching costs incurred by depositors in the case of bank insolvency cannot be eliminated. Any competitive equilibrium with solvency risk and tight collateral requirements representing the optimal monetary policy, i.e. liquidity risk and default penalties for bankers, yields welfare which is generally lower than the second-best welfare due to default penalties imposed on bankers and lost monitoring activities by illiquid bankers.

The paper is structured as follows: Section 2 relates our work to the existing literature. Section 3 introduces the model and discusses the optimal choices of the individual agents. Section 4 provides the equilibrium analysis, while Section 5 outlines the optimal monetary policy. Subsequently, Section 6 provides the welfare comparison with today's monetary system, and Section 7 investigates the dynamic effects of our model. Section 8 concludes.

2 Relation to the Literature

The introduction of a central bank digital currency is a widely debated issue (Barontini and Holden, 2019). Of all the forms of CBDCs discussed, we focus in this paper on a near substitute for bank deposits with equivalent properties as a medium of exchange.[6] Moreover, similar to reserves held by banks with the central bank, the central bank digital currency held by the public is interest-bearing. Then, the only difference between banks and the public is that banks can borrow national currency from the central bank, while the public cannot.

A series of papers discusses the pros and cons of such central bank digital currencies. The main advantages are considered to be a disciplining effect on commercial banks (Berentsen and Schär, 2018), financial inclusion and an increase of financial stability (Ricks et al., 2018; Berentsen and Schär, 2018), as well as a better conduct of monetary policy (Bordo and Levin, 2017). However, Cecchetti and Schoenholtz (2018) argue that a CBDC may also generate financial instability. Among others, Engert et al. (2017) claim that a central bank digital currency could improve competitiveness in payments. Similarly, Kahn et al. (2018) consider the mitigation of competition problems in the banking sector to be the strongest argument in favor of introducing a CBDC. A comprehensive overview of the potential implications of central banks issuing digital currencies for the public can be found in Pichler et al. (2020).

A few theoretical papers have already assessed possible effects of a central bank digital currency. Andolfatto (2018) shows that in an overlapping generation framework with imperfectly competitive banks, a central bank digital currency not only increases financial inclusion but also raises deposit rates through increased competition. This positive competition effect is also at work in Chiu et al. (2019), who calibrate their model for the US economy and find that, subject to suitable interest rate setting, a central bank digital currency may raise bank lending and output significantly. Barrdear and Kumhof (2016) develop a dynamic stochastic general equilibrium model to show how central bank digital currencies lower the real policy rate and thereby stimulate the economy. Similarly, Keister and Sanches (2019) find a central bank digital currency to be generally welfare-improving, while noting that there may also be instances in which the funding costs of banks increase, so that lending and ultimately welfare are reduced. The introduction of a CBDC would also pose technical and organizational challenges. The former are addressed by Böhme (2019) and Auer and Böhme (2020), while the latter are discussed by Bindseil (2019).

This paper relates to the growing literature discussing the creation of money by private agents. Models of bank money creation have been developed by Faure and Gersbach (2017) and Benigno and Robatto (2019), which both feature macroeconomic risk. In this model, we focus instead on idiosyncratic risk, while introducing a second form of money, the CBDC, and modeling moral hazard on the part of bankers and bank borrowers.

As we compare monetary systems with and without CBDC, our work is also closely connected to the literature on monetary architectures and the equivalence of monies. Brunnermeier and Niepelt (2019), for example, establish some general conditions for the equivalence of public and private money, without focusing on particular institutional arrangements that rule the process of money creation. Faure and Gersbach (2018), in turn, compare monetary architectures in which money is solely created by the central bank with today's monetary system, which relies particularly on private money creation in the form of deposit issuance by banks.

[6] An overview of possible types of central bank digital currencies can be found in Bech and Garratt (2017) and Kumhof and Noone (2018).

In our model, bank borrowers, represented by firms, are prone to moral hazard. We introduce a monitoring technology for bankers in the spirit of Holmstrom and Tirole (1997) that prevents any opportunistic behavior by firms. We therefore relate to a large literature that subscribes to this interpretation of monitoring and, specifically, its application to banking (see, for example, Gersbach and Rochet (2017)). Moreover, we introduce moral hazard on the part of bankers, allowing them to engage in costly monitoring or shirk. Note that in our model depositors do not need to monitor bankers as in the classic paper by Calomiris and Kahn (1991) since they can always switch to CBDC and are not impacted by bank defaults.

3 Model

3.1 Macroeconomic environment

The model features four agents: households, firms, bankers, and a central bank. Households and bankers are endowed with a capital good, which is used by firms to produce a unique consumption good. We consider a monetary economy where transactions are settled instantaneously by using bank deposits or CBDC.[7] The latter only enters the economy when depositors switch from private banks to the central bank. Bankers grant loans financed with equity and deposits and can monitor borrowers. Loans are demanded by firms that are penniless and need to finance the acquisition of the capital good in the markets instantaneously, i.e. before output is produced and sold. Monitoring of firms increases their expected productivity and ultimately their expected loan repayment, but it also requires costly efforts on the part of bankers. Markets are competitive. By assuming that each banker is matched with one firm and one household, we can account for idiosyncratic risk.

Bankers face runs from households if the latter prefer CBDC to deposits or bankers become insolvent. Households can execute their deposit transfers at any time without the consent of bankers or the central bank. Thus, the demand for reserves by the individual banker may exceed the collateral capacity determined by the central bank. Then the banker will become illiquid and default. In the case of illiquidity, the central bank seizes all available bank assets and the banker faces a default penalty. Firms are exposed to idiosyncratic productivity shocks, so that sufficiently high bank leverage may expose banks to solvency risk, i.e. the revenues from loan financing are insufficient to service the liabilities vis-à-vis depositors and the central bank. Depending on the returns from the assets seized, the central bank generates losses or profits which, as the central bank operates under a balanced budget, are financed through taxes or distributed by using transfers.

3.2 Summary of events and notation

We model a monetary economy in which transactions are settled instantaneously so that the timing of events is of great importance for our analysis. Until Section 7 we consider a static framework with the following three subsequent stages, summarized in Figure 2.

[7] Firms are subject to limited commitment to repayment, which can be overcome by bank loans, securing repayment by bankers and allowing firms to acquire capital goods instantaneously using money in the form of bank deposits or CBDC.

Stage I. The central bank sets the loan rates for reserves, the deposit rates for reserves and the CBDC, and by setting a haircut determines the valuation of bankers' collateral, i.e. the loans granted by bankers to firms. Each banker is matched with one firm and one household. The banker provides the firm with loan financing and decides on future monitoring activities. The firm uses the deposits acquired to purchase capital good on the markets. Bankers use all their deposits for the equity financing of banking operations. As capital good is sold to firms, bankers may experience a CBDC-induced bank run if the matched household prefers CBDC to deposits and thus initially opens an account with the central bank instead of the matched banker. If the collateral capacity of a banker is insufficient to obtain the reserves required to service the deposit transfer to the central bank, the banker will become illiquid and default. The assets of the respective banker are then seized by the central bank, and the banker will face a default penalty imposed by the central bank.

Stage II. Any liquid banker can execute the monitoring activity previously decided on. The idiosyncratic productivity shocks realize and firms transform the capital good into the consumption good. Bankers may face insolvency, in which case depositors may transfer their funds to the central bank. If the household possessing funds with an insolvent banker chooses to hold CBDC instead of deposits, the respective banker will only default on the central bank. If the banker's collateral capacity is insufficient to cover the liabilities vis-à-vis the central bank, the banker will face a default penalty imposed by the central bank.

Stage III. Solvent bankers credit deposits with interest and pay out dividends on the equity financing provided. Central bank losses are financed through taxes, while profits are distributed by using transfers. Households and bankers use their funds to purchase the consumption good on the markets. Firms use the revenues from sales to meet their repayment obligations on the outstanding loans. Similarly, bankers repay their borrowed reserves to the central bank.

Stage I	**Stage II**	**Stage III**
• Monetary policy decided	• Monitoring by banks	• Interest payments
• Loan financing by banks	• Production by firms	• Taxes and transfers
• Capital-good market	• Bank insolvency	• Consumption-good market
• CBDC-induced runs	• Insolvency-caused runs	• Loan repayment

Figure 2: Summary of Events

From the perspective of the individual bank, a bank run is caused either by a household preferring CBDC to deposits or by bank insolvency, which, in its turn, will result from a negative productivity shock for the financed firm in the presence of sufficiently high bank leverage. Hence, for each triplet (banker, firm and household), the returns on deposits, loans, and equity can at most depend on the type of household, the idiosyncratic productivity shock for the firm, and bank leverage. For the subsequent description of the model it will be useful here to formally introduce the multivariate state $\mathbf{z} := (\varphi, h, s) \in \mathcal{Z} := [1, +\infty) \times \mathcal{H} \times \mathcal{S}$, with φ denoting the leverage of a representative bank, $h \in \mathcal{H} := \{\underline{h}, \overline{h}\}$ denoting the type of household, where $\underline{h}$ ($\overline{h}$) indexes a household that initially opens an account with the central bank (with the matched banker) and $s \in \mathcal{S} := \{\underline{s}, \overline{s}\}$ denotes the idiosyncratic productivity shock for the matched firm, where $\underline{s}$ ($\overline{s}$) indexes a negative (positive) productivity shock.

In the following subsections we introduce the optimization problems of the individual agents, outline the optimal choices, and characterize the various equilibria. The proofs relating to the results of the following sections can be found in Appendix A.

3.3 Households

There is a continuum of households with unit mass. A mass $\mu \in [0, 1]$ of households initially opens an account with the central bank, while the residual mass $1 - \mu$ of households opens an account with private bankers. Each type of household is identical with respect to its behavior on the markets, so that we can focus on a representative household for each type. The household $h \in \mathcal{H}$ maximizes utility, which we assume to be linear and strictly increasing in consumption, and is endowed with capital good $K > 0$, which is sold on the markets to firms at the nominal price $Q > 0$. Depending on where the household initially opens an account—with the central bank or with a private banker—the proceeds from capital good sales, QK, are held as CBDC, denoted by $D^h_{CB} \geq 0$, or as deposits, denoted by $D^h \geq 0$. Based on the previous outline, it holds that $D^h_{CB} = QK\mathbb{1}\{h = \underline{h}\}$ and $D^h = QK\mathbb{1}\{h = \overline{h}\}$.

Households face a portfolio allocation problem, since deposits with bankers are subject to a potentially stochastic nominal gross return $R^D_{\mathbf{z}} \geq 0$, while the holdings of CBDC are credited with interest according to the deterministic nominal gross rate $R^D_{CB} > 0$ set by the central bank. Depending on the realized state $\mathbf{z} \in \mathcal{Z}$, each household $h \in \mathcal{H}$ can choose to hold the funds $D^h_{CB} + D^h = QK$ as CBDC, denoted by $D^h_{CB,\mathbf{z}} \geq 0$, or as deposits, denoted by $D^h_{\mathbf{z}} \geq 0$.[8] Households own firms which operate under limited liability and without equity financing, so that households receive firm profits Π^f as dividends.[9] After accounting for taxes and transfers T^h, household $h \in \mathcal{H}$ uses the funds credited with interest $D^h_{CB,\mathbf{z}} R^D_{CB} + D^h_{\mathbf{z}} R^D_{\mathbf{z}}$, and firm profits Π^f, to finance the purchase of consumption good $C^h_{\mathbf{z}}$ from firms on the markets at the nominal price $P > 0$.[10] As utility is strictly increasing in consumption, the budget constraint is binding and given by $PC^h_{\mathbf{z}} = D^h_{CB,\mathbf{z}} R^D_{CB} + D^h_{\mathbf{z}} R^D_{\mathbf{z}} + \Pi^f + T^h$.

Any transfer of funds from a private bank to the central bank, or vice versa, is associated with costs which, in our model, take the form of a non-monetary utility loss $\nu > 0$.[11] Thus, based on the previous outline, household $h \in \mathcal{H}$ faces for each state $\mathbf{z} \in \mathcal{Z}$ the portfolio allocation problem

$$\max_{D^h_{CB,\mathbf{z}} \geq 0} D^h_{CB,\mathbf{z}} r^D_{CB} + (QK - D^h_{CB,\mathbf{z}}) r^D_{\mathbf{z}} - \nu \mathbb{1}\{D^h_{CB,\mathbf{z}} \neq D^h_{CB}\},$$

where interest rates, denoted by lower-case letters, are expressed in terms of the consumption

[8] We assume that a transfer of funds between private bankers is at least as costly as a transfer of funds to the central bank. So, even if we neglect solvency risk, households can never be better off by transferring deposits to another banker than the central bank.

[9] Without macroeconomic risk, the aggregate firm profits do not depend on firm-specific productivity shocks.

[10] Since there is no aggregate risk, the price of the consumption good will be deterministic.

[11] Such losses can be justified with the effort involved in engineering a transfer, e.g. account opening and closing.

good, which we define as the numeraire of the economy.[12] A household will only shift funds between a private banker and the central bank if the alternative money yields excess returns leading to a utility gain sufficient to offset the utility loss resulting from the transfer of funds. Thus, the optimal choice of households between deposits and CBDC is of a knife-edge type. If a household is indifferent between deposits and CBDC, we assume that it will stay with its initial choice. The following lemma summarizes the optimal choice for both types of household. In what follows, we use the notation $\tilde{\nu} := \nu/(QK)$.

Lemma 1 (Optimal Choice of Households)
$D^{\overline{h}}_{CB,\mathbf{z}} = QK$ iff $r^D_{\mathbf{z}} < r^D_{CB} - \tilde{\nu}$, $D^{\underline{h}}_{CB,\mathbf{z}} = QK$ iff $r^D_{\mathbf{z}} \leq r^D_{CB} + \tilde{\nu}$, and $D^h_{CB,\mathbf{z}} = 0$ otherwise.

3.4 Firms

Firms operate with identical production functions and exist in a continuum with unit mass, so that we can focus on a representative firm. To produce the consumption good, the firm purchases capital good $K^f \geq 0$ on the markets from households and bankers at the nominal price $Q > 0$. The firm operates without equity financing, relying on external financing in the form of bank loans $L^f = QK^f$ to finance the acquisition of capital good. The loans are subject to repayment determined by the potentially stochastic nominal gross loan rate $R^L_s > 0$.[13]

We assume that the firm operates with limited liability, so that in the case of default, the matched bank can never seize more than the available production output $Y^f_s = A_s K^f$, where A_s represents the idiosyncratic productivity of the firm. Since $\underline{s}$ ($\overline{s}$) represents a negative (positive) productivity shock, it holds that $0 \leq A_{\underline{s}} \leq A_{\overline{s}}$. The expected productivity of the firm depends on the monitoring activities $m(h)$ of the matched banker, which may vary with the type of matched household. We assume that the idiosyncratic productivity shock is distributed with probabilities $\eta_{s|m(h)} \in (0,1)$, where $m(h) \in \{0,1\}$ denotes the monitoring decision of the matched banker, with value zero (one) representing shirking (monitoring). In our model, monitoring increases the likelihood of a positive productivity shock, i.e. $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0} > 0$. In what follows, we denote the monitoring activities by $\underline{m} := m(\underline{h})$ and $\overline{m} := m(\overline{h})$. To indicate that the firm's expectation depends on the banker's monitoring activities, we index the expectation operator $\mathbb{E}[\cdot]$ by the monitoring decisions $\mathbf{m} := (\underline{m}, \overline{m})$. The production output is sold on the markets to households and bankers at the nominal price $P > 0$. The firm maximizes profits, so that its optimization problem in real terms is given by

$$\max_{K^f \geq 0} \mathbb{E}_{\mathbf{m}}[\max\{A_s - r^L_s Q, 0\}] K^f,$$

where the loan rate, denoted by lower-case letters, is measured in terms of the consumption good, i.e. $r^L_s := R^L_s / P$. Whenever the firm faces excess returns from production in one of the states, there exists no optimal finite demand for capital good as, at least after some critical level, the firm's profits will grow with the amount of capital good. The optimal choice of the firm is summarized in the following lemma:

[12] Hence, $r^D_{CB} := R^D_{CB}/P$ and $r^D_{\mathbf{z}} := R^D_{\mathbf{z}}/P$ for all $\mathbf{z} \in \mathcal{Z}$.

[13] We can assume, without loss of generality, that the loan rate varies at most with the firm's idiosyncratic productivity shock $s \in \mathcal{S}$, i.e. $R^L_{\mathbf{z}} = R^L_s$ for all $\mathbf{z} \in \mathcal{Z}$, as this represents an equilibrium outcome.

Lemma 2 (Optimal Choice of the Firm)
$K^f = +\infty$ *iff* $A_s > r^L_s Q$ *for some* $s \in \mathcal{S}$*, and* $K^f \in [0, +\infty)$ *otherwise.*

3.5 Central bank

The central bank has three instruments for conducting monetary policy: interest rates on reserves and the CBDC, collateral requirements, and default penalties on bankers. The details are as follows: The central bank lends reserves used by bankers to hold reserve deposits or to service deposit transfers. Bankers' reserve holdings and the public's CBDC holdings are both credited with the same (deterministic) nominal gross interest rate $R^D_{CB} > 0$[14], while reserve loans lead to a repayment obligation on the part of bankers that is determined by the (deterministic) nominal gross loan rate $R^L_{CB} > 0$. For simplicity, we assume that both interest rates are equal.[15]

Assumption 1 (Central Bank Rates)
$R^D_{CB} = R^L_{CB}$.

Deposit transfers are settled using reserves that the individual banker can borrow from the central bank while depositing assets as collateral. The collateral capacity of the individual banker is given by ΨL^b, where L^b denotes the loan financing provided by the banker to the matched firm and $\Psi \geq 0$ represents the deterministic nominal haircut chosen by the central bank.

Every household can transfer funds from a banker to the central bank, i.e. convert deposits into CBDC, at any time, without the consent of the respective banker or the central bank. Thus, the deposit transfers may lead to a banker having liabilities vis-à-vis the central bank that exceed the banker's collateral capacity. If the collateral capacity is insufficient to cover the repayment obligation on the reserve loan $L^b_{CB,\mathbf{z}}$ required to settle deposit transfers, the respective banker will become illiquid and default, in which case the central bank seizes all available assets and imposes a default penalty on the banker.[16] The latter scales with any outstanding claim of the central bank in excess of the banker's collateral capacity. Specifically, the banker experiences a utility loss in the form of $\phi \max\{r^D_{CB} L^b_{CB,\mathbf{z}} - \psi L^b, 0\}$, where $\phi > 0$ represents a scaling parameter also chosen by the central bank and $\psi := \Psi/P$ denotes the haircut in terms of the consumption good.

As the central bank operates under a balanced budget, its losses are financed through taxes while its profits are distributed by using transfers. In the following, we denote aggregate taxes and transfers in nominal terms by T and nominal central bank profits and losses by Π^{CB}.

[14]Introducing two different deposit rates for reserves and the CBDC, while preserving the unrestricted right of converting deposits leads to arbitrage opportunities for bankers.

[15]A spread between central bank rates can be accommodated in our framework and does not alter our results qualitatively, as it generates central bank profits which, assuming a balanced budget, are distributed to households and bankers before the purchase of the consumption good.

[16]Note that we abstract here from the possibility of interbank borrowing, which constitutes for the individual banker an alternative way of obtaining liquidity. However, in the case of bank insolvency interbank borrowing is not effective in reducing the respective banker's liabilities vis-á-vis the central bank and the resulting default penalties. Thus, integrating an interbank market into our framework does not impair the subsequently illustrated effect of monetary policy on bankers' monitoring incentives.

3.6 Bankers

There is a continuum of identical bankers with unit mass, so that we can focus on a representative banker. Each banker maximizes utility, which is linear and strictly increasing in consumption, and is endowed with capital good $E > 0$, which is sold on the markets to firms at the nominal price $Q > 0$. The banker can decide whether to open an account with the central bank and hold the proceeds from capital good sales as CBDC or to conduct banking operations with limited liability and financed with bank equity $E^b = QE$. If indifferent, the banker is assumed to engage in banking operations. In this case, the banker supplies loan financing $L^b \geq E^b$ to the matched firm, which, at the outset, is completely funded with deposits. As soon as the banker has sold the endowment of capital good and received deposits in return, all funds are used to provide equity financing for the banking operations. Since the amount of equity financing is fixed, the loan supply implicitly determines the leverage $\varphi := L^b/E^b$, which must comply with a regulatory leverage constraint, i.e. $\varphi \leq \varphi^r$, where $\varphi^r \in [1, +\infty)$.

The banker can demand reserves $L^b_{CB,\mathbf{z}} \geq 0$ from the central bank, either to hold reserves $D^b_{CB,\mathbf{z}} \geq 0$ with the central bank or to service deposit transfers. Thus, for any state $\mathbf{z} \in \mathcal{Z}$, the balance sheet identity $L^b + D^b_{CB,\mathbf{z}} = D^b_{\mathbf{z}} + L^b_{CB,\mathbf{z}} + E^b$ applies, where $D^b_{\mathbf{z}}$ denotes the total supply of deposits to the matched household. For state $\mathbf{z} \in \mathcal{Z}$, the real returns on equity are therefore given by $r^{E,+}_{\mathbf{z}} E^b$, where $r^{E,+}_{\mathbf{z}} := \max\{r^L_s L^b + r^D_{CB} D^b_{CB,\mathbf{z}} - r^D_{\mathbf{z}} D^b_{\mathbf{z}} - r^D_{CB} L^b_{CB,\mathbf{z}}, 0\}/E^b$. If the repayment obligations on reserve loans, $r^D_{CB} L^b_{CB,\mathbf{z}}$, exceed collateral capacity, ψL^b, the banker will become illiquid and default, in which case the central bank seizes all available assets and imposes penalty $r^{E,-}_{\mathbf{z}} E^b$ on the banker, where $r^{E,-}_{\mathbf{z}} := \phi \max\{r^D_{CB} L^b_{CB,\mathbf{z}} - \psi L^b, 0\}/E^b$. The assumption of competitive markets implies that interest rates must form in such a way that the banker generates no returns in excess of the outside option, i.e. holding CBDC at the central bank, which yields the return $r^D_{CB} QE$. Arbitrage opportunities on the deposit market, and hence excess returns for the banker, are only ruled out if in each state without default the deposit rate equals the central bank rate, as expressed in the following lemma. Otherwise, the price-taking behavior imposed on the banker is not incentive-compatible, i.e. by setting a deposit rate different from the one prevailing on the deposit market, the banker can achieve excess returns without risk.

Lemma 3 (Deposit Rate)
For any state $\mathbf{z} \in \mathcal{Z}$, where the banker does not default, it holds that $r^D_{\mathbf{z}} = r^D_{CB}$.

From Lemma 3, the positive switching costs, $\nu > 0$, and the fact that the deposit rate falls short of the central bank rate if the banker defaults, we know that households that initially opens an account with the central bank will never transfer their funds to a banker. Thus, the banker does not experience any deposit inflows. Using Assumption 1, which states the equality of central bank rates, we can then, without loss of generality, assume that the banker does not hold any reserve deposits, i.e. $D^b_{CB,\mathbf{z}} = 0$.

Furthermore, the matching of one banker and one household enables us to express the demand for reserve loans as $L^b_{CB,\mathbf{z}} = \xi_{\mathbf{z}}(L^b - E^b)$, where $\xi_{\mathbf{z}} = \max\{\xi_h, \xi_{\varphi,s}\}$ represents the bank run indicator, with $\xi_h = \mathbb{1}\{h = \underline{h}\}$ indicating a CBDC-induced bank run and $\xi_{\varphi,s} \in \{0, 1\}$ indicating a run following bank insolvency. From balance sheet identity, we can then infer that the supply of deposits is given by the residual, i.e. $D^b_{\mathbf{z}} = (1-\xi_{\mathbf{z}})(L^b - E^b)$. The banker faces solvency risk if and only if, in the presence of a negative productivity shock for the financed firm ($s = \underline{s}$), the returns from loan financing, $r^L_{\underline{s}} L^b$, are not sufficient to cover the liabilities vis-à-vis the matched household as the only potential depositor and the central bank, $r^D_{CB}(L^b - E^b)$, i.e.

$$r^L_{\underline{s}}\varphi < r^D_{CB}(\varphi - 1) \quad \Leftrightarrow \quad \varphi > \varphi^S := \frac{r^D_{CB}}{r^D_{CB} - r^L_{\underline{s}}} > 0,$$

where we have used the definition of the leverage, $\varphi = L^b/E^b$. We define $\varphi^S := +\infty$ if $r^D_{CB} \leq r^L_{\underline{s}}$. Depositors incur switching costs, so bank insolvency does not necessarily trigger a bank run. Instead, the matched household may prefer a bail-in, i.e. stay with the banker and accept a deposit rate that is lower than the central bank rate if the deposit rate is still sufficiently high for the switching costs associated with a transfer of funds to the central bank to lead to a higher utility loss for the household. Specifically, depositors will shift their funds to the central bank if and only if

$$r^L_{\underline{s}}\varphi < (r^D_{CB} - \tilde{\nu})(\varphi - 1) \quad \Leftrightarrow \quad \varphi > \varphi^R := \frac{r^D_{CB} - \tilde{\nu}}{r^D_{CB} - \tilde{\nu} - r^L_{\underline{s}}} > 0.$$

We define $\varphi^R := +\infty$ if $r^D_{CB} - \tilde{\nu} \leq r^L_{\underline{s}}$. Thus, bank runs due to insolvency occur if and only if the financed firm incurs a negative productivity shock and bank leverage is sufficiently high to incentivize depositors to shift their funds to the central bank, i.e. $\xi_{\varphi,s} = \mathbb{1}\{s = \underline{s} \wedge \varphi > \varphi^R\}$. Accordingly, we can characterize the bank run indicator as $\xi_{\mathbf{z}} = \mathbb{1}\{h = \underline{h} \vee (s = \underline{s} \wedge \varphi > \varphi^R)\}$. The banker faces liquidity risk if and only if the repayment obligation on central bank loans, $r^D_{CB} L^b_{CB,\mathbf{z}}$, exceeds the collateral capacity, ψL^b, determined by the central bank, i.e.

$$\psi\varphi < r^D_{CB}(\varphi - 1) \quad \Leftrightarrow \quad \varphi > \varphi^L := \frac{r^D_{CB}}{r^D_{CB} - \psi} > 0,$$

where we have used the definition of the leverage, $\varphi = L^b/E^b$ and the fact that $L^b_{CB,\mathbf{z}} = \xi_{\mathbf{z}}(L^b - E^b)$, with $\xi_{\mathbf{z}} \in \{0,1\}$. We define $\varphi^L := +\infty$ if $r^D_{CB} \leq \psi$. Using our previous results, we can further characterize equity returns and default penalties as $r^{E,+}_{\mathbf{z}} = \max\{(r^L_s - r^D_{CB})\varphi + r^D_{CB}, 0\}$ and $r^{E,-}_{\mathbf{z}} = \phi \max\{(\xi_{\mathbf{z}} r^D_{CB} - \psi)\varphi + \xi_{\mathbf{z}} r^D_{CB}\}$, respectively. Note that the banker is only exposed to equity returns if the banker is not facing illiquidity, i.e. if there is no CBDC-induced bank run ($h = \overline{h}$) or there is a CBDC-induced bank run but the leverage is not sufficiently high to cause liquidity risk ($h = \underline{h}$ and $\varphi \leq \varphi^L$). The banker also decides on the monitoring activities $m(h) \in \{0,1\}$, which may depend on the occurrence of a CBDC-induced bank run or, equivalently, may vary with the type of matched household $h \in \mathcal{H}$. To indicate that the banker's expectation depends on the monitoring activities, we index the expectation operator $\mathbb{E}[\cdot]$ by the monitoring decisions $\mathbf{m} := (\underline{m}, \overline{m})$, where we again use $\underline{m} := m(\underline{h})$ and $\overline{m} := m(\overline{h})$ as a shortcut. Monitoring requires effort on the part of the banker, which, in our model, takes the form of a non-monetary utility loss $\kappa > 0$ that scales with the amount of loan financing L^b.[17] The banker uses the returns on equity, $\zeta_{\mathbf{z}} R^{E,+}_{\mathbf{z}} E^b$, with $\zeta_{\mathbf{z}} = 1 - \mathbb{1}\{h = \underline{h} \wedge \varphi > \varphi^L\}$ being

[17] The assumption that monitoring efforts scale with the amount of loan financing is technical in nature, as it simplifies the analysis of the banker's optimization problem.

the liquidity indicator, to finance the purchase of consumption good $C^b_{\mathbf{z}}$ on the markets from firms at the nominal price $P > 0$. As utility is strictly increasing in consumption, the budget constraint is binding and given by $PC^b_{\mathbf{z}} = \zeta_{\mathbf{z}} R^{E,+}_{\mathbf{z}} E^b$. The optimization problem of the banker in real terms is then given by

$$\max_{\substack{\varphi\in[1,\varphi^r],\\ m(h)\in\{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} r^{E,+}_{\mathbf{z}} - r^{E,-}_{\mathbf{z}} - m(h)\kappa\varphi]QE.$$

The following lemma summarizes the banker's optimal choice.

Lemma 4 (Optimal Choice of the Banker)
The banker chooses leverage φ and monitoring activities $m(h)$, with $h \in \mathcal{H}$, such that

(i) $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ *iff*

$$\mathbb{E}_{\mathbf{m}}[r^L_s] = r^D_{CB} + \overline{m}\kappa,$$

where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(r^L_{\overline{s}} - r^L_{\underline{s}}) \geq \kappa\}$,

(ii) $\varphi^L < \varphi = \varphi^r \leq \varphi^S$ *iff* $\phi \in (0,1)$ *and*

$$\mathbb{E}_{\overline{m}}[r^L_s | h = \overline{h}] = r^D_{CB}\left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \overline{m}\kappa,$$

where $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle(r^L_{\overline{s}} - r^L_{\underline{s}}) \geq \kappa\}$,

(iii) $\varphi^S < \varphi = \varphi^r \leq \varphi^L$ *iff*

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[r^L_{\overline{s}} - r^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$,

(iv) $\max\{\varphi^S, \varphi^L\} < \varphi = \varphi^r \leq \varphi^R$ *iff* $\mu\phi < \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$ *and*

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

with $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle[r^L_{\overline{s}} - r^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$,

(v) $\max\{\varphi^S, \varphi^L, \varphi^R\} < \varphi = \varphi^r$ *iff* $\phi \in (0,1)$ *and*

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

where $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle[r^L_{\overline{s}} - \phi\psi - r^D_{CB}(1-\phi)(\varphi^r - 1)/\varphi^r] \geq \kappa\}$.

Note that the loan rates required to incentivize the banker to shoulder on liquidity risk are increasing in the mass of households initially opening an account with the central bank, denoted by μ, and the default penalty per unit of supplied loan financing, denoted by $\phi\epsilon/\varphi$, where

$\epsilon := r^D_{CB}(\varphi - 1) - \psi\varphi$. Thus, the higher the risk of illiquidity and the higher the utility loss due to default penalties in the case of illiquidity, the higher the returns required from loan financing. Similarly, in the presence of solvency risk, the loan rates decrease with the probability of a positive productivity shock, denoted by $\eta_{\bar{s}|\overline{m}}$. Thus, the lower the probability of bank insolvency, the lower the returns from loan financing required to incentivize the banker to shoulder on solvency risk.

4 Equilibrium Analysis

4.1 Equilibrium definition

In our subsequent analysis we focus on competitive equilibria, which are introduced hereafter. In what follows, we denote the expected consumption of the banker and the household using $C^b := \mathbb{E}_{\mathbf{m}}[C^b_{\mathbf{z}}]$ and $C^h := \mathbb{E}_{\mathbf{m}}[C^h_{\mathbf{z}}]$, with $h \in \mathcal{H}$, respectively. Note that expectations are taken at the first stage, when monetary policy has been decided on and all interest rates are known. Due to the law of large numbers, the aggregate consumption of bankers and households is then given by C^b and $(1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, respectively. Also due to the law of large numbers, aggregate production equals expected production, denoted by $Y^f := \mathbb{E}_{\mathbf{m}}[A_s]K^f$, and aggregate firm profits equal expected firm profits, denoted in real terms by $\pi^f := \mathbb{E}_{\mathbf{m}}[\max\{A_s - r^L_s Q, 0\}]K^f$.

Definition 1 (Competitive Equilibrium)
Given a monetary policy R^D_{CB}, Ψ and ϕ, a competitive equilibrium is a set of interest rates $\{R^D_{\mathbf{z}}, R^L_s\}_{\mathbf{z}\in\mathcal{Z}}$, prices $\{Q, P\}$, and choices $\{D^h_{CB,\mathbf{z}}\}_{\mathbf{z}\in\mathcal{Z}}$, K^f, φ and $m(h)$, with $h \in \mathcal{H}$, such that

(i) given $\{R^D_{CB}, R^D_{\mathbf{z}}, Q, P\}_{\mathbf{z}\in\mathcal{Z}}$, choices $\{D^h_{CB,\mathbf{z}}\}_{\mathbf{z}\in\mathcal{Z}}$ maximize the utility of household $h \in \mathcal{H}$,

(ii) given $\{R^L_s, Q, P\}_{s\in\mathcal{S}}$, choice K^f maximizes the profits of the firm,

(iii) given $\{R^D_{CB}, R^D_{\mathbf{z}}, R^L_s, \Psi, \phi, Q, P\}_{\mathbf{z}\in\mathcal{Z}}$, choices φ and $m(h)$, with $h \in \mathcal{H}$, maximize the utility of the banker,

(iv) the good markets clear, i.e. $K^f = K + E$ and $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, and

(v) the asset markets clear, i.e. $L^b = L^f$.

Note that the asset markets for deposits, CBDC, equity, and reserves clear by construction of the model. Thus, when analyzing competitive equilibria, we only have to take the clearing of the markets for loans, capital good, and consumption good into account.

4.2 Equilibrium properties

First we highlight some general properties of all competitive equilibria in our framework and then proceed to a characterization of the various possible equilibria that differ in terms of bankers' risk exposure. Since in equilibrium the market for capital good clears, i.e. $K^f = K + E$, loan demand is determined and given by $L^f = Q(K + E)$. Loan supply then follows from the clearing of the loan market, i.e. $L^b = L^f$. As the bankers' equity financing is fixed, $E^b = QE$, the equilibrium leverage is given by $\varphi = (K + E)/E$. The regulatory leverage constraint must thus satisfy $\varphi^r \geq (K+E)/E$. From Lemma 4 we know that in any environment where bankers are exposed to risk, bankers will choose $\varphi = \varphi^r$, so that the latter inequality must be binding in equilibrium, i.e. $\varphi^r = (K + E)/E$.

In equilibrium, deposit rates never exceed the central bank rate. Specifically, as stated in Lemma 3, the deposit rate paid by the banker without default equals the central bank rate and will only fall short of the central bank rate if the banker defaults due to illiquidity or insolvency. Thus, due to positive switching costs, households that have initially opened an account with the central bank will never transfer their funds to a private banker. Households that have initially opened an account with a banker will transfer their funds to the central bank if and only if the respective banker defaults due to insolvency. However, one remark is in order: If an insolvent banker can pay a deposit rate that is sufficiently high for the switching costs related to a deposit transfer to lead to a higher utility loss for the matched household, depositors will accept a bail-in in the case of bank insolvency. This specific case arises if bank leverage is sufficiently low, i.e. $\varphi \leq \varphi^R$. CBDC holdings thus satisfy $D^h_{CB,\mathbf{z}} = \xi_{\mathbf{z}} QK$, where $\xi_{\mathbf{z}} = \mathbb{1}\{h = \underline{h} \vee (s = \underline{s} \wedge \varphi > \varphi^R)\}$ is the measure of households. Deposit holdings then represent the residual, i.e. $D^h_{\mathbf{z}} = (1 - \xi_{\mathbf{z}})QK$.

We have specified all equilibrium choices except the banker's monitoring decision, which is described when we characterize the various possible equilibria in our framework. For each equilibrium we provide existence conditions and utilitarian welfare, which, based on our assumption of linear utility, comprises aggregate consumption, utility losses due to monitoring, default penalties, and switching costs emerging from deposit transfers. The prices and interest rates prevailing in each equilibrium can be found in the respective proof (see Appendix A).

Note that, in the presence of liquidity risk, the banker's optimality condition relates the real central bank rate r^D_{CB} to the real haircut ψ. As, besides the penalty parameter ϕ, the central bank also chooses the nominal central bank rate R^D_{CB} and the nominal haircut Ψ, both influencing prices in the economy, we can interpret monetary policy as setting the real central bank rate or, alternatively, as setting the real haircut. In the following, we adopt the latter view, so that various equilibrium conditions include the real haircut, which is at the discretion of the central bank.

First consider the situation where bankers face neither a liquidity risk nor a solvency risk. Banking operations, and hence deposits, are safe, so there are no default penalties for bankers and no deposit transfers. Then welfare simply comprises aggregate consumption and potential utility losses due to monitoring. The banker will monitor if and only if the expected productivity gain exceeds the utility loss due to monitoring. Given the financing of banking operations via external funds, such an equilibrium without risk exists if and only if the collateral requirements of the central bank are sufficiently loose, thus not exposing bankers to liqudity risk, and the productivity losses induced by a negative productivity shock are sufficiently small, thus ruling out solvency risk.

Proposition 1 (Equilibrium without Risk)
There exists a unique equilibrium without risk iff

$$\frac{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q}{1 + E/K} \leq \min\{\psi Q, A_{\underline{s}}\},$$

and it yields welfare $W = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K + E)$, *where the monitoring decision is given by* $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q\}$.

Note that, in general, prices follow from the banker's optimality condition, provided in Lemma 4. For example, if bankers face neither a liquidity risk nor a solvency risk, i.e. $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$, it holds that $\mathbb{E}_{\mathbf{m}}[r_s^L] = r_{CB}^D + \overline{m}\kappa$, where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(r_{\overline{s}}^L - r_{\underline{s}}^L) \geq \kappa\}$. As shown in the proof of Proposition 1 (see Appendix A), in equilibrium loan rates are linked to firm productivity, i.e. $A_s = r_s^L Q$, with $s \in \mathcal{S}$. Hence, the banker's optimality condition in nominal terms reads as

$$\frac{\mathbb{E}_{\mathbf{m}}[A_s]}{Q} = \frac{R_{CB}^D}{P} + \overline{m}\kappa,$$

which fully characterizes the prices in our economy. Thus, given a capital good price Q, the consumption good price P is increasing in the central bank rate R_{CB}^D, the monitoring efforts κ (if bankers monitor, i.e. $\overline{m} = 1$), and decreasing in aggregate productivity $\mathbb{E}_{\mathbf{m}}[A_s]$. For the following cases, in which bankers face risk, the price relationships can be derived by the same procedure. In any situation where bankers face liquidity risk, default penalties and hence the nominal haircut Ψ chosen by the central bank will influence the prices in the economy, too.

Second, consider the situation where bankers face a liquidity risk but no solvency risk. Thus, the central bank will adopt tight collateral requirements, so that, in the case of a CBDC-induced bank run, bankers will face a repayment obligation towards the central bank that exceeds their collateral capacity. However, productivity shocks are moderate in this situation, so that bankers not facing a CBDC-induced bank run will remain solvent, even if productivity is low. As in any equilibrium without risk, liquid bankers will monitor if and only if the expected productivity gain exceeds the utility loss due to monitoring. The assets of bankers who become illiquid and default are seized by the central bank, so the respective bankers have no incentive to monitor, ultimately lowering aggregate production output over and against the equilibrium without risk. As liquid banks face no solvency risk, depositors have no incentive to transfer funds to the central bank, so there are no switching costs. Welfare thus comprises aggregate consumption, utility losses due to monitoring, and default penalties.

Proposition 2 (Equilibrium with Liquidity Risk)
There exists a unique equilibrium with liquidity risk iff $\varphi^r = (K + E)/E$, $\phi \in (0, 1)$ *and*

$$\psi Q < \frac{(1 - \mu)(\mathbb{E}_{\overline{m}}[A_s|h = \overline{h}] - \overline{m}\kappa Q)}{1 - \mu + E/K} \leq A_{\underline{s}},$$

and it yields welfare $W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - \mu\phi\epsilon\}(K+E)$, *where the monitoring decision* *is given by* $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q\}$ *and it holds that*

$$\epsilon = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q) - \psi Q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}.$$

Third, consider the situation where bankers face a solvency risk but no liquidity risk. No bank run due to either a household preferring CBDC to deposits or bank insolvency will lead to a default penalty for the banker, because the collateral capacity determined by the central bank suffices to cover any liability towards the central bank emerging from deposit transfers. Solvency risk arises when the productivity losses due to a negative productivity shock are sufficiently large for the revenues from loan financing to be insufficient to meet the liabilities towards the matched household or the central bank. A CBDC-induced bank run does not alter the size of bank liabilities, as the deposit rate and the central bank rate equal without bank default. Hence, the banker's monitoring decision is independent of the type of matched household, or equivalently, the occurrence of a CBDC-induced bank run. Households possessing deposits with insolvent bankers will only transfer their funds to the central bank if a bail-in leads to a higher utility loss than transferring their funds to the central bank and incurring switching costs. Hence, whenever the switching costs ν are lower than a critical level ν^*, households possessing deposits with insolvent bankers will transfer their funds to the central bank. Since bankers face no illiquidity, utilitarian welfare comprises aggregate consumption, utility losses on the part of bankers due to monitoring, and switching costs on the part of depositors.

Proposition 3 (Equilibrium with Solvency Risk)
There exists a unique equilibrium with solvency risk iff $\varphi^r = (K+E)/E$ *and*

$$A_{\underline{s}} < \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}} + E/K} \leq \psi Q$$

and it yields welfare $W^S = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\nu < \nu^*\}$*, where the monitoring decision is given by* $\underline{m} = \overline{m} = \mathbb{1}\{\triangle A_{\overline{s}} \geq \kappa Q(1+\eta_{\overline{s}|0}K/E)\}$ *and the critical switching cost level satisfies*

$$\nu^* = \left(\frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}} + E/K} - A_{\underline{s}}\right)(K+E).$$

Finally, we consider the situation where bankers face both liquidity risk and solvency risk. Negative productivity shocks lead to low loan repayments, which are insufficient for the banker to meet the obligations vis-à-vis the matched household or the central bank. In addition, the central bank imposes tight collateral requirements, such that, if the banker is exposed to a bank run, the resulting liability towards the central bank will exceed collateral capacity. Bankers then default due either to illiquidity or insolvency. Due to switching costs on the part of depositors, bank insolvency does not necessarily trigger a bank run. Only if the switching costs are sufficiently low will households possessing accounts with insolvent bankers shift their funds to the central bank. Thus if switching costs are sufficiently low the banker will incur the same default penalty in the case of insolvency as in the case of illiquidity.

Compared to the situation where bankers face only a solvency risk, the mass of defaulting bankers will increase due to illiquidity after a CBDC-induced bank run. As illiquid bankers do not monitor, the mass of bankers potentially monitoring will decrease over and against the situation where bankers only face a solvency risk. With tight collateral requirements, bankers will face not only a liquidity risk but also default penalties that bankers incur in the case of illiquidity or insolvency. While bankers cannot influence the likelihood of a CBDC-induced bank run, they can monitor borrowers in order to increase the likelihood of a positive productivity shock and ultimately decrease the likelihood of bank insolvency. If depositors switch to the central bank in the case of bank insolvency, monitoring will decrease the expected default penalties. Thus, tight collateral requirements can incentivize bankers to start monitoring.

The following proposition characterizes the equilibrium with both liquidity risk and solvency risk, where depositors accept a bail-in if the respective banker becomes insolvent. Thus, tight collateral requirements do not lead to default penalties in the case of bank insolvency and therefore only indirectly affect the monitoring incentives, as default penalties also influence prices in the economy. Utilitarian welfare comprises aggregate consumption, potential utility losses due to monitoring, and default penalties, but not switching costs.

Proposition 4 (Equilibrium with Liquidity and Solvency Risk and with Bail-in) *There exists a unique equilibrium with liquidity and solvency risk and with bail-in iff* $\varphi^r = (K+E)/E$, $\mu\phi < \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$, *and*

$$\max\left\{A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \psi Q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

$$\leq A_{\underline{s}} + \frac{\nu}{K+E} + \frac{\mu\phi(A_{\underline{s}} + \nu/(K+E) - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

and it yields welfare $W_B^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - \mu\phi\epsilon\}(K+E)$, *where the monitoring decision is given by* $\underline{m} = 0$ *and* $\overline{m} = 1$ *iff*

$$\triangle A_{\overline{s}} - \frac{\triangle\mu\phi\psi Q}{\mu\phi + E/K} \geq \kappa Q\left[1 + \frac{(1-\mu)\eta_{\overline{s}|0}}{\mu\phi + E/K}\right],$$

and it holds that

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}.$$

The following proposition describes the equilibrium with liquidity risk and solvency risk and with deposit transfers of households in the case of bank insolvency. Thus, switching costs are sufficiently low for depositors to prefer switching to the central bank rather than keeping deposits with an insolvent banker and accepting a bail-in. As a consequence, tight collateral requirements lead to default penalties in the case of bank insolvency and directly affect the monitoring incentives of bankers. The monitoring decision depends, as before, on the expected productivity gain and the utility losses due to monitoring, but now also include the expected reduction of default penalties, as monitoring reduces the likelihood of bank insolvency. Utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring and default penalties, and switching costs on the part of depositors.

Proposition 5 (Equilibrium with Liquidity and Solvency Risk and without Bail-in) *There exists a unique equilibrium with liquidity and solvency risk and without bail-in iff* $\varphi^r = (K+E)/E$, $\phi \in (0,1)$ *and*

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} + \frac{\nu}{K+E} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \psi Q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

and it yields welfare $W^{LS}_{NB} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\epsilon\}(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu$, *where the monitoring decision is given by* $\underline{m} = 0$ *and* $\overline{m} = 1$ *iff*

$$\triangle A_{\overline{s}} - \triangle\phi\psi Q \frac{1+E/K}{\phi + E/K} \geq \kappa Q\left[1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right],$$

and it holds that

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}.$$

5 Optimal Monetary Policy

The central bank aims at maximizing utilitarian welfare in the economy by setting the nominal central bank rate $R^D_{CB} > 0$, the nominal haircut $\Psi \geq 0$, and the penalty parameter $\phi > 0$. The central bank determines the collateral capacity of bankers and thus decides on bankers' exposure to liquidity risk and default penalties. As the latter can influence bankers' monitoring decisions, the central bank can use its collateral framework to improve monitoring activities in the economy.

5.1 Necessary conditions for tight collateral requirements

In our model, bankers are exposed to two types of risk. Bankers may experience (a) a CBDC-induced bank run leading to illiquidity if the central bank sets tight collateral requirements, and (b) low loan repayment as a consequence of a negative productivity shock for the financed firm leading to insolvency if leverage is sufficiently high. The monitoring of bankers can only influence the likelihood of a positive productivity shock for the financed firm, but not the likelihood of a CBDC-induced bank run. Thus, default penalties can only influence bankers' monitoring decisions if there is a solvency risk. If, independently of the productivity shock for the financed firm, a banker is able to service the liabilities towards the matched household or the central bank it will never be optimal to apply tight collateral requirements. In such a case, tight collateral requirements would expose bankers to default penalties and potentially even reduce monitoring activities if some banks become illiquid, but they would have no positive effects. Hence, without solvency risk, tight collateral requirements lead to welfare loss due to default penalties for bankers and, if liquid bankers monitor, due to lower aggregate production.

But with solvency risk and sufficiently low switching costs, tight collateral requirements can improve bankers' monitoring activities, as bank insolvency triggers a bank run and ultimately exposes the respective banker to a default penalty. The likelihood of insolvency induced by a negative productivity shock for the financed firm can be reduced through monitoring. Thus, tight collateral requirements prevent bankers from shirking. However, since tight collateral requirements lead to illiquidity following a CBDC-induced bank run, there are also negative consequences, particularly utility losses on the part of bankers due to default penalties. On that account, tight collateral requirements are only optimal if the aggregate productivity gains resulting from the improved monitoring activities of liquid bankers are sufficient to offset the monitoring efforts and the default penalties of illiquid and insolvent bankers, as stated in the following proposition.

Proposition 6 (Optimal Monetary Policy)
Tight collateral requirements, i.e. $R_{CB}^D K > \Psi(K+E) \geq 0$ *and* $\phi > 0$*, are optimal, if bankers shirk with loose collateral requirements, i.e.* $\triangle A_{\overline{s}} < \kappa Q(1+\eta_{\overline{s}|0}K/E)$*, if tight collateral requirements incentivize bankers to monitor, i.e. there exists* $R_{CB}^D > 0$*,* $\Psi \geq 0$ *and* $\phi \in (0,1)$*, such that*

$$\triangle A_{\overline{s}} - \triangle\phi\psi Q \frac{1+E/K}{\phi+E/K} \geq \kappa Q\left[1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right],$$

if banking with liquidity risk and solvency risk is viable (left hand side of inequality) and if implementing tight collateral requirements is welfare improving (right hand side of inequality), i.e.

$$\underline{\chi}(\phi,\psi) < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K} < \overline{\chi}(\phi,\psi),$$

where

$$\underline{\chi}(\phi,\psi) := \max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}}+\nu/(K+E)-\psi Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}, \psi Q\right\}$$

and

$$\overline{\chi}(\phi,\psi) := \psi Q + \{(1-\mu)[\triangle(A_{\overline{s}}-A_{\underline{s}}) - \kappa Q - (\eta_{\underline{s}|1}-\eta_{\underline{s}|0}\mathbb{1}\{\nu<\nu^*\})\nu/(K+E)]\}$$
$$\times\{[(1-\mu)\eta_{\overline{s}|1}+E/K]^{-1}+[(1-\mu)\eta_{\underline{s}|1}+\mu]^{-1}\phi^{-1}\},$$

with the critical switching cost level $\nu^* = [\eta_{\overline{s}|0}A_{\overline{s}}/(\eta_{\overline{s}|0}+E/K) - A_{\underline{s}}](K+E)$*. Otherwise, loose collateral requirements are optimal, i.e.* $\Psi(K+E) \geq R_{CB}^D K > 0$ *and* $\phi > 0$*.*

5.2 The central bank's optimization problem

In our model, the central bank has to choose between loose and tight collateral requirements. As shown in Proposition 6, the central bank will only choose tight collateral requirements exposing bankers to liquidity risk and default penalties if bankers' monitoring incentives can be improved, banking with liquidity risk and solvency risk is viable, and tight collateral requirements are welfare-improving, i.e. productivity gains following from bankers' improved monitoring incentives offset monitoring effort and default penalties. We can show that when tight collateral requirements are optimal, choosing optimal monetary policy essentially boils down to choosing the default penalty parameter ϕ. The nominal central bank rate R^D_{CB} and the nominal haircut Ψ, both influencing the prices in the economy, are then chosen to ensure that the real haircut ψ satisfies a pre-specified condition, which itself varies with the default penalty parameter ϕ. The details are summarized in the following lemma, which follows directly from Proposition 6.

Lemma 5 (Optimal Monetary Policy)
If tight collateral requirements are optimal (see Proposition 6), the optimal default penalty parameter $\hat{\phi}$ satisfies

$$\hat{\phi} \in \arg\min_{\phi \in (0,1)} \phi \tilde{\epsilon}(\phi) \quad \textit{subject to} \quad \max\{\underline{\gamma}_1(\phi), \underline{\gamma}_2\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K},$$

$$\underline{\gamma}_1(\phi) = A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}, \underline{\gamma}_2 = 0,$$

$$\overline{\gamma}_1 = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \quad \textit{and} \quad \overline{\gamma}_2(\phi) = \frac{\phi + E/K}{1 + E/K}\left[\frac{A_{\overline{s}}}{\phi} - \frac{\kappa Q}{\triangle \phi}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right)\right].$$

The optimal central bank rate $\hat{R}^D_{CB} > 0$ and the optimal haircut $\hat{\Psi} \geq 0$ satisfy

$$\hat{R}^D_{CB} K > \hat{\Psi}(K+E) \geq 0 \quad \textit{and} \quad \hat{\Psi} Q = P(\hat{R}^D_{CB}, \hat{\Psi}, \hat{\phi}) \min\{\overline{\gamma}_1, \overline{\gamma}_2(\hat{\phi})\},$$

where

$$P(\hat{R}^D_{CB}, \hat{\Psi}, \hat{\phi}) = \frac{\hat{R}^D_{CB}\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu][1 + (1-\hat{\phi})/\varphi^r]\} - [(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\hat{\Psi}}{(1-\mu)\left(\eta_{\overline{s}|1} A_{\overline{s}}/Q - \kappa\right)}.$$

5.3 An explicit solution

In the following, we provide sufficient conditions for tight collateral requirements and characterize optimal monetary policy explicitly in the case where negative productivity shocks are extreme and there are no switching costs. Most notably, in such an environment the central bank will choose optimally any positive nominal central bank rate while setting the nominal haircut to zero and using the default penalty parameter to incentivize bankers to monitor. The following corollary provides the details:

Corollary 1 (Optimal Monetary Policy)
Suppose $\kappa Q < \triangle A_{\overline{s}} < \kappa Q(1 + \eta_{\overline{s}|0}K/E)$. *Then, if negative productivity shocks are extreme, i.e.* $A_{\underline{s}} = 0$, *if there are no switching costs, i.e.* $\nu = 0$, *and if the risk exposure of bankers is low, i.e.* $\mu > 0$ *and* $\eta_{\underline{s}|1} > 0$ *are sufficiently small, such that* $\mu\eta_{\overline{s}|1} < \eta_{\overline{s}|0}$ *and*

$$\frac{(1-\mu)\eta_{\overline{s}|0}A_{\overline{s}}}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}},$$

then tight collateral requirements are optimal, and the optimal monetary policy satisfies

$$\hat{R}^D_{CB} > 0, \quad \hat{\Psi} = 0, \quad \text{and} \quad \hat{\phi} = \frac{\kappa Q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa Q)E/K}{\triangle A_{\overline{s}} - \kappa Q[(1-\mu)\eta_{\underline{s}|0} + \mu]}.$$

Note that, with the imposed condition $\triangle A_{\overline{s}} > \kappa Q$, the optimal default penalty parameter decreases with the equity-to-deposits ratio E/K. The higher the equity financing of banks, the more returns from monitoring can be skimmed by bankers and, thus, the higher are the incentives of bankers to monitor. Hence, bankers are bound to be less incentivized through the use of default penalties. The optimal default penalty parameter $\hat{\phi}$ increases with the probability of a positive productivity shock without monitoring, denoted by $\eta_{\overline{s}|0}$. Clearly, the higher the probability of a positive productivity shock without monitoring, the lower the returns from monitoring and, hence, the lower the incentives for bankers to engage in costly monitoring. As a consequence, default penalties required to incentivize bankers to monitor must increase.

5.4 Social planner solutions

As outlined before, through the use of the collateral framework, the central bank can under certain conditions incentivize bankers to monitor and thereby induce a welfare gain. However, it needs to be clarified how utilitarian welfare in a competitive equilibrium with an optimal monetary policy compares to the first-best (second-best) utilitarian welfare achieved by a unconstrained (constrained) social planner. The unconstrained social planner has complete information about agents' activities, and can reallocate the endowments of capital good among households and bankers as well as impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states. The constrained social planner, in turn, has incomplete information about agents' activities and cannot observe bankers' monitoring activities. The constrained social planner can only impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states but not reallocate the endowments of the capital good. The first-best and second-best welfare are analyzed in the presence of loose collateral requirements, i.e. bankers face no liquidity risk and default penalties. Note that welfare in such an environment is maximized if households do not incur switching costs and the welfare gain due to the productivity increase induced by monitoring offsets bankers' utility losses due to monitoring.

When bankers face no risk, households do not incur switching costs and bankers' monitoring decision maximizes welfare, i.e. bankers monitor if and only if the welfare gain due to the productivity increase induced by monitoring $\triangle(A_{\overline{s}} - A_{\underline{s}})(K + E)$ offsets the utility losses due to monitoring $\kappa Q(K + E)$ (see Proposition 1). Thus any competitive equilibrium without a risk for bankers yields the first-best welfare. From Proposition 3 it follows that also any competitive equilibrium with a solvency risk but no liquidity risk for bankers yields the first-best welfare if households accept a bail-in the case of bank insolvency, i.e. switching costs are sufficiently high so that $\nu \geq \nu^*$, with ν^* provided in Proposition 3, and bankers' monitoring decision is welfare-maximizing, i.e. $\triangle A_{\overline{s}} \geq \kappa Q(1+\eta_{\underline{s}|0}K/E)$ if and only if $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$.

Proposition 7 (Competitive Equilibrium without Liquidity Risk)
When bankers face no risk, the competitive equilibrium yields the first-best welfare. Households do not incur switching costs and bankers' monitoring decision maximizes welfare, i.e. bankers monitor if the welfare gain due to the productivity increase induced by monitoring offsets bankers' utility losses due to monitoring, i.e. $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$.

When bankers face a solvency risk, the competitive equilibrium yields the first-best welfare if (a) depositors accept a bail-in in the case of bank insolvency, i.e. switching costs are sufficiently high so that $\nu \geq \nu^*$*, with* ν^* *provided in Proposition 3, and (b) monitoring by bankers maximizes welfare, i.e.* $\triangle A_{\overline{s}} \geq \kappa Q(1+\eta_{\underline{s}|0}K/E)$ *if and only if* $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$.

Note that when bankers face no risk, the competitive equilibrium yields the first-best welfare as there are no switching costs incurred by households and bankers' monitoring decision maximizes welfare. Thus the unconstrained social planner can always achieve the first-best welfare by reallocating households' and bankers' endowments of the capital good, so that bankers face no solvency risk.

Proposition 8 (Unconstrained Social Planner Solution)
The social planner can always achieve the first-best welfare by reallocating the capital good between households and bankers, so that bankers are not exposed to a solvency risk.

We now turn to the equilibrium implemented by a constrained social planner which has perfect but incomplete information about agents' activities and in particular cannot observe bankers' monitoring activities. In contrast to the unconstrained social planner, the constrained social planner can only impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states. The constrained social planner can therefore not eliminate any solvency risk faced by bankers, and the potential switching costs in the case of bank insolvency on the part of households, but use contingent taxes and transfers to ensure that bankers' monitoring decision is welfare-maximizing. On that account, we assume that the constrained social planner imposes (distributes) taxes (transfers) depending on a bank's observed loan returns or, equivalently, the idiosyncratic productivity shock for the financed firm. Thus we denote these taxes (transfers) in real terms by $\tau_s := T_s/P$.

Based on the previous remarks, the constrained social planner does not need to apply any taxes or transfers if bankers do not face a risk, since the competitive equilibrium yields the first-best welfare. Similarly, the constrained social planner does not need to become active if bankers face a solvency risk, but households accept a bail-in in the case of bank insolvency and bankers' monitoring decision is welfare-maximizing as stated in Proposition 7. Note that in any environment with a solvency risk for bankers, inefficiencies compared to the first-best welfare can arise for two reasons: Either because households' switching costs are sufficiently low so that they convert deposits in the case of bank insolvency and thus incur utility losses or because bankers' monitoring decision is not welfare-maximizing. The constrained social planner can, in contrast to the unconstrained social planner, not eliminate solvency risk for bankers and thus not avoid households incurring switching costs. However, the constrained social planner can use the contingent taxes and transfers to align bankers' monitoring incentives with the objective of maximizing utilitarian welfare. With contingent taxes and transfers, bankers' optimization problem in real terms is given by

$$\max_{\substack{\varphi\in[1,\varphi^r],\\ m(h)\in\{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} r_{\mathbf{z}}^{E,+} - r_{\mathbf{z}}^{E,-} - m(h)\kappa\varphi + \tau_s\varphi]QE.$$

Following the proof of Lemma 4, in the presence of solvency risk bankers' monitoring decision reads as $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[r_{\overline{s}}^L - r_{CB}^D(\varphi^r - 1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}}\}$, where we have assumed, without loss of generality, $\tau_{\underline{s}} = 0$. It is irrelevant whether the constrained social planner distributes transfers to bankers that monitor, imposes taxes on bankers that do not monitor or both. Following the proof of Proposition 3, we can state that in equilibrium bankers' monitoring decision is given by

$$\underline{m} = \overline{m} = \mathbb{1}\{\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\underline{s}|0}K/E) - \triangle\tau_{\overline{s}}Q\}.$$

The aim of the constrained social planner is then to choose $\tau_{\overline{s}}$, so that bankers' monitoring activity equals the welfare-maximizing monitoring activity $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\overline{s}}) \geq \kappa Q\}$. The details are stated in Proposition 9. If switching costs are small so that $\nu < \nu^*$, with ν^* provided in Proposition 3, the constrained social planner can only implement the second-best welfare, as households convert deposits into CBDC in the case of bank insolvency and thus incur switching costs.

Proposition 9 (Constrained Social Planner Solution)
When bankers face a solvency risk, the constrained social planner maximizes welfare by applying the contingent taxes and transfers of the form $\tau_{\underline{s}} = 0$ *and*

$$\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0}K/E)/\triangle - A_{\overline{s}}/Q, 0\}.$$

If switching costs are sufficiently high, so that households accept a bail-in in the case of bank insolvency, i.e. $\nu \geq \nu^*$*, with* ν^* *provided in Proposition 3, the constrained social planner can achieve the first-best welfare. Otherwise, the constrained social planner can only achieve the second-best welfare, as solvency risk for bankers and the resulting switching costs for households cannot be eliminated.*

The question which remains to be answered is how utilitarian welfare in a competitive equilibrium with a central bank that aims at maximizing welfare, through the use of its collateral framework, compares to the first-best and second-best welfare. First, note that the central bank can, under certain circumstances, incentivize bankers to monitor, when exposing them to liquidity risk and default penalties. While monitoring leads to a welfare gain through the induced productivity increase, the imposed default penalties and the lost monitoring activities by illiquid bankers yield a welfare loss. On that account, the central bank can in general only implement a third-best welfare, as stated in the following Proposition:

Proposition 10 (Competitive Equilibrium with Liquidity Risk)
Suppose bankers face a solvency risk and switching costs are sufficiently low, so that households convert deposits into CBDC in the case of bank insolvency, i.e. $\nu < \nu^*$ with ν^* provided in Proposition 3. If it is optimal for the central bank to apply tight collateral requirements (for the necessary conditions see Proposition 6), the resulting welfare is in general only third-best and the welfare loss compared to the second-best welfare is given by

$$-\mu[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa Q](K+E) - [(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})(K+E),$$

where $\hat{\phi}$ follows from Lemma 5. If $\mu \to 0$ and $\eta_{\underline{s},1} \to 0$, utilitarian welfare in a competitive equilibrium with tight collateral requirements as optimal monetary policy approaches the second-best welfare and, with negligible switching costs, i.e. $\nu \to 0$, the first-best welfare.

6 Comparison with Today's Monetary System

In today's monetary system, bank deposits are the predominant form of money. They are often insured, for instance by governmental guarantees. Thus, in the case of bank insolvency, depositors generally do not have to convert deposits into cash or into any other safe asset. Nor do bankers face penalties in the case of their bank defaulting and claims being made on the deposit insurance. A monetary system with CBDC as the only legal tender and no deposit insurance scheme is equivalent to today's monetary system in terms of the real allocation in the economy, if there are no switching costs associated with converting deposits into CBDC and bankers do not face penalties in the case of default. Hence, within our framework we can replicate the real allocation emerging in today's monetary system by setting switching costs to zero, i.e. $\nu = 0$, and by focusing on loose collateral requirements, i.e. $\Psi(K+E) \geq R^D_{CB}K$.

As outlined in Section 5, introducing a central bank digital currency and abolishing deposit insurances while establishing the unrestricted right of converting deposits into CBDC may enable the central bank, through the use of its collateral framework, to improve bankers' monitoring incentives. However, this effect of monetary policy only exists in the presence of solvency risk. Without solvency risk, households holding deposits will never shift their funds to the central bank, so there are no switching costs, and the alternative system yields the same welfare as today's monetary system. The same result applies if bankers face a solvency risk but households face sufficiently high switching costs to ensure that, in the case of bank insolvency, they will accept a bail-in and not transfer funds to the central bank.

Finally, consider the situation where bankers face a solvency risk and switching costs are sufficiently low for households holding deposits with insolvent bankers not to accept a bail-in and thus to shift their funds to the central bank. If loose collateral requirements ruling out liquidity risk and default penalties for bankers, are optimal, the alternative monetary system will yield a welfare loss compared to today's monetary system due to positive switching costs on the part of depositors. In the extreme case where there are no switching costs, the alternative system with loose collateral requirements will yield the same welfare as today's monetary system. When tight collateral requirements are optimal, i.e. when bankers' monitoring activities can be improved through default penalties and the resulting productivity gains offset utility losses due to monitoring efforts and default penalties, the institutional changes will lead, with sufficiently low switching costs, to a welfare gain over and against today's monetary system. Hence, introducing an interest-bearing central bank digital currency, as a medium of exchange equivalent to bank deposits, abolishing deposit insurances and establishing the unrestricted right of converting deposits into CBDC will only entail welfare losses if bankers face a solvency risk and bankers' monitoring incentives cannot be improved through tight collateral requirements. The previous observations are summarized in the following proposition:

Proposition 11 (Comparison with Today's Monetary System)
Without solvency risk or with solvency risk and bail-ins, a CBDC will never entail welfare losses compared with today's monetary system. With solvency risk and no bail-ins, a CBDC will lead to a welfare gain compared with today's monetary system if tight collateral requirements are optimal and switching costs are sufficiently low; otherwise, a CBDC will entail a welfare loss due to positive switching costs on the part of depositors.

7 A Dynamic Perspective

We now consider a dynamic version of our model with discrete time, denoted by $t \in \mathbb{N}_0$. In particular, we focus on an endowment economy where households and bankers do not save and receive the same endowment $K > 0$ and $E > 0$, respectively, at the beginning of each period. Each period can be separated into the three stages of our static framework. Moreover, we focus on the particular case of sufficiently small switching costs ν, where bank insolvency will trigger a bank run. We use this simple setup to illustrate the fundamental forces at work.

First note that, as stated in the following proposition, the mass of households possessing accounts with the central bank only changes over time if bankers face a solvency risk. In the case of insolvency, a household possessing deposits with the respective banker will transfer the funds to the central bank. Due to positive switching costs and the fact that deposit rates never exceed the central bank rate, households, once they have opened an account with the central bank, continue to hold CBDC. Thus, with solvency risk, the mass of households holding CBDC will increase over time. Without solvency risk, the mass of households possessing an account with the central bank will remain constant over time.

Proposition 12 (Households with Central Bank Accounts)
The mass of households possessing an account with the central bank evolves according to $\mu_{t+1} = \mu_0$ without solvency risk, and according to $\mu_{t+1} = (1 - \mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$ with solvency risk.

In our model, the mass of households possessing an account with the central bank is closely connected to the mass of defaulting bankers, as outlined in the following proposition:

Proposition 13 (Bank Default)
The mass of defaulting bankers is given by $\sigma_t = \mu_0$ *if only liquidity risk is present,* $\sigma_t = \eta_{\underline{s}|\overline{m}}$ *if only solvency risk is present, and* $\sigma_t = (1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$ *if both liquidity risk and solvency risk are present.*

From Proposition 5 we can infer that an equilibrium with liquidity risk, following from tight collateral requirements, solvency risk, and no bail-ins, can at most exist for a finite period of time. Specifically, note that there exists no sequence $\{\psi_t\}_{t\in\mathbb{N}_0}$ such that for all $t \in \mathbb{N}_0$

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t]\phi(A_{\underline{s}} + \frac{\nu}{K+E} - \psi_t Q)}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K}, \psi_t Q\right\} < \frac{(1-\mu_t)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K},$$

where $\mu_{t+1} = (1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$. With solvency risk, the mass of households possessing accounts with the central bank converges to one, i.e. $\lim_{t\to\infty}\mu_t = 1$, such that the right-hand side approaches zero while the left-hand side remains positive for any $\psi_t \geq 0$. Hence, with constant endowments of households and bankers, tight collateral requirements can only be maintained for a finite period of time. After this period, tight collateral requirements would render banking non-viable in our economy. As a consequence, the central bank can only use its collateral framework to improve monitoring activities by bankers for a finite period of time without rendering banking non-viable. We summarize this observation in the following proposition:

Proposition 14 (Viability of Banking)
Suppose bankers face both solvency risk and liquidity risk, i.e. the central bank applies tight collateral requirements. Then there exists a period $\tilde{t} \in \mathbb{N}_0$ *subsequent to which banking will be non-viable.*

The corollary implies that the central bank faces a dilemma over time when it introduces a central bank digital currency. To induce monitoring by bankers, tight collateral requirements would be needed, but at some point this renders banking non-viable since bankers face a growing liquidity risk that reduces their chances of earning sufficient returns on their endowments in the good state and of offsetting utility losses when they default. As a consequence, the central bank will optimally choose loose collateral requirements and stop punishing default by banks, so that monitoring ceases.

8 Discussion and Conclusion

While a CBDC may entail various benefits for society, such as financial inclusion or higher deposit rates resulting from increased competition among banks, they also entail risks for the banking system, potentially impairing the viability of banking or causing financial instability. Thus, the integration of a CBDC into our current monetary system poses several challenges to policymakers, and the economic consequences of such a new form of national currency are still unclear.

We examine how the introduction of an interest-bearing central bank digital currency (CBDC) impacts bank activities and monetary policy. As depositors can switch from bank deposits to CBDC as a safe medium of exchange at any time, banks face digital runs, either because depositors have a preference for CBDC or because they fear bank insolvency. By setting appropriate collateral requirements (and default penalties) optimal monetary policy can initially increase monitoring incentives for bankers, which leads to higher aggregate productivity. We provide necessary conditions for the optimality of tight collateral requirements and characterize the optimal monetary policy explicitly under specific assumptions on firm productivity and switching costs.

As the mass of households holding CBDC increases, monetary policy with tight collateral requirements generating liquidity risk for banks and exposing bankers to default penalties would after some time render banking non-viable, thus prompting the central bank to deviate from these policies. Under these circumstances, monitoring incentives will revert to low levels. Hence, the central bank faces a dilemma when introducing a central bank digital currency. While in the short term tight collateral requirements can be used to incentivize bankers to monitor, in the long term they will endanger the viability of banking. Introducing a central bank digital currency therefore involves risks for the entire banking system. Since banks' liquidity demand is likely to rise with a CBDC, the rules for liquidity provisions by the central bank, including the collateral framework, come to the fore.

We also compare this alternative monetary system (CBDC and no deposit insurance) with the current monetary system, where bank deposits are the principal form of money, often insured by such things as governmental guarantees. Most notably, without switching costs and with an optimal monetary policy, a CBDC will never entail welfare losses over and against today's monetary system. However, it may enable the central bank, through the use of its collateral framework, to improve the monitoring incentives for bankers and ultimately to increase welfare.

We compare welfare in a competitive equilibrium with welfare achieved by an unconstrained and constrained social planner. The unconstrained social planner has complete information about agents' activities. Any competitive equilibrium without solvency risk and with loose collateral requirements representing the optimal monetary policy yields the first-best welfare. By reallocating endowments between agents the unconstrained social planner can achieve the first-best welfare as solvency risk for bankers is ruled out, which guarantees a welfare-maximizing monitoring decision by bankers and avoids switching costs incurred by depositors in the case of bank insolvency. The constrained social planner has limited information about agents' activities and is restricted to taxes and transfers contingent on idiosyncratic states. In contrast to the unconstrained social planner, the constrained social planner can only achieve the second-best welfare: Bankers' monitoring decision can be aligned with the objective of maximizing welfare but solvency risk for bankers and thus switching costs incurred by depositors in the case of bank insolvency cannot be eliminated. Any competitive equilibrium with solvency risk and tight collateral requirements representing the optimal monetary policy, i.e. liquidity risk and default penalties for bankers, yields welfare which is generally lower than the second-best welfare due to default penalties imposed on bankers and lost monitoring activities by illiquid bankers.

Several features of our model can be studied in greater detail and are of particular interest when further analyzing the economic consequences of central bank digital currencies. First, we have focused on a simple dynamic version of our model to illustrate the fundamental forces at work, avoiding in particular capital accumulation by households and bankers. Modeling and analyzing a dynamic version of our framework, which allows for capital accumulation by agents, would help to draw a complete picture of the long-term effects following the introduction of a central bank digital currency. Second, we focused on a particular institutional rule that enables agents to convert bank deposits into CBDC any time, specifically without the consent of the respective banker or the central bank. While this institutional setup enables the central bank to expose bankers to default penalties and ultimately to improve bankers' monitoring incentives, a comparison with other institutional setups has yet to be made. On this account, an in-depth study of various institutional rules accompanying the introduction of a central bank digital currency is urgently required. Third, our framework abstracts from the interbank market, which may however allow individual banks, whose solvency is not questioned, facing CBDC-induced bank runs to avoid illiquidity by borrowing from other banks. Whereas in the present paper we only provide an intuition of the impact of the interbank market, a more analytical analysis may be valuable.

References

Andolfatto, D. (2018). Assessing the impact of central bank digital currency on private banks. *Federal Reserve Bank of St. Louis Working Paper*, 026C.

Auer, R. and Böhme, R. (2020). The technology of retail central bank digital currency. *Bank for International Settlements Quarterly Review*, March 2020:85–100.

Barontini, C. and Holden, H. (2019). Proceeding with caution-a survey on central bank digital currency. *Bank for International Settlements Paper*, 101.

Barrdear, J. and Kumhof, M. (2016). The macroeconomics of central bank issued digital currencies. *Bank of England Staff Working Paper*, 605.

Bech, M. L. and Garratt, R. (2017). Central bank cryptocurrencies. *Bank for International Settlements Quarterly Review*, September 2017:55–70.

Benigno, P. and Robatto, R. (2019). Private money creation, liquidity crises, and government interventions. *Journal of Monetary Economics*, 106:42–58.

Berentsen, A. and Schär, F. (2018). The case for central bank electronic money and the non-case for central bank cryptocurrencies. *Federal Reserve Bank of St. Louis Review*, 100(2):97–106.

Bindseil, U. (2004). *Monetary policy implementation: Theory, past, and present.* Oxford University Press on Demand.

Bindseil, U. (2019). Central bank digital currency: Financial system implications and control. *International Journal of Political Economy*, 48(4):303–335.

Bindseil, U., Corsi, M., Sahel, B., and Visser, A. (2017). The Eurosystem collateral framework explained. *European Central Bank Occasional Paper*, 189.

Böhme, R. (2019). The technology behind CBDCs: State of the art, design options, and implications. *Presentation at the BIS-OeNB Workshop on Digital Currencies, Central Banks and the Blockchain: Policy Implications*, Vienna, 25 October, 2019, available at: `https://www.oenb.at/dam/jcr:0b5d72a7-e154-4495-8084-098f470325ff/BOEHME.pdf`, accessed 24 February, 2020.

Bordo, M. D. and Levin, A. T. (2017). Central bank digital currency and the future of monetary policy. *National Bureau of Economic Research Working Paper*, 23711.

Brown, M., Hentschel, N., Mettler, H., and Stix, H. (2020). Financial innovation, payment choice and cash demand - causal evidence from the staggered introduction of contactless debit cards. *SSRN*, April 2020, available at `https://ssrn.com/abstract=3582388` or `http://dx.doi.org/10.2139/ssrn.3582388`,accessed 15 June, 2020.

Brunnermeier, M. K. and Niepelt, D. (2019). On the equivalence of public and private money. *Journal of Monetary Economics*, 106:27–41.

Calomiris, C. W. and Kahn, C. M. (1991). The role of demandable debt in structuring optimal banking arrangements. *The American Economic Review*, 81(3):497–513.

Cecchetti, S. G. and Schoenholtz, K. (2018). Universal central bank digital currency? *Blogpost, Money & Banking*, 22 April, 2018.

Chiu, J., Davoodalhosseini, S. M., Hua Jiang, J., and Zhu, Y. (2019). Central bank digital currency and banking. *Bank of Canada Staff Working Paper*, 20.

Engert, W., Fung, B. S., et al. (2017). Central bank digital currency: Motivations and implications. *Bank of Canada Staff Working Paper*, 16.

Faure, S. and Gersbach, H. (2017). Money creation and destruction. *CESifo Working Paper*, 6565.

Faure, S. and Gersbach, H. (2018). Money creation in different architectures. *CEPR Discussion Paper*, 13156.

Gersbach, H. and Rochet, J.-C. (2017). Capital regulation and credit fluctuations. *Journal of Monetary Economics*, 90:113–124.

Hicks, J. R. (1967). *Critical Essays in Monetary Theory.* Clarendon Press, Oxford.

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics*, 112(3):663–691.

Kahn, C. M., Rivadeneyra, F., and Wong, T.-N. (2018). Should the central bank issue e-money? *Bank of Canada Staff Working Paper*, 58.

Keister, T. and Sanches, D. (2019). Should central banks issue digital currency? *Federal Reserve Bank of Philadelphia Working Paper*, 26.

Kumhof, M. and Noone, C. (2018). Central bank digital currencies—design principles and balance sheet implications. *Bank of England Staff Working Paper*, 725.

Pichler, P., Summer, M., and Weber, B. (2020). Does digitalization require central bank digital currencies for the general public? *Monetary Policy & the Economy*, 2019(Q4):40–56.

Ricks, M., Crawford, J., and Menand, L. (2018). A public option for bank accounts (or central banking for all). *UC Hastings Research Paper*, 287.

A Proofs

Proof of Lemma 1. Note that the initial CBDC holdings satisfy $D^h_{CB} = QK\mathbb{1}\{h = \underline{h}\}$. Thus, the household that initially opens an account with a banker ($h = \overline{h}$) transfers deposits to the central bank iff the excess returns on CBDC suffice to offset the switching costs, that is $\nu < QK(r^D_{CB} - r^D_{\mathbf{z}})$. Using the notation $\tilde{\nu} := \nu/(QK)$, the latter condition translates into $r^D_{\mathbf{z}} < r^D_{CB} - \tilde{\nu}$. Similarly, the household that initially opens an account with the central bank ($h = \underline{h}$) will keep the funds at the central bank iff $QK(r^D_{\mathbf{z}} - r^D_{CB}) \leq \nu$ or, equivalently, $r^D_{\mathbf{z}} \leq r^D_{CB} + \tilde{\nu}$. ■

Proof of Lemma 2. Suppose $A_s > r^L_s Q$ for some state $s \in \mathcal{S}$. As the firm operates with limited liability, its expected profits grow with the amount of capital good K^f, starting at some critical level. Thus, there exists no optimal finite demand for capital good. We denote this case by $K^f = +\infty$. Instead, if $A_s \leq r^L_s Q$ for all states $s \in \mathcal{S}$, the firm will generate zero profits for any production input in each state and is thus indifferent in its demand for capital good $K^f \in [0, +\infty)$. ■

Proof of Lemma 3. Note that deposit flows are matched by reserve flows and reserves are credited with the real interest rate r^D_{CB}. To rule out arbitrage opportunities and thus excess returns for the banker, the deposit rate must satisfy $r^D_{\mathbf{z}} = r^D_{CB}$ for any state $\mathbf{z} \in \mathcal{Z}$ where the banker does not default. Suppose $r^D_{\mathbf{z}} > r^D_{CB}$. Then the banker benefits by setting a deposit rate $\tilde{r}^D_{\mathbf{z}} < r^D_{\mathbf{z}}$, as it would not be matched with any household, but finances loans with equity and central bank loans, where the latter are subject to the repayment rate $r^D_{CB} < r^D_{\mathbf{z}}$.

Similarly, suppose $r^D_{CB} > r^D_{\mathbf{z}}$. Then the banker profits from setting a deposit rate $\tilde{r}^D_{\mathbf{z}} < r^D_{CB}$ and $\tilde{r}^D_{\mathbf{z}} > r^D_{\mathbf{z}}$, as all households of type $h = \overline{h}$ would initially open an account with this banker. The latter then generates riskless profits due to the interest rate spread $r^D_{CB} - \tilde{r}^D_{\mathbf{z}} > 0$. ■

Proof Lemma 4. We address each case separately. (i) Consider the situation where the banker faces neither liquidity risk nor solvency risk, i.e. $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$. Then the banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\mathbb{E}_1[(r^L_s - r^D_{CB})\varphi + r^D_{CB}|h]QE \geq \mathbb{E}_0[(r^L_s - r^D_{CB})\varphi + r^D_{CB}|h]QE + \kappa\varphi QE$$

or, equivalently, $\mathbb{E}_1[r^L_s|h] - \mathbb{E}_0[r^L_s|h] \geq \kappa$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, we can state that the banker will monitor independently of the type of household, i.e. $\underline{m} = \overline{m} = 1$, iff $\triangle(r^L_{\overline{s}} - r^L_{\underline{s}}) \geq \kappa$. The banker's expected utility from conducting banking operations with a leverage $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ is given by $\mathbb{E}_{\mathbf{m}}[(r^L_s - r^D_{CB})\varphi + r^D_{CB} - m(h)\kappa\varphi]QE$. Due to competitive markets, the utility expected from banking must equal the utility from holding CBDC, i.e. $r^D_{CB}QE$. Using the fact that $\underline{m} = \overline{m}$, the banker will thus only choose a leverage ratio $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ if $\mathbb{E}_{\mathbf{m}}[r^L_s] = r^D_{CB} + \overline{m}\kappa$.

(ii) Consider the situation where the banker faces a liquidity risk but no solvency risk, i.e. $\varphi^L < \varphi \leq \varphi^S$. The banker will monitor iff the matched household opens an account with the banker, as otherwise the banker will become illiquid and defaults. Thus it holds that $\underline{m} = 0$.

For $h = \overline{h}$, the banker will monitor, i.e. $\overline{m} = 1$, iff

$$\mathbb{E}_1[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D | h = \overline{h}]QE \geq \mathbb{E}_0[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D | h = \overline{h}]QE + \kappa\varphi QE$$

or, equivalently, $\mathbb{E}_1[r_s^L | h = \overline{h}] - \mathbb{E}_0[r_s^L | h = \overline{h}] \geq \kappa$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, we can state that the banker will monitor iff $h = \overline{h}$ and $\triangle(r_{\overline{s}}^L - r_{\underline{s}}^L) \geq \kappa$. Using the fact that $\underline{m} = 0$, we know that the utility expected from conducting banking operations is given by

$$\{(1-\mu)\mathbb{E}_{\overline{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D | h = \overline{h}] - (1-\mu)\overline{m}\kappa\varphi - \mu\phi[(r_{CB}^D - \psi)\varphi - r_{CB}^D]\}QE.$$

Due to competitive markets, the latter must equal the utility from holding CBDC, i.e $r_{CB}^D QE$. Thus, the banker will only choose $\varphi^L < \varphi \leq \varphi^S$ with $\varphi < \varphi^r$ if

$$\mathbb{E}_{\overline{m}}[r_s^L | h = \overline{h}] = r_{CB}^D \left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi}\right) + \frac{\mu\phi}{1-\mu}\left(r_{CB}^D \frac{\varphi - 1}{\varphi} - \psi\right) + \overline{m}\kappa,$$

and there is no incentive for the banker to adjust the supply of loans, i.e.

$$(1-\mu)\mathbb{E}_{\overline{m}}[r_s^L - r_{CB}^D - \overline{m}\kappa | h = \overline{h}] - \mu\phi(r_{CB}^D - \psi) = 0,$$

which, however, contradicts the former equation. Hence, the banker will only choose a leverage $\varphi^L < \varphi \leq \varphi^S$ if $\varphi = \varphi^r$,

$$\mathbb{E}_{\overline{m}}[r_s^L | h = \overline{h}] = r_{CB}^D \left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(r_{CB}^D \frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \overline{m}\kappa$$

and

$$(1-\mu)\mathbb{E}_{\overline{m}}[r_s^L - r_{CB}^D - \overline{m}\kappa | h = \overline{h}] - \mu\phi(r_{CB}^D - \psi) > 0,$$

which with the previous equation translates into $\phi < 1$.

(iii) Consider the situation where the banker faces a solvency risk but no liquidity risk, i.e. $\varphi^S < \varphi \leq \varphi^L$. The banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\eta_{\overline{s}|1}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D]QE \geq \eta_{\overline{s}|0}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D]QE + \kappa\varphi QE,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, can be rewritten as $\triangle[r_{\overline{s}}^L - r_{CB}^D(\varphi - 1)/\varphi] \geq \kappa$. Using the fact that the banker's monitoring decision does not depend on the type of matched household, i.e. $\underline{m} = \overline{m}$, we know that the banker's expected utility from conducting banking operations is given by $\{\eta_{\overline{s}|\overline{m}}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D] - \overline{m}\kappa\varphi\}QE$. Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e. $r_{CB}^D QE$. Thus, with $\underline{m} = \overline{m}$ we can deduce that the banker will choose $\varphi^S < \varphi \leq \varphi^L$ with $\varphi < \varphi^r$ if

$$r_{\overline{s}}^L = r_{CB}^D \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to adjust the supply of loans, i.e. $\eta_{\overline{s}|\overline{m}}(r_{\overline{s}}^L - r_{CB}^D) - \overline{m}\kappa = 0$, which, however, contradicts the former equation. Hence the banker will only choose a leverage $\varphi^S < \varphi \leq \varphi^L$ if $\varphi = \varphi^r$,

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and $\eta_{\overline{s}|\overline{m}}(r^L_{\overline{s}} - r^D_{CB}) - \overline{m}\kappa > 0$, which follows directly from the previous equation.

(iv) Consider the situation where the banker faces both a liquidity risk and a solvency risk, i.e. $\varphi > \max\{\varphi^L, \varphi^S\}$. In the case of bank insolvency, depositors will prefer a bail-in over a transfer of funds to the central bank, i.e. $\varphi \leq \varphi^R$. The banker will monitor iff matched with a household that opens an account with the banker ($h = \overline{h}$), as otherwise the banker will become illiquid and defaults. Thus it holds that $\underline{m} = 0$. In addition, we can state $\overline{m} = 1$ iff $\eta_{\overline{s}|1}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \geq \eta_{\overline{s}|0}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE + \kappa\varphi QE$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, the latter inequality can be rewritten as $\triangle[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \geq \kappa\varphi QE$ or, equivalently, $\triangle[r^L_{\overline{s}} - r^D_{CB}(\varphi - 1)/\varphi] \geq \kappa$. Using the fact that $\underline{m} = 0$ and the fact that default penalties will only arise if the banker is matched with a household that opens an account with the central bank ($h = \underline{h}$) but not in the case of bank insolvency as the matched household prefers a bail-in to a transfer of funds, the banker's expected utility from conducting banking operations is given by

$$\{(1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}] - (1-\mu)\overline{m}\kappa\varphi - \mu\phi[r^D_{CB}(\varphi - 1) - \psi\varphi]\}QE.$$

Due to competitive markets, the expected utility from conducting banking operations must equal the utility from holding CBDC, i.e. $r^D_{CB}QE$. Thus the banker will only choose $\max\{\varphi^L, \varphi^S\} < \varphi$ with $\varphi < \varphi^r$ if

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi - 1}{\varphi} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to adjust the supply of loans, i.e.

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}r^D_{CB} - \overline{m}\kappa) - \mu\phi(r^D_{CB} - \psi) = 0,$$

which, however, contradicts the former equation. Hence the banker will only choose $\max\{\varphi^L, \varphi^S\} < \varphi$ if $\varphi = \varphi^r$,

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and $(1-\mu)(\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}r^D_{CB} - \overline{m}\kappa) - \mu\phi(r^D_{CB} - \psi) > 0$, which, with the previous equation, translates into $\mu\phi < \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$.

(v) Consider the situation where the banker faces both a liquidity risk and a solvency risk, i.e. $\varphi > \max\{\varphi^L, \varphi^S\}$, and in the case of bank insolvency depositors transfer their funds to the central bank, i.e. $\varphi > \varphi^R$. The banker will monitor iff matched with a household that opens an account with the banker ($h = \overline{h}$) as otherwise the banker will become illiquid and default.

Thus it holds that $\underline{m} = 0$. In addition, we can state that $\overline{m} = 1$ iff

$$\eta_{\overline{s}|1}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE - \eta_{\underline{s}|1}\phi[(r^D_{CB} - \psi)\varphi - r^D_{CB}]QE$$
$$\geq \eta_{\overline{s}|0}[(r^L_{\overline{s}} - r^D_{CB})(\varphi - 1) + r^L_{\overline{s}}]QE - \eta_{\underline{s}|0}\phi[(r^D_{CB} - \psi)\varphi - r^D_{CB}]QE + \kappa\varphi QE.$$

Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, the latter inequality can be rewritten as

$$\triangle[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE - \triangle\phi[(\psi - r^D_{CB})\varphi + r^D_{CB}]QE \geq \kappa\varphi QE,$$

or equivalently, $\triangle[r^L_{\overline{s}} - \phi\psi - r^D_{CB}(1-\phi)(\varphi-1)/\varphi] \geq \kappa$. The banker's expected utility from conducting banking operations is given by

$$\{(1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}] - (1-\mu)\overline{m}\kappa\varphi - \phi[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu][r^D_{CB}(\varphi - 1) - \psi\varphi]\}QE.$$

Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e. $r^D_{CB}QE$. Thus the banker will only choose $\max\{\varphi^L, \varphi^S, \varphi^R\} < \varphi$ with $\varphi < \varphi^r$ if

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi - 1}{\varphi} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

and there is no incentive to adjust the supply of loans, i.e.

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}r^D_{CB} - \overline{m}\kappa) - \phi[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](r^D_{CB} - \psi) = 0,$$

which, however, contradicts the former equation. Thus the banker will only choose $\max\{\varphi^L, \varphi^S, \varphi^R\} < \varphi$ if $\varphi = \varphi^r$,

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and $(1-\mu)(\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}r^D_{CB} - \overline{m}\kappa) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(r^D_{CB} - \psi) > 0$, which, with the previous equation, translates into $\phi < 1$. So far, we have established the conditions for the banker's choice of leverage and monitoring. Since the previous conditions are mutually exclusive, these conditions are necessary and sufficient. ■

Proof of Proposition 1. Consider the situation where bankers face neither a liquidity risk nor a solvency risk, i.e. $\varphi \leq \min\{\varphi^L, \varphi^S\}$. From Lemma 4 we know that the banker will choose leverage $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ iff

$$\mathbb{E}_{\mathbf{m}}[r^L_s] = r^D_{CB} + \overline{m}\kappa. \tag{A.1}$$

Furthermore, the banker's monitoring decision is independent of the type of household and given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(r^L_{\overline{s}} - r^L_{\underline{s}}) \geq \kappa\}$. As banks are not defaulting, the central bank makes zero profits, i.e. $\pi^{CB} = 0$, where $\pi^{CB} := \Pi^{CB}/P$ denotes central bank profits in terms of the consumption good. Moreover, in equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r_s^L Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = r_s^L Q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e. $\pi^f = 0$. Since the central bank and firms make zero profits, there are no taxes and transfers, i.e. $\tau = 0$, where $\tau := T/P$ denotes taxes and transfers in terms of the consumption good.

Without taxes and transfers and zero firm profits, the expected consumption of the banker and the household is given by $C^b = \mathbb{E}_{\mathbf{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D]QE$ and $C^h = r_{CB}^D QK$, with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) = \mathbb{E}_{\mathbf{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D]QE + r_{CB}^D QK,$$

which, using the equilibrium leverage $\varphi = (K+E)/E$, translates into

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) = \mathbb{E}_{\mathbf{m}}[r_s^L]Q(K+E)$$

and finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r_s^L]Q$, which is satisfied as $A_s = r_s^L Q$ for all $s \in \mathcal{S}$. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q\}$.

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption and potential utility losses on the part of bankers due to monitoring. Note that, if the banker faces no risk, there will be no default penalties for bankers and no switching costs on the part of depositors. Using the fact that $\underline{m} = \overline{m}$, utilitarian welfare is given by

$$W = \mathbb{E}_{\mathbf{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D]QE - \overline{m}\kappa\varphi QE + r_{CB}^D QK.$$

Making use of equilibrium conditions $\varphi = (K+E)/E$ and $A_s = r_s^L Q$, with $s \in \mathcal{S}$, utilitarian welfare further simplifies to $W = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K+E)$.

Liquidity risk and solvency risk are ruled out iff $\varphi \leq \min\{\varphi^L, \varphi^S\}$. Using the definition of φ^L and φ^S, we can state that the banker does not face any risk iff

$$\varphi \leq \min\left\{\frac{r_{CB}^D}{r_{CB}^D - \psi}, \frac{r_{CB}^D}{r_{CB}^D - r_{\underline{s}}^L}\right\}.$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that liquidity risk and solvency risk are ruled out iff

$$\frac{K+E}{E} \leq \min\left\{\frac{r_{CB}^D}{r_{CB}^D - \psi}, \frac{r_{CB}^D}{r_{CB}^D - r_{\underline{s}}^L}\right\} \quad \Leftrightarrow \quad \frac{K}{K+E} \leq \min\left\{\frac{\psi}{r_{CB}^D}, \frac{r_{\underline{s}}^L}{r_{CB}^D}\right\}.$$

Using equilibrium condition $A_s = r_s^L Q$, with $s \in \mathcal{S}$, and the fact that based on (A.1), in equilibrium, the real central bank rate satisfies $r_{CB}^D = \mathbb{E}_{\mathbf{m}}[r_s^L] - \overline{m}\kappa$, we know that the banker will face neither a liquidity risk nor a solvency risk iff

$$\frac{K}{K+E} \leq \frac{\min\{\psi Q, A_{\underline{s}}\}}{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q} \quad \Leftrightarrow \quad \frac{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q}{1 + E/K} \leq \min\{\psi Q, A_{\underline{s}}\}.$$

■

Proof of Proposition 2. Consider the situation where bankers face a liquidity risk but no solvency risk, i.e. $\varphi^L < \varphi \leq \varphi^S$. From Lemma 4 we know that the banker will choose leverage $\varphi^L < \varphi \leq \varphi^S$ iff $\varphi = \varphi^r$, $\phi < 1$ and

$$\mathbb{E}_{\overline{m}}[r_s^L|h = \overline{h}] = r_{CB}^D\left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(r_{CB}^D\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \overline{m}\kappa. \tag{A.2}$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with liquidity risk but without solvency risk exists only if $\varphi^r = (K+E)/E$. Furthermore, the banker will only monitor if the matched household opens an account with the banker ($h = \overline{h}$), as otherwise the banker will become illiquid and default, i.e. $\underline{m} = 0$, and $\overline{m} = \mathbb{1}\{\triangle(r_{\overline{s}}^L - r_{\underline{s}}^L) \geq \kappa\}$. As banks are defaulting due to illiquidity when they are matched with a household that opens an account with the central bank ($h = \underline{h}$), the central bank's profits or losses in terms of the consumption are given by

$$\pi^{CB} = \mu[\mathbb{E}_0[r_s^L|h = \underline{h}]L^b - r_{CB}^D(L^b - E^b)] = \mu[\mathbb{E}_0[r_s^L|h = \underline{h}]Q(K+E) - r_{CB}^D QK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$ and the fact that bankers who become illiquid and default do not monitor, i.e. $\underline{m} = 0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r_s^L Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = r_s^L Q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e. $\pi^f = 0$. Since the central bank operates under a balanced budget, it holds that $\pi^{CB} = \tau$.

With zero firm profits, the expected consumption of the banker and the household is given by

$$C^b = (1-\mu)\mathbb{E}_{\overline{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D|h = \overline{h}]QE \quad \text{and} \quad C^h = r_{CB}^D QK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =&(1-\mu)\mathbb{E}_{\overline{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D|h = \overline{h}]QE \\ &+ (1-\mu)(r_{CB}^D QK + \tau^{\overline{h}}) + \mu(r_{CB}^D QK + \tau^{\underline{h}}),\end{aligned}$$

which, using the equilibrium leverage $\varphi = (K+E)/E$, translates into

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) =(1-\mu)\mathbb{E}_{\overline{m}}[r_s^L|h = \overline{h}]Q(K+E) + (1-\mu)\tau^{\overline{h}} + \mu(r_{CB}^D QK + \tau^{\underline{h}}).$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB}$, it follows that

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =&(1-\mu)\mathbb{E}_{\overline{m}}[r_s^L|h = \overline{h}]Q(K+E) \\ &+ \mu r_{CB}^D QK + \mu[\mathbb{E}_0[r_s^L|h = \underline{h}]Q(K+E) - r_{CB}^D QK],\end{aligned}$$

which finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r_s^L]Q$, which is satisfied as $A_s = r_s^L Q$ for all $s \in \mathcal{S}$. The banker's monitoring decision, if matched with a household that opens an account with the banker ($h = \overline{h}$), is given by $\overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q\}$.

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and bankers' default penalties. Note that, if the banker faces liquidity risk but no solvency risk, there are no switching costs on the part of depositors. Using the fact that $\underline{m} = 0$, the welfare is given by

$$W^L = (1-\mu)\mathbb{E}_{\overline{m}}[(r_s^L - r_{CB}^D)\varphi + r_{CB}^D | h = \overline{h}]QE - (1-\mu)\overline{m}\kappa\varphi QE$$

$$- \mu\phi[r_{CB}^D(\varphi - 1) - \psi\varphi]QE + r_{CB}^D QK + (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}},$$

which, using equilibrium leverage $\varphi = (K+E)/E$, further simplifies to

$$W^L = (1-\mu)\mathbb{E}_{\overline{m}}[r_s^L | h = \overline{h}]Q(K+E) - (1-\mu)\overline{m}\kappa\varphi QE$$

$$- \mu\phi[r_{CB}^D QK - \psi Q(K+E)] + (1-\mu)\tau^{\overline{h}} + \mu(r_{CB}^D QK + \tau^{\underline{h}}).$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB}$, utilitarian welfare is given by

$$W^L = (1-\mu)\mathbb{E}_{\overline{m}}[r_s^L | h = \overline{h}]Q(K+E) - (1-\mu)\overline{m}\kappa\varphi QE - \mu\phi[r_{CB}^D QK - \psi Q(K+E)]$$

$$+ \mu r_{CB}^D QK + \mu[\mathbb{E}_0[r_s^L | h = \underline{h}]Q(K+E) - r_{CB}^D QK],$$

which, using $A_s = r_s^L Q$, with $s \in \mathcal{S}$, then reads as

$$W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q\}(K+E) - \mu\phi[r_{CB}^D QK - \psi Q(K+E)].$$

To fully characterize utilitarian welfare, we derive in the following the real central bank rate r_{CB}^D prevailing in equilibrium. First note that, using the equilibrium condition $A_s = r_s^L Q$, with $s \in \mathcal{S}$, (A.2) can be rewritten as

$$\mathbb{E}_{\overline{m}}[A_s | h = \overline{h}] = r_{CB}^D Q\left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(r_{CB}^D Q\frac{\varphi^r - 1}{\varphi^r} - \psi Q\right) + \overline{m}\kappa Q.$$

Rearranging yields

$$(1-\mu)(\mathbb{E}_{\overline{m}}[A_s | h = \overline{h}] - \overline{m}\kappa Q) + \mu\phi\psi Q = r_{CB}^D Q(1 - \mu + \mu\phi + \mu(1-\phi)/\varphi^r),$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$r_{CB}^D Q = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s | h = \overline{h}] - \overline{m}\kappa Q) + \mu\phi\psi Q}{(1-\mu) + \mu\phi + \mu(1-\phi)/\varphi^r}. \tag{A.3}$$

Thus, it holds that

$$r_{CB}^{D}QK - \psi Q(K+E) = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)K + \mu\phi\psi QK}{(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r} - \psi Q(K+E)$$

$$= \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)K - \psi Q[(1-\mu)K+E]}{(1-\mu)+\mu\phi+\mu(1-\phi)E/(K+E)}$$

$$= \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q) - \psi Q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}(K+E).$$

Hence welfare is given by $W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - \mu\phi\epsilon\}(K+E)$, where

$$\epsilon := \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q) - \psi Q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}.$$

Liquidity risk exists iff $\varphi > \varphi^L$, while solvency risk is ruled out iff $\varphi \leq \varphi^S$. Using the definition of φ^L and φ^S and equilibrium leverage $\varphi = (K+E)/E$, we can state that the banker will face a liquidity risk but no solvency risk iff

$$\frac{r_{CB}^D}{r_{CB}^D - \psi} < \varphi \leq \frac{r_{CB}^D}{r_{CB}^D - r_{\underline{s}}^L} \quad \Leftrightarrow \quad \frac{\psi}{r_{CB}^D} < \frac{K}{K+E} \leq \frac{r_{\underline{s}}^L}{r_{CB}^D}.$$

Using the equilibrium condition (A.3), we know that

$$\frac{\psi}{r_{CB}^D} < \frac{K}{K+E} \quad \Leftrightarrow \quad \frac{\psi Q[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]}{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)+\mu\phi\psi Q} < \frac{K}{K+E},$$

where the latter can be further rearranged to give

$$\psi Q(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] < [(1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)+\mu\phi\psi Q]K.$$

Note that, using $\varphi^r = (K+E)/E$,

$$(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] = [(1-\mu)+\mu\phi](K+E)+\mu(1-\phi)E$$

$$= (1-\mu+\mu\phi)K+E.$$

Hence, the previous inequality translates into

$$\psi Q[(1-\mu+\mu\phi)K+E] < [(1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)+\mu\phi\psi Q]K$$

$$\Leftrightarrow \quad \psi Q[(1-\mu)K+E] < (1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)K$$

$$\Leftrightarrow \quad \psi Q(1-\mu+E/K) < (1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)$$

$$\Leftrightarrow \quad \psi Q < \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)}{1-\mu+E/K}.$$

Similarly, using the equilibrium conditions $A_s = r_s^L Q$, with $s \in \mathcal{S}$, and (A.3), we know that the banker will face no solvency risk iff

$$\frac{K}{K+E} \leq \frac{r_{\underline{s}}^L}{r_{CB}^D} \quad \Leftrightarrow \quad \frac{K}{K+E} \leq \frac{A_{\underline{s}}[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]}{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)+\mu\phi\psi Q},$$

which can be rearranged to

$$[(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)+\mu\phi\psi Q]K \leq A_{\underline{s}}(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]$$

and using $(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] = (1-\mu+\mu\phi)K+E$, as previously derived, further simplifies to

$$\mu\phi\psi QK \leq A_{\underline{s}}[(1-\mu+\mu\phi)K+E]-(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)K$$

or, equivalently,

$$\psi Q \leq \frac{A_{\underline{s}}}{\mu\phi}(1-\mu+\mu\phi+E/K)-\frac{1-\mu}{\mu\phi}(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q).$$

As shown before, the liquidity risk and solvency risk condition both constrain the real haircut. Thus we can verify when the liquidity risk condition will be stricter than the solvency risk condition, i.e.

$$\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K} \leq \frac{A_{\underline{s}}}{\mu\phi}(1-\mu+\mu\phi+E/K)-\frac{1-\mu}{\mu\phi}(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q),$$

which can be rearranged to

$$\mu\phi\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K} \leq A_{\underline{s}}(1-\mu+\mu\phi+E/K)-(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)$$

and

$$\mu\phi\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K}+(1-\mu+E/K)\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K} \leq A_{\underline{s}}(1-\mu+\mu\phi+E/K).$$

Further rearranging yields

$$(1-\mu+\mu\phi+E/K)\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K} \leq A_{\underline{s}}(1-\mu+\mu\phi+E/K)$$

$$\Leftrightarrow \quad \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa Q)}{1-\mu+E/K} \leq A_{\underline{s}}.$$

Note that with the liquidity risk condition we can deduce $\psi Q < A_{\underline{s}}$. For $\psi Q \geq A_{\underline{s}}$, we can show that the solvency risk condition contradicts the liquidity risk condition. Thus the banker will face a liquidity risk but no solvency risk iff

$$\psi Q < \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa Q)}{1-\mu+E/K} \leq A_{\underline{s}}.$$

■

Proof of Proposition 3. Consider the situation where bankers face a solvency risk but no liquidity risk, i.e. $\varphi^S < \varphi \leq \varphi^L$. From Lemma 4 we know that the banker will choose a leverage $\varphi^S < \varphi \leq \varphi^L$ iff $\varphi = \varphi^r$ and

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}. \tag{A.4}$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with solvency risk but without liquidity risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[r^L_{\overline{s}} - r^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$. As banks are only defaulting due to insolvency, i.e. when the financed firm incurs a negative productivity shock, the central bank's losses in real terms are given by

$$\pi^{CB} = \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}L^b - r^D_{CB}(L^b - E^b)] = \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}Q(K+E) - r^D_{CB}QK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$, and the fact that the banker's monitoring decision is independent of the type of household, i.e. $\underline{m} = \overline{m}$. In equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r^L_s Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e. $\pi^f = 0$. Since the central bank operates under a balanced budget, it holds that $\pi^{CB} = \tau$.

With zero firm profits, the expected consumption by the banker and the household is given by

$$C^b = \eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \quad \text{and} \quad C^h = r^D_{CB}QK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =& \eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \\ &+ (1-\mu)(r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}).\end{aligned}$$

Using equilibrium leverage $\varphi = (K+E)/E$, the latter equation translates into

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) = \eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K+E) + (1-\mu)(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\overline{h}}) + \mu(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\underline{h}}).$$

From $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB}$ we can deduce

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) = \eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K+E) + \eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}Q(K+E) - r^D_{CB}QK],$$

which finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r_s^L]Q$, which is satisfied as $A_s = r_s^L Q$ for all $s \in \mathcal{S}$. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - r_{CB}^D QK/(K+E)] \geq \kappa Q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank rate r_{CB}^D prevailing in equilibrium. First note that, using equilibrium condition $A_s = r_s^L Q$, with $s \in \mathcal{S}$, (A.4) can be rewritten as

$$A_{\overline{s}} = r_{CB}^D Q \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}} \frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q = r_{CB}^D Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r),$$

such that we can finally deduce that, in equilibrium, the real central bank rate satisfies

$$r_{CB}^D Q = \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r}. \tag{A.5}$$

We can then state that the banker will monitor, independently of the type of matched household iff

$$\triangle \left[A_{\overline{s}} - \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r} \frac{K}{K+E}\right] \geq \kappa Q.$$

Rearranging yields

$$\triangle \left[A_{\overline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) - (\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)K\right] \geq \kappa Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E).$$

Note that, using $\varphi^r = (K+E)/K$,

$$(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}}(K+E) + \eta_{\underline{s}|\overline{m}} E = E + \eta_{\overline{s}|\overline{m}} K.$$

Thus, the latter inequality reads as

$$\begin{aligned} &\triangle \left[A_{\overline{s}}(E + \eta_{\overline{s}|\overline{m}} K) - (\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)K\right] \geq \kappa Q(E + \eta_{\overline{s}|\overline{m}} K) \\ \Leftrightarrow \quad &\triangle A_{\overline{s}} E \geq \kappa Q(E + \eta_{\overline{s}|\overline{m}} K - \triangle \overline{m} K). \end{aligned}$$

Exploiting $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$ and setting $\overline{m} = 1$, as the condition, if satisfied, implies monitoring, we know that the banker will monitor iff

$$\triangle A_{\overline{s}} E \geq \kappa Q[E + \eta_{\overline{s}|1} K - (\eta_{\overline{s}|1} - \eta_{\overline{s}|0})K] \quad \Leftrightarrow \quad \triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0} K/E).$$

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and potential switching costs on the part of depositors. Note that, if the banker faces no liquidity risk, there are no default penalties for bankers. Whether depositors transfer funds to the central bank if the respective banker becomes insolvent depends on the leverage. Specifically, depositors will switch in the case of insolvency iff $\varphi^R < \varphi$. Utilitarian welfare is then given by

$$W^S = \eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE - \overline{m}\kappa\varphi QE$$
$$+ (1-\mu)(r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R < \varphi\},$$

which, using equilibrium leverage $\varphi = (K+E)/E$, further simplifies to

$$W^S = \eta_{\overline{s}|\overline{m}}(r^L_{\overline{s}} - \overline{m}\kappa)Q(K+E) + (1-\mu)(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\overline{h}})$$
$$+ \mu(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\underline{h}}) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R < \varphi\}.$$

Note that only a mass $\eta_{\underline{s}|\overline{m}}$ of households that initially open an account with a banker (mass $1-\mu$) will potentially transfer funds and thus incur switching costs. With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB}$, welfare is given by

$$W^S = \eta_{\overline{s}|\overline{m}}(r^L_{\overline{s}} - \overline{m}\kappa)Q(K+E) + \eta_{\underline{s}|\overline{m}}r^D_{CB}QK$$
$$+ \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}Q(K+E) - r^D_{CB}QK] - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R < \varphi\},$$

which, using $A_s = r^L_s Q$, with $s \in \mathcal{S}$, then reads as

$$W^S = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R < \varphi\}.$$

We will specify the switching condition $\varphi^R < \varphi$ at a later stage. First note that liquidity risk is ruled out iff $\varphi \leq \varphi^L$, while solvency risk exists iff $\varphi > \varphi^S$. Using the definition of φ^L and φ^S, we can state that the banker will face a solvency risk but no liquidity risk iff

$$\frac{r^D_{CB}}{r^D_{CB} - r^L_{\underline{s}}} < \varphi \leq \frac{r^D_{CB}}{r^D_{CB} - \psi}.$$

Using equilibrium leverage $\varphi = (K+E)/E$, the latter inequalities translate into

$$\frac{r^D_{CB}}{r^D_{CB} - r^L_{\underline{s}}} < \frac{K+E}{E} \leq \frac{r^D_{CB}}{r^D_{CB} - \psi} \quad \Leftrightarrow \quad \frac{r^L_{\underline{s}}}{r^D_{CB}} < \frac{K}{K+E} \leq \frac{\psi}{r^D_{CB}}.$$

Using equilibrium conditions $A_s = r^L_s Q$, with $s \in \mathcal{S}$, and (A.5), we know that the banker will face a solvency risk but not liquidity risk iff

$$\frac{A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)}{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q} < \frac{K}{K+E} \leq \frac{\psi Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)}{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q},$$

which can be rewritten as

$$A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) < (\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \leq \psi Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E).$$

Note that, using $\varphi^r = (K+E)/E$, it holds that $(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}}K + E$. Then the latter inequalities read as

$$A_{\underline{s}}(\eta_{\overline{s}|\overline{m}}K+E)<(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)K\leq\psi Q(\eta_{\overline{s}|\overline{m}}K+E)$$

or, equivalently,

$$A_{\underline{s}}<\frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}}+E/K}\leq\psi Q.$$

Finally, we need to specify when bank insolvency will trigger a bank run, i.e. when $\varphi^R<\varphi$. Using equilibrium leverage $\varphi=(K+E)/E$ and the definition of φ^R, we know that $\varphi^R<\varphi$ iff

$$\frac{r^D_{CB}-\tilde{\nu}}{r^D_{CB}-\tilde{\nu}-r^L_{\underline{s}}}<\frac{K+E}{E}\quad\Leftrightarrow\quad\frac{r^L_{\underline{s}}}{r^D_{CB}-\tilde{\nu}}<\frac{K}{K+E},$$

where $\tilde{\nu}:=\nu/(QK)$. Using the equilibrium conditions $A_s=r^L_sQ$, with $s\in\mathcal{S}$, and (A.5), the latter inequality translates into

$$\frac{A_{\underline{s}}}{r^D_{CB}Q-\nu/K}<\frac{K}{K+E}\quad\Leftrightarrow\quad\frac{A_{\underline{s}}(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)}{(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)-\nu(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)/K}<\frac{K}{K+E}.$$

Rearranging yields

$$\begin{aligned}
&A_{\underline{s}}(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)<[(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)-\nu(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)/K]K\\
\Leftrightarrow\quad&A_{\underline{s}}(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)<(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)K-\nu(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)\\
\Leftrightarrow\quad&\nu<\frac{(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)K}{\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r}-A_{\underline{s}}(K+E)\\
\Leftrightarrow\quad&\nu<\left(\frac{(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q)K}{(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)}-A_{\underline{s}}\right)(K+E),
\end{aligned}$$

which, using $(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)=\eta_{\overline{s}|\overline{m}}K+E$, leads to

$$\nu<\nu^*:=\left(\frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}}+E/K}-A_{\underline{s}}\right)(K+E).$$

Thus the utilitarian welfare is given by $W^S=(\mathbb{E}_{\mathbf{m}}[A_s]-\overline{m}\kappa Q)(K+E)-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\nu<\nu^*\}$.
■

Proof of Proposition 4. Consider the situation where bankers face both a solvency and a liquidity risk, i.e $\max\{\varphi^L,\varphi^S\}<\varphi$. However, bank insolvency will not trigger a bank run, i.e. $\varphi\leq\varphi^R$. Then we know from Lemma 4 that the banker will choose $\max\{\varphi^L,\varphi^S\}<\varphi\leq\varphi^R$ iff $\varphi=\varphi^r$, $\mu\phi<\mu+(1-\mu)\eta_{\underline{s}|\overline{m}}$ and

$$r^L_{\overline{s}}=r^D_{CB}\left(1+\frac{(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right)+\frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r-1}{\varphi^r}-\psi\right)+\frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}.\tag{A.6}$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with liquidity risk and solvency risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = 0$ and $\overline{m} = \mathbb{1}\{\triangle[r^L_{\overline{s}} - r^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$. Banks are defaulting due to illiquidity and insolvency, such that the central bank's profits and losses in terms of the consumption good are given by

$$\begin{aligned}\pi^{CB} &= [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]L^b - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}(L^b - E^b)\\ &= [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]Q(K+E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}QK,\end{aligned}$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$ and the fact that the banker will not monitor if the matched household initially opens an account with the central bank ($h = \underline{h}$), i.e. $\underline{m} = 0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r^L_s Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e. $\pi^f = 0$. Since the central bank operates under a balanced budget, the taxes and transfers in real terms are given by $\tau = \pi^{CB}$.

With zero firm profits, expected consumption by the banker and the household is given by

$$C^b = (1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \quad \text{and} \quad C^h = r^D_{CB}QK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =&(1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE\\ &+ (1-\mu)(r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}),\end{aligned}$$

which, using equilibrium leverage $\varphi = (K+E)/E$, further simplifies to

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) =(1-\mu)\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K+E) + (1-\mu)(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}),$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{cb}$, the latter translates into

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =&(1-\mu)\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}QK\\ &+ [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]Q(K+E) - [\mu + (1-\mu)\eta_{\underline{s}|\overline{m}}]r^D_{CB}QK\end{aligned}$$

and finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r^L_s]Q$, which is satisfied as $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. The banker's monitoring decision is given by $\underline{m} = 0$ and $\overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - r^D_{CB}QK/(K+E)] \geq \kappa Q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank rate r^D_{CB} prevailing in equilibrium. First note that, using $A_s = r^L_s Q$, with $s \in \mathcal{S}$, (A.6) can be rewritten as

$$A_{\overline{s}} = r_{CB}^D Q \left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}} \frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}} \left(r_{CB}^D Q \frac{\varphi^r - 1}{\varphi^r} - \psi Q\right) + \frac{\overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging the latter equation yields

$$(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q = r_{CB}^D Q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu(1-\phi)]/\varphi^r\},$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$r_{CB}^D Q = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r}. \tag{A.7}$$

With (A.7), we can state that the banker will monitor iff $h = \overline{h}$ and

$$\triangle \left[A_{\overline{s}} - \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r} \frac{K}{K+E}\right] \geq \kappa Q.$$

Note that, using $\varphi^r = (K+E)/E$,

$$\begin{aligned}
&\{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r\}(K+E) \\
&\qquad = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi](K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu(1-\phi)]E \\
&\qquad = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E.
\end{aligned}$$

Then the latter inequality translates into

$$\triangle A_{\overline{s}} - \triangle \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K} \geq \kappa Q.$$

Setting $\overline{m} = 1$, as the condition, if satisfied, implies monitoring, and further rearranging yields

$$\begin{aligned}
&\triangle A_{\overline{s}}[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K] - \triangle[(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q) + \mu\phi\psi Q] \\
&\qquad\qquad \geq \kappa Q[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K] \\
\Leftrightarrow\quad &\triangle A_{\overline{s}}(\mu\phi + E/K) - \triangle\mu\phi\psi Q \geq \kappa Q[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K - (1-\mu)\triangle],
\end{aligned}$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, finally reads as

$$\triangle A_{\overline{s}} - \frac{\triangle\mu\phi\psi Q}{\mu\phi + E/K} \geq \kappa Q \left[1 + \frac{(1-\mu)\eta_{\overline{s}|0}}{\mu\phi + E/K}\right].$$

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and default penalties. Note that, as bank insolvency does not trigger a bank run, there are no switching costs for depositors. Of course, the latter requires $\varphi \leq \varphi^R$, which we will further specify at a later stage. Utilitarian welfare with liquidity and solvency risk and with bail-in in the case of bank insolvency, is then given by

$$W_B^{LS} = (1-\mu)\{\eta_{\overline{s}|\overline{m}}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D] - \overline{m}\kappa\varphi - \mu\phi[r_{CB}^D(\varphi - 1) - \psi\varphi]\}QE$$

$$+ (1-\mu)(r_{CB}^D QK + \tau^{\overline{h}}) + \mu(r_{CB}^D QK + \tau^{\underline{h}}),$$

which, using equilibrium leverage $\varphi = (K+E)/E$, translates into

$$W_B^{LS} = (1-\mu)(\eta_{\overline{s}|\overline{m}} r_{\overline{s}}^L - \overline{m}\kappa)Q(K+E) - \mu\phi[r_{CB}^D QK - \psi Q(K+E)]$$

$$+ (1-\mu)(\eta_{\underline{s}|\overline{m}} r_{CB}^D QK + \tau^{\overline{h}}) + \mu(r_{CB}^D QK + \tau^{\underline{h}}),$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{cb}$, welfare is given by

$$W_B^{LS} = (1-\mu)(\eta_{\overline{s}|\overline{m}} r_{\overline{s}}^L - \overline{m}\kappa)Q(K+E) - \mu\phi[r_{CB}^D QK - \psi Q(K+E)] + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu] r_{CB}^D QK$$

$$+ [\mu\mathbb{E}_0[r_s^L | h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}} r_{\underline{s}}^L]Q(K+E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu] r_{CB}^D QK,$$

which, using $A_s = r_s^L Q$, with $s \in \mathcal{S}$, reads as

$$W_B^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q\}(K+E) - \mu\phi[r_{CB}^D QK - \psi Q(K+E)],$$

where, with equilibrium condition (A.7),

$$r_{CB}^D QK - \psi Q(K+E) = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r} K - \psi Q(K+E)$$

$$= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)K - \psi Q[E + (1-\mu)\eta_{\overline{s}|\overline{m}} K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}} E/(K+E) + \mu\phi + \mu(1-\phi)E/(K+E)}$$

$$= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}(K+E).$$

Hence, welfare is given by $W_B^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - \mu\phi\epsilon\}(K+E)$, where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}.$$

Liquidity risk exists iff $\varphi^L < \varphi$, while solvency risk exists iff $\varphi^S < \varphi$. Using the definition of φ^L and φ^S and the equilibrium leverage $\varphi = (K+E)/E$, we can state that the banker will face liquidity risk and solvency risk iff

$$\max\left\{\frac{r_{CB}^D}{r_{CB}^D - \psi}, \frac{r_{CB}^D}{r_{CB}^D - r_{\underline{s}}^L}\right\} < \frac{K+E}{E} \quad \Leftrightarrow \quad \max\left\{\frac{\psi}{r_{CB}^D}, \frac{r_{\underline{s}}^L}{r_{CB}^D}\right\} < \frac{K}{K+E}.$$

Using equilibrium conditions $A_s = r_s^L Q$, with $s \in \mathcal{S}$, and (A.7), we know that the banker will face liquidity risk and solvency risk iff

$$\frac{\max\{\psi Q, A_{\underline{s}}\}[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q} < \frac{K}{K+E}.$$

The liquidity risk condition

$$\psi Q\frac{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q} < \frac{K}{K+E}.$$

can be further rearranged as

$$\psi Q(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]$$
$$< K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q].$$

Note that, using $\varphi^r = (K+E)/E$,

$$(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]$$
$$= (1-\mu)\eta_{\overline{s}|\overline{m}}(K+E) + (1-\mu)\eta_{\underline{s}|\overline{m}}E + \mu\phi(K+E) + \mu(1-\phi)E$$
$$= [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E.$$

Then the latter inequality translates into

$$\psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}}K + \mu\phi K + E] < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q]$$
$$\Leftrightarrow \quad \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}}K + E] < K(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)$$
$$\Leftrightarrow \quad \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K] < (1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)$$
$$\Leftrightarrow \quad \psi Q < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

The solvency risk condition is given by

$$A_{\underline{s}}\frac{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q} < \frac{K}{K+E}.$$

and can be rearranged to

$$A_{\underline{s}}(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]$$
$$< K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q].$$

Using, as before,

$$(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r] = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E,$$

we obtain

$$\begin{aligned}
& A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}}K + \mu\phi K + E] < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q] \\
\Leftrightarrow\quad & A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}}K + E] + \mu\phi(A_{\underline{s}} - \psi Q)K < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)] \\
\Leftrightarrow\quad & A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K] + \mu\phi(A_{\underline{s}} - \psi Q) < [(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)] \\
\Leftrightarrow\quad & A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.
\end{aligned}$$

Thus the banker will face liquidity risk and solvency risk iff

$$\max\left\{A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \psi Q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Bank insolvency does not trigger a bank run iff $\varphi \leq \varphi^R$, which, using equilibrium leverage $\varphi = (K+E)/E$ and the definition of φ^R, translates into

$$\frac{K+E}{E} \leq \frac{r^D_{CB} - \tilde{\nu}}{r^D_{CB} - \tilde{\nu} - r^L_{\underline{s}}} \quad \Leftrightarrow \quad \frac{K}{K+E} \leq \frac{r^L_{\underline{s}}}{r^D_{CB} - \tilde{\nu}},$$

where $\tilde{\nu} := \nu/(QK)$. Using equilibrium conditions $A_s = r^L_s Q$, with $s \in \mathcal{S}$, and (A.7), we can state $\varphi \leq \varphi^R$ iff

$$\frac{K}{K+E} \leq \frac{A_{\underline{s}}\alpha}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + \mu\phi\psi Q - \nu\alpha/K},$$

where $\alpha := (1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r$. Rearranging yields

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \leq A_{\underline{s}}\alpha(K+E) + \nu\alpha - \mu\phi\psi QK.$$

Using $\alpha(K+E) = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E$ as before, the latter inequality translates into

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \leq \left(A_{\underline{s}} + \frac{\nu}{K+E}\right)\{[(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E\} - \mu\phi\psi QK,$$

which can be rewritten as

$$\frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} \leq A_{\underline{s}} + \frac{\nu}{K+E} + \frac{\mu\phi(A_{\underline{s}} + \nu/(K+E) - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

■

Proof of Proposition 5. Consider the situation where bankers face both a liquidity and a solvency risk and bank insolvency triggers a bank run, i.e. $\min\{\varphi^L, \varphi^S, \varphi^R\} < \varphi$. Then we know from Lemma 4 that the banker will choose $\max\{\varphi^L, \varphi^S, \varphi^R\} < \varphi$ iff $\varphi = \varphi^r$, $\phi < 1$ and

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}, \quad \text{(A.8)}$$

Using equilibrium leverage, we know that such an equilibrium with liquidity risk and solvency risk only exists if $\varphi^r = (K + E)/E$. Furthermore, the banker will monitor iff matched with a household that initially holds deposits ($h = \overline{h}$) and iF $\triangle[r^L_{\overline{s}} - \phi\psi - r^D_{CB}(1-\phi)(\varphi^r - 1)/\varphi^r] \geq \kappa$. Banks are defaulting due to illiquidity and insolvency, such that the central bank's profits and losses in terms of the consumption good are given by

$$\begin{aligned}\pi^{CB} &= [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]L^b - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}(L^b - E^b)\\ &= [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]Q(K + E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}QK,\end{aligned}$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$, and the fact that the banker will not monitor if the matched household initially opens an account with the central bank ($h = \underline{h}$), i.e. $\underline{m} = 0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r^L_s Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers it must hold that $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. Hence, firms make zero profits, i.e. $\pi^f = 0$. Since the central bank operates under a balanced budget, taxes and transfers in real terms are given by $\tau = \pi^{CB}$.

With zero firm profits, expected consumption by the banker and the household is given by

$$C^b = (1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \quad \text{and} \quad C^h = r^D_{CB}QK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K + E) =& (1-\mu)\eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE\\ &+ (1-\mu)(r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}),\end{aligned}$$

which, using equilibrium leverage $\varphi = (K + E)/E$, further simplifies to

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K + E) =& (1-\mu)\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K + E)\\ &+ (1-\mu)(\eta_{\underline{s}|\overline{m}}r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}),\end{aligned}$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{cb}$, the latter translates into

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K + E) =& (1-\mu)\eta_{\overline{s}|\overline{m}}r^L_{\overline{s}}Q(K + E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r^D_{CB}QK\\ &+ [\mu\mathbb{E}_0[r^L_s|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]Q(K + E) - [\mu + (1-\mu)\eta_{\underline{s}|\overline{m}}]r^D_{CB}QK\end{aligned}$$

and finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r^L_s]Q$, which is satisfied as $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. The banker will monitor iff $h = \overline{h}$ and $\triangle[A_{\overline{s}} - \phi\psi Q - r^D_{CB}Q(1-\phi)K/(K+E)] \geq \kappa Q$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank rate prevailing in equilibrium. First note that, using equilibrium condition $A_s = r^L_s Q$, with $s \in \mathcal{S}$, (A.8) can be rewritten as

$$A_{\overline{s}} = r^D_{CB}Q\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}Q\frac{\varphi^r - 1}{\varphi^r} - \psi Q\right) + \frac{\overline{m}\kappa Q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\begin{aligned}&(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q \\ &\qquad = r^D_{CB}Q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r\}\end{aligned}$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$r^D_{CB}Q = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r}. \tag{A.9}$$

Using (A.9), we can state that the banker will monitor iff $h = \overline{h}$ and

$$\triangle\left[A_{\overline{s}} - \phi\psi Q - \frac{\{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q\}(1-\phi)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r}\frac{K}{K+E}\right] \geq \kappa Q.$$

Note that, using $\varphi^r = (K+E)/E$,

$$\begin{aligned}&\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r\}(K+E) \\ &\qquad = \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}(K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E \\ &\qquad = \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}K + E.\end{aligned}$$

Then the latter inequality translates into

$$\triangle\left[A_{\overline{s}} - \phi\psi Q - \frac{\{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q\}(1-\phi)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}\right] \geq \kappa Q.$$

Rearranging yields

$$\begin{aligned}&\triangle(A_{\overline{s}} - \phi\psi Q)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K\} \\ &\quad - \triangle(1-\phi)\{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q\} \\ &\qquad\qquad \geq \kappa Q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K\},\end{aligned}$$

which, in turn, simplifies to

$$\triangle A_{\overline{s}}(\phi+E/K)-\triangle\psi Q(1+E/K)\geq\kappa Q\{(1-\mu)\eta_{\overline{s}|\overline{m}}+[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]\phi+E/K\}$$
$$-\kappa Q(1-\phi)(1-\mu)\triangle\overline{m}.$$

Setting $\overline{m}=1$, as the condition, if satisfied, implies monitoring, and using $\triangle:=\eta_{\overline{s}|1}-\eta_{\overline{s}|0}$ yields

$$\triangle A_{\overline{s}}(\phi+E/K)-\triangle\phi\psi Q(1+E/K)\geq\kappa Q[\phi+E/K+(1-\phi)(1-\mu)\eta_{\overline{s}|0}],$$

which then translates into

$$\triangle A_{\overline{s}}-\triangle\phi\psi Q\frac{1+E/K}{\phi+E/K}\geq\kappa Q\left[1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right].$$

Utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring and default penalties, and switching costs on the part of depositors. Utilitarian welfare with liquidity and solvency risk and without bail-ins in case of bank insolvency is then given by

$$W_{NB}^{LS}=(1-\mu)\{\eta_{\overline{s}|\overline{m}}[(r_{\overline{s}}^{L}-r_{CB}^{D})\varphi+r_{CB}^{D}]-\overline{m}\kappa\varphi-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]\phi[r_{CB}^{D}(\varphi-1)-\psi\varphi]\}QE$$
$$+(1-\mu)(r_{CB}^{D}QK+\tau^{\overline{h}})+\mu(r_{CB}^{D}QK+\tau^{\underline{h}})-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

which, using equilibrium leverage $\varphi=(K+E)/E$, reads as

$$W_{NB}^{LS}=(1-\mu)\eta_{\overline{s}|\overline{m}}r_{\overline{s}}^{L}Q(K+E)-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]\phi[r_{CB}^{D}QK-\psi Q(K+E)]$$
$$+(1-\mu)(\eta_{\underline{s}|\overline{m}}r_{CB}^{D}QK+\tau^{\overline{h}})+\mu(r_{CB}^{D}QK+\tau^{\underline{h}})-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

With $\tau=(1-\mu)\tau^{\overline{h}}+\mu\tau^{\underline{h}}$ and $\tau=\pi^{cb}$, the welfare is given by

$$W_{B}^{LS}=(1-\mu)\eta_{\overline{s}|\overline{m}}r_{\overline{s}}^{L}Q(K+E)-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]\{\phi[r_{CB}^{D}QK-\psi Q(K+E)]+r_{CB}^{D}QK\}$$
$$+[\mu\mathbb{E}_0[r_s^L|h=\underline{h}]+(1-\mu)\eta_{\underline{s}|\overline{m}}r_{\underline{s}}^{L}]Q(K+E)-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]r_{CB}^{D}QK,$$

which, using $A_s=r_s^LQ$, with $s\in\mathcal{S}$, reads as

$$W_{B}^{LS}=\{\mathbb{E}_{\mathbf{m}}[A_s]-(1-\mu)\overline{m}\kappa Q\}(K+E)-\mu\phi[r_{CB}^{D}QK-\psi Q(K+E)],$$

where, with the equilibrium condition (A.9),

$$
\begin{aligned}
& r_{CB}^{D}QK - \psi Q(K+E) \\
&\quad = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^{r}} K - \psi Q(K+E) \\
&\quad = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K - \psi Q[E + (1-\mu)\eta_{\overline{s}|\overline{m}}K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E/(K+E)} \\
&\quad = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}(K+E).
\end{aligned}
$$

The welfare is then given by

$$W_{NB}^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa Q - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\epsilon\}(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}.$$

In the case of bank insolvency, depositors will shift their funds to the central bank iff $\varphi^R < \varphi$, which we further specify at a later stage.

Liquidity risk exists iff $\varphi^L < \varphi$, while solvency risk exists iff $\varphi^S < \varphi$. Using the definition of φ^L and φ^S and equilibrium leverage $\varphi = (K+E)/E$, we can state that bankers face liquidity risk and solvency risk iff

$$\max\left\{\frac{r_{CB}^{D}}{r_{CB}^{D} - \psi}, \frac{r_{CB}^{D}}{r_{CB}^{D} - r_{\underline{s}}^{L}}\right\} < \frac{K+E}{E} \quad \Leftrightarrow \quad \max\left\{\frac{\psi}{r_{CB}^{D}}, \frac{r_{\underline{s}}^{L}}{r_{CB}^{D}}\right\} < \frac{K}{K+E}.$$

Using the equilibrium conditions $A_s = r_s^L Q$, with $s \in \mathcal{S}$, and (A.9), we know that the banker will face liquidity risk and solvency risk iff

$$\frac{\max\{\psi Q, A_{\underline{s}}\}\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^{r} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q} < \frac{K}{K+E}.$$

The liquidity risk condition is given by

$$\psi Q \frac{\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^{r} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q} < \frac{K}{K+E},$$

which can be rearranged to

$$
\begin{aligned}
& \psi Q(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^{r} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\} \\
&\qquad < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q].
\end{aligned}
$$

Note that, using $\varphi^r = (K + E/K)$,

$$(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}$$

$$= (1-\mu)\eta_{\overline{s}|\overline{m}}(K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(K+E)$$

$$= \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}K + E.$$

Then the latter inequality translates into

$$\psi Q\{(1-\mu)\eta_{\overline{s}|\overline{m}}K + E\} < (1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \quad \Leftrightarrow \quad \psi Q < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Following the same procedure, the solvency risk condition

$$A_{\underline{s}}\frac{\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q} < \frac{K}{K+E},$$

can be rearranged such that

$$A_{\underline{s}}(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}$$

$$< K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q],$$

and finally reads as

$$A_{\underline{s}} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Thus the banker will face both liquidity risk and solvency risk iff

$$\max\left\{A_{\underline{s}} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \psi Q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Finally, bank insolvency will trigger a bank run iff $\varphi^R < \varphi$, which, using equilibrium leverage $\varphi = (K+E)/E$ and the definition of φ^R, translates into

$$\frac{r^D_{CB} - \tilde{\nu}}{r^D_{CB} - \tilde{\nu} - r^L_{\underline{s}}} < \frac{K+E}{E} \quad \Leftrightarrow \quad \frac{r^L_{\underline{s}}}{r^D_{CB} - \tilde{\nu}} < \frac{K}{K+E}.$$

Using equilibrium conditions $A_s = r^L_s Q$, with $s \in \mathcal{S}$, and (A.9), we can state $\varphi^R < \varphi$ iff

$$\frac{A_{\underline{s}}\alpha}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi Q - \nu\alpha/K} < \frac{K}{K+E},$$

where $\alpha := (1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r$. Rearranging yields

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \leq A_{\underline{s}}\alpha(K+E) + \nu\alpha - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi QK.$$

Using $\alpha(K+E) = [(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}\phi + \mu\phi]K + E$ as before, the latter inequality translates into

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)K \leq \left(A_{\underline{s}} + \frac{\nu}{K+E}\right)\{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}\phi + \mu\phi]K + E\}$$

$$- [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\psi QK.$$

which can be rewritten as

$$\frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} \leq A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} + \nu/(K+E) - \psi Q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

■

Proof of Proposition 6. First note that if the banker does not face a solvency risk, it is never optimal for the central bank to trigger liquidity risk by setting tight collateral requirements. The latter will only lead to illiquidity following a CBDC-induced bank run, while it does not have any positive effect. In particular, monitoring incentives for liquid bankers remain identical to the situation with loose collateral requirements. We denote the monitoring decision of the banker without solvency risk and without liquidity risk by $\mathbf{m} = (\underline{m}, \overline{m})$. From Proposition 1 we know that $\underline{m} = \overline{m}$. Similarly, the monitoring decision without solvency risk, but with liquidity risk due to tight collateral requirements, is denoted by $\mathbf{m}_L = (\underline{m}_L, \overline{m}_L)$. Illiquid bankers will not monitor, such that $\underline{m}_L = 0$. From Proposition 2 we know that liquid bankers have the same monitoring incentives as in the situation with loose collateral requirements, such that $\overline{m}_L = \overline{m}$. Without solvency risk, the change in welfare induced by tight collateral requirements is given by

$$W - W^L = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K+E) - [\mathbb{E}_{\mathbf{m}_L}[A_s] - (1-\mu)\overline{m}_L\kappa Q - \mu\phi\epsilon](K+E),$$

where according to Proposition 2

$$\epsilon = \frac{(1-\mu)(\mathbb{E}_{\overline{m}_L}[A_s|h=\overline{h}] - \overline{m}_L\kappa Q) - \psi Q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}.$$

Using $\overline{m} = \overline{m}_L$, it follows that

$$W - W^L = \mu(\mathbb{E}_{\overline{m}}[A_s|h=\underline{h}] - \mathbb{E}_0[A_s|h=\underline{h}] - \overline{m}\kappa Q + \phi\epsilon)(K+E) \geq 0.$$

Accordingly, tight collateral requirements, i.e. $\phi > 0$ and $R^D_{CB}K > \Psi(K+E)$, exposing the banker to liquidity risk and default penalties, are never optimal if there is no solvency risk.

In what follows we focus on the situation where the banker faces a solvency risk. Using the existence conditions provided in Proposition 3, bankers will face a solvency risk iff

$$A_{\underline{s}} < \frac{\eta_{\overline{s}|\overline{m}_S}A_{\overline{s}} - \overline{m}_S\kappa Q}{\eta_{\overline{s}|\overline{m}_S} + E/K},$$

where the banker's monitoring decision in the presence of solvency risk and in the absence of liquidity risk is denoted by $\mathbf{m}_S = (\underline{m}_S, \overline{m}_S)$. From Proposition 3 we know that $\underline{m}_S = \overline{m}_S$. Clearly, if bankers monitor without being exposed to liquidity risk and the ensuing default penalties, i.e. $\overline{m}_S = 1$, tight collateral requirements are never optimal, i.e. they induce a welfare loss as bankers face default penalties and bankers that become illiquid do not monitor. Thus, tight collateral requirements can only induce a welfare gain if bankers shirk without liquidity risk. According to Proposition 3, this translates into the condition $\triangle A_{\overline{s}} < \kappa Q(1+\eta_{\overline{s}|0}K/E)$. We denote the monitoring decision of the banker in the presence of solvency risk and liquidity risk by $\mathbf{m}_{LS} = (\underline{m}_{LS}, \overline{m}_{LS})$. Illiquid bankers do not monitor, so $\underline{m}_{LS} = 0$. According to Proposition 5, in the presence of both solvency risk and liquidity risk liquid bankers will only monitor, i.e. $\overline{m}_{LS} = 1$, if there exists $\phi \in (0,1)$ and $\psi \geq 0$ such that

$$\triangle A_{\overline{s}} - \triangle\phi\psi Q\frac{1+E/K}{\phi+E/K} \geq \kappa Q\left[1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right].$$

Note that, based on the existence conditions provided in Proposition 5, the default penalty parameter ϕ must be smaller than one. In what follows, we assume that the latter inequality holds, i.e. $\overline{m}_{LS} = 1$. Based on Proposition 5, an equilibrium with solvency risk and liquidity risk and without bail-ins in the case of bank insolvency will then exist iff

$$\underline{\chi}(\phi,\psi) < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}$$

where

$$\underline{\chi}(\phi,\psi) := \max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}} + \nu/(K+E) - \psi Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}, \psi Q\right\}.$$

Note that the latter inequality is sufficient for the previously introduced existence condition of an equilibrium with solvency risk only where bankers do not monitor, i.e. $\overline{m}_S = 0$. Using $\overline{m}_{LS} = 1$ and $\overline{m}_S = 0$, in the presence of a solvency risk the welfare change induced by tight collateral requirements is given by

$$\begin{aligned} W^{LS}_{NB} - W^S = &\{\mathbb{E}_{\mathbf{m}_{LS}}[A_s] - (1-\mu)\overline{m}_{LS}\kappa Q - [(1-\mu)\eta_{\underline{s}|\overline{m}_{LS}} + \mu]\phi\}(K+E) \\ &- (1-\mu)\eta_{\underline{s}|\overline{m}_{LS}}\nu - (\mathbb{E}_{\mathbf{m}_S}[A_s] - \overline{m}_S\kappa Q)(K+E) + (1-\mu)\eta_{\underline{s}|\overline{m}_S}\nu\mathbb{1}\{\nu < \nu^*\}, \end{aligned}$$

where, based on Proposition 5,

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}_{LS}}A_{\overline{s}} - \kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|\overline{m}_{LS}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}_{LS}} + [(1-\mu)\eta_{\underline{s}|\overline{m}_{LS}} + \mu]\phi + E/K}$$

and, following Proposition 3,

$$\nu^* := \left(\frac{\eta_{\overline{s}|\overline{m}_S}A_{\overline{s}}}{\eta_{\overline{s}|\overline{m}_S} + E/K} - A_{\underline{s}}\right)(K+E).$$

Setting $\mathbf{m}_S = (0,0)$ and $\mathbf{m}_{LS} = (0,1)$, it follows that

$$W_{NB}^{LS} - W^S = \{\mathbb{E}_{(0,1)}[A_s] - (1-\mu)\kappa Q - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\}(K+E)$$

$$- (1-\mu)\eta_{\underline{s}|1}\nu - \mathbb{E}_{(0,0)}[A_s](K+E) + (1-\mu)\eta_{\underline{s}|0}\nu\mathbb{1}\{\nu < \nu^*\},$$

where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}$$

and

$$\nu^* := \left(\frac{\eta_{\overline{s}|0}A_{\overline{s}}}{\eta_{\overline{s}|0} + E/K} - A_{\underline{s}}\right)(K+E).$$

Note that

$$\mathbb{E}_{(0,1)}[A_s] = \mu[\eta_{\underline{s}|0}A_{\underline{s}} + \eta_{\overline{s}|0}A_{\overline{s}}] + (1-\mu)[\eta_{\underline{s}|1}A_{\underline{s}} + \eta_{\overline{s}|1}A_{\overline{s}}]$$

and

$$\mathbb{E}_{(0,0)}[A_s] = \eta_{\underline{s}|0}A_{\underline{s}} + \eta_{\overline{s}|0}A_{\overline{s}}.$$

Then the welfare change, induced by tight collateral requirements, is given by

$$W_{NB}^{LS} - W^S = (1-\mu)[(\eta_{\overline{s}|1} - \eta_{\overline{s}|0})A_{\overline{s}} + (\eta_{\underline{s}|1} - \eta_{\underline{s}|0})A_{\underline{s}} - \kappa Q](K+E)$$

$$- [(1-\mu) + \mu]\phi\epsilon(K+E) - (1-\mu)(\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, further simplifies to

$$W_{NB}^{LS} - W^S = \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa Q] - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\epsilon\}(K+E)$$

$$- (1-\mu)(\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu.$$

Tight collateral requirements are welfare-improving if $W_{NB}^{LS} - W^S > 0$ or equivalently

$$(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa Q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)] > [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\epsilon.$$

Using the structure of ϵ, the latter inequality translates into

$$\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K\}$$

$$\times \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa Q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\}$$

$$> [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \psi Q[(1-\mu)\eta_{\overline{s}|1} + E/K]\},$$

which can be further rearranged as

$$\left\{1+\frac{(1-\mu)\eta_{\overline{s}|1}+E/K}{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi}\right\}\{(1-\mu)[\triangle(A_{\overline{s}}-A_{\underline{s}})-\kappa Q-(\eta_{\underline{s}|1}-\eta_{\underline{s}|0}\mathbb{1}\{\nu<\nu^*\})\nu/(K+E)]\}$$

$$>(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)-\psi Q[(1-\mu)\eta_{\overline{s}|1}+E/K].$$

and finally reads as

$$\{[(1-\mu)\eta_{\overline{s}|1}+E/K]^{-1}+[(1-\mu)\eta_{\underline{s}|1}+\mu]^{-1}\phi^{-1}\}$$

$$\times\{(1-\mu)[\triangle(A_{\overline{s}}-A_{\underline{s}})-\kappa Q-(\eta_{\underline{s}|1}-\eta_{\underline{s}|0}\mathbb{1}\{\nu<\nu^*\})\nu/(K+E)]\}$$

$$>\frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}-\psi Q.$$

Using the definition

$$\overline{\chi}(\phi,\psi):=\psi Q+\{(1-\mu)[\triangle(A_{\overline{s}}-A_{\underline{s}})-\kappa Q-(\eta_{\underline{s}|1}-\eta_{\underline{s}|0}\mathbb{1}\{\nu<\nu^*\})\nu/(K+E)]\}$$

$$\times\{[(1-\mu)\eta_{\overline{s}|1}+E/K]^{-1}+[(1-\mu)\eta_{\underline{s}|1}+\mu]^{-1}\phi^{-1}\},$$

we can state that tight collateral requirements will induce a welfare gain if

$$\frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}<\overline{\chi}(\phi,\psi).$$

■

Proof of Lemma 5. If tight collateral requirements are optimal, i.e. the conditions stated in Proposition 6 apply, the optimization problem of the central bank is given by

$$\max_{R^D_{CB}>0,\Psi\geq0,\phi>0} W^{LS}_{NB}-W^S \tag{A.10}$$

subject to

$$\underline{\chi}(\phi,\psi)<\frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K} \tag{A.11}$$

$$\kappa Q\left[1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right]\leq\triangle A_{\overline{s}}-\triangle\phi\psi Q\frac{1+E/K}{\phi+E/K}. \tag{A.12}$$

Note that

$$W^{LS}_{NB}-W^S=\{(1-\mu)[\triangle(A_{\overline{s}}-A_{\underline{s}})-\kappa Q]-[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi\epsilon(\phi,\psi)\}(K+E),$$

where

$$\epsilon(\phi,\psi)=\frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)-\psi Q[(1-\mu)\eta_{\overline{s}|1}+E/K]}{(1-\mu)\eta_{\overline{s}|1}+[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi+E/K}.$$

Hence the optimization problem of the central bank can be rewritten as

$$\min_{R^D_{CB}>0,\Psi\geq 0,\phi>0} \phi\epsilon(\phi,\psi) \quad \text{subject to} \quad \text{(A.11)} \quad \text{and} \quad \text{(A.12)}.$$

Next we analyze the constraint (A.11), which reads as

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}}+\nu/(K+E)-\psi Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}, \psi Q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}.$$

Clearly, if $A_{\underline{s}} + \nu/(K+E) \leq \psi Q$, the condition translates into

$$\psi Q < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}.$$

For $\psi Q < A_{\underline{s}} + \nu/(K+E)$, the condition reads as

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}}+\nu/(K+E)-\psi Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K} < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K},$$

which can be rearranged to

$$[(1-\mu)\eta_{\overline{s}|1}+E/K][A_{\underline{s}}+\nu/(K+E)] + \{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}}+\nu/(K+E)-\psi Q)\}$$
$$< (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q).$$

Further rearranging yields

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1}+E/K][A_{\underline{s}}+\nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi} < \psi Q.$$

Thus the real haircut must satisfy

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1}+E/K][A_{\underline{s}}+\nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi}$$
$$< \psi Q < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa Q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}. \quad \text{(A.13)}$$

Now we focus on constraint (A.12), which can be rewritten as

$$\psi Q \leq \frac{\phi+E/K}{1+E/K}\left[A_{\overline{s}} - \frac{\kappa Q}{\triangle}\left(1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right)\right]. \quad \text{(A.14)}$$

Note that the real haircut can only be non-negative, i.e. $\psi Q \geq 0$, as the central bank chooses a non-negative nominal haircut $\Psi \geq 0$. From Proposition 6 we know that, if tight collateral requirements are optimal, there exists a $\tilde{\phi} \in (0,1)$ such that the right-hand side of the latter inequality is zero. Moreover, the right-hand side of the latter inequality is increasing with ϕ. Furthermore, note that $\phi\epsilon(\phi,\psi)$ is decreasing with ψQ. Hence the central bank will choose the highest possible real haircut satisfying (A.13) and (A.14). Thus we define the lower bounds

$$\underline{\gamma}_1(\phi) := A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi},$$

and $\underline{\gamma}_2 := 0$, and the upper bounds

$$\overline{\gamma}_1 := \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \quad \text{and} \quad \overline{\gamma}_2(\phi) := \frac{\phi + E/K}{1+E/K}\left[\frac{A_{\overline{s}}}{\phi} - \frac{\kappa Q}{\triangle\phi}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right)\right].$$

Then we can rewrite the central bank's optimization problem as

$$\min_{\phi\in(0,1)} \phi\tilde{\epsilon}(\phi) \quad \text{subject to} \quad \max\{\underline{\gamma}_1(\phi), \underline{\gamma}_2\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.$$

The nominal central bank rate $R^D_{CB} > 0$ and the nominal haircut $\Psi \geq 0$ must then be chosen such that $R^D_{CB}K > \Psi(K+E) \geq 0$ and $\psi Q = \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}$. The latter condition can be rewritten in nominal terms as $\Psi Q = P\min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}$. Next we can show that, given a capital good price Q, the nominal output good price depends on the nominal central bank rate, the nominal haircut, and the default penalty parameter, i.e. $P = P(R^D_{CB}, \Psi, \phi)$. First note that from (vi) of Lemma 4 it follows

$$r^L_{\overline{s}} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

which, using equilibrium condition $A_s = r^L_s Q$, with $s \in \mathcal{S}$, translates into

$$\frac{A_{\overline{s}}}{Q} = r^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(r^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}.$$

Setting $\overline{m} = 1$, as bankers monitor with tight collateral requirements, and rearranging further yields

$$\begin{aligned}(1-\mu)\left(\eta_{\overline{s}|1}A_{\overline{s}}/Q - \kappa\right) = r^D_{CB}\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]/\varphi^r\} \\ + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi[r^D_{CB}(\varphi^r - 1)/\varphi^r - \psi].\end{aligned}$$

Rewriting in nominal terms yields

$$\begin{aligned}P(1-\mu)\left(\eta_{\overline{s}|1}A_{\overline{s}}/Q - \kappa\right) = R^D_{CB}\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]/\varphi^r\} \\ + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi[R^D_{CB}(\varphi^r - 1)/\varphi^r - \Psi].\end{aligned}$$

and finally reads as

$$P = \frac{R_{CB}^D\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu][1 + (1-\phi)/\varphi^r]\} - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\Psi}{(1-\mu)\left(\eta_{\overline{s}|1} A_{\overline{s}}/Q - \kappa\right)},$$

such that $P = P(R_{CB}^D, \Psi, \phi)$. ■

Proof of Corollary 1. First note that with a high bank leverage, i.e. $E/K \to 0$, we know from Proposition 3 that without tight collateral requirements, and thus without liquidity risk and default penalties, bankers will shirk as $\triangle A_{\overline{s}} < \lim_{E/K\to 0} \kappa Q(1 + \eta_{\overline{s}|0} K/E) = +\infty$. Second, note that if $\triangle A_{\overline{s}} > \kappa Q$, we know there exists a $\tilde{\phi} \in (0,1)$ and $\tilde{\Psi} = 0$, such that

$$\triangle A_{\overline{s}} = \kappa Q \left[1 + \frac{(1-\tilde{\phi})(1-\mu)\eta_{\overline{s}|0}}{\tilde{\phi} + E/K}\right].$$

From Proposition 6, we know that, with $A_{\underline{s}} = \nu = 0$, it follows $\underline{\chi}(\phi,\psi) = \psi Q$ and

$$\overline{\chi}(\phi,\psi) = \psi Q + (1-\mu)[\triangle A_{\overline{s}} - \kappa Q]\left\{\frac{1}{(1-\mu)\eta_{\overline{s}|1} + E/K} + \frac{1}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}\right\}.$$

Note that for $\mu, \eta_{\underline{s}|1} \to 0$, the monetary policy $\tilde{\phi} \in (0,1)$ and $\tilde{\Psi} = 0$, which, as shown before, incentivizes bankers to monitor, yields $\underline{\chi}(\tilde{\phi},\tilde{\psi}) = 0$ and $\overline{\chi}(\tilde{\phi},\tilde{\psi}) = +\infty$. Thus, banking with liquidity risk and solvency risk is viable, and tight collateral requirements are welfare-improving. Hence, for a sufficiently small risk exposure of bankers, i.e. small μ and $\eta_{\underline{s}|1}$, it is optimal for the central bank to apply tight collateral requirements exposing bankers to liquidity risk and default penalties. Specifically, we require that

$$\frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \psi Q + \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} + \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}} \tag{A.15}$$

where $\hat{\phi}$ represents the optimal default penalty parameter.

With Lemma 5 we now deduce that the optimal monetary policy $\hat{R}_{CB}^D > 0$, $\hat{\Psi} \geq 0$ and $\hat{\phi} > 0$ satisfies

$$\hat{\phi} \in \arg\min_{\phi \in [\tilde{\phi},1)} \phi\tilde{\epsilon}(\phi) \quad \text{subject to} \quad \max\{\underline{\gamma}_1, \underline{\gamma}_2(\phi)\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where, using $A_{\underline{s}} = \nu = 0$, it follows that

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K},$$

$$\underline{\gamma}_1(\phi) = -\frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi} < 0, \quad \underline{\gamma}_2 = 0,$$

$$\overline{\gamma}_1 = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \quad \text{and} \quad \overline{\gamma}_2(\phi) = \frac{\phi + E/K}{1+E/K}\left[A_{\overline{s}} - \frac{\kappa Q}{\triangle}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right].$$

Note that in the limit the constraint $\max\{\underline{\gamma}_1, \underline{\gamma}_2(\phi)\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}$ is always satisfied, so the central bank faces an unconstrained optimization problem. If μ is sufficiently small, such that $\mu\eta_{\overline{s}|1} < \eta_{\overline{s}|0}$, we can deduce that $\overline{\gamma}_1 > \overline{\gamma}_2(\phi)$ for all $\phi \in (0,1)$. Hence the optimization problem reads as

$$\hat{\phi} \in \arg \min_{\phi \in [\tilde{\phi},1)} \phi\tilde{\epsilon}(\phi),$$

where, using $\overline{\gamma}_1 > \overline{\gamma}_2(\phi)$, it follows that

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \overline{\gamma}_2(\phi)[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.$$

Taking the derivative of $\phi\tilde{\epsilon}(\phi)$ with respect to ϕ yields

$$\begin{aligned}
\frac{\partial \phi\tilde{\epsilon}(\phi)}{\partial \phi} &= \tilde{\epsilon}(\phi) + \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \left[\overline{\gamma}_2(\phi) + \phi\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi}\right][(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K} \\
&\quad - \frac{\{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q) - \overline{\gamma}_2(\phi)[(1-\mu)\eta_{\overline{s}|1} + E/K]\}[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}{\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K\}^2} \\
&= \tilde{\epsilon}(\phi)\left(2 - \frac{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}\right) \\
&\quad - \frac{(1-\mu)\eta_{\overline{s}|1}\phi\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi}}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.
\end{aligned}$$

Note that the derivative of $\overline{\gamma}_2(\phi)$ with respect to ϕ is given by

$$\begin{aligned}
\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi} &= \frac{\phi(1+E/K) - (\phi + E/K)(1+E/K)}{\phi^2(1+E/K)^2}\left[A_{\overline{s}} - \frac{\kappa Q}{\triangle}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right] \\
&\quad + \frac{\phi + E/K}{1+E/K}\left[\frac{\kappa Q}{\triangle}\frac{(-\phi)(1-\mu)\eta_{\overline{s}|0} - (1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi^2}\right] \\
&= -\frac{E/K}{\phi^2(1+E/K)}\left[A_{\overline{s}} - \frac{\kappa Q}{\triangle}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right] \\
&\quad - \frac{\phi + E/K}{1+E/K}\left[\frac{\kappa Q}{\triangle}\frac{(1-\mu)\eta_{\overline{s}|0}}{\phi^2}\right].
\end{aligned}$$

Note that $\overline{\gamma}_2(\phi) \geq 0$ for all $\phi \geq \tilde{\phi}$. Thus for all $\phi \geq \tilde{\phi}$ we know that the first derivative of $\overline{\gamma}_2(\phi)$ with respect to ϕ is negative and $\tilde{\epsilon}(\phi) > 0$ for all $\phi \in (0,1)$, so we can conclude that the derivative of $\phi\tilde{\epsilon}(\phi)$ is positive and hence the optimal monetary policy is characterized by $\hat{\phi} = \tilde{\phi}$,

$\hat{\Psi} = \tilde{\Psi} = 0$ and $\hat{R}^D_{CB} > 0$. Note that $\tilde{\phi}$ is determined by the equation

$$\triangle A_{\overline{s}} = \kappa Q \left[1 + \frac{(1-\tilde{\phi})(1-\mu)\eta_{\overline{s}|0}}{\tilde{\phi} + E/K}\right],$$

which can be rearranged to

$$\tilde{\phi}[\triangle A_{\overline{s}} - \kappa Q + \kappa Q(1-\mu)\eta_{\overline{s}|0}] = \kappa Q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa Q)E/K,$$

which finally reads as

$$\tilde{\phi} = \frac{\kappa Q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa Q)E/K}{\triangle A_{\overline{s}} - \kappa Q[(1-\mu)\eta_{\underline{s}|0} + \mu]}.$$

Given optimal monetary policy, constraint (A.15) translates into

$$\frac{(1-\mu)\eta_{\overline{s}|0}A_{\overline{s}}}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}}.$$

■

Proof of Proposition 7. Focusing on competitive equilibria without a liquidity risk for bankers, note that the first-best utilitarian welfare is achieved if households do not incur switching costs and bankers monitor if the welfare gain through the induced productivity increase offsets the bankers' utility losses due to monitoring. In any competitive equilibrium without a solvency risk for bankers, households do not incur switching costs as they are not exposed to bank insolvency. In addition, bankers' monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q\}$ (see Propositon 1) and thus is welfare-maximizing because bankers monitor if the welfare gain due to the productivity increase induced by monitoring $\triangle(A_{\overline{s}} - A_{\underline{s}})(K+E)$ offsets the utility losses for bankers due to monitoring $\kappa Q(K+E)$. Thus any competitive equilibrium without a risk for bankers yields the first-best welfare. From Proposition 3 we know that the first-best welfare is also achieved in any competitive equilibrium with a solvency risk for bankers if the following two conditions are met: First, households do not convert their deposits into CBDC in the case of a bank insolvency and thus do not incur switching costs. In other words, households accept a bail-in, which occurs if switching costs are sufficiently high, i.e. $\nu \geq \nu^*$, with ν^* provided in Proposition 3. Second, bankers' monitoring decision must be welfare maximizing, i.e. bankers should monitor only if the welfare gain due to the productivity increase, $\triangle(A_{\overline{s}} - A_{\underline{s}})(K+E)$, induced by monitoring offsets bankers' utility losses due to monitoring, $\kappa Q(K+E)$. With Proposition 3 we know that it must hold $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$ if and only if $\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E)$. ■

Proof of Proposition 8. As the social planner can reallocate the capital good among households and bankers, Proposition 8 follows directly from Proposition 7 stating that any competitive equilibrium without solvency risk yields the first-best welfare, as households do not incur switching costs and bankers' monitoring decision is welfare-maximizing. ■

Proof of Proposition 9. Note that with a solvency risk faced by bankers, the constrained social planner can only align the bankers' monitoring incentives with the objective of maximizing utilitarian welfare but not eliminate solvency risk for bankers and thus not the switching costs faced by households in the case of bank insolvency. It is then the aim of the constrained social planner through the use of taxes and transfers depending on the idiosyncratic productivity shock for the financed firm to align bankers' monitoring decision with the objective of maximizing welfare. With contingent taxes and transfers τ_s, the banker's optimization problem in real terms is given by

$$\max_{\substack{\varphi \in [1,\varphi^r], \\ m(h) \in \{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} r_{\mathbf{z}}^{E,+} - r_{\mathbf{z}}^{E,-} - m(h)\kappa\varphi + \tau_s \varphi]QE.$$

Consider the situation where the banker faces a solvency risk but no liquidity risk, i.e. $\varphi^S < \varphi \leq \varphi^L$. The banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\eta_{\overline{s}|1}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D]QE + \mathbb{E}_1[\tau_s|h]\varphi QE$$
$$\geq \eta_{\overline{s}|0}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D]QE + \mathbb{E}_0[\tau_s|h]\varphi QE + \kappa\varphi QE,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, can be rewritten as $\triangle[r_{\overline{s}}^L - r_{CB}^D(\varphi - 1)/\varphi] \geq \kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}}$. Using the fact that the banker's monitoring decision does not depend on the type of matched household, i.e. $\underline{m} = \overline{m}$, we know that the banker's expected utility from conducting banking operations is given by $\{\eta_{\overline{s}|\overline{m}}[(r_{\overline{s}}^L - r_{CB}^D)\varphi + r_{CB}^D] - \overline{m}\kappa\varphi + \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi\}QE$. Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e. $r_{CB}^D QE$. Thus, with $\underline{m} = \overline{m}$ we can deduce that the banker will choose $\varphi^S < \varphi \leq \varphi^L$ with $\varphi < \varphi^r$ if

$$r_{\overline{s}}^L = r_{CB}^D \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}} \frac{1}{\varphi}\right) + \frac{\overline{m}\kappa - \mathbb{E}_{\mathbf{m}}[\tau_s]}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to adjust the supply of loans, i.e. $\eta_{\overline{s}|\overline{m}}(r_{\overline{s}}^L - r_{CB}^D) - \overline{m}\kappa + \mathbb{E}_{\mathbf{m}}[\tau_s] = 0$, which, however, contradicts the former equation. Hence the banker will only choose a leverage $\varphi^S < \varphi \leq \varphi^L$ if $\varphi = \varphi^r$,

$$r_{\overline{s}}^L = r_{CB}^D \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}} \frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa - \mathbb{E}_{\mathbf{m}}[\tau_s]}{\eta_{\overline{s}|\overline{m}}}$$

and $\eta_{\overline{s}|\overline{m}}(r_{\overline{s}}^L - r_{CB}^D) - \overline{m}\kappa + \mathbb{E}_{\mathbf{m}}[\tau_s] > 0$, which follows directly from the previous equation.

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with solvency risk but without liquidity risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[r_{\overline{s}}^L - r_{CB}^D(\varphi^r - 1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}}\}$. As banks are only defaulting due to insolvency, i.e. when the financed firm incurs a negative productivity shock, the central bank's losses in real terms are given by

$$\pi^{CB} = \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}L^b - r^D_{CB}(L^b - E^b)] = \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}Q(K+E) - r^D_{CB}QK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$, and the fact that the banker's monitoring decision is independent of the type of household, i.e. $\underline{m} = \overline{m}$. Moreover, in equilibrium, the demand for capital good is finite, such that, with Lemma 2, we can deduce $A_s \leq r^L_s Q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold $A_s = r^L sQ$ for all $s \in \mathcal{S}$. Hence, firms make zero profits, i.e. $\pi^f = 0$. With contingent taxes and transfers implemented by the social planner and a balanced budget for the central bank, it holds that $\pi^{CB} - \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi QE = \tau$.

With zero firm profits, the expected consumption by the banker and the household is given by

$$C^b = \eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE \quad \text{and} \quad C^h = r^D_{CB}QK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. Market clearing for the consumption good, $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, yields

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =& \eta_{\overline{s}|\overline{m}}[(r^L_{\overline{s}} - r^D_{CB})\varphi + r^D_{CB}]QE - \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi QE \\ &+ (1-\mu)(r^D_{CB}QK + \tau^{\overline{h}}) + \mu(r^D_{CB}QK + \tau^{\underline{h}}).\end{aligned}$$

Using equilibrium leverage $\varphi = (K+E)/E$, the latter equation translates into

$$\begin{aligned}\mathbb{E}_{\mathbf{m}}[A_s](K+E) =& \eta_{\overline{s}|\overline{m}} r^L_{\overline{s}} Q(K+E) + \mathbb{E}_{\mathbf{m}}[\tau_s]Q(K+E) \\ &+ (1-\mu)(\eta_{\underline{s}|\overline{m}} r^D_{CB}QK + \tau^{\overline{h}}) + \mu(\eta_{\underline{s}|\overline{m}} r^D_{CB}QK + \tau^{\underline{h}}).\end{aligned}$$

From $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB} + \mathbb{E}_{\mathbf{m}}$ we can deduce

$$\mathbb{E}_{\mathbf{m}}[A_s](K+E) = \eta_{\overline{s}|\overline{m}} r^L_{\overline{s}} Q(K+E) + \eta_{\underline{s}|\overline{m}} r^D_{CB}QK + \eta_{\underline{s}|\overline{m}}[r^L_{\underline{s}}Q(K+E) - r^D_{CB}QK],$$

which finally reads as $\mathbb{E}_{\mathbf{m}}[A_s] = \mathbb{E}_{\mathbf{m}}[r^L_s]Q$, which is satisfied as $A_s = r^L_s Q$ for all $s \in \mathcal{S}$. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - r^D_{CB}QK/(K+E)] \geq (\kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}})Q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank rate r^D_{CB} prevailing in equilibrium. First note that, using equilibrium condition $A_s = r^L_s Q$, with $s \in \mathcal{S}$, (A.4) can be rewritten as

$$A_{\overline{s}} = r^D_{CB}Q\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa Q - \mathbb{E}_{\mathbf{m}}[\tau_s]Q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa Q + \mathbb{E}_{\mathbf{m}}[\tau_s]Q = r^D_{CB}Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r),$$

such that we can finally deduce that, in equilibrium, the real central bank rate satisfies

$$r_{CB}^D Q = \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q + \mathbb{E}_{\mathbf{m}}[\tau_s]Q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r}. \tag{A.16}$$

We can then state that the banker will monitor, independently of the type of matched household iff

$$\triangle \left[A_{\overline{s}} - \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q + \mathbb{E}_{\mathbf{m}}[\tau_s]Q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r} \frac{K}{K+E} \right] \geq (\kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}})Q.$$

Rearranging yields

$$\begin{aligned} &\triangle \left[A_{\overline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) - (\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q + \mathbb{E}_{\mathbf{m}}[\tau_s]Q)K \right] \\ &\qquad\qquad \geq (\kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}})Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E). \end{aligned}$$

Note that, using $\varphi^r = (K+E)/K$,

$$(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}}(K+E) + \eta_{\underline{s}|\overline{m}}E = E + \eta_{\overline{s}|\overline{m}}K.$$

Thus, the latter inequality reads as

$$\begin{aligned} &\triangle \left[A_{\overline{s}}(E + \eta_{\overline{s}|\overline{m}}K) - (\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q + \mathbb{E}_{\mathbf{m}}[\tau_s]Q)K \right] \geq (\kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}})Q(E + \eta_{\overline{s}|\overline{m}}K) \\ \Leftrightarrow \quad &\triangle A_{\overline{s}}E \geq \kappa Q(E + \eta_{\overline{s}|\overline{m}}K - \triangle\overline{m}K) + \triangle\mathbb{E}_{\mathbf{m}}[\tau_s]QK - \triangle(\tau_{\overline{s}} - \tau_{\underline{s}})Q(E + \eta_{\overline{s}|\overline{m}}K). \end{aligned}$$

Exploiting $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$ and setting $\overline{m} = 1$, as the condition, if satisfied, implies monitoring, we know that the banker will monitor iff

$$\begin{aligned} &\triangle A_{\overline{s}}E \geq \kappa Q[E + \eta_{\overline{s}|1}K - (\eta_{\overline{s}|1} - \eta_{\overline{s}|0})K] + \triangle(\eta_{\overline{s}|1}\tau_{\overline{s}} + \eta_{\underline{s}|1}\tau_{\underline{s}})QK - \triangle(\tau_{\overline{s}} - \tau_{\underline{s}})Q(E + \eta_{\overline{s}|1}K) \\ \Leftrightarrow \quad &\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E) - \triangle\tau_{\overline{s}}Q + \triangle\tau_{\underline{s}}Q(1 + K/E). \end{aligned}$$

Without loss of generality, we can set $\tau_{\underline{s}} = 0$ as it is irrelevant whether the constrained social planner imposes taxes on the non-monitoring bankers, distributes transfer to the monitoring bankers or both. Thus the constrained social planner must choose $\tau_{\overline{s}}$ such that $\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E) - \triangle\tau_{\overline{s}}Q$ if and only if $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$. In what follows, we show that it always holds $\triangle(A_{\underline{s}} - A_{\underline{s}}) \geq \kappa Q$ if $\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E)$. Thus, whenever the banker monitors in the presence of solvency risk, monitoring is also welfare-maximizing. In other words, the constrained social planner must never apply taxes to prevent the banker from monitoring because it would be not welfare-maximizing. Suppose the banker faces solvency risk and is monitoring without any taxes or transfers applied by the constrained social planner. Then we know from Proposition 3 that it holds

$$\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E) \quad \text{and} \quad A_{\underline{s}} < \frac{\eta_{\overline{s}|1}A_{\overline{s}} - \kappa Q}{\eta_{\overline{s}|1} + E/K}.$$

The latter inequality can be rearranged to

$$
\begin{aligned}
& \kappa Q < \eta_{\overline{s}|1} A_{\overline{s}} - (\eta_{\overline{s}|1} + E/K) A_{\overline{s}} \\
\Leftrightarrow \quad & \kappa Q < \eta_{\overline{s}|1}(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}} E/K \\
\Leftrightarrow \quad & \kappa Q < (\eta_{\overline{s}|1} - \eta_{\overline{s}|0})(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}} E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}})
\end{aligned}
$$

and finally, with the notation $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, reads as

$$
\kappa Q < \triangle(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}} E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}}). \tag{A.17}
$$

Suppose now that monitoring by bankers is not welfare-maximizing, i.e. $\triangle(A_{\overline{s}} - A_{\underline{s}}) < \kappa Q$. Then we know with A.17 that it must hold

$$
-A_{\overline{s}} E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}}) > 0 \quad \Leftrightarrow \quad K/E > \frac{A_{\underline{s}}}{\eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}})}. \tag{A.18}
$$

Since the banker is monitoring, we know it holds that

$$
\triangle A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0} K/E),
$$

which with the inequality A.18 implies

$$
\begin{aligned}
& \triangle A_{\overline{s}} > \kappa Q \left(1 + \eta_{\overline{s}|0} \frac{A_{\underline{s}}}{\eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}})}\right) \\
\Leftrightarrow \qquad & \triangle A_{\overline{s}} > \kappa Q \left(1 + \frac{A_{\underline{s}}}{(A_{\overline{s}} - A_{\underline{s}})}\right) \\
\Leftrightarrow \qquad & \triangle A_{\overline{s}}(A_{\overline{s}} - A_{\underline{s}}) > \kappa Q \left(A_{\overline{s}} - A_{\underline{s}} + A_{\underline{s}}\right) \\
\Leftrightarrow \qquad & \triangle(A_{\overline{s}} - A_{\underline{s}}) > \kappa Q,
\end{aligned}
$$

where the latter represents a contradiction to the previous assumption of monitoring not being welfare-maximizing. Accordingly, whenever the banker monitors in the presence of solvency risk, monitoring is also welfare-maximizing. As a consequence, we know that the constrained social planner must only apply contingent taxes and transfers if the banker faces solvency risk and does not monitor, i.e. $\triangle A_{\overline{s}} < \kappa Q(1 + \eta_{\overline{s}|0} K/E)$, although monitoring would be welfare-maximizing, i.e. $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$. If we assume that the banker chooses in the case of indifference the welfare-maximizing monitoring activity, the optimal transfer for monitoring bankers set by the constrained social planner satisfies

$$
\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0} K/E)/\triangle - A_{\overline{s}}/Q, 0\}.
$$

We still need to check whether the transfer applied by the constrained social planner is feasible.

As in our model only households are taxed, the following constraint applies:

$$
\begin{aligned}
& r_{CB}^D QK + \tau \geq 0 \\
\Leftrightarrow \quad & r_{CB}^D QK + \pi^{CB} - \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi QE \geq 0 \\
\Leftrightarrow \quad & r_{CB}^D QK + \eta_{\underline{s}|\overline{m}}[r_{\underline{s}}^L Q(K+E) - r_{CB}^D QK] - \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} Q(K+E) \geq 0 \\
\Leftrightarrow \quad & r_{CB}^D(\varphi-1)/\varphi + \eta_{\underline{s}|\overline{m}}[r_{\underline{s}}^L - r_{CB}^D(\varphi-1)/\varphi] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \\
\Leftrightarrow \quad & \eta_{\underline{s}|\overline{m}} r_{\underline{s}}^L + \eta_{\overline{s}|\overline{m}} r_{CB}^D(\varphi-1)/\varphi \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \\
\Leftrightarrow \quad & \eta_{\underline{s}|\overline{m}} r_{\underline{s}}^L + \eta_{\overline{s}|\overline{m}} r_{\overline{s}}^L - \eta_{\overline{s}|\overline{m}}[r_{\overline{s}}^L - r_{CB}^D(\varphi-1)/\varphi] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}}.
\end{aligned}
$$

As the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[r_{\overline{s}}^L - r_{CB}^D(\varphi^r-1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}}\}$, we can also express the optimal tax applied by the constrained social planner as

$$
\tau_{\overline{s}} = \kappa/\triangle - r_{\overline{s}}^L + r_{CB}^D(\varphi-1)/\varphi,
$$

which then can be rewritten as $r_{\overline{s}}^L - r_{CB}^D(\varphi-1)/\varphi = \kappa/\triangle - \tau_{\overline{s}}$. Then the latter inequality translates into

$$
\mathbb{E}_{\mathbf{m}}[r_s^L] - \eta_{\overline{s}|\overline{m}}[\kappa/\triangle - \tau_{\overline{s}}] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \qquad \Leftrightarrow \qquad \triangle\mathbb{E}_{\mathbf{m}}[r_s^L] \geq \eta_{\overline{s}|\overline{m}}\kappa.
$$

Setting $\underline{m} = \overline{m} = 1$ and using $A_s = r_s^L Q$, with $s \in \mathcal{S}$, the latter condition reads as

$$
\begin{aligned}
& \triangle(\eta_{\overline{s}|1} A_{\overline{s}} + \eta_{\underline{s}|1} A_{\underline{s}}) \geq \eta_{\overline{s}|1}\kappa Q \\
\Leftrightarrow \quad & \triangle[\eta_{\overline{s}|1}(A_{\overline{s}} - A_{\overline{s}}) + A_{\underline{s}}] \geq \eta_{\overline{s}|1}\kappa Q \\
\Leftrightarrow \quad & \eta_{\overline{s}|1}[\triangle(A_{\overline{s}} - A_{\overline{s}}) - \kappa Q] \geq -\triangle A_{\underline{s}},
\end{aligned}
$$

which holds true as monitoring by bankers was assumed to be welfare-maximizing, i.e. $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa Q$. ■

Proof of Proposition 10. Suppose bankers face solvency risk. Then we know from Proposition 9 and 3 that utilitarian welfare achieved by the constrained social planner is given by

$$
W^{CS} = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa Q)(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\nu < \nu^*\},
$$

where $\underline{m} = \overline{m} = \mathbb{1}\{A_{\overline{s}} \geq \kappa Q(1 + \eta_{\overline{s}|0}K/E) - \triangle\tau_{\overline{s}}Q\}$ with $\tau_{\overline{s}}$ satisfying

$$
\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0}K/E)/\triangle - A_{\overline{s}}/Q, 0\}.
$$

Now suppose tight collateral requirements are optimal (see Proposition 6), where the optimal default penalty parameter $\hat{\phi}$ follows from Lemma 5. With Proposition 6 we can immediately deduce that this implies monitoring is welfare-maximizing, i.e. $\triangle(A_{\overline{s}} - A_{\underline{s}})$. The constrained social planner will therefore implement contingent transfers $\tau_{\overline{s}}$, so that $\underline{m} = \overline{m} = 1$ and welfare is given by

$$W^{CS} = (\mathbb{E}_1[A_s] - \kappa Q)(K + E) - (1 - \mu)\eta_{\underline{s}|1}\nu\mathbb{1}\{\nu < \nu^*\}.$$

Then we know from Proposition 5 that the welfare in the competitive equilibrium with optimal monetary policy is given by

$$W_{NB}^{LS} = \{\mu\mathbb{E}_0[A_s] + (1 - \mu)\mathbb{E}_1[A_s] - (1 - \mu)\kappa Q - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E) - (1 - \mu)\eta_{\underline{s}|1}\nu.$$

Suppose $\nu < \nu^*$, with ν^* provided in Proposition 3, so that households convert deposits in the case of bank insolvency. Then, the difference between utilitarian welfare in the competitive equilibrium with optimal monetary policy and second-best welfare is given by

$$\mu(\mathbb{E}_1[A_s] - \mathbb{E}_0[A_s] - \kappa Q)(K + E) - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E),$$

which, using $\mathbb{E}_1[A_s] - \mathbb{E}_0[A_s] = \triangle(A_{\overline{s}} - A_{\underline{s}})$, translates into

$$\mu[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa Q](K + E) - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E).$$

Clearly, for any $\mu > 0$ there is a welfare loss in the competitive equilibrium with optimal monetary policy compared to the constrained social planner solution due to lost monitoring activities by illiquid bankers. In addition, there is a welfare loss due to the imposed default penalties. For $\mu \to 0$ and $\eta_{\underline{s}|1} \to 0$ utilitarian welfare in the competitive equilibrium with optimal monetary policy approaches utilitarian welfare achieved by the constrained social planner. If in addition switching costs are negligible, i.e. $\nu \to 0$, welfare in the competitive equilibrium approaches the first-best welfare. ■

Proof of Proposition 11. Note that using our framework, which features a CBDC and no deposit insurance scheme, we can replicate the real allocation emerging in today's monetary system, with deposits as the only medium of exchange and a deposit insurance scheme if there are no costs for converting deposits into CBDC and there are no penalties for bankers if they default on liabilities towards the central bank. Thus using our framework, we can replicate the real allocation emerging in today's monetary system by setting switching costs to zero, i.e. $\nu = 0$, and focusing on loose collateral requirements, i.e. $\Psi(K + E) \geq R_{CB}^{D}K$, which rules out default penalties for bankers.

Without solvency risk, households will never transfer funds from private bankers to the central bank. Thus, even if there are costs for converting deposits into CBDC, the alternative system with a CBDC and no deposit insurance scheme yields the same welfare as today's monetary system. We obtain the same result if bankers face a solvency risk but switching costs are sufficiently high, such that households holding deposits with an insolvent banker accept a bail-in and do not transfer funds to the central bank.

Last, consider the situation where bankers face a solvency risk and switching costs are sufficiently low, so that households holding deposits with insolvent bankers will not accept a bail-in and shift their funds to the central bank. Then, if loose collateral requirements are optimal, the alternative monetary system yields a welfare loss compared to today's monetary system, due to switching costs on the part of depositors. If tight collateral requirements are optimal, the alternative system may yield a welfare gain compared to today's monetary system if the switching costs are sufficiently low. ■

Proof of Proposition 12. Note that we assume sufficiently small switching costs ν, such that bank insolvency will trigger a bank run. Households will then only transfer their funds from a banker to the central bank if the respective banker defaults due to insolvency. Otherwise, the mass of households holding accounts with the central bank will stay constant over time, i.e. $\mu_{t+1} = \mu_0$. In turn, if bankers face a solvency risk, i.e. they will default if the financed firm experiences a negative productivity shock ($s = \underline{s}$), a household which has a deposit with a banker will shift the funds to the central bank and, due to positive switching costs, stays with the central bank in the following periods. Thus, with solvency risk the mass of households holding accounts with the central bank will evolve in accordance with $\mu_{t+1} = (1-\mu_t)\eta_{\overline{s}|\overline{m}}+\mu_t$. ■

Proof of Proposition 13. We denote the mass of defaulting bankers in period $t \in \mathbb{N}_0$ by σ_t. Without liquidity and solvency risk, no banker will default, i.e. $\sigma_t = 0$. With liquidity risk only, bankers experiencing a CBDC-induced bank run or, equivalently, matched with a household that holds an account with the central bank, will default. The mass of such households in the economy is given by $\mu_t = \mu_0$ and stays constant over time as there is no solvency risk (see Proposition 12). Thus, with liquidity risk only, the mass of defaulting bankers is constant and is given by $\sigma_t = \mu_0$. With solvency risk only, bankers will default if the financed firm incurs a negative productivity shock ($s = \underline{s}$), which occurs with probability $\eta_{\underline{s}|\overline{m}}$. Note that with solvency risk only, the monitoring decision is independent of the type of matched household, as stated in Proposition 3. Thus, with solvency risk only, the mass of defaulting bankers is constant and is given by $\sigma_t = \eta_{\underline{s}|\overline{m}}$. With liquidity risk *and* solvency risk, the mass of defaulting bankers is given by $\sigma_t = \mu_t + (1 - \mu_t)\eta_{\underline{s}|\overline{m}}$, for the reasons stated above. From Proposition 12, we know that with solvency risk, the mass of households possessing accounts with the central bank is converging to one, i.e. $\lim_{t\to\infty} \mu_t = 1$, such that in the presence of both liquidity and solvency risk the mass of defaulting bankers also approaches one, i.e. $\lim_{t\to\infty} \sigma_t = 1$. ■

Proof of Proposition 14. From Proposition 5 we know that an equilibrium with liquidity risk following from tight collateral requirements, solvency risk, and no bail-ins, will exist iff

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t]\phi(A_{\underline{s}} - \psi_t Q) + \frac{\nu}{K+E}}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K}, \psi_t Q\right\} < \frac{(1-\mu_t)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa Q)}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K},$$

where $\mu_{t+1} = (1 - \mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$. Specifically, note that there exists no sequence $\{\psi_t\}_{t\in\mathbb{N}_0}$ such that for all $t \in \mathbb{N}_0$ the above inequality is satisfied: With solvency risk, the mass of households holding accounts with the central bank converges to one, i.e. $\lim_{t\to\infty} \mu_t = 1$, such that the right-hand side approaches zero while the left-hand side remains positive for any $\psi_t \geq 0$. Hence, with constant endowments of households and bankers, tight collateral requirements can only be maintained for a finite period of time without rendering banking non-viable, i.e. there exists a period $\tilde{t} \in \mathbb{N}_0$ subsequent to which tight collateral requirements will lead to non-viability of banking. ■