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APPOINTED LEARNING FOR THE COMMON GOOD: OPTIMAL COMMITTEE SIZE AND EFFICIENT REWARDS

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JEL Classification: C72, D71, D8

Keywords: Voting - Committee - Cost sharing - Information acquisition - Reward scheme - Monetary transfers - Majority rule

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Appointed Learning for the Common Good: Optimal Committee Size and Efficient Rewards*

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A population of identical individuals must choose one of two alternatives under uncertainty about what the right alternative is. Individuals can gather information of increasing accuracy at an increasing convex utility cost. For such a setup, we analyze how vote delegation to a committee and suitable monetary transfers for its members can ensure that high or optimal levels of information are (jointly) acquired. Our main insight is that to maximize the probability of choosing the right alternative committee size must be small, no matter whether information acquisition costs are private or not. Our analysis and results cover two polar cases—information costs are either private or public—and unravel both the potential and the limitations of monetary transfers in committee design.

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1 Introduction

Motivation

From a theoretical and empirical viewpoint alike, it has been well understood for a long time that agents have strong incentives to free-ride on others' efforts to find out which alternative should be implemented for the common good (Downs, 1957). Incentives to become informed before voting are low because a single individual bears the cost of acquiring the information, but his/her probability of affecting the outcome (i.e., his/her probability of being pivotal) is small, particularly in large populations. The problem that there is too little information at both the individual and the aggregate level is pervasive both in politics (Borgonovi et al., 2010; Elenbaas et al., 2012; Stephenson, 2010) and in corporate governance (Alam et al., 2014; De Haan and Vlahu, 2016; Reid, 1984; Yeoh, 2000). It is also salient for numerous panels of appointed experts such as scientific referees, juries, public procurement committees, monetary policy committees, and hiring committees (see e.g. Gersbach and Hahn, 2012; Persico, 2004; Reis, 2013). How should committees of (expert) decision-makers be designed to ensure that a sufficiently high level of information is acquired and later expressed through voting?

Committee delegation and monetary payments

An intuitive way of remedying the underinvestment in information in all of the above scenarios is to increase the chances for agents to be pivotal and/or to reduce the private costs associated with information acquisition. In exploring this double avenue our main goal is to advance knowledge about the design of committees of decision-makers appointed from a larger population. To do so, we investigate a family of mechanisms that first determine a committee with a certain size and second set a suitable reward scheme for its members. A *committee* is a (randomly) chosen subset of agents, all of whom are given the exclusive right to vote. A *reward scheme* is a population-wide, budget-balanced vector of (positive and negative) monetary transfers.

While it is obvious that monetary payments *can* have a positive effect on agents' incentives to acquire information, little is known about how reward schemes and committee size should be optimally selected jointly by a community to learn what alternative is the best for the common good. Hence, this is the focus of the paper. This means that we abstract from all other variables that can influence optimal committee design (see e.g., Gerling et al., 2005; Persico, 2004). For instance, we do not consider the voting rule as a design variable, and stick to the widely used majority rule. Assuming neutrality, i.e., treating both alternatives equally, is often a requirement in many voting situations, especially in the absence of a status quo. Although we proceed with a polity as our default setup, our insights readily extend to any of the situations described above.

Our setup

For the analysis we build on Martinelli (2006). We assume that there are two ex-ante equally likely alternatives, A and B, and that each agent (of the committee) receives an informative signal of quality $\frac{1}{2} + x$ about the state of the world (i.e., the signal about the state of the world is correct with probability $\frac{1}{2} + x$). Variable x is chosen by the agent himself or herself and has an associated cost in utility terms equal to $c(x)$. Conditional on knowing the state of the world, either all agents of the entire population would like to implement A, or all of them would like to implement B. Hence, while all the agents share the same preferences, they differ in the information they possess. This has major consequences for outcomes, as it may affect which alternative the agents will vote for. We do not consider the possibility of communication, so voting (through the majority rule) is the only way in which agents can act on their information. This is a reasonable assumption for a number of setups, including, but not limited to, groups of citizens who have been randomly chosen from the citizenry, nodes in a blockchain, and scientific reviewers. In general, the feature that committee members cannot communicate with each other can be adopted as part of the committee design. We also assume informative voting, i.e., individuals do not strategize in the use of the information they get, but simply translate it into a vote.[1]

A preview of our results

We consider two distinct approaches, which serve as polar cases for the analysis of committee design with side payments. First, we assume that although information costs are privately incurred, they are verifiable and contractible. Assuming in addition that all incurred information costs are divided equally among *all* agents independently of the individual effort exerted by each of them and regardless of whether the agents belong to the committee or not, we show that *optimal* committee size is one, i.e., one-member committees implement the right alternative with highest probability. This result holds provided that the information acquisition cost function is a monomial of degree two at most. For information acquisition cost functions that are monomials of a degree higher than two, optimal committees have more than one member. For general information acquisition cost functions, these results mean that high (low) convexity of the information acquisition cost function is associated with large (small) committees.

In our second approach, we assume that contracts cannot be written that are contingent on the information costs incurred by the agents. We also assume that it is impossible to condition rewards on the correct state of the world.[2] This means that rewards can only be conditioned on the voting pattern. To channel the incentives of the agents and avoid them free-riding on each other, we then show that it suffices to consider the following class of reward schemes: Each member of the committee will receive a (positive or negative) transfer that depends on

[1]For most of our analysis this can be done without loss of generality. From Austen-Smith and Banks (1996) we know that informative voting is rational when the prior is symmetric and the voting rule is simple majority. Some papers have analyzed the validity of informative voting (see e.g. Grosser and Seebauer, 2016) from an empirical perspective.

[2]This typically occurs when the correct state of the world is revealed much later than payments need to be executed or when it is never revealed independently of the committee decision, since it is precisely the task of an expert committee to find out the state of the world. Moreover, even if the state of the world may be observable, it may not be contractible, as has been argued in the incomplete contract literature.

the vote tally difference between the two alternatives obtained after all committee members have cast their votes. The particular structure of the reward scheme to be chosen depends on whether $c'(1/2)$ is finite or infinite, and hence on whether full learning is possible or not. In either case we require the rewards to be uniformly financed by *all* members of the population. Then we show that an adequate reward scheme guarantees that the right alternative is implemented with probability one (when full learning is possible) or that such probability converges to one with population size (when full learning is impossible). As with cost sharing, committees must be small, whether in absolute terms (when full learning is possible) or in relative terms (when full learning is impossible). At all events, committee size can never be smaller than three.

Our results thus rationalize the use of monetary transfers in small committees whose members are appointed from a larger population and care about the common good. Financing is easily guaranteed when taxation can be enforced (whether *de jure* by some external authority or *de facto* using some standard folk-theorem argument), in which case we do not need to be concerned with participation incentive constraints (i.e., with individual rationality). Even if such concerns matter, we show that provided there are enough citizens individuals who are not committee members will prefer to take part in the mechanism (at the interim stage in which they do not know the state of the world but know that they are not part of the committee) rather than leaving the problem altogether. At the interim stage, committee members for their part are content with being part of the committee.[3]

Extensions and further results

The results we derive in our baseline setup carry over to more general setups that include asymmetric priors, asymmetric preferences, and private values. Additionally, the good properties of our family of mechanisms can be maintained under the restriction that all citizens—and not just committee members—must keep their right to vote. To show this, we add a second voting stage to the mechanisms considered, which therefore take the form of Assessment Voting (Gersbach et al., 2019). This is a two-round mechanism that splits the population into two groups that vote sequentially, with all the individuals in one group voting simultaneously; the first voting group can be thought of as the committee (or *assessment group, AG*). All individuals cast one vote, members of the second voting round know the outcome of the first voting round before they vote, and the alternative that receives more votes in the two voting rounds combined is implemented. To be consistent with our framework, we allow all citizens to acquire costly information about the state of the world, no matter which round they vote in. Finally, we also investigate different cost sharing rules for setups in which costs are verifiable and can be shared, and compare them to the cost sharing rule according to which citizens who do not belong to the committee uniformly finance the information acquisition costs incurred by committee members.

[3]Under cost sharing, all individuals expect the same positive utility, which is never lower than the utility when each alternative is chosen with the same probability. Hence, if the outside option entails a lower utility than the latter, all the citizens will participate in the mechanism.

Organization of the paper

The paper is organized as follows: In Section 2 we discuss the papers that are most relevant for our model. In Section 3 we introduce our model and set up notation. In Section 4 we discuss the case of cost sharing. In Section 5 we discuss the case in which cost sharing is not feasible and reward schemes are used. In Section 6 we consider that citizens cannot be deprived of their right to vote and discuss different cost sharing rules. In Section 7 we investigate some extensions of our baseline setup. Section 8 concludes. The proofs are in the Appendix.

2 Literature

Our paper is related to several strands of the literature.

Condorcet jury theorem

The *broad* literature on the Condorcet jury theorem (see Austen-Smith and Banks, 1996; Castanheira, 2003; Condorcet, 1785; Feddersen and Pesendorfer, 1996; Gratton, 2014; Krishna and Morgan, 2012; Ladha, 1992; Martinelli, 2006; Persico, 2004; Razin, 2003; Triossi, 2013; Young, 1995) investigates whether elections can aggregate the information that is dispersed in the electorate. We contribute to this literature by showing how monetary transfers and vote delegation to a committee can be used jointly to overcome low levels of information acquisition arising when the accuracy of information is determined endogenously by each individual at variable cost.

Rational ignorance and the common good in elections

From the above literature, the paper that is closest to ours is Martinelli (2006). Its main result is that for suitable cost acquisition functions, rational voters reluctant to incur information acquisition costs are still able to make good electoral decisions collectively in the case of large populations. Although each individual acquires very little information (i.e., individuals are rationally ignorant), the aggregate level of information is much higher (i.e., the community of individuals is *not* ignorant) and with high probability leads to the implementation of the right alternative. We complement Martinelli (2006) in that we focus on societies (and committees) of any size that are unable to implement the correct alternative with high probability through normal elections. In such cases, our results suggest *(a)* delegating the voting power to a committee made up of a small number of citizens and *(b)* channeling adequate transfers to the committee members from the rest of society that incentivize the former to acquire high levels of information and express them through voting.[4]

Delegating to a committee

Our family of mechanisms is characterized by the feature that voting power is *fully* delegated to

[4]The possibility that vote delegation to a small committee could be better than one-round universal suffrage has already been pointed out by Martinelli (2006) through an example with a quadratic cost function. However, he does not consider side-payments to sustain the desirable outcome in equilibrium.

a committee. The rationale for doing this is that it increases the committee members' pivotality. At least since Gilligan and Krehbiel (1987), it is well known that the parent body (in this case, the rest of the population) should have limited rights to amend the decision taken by a committee. This is in line with our insights. Yet, we also show that as long as committee members vote before the rest of the population exclusive voting rights may not be necessary. Other potential benefits associated with higher pivotality, and hence with our family of mechanisms, includes protection against special interests (see e.g. Louis-Sidois and Musolff, 2020).[5]

Monetary transfers and reward schemes

The second main feature of the family of mechanisms we consider is that they incorporate monetary transfers. It is intuitive that monetary transfers *can* affect voters' behavior in voting and, more generally, any agent's behavior in a strategic situation. Winter (2004) argues that even if all agents are ex ante equal, monetary transfers in the form of asymmetric reward schemes can prompt agents to exert socially efficient effort levels (see also Bernstein and Winter, 2012). Our paper shares this insight in the case of voting: Giving rewards to members of the committee only may be desirable for the common good. More recently, Azriele (2018) has considered a setup that is similar to ours, except that he is not constrained by voting with the majority rule. He then shows that optimal contracts between the experts and a decision-maker discriminate between the experts and entail transfers that cannot be contingent on the true state of the world. We share this view in our analysis of reward schemes (see also Dal Bo, 2007).

Acquiring costly information that benefits everybody and can be used later via voting is a public good. Hence the positive externalities associated with acquiring information may lead to an underprovision of information. For general public goods, it is known that introducing monetary transfers from some agents to others can introduce negative externalities that compensate the positive externalities set out above (Morgan, 2000). For committee design, Persico (2004) analyzes how (huge) side-payments can be used to ensure full information acquisition in committees, when this is feasible. We complement his results by *(i)* emphasizing that large punishments are only needed off equilibrium, *(ii)* focusing on arbitrary information acquisition cost functions, and *(iii)* considering reward schemes that feature the minimal monetary transfers needed—together with the incentives linked to pivotality—to ensure high levels of information acquisition.[6] The latter is important for practical implementation, as high monetary transfers can crowd out the direct price effect provided by monetary transfers (see e.g. Gneezy et al., 2011; Meier, 2007).

Optimal committee size

Some papers have examined optimal committee size by analyzing how voting rules should be

[5]When small committees are exposed to external influence, outside payments trying to sway (voting) behavior may be directed at committee members. This has been thoroughly studied both from a theoretical and empirical perspective in the case of lobbying groups trying to buy the vote of legislators (see e.g. Austen-Smith and Wright, 1994; Felgenhauer and Grüner, 2008; Wright, 1990).

[6]From a technical perspective, our model and approach are very different from that of Persico (2004). For instance, we impose the use of the majority rule and provide a full account of symmetric equilibria for this case.

designed not only to aggregate dispersed information efficiently but also to induce agents to acquire sufficient information. By letting the vote threshold vary that is needed to upset a status quo, Persico (2004) also shows that both the threshold and the committee size relative to the total population should be determined by the information level acquired in (the symmetric) equilibrium and thus be generically above a half, but typically lower than one. Like Persico (2004), we focus on environments where information is a public good and investigate a family of mechanisms characterized by an additional design variable beyond committee size. While Persico (2004) considers the voting threshold, we stick to the majority rule and introduce transfer schemes as part of the design. Our results are then in sharp contrast. With either cost sharing or reward schemes, it is typically optimal to have committees that are small, at least in relative terms.

Committees are paramount for the functioning of representative democracies, and their (optimal) size depends on a number of variables, e.g., representativeness. In the case of the US Senate, for instance, the size of standing committees typically ranges from 12 to 29, while in the House of Representatives it ranges between 11 and 61.[7] In the corporate world, a board of directors often takes the most important decisions for a firm. The size of such a board ranges from 3 to more than 30.[8] Its size (and composition) also depends on several variables, e.g., disciplining its members and providing them with the incentives not to exploit private information (Raheja, 2005). In general, there is mixed evidence about whether larger boards of directors lead to better firm performance (Wang et al., 2009). For their part, juries typically range from 3 jurors to 12, editors usually ask for a report about a submitted paper from between 1 and 5 reviewers, while hiring committees can entail a few members of a firm/department or, on occasion, many more. According to Erhart and Vasquez-Paz (2007), the optimal size for monetary committees is between 5 and 9 (see also Blinder, 2007). In turn, larger public procurement committees may yield better (financial) decisions (Zábojníková, 2016). From a general perspective, different optimal sizes for panels of experts obey different rationales, e.g., minimizing the impact of career concerns on decisions (see e.g., Hahn, 2017; Levy, 2007). Our results rationalize the use of small committees when side-payments are possible and committee members are given the exclusive right to vote.

Mechanism design

Our paper is also related to a strand of the mechanism design literature that studies optimal information acquisition mechanisms for committees. In Gerardi and Yariv (2008), there is the possibility of communication and the mechanism designer who participates in the decision chooses both the optimal size of a committee and the way individual decisions translate into the final outcome. Gershkov and Szentes (2009) characterize the optimal stopping rule in a sequential mechanism in which a randomly selected voter is asked in each round to acquire some

[7]See `https://www.llsdc.org/assets/sourcebook/crs-93-920.pdf`, retrieved 29 January 2020.
[8]See `https://www.investopedia.com/articles/analyst/03/111903.asp`, retrieved 29 January 2020.

information.[9] Koriyama and Szentes (2009) study the minimum size of committees that guarantees ex-ante efficiency. We differ from these papers in that we allow side-payments and consider different (continuous) levels of information acquisition. This leads to results about optimal size that are not in accord with the above papers, as we suggest a lower size.

Agency problems

For the analysis of the static game(s) underlying our family of mechanisms, we assume that there is preference homogeneity—within the committee and between the committee and the rest of population.[10] Hence there are no incentives for committee members to manipulate or withhold information. The latter feature is common in agency models in which one or several principals try to influence the decision of a body of voters through information transmission (see e.g. Alonso and Câmara, 2016; Chan et al., 2019; Hagenbach et al., 2014; Jackson and Tan, 2013). Our paper contrasts with these articles. In our common-value setup introducing side-payments can be welfare-enhancing.

Experimental evidence for committees

In our setup, all citizens vote according to their signal, regardless of accuracy; this is called informative voting. From an empirical perspective, many papers have analyzed the validity of this behavior (we refer to Grosser and Seebauer, 2016). Very recently, Kawamura and Vlaseros (2017) have shown that some voters tend to disregard the (private) signals they have obtained and use public signals (expert opinions). This, in turn, decreases information aggregation. The two approaches we consider, cost sharing and rewards, increase individual signal precision. This could help voters to ignore (noisy) public signals and lead to higher levels of aggregate information.

Real-world applications and computer science

The mechanism we propose—vote delegation to a committee, coupled with a transfer scheme—could be used instead of population-wide referenda (as an *online voting* procedure) in the case of large populations if committee members were chosen at random from the entire population. Thus we contribute to the growing strand of research attempting to find mechanisms for collective democratic decisions that can correct some inefficiencies of the decision-making procedures currently in place (see e.g. Gersbach et al., 2019; Lalley and Weyl, 2018).[11] As far as side-payments are concerned, central (governmental) authorities have the power to execute them. Alternatively, payments could be implemented via Smart Contracts in the absence of such an authority. As for the composition of the committee, it is clear that its members should be chosen in accordance with a fair randomization device. Micali and Cheng (2017) argue that a verifiable random function can be implemented via the blockchain.

[9] A more general setting of multiparty computation is analyzed in Smorodinsky and Tennenholtz (2006).

[10] According to Cai (2009), preference heterogeneity within the committee calls for larger committees.

[11] In Section 6.1, we analyze how Assessment Voting (Gersbach et al., 2019) would work in a common-value setup in which information about the state of the world is costly.

In the computer science literature, assigning full voting power to a fraction of the population is known as *random sample voting (RSV)*—see Chaum (2016). The latter paper suggests such a voting procedure and proposes a protocol for implementing it that ensures the randomness of voter selection, non-manipulability, verifiability, and anonymity. Subsequently, Basin et al. (2018) have developed a provably secure protocol for RSV, which could also be adapted to our mechanism. A recent paper by Meir et al. (2020) studies the performance of random committees of representatives with arbitrary population size that must vote on a number of binary issues. Our paper shows that when coupled with schemes that ensure payments, RSV can yield good decisions even when citizens must exert effort to become informed.

Another distinct application of our mechanism with a random committee could be blockchains (Atzori, 2015; Beck et al., 2018; Paech, 2017). Governance has become a central issue for any blockchain because its participants not only have to achieve consensus on the validity of transactions, but the evolution of the blockchain itself requires repeated collective decisions beyond the decisions that are taken implicitly by forks. Our paper suggests a simple mechanism that could be used to improve the governance of blockchains.

3 Model

For our analysis we build on Martinelli (2006). There is a population of $2n+1$ individuals (or agents) who must choose between two alternatives, say A and B. One alternative is the right one, i.e., it yields higher utility for all individuals than the other alternative. For simplicity, we normalize the utility obtained from the right alternative to one, and the utility obtained from the wrong alternative to zero. However, the right alternative is unknown, as it depends on the (unknown) state of the world. If the state of the world w is A (B), then A (B) is the right alternative. We assume that the ex-ante probabilities that either alternative is the right one are equal to $\frac{1}{2}$. Individuals can nonetheless obtain a signal about the likelihood of the right alternative, i.e., a signal about the state of the world. Specifically, there is an *(information acquisition) cost function*

$$c : \left[0, \frac{1}{2}\right] \to \mathbb{R}_+ \cup \{\infty\},$$

where $c(x)$ must be interpreted as the cost in utility units for each voter of receiving a signal $s_i \in \{A, B\}$ about the state of the world of quality $1/2 + x$ ($x \in [0, \frac{1}{2}]$). The latter means that for $y \in \{A, B\}$,

$$Prob\big[s_i = y | w = y\big] = \frac{1}{2} + x.$$

We assume that signals are stochastically independent across individuals and that $c(x)$ is increasing. Note that $c(x)$ can be infinity, in which case an individual cannot inform himself or herself with certainty about the state of the world.

There is also a committee, which is a subset of $2m + 1$ individuals chosen randomly from the general population. This means that $m \leq n$. Each member i of the committee can decide about his/her own level of information, which we denote by x_i. Once committee members have received their signals, all of them simultaneously vote for one of the alternatives. The alternative that receives at least $m + 1$ votes is implemented. We also assume that no abstention occurs. While this is a very strong assumption to impose on the entire population if n is large, it is not very demanding for low values of m.[12] With no abstention, the alternative with the largest number of votes is implemented.

Finally, there is a monetary transfer scheme (to be specified later) according to which total payoffs are realized. A *transfer scheme* is a vector $(v_i)_{i=1}^{2n+1} \in \mathbb{R}^{2n+1}$ for all members of the population, with the property that it is budget balanced, i.e., $\sum_{i=1}^{2n+1} v_i = 0$. We assume that individuals have linear utility in money. This means that when alternative y is implemented and the right alternative is alternative z the total utility that individual i derives is

$$u_i(z, y, x_i, v_i) := \mathbb{1}_z(y) - c(x_i) + v_i,$$

where $\mathbb{1}_z(y) = 1$ if $z = y$ and $\mathbb{1}_z(y) = 0$ otherwise.

To sum up, we analyze a mechanism that specifies the following sequence of events:

1. The committee is formed.
2. Committee members decide how much information they want to acquire.
3. Committee members cast a vote.
4. The state of the world is realized.
5. A transfer scheme is applied, and total payoffs are realized.

We assume that individuals always vote in favor of the alternative that is interim most likely, given their signal. If both alternatives are equally likely, they vote according to their signal. When the signal is informative, voting in favor of the other alternative is weakly dominated (in expected terms). When the signal is uninformative (i.e., the only information the agents have is the prior), this assumption rules out implausible equilibria.[13]

Following all the above, the (static) game we consider is one in which all committee members compose the player set and the strategy set of each player is the interval $[0, 1/2]$. We denote this game by $\mathcal{G}^m$, as the committee is made up of $2m + 1$ members. For the analysis, we focus

[12] Abstention in a costly information acquisition setting with continuous signals and cost functions is discussed in McMurray (2013) and Oliveros (2013).

[13] This assumption is of a technical nature and differs from Martinelli (2006). It rules out undesirable equilibria, with or without transfer schemes, such as the one where no agent procures information and all agents vote for a given alternative.

on *symmetric* Nash equilibria, i.e., we assume that all players choose the same information level x, with $x \in [0, 1/2]$.[14] We stress that a citizen chooses the level of information s/he wants to acquire, but s/he does *not* choose the alternative s/he votes for. This follows from the assumption that after each agent i has received a (possibly uninformative) signal, s/he simply follows the recommendation given by the signal.

Finally, we make some assumptions on the cost function $c(\cdot)$ beyond the fact that is strictly increasing in $(0, 1/2)$. First, $c(\cdot)$ is twice continuously differentiable in the interval $(0, \frac{1}{2})$. This assumption is of a technical nature and simply facilitates the analysis. Second, $c(\cdot)$ is strictly convex. This rules out multiplicity of symmetric equilibria with cost sharing. Third, $c(0) = 0$, so acquiring zero information is costless. Fourth, $c'(0) = 0$. This last assumption guarantees the existence of equilibria with positive information acquisition.[15]

4 Cost Sharing

In this section we examine perfect cost sharing, i.e., we consider that for each $i \in \{1, \ldots, 2n+1\}$,

$$v_i = c(x_i) - \frac{1}{2n+1} \cdot \sum_{j=1}^{2m+1} c(x_j), \tag{1}$$

where we assume that $x_j = 0$ if individual j does not belong to a committee. This means that regardless of how much costly information one member of committee acquires, the cost of doing so is distributed equally among all members of the population, not just committee members. Thus, it must be the case that such costs are observable (and contractible). For some applications, observability of costs is a realistic assumption, blockchains, for example. Information costs are also (approximately) observable if members of the committee must produce a written assessment (audit or software tool) about which alternative they think is desirable. This is the case for scientific referees and, often, for members of a hiring committee.

Let us consider the decision of a committee member i about what effort level x_i to exert in order to increase the precision of his/her signal, which comes at cost $c(x_i)$. Due to our focus on symmetric equilibria, let any other member j of the committee choose x, with $x \in [0, 1/2]$. Then, according to Equation (1), the expected payoff of individual i, which depends on variable x_i

[14]We assume that transfers are made from individuals who are not committee members to committee members. This means that a symmetric equilibrium of our game does not necessarily correspond to a (non-symmetric) equilibrium of the game underlying the mechanism in which all individuals of the society can acquire information and all of them have the right to vote.

[15]The mechanisms we propose are still useful if $c'(0) > 0$, as they could be used to increase the pivotality of the committee members in order to generate positive information acquisition in equilibrium. However, we do not address such cost functions or investigate the extent to which our mechanism can improve outcome precision.

(given x), is equal to

$$U_i(x_i|x) = P_x[\text{tie}] \cdot \left(\frac{1}{2} + x_i\right) - \frac{c(x_i) + \sum_{j=1, j\neq i}^{2m+1} c(x)}{2n+1} + \chi. \tag{2}$$

In the above equation, $P_x[\text{tie}]$ is the probability that the vote of all other committee members will yield a tie. This is the only case in which i's vote matters for the outcome and hence for expected utility from the implementation of the decision. The third term in Equation (2), denoted by χ, captures the expected utility of the outcome when there is no tie. This part is independent of x. Accordingly, for all $x_i \in (0, 1/2)$

$$U_i'(x_i) = P_x[\text{tie}] - \frac{c'(x_i)}{2n+1} \tag{3}$$

and

$$U_i''(x_i) = -\frac{c''(x_i)}{2n+1} < 0. \tag{4}$$

When all committee members other than individual i choose information level x, the probability that their votes yield a tie is equal to

$$P_x[\text{tie}] = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{2} - x\right)^m \cdot \left(\frac{1}{2} + x\right)^m & \text{if } m > 0. \end{cases} \tag{5}$$

From Equations (3)–(5), if we take $x_i = x$, we obtain the first-order condition that is necessary and sufficient for a strategy profile $(x, \ldots, x)$, with $x \in (0, 1/2)$, to be a symmetric equilibrium of $\mathcal{G}^m$, namely,

$$\frac{c'(x)}{2n+1} = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m & \text{if } m > 0. \end{cases} \tag{6}$$

Since $c'(0) = 0$, it is clear that the strategy profile in which no citizen acquires information, namely $(0, \ldots, 0)$, cannot be an equilibrium of $\mathcal{G}^m$ for any $m \geq 0$. Then, if $m = 0$, either Equation (6) has one solution, say $x \in (0, 1/2)$, or none. In the former case, the strategy profile $(x, \ldots, x)$ is a symmetric equilibrium. In the latter case, the strategy profile in which all committee members acquire full information, namely $(1/2, \ldots, 1/2)$, is an equilibrium. In either case, $\mathcal{G}^0$ has no more equilibria since $c''(\cdot) > 0$. As for case $m \geq 1$, if we let

$$\theta(x) := \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m - \frac{c'(x)}{2n+1},$$

we then obtain $\theta(0) > 0$, $\theta(1/2) < 0$, and $\theta'(x) < 0$. Thus Equation (6) has a unique solution, which in turn determines the only symmetric equilibrium of game $\mathcal{G}^m$. Henceforth, we use $x_m^*(n)$ to denote the unique symmetric equilibrium of $\mathcal{G}^m$ when the committee is made up of $2m+1$ voters and the population consists of $2n+1$ individuals, with $0 \leq m \leq n$.

Here an important remark is enabling us to connect our results to those of Martinelli (2006). Our setup in this section—but not the one in Section 5—is in a one-to-one correspondence with a setup in which a society of $2m+1$ individuals have the right to vote and each of them must individually bear the information acquisition costs using the cost function $\tilde{c}(x) := c(x)/(2n+1)$. In this case, an interior equilibrium without cost sharing is pinned down by

$$\tilde{c}'(x) = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m & \text{if } m > 0. \end{cases} \tag{7}$$

This means that an increase in the total number of citizens in our setup is equivalent to an across-the-board decrease of information acquisition costs in a setup without cost sharing. We stress that the homogeneous decrease of $\tilde{c}(x)$ for all $x \in [0, 1/2]$ does *not* alter the shape of the information acquisition cost function $\tilde{c}(\cdot)$ compared to $c(\cdot)$. In a similar vein, one could consider a variant of our mechanism in which costs are shared only among members of the committee and not among all members of the society. In such a case, it suffices to consider the cost function $\hat{c}(x) := c(x) \cdot (2m+1)/(2n+1)$ and then the analogue of Equation (7). This and other cost sharing rules are analyzed in Section 6.2.

Focusing on our setup with cost sharing among all individuals of the population, we start by showing that the equilibrium individual information acquisition level is decreasing in the number of members of the committee, and that it typically converges to zero as the number of committee members goes to infinity. Recall that it must be the case that $m \leq n$ at all times, i.e., committee size can never be larger than total population size.

Lemma 1. *For any $m, n \in \mathbb{N}$ such that $m+1 \leq n$, $x^*_{m+1}(n) < x^*_m(n)$.*

Proof. See Appendix. □

Lemma 2. *There is a subsequence $(x^*_m(f(m)))_{m=0}^{\infty}$ of $((x^*_m(n))_{m=0}^{n})_{n=1}^{\infty}$ that converges to zero, provided that $f(m)/\sqrt{m}$ is sub-exponential.*

Proof. See Appendix. □

We stress that for the above lemmas to hold, it suffices for $c(\cdot)$ to be convex (besides the other technical assumptions that we made). On the one hand, Lemma 1 is not surprising. As we increase the number of committee members, the probability that an individual member will be pivotal (and will thus break a tie) becomes smaller, all else being equal. Since $c(\cdot)$ is convex, this translates into a lower value of the information acquisition equilibrium level for each of the committee members. One can then easily verify that for one-member committees ($m = 0$), full information acquisition is not generically attained, at least for low values of n. One example is given in Table 1 below. For their part, committees with more than one agent will never reach full information about the state of the world at the aggregate level, no matter how many members there are in the entire society and even if full information can be attained at a finite (disutility) cost. This is because in these situations, a single individual is never pivotal if all other individuals agree with each other about what the best alternative is and vote accordingly.

On the other hand, because the probability of being pivotal converges to zero as the number of committee members goes to infinity, the information acquisition equilibrium level also goes to zero. For the latter to happen, however, it is essential that committee size relative to total population does not converge to zero too fast, as required by the condition on $f(m)$. Otherwise, due to equal sharing, the effective costs of acquiring information for a single committee member become so small that positive levels of information acquisition can be sustained in equilibrium. For example, consider the extreme case in which m is kept constant at zero while n converges to infinity. Then, for sufficiently large n, there is no interior solution for the maximization problem faced by the only committee member, who will inform himself/herself to the maximum level (i.e., s/he will choose $x^* = 1/2$).

Lemma 1 shows that reducing the size of the committee but keeping total population size constant prompts every committee member to acquire higher levels of information. This is because the chances of being pivotal increase for each committee member, all else being equal. Another possibility is to keep the size of the committee constant and increase the size of the population. Doing so also prompts every committee member to acquire higher levels of information. The reason is that information acquisition costs decline for each member of the committee, all else being equal. This result follows easily from Equation (6) and the fact that $c(\cdot)$ is a strictly convex function and is formalized next without a proof.

Lemma 3. *For any $m, n \in \mathbb{N}$ such that $m \leq n$, $x_m^*(n) < x_m^*(n+1)$.*

Lemmas 1 and 3 identify two ways of increasing the level of information a single member of the committee acquires. The following lemma compares both ways in terms of the extent to which the individual information level increases.

Lemma 4. *For any $m, n \in \{2, 3, \ldots\}$, it holds that $x_{m-1}^*(n) > x_m^*(n+1)$, provided that*

$$n \geq 2m. \tag{8}$$

Proof. See Appendix. □

Consider a society with a committee made up of at least three members but involving fewer than half of the individuals of the society. Lemma 4 shows that, for such a society, the effect on the individual information level acquired of removing two members from the committee (but not from the society) is larger than the effect of adding two individuals to the society (but not to the committee). That is, the pivotality increase that results from having committee size go down from $2m + 1$ to $2m - 1$ dominates the decrease in costs that results from having population size go up from $2n+1$ to $2n+3$. It is worth noting that we cannot relax Condition (8) in Lemma 4. This is shown in the following example:

Example 1. *Consider $m = 4$, $n = 5$, and $c(x) = 4x^2$. Then,*

$$x_3(5) \approx 0.22215 < 0.236881 \approx x_4(6).$$

What are the implications of Lemmas 1–4 for the likelihood of taking the right decision and for welfare? To answer these questions, we focus for simplicity on cost functions of the kind $c(x) = ax^b$, with $a, b > 0$, although our insights extend to more general specifications of the information acquisition cost function. Parameter a measures information acquisition costs in (dis)utility terms. The higher it is, the more costly it is for an individual to inform himself or herself. For its part, parameter b measures the degree of convexity of the cost function. The higher b is, the more convex such a function will be. Note that $c''(1/2) < \infty$, so that full information is possible, and that

$$c''(0) = 0 \Leftrightarrow b > 2. \tag{9}$$

If $c''(0) = 0$, which happens when the information acquisition function is sufficiently convex, then the information acquisition costs are very low for levels of information very close to zero. According to Martinelli (2006) (see also Downs, 1957), this could the case when the costs of "paying a little attention" are low because citizens (or committee members) are *"involuntarily exposed to a flow of political information in the course of everyday activities"*. If $c''(0) > 0$, which happens when the information acquisition function is *not* sufficiently convex, these costs are significantly higher.

First, let us use Q_m to denote the probability that the right alternative is implemented if the committee is made up of $2m+1$ individuals. One can easily verify that

$$Q_m = Q_m(x_m^*) := \sum_{i=m+1}^{2m+1} \binom{2m+1}{i} \cdot \left(\frac{1}{2} + x_m^*\right)^i \cdot \left(\frac{1}{2} - x_m^*\right)^{2m+1-i}. \tag{10}$$

It then turns out that Q_m is maximal for $m = 0$ if the cost function is moderately convex.

Theorem 1. *Let the information acquisition cost function be $c(x) = ax^b$, with $0 < a$ and $1 < b \leq 2$. Then $Q_0 > Q_m$ for all $m > 0$.*

Proof. See Appendix. □

The above result shows that the probability of choosing the right alternative for $m = 0$ (in which case the committee is made up of one individual) is larger than for any $m > 0$ (in which case the committee consists of at least three individuals). Theorem 1 thus suggests that as far as cost functions that are moderately convex are concerned and provided that cost sharing is feasible, the committee should be made up of only one member *if* the goal is to implement the right alternative with the highest possible probability.[16] The proof of Theorem 1 is based on finding a suitable *upper* bound derived for the function $Q_m(x)$.

The next proposition shows that the result of Theorem 1 is *not* a property of information acquisition cost functions being convex.

Proposition 1. *Let the information acquisition cost function be $c(x) = ax^b$, with $a > 0$ and $b > 2$. Then,*

(i) there exist $a^*(b) > 0$ and $m^*(b) > 0$ such that for any $a > a^*(b)$ and $m > m^*(b)$,

$$Q_0 < Q_m.$$

(ii) if, moreover, $2^{\frac{b}{b-1}} < 3$, then there exists $a^*(b) > 0$ such that

$$Q_0 < Q_1 \Longleftrightarrow a > a^*(b).$$

Proof. See Appendix. □

The proof of Proposition 1 is based on a suitable *lower* bound for function $Q_m(x)$. On the one hand, Part *(i)* of Proposition 1 is concerned with the comparison between a one-member committee and committees with sufficiently many members, and it is therefore only informative in situations where the latter are feasible. In such cases, a *sufficient* condition for large committees to outperform a one-member committee in the implementation of the correct alternative is that parameter b be larger than two—so $c''(0) = 0$, see (9)—and parameter a be sufficiently large. The former condition requires that the information acquisition cost function is sufficiently convex, while the latter condition requires information costs to be sufficiently large in their extent.

On the other hand, note that $2^{\frac{b}{b-1}} < 3$ if and only if $b > \frac{\log_2 3}{\log_2 3 - 1} \approx 2.71$. Using the equivalence in (9), it then follows from Part *(ii)* of Proposition 1 that for Q_0 to be larger than Q_1 it is not necessary that $c''(0) = 0$. This result is important when committees must be small, say, for operational reasons. For instance, this is the case of reviewers in the refereeing process, all of which need to produce a report that will be read by the editor in charge.

To sum up, Theorem 1 and Proposition 1 suggest that, in general, moderate convexity of the information acquisition cost function is not only *sufficient* for one-member committees to be optimal regarding implementation of the correct alternative, but it is also *necessary* if information acquisition costs are large.

It is interesting to compare these results with Martinelli (2006). To do so, let $c(x) = ax^b$. First, consider that $b \leq 2$. Martinelli (2006) shows that the probability that the right alternative is implemented by a large population of individuals is bounded away from one. Theorem 1

[16]Of course, there are reasons (outside of our model) to be concerned with one-member committees, such as the possibility that the only committee member acts as a dictator.

complements this result for *all* population sizes by showing that full delegation of voting power to a one-member committee can overcome the difficulties that normal voting has in implementing the right alternative with (arbitrarily) high probability. If costs can be further shared among all members of the population, the only committee member attains much higher levels of information acquisition and even inform herself/himself perfectly if the population is very large (and full learning is possible). Second, consider that $b > 2$, which guarantees $c''(0) = 0$ and disregards all information acquisition functions that are either linear or moderately convex. Martinelli (2006) shows that despite the fact that the level of information acquired by each individual goes to zero, the probability of selecting the right alternative converges to one as the number of voters tends to infinity. Our results offer new insights. If committee size cannot become arbitrarily large, the turning point determining whether or not delegating all voting power to a single voter is optimal (in terms of the probability of implementing the right alternative) may not be $c''(0) = 0$.[17]

It is also useful to interpret Theorem 1 and Proposition 1, as well as Lemmas 1–4, in the context of the Condorcet Jury Theorem. According to the latter, if jury size increases, the probability that the right alternative is chosen converges to one, provided that each member is not completely uninformed. This does not necessarily occur in our model because we endogenize the information acquisition level, which becomes smaller as we increase committee size (i.e., jury size), all else being equal.

In our previous analysis, we have focused on the probability of choosing the right alternative and disregarded the societal costs of acquiring the information that are necessary to achieve this. Incurring very large information costs might be objectionable from a welfare perspective. To discuss whether committee size should be chosen from a welfare perspective as prescribed by Theorem 1 and Proposition 1, consider now the expected average per-capita utility under our mechanism, which we define as *welfare* and denote as[18]

$$W_m = Q_m - \frac{2m+1}{2n+1} \cdot c(x_m^*). \tag{11}$$

The above definition of welfare takes into account both the probability of selecting the right alternative and the information acquisition costs that have to be incurred. To find the optimal size of the committee from the perspective of a social planner seeking to maximize welfare, we have to solve

$$\arg\max_{m \in \mathbb{N}} W_m.$$

Finding the general solution to the above problem is difficult. Yet we can prove a result that shows that under cost sharing maximizing the probability of choosing the right alternative and maximizing welfare can lead to different committee size.

[17] As discussed in Martinelli (2006), welfare-optimal committee size depends generically on the shape of $c(\cdot)$ for all information levels.

[18] To avoid cumbersome notation, we drop the dependence on n.

Theorem 2. *Let the information acquisition cost function be $c(x) = ax^b$, with $b > 1$. Then there is $a^*(b) > 0$ such that if $0 < a \leq a^*(b)$, $W_0 > W_m$ for any integer $0 < m \leq n$.*

Proof. See Appendix. □

Accordingly, for a given population, if costs are very low, choosing a committee of one member maximizes welfare, regardless of the degree of convexity of the information acquisition function as captured by parameter b. This contrasts with the choice of committee size when the goal is to maximize the probability of choosing the right alternative regardless of the costs of acquiring information. As shown in Theorem 1 and Proposition 1, the value of parameter b capturing the degree of convexity of the information acquisition function can have major consequences for optimal committee size if the goal is to maximize the probability of implementing the right alternative no matter the costs needed to acquire information. Depending on the context, information acquisition costs may be relevant or not for the design of the committee.

As an immediate corollary from Theorem 2 we obtain the following result by using Equation (7).

Corollary 1. *Let the information acquisition cost function be $c(x) = ax^b$, with $b > 1$. Then, for every $\delta > 0$ there is $\tilde{n}(\delta)$ such that if $n \geq \tilde{n}(\delta)$,*

$$|W_0 - W_m^*(n)| < \delta,$$

where

$$W_m^*(n) = \max_{0 \leq m \leq n} W_m.$$

That is, for arbitrarily large populations, delegating to a committee made up of one member yields the highest welfare (asymptotically) that any committee size can attain. In fact, doing so implements the right alternative with probability one, while total—and, hence, average—costs converge to zero with population size.

Next, we consider an example designed to illustrate the *extent* of the effect of committee vote delegation and cost sharing on information acquisition levels. The number of voters is $2n+1 = 21$ and the cost function is $c(x) = 30x^2$ (see Table 1). We see that welfare is maximal when the size of the committee is minimal, i.e., when $m = 0$ and the committee consists of only one member. Moreover, the likelihood of selecting the right alternative is monotonically decreasing in the size of the committee, and so is welfare.[19,20] The example also illustrates the differences between

[19] A proof of the general statement that welfare is maximized for one-member committees would require better lower and/or upper bounds for function $Q_m(x)$. The bounds used in the proofs of Theorem 1 and Proposition 1 do not suffice.

[20] It is not necessarily the case that Q_m is either monotonically increasing or monotonically decreasing as a function of m, even when $Q_0 < 1$. For instance, let $n = 7, a = 48, b = 4.1$. Then, one can verify that $(Q_m)_{m=0}^7 = (0.935889, 0.897275, , 0.906207, 0.914927, 0.922109, 0.928004, 0.932914, 0.937072)$. In this same example, $(W_m)_{m=0}^7 = (0.829575, 0.827072, 0.84285, 0.85566, 0.865888, 0.874251, 0.881252, 0.887226)$. That is, welfare is neither monotonic.

our setup and that of Martinelli (2006). Assume that m (and, hence, n) becomes arbitrarily large but that no cost sharing is possible and all individuals vote simultaneously after each of them has acquired the equilibrium information level. Then the probability that the right alternative is implemented converges to 0.69. With 21 members and cost sharing, by contrast, such probability is 0.85. The difference between the two cases arises because *(a)* pivotality is higher in a committee than in the entire population and *(b)* information acquisition costs are spread across all citizens.

Committee size ($2m+1$)	Prob. of right alternative (Q_m)	Information level (x^*_m)	Welfare (W_m)
1	0.85	0.35	0.675
3	0.728586	0.157611	0.622124
5	0.711866	0.117219	0.61372
7	0.705144	0.097392	0.610292
9	0.701521	0.085089	0.608433
11	0.699256	0.0765101	0.607268
13	0.697706	0.0700913	0.606469
15	0.696579	0.0650562	0.605887
17	0.695723	0.0609701	0.605444
19	0.69505	0.0575681	0.605096
21	0.694508	0.0546785	0.604815

Table 1: Cost function $30x^2$ and total number of individuals 21.

Finally, let $(x, \dots, x)$ be an equilibrium of our setup with vote delegation and cost sharing, Then consider the case in which no delegation occurs and information acquisition costs are fully incurred privately. Can there be an asymmetric equilibrium in the latter setup in which $2m+1$ individuals out of the total $2n+1$ individuals of the population incur information acquisition costs x and the latter do not inform themselves and either abstain or vote according to their (uninformative) signal? Since $c'(0) = 0$, the answer is trivially negative. This means that vote delegation to a committee (and cost sharing) modifies the outcomes that can be attained through normal elections, and we have argued that these two features combined can in fact *significantly* improve upon normal elections.

5 Reward Schemes

In this section we assume that the costs incurred by individuals to acquire information are private. This makes it impossible to use the cost sharing scheme that we have analyzed in the previous section, as *ceteris paribus* every individual has the incentive to claim the highest possible cost. In such circumstances, alternative ways that can induce members of the committee to acquire information have to be found. If voting takes place privately, as is customary in democratic societies, the only public information that is available is the voting tally, i.e., the number of votes for alternatives A and B. As we argued in the Introduction, we proceed with the assumption that conditions on whether the correct state of the world occur are *not* possible.[21]

In the following, we show that adequately contracting on the vote tally suffices to prompt the committee members to either inform themselves fully or ensure that the aggregate level of information converges to the highest possible one. Roughly speaking, contracting on such information guarantees that, on equilibrium, the incentives linked to pivotality are either zero or go to zero with committee size. The only incentives that matter then are those linked to the side-payments contingent on vote tally, which then lead to desirable outcomes.

To elaborate, let k denote henceforth the vote tally difference between alternatives A and B, with $k \in \{-2m-1, \ldots, 2m+1\}$. That is, if $k > 0$, alternative A received k more votes from the committee members than alternative B in the voting round. For each m we then consider functions of the following type:

$$\begin{aligned} t^m : \{-2m-1, \ldots, 2m+1\} &\to \mathbb{R} \\ k &\to t^m(k), \end{aligned}$$

with

$$t^m(k) = t^m(-k) \text{ for } k \in \{-2m-1, \ldots, 2m+1\}\,. \tag{12}$$

Any such function assigns a (positive or negative) reward to any outcome based on the absolute difference in terms of votes between alternatives, and hence regardless of which alternative receives the most votes. We stress that because there is no abstention, only odd numbers of the set $\{-2m-1, \ldots, 2m+1\}$ can occur as vote tally difference.

In Section 4 we considered transfer schemes based on the observed information acquisition costs, and then analyzed the resulting mechanisms based on such transfer schemes. Here we consider transfer schemes which are *not* based on the unobservable information acquisition costs. More specifically, for each $i \in \{1, \ldots, 2n+1\}$, we let

$$v_i = \begin{cases} t^m(k) & \text{if } i \text{ is committee member,} \\ -\frac{2m+1}{2n+1} \cdot t^m(k) & \text{if } i \text{ is not committee member.} \end{cases} \tag{13}$$

That is, if the vote difference between alternatives is k in absolute value, all the members of the committee are given the same reward $t^m(k)$. If $t^m(k) > 0$, this reward is financed homogeneously by the rest of the population. If $t^m(k) < 0$, the committee members subsidize the rest of the population.

Next we analyze our mechanism—vote delegation to a committee plus transfers—when the transfer scheme is defined as in (13). The latter is generically called *threshold scheme*, or *TS*

[21] Our analysis sidesteps career concerns and places the focus on the incentives for acquiring costly information for the common good.

for short. The only information a TS uses beyond the vote tally is whether a given individual belongs to the committee or not. We show that if we choose $t^m(\cdot)$ in a certain way, these incentives induce individuals to acquire sufficiently informative signals about the state of the world so that the right alternative is implemented with probability (close to) one. This means that a TS can act as a device coordinating committee members to acquire a higher level of information.

With a TS, the expected utility of any member i of the committee (for a given size $2m+1$) when s/he chooses information acquisition level $x_i \in [0, 1/2]$ and all the other $2m$ members of the committee choose $x \in [0, 1/2]$, as demanded by our notion of symmetric equilibrium, is

$$\begin{aligned} U_i(x_i|x) = & \binom{2m}{m} \cdot \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m \left(\frac{1}{2}+x_i\right) - c(x_i) + \chi \\ & + \sum_{k=1}^{m} \binom{2m}{m+k} \left(\frac{1}{2}+x\right)^{m+k} \left(\frac{1}{2}-x\right)^{m-k} \left[\left(\frac{1}{2}+x_i\right) \cdot t^m(2k+1) + \left(\frac{1}{2}-x_i\right) \cdot t^m(2k-1)\right] \\ & + \sum_{k=1}^{m} \binom{2m}{m+k} \left(\frac{1}{2}+x\right)^{m-k} \left(\frac{1}{2}-x\right)^{m+k} \left[\left(\frac{1}{2}+x_i\right) \cdot t^m(2k-1) + \left(\frac{1}{2}-x_i\right) \cdot t^m(2k+1)\right] \\ & + \binom{2m}{m} \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m \cdot t^m(1). \end{aligned} \tag{14}$$

where χ is independent of x_i. If $t^m(k) = 0$ for all $k \in \{0, \ldots, m\}$, Equation (14) reduces to the case of standard voting with turnout equal to $2m+1$.[22] Recall that we are assuming (12). Therefore, if we define

$$\phi^m(k) := t^m(2k+1) - t^m(2k-1) \text{ for all } k \in \{0, \ldots, m\}$$

and

$$\phi^m(0) := t^m(1) = t^m(-1),$$

it follows that for all $x_i \in (0, 1/2)$,

$$\begin{aligned} U_i'(x_i|x) = & \binom{2m}{m} \cdot \left(\frac{1}{4}-x^2\right)^m - c'(x_i) \\ & + \sum_{k=1}^{m} \binom{2m}{m+k} \cdot \phi^m(k) \cdot \left(\frac{1}{4}-x^2\right)^{m-k} \cdot \left[\left(\frac{1}{2}+x\right)^{2k} - \left(\frac{1}{2}-x\right)^{2k}\right] \end{aligned}$$

and

$$U_i''(x_i|x) = -c''(x_i) < 0.$$

Accordingly, a necessary and sufficient condition for a strategy profile $(x, \ldots, x)$, with $x \in$

[22]Note that this is the case analyzed by Martinelli (2006).

$(0, 1/2)$, to be an equilibrium of $\mathcal{G}^m$ is that

$$c'(x) = \overbrace{\binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m}^{:=A(x)} \\ \underbrace{+ \sum_{k=1}^{m} \binom{2m}{m+k} \cdot \phi^m(k) \cdot \left(\frac{1}{4} - x^2\right)^{m-k} \cdot \left[\left(\frac{1}{2} + x\right)^{2k} - \left(\frac{1}{2} - x\right)^{2k}\right]}_{:=B(x)}. \tag{15}$$

If $m > 0$ two important observations follow readily from the above expression. First, since $c'(0) = 0$, if $\phi^m(k) = 0$ for all $k \in \{1, \ldots, m\}$, then Equation (15) has exactly one solution, say x. The reason is that $c'(\cdot)$ is increasing and $A(\cdot)$ is decreasing. In fact, $(x, \ldots, x)$ is the only symmetric equilibrium of $\mathcal{G}^m$. This captures one-round voting within a committee. When $\phi^m(k) \neq 0$ for some $k \in \{1, \ldots, m\}$, by contrast, neither uniqueness nor existence of (interior) symmetric equilibria are guaranteed in general. Clearly, because $c'(0) = 0$ and $A(0) + B(0) > 0$, the strategy profile where no committee member acquires information, namely $(0, \ldots, 0)$, cannot be an equilibrium. If

$$A(x) + B(x) > c'(x) \text{ for all } x \in (0, 1/2), \tag{16}$$

the only symmetric equilibrium of $\mathcal{G}^m$ is the strategy profile in which all committee agents acquire full information, namely $(1/2, \ldots, 1/2)$. If Equation (16) does not hold, then there is at least one (interior) symmetric equilibrium $(x, \ldots, x)$, with $x \in (0, 1/2)$. But there may be other equilibria, including $(1/2, \ldots, 1/2)$.

The second important observation from (15) is that a TS matters for equilibrium up to a constant, since the value of $\phi^m(0)$ does not affect the incentives of the committee members to acquire information. This is because in the event that a citizen breaks a tie, s/he will create a difference of one vote no matter which alternative s/he votes for. While this is relevant for the utility citizens derive from the alternative implemented, it does not affect the incentives to be informed conditional on being pivotal. This is reminiscent of the swing voters' curse (Feddersen and Pesendorfer, 1996; Herrera et al., 2019a,b). Henceforth, for each integer $m \geq 0$, we consider functions such as

$$\begin{aligned} \phi^m : \{0, 1, \ldots, m\} &\to \mathbb{R} \\ k &\to \phi^m(k). \end{aligned}$$

On the one hand, $\phi^m(k)$, with $k > 0$, rewards (or punishes) a *marginal vote* even if it does not break a tie. Such rewards may arise in normal elections if voters care about the victory margin, and they might be significant (Herrera et al., 2019b). In our case, these rewards are chosen by design. Breaking a tie, on the other hand, is rewarded (or punished) by $\phi^m(0)$, which is added to the benefits associated with pivotality.

For the subsequent analysis we proceed with a general information acquisition cost function $c(\cdot)$ and distinguish two cases, depending on the value of $c'(1/2)$.

5.1 Full information

First we consider the case where $c'(1/2) < \infty$. This means that it is theoretically possible to find out with full precision what the right alternative is (at a finite cost). For this case, we start by considering the following reward scheme:

$$\phi^m(k) = \begin{cases} c'(1/2) & \text{if } k = m, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The TS defined in (17) only rewards decisions that are reached unanimously within the committee. If they reach unanimity, each of the committee members is given a reward that amounts to $c'(1/2)$. A unanimous decision can opt for the correct alternative or the incorrect one. Unless all the members of the committee inform themselves perfectly, both options will occur in equilibrium.

We start by showing the following result:

Proposition 2. *Suppose that the reward scheme is defined by* (17) *for a given integer* $m \geq 1$. *Then the strategy profile in which all members of the committee acquire complete information,* $(1/2, \dots, 1/2)$, *is an equilibrium of* $\mathcal{G}^m$.

Proof. See Appendix. □

As a matter of fact, from the proof of Proposition 2 one immediately sees that provided that $c'(1/2) < \infty$, a necessary and sufficient condition for $(1/2, \dots, 1/2)$ to be an equilibrium of $\mathcal{G}^m$ is

$$\phi^m(m) \geq c'\left(\frac{1}{2}\right). \tag{18}$$

That is, the TS must reward the marginal effort of acquiring another piece of information beyond x, which is highest when x is (infinitely) close to $1/2$ since $c(\cdot)$ is convex. Hence, full information can be attained in equilibrium if and only if inequality (18) holds. Assuming that $\phi^m(k) \geq 0$ for all $k \in \{0, \dots, m-1\}$, this means that the transfers that members who are not part of the committee must pay to the members of the committee in order to ensure full information acquisition can never be lower than

$$\frac{2m+1}{2(n-m)} \cdot c'\left(\frac{1}{2}\right). \tag{19}$$

In Proposition 2 we have assumed $m \geq 1$, so the committee consists of at least three members. If $m = 0$, and hence the committee consists of only one member, the reward scheme defined in (17) has no bearing on the unique committee member's calculus for maximizing his/her utility (see Equation (14)), which is accordingly

$$max_{x\in[0,1/2]}[2x - c(x)].$$

The solution to the above problem is generically different from 1/2. Why is the case with more committee members different? If there are at least three members in the committee, all the incentives for becoming informed due to (voting) pivotality disappear in the strategy profile of Proposition 2 is played. This is because all individuals inform themselves perfectly, so they all vote for the right alternative. As a consequence, no single individual can change the voting outcome (which uses the majority rule). The only incentives left for informing oneself are those linked to the TS. If $\phi^m(m) \geq c'(1/2)$, the convexity of $c(\cdot)$ guarantees that the strategy profile of Proposition 2 is an equilibrium. Note that in this equilibrium all three members of the committee inform themselves fully, so the equilibrium is inefficient in that duplicate information is acquired.

While Proposition 2 ensures that full information acquisition is an equilibrium, there may be other equilibria with partial information acquisition. This is shown next and warns us that ill-designed transfer may have serious consequences for outcomes.

Example 2. *Consider that* $c(x) = 3x^3 + 2x^2$ *and* $m = 2$*, and assume that the TS defined in* (17) *is used. Then game* $\mathcal{G}^m$ *has two equilibria: one equilibrium in which all citizens choose* $x = \frac{1}{2}$ *and another equilibrium in which all citizens choose* $x \approx 0.24$*. Figure 1 illustrates this multiplicity of equilibria by plotting function* $c'(x) - (A(x) + B(x))$*—see Equation* (15)*—when the TS defined in* (17) *is used, as well as when other rewards for unanimity are considered.*

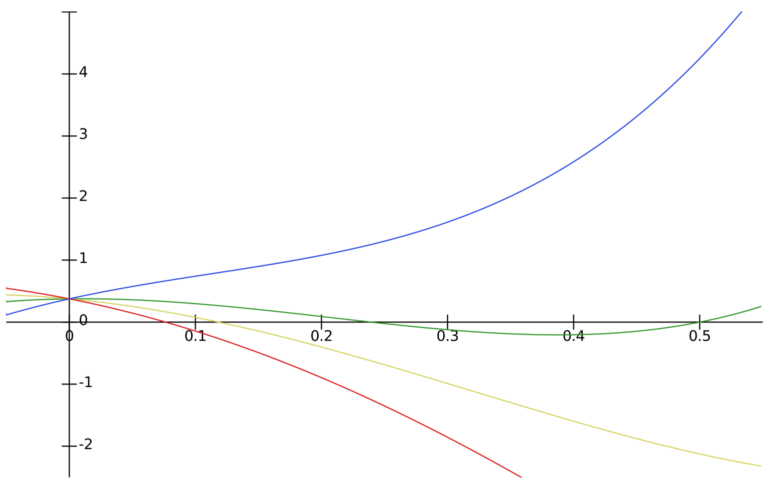

Figure 1: Function $(A(x)+B(x))-c'(x)$—see Equation (15)—when $\phi^2(0) = \phi^2(1) = 0$ and the reward for unanimity, $\phi^m(k)$, takes different values. The blue curve corresponds to $\phi^2(2) = 2c'(\frac{1}{2})$, the green curve corresponds to $\phi^2(2) = c'(\frac{1}{2})$ (see Example 2), the yellow curve corresponds to $\phi^2(2) = \frac{1}{2}c'(\frac{1}{2})$, and the red curve corresponds to no rewards, i.e., $\phi^2(2) = 0$. An interior equilibrium exists when the curve intersects the x-axis between 0 and 1/2. A full information equilibrium exists when the curve is not negative at $x = 1/2$.

To deal with uniqueness of (symmetric) equilibria, consider now the following reward scheme:

$$\phi^m(k) = \begin{cases} \frac{4^{m-1}}{m} \cdot r_m & \text{if } k = m, \\ 0 & \text{otherwise,} \end{cases} \tag{20}$$

where $(r_m)_{m=0}^n$ is a certain increasing finite sequence that depends on $c(\cdot)$ satisfying that the infinite sequence $((r_m)_{m=0}^n)_{n=0}^\infty$ is bounded from above, say by some $r > 0$, and that for all $m \geq 1$,[23]

$$\frac{4^{m-1}}{m} \cdot r_m \geq c'\left(\frac{1}{2}\right).$$

We obtain the following result:

Theorem 3. *Suppose the reward scheme is defined by* (20) *for a given integer* $m \geq 1$. *Then the strategy profile in which all members of the committee acquire complete information,* $(1/2, \ldots, 1/2)$, *is the only symmetric equilibrium of* $\mathcal{G}^m$.

Proof. See Appendix. □

The above theorem shows that regardless of the size of the committee, one can always find finite rewards for its members that induce *each* of them to acquire full information. While this is intuitive for arbitrarily large rewards, Theorem 3 does more, as it gives an upper bound to the reward that must be given to each committee member for reaching a unanimous decision. Using the fact that $(r_m)_{m\geq 1}$ is bounded from above by r, the bound to individual rewards is

$$\frac{4^{m-1}}{m} \cdot r.$$

This means that the amount that each individual who is *not* a member of the committee needs to contribute is, in turn, bounded by

$$\frac{4^{m-1} \cdot r}{2(n-m)}.$$

Hence, as long as m can be chosen so that $m = \Theta(\ln n)$, individual contributions to the rewards for committee members do not grow unbounded. This means that if full information acquisition is possible, a *sufficient* condition for committees to implement the correct alternative with probability one through a feasible reward scheme is that the size of the committee grow logarithmically with the total population. This is a result that is valuable from a positive perspective, since committees formed from a parent body may have many different sizes based on a variety of rationales. On the other hand, it is straightforward to see that as far as the TS defined in (20) is concerned, it is always preferable from a welfare perspective to choose $m = 1$ over any $m > 1$. The reason is that both options set committees that implement the correct alternative

[23] See the proof of Theorem 3 for details about sequence $((r_m)_{m=0}^n)_{n=0}^\infty$.

with probability one, but $m = 1$ involves the lowest aggregate information costs and hence the lowest transfers, all else being equal.[24]

In the following we show that it is possible to compensate the committee members for the costs associated with acquiring full information and, at the same time, attain the bound given in (18). To this end, consider the following reward scheme:

$$\phi^m(k) = \begin{cases} c'\left(\frac{1}{2}\right) + \varepsilon & \text{if } k = m, \\ t_m > 0 & \text{if } k = k', \\ -t_m < 0 & \text{if } k = 0, \\ 0 & \text{otherwise}, \end{cases} \tag{21}$$

where $k' \in \{1, \ldots, m-1\}$ and $\varepsilon > 0$, and t_m is a parameter that needs to be determined. Note that this requires $m \geq 2$, i.e., the committee must consist of at least five members.

We obtain the following result:

Proposition 3. *Suppose that the reward scheme is defined by* (21) *for a given integer* $m \geq 2$*. Then the strategy profile in which all members of the committee acquire complete information,* $(1/2, \ldots, 1/2)$*, is the only symmetric equilibrium of* $\mathcal{G}^m$ *provided that parameter* t_m *is chosen to be sufficiently large.*

Proof. See Appendix. □

The above result guarantees that the bound defined by (18) is attainable in the limit, so it is (asymptotically) tight if we take $\varepsilon \to 0$. This means, in turn, that there is no other TS that is more efficient than the one defined by (21) given that, in any equilibrium, *(i)* it implements the right alternative with higher probability, *(ii)* it entails lower transfers to committee members that cover their information acquisition costs, and *(iii)* it gives zero transfers if unanimity is short by two votes. The latter occurs if one individual deviates from full information or makes a mistake, and can be seen as a fairness property that ensures that unilateral deviations from the desired outcome have dramatic consequences. In fact, note that we can set $k' = 1$, in which case the committee members must only pay t_m if the difference between the votes cast for either alternative is one. This is very unlikely for large committees even when its members make mistakes. It remains for future research to establish whether the same efficiency result can be attained without fines (i.e., without the possibility of committee members making monetary transfers to the rest of the population).

We emphasize that to ensure the outcome of Proposition 3, the committee must consist of at least five members. Then the TS defined by (21) works by setting very large punishments if slim

[24]If there are arbitrarily low, but positive deadweight redistribution costs, transfers should be kept to a minimum, all else being equal.

majorities are reached. While this will not be the case on the equilibrium path (of the unique equilibrium), such a promise is crucial off the equilibrium path to make sure that other equilibria do not exist beyond the full information acquisition equilibrium.

Proposition 3 bears some resemblance to a finding by Persico (2004), but it differs in some important respects.[25] Both results show that arbitrarily large rewards or punishments can trivially ensure full information, provided that it is feasible. This is because in such cases, incentives related to pivotality vanish altogether (in the limit), and individuals focus entirely on avoiding such punishments. While in Persico (2004) these large rewards must be given in equilibrium, we show that large punishments are only needed off equilibrium. Proposition 3 can then be seen as a complement of Persico (2004) in that it determines a tight bound for the transfers to be given to the committee members, and it does so for arbitrary information acquisition functions. Another noteworthy difference from Persico (2004) is that we consider transfers between the committee members and the rest of the population, and not between the majority members of the committee and the minority members of the committee.

As set out above, the extent of transfers also plays an important role not only for welfare but also for the incentives of citizens to participate in the mechanism. Recall that for a given $m \geq 1$, the committee consists of $2m + 1$ members. Let us now assume that each of the committee members is given a positive transfer in equilibrium, say r_m, satisfying

$$r_m \geq c'\left(\frac{1}{2}\right). \tag{22}$$

Because of (13), this means that every other individual in the society must incur a disutility equal to $-\frac{(2m+1)r_m}{2(n-m)}$. Then consider a mechanism based on a TS that guarantees the unique full information equilibrium outcome.[26] In such a case, every individual of the population expects utility 1 from the alternative being implemented.

We proceed on the assumptions that *(i)* not participating in the mechanism requires abandoning the population altogether, which yields utility $u < 1$, and *(ii)* refusing to be a committee member but not a member of the population simply entails another citizen being a committee member. Part *(i)* prevents individuals from free-riding completely on the alternative eventually implemented. In democratic societies, to name but a paramount example of where our mechanism can be applied, taxation can be enforced by law, so participation constraints can be ignored. Part *(ii)* ensures that leaving the mechanisms does not affect the probability that the right alternative is implemented, and it therefore provides the higher incentives (if any) for a citizen who is a member of the committee to exit the committee (but not the population), all else being equal in the case of full information.

[25] See Section 7 in Persico (2004).

[26] A limit argument based on the logic for full information equilibria can be applied for the case of asymptotically full information equilibria.

Accordingly, for those who are not committee members to be willing to participate in the mechanism and not leave the population altogether, the interim participation condition (i.e., the participation condition after the committee has been constituted but before the state of the world is realized) is

$$u < 1 - \frac{2m+1}{2(n-m)} \cdot r_m.$$

The above individual rationality condition is satisfied if n is large enough for a given $m \geq 1$. As for the committee members, they neither want to exit the committee nor the population from an interim perspective. From (22) it follows that

$$1 - c\left(\frac{1}{2}\right) + r_m \geq 1 + c\left(\frac{1}{2}\right) > 1 > u.$$

The first inequality is due to the fact that $c'(\cdot)$ is increasing and $c(0) = 0$, so

$$\frac{1}{2} \cdot c'\left(\frac{1}{2}\right) = c(0) + c'\left(\frac{1}{2}\right) \cdot \frac{1}{2} > c\left(\frac{1}{2}\right).$$

Finally, from an ex ante perspective (i.e., before the committee is formed), it is hence also clear that all individuals will want to participate in the mechanism, provided of course that the population is large enough.

5.2 Partial information

Next we consider $c'(1/2) = \infty$, in which case one single individual cannot inform himself or herself perfectly about the state of the world. This implies, in turn, that no population with a finite number of individuals can learn about the state of the world with probability one. Accordingly, only asymptotic results are possible. For this case, we assume that

$$c''(0) < \infty. \tag{23}$$

This (weak) condition guarantees that positive levels of information acquisition can in principle be attained. Then, for any $\varepsilon > 0$, we consider the following reward scheme:

$$\phi^m(k) = \begin{cases} m^{\frac{1}{2}+\epsilon} \cdot g_n(m) & \text{if } k = 1, \\ 0 & \text{otherwise,} \end{cases} \tag{24}$$

where for all n, with $m \leq n$,

$$g_n(m) \leq g_n(m-1) < \infty. \tag{25}$$

and

$$\lim_{m,n\to\infty} g_n(m) = c''(0) < \infty. \tag{26}$$

The TS just defined rewards equally any difference in votes that is larger than one. We refer to the proof of Proposition 4 (see below) for more details. Here it suffices to note that thanks to (23) the reward scheme defined in (24) is well defined. If (23) did not hold, then (25) and (26) would require that committee members receive an infinite amount of money.

The next result shows that the above reward scheme guarantees that, as m (and hence n) goes to infinity, the mechanism chooses the right alternative with probability one asymptotically. Because we are concerned with a limit result, we assume that for any given $m \in \mathbb{N}$ such that the committee consists of $2m+1$ members, total population amounts to $2f(m)+1$, with $f(m) \geq m$. Then statements about convergence when m grows require n to grow exactly as described by $f(m)$. In particular, we can assume that

$$\lim_{m\to\infty} \frac{m}{f(m)} \cdot m^{\frac{1}{2}+\varepsilon} = \lim_{m\to\infty} \frac{m^{\frac{1}{2}+\varepsilon}}{\frac{2f(m)+1}{2m+1}} = \lambda, \tag{27}$$

with $0 \leq \lambda < 1$. In particular, relative committee size converges to zero as total population grows. Assuming (27), we also obtain

$$0 \leq \lim_{m\to\infty} \frac{(2m+1)\cdot\phi^m(1)}{2(f(m)-m)} = \lambda \cdot c''(0) < c''(0),$$

where the last inequality holds due to (23). The latter condition specifies the (limit) amount that individuals who are not members of the committee need to pay in the case of very large populations when the reward scheme defined by (24) is used.

We can now present the next result.

Theorem 4. *Assume that* (23) *holds and that the reward scheme is defined by* (24) *for any integer* $m \geq 1$. *Then, for any sequence* $(x_m(f(m)))_{m\geq 1}$ *where* $(x_m(f(m),\dots,x_m(f(m))$ *is a symmetric equilibrium of* $\mathcal{G}^m$,

$$\lim_{m\to\infty} Q_m = 1.$$

Proof. See Appendix. □

That is, although the information level acquired in any symmetric equilibrium tends to zero as committee size tends to infinity, it remains large enough to ensure that the probability of the right alternative being chosen by a majority within the committee converges to one. This result holds regardless of the relative size of the committee compared to total population. Yet, in the particular case where $n = f(m)$, the corresponding TS sets transfers that are bounded, regardless of the size of the committee. For any information acquisition cost function $c(\cdot)$, adequate reward schemes can therefore be used to ensure that large populations attain (close to) full information.

As for welfare, one can easily verify from the proof of Theorem 4 that provided the committee is sufficiently large in absolute terms (but not in terms relative to total population), then the right alternative is implemented with any desirably high probability. Moreover, if we assume (27) individual transfers made by agents who do not belong to the committee converge to zero if we take arbitrarily large populations and keep the size of the committee constant.[27] This means that there is no other mechanism of the type considered in this section that is more (asymptotically) efficient for large populations than the one that uses the TS defined by (24).

6 Assessment Voting and Other Cost Sharing Rules

In this section we do two things. First, we discuss how to take account of the fact that in some circumstances citizens cannot be deprived of their voting rights. Second, we discuss different cost sharing rules when information acquisition costs are observable and contractible.

6.1 Two voting rounds

Suppose that, in order to implement an alternative, every individual must be given the right to vote. Departing from the setup in the previous sections, *one* possible way to do so is for individuals who are not part of the committee to (simultaneously) vote in a voting round after the result of the first voting round by committee members has been made public. The votes collected by each alternative in both rounds are added, and the alternative with more votes is implemented (recall that there cannot be ties if all citizens vote). This mechanism is called Assessment Voting (in short AV, Gersbach et al., 2019). An additional assumption is that committee members are chosen randomly from the population. AV is particularly appealing in our setup when standard democratic desiderata—such as one person, one vote—need to be imposed on the voting procedure.

We claim that if a strategy profile $(x, \ldots, x)$, with $x \in (0, 1/2]$, is an equilibrium of the *static* game where only committee members vote, the strategy profile in which *(i)* all members of the committee choose x information acquisition and *(ii)* all other citizens choose zero information acquisition and vote for the alternative that collected more votes in the first round is a sequential equilibrium of the *dynamic* game underlying AV, provided that total population is sufficiently large and that any individual with the same information in the second round votes in favor of the same alternative (with probability one). This means that with AV in the case of very large population, committee members become the only experts in the population. The citizens who are not committee members anticipate that they will not be pivotal and do not acquire any information. This result on AV holds both when cost sharing (see Section 4) and TS (see Section 5) are used for the first voting round. As for the second voting round in AV, we rule out the possibility of side-payments.[28]

[27] In this case, λ as defined in (27) is arbitrarily close to zero.

[28] The payments made in the first voting round are sunk when the second voting round starts.

We now need to show that the above claim is correct. Assuming behavior of the second-round citizens as described above, the problem faced by the first-round citizens—the (expert) committee members—is the same as in the baseline setup with only one round. This is because the right alternative is implemented if and only if it obtains the most votes in the first voting round. As for the second round, note that the individuals with the right to vote in this round must use Bayes' rule to find the posterior probability that alternative A is the right one. This posterior belief, denoted by q, depends on x and m (the committee size) as well as on the difference in votes between the two alternatives d (with $d \in \{-2m-1, \dots, 2m+1\}$), and it satisfies

$$q = q(x, m, d) \begin{cases} > \frac{1}{2} & \text{if } d > 0, \\ < \frac{1}{2} & \text{if } d < 0. \end{cases}$$

The above inequalities hold because the initial prior is 1/2, committee size is odd, and we are assuming that committee members vote informatively. We stress that $d > 0$ ($d < 0$) means that alternative A received more (less) votes than alternative B in the first voting round.

Then we have the following result:

Proposition 4. *Given x, m, q, d, in any Nash equilibrium of the game underlying the second round of AV, it must be the case that if n is sufficiently large, all citizens with the right to vote in the second round (a) gather zero information and (b) vote for the alternative that received more votes in the first (committee) voting round.*

Proof. See Appendix. □

This demonstrates that our previous claim was correct and accordingly that our insights from the previous sections carry over to AV. Proposition 4 adds to the (nice) properties of AV (Gersbach et al., 2019) as a potential decision-making procedure.

6.2 Cost sharing rules

In this section we consider different cost sharing rules that further highlight the role of information acquisition in committees. Assume that if a committee member acquires information level x, with $x \in [0, 1/2]$, the costs s/he privately incurs are $g \cdot c(x)$. We assume that the individuals who are not part of the committee uniformly finance the subsidies to the committee members. The cost sharing setup analyzed in Section 4 assumes $g = 1/(2n+1)$, while Martinelli (2006) assumes $g = 1$. Assuming that cost sharing is only done by individuals in the committee entails $g = 1/(2m+1)$. For a general $g \in [(1/(2n+1), 1]$, the information level associated with the only symmetric equilibrium can be identified by considering the following equation:

$$g \cdot c'(x) = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m & \text{if } m > 0. \end{cases} \tag{28}$$

We use $x^*_m(n, g)$ to denote the solution to the above equation for the above example—see Table 1. The graph of $x^*_m(n, g)$ for different (positive) values of g is depicted in Figure 2.

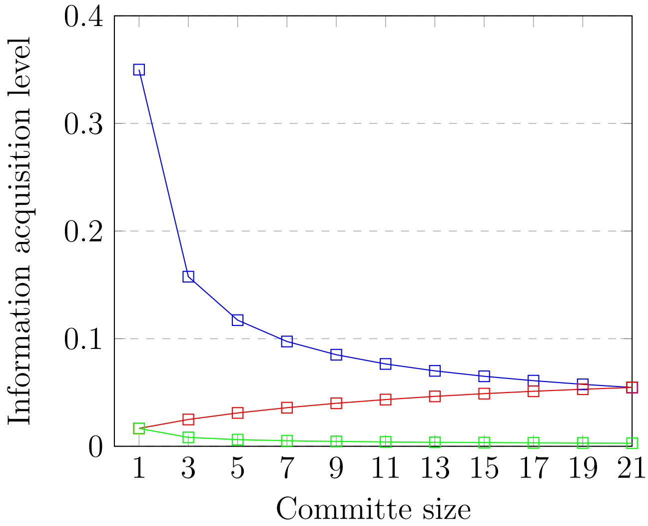

Figure 2: Different values of information acquisition $x^*(m, g)$, for $g = 1/(2n+1)$ (blue line), $g = 1/(2m+1)$ (red line), and $g = 1$ (green line).

Sharing individual information acquisition costs among all the members of a larger community triggers the acquisition of information for each individual more than other cost sharing rules prescribing that individuals incur a higher share of their private costs. This is why the three curves of Figure 2 do not cross (they coincide at some points). A second observation from Figure 2 figure is that sharing costs has a dramatic effect on the individual information acquisition levels. This is because the equation that pins down this level—see Equation (28)—is highly non-lineal in g. Particularly for small committees, but also for large ones, being able to share costs among all individuals is a desirable property as far as the probability of implementing the right alternative is concerned.

A third remarkable feature of Figure 2 concerns the case where $g = 1/(2m + 1)$. In this case, increasing committee size has two (partial equilibrium) effects: On the one hand, it reduces the chances for every committee member to be pivotal; on the other, it reduces the individual costs of information acquisition. In the above example, the latter effect dominates over the former, and hence the red line is increasing in committee size. However, as shown in Figure 3, this is not a general property of this particular cost sharing scheme. According to this figure, individual information levels when costs are highly convex and are shared *only* among members of the committee are not monotonic in the size of the committee. Whether non-monotonicity of information acquisition levels in committee size is a general property of highly convex information acquisition cost functions would require further scrutiny.

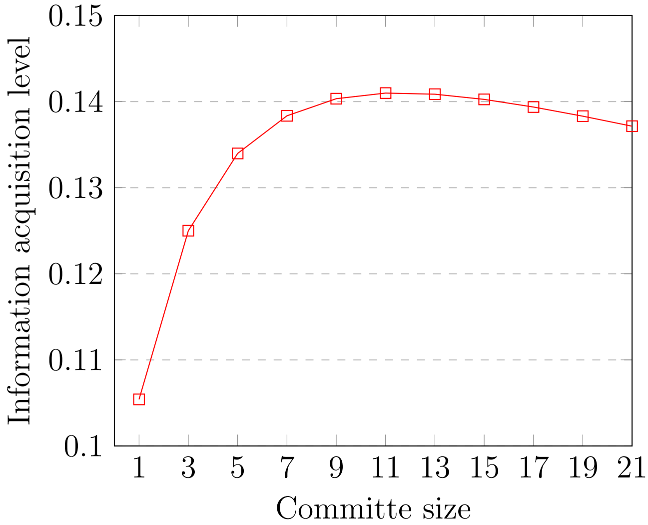

Figure 3: Information acquisition level $x^*(m, g)$ for $g = 1/(2m + 1)$ when $c(x) = 30x^3$.

The rule used for sharing the costs of acquiring information can significantly affect the incentives for individuals to acquire information and thus the optimal design of the committee, in particular its size. Figure 3 also reveals that particular cost sharing rules, such as $g = 1/(2m+1)$, can lead to a setup that cannot be subsumed into that of Martinelli (2006). It is also worth noting that unlike the cost sharing rule $g = 1/(2n+1)$, using $g = 1/(2m+1)$ to share the costs of information acquisition affects the (individual rationality) incentives to become part of the committee. If this cannot be enforced by other means, using $g = 1/(2m + 1)$ may not be feasible.

Finally, it is worth discussing what happens with negative or zero values of g. If $g < 0$, in particular, an individual not only does not incur a disutility cost for acquiring information, but s/he is rewarded for any marginal increase in the information level s/he acquires.[29] There are at least two drawbacks associated with such payment schemes. First, unlike $g = 1/(2n + 1)$, it does not treat all individuals of the society equally from an ex-post perspective. All else being equal, this is a desirable property for a population of identical individuals. Second, suppose that $c(1/2) = \infty$. In this case, all individuals of the (finite) population must be committed to making an infinite monetary payment to the committee members, which is obviously unfeasible. As an alternative, one can impose a cap on such transfers. However, if the same payment scheme must be used for different information acquisition cost functions, imposing such a cap might yield worse outcomes than the case where the cost-sharing rule $g = 1/(2n + 1)$ is used.[30]

[29]Similar criticisms apply when $g = 0$.

[30]In Section 5 we use transfer schemes that resemble cost sharing with $g < 0$ because costs are unobservable. With observable costs, one can aim at goals such ex post fairness.

7 Extensions

In this section we analyze some extensions of our baseline setup and discuss how they affect our results both for cost sharing (see the analysis in Section 4) and TS (see the analysis in Section 5).

7.1 Asymmetric priors

Let us assume that the ex ante probability that A is the right alternative is p, with $p \in [1/2, 1)$. In our baseline setup we have assumed $p = 1/2$. In general, if committee member i receives a signal of quality $\frac{1}{2} + x_i$, which comes at private cost $c(x_i)$, then the posteriors that alternative A is the right alternative are as follows: If the private signal that citizen i receives is A,

$$Prob\Big[w = A \Big| s_i = A\Big] = \frac{y_i \cdot p}{y_i \cdot p + (1 - y_i) \cdot (1 - p)},$$

where $y_i := 1/2 + x_i$. If the private signal that citizen i receives is B,

$$Prob\Big[w = A \Big| s_i = B\Big] = \frac{(1 - y_i) \cdot p}{y_i \cdot (1 - p) + (1 - y_i) \cdot p}.$$

For our analysis of asymmetric priors, we assume informative voting. That is, committee members vote for implementing alternative A (alternative B) if their posterior in favor of alternative A (alternative B) is higher than 1/2, with ties being broken in favor of the private signal. This means that if committee member i receives signal A, s/he votes for alternative A. By contrast, if committee member i receives signal B, s/he votes for alternative B if and only if $x_i \geq p - \frac{1}{2}$.

The consequences of the above remarks are as follows: First, if committee member i chooses quality signal $x_i \in [0, 1/2]$ such that

$$x_i < p - \frac{1}{2},$$

s/he always votes for alternative A, no matter what signal is received. Because signals are costly, this means that acquiring any such information level is dominated (in expected terms) by acquiring zero information. Second, if committee member i chooses quality signal $x_i \in [0, 1/2]$ such that

$$x_i \geq p - \frac{1}{2},$$

then s/he always follows the recommendation of the signal. As far as the analysis of our (family of) mechanisms is concerned, this means that the actual action space for committee member i is

$$\{0\} \cup \left[p - \frac{1}{2}, \frac{1}{2}\right].$$

instead of $[0, 1/2]$. Accordingly, if full information acquisition is the only symmetric equilibrium of the underlying game with symmetric priors, it is also the only symmetric equilibrium with asymmetric priors. This property applies for TS when full information is attainable (see the analysis in Section 5.1).

Now consider that there is an interior symmetric equilibrium $(x, \ldots, x)$ with symmetric priors. Then it is an equilibrium for asymmetric priors too, provided that $x \geq p - 1/2$. The latter condition holds if priors are not very asymmetric.[31] If $x < p - 1/2$, by contrast, either $(0, \ldots, 0)$ or $(p - 1/2, \ldots, p - 1/2)$ are symmetric equilibrium, depending on $c(\cdot)$ and the scheme regulating the side-payments.[32] In the following, we elaborate on the cases of cost sharing and TS.

First, in the case of cost sharing (see Section 4), either $(0, \ldots, 0)$ or $(p - 1/2, \ldots, p - 1/2)$ must be the only symmetric equilibrium with asymmetric priors. This means that asymmetric priors can induce committee members not to gather any information at all, despite the fact that $c(0) = c'(0) = 0$. Such a property of priors is not surprising. If, from an ex-ante perspective, there is agreement that one alternative is much more likely than the other, citizens may not acquire further information to achieve more certainty about the state of the world if doing so is sufficiently costly.

Second, for the TS analyzed in Section 5, the probability that the right alternative is implemented is one (when full information is attainable, see Section 5.1) or arbitrarily close to one if we increase committee size m sufficiently (when full information is not attainable, see Section 5.2). Hence, in equilibrium the probability of a tie must either be zero or converge to zero with m, respectively. Therefore the incentives related to pivotality then vanish on and off equilibrium, and only those associated with the TS survive (in the limit). Accordingly, if the rewards given to committee members are sufficiently large (taking into account the value of p), it must be the case that $(p - 1/2, \ldots, p - 1/2)$ is the only equilibrium. We stress that p is exogenously given, so it is independent of $c(\cdot)$ and m.

7.2 Asymmetric preferences

Suppose that for any individual i of the population,

$$U_i(A|w = A) = 1 \quad \text{and} \quad U_i(B|w = A) = 0,$$

while

$$U_i(B|w = B) = \theta \quad \text{and} \quad U_i(A|w = B) = 0,$$

where $\theta \in (0, 1]$. That is, the error of choosing alternative B when A is the right alternative is equally or more serious than choosing alternative A when B is the right alternative. In the baseline setup, we have assumed $\theta = 1$, i.e., both error types are equally serious.

Suppose now that some individual i's belief that A is the right alternative is ρ, with $\rho \in [0, 1]$.

[31] Note that p is an exogenous parameter and that x is determined independently of the actual value of x.

[32] It could be the case that neither of the two strategy profiles are equilibria.

Then s/he (individually) prefers to choose alternative A if and only if

$$\rho \cdot 1 + (1-\rho) \cdot 0 \geq \rho \cdot 0 + (1-\rho) \cdot \theta,$$

which can be rearranged as

$$\rho \geq \frac{\theta}{1+\theta} := \theta^*.$$

Parameter θ^* is usually called the (individual) threshold of reasonable doubt (Feddersen and Pesendorfer, 1998). If $\theta = 1$, we have $\theta^* = 1/2$. If $\theta = 0$, we have $\theta^* = 0$.

To see how our results depend on the value of θ^* (or θ), we maintain our focus on symmetric equilibria. Let $(x, \dots, x)$, with $x \in (0, 1/2)$, be a strategy profile in which all citizens choose information level x.[33] Then for any committee member i, the benefits linked to pivotality (i.e., excluding the side-payments) when s/he chooses x_i and everybody else chooses x are

$$\frac{1}{2} \cdot P_x[\text{tie}|w = A] \cdot \left(\frac{1}{2} + x_i\right) + \frac{1}{2} \cdot P_x[\text{tie}|w = B] \cdot \left(\frac{1}{2} + x_i\right) \cdot \theta.$$

Given that the probability of a tie is the same in both states of the world, the above expression can be rewritten as

$$\frac{1}{2(1-\theta^*)} \cdot P_x[\text{tie}] \cdot \left(\frac{1}{2} + x_i\right).$$

To see the effect of introducing $\theta \neq 1$ on equilibrium behavior, we distinguish the case of cost sharing from the case where a TS is used. On the one hand, in the case of cost sharing (see Section 4), introducing θ^* is equivalent to having each individual care equally about the right alternative in both states of the world and multiplying the cost function by $2(1-\theta^*)$. In this case, preferences are symmetric and the cost function would be

$$\hat{c}(x) = 2(1-\theta^*) \cdot c(x). \tag{29}$$

This follows from (6). Hence, lower reasonable doubts (i.e., lower values of θ^*) yield lower levels of information acquisition, all else being equal. In fact, note that in Equation (29), changing θ^* does not change the shape of the cost function. The effect of changes of this kind on whether one-member committees yield the highest probability of implementing the right alternative has been indicated in Proposition 1.

On the other hand, in the case where some reward scheme t^m is used, introducing θ^* is equivalent to having each individual care equally about the right alternative in both states of the world, considering $\hat{c}(\cdot)$ as defined in (29), and additionally considering a reward scheme

$$\hat{t}^m(k) = 2(1-\theta^*) \cdot t^m(k) \text{ for all } k \in \{1, \dots, n\}.$$

[33] For full information equilibria, there are no incentives linked to pivotality if the committee consists of at least three members. This means that asymmetric preferences have no bearing on our results in such cases.

This follows from (15). One can verify that the statements of Propositions 2–4 remain valid in this setup, regardless of the value of θ^*. This is because pivotality is either zero (when full information is attainable) or the incentives that it carries for committee members are ignored as is shown in the proof of our (sufficient) results. Hence, lower reasonable doubts—i.e., lower values of θ^*—do not affect information acquisition levels when the TS considered in Section 5 are used, all else being equal.

7.3 Private values

Suppose that an individual can be of three types, A-partisan, B-partisan, and non-partisan. Assuming that types are private, let p_A (p_B) be the ex-ante probability that an individual is A-partisan (B-partisan), while an individual is non-partisan with probability $1 - p_A - p_B$. The ex-ante distribution of types is assumed to be common knowledge. Partisan voters always vote for their preferred alternative, so they do not acquire any information because doing so is costly. For our analysis, we assume that members of the committee are randomly chosen from the population. Then, from the perspective of a committee member i who is non-partisan, the probability that all other members of the committee yield a tie (assuming that non-partisans choose information acquisition level x) is

$$\begin{aligned} P_x\left[pivotal\right] = &\sum_{\substack{n_A, n_B \leq m \\ 0 \leq n_A + n_B \leq 2m}} \frac{F(n_A, n_B, p_A, p_B)}{2} \cdot \left(\frac{1}{2} + x\right)^{m-n_A} \cdot \left(\frac{1}{2} - x\right)^{m-n_B} \\ &+ \sum_{\substack{n_A, n_B \leq m \\ 0 \leq n_A + n_B \leq 2m}} \frac{F(n_A, n_B, p_A, p_B)}{2} \cdot \left(\frac{1}{2} - x\right)^{m-n_A} \cdot \left(\frac{1}{2} + x\right)^{m-n_B}, \end{aligned}$$

where

$$F(n_A, n_B, p_A, p_B) := \binom{2m}{n_A} \cdot \binom{2m - n_A}{n_B} \cdot p_A^{n_A} \cdot p_B^{n_B} \cdot (1 - p_A - p_B)^{2m - n_A - n_B} \cdot \binom{2m - n_A - n_B}{m - n_A}.$$

Accordingly,

$$P_x\left[pivotal\right] = G(n_A, n_B, p_A, p_B, x) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m,$$

where

$$\begin{aligned} &G(n_A, n_B, p_A, p_B, x) \\ :=&\frac{1}{\binom{2m}{m}} \cdot \sum_{\substack{n_A, n_B \\ 0 \leq n_A + n_B \leq 2m}} \frac{F(n_A, n_B, p_A, p_B)}{2} \left(\left(\frac{1}{2} + x\right)^{-n_A} \left(\frac{1}{2} - x\right)^{-n_B} + \left(\frac{1}{2} - x\right)^{-n_A} \left(\frac{1}{2} + x\right)^{-n_B} \right). \end{aligned}$$

One can verify that for $m \geq 1$, $P_{\frac{1}{2}}[pivotal] = 0$ if $p_A = p_B = 0$, while $P_{\frac{1}{2}}[pivotal] > 0$ if $p_A, p_B > 0$. Hence, introducing partisan voters increases the chances that an equilibrium with full information acquisition exists, all else being equal (and assuming that the majority rule is used). This is because partisan voters introduce incentives to inform linked to pivotality that are otherwise absent when there are only non-partisan voters who inform themselves fully. Whether other equilibria can be introduced with partisan voters would require a more thorough analysis of the above expressions.

8 Conclusion

We have analyzed the problem of acquiring costly information and expressing it through voting with the majority rule in a common-value setup. The main novelty of our approach is that we have simultaneously introduced monetary transfers and vote delegation to a committee formed from a parent body, say, the electorate. We have distinguished two main scenarios, depending on whether it is feasible to share the information acquisition costs among all agents of the population (when it is not, we must consider certain reward schemes). The two scenarios reflect two distinct (and polar) cases. Our main insight for both scenarios is that if it is possible to reward committee members adequately it might be better to delegate information acquisition and voting rights to small committees instead of the entire electorate. This maximizes the probability of implementing the right alternative.

It is intuitive that using side-payments can incentivize agents to acquire (more) costly information. Yet our thorough analysis not only sheds new light on the optimal (and suboptimal) use of monetary incentives in combination with variable committee size but also yields insights about the role of monetary transfers that are relevant from a positive perspective.

Finally, there are avenues for future research. For instance, one could characterize all reward schemes that maximize the probability of selecting the socially optimal alternative.

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Appendix

Proof of Lemma 1. Throughout the proof, we let $m, n \geq 0$ be integer numbers such that $m+1 \leq n$, and denote $x_m^* := x_m^*(n)$ and $x_{m+1}^* := x_{m+1}^*(n)$. Then recall the first-order condition (6) that pins down an interior equilibrium solution, namely

$$\frac{c'(x)}{2n+1} = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{2} - x\right)^m \cdot \left(\frac{1}{2} + x\right)^m & \text{if } m > 0. \end{cases} \tag{30}$$

If $x = 0$ ($x = 1/2$), i.e., if there is a corner solution, then the left-hand side of (30) must be larger than or equal to (smaller than or equal to) the right-hand side. Because $c'(x), c''(x) > 0$ and $c'(0) = 0$, we have

$$0 < x_0 \leq \frac{1}{2}$$

and, for all $m > 0$,

$$0 < x_m < \frac{1}{2}.$$

If $x_0^* = 1/2$, the result of the lemma holds immediately. Hence, let $m \geq 0$ and assume that

$$0 < x_m^*, x_{m+1}^* < 1/2. \tag{31}$$

Suppose now that

$$x_m^* \leq x_{m+1}^*. \tag{32}$$

Then (31) implies that

$$\binom{2m}{m} \cdot \left(1/4 - (x_m^*)^2\right)^m \leq \frac{c'(x_m^*)}{2n+1} \leq \frac{c'(x_{m+1}^*)}{2n+1} \leq \binom{2m+2}{m+1} \cdot \left(1/4 - (x_{m+1}^*)^2\right)^{m+1},$$

where the second inequality holds since $c(\cdot)$ is a convex function and therefore $c'(\cdot)$ is increasing. Because $x_m^* \leq x_{m+1}^*$, the above inequality implies

$$4 \leq \frac{(2m+1)(2m+2)}{(m+1)^2}.$$

However, the latter inequality does not hold for any $m \geq 0$, since $4(m+1)(m+1) \leq (2m+1)(2m+2)$ is equivalent to $2m+2 \leq 0$. Hence, (32) cannot hold, and it must therefore be the case that $x_{m+1}^* < x_m^*$. This completes the proof of the lemma.

□

Proof of Lemma 2. For a fixed $n \in \mathbb{N}$, we know from Lemma 1 that $(x_m^*(n))_{m=0}^n$ is decreasing. We stress that we are assuming that $m \leq n$, since the committee cannot be larger than the total population. Consider now the following sequence (of sequences):

$$((x_m^*(n))_{m=0}^n)_{n=1}^\infty.$$

For each $m \in \mathbb{N}$, let $f(m)$ be any (weakly) increasing function such that

$$m \leq f(m) \tag{33}$$

and for any $\varepsilon > 0$,

$$\lim_{m\to\infty}\left[\frac{f(m)}{\sqrt{m}\cdot(1+\varepsilon)^m}\right] = 0. \tag{34}$$

Then we can define the following sub-sequence

$$(z_m)_{m=0}^\infty := (x_m^*(f(m)))_{m=0}^\infty.$$

If $(z_m)_{m=0}^\infty$ is bounded from below by a constant $x^* > 0$, then it must be the case that for all $m > 0$ (see (30) in the proof of Lemma 1),

$$0 < c'(x^*) < (2f(m)+1)\cdot\binom{2m}{m}\cdot\left(\frac{1}{2}-x^*\right)^m\cdot\left(\frac{1}{2}+x^*\right)^m, \tag{35}$$

where the inequalities hold because no committee member will choose in equilibrium either the minimum level of information acquisition, namely 0, or the maximum level, namely 1/2, respectively. Then note that

$$\begin{aligned}
&\lim_{m\to\infty}\left[(2f(m)+1)\cdot\binom{2m}{m}\cdot\left(\frac{1}{2}-x^*\right)^m\cdot\left(\frac{1}{2}+x^*\right)^m\right]\\
=&\frac{\sqrt{2}}{\pi}\cdot\lim_{m\to\infty}\left[(2f(m)+1)\cdot\frac{4^m}{\sqrt{m}}\cdot\left(\frac{1}{2}-x^*\right)^m\cdot\left(\frac{1}{2}+x^*\right)^m\right]\\
=&\frac{\sqrt{2}}{\pi}\cdot\lim_{m\to\infty}\left[\frac{2f(m)+1}{\sqrt{m}}\cdot\left(1-(2x^*)^2\right)^m\right]\\
=&\frac{2\sqrt{2}}{\pi}\cdot\lim_{m\to\infty}\left[\frac{f(m)}{\sqrt{m}\cdot\left(\frac{1}{1-(2x^*)^2}\right)^m}\right] = 0,
\end{aligned}$$

where the first equality holds by Stirling's approximation and the last equality is implied by (33) and (34). However, this contradicts (35), so $(z_m)_{m=0}^\infty$ is not bounded from below by a strictly positive real number. Accordingly, there is a subsequence of $(z_m)_{m=0}^\infty$ that converges to zero. This completes the proof of the lemma.

□

Proof of Lemma 4. The proof is by contradiction. Hence, suppose that

$$x^*_{m-1}(n) \leq x^*_m(n+1). \tag{36}$$

Recall that for given $\tilde{n}, \tilde{m}$, with $1 < \tilde{m} \leq \tilde{n}$, we know from (6) that

$$\frac{c'(x^*_{\tilde{m}}(\tilde{n}))}{2\tilde{n}+1} = \binom{2\tilde{m}}{\tilde{m}} \cdot \left(\frac{1}{4} - (x^*_{\tilde{m}}(\tilde{n}))^2\right)^{\tilde{m}}. \tag{37}$$

Because $c'(\cdot)$ is increasing, it must be the case that

$$c'(x^*_{m-1}(n)) \leq c'(x^*_m(n+1)),$$

which, by using (37), is equivalently written as

$$(2n+1) \cdot \binom{2(m-1)}{m-1} \cdot \left(\frac{1}{4} - \left(x^*_{m-1}(n)\right)^2\right)^{m-1} \leq (2n+3) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - (x^*_m(n+1))^2\right)^m. \tag{38}$$

However, we claim (and show below) that inequality (38) cannot hold. This implies that (36) cannot hold either, so

$$x^*_{m-1}(n) > x^*_m(n+1).$$

The latter proves the statement of the lemma. To show the above claim, note on the one hand that

$$\begin{aligned}
&(2n+3) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - (x^*_m(n+1))^2\right)^m \\
<&(2n+3) \cdot \binom{2(m-1)}{m-1} \cdot \frac{(2m-1)(2m)}{m^2} \cdot \left(\frac{1}{4} - (x^*_m(n+1))^2\right)^{m-1} \cdot \frac{1}{4} \\
\leq&(2n+3) \cdot \binom{2(m-1)}{m-1} \cdot \frac{(2m-1)(2m)}{m^2} \cdot \left(\frac{1}{4} - \left(x^*_{m-1}(n)\right)^2\right)^{m-1} \cdot \frac{1}{4}
\end{aligned} \tag{39}$$

where the first inequality holds since $x^*_m(n+1) > 0$ and the second inequality holds by (36). On the other hand, assuming that $m > 1$ and $n > 2m$, one can verify that it must be the case that

$$4(2n+1)m^2 \geq (2n+3)(2m-1)(2m). \tag{40}$$

Combining (39) and (40) shows that inequality (38) cannot hold. This completes the proof of the lemma.

□

Proof of Theorem 1. Throughout the proof, we let $m, n \geq 0$ be integer numbers such that $m + 1 \leq n$, and then denote $x^*_m := x^*_m(n)$ and $x^*_{m+1} := x^*_{m+1}(n)$. We also use the notation $Q_m(x)$ to denote the probability that the right alternative is implemented when *all* the committee members incur information cost $x \in [0, 1/2]$. That is,

$$Q_m(x) := \sum_{i=m+1}^{2m+1} \binom{2m+1}{i} \cdot \left(\frac{1}{2} + x\right)^i \cdot \left(\frac{1}{2} - x\right)^{2m+1-i}. \tag{41}$$

Recall from (41) that $Q_m = Q_m(x_m^*)$ considers the case where x_m^* is the information acquisition level chosen in (the unique) equilibrium. We assume that $Q_0 < 1$, since otherwise the statement of the proposition holds trivially due to Lemma 1. This means, in particular, that we can focus henceforth on the case where, for all $m \geq 0$,

$$\frac{1}{2} > x_m^* > x_{m+1}^* > 0.$$

Now consider Equation (6) when $c(x) = ax^b$, which determines the level x_m^* of information that is acquired by each member of the committee in the unique equilibrium, namely

$$\frac{ab \cdot x^{b-1}}{2n+1} = \begin{cases} 1 & \text{if } m = 0, \\ \binom{2m}{m} \cdot \left(\frac{1}{2} - x\right)^m \cdot \left(\frac{1}{2} + x\right)^m & \text{if } m > 0. \end{cases} \tag{42}$$

Then

$$x_0^* = \left(\frac{2n+1}{ba}\right)^{\frac{1}{b-1}} < \frac{1}{2} \tag{43}$$

and, for $m > 0$,

$$x_m^* = \left(\frac{2n+1}{ab}\binom{2m}{m} \cdot \left(\frac{1}{2} - x_m^*\right)^m \left(\frac{1}{2} + x_m^*\right)^m\right)^{\frac{1}{b-1}}. \tag{44}$$

At times it will be convenient to use the variable $z := \frac{1}{2} + x$ instead of x, as well as $z_m := \frac{1}{2} + x_m$, and furthermore to introduce function

$$T_m(z) := Q_m(z - 1/2) \tag{45}$$

The reason is that $\frac{dT_m(z)}{dz}$ and $\frac{dQ_m(x)}{dx}$ have the same sign, which is what we are interested in. That is,

$$T_m(z) := \sum_{i=m+1}^{2m+1} \binom{2m+1}{i} \cdot z^i (1-z)^{2m+1-i}. \tag{46}$$

Then, using (46), we obtain that for all $m \geq 0$,

$$\begin{aligned}
\frac{dT_m(z)}{dz} &= \sum_{i=m+1}^{2m+1} \binom{2m+1}{i} \cdot \left(iz^{i-1}(1-z)^{2m-i+1} + (2m+1-i)(-1)z^i(1-z)^{2m-i}\right) \\
&= \sum_{i=m+1}^{2m+1} \frac{(2m+1)!}{(i-1)!(2m+1-i)!} \cdot z^{i-1}(1-z)^{2m-i+1} - \sum_{j=m+1}^{2m} \frac{(2m+1)!}{j!(2m-j)!} \cdot z^j(1-z)^{2m-j} \\
&= (2m+1) \cdot \sum_{i=m+1}^{2m+1} \frac{(2m)!}{(i-1)!(2m+1-i)!} \cdot z^{i-1}(1-z)^{2m-i+1} \\
&- (2m+1) \sum_{j=m+1}^{2m} \frac{(2m)!}{j!(2m-j)!} \cdot z^j(1-z)^{2m-j} \\
&= (2m+1) \cdot \sum_{j=m}^{2m} \frac{(2m)!}{j!(2m-j)!} \cdot z^j(1-z)^{2m-j} \\
&- (2m+1) \sum_{j=m+1}^{2m} \frac{(2m)!}{j!(2m-j)!} \cdot z^j(1-z)^{2m-j} = (2m+1) \cdot \binom{2m}{m} \cdot z^m(1-z)^m, \qquad (47)
\end{aligned}$$

where the penultimate equality is obtained by the change of variables $j = i - 1$. From (47) it follows that

$$0 < \frac{dT_m(z)}{dz} \leq (2m+1) \cdot \underbrace{\binom{2m}{m} \cdot \left(\frac{1}{2}\right)^{2m}}_{:=l_m} \qquad (48)$$

for any $z \in [1/2, 1]$, since $z(1-z) \leq 1/4$. Because function $T_m(z)$ is continuously differentiable on $[1/2, 1]$, by the mean value theorem there exists $z^* \in [1/2, z_m]$ such that

$$\frac{dT_m(z^*)}{dz} = \frac{T_m(z_m) - T_m(1/2)}{z_m - 1/2}. \qquad (49)$$

Using (48)–(49), it follows that

$$T_m(z_m) \leq T_m(1/2) + \left(z_m - \frac{1}{2}\right) \cdot (2m+1) \cdot \binom{2m}{m} \cdot \left(\frac{1}{2}\right)^{2m} = \frac{1}{2} + (2m+1) \cdot x_m^* l_m. \qquad (50)$$

We note that for the last equality we have used the fact that $T_m(1/2) = Q_m(0) = 1/2$, which follows from the symmetry of the binomial distribution. It is straightforward to verify that

$$T_0(z_0) = \frac{1}{2} + x_0^*.$$

Together with (50), it follows from the above equation that a sufficient condition for $Q_0 > Q_m$ ($m > 0$) is that

$$\frac{1}{2} + (2m+1) \cdot x_m^* l_m < \frac{1}{2} + x_0^*,$$

which, using equations (43) and (44), can be rewritten as

$$(2m+1)^{b-1} \cdot \binom{2m}{m} \cdot \left(\frac{1}{2} - x_m^*\right)^m \left(\frac{1}{2} + x_m^*\right)^m \cdot l_m^{b-1} < 1. \tag{51}$$

Moreover, since $x_m^* > 0$,

$$\binom{2m}{m} \cdot \left(\frac{1}{2} - x_m^*\right)^m \left(\frac{1}{2} + x_m^*\right)^m < l_m.$$

Hence, inequality (51) is implied by the following inequality

$$(2m+1)^{b-1} \cdot l_m^b < 1.$$

or, equivalently, by

$$l_m^{\frac{b}{b-1}} \cdot (2m+1) < 1. \tag{52}$$

It therefore remains to prove inequality (52), which we turn to in the remainder of the proof. Since $b - 1 \leq 1$, a sufficient condition for inequality (52) is

$$l_m^2 < \frac{1}{2m+1}. \tag{53}$$

Then note that

$$l_m = \frac{(2m)!}{m!m!} \cdot \left(\frac{1}{2}\right)^{2m} \leq \frac{e(2m)^{2m+0.5}e^{-2m}}{\sqrt{2\pi}m^{m+0.5}e^{-m}\sqrt{2\pi}m^{m+0.5}e^{-m}} \frac{1}{2^{2m}} = \frac{e}{\pi\sqrt{2m}}, \tag{54}$$

where the inequality holds by using the following lower and upper bounds of factorial numbers (see Robbins, 1955),

$$\sqrt{2\pi}j^{j+0.5}e^{-j} \leq j! \leq ej^{j+0.5}e^{-j}. \tag{55}$$

In turn, using (54), a sufficient condition for (57) is

$$\left(\frac{e}{\pi}\right)^2 < \frac{2m}{2m+1}. \tag{56}$$

If $m > 1$, then

$$\left(\frac{e}{\pi}\right)^2 < \frac{4}{5} \leq \frac{2m}{2m+1},$$

and hence,

$$Q_0 > Q_m \text{ for all } m > 1.$$

It thus remains to show that $Q_0 > Q_1$. Since $l_1 = 1/2$,

$$l_1^2 = \frac{1}{4} < \frac{1}{3}, \tag{57}$$

and thus (57) holds. This completes the proof of the Theorem.

□

Proof of Proposition 1. We consider $c(x) = ax^b$ and assume $b > 2$. We use the following shortcuts:

$$t(a) := \frac{2n+1}{ab} \quad \text{and} \quad \alpha := \frac{1}{b-1}.$$

Hence,

$$b > 2 \iff 0 < \alpha < 1. \tag{58}$$

and

$$t'(a) < 0 \quad \text{and} \quad \lim_{a\to\infty} t^\alpha(a) = 0. \tag{59}$$

Using (6), we obtain

$$x_0 = t^\alpha(a) > 0$$

and that x_m solves the following equation:

$$x_m = t^\alpha(a) \cdot \left[\binom{2m}{m} \cdot \left(\frac{1}{2} - x_m\right)^m \cdot \left(\frac{1}{2} + x_m\right)^m\right]^\alpha = x_0 \cdot \left[\binom{2m}{m}\left(\frac{1}{4} - x_m^2\right)^m\right]^\alpha. \tag{60}$$

In particular, using implicit derivation, we obtain

$$\frac{\partial x_m}{\partial x_0} = \frac{\left[\binom{2m}{m}\left(\frac{1}{4} - x_m^2\right)\right]^\alpha}{1 + 2\alpha x_m \cdot \left[\binom{2m}{m}\left(\frac{1}{4} - x_m^2\right)\right]^{\alpha-1}} > 0. \tag{61}$$

Using $b > 2$ and (58), the claim of the proposition can be equivalently stated as requiring that for any α with the property that $\alpha < 1$, there exists $t^*(\alpha) > 0$ such that

$$Q_0 = \frac{1}{2} + x_0 < \sum_{k=m+1}^{2m+1} \binom{2m+1}{k} \left(\frac{1}{2} + x_m\right)^k \left(\frac{1}{2} - x_m\right)^{2m+1-k} = Q_m \quad \text{for any } t(a) < t^*(\alpha). \tag{62}$$

From (45) and (47)—see the proof of Theorem 1—, we know that for any integer $m \geq 0$,

$$Q_m'(x) = (2m+1) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m.$$

Clearly, for all $x \in (0, 1/2)$, it is the case that $Q_m'(x) > 0$ and $Q_m''(x) < 0$. Hence,

$$\frac{1}{2} + x_m \cdot (2m+1) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - x_m^2\right)^m = Q_m(0) + x_m \cdot Q_m'(x_m) < Q_m(x_m).$$

This means that inequality (62) is implied by

$$x_0 < x_m \cdot (2m+1) \cdot \binom{2m}{m} \cdot \left(\frac{1}{4} - x_m^2\right)^m. \tag{63}$$

Using (60), inequality (63) is equivalent to

$$1 < (2m+1) \cdot \left[\binom{2m}{m} \cdot \left(\frac{1}{4} - x_m^2\right)^m\right]^{\alpha+1}.$$

We stress that if the above inequality holds, so does inequality (62). Now let

$$f(x) := (2m+1) \cdot \left[\binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m\right]^{\alpha+1}.$$

Clearly, $f'(x) < 0$. In addition, we claim that there is a positive integer m^* such that for all $m \geq m^*$,

$$f(0) = (2m+1) \cdot \left[\binom{2m}{m} \cdot \left(\frac{1}{4}\right)^m\right]^{\alpha+1} > 1. \tag{64}$$

On the one hand, from (55)—see the proof of Theorem 1—, we obtain

$$\binom{2m}{m} \cdot \left(\frac{1}{4}\right)^m \geq \left(\frac{2\sqrt{\pi}}{e^2\sqrt{m}}\right). \tag{65}$$

On the other hand, (58) implies $\alpha < 1$, so we have

$$\lim_{m\to\infty} (2m+1) \cdot \left(\frac{2\sqrt{\pi}}{e^2\sqrt{m}}\right)^{1+\alpha} = +\infty. \tag{66}$$

The combination of (65) and (66) yields

$$\lim_{m\to\infty} f(0) = \infty.$$

Therefore the claim in (64) is correct. Next, assume that

$$\lim_{a\to 0} x_m = L > 0.$$

Then, by using (59) and (60) we obtain

$$0 < L = 0 \cdot \left[\binom{2m}{m}\left(\frac{1}{4} - L^2\right)\right]^{\alpha} = 0,$$

which is a contradiction. Hence,

$$\lim_{a\to 0} x_m = 0. \tag{67}$$

Finally, from (59), (64), and (67), we obtain that inequality (62) holds.

Next, if we assume

$$3 > 2^{\frac{b}{b-1}}. \tag{68}$$

we can obtain its equivalent

$$\left(\frac{1}{2}\right)^{\alpha}\frac{3}{2} > 1. \tag{69}$$

Using (6), we obtain

$$x_0 = t^{\alpha}(a) > 0$$

and that x_1 solves the following equation:

$$x_1 = t^{\alpha}(a)\cdot\left(2\left(\frac{1}{2}+x_1\right)\left(\frac{1}{2}-x_1\right)\right)^{\alpha} = x_0\cdot\left(\frac{1}{2}-2x_1^2\right)^{\alpha}. \tag{70}$$

In particular, using implicit derivation, we obtain

$$\frac{\partial x_1}{\partial x_0} = \frac{\left(\frac{1}{2}-2x_1^2\right)^{\alpha}}{1+\left(\frac{1}{2}-2x_1^2\right)^{\alpha-1}\cdot 4x_1} > 0. \tag{71}$$

Using (68) and (69), the claim of the proposition can be equivalently stated as requiring that for any α with the property that

$$\left(\frac{1}{2}\right)^{\alpha}\frac{3}{2} > 1,$$

there exists $t^*(\alpha) > 0$ such that

$$Q_0 = \frac{1}{2}+x_0 < 3\left(\frac{1}{2}+x_1\right)^2\left(\frac{1}{2}-x_1\right)+\left(\frac{1}{2}+x_1\right)^3 = Q_1 \quad \text{for any } t(a)\le t^*(\alpha). \tag{72}$$

The latter inequality is equivalent to

$$x_0 < x_1\cdot\left(\frac{3}{2}-2x_1^2\right). \tag{73}$$

Using (70) and (73), we obtain a further equivalent formulation of (73), namely

$$1 < \left(\frac{1}{2}-2x_1^2\right)^{\alpha}\left(\frac{3}{2}-2x_1^2\right).$$

Now let

$$f(x) := \left(\frac{1}{2}-2x^2\right)^{\alpha}\left(\frac{3}{2}-2x^2\right)$$

Clearly, $f'(x) < 0$. Moreover, note that (68) and (69) imply that

$$f(0) = \left(\frac{1}{2}\right)^{\alpha}\frac{3}{2} > 1. \tag{74}$$

Next, assume that

$$\lim_{a\to 0} x_1 = L > 0.$$

Then, by using (70) we obtain

$$0 < L = 0 \cdot \left(\frac{1}{2} - 2L^2\right)^{\alpha} = 0,$$

which is a contradiction. Hence,

$$\lim_{a\to 0} x_1 = 0. \tag{75}$$

Finally, from (74), and (75), we obtain that inequality (72) holds. Together with (71), this completes the proof of the proposition.

□

Proof of Theorem 2. Recall that for each $m \geq 0$, welfare is defined (in Equation (11)) as

$$W_m = Q_m(x_m^*(n)) - \frac{2m+1}{2n+1} \cdot c(x_m^*(n)), \tag{76}$$

where $Q_m := Q_m(x_m^*)$ is the equilibrium probability of implementing the right alternative and $x_m^* := x_m^*(n)$ is the equilibrium acquisition level. From Equation (6) we know that if $c(x) = ax^b$, then

$$x_0^* = \min\left\{\frac{1}{2}, \left(\frac{2n+1}{ab}\right)^{\frac{1}{b-1}}\right\} \tag{77}$$

and if $m > 0$,

$$\frac{ab(x_m^*)^{b-1}}{2n+1} = \binom{2m}{m} \cdot \left(\frac{1}{4} - (x_m^*)^2\right)^m \tag{78}$$

First, it is clear that if parameter a is sufficiently low, Equation (77) implies

$$x_0^* = \frac{1}{2} \tag{79}$$

and hence

$$Q_0 = 1. \tag{80}$$

On the other hand, for each $m > 0$ define

$$L_m := \lim_{a\to 0} x_m^* \leq \frac{1}{2}.$$

By Lemma 3, the limit is well defined (we stress that decreasing a is the same as increasing n). Then Equation (78) implies

$$0 = \binom{2m}{m} \cdot \left(\frac{1}{4} - L_m^2\right)^m,$$

so

$$L_m = \frac{1}{2} \quad \text{for all } m > 0. \tag{81}$$

Using Equation (76) and Equations (79)–(81), it follows that for each $m > 0$

$$\begin{aligned}
\lim_{a\to 0}\left[W_0 - W_m\right] &= \lim_{a\to 0}(1 - Q_m) + \lim_{a\to 0} a \cdot \lim_{a\to 0}\frac{1}{2n+1}\cdot\left((2m+1)\cdot(x_m^*)^b - \left(\frac{1}{2}\right)^b\right)\\
&\geq \lim_{a\to 0} a \cdot \lim_{a\to 0}\frac{1}{2n+1}\cdot\left((2m+1)\cdot(x_m^*)^b - \left(\frac{1}{2}\right)^b\right)\\
&= \lim_{a\to 0} a \cdot \frac{2m}{2n+1}\cdot\left(\frac{1}{2}\right)^b \geq \left[\frac{2}{2n+1}\cdot\left(\frac{1}{2}\right)^b\right]\cdot\lim_{a\to 0} a.
\end{aligned}$$

This means that there is $a^*(b) > 0$ such that

$$W_0 - W_m > 1 \quad \text{for all } m > 0.$$

□

Proof of Proposition 2. Consider any committee member i. Then suppose that all other $2m$ members of the committee acquire precise information about the right alternative, that is, they all choose information level $x = 1/2$. Then, provided that $m \geq 1$, individual i's utility simplifies to (see Equation (14))

$$U_i(x_i) = \left(\frac{1}{2} + x_i\right)\cdot r - c(x_i).$$

Note that $m \geq 1$ guarantees that the committee consists of at least three members, so individual i is not pivotal in the voting round because the other two committee members are perfectly informed and thus vote for the same alternative. Then,

$$U_i'(x_i) = r - c'(x_i) \geq r - c'\left(\frac{1}{2}\right) \geq 0,$$

where the first inequality holds because $c(\cdot)$ is a convex function and the second inequality holds by assumption. This means that the derivative $U_i(x_i)$ is positive in the interval $[0, 1/2]$, so $U_i(x_i)$ is maximized for $x^* = \frac{1}{2}$. □

Proof of Theorem 3. Recall that we focus on the case $m \geq 1$. Consider some committee member i, and let all the other $2m$ members of the committee use strategy $x \in [0, 1/2]$, as demanded by our notion of symmetric equilibrium. Then individual i's expected utility when s/he chooses

$x_i \in [0, 1/2]$ is

$$U_i(x_i) = \binom{2m}{m} \cdot \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m \left(\frac{1}{2}+x_i\right) - c(x_i)$$
$$+ \left(\frac{1}{2}+x\right)^{2m} \left(\frac{1}{2}+x_i\right) \cdot r_m + \left(\frac{1}{2}-x\right)^{2m} \left(\frac{1}{2}-x_i\right) \cdot r_m + \chi. \tag{82}$$

We stress that $r_m \geq 0$ is some constant that depends on m (to be determined below) which is independent of x. Then,

$$U_i'(x_i) = \underbrace{\binom{2m}{m} \cdot \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m + r_m \cdot \left[\left(\frac{1}{2}+x\right)^{2m} - \left(\frac{1}{2}-x\right)^{2m}\right]}_{:=D_m(x)} - c'(x_i). \tag{83}$$

Since, by assumptions, $c'(0) = 0$, then

$$U_i'(0) = \binom{2m}{m} \cdot \left(\frac{1}{2}\right)^{2m} - c'(0) = \binom{2m}{m} \cdot \left(\frac{1}{2}\right)^{2m} > 0.$$

This means that the (symmetric) strategy profile where committee members acquire no information cannot be an equilibrium. If there is an (interior) symmetric equilibrium defined by x_m^* with less than full information acquisition, i.e.,

$$0 < x_m^* < \frac{1}{2}, \tag{84}$$

then it must be the case that $U_i'(x_m^*) = 0$, or, equivalently, that

$$D_m(x_m^*) = c'(x_m^*). \tag{85}$$

Now let $y_m^* \in (0, 1/2)$ denote the (interior) equilibrium information acquisition level if there were no rewards (or cost sharing), i.e. if $r_m = 0$, namely

$$\binom{2m}{m} \cdot \left(\frac{1}{2}-y_m^*\right)^m \left(\frac{1}{2}+y_m^*\right)^m = c'(y_m^*). \tag{86}$$

From the regularity assumptions made on $c(\cdot)$, we know that y_m^* is well-defined and unique. Then note that for all $x < y_m^*$,

$$D_m(x) - c'(x) \geq \binom{2m}{m} \cdot \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m - c'(x)$$
$$> \binom{2m}{m} \cdot \left(\frac{1}{2}+y_m^*\right)^m \left(\frac{1}{2}-y_m^*\right)^m - c'(y_m^*) = 0,$$

where the first inequality holds since $r_m \geq 0$ and $x \geq 0$, and the second inequality holds because $c'(\cdot)$ is a strictly increasing function. Note that for the second inequality we have used the fact that

$$\binom{2m}{m} \cdot \left(\frac{1}{2}+x\right)^m \left(\frac{1}{2}-x\right)^m = \binom{2m}{m} \cdot \left(\frac{1}{2}-x^2\right)^m$$

is decreasing for $x \in [0, 1/2]$. To sum up, it must be the case that

$$x_m^* \geq y_m^*. \tag{87}$$

Moreover, for all $x \in [0, 1/2]$,

$$\begin{aligned} D_m(x) &\geq r \cdot \left[\left(\frac{1}{2}+x\right)^{2m} - \left(\frac{1}{2}-x\right)^{2m}\right] = r \cdot \left(\sum_{j=0}^{m-1} \binom{2m}{2j+1} \cdot \left(\frac{1}{2}\right)^{2(m-j)-1} x^{2j+1}\right) \\ &\geq r \cdot \frac{2m}{2^{2m-1}} \cdot x. \end{aligned} \tag{88}$$

Next, define s_m such that

$$s_m \in \arg\max_{q \geq y_m^*} \frac{c'(q)}{q}$$

and

$$s_m \leq y \text{ for all } y \in \arg\max_{q \geq y_m^*} \frac{c'(q)}{q}.$$

Due to the assumptions on $c(\cdot)$ and the fact that $y_m^* > 0$, it must be the case that s_m is well-defined and unique. From (87) and the definition of s_m it follows, in particular, that

$$\frac{c'(x_m^*)}{x_m^*} \leq \frac{c'(s_m)}{s_m}. \tag{89}$$

Similarly to the proof of Lemma 1, one can verify that y_m^* decreases with m (and converges to zero), and hence s_m also decreases with m. This means that $(s_m)_{m=1}^{\infty}$ has a limit, which we denote by s. We recall that it must always be the case that $n \geq m$, i.e., the number of citizens is at least as large as the number of committee members. If $s > 0$, then for any integer $m \geq 1$,

$$\frac{c'(s_m)}{s_m} \leq \frac{c'(s)}{s} < \infty$$

If $s = 0$, then for any integer $m \geq 1$,

$$\frac{c'(s_m)}{s_m} \leq \lim_{s \to 0} \frac{c'(s)}{s} = c''(0) < \infty,$$

where the latter inequality holds by assumption. In either case, the sequence

$$\left(\frac{c'(s_m)}{s_m}\right)_{m \geq 1}$$

is bounded from above by a constant (that depends only on $c(\cdot)$).

Finally, let

$$r_m := \frac{2^{2m-1}}{2m} \cdot \frac{c'(s_m)}{s_m}.$$

From (83), (84), and (88), we obtain

$$D_m(x_m^*) > r_m \cdot \frac{2m}{2^{2m-1}} \cdot x_m^* = x_m^* \cdot \frac{c'(s_m)}{s_m} \geq c'(x_m^*).$$

However, this contradicts (85). Hence there cannot be a symmetric equilibrium in which committee members inform themselves partially. It therefore remains to show that $x_m^* = 1/2$ is an equilibrium. This follows from Proposition 2, if we define

$$t_m := \max\left\{r_m, c'\left(\frac{1}{2}\right)\right\}.$$

□

Proof of Proposition 3. Let $k' \in \{1, \dots, m-1\}$, and hence assume that $m > 1$. From (15), we know that the strategy profile $(x, \dots, x)$, with $x \in (0, 1/2)$, is an equilibrium of $\mathcal{G}^m$ only if

$$\begin{aligned} c'(x) = &\binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m \\ &+ \phi^m(m) \cdot \left[\left(\frac{1}{2} + x\right)^{2m} - \left(\frac{1}{2} - x\right)^{2m}\right] \\ &+ \binom{2m}{m+k'} \cdot \phi^m(k') \cdot \left(\frac{1}{4} - x^2\right)^{m-k'} \cdot \left[\left(\frac{1}{2} + x\right)^{2k'} - \left(\frac{1}{2} - x\right)^{2k'}\right], \end{aligned} \tag{90}$$

where

$$\phi^m(m) = c'(1/2) + \varepsilon,$$

for some $\varepsilon > 0$, and we let

$$\phi^m(k') = z_l,$$

for some $l \geq 1$. We assume that $(z_l)_{l \geq 1}$ is any increasing sequence such that

$$\lim_{l \to \infty} z_l = \infty, \tag{91}$$

and that (90) has a solution for each $l \geq 1$, which we denote as x_l, with $x_l \in (0, 1/2)$. Then

$$\begin{aligned}
\underbrace{\lim_{l\to\infty} c'(x_l)}_{:=L_0} &= \underbrace{\lim_{l\to\infty} \binom{2m}{m} \cdot \left(\frac{1}{4} - x_l^2\right)^m}_{:=L_1} \\
&+ \underbrace{\lim_{l\to\infty} \phi^m(m) \cdot \left[\left(\frac{1}{2} + x_l\right)^{2m} - \left(\frac{1}{2} - x_l\right)^{2m}\right]}_{:=L_2} \\
&+ \underbrace{\lim_{l\to\infty} \binom{2m}{m+k'} \cdot \phi^m(k') \cdot \left(\frac{1}{4} - x^2\right)^{m-k'} \cdot \left[\left(\frac{1}{2} + x_l\right)^{2k'} - \left(\frac{1}{2} - x_l\right)^{2k'}\right]}_{:=L_3}. \qquad (92)
\end{aligned}$$

Clearly, $L_0, L_1, L_2, L_3 \geq 0$. On the one hand, assume that

$$\lim_{l\to\infty} x_l = x_L > 0. \qquad (93)$$

Then, by our regularity assumptions,

$$L_0 = c'(x_L) < \infty,$$

which implies

$$L_3 < \infty.$$

Due to (94),

$$\lim_{l\to\infty} \left[\left(\frac{1}{2} + x_l\right)^{2k'} - \left(\frac{1}{2} - x_l\right)^{2k'}\right] = \left[\left(\frac{1}{2} + x_L\right)^{2k'} - \left(\frac{1}{2} - x_L\right)^{2k'}\right] > 0,$$

so it must be the case that

$$\lim_{l\to\infty} z_l \cdot \lim_{l\to\infty} \left(\frac{1}{4} - x_l^2\right)^{m-k'} < \infty.$$

A necessary condition for the above inequality to hold is

$$\lim_{l\to\infty} \left(\frac{1}{4} - x_l^2\right)^{m-k'} = 0,$$

which holds only if

$$x_L = \frac{1}{2}.$$

But this implies

$$L_0 = c'\left(\frac{1}{2}\right) < c'\left(\frac{1}{2}\right) + \varepsilon \leq L_1 + L_2 + L_3,$$

which is in contradiction to (92). On the other hand, assume that

$$\lim_{l\to\infty} x_l = x_L = 0. \tag{94}$$

Since $c'(0) = 0$,

$$0 = L_0 < \binom{2m}{m} \cdot \left(\frac{1}{4}\right)^m = L_1 \leq L_1 + L_2 + L_3,$$

which is also in contradiction to (92). That is, we have proved that a sequence $(z_l)_{l\geq 1}$ with the properties imposed above cannot exist. This means that if $\phi^m(k')$ is sufficiently large, there is no symmetric equilibrium in which individuals choose $x \in (0, 1/2)$. Since $c'(0) = 0$, we also know that there is no equilibrium in which no information is acquired. Finally, Theorem 3 guarantees that full information can be sustained in equilibrium. This completes the proof of the proposition. □

Proof of Theorem 4. From (24) we know that the strategy profile $(x, \ldots, x)$, with $x \in (0, 1/2)$, is an equilibrium of $\mathcal{G}^m$ if and only if

$$c'(x) = \binom{2m}{m} \cdot \left(\frac{1}{4} - x^2\right)^m + \binom{2m}{m+1} \cdot \phi^m(1) \cdot \left(\frac{1}{4} - x^2\right)^{m-1} \cdot 2x, \tag{95}$$

where $\phi^m(1)$ is a constant that will be determined below and does not depend on x. Since $c'(0) = 0$ and $c'(1/2) = \infty$, there is no equilibrium with zero information acquisition and/or with full information acquisition for any $m \geq 1$. Also for any $m \geq 1$, there is (at least one equilibrium). Let $(x_m)_{m\geq 1}$ by any sequence where x_m is a solution of Equation (95) (for the corresponding m).

The remainder of the proof proceeds in several steps. First, suppose that for *all* sufficiently large committee sizes m,

$$x_m \geq \underline{x}_m := \sqrt{\frac{d_m \log m}{m}}, \tag{96}$$

where $(d_m)_{m\geq 0}$, with $d_m > 0$ for all $m \geq 1$, are defined below. We claim that if the elements of the sequence $(d_m)_{m\geq 0}$ are chosen to be sufficiently low but positive, the probability of the right alternative being implemented goes to one as committee size m grows unboundedly, i.e.,

$$\lim_{m\to\infty} Q_m(x_m) = 1. \tag{97}$$

Clearly, because $Q_m(x)$ is increasing in x (for a given m), it suffices to assume $x_m = \underline{x}_m$ for sufficiently large m (see below). To show the claim, for any $m \geq 1$, let $X_1, \cdots, X_{2m+1}$ be i.i.d. Bernoulli random variables with parameter $\frac{1}{2} + \underline{x}_m$. In addition, define

$$S_m := \sum_{l=1}^{2m+1} X_l.$$

Then the probability that the right alternative is *not* implemented is equal to

$$P\left[S_m \leq m\right].$$

By linearity of expectation,

$$\mathbb{E}(S_m) = (2m+1) \cdot \left(\frac{1}{2} + \underline{x}_m\right), \tag{98}$$

so

$$m - \mathbb{E}(S_m) = m - (2m+1) \cdot \left(\frac{1}{2} + \underline{x}_m\right) < -2m \cdot \underline{x}_m < 0. \tag{99}$$

Then,

$$\begin{aligned} P[S_m \leq m] &\leq P\Big[S_m - \mathbb{E}(S_m) \leq -2m \cdot \underline{x}_m\Big] \leq P\Big[\big|S_m - \mathbb{E}(S_m)\big| \geq 2m \cdot \underline{x}_m\Big] \\ &\leq 2\exp\left(-\frac{2\big(2m \cdot \underline{x}_m\big)^2}{2m+1}\right) = 2\exp\left(-\frac{8m^2}{2m+1} \cdot \left(\frac{d_m \log m}{m}\right)\right) \leq 2m^{-\varepsilon}, \end{aligned}$$

where the second inequality holds due to (98) and (99), the third inequality is Hoeffding's inequality (see Hoeffding, 1963), the equality follows from the definition of $\underline{x}_m$ (see (96)), and the last inequality is explained as follows: Given $\varepsilon > 0$, there is large enough $m^*(\varepsilon)$ such that if we let

$$d_m = d := \frac{\varepsilon}{4}\ , \text{ for all } m \geq m^*(\varepsilon), \tag{100}$$

it must be the case that

$$m^{\varepsilon} \leq \exp\left(\frac{8m^2}{2m+1} \cdot \left(\frac{d \log m}{m}\right)\right) \text{ for any } m \geq m^*(\varepsilon). \tag{101}$$

This is because

$$\lim_{m\to\infty} \exp\left(\frac{8m^2}{2m+1} \cdot \left(\frac{d \log m}{m}\right)\right) / m^{\varepsilon} = \lim_{m\to\infty} m^{4d}/m^{\varepsilon} = 1.$$

We have therefore proved that the above claim is correct: It suffices to take $\varepsilon \to 0$.

Second, on the one hand,

$$\lim_{m\to\infty}\frac{\left(\frac{1}{4}-\frac{d_m\log m}{m}\right)^{m-1}}{\left(\frac{1}{4}\right)^{m-1}}m^{\varepsilon}=\lim_{m\to\infty}\left(1-\frac{4d_m\log m}{m}\right)^{m-1}m^{\varepsilon}$$
$$=\lim_{m\to\infty}\left(1-\frac{4d_m\log m}{m}\right)^{m}\cdot\lim_{m\to\infty}\left(1-\frac{4d_m\log m}{m}\right)^{-1}m^{\varepsilon}$$
$$=\lim_{m\to\infty}\left(1-\frac{4d_m\log m}{m}\right)^{m}m^{\varepsilon}=\lim_{m\to\infty}\left(1-\frac{4d_m\log m}{m}\right)^{\frac{m}{4d_m\log m}\cdot 4d_m\log m}m^{\varepsilon}$$
$$=\lim_{m\to\infty}\left(\frac{1}{e}\right)^{4d_m\log m}=\lim_{m\to\infty}m^{-4d_m}m^{\varepsilon}=1. \tag{102}$$

On the other hand, by applying Stirling's approximation

$$\lim_{m\to\infty}\binom{2m}{m+1}\left(\frac{1}{4}\right)^{m-1}=\lim_{m\to\infty}\frac{(2m)!}{(m+1)!(m-1)!}\left(\frac{1}{4}\right)^{m-1}$$
$$=\lim_{m\to\infty}\frac{1}{\sqrt{\pi}}\cdot\sqrt{\frac{m}{m^2-1}}\cdot\frac{m^{2m}}{(m+1)^{m+1}(m-1)^{m-1}}$$
$$=\frac{1}{\sqrt{\pi}}\cdot\lim_{m\to\infty}\frac{1}{\sqrt{m}}. \tag{103}$$

By combining (102) and (103), we therefore obtain

$$\lim_{m\to\infty}\binom{2m}{m+1}\left(\frac{1}{4}-\frac{d_m\log m}{m}\right)^{m-1}=\lim_{m\to\infty}\binom{2m}{m+1}\left(\frac{1}{4}\right)^{m-1}\cdot\lim_{m\to\infty}\frac{\left(\frac{1}{4}-\frac{d_m\log m}{m}\right)^{m-1}}{\left(\frac{1}{4}\right)^{m-1}}$$
$$=\frac{1}{\sqrt{\pi}}\cdot\lim_{m\to\infty}m^{-\varepsilon}\cdot\lim_{m\to\infty}\frac{1}{\sqrt{m}}$$
$$=\frac{1}{\sqrt{\pi}}\cdot\lim_{m\to\infty}\frac{1}{m^{\frac{1}{2}+\varepsilon}}. \tag{104}$$

Third and last, for each $\varepsilon>0$, let

$$\phi^m(1):=\frac{c'(s_m)}{s_m}\cdot m^{\frac{1}{2}+\varepsilon}$$

where

$$s_m\in\arg\max_{q\leq \underline{x}_m}\frac{c'(q)}{q}$$

and

$$s_m\leq y \text{ for all } y\in\arg\max_{q\leq \underline{x}_m}\frac{c'(q)}{q}.$$

In particular, it must be the case that

$$\frac{c'(\underline{x}_m)}{\underline{x}_m}\leq\frac{c'(s_m)}{s_m}<\infty. \tag{105}$$

The strict inequality holds because

$$\lim_{s\to 0}\frac{c'(s)}{s}=c''(0)<\infty.$$

Additionally, note that $(x_m)_{m\geq m^*(\varepsilon)}$ is decreasing, and therefore so is $(s_m)_{m\geq m^*(\varepsilon)}$. Then,

$$\begin{aligned}
&\lim_{m\to\infty}\binom{2m}{m}\cdot\left(\frac{1}{4}-x_m^2\right)^m+\binom{2m}{m+1}\cdot\phi^m(1)\cdot\left(\frac{1}{4}-x_m^2\right)^{m-1}\cdot 2x_m\\
\geq&\lim_{m\to\infty}\binom{2m}{m+1}\cdot\left(\frac{1}{4}-x_m^2\right)^{m-1}\cdot 2\lim_{m\to\infty}m^{\frac{1}{2}+\varepsilon}\cdot\lim_{m\to\infty}\frac{c'(s_m)}{s_m}\cdot x_m\\
\geq&\lim_{m\to\infty}\binom{2m}{m+1}\cdot\left(\frac{1}{4}-x_m^2\right)^{m-1}\cdot 2\lim_{m\to\infty}m^{\frac{1}{2}+\varepsilon}\cdot 2\lim_{m\to\infty}c'(\underline{x}_m)\\
=&\frac{2}{\sqrt{\pi}}\cdot\lim_{m\to\infty}c'(\underline{x}_m)>\lim_{m\to\infty}c'(x_m),
\end{aligned}\tag{106}$$

where the second inequality holds by (105) and the equality holds by (104). However, (106) contradicts the fact that x_m satisfies (95) for sufficiently large m. That is, for any $m\geq m^*(\varepsilon)$, it must be the case that

$$x_m\geq\underline{x}_m.$$

Finally, taking $\varepsilon\to 0$ and using Equation (97) completes the proof of the Theorem.

□

Proof of Proposition 4. Here we focus on the decision of the second voting round, in which $2n-2m$ individuals vote after the outcome of the first voting round by the committee has been made public. We let d denote the *difference* in votes that alternative A received from the committee members compared to alternative B. Note that d is an odd number (assuming all the committee members vote) and that it may be positive or negative ($d\in\{-2m-1,\ldots,2m+1\}$). This means that in the second voting round alternative A must receive at least $d+1$ more votes than alternative B if it is to be implemented. Like in the first voting round, we focus in the second round on symmetric equilibrium. That is, we assume that all individuals acquire the same level of information $x^{**}(n)\in[0,1]$ and that, conditional on the signal they obtain (and given their posterior q obtained after the first voting round), they vote for the same alternative (with probability one). We also assume that there is no abstention in the second round and that

$$2n-2m\geq 2m+4,\tag{107}$$

which is clearly satisfied if n is large enough. Condition (107) ensures that individuals who are not members of the committee can undo any decision taken by members of the committee (for appropriate voting behavior which does not require unanimity). Without loss of generality, we assume that $d>0$, and hence $q>1/2$. That is, A received more votes in the first round and individuals with the right to vote in the second round believe that alternative A is the right alternative with probability larger than 1/2.

Recall that if $x < 1/2$, in equilibrium all individuals in the second round must equate the pivotal probability with the increases in disutility derived from acquiring more information than x—see Equation (6) (for cost sharing) and Equation (15) (for TS). We distinguish two cases. First, assume that given x, m, d, and q, all voters in the second round vote for the same alternative. Due to (107), no single individual is pivotal, so it must be the case that $x = 0$. This means that individuals voting in the second round receive no informative signal, so the only voting behavior that is consistent with equilibrium is to vote according to the posterior $q > \frac{1}{2}$ in favor of alternative A.

Second, assume that given x, m, d, and q, all voters in the second round vote for different alternatives depending on the signal. That is, those who receive an A signal vote for alternative A, those who receive a B signal vote for alternative B. Now let i denote one individual with the right to vote in the second round. One can easily see that (excluding i's vote) the probability that there is a tie is then

$$\begin{aligned} P_x[\text{tie}] = q \cdot &\left[\binom{2(n-m)-1}{(n-m)-\frac{d+1}{2}} \cdot \left(\frac{1}{2}+x\right)^{(n-m)-\frac{d+1}{2}} \cdot \left(\frac{1}{2}-x\right)^{(n-m)+\frac{d-1}{2}} \right] \\ + (1-q) \cdot &\left[\binom{2(n-m)-1}{(n-m)-\frac{d+1}{2}} \cdot \left(\frac{1}{2}-x\right)^{(n-m)-\frac{d+1}{2}} \cdot \left(\frac{1}{2}+x\right)^{(n-m)+\frac{d-1}{2}} \right]. \end{aligned} \tag{108}$$

One can easily verify that for fixed x, m, d, and q,

$$\lim_{n\to\infty} P_x[tie] = 0.$$

Hence,

$$\lim_{n\to\infty} x^{**}(n) = 0.$$

Then using the same logic as in Section 7.1, we can see that if n is sufficiently large, any individual with the right to vote in the second round believes, given the posterior q and his/her own signal of accuracy $x^{**}(n) \approx 0$, that A is the right alternative with probability higher than 1/2. This is because $q > 1/2$ is independent of n. But this contradicts the assumption that individuals acquire positive levels of information in equilibrium.

To sum up, for large n, in any equilibrium of the game underlying AV, it must be the case that all individuals in the second round acquire zero information and simply vote for the alternative that received more votes in the committee voting round, provided that its members chose a positive level of information acquisition.

□