DISCUSSION PAPER SERIES

DP15297

GLOBALIZATION AND PANDEMICS

Pol Antras, Stephen J. Redding and Esteban Rossi-Hansberg

INTERNATIONAL TRADE AND REGIONAL ECONOMICS

MACROECONOMICS AND GROWTH

ISSN 0265-8003

GLOBALIZATION AND PANDEMICS

Pol Antras, Stephen J. Redding and Esteban Rossi-Hansberg

Discussion Paper DP15297
Published 16 September 2020
Submitted 14 September 2020

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

GLOBALIZATION AND PANDEMICS

Abstract

We develop a model of human interaction to analyze the relationship between globalization and pandemics. Our framework provides joint microfoundations for the gravity equation for international trade and the Susceptible-Infected-Recovered (SIR) model of disease dynamics. We show that there are cross-country epidemiological externalities, such that whether a global pandemic breaks out depends critically on the disease environment in the country with the highest rates of domestic infection. A deepening of global integration can either increase or decrease the range of parameters for which a pandemic occurs, and can generate multiple waves of infection when a single wave would otherwise occur in the closed economy. If agents do not internalize the threat of infection, larger deaths in a more unhealthy country raise its relative wage, thus generating a form of general equilibrium social distancing. Once agents internalize the threat of infection, the more unhealthy country typically experiences a reduction in its relative wage through individual-level social distancing. Incorporating these individual-level responses is central to generating large reductions in the ratio of trade to output and implies that the pandemic has substantial effects on aggregate welfare, through both deaths and reduced gains from trade.

JEL Classification: F15, F23, I10

Keywords: Globalization, Pandemics, Gravity Equation, SIR model

Pol Antras - pantras@fas.harvard.edu
Harvard University and CEPR

Stephen J. Redding - reddings@princeton.edu
Princeton University and CEPR

Esteban Rossi-Hansberg - erossi@princeton.edu
Princeton University and CEPR

Acknowledgements
We thank Elena Aguilar, Maxim Alekseev, Gordon Ji, Daniel Ramos, Sean Zhang, and Shuhan Zou for research assistance. We have received valuable comments from participants at the Sardinia Empirical Trade Conference and Purdue University. All errors, opinions and omissions are our own.

Globalization and Pandemics*

Pol Antràs
Harvard University and NBER
pantras@fas.harvard.edu

Stephen J. Redding
Princeton University and NBER
reddings@princeton.edu.

Esteban Rossi-Hansberg
Princeton University and NBER
erossi@princeton.edu

September 14, 2020

Abstract

We develop a model of human interaction to analyze the relationship between globalization and pandemics. Our framework provides joint microfoundations for the gravity equation for international trade and the Susceptible-Infected-Recovered (SIR) model of disease dynamics. We show that there are cross-country epidemiological externalities, such that whether a global pandemic breaks out depends critically on the disease environment in the country with the highest rates of domestic infection. A deepening of global integration can either increase or decrease the range of parameters for which a pandemic occurs, and can generate multiple waves of infection when a single wave would otherwise occur in the closed economy. If agents do not internalize the threat of infection, larger deaths in a more unhealthy country raise its relative wage, thus generating a form of general equilibrium social distancing. Once agents internalize the threat of infection, the more unhealthy country typically experiences a reduction in its relative wage through individual-level social distancing. Incorporating these individual-level responses is central to generating large reductions in the ratio of trade to output and implies that the pandemic has substantial effects on aggregate welfare, through both deaths and reduced gains from trade.

KEYWORDS: Globalization, Pandemics, Gravity Equation, SIR Model
JEL: F15, F23, I10

*We thank Elena Aguilar, Maxim Alekseev, Gordon Ji, Daniel Ramos, Sean Zhang, and Shuhan Zou for research assistance. We have received valuable comments from participants at the Sardinia Empirical Trade Conference and Purdue University. All errors, opinions and omissions are our own.

> "As to foreign trade, there needs little to be said. The trading nations of Europe were all afraid of us; no port of France, or Holland, or Spain, or Italy would admit our ships or correspond with us." (*A Journal of the Plague Year*, Daniel Defoe, 1665)

1 Introduction

Throughout human history, globalization and pandemics have been closely intertwined. The Black Death arrived in Europe in October 1347 when twelve ships from the Black Sea docked at the Sicilian port of Messina – the word quarantine originates from the Italian word for a forty-day period of isolation required of ships and their crews during the Black Death pandemic. Much more recently, on January 21, 2020, the first human-to-human infections of COVID-19 in Europe are presumed to have taken place in Starnberg, Germany, when a local car parts supplier (Webasto) organized a training session with a Chinese colleague from its operation in Wuhan, China. These examples are by no means unique; accounts of contagion through international business travel abound. In this paper we study the interplay between human interactions – motivated by an economically integrated world – and the prevalence and severity of pandemics.

We develop a conceptual framework to shed light on a number of central questions about the two-way interaction between trade and pandemics. Does a globalized world make societies more vulnerable to pandemics? To what extent are disease dynamics different in closed and open economies? What are the implications of pandemics for the volume and pattern of international trade? How do these changes in the volume and pattern of international trade in turn influence the spread of the disease? To what extent are there externalities between the health policies of different countries in the open economy equilibrium? Will the threat of future pandemics have a permanent impact on the nature of globalization?

Our conceptual framework combines the canonical model of international trade from economics (the gravity equation) with the seminal model of the spread of infectious diseases from epidemiology (the Susceptible-Infected-Recovered or SIR model). We provide joint microfoundations for these relationships in a single underlying theory in which both international trade and the spread of disease are driven by human interactions. Through jointly modelling these two phenomena, we highlight a number of interrelationships between them. On the one hand, the contact rate among individuals, which is a central parameter in benchmark epidemiology models, is endogenous in our framework, and responds to both economic forces (e.g., the gains from international trade) and to the dynamics of the pandemic (e.g., the perceived health risk associated with international travel). On the other hand, we study how the emergence of a pandemic and the perceived risk of future outbreaks shapes the dynamics of international trade, and the net gains from international trade once the death toll from the pandemic is taken into account.

We consider an economic setting – described in Section 2 – in which agents in each country consume differentiated varieties and choose the measures of these varieties to source from home and abroad. We suppose that sourcing varieties is costly, both in terms of the fixed costs of meeting with other agents that sell varieties – an activity that involves intranational or international travel – and the variable costs of shipping varieties. Within this environment, the measures of varieties sourced at home and abroad are endogenously determined by trade frictions, country sizes, and the state of a pandemic, thus determining the intensity with which agents meet one another. If a healthy (susceptible) agent meets an infected agent, the probability that the disease is transmitted between them depends on the local epidemiological environment where the meeting takes place. This contagion risk associated with the local epidemiological environment is in turn shaped by local climate, by local social and cultural norms, and also by local health policies. Therefore, since domestic agents meet with other agents at home and abroad, the rate at which they are infected by the disease depends not only on their home health policies but also on those abroad.

To build intuition, we begin in Sections 3 and 4 by assuming that infection does not affect the ability of agents to produce and trade, and that agents are unaware of the threat of the infection, which implies that they do not have an incentive to alter their individual behavior (though, in Section 4, we allow the pandemic to cause deaths). In such a case, we show that human interactions and trade flows are characterized by gravity equations that feature origin characteristics, destination characteristics and measures of bilateral trade frictions. Using these gravity equations, we show that the welfare gains from trade can be written in terms of certain sufficient statistics, namely the domestic trade share, the change in a country's population (i.e., deaths) that can ascribed to trade integration, and model parameters. This is similar to the celebrated Arkolakis et al. (2012) formula for the gains from trade, but how trade shares map into welfare changes now depends on a wider range of model parameters than the conventional elasticity of trade with respect to trade costs. These gravity equations also determine the dynamics of the pandemic, which take a similar form to those of multi-group SIR model, but one in which the intensity of interactions between the different groups is endogenously determined by international trade, and potentially evolves over the course of the disease outbreak due to general-equilibrium effects. We find that these disease dynamics differ systematically between the open-economy case and the closed-economy case. In particular, in the open economy, the condition for a pandemic to be self-sustaining (i.e., $\mathcal{R}_0^{Open} > 1$, where $\mathcal{R}_0^{Open}$ is the global basic reproduction number) depends critically on the epidemiological environment in the country with the highest rates of domestic infection.

We show that globalization and pandemics interact in a number of subtle ways. First, we demonstrate that the dynamics of the disease are significantly impacted by the degree of trade openness. More specifically, we show that a decline in any international trade or mobility friction reduces the rates at which agents from the same country meet one another and increases the rates at which agents from different countries meet one another. If countries are sufficiently symmetric in all respects, a decline in any (symmetric) international trade friction also leads to an overall increase in the total number of human interactions (domestic plus foreign). As a result, whenever countries are sufficiently symmetric, a decline in any (symmetric) international trade friction *increases* the range of parameters for which a global pandemic occurs. More precisely, even if an epidemic would not be self-sustaining in either of the two symmetric countries in the closed economy (because

$\mathcal{R}_0^{Closed} < 1$), it can be self-sustaining in an open economy ($\mathcal{R}_0^{Open} > 1$), because of the enhanced rate of interactions between agents in the open economy.

In contrast, if countries are sufficiently different from one another in terms of some of their primitive epidemiological parameters (i.e., the exogenous component of the infection rate or the recovery rate from the disease), a decline in any international trade friction can have the opposite effect of *decreasing* the range of parameters for which a global pandemic occurs. This situation arises because the condition for the pandemic to be sustaining in the open economy depends critically on the domestic rate of infections in the country with the worst disease environment. As a result, when one country has a much worse disease environment than the other, trade liberalization can reduce the share of that country's interactions that occur in this worse disease environment, thereby taking the global economy below the threshold for a pandemic to be self-sustaining for the world as a whole. Hence, in this case, on top of the negative effect on income, tightening trade or mobility restrictions can worsen the spread of the disease in all countries, including the relatively healthy one.

More generally, when a pandemic occurs in the open economy, we show that its properties are influenced by the disease environments in all countries, and can display significantly richer dynamics than in the standard closed-economy SIR model. For instance, even without lockdowns, multiple waves of infection can occur in the open economy, when there would only be a single wave in each country in the closed economy.

All the results discussed so far hold even in an environment in which the pandemic causes no deaths (or dead individuals are immediately replaced by newborn individuals). When we allow in Section 4 for the pandemic to cause deaths and thus a decline in population, we obtain additional general-equilibrium effects. In this case, for instance, a country with a worse disease environment tends to experience a larger reduction in population and labor supply, which in turn leads to an increase in its relative wage. This wage increase reduces the share of interactions that occur in that country's bad disease environment, and increases the share that occur in better disease environments, which again can take the global economy below the threshold for a pandemic to be self-sustaining. Therefore, the general equilibrium effects of the pandemic on wages and trade patterns induce a form of "general-equilibrium social distancing" from bad disease environments that operates even in the absence purposeful social distancing motivated by health risks.[1]

In Section 5, we allow individuals to internalize the threat of infection and optimally adjust their behavior depending on the observed state of the pandemic. As in recent work (see Farboodi et al., 2020), it proves useful to assume that agents are uncertain about their own health status, and simply infer their health risk from the shares of their country's population with different health status (something they can infer from data on pandemic-related deaths). Technically, this turns the problem faced by agents into a dynamic optimal control problem in which the number of varieties that agents source from each country responds directly to the relative severity of the disease in

[1]Similar effects would operate if infections reduced the productivity of agents in the labor market, in addition to their effects on mortality.

each country. As in recent closed-economy models of social distancing (such as Farboodi et al., 2020, or Toxvaerd et al., 2020), these behavioral responses reduce human interactions, and thereby tend to flatten the curve of infections. In contrast to these closed-economy setups, these behavioral responses now have international general equilibrium implications. In both countries, agents skew their interactions away from the relatively unhealthy country, which leads to the largest falls in the ratio of trade to income in the relatively healthier country. This redirection of interactions reduces the relative demand for the unhealthy country's goods, which in turn reduces its relative wage, thereby having the opposite effect to the reduction in its relative labor supply from greater death. Depending of the timing of the wave of infections in each country, which country has more infections than the other can change over the course of the pandemic, thereby reversing this pattern of changes in trade openness and relative wages over time. We show that introducing these individual-level responses is central to generating large reductions in the ratio of trade to output and implies that the pandemic has substantial effects on aggregate welfare, through both deaths and reduced gains from trade.

Finally, we consider an extension of our dynamic framework in which there are adjustment costs of establishing the human interactions needed to sustain trade. In the presence of these adjustment costs, households react less aggressively to the pandemic and their reaction is smoother, which leads to a faster and more severe pandemic with a greater total number of deaths, but less pronounced temporary reductions in real income and trade. In deciding to accumulate contacts, households now anticipate the costs incurred in adjusting these contacts during a pandemic, although in practice with symmetric adjustment costs we find that these anticipatory effects are negligible.

Throughout the paper, we use as our core setup an economy with two countries where agents can interact across borders but are subject to trade and migration frictions. Most of our results can be easily extended to contexts with multiple regions or even a continuum of them. We focus on international trade as our main application because of the close relationship between trade and pandemics throughout human history. Nevertheless, these extensions could be used to flexibly study interactions across regions within countries or neighborhoods in a city. Ultimately, the decision of which stores to patronize in a city, and how these decisions affect local disease dynamics, is shaped by many of the same economic trade-offs that we study in an international context in this paper.

Our paper connects with several strands of existing research. Within the international trade literature, we build on the voluminous gravity equation literature, which includes, among many others, the work of Anderson (1979), Anderson and Van Wincoop (2003), Eaton and Kortum (2003), Chaney (2008), Helpman et al. (2008), Arkolakis (2010), Allen and Arkolakis (2014), and Allen et al. (2020). As in the work of Chaney (2008) and Helpman et al. (2008), international trade frictions affect both the extensive and intensive margin of trade, but our model features selection into importing rather than selection into exporting (as in Antràs et al., 2017) and, more importantly, it emphasizes human interactions among buyers and sellers. In that latter respect, we connect with the work on the diffusion of information in networks, which has been applied to a trade context by Chaney (2014). By endogenizing the interplay between globalization and pandemics, we study the nature and size of trade-induced welfare losses associated with disease transmission, thereby contributing to the very active recent literature on quantifying the gains from international trade (see, for instance, Eaton and Kortum, 2002, Arkolakis et al., 2012, Melitz and Redding, 2014, Costinot and Rodriguez-Clare, 2015, Ossa, 2015).

Although our model is admittedly abstract, we believe that it captures the role of international business travel in greasing the wheels of international trade. With this interpretation, our model connects with an empirical literature that has studied the role of international business travel in facilitating international trade (see Cristea, 2011, Blonigen and Cristea, 2015, and Startz, 2018), and more generally, in fostering economic development (see Hovhannisyan and Keller, 2015, Campante and Yanagizawa-Drott, 2018). Our simple microfounded model of trade through human interaction provides a natural rationalization for a gravity equation in international trade and shows how different types of trade frictions affect the extensive and intensive margins of trade.

Our paper also builds on the literature developing epidemiological models of disease spread, starting with the seminal work of Kermack and McKendrick (1927, 1932). More specifically, our multi-country SIR model shares many features with multigroup models of disease transmission, as in the work, among others, of Hethcote (1978), Hethcote and Thieme (1985), van den Driessche and Watmough (2002), and Magal et al. (2016).[2] A key difference is that the interaction between groups is endogenously determined by the gravity structure of international trade. The recent COVID-19 pandemic has triggered a remarkable explosion of work by economists studying the spread of the disease (see, for instance, Fernández-Villaverde and Jones, 2020) and exploring the implications of several types of policies (see, for instance, Alvarez et al., 2020, Acemoglu et al., 2020, Atkeson, 2020, or Jones et al., 2020). Within this literature, a few papers have explored the spatial dimension of the COVID-19 pandemic by simulating multi-group SIR models applied to various urban and regional contexts (see, among others, Argente et al., 2020, Bisin and Moro, 2020, Cuñat and Zymek, 2020, Birge et al., 2020, and Fajgelbaum et al., 2020). Our paper also connects with a subset of that literature, exemplified by the work of Alfaro et al. (2020), Farboodi et al. (2020), Fenichel et al. (2011), and Toxvaerd (2020) that has studied how the behavioral response of agents (e.g., social distancing) affects the spread and persistence of pandemics. Whereas most of this research is concerned with COVID-19 and adopts a simulation approach, our main goal is to develop a model of human interaction that jointly provides a microfoundation for a gravity equation and multi-group SIR dynamics, and can be used to analytically characterize the two-way relationship between globalization and pandemics in general.

Our work is also related to a literature in economic history that has emphasized the role of international trade in the transmission of disease. For the case of the Black Death, Christakos et al. (2005), Boerner and Severgnini (2014), Ricci et al. (2017), and Jedwab et al. (2019) argue that trade routes are central to understanding the spread of the plague through medieval Europe. In a review of a broader range of infection diseases, Saker et al. (2002) argue that globalization

[2]See Hetchote (2000) and Brauer and Castillo-Chavez (2012) for very useful reviews of mathematical modelling in epidemiology, and Ellison (2020) for an economist's overview of SIR models with heterogeneity.

has often played a pivotal role in disease transmission. The recent COVID-19 pandemic has also provided numerous examples of the spread of the virus through business travel.[3]

The rest of the paper is structured as follows. In Section 2, we present our baseline gravity-style model of international trade with endogenous intranational and international human interactions. In Section 3, we consider a first variant of the dynamics of disease spread in which the rate of contact between agents (though endogenous) is time-invariant during the pandemic. In Section 4, we incorporate labor supply responses to the pandemic, which affect the path of relative wages (and thus the rate of contact of agents within and across countries) during the pandemic. In Section 5, we incorporate individual behavioral responses motivated by agents adjusting their desired number of human interactions in response to their fear of being infected by the disease. We offer some concluding remarks in Section 6.

2 Baseline Economic Model

We begin by developing a stylized model of the global economy in which international trade is sustained by human interactions. Our baseline model is a simple two-country world, in which countries use labor to produce differentiated goods that are exchanged in competitive markets via human interactions. In Section 2.3, we outline how our model can be easily extended to settings featuring (i) multiple countries, (ii) intermediate inputs, and (iii) scale economies and imperfect competition.

2.1 Environment

Consider a world with two locations: East and West, indexed by i or j. We denote by $\mathcal{J}$ the set of countries in the world, so for now $\mathcal{J} = \{East, West\}$. Location $i \in \mathcal{J}$ is inhabited by a continuum of measure L_i of households, and each household is endowed with the ability to produce a differentiated variety using labor as the only input in production. We denote by w_i the wage rate in country i.

Trade is costly. There are iceberg bilateral trade cost $\tau_{ij} = t_{ij} \times (d_{ij})^{\delta}$, when shipping from j back to i, where $d_{ij} \geq 1$ is the symmetric distance between i and j, and t_{ij} is a man-made additional trade friction imposed by i on imports from country j. We let these man-made trade costs be potentially asymmetric reflecting the fact that one country may impose higher restrictions to trade (e.g., tariffs, or delays in goods clearing customs) than the other country. For simplicity,

[3]A well-known example in the U.S. is the conference held by biotech company Biogen in Boston, Massachusetts on February 26 and 27, and attended by 175 executive managers, who spread the covid-19 virus to at least six states, the District of Columbia and three European countries, and caused close to 100 infections in Massachusetts alone http://www.nytimes.com/2020/04/12/us/coronavirus-biogen-boston-superspreader.html). Another example is Steve Walsh, the so-called British "super spreader," who is linked to at least 11 new infections of COVID-19, and who caught the disease in Singapore, while he attended a sales conference in late January of 2020 (see https://www.washingtonpost.com/world/europe/british-coronavirus-super-spreader-may-have-infected-at-least-11-people-in-three-countries/2020/02/10/016e9842-4c14-11ea-967b-e074d302c7d4_story.html). The initial spread of COVID-19 to Iran and Nigeria has also been tied to international business travel.

there are no man-made frictions to internal shipments, so $t_{ii} = 1$ and $\tau_{ii} = (d_{ii})^{\delta}$, where $d_{ii} < d_{ij}$ can be interpreted as the average internal distance in country $i = East, West$.

Each household is formed by two individuals. One of these individuals – the *seller* – is in charge of producing and selling the household-specific differentiated variety from their home, while the other individual – the *buyer* – is in charge of procuring varieties for consumption from other households in each of the two locations. We let all households in country i be equally productive in manufacturing varieties, with one unit of labor delivering Z_i units of goods. Goods markets are competitive and sellers make their goods available at marginal cost. Households have CES preferences over differentiated varieties, with an elasticity of substitution $\sigma > 1$ regardless of the origin of these varieties, and they derive disutility from the buyer spending time away from home. More specifically, a household in country i incurs a utility cost

$$c_{ij}(n_{ij}) = \frac{c}{\phi} \times \mu_{ij} \times (d_{ij})^{\rho} \times (n_{ij})^{\phi}, \tag{1}$$

whenever the household's buyer secures n_{ij} varieties from location j, at a distance $d_{ij} \geq 1$ from i. The parameter μ_{ij} captures (possibly asymmetric) travel restrictions imposed by country j's government on visitors from i. The parameter c governs the cost of travel and we assume it is large enough to ensure an interior solution in which $n_{ij} \leq L_j$ for all i and $j \in \mathcal{J}$. We assume that whenever $n_{ij} < L_j$, the set of varieties procured from j are chosen at random, so if all households from i procure n_{ij} from j, each household's variety in j will be consumed by a fraction n_{ij}/L_j of households from i.[4]

Welfare of households in location i is then given by

$$W_i = \left(\sum\nolimits_{j \in \mathcal{J}} \int_0^{n_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}} - \frac{c}{\phi} \sum\nolimits_{j \in \mathcal{J}} \mu_{ij} (d_{ij})^{\rho} \times (n_{ij})^{\phi}, \tag{2}$$

where $q_{ij}(k)$ is the quantity consumed in location i of the variety produced in location j by household k.

2.2 Equilibrium

Let us first consider consumption choices in a given household for a given n_{ij}. Maximizing (2) subject to the households' budget constraint, we obtain:

$$q_{ij} = \frac{w_i}{(P_i)^{1-\sigma}} \left(\frac{\tau_{ij} w_j}{Z_j} \right)^{-\sigma}, \tag{3}$$

[4] It may seem arbitrary that it is buyers rather than sellers who are assumed to travel. In section 2.3, we offer an interpretation of the model in which trade is in intermediate inputs and the buyer travels in order to procure the parts of components necessary for the household to produce a final consumption good. In section 2.3, we also consider the case in which travel costs are in terms of labor, rather than a utility cost. Finally, in that same section 2.3, we also explore a variant of the model in which it is sellers rather than buyers who travel, as is often implicitly assumed in standard models of firm participation in trade (cf., Melitz, 2003).

where w_i is household income, w_j/Z_j is the common free-on-board price of all varieties produced in location j, τ_{ij} are trade costs when shipping from j to i, and P_i is a price index given by

$$P_i = \left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{4}$$

Multiplying equation (3) by $(q_{ij})^{(\sigma-1)/\sigma}$, summing across locations, and rearranging, it is straightforward to show that

$$Q_i = \left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\,(q_{ij})^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} = \frac{w_i}{P_i}, \tag{5}$$

so real consumption equals real income.

In order to characterize each household's choice of n_{ij}, we first plug (3) and (4) into (2) to obtain

$$W_i = w_i\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}} - \frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}}\mu_{ij}\,(d_{ij})^{\rho}\times(n_{ij})^{\phi}. \tag{6}$$

The first order condition associated with the choice of n_{ij} delivers (after plugging in (5)):

$$n_{ij} = \left(c\,(\sigma-1)\,\mu_{ij}\right)^{-1/(\phi-1)}(d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}. \tag{7}$$

Notice that bilateral human interactions follow a 'gravity-style' equation that is log-separable in origin and destination terms, and a composite of bilateral trade frictions. Evidently, natural and man-made barriers to trade (d_{ij}, t_{ij}) and to labor mobility (μ_{ij}) will tend to reduce the number of human interactions sought by agents from country i in country j. As we show in Appendix A.1, for the second-order conditions to be met for all values of μ_{ij}, d_{ij}, and t_{ij}, we need to impose $\phi > 1/(\sigma-1)$ and $\sigma > 2$.

Bilateral import flows by country i from country j are in turn given by

$$X_{ij} = n_{ij}p_{ij}q_{ij}L_i = \left(c\,(\sigma-1)\,\mu_{ij}\right)^{-\frac{1}{\phi-1}}(d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{P_i}\right)^{\frac{1}{\phi-1}} w_iL_i. \tag{8}$$

Notice that the trade shares can be written as

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}}X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}}\times(\mu_{ij})^{-\frac{1}{\phi-1}}(d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}(t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}}(\mu_{i\ell})^{-\frac{1}{\phi-1}}(d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}(t_{i\ell}w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}}, \tag{9}$$

and are thus log-separable in an origin-specific term S_j, a destination-specific term Θ_i, and a composite bilateral trade friction term given by:[5]

$$(\Gamma_{ij})^{-\varepsilon} = (\mu_{ij})^{-\frac{1}{\phi-1}}(d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}(t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}, \tag{10}$$

[5] More specifically, $S_j = (w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}}$ and $\Theta_i = \sum_{\ell\in\mathcal{J}}(\mu_{i\ell})^{-\frac{1}{\phi-1}}(d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}(t_{i\ell}w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}$.

which encompasses mobility frictions (μ_{ij}), transport costs (d_{ij}) and trade frictions (t_{ij}).

Following Head and Mayer (2014), it then follows that bilateral trade flows in (8) also follow a standard gravity equation

$$X_{ij} = \frac{X_i}{\Phi_i}\frac{Y_j}{\Omega_j}(\Gamma_{ij})^{-\varepsilon},$$

where X_i is total spending in country i, Y_j is country j's value of production, and

$$\Phi_i = \sum\nolimits_{j\in\mathcal{J}} \frac{Y_j}{\Omega_j}(\Gamma_{ij})^{-\varepsilon}; \qquad \Omega_j = \sum\nolimits_{i\in\mathcal{J}} \frac{X_i}{\Phi_i}(\Gamma_{ji})^{-\varepsilon}.$$

Notice that the distance elasticity is affected by the standard substitutability σ, but also by the traveling cost elasticity ρ, and by the convexity ϕ of the traveling costs. It is clear that both $\rho > 0$ and $\phi > 1$ increase the distance elasticity relative to a standard Armington model (in which the distance elasticity would be given by $\delta(\sigma - 1)$). The other man-made bilateral frictions also naturally depress trade flows.[6]

We next solve for the price index and household welfare in each country. Invoking equation (5), plugging (3) and (7), and simplifying delivers

$$P_i = \left(\frac{w_i}{c(\sigma-1)}\right)^{-\frac{1}{\phi(\sigma-1)-1}} \left(\sum\nolimits_{j\in\mathcal{J}} (\Gamma_{ij})^{-\varepsilon}(w_j/Z_j)^{-\frac{(\sigma-1)\phi}{\phi-1}}\right)^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}}. \tag{11}$$

Going back to the expression for welfare in (2), and plugging (5), (7) and (11), we then find

$$W_i = \frac{\phi(\sigma-1)-1}{\phi(\sigma-1)}\frac{w_i}{P_i}, \tag{12}$$

which combined with (9) implies that aggregate welfare is given by

$$W_i L_i = \frac{\phi(\sigma-1)-1}{\phi(\sigma-1)} \times (\pi_{ii})^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}} \times \left(\frac{(Z_i)^{\phi(\sigma-1)}}{c(\sigma-1)}(\Gamma_{ii})^{-\varepsilon(\phi-1)}\right)^{\frac{1}{\phi(\sigma-1)-1}} L_i. \tag{13}$$

This formula is a variant of the Arkolakis et al. (2012) welfare formula indicating that, with estimates of ϕ and σ at hand, one could compute the change in welfare associated with a shift to autarky only with information on the domestic trade share π_{ii}. A key difference relative to their contribution, however, is that the combination of ϕ and σ relevant for welfare cannot easily be backed out from estimation of a 'trade elasticity' (see equation (10)). Later, when we allow trade to affect the transmission of disease and this disease to affect mortality, a further difference will be that the effect of trade on aggregate welfare will also depend on its effect on mortality (via changes in L_i).

[6] It is also worth noting that when $\mu_{ij} = \mu_{ji}$ and $t_{ji} = t_{ij}$, this gravity equation is fully symmetric, and

$$\Phi_i = \Omega_i = \sum\nolimits_j S_j \phi_{ij} = \sum\nolimits_j (w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}.$$

We conclude our description of the equilibrium of our model by discussing the determination of equilibrium wages. For that, it is simplest to just invoke the equality between income and spending in each country, that is $\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i$, which plugging in (9), can be written as

$$\pi_{ii}(w_i, w_j) \times w_i L_i + \pi_{ji}(w_i, w_j) \times w_j L_j = w_i L_i, \tag{14}$$

where $\pi_{ii}(w_i, w_j)$ and $\pi_{ji}(w_i, w_j)$ are given in equation (9). This pair of equations (one for i and one for j) allow us to solve for w_i and w_j as a function of the unique distance d_{ij}, the pair of mobility restriction parameters μ_{ij} and μ_{ji}, the pair of man-made trade barriers t_{ij} and t_{ji}, and the parameters ϕ, σ, δ, and ρ. Setting one of the country's wages as the numéraire, the general equilibrium only requires solving one of these non-linear equations in (14). Once one has solved for this (relative) wage, it is straightforward to solve for trade flows and for the flow of buyers across locations, as well as for the implied welfare levels.

Note that the general-equilibrium condition in (14) is identical to that obtained in standard gravity models, so from the results in Alvarez and Lucas (2007), Allen and Arkolakis (2014), or Allen et al. (2020), we can conclude that:[7]

Proposition 1 *As long as trade frictions Γ_{ij} are bounded, there exists a unique vector of equilibrium wages $\mathbf{w}^* = (w_i, w_j) \in \mathbb{R}^2_{++}$ that solves the system of equations in (14).*

Using the implicit-function theorem, it is also straightforward to see that the relative wage w_j/w_i will be increasing in L_i, Γ_{ii}, Γ_{ji}, and Z_j, while it will be decreasing in L_j, Γ_{jj}, Γ_{ij}, and Z_i.

Given the vector of equilibrium wages $\mathbf{w} = (w_i, w_j)$, we are particularly interested in studying how changes in trade frictions (d_{ij}, t_{ij}, or μ_{ij}) affect the rate of human-to-human interactions at home, abroad and worldwide. Note that, combining equations (3), (8), and (9), we can express

$$n_{ij}(\mathbf{w}) = \left(\frac{t_{ij} (d_{ij})^{\delta} w_j}{P_i(\mathbf{w}) Z_j} \right)^{\sigma - 1} \pi_{ij}(\mathbf{w}), \tag{15}$$

where $\pi_{ij}(\mathbf{w})$ is given in (9) and $P_i(\mathbf{w})$ in (11). Studying how $n_{ii}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ are shaped by the primitive parameters of the model is complicated by the general equilibrium nature of our model, but in Appendix A.2 we are able to show that:

Proposition 2 *A decline in any international trade or mobility friction $(d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji})$ leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).*

In words, despite the fact that changes in trade and mobility frictions obviously impact equilibrium relative wages, the more open are economies to the flow of goods and people across borders,

[7] In Alvarez and Lucas (2007), uniqueness requires some additional (mild) assumptions due to the existence of an intermediate-input sector. Because our model features no intermediate inputs, we just need to assume that trade frictions remain bounded.

the larger will be international interactions and the lower will be domestic interactions.

We can also study the effect of reductions in international trade and mobility frictions on the *overall* measure of varieties consumed by each household, which also corresponds to the number of human interactions experienced by each household's buyer (i.e., $n_{ii} + n_{ij}$). Similarly, we can also study the total number of human interactions carried out by each household's seller (i.e., $n_{ii}+n_{ji}$).[8] General equilibrium forces complicate this comparative static, but we are able to show that (see Appendix A.3).

Proposition 3 *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then, a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.*

The assumption of full symmetry is extreme, but the result of course continues to hold true if country asymmetries are small and trade frictions are not too asymmetric across countries. Furthermore, exhaustive numerical simulations suggest that the result continues to hold true for arbitrarily asymmetric declines in trade frictions, as long as countries are symmetric in size ($L_i = L$) and in technology ($Z_i = Z$).

Reverting back to our general equilibrium with arbitrary country asymmetries, we can also derive results for how changes in the labor force in either country affect the per-household measure of interactions at home and abroad. More specifically, from equation (14), it is straightforward to see that the relative wage w_j/w_i is monotonic in the ratio L_i/L_j. Furthermore, working with equations (7) and (11), we can establish (see Appendix A.4 for a proof):

Proposition 4 *A decrease in the relative size of country i's population leads to a decrease in the rates n_{ii} and n_{ji} at which individuals from all countries will meet individuals in country i, and to an increase in the rates n_{jj} and n_{ij} at which individuals from all countries will meet individuals in the other country j.*

This result will prove useful in Section 4, where we study how general equilibrium forces partly shape the dynamics of an epidemic. For instance, if the epidemic affects labor supply disproportionately in one of the countries, then the implied increase in that country's relative wage will induce a form of general equilibrium social distancing, as it will incentivize home buyers to avoid that country, even without social distancing motivated by health risks.

2.3 Extensions

Our baseline economic model is special among many dimensions, so it is important to discuss the robustness of some of the key insights we take away from our economic model. Because the gravity equation of international trade can be derived under a variety of economic environments and market

[8] Note that despite us modeling a frictionless labor market, the assumed symmetry of all households implies that no household has any incentive to hire anybody to buy or sell goods on its behalf.

structures, it is perhaps not too surprising that many of the key features of our model carry over to alternative environments featuring multiple countries, intermediate input trade, scale economies and imperfect competition. We next briefly describe four extensions of our model, but we leave all mathematical details to the Appendix (see Appendix C).

Some readers might object to the fact that, in our baseline model, production uses labor, while the traveling cost is specified in terms of a utility cost. We make this assumption to identify international travel with specific members of the household, which facilitates a more transparent transition to a model of disease transmission driven by human-to-human interactions. Nevertheless, in terms of the mechanics of our economic model, this assumption is innocuous. More specifically, in the first extension studied in Appendix C, we show that Propositions 1 through 4 continue to hold whenever travel costs in equation (1) are specified in terms of labor rather than being modelled as a utility cost. In fact, this version of the model is isomorphic to our baseline model above, except for a slightly different expression for the equilibrium price index P_i.

The assumption that households travel internationally to procure consumption goods may seem unrealistic. Indeed, international business travel may be better thought as being a valuable input when firms need specialized inputs and seek potential providers of those inputs in various countries. Fortunately, it is straightforward to re-interpret our model along those lines by assuming that the differentiated varieties produced by households are intermediate inputs, which all households combine into a homogeneous final good, which in equilibrium is not traded. The details of this re-interpretation are worked out in the second extension studied in Appendix C.

Returning to our baseline economic model, in Appendix C we next derive our key equilibrium conditions for a world economy with multiple countries. In fact, all the equations above, except for the labor-market clearing condition (14) apply to that multi-country environment once the set of countries $\mathcal{J}$ is re-defined to include multiple countries. The labor-market condition is in turn simply given by $\sum_{j\in\mathcal{J}} \pi_{ij}(\mathbf{w})\, w_j L_j = w_i L_i$, where $\pi_{ij}(\mathbf{w})$ is defined in (9). Similarly, the model is also easily adaptable to the case in which there is a continuum of locations $i \in \Omega$, where Ω is a closed and bounded set of a finite-dimensional Euclidean space. The equilibrium conditions are again unaltered, with integrals replacing summation operators throughout.

Finally, in Appendix C we explore a variant of our model in which it is the household's seller rather than the buyer who travels to other locations. We model this via a framework featuring scale economies, monopolistic competition and fixed cost of exporting, as in the literature on selection into exporting emanating from the seminal work of Melitz (2003), except that the seller fixed costs are a function of the measure of buyers reached in a destination market. Again, Propositions 1 through 4 continue to hold in such an environment.

3 A Two-Country SIR Model with Time-Invariant Interactions

So far, we have just characterized a static (steady-state) model of international trade supported by international travel. Now let us consider the case in which the model above describes a standard "day" in the household. More specifically, in the morning the buyer in each household in i leaves the house and visits n_{ii} sellers in i and n_{ij} sellers in j, procuring goods from each of those households. For simplicity, assume that buyers do not travel together or otherwise meet each other. While the buyer visits other households and procures goods, the seller in each household sells its own goods to visitors to their household. There will be n_{ii} domestic visitors and n_{ji} foreign visitors. In the evening, the two members of the household reunite.

3.1 Preliminaries

With this background in mind, consider now the dynamics of contagion. As in the standard epidemiological model, we divide the population at each point in time into **S**usceptible households, **I**nfectious households, and **R**ecovered households (we will incorporate deaths in the next section). We think of the health status as being a household characteristic, implicitly assuming a perfect rate of transmission within the household (they enjoy a passionate marriage), and also that recovery is experienced contemporaneously by all household members. For simplicity, we ignore the possibility that a vaccine puts an end to an epidemic before herd immunity is achieved.

In this section we seek to study the dynamics of a two-country SIR model in which the pandemic only generates cross-country externalities via contagion (and not via terms of trade effects), and in which households do not exert any pandemic-motivated social distancing. Hence, we assume that the infection has no effect on the ability to work and trade or mortality, and that agents are unaware of the threat of infection and their health status, which implies that they have no incentive to change their individual behavior. Labor supply and aggregate income are constant in each country and over time, because there are no deaths and households have no incentive to social distance. We relax these assumptions in Section 4, where we allow for deaths from the disease, but assume that agents remain unaware of the threat of infection and their health status, and hence continue to have no incentive to change their individual behavior. The result is a model in which the dynamics of the pandemic affect the evolution of the labor supply and aggregate income in each country. In Section 5 we go further and assume that agents understand that if they become infected, they have a positive probability of dying (an event that they, of course, do notice!). The possibility of dying generates behavioral responses to prevent contagion by reducing interactions.

In sum, the goal of this section is to understand how cross-country interactions motivated by economic incentives affect the spread of a pandemic in a world in which these interactions are *time-invariant* during the pandemic. It is important to emphasize, however, that the fixed measure of interactions chosen by each household is still endogenously shaped by the primitive parameters of our model, as described in Section 2. We will be particularly interested in studying the incidence and dynamics of the pandemic for different levels of trade integration, and different values of the primitive epidemiological parameters (the contagion rate conditional on a number of interactions and the recovery rate) in each country.

3.2 The Dynamic System

As argued above, for now, the population, technology and relative wage will all be time-invariant, so we can treat n_{ii}, n_{ij}, n_{ji} and n_{jj} as fixed parameters (though obviously their constant level is shaped by the primitives of the model).

The share of households of each type evolve according to the following laws of motion (we ignore time subscripts for now to keep the notation tidy):

$$\dot{S}_i = -2n_{ii} \times \alpha_i \times S_i \times I_i - n_{ij} \times \alpha_j \times S_i \times I_j - n_{ji} \times \alpha_i \times S_i \times I_j \quad (16)$$

$$\dot{I}_i = 2n_{ii} \times \alpha_i \times S_i \times I_i + n_{ij} \times \alpha_j \times S_i \times I_j + n_{ji} \times \alpha_i \times S_i \times I_j - \gamma_i I_i \quad (17)$$

$$\dot{R}_i = \gamma_i I_i \quad (18)$$

To better understand this system, focus first on how infections grow in equation (17). The first term $2n_{ii} \times \alpha_i \times S_i \times I_i$ in this equation captures newly infected households in country i. Sellers in i receive (in expectation) n_{ii} domestic buyers, while buyers meet up with n_{ii} domestic sellers. The household thus jointly has $2n_{ii}$ domestic contacts. In those encounters, a new infection occurs with probability α_i whenever one of the agents is susceptible (which occurs with probability S_i) and the other agent is infectious (which occurs with probability I_i).[9] The second term of equation (17) reflects new infections of country i's households that occur in the foreign country when susceptible buyers from i (of which there are S_i) visit foreign households with infectious sellers. There are n_{ij} of those meetings, leading to an new infection with probability α_j whenever the foreign seller is infectious (which occurs with probability I_j). Finally, the third term in (17) reflects new infections associated with susceptible sellers in country i receiving infectious buyers from abroad (country j). Each susceptible domestic buyer (constituting a share S_i of i's population) has n_{ji} such meetings, which cause an infection with probability α_i whenever the foreign buyer is infectious (which occurs with probability I_j). The final term in equation (17) simply captures the rate at which infectious individuals recover (γ_i), and note that we assume that this recovery rate only depends on the country in which infected agents reside, regardless of where they got infected.

Once the equation determining the dynamics of new infections is determined, the one determining the change of susceptible agents in (16) is straightforward to understand, as it just reflects a decline in the susceptible population commensurate with new infections. Finally, equation (18) governs the transition from infectious households to recovered households.

In Section B of the Appendix, we provide further details on the numerical simulations of the two-country SIR model that we use in the figures below to illustrate our results, including a justification for the parameter values we use.

[9] In summing the buyer and seller domestic contact rates to obtain a domestic contact rate of $2n_{ii}$ for the household, we use the property of continuous time that there is zero probability that the buyer and seller are simultaneously infected at exactly the same instant.

3.3 The Closed-Economy Case

Our model reduces to a standard SIR model when there is no movement of people across countries, and thus no international trade. In such a case, the system in (16)-(18) reduces to

$$\begin{aligned}\dot{S}_i &= -\beta_i \times S_i \times I_i \\ \dot{I}_i &= \beta_i \times S_i \times I_i - \gamma_i I_i \\ \dot{R}_i &= \gamma_i I_i\end{aligned}$$

where $\beta_i = 2n_{ii}$ is the so-called contact rate. The dynamics of this system have been studied extensively since the pioneering work of Kermack and McKendrick (1927, 1932). Suppose that at some time t_0, there is an outbreak of a disease which leads to initial infections $I_i(t_0) = \varepsilon > 0$, where ε is small. Because ε is small, $S_i(t_0)$ is very close to 1, and from the second equation, we have the standard result that if the so-called *basic reproduction number* $\mathcal{R}_{0i} = \beta_i/\gamma_i$ is less than one, then, $\dot{I}_i(t) < 0$ for all $t > t_0$, and the infection quickly dies out. In other words, when $\mathcal{R}_{0i} = \beta_i/\gamma_i < 1$ an *epidemic-free* equilibrium is globally stable. If instead $\mathcal{R}_{0i} = \beta_i/\gamma_i > 1$, the number of new infections necessarily rises initially and the share of susceptible households declines until the system reaches a period t^* at which $S_i(t^*) = \gamma_i/\beta_i$, after which infections decline and eventually go to 0. The steady-state values of $S_i(\infty)$ in this *epidemic* equilibrium is determined by the solution to this simple non-linear equation:[10]

$$\ln S_i(\infty) = -\frac{\beta_i}{\gamma_i}(1 - S_i(\infty)). \tag{19}$$

This equation admits a unique solution with $1 > S_i(\infty) > 0$.[11] Furthermore, because $S_i(\infty) < \gamma_i/\beta_i$ (since $S_i(t^*) = \gamma_i/\beta_i$ at the peak of infections), differentiation of (19) implies that the steady-state share of susceptible households $S_i(\infty)$ is necessarily decreasing in $\mathcal{R}_{0i}$. In sum, in the closed-economy case, $S_i(\infty) = 1$ as long as $\mathcal{R}_{0i} \leq 1$, but when $\mathcal{R}_{0i} > 1$, the higher is $\mathcal{R}_{0i}$, the lower is $S_i(\infty)$, and the more people will have been infected by the end of the epidemic.

3.4 The Open-Economy Case

We can now return to the two-country system in (16)-(18). We first explore the conditions under which a pandemic-free equilibrium is stable, and infections quickly die out worldwide, regardless of where the disease originated. For that purpose, it suffices to focus on the laws of motion for

[10]To see this, begin by writing

$$\frac{\dot{S}_i}{S_i} = -\beta_i I_i = -\frac{\beta_i}{\gamma_i}\dot{R}(i).$$

Now taking logs and integrating, and imposing $I_i(\infty) = 0$, delivers

$$\ln S_i(\infty) - \ln S_i(t_0) = -\frac{\beta_i}{\gamma_i}(1 - S_i(\infty) - R_i(0)).$$

Finally, imposing $\ln S_i(t_0) \simeq 0$ and $R_i(t_0) \simeq 0$, we obtain equation (19).

[11]Equation (19) is obviously also satisified when $S_i(\infty) = 1$, but this equilibrium is not stable when $\mathcal{R}_{0i} > 1$.

(S_i, S_j, I_i, I_j) evaluated at the pandemic-free equilibrium, in which $S_i = S_j \simeq 1$ and $I_i = I_j \simeq 0$. The Jacobian of this system is given by

$$J = \begin{bmatrix} 0 & 0 & -2\alpha_i n_{ii} & -(\alpha_j n_{ij} + \alpha_i n_{ji}) \\ 0 & 0 & -(\alpha_j n_{ij} + \alpha_i n_{ji}) & -2\alpha_j n_{jj} \\ 0 & 0 & 2\alpha_i n_{ii} - \gamma_i & \alpha_j n_{ij} + \alpha_i n_{ji} \\ 0 & 0 & \alpha_j n_{ij} + \alpha_i n_{ji} & 2\alpha_j n_{jj} - \gamma_j \end{bmatrix},$$

and the largest positive eigenvalue of this matrix (see Appendix D) is given by

$$\lambda_{\max} = \frac{1}{2}(2\alpha_i n_{ii} - \gamma_i) + \frac{1}{2}(2\alpha_j n_{jj} - \gamma_j) + \frac{1}{2}\sqrt{4(\alpha_j n_{ij} + \alpha_i n_{ji})^2 + ((2\alpha_i n_{ii} - \gamma_i) - (2\alpha_j n_{jj} - \gamma_j))^2}.$$

Since we are interested in finding necessary conditions for stability of this equilibrium (i.e., $\lambda_{\max} < 0$), and noting that $\lambda_{\max}$ is increasing in n_{ij} and n_{ji}, we have that

$$\lambda_{\max} \geq \lambda_{\max}|_{n_{ij}=n_{ji}=0} = \max\{2\alpha_i n_{ii} - \gamma_i, 2\alpha_j n_{jj} - \gamma_{jj}\}. \tag{20}$$

As a result, a pandemic-free equilibrium can only be stable whenever $2\alpha_i n_{ii}/\gamma_i \leq 1$ and $2\alpha_j n_{jj}/\gamma_{jj} \leq 1$. In words, if the reproduction number $\mathcal{R}_{0i}$ based only on domestic interactions (but evaluated at the world equilibrium value of n_{ii}) is higher than 1 in *any* country, the pandemic-free equilibrium is necessarily unstable, and the world will experience at least one period of rising infections along the dynamics of the pandemic. This result highlights the externalities that countries exert on other countries when the disease is not under control purely based on the domestic interactions of agents.

It is interesting to note that we achieve the exact same result when studying the global reproduction number $\mathcal{R}_0$ associated with the world equilibrium dynamics. Remember that $\mathcal{R}_0$ is defined as the expected number of secondary cases produced by a single (typical) infection starting from a completely susceptible population. Because our model maps directly to multigroup models of disease transmission, we can invoke (and verify) results from that literature to provide an alternative analysis of the stability of the pandemic-free equilibrium in our two-country dynamic system (cf., Hethcote, 1978, Hethcote and Thieme, 1985, van den Driessche and Watmough, 2002, Magal et al. 2016). In particular, it is a well-known fact that the pandemic-free equilibrium is necessarily stable if $\mathcal{R}_0 < 1$. In order to compute $\mathcal{R}_0$, we follow the approach in Diekmann et al. (1990), and write the two equations determining the dynamics of infections as

$$\begin{bmatrix} \dot{I}_i \\ \dot{I}_j \end{bmatrix} = \underbrace{\begin{bmatrix} 2\alpha_i n_{ii} S_i & (\alpha_j n_{ij} + \alpha_i n_{ji}) S_i \\ (\alpha_j n_{ij} + \alpha_i n_{ji}) S_j & 2\alpha_j n_{jj} S_j \end{bmatrix}}_{F} \begin{bmatrix} I_i \\ I_j \end{bmatrix} - \underbrace{\begin{bmatrix} \gamma_i & 0 \\ 0 & \gamma_j \end{bmatrix}}_{V} \begin{bmatrix} I_i \\ I_j \end{bmatrix}.$$

The *next generation matrix* FV^{-1} (evaluated at $t = t_0$, and thus $S_i(t_0) = S_j(t_0) \simeq 1$) is given by

$$FV^{-1} = \begin{bmatrix} 2\alpha_i n_{ii}/\gamma_i & (\alpha_j n_{ij} + \alpha_i n_{ji})/\gamma_j \\ (\alpha_j n_{ij} + \alpha_i n_{ji})/\gamma_i & 2\alpha_j n_{jj}/\gamma_j \end{bmatrix}.$$

From the results in Diekmann et al. (1990), we thus have that

$$\mathcal{R}_0 = \rho\left(FV^{-1}\right),$$

where $\rho\left(FV^{-1}\right)$ is the spectral radius of the next generation matrix. In our case, this is given by

$$\mathcal{R}_0 = \frac{1}{2}\left(\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j}\right) + \frac{1}{2}\sqrt{\left(\frac{2\alpha_i n_{ii}}{\gamma_i} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right)^2 + 4\frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}}. \tag{21}$$

As in the case of $\lambda_{\max}$ in equation (20), we have that $\mathcal{R}_0$ is nondecreasing in n_{ij} and n_{ji}, and thus

$$\mathcal{R}_0 \geq \mathcal{R}_0|_{n_{ij}=n_{ji}=0} = \max\left\{\frac{2\alpha_i n_{ii}}{\gamma_i}, \frac{2\alpha_j n_{jj}}{\gamma_j}\right\}. \tag{22}$$

This confirms again that the disease can only be contained (that is, the pandemic-free equilibrium is stable) only if both countries' disease reproduction rate based on their domestic interactions is less than one.[12] Therefore, even if a country has a strict disease environment that would prevent an epidemic under autarky, it may be drawn into a world pandemic in the open economy equilibrium, if its trade partner has a lax disease environment, as measured by its open economy domestic reproduction rate.

Having described the existence and stability of a pandemic-free equilibrium, we next turn to a situation in which $\mathcal{R}_0 > 1$ and the resulting contagion dynamics lead to a pandemic. Building on the existing literature on multigroup models of disease transmission, it is well known that whenever the global reproduction rate satisfies $\mathcal{R}_0 > 1$, there exists a unique asymptotically globally stable 'pandemic' equilibrium in which the growth in the share of *worldwide* infected households necessarily increases for a period of time, and then declines to a point at which infections vanish and the share of susceptible households in the population in each country $(S_i(\infty), S_j(\infty))$ takes a value strictly between 0 and 1 (see, for instance, Hethcote, 1978).[13] Starting from equations (16)-(18), and going through analogous derivations as in the closed-economy case (see Appendix D), we obtain the following system of nonlinear equations pinning down the steady-state values $(S_i(\infty), S_j(\infty))$

[12] Although the expressions for $\lambda_{\max}$ and $\mathcal{R}_0$ appear different, it is straightforward to show that a necessary condition for *both* $\lambda_{\max} < 0$ and $\mathcal{R}_0 < 1$ is

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} + \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j} < 1.$$

If either $2\alpha_i n_{ii}/\gamma_i > 1$ or $2\alpha_j n_{jj}/\gamma_j > 1$, this condition cannot possibly hold.

[13] Proving global stability of the endemic equilibrium is challenging for some variants of the SIR model, but for the simple one in (16)-(18), featuring permanent immunity and no vital dynamics, global stability of the endemic equilibrium is implied by the results in Hethcote (1978), particularly section 6.

of the share of susceptible households in each country in that pandemic equilibrium:

$$\ln S_i(\infty) = -\frac{2\alpha_i n_{ii}}{\gamma_i}(1 - S_i(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(1 - S_j(\infty)) \quad (23)$$

$$\ln S_j(\infty) = -\frac{2\alpha_j n_{jj}}{\gamma_j}(1 - S_j(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}(1 - S_i(\infty)). \quad (24)$$

Although we cannot solve this system in closed form, we can easily derive some comparative statics. In particular, total differentiating we find that the steady-state values of S_i and S_j are decreasing in n_{ii}, n_{jj}, n_{ij}, and n_{ji}, and are increasing in γ_i and γ_j (see Appendix D).

We summarize these results in this section with the following proposition (see Appendix D for a proof):

Proposition 5 *Assume that there is trade between the two countries (i.e., $\alpha_j n_{ij} + \alpha_i n_{ji} > 0$), which implies that the next generation matrix FV^{-1} is irreducible. If $\mathcal{R}_0 \leq 1$, the no-pandemic equilibrium is the unique stable equilibrium. If $\mathcal{R}_0 > 1$, the no-pandemic equilibrium is unstable, and there exists a unique stable endemic equilibrium with a steady-state featuring no infections ($I_i(\infty) = I_j(\infty) = 0$) and shares of susceptible agents $S_i(\infty) \in (0,1)$ and $S_j(\infty) \in (0,1)$ that satisfy equations (23) and (24).*

In Figure 1, we illustrate these analytical results by holding the infection rate in Country 1 (α_1) constant and varying the infection rate in Country 2 (α_2). The starting point is two identical countries with a common infection rate of $\alpha_1 = \alpha_2 = 0.04$. The rest of the parameter values are described in Appendix B. For this initial common infection rate, the global reproduction number is $\mathcal{R}_0 = 0.75$, and the open economy domestic reproduction rates are $\mathcal{R}_{01} = \mathcal{R}_{02} = 0.46$. Thus, the initial infection quickly dies out and there is no global pandemic. The fraction of recovered agents in the long run, $R_i(\infty)$, which is equal to the cumulative number of infected agents in the absence of deaths, is essentially zero in both countries. The left panel of Figure 1 plots $R_i(\infty)$ as a function of $\mathcal{R}_0$ as we progressively increase α_2 from 0.04 to 0.10. The value of $\mathcal{R}_0$ is monotone in α_2 and increases from 0.75 to 1.46. Hence, as the exogenous infection rate of Country 2 increases, the global reproduction rate increases beyond the critical value of 1, and the world experiences a global pandemic. Note how the fraction of the cumulative number of recovered agents rises rapidly once $\mathcal{R}_0$ increases beyond 1 and both countries go through increasingly severe pandemics. Note also the importance of cross-country contagion in the open economy. Even though nothing is changing in the domestic characteristics of Country 1, it is dramatically affected by the worsening conditions in Country 2. The right panel shows the evolution of the pandemic in Country 1 for different levels of severity of the disease environment in Country 2.[14] The most severe and rapid pandemics are associated with the highest values of α_2 (the lightest curve in the graph). As α_2 declines and $\mathcal{R}_0$ falls and crosses the value of 1, the evolution of inflections flattens and becomes longer, until the pandemic eventually disappears.

[14]The color of each curve, correspond to the colors of the points in the left panel.

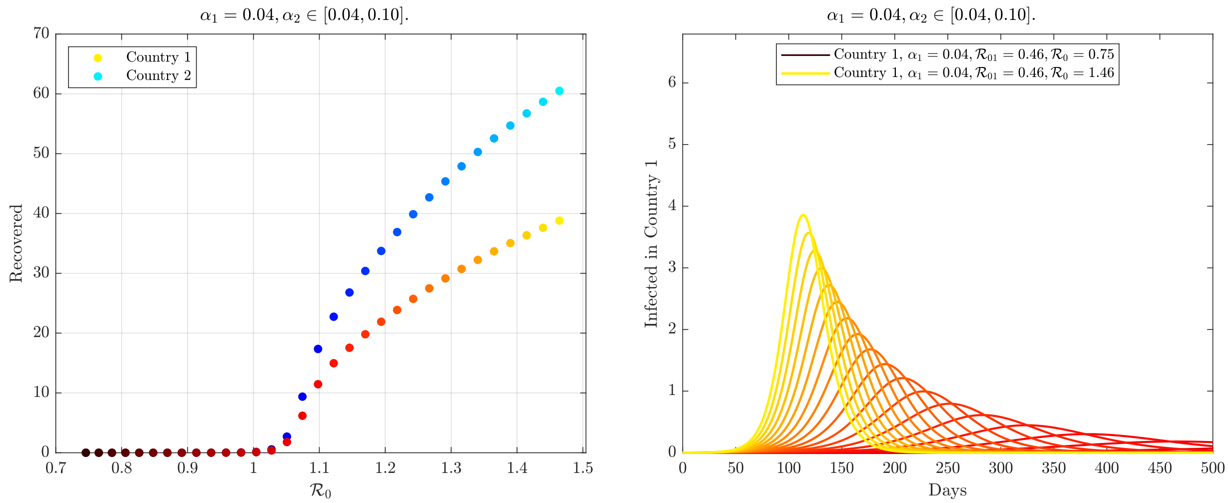

Figure 1: The Impact of Changes in the Exogenous Infection Rate in Country 2, α_2

The value of $\mathcal{R}_0$ is critical to determine the stability of a pandemic-free equilibrium. However, it is worth emphasizing that is not critical to determine the existence of a pandemic cycle in *each* country. For values of $\mathcal{R}_0$ close enough to 1, an individual country can experience a pandemic, even if the world as a whole does not, if the declining number of cases in the other country is sufficiently large. Similarly, even if $\mathcal{R}_0 > 1$, some countries might not experience a pandemic when $\mathcal{R}_0$ is close enough to 1, even if the world economy as a whole does, since cases might be rising sufficiently fast in the other country. In Figure 1, in fact, cases rise slowly when the economy crosses the $\mathcal{R}_0 = 1$ threshold. At that point, pandemics are small and happen only in the sick country, while the number of cases in the healthy country remain essentially steady. The peak of infections in both countries is a smooth function of the value of α_2.

3.5 Trade Integration and Global Pandemics

We now turn to the question of how globalization affects the prevalence and severity of a pandemic in both countries. In terms of the stability of a pandemic-free equilibrium, inspection of equations (20) and (22) might lead one to infer that avoiding a pandemic is always more difficult in a globalized world. On the one hand, it is obvious that, for given positive values of n_{ii} and n_{jj}, if the ratio α_i/γ_j is sufficiently high in *any* country in the world, a global pandemic affecting *all* countries cannot be avoided, even though the country with the lower ratio α_i/γ_j might well have avoided it under autarky. On the other hand, it would seem that even when $\alpha_i = \alpha_j$ and $\gamma_i = \gamma_j$, the max operator in (20) and (22) implies that the pandemic-free equilibrium is less likely to be stable in the open economy. It is important to emphasize, however, that n_{ii} and n_{jj} are endogenous objects and will naturally be lower, the lower are trade frictions, as formalized in Proposition 2. Still, it seems intuitive that globalization will typically foster more human interactions, as these are necessary to materialize the gains associated with trade integration, and that this will generally make it easier for pandemics to occur.

To explore this more formally, let us first consider a fully symmetric world in which all primitives of the model (population size, technology, trade barriers, recovery rates, etc.) are common in both countries, so that we have $n_{ii} = n_{jj} = n_{dom}$, $n_{ij} = n_{ji} = n_{for}$, $\alpha_i = \alpha_j = \alpha$, and $\gamma_i = \gamma_j = \gamma$. In such a case, we have

$$\lambda_{\max} = 2\alpha \left(n_{dom} + n_{for}\right) - \gamma; \qquad \mathcal{R}_0 = \frac{2\alpha \left(n_{dom} + n_{for}\right)}{\gamma},$$

and it thus follows immediately from Proposition 3 that a decline in any (symmetric) international trade friction increases $\mathcal{R}_0$ and thus decreases the range of parameters for which a pandemic-free equilibrium is stable. Furthermore, in this same symmetric case, the steady-state share of susceptible households in the population is identical in both countries and implicitly given by

$$\ln S_i(\infty) = -\frac{2\alpha \left(n_{dom} + n_{for}\right)}{\gamma} \left(1 - S_i(\infty)\right),$$

and thus not only the frequency but also the severity of the pandemic are higher the lower are (symmetric) trade frictions.

We summarize these results as follows:

Proposition 6 *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, $\Gamma_{ij} = \Gamma$, $\alpha_i = \alpha_j$, and $\gamma_i = \gamma$ for all i. Then, a decline in any (symmetric) international trade friction: (i) increases $\mathcal{R}_0$, thus decreasing the range of parameters for which a pandemic-free equilibrium is stable, and (ii) increases the share of each country's population that becomes infected during the pandemic when $\mathcal{R}_0 > 1$.*

Although we have so far focused on a fully symmetric case, the main results in this Proposition continue to hold true even if countries are not perfectly symmetric. More generally, and as noted in footnote 12, a necessary condition for the pandemic-free equilibrium to be stable is

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} + \frac{\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2}{\gamma_i \gamma_j} < 1, \tag{25}$$

and thus what is key for the effects of reductions of trade and mobility barriers on the occurrence of pandemics is whether the left-hand-side of this expression increases or declines with those reductions in barriers.

Figure 2 illustrates part (i) of Proposition 6 for a case in which we introduce an asymmetry in the exogenous infection rate across countries but the parameter condition in (25) is still satisfied. We let $\alpha_1 = 0.04$ and $\alpha_2 = 0.07$ and study the cumulative number of recovered agents when we increase symmetric international trade frictions (t_{ij}, left panel) and mobility frictions (μ_{ij}, right panel). The first point on both graphs, when $t_{12} = t_{21} = \mu_{12} = \mu_{21} = 1$, is one of the cases we studied in Figure 1. The large infection rate in Country 2 generates a pandemic in both countries. Globalization is essential to generate this pandemic. As both graphs illustrate, as we increase either tariffs or mobility restrictions, global interactions decline, and the total number of recovered agents decreases. Eventually, when the world is sufficiently isolated, the pandemic disappears and the pandemic-free equilibrium becomes stable. In both graphs, the value of $\mathcal{R}_0$ (plotted in orange and measured in the right axis) declines smoothly with frictions. The vertical line in the figure indicates the value of tariffs or mobility frictions, respectively, corresponding to $\mathcal{R}_0 = 1$.[15] Clearly, both types of barriers generate similar qualitative reductions in $R_i(\infty)$, although for this specific set of parameter values, the migration restrictions needed to eliminate the pandemic are larger than the corresponding trade frictions.

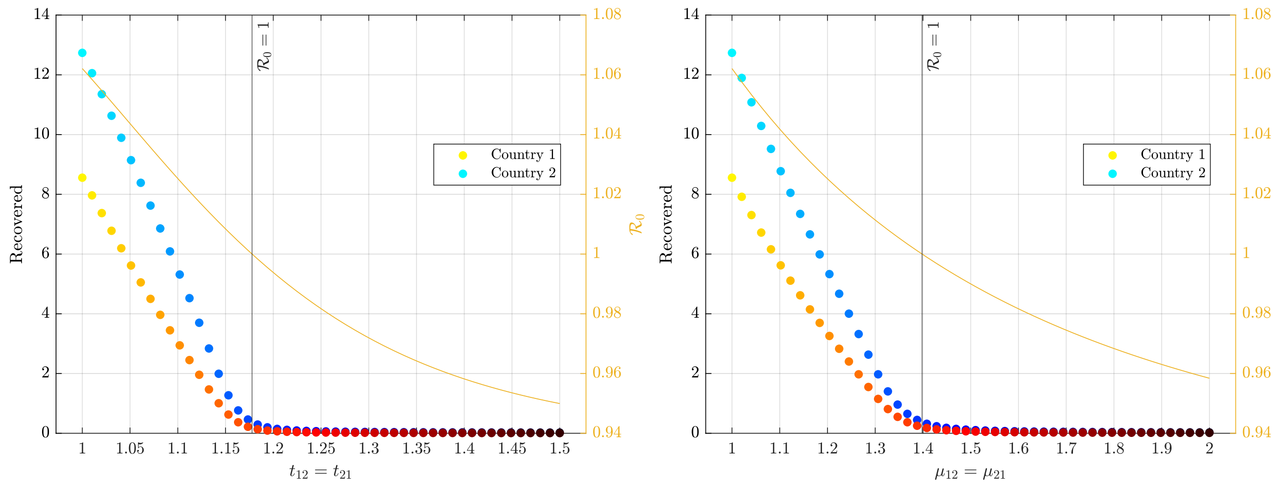

Figure 2: The Impact of Changes in Trade (left) and Mobility (right) Frictions

Figure 3 illustrates part (ii) of Proposition 6 by depicting the evolution of the fraction of agents infected for different levels of trade frictions. It corresponds to the exercise on the left panel of Figure 2 (with $\alpha_2 = 0.07 > 0.04 = \alpha_1$), so the lightest curves represent the evolution of the fraction of infected for the case with free trade ($t_{12} = t_{21} = 1$), and the darkest curves represent the case when $t_{12} = t_{21} = 1.5$. Clearly, as we increase tariffs, the epidemic in both countries becomes less severe and prolonged. The peak of the infection curve declines monotonically, as does the total number of recovered agents. Eventually, although impossible to appreciate in the graph, high tariffs eliminate the pandemic altogether and infections decline monotonically from their initial value.

Although in most cases condition (25) becomes tighter the lower are trade and mobility barriers, it is instructive to explore scenarios in which greater integration may actually *reduce* the risk of a pandemic. Suppose, in particular, that country j is a much lower risk environment, in the sense that α_j is very low – so infections are very rare – and γ_j is very high – so infected households

[15] Note that the value of $R_i(\infty)$, does not become zero for either country right at the point where tariffs or mobility frictions lead $\mathcal{R}_0$ to become greater than one. The reason is that even though one of the countries necessarily avoids a pandemic, it lingers close to its initial value of infections for a long time, which accumulates to a positive cumulative number of recovered agents.

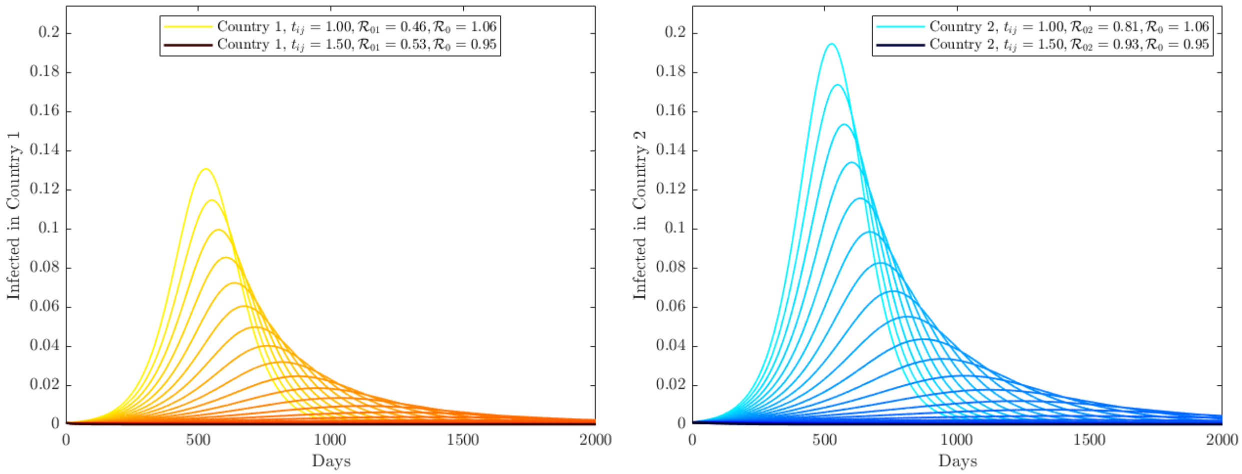

Figure 3: The Impact of Changes in Trade Frictions on the Evolution of Infections

quickly recover in that country. In the limiting case $\alpha_j \to 0$, condition (25) reduces to

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{1}{\gamma_j}\frac{(\alpha_i n_{ji})^2}{\gamma_i} < 1.$$

For a high value of γ_j, it is then straightforward to see that the fall in country i's domestic interactions n_{ii} associated with a reduction in international barriers makes this constraint laxer, even if n_{ji} goes up with that liberalization. In those situations it is perfectly possible for a pandemic-free equilibrium worldwide to only be stable when barriers are low. The intuition for this result is straightforward. In such a scenario, globalization makes it economically appealing for agents from a high-risk country to increase their interactions with agents in a low-risk country, and despite the fact that overall interactions by these agents may increase, the reduction in domestic interactions in their own high-risk environment is sufficient to maintain the disease in check.

More generally, beyond this limiting case, if countries differ enough in their epidemiological parameters, even when $\mathcal{R}_0 > 1$, it may well be the case that a decline in international trade frictions actually ameliorates the pandemic by incentivizing agents in the high-risk country to shift more of their interactions to the low-risk country.

We summarize this result as follows:

Proposition 7 *When the contagion rate α_i and the recovery rate γ_i vary sufficiently across countries, a decline in any international trade friction (i) decreases $\mathcal{R}_0$, thus increasing the range of parameters for which a pandemic-free equilibrium is stable, and (ii) when $\mathcal{R}_0 > 1$, it reduces the share of the population in the high-risk (high α_i, low γ_i) country that becomes infected during the pandemic, and it may also reduce the share of the population in the low-risk (low α_i, high γ_i) country that become infected during the pandemic.*

An interesting implication of the last statement of Proposition 7 is that although it would seem intuitive that a healthy country should impose high restrictions to the inflow of individuals from a high-risk country where a disease has just broken out, in some cases such restrictions may in fact contribute to the spread of the disease in the high-risk country, which may then make a global pandemic inevitable unless mobility restrictions are set at prohibitive levels.

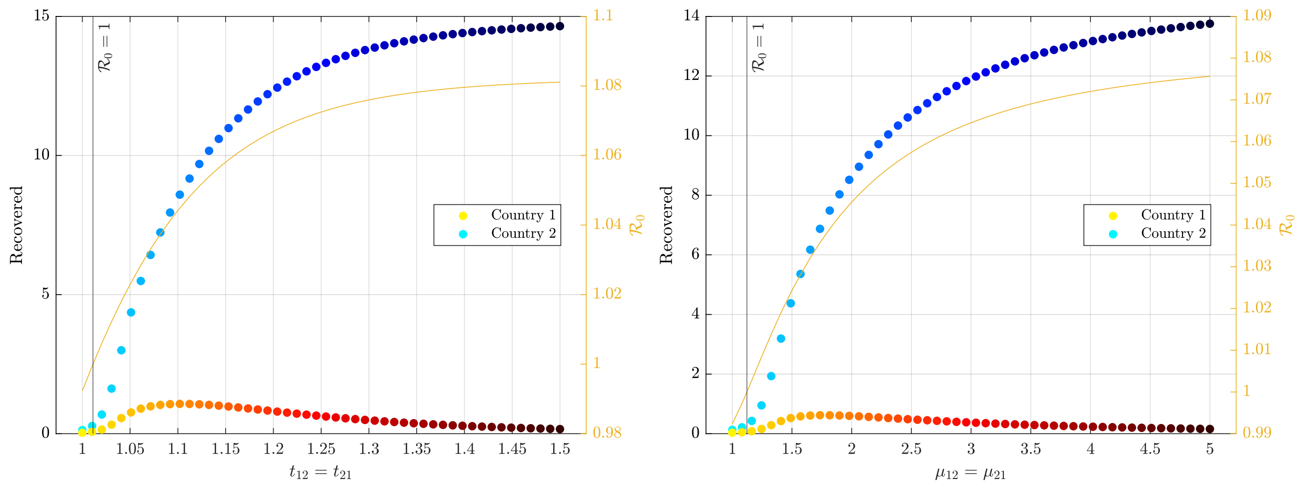

Figure 4: The Impact of Changes in Trade (left) and Mobility (right) Frictions with Large Differences in Infection Rates Across Countries ($\alpha_1 = 0.008$ and $\alpha_2 = 0.052$)

Figure 4 presents examples in which increases in trade and mobility barriers eliminate the possibility of a pandemic-free equilibrium (as predicted by part (i) of Proposition 7). As we argued above, to generate these examples we need large differences in exogenous infection rates. The figure makes the exogenous infection rate in the healthy country, Country 1, extremely small at $\alpha_1 = 0.008$, and sets $\alpha_2 = 0.052$ (a standard value).[16] In both panels, increases in frictions now lead to increases in $\mathcal{R}_0$ (again depicted in orange and measured in the right axis). Without frictions the pandemic-free equilibrium is stable. Agents in Country 2 interact sufficiently with the healthier Country 1, which helps them avoid the pandemic. As both economies impose more frictions, domestic interactions increase rapidly, while foreign interactions drop. This is bad news for Country 2, since its larger infection rate now leads to a pandemic. Perhaps surprisingly, it is also bad news for Country 1 since, although it interacts less with Country 2, it does so sufficiently to experience a pandemic. Larger frictions, which decrease aggregate income in both countries smoothly, also worsen the pandemic in both countries, at least when frictions are not too large; a clear case for free trade and mobility. Of course, as frictions increase further, eventually they isolate Country 1 sufficiently and so the severity of its local pandemic declines. In autarky, Country 1 avoids the pandemic completely, but at a large cost in the income of both countries. In contrast, higher frictions always worsen the pandemic in Country 2. Contacts with the healthy country are

[16] Relative to the baseline parameters the example also lowers c to 0.1 and ϕ to 1.5. These additional changes increase the overall number of domestic and foreign interactions.

always beneficial, since they dilute interactions with locals, which are more risky.

In Figure 5 we illustrate part (ii) of Proposition 7 for the case with high differences in exogenous infection rates across countries that we presented in Figure 4. We focus on three specific exercises: A case with free trade where $t_{12} = t_{21} = 1$, another with intermediate tariffs where $t_{12} = t_{21} = 1.2$, and a third one where countries are in autarky. With free trade, there is no pandemic in either country. As we increase trade frictions, a pandemic develops in both countries, although it is much more severe in Country 2, the country with the higher exogenous infection rate. Still, the pandemic in Country 1 ends up infecting around 1% of the population. Moving to autarky eliminates the pandemic for Country 1, but makes it even more severe, faster, and with a higher peak, in Country 2. Closing borders helps the healthy country eliminate the pandemic only if trade is completely eliminated, and at the cost of a much more severe pandemic in Country 2 and larger income losses for everyone. Although Figure 5 uses countries of identical size and studies the case of changes in symmetric tariffs, we obtain very similar results when countries are asymmetric, or when Country 1, the healthy country, is the only country closing its borders. Similar examples can also be generated when considering mobility rather than trade frictions, as in the right panel of Figure 4. The essential ingredient for declines in international frictions to ameliorate the pandemic, on top of increasing incomes, is for countries to exhibit large asymmetries in epidemiological conditions.

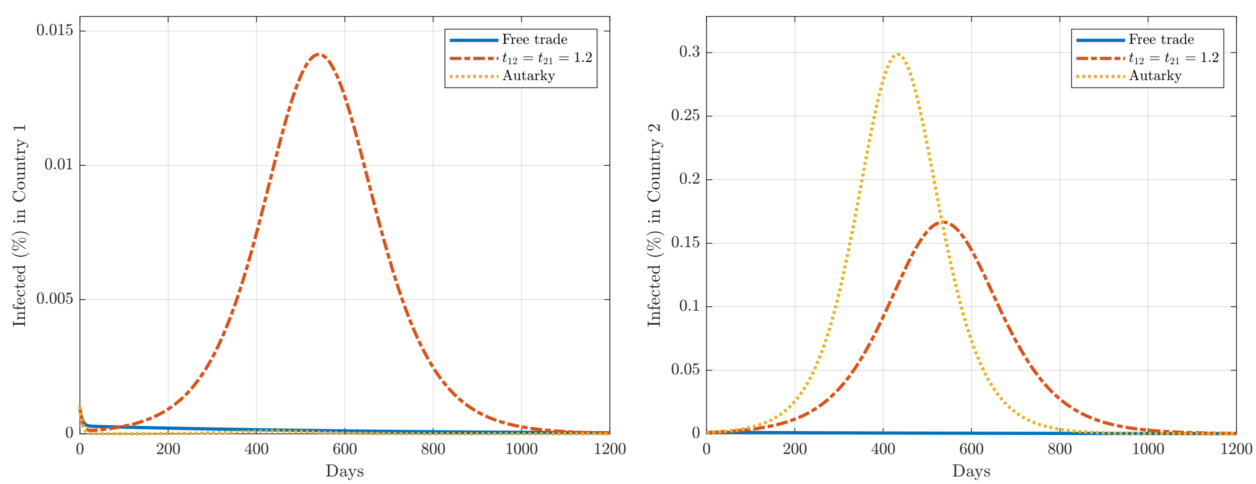

Figure 5: Evolution of Infections under Free Trade, Intermediate Trade Frictions, and Autarky with Large Differences in Infection Rates Across Countries ($\alpha_1 = 0.008$ and $\alpha_2 = 0.052$)

3.6 Transitional Dynamics: A Second Wave

When $\mathcal{R}_0 > 1$ and the world economy converges to the pandemic steady-state equilibrium in equations (23) and (24), convergence to that steady-state may entail significantly richer dynamics than in the closed-economy SIR model. In particular, in the open economy, integrating the dynamics of infections in each country using the initial conditions $S_i(0) = S_j(0) = 1$ and $R_i(0) = R_j(0) = 0$, we have the following closed-form solutions for infections in each country at each point in time (I_{it}, I_{jt}) as a function of susceptibles in each country ($S_i(t)$, $S_j(t)$):

$$I_i(t) = 1 - S_i(t) + \frac{\log S_i(t) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{2\alpha_j n_{jj}} \log S_j(t)}{\frac{2\alpha_i n_{ii}}{\gamma_i} - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{2\alpha_j n_{jj}} \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}}, \tag{26}$$

$$I_j(t) = 1 - S_j(t) + \frac{\log S_j(t) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{2\alpha_i n_{ii}} \log S_i(t)}{\frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{2\alpha_i n_{ii}} \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}}. \tag{27}$$

In the closed economy, there is necessarily a single wave of infections in the absence of a lockdown or other time-varying health policies. In contrast, in the open economy, it becomes possible for a country to experience multiple waves of infections, even in the absence of lockdowns or other time-varying health policies. From equations (26) and (27), the rate of growth of infections in each country is highest when $S_i(t) = S_j(t) = 1$, and declines as the number of susceptibles in each country falls, but the decline with $S_i(t)$ occurs at a different rate from the decline with $S_j(t)$. It is this difference that creates the possibility of multiple waves. If one country has a wham-bam epidemic that is over very quickly in the closed economy, while the other country has an epidemic that builds slowly in the closed economy, this creates the possibility for the country with the quick epidemic in the closed economy to have multiple peaks of infections in the open economy. The first peak reflects the rapid explosion of infections in that country, which dissipates quickly. The second peak, which is in general smaller, reflects the evolution of the pandemic in its trading partner.

In Figure 6 we provide an example of such a case, in which Country 1 experiences two waves of infections in the open economy, whereas Country 2 experiences a single, more prolonged and severe, wave. Country 1 features a large value of α_1, but also a large value of γ_1. Thus, although the infection rate is large, people remain contagious only briefly (perhaps because of a good contact tracing program). The resulting domestic reproduction rate $\mathcal{R}_{01} = 1.08$ and the resulting first peak of the pandemic is relatively small and quick. Since Country 1 is assumed ten times smaller than Country 2, its small initial pandemic has no significant effect on Country 2. There, the infection rate is much smaller, but the disease remains contagious for much longer, leading to a larger $\mathcal{R}_{01} = 1.66$, which also results in a global reproduction number $\mathcal{R}_0 = 1.66$.[17] The result is a more protracted but also much longer singled-peaked pandemic in Country 2. This large pandemic does affect the smaller country through international economic interactions. The large country amounts for many of the interactions of the small country, which leads to the second wave of the pandemic in Country 2. Essential for this example is that countries have very different timings for their own pandemics in autarky, but also that in the open economy the relationship is very asymmetric, with the small country having little effect on the large country but the large country influencing the small country significantly. If the interactions are large enough in both directions, both countries will end up with a synchronized pandemic with only one peak.

[17] The parameter values used in the exercise are $\sigma = 4.5$, $L_1 = 2$, $L_2 = 20$, $d_{12} = d_{11}$, $c = 0.12$, $\alpha_1 = 0.69$, $\alpha_2 = 0.09$, $\gamma_1 = 2.1$ and $\gamma_2 = 0.18$. All other values are identical to the baseline case.

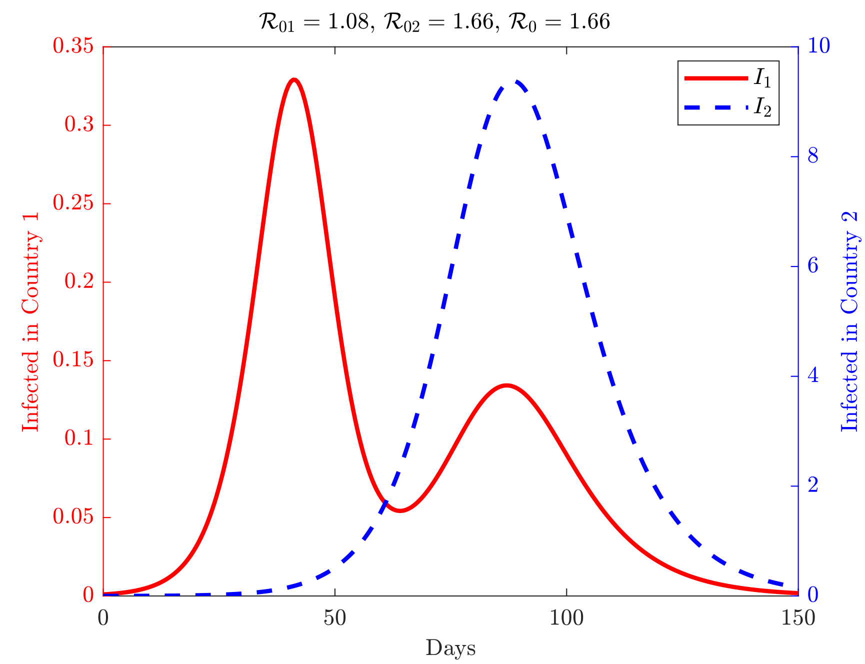

Figure 6: Multiple Waves of Infection in the Open Economy

4 General-Equilibrium Induced Responses

In this section, we allow the infection to affect mortality, but continue to assume that agents are unaware of the threat of infection.[18] There are two main implications of introducing deaths. First, the pandemic will now affect aggregate income (and thus welfare) in both countries, as households that die as a result of the pandemic will forego the net present discounted value of their future lifetime utility, which in our model is proportional to real income. Second, because deaths are not immediately replaced by new inflows into the labor force, the pandemic will affect labor supply and aggregate demand in each country, and this will impact equilibrium relative wages and real income.[19] In Section 5, we further generalize the analysis to allow individuals to internalize the threat of infection and incorporate behavioral responses.

With this new assumption, the shares of households of each type evolve according to the following laws of motion (where we again ignore time subscripts to keep the notation tidy):

$$\dot{S}_i = -2n_{ii}(\mathbf{w}) \times \alpha_i \times S_i \times I_i - [n_{ij}(\mathbf{w}) \times \alpha_j + n_{ji}(\mathbf{w}) \times \alpha_i] \times S_i \times I_j \quad (28)$$

$$\dot{I}_i = 2n_{ii}(\mathbf{w}) \times \alpha_i \times S_i \times I_i + [n_{ij}(\mathbf{w}) \times \alpha_j + n_{ji}(\mathbf{w}) \times \alpha_i] \times S_i \times I_j - (\gamma_i + \eta_i) I_i \quad (29)$$

$$\dot{R}_i = \gamma_i I_i \quad (30)$$

$$\dot{D}_i = \eta_i I_i \quad (31)$$

[18] We implicitly assume that if one of the household members dies, the other one does too. So it is not only a passionate marriage, but also a *romantic* one (in the narrow sense of the word).

[19] We could easily introduce a set of agents that are symptomatic infected agents who also reduce their labor supply, but that would complicate the analysis and blur the comparison with the results in the previous section.

There are two main differences between this dynamic system and the one above in (16)-(18). First, we now have four types of agents, as some infected agents transition to death rather than recovery. The rate at which infected agents die is given by η_i, and as in the case of the rate of recovery γ_i, it only depends on the country in which infected agents reside, and not on where they got infected. Second, we now need to make explicit the dependence of the contact rates $n_{ii}(\mathbf{w})$, $n_{ij}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ on the vector of equilibrium wages $\mathbf{w}$. As the changes in each country's population caused by deaths affect wages, these contact rates are no longer time invariant, and evolve endogenously over the course of the pandemic. In particular, the equilibrium wage vector is determined by the following goods market clearing condition:

$$\sum\nolimits_{j \in \mathcal{J}} \pi_{ji}(\mathbf{w})\, w_j (1 - D_j)\, L_j = w_i (1 - D_i)\, L_i,$$

where remember that $\pi_{ij}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ are given by (9) and (15), respectively.

We now show that this endogeneity of wages introduces a form of general equilibrium social distancing into the model. In particular, if the country with a worse disease environment experiences more deaths, its relative wage will rise. As this country's relative wage increases, its varieties become relatively less attractive to agents in the country with the better disease environment. Therefore, purely from the general equilibrium force of changes in relative labor supplies, agents in the healthy country engage in a form of endogenous social distancing, in which they skew their interactions away from the country with a worse disease environment, as summarized in the following proposition (see Appendix A.8 for a proof):

Proposition 8 *If country j experiences more deaths than country i, the resulting change in relative wages (w_j/w_i) leads country i to reduce its interactions with country j and increase its interactions with itself (general equilibrium social distancing).*

It is worth stressing that even if one of the countries features more favorable primitive health parameters than the other one, which country appears *de facto* more unhealthy can change over the course of the pandemic if the two countries' waves of infection are staggered in time. In the initial stages of the pandemic one country may experience a larger relative reduction in its labor supply (leading to endogenous social distancing in the other country), while in the later stages of the pandemic the other country experiences a larger relative reduction in its labor supply (leading to the opposite pattern of endogenous social distancing).[20]

Another straightforward implication of explicitly modeling deaths is that they naturally affect aggregate income in both countries. More specifically, whenever changes in trade or mobility barriers affect population, aggregate real income $(w_i L_i / P_i)$ and aggregate welfare $(W_i L_i)$ are directly impacted by trade-induced changes in population. Because around $\mathcal{R}_0 = 1$ deaths are particularly

[20] Although we have established this general equilibrium social distancing mechanism using death as the source of changes in relative labor supplies, if the disease also were to reduce the productivity of workers while they are infected, this additional source of labor supply movements would naturally exacerbate the general equilibrium interactions between countries.

responsive to changes in trade frictions, this effect is not necessarily negligible when evaluating the welfare implications of trade in a world with global pandemics.

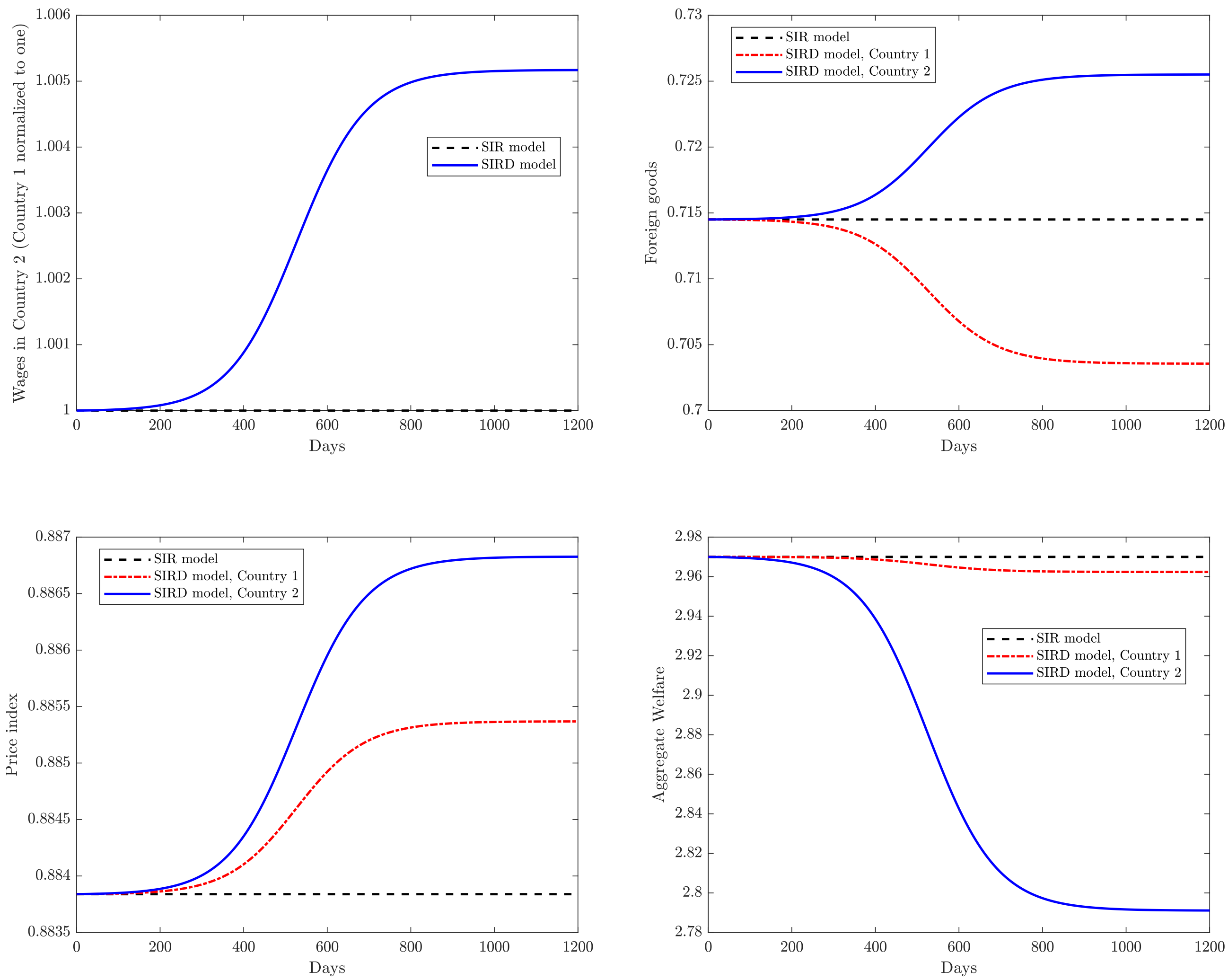

Figure 7: General Equilibrium Induced Social Distancing

We close this section by illustrating the result in Proposition 8 with a numerical example, where we let $\eta_1/(\eta_1 + \gamma_1) = 0.01$ and $\eta_2/(\eta_2 + \gamma_2) = 0.50$. Namely, we let the death rate among the infected be 1% in Country 1 and, an admittedly extreme, 50% in Country 2. The large difference in death rates amplifies the general equilibrium effects. The rest of the parameters are set to their baseline, symmetric values, across countries. Figure 7 presents the results. We denote by 'SIRD model' the case in which we incorporate deaths. For comparison purposes, we also present results for the case when $\eta_1 = \eta_2 = 0$, which we label the 'SIR model.' The larger death rate in Country 2 leads to a relative decrease in its labor supply, which increases relative wages, as illustrated in the top-left panel. Since the countries are otherwise symmetric and we chose the wage of Country 1 as the numéraire, only the wage of Country 2 increases above one in the case where death rates are positive. The resulting increase in relative wage is small (0.5%) even though about 6% of agents end up dying in Country 2. Labor supply falls, but so does the aggregate demand for goods in that country. The larger wage in Country 2 implies that both countries bias their consumption towards Country 1 varieties. As the top-right panel illustrates, the consumption of foreign varieties increases in Country 2 but falls in Country 1. Of course, we see the opposite effect on domestic varieties although the adjustments are smaller. Ultimately, agents in both countries consume less varieties, which increases the price index in both countries, although by more in Country 2 (see the bottom-left panel). Real income falls in Country 1, both per capita and in aggregate, because of this increase in the price index. In contrast, in Country 2, real income per capita rises, because the wage increases by more than the price index. Nevertheless, aggregate real income falls as result of the reduction in labor supply from deaths (bottom-right panel).

5 Behavioral Responses

Up to this point, we have assumed that agents do not change their behavior during the pandemic, unless changes in relative wages induce them to do so. Implicitly, we were assuming that although households may observe that other households are dying, they do not understand the underlying cause of those deaths and go on with their lives.

In this section, we instead consider the more realistic (but also more complicated) case in which households realize that the deaths they observe are related to the outbreak of a pandemic. Following the approach in Farboodi et al. (2020), we continue to assume, however, that all infected individuals are asymptomatic, in the sense that household behavior is independent of their specific health status, though their actual behavior is shaped by their expectation of the probability with which they are susceptible, infected, or recovered households. How is that expectation formed? A natural assumption is that agents have rational expectations and that their belief of the probability with which they have a specific health status is equal to the share of the population in their country with that particular health status.[21]

We denote the individual beliefs of the probability of being infected, susceptible recovered or dead with lowercase letters, except for their belief of their death rate, which we denote by $k_i(t)$ (instead of $d_i(t)$) to avoid a confusion with the notation we used for distance. The maximization

[21] This may raise the question among some readers as to how households are able to form this belief if, according to our assumptions, nobody observes their own health status. It suffices to assume, however, that agents have common knowledge of all parameters of the model, and form rational expectations of the path of the pandemic. For the latter, it suffices to assume that agents observe pandemic-related deaths at the outbreak of the disease. More specifically, at $t = 0$, notice from equation (31) that (i) I_{i0} can be obtained from $I_{i0} = D_0\eta_i$ since $D_{-1} \simeq 0$; (ii) $R_{i0} \simeq 0$; and (iii) S_{it} is then trivially $S_{i0} = 1 - I_{i0} - R_{i0} - D_{i0}$. With this initial condition, agents can solve for the future path of the pandemic using rational expectations.

problem of the individual, for known $i_i(0)$, $s_i(0)$ and $k_i(0)=0$, is given by

$$\begin{aligned} W_i^s(0) = \max_{n_{ii}(\cdot),n_{ij}(\cdot)} \quad & \int_0^\infty e^{-\xi t}\left[\left[Q_i\left(n_{ii}(t), n_{ij}(t)\right) - C_i\left(n_{ii}(t), n_{ij}(t)\right)\right]\left(1-k_i(t)\right)\right] dt \\ \text{s.t.} \quad & \dot{s}_i(t) = -s_i(t)\left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t)\right], \\ & \dot{i}_i(t) = s_i(t)\left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t)\right] \\ & \qquad - \left(\gamma_i + \eta_i\right) i_i(t), \\ & \dot{k}_i(t) = \eta_i i_i(t), \end{aligned}$$

where ξ is the rate of time preference, and where from equation (6),

$$Q_i\left(n_{ii}(t), n_{ij}(t)\right) = w_i(t)\left(\sum_{j\in\mathcal{J}} n_{ij}(t)\left(\frac{\tau_{ij} w_j(t)}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}},$$

and

$$C_i\left(n_{ii}(t), n_{ij}(t)\right) = \frac{c}{\phi}\sum_{j\in\mathcal{J}} \mu_{ij}\left(d_{ij}\right)^{\rho} \times \left(n_{ij}(t)\right)^{\phi}.$$

Notice that we denote with an asterisk variables chosen by *other* households that affect the dynamics of infection of a given household.[22] In equilibrium, aggregate consistency implies that $i_i(t) = I_i(t)$, $s_i(t) = S_i(t)$, and $k_i(t) = D_i(t)$. Implicitly, we are assuming that agents decide their optimal path of $n_{ii}(\cdot)$ and $n_{ij}(\cdot)$ at period zero and commit to following it. Otherwise, without commitment, at some future period and conditional on being alive, agents would want to reoptimize their choices by solving the problem above but setting $k_i(t) = 0$.[23]

The Hamiltonian of the problem faced by each household is given by

$$\begin{aligned} & H(s, i, n_{ii}, n_{ij}, \theta^i, \theta^s, \theta^k) \\ = \; & \left[Q_i\left(n_{ii}(t), n_{ij}(t)\right) - C_i\left(n_{ii}(t), n_{ij}(t)\right)\right]\left(1-k_i(t)\right)e^{-\xi t} \\ & -\theta_i^s(t)\, s_i(t)\left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t)\right] \\ & +\theta_i^i(t)\left[s_i(t)\left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t)\right] - \left(\gamma_i + \eta_i\right) i_i(t)\right] \\ & +\theta_i^k(t)\, \eta_i i_i(t). \end{aligned}$$

Hence, the optimality condition with respect to the choice of n_{ij} is

$$\left[\frac{\partial Q_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}(t)} - \frac{\partial C_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}(t)}\right]\left(1-k_i(t)\right)e^{-\xi t} = \left[\theta_i^s(t) - \theta_i^i(t)\right] s_i(t)\, \alpha_j I_j(t), \quad (32)$$

[22] For instance, though the aggregate domestic rate of contact in i is $2\alpha_i n_{ii}$, a household has no control over how many buyers visit the household's seller, so the household only controls the rate $\alpha_i n_{ii}$ of contacts generated by the household's buyer.

[23] The reason for this is that the probability of deaths acts like non-exponential discounting in the value function solved by agents, and it is well-understood that non-exponential discounting creates a wedge between the solution of dynamic problems with and without commitment. Farboodi et al. (2020) bypass this issue by assuming that, instead of foregoing future utility when dying, agents pay a one-time utility cost (or value of life) at the moment they die.

while the optimality conditions associated with the co-state variables are given by:

$$-\dot{\theta}_i^s(t) = -\left[\theta_i^s(t) - \theta_i^i(t)\right]\left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t)\right], \quad (33)$$

$$-\dot{\theta}_i^i(t) = \eta_i \theta_i^k(t) - (\gamma_i + \eta_i)\,\theta_i^i(t), \quad (34)$$

$$-\dot{\theta}_i^k(t) = -\left[Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t))\right] e^{-\xi t}. \quad (35)$$

Finally, the transversality conditions are

$$\lim_{t\to\infty} \theta_i^i(t)\, i_i(t) = 0,$$
$$\lim_{t\to\infty} \theta_i^s(t)\, s_i(t) = 0,$$
$$\lim_{t\to\infty} \theta_i^k(t)\, k_i(t) = 0.$$

To complete the description of the model, we need to specify the general equilibrium determination of wages. As in the version of our model with deaths in Section 4, we again have that wages are determined by the following goods market clearing condition:

$$\sum_{j\in\mathcal{J}} \pi_{ji}(\mathbf{w},t) \times w_j(t) \times (1 - D_j(t))\, L_j = w_i(t) \times (1 - D_i(t)) \times L_i.$$

Importantly, however, the trade shares $\pi_{ji}(\mathbf{w},t)$ are now impacted by the fact that the level of interactions $n_{ij}(t)$ are directly affected by the dynamics of the pandemic. Still, computationally, it is straightforward to solve for a dynamic equilibrium in which $\pi_{ij}(\mathbf{w},t) = X_{ij}(t) / \sum_{\ell\in\mathcal{J}} X_{i\ell}(t)$, and $X_{ij}(t) = n_{ij}(t)\, p_{ij}(t)\, q_{ij}(t)\,(1 - D_i(t))\, L_i$. More specifically, the dynamic model can be solved through a backward shooting algorithm (see Appendix E for details).

This is obviously a rather complicated system characterized by several differential equations, and two (static) optimality conditions for the choices of n_{ii} and n_{ij} in each country. Nevertheless, we are able to show analytically that the solution to this problem necessarily involves individual-level social distancing. In the absence of a pandemic, households equate the marginal utility from sourcing varieties from each location to the marginal cost of sourcing those varieties. During a pandemic, households internalize that the interactions involved in sourcing varieties expose them to infection, which leads them to reduce interactions until the marginal utility from those interactions exceeds the marginal cost, as summarized in the following proposition (proven in Appendix A.9).

Proposition 9 *Along the transition path, $\theta_i^s(t) - \theta_i^i(t) \geq 0$ for all t, which implies:*

$$\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}(t)} > \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}(t)}, \qquad \textit{as long as } I_j(t) > 0.$$

An implication of this result is that the pandemic generically has a larger impact on foreign interactions than on domestic interactions. This implication can been seen by re-arranging the optimality condition (32) and substituting for the marginal utility and marginal cost for interactions:

$$\frac{1}{n_{ij}}\frac{n_{ij}q_{ij}^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell\in\mathcal{J}}n_{i\ell}q_{i\ell}^{\frac{\sigma-1}{\sigma}}}Q_i=\frac{1}{n_{ij}}c\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}+\frac{\left[\theta_i^s\left(t\right)-\theta_i^i\left(t\right)\right]s_i\left(t\right)\alpha_jI_j\left(t\right)}{(1-k_i\left(t\right))e^{-\xi t}},$$

where the term on the left-hand side is the marginal utility from interactions; the first term on the right-hand side is the marginal cost of interactions; and the second term on the right-hand side is the wedge capturing the threat of infection. As foreign interactions are generically a smaller share of the consumption index than domestic interactions, the fraction on the left-hand side is generically smaller for foreign interactions ($i\neq j$). Therefore, as a pandemic emerges and the threat of infection becomes positive, a larger reduction in n_{ij} is generically needed for foreign interactions, in order to raise the marginal utility on the left-hand side until it is equal to the marginal cost plus the positive wedge capturing the threat of infections on the right-hand side.

We now illustrate some of these implications of behavioral responses for the case of symmetric countries. We use the baseline parameters with $\alpha_i=0.1$, $\gamma_i+\eta_i=0.2$, and $\eta_i/\left(\eta_i+\gamma_i\right)=0.0062$ (a 0.62% death rate among those infected) for all i. We also show a specification with half the death rate of $\eta_i/\left(\eta_i+\gamma_i\right)=0.003$ for all i, as well as the case without behavioral responses from the previous section. As we choose the wage in one country as the numéraire, with symmetric countries, the relative wage is also equal to one and constant over time. In the absence of any behavioral responses, this constant relative wage implies that both the mass of varieties and price index are constant over time, as shown in the Proof of Proposition A.4. In contrast, in the presence of behavioral responses, households reduce the intensity of their interactions in response to the threat of infection, which leads to changes in the mass of varieties and the price index over time.

In the top-left panel of Figure 8, we show the percentage of individuals infected in Country 2 for all three specifications (with symmetry the figure for Country 1 is identical). Households' behavioral response of reducing interactions leads to a "flattening of the curve of the pandemic," such that the pandemic has lower peak and lower cumulative infections, but takes longer to subside. Clearly, the larger the death rate, the stronger the behavioral response and the flatter the resulting curve of infections. The top-right panel in Figure 8 presents the resulting evolution of cumulative deaths in Country 2. Behavioral responses delay and reduce total deaths, with the level (and proportional reduction) larger, the larger the death rate. Naturally, the behavioral response and the associated reductions in the number of deaths come at an economic cost for survivors. As the bottom-left panel shows, the reductions in the number of purchased domestic and foreign varieties increase the price index in each country, which results in a corresponding decline in real income. This increase in the price index, and reduction in real income, is larger the stronger the behavioral response, and hence increases with the death rate. Finally, the bottom-right panel displays the trade over GDP ratio (calculated as imports plus exports over GDP). In the example, trade/GDP falls from about 0.45 to less than 0.25 when the death rate is 0.3%, and to 0.17 when the death rate is 0.62%. Therefore, the flattening of the curve of infections and reduction in the number of deaths comes at the cost of lower trade and real income. Of course, behavioral responses are ex-ante privately optimal, so it is not surprising that they improve individual welfare.[24]

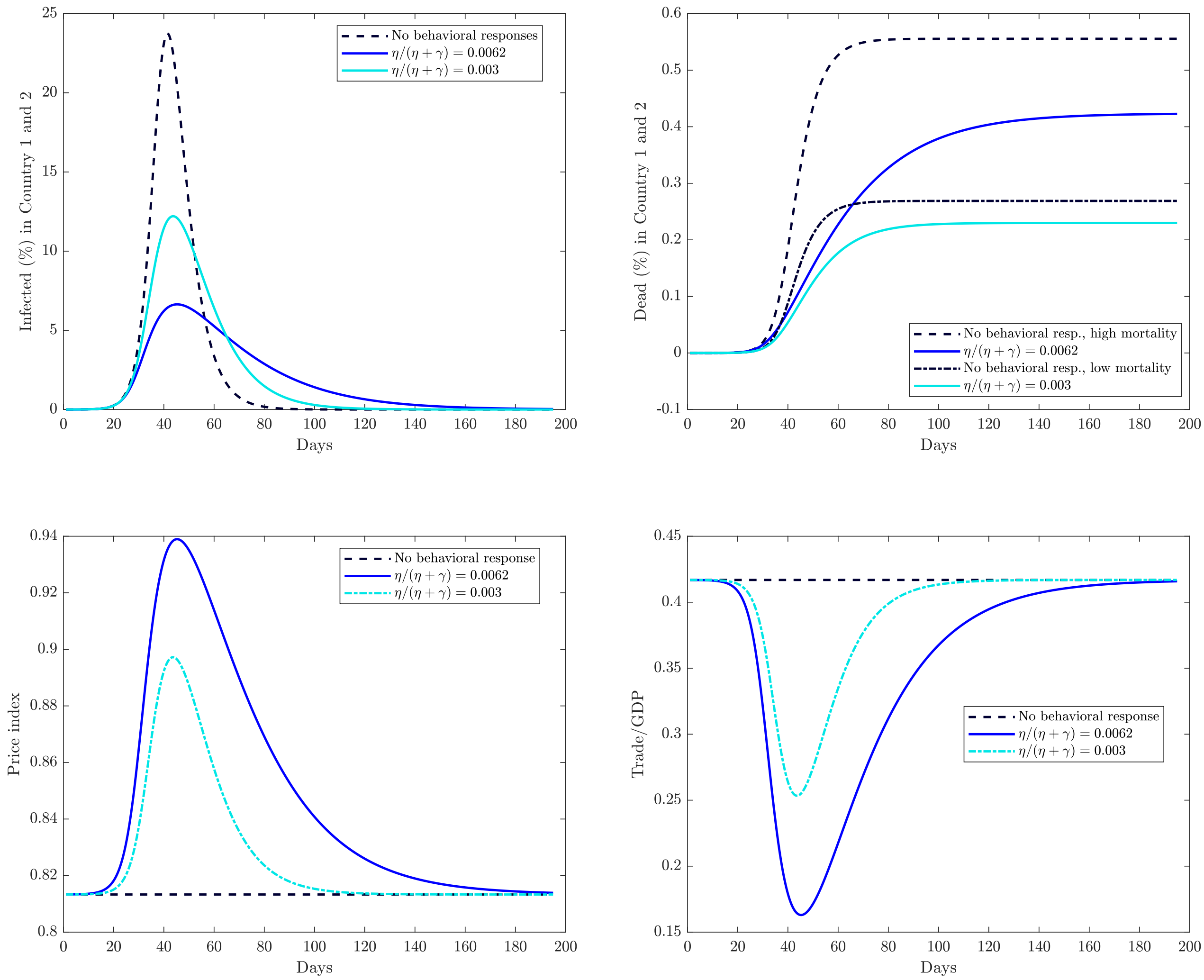

Figure 8: Behavioural Responses with Symmetric Countries for Various Death Rates

The presence of behavioral responses in the model thus leads to endogenous social distancing that has both economic and epidemiological implications. Households keep reducing interactions until the (monotonically decreasing) reproduction number, $\mathcal{R}_0 \times S_i(t)$, falls below 1. Once this reproduction number crosses that threshold, interactions start growing again, as herd immunity reduces the number of infections. An implication is that the magnitude of households' behavioral responses depends crucially on the value of $\mathcal{R}_0$. The larger this value, the larger the resulting behavioral response. Furthermore, the model with behavioral responses results in reproduction numbers that linger closer to one as economic activity endogenously recovers, once the worst of the

[24]It is worth stressing that these responses are not necessarily socially optimal due to the externalities that agents exert on other agents when traveling.

pandemic has passed.

The value of mobility and trade frictions plays an important role in shaping the magnitude and pattern of behavioral responses. First, with symmetric countries, higher mobility and trade frictions imply a reduction in the overall volume of human interactions, which leaves less scope for behavioral responses. Second, higher mobility and trade frictions imply that more of the burden of adjustment falls on domestic rather than foreign transactions. In Figure 9, we show the evolution of the trade/GDP ratios for symmetric countries for two different levels of mobility (left panel) and trade (right panel) frictions and the baseline values of our other parameters. As discussed above, in the symmetric case without behavioral responses, all human contacts $n_{ii}(t)$ and $n_{ij}(t)$ are constant in time, which implies that mobility and trade frictions only reduce the level of the trade/GDP ratios. Once we incorporate behavioral responses, trade/GDP follows the trajectory of the pandemic. The larger value of trade frictions reduces trade openness, which dampens the absolute magnitude of the behavioral response, although trade openness can end up falling to quite low levels. In this example with 10% trade frictions, ($t_{12} = t_{21} = 1.1$), trade essentially falls to zero in the most severe phase of the pandemic. For each level of trade frictions, behavioral responses reduce the total number of deaths, and for the parameter values considered here, higher trade and mobility frictions also reduce the total number of deaths.

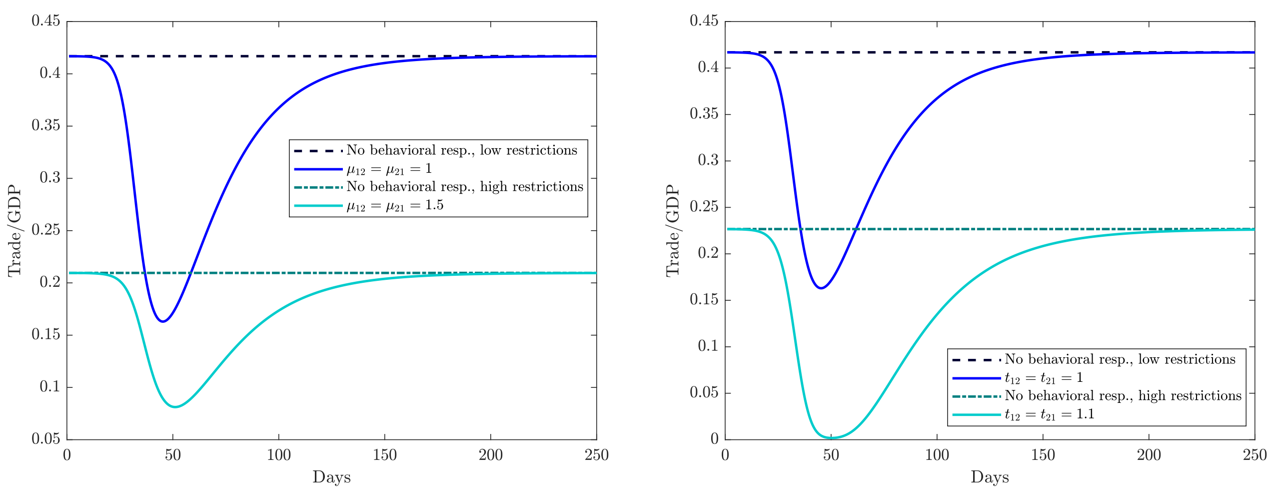

Figure 9: The Effect of Mobility and Trade Frictions on Trade/GDP with Behavioural Responses

We next illustrate some of the implications of our model when countries are asymmetric. We focus on a case in which countries differ in their mortality rate, where remember that we assume that mortality is determined by the country in which a household lives rather than the country in which it was infected. We let Country 1 have a relatively low mortality rate of 0.3% and we leave the mortality rate of Country 2 at the higher baseline value of 0.62%. Figure 10 presents the results. The top-left panel shows the percent of infections in each country. As benchmarks, we also display the average of infections in the two countries, as well as infections in the case of two symmetric countries with an average mortality rate of 0.46% (the mean of 0.3 and 0.62%). There is a stronger behavioral response in the high-mortality Country 2 because households internalize the greater risk that infection leads to death, which results in a "flatter" curve of infections in this country. The low-mortality Country 1 ends up with about 10% higher total infections, because of its more subdued behavioral response. However, its lower mortality rate implies that it ends up with only about half the total number of deaths. This asymmetric behavioral response implies that Country 1 is a relatively dangerous destination for doing business in the early stages of the pandemic, but a relatively safe destination in the later stages of the pandemic, since it reaches herd immunity faster. Comparing the average response for the world with asymmetric countries to the response in the symmetric case with average mortality rates illustrates the implied aggregate effects from differences across countries in mortality rates. In the asymmetric case, the world's infection curve is marginally flatter than in a symmetric world with average mortality rates.

The top-right panel in Figure 10 displays Country 1's relative wage. As a result of the smaller behavioral response in this lower mortality country, there is a greater risk of infection in Country 1 in the early stages of the pandemic, which leads to a decline in demand for this country's varieties and a fall in its relative wage. Once Country 1's infection rate falls, demand for its varieties recovers, and hence so does its wage. Eventually, once Country 1's infection rate falls below that of Country 2, it becomes the relatively safe environment in which to source varieties, and its relative wage rises temporarily above one, before falling back to one as the pandemic ends. Therefore, these behavioral responses in general equilibrium with asymmetric countries lead to demand effects that reduce the relative wage of the country with a relatively higher infection rate. In addition, as shown in the previous section, there is another general equilibrium effect from changes in relative labor supply. A country with a higher death rate experiences a reduction in its relative labor supply, which leads to an increase in its relative wage. The top-right panel of Figure 10 shows the balance of these forces, and demonstrates that relative demand effects generally dominates and overturns the result in Section 4 linking higher death rates to higher relative wages.

As before, the stronger behavioral response in Country 2 as a result of its higher mortality rate comes with greater economic costs. Country 2's reduction in domestic and foreign purchases raises its price index and reduces its real income. The effect on the price index in Country 1 is more nuanced. Country 1 also reduces domestic and foreign interactions, which tends to increase its price index. However, the decline in its relative wage during the first part of the pandemic reduces the price of domestic varieties. The bottom-left panel in Figure 10 shows how these forces result in a price index with multiple peaks. Overall, the effect of the pandemic on the real income of Country 1 is negative but substantially smaller in magnitude than in Country 2. As shown in the bottom-right panel, the reduction in human interactions from social distancing reduces trade openness dramatically, particularly in Country 2, where behavioral responses are stronger. The asymmetry in mortality rates between the two countries initially leads to a larger reduction in trade openness than in a symmetric world with average mortality rates, in part because the behavioral response of Country 2 is particularly strong in the earlier phases of the pandemic. Later in the pandemic,

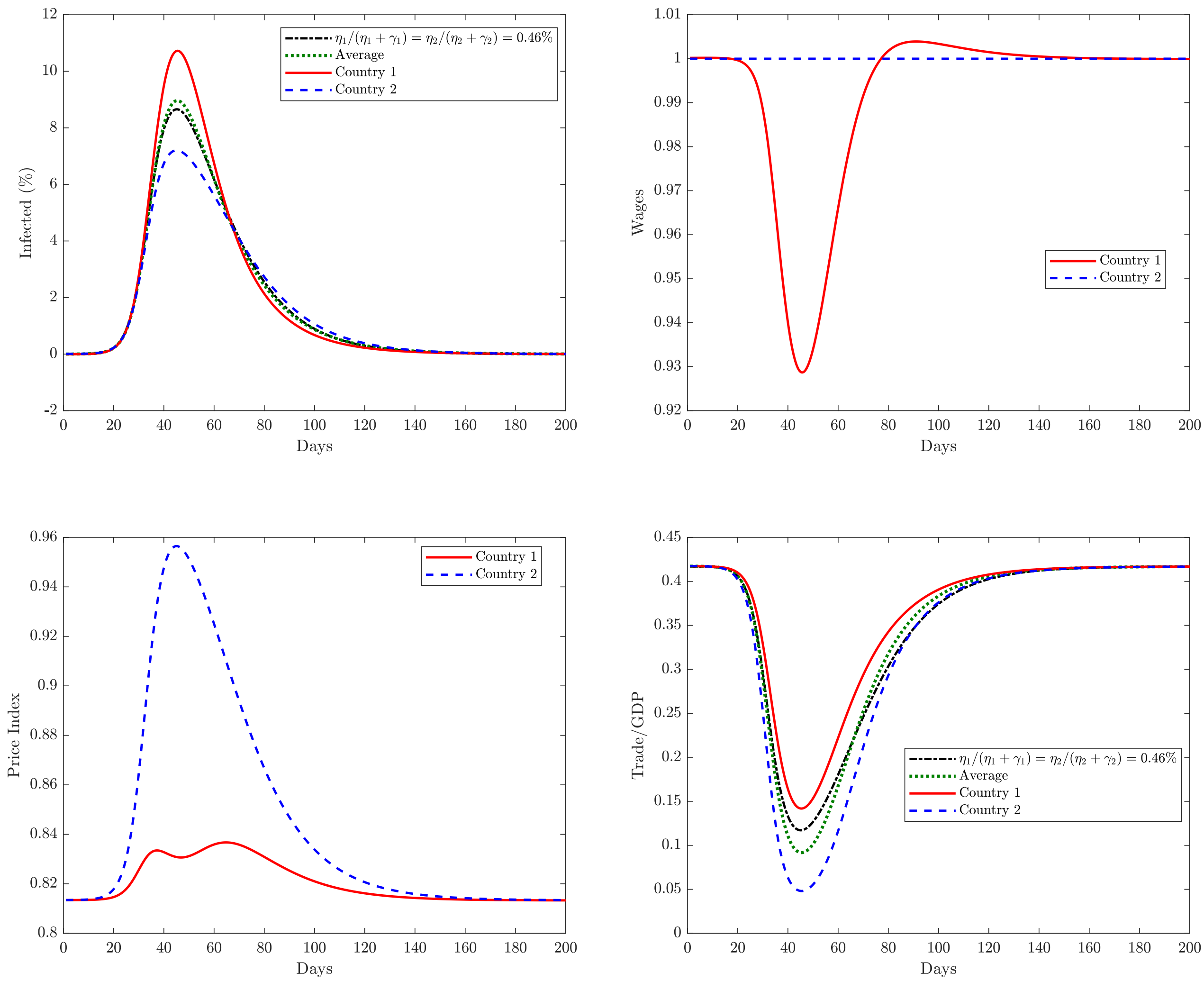

Figure 10: Behavioural Responses with Asymmetric Mortality Rates

the asymmetric case has higher trade openness than in a symmetric world, because the initially subdued behavioral response of Country 1 creates a more pronounced and faster wave of infections.

Adjustment Costs and the Risk of a Pandemic

Despite the potential for significant disruptions in international trade during a pandemic, a clear implication of the first-order condition (32) is that as long as $I_i(t) = I_j(t) = 0$, human interactions are at the same level as in a world without the potential for pandemics. In other words, although we have generated rich dynamics of international trade during a pandemic, as soon as this pandemic is overcome (via herd immunity or the arrival of a vaccine), our model predicts that life immediately goes back to normal. We next explore an extension of our model that explores the robustness of this notion of a rapid V-shape recovery in economic activity and international trade flows after a global pandemic.

The main novel feature we introduce is adjustment costs associated with changes in the measures of human contacts $n_{ii}(t)$ and $n_{ij}(t)$. More specifically, we assume that whenever a household wants to change the measure of contacts $n_{ij}(t)$, it needs to pay a cost $\psi_1 |\dot{n}_{ij}(t)|^{\psi_2}$, where $\psi_1 > 0$ and $\psi_2 > 1$. An analogous adjustment cost function applies to changes in domestic interactions n_{ii}. Notice that this formulation assumes that the cost of reducing or increasing the number of contacts are symmetric. This leads to the following modified first-order condition for the choice of n_{ij} at any point in time t_0 (an analogous condition holds for n_{ii}):

$$\int_{t_0}^{\infty} e^{-\xi t} \left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} \right] (1 - k_i(t)) dt$$
$$= \int_{t_0}^{\infty} e^{-\xi t} \left[\theta_i^s(t_0) - \theta_i^i(t_0) \right] s_i(t_0) a_j I_j(t_0) dt + e^{-\xi t_0} \psi_1 \psi_2 |\dot{n}_{ij}(t_0)|^{\psi_2 - 1} (1 - k_i(t_0)).$$

Since dead individuals do not pay adjustment costs, equation (35) becomes

$$-\dot{\theta}_i^k(t) = -\left[Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t)) - \psi_1 (|\dot{n}_{ii}(t)|^{\psi_2} + |\dot{n}_{ij}(t)|^{\psi_2}) \right] e^{-\xi t}.$$

The rest of the system is as before with the added feature that the values of $n_{ii}(t)$ and $n_{ij}(t)$ are now state variables, with exogenous initial conditions $n_{ii}(0)$ and $n_{ij}(0)$.[25]

As the first-order condition makes evident, the choice of $\dot{n}_{ij}(t_0)$ now affects the values of $n_{ii}(t)$ and $n_{ij}(t)$ in the future directly and not only through its impact on the pandemic (and the corresponding co-state variables $\theta_i^s(t_0)$ and $\theta_i^i(t_0)$). This has two important implications. First, adjustment costs imply that agents will react less aggressively to a pandemic and overall their reaction will be smoother. Of course, the counterpart is that their endogenous response will attenuate the flattening of the curve of infections associated with behavioral responses. Second, if households anticipate that the probability of a future pandemic is $\lambda > 0$, the growth in the resurgence of human interactions will be slower than in the world in which the perceived probability of a future pandemic is 0, and the more so the larger is λ. As a result, if due to recency effects, households perceive a particularly high risk of future pandemics in the aftermath of a pandemic, this could slow the recovery of international trade flows after a pandemic occurs.

Figure 11 presents a numerical example of an economy with symmetric countries, behavioral responses, and adjustment costs. The figure uses the baseline parameters from the previous section for symmetric countries, together with $\psi_1 = 1$ and $\psi_2 = 4$ for the adjustment cost parameters. The left-panel shows the evolution of foreign varieties consumed, $n_{ij}(t)$, and compares it with the case with no adjustment costs ($\psi_1 = 0$). Clearly, adjustment costs reduce the magnitude of the behavioral response. Not only do agents take longer to start the adjustment, but the adjustment is substantially smaller. In computing this example we assume that the pandemic never repeats itself. Hence, eventually the number of varieties consumed is the same as in the behavioral case without

[25] Alternatively we can use terminal conditions. This is what we do in the numerical exercise below where we assume that a pandemic ends, and never happens again, after some large time period T.

adjustment costs. We use this value as the terminal condition and compare the resulting initial $n_{ij}(1)$. Anticipatory effects, agents adjusting in anticipation of a pandemic, imply that the initial value should be smaller than the terminal one. Figure 11 shows no indication that these effects are significant. Although $n_{ij}(1) < n_{ij}(T)$, the effect is negligible and cannot be perceived in the graph. This is the case, even though the effect on the evolution of domestic and foreign contacts is fairly large. This pattern of results is consistent with the view that economies will quickly return to normal after the pandemic, although with the caveat that we have here assumed that adjustment costs are symmetric and that the pandemic does not affect agents' beliefs of the probability of future pandemics. The right panel of Figure 11 presents the corresponding evolution of infections with and without adjustment costs. As discussed above, the milder and delayed behavioral response in the case with adjustment costs leads to a faster increase in the number of infections. It also leads to a corresponding faster decline, since herd immunity starts reducing the number of infections earlier. The result is a faster, but more severe, pandemic with more overall deaths, but less pronounced temporary reductions in real income and trade.

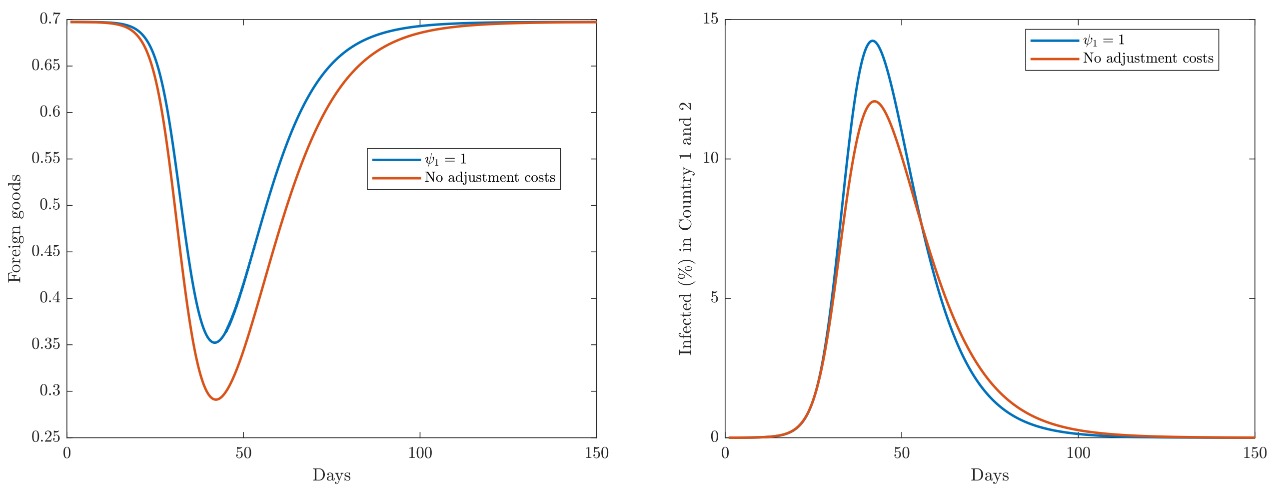

Figure 11: Behavioural Responses with Adjustment Costs

6 Conclusions

Although globalization brings aggregate economic gains, it is often argued that it also makes societies more vulnerable to disease contagion. In this paper, we develop a model of human interaction to analyze the relationship between globalization and pandemics. We jointly microfound both the canonical model of international trade from economics (the gravity equation) with the seminal model pandemics from epidemiology (the Susceptible-Infected-Recovered (SIR) model) using a theory of human interaction. Through jointly modelling these two phenomena, we highlight a number of interactions between them. On the one hand, the contact rate among individuals, which is a central parameter in benchmark epidemiology models, is endogenous in our framework, and responds to both economic forces (e.g., the gains from international trade) and to the dynamics of the pandemic (e.g., the perceived health risk associated with international travel). On the other hand, we study how the emergence of a pandemic and the perceived risk of future outbreaks shapes the dynamics of international trade, and the net gains from international trade once the death toll from the pandemic is taken into account.

We begin by considering the case in which the disease does not affect the ability of agents to produce and trade, and agents are unaware of the threat of infection, which implies that they do not have an incentive to alter their individual behavior. Even in this case, globalization influences the dynamics of the disease, because it changes patterns of human interaction. We show that there are cross-country epidemiological externalities, such that whether a pandemic occurs in the open economy depends critically on the disease environment in the country with the highest rate of domestic infection. If countries are symmetric, a decline in any (symmetric) international trade friction also leads to an overall increase in the total number of human interactions (domestic plus foreign), which *increases* the range of parameters where a pandemic occurs. In this case, even if an epidemic would not be self-sustaining in a country in the closed economy, it can be self-sustaining in an open economy. In contrast, if countries are sufficiently different from one another in terms of their primitive epidemiological parameters (e.g., as a result of different health policies), a decline in any international trade friction can have the opposite effect of *decreasing* the range of parameters where a pandemic occurs. In this case where one country has a much worse disease environment than the other, trade liberalization can reduce the share of that country's interactions that occur in this bad disease environment, thereby taking the global economy below the threshold for a pandemic to be self-sustaining. In the presence of differences in the timing of infections, multiple waves of infection can occur in the open economy, when there would be a single wave in the closed economy.

We next allow the infection to cause deaths (or reduce productivity in the labor market), but assume that agents remain unaware of the threat of infection, and hence continue to have no incentive to alter their individual behavior. In this case, a country with a worse disease environment experiences a larger reduction in labor supply, which in turn leads to an increase in its relative wage. This wage increase reduces the share of interactions that occur in that country's bad disease environment and increases the share that occur in better disease environments, which again can take the global economy below the threshold for a pandemic to be self-sustaining. Therefore, the general equilibrium effects of the pandemic on wages and trade patterns induce a form of "general equilibrium social distancing" from bad disease environments that operates even in the absence of purposeful social distancing motivated by health risks.

We then allow individuals to become aware of the threat of infections and optimally adjust their behavior depending on the observed state of the pandemic. In this case, agents are not willing to interact as much with the unhealthy country thereby decreasing its relative wage. Overall, we find that behavioral responses lead to amplified reductions in international trade and income, but save lives. Adding adjustment costs of establishing the human interactions needed to sustain trade delays and diminishes these behavioral responses.

Although we have argued that our results are robust to alternative specifications of our model of international trade, our theoretical framework is still missing a number or realistic features. For example, in future work it would be interesting to explore the implications of allowing for cross-sectoral heterogeneity in the importance of face-to-face interactions for sustaining international trade. Similarly, and although we have studied the effects of various parameters that are at least partly shaped by government policies, it would be fruitful to more thoroughly study optimal policy in our framework. We leave these extensions for future work.

References

Acemoglu, Daron, Victor Chernozhukov, Iván Werning, Michael D. Whinston (2020), "Optimal Targeted Lockdowns in a Multi-Group SIR Model, NBER Working Paper No. 27102.

Alfaro, Laura, Ester Faia, Nora Lamersdorf, and Farzad Saidi (2020), "Social Interactions in Pandemics: Fear, Altruism, and Reciprocity," NBER Working Paper No. 27134.

Allen, Treb, and Costas Arkolakis (2014), "Trade and the Topography of the Spatial Economy," *Quarterly Journal of Economics* 129, no. 3: 1085-1140.

Allen, Treb, Costas Arkolakis, and Yuta Takahashi (2020), "Universal Gravity," *Journal of Political Economy* 128, no. 2: pp. 393–433.

Alvarez, Fernando, David Argente, and Francesco Lippi (2020), "A Simple Planning Problem for COVID-19 Lockdown," NBER Working Paper No. 26981.

Alvarez, Fernando, and Robert E. Lucas Jr. (2007), "General Equilibrium Analysis of the Eaton–Kortum model of International Trade," *Journal of Monetary Economics* 54, no. 6 (2007): 1726-1768.

Anderson, James E. (1979), "A Theoretical Foundation for the Gravity Equation," *American Economic Review*, 69(1), pp.106-116.

Anderson, James E. and Eric Van Wincoop (2003), "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, 93(1), pp.170-192.

Antràs, Pol, Teresa C. Fort and Felix Tintelnot (2017) "The Margins of Global Sourcing: Theory and Evidence from US Firms," *American Economic Review*, 107(9), 2514-64.

Argente, David O., Chang-Tai Hsieh, and Munseob Lee (2020), "The Cost of Privacy: Welfare Effect of the Disclosure of COVID-19 Cases," NBER Working Paper No.27220.

Arkolakis, Costas (2010), "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy* 118, no. 6, pp: 1151-1199.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare (2012), "New trade models, same old gains?" *American Economic Review* 102.1: 94-130.

Atkeson, Andrew (2020), "What Will Be the Economic Impact of COVID-19 in the US? Rough Estimates of Disease Scenarios," NBER Working Paper No. 26867.

Birge, John R., Ozan Candogan, and Yiding Feng (2020), "Controlling Epidemic Spread: Reducing Economic Losses with Targeted Closures," Becker-Friedman Institute Working Paper No. 2020-57.

Bisin, Alberto, and Andrea Moro (2020), "Learning Epidemiology by Doing: The Empirical Implications of a Spatial SIR Model with Behavioral Responses," mimeo New York University.

Blonigen, Bruce A. and Anca D. Cristea (2015), "Air Service and Urban Growth: Evidence from a Quasi-Natural Policy Experiment," *Journal of Urban Economics* 86, 128–146.

Boerner, Lars and Battista Severgnini (2014) "Epidemic Trade," *Economic History Working Papers*, 212, London School of Economics.

Brauer, Fred, and Carlos Castillo-Chavez (2012), *Mathematical Models in Population Biology and Epidemiology*, Vol. 2. New York: Springer, 2012.

Campante, Filipe, and David Yanagizawa-Drott (2018), "Long-Range Growth: Economic Development in the Global Network of Air Links," *Quarterly Journal of Economics* 133, no. 3 (2018): 1395-1458.

Chaney, Thomas (2008), "Distorted Gravity: the Intensive and Extensive Margins of International Trade," *American Economic Review* 98, no. 4: 1707-21.

Chaney, Thomas (2014), "The Network Structure of International Trade," *American Economic Review* 104, no. 11: 3600-3634.

Christakos, George, Ricardo A. Olea, Marc L. Serre, Hwa-Lung Yu and Lin-Lin Wang (2005) *Interdisciplinary Public Health Reasoning and Epidemic Modelling: The Case of Black Death*, Amsterdam: Springer.

Costinot, Arnaud, and Andrés Rodríguez-Clare (2015), "Trade Theory with Numbers: Quantifying the Consequences of Globalization" in *Handbook of International Economics*, vol. 4: pp. 197-261.

Cristea, Anca D. (2011), "Buyer-Seller Relationships in International Trade: Evidence from U.S. States' Exports and Business-Class Travel," *Journal of International Economics* 84, no. 2: 207-220.

Cuñat, Alejandro, and Robert Zymek (2020), "The (Structural) Gravity of Epidemics" mimeo University of Vienna.

Diekmann, O., J.A.P. Heesterbeek, J.A.J. Metz (1990), "On the Definition and the Computation of the Basic Reproduction Ratio R_0 in Models for Infectious Diseases in Heterogeneous Populations," *Journal of Mathematical Biology* 28, 365-382.

Ellison, Glenn (2020), "Implications of Heterogeneous SIR Models for Analyses of COVID-19," NBER Working Paper No. 27373.

Eaton, Jonathan and Samuel Kortum, (2002), "Technology, Geography, and Trade," *Econometrica*, 70:5, 1741-1779.

Fajgelbaum, Pablo D., Amit Khandelwal, Wookun Kim, Cristiano Mantovani, and Edouard Schaal (2020), "Optimal Lockdown in a Commuting Network," mimeo UCLA.

Farboodi, Maryam, Gregor Jarosch, and Robert Shimer (2020), "Internal and External Effects of Social Distancing in a Pandemic," NBER Working Paper 27059.

Fenichel, Eli P., Carlos Castillo-Chavez, M. Graziano Ceddia, Gerardo Chowell, Paula A. Gonzalez Parra, Graham J. Hickling, Garth Holloway et al. (2011), "Adaptive Human Behavior in Epidemiological Models," *Proceedings of the National Academy of Sciences* 108, no. 15: 6306-6311.

Fernández-Villaverde, Jesús and Chad Jones (2020), "Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities," NBER Working Paper No. 27128.

Giannone, Elisa, Nuno Paixao, and Xinle Pang (2020), "Pandemic in an Inter-regional Model: Optimal Lockdown Policies," mimeo Penn State University.

Head, Keith, and Philippe Mayer (2014), "Gravity Equations: Workhorse, Toolkit, and Cookbook," Ch. 3 in *Handbook of International Economics*, Gopinath, Gita, Elhanan Helpman and Kenneth Rogoff (Eds), Vol. 4.

Helpman, Elhanan, Marc Melitz, and Yona Rubinstein (2008), "Estimating Trade Flows: Trading Partners and Trading Volumes," *Quarterly Journal of Economics* 123, no. 2: 441-487.

Hethcote, Herbert W. (1978), "An Immunization Model for a Heterogeneous Population," *Theoretical Population Biology* 14, no. 3: 338-349.

Hethcote, Herbert W (2000), "The Mathematics of Infectious Diseases," *SIAM Review*, 42(4), pp.599-653.

Hethcote, H.W. and Thieme, H.R., (1985), "Stability of the Endemic Equilibrium in Epidemic Models with Subpopulations," *Mathematical Biosciences*, 75(2), pp. 205-227.

Hovhannisyan, Nune and Wolfgang Keller (2015), "International Business Travel: An Engine of Innovation?, *Journal of Economic Growth*, 20:75–104.

Kermack, William Ogilvy and A. G. McKendrick, (1927), "A Contribution to the Mathematical Theory of Epidemics, Part I," *Proceedings of the Royal Society of London.* Series A, 115 (772), pp. 700–721.

Kermack, William Ogilvy and A. G. McKendrick, (1932), "Contributions to the Mathematical Theory of Epidemics. II – The Problem of Endemicity," *Proceedings of the Royal Society of London.* Series A, 138 (834), pp. 55–83.

Jedwab, Remi, Noel D. Johnson and Mark Koyama (2019) "Pandemics, Places, and Populations: Evidence from the Black Death," George Washington University, mimeograph.

Jones, Callum J, Thomas Philippon, and Venky Venkateswaran (2020), "Optimal Mitigation Policies in a Pandemic: Social Distancing and Working from Home," NBER Working Paper 26984.

Magal, Pierre, Seydi Ousmane, and Glenn Webb (2016) "Final Size of an Epidemic for a Two-group SIR Model," *Society for Industrial and Applied Mathematics*, 76(5), 2042-2059.

Melitz, Marc J. (2003), "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica* 71, no. 6 (2003): 1695-1725.

Melitz, Marc J., and Stephen J Redding (2014), "Missing Gains from Trade?" *American Economic Review* 104 (5): pp. 317-21.

Ossa, Ralph (2015), "Why Trade Matters After All," *Journal of International Economics* 97(2): pp. 266-277.

Ricci P. H. Yue, Harry F. Lee and Connor Y. H. Wu (2017) "Trade Routes and Plague Transmission in Pre-industrial Europe," *Nature*, Scientific Reports, 7, 12973, 1-10.

Saker, Lance, Kelley Lee, Barbara Cannito, Anna Gilmore and Diarmid Campbell-Lendrum (2004) "Globalization and Infectious Diseases: A Review of the Linkages," *Special Topics in Social, Economic and Behavioural (SEB) Research*, Geneva: World Health Organization.

Startz, Meredith (2018), "The Value of Face-To-Face: Search and Contracting Problems in Nigerian Trade," mimeo Stanford University.

Toxvaerd, Flavio (2020), "Equilibrium Social Distancing," Cambridge-INET Working Paper Series No: 2020/08.

Van den Driessche, Pauline, and James Watmough (2002), "Reproduction Numbers and Sub-Threshold Endemic Equilibria for Compartmental Models of Disease Transmission," *Mathematical Biosciences* 180, no. 1-2, pp. 29-48.

A Theoretical Appendix

A.1 Second-Order Conditions for Choice of n_{ij}

From equation (6), we obtain, for all $j \in \mathcal{J}$,

$$\begin{aligned}
\frac{\partial W(i)}{\partial n_{ij}} &= \frac{w_i}{(\sigma-1)} \left(\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}-1} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} - c\mu_{ij} (d_{ij})^{\rho} (n_{ij})^{\phi-1} \;; \\
\frac{\partial W(i)}{\partial (n_{ij})^2} &= \frac{w_i}{(\sigma-1)} \left(\frac{2-\sigma}{\sigma-1}\right) \left(\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}-2} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \\
&\quad - (\phi-1)\, c\mu_{ij} (d_{ij})^{\rho} \times (n_{ij})^{\phi-2} \\
&= \left(\frac{2-\sigma}{\sigma-1}\right) \left(\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \right)^{-1} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} c\mu_{ij} (d_{ij})^{\rho} \times (n_{ij})^{\phi-1} \\
&\quad - (\phi-1)\, c\mu_{ij} (d_{ij})^{\rho} \times (n_{ij})^{\phi-2} \\
&= c\mu_{ij} (d_{ij})^{\rho} \times (n_{ij})^{\phi-2} \left[\left(\frac{1}{(\sigma-1)} - 1\right) \left(\frac{n_{ij} \frac{\tau_{ij} w_j}{Z_j}}{\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma}} \right)^{1-\sigma} - (\phi-1) \right]; \\
\frac{\partial^2 W(i)}{\partial n_{ij} \partial n_{ii}} &= \frac{w_i}{(\sigma-1)} \left(\frac{2-\sigma}{\sigma-1}\right) \left(\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}-2} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \left(\frac{\tau_{ii} w_i}{Z_i}\right)^{1-\sigma} .
\end{aligned}$$

Notice that $\frac{\partial W(i)}{\partial (n_{ij})^2} < 0$ if only if:

$$\left(\frac{2-\sigma}{\sigma-1}\right) \left(\frac{n_{ij} \frac{\tau_{ij} w_j}{Z_j}}{\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma}} \right)^{1-\sigma} < (\phi-1)\,,$$

so this condition could be violated for large enough τ_{ij}, unless $\sigma > 2$, in which case the condition is surely satisfied as long as $\phi(\sigma-1) > 1$.

Next note that

$$\left(\frac{\partial^2 W(i)}{\partial n_{ij} \partial n_{ii}}\right)^2 = \left(\frac{w_i}{\sigma-1} \frac{2-\sigma}{\sigma-1} \left(\sum_{j\in\mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}-2} \left(\frac{\tau_{ij} w_j}{Z_j}\right)^{1-\sigma} \left(\frac{\tau_{ii} w_i}{Z_i}\right)^{1-\sigma} \right)^2 = \Xi^2$$

and

$$\frac{\partial W(i)}{\partial (n_{ii})^2}\frac{\partial W(i)}{\partial (n_{ij})^2} = \left(\begin{array}{c} \frac{1}{(\sigma-1)}\frac{2-\sigma}{\sigma-1}w_i\left(\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2}\left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma}\left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma} \\ -(\phi-1)\,c\mu_{ii}(d_{ii})^{\rho}\times(n_{ii})^{\phi-2} \end{array}\right)$$
$$\times\left(\begin{array}{c} \frac{1}{(\sigma-1)}\frac{2-\sigma}{\sigma-1}w_i\left(\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma} \\ -(\phi-1)\,c\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi-2} \end{array}\right)$$
$$= \Xi^2 - \varkappa_{ij}^{i} - \varkappa_{ij}^{j} + \varpi_{ij},$$

where $\varkappa_{ij}^{i} < 0$ and $\varkappa_{ij}^{j} < 0$, and $\varpi_{ij} > 0$, whenever $\sigma > 2$ and $\phi > 1$.

In sum, when $\sigma > 2$ and $\phi(\sigma-1) > 0$, we have

$$\frac{\partial W(i)}{\partial (n_{ii})^2}\frac{\partial W(i)}{\partial (n_{ij})^2} > \left(\frac{\partial^2 W(i)}{\partial n_{ij}\partial n_{ii}}\right)^2,$$

and the second-order conditions are met.

A.2 Proof of Proposition 2

Proof of part a):

From equation (7), we can write

$$n_{ii}(\mathbf{w}) = (c(\sigma-1)\mu_{ii})^{-1/(\phi-1)}(d_{ii})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ii}}{Z_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{-\frac{\sigma-2}{\phi-1}},$$

but remember from (13) that

$$\frac{w_i}{P_i} = (\pi_{ii})^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}} \times \left(\frac{(Z_i)^{\phi(\sigma-1)}}{c(\sigma-1)}(\Gamma_{ii})^{-\varepsilon(\phi-1)}\right)^{\frac{1}{\phi(\sigma-1)-1}}.$$

This implies that, in order to study the effect of international trade frictions on $n_{ii}(\mathbf{w})$, it suffices to study their effect on π_{ii}, with the dependence of n_{ii} on π_{ii} being monotonically positive. Now from

$$\pi_{ii} = \frac{(w_i/Z_i)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\Gamma_{ii})^{-\varepsilon}}{\sum_{\ell\in\mathcal{J}}(w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\Gamma_{i\ell})^{-\varepsilon}},$$

it is clear that the impact effect of a lower $\Gamma_{i\ell}$ is to decrease π_{ii} and thus to decrease n_{ii}. To take into account general-equilibrium forces, we can write equation (14) as

$$\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}}{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}+(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ij})^{-\varepsilon}}L_i+\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}{(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{jj})^{-\varepsilon}+(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}\omega L_j=L_i, \tag{A.1}$$

where $\omega \equiv w_j/w_i$ is the relative wage in country j. From this equation, it is easy to see that if Γ_{ij} falls, ω cannot possibly decrease. If it did, both terms in the left-hand-side of (A.1) would fall. But if ω goes up, then π_{ii} goes up *by more* than as implied by the direct fall in Γ_{ij}. Similarly, if Γ_{ji} falls, π_{ij} falls on impact, so ω needs to increase to re-equilibrate the labor market, and again π_{ii} must decline.

Because the results above hold for Γ_{ij} and Γ_{ji}, they must hold for any of the constituents of those composite parameters.

Proof of part b):

Note from equations (2), (5), and (12) that

$$\frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}}\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi}=\frac{1}{\phi(\sigma-1)}\frac{w_i}{P_i}.$$

In part a) of the proof, we have established that when any international trade friction decreases, π_{ii} goes down, and from (13), w_i/P_i goes up. Thus, $\mu_{ii}(d_{ii})^{\rho}\times(n_{ii})^{\phi}+\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi}$ goes up when any international trade friction decreases. But because n_{ii} goes down and μ_{ij} and d_{ij} (weakly) go down, it must be the case that n_{ij} increases.

A.3 Proof of Proposition 3

We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii}(d_{ii})^{\rho}\times(n_{ii})^{\phi}+\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi}.$$

Differentiating:

$$\phi\left[\mu_{ii}(d_{ii})^{\rho}\times(n_{ii})^{\phi-1}\underbrace{dn_{ii}}_{<0}+\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi-1}dn_{ij}\right]+\underbrace{d\left(\mu_{ij}(d_{ij})^{\rho}\right)}_{\leq 0}\times(n_{ij})^{\phi}>0. \tag{A.2}$$

Clearly, we must have

$$\mu_{ii}(d_{ii})^{\rho}\times(n_{ii})^{\phi-1}dn_{ii}+\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi-1}dn_{ij}>0.$$

So if

$$\mu_{ii}(d_{ii})^{\rho}(n_{ii})^{\phi-1}>\mu_{ij}(d_{ij})^{\rho}\times(n_{ij})^{\phi-1},$$

we must have

$$dn_{ij} > -dn_{ii},$$

which would prove the Proposition.

Now, from the FOC for the choice of n's, that is equation (7),

$$\begin{aligned}\mu_{ii}\left(d_{ii}\right)^{\rho}\left(n_{ii}\right)^{\phi-1} &= \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}\frac{\left(P_i\right)^{\frac{\sigma-1}{(\phi-1)}}}{(\sigma-1)\,c}\times\left(\frac{\left(d_{ii}\right)^{\delta}t_{ii}w_i}{Z_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\\ \mu_{ij}\left(d_{ij}\right)^{\rho}\left(n_{ij}\right)^{\phi-1} &= \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}\frac{\left(P_i\right)^{\frac{\sigma-1}{(\phi-1)}}}{(\sigma-1)\,c}\times\left(\frac{\left(d_{ij}\right)^{\delta}t_{ij}w_j}{Z_j}\right)^{-\frac{\sigma-1}{(\phi-1)}},\end{aligned}$$

so a sufficient condition for the result is

$$\frac{\left(d_{ii}\right)^{\delta}t_{ii}w_i}{Z_i} < \frac{\left(d_{ij}\right)^{\delta}t_{ij}w_j}{Z_j}.$$

This amounts to prices for domestic varieties being lower than prices for foreign varieties. This makes sense, in such a case, desired quantities of domestic varieties will be higher, and the marginal benefit of getting more of them will be higher.

Note finally that with full symmetry, we must have $w_i = w_j$ and $Z_j = Z_i$, and the condition above trivially holds since $t_{ij} > t_{ii}$ and $d_{ij} > d_{ii}$.

A.4 Proof of Proposition 4

Note from equation (11), that we can write

$$\begin{aligned}\frac{w_i}{P_i} &= const\times\left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ii}\right)^{-\varepsilon}+\left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{\frac{(\phi-1)}{\phi(\sigma-1)-1}}\\ \frac{w_j}{P_i} &= const\times\omega\left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ii}\right)^{-\varepsilon}+\left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{\frac{(\phi-1)}{\phi(\sigma-1)-1}}\end{aligned}$$

where $\omega = w_j/w_i$. Plugging in (7), we have

$$n_{ii} = const\times\left(\frac{w_i}{P_i}\right)^{-\frac{\sigma-2}{\phi-1}}\times\left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ii}\right)^{-\varepsilon}+\left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}},$$

and thus n_{ii} increases in ω. Next, note

$$\begin{aligned} n_{ij} &= const \times \left(\frac{w_j}{P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)} \\ &= const \times \omega^{-\frac{\sigma-1}{(\phi-1)}} \left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ii})^{-\varepsilon} + \left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ij})^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}} \end{aligned}$$

The effect of ω may look ambiguous, but in fact we have that n_{ij} decreases if ω goes up. To see this, note that

$$\frac{\partial \omega^{-a}\left(b + c\omega^{-d}\right)^{-g}}{\partial \omega} = -\frac{(a-dg)\,c + ab\omega^d}{\left(\frac{1}{\omega^d}\left(c + b\omega^d\right)\right)^g \omega^a \omega \left(c + b\omega^d\right)},$$

which is negative if $a - dg > 0$. But here we have

$$a - dg = \frac{\sigma-1}{(\phi-1)} - \frac{\phi(\sigma-1)}{\phi-1}\frac{\sigma-2}{\phi(\sigma-1)-1} = \frac{\sigma-1}{\phi(\sigma-1)-1} > 0.$$

In sum, n_{ij} decreases in ω. Because an increase in L_i/L_j increases in ω (from straightforward use of the implicit function theorem to (14)), the Proposition follows.

Notice also that

$$\begin{aligned} n_{ji} &= const \times \left(\frac{w_i}{P_j}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_j}{P_j}\right)^{1/(\phi-1)} \\ &= const \times \omega^{\frac{\sigma-1}{(\phi-1)}} \left(\left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{jj})^{-\varepsilon} + \left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ji})^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}}, \end{aligned}$$

and by an analogous argument above, we have that n_{ji} increases in ω, and thus an increase in population in i leads to an increase n_{ji} (while also decreasing n_{jj}).

A.5 Proof of Proposition 5

See main text and Online Appendix D. Here we just discuss the derivation of the system of equations in (23)-(24), and derive the comparative statics mentioned in the main text.

We begin with the law of motion for susceptible agents in each country in equation (16):

$$\begin{aligned} \dot{S}_i &= -2\alpha_i n_{ii} \times S_i \times I_i - \alpha_j n_{ij} \times S_i \times I_j - \alpha_i n_{ji} \times S_i \times I_j \\ \dot{S}_j &= -2\alpha_j n_{jj} \times S_j \times I_j - \alpha_j n_{ij} \times S_j \times I_i - \alpha_i n_{ji} \times S_j \times I_i \end{aligned}$$

Dividing by the own share of susceptibles, and plugging the expression for $\dot{R}_i$ and $\dot{R}_j$ in (18), we obtain

$$\begin{aligned}
\frac{\dot{S}_i}{S_i} &= -\frac{2\alpha_i n_{ii}}{\gamma_i}\dot{R}_i - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\dot{R}_j \\
\frac{\dot{S}_j}{S_j} &= -\frac{2\alpha_j n_{jj}}{\gamma_j}\dot{R}_j - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\dot{R}_i.
\end{aligned}$$

Turning the growth rate in the left-hand-side to a log-difference, and integrating we get

$$\begin{aligned}
\ln S_i(t) - \ln S_i(0) &= -\frac{2\alpha_i n_{ii}}{\gamma_i}\left(R_i(t) - R_i(0)\right) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left(R_j(t) - R_j(0)\right) \\
\ln S_j(t) - \ln S_j(0) &= -\frac{2\alpha_j n_{jj}}{\gamma_j}\left(R_j(t) - R_j(0)\right) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\left(R_i(t) - R_j(0)\right)
\end{aligned}$$

Finally, noting $S_i(0) \simeq 1$ and $R_i(0) \simeq 1$, and $R_i(\infty) = 1 - S_i(\infty)$ (since $I_i(\infty) = 0$), we obtain the system in (23)-(24), that is:

$$\begin{aligned}
\ln S_i(\infty) &= -\frac{2\alpha_i n_{ii}}{\gamma_i}\left(1 - S_i(\infty)\right) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left(1 - S_j(\infty)\right) \\
\ln S_j(\infty) &= -\frac{2\alpha_j n_{jj}}{\gamma_j}\left(1 - S_j(\infty)\right) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\left(1 - S_i(\infty)\right)
\end{aligned}$$

Although we cannot solve the system in closed-form, we can derive some comparative statics. In particular, total differentiating we find

$$\begin{aligned}
& \frac{1}{S_i(\infty)}dS_i(\infty) - \frac{2\alpha_i n_{ii}}{\gamma_i}dS_i(\infty) + \left(1 - S_i(\infty)\right)d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right) \\
=& \left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\right)dS_j(\infty) - d\left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\right)\left(1 - S_j(\infty)\right) \\
& \frac{1}{S_j(\infty)}dS_j(\infty) - \frac{2\alpha_j n_{jj}}{\gamma_j}dS_j(\infty) + \left(1 - S_j(\infty)\right)d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right) \\
=& \left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\right)dS_i(\infty) - d\left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\right)\left(1 - S_i(\infty)\right)
\end{aligned}$$

Solving

$$\begin{aligned}
dS_i(\infty) &= -\frac{\left[\begin{array}{c} \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\right) + \left(1 - S_j(\infty)\right)d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\right) \\ + \left(\frac{1}{S_j(\infty)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right)\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\right)\left(1 - S_j(\infty)\right) + \left(1 - S_i(\infty)\right)d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\right) \end{array}\right]}{\left(\frac{1}{S_i(\infty)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(\infty)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right) - \frac{(\alpha_j n_{ij}+\alpha_i n_{ji})^2}{\gamma_i\gamma_j}} \\
dS_j(\infty) &= -\frac{\left[\begin{array}{c} \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\right) + \left(1 - S_i(\infty)\right)d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\right) \\ + \left(\frac{1}{S_i(\infty)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\right)\left(1 - S_i(\infty)\right) + \left(1 - S_j(\infty)\right)d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\right) \end{array}\right]}{\left(\frac{1}{S_j(\infty)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right)\left(\frac{1}{S_i(\infty)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right) - \frac{(\alpha_j n_{ij}+\alpha_i n_{ji})^2}{\gamma_i\gamma_j}}
\end{aligned}$$

Next, note that because new infections eventually go to zero, there have to be (at least) two peaks of infection (t_i^* and t_j^*) defined by $\dot{I}_i(t_i^*) = \dot{I}_j\left(t_j^*\right) = 0$. Whenever there are more than two peaks in one country, should set t_i^* and t_j^* to the latest periods for which $\dot{I}_i(t_i^*) = \dot{I}_j\left(t_j^*\right) = 0$.

Now we have two cases to consider:

- **Case 1: $t_i^* \geq t_j^*$.** Then $\dot{I}_i(t_i^*) = 0 > \dot{I}_j\left(t_j^*\right)$ and

$$\begin{aligned} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i(t_i^*) + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_i(t_i^*) \times \frac{I_j(t_i^*)}{I_i(t_i^*)} &= 1 \\ \frac{2\alpha_j n_{jj}}{\gamma_j} S_j(t_i^*) + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_j(t_i^*) \times \frac{I_i(t_i^*)}{I_j(t_i^*)} &\leq 1 \end{aligned}$$

  and thus

$$\left(\frac{1}{S_i(t_i^*)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(t_i^*)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right) \geq \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j} \times \frac{I_j(t_i^*)}{I_i(t_i^*)} \times \frac{I_i(t_i^*)}{I_j(t_i^*)} = \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}$$

  But at $S_i(t_i^*) > S_i(\infty)$ and $S_j(t_i^*) > S_i(\infty)$, so we must have $\frac{2\alpha_i n_{ii}}{\gamma_i} S_i(\infty) \leq 1$ and $\frac{2\alpha_j n_{jj}}{\gamma_j} S_j(\infty) \leq 1$, as well as

$$\left(\frac{1}{S_i(\infty)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(\infty)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right) \geq \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}.$$

- **Case 2: $t_j^* \geq t_j^*$.** Then $\dot{I}_j\left(t_j^*\right) = 0 > \dot{I}_i(t_i^*)$ and

$$\begin{aligned} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i\left(t_j^*\right) + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_i\left(t_j^*\right) \times \frac{I_j\left(t_j^*\right)}{I_i\left(t_j^*\right)} &\leq 1 \\ \frac{2\alpha_j n_{jj}}{\gamma_j} S_j\left(t_j^*\right) + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_j\left(t_j^*\right) \times \frac{I_i\left(t_j^*\right)}{I_j\left(t_j^*\right)} &= 1 \end{aligned}$$

  and thus

$$\left(\frac{1}{S_i\left(t_j^*\right)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j\left(t_j^*\right)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right) \geq \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j} \times \frac{I_j\left(t_j^*\right)}{I_i\left(t_j^*\right)} \times \frac{I_i\left(t_j^*\right)}{I_j\left(t_j^*\right)} = \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}$$

  But $S_i\left(t_j^*\right) > S_i(\infty)$ and $S_j\left(t_j^*\right) > S_i(\infty)$, so we must again have $\frac{2\alpha_i n_{ii}}{\gamma_i} S_i(\infty) \leq 1$ and $\frac{2\alpha_j n_{jj}}{\gamma_j} S_j(\infty) \leq 1$, as well as

$$\left(\frac{1}{S_i(\infty)} - \frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(\infty)} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right) \geq \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}$$

Going back to the system, this means that an increase in any n or a decrease in any γ will decrease the steady-state values for $S_i(\infty)$ and $S_j(\infty)$, and thus increase infections everywhere.

A.6 Proof of Proposition 6

See main text. In particular, the result is an immediate corollary of Proposition 3.

A.7 Proof of Proposition 7

See main text.

A.8 Proof of Proposition 8

The goods market clearing condition with deaths defines the following implicit function:

$$\Lambda_i = \left[\begin{array}{c} \frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}}{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}+(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ij})^{-\varepsilon}}(1-D_i)L_i \\ +\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}{(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{jj})^{-\varepsilon}+(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}\omega(1-D_j)L_j-(1-D_i)L_i \end{array} \right] = 0.$$

Taking partial derivatives of this implicit function, we have:

$$\frac{\partial \Lambda_i}{\partial D_i} > 0, \qquad \frac{\partial \Lambda_i}{\partial D_j} < 0, \qquad \frac{\partial \Lambda_i}{\partial \omega} > 0.$$

Therefore, from the implicit function theorem, we have the following comparative statics of the relative wage with respect to deaths in the two countries:

$$\frac{d\omega}{dD_i} = -\frac{\partial \Lambda_i/\partial D_i}{\partial \Lambda_i/\partial \omega} < 0, \qquad \frac{d\omega}{dD_j} = -\frac{\partial \Lambda_i/\partial D_j}{\partial \Lambda_i/\partial \omega} > 0. \tag{A.3}$$

We now combine these results above with the comparative statics of bilateral interactions with respect to the relative wage (ω) from Proposition 4. In particular, from the proof of that proposition, we have the following results:

$$\frac{dn_{ii}}{d\omega} > 0, \qquad \frac{dn_{ij}}{d\omega} < 0. \tag{A.4}$$

Combining these two sets of relationships (A.3) and (A.4), we have the following results stated in the proposition:

$$\frac{dn_{ii}}{dD_i} = \underbrace{\frac{dn_{ii}}{d\omega}}_{>0}\underbrace{\frac{d\omega}{dD_i}}_{<0} < 0, \qquad \frac{dn_{ii}}{dD_j} = \underbrace{\frac{dn_{ii}}{d\omega}}_{>0}\underbrace{\frac{d\omega}{dD_j}}_{>0} > 0.$$

$$\frac{dn_{ij}}{dD_i} = \underbrace{\frac{dn_{ij}}{d\omega}}_{<0}\underbrace{\frac{d\omega}{dD_i}}_{<0} > 0, \qquad \frac{dn_{ij}}{dD_j} = \underbrace{\frac{dn_{ij}}{d\omega}}_{<0}\underbrace{\frac{d\omega}{dD_j}}_{>0} < 0.$$

A.9 Proof of Proposition 9

Because $Q_i(n_{ii}(t), n_{ij}(t)) \geq C_i(n_{ii}(t), n_{ij}(t))$, from equation (35), we must have $\dot{\theta}_i^k(t) \geq 0$ at all t. This in turn implies that we must have $\theta_i^k(t) \leq 0$ at all t for the transversality condition to be met (i.e., convergence to 0 from below).

We next show that $\dot{\theta}_i^i(t) \geq 0$ and $\theta_i^i(t) \leq 0$ for all t. First note that we must have

$$\eta_i \theta_i^k(t) < (\gamma_i + \eta_i)\, \theta_i^i(t)$$

and thus (from equation (34)) $\dot{\theta}_i^i(t) > 0$ for all t. To see this, note that if instead we had

$$\eta_i \theta_i^k(t_0) > (\gamma_i + \eta_i)\, \theta_i^i(t_0),$$

at any time t_0, then $\dot{\theta}_i^i(t_0) < 0 < \dot{\theta}_k^i(t_0)$ so this inequality would continue to hold for all $t_0 > t$. But then we would have $\dot{\theta}_i^i(t) < 0$ for all $t > t_0$, and for $\theta_i^i(t)$ to meet its transversality condition, we would need to have $\theta_i^i(t) > 0$ at all $t > t_0$. But if $\theta_i^i(t) > 0$ and $\theta_i^k(t) \leq 0$ for $t > t_0$, it is clear from equation (34) that $\dot{\theta}_i^i(t) > 0$ for $t > t_0$, which is a contradiction. In sum, $\dot{\theta}_i^i(t) > 0$ for all t. But then for $\theta_i^i(t)$ to meet its transversality condition (from below), we need $\theta_i^i(t) \leq 0$ for all t.

Finally, to show that show that $\theta_i^s(t) > \theta_i^i(t)$ for all t, suppose that $\theta_i^s(t_0) < \theta_i^i(t_0)$ for some t_0. From equation (33), this would imply $\dot{\theta}_i^s(t_0) < 0$. But because $\dot{\theta}_i^i(t) > 0$ for all t, we would continue to have $\theta_i^s(t) < \theta_i^i(t)$ for all $t > t_0$, and thus $\dot{\theta}_i^s(t) < 0$ for all $t > t_0$. This would imply that, for $t > t_0$, $\theta_i^s(t)$ would converge to its steady-state value of 0 from above, i.e., $\theta_i^s(t) > 0$ for $t > t_0$. But because $\theta_i^i(t) \leq 0$ for all t, from equation (33), we would have $\dot{\theta}_i^s(t) > 0$ for $t > t_0$, which is a contradiction. In sum, we must have $\theta_i^s(t) > \theta_i^i(t)$ for all t.

B Simulation Appendix

In this section of the Appendix, we discuss our choice of parameter values. A description of the computational algorithms used is presented in Appendix E. The simulation presented in the main text are supposed to be illustrative rather than a detailed calibration for a specific circumstance. Nevertheless, the baseline calibration adopts the central values of the epidemiology parameters in Fernández-Villaverde and Jones (2020). For example, in Figure 1 we set the value of the exogenous component of the infection rate in the healthy country, $\alpha_1 = 0.04$, and we vary the value for the sick country between $\alpha_2 \in [0.04, 0.1]$. Using the equilibrium values of interactions, this leads to a value of $2n_{ii}\alpha_i + n_{ij}\alpha_j + n_{ji}\alpha_i$ (the actual infection rate in Country i if $I_i = I_j$) in the range $[0.15, 0.20]$ in Country 1 and $[0.15, 0.33]$ in Country 2, well in the range of values estimated in Fernández-Villaverde and Jones (2020). We also set $\gamma_i = 0.2$, which implies an infectious period of 5 days.

The economic model also involves a number of parameters. We set the elasticity of substitution $\sigma = 5$, a central value in the trade literature (Costinot and Rodríguez-Clare, 2015), and normalize productivity $Z_i = 1$ for all i. We also set Country size $L_i = 3$ when countries are symmetric. We choose values so that the choice of trading partners n_{ij} is never constrained. We choose a baseline value for the elasticity of the cost of consuming more varieties in a region of $\phi = 2$. Hence, the second order conditions discussed in the text are satisfied since $\phi > 1/(\sigma - 1)$. Note that we also require $\phi > 1$. We eliminate all man-made frictions in the baseline, so $t_{ij} = \mu_{ij} = 1$ for all i, j, and let $d_{ij} = 1.1$ for $i \neq j$ and 1 otherwise. We set to one the elasticity of trade costs with respect to distance, so $\delta = 1$. Finally we set the level of the cost of creating contacts, $c = 0.15$, which guarantees that equilibrium contacts are always in an interior solution. Of course, in the main text we show a number of exercises in which we change these parameter values, and in particular introduce trade and mobility frictions. Whenever we vary from the parameter values mentioned above we state that in the discussion of the relevant graph.

Globalization and Pandemics

Pol Antràs, Stephen J. Redding, and Esteban Rossi-Hansberg

Online Appendix (Not for Publication)

C Extensions of Economic Model

In this Appendix, we flesh out some of the details of the four extensions of our framework mentioned in Section 2.3 of the main text.

C.1 Traveling Costs in Terms of Labor

If traveling costs are specified in terms of labor (rather than utility), welfare at the household level depends only on consumption

$$W_i = \left(\sum_{j \in \mathcal{J}} \int_0^{n_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}},$$

and the implied demand (for a given n_{ii} and n_{ij}) is given by

$$q_{ij}(k) = \left(\frac{p_{ij}}{P_i} \right)^{-\sigma} \frac{\mathcal{I}_i}{P_i},$$

where $\mathcal{I}_i$ is household income, which is given by

$$\mathcal{I}_i = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} \right),$$

since the household now needs to hire labor to be able to secure final-good differentiated varieties, and where

$$P_i = \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Welfare can therefore be rewritten as

$$W_i = \frac{\mathcal{I}_i}{P_i} = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} \right) \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

The first-order condition for the choice of n_{ij} delivers:

$$n_{ij} = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i} \right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}}$$

Bilateral import flows by country i from country j are given by

$$X_{ij} = n_{ij} p_{ij} q_{ij} L_i = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i} \right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} \mathcal{I}_i L_i,$$

and the trade share can be written as

$$\pi_{ij} = \frac{X_{ij}}{\sum_{l\in\mathcal{J}} X_{il}} = \frac{\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{ij}^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{l\in\mathcal{J}} \left(\frac{w_l}{Z_l}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{il}^{-\frac{1}{\phi-1}} d_{il}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{il}^{-\frac{\phi(\sigma-1)}{\phi-1}}} = \frac{S_j}{\Phi_i} \times \Gamma_{ij}^{-\varepsilon},$$

where

$$\Gamma_{ij}^{-\varepsilon} = \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_i^{-\frac{\phi(\sigma-1)}{\phi-1}},$$

which is identical to equation (9) applying to our baseline model with traveling costs in terms of labor.

The price index is in turn given by

$$P_i = (c(\sigma-1))^{\frac{1}{\phi(\sigma-1)}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{-\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

and using this expression together for the one for π_{ij}, one can verify that we can write

$$n_{ij} = \left(\frac{t_{ij} d_{ij}^{\delta} w_j}{Z_j P_i}\right)^{\sigma-1} \pi_{ij},$$

just as in equation (15) of the main text.

Plugging this expression back into the budget constraint yields

$$\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i,$$

and a resulting price index equal to

$$P_i = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

which is only slightly different than expression (11) in the main text,

The labor-market conditions are given by

$$\pi_{ii}\mathcal{I}_i L_i + \pi_{ji}\mathcal{I}_j L_j = \mathcal{I}_i L_i$$

or, equivalently,

$$\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i,$$

just as in the main text, and remember that the expressions for π_{ii} and π_{ji} are also left unchanged.

We next turn to verifying that Propositions 1 through 4 in the main text continue to hold whenever travel costs in equation (1) are specified in terms of labor rather than being modelled as a utility cost.

Proposition 1': *As long as trade frictions (Γ_{ij}) are bounded, there exists a unique vector of equilibrium wages $w^* = (w_i, w_j) \in R^2_{++}$ that solves the system of equations above.*

Proof. By results in standard gravity models in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020). ∎

Proposition 2': *A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).*

Proof. (a) Given that $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i$,

$$n_{ii} = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{\frac{1}{\phi-1}} \left(\frac{t_{ii} w_i}{Z_i P_i}\right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ii}^{-\frac{1}{\phi-1}} d_{ii}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$$

Then

$$\frac{P_i}{w_i} = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

where $\omega = w_j/w_i$ is the relative wage in country j .

Note that the labor constraint can be rewritten as

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i + \frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i$$

Consider a case when Γ_{ij} decreases, while other Γ_{kl} remain constant. That means that the first term in the sum goes down, while the second term is constant. For the equality to hold, ω should increase. After re-equilibration, the second term in the sum increased, which means that the first term decreased. This means that P_i/w_i decreased, and n_{ii} as well.

Consider now a case when Γ_{ji} decreases, while other Γ_{kl} remain constant. The second term increases, so ω needs to go down to equilibrate the model. That means that the first term decreases, and P_i/w_i and n_{ii} decrease by extension.

Therefore, whenever one decreases any international friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$), Γ_{ij} or Γ_{ji} goes down, and, hence, n_{ii} and n_{jj} go down.

(b) Note that

$$\frac{\mathcal{I}_i}{w_i} = 1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi}$$

Since $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1)} w_i$, the left-hand side is constant. Since n_{ii} and n_{jj} decrease, n_{ij} and n_{ji} must increase. ∎

Proposition 3': *Suppose that countries are symmetric, in the sense that* $L_i = L$*,* $Z_i = Z$*, and* $\Gamma_{ij} = \Gamma$ *for all* i*. Then a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions (*$n_{dom} + n_{for}$*) experienced by both household buyers and household sellers.*

Proof. We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} = \frac{1}{\phi(\sigma-1)+1}$$

Differentiating yields

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \phi \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1} dn_{ij} + \phi n_{ij}^{\phi} \underbrace{d\left(\mu_{ij} d_{ij}^{\rho}\right)}_{\leq 0} = 0$$

Hence,

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \phi \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1} dn_{ij} \geq 0,$$

and if $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1}$, then $dn_{ij} > -dn_{ii}$.

From the FOC for the choice of n_{ii} and n_{ij},

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} = \frac{1}{c(\sigma-1)} \frac{\mathcal{I}_i}{w_i} \left(\frac{p_{ii}}{P_i}\right)^{1-\sigma}$$

$$\mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1} = \frac{1}{c(\sigma-1)} \frac{\mathcal{I}_i}{w_i} \left(\frac{p_{ij}}{P_i}\right)^{1-\sigma}$$

Therefore, $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1}$ is satisfied if and only if $p_{ii} < p_{ij}$.

When countries are symmetric, this holds trivially because of international trade costs $t_{ij} > t_{ii}$ and $d_{ij} > d_{ii}$. Hence, $dn_{ij} > -dn_{ii}$, and $n_{dom} + n_{for}$ increases. ■

Proposition 4': *An increase in the relative size of country* i*'s population leads to an increase in the rate* n_{ii} *at which individuals from* i *will meet individuals in their own country, and to a decrease in the rate* n_{ij} *at which individuals will meet individuals abroad.*

Proof. Consider again

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i + \frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i$$

An increase in L_i makes the left-hand side smaller then the right-hand side. Therefore, ω grows to re-equilibrate. Then

$$\frac{P_i}{w_i} = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

increases, and $n_{ii} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$ increases with it.

Since

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} = \frac{1}{\phi(\sigma-1)+1},$$

n_{ij} decreases.

Therefore, following a growth in population L_i, n_{ii} increases while n_{ij} decreases. ■

C.2 International Sourcing of Inputs

The assumption that households travel internationally to procure final goods may seem unrealistic. Perhaps international travel is better thought as being a valuable input when firms need specialized inputs and seek potential providers of those inputs in various countries. It is straightforward to re-interpret our model along those lines. In particular, suppose now that all households in country i produce a homogeneous final good but also produce differentiated intermediate input varieties. The household's final good is produced combining a bundle of the intermediate inputs produced by other households. Technology for producing the final good is given by

$$Q_i = \left(\sum\nolimits_{j\in\mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I(k)^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}}$$

and this final good is not traded (this is without loss of generality if households are homogeneous in each country and trade costs for final goods are large enough). Household welfare is linear in consumption of the final good and is reduced by the disutility cost of a household's member having to travel to secure intermediate inputs. In particular, we have

$$W_i = \left(\sum\nolimits_{j\in\mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I(k)^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}} - \frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}} \mu_{ij}(d_{ij})^{\rho} \times \left(n_{ij}^I\right)^{\phi}.$$

Under this model is isomorphic to the one above, except that trade will be in intermediate inputs rather than in final goods.

C.3 Multi-Country Model

We next consider a version of our model with a world economy featuring multiple countries. It should be clear that all our equilibrium conditions, except for the labor-market clearing condition (14) apply to that multi-country environment once the set of countries $\mathcal{J}$ is redefined to include multiple countries. The labor-market condition is in turn simply given by

$$\sum\nolimits_{j\in\mathcal{J}} \pi_{ij}\left(\mathbf{w}\right) w_j L_j = w_i L_i,$$

where $\pi_{ij}\left(\mathbf{w}\right)$ is defined in (9) for an arbitrary set of countries $\mathcal{J}$. Similarly, the model is also easily adaptable to the case in which there is a continuum of locations $i \in \Omega$, where Ω is a closed and bounded set of a finite dimensional Euclidean space. The equilibrium conditions are again unaltered, with integrals replacing summation operators throughout.

From the results in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020), it is clear that Proposition 1 in the main text on existence and uniqueness will continue to hold. In the presence of arbitrary asymmetries across countries, it is hard however to derive crisp comparative static results of the type in Propositions 2 and 4. Nevertheless, our result in Proposition 3 regarding the positive effect of declines of trade and mobility barriers on the overall level of human interactions between symmetric countries is easily generalizable to the case of many countries (details available upon request - future versions of the paper will include an Online Appendix with the details).

C.4 Traveling Salesman Model

Finally, we explore a variant of our model in which it is the household's seller rather than the buyer who travels to other locations. We model this via a framework featuring scale economies, monopolistic competition and fixed cost of exporting, as in the literature on selection into exporting emanating from the seminal work of Melitz (2003), except that the fixed costs of selling are defined at the buyer level rather than at the country level, as in the work of Arkolakis (2010).

On the consumption side, households maximize their utility, given by

$$W_i = \left(\sum_{j\in\mathcal{J}} \int_0^{\eta_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}},$$

where η_{ij} is the measure of varieties available to them, subject to the household budget constraint. This yields

$$q_{ij}(k) = \left(\frac{p_{ij}}{P_i}\right)^{-\sigma} \frac{\mathcal{I}_i}{P_i},$$

where $\mathcal{I}_i$ is household income and the price index is

$$P_i = \left(\sum_{j\in\mathcal{J}} \eta_{ij} p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Household sellers in country j produce N_j varieties and make them available to n_{ij} consumers. Both N_j and n_{ij} are endogenous and pinned down as part of the equilibrium. Note that because there are L_i and L_j households in i and j, respectively, the measure of varieties available from j to consumers in i is given by $\eta_{ij} = n_{ij} N_j L_j / L_i$ (where implicitly we assume that which $n_{ij} < L_j$

consumers in j get access to a seller's varieties is chosen at random).

The level of output and price of each variety, as well as the measure of consumers n_{ij} sellers reach out to follows from profit maximization:

$$\max_{n_{ij},p_{ij}} n_{ij}\left(p_{ij}-\frac{\tau_{ij}w_j}{Z_j}\right)q_{ij}-w_j\frac{c}{\phi}\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi},$$

where again n_{ij} is the number of customers served, and where the remaining parameters are analogous to those in our baseline model.

Sellers naturally charge a constant markup over marginal cost,

$$p_{ij}=\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_j}{Z_j},$$

so the choice of n_{ij} boilds down to

$$\max_{n_{ij}} \frac{n_{ij}}{\sigma}p_{ij}q_{ij}-w_j\frac{c}{\phi}\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}.$$

The first-order condition of this problem yields

$$\frac{p_{ij}q_{ij}}{\sigma}=w_jc\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}\Rightarrow n_{ij}=\left(\frac{p_{ij}q_{ij}}{c\sigma\mu_{ij}d_{ij}^{\rho}w_j}\right)^{\frac{1}{\phi-1}}$$

Developing a new variety costs w_if. Hence, by free entry, $\sum_k \Pi_{ki}=w_if$, and the zero-profit condition also entails $\mathcal{I}_i=w_i$. As a result, we can express n_{ij} as

$$n_{ij}=(c\sigma)^{-\frac{1}{\phi-1}}\mu_{ij}^{-\frac{1}{\phi-1}}d_{ij}^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{\sigma}{\sigma-1}\frac{t_{ij}w_j}{P_iZ_j}\right)^{-\frac{\sigma-1}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}.$$

With this expression at hand, we can compute the import volume of country i from country j:

$$\begin{aligned}X_{ij}&=\eta_{ij}p_{ij}q_{ij}L_i=n_{ij}p_{ij}q_{ij}N_jL_j\\&=w_jc\sigma\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}N_jL_j\\&=(c\sigma)^{-\frac{1}{\phi-1}}\mu_{ij}^{-\frac{1}{\phi-1}}d_{ij}^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(\frac{\sigma}{\sigma-1}\frac{t_{ij}w_j}{P_iZ_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}w_iN_jL_j\\&=(c\sigma)^{-\frac{1}{\phi-1}}\Gamma_{ij}^{-\varepsilon}\left(\frac{\sigma}{\sigma-1}\frac{w_j}{P_iZ_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}w_iN_jL_j\end{aligned}$$

Hence, the share of country j in country i's import is

$$\pi_{ij}=\frac{\Gamma_{ij}^{-\varepsilon}\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}w_j^{-\frac{1}{\phi-1}}N_jL_j}{\sum_k\Gamma_{ik}^{-\varepsilon}\left(\frac{w_k}{Z_k}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}w_k^{-\frac{1}{\phi-1}}N_kL_k}.$$

Solving for price index yields

$$w_iL_i = \sum_j X_{ij}$$

$$w_iL_i = \sum_j (c\sigma)^{-\frac{1}{\phi-1}}\Gamma_{ij}^{-\varepsilon}\left(\frac{\sigma}{\sigma-1}\frac{w_j}{P_iZ_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}} w_iN_jL_j$$

$$P_i = \frac{\sigma}{\sigma-1}(c\sigma)^{\frac{1}{\phi(\sigma-1)}}L_i^{\frac{\phi-1}{\phi(\sigma-1)}}\left(\sum_j \Gamma_{ij}^{-\varepsilon}\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}} N_jL_j\right)^{-\frac{\phi-1}{\phi(\sigma-1)}}.$$

We can next study the choice of the number of varities N_j produced by sellers. Profits of sellers are given by

$$\Pi_{ij} = \frac{\phi-1}{\phi}\frac{n_{ij}p_{ij}q_{ij}}{\sigma} = \frac{\phi-1}{\phi}\frac{X_{ij}}{\sigma N_jL_j},$$

so the zero-profit condition implies

$$\sum_k \Pi_{ki} = w_if \Rightarrow \frac{\phi-1}{\phi}\frac{1}{\sigma N_iL_i}\sum_k X_{ki} = w_if.$$

Since $\sum_k X_{ki} = w_iL_i$,

$$\frac{\phi-1}{\phi}\frac{w_iL_i}{\sigma N_iL_i} = w_if \Rightarrow N_i = \frac{\phi-1}{\phi\sigma f}$$

Hence, the number of varieties is constant and independent of many of the parameters of the model.

We finally turn to the general equilibrium of the model, which is associated with the condition:

$$\pi_{ii}w_iL_i + \pi_{ji}w_jL_j = w_iL_i$$

Plugging in the expressions for trade shares yields

$$\frac{\Gamma_{ii}^{-\varepsilon}\left(\frac{w_i}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_i^{-\frac{1}{\phi-1}}L_i}{\sum_k\left(\Gamma_{ik}^{-\varepsilon}\frac{w_k}{Z_k}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_k^{-\frac{1}{\phi-1}}L_k}w_iL_i + \frac{L_i\left(\frac{w_i}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_i^{-\frac{1}{\phi-1}}\Gamma_{ji}^{-\varepsilon}}{\sum_k L_k\left(\frac{w_k}{Z_k}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_k^{-\frac{1}{\phi-1}}\Gamma_{jk}^{-\varepsilon}}w_jL_j = w_iL_i.$$

We are now ready to state and proof results analogous to those in Propositions 1 and 4 in the main text.

Proposition 1": As long as trade frictions (Γ_{ij}) are bounded, there exists a unique vector of equilibrium wages $\mathbf{w}^* = (w_i, w_j) \in \mathbb{R}^2_{++}$ that solves the system of equations above.

Proof. This follows again from results in standard gravity models in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020), and the fact that if there exists a unique wage vector, the remaining equilibrium variables in this single-sector economy are uniquely determined. ■

Proposition 2": A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).

Proof. **(a)** First, note that

$$n_{ii} = \xi \mu_{ii}^{-\frac{1}{\phi-1}} d_{ii}^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ii} w_i}{P_i Z_i}\right)^{-\frac{\sigma-1}{\phi-1}} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$$

Then

$$\frac{P_i}{w_i} = const \times L_i^{\frac{\phi-1}{\phi(\sigma-1)}} \left(L_i \Gamma_{ii}^{-\varepsilon} Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} + L_j \Gamma_{ij}^{-\varepsilon} \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}}$$

where $\omega = w_j/w_i$ is the relative wage in country j .

Note that the equilibrium equations can be rewritten as

$$\frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i \tag{C.1}$$

$$+ \frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i \tag{C.2}$$

Consider a case when Γ_{ij} decreases, while other Γ_{kl} remain constant. That means that the first term in the sum goes down, while the second term is constant. For the equality to hold, ω should increase. After re-equilibration, the second term in the sum increased, which means that the first term decreased. This means that P_i/w_i decreased, and n_{ii} as well.

Consider now a case when Γ_{ji} decreases, while other Γ_{kl} remain constant. The second term increases, so ω needs to go down to equilibrate the model. That means that the first term decreases, and P_i/w_i and n_{ii} decrease by extension.

Therefore, whenever one decreases any international friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$), Γ_{ij} or Γ_{ji} goes down, and, hence, n_{ii} and n_{jj} go down.

(b) Note that $\Pi_{ii} + \Pi_{ji} = w_i f$. That can be rewritten as

$$\frac{\phi-1}{\phi} \frac{n_{ii} p_{ii} q_{ii}}{w_i \sigma} + \frac{\phi-1}{\phi} \frac{n_{ji} p_{ji} q_{ji}}{w_i \sigma} = f$$

Using the FOC for n_{ij}, that yields

$$\frac{\phi-1}{\phi} c \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \frac{\phi-1}{\phi} c \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = f$$

Since n_{ii} and n_{jj} decrease and frictions do not increase, n_{ij} and n_{ji} have to increase. ■

Proposition 3": Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.

Proof. We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = const$$

Differentiating yields

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \phi \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} dn_{ji} + n_{ji}^{\phi} \underbrace{d\left(\mu_{ji} d_{ji}^{\rho}\right)}_{\leq 0} = 0$$

Hence,

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} dn_{ji} \geq 0,$$

and if $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1}$, then $dn_{ji} > -dn_{ii}$.

From the FOC for the choice of n_{ii} and n_{ji},

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} = const \times \frac{p_{ii} q_{ii}}{w_i} = const \times \left(\frac{p_{ii}}{P_i}\right)^{1-\sigma}$$

$$\mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} = const \times \frac{p_{ji} q_{ji}}{w_i} = const \times \left(\frac{p_{ji}}{P_j}\right)^{1-\sigma} \left(\frac{w_j}{w_i}\right)$$

Since the countries are symmetrics, $P_i = P_j$ and $w_i = w_j$, so the inequality is satisied if and only if $p_{ii} < p_{ji}$.

When countries are symmetric, this holds trivially because of international trade costs $t_{ji} > t_{ii}$ and $d_{ji} > d_{ii}$. Hence, $dn_{ji} > -dn_{ii}$, and $n_{dom} + n_{for}$ increases. ■

Proposition 4": An increase in the relative size of country i's population leads to an increase in the rate n_{ii} at which individuals from i will meet individuals in their own country, and to a decrease in the rate n_{ji} at which individuals will meet individuals abroad.

Proof. Consider again

$$\frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i \tag{C.3}$$

$$+ \frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i \tag{C.4}$$

An increase in L_i makes the left-hand side smaller then the right-hand side. Therefore, ω grows to re-equilibrate. Then

$$\frac{P_i}{w_i} = const \times L_i^{\frac{\phi-1}{\phi(\sigma-1)}} \left(L_i \Gamma_{ii}^{-\varepsilon} Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} + L_j \Gamma_{ij}^{-\varepsilon} \left(\frac{Z_j}{\omega} \right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \right)^{-\frac{\phi-1}{\phi(\sigma-1)}}$$

increases, and $n_{ii} = const \times \left(\frac{P_i}{w_i} \right)^{\frac{\sigma-1}{\phi-1}}$ increases with it.

Since

$$\frac{\phi-1}{\phi} c\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \frac{\phi-1}{\phi} c\mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = f,$$

n_{ji} decreases.

Therefore, following a growth in population L_i, n_{ii} increases while n_{ji} decreases. ■

D Proof of Proposition 5

Proposition 5: *Assume that there is trade between the two countries (i.e., $\alpha_j n_{ij} + \alpha_i n_{ji} > 0$), which implies that the next generation matrix FV^{-1} is irreducible. If $\mathcal{R}_0 \leq 1$, the no-pandemic equilibrium is the unique stable equilibrium. If $\mathcal{R}_0 > 1$, the no-pandemic equilibrium is unstable, and there exists a unique stable endemic equilibrium.*

Proof. The proof of existence and uniqueness, depending on whether $\mathcal{R}_0 \leq 1$ or $\mathcal{R}_0 > 1$, follows standard arguments for a two-group SIR model, as in Magal et al. (2016). We proceed in the following steps. **(A)** The system of dynamic equations for susceptibles, infected and recovered is given by:

$$\dot{S}_i(t) = -2\alpha_i n_{ii} S_i(t) I_i(t) - \alpha_j n_{ij} S_i(t) I_j(t) - \alpha_i n_{ji} S_i(t) I_j(t), \tag{D.1}$$

$$\dot{S}_j(t) = -2\alpha_j n_{jj} S_j(t) I_j(t) - \alpha_i n_{ji} S_j(t) I_i(t) - \alpha_j n_{ij} S_j(t) I_i(t), \tag{D.2}$$

$$\dot{I}_i(t) = 2\alpha_i n_{ii} S_i(t) I_i(t) + \alpha_j n_{ij} S_i(t) I_j(t) + \alpha_i n_{ji} S_i(t) I_j(t) - \gamma_i I_i(t), \tag{D.3}$$

$$\dot{I}_j(t) = 2\alpha_j n_{jj} S_j(t) I_j(t) + \alpha_i n_{ji} S_j(t) I_i(t) + \alpha_j n_{ij} S_j(t) I_i(t) - \gamma_j I_j(t), \tag{D.4}$$

$$\dot{R}_i(t) = \gamma_i I_i(t), \tag{D.5}$$

$$\dot{R}_j(t) = \gamma_j I_j(t). \tag{D.6}$$

Note that we can re-write the dynamic equations for infections (D.3) and (D.4) as:

$$\begin{bmatrix} \dot{I}_i(t) \\ \dot{I}_j(t) \end{bmatrix} = \left\{ \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i(t) & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i(t) \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_j(t) & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j(t) \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} \gamma_i I_i(t) \\ \gamma_j I_j(t) \end{bmatrix}. \tag{D.7}$$

The properties of this dynamic system depend crucially on the properties of the matrix B:

$$B \equiv \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i(t) & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i(t) \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_j(t) & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j(t) \end{bmatrix}.$$

We assume that there is trade between the two countries:

$$\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} > 0, \qquad \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} > 0,$$

which implies that the matrix B is irreducible for all strictly positive susceptibles $S_i(t), S_j(t) > 0$.
(B) Re-writing equations (D.1) and (D.2) in proportional changes, and using equations (D.5) and (D.6), we have:

$$\frac{\dot{S}_i(t)}{S_i(t)} = -\frac{2\alpha_i n_{ii}}{\gamma_i}\dot{R}_i(t) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\dot{R}_j(t),$$

$$\frac{\dot{S}_j(t)}{S_j(t)} = -\frac{2\alpha_j n_{jj}}{\gamma_j}\dot{R}_j(t) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\dot{R}_i(t).$$

Integrating from 0 to t, we have:

$$\log S_i(t) - \log S_i(0) = -\frac{2\alpha_i n_{ii}}{\gamma_i}(R_i(t) - R_i(0)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(R_j(t) - R_j(0)),$$

$$\log S_j(t) - \ln S_j(0) = -\frac{2\alpha_j n_{jj}}{\gamma_j}(R_j(t) - R_j(0)) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}(R_i(t) - R_i(0)).$$

Using the accounting identities, $S_i(t) + I_i(t) + R_i(t) = 1$ and $S_j(t) + I_j(t) + R_j(t) = 1$, we obtain:

$$\log S_i(t) - \log S_i(0) = \frac{2\alpha_i n_{ii}}{\gamma_i}\left[(S_i(t) + I_i(t)) - (S_i(0) + I_i(0))\right] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left[(S_j(t) + I_j(t)) - (S_j(0) + I_j(0))\right],$$

$$\log S_j(t) - \ln S_j(0) = \frac{2\alpha_j n_{jj}}{\gamma_j}\left[(S_j(t) + I_j(t)) - (S_j(0) + I_j(0))\right] + \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\left[(S_i(t) + I_i(t)) - (S_i(0) + I_i(0))\right].$$

In steady-state as $t \to \infty$, we have $I_i(\infty) = I_j(\infty) = 0$, and hence:

$$S_i(\infty) = S_i(0)\exp\left[\frac{2\alpha_i n_{ii}}{\gamma_i}\left[S_i(\infty) - V_i\right] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left[S_j(\infty) - V_j\right]\right], \tag{D.8}$$

$$S_j(\infty) = S_j(0)\exp\left[\frac{2\alpha_j n_{jj}}{\gamma_j}\left[S_j(\infty) - V_j\right] + \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\left[S_i(\infty) - V_i\right]\right], \tag{D.9}$$

where $V_i \equiv S_i(0) + I_i(0)$ and $V_j(0) \equiv S_j(0) + I_j(0)$. We now define the following notation:

$$X \leq Y \quad \Leftrightarrow \quad X_k \leq Y_k \text{ for all } k \in \{i, j\},$$

$$X < Y \quad \Leftrightarrow \quad X \leq Y \text{ and } X_k < Y_k \text{ for some } k \in \{i, j\},$$

$$X \ll Y \quad \Leftrightarrow \quad X_k < Y_k \text{ for all } k \in \{i, j\},$$

and represent the system (D.8)-(D.9) as the following map:

$$X = T(X),$$

$$\begin{pmatrix} x_i \\ x_j \end{pmatrix} = T \begin{pmatrix} x_i \\ x_j \end{pmatrix} = \begin{pmatrix} T_i (x_i, x_j) \\ T_j (x_i, x_j) \end{pmatrix},$$

with

$$T_i (x_i, x_j) = S_i (0) \exp \left[\frac{2\alpha_i n_{ii}}{\gamma_i} [x_i - V_i] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} [x_j - V_j] \right],$$

$$T_j (x_i, x_j) = S_j (0) \exp \left[\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} [x_i - V_i] + \frac{2\alpha_j n_{jj}}{\gamma_j} [x_j - V_j] \right].$$

(C) Using this notation, we begin by establishing that all the fixed points of T in $[0, S(0)]$ are contained in the smaller interval $[S^-, S^+]$. To establish this result, note that T is monotone in increasing, which implies that:

$$X \leq Y \quad \Rightarrow \quad T(X) \leq T(Y).$$

Using our assumption of positive trade, $\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} > 0$ and $\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} > 0$, this implies:

$$X \ll Y \quad T(X) \ll T(Y).$$

For $S(0) \gg 0$, and using the definitions of V_i and V_j above, this implies:

$$0 \ll T(0) < T(S(0)) < S(0).$$

Therefore, by induction arguments, we have the following result for each $n \geq 1$:

$$0 \ll T(0) \cdots \ll T^n (0) \ll T^{n+1} (0) \leq T^{n+1} (S(0)) < \cdots T^n (S(0)) < S(0).$$

By taking the limit as n does to $+\infty$, we obtain:

$$0 \ll \lim_{n \to +\infty} T^n (0) =: S^- \leq S^+ := \lim_{n \to +\infty} T^n (S(0)) < S(0).$$

Then, by continuity of T, we have:

$$T\left(S^-\right) = S^- \quad \text{and} \quad T\left(S^+\right) = S^+.$$

(D) We next establish that if $S^- < S^+$ then $S^- \ll S^+$. This property follows from our assumption that the matrix B above is irreducible. Assume, for example, that $S_i^- < S_i^+$. Then, since $\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} > 0$, we have:

$$S_j^- = T_j \left(S_i^-, S_j^-\right) \leq T_j \left(S_i^-, S_j^+\right) < T_2 \left(S_i^+, S_j^+\right) = S_j^+.$$

Hence,

$$S_i^- < S_i^+ \Rightarrow S_j^- < S_j^+.$$

By the same argument, $\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}>0$ implies,

$$S_j^- < S_j^+ \Rightarrow S_i^- < S_i^+.$$

(E) We now establish the following result for $\lambda > 1$ and $X \gg 0$:

$$T\left(\lambda X + S^-\right) - T\left(S^-\right) \gg \lambda\left[T\left(X+S^-\right) - T\left(S^-\right)\right].$$

Note that we can write the left-hand side of this inequality as follows:

$$T\left(\lambda X + S^-\right) - T\left(S^-\right) = \int_0^1 DT\left(l\lambda X + S^-\right)(\lambda X)\,dl = \lambda \int_0^1 DT\left(l\lambda X + S^-\right) X dl,$$

where the differential of T is given by:

$$DT(X) = \begin{pmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} T_i(x_i, x_j) & \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j} T_i(x_i, x_j) \\ \frac{\alpha_i n_{ji}+\alpha_j n_{ij}}{\gamma_i} T_j(x_i, x_j) & \frac{2\alpha_j n_{jj}}{\gamma_j} T_j(x_i, x_j) \end{pmatrix}. \tag{D.10}$$

Since $\lambda > 1$ and $X \gg 0$, we have:

$$DT\left(l\lambda X + S^-\right) X \gg DT\left(lX + S^-\right) X \qquad \forall \quad l \in [0,1].$$

It follows that:

$$\begin{aligned} T\left(\lambda X + S^-\right) - T\left(S^-\right) &\gg \lambda \int_0^1 DT\left(lX + S^-\right) X dl, \\ &= \lambda\left[T\left(X+S^-\right) - T\left(S^-\right)\right], \end{aligned}$$

which establishes the result.

(F) We now show that the map T has at most two equilibria such that either:

(i) $S^- = S^+$ and T has only one equilibrium in $[0, S(0)]$;

(ii) $S^- \ll S^+$ and the only equilibria of T in $[0, S(0)]$ are S^- and S^+.

We prove this result by contradiction. Assume that $S^- \neq S^+$. Then $S^- < S^+$, which implies $S^- \ll S^+$. Now suppose that there exists $\bar{X} \in [S^-, S^+]$ a fixed point T such that:

$$S^- \neq \bar{X} \qquad \text{and} \qquad \bar{X} \neq S^+.$$

Then, by using the same arguments as in **(D)** above, we have:

$$S^- \ll \bar{X} \ll S^+.$$

Now define:

$$\gamma := \sup\left\{\lambda \geq 1 : \lambda\left(\bar{X} - S^-\right) + S^- \leq S^+\right\}. \tag{D.11}$$

Since $\bar{X} \ll S^{+}$ this implies that

$$\gamma > 1.$$

We have

$$\gamma\left(\bar{X} - S^{-}\right) + S^{-} \leq S^{+},$$

and, by applying T to both sides of this inequality, we obtain:

$$T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) \leq S^{+}.$$

Now, using **(E)**, we have:

$$\begin{aligned} T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right) &\gg \gamma\left[T\left(\left(\bar{X} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right)\right], \\ &= \gamma\left[T\left(\bar{X}\right) - T\left(S^{-}\right)\right], \\ &= \gamma\left[\bar{X} - S^{-}\right]. \end{aligned}$$

Therefore we have shown that:

$$S^{+} \geq T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) \gg \gamma\left[\bar{X} - S^{-}\right],$$

which contradicts the definition of gamma as the supremum of the set in equation (D.11), since $S^{-} \geq 0$. Therefore we cannot have another fixed point $\bar{X} \in \left[S^{-}, S^{+}\right]$.
(G) Now consider the case where:

$$S^{-} \ll S^{+}.$$

In this case of two equilibria, the differential of T can be written as:

$$DT\left(S^{\pm}\right) = B\left(S_i^{\pm}\right) = \begin{pmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^{\pm} & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i^{\pm} \\ \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} S_j^{\pm} & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^{\pm} \end{pmatrix}.$$

(H) We now establish the following property of the spectral radius of the matrices $DT\left(S^{-}\right)$ and $DT\left(S^{+}\right)$:

$$\rho\left(DT\left(S^{-}\right)\right) < 1 < \rho\left(DT\left(S^{+}\right)\right).$$

To prove this result, note that:

$$\begin{aligned} S^{+} - S^{-} &= T\left(S^{+}\right) - T\left(S^{-}\right), \\ &= T\left(\left(S^{+} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right), \\ &= \int_0^1 DT\left(l\left(S^{+} - S^{-}\right) + S^{-}\right)\left(S^{+} - S^{-}\right) dl. \end{aligned}$$

Since $S^+ - S^- \gg 0$, we also have:

$$\begin{aligned} DT\left(S^+\right)\left(S^+ - S^-\right) &\gg \int_0^1 DT\left(l\left(S^+ - S^-\right) + S^-\right)\left(S^+ - S^-\right) dl, \\ &\gg DT\left(S^-\right)\left(S^+ - S^-\right). \end{aligned}$$

Combining these two results, we obtain:

$$DT\left(S^+\right)\left(S^+ - S^-\right) \gg \left(S^+ - S^-\right) \gg DT\left(S^-\right)\left(S^+ - S^-\right). \tag{D.12}$$

which can be equivalently written as:

$$\left[DT\left(S^+\right) - I\right]\left(S^+ - S^-\right) > 0,$$

$$\left[DT\left(S^+\right) - \xi^+ I\right]\left(S^+ - S^-\right) = 0, \qquad \xi^+ > 1,$$

and

$$\left[DT\left(S^-\right) - I\right]\left(S^+ - S^-\right) < 0,$$

$$\left[DT\left(S^-\right) - \xi^- I\right]\left(S^+ - S^-\right) = 0, \qquad \xi^- < 1,$$

where I is the identity matrix. Noting that the matrices $DT\left(S^+\right)$ and $DT\left(S^-\right)$ are non-negative and irreducible, the Perron-Frobenius theorem implies:

$$\xi^- = \rho\left(DT\left(S^-\right)\right) < 1 < \rho\left(DT\left(S^+\right)\right) = \xi^+.$$

(I) We now solve explicitly for the spectral radius of the matrices $DT\left(S^\pm\right)$. We find the eigenvalues of the matrix $DT\left(S^\pm\right)$ by solving the characteristic equation:

$$\left|DT\left(S^\pm\right) - \xi^\pm I\right| = \left|\begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_j^\pm \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_i^\pm & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm \end{bmatrix} - \begin{bmatrix} \xi^\pm & 0 \\ 0 & \xi^\pm \end{bmatrix}\right| = 0.$$

The characteristic polynomial is:

$$\left(\xi^\pm\right)^2 - \left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm + \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right)\xi^\pm + \left(\frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} S_i^\pm S_j^\pm - \frac{\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2}{\gamma_i \gamma_j} S_i^\pm S_j^\pm\right) = 0.$$

The spectral radius is the largest eigenvalue:

$$\rho\left(DT\left(S^\pm\right)\right) = \frac{1}{2}\left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm + \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right) + \frac{1}{2}\sqrt{\left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm - \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right)^2 + 4\frac{\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2}{\gamma_i \gamma_j} S_i^\pm S_j^\pm}.$$

(J) We now use the results in **(H)** and **(I)** to examine the local stability of the two steady-state equilibria. From the dynamics of infections in equation (D.7), we have:

$$\begin{bmatrix} \dot{I}_i^{\pm} \\ \dot{I}_j^{\pm} \end{bmatrix} = \left\{ \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^{\pm} & \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j} S_j^{\pm} \\ \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i} S_i^{\pm} & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^{\pm} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} I_i^{\pm} \\ I_j^{\pm} \end{bmatrix}. \tag{D.13}$$

Therefore the spectral radius of the matrix $DT(S^{\pm})$ corresponds to the global $\mathcal{R}_0$ that determines the local stability of the two steady-state equilibria. As we have shown that $\rho(DT(S^{+})) > 1$, the steady-state S^{+} is locally unstable. As we have shown that $\rho(DT(S^{-})) < 1$, the steady-state S^{-} is locally stable.

■

E Computational Appendix

In this Appendix we describe the algorithms we use to do the numerical simulations in each section of the paper.

E.1 A Two-Country SIR Model with Time-Invariant Interactions

Solution Algorithm

1. Compute the value of n_{ii}, n_{ij}, n_{ij}, and n_{jj} as the outcome of the equilibrium that solves

$$n_{ij} = (c(\sigma-1)\mu_{ij})^{-1/(\phi-1)} (d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij}w_j}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i,$$

   where π_{ij} is given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} (d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{i\ell} w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}},$$

   corresponding to equation (9) in the paper. Call them $\bar{n}_{ii}$, $\bar{n}_{ij}$, $\bar{n}_{ij}$, and $\bar{n}_{jj}$. Provided population, technology, and relative wages are time invariant, these quantities will be fixed.

2. Set $I_i(0) = 0.1 \times 10^{-4}$, $S_i(0) = 1 - I_i(0)$, and $R_i(0) = 0$ for all i. For each $t \in [1, T]$ solve the following system of equations:

$$\begin{bmatrix} S_i(t+1) \\ I_i(t+1) \\ R_i(t+1) \\ S_j(t+1) \\ I_j(t+1) \\ R_j(t+1) \end{bmatrix} = \begin{bmatrix} -\Omega_i & \Omega_i & 0 & 0 & 0 & 0 \\ 0 & -\gamma_i & \gamma_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Omega_j & \Omega_j & 0 \\ 0 & 0 & 0 & 0 & -\gamma_j & \gamma_j \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times (1/step) \times \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \end{bmatrix} + \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \end{bmatrix}$$

where

$$\Omega_i = \alpha_i \times 2\bar{n}_{ii} \times I_i(t) + \alpha_j \times \bar{n}_{ij} \times I_j(t) + \alpha_i \times \bar{n}_{ji} \times I_j(t).$$

This system corresponds to equations $(16)-(18)$ in the paper. The variable *step* marks the number of steps taken within each time period, in this section we use $step = 2$.

Associated Figures

This section in the paper uses three sets of parameters. Figures 1, 2, and 3 present a general specification in which international trade favors the onset of a pandemic, with standard parameters as listed in Table E.1 for Figure 1 and Table E.2 for Figures 2 and 3. Figures 4 and 5 look at an example in which free trade prevents the onset of a pandemic, using parameters listed in Table E.3. Figure 6 presents the possibility of second waves of infection, using parameters listed in Table E.4.

If no other mention is made, trade frictions are set at baseline values $\mu_{ij} = \mu_{ji} = 1$, $t_{ij} = t_{ji} = 1$, $d_{ij} = d_{ij} = 1.1$. Some of these figures study changes in trade frictions moving one of these parameters. All other parameters are kept at baseline value.

Table E.1: Baseline parameters - Figure 1 in draft.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1	0.04
α_2	$\{0.04, 0.10\}$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

In order to obtain the result described for the second set of parameters, $\phi = 1.5$ is crucial. The only other difference with respect to the general scenario is a decrease of c to 0.1. This is not necessary: the qualitative result also holds for $c = 0.15$ but it was originally changed so that n_{ii} would be approximately the same in both cases.

There are more parameters that will generate a second wave of infections. The ones presented here were picked to obtain *reasonable* values for R^{0i} and R^0. What is essential for this feature to occur is that both countries have different timings for their own pandemics in autarky. One (small) country has very fast contagion rates (α) and very short recovery periods (high γ), while in the other (big) country the disease must progress much slower so that when the cycle starts it will drag the first country with it once again. The difference in size is there so that when the small country goes through its first cycle, the big country will remain mostly unaffected.

Table E.2: Baseline parameters - Figures 2, 3 in draft.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1, α_2	$0.04, 0.07$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

Table E.3: "*Better trade*" parameters - Figures 4, 5 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
$\alpha_1,\ \alpha_2$	$0.04, 0.07$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

E.2 General-Equilibrium Induced Responses

Solution Algorithm

1. Compute the value of $n_{ii}(0)$, $n_{ij}(0)$, $n_{ij}(0)$, and $n_{jj}(0)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma-1\right)\mu_{ij}\right)^{-1/(\phi-1)}\left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii}w_iL_i(1-D_i(t)) + \pi_{ji}w_jL_j(1-D_j(t)) = w_iL_i(1-D_i(t)),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}}X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}}\times(\mu_{ij})^{-\frac{1}{\phi-1}}\left(d_{ij}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(t_{ij}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}}\left(\mu_{i\ell}\right)^{-\frac{1}{\phi-1}}\left(d_{i\ell}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(t_{i\ell}w_\ell/Z_\ell\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}},$$

Table E.4: Second-wave parameters - Figure 6 in draft

Parameter	Value
σ	4.5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$2, 20$
$d_{12} = d_{21}$	1
δ	1
ρ	1
c	0.12
α_1, α_2	$0.69, 0.09$
β_1, β_2	$2.29, 0.30$
γ_1, γ_2	$2.1, 0.18$

corresponding to equation (9) in the paper. These values are no longer fixed and will evolve as the pandemic progresses.

2. Set $I_i(0) = 0.1 \times 10^{-4}$, $S_i(0) = 1 - I_i(0)$, and $R_i(0) = 0$ for all i. For each $t \in [1, T]$:

   (a) Solve the following system of equations:

$$\begin{bmatrix} S_i(t+1) \\ I_i(t+1) \\ R_i(t+1) \\ D_i(t+1) \\ S_j(t+1) \\ I_j(t+1) \\ R_j(t+1) \\ D_j(t+1) \end{bmatrix} = \begin{bmatrix} -\Omega_i & \Omega_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa_i & \gamma_i & \eta_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega_j & \Omega_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa_j & \gamma_j & \eta_j \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times (1/step) \times \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ D_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \\ D_j(t) \end{bmatrix} + \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ D_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \\ D_j(t) \end{bmatrix}$$

   where $\kappa_i = \gamma_i + \eta_i$ and

$$\Omega_i = \alpha_i \times 2n_{ii}(t) \times I_i(t) + \alpha_j \times n_{ij}(t) \times I_j(t) + \alpha_i \times n_{ji}(t) \times I_j(t).$$

   This system corresponds to equations (28) – (31) in the paper. The variable *step* marks the number of steps taken within each time period, in this section we use $step = 2$.

   (b) Update $n_{ij}(t+1)$ and $w_i(t+1)$ as the values that solve:

$$n_{ij}(t+1) = (c(\sigma-1)\mu_{ij})^{-1/(\phi-1)} (d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij} w_j(t+1)}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i(t+1)}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii} w_i(t+1) L_i(1 - D_i(t+1)) + \pi_{ji} w_j(t+1) L_j(1 - D_j(t+1)) = w_i(t+1) L_i(1 - D_i(t+1)).$$

Associated Figures

This section in the paper is associated with Figure 7, which uses the parameters described in Table E.5. These correspond to the first set of parameters in the previous section (associated to Figures 1, 2, and 3). The duration of the disease remains the same, as the exit rate from the infected stage $(\gamma_i + \eta_i)$ is unchanged, but now both countries experience deaths, with one of them having a much higher death rate than the other (η_i marks the entry into the dead stage, so $\eta_i/(\gamma_i + \eta_i)$ marks how many of those that were infected will end up dying).

Table E.5: Section 4 parameters - Figure 7.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1, α_2	$0.04, 0.07$
$(\gamma_i + \eta_i)$	0.20
$\eta_i/(\gamma_i + \eta_i)$	$0.01, 0.50$

E.3 Behavioral Responses - Symmetric Case

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and $T = 10,000$. Guess $D(\infty) = \mathcal{D}_i$.

2. Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma-1\right)\mu_{ij}\right)^{-1/(\phi-1)}\left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii}w_iL_i(1-\mathcal{D}_i) + \pi_{ji}w_jL_j(1-\mathcal{D}_j) = w_iL_i(1-\mathcal{D}_i),$$

where π_{ij} is given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}}X_{i\ell}} = \frac{\left(w_j/Z_j\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\times\left(\mu_{ij}\right)^{-\frac{1}{\phi-1}}\left(d_{ij}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(t_{ij}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}}\left(\mu_{i\ell}\right)^{-\frac{1}{\phi-1}}\left(d_{i\ell}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(t_{i\ell}w_\ell/Z_\ell\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

3. Transversality conditions are satisfied if

$$\lim_{t\to\infty} \theta_i^k(t) = 0$$
$$\lim_{t\to\infty} \theta_i^i(t) = 0$$
$$\lim_{t\to\infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = \theta_i^s(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the Lagrange multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - [Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))]\, e^{-\xi t}\Delta t$$
$$\theta_i^i(t) = \frac{1}{1+(\gamma_i+\eta_i)\Delta t}[\eta_i \theta_i^k(t)\Delta t + \theta_i^i(t+1)]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

4. Set $I_i(T) = 10^{-6}$, $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i+\eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}}\right](1-\mathcal{D}_i)e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)]S_i(T)\alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

5. For each $t \in [T, 0]$ solve the following system of equations, where all values evaluated at $t+1$ are known, to obtain values at t:

$$\theta_i^s(t+1) - \theta_i^s(t) = [\theta_i^s(t) - \theta_i^i(t)][2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t$$
$$\theta_i^s(t+1) - \theta_i^i(t) = (\gamma_i+\eta_i)\theta_i^i(t)\Delta t - \eta_i \theta_i^k(t)\Delta t$$
$$\theta_k^s(t+1) - \theta_i^k(t) = [Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t))]\, e^{-\xi t}\Delta t$$
$$I_i(t+1) - I_i(t) = S_i(t)[2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t - (\gamma_i+\eta_i)I_i(t)\Delta t$$
$$S_i(t+1) - S_i(t) = -S_i(t)[2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t$$
$$D_i(t+1) - D_i(t) = \eta_i I_i(t)\Delta t$$

and where $n_{i\cdot}(t)$ is again obtained as the value that solves:

$$\left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}}\right](1-D_i(t))e^{-\xi t} = [\theta_i^s(t) - \theta_i^i(t)]S_i(t)\alpha_j I_j(t).$$

These correspond to equations (32)-(35) in the paper plus the equations determining the evolution of the epidemiological variables, once we have imposed equilibrium conditions.

6. Repeat for all periods until $I(t)$ reaches the desired initial condition, that is, $I(t) = 10^{-5}$. If at this t we have $|D(t)| < 10^{-5}$ stop. Otherwise, adjust guess $\mathcal{D}_i$.

Associated Figures

This section in the paper is associated with Figures 8 and 9, which uses the parameters described in Table E.6.

Table E.6: Behavioral response parameters - Figures 8, 9 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.0062, 0.0062$
Δt	$1/5$
ξ	$0.05/(365 \times \Delta t)$

The initial guess used in the code Figure 8 is $\mathcal{D}_i = 0.0022$, and the initial guess for Figure 9 is $\mathcal{D}_i = 0.004$.

E.4 Behavioral Responses - Asymmetric Case

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and $T = 10{,}000$. Guess $D_1(\infty) = \mathcal{D}_1$. Fix $I_1(T) = 10^{-7}$.

2. Generate a grid for $D_2(\infty) = \mathcal{D}_2$ wide enough to contain the solution (use solution without behavioral responses as an upper bound for this guess). For each of the points in this grid

   (a) Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = (c\,(\sigma-1)\,\mu_{ij})^{-1/(\phi-1)}\,(d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii}w_iL_i(1-\mathcal{D}_i) + \pi_{ji}w_jL_j(1-\mathcal{D}_j) = w_iL_i(1-\mathcal{D}_i),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell \in \mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell \in \mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} (d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{i\ell} w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

(b) Transversality conditions are satisfied if

$$\lim_{t\to\infty} \theta_i^k(t) = 0$$
$$\lim_{t\to\infty} \theta_i^i(t) = 0$$
$$\lim_{t\to\infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = \theta_i^s(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - [Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))]\, e^{-\xi t} \Delta t$$
$$\theta_i^i(t) = \frac{1}{1+(\gamma_i+\eta_i)\Delta t}[\eta_i \theta_i^k(t)\Delta t + \theta_i^i(t+1)]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

(c) Guess a value for $I_2(T)$. Set $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i+\eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}}\right](1-\mathcal{D}_i)e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)] S_i(T) \alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

i. Given a value for $I_2(T)$, for each $t \in [T, 0]$ solve the following system of equations, where all values evaluated at $t+1$ are known, to obtain values at t:

$$\theta_i^s(t+1) - \theta_i^s(t) = [\theta_i^s(t) - \theta_i^i(t)][2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)]\Delta t$$
$$\theta_i^s(t+1) - \theta_i^i(t) = (\gamma_i + \eta_i)\theta_i^i(t)\Delta t - \eta_i \theta_i^k(t)\Delta t$$
$$\theta_k^s(t+1) - \theta_i^k(t) = [Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t))]\, e^{-\xi t} \Delta t$$
$$I_i(t+1) - I_i(t) = S_i(t)[2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)]\Delta t - (\gamma_i + \eta_i) I_i(t)\Delta t$$
$$S_i(t+1) - S_i(t) = -S_i(t)[2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)]\Delta t$$
$$D_i(t+1) - D_i(t) = \eta_i I_i(t)\Delta t$$

and where $n_{i\cdot}(t)$ is again obtained as the value that solves:

$$\left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}}\right](1-D_i(t))e^{-\xi t} = [\theta_i^s(t)-\theta_i^i(t)]S_i(t)\alpha_j I_j(t).$$

These correspond to equations (32)-(35) in the paper plus the equations determining the evolution of the epidemiological variables, once we have imposed equilibrium conditions.

ii. Given a particular grid, two adjacent guesses of $\mathcal{D}_2$ may lead to diverging paths for I_i. If this is the case, pick the two guesses that split the paths between those diverging upwards and downwards and re-draw a finer grid for $\mathcal{D}_2$ within these bounds.

iii. Repeat for all periods until $I_i(t)$ reaches the desired initial condition, that is, $I(t) = 10^{-5}$ and $I_i(t) < I_i(t+1)$ in a flat line (meaning it does not diverge to plus or minus infinity). If at this t we have $D_1(t) = D_2(t)$ go back to outside layer of the loop. Otherwise, adjust guess $I_2(T)$.

3. If at this t we have $|D_i(t)| < 10^{-5}$ stop. Otherwise, adjust guess $\mathcal{D}_1$.

Associated Figures

This section in the paper is associated with Figure 10, which uses the parameters described in Table E.7.

Table E.7: Behavioral response parameters - Figure 10 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.003, 0.0062$
Δt	1/3
ξ	$0.05/(365 \times \Delta t)$

Notes about the Algorithm

This algorithm is not closed, as it still requires a mechanism that will automatically define which are the bounds for $\mathcal{D}_2$ in step 2(c)ii.

E.5 Adjustment Costs and the Risk of a Pandemic

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and T = 10,000. Guess $D(\infty) = \mathcal{D}_i$.

2. Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma-1\right)\mu_{ij}\right)^{-1/(\phi-1)}\left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}}\left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$
$$\pi_{ii}w_iL_i(1-\mathcal{D}_i) + \pi_{ji}w_jL_j(1-\mathcal{D}_j) = w_iL_i(1-\mathcal{D}_i),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} (d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{i\ell}w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

3. Transversality conditions are satisfied if

$$\lim_{t\to\infty} \theta_i^k(t) = 0$$
$$\lim_{t\to\infty} \theta_i^i(t) = 0$$
$$\lim_{t\to\infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - \left[Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))\right] e^{-\xi t}\Delta t$$
$$\theta_i^i(t) = \frac{1}{1+(\gamma_i+\eta_i)\Delta t}\left[\eta_i\theta_i^k(t)\Delta t + \theta_i^i(t+1)\right]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

4. Set $I_i(T) = 10^{-7}$, $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i+\eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}}\right](1-\mathcal{D}_i)e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)]S_i(T)\alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

5. For each $\tau - 1 \in [T, 0]$ solve the following system of equations, where all values evaluated at τ are known and we have imposed perfect symmetry between countries, to obtain values at t:

$$\begin{aligned}
\theta^s(\tau) - \theta^s(\tau-1) &= [\theta^s(\tau) - \theta^i(\tau)][2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau)I(\tau)]\Delta\tau \\
\theta^s(\tau) - \theta^s(\tau-1) &= (\gamma+\eta)\theta^i(\tau)\Delta\tau - \eta\theta^k(\tau)\Delta\tau \\
\theta^k(\tau) - \theta^k(\tau-1) &= \left[Q(n_j(\tau), n_j(\tau)) - C(n_i(\tau), n_j(\tau)) - \psi_1(|g_{ii}\,(t)|^{\psi_2} + |g_{ij}\,(t)|^{\psi_2})\right] e^{-\xi\tau}\Delta\tau \\
I(\tau) - I(\tau-1) &= S(\tau)[2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau)I(\tau)]\Delta\tau - (\gamma+\eta)I(\tau)\Delta\tau \\
S(\tau) - S(\tau-1) &= -S(\tau)[2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau))I(\tau)]\Delta\tau \\
D(\tau) - D(\tau-1) &= \eta I(\tau)\Delta\tau
\end{aligned}$$

and where $n_{i\cdot}(\tau)$ is obtained as $n_{i\cdot}(\tau+1) - g_{i\cdot}(\tau) \times \Delta t$ for the value of $g_{i\cdot}(\tau)$ that solves:

$$\begin{aligned}
& e^{-\xi\tau}\left[\frac{\partial Q_i}{\partial n_{ij}}(n_{ij}(\tau)) - \frac{\partial C_i}{\partial n_{ij}}(n_{ij}(\tau))\right] \times (1 - D(\tau)) \\
& + \sum_{t=\tau+1}^{\infty} e^{-\xi t}\left[\frac{\partial Q_i}{\partial n_{ij}}(n_{ij}(t)) - \frac{\partial C_i}{\partial n_{ij}}(n_{ij}(t))\right] \times (1 - D(t)) \\
& - (\theta^s(\tau) - \theta^i(\tau)) \times S(\tau) \times \alpha \times I(\tau) - \sum_{t=\tau+1}^{\infty} (\theta^s(t) - \theta^i(t)) \times S(t) \times \alpha \times I(t) \\
& - \psi_1\psi_2 |\frac{n_{ij}(\tau+1) - n_{ij}(\tau)}{\Delta\tau}|^{\psi_2 - 1} \times (1 - D(\tau))e^{-\xi\tau} \\
& = 0.
\end{aligned}$$

Note that, in contrast to the other cases above, we compute changes as happening between τ and $\tau - 1$, rather $\tau + 1$ and τ. This makes the system easier to solve backwards, although the difference in solutions is negligible for small enough step size.

6. Repeat for all periods until $I(\tau)$ reaches the desired initial condition, that is, $I(\tau) = 10^{-5}$. If at this τ we have $D(\tau) = 0$ stop. Otherwise, adjust guess $\mathcal{D}_i$.

Associated Figures

This section in the paper is associated with Figure 11, which uses the parameters described in Table E.8.

Table E.8: Behavioral response parameters - Figure 10 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.0062, 0.0062$
ξ	$0.05/(365 \times \Delta t)$
ψ_1	1
ψ_2	4
Δt	$1/10$