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## **THE SOCIAL VALUE OF DEBT IN THE MARKET FOR CORPORATE CONTROL**

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## Abstract

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# The Social Value of Debt in the Market for Corporate Control\*

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## Abstract

How should bidders finance tender offers when the objective of the takeover is to improve incentives? In such a setting, debt finance has benefits even when bidders have deep pockets: It amplifies incentive gains, imposes *Pareto* sharing on bidders and free-riding target shareholders, and makes bidding competition more efficient. High leverage, independent of financing needs, can be privately and socially optimal. Although takeover debt dilutes target shareholders, they may benefit most from it, especially when bidding is competitive.

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First, LBOs changed the incentives of managers by providing them with substantial equity stakes . . . Second, the high amount of debt . . . imposed strong financial discipline on company management . . . Third, leveraged buyout sponsors . . . governed the companies they purchased. The boards of the LBO companies were small and dominated by investors with substantial equity stakes. (Holmstrom and Kaplan, 2001, p.128)

“Leverage” refers to the fact that the company being purchased is forced to pay for . . . its own acquisition. . . If this sounds like an odd arrangement, that’s because it is. (Kosman, 2012, para.8).

## 1 Introduction

Many view the takeover wave of the 1980s, in retrospect, as a watershed in the history of corporate governance (Shleifer and Vishny, 1990; Holmstrom and Kaplan, 2001). It manifested Manne (1965)’s vision of a market for corporate control where takeovers address incentive conflicts, giving birth to the private equity (PE) industry that has since become a pillar of corporate governance. A controversial characteristic of these takeovers is how highly leveraged they are, at times imposing debt-to-capital ratios of up to 90 percent on the target companies. Why this is so and whether this is desirable has been the subject of much debate among policy-makers and academics.

This question is again receiving increased attention as the role of PE funds in the economy is growing. By some estimates, PE firms in the U.S. managed up to \$7 trillion in assets in 2018, and in some industries, like retail, nearly all recent bankruptcy events involved PE-owned companies (Appelbaum and Batt, 2018; Scigliuzzo, Butler, and Bakewell, 2019). Such developments have rattled policy-makers, prompting regulatory initiatives in the European Union and the “Stop Wall Street Looting Act” sponsored by U.S. Senator Elizabeth Warren.<sup>1</sup>

PE advocates endorse buyouts as efficient transactions that create high-powered incentives through leverage and concentrated ownership. Critics argue that PE firms benefit from leverage at the expense of other stakeholders in the firms and externalize the downside, and that such rent-seeking generates inefficiently high leverage. What drives buyout leverage at the firm level matters also at the macroeconomic level if the aggregate debt taken on by the PE industry engenders systemic risks. In this paper, we set out to study this “incentives or rent-seeking” question theoretically.

**Incentive-benefit theory.** According to the dominant theory (pioneered by Jensen, 1986), debt serves to improve incentives. Debt causes equity to be levered and allows

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<sup>1</sup><https://www.congress.gov/bill/116th-congress/senate-bill/2155>.

it to be concentrated, which creates high-powered incentives and discipline to pay out free cash flow. This incentive-benefit theory is persuasive—but because it views debt only as optimizing the *post-takeover* ownership and capital structure, it suffers from a few caveats:

First, the theory does not require the takeover itself to be leveraged. The bidder can raise debt after (and in management buyouts even before) the takeover. Second, even if it were a matter of convenience to leverage the takeover, that leverage should depend on similar variables as corporate leverage in general; but this prediction lacks empirical support (Axelsson et al., 2013). Third and relatedly, the theory cannot explain why buyout leverage tends to be higher than general corporate leverage.

The first caveat is not merely nitpicking or semantics. While the incentive-benefit theory and empirical studies have rebutted many initial objections to leveraged buyouts, a persistent concern relates to the *way* the debt is raised (as opposed to only the debt levels): *bidders* raise funds by collateralizing *target* assets. Kosman (2009, p.3) compares this so-called bootstrapping to mortgage finance with the twist that “while we pay our mortgages, PE firms had the companies they bought take the loans, making *them* responsible for repayment.” A testimony by Eileen Appelbaum<sup>2</sup> in a recent congressional hearing summarizes why this practice is contentious (America for Sale? An Examination of the Practices of Private Funds, 2019, p.4):

[I]t is the company, not the PE fund that owns it, that is obligated to repay this debt . . . The [PE] firm will lose at most its equity investment in the portfolio company, and often this has already been repaid via fees the PE firm collects from the company. The PE firm has little to no skin in the game; it’s the company . . . that the use of leverage . . . put at risk.

The point she denounces is that bootstrapping grants bidders *limited liability* with respect to the takeover debt. This poses a conundrum for incentive-benefit theory, as standard principal-agent theory considers limited liability detrimental to incentives. Indeed, issuing deal-by-deal debt at the PE fund level or through intermediate holding companies would give PE firms, in Appelbaum’s words, more “skin in the game.” Bootstrapping insinuates a less benign motivation for the debt and largely underlies the predatory image of “raiders.”

**Rent-extraction theory.** Müller and Panunzi (2004) justify bootstrapping. Unlike the incentive-benefit theory, they focus on frictions in the takeover *process*. Following Grossman and Hart (1980), they examine how free-riding behavior among dispersed

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<sup>2</sup>Appelbaum is co-director of the Center for Economic Policy Research (CEPR) and co-author of *Private Equity at Work: When Wall Street Manages Main Street*, which was a finalist in 2016 for the Academy of Management’s George R. Terry Book Award.

target shareholders can render takeover bids unprofitable, and show that this friction creates a role for takeover debt: Bootstrapping dilutes target shareholders who retain their shares by the amount the bidder receives from lenders for her bid. This shifts rents to bidders, thus allowing them to overcome the free-rider problem.

The “eureka” effect here is to see that bootstrapping can be necessary and socially valuable. Still, the theory does not dispel normative concerns. Conditional on a bid, bootstrapping seems to be at best redistributive. Indeed, Müller and Panunzi (2004, Section VI.B) caution that, in light of bankruptcy costs, the rent-extraction objective tempts bidders to go for inefficiently high leverage. This resonates with Appelbaum’s criticism and could warrant a legal cap on bootstrapping, forcing bidders to raise any (incentive-motivated) additional debt in ways that do not afford the rent-extraction benefit.

That being said, the theory seems at odds with the empirical evidence: The logic that takeover debt maps into bidder profit does not square with the fact that takeover gains in the 1980s went primarily to the target shareholders (e.g., Jarrell et al., 1988). The theory further predicts that bidder competition reduces takeover leverage. One explanation for the drop in bidder returns in the late 1980s is increased competition (Holmstrom and Kaplan, 2001, p.128). Yet, debt levels did not fall and, if anything, became less sustainable (Kaplan and Stein, 1993; Andrade and Kaplan, 1998).<sup>3</sup>

**Current paper.** We integrate the two aforementioned perspectives that (i) buyouts aim to improve incentives in the (post-takeover) target firms and (ii) leverage can be used to extract rents. Our theory adds to the Grossman and Hart (1980) framework a pre-bid stage in which the bidder can choose financing and a post-takeover stage in which the incentives to create value depend on the ownership and capital structure. We abstract from wealth constraints, and so financing is purely driven by frictions, not endowments.

We show that integrating the incentive-benefit and rent-extraction arguments can tie up the different loose ends discussed above, providing both a stronger normative justification for bootstrapping and predictions that better match empirical patterns. On the one hand, the rent-extraction motive requires that the takeover itself is leveraged and causes demand for debt to exceed that in conventional (i.e., non-takeover) financing situations. On the other hand, the incentive effects entail that target share-

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<sup>3</sup>Müller and Panunzi remark on the difficulty of explaining such patterns with their model:

“[A] minimal amount of debt equal to the raider’s transaction cost might be sufficient to ensure that the takeover takes place. Indeed, if debt is costly and the raider’s profit is limited due to bidding competition, it is precisely this minimal amount of debt that is optimal. Hence, while our model provides a role for debt in takeovers, it cannot explain LBO-style debt levels.” (Müller and Panunzi, 2004, p.1220).

holders *gain* from *more* takeover debt (under reasonable conditions) and that bidder competition *increases* the levels of takeover debt. As a result, high takeover leverage can be *Pareto-dominant* and can *predominantly* benefit target shareholders. Caps on bootstrapping strictly reduce welfare in our model.

These conclusions are driven by three benefits of debt financing that arise despite our abstracting from outside financing needs:

- I. **Efficiency gains.** Without means to extract (part of) the value improvement, a bidder does not gain from improving incentives by acquiring target shares. While she can extract rents upfront by pledging post-takeover value to lenders, lenders only participate if she is trusted to create enough value. *To obtain more debt, she must hence concentrate more equity to improve her incentives.* Thus, the bidder’s demand for takeover debt leads to greater incentive improvements. Or putting it more provocatively, bootstrapping increases value.
- II. **Pareto gains.** The bidder’s debt capacity—and therefore her ability to extract rents—is incentive-constrained: she must leave a wedge between debt and firm value such that equity is sufficiently “in the money” to avoid a debt overhang. This wedge is the post-takeover share value and thus what she must pay target shareholders in the tender offer. The wedge increases with debt under common formulations of the incentive problem, *in which case debt constraints act as a Pareto sharing rule.*
- III. **Efficient competition.** On the one hand, since takeover debt enables bidders to better recoup the costs of improving value, it raises their reservation prices. *This makes competition fiercer.* On the other hand, insofar as free-riding target shareholders share in the additional value induced by leverage, the debt level that maximizes a bidder’s profit is generally below her maximum debt capacity. *Competition forces bidders to employ more than their privately optimal level of debt,* which increases efficiency and benefits target shareholders.

Our analysis connects the literature on the incentive benefits of debt (à la [Jensen \(1986\)](#) or [Innes \(1990\)](#)) with the literature on free-riding in tender offers (particularly, [Burkart et al. \(1998\)](#) and [Müller and Panunzi \(2004\)](#)). It is therefore informative to explain the novelty of the above three results in the context of these literatures.

**Contribution to the incentive theory of leveraged buyouts.** The role of debt in the standard theory of buyouts is the same as in standard capital structure theory: achieving the second-best outcome under wealth constraints.<sup>4</sup> Limiting wealth in our

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<sup>4</sup>In our model, bidders are capable of establishing the first-best post-takeover incentive structure by self-financing the entire takeover.



model would add this role to debt, but that additional constraint is slack when the optimal rent-extraction level of debt exceeds the need for outside financing. This can explain systematic differences between buyout leverage and corporate leverage.

In fact, as the bidder bootstraps target assets to extract rents (rather than being in need of cash), her own financing contribution can easily be *negative* in our model. That is, the funds she raises from outside investors can exceed the total consideration she pays target shareholders. By contrast, in standard models, wealth constraints are binding: the bidder optimally uses as much of her own funds (as little outside funds) as possible. Negative financing contributions do not signify a “free lunch.” Outside financing is conditional on the bidder receiving an equity stake that incentivizes her to create value. Thus, whether or not she invests financial capital, the rents she extracts are returns to *human* capital. In a sense, her limited liability with respect to the debt is analogous to that of managers.

Debt has a novel incentive role in our model: to qualify for more debt (to extract rents), the bidder must increase her equity stake to prevent a debt overhang problem. This interplay between debt overhang (Myers, 1977) and equity concentration (Jensen and Meckling, 1976) generates an ownership-leverage relationship that pins down the (required) bidder incentives for every (feasible) debt level. This relationship governs how the bidder balances (i) leveraging up to “raid” the target against (ii) managerial incentives and debt-equity conflicts in the post-takeover firm.

It is known that leveraged buyouts significantly increase the management’s equity exposure and that active shareholders with large equity stakes come to dominate the board of directors (Kaplan, 1989). The leverage-ownership relationship in our model implies that the post-buyout ownership of such insiders increases with buyout debt. By contrast, equity is always fully concentrated in classic incentive theories of debt (e.g., Innes, 1990).

**Contribution to the literature on tender offers.** Following Grossman and Hart (1980), the key question in this literature is whether free-riding undermines takeovers of widely held firms. Since free-riding is a failure of target shareholders to collectively bargain with bidders for mutual benefit, Grossman and Hart argue that bidders need means to “unilaterally” exclude target shareholders from part of the takeover gains. Identifying exclusion mechanisms is an underlying theme of this literature. The main known mechanisms are dilution (Grossman and Hart, 1980), toeholds (Shleifer and Vishny, 1986; Kyle and Vila, 1991), and leverage (Müller and Panunzi, 2004).<sup>5</sup> There are few comparisons since the mechanisms are economically equivalent in standard

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<sup>5</sup>Freeze-out mergers offer an alternative mechanism (Yarrow, 1985; Amihud et al., 2004), but this mechanism is not robust to legal or strategic uncertainty (Müller and Panunzi, 2004; Dalkir, Dalkir, and Levit, 2019).

tender offer models. To the contrary, it is typically emphasized how *similar* they are (e.g., Müller and Panunzi, 2004; Burkart and Lee, 2015).

The Achilles heel of exclusion mechanisms is that they harm target shareholders conditional on a bid. Target shareholders therefore prefer limits on exclusion even if those deter some takeover bids—such as disclosure rules that limit bidders’ ability to buy toeholds or investor protection laws that limit controlling shareholders’ power to expropriate minority shareholders. We show that takeover leverage may be uniquely exempt from this caveat, because it is not a “unilateral” mechanism.

Debt finance requires lender participation. In the presence of incentive problems, this creates financing constraints. These constraints impose a limit on exclusion and thus a “sharing rule” between bidders and target shareholders—despite the latter’s failure to bargain. By generating incentive gains *and* a sharing rule, acquisition debt *benefits* target shareholders even conditional on a bid (under conditions that we show to be satisfied in standard specifications of the incentive problem). In short, takeover leverage can simultaneously *improve incentives* and *neutralize the free-rider problem*. We would argue that this is why debt is crucial to the functioning of the market for corporate control.

Our analysis marries the frameworks of Burkart, Gromb, and Panunzi (1998) and Müller and Panunzi (2004). Allowing for takeover leverage qualifies Burkart, Gromb, and Panunzi’s conclusion that the bidder acquires the smallest possible equity stake. Endogenizing the post-takeover firm value qualifies Müller and Panunzi’s conclusion that takeover leverage harms target shareholders and therefore decreases with bidder competition. We note that the extension in Section 6 of the discussion paper version (Müller and Panunzi, 2003), not included in the published article, is closely related to our analysis. It shows that moral hazard constrains debt financing but does not trace out the ramifications for efficiency, surplus division, and bidder competition.

**Other papers.** Burkart, Gromb, Müller, and Panunzi (2014) examine how investor protection laws impact (the financing of) tender offers by wealth-constrained bidders. Axelsson, Stromberg, and Weissbach (2009) study optimal contracts between private equity firms and passive capital providers (limited partners). Malenko and Malenko (2015) propose an explanation for the high levels of buyout leverage based on buyout firms’ reputational concerns vis-à-vis lenders. Last, several papers explore the role of debt in bidding contests. The are discussed in Section 3.3.

## 2 Financing Incentive-Improving Tender Offers

We present a simple tender offer model with financing in which the *source* of takeover gains is an improvement in incentives while the *distribution* of those gains is subject to the free-rider problem. To our knowledge, this is the first tender offer model in the tradition of Grossman and Hart (1980) in which debt and (outside) equity financing both play a critical role.

### 2.1 Model

**Source of takeover gains.** Consider a widely held firm (“target”) facing a single potential acquirer (“bidder”). If the bidder acquires control, she can generate a value improvement  $V(e) \geq 0$ , relative to the value under the current management, which is normalized to zero. Generating value requires unobservable effort  $e$ , which imposes a private cost  $C(e)$  on the bidder.

We assume that the value improvement function  $V$  is linear in effort,  $V(e) = \theta e$ , where  $\theta$  is the marginal return to effort. The cost function  $C$  is twice differentiable, strictly increasing, and strictly convex, i.e.,  $C'(e) > 0$  and  $C''(e) > 0$  for all  $e \geq 0$ . We further assume  $C(0) = 0$ ,  $\lim_{e \rightarrow 0} C'(e) = 0$ , and  $\lim_{e \rightarrow +\infty} C'(e) = +\infty$  to restrict attention to takeovers that (would) have strictly positive but finite value. Both the value improvement function and the cost function are common knowledge, but effort is unobservable. Our focus on linear  $V$  is without loss of generality in that all results can be directly translated to concave value improvement functions.<sup>6</sup>

We can alternatively model the post-takeover incentive problem as private benefit extraction (as in Burkart et al., 1998). This would add a source of bidder gains without altering the key insights; the incentive role of takeover debt would be to decrease extraction. Also, while we model only *post*-takeover effort, all findings apply equally to efforts in preparation of a bid (such as assessing target suitability and identifying potential value improvements) as long as effort is unobservable.

**Division of takeover gains.** To gain control, the bidder must purchase at least half of the target shares by way of a tender offer. The incumbent management is assumed to be unwilling or unable to counterbid; alternatively, it may be part of the investor group that makes the offer to buy out the current shareholders.

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<sup>6</sup>Suppose  $V : [0, +\infty) \rightarrow \mathbb{R}$  is a twice differentiable, strictly increasing, and concave function. The game we consider is isomorphic to a game in which the bidder, instead of choosing  $e$ , chooses  $y$  where  $\theta y = V(e)$ . In the latter game, the bidder’s post-takeover objective function is  $\alpha[\theta y - D]^+ - C(V^{-1}(\theta y))$ , where  $V^{-1}$  denotes the inverse function of  $V$ . Since the inverse of a strictly increasing, strictly concave function is a strictly increasing, strictly convex function, the composition  $C \circ V^{-1}$  satisfies the assumptions postulated for  $C$  in our model.

Each target shareholder is non-pivotal for the tender offer outcome. The resulting free-riding behavior frustrates the takeover unless the bidder has means to “exclude” target shareholders from part of the post-takeover value (Grossman and Hart, 1980). We focus on the exclusion mechanism identified by Müller and Panunzi (2004): debt collateralized with target assets. Since debt is senior, shareholders are excluded from future cash flow pledged to the lenders, while the bidder extracts the present value of those cash flows in the form of a loan prior to the bid.

Specifically, we allow the bidder, though wealth-unconstrained, to involve outside funding for the bid in the form of debt and equity. She can choose to pledge a fraction  $(1 - \gamma) \in [0, 1]$  of the cash flow from the acquired target shares to outside investors in exchange for some amount  $F^E$  of equity financing. Similarly, she can promise outside creditors a debt repayment  $D \geq 0$  in exchange for some amount  $F^D$  of debt financing. We normalize pre-takeover firm value and leverage as well as discount rates to zero. We abstract from exclusion mechanisms other than debt. So a profitable bid requires that  $F^D > 0$  and that the debt is raised through a *bootstrap* acquisition. It is without loss of generality to ignore “non-bootstrapped” debt in our model.

**Sequence of events.** Our model has three stages. In stage 1, the bidder makes a take-it-or-leave-it cash bid to acquire target shares at a price  $p$  per share and chooses how to finance the bid. The financing is publicly observable. The bid is conditional, that is, it becomes void if less than half of the shares are tendered.

In stage 2, target shareholders non-cooperatively decide whether to tender their shares. The shareholders are homogeneous and atomistic such that no one is pivotal. Specifically, we assume a unit mass of shares dispersed among an infinite number of shareholders whose individual holdings are equal and indivisible.<sup>7</sup> Shareholder  $i$ 's tendering strategy maps the offer terms into a probability that she tenders her shares,  $\beta_i : (\gamma, F^E, D, F^D, p) \rightarrow [0, 1]$ . Without loss of generality, we will focus on symmetric strategies and omit the index  $i$ . So, by the law of large numbers,  $\beta$  shares are traded in a successful bid.

In stage 3, if less than half the shares are tendered, the takeover fails. Otherwise, the bidder pays  $\beta p$  for the fraction  $\beta$  of shares tendered and obtains control. Net of the fraction  $\gamma$  financed by outside investors, the bidder then owns the “inside” equity stake  $\alpha \equiv \gamma\beta$ , and chooses her effort level  $e \geq 0$  to maximize her post-takeover payoff  $U(\alpha, D, e)$ . So, her post-takeover strategy is a function  $e : (\alpha, D) \rightarrow \mathbb{R}^+$ . Finally, the firm value and all payoffs are realized (see Figure 1).

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<sup>7</sup>These assumptions are standard in tender offer models exploring the free-rider problem. When they are relaxed, Grossman and Hart (1980)'s result that target shareholders extract all the gains in security benefits becomes diluted (Holmström and Nalebuff, 1992).

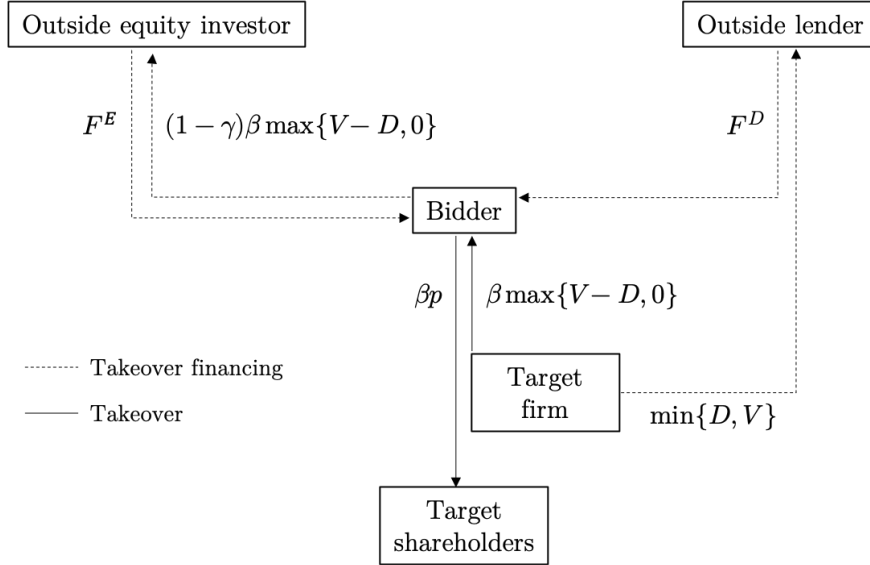


Figure 1: This summarizes the payments vis-à-vis a successful bidder in our model. Consider a management buyout as illustration: Incumbent managers and a buyout firm together are the “bidder,” limited partners in the buyout fund are the “outside equity investor,” and bondholders or a loan syndicate are the “outside lender.” The fact that debt funds are disbursed to the bidder but repaid directly by the target firm is the key effect of bootstrapping.

**Interpretation** A leveraged buyout, whether hostile or management-led, is usually carried out by a group of investors that may comprise incumbent management and a private equity firm, or a consortium of private equity firms. These investors take not only large equity positions<sup>8</sup> in the target firm but also active roles in management or the board after the takeover (Kaplan and Stromberg, 2009, p.130f). They are represented by the “bidder” in our model.

Private equity firms raise equity funding for the buyouts through private equity funds. This funding is typically provided by institutional investors, such as pension funds, endowments, and insurance companies (Kaplan and Stromberg, 2009, p.123f). These so-called *limited partners*—unlike the private equity firms who are known as *general partners*—do not take on an active role in the post-takeover firms. They can be thought of as the “outside equity investor” in our model.

When a specific buyout deal materializes, private equity firms contribute some of the capital from the private equity funds as equity to finance the buyout. This equity financing is further complemented with debt financing from banks or bond investors.

<sup>8</sup>In the case of private equity firms, part of the equity exposure comes from the “carried interest” they earn when their private equity funds perform well. We abstract from this compensation feature in our model.

The debt makes up the lion’s share of the financing, covering 60 to 90 percent of the acquisition price (Kaplan and Stromberg, 2009, p.124f). The third parties providing the debt financing are the “outside lender” in our model.

Unlike the equity, the debt is raised at *deal* (rather than fund) level. This allows it to be collateralized by the assets of the *target firm* through a bootstrap acquisition (see Müller and Panunzi, 2004, Section II). In a first step, a shell company is created and funded from the aforementioned sources of buyout financing. This shell company makes a bid to acquire a majority of the target shares. If successful, the second step is to legally merge the target firm with the shell company such that the former’s assets are matched with the latter’s debt. Consequently, all equity investors receive, in our model notation, fractions of  $[V(e) - D]^+$ . Without the second step, “shell company” shareholders and “target” shareholders would, respectively, receive  $[\beta V(e) - D]^+$  and  $(1 - \beta)V(e)$  instead.

The equity stake of the *active* investor group in the merged company is a function of the fraction  $1 - \gamma$  of outside equity financing and the equity share  $\beta$  tendered by the original target shareholders:  $\alpha = \gamma\beta$ . With the sole exception of  $\alpha = 1$ , for every  $\alpha < 1$ , the roles of  $\beta$  and  $\gamma$  are somewhat interchangeable; though, a given  $\gamma$  implies a lower bound on  $\alpha$ , namely  $\alpha \geq \gamma/2$ , since a successful takeover requires  $\beta \geq 1/2$ . Still, since  $\gamma \in [0, 1]$ , the bidder can implement any  $\alpha \in [0, 1]$  in our model. In practice, all initial shareholders are bought out in public-to-private buyouts ( $\beta = 1$ ), whereas in cash-outs, selling shareholders retain shares in the post-takeover firm ( $\beta < 1$ ). While the various real-world cases are subsumed in our model, distinguishing between them is not important as only  $\alpha$  matters for our results.

We should note that our model admits takeovers with  $\alpha \rightarrow 0$ . In fact, for  $D \rightarrow 0$ , the optimal  $\alpha$  converges to 0. Also, we allow  $\alpha$  to be fully optimized at the deal level, while it is in practice to some degree pre-determined by the financial structure of the private equity funds. These modelling choices are a matter of analytical convenience. Empirically, the median equity stake of the post-takeover management team is about 16 percent (Kaplan and Stromberg, 2009, p.131), which excludes the equity stake and carried interest of the private equity firm. The model implication that little debt pushes the optimal  $\alpha$  (and hence the value improvement) to 0 is best interpreted to the effect that a buyout without leverage is not lucrative.

## 2.2 Equilibrium

We derive the equilibrium by backward induction in three subsections corresponding to the stages of the game. We focus on the bidder’s equity stake  $\alpha$  and takeover debt  $D$ , which characterize the post-takeover ownership and capital structure. Unlike in

typical financing models, there are no wealth constraints that call for outside funds. Financing decisions are purely driven by the interaction between financing frictions, tendering decisions, and effort choice.

### 2.2.1 Effort choice

After a successful bid, the bidder's equity stake is  $\alpha$  and the target firm assumes the acquisition debt (of face value)  $D$ . The bidder then chooses effort  $e$  to maximize the value of her equity stake in the levered firm net of private effort costs,  $U(\alpha, D, e) \equiv \alpha[V(e) - D]^+ - C(e)$ .

This objective function is not globally concave in  $e$ . Let  $e_D$  satisfy  $V(e_D) = D$ . For  $e \in [0, e_D)$ , equity is “out of the money” because  $V(e) < D$ , and so  $U(\alpha, D, e) = -C(e)$  which is strictly decreasing in  $e$ . For  $e \geq e_D$ ,  $U(\alpha, D, e) = \alpha[V(e) - D] - C(e)$  since equity is “in the money.” Under our assumptions about  $V$  and  $C$ , this is strictly concave and the first-order condition,  $\alpha V'(e) = C'(e)$ , has a unique, strictly positive solution, hereafter denoted by  $e^+(\alpha)$ .

Because  $U(\alpha, D, e)$  is not globally concave,  $e^+(\alpha)$  need not be a global optimum. Specifically, given that  $\frac{\partial U}{\partial e} < 0$  for  $e \in [0, e_D)$ , it is possible that  $U(\alpha, D, e^+(\alpha)) < 0$ . If so, the bidder's optimal effort is  $e = 0$ . To summarize the above arguments:

**Lemma 1.** *The bidder's optimal effort is  $e^*(\alpha, D) = e^+(\alpha) > 0$  if*

$$\alpha[V(e^+(\alpha)) - D] - C(e^+(\alpha)) \geq 0 \quad (1)$$

where  $e^+(\alpha)$  is the solution to

$$\alpha V'(e^+(\alpha)) = C'(e^+(\alpha)) \quad (2)$$

Otherwise, she makes no effort to improve value, i.e.,  $e^*(\alpha, D) = 0$ .

Lemma 1 replicates established wisdom within our takeover setting. Outside debt can lead to a debt overhang that undermines a (controlling) shareholder's incentives to improve firm value (Myers, 1977). Here, this occurs when condition (1) is violated. Outside equity dilutes the incentives of “inside” shareholders to increase firm value (Jensen and Meckling, 1976). Firm value thus increases in ownership concentration. Indeed, conditional on (1), effort  $e^+(\alpha)$  and firm value  $V(e^+(\alpha))$  are increasing in  $\alpha$  (by the envelope theorem).

The novel element of Lemma 1 is that these two effects interact in condition (1). Whether a debt overhang problem emerges depends not only on the debt level  $D$  but also on the level of ownership concentration  $\alpha$ . The intuition is simple: The bidder's

incentives derive from a *levered equity stake*  $\alpha[V(e^+(\alpha)) - D]$ . While  $D$  lowers the total value of equity,  $\alpha$  determines the bidder's share of that total value. This has the implication that a given debt level is less likely to undermine the bidder's incentives if she owns more equity. Or put differently, a firm with more concentrated ownership can sustain a higher level of (incentive-compatible) debt. This interaction between  $\alpha$  and  $D$  will be crucial.

### 2.2.2 Tendering decisions

As Lemma 1 indicates, the first-best structure is fully concentrated ownership and no debt, i.e.,  $(\alpha, D) = (1, 0)$ .<sup>9</sup> An ideal market for corporate control would restore this structure. We discuss next how free-riding behavior by dispersed target shareholders distorts bidders' preferences regarding  $\alpha$  and  $D$ .

Suppose target shareholders face a cash bid  $p$  (partially) financed with debt  $D$ . Being non-pivotal, an individual shareholder  $i$  tenders only if  $p \geq V(e^*(\hat{\alpha}_i, D))$  where  $\hat{\alpha}_i$  denotes  $i$ 's belief about the bidder's post-takeover equity stake. Because tendering decisions depend on individual beliefs, no dominant-strategy equilibrium exists. In a rational expectations equilibrium, beliefs are consistent with the outcome, so shareholders tender only if

$$p \geq [V(e^*(\alpha, D)) - D]^+. \quad (3)$$

That is, target shareholders tender their shares only if they extract (at least) the full increase in share value that the bidder will generate. This is known as the free-rider condition.

Previous work has analyzed two special cases of (3). Müller and Panunzi (2004) study a model with exogenous post-takeover values where (3) becomes  $p \geq (V - D)^+$ . In this setting, the bidder wants to maximize  $D$ . In contrast, Burkart, Gromb, and Panunzi (1998) consider endogenous post-takeover values but abstract from debt. In this case, (3) reduces to  $p \geq V(e^*(\alpha, 0))$ , and the bidder wants to minimize  $\alpha$ . Both cases highlight that the bidder seeks to decrease the right-hand side of (3)—i.e., the post-takeover share value—which target shareholders extract through the bid price. We show that the more general case, in which  $D$  and  $\alpha$  are jointly chosen, overturns the predictions derived from these special cases.

Before we characterize the stage-2 subgame equilibrium, note that (3) is merely a necessary condition for a successful bid; a failed bid, in which an insufficient number of shares is tendered, can always be supported as a self-fulfilling equilibrium outcome.

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<sup>9</sup>This is the only structure that leads to the first-best outcome for every admissible specification of  $V$  and  $C$ . For any  $D > 0$ , there exist admissible  $V$  and  $C$  such that (1) is violated.



To focus on the interesting case, we assume that shareholders always tender when the free-rider condition is weakly satisfied, thereby selecting the Pareto-dominant success equilibrium whenever it exists.

Denote the post-takeover share value that the bidder will create for a given stake  $\alpha$  and debt  $D$  by  $E(\alpha, D)$  and her *equilibrium* post-takeover equity stake by  $\alpha^*(p, D)$ . Since a successful bid implies that  $\beta \in [1/2, 1]$  shares are tendered, the bidder's post-takeover stake  $\alpha$  lies in the interval  $[\gamma/2, \gamma]$  for a given outside equity financing share  $1 - \gamma$ . Hence, the post-takeover share value must lie between  $E(\gamma/2, D)$  and  $E(\gamma, D)$ . In the subsequent lemma, we omit describing the subgame equilibrium for bids that can be ruled out a priori: bids that fail for any set of beliefs ( $p < E(\gamma/2, D)$ ) and bids that could be undercut without affecting any other decision ( $p > E(\gamma, D)$ ).

**Lemma 2.** *Any bid  $p \in [E(\gamma/2, D), E(\gamma, D)]$  succeeds, and  $\alpha^*(p, D) = \alpha_p$  where  $\alpha_p$  satisfies  $p = E(\alpha_p, D)$ .*

*Proof.* For every  $p \in [E(\gamma/2, D), E(\gamma, D)]$ , there exists a unique  $\alpha_p \in [\gamma/2, \gamma]$  such that  $E(\alpha_p, D) = p$ . Every shareholder tenders for  $\hat{\alpha}_i < \alpha_p$ , retains her shares for  $\hat{\alpha} > \alpha_p$ , and is indifferent between tendering and retaining for  $\hat{\alpha} = \alpha_p$ .  $\square$

Target shareholders are willing to sell shares until the post-takeover share value, which increases with the bidder's stake, reaches the bid price. As in [Burkart, Gromb, and Panunzi \(1998\)](#), supply is hence upward-sloping: the fraction of shares tendered increases with the price. In equilibrium, the bidder ends up with the stake for which the free-rider condition (3) holds with equality.<sup>10</sup>

### 2.2.3 Bid and financing

The bidder's ex ante profit is  $\alpha E(\alpha_p, D) - \beta p - C(e) + F^E + F^D$ . It comprises the value of the equity stake she expects to acquire, less effort cost and takeover payment, and outside funds she raises for the bid. She maximizes this by choosing the bid  $p$ , outside equity financing  $\{\gamma, F^E\}$ , and debt financing  $\{D, F^D\}$  subject to (1), (2), (3), and the following participation constraints: Outside equity investors demand

$$F^E \leq \beta(1 - \gamma)E(\alpha_p, D). \quad (4)$$

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<sup>10</sup>Though the outcome is pinned down, the equilibrium strategy profile is not necessarily unique. The outcome obtains when each shareholder tenders with probability  $\beta_p \equiv \alpha_p/\gamma$ , but also when mass  $\beta_p$  of shareholders tenders with certainty while all others keep their shares.

Outside lenders demand  $F^D \leq \min[D, V(e)]$  which, since debt overhang constraint (1) requires  $V(e) > D$ , reduces to

$$F^D \leq D. \quad (5)$$

We assume perfect competition among outside financiers such that they merely break even. Hence, (4) and (5) hold with equality. Substituting these binding participation constraints in the bidder's ex ante profit yields  $\beta[E(\alpha, D) - p] - C(e) + D$ .

Recall from Lemma 2 that free-rider condition (3) is endogenously binding; target shareholders tender  $\alpha_p$  shares such that  $E(\alpha_p, D) = p$ . Recall further from Lemma 1 that, conditional on (1), post-takeover effort is  $e^+(\alpha)$ , which satisfies (2). To demarcate the new element of our analysis from existing results, we first state how these constraints—binding free-rider condition (3) and first-order condition (2) for effort—affect the bidder. Plugging these constraints into her ex ante profit gives

$$D - C(e^+(\alpha)). \quad (6)$$

This replicates the known insights that debt  $D$  enables the bidder to extract private gains and that a larger equity stake  $\alpha$  is unattractive because it induces her to incur higher effort costs, while all gains in share value accrue to target shareholders. This also shows that the bidder's ex ante problem essentially reduces to choosing the post-takeover ownership and capital structure  $(\alpha, D)$ .<sup>11</sup>

The new element is the joint restriction that debt overhang constraint (1) imposes on  $D$  and  $\alpha$ . This constraint cannot be slack at the optimum. Otherwise, the bidder could lower  $\alpha$  while preserving  $D$ . This would increase her profit, as (6) shows. Using the binding constraint (1) to replace  $D$  in (6) collapses the bidder's stage-0 choices to a univariate optimization problem:

$$\max_{\alpha \in [1/2, 1]} V(e^+(\alpha)) - C(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}. \quad (P)$$

In Section 3, we use this representation of the problem to study the role of debt. Before doing so, we conclude this section by establishing equilibrium existence (though not uniqueness).

**Lemma 3.** *If the bidder's profit under (P) is negative, she makes no bid. Otherwise, she succeeds with a bid such that (1)-(5) bind and  $\alpha$  solves (P).*

<sup>11</sup>This is why it is without loss of generality to abstract from cash-equity bids and restricted bids. The same objective function obtains (i) for cash-equity bids with  $1 - \alpha$  being the fraction of post-takeover equity offered to target shareholders as payment combined with cash or (ii) for cash bids in which the number of shares the bidder offers to acquire is restricted to  $\alpha$ .

*Proof.* The objective function is continuous in  $\alpha$  and its domain is compact. Hence there exists an  $\alpha \in [1/2, 1]$  that solves (P). If the profit under this solution is positive, the bidder makes a successful bid. Otherwise, she abstains from a takeover.  $\square$

### 3 Roles of Debt in Control Reallocation

This section presents our main results. In interpreting these results, it is important to bear in mind that our model abstracts from wealth constraints; the bidder is able to implement the first-best (post-takeover) incentive structure by fully self-financing a takeover, with no need for debt. What keeps her from doing so are frictions in the takeover process. Our results hence speak to how takeover debt improves the process of reallocating control from dispersed shareholders to bidders, as opposed to only the resultant control allocation. The next subsections identify in turn three effects that debt has on this process.

#### 3.1 Ownership-leverage relationship

We begin by considering the effect of takeover debt on total surplus. In our model, the social surplus created by a successful takeover is  $\mathcal{W}(\alpha) \equiv V(e^+(\alpha)) - C(e^+(\alpha))$ . While this expression depends only on the bidder's post-takeover equity stake  $\alpha$ , the latter is linked to debt  $D$  through debt overhang constraint (1), which is binding in equilibrium. Solving the binding constraint for  $D$  yields

$$D = V(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}. \quad (1^*)$$

As shown in the proof of the next result,  $D$  as defined in (1\*) is a strictly increasing function of  $\alpha \in [1/2, 1]$ . This *ownership-leverage function* has the following intuition: for a bid to remain feasible, a higher debt level  $D$  calls for a larger bidder stake  $\alpha$  to avoid a debt overhang problem. The larger  $\alpha$  leads in turn to a higher surplus  $\mathcal{W}(\alpha)$ .

**Proposition 1.** *Takeover debt increases total takeover gains.*

*Proof.* Section B of the Appendix.  $\square$

Proposition 1 is somewhat surprising because the primary role of takeover debt is to shift rents from target shareholders to bidders. Indeed, in Müller and Panunzi (2004), increasing leverage conditional on a bid is purely redistributive, and socially inefficient in their model extension with exogenous bankruptcy costs. Proposition 1 differs because the post-takeover value improvement in our model depends on bidder incentives. This incentive problem creates two countervailing effects.

On the equity side, the fact that owning a larger stake creates stronger incentives to create value is a *disincentive* to buy shares when faced with the free-rider problem. While the bidder is more incentivized to provide costly effort when acquiring a larger stake, target shareholders extract the resulting value increase through the bid price. All else equal, the bidder therefore prefers low  $\alpha$  so that a takeover at best partially restores incentives in the target firm.

On the debt side, lenders that are concerned about the bidder’s incentives restrict how much they are willing to lend. By the ownership-leverage function, the bidder’s debt capacity increases in her equity stake: To obtain a larger loan, she must commit to generate more value. Acquiring more equity, which mitigates the agency problem, is that commitment. To the extent that the bidder wants to raise debt, this need for commitment prevails over the aforementioned preference for low  $\alpha$ . (In Appendix E.2 we provide an example in which leverage restores first-best incentives in our model.)

This is a notable qualification of the prediction in [Burkart, Gromb, and Panunzi \(1998\)](#) that bidders buy as few shares as needed to gain control when value creation is endogenous, and replaces it with the following prediction: Post-takeover ownership concentration increases with takeover debt, which in turn increases total gains.<sup>12</sup> It is known that, after buyouts, executives own more equity and active investors dominate the boards ([Kaplan, 1989](#)). According to our theory, ownership by such post-buyout “insiders” increases with buyout debt. Also, total buyout gains increase with buyout debt, but this link should be weaker when post-buyout insider ownership is controlled for.

Another interesting model implication is that the bidder’s financing contribution can be *negative* even though she receives a post-takeover equity stake. In fact, it can be that an increase in debt increases the equity stake she receives—while reducing her financing contribution. This does not, however, signify a “free lunch.” Any profit she earns is compensation for effort she invests into improving the firm value. Indeed, a positive equity stake coupled with a negative financing contribution is best compared to a linear compensation contract that gives the bidder equity incentives plus a cash salary. In this case, leveraged buyouts amount to “hiring” the bidder as new management, or to “realigning” the compensation of incumbent management if the latter is involved in the buyout. (We provide examples in Appendix E.)

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<sup>12</sup>By contrast, in an incentive theory with wealth constraints à la [Innes \(1990\)](#), the “insider” or bidder *always* ends up with 100 percent of the equity. Consider a bidder financed by outside equity and debt. By [Innes \(1990\)](#), swapping outside debt for outside equity increases both incentives and pledgeable income. Such a swap is, therefore, always feasible and profitable.

### 3.2 Debt constraint as sharing rule

We now examine how the surplus  $\mathcal{W}(\alpha)$  is divided between the bidder and the target shareholders. This too is determined by the ownership-leverage function (1\*). Note that (1\*) implies an equity value of  $V(e^+(\alpha)) - D = \frac{C^+(\alpha)}{\alpha}$ , where  $C^+(\alpha) \equiv C(e^+(\alpha))$  for brevity. Furthermore, note that the bidder's profit function in (P) can be written as

$$\mathcal{W}(\alpha) = \frac{C^+(\alpha)}{\alpha}.$$

As is characteristic of the free-rider problem, her profit is the total surplus less the post-takeover equity value, which target shareholders extract through the bid price. In our model, this equity value is determined by (1\*): It is the wedge the bidder has to leave between firm value and debt such that equity is sufficiently “in the money” to prevent a debt overhang problem. How the wedge  $\frac{C^+(\alpha)}{\alpha}$  varies with  $\alpha$  determines how the increase in  $\mathcal{W}(\alpha)$  induced by more debt is divided between the bidder and the target shareholders.

There are two countervailing effects. On one hand, if we hold the effort cost (the numerator) fixed, the wedge decreases in  $\alpha$ . Equity incentives depend on total equity value and equity concentration, which creates a form of “incentive substitutability”: a controlling shareholder with a larger stake  $\alpha$  can reduce total equity value  $V(\bar{e}) - D$  more without creating a debt overhang problem.

On the other hand, subject to (1\*), the optimal “in-the-money” effort  $e^+(\alpha)$  and associated cost  $C^+(\alpha)$  increase in  $\alpha$ . Intuitively, if one requires the bidder to acquire a larger equity stake  $\alpha$  as an incentive for her to improve firm value more, any accompanying increase in debt  $D$  must avoid discouraging the *higher* effort necessary for a larger improvement. This effect causes the wedge to increase in  $\alpha$ .

If the second effect is so strong that  $\frac{\partial C^+(\hat{\alpha})}{\partial \alpha} \geq 0$  at a given  $\hat{\alpha}$ , target shareholders weakly benefit from a marginal increase in takeover debt. Whether this is the case, given a linear value improvement function, depends on the shape of the cost function at  $e^+(\hat{\alpha})$ .<sup>13</sup>

**Lemma 4.**  $\frac{\partial C^+(\alpha)}{\partial \alpha} \geq 0$  if and only if  $\frac{C'(e^+(\alpha))}{C''(e^+(\alpha))} \geq \frac{C(e^+(\alpha))}{C'(e^+(\alpha))}$ .

*Proof.* Section C of the Appendix. □

This condition is not very restrictive. For example, every weakly log-concave cost function satisfies this condition *globally* (i.e., at all  $e$ ). This includes such commonly applied functional families as power functions  $C(e) = \frac{c}{n}e^n$  and exponential functions

<sup>13</sup>More generally, if permitting concave value improvement functions, it depends on the concavity of the post-takeover objective function.

$C(e) = \exp(e) - c$ . In some cases the gains to target shareholders are strictly positive (i.e., the condition in Lemma 4 holds strictly). So bidders' use of takeover debt can be *Pareto*-improving. Empirically, this amounts to the following prediction:

**Proposition 2.** *Takeover debt can increase the bid price.*

*Proof.* Section C of the Appendix. □

This too stands in stark contrast to the predictions in Müller and Panunzi (2004) where takeover leverage is a pure wealth transfer and *strictly decreases* the bid price. Like Proposition 1, Proposition 2 is a consequence of the post-takeover value depending on incentives, which creates an agency problem between bidder and lenders. This agency problem limits the bidder's debt capacity and thus the gains she can extract. In other words, the financing constraint imposes a "sharing rule" that splits the surplus between bidder and target shareholders.

It is also instructive to contrast this result with the role the debt constraint plays in incentive-based models with wealth constraints. A standard model focuses on how debt capacity  $D$  measures up against the need for outside funds. In our model, which abstracts from wealth constraints, the incentive problem is paired with the free-rider problem instead. As mentioned, the focus here is on how debt capacity  $D$  determines what the bidder extracts from the takeover and the residual value  $V - D$  she has to "leave on the table" to target shareholders.

In our view, Proposition 2 is significant in two ways: First, it sets debt apart from other exclusion mechanisms. Whether exclusion mechanisms restore takeovers as an efficient governance instrument is a central theme in the tender offer literature. The problem is that target shareholders prefer limits on exclusion, even if those prevent some takeovers, because it redistributes takeover gains away from them. Proposition 2 says that this caveat need not apply to takeover leverage when taking into account that value creation is endogenous. This endows debt with a unique potential to overcome the free-rider problem and restore the market for corporate control.

Second, Proposition 2 can reconcile the notion of debt as an exclusion mechanism with the empirical association of high takeover leverage and high target shareholder returns, such as during the 1980s takeover wave (Jensen, 1988). The fact that this is achieved by incorporating incentives—the key element of standard buyout theories—into the model makes this result particularly appealing. (Appendix E.1 provides examples with very high leverage ratios that benefit target shareholders.) In the next subsection we show that bidder competition reinforces the positive link between takeover debt and target (shareholder) returns.

### 3.3 Leveraging competition

Consider two competing bidders  $i \in \{1, 2\}$ . To win control of the target, either bidder must outbid her rival with an offer satisfying the free-rider condition. We present two results in this setting: (i) the winner's use of debt depends on the loser's reservation price and (ii) reservation prices are higher thanks to debt.

#### 3.3.1 The effect of competition on debt

We must preface this analysis with a corollary of Proposition 2: If target shareholders capture part of the additional gains induced by takeover debt, it need not be optimal for a (single) bidder to demand the maximum feasible debt amount. Let  $D(\alpha)$  denote the strictly increasing ownership-leverage function (1\*). A single bidder chooses the pair  $(\alpha, D(\alpha))$  that maximizes  $\mathcal{W}(\alpha) - \frac{C^+(\alpha)}{\alpha}$  (Section 3.2). Her maximum feasible debt level is given by the largest  $(\alpha, D(\alpha))$ -values for which  $\mathcal{W}(\alpha) - \frac{C^+(\alpha)}{\alpha} = 0$  or by the corner values  $(1, D(1))$ . Hence, the bidder does not generally max out on debt if the solution to (P) is  $\alpha^* < 1$  (except in the knife-edge case where she exactly breaks even with  $\alpha^*$ ).

This begs the question whether a bidder increases or decreases debt in response to competition. Without loss of generality, let bidder 1 have the higher reservation price and win the contest. Let  $\bar{p}_2$  denote bidder 2's reservation price. Under competition, bidder 1's stage-1 optimization problem features the added constraint that she must outbid her rival:

$$p_1 \geq \bar{p}_2 \tag{7}$$

Denote her profit-maximizing bid in the absence of competition by  $(\alpha_1^*, D(\alpha_1^*))$ , with the bid price  $p_1^*$ . Assume  $\alpha_1^* < 1$ . We restrict our attention to effective competition, that is,  $\bar{p}_2 > p_1^*$ . Otherwise, (7) is irrelevant.

Suppose bidder 1's offer matches bidder 2's reservation price so that (7) is strictly binding:  $p_1 = \bar{p}_2$ .<sup>14</sup> Focusing on interior solutions, where bidder 1 gets  $\alpha < 1$  shares, recall from Lemma 2 that free-rider condition (3) is, endogenously, strictly binding. (We analyze the corner solution  $\alpha = 1$  in the proof of the next result.) Solving the binding free-rider condition with  $p_1 = \bar{p}_2$  for  $D$  yields

$$D = V(e^+(\alpha)) - \bar{p}_2. \tag{8}$$

This represents debt as a function of ownership concentration. We henceforth denote

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<sup>14</sup>Since the objective function in (P) can be non-monotonic in  $\alpha$ , there are cases in which bidder 1 prefers to offer *strictly* more than  $\bar{p}_2$ . The arguments that follow in the text also apply to those cases, with  $\bar{p}_2$  replaced by  $\bar{p}_2^+ = \bar{p}_2 + \Delta$  for some  $\Delta > 0$ .

this function as  $D^c(\alpha)$ . It represents all  $(\alpha, D)$  for which free-rider condition (3) and competition constraint (7) strictly bind. By contrast,  $D(\alpha)$  represents all  $(\alpha, D)$  for which debt overhang constraint (1) strictly binds.

Like  $D(\alpha)$ ,  $D^c(\alpha)$  is strictly increasing but for an entirely different reason: It tells us that bidder 1 can create a given post-takeover share value—i.e., target shareholder wealth—by combining *either* high debt levels with high firm values (induced by high bidder stakes) *or* low debt levels with low firm values (induced by low bidder stakes).

Since all  $(\alpha, D)$  defined by  $D^c(\alpha)$  yield the same payoff  $\bar{p}_2$  for target shareholders, bidder 1's profit subject to (8) is

$$\mathcal{W}(\alpha) - \bar{p}_2$$

which strictly increases with  $\alpha$ . It is thus optimal to match a rival using the highest  $(\alpha, D^c(\alpha))$ -pair that is *feasible*, that is, does not violate the debt overhang constraint—which is given by the highest  $\alpha$  satisfying  $D^c(\alpha) \leq D(\alpha)$ , hereafter denoted as  $\alpha_1^{**}$ . If the previous inequality is slack at the no-competition optimum  $\alpha_1^*$ , then  $\alpha_1^{**} > \alpha_1^*$ . This is indeed the case:  $D^c(\alpha_1^*) = V(e^+(\alpha_1^*)) - \bar{p}_2 < V(e^+(\alpha_1^*)) - p_1^* = D(\alpha_1^*)$  with the the inequality following from effective competition ( $\bar{p}_2 > p_1^*$ ).

**Proposition 3.** *Fiercer competition leads to higher levels of takeover debt and larger post-takeover target firm values.*

*Proof.* Section D of the Appendix. □

The intuition is simply that, conditional on conceding a *given* target shareholder wealth  $\bar{p}_2$ , a bidder is best off maximizing takeover surplus. To this end, she increases her incentives to improve post-takeover firm value by raising  $\alpha$ , while also increasing  $D$  to extract enough of the additional gains to keep post-takeover share value at  $\bar{p}_2$ —until debt overhang constraint (1) binds (or she reaches  $\alpha = 1$ ).

Proposition 3 also stands out in light of existing theories. In Müller and Panunzi (2004), the post-takeover value is exogenous and competition reduces takeover debt. In incentive-based models with wealth constraints, competition increases a bidder's outside financing need, which pushes her away from the *second-best* (post-takeover) incentive structure. In our model, bidders generally do *not* aim for the strongest feasible (post-takeover) incentive structure due to the free-rider problem. Competition forces them to pursue more high-powered incentive structures to remain competitive, which necessitates more leverage.

The impact of bidder competition on profits is the conventional one: The added constraint (7), when binding, reduces bidder profits. Given overall surplus increases,



target shareholders gain. Hence, contrary to Müller and Panunzi (2004), Proposition 3 reconciles (i) bidder competition with high takeover leverage and (ii) high takeover leverage with low bidder returns, consistent with the following account of the 1980s buyout wave (Holmstrom and Kaplan, 2001, p.128f):

The leveraged buyout experience was different in the latter half of the 1980s. Roughly one-third of the leveraged buyouts completed after 1985 subsequently defaulted on their debt, some spectacularly...

But even for the late 1980s, the evidence is supportive of the efficiency story. The reason for the defaults was not that profits didn't improve, but that they didn't improve by enough to pay off the enormous quantities of debt that had been taken on...

The likely answer is that the success of the LBOs of the early 1980s attracted entrants and capital... As a result, much of the benefit of the improved discipline, incentives, and governance accrued to the selling shareholders rather than to the post-buyout LBO investors. The combined gains remained positive, but the distribution changed.

At the extreme in our model, if the bidders are equally competitive, the winner raises her maximum feasible debt amount but *all* of the surplus goes to target shareholders—even though the debt serves as an exclusion mechanism.

### 3.3.2 The effect of debt on competition

We now examine how a bidder's reservation price depends on her own use of takeover debt. Recall from Section 2.2.3 that, subject to the first-order condition for optimal effort (2) and free-rider condition (3) strictly binding, the bidder's profit is reduced to  $D - C(e^+(\alpha))$ . Instead of considering the debt level that maximizes this profit for a given  $\alpha$ —i.e., at which debt overhang constraint (1) strictly binds—consider now the debt level at which the bidder merely breaks even:

$$D = C(e^+(\alpha)). \tag{9}$$

This also describes debt as a strictly increasing function of ownership concentration, hereafter denoted by  $D^0(\alpha)$ . It tells us the most the bidder may borrow for a given equity purchase to avoid profits, or conversely, the most equity she can buy for given debt to avoid losses. The implied share value is the break-even price for a given  $\alpha$ :  $p^0(\alpha) = V(e^+(\alpha)) - C(e^+(\alpha)) = \mathcal{W}(\alpha)$ . Because  $\mathcal{W}'(\alpha) > 0$ , the reservation price is the break-even price under the highest feasible  $(\alpha, D^0(\alpha))$ -pair. The latter is pinned down by the highest  $\alpha$  that satisfies  $D^0(\alpha) \leq D(\alpha)$ , hereafter denoted by  $\bar{\alpha}$ .

To state the importance of takeover debt for a bidder’s reservation price, consider the impact of an *exogenous* debt limit  $D \leq \bar{D}$ . If  $\bar{D} < D^0(\bar{\alpha})$ , the debt limit reduces the reservation price to  $p^0(\bar{\alpha})$  where  $\bar{\alpha}$  solves (9) for the debt level  $\bar{D}$ . When applied to bidder 2 in our competition setting, this argument implies the following result.

**Proposition 4.** *Takeover debt makes competition fiercer.*

The purpose of debt under the break-even condition (9) is to compensate the bidder for her costs. The ability to recover costs is essential to the bidder’s willingness to generate surplus, and the maximum surplus she is willing to generate determines her reservation price. To sum up Propositions 3 and 4: Debt makes bidders more competitive and the increased competition forces winners to take on higher, more efficient levels of debt—to the benefit of target shareholders.

The *pro*-competitive impact of debt in our setting stands out against the existing literature. In Chowdhry and Nanda (1993), bidders use debt as a commitment to bid aggressively in order to deter entry by potential competitors. Extending results from Hansen (1985) and Rhodes-Kropf and Viswanathan (2000), DeMarzo, Kremer, and Skrzypacz (2005) show that bidders prefer to compete by leaving sellers with “flatter securities” (like debt or cash) because it reduces expected seller revenues.

## 4 Conclusion

The question of why firms use debt and how much they should use is one of the classic questions in finance. Although much is by now understood, the question still sparks debate in areas where leverage seems “excessive,” such as in the financial sector or in leveraged buyouts. When the latter first emerged in the 1980s, the U.S. Senate held hearings during which then-SEC Chairman Alan Greenspan cautioned ([Leveraged Buyouts and Corporate Debt, 1989](#), p.17),

[T]he extent of the leverage involved is worrisome, in the sense that while one may say the restructuring is a plus, how it is financed is a different question and something which I find disturbing . . . If, for example, all of this restructuring were done with equity, rather than leveraged buyouts, I frankly would feel considerably more comfortable.

The narrative that prevailed is that leveraging the takeover optimizes the incentives with which the firm is managed after the takeover. Although persuasive, this theory leaves several questions open: If implementing optimal capital structure, why are the leverage ratios in buyouts so much *higher* than in firms, at times reaching up to 90

percent of total capital? If debt serves to discipline and incentivize those who control the post-takeover firm, why do bidders raise it in such a manner as to *eschew* liability (“bootstrapping”)? Unanswered, these questions stir up the suspicion that the debt serves a less benign purpose. Such criticism is resurfacing as private equity activity is growing (Kosman, 2009; Appelbaum and Batt, 2014), echoing Greenspan’s unease.

In this paper, we offer answers to these questions by merging the incentive theory of leveraged buyouts with the literature on the free-rider problem in tender offers. In this integrated framework, “excessive” levels of takeover debt (beyond funding needs) are Pareto-dominant outcomes. While this does not refute that buyout leverage can have a dark side, it offers an argument for why highly leveraged *bootstrap* acquisitions may be a socially efficient transaction design. According to our theory, *takeover* debt is special in that it simultaneously improves incentives *and* counteracts free-riding. This suggests that debt is crucial to a functioning market for corporate control and may explain why leverage plays such an outsize role in buyouts, especially in public-to-private transactions.

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## Appendix

### A Auxiliary results

For reference, we state the following result from one variable calculus (e.g., [Rudin \(1964, p. 114\)](#)):

**Lemma A.1.** *Let  $f : (0, +\infty) \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) > 0$  for all  $x \in (0, +\infty)$ . Then  $f$  is strictly increasing on  $(0, +\infty)$  and has a differentiable inverse function  $g$  with*

$$g'(f(x)) = \frac{1}{f'(x)}$$

for all  $x \in (0, +\infty)$ . If  $f : (0, +\infty) \rightarrow \mathbb{R}$  is twice differentiable and such that  $f''(x) > 0$  for all  $x \in (0, +\infty)$  then its inverse  $g$  is also twice differentiable and we have

$$g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

for all  $x \in (0, +\infty)$ .

We now derive two auxiliary results.

**Lemma A.2.** *There is a unique differentiable function  $e : [1/2, 1] \rightarrow \mathbb{R}_{\geq 0}$  such that  $\alpha V'(e(\alpha)) = C'(e(\alpha))$  for all  $\alpha \in [1/2, 1]$  and such that  $e'(\alpha) > 0$  for all  $\alpha \in (1/2, 1)$ . If moreover  $C'''(e)$  exists for all  $e > 0$ , then  $e$  is twice differentiable.*

*Proof.* Define a function  $H : (0, +\infty) \rightarrow \mathbb{R}$  by  $H(e) = \frac{C'(e)}{\theta}$ . Clearly

$$H'(e) = \frac{C''(e)}{\theta} > 0$$

for all  $e > 0$  by our assumption that  $C''(e) > 0$  for all  $e \geq 0$ . Thus  $H$  satisfies the premises of Lemma A.1, and hence there is a differentiable function  $G$  such that  $G(H(e)) = e$  for all  $e > 0$  and  $H(G(y)) = y$  for all  $y$  in the range of  $H$ . From our assumptions  $\lim_{e \rightarrow 0} C'(e) = 0$  and  $\lim_{e \rightarrow +\infty} C'(e) = +\infty$  and the fact that  $H$  is continuous, it follows that  $[1/2, 1]$  is a subset of the range of  $H$ , i.e.,  $[1/2, 1] \subseteq H((0, +\infty))$ . Hence we may define  $e : [1/2, 1] \rightarrow (0, +\infty)$  by  $e(\alpha) := G(\alpha)$  for all  $\alpha \in [1/2, 1]$ . Then  $\frac{C'(e(\alpha))}{\theta} = H(e(\alpha)) = H(G(\alpha)) = \alpha$  for all  $\alpha \in [1/2, 1]$  and the first part of the claim follows. Let  $\alpha \in (1/2, 1)$  and  $e > 0$  be such that  $H(e) = \alpha$ , applying Lemma A.1 once again then yields

$$e'(\alpha) = e'(H(e)) = \frac{1}{H'(e)} = \frac{\theta}{C''(e)} > 0.$$

Moreover if  $C$  is thrice differentiable we have that

$$e''(\alpha) = e''(H(e)) = -\frac{H''(e)}{(H'(e))^3} = -\theta^2 \frac{C'''(e)}{[C''(e)]^3}.$$

□

## B Proof of Proposition 1

Equation (1\*) defines the equilibrium debt level  $\bar{D}(\alpha) \equiv V(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}$ . Now,

$$\begin{aligned} \bar{D}'(\alpha) &= V'(e^+(\alpha))e^{+'}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))e^{+'}(\alpha) \\ &= (V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha)))e^{+'}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) \\ &= \frac{1}{\alpha^2}C(e^+(\alpha)) > 0. \end{aligned}$$

The third equality holds because  $\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$  by (2). The fact that  $\bar{D}(\alpha)$  is strictly increasing implies the same for its inverse function. Last, note that

$W(\alpha)$  is strictly increasing in  $\alpha$  with the first-best outcome being attained for  $\alpha = 1$ .

## C Proof of Proposition 2

*Target shareholder gains.* Target shareholders benefit from higher  $\alpha$  if

$$\begin{aligned} \frac{d}{d\alpha} \frac{C(e^+(\alpha))}{\alpha} &= \frac{C'(e^+(\alpha))e^{+'}(\alpha)}{\alpha} - \frac{C(e^+(\alpha))}{\alpha^2} \\ &= \frac{\theta}{\alpha} \left[ \frac{C'(e^+(\alpha))}{C''(e^+(\alpha))} - \frac{C(e^+(\alpha))}{C'(e^+(\alpha))} \right] \geq 0. \end{aligned}$$

The second equality holds by Lemma A.1, whereby if  $e^+(\alpha) > 0$ , then  $e^{+'}(\alpha) = \frac{\theta}{C''(e^+(\alpha))}$ . A sufficient condition for the inequality to hold (globally) is log-concavity of  $C$ , i.e.,  $C(e)C'''(e) \leq [C''(e)]^2$  for all  $e > 0$ . Power functions satisfy this property.

*Bidder gains.* The bidder's profit,  $\pi^B(\alpha) \equiv V(e^+(\alpha)) - [1 + \frac{1}{r}] C(e^+(\alpha))$ , is strictly increasing in  $\alpha$  if

$$\begin{aligned} \frac{d\pi^B(\alpha)}{d\alpha} &= V'(e^+(\alpha))e^{+'}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))e^{+'}(\alpha) - C'(e^+(\alpha))e^{+'}(\alpha) \\ &= \left[ V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha)) \right] e^{+'}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - C'(e^+(\alpha))e^{+'}(\alpha) \\ &= \frac{1}{\alpha^2}C(e^+(\alpha)) - C'(e^+(\alpha))e^{+'}(\alpha) \\ &= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))\theta}{C''(e^+(\alpha))} \\ &= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))C'(e^+(\alpha))}{C''(e^+(\alpha))\alpha} \\ &= \frac{1}{\alpha} \left( \frac{C(e^+(\alpha))}{\alpha} - \frac{[C'(e^+(\alpha))]^2}{C''(e^+(\alpha))} \right) > 0 \end{aligned}$$

The second equality is obtained by rearranging terms. The third equality holds since  $\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$  by (2). The fourth equality follows from Lemma A.2. The fifth equality holds because  $\alpha\theta = C'(e^+(\alpha))$  by (2). A sufficient condition for the last inequality to be satisfied (globally) is that

$$\frac{1}{\alpha} \left( \frac{C(e)}{\alpha} - \frac{[C'(e)]^2}{C''(e)} \right) > \frac{1}{\alpha} \left( C(e) - \frac{[C'(e)]^2}{C''(e)} \right) \geq 0$$

for all  $e > 0$ . The strict inequality holds for all  $\alpha < 1$ . The last weak inequality holds if  $C$  is log-convex, i.e., if  $C(e)C'''(e) \geq [C''(e)]^2$  for all  $e > 0$ . Exponential functions satisfy this property.

Appendix E uses the two families of functions identified above as examples for



which Pareto-improving use of debt occurs. It is worth emphasizing that the above sufficient conditions, which the examples satisfy, are stronger than needed for Pareto improvements to be feasible.

## D Proof of Proposition 3

The proposition is mainly proved in the text. What remains is (i) to analyze corner solutions and (ii) to prove, by iteration, that bidder 1's takeover debt *continuously* (weakly) increases with bidder 2's reservation price.

**Corner solution.** Suppose bidder 2's reservation price is such that, bidder 1 could match it using an offer that leads to  $\alpha = 1$ . At  $\alpha = 1$ , the free-rider condition can be slack in equilibrium. Still, because she buys all shares at a price equal to  $\bar{p}_2$ , her profit is  $\mathcal{W}(1) - \bar{p}_2$ , which equals the maximum value of the profit function  $\mathcal{W}(\alpha) - \bar{p}_2$  used in the arguments in the text. Thus, the result that bidder 2's presence increases bidder 1's takeover debt, if  $\alpha_1^* < 1$ , is valid also when the winning bid is a corner solution. Once in the corner solution, bidder 2 can meet any further increase in  $\bar{p}_2$  by decreasing debt but, equivalently, also by increasing  $p_1$  without a change in debt.

**Proof by iteration.** The arguments in the text establish that effective competition by bidder 2 increases bidder 1's takeover debt *relative to the single-bidder case*. Now suppose that, for a given  $\bar{p}_2$ , bidder 1's winning bid involves  $\alpha^{**} < 1$ . Would bidder 1 increase takeover debt further if bidder 2's reservation price increased to  $\bar{p}_2^\xi > \bar{p}_2$ ? One can show that this is the case by renaming  $\alpha^{**}$  as  $\alpha^*$ ,  $\bar{p}_2$  as  $p_1^*$ , and  $\bar{p}_2^\xi$  as  $\bar{p}_2$  and then reiterating the arguments in the text. Important to this iterative procedure is the observation that the debt overhang constraint strictly binds under any optimal non-corner winning bid; for example, if  $\alpha^{**} < 1$ , then  $D^c(\alpha^{**}) = D(\alpha^{**})$ .

## E Examples

**Example E.1** (Power functions). Let  $V(e) \equiv \theta e$  and  $C(e) \equiv \frac{c}{n}e^n$  where  $\theta > 0$ ,  $c > 0$  and  $n \in \mathbb{N}$  are exogenous parameters. These functions satisfy all our assumptions. It can also be shown that they generate unique solutions to (P) (proof available upon request). So, if the bidder's profit is positive under the solution to (P), there exists a unique  $\langle D, \alpha, p, e \rangle$  such that  $\alpha V'(e) = C'(e)$ ,  $p = V(e) - D$ ,  $\alpha D = \alpha V(e) - C(e)$ , and  $\alpha \in [1/2, 1]$  satisfying  $\alpha \in \{1/2, 1\}$  or the ex ante first-order condition for (P),

$$\frac{1}{\alpha^2}C(e^+(\alpha)) = C'(e^+(\alpha))e^{+'}(\alpha). \quad (\text{E.1})$$

The specific functional form allows us to express  $\langle D, \alpha, p, e \rangle$  in closed form. The first-order condition for effort  $\alpha V'(e) = C'(e)$  yields  $e = \left(\frac{\alpha\theta}{c}\right)^{\frac{1}{n-1}}$ . The equilibrium stake  $\alpha$  solves (E.1). One can show that this condition holds if and only if

$$\theta e^{+'}(\alpha) \left( \frac{n-1}{n} - \alpha \right) = 0,$$

which in turn holds if and only if  $\alpha = 0$  (since  $e^{+'}(0) = 0$ ) or  $\alpha = \frac{n-1}{n}$ . Of these, only  $\alpha = \frac{n-1}{n}$  is admissible as a solution to (P). It is straightforward to verify that

$$D = \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}$$

and

$$p = \frac{\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}.$$

Furthermore, the bidder's profit under the solution to (P) is positive since

$$\begin{aligned} D - C(e^+(\alpha)) &= \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} - \frac{(n-1)\theta}{n^2} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} \\ &= \theta \left( \frac{n-1}{n} \right)^2 \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} \geq 0. \end{aligned}$$

To sum up, there is a unique equilibrium in which

$$\langle D, \alpha, p, e \rangle = \left\langle \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}, \frac{n-1}{n}, \frac{\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}, \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} \right\rangle.$$

Given log-concavity of power functions for all  $n \in \mathbb{N}$ , (more) debt always increases the post-takeover share value and target shareholder wealth (cf. proof of Proposition 2). Equilibrium leverage can be very high. The debt-equity ratio is  $D/p = n - 1$ . So, for  $n = 5$ , the debt-equity ratio is 4, i.e., the debt-to-capital ratio is 80 percent. Note also that the bidder's financing contribution is negative,

$$\alpha p - D = \frac{(n-1)\theta}{n^2} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} - \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} < 0,$$

for all  $n > 1$ . ◁

**Example E.2** (Exponential functions.). Let  $V(e) \equiv \theta e$  and  $C(e) \equiv \exp(e)$  with  $\theta > \exp(2)$ . These functions satisfy all our assumptions, and can be shown to entail unique solutions to (P) (proof available upon request). If the bidder's profit is posi-

tive under (P), there is a unique  $\langle D, \alpha, p, e \rangle$  such that  $\alpha V'(e) = C'(e)$ ,  $p = V(e) - D$ ,  $\alpha D = \alpha V(e) - C(e)$ , and  $\alpha \in [1/2, 1]$  either satisfying the ex ante first-order condition (E.1) or  $\alpha \in \{1/2, 1\}$ . The post-takeover first-order condition  $\alpha V'(e) = C'(e)$  yields  $e^+(\alpha) = \ln(\alpha\theta)$ , which is strictly positive given  $\alpha\theta > \frac{\exp(2)}{2} > 1$ . Substituting  $e^+(\alpha)$  into the profit function of (P) yields

$$\theta \ln(\alpha\theta) - (1 + 1/\alpha)\alpha\theta.$$

Differentiating with respect to  $\alpha$  yields  $\theta(1/\alpha - 1)$ , which is strictly positive for all  $\alpha \in [1/2, 1)$ . Thus,  $\alpha = 1$  is the unique solution to (P). It is straightforward to verify that

$$D = \theta \ln(\theta) - \theta$$

and

$$p = \theta.$$

Furthermore, the bidder's profit is

$$D - C(e^+(1)) = \theta(\ln(\theta) - 2),$$

which is positive since  $\theta > \exp(2)$  implies  $\ln(\theta) > 2$ . To summarize, there is a unique equilibrium in which

$$\langle D, \alpha, p, e \rangle = \langle \theta \ln(\theta) - \theta, 1, \theta, \ln(\theta) \rangle.$$

Given the exponential function is weakly log-concave, the use of debt is weakly Pareto-improving. Note that  $\alpha = 1$  in equilibrium such that the first-best incentives in our model are restored. Note also that the equilibrium debt-equity ratio is  $D/p = \ln(\theta) - 1$ . For example, if  $\theta = \exp(5)$ , the debt-equity ratio is 4. The bidder's financing contribution is

$$\alpha p - D = 2\theta - \theta \ln(\theta).$$

This is negative for all  $\theta > \exp(2)$ . ◁