DISCUSSION PAPER SERIES

DP15239

HEDGING MACROECONOMIC AND FINANCIAL UNCERTAINTY AND VOLATILITY

Ian Dew-Becker, Stefano W Giglio and Bryan Kelly

FINANCIAL ECONOMICS

MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

HEDGING MACROECONOMIC AND FINANCIAL UNCERTAINTY AND VOLATILITY

Ian Dew-Becker, Stefano W Giglio and Bryan Kelly

Discussion Paper DP15239
Published 31 August 2020
Submitted 27 August 2020

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

HEDGING MACROECONOMIC AND FINANCIAL UNCERTAINTY AND VOLATILITY

Abstract

We study the pricing of shocks to uncertainty and volatility using a wide-ranging set of options contracts covering a variety of different markets. If uncertainty shocks are viewed as bad by investors, they should carry negative risk premia. Empirically, however, uncertainty risk premia are positive in most markets. Instead, it is the realization of large shocks to fundamentals that has historically carried a negative premium. In other words, we find that the return premium for gamma is negative while that for vega is positive. These results imply that it is jumps, for which exposure is measured by gamma, not forward-looking uncertainty shocks, measured by vega, that drive investors' marginal utility. In further support of the jump interpretation, the return patterns are more extreme for deeper out of the money options.

JEL Classification: N/A

Keywords: N/A

Ian Dew-Becker - ian.dewbecker@gmail.com
Northwestern University

Stefano W Giglio - stefano.giglio@yale.edu
Yale University and CEPR

Bryan Kelly - bryan.kelly@yale.edu
Yale

Hedging macroeconomic and financial uncertainty and volatility*

Ian Dew-Becker, Stefano Giglio, and Bryan Kelly

June 11, 2020

Abstract

We study the pricing of shocks to uncertainty and volatility using a wide-ranging set of options contracts covering a variety of different markets. If uncertainty shocks are viewed as bad by investors, they should carry negative risk premia. Empirically, however, uncertainty risk premia are positive in most markets. Instead, it is the *realization* of large shocks to fundamentals that has historically carried a negative premium. In other words, we find that the return premium for gamma is negative while that for vega is positive. These results imply that it is jumps, for which exposure is measured by gamma, not forward-looking uncertainty shocks, measured by vega, that drive investors' marginal utility. In further support of the jump interpretation, the return patterns are more extreme for deeper out of the money options.

1 Introduction

Background

It is well established that a wide range of measures of economic volatility and uncertainty vary over time. Uncertainty about all features of the aggregate economy, including productivity, the level of the stock market, inflation, interest rates, and energy prices, varies substantially, and often as the direct result of policy choices. It is therefore important to

*Dew-Becker: Northwestern University. Giglio: Yale University. Kelly: Yale University. We appreciate helpful comments from Dmitry Muravyev, Federico Gavazzoni, Nina Boyarchenko, Vito Gala, Alex Hsu, Ivan Shaliastovich, Emil Siriwardane, and seminar participants at Kellogg, CITE, Syracuse, Yale, the University of Illinois, the Federal Reserve Board, UT Austin, LBS, LSE, Columbia, Queen Mary, FIRS, WFA, INSEAD, SITE, the NBER, UIUC, the MFA, Temple, the AEA, UBC, the CBOE, the Federal Reserve Bank of Chicago, and the Macro Finance Society.

understand how uncertainty affects the economy, both to reveal the basic drivers of economic fluctuations and also to guide policymakers.

There are numerous theories, both in macro and finance, that explore the relationship between uncertainty and real activity. This literature highlights that causation runs in both directions, so even the sign of the relationship between the two is ambiguous in many cases.[1] The empirical literature studying uncertainty in macroeconomics has focused almost entirely on analyzing raw correlations or using vector autoregressions (VAR) with varying identifying assumptions, and thus far it has not resolved the question of whether uncertainty is contractionary in either the short- or long-run – i.e. whether uncertainty is typically good or bad.

Parallel to the macro literature, there is a long-running literature in finance that studies how uncertainty and volatility are priced in financial markets. That literature distinguishes between the pricing of shocks to uncertainty about the future – i.e. shocks to conditional variances or implied volatilities – and realized volatility, or the actual occurrence of jumps. Constantinides, Jackwerth, and Savov (2013) and Cremers, Halling, and Weinbaum (2015), for example, study the pricing of uncertainty and jump risk looking at option portfolios with different vega (implied volatility) and gamma (realized volatility or jump) exposure.

Contribution and Methods

This paper takes a *finance* approach to evaluating the effects of uncertainty shocks, building on the work of Constantinides, Jackwerth, and Savov (2013), Cremers, Halling, and Weinbaum (2015), and Dew-Becker et al. (2017). Instead of studying a VAR with all of the associated identification challenges, as in the macro literature, we use one of the key insights of the finance literature, that financial markets provide a direct window on how investors perceive shocks.[2] The main contribution of this paper relative to past work is to use options across a wide range of underlyings and maturities to measure the risk premia associated with shocks to uncertainty and to realized volatility. Those premia can furthermore be used to construct implied premia on shocks to major macro uncertainty indexes and hence shed light on the question of how uncertainty shocks affect the real economy.

If investors are willing to accept negative average returns on portfolios that hedge uncertainty shocks, as they would on an insurance contract, that implies that they view uncer-

[1]For example, see Schwert (1989), Caballero (1999), Bloom (2009), Schwert (2011), Pastor and Veronesi (2009), Bachmann and Moscarini (2012), and a summary discussion in Bloom et al. (2017) about the potentially expansionary effects of uncertainty shocks. In finance, see the finance literature on good and bad uncertainty, e.g. Bekaert, Engstrom, and Ermolov (2015) and Segal, Shaliastovich, and Yaron (2015).

[2]To be clear, the analysis of risk premia does not identify structural shocks; it only reveals the correlation of innovations in marginal utility with reduced-form innovations to uncertainty (since there is no structural identification here, we will use the terms "shock" and "innovation" interchangeably).

tainty as being bad in that it rises in high marginal utility states. On the other hand, if the hedging portfolios have positive average returns, then investors view uncertainty as typically rising in low marginal utility (good) states. So rather than running sophisticated regressions of output on uncertainty, we follow the finance tradition of letting investors speak to the question.

While there is a large literature that estimates the risk premia for uncertainty about the S&P 500 based on the pricing of options,[3] recent evidence shows that aggregate uncertainty has multiple dimensions beyond the financial uncertainty captured by the S&P 500 (Ludvigson, Ma, and Ng (2015); Baker, Bloom, and Davis (2015)). This paper contributes to the literature by estimating risk premia associated with uncertainty and realized volatility (jumps) in 19 different markets covering a range of features of the economy, including financial conditions, inflation, and the prices of real assets. The broad range allows the analysis to uncover consistent patterns in investors' attitudes to different types of uncertainty. We also use all the options together to construct hedging portfolios for aggregate uncertainty measures developed in the literature – in Jurado, Ludvigson, and Ng (JLN; 2015) and the economic policy uncertainty (EPU) index of Baker, Bloom, and Davis (2015). Fitting those indexes actually requires using more than just the S&P 500 – the results show that to span uncertainty about the real economy, it is important to have implied volatilities for real assets, like energies and metals, underscoring the value of the breadth of our dataset.

In each of the 19 markets, we construct straddles and strangles at maturities of one to five months, and measure two-week holding period returns. We show, both theoretically and empirically, that the different maturities have different loadings on the underlying risks, allowing estimation of risk premia using standard factor models. We examine risk premia for two types of shocks – to uncertainty, and to realized volatility (jumps). An uncertainty shock represents an increase in the dispersion of agents' conditional distribution for future outcomes, and an option's exposure to uncertainty shocks is measured (approximately) by its vega. The second shock is to the *realization* of large outcomes, i.e. exposure to realized volatility, or gamma (formally, exposure to squared returns).

Vega and gamma – exposures to implied and realized volatility – have a formal link to theoretical models. Whereas uncertainty in models is a forward-looking conditional variance, realized volatility is a contemporaneous sample variance. That is, for some shock ε, with $var_t(\varepsilon_{t+1}) = \sigma_t^2$, uncertainty is σ_t^2, while volatility is ε_t^2. Vega is literally the exposure of an option to σ_t^2, while gamma is exposure to ε_t^2. The distinction between σ_t^2 and ε_t^2 is crucial from a theoretical point of view: models in which forward-looking uncertainty matters for

[3]See Egloff, Leippold, and Wu (2010), Dew-Becker et al. (2017), Van Binsbergen and Koijen (2017), Andries et al. (2015), and Ait-Sahalia, Karaman, and Mancini (2015).

the economy have predictions about σ_t^2 but not about ε_t^2.

To summarize, then, the basic method in the paper is to measure risk premia on implied and realized volatility (jumps), or vega and gamma, using a typical factor pricing model on a panel of option returns across maturities, strikes, and numerous different underlyings. The estimated premia are then used to infer the relationship of marginal utility with uncertainty and realized volatility, both for specific underlyings and also for prominent macro uncertainty indexes.

Results

The main results focus on straddles because the options in the portfolio are initially at the money and hence most liquid. The empirical analysis yields three key findings. **First**, across 19 option markets, the risk premium for hedging uncertainty shocks – vega – is in the majority of cases *positive*. For nonfinancial underlyings and the JLN macro and inflation uncertainty indexes, the premia are statistically and economically significantly positive, with Sharpe ratios near 0.5. The results imply that investors in these markets view periods of high uncertainty about the real economy as being good on average. For the financial sector (including the S&P 500) and the JLN financial uncertainty and EPU index, the premium on uncertainty is not clearly distinguishable from zero.

The **second** empirical result runs in the opposite direction: consistently across both the financial and real sectors of the economy, portfolios that hedge realized volatility, or jumps, earn statistically and economically significantly negative returns. Investors on average therefore view periods in which shocks to fundamentals themselves are large as being bad.

It is well known that both volatility and uncertainty are countercyclical, but their overall correlation is not as high as one might expect – only about 65 percent on average across markets – and the average correlation between their innovations is only 0.2. The results here show that innovations in realized volatility identify the states of the world that investors view as actually negative, whereas surprise increases in implied volatility – which is high in other, mostly unrelated, states of the world – are not on average perceived as bad.

Our findings for realized volatility contribute to the growing literature studying skewness risk in the economy: if shocks to the economy (i.e. aggregate consumption) are skewed to the left, then large shocks tend to be bad.[4] An explanation for the pricing of realized volatility could then simply be that hedging realized volatility helps hedge downward jumps and disasters in aggregate consumption. If it is truly jumps that drive pricing, then we would expect that the negative returns on options would be larger for options that are farther out

[4]See Barro (2006), Bloom, Guvenen, and Salgado (2016), Seo and Wachter (2018a,b), Siriwardane (2015) and Berger, Dew-Becker, and Giglio (2018). Dew-Becker, Tahbaz-Salehi, and Vedolin (2019) provide a structural model for the source of aggregate skewness.

of the money. To test the hypothesis that the pricing is compensation for jump risk, we extend the baseline results to examine returns on strangles, which are like straddles, in holding both a put and a call, but in which both options are out of the money at inception. Relative to straddles, strangles only have positive payoffs for relatively large movements in the underlying.

Our **third** result is that the gamma/jump premia for strangles are about twice as large as those for straddles, formalizing the idea that it is jumps, rather than small (or diffusive) movements in underlying prices that investors are averse to. As with the results for straddles, the result that deeper out-of-the-money options have more negative returns is well known for the S&P 500. Our results are novel for showing that the same result appears in a wide range of markets, including those linked to the real economy.

Because the variance risk premium is robustly negative across many markets, jumps – which drive surprises in realized volatility – tend to be robustly viewed as bad events by investors, regardless of where they occur. According to asset prices, what policymakers should focus on, rather than uncertainty about the future (the possibility that something extreme *might* happen), is the *realization* of extreme (typically negative) events. For investors, the results imply that the mean-variance efficient portfolio among the assets we study is short gamma – jump risk – and either neutral to or long vega (exposure to implied volatility), and we show that large Sharpe ratios are available when buying vega and selling gamma across many markets. In the paper, we also build a simple extension of the standard long-run risk model of Bansal and Yaron (2004) that shows how our results can arise in equilibrium.

Relationship with past work

The paper is related to two main strands of literature. The first studies the relationship between uncertainty and the macroeconomy. Numerous channels have been proposed through which uncertainty about various aspects of the aggregate economy may have real effects, but the models do not generate a uniform prediction that uncertainty shocks are necessarily contractionary.[5] Our results are more consistent with the expansionary forces present in the models. There are also models with joint or reverse causation, such as Pastor and Veronesi (2009) and Bachmann and Moscarini (2012).[6] The related empirical literature tries to measure whether uncertainty does in fact have contractionary effects, finding often conflicting results.[7]

[5] See Basu and Bundick (2017), Bloom (2009), Bloom et al. (2017), Leduc and Liu (2015), Gourio (2013), Gilchrist and Williams (2005) and Bloom et al. (2017).

[6] See also Decker, D'erasmo and Boedo (2016), Berger and Vavra (2013), Ilut, Kehrig and Schneider (2015), Kozlowski, Veldkamp, and Venkateswaran (2016), Cesa-Bianchi, Pesaran, and Rebucci (2018) and Diercks, Hsu, and Tamoni (2019).

[7] For example, Schwert (1989), Schwert (2011), Berger, Dew-Becker, and Giglio (2017), Bretscher, Hsu,

This paper builds on that work from a finance perspective, by providing measures of risk premia that indicate how investors perceive the effects of aggregate uncertainty shocks across many markets. The finance perspective of this paper means that the methods and data are very different from papers that have instead used a macroeconomic approach to the question. For example, Berger, Dew-Becker, and Giglio (2018) estimate a structural vector autoregression as is common in the macroeconomics literature to try and understand the effect of uncertainty shocks on the economy. This paper – while trying to answer a similar question – takes a financial economics approach, studying risk premia, and requiring none of the VAR identifying assumptions.

As discussed above, Constantinides, Jackwerth, and Savov (2013) and Cremers, Halling, and Weinbaum (2015) are important precedents in the finance literature for studying the pricing of shocks to uncertainty and volatility. We build on Constantinides, Jackwerth, and Savov (2013) in that we also examine factor risk premia estimated from option returns, with the innovation that we look across a broader range of markets. Our analysis uses methods similar to that paper and also to those of Cremers, Halling, and Weinbaum (2015), in that we study both a factor model and replicating portfolios. We differ from Cremers, Halling, and Weinbaum (2015) in that we use option returns to measure risk premia, rather than projecting stock returns onto uncertainty and volatility factors. Because stock returns are driven by so many different risk factors, options can be useful for helping to isolate underlying risks relatively precisely. That difference can help explain differences between the results obtained by us and Constantinides, Jackwerth, and Savov (2013) relative to Cremers, Halling, and Weinbaum (2015).

The paper also draws on a literature in finance estimating the pricing of volatility (ε^2) risk. The past literature almost exclusively studies the S&P 500, and it in general studies just the variance risk premium, which is the pricing of realized volatility (as measured by the average gap between option-implied and realized volatility).[8] In addition to studying a much broader range of markets, our contribution is to also isolate the premium on *implied* volatility.

The remainder of the paper is organized as follows. Section 2 describes the data and

---

and Tamoni (2019), Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2015), Baker, Bloom, and Davis (2015), Bachmann and Bayer (2013), and Alexopoulos and Cohen (2009). For papers on different types of uncertainty, see also Bretscher, Schmid, and Vedolin (2018), Elder and Serletis (2010), Darby et al. (1999), Huizinga (1993) and Elder (2004).

[8]For example, see Ait-Sahalia, Karaman, and Mancini (2015), Bollerslev and Todorov (2011), Andersen, Fusari, and Todorov (2015, 2017), Dew-Becker et al. (2016), Constantinides, Jackwerth, and Savov (2013), Cremers, Halling, and Weinbaum(2015), and Farago and Tedongap (2018) for work on the S&P500. A few papers have studied specific markets, like Bakshi, Kapadia, and Madan (2003), Mueller, Vedolin, and Yen (2017), Prokopczuk et al. (2017), Trolle and Schwartz (2010).

its basic characteristics. Our main results on the cost of hedging uncertainty and volatility shocks are in section 3. We then provide a theoretical derivation of the risk exposures of the options in section 4 and use it to construct replicating portfolios. Section 5 reports the cost of hedging macroeconomic uncertainty and realized volatility, combining all 19 markets together. Section 6 presents robustness results and section 7 concludes.

2 Measures of uncertainty and realized volatility

This section describes our main data sources and then examines various measures of uncertainty and realized volatility.

2.1 Data

2.1.1 Options and futures

We obtain data on prices of financial and commodity futures and options from the end-of-day database from the CME Group, which reports closing settlement prices, volume, and open interest over the period 1983–2015. Each market includes both futures and options, with the options written on the futures. The futures may be cash- or physically settled, while the options settle into futures. As an example, a crude oil call option gives its holder the right to buy a crude oil future at the strike price. The underlying crude oil future is itself physically settled – if held to maturity, the buyer must take delivery of oil.[9]

To be included in the analysis, contracts are required to have least 15 years of data and maturities for options extending to at least six months, which leaves 14 commodity and 5 financial underlyings. The final contracts included in the data set have 18 to 31 years of data. A number of standard filters are applied to the data to reduce noise and eliminate outliers. Those filters are described in appendix A.1.

We calculate implied volatility for all of the options using the Black–Scholes (1973) model (technically, the Black (1976) model for the case of futures).[10] Unless otherwise specified, implied volatility is calculated at the five-month maturity. We take this value as the benchmark measure of uncertainty in each market. In general, longer maturities are naturally more tightly linked to long-lived economic decisions, like physical investments. We

[9]The underlying futures in general expire in the same month as the option. Crude oil options, for example, currently expire three business days before the underlying future.

[10]The majority of the options that we study have American exercise, while the Black model technically refers to European options. We examine IVs calculated assuming both exercise styles (we calculate American IVs using a binomial tree) and obtain nearly identical results. Since there are no dividends on futures contracts, early exercise is only rarely optimal for the options studied here.

do not go past five months because there is less trade and liquidity at longer maturities, making prices less reliable.

Implied volatilities extracted from options reflect market's uncertainty about future returns, but they also contain a risk premium, which can potentially vary over time. However, even in the presence of that risk premium, implied volatilities appear to provide very good summaries of the available information in the data for forecasting future volatility, driving out other standard uncertainty measures from forecasting regressions. Appendix A.2 compares implied volatilities to regression-based forecasts of future volatilities and shows that they are all over 90 percent correlated (with an average correlation of 97 percent), indicating that option-implied volatility is a good, if not perfect, proxy for true (physical) uncertainty. For that reason, in what follows we refer to implied volatility and uncertainty interchangeably, with the understanding that deviations due to time-varying risk premia are quantitatively small at the monthly frequencies we focus on.[11]

2.2 The time series of implied volatility

Figure 1 plots option implied volatility for three major futures: the S&P 500, crude oil, and US Treasury bonds. The implied volatilities clearly share common variation; for example, all rise around 1991, 2001, and 2008. On the other hand, they also have substantial independent variation. Their overall correlations (also reported in the figure) are only in the range 0.5–0.6.

Table 1 reports pairwise correlations of implied volatility across the 19 underlyings. The largest correlations in implied volatility are among similar underlyings – crude and heating oil, the agricultural products, gold and silver, and the British Pound and Swiss Franc. Correlations outside those groups are notably smaller, in many cases close to zero. The largest eigenvalue of the correlation matrix explains 46 percent of the total variation. The remaining eigenvalues are much smaller, though – even the second largest only explains 16 percent of the total variation. Eight eigenvalues are required to explain 90 percent of the total variation in the IVs, which is perhaps a reasonable estimate of the number of independent components in the data.

The common variation in the implied volatilities is much larger than the common variation in the underlying futures returns. The largest principal component for the futures returns explains less than half as much variation – 19 percent versus 46. In other words, while the individual futures prices may be driven by idiosyncratic shocks, or their correlations with each other might change over time, masking common variation, investor *uncertainty* about

[11] See also Bekaert, Hoerova, and Lo Duca (2013) for an analysis of the variation in risk premia in implied volatilities.

futures returns has a substantial degree of commonality across markets (similar to Herskovic et al. (2016)), showing that we are not studying uncertainty that is purely idiosyncratic and isolated to individual futures markets. The table below formalizes that result, reporting the variation explained by the first eigenvalue for implied volatility, realized volatility (discussed below), and the underlying futures returns, along with bootstrapped 95-percent confidence bands.

Fraction of variation explained by largest eigenvalue

	IV	RV	Futures return
Largest Eigenvalue (% explained)	45.9%	28.1%	19.1%
95% Bootstrap CI	37.3%	23.7%	16.7%
	49.5%	41.8%	21.2%

2.3 Relationship between implied volatility and macro uncertainty indexes

Our ultimate goal is to understand the pricing of economic uncertainty. We therefore want to check whether the implied volatilities in the futures markets we study are related to other prominent measures of uncertainty. Figure 2 quantifies how well the 19 IVs can replicate two well-known macro uncertainty indexes: the JLN index from Jurado, Ludvigson, and Ng (2015) and the EPU index of Baker, Bloom, and Davis (2015) (see section 5 for a more detailed description of the indexes). Figure 2 plots the time series of the JLN indexes and EPU index against the fitted values from their projection onto the 19 implied volatilities. The right-hand panels plot the pairwise correlations of the implied volatilities in the individual markets with the fitted uncertainty. For financials, the correlation with S&P 500 implied volatility is 97 percent. The next highest correlation is only 68 percent, for Treasury bonds. So figure 2 shows that fitted financial uncertainty is very nearly equivalent to S&P 500 implied volatility.[12]

The second row plots fitted uncertainty for real variables. In this case, gold, copper, crude oil, and heating oil are the most important contributors. The third row shows similar results for the price component of JLN uncertainty. Uncertainty about the real economy and

[12]The strong fit the S&P 500 implied volatility is not simply due to the fact that S&P 500 returns are included in the JLN construction. The results are similar when all variables involving the S&P 500 index (returns, dividends, etc.) are dropped.

inflation are therefore driven by similar factors, and those factors are notably distinct from financial uncertainty, which shows why having a broad range of IVs, and looking at markets beyond the S&P 500, is important.

The bottom panels plot results for the EPU index. The highest pairwise correlations are with financial IVs, Treasuries, gold, the S&P 500, and currencies. That implies that the EPU index measures a similar type of uncertainty as other financial uncertainty measures, perhaps because news coverage often focuses on financial markets.[13]

3 The cost of hedging uncertainty and volatility

In this section we present the main results of the paper: we estimate the cost of hedging shocks to volatility and uncertainty using option portfolios.

We compute the cost of hedging a shock as the negative of the average excess return (risk premium) on the portfolio that hedges that shock. We report all risk premia in terms of Sharpe ratios, which reveal the compensation for bearing a risk (or the cost of hedging it) per unit of risk, and are therefore more easily comparable across markets. The option returns are highly skewed, so an investor here would care about more than just the Sharpe ratio; we use it simply as a device for holding effective leverage constant across markets. For reference, the historical Sharpe ratio of US equities in our sample is 0.52.

We estimate risk premia for implied and realized volatility using a standard linear factor model, and we use straddle returns of different maturities as test assets. Typical factor models use a small number of aggregate factors. Here, though, we are interested in the price of risk for shocks to all 19 types of uncertainty. We therefore estimate market-specific factor models. This is similar to the common practice of pricing equities with equity-specific factors, bonds with bond factors, currencies with currency factors, etc.[14]

The cost of hedging a risk has a simple but important economic interpretation: it measures the extent to which the risk is "bad" with respect to state prices or marginal utility. Consider a factor X and an asset with returns R_X that hedges it, in the sense that R_X varies one-for-one (and is perfectly correlated) with *innovations to X*. Then if M represents the stochastic discount factor,

$$E\left[\frac{R_{X,t+1}-R_f}{std_t\left(R_{X,t+1}\right)}\right] = -cov\left(M_{t+1}-E_tM_{t+1}, \frac{X_{t+1}-E_tX_{t+1}}{std_t\left(X_{t+1}\right)}\right)R_f, \tag{1}$$

[13]To account for possible overfitting due to the fact that we have 19 explanatory variables, we experimented with lasso and variable selection based on information criteria. The results were highly similar in all cases.

[14]The analysis is similar to those of Jones (2006) and Constantinides, Jackwerth, and Savov (2013).

where R_f is the gross risk-free rate, which we treat as constant for the sake of exposition, E_t is the expectation operator, and std_t is the standard deviation conditional on date-t information. The equation says that the negative of the risk premium on a portfolio that hedges the risk X measures the covariance of innovations in X_{t+1} with state prices. More generally, when the correlation between R_X and innovations in X is less than 1, $E[R_X - R_f]$ measures the covariance of state prices with the part of innovations to X that is spanned by R_X. So if the premium $E[R_X - R_f]$ is negative, times when R_X (and hence X) rise are bad times, in which state prices are high. The factor model and subsequent analysis will deliver estimated Sharpe ratios for the various risk factors we study.

Finally, as we review in Appendix A.3, the risk premia estimated from linear factor models correspond to the average excess returns of portfolios that isolate each risk (that is, each portfolio has beta of 1 with respect to one risk factor, and 0 with respect to all other factors). These portfolios are precisely those portfolios that allow investor to change risk exposure to any factor and that factor only; we refer to them as factor-hedging portfolios.

3.1 Method

3.1.1 Factor model specification

For each market we estimate a time-series model of the form

$$r_{i,n,t} = a_{i,n} + \beta_{i,n}^{f} \frac{f_{i,t}}{IV_{i,t-1}} + \beta_{i,n}^{f^2} \frac{1}{2} \left(\frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \beta_{i,n}^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}} + \varepsilon_{i,n,t}, \tag{2}$$

where $f_{i,t}$ is the futures return for underlying i and $\Delta IV_{i,t}$ is the change in the five-month at-the-money implied volatility for underlying i. $r_{i,n,t}$ is a return on each of the N test assets (straddles and strangles, described in greater detail below).

The underlying futures return $f_{i,t}$ controls for any exposure of the test assets to the underlying, though in general that loading will be small, given that we use as test assets portfolios with payoffs that are symmetric in the value of the underlying. Much more important is the fact that straddles and strangles have nonlinear exposures to the futures return. $(f_{i,t}/IV_{i,t-1})^2$ captures that nonlinearity. $\beta_{i,n}^{f^2}$ will be interpreted as the exposure of the options to realized volatility.[15] Finally, the third factor is the change in the at-the-money implied volatility for the specific market at the five-month maturity, representing an uncertainty shock in that market.[16]

[15]The results are similar when the second factor is the absolute value of the futures return or when it is measured as the sum of squared daily returns over the return period.

[16]Since the IVs may be measured with error, we construct this factor by regressing available implied

The three factors are scaled by lagged implied volatility for two reasons. First, this helps control heteroskedasticity. Intuitively, the factors are measuring innovations in standard deviation units, so that we are pricing based on how much the underlying moves relative to what investors expected. The second reason will be demonstrated in the next section: it is what the Black–Scholes model implies for the exposures of straddles and strangles. That is, the option portfolios yield exposure to the scaled factors used here, rather than, for example, the raw futures return (and raw futures return squared). So while the analysis in this section does not rely on Black-Scholes, this scaling will be useful for interpreting the results.

We estimate a standard linear specification for the risk premia,

$$E\left[r_{i,n,t}\right] = \gamma_i^f \beta_{i,n}^f Std\left(\frac{f_{i,t}}{IV_{i,t-1}}\right) + \gamma_i^{f^2} \beta_{i,n}^{f^2} Std\left(\left(\frac{f_{i,t}}{IV_{i,t-1}}\right)^2\right) + \gamma_i^{\Delta IV} \beta_{i,n}^{\Delta IV} Std\left(\frac{\Delta IV_{i,t}}{IV_{i,t-1}}\right) + \alpha_{i,n} \quad (3)$$

where $\alpha_{i,n}$ is a fitting error, using standard two-step cross-sectional regressions. The γ coefficients represent the risk premia that are earned by investments that provide direct exposure to the factors. Due to the scaling by standard deviations, the γ's are the Sharpe ratios of the hedging portfolios for each factor constructed using the test assets.[17]

3.1.2 Test assets

Our main results are for two-week returns on straddles with maturities between one and five months.[18] A straddle is a portfolio holding a put and a call with the same maturity and strike; we specifically study zero-delta straddles, with the strike set so that the Black–Scholes delta of the portfolio is zero. The final payoff of a zero-delta straddle depends on the absolute value of the return on the underlying, meaning that they have symmetrical exposures to positive and negative returns,. For the remainder of the paper, we refer to zero-delta straddles simply as straddles (that is, we only work with zero-delta straddles).

volatilities on maturity for each underlying and date and then taking the fitted value from that regression at the five-month maturity.

[17]While $f_{i,t}^2$ and $\Delta IV_{i,t}$ are nontradable factors, $f_{i,t}$ itself is tradable, so we include it as a test asset, yielding the additional restriction $E\left[f_{i,t}/IV_{i,t-1}\right] = \gamma_i^f Std(f_{i,t}/IV_{i,t-1})$ (see Cochrane (2005)).

[18]Past work on option returns and volatility risk premia has examined returns at frequencies of anywhere from a day (e.g. Andries et al. (2017)), to holding to maturity (Bakshi and Kapadia (2003)). The precision of estimates of the riskiness of the straddles is, all else equal, expected to be higher with shorter windows. On the other hand, shorter windows cause any measurement error in option prices to have larger effects.

Some of the existing literature, beginning with Bakshi and Kapadia (2003), examines delta-hedged returns. Even with delta hedging, the higher-order risk exposures of the straddles change substantially as the price of the underlying changes over time.

Another alternative is to examine returns on synthetic variance swaps. Synthetic variance swap prices are constructed using the full range of strikes, so they require much more data than straddles. The markets we study do not all have liquid options at extreme strikes and multiple maturities, so we focus on straddles, which just require liquidity near the money.

Straddles give investors exposure both to realized and implied volatility. They are exposed to realized volatility because the final payoff of the portfolio is a function of the absolute value of the underlying futures return. But when a straddle is sold before maturity (as in our case, since we use two-week holding period returns), the sale price will also depend on expected future volatility, meaning that straddles can give exposure to uncertainty shocks. Since the options in the straddle are at the money at inception, a straddle is the most liquid zero-delta portfolio we can construct.

In principle, it is also possible to estimate the factor risk premia using other assets, like stock or bond returns (e.g. Cremers, Halling, and Weinbaum (2015)). We focus on option returns because they depend directly on realized volatility and uncertainty – which is why they are used to construct implied volatility measures – whereas for other assets the connection is less clear (many other factors affect their returns) and there could be nontrivial problems with exposures shifting over time. We show below that under the simple Black–Scholes benchmark, the factor loadings will be constant.

3.2 Empirical results

3.2.1 Hedging uncertainty shocks

The dotted red series in figure 3 plots estimated risk premia and confidence bands for the realized and implied volatility factors – $\gamma_i^{f^2}$ and $\gamma_i^{\Delta IV}$, respectively – using straddles as test assets. Again, the risk premia should be interpreted as annualized Sharpe ratios, since they are scaled to measure average annualized returns per unit of annualized standard deviation. The top panel plots premia for implied volatility and the bottom panel realized volatility. The boxes are point estimates while the bars represent 95-percent confidence bands based on a block bootstrap.[19]

Across the top panel, implied volatility shocks carry zero or even positive premia. For financials, the average Sharpe ratios tend to be near zero or weakly negative. The S&P 500 has a positive premium, consistent with results for variance swaps discussed extensively in Dew-Becker et al. (2016). That result is not completely robust here, however – something we discuss further below – but there is certainly no evidence of a significantly negative premium for S&P 500 uncertainty. For the nonfinancials, on the other hand, all 14 sample Sharpe ratios are actually positive, and five of those are individually statistically significant. Overall,

[19]The bootstrap is constructed with 50-day blocks and 5000 replications. It is used to account for the fact that the returns use overlapping windows. Hansen–Hodrick type standard errors are not feasible here due to the fact that observations in the data are not equally spaced in time. The block bootstrap additionally accounts for other sources of serial correlation in the returns, such as time-varying risk premia.

for only one out of 19 contracts, the British Pound, do we find a significantly negative Sharpe ratio.

To formally estimate the average risk premia across contracts, we use a random effects model, which yields an estimate of the population mean risk premium while simultaneously accounting for the fact that each of the sample Sharpe ratios is estimated with error, and that the errors are potentially correlated across contracts (see appendix A.5).

For both nonfinancials and all markets overall, the estimated population mean Sharpe ratio is statistically and economically significantly positive, while for financials it is close to zero. The group-level means have the advantage of being much more precisely estimated than the Sharpe ratios for the markets individually. They show that on average, instead of there being a cost to hedging uncertainty shocks, the factor risk premium for uncertainty shocks is actually positive. For nonfinancials, the average Sharpe ratio is 0.43, and the lower end of the 95-percent confidence interval is 0.13. For the overall mean, the corresponding numbers are 0.32 and 0.08, so the average Sharpe ratios are significantly positive in both cases. The top panel of table 3 reports the estimated average Sharpe ratios for financials and nonfinancials, and, in the third column, their difference, and shows that the difference in risk premia between the two groups is not statistically significant.

The top panel of figure 3 contains our key results on the risk premium for uncertainty. It shows that across the board, risk premia for uncertainty are indistinguishable from zero or, if anything, somewhat positive. The results allow us to quantify the overall correlation between uncertainty and marginal utility. For financial underlyings, including the S&P 500, the zero or very weakly negative risk premium implies that the correlation is close to zero. For the nonfinancial underlyings, which are closely linked to the JLN real and price uncertainty series, the results imply that the correlation is positive.

3.2.2 Hedging realized volatility shocks

The bottom panel of figure 3 reports risk premia for realized volatility across the 19 markets, representing our second main result. The numbers are drastically different from those for IV. Whereas implied volatility has earned a zero or even positive premium, the realized volatility premia are almost all estimated to be negative. For the S&P 500, this result is well known and is referred to as the variance risk premium. The S&P 500 realized volatility risk premium is most negative, at -1.26 – the premium for selling insurance against shocks to realized volatility is more than *twice* as large as the premium on the stock market over the same period. For the other financial underlyings, the premium on realized volatility is not statistically significantly negative. For the nonfinancials, 11 of 14 estimated premia are negative (6 significantly).

Looking at the category means, in this case all three estimates – financials, nonfinancials, and all assets – are negative. The values are on the edge of statistically significant for the nonfinancials and the overall mean, with confidence bands just barely encompassing zero. The point estimate for the overall mean Sharpe ratio is -0.26 and the upper end of the 95-percent confidence interval is 0.04. Those values are almost the same as what we obtain for uncertainty, but with the opposite sign. As with uncertainty, table 3 shows that the difference between financials and nonfinancials is not statistically significant.

In sum, in stark contrast to the results for hedging uncertainty, the bottom panel of figure 3 shows that there has historically been, consistently across markets, an economically significant cost to hedge realized volatility. Contracts that, rather than loading on changes in implied volatility, load on actual realized squared returns, earn negative Sharpe ratios with magnitudes up to twice as large as that for the overall stock market. So while uncertainty is viewed as neutral or even good on average, *realized volatility* or *jumps* – the realization of large squared returns – is viewed as mostly bad, for both financials and nonfinancials.

3.2.3 Goodness of fit

Figure 4 reports a scatter plot of realized returns on the various straddle returns against the fitted returns from the model. The figure shows that there is a wide spread in realized returns that the model is able to capture. In addition, there are no large outliers. Table A.1 in the appendix reports the p-values of the χ^2 test of the model based on the squared fitting errors (bootstrapped following Constantinides, Jackwerth, and Savov (2013)). That test is very stringent, especially when the fitting errors are small on average, since they are scaled by their sample variance. That said, the test rejects in only three of the 19 markets. The p-value for the S&P 500 is 0.22, similar to the one obtained by Constantinides, Jackwerth, and Savov (2013). The fact that the model is rejected for only one of the 14 nonfinancials suggests that the results for nonfinancials – where the differences in the pricing of implied and realized volatility are most pronounced – should be most reliable. The test rejects for two of the five financial underlyings, which implies that they are more likely to have specification error.

3.3 Interpretation of the results

How can realized volatility have a negative price of risk, while uncertainty have a positive one? Key to understanding this distinction is noticing that realized volatility (which is computed by squaring shocks) is strongly dominated by large price movements like jumps, which our empirical results suggest tend to be bad for investors on average. So it is easy to see how investors might dislike realized volatility, as it captures the occurrence of a large, bad shock.

On the other hand, innovations in implied volatility are driven by changes in the perceived uncertainty about good and bad potential events: so a higher probability of a bad jump will increase uncertainty, but a higher probability of a good event (e.g., a new technology) will *also* increase uncertainty. Our results show that on net, investors seem to perceive increases in uncertainty as being associated with good states of the world.

Section A.11 in the appendix formalizes this idea, describing a simple extension of the standard long-run risk model of Bansal and Yaron (2004) that is consistent with our results on the pricing of both volatility and uncertainty shocks.

Finally, it is valuable to compare our analysis with some closely related past work. As discussed above, both Constantinides, Jackwerth, and Savov (2013; CJS) and Cremers, Halling, and Weinbaum (2015; CHW) also examine the pricing of uncertainty and realized volatility in the S&P 500 using factor models. While we cannot compare our full range of results with theirs, we can at least see how those for the S&P 500 compare.

The analysis of CJS is closest to us, as they also use option portfolios as test assets. In table 8, they report a premium of approximately zero for shocks to uncertainty and a large negative premium for realized volatility for the S&P 500. So consistent with us, they find much stronger pricing of realized than implied volatility, though their uncertainty premium is less positive. CHW, instead, use the cross-section of equities as their test assets and find a more strongly negative premium for uncertainty. However, they also report returns on an uncertainty hedging portfolio, which aligns very closely with our analysis in the next section (see their table 1). In that case, their results are quantitatively highly similar to ours. We discuss this observation further below.

3.4 Is realized volatility about jumps? Evidence from strangles

Similar to others (e.g. Cremers, Halling, and Weinbaum (CHW; 2015)), we have argued thus far that the exposures to squared returns on the underlying – or gamma – represent exposure to jump risk. While CHW focused on straddles, we further test the hypothesis that the premia are for jumps by examining returns on strangles. A strangle is, like a straddle, a portfolio long a put and a call, with the delta set to zero here by construction. However, in the case of a strangle, the two options are out-of-the-money, with different strikes, rather than both having the same strike. So whereas the final payoff of a straddle depends on the absolute value of the change in the underlying, a strangle only pays off if the underlying moves sufficiently far from its initial value (with that required distance being a choice variable).

We examine returns on strangles where the put and call strikes are 1 standard deviation unit (scaling by time to maturity) from the forward price when the portfolio is formed, so they only have positive payoffs at maturity if the underlying moves further than that. As with the straddles, we examine two-week returns.

Figure 5 replicates figure 3 for the case of strangles. For the uncertainty risk premia, the results are qualitatively and quantitatively similar to those for straddles: for financials the premium is close to zero, and for nonfinancials it is 0.42.

It is for the RV/gamma risk premia that we find a substantial difference, representing our third main result. Across the various markets, the premia are generally *twice* as large for strangles as for straddles. Every single point estimate is now negative, and only one confidence band contains zero. For financial underlyings, the average premium is now statistically significant, at -1.54. For nonfinancials and all assets combined, the means are both -1.48 and -1.5 respectively.

These results show that it is really the tail of the distribution that drives the RV results. The finding that deep out-of-the-money options have the largest premia is well known for the S&P 500. This paper is novel for showing that the relationship of the gamma premium with moneyness in fact holds across all the markets that we study (and it is strikingly different from the patterns on uncertainty).

To sum up, figures 3 and 5 contain our three main results. Pervasively across markets, premia related to vega (uncertainty) are zero or positive, while premia for gamma (jump risk) are significantly negative. Furthermore, the jump risk premia are largest for out-of-the-money options. Economically, the results show that it is periods with extreme shocks – realized volatility or jumps – that investors are averse to, rather than simple increases in forward-looking uncertainty.

4 Theoretical risk exposures of straddles and strangles

We argued heuristically above that straddles and strangles are natural test assets for a factor model involving realized and implied volatility since they have zero delta and payoffs that are convex in the underlying return. This section formalizes that intuition by calculating the theoretical exposures of options of different maturities to those shocks, following the analysis of Cremers, Halling, and Weinbaum (2015). Similar to them, we then show that we can construct replicating portfolios that, under the theory, should provide direct exposure to shocks to either implied or realized volatility. Formally, under the Black-Scholes model, one portfolio has positive vega and zero gamma, and the other has positive gamma and zero vega. These portfolios give an alternative, and in some sense more direct, way of measuring the risk premia.

4.1 Return exposures

The exposures of the portfolios studied above to the risk factors we use in our linear factor model can be approximated theoretically using the Black–Scholes model, as in Coval and Shumway (2001), Bakshi and Kapadia (2003), and Cremers, Halling, and Weinbaum (2015). Appendix A.4 shows that the partial derivatives of the zero-delta straddle and strangle return with respect to the underlying futures return, f, its square, and the change in volatility, can be approximated as

$$\frac{\partial r_{n,t}}{\partial f_t} \approx 0, \tag{4}$$

$$\frac{\partial^2 r_{n,t}}{\partial \left(f_t/\sigma_{t-1}\right)^2} \approx n^{-1}, \tag{5}$$

$$\frac{\partial r_{n,t}}{\partial \left(\Delta\sigma_t/\sigma_{t-1}\right)} \approx 1, \tag{6}$$

where $r_{n,t}$ is the return on date t of a straddle or strangle with maturity n, f_t is the return on the underlying future, σ_t is the implied volatility of the underlying, and Δ is the first-difference operator.[20]

It is perhaps surprising at first that the exposures are the same for both straddles and strangles. Intuitively, the two types of portfolios have the same exposures up to the second order – where they differ is in their higher-order exposures, which are naturally larger for the strangles. The first partial derivative says that the straddles and strangles have close to zero local exposure to the futures return. The second line says that the exposure of the options to *squared* returns on the underlying – realized volatility – is approximately inversely proportional to time to maturity. The third line shows that they are also exposed to changes in expected future volatility, through $\frac{\Delta\sigma_t}{\sigma_{t-1}}$, and that exposure is approximately constant across maturities.

To see how the risk exposures differ in their higher order terms, figure A.4 in the appendix plots the return on a straddle and a 1-standard-deviation strangle as a function of the change in the price of the underlying. One can see how the two curves are not just tangent at zero,

[20] We ignore here the fact that options at different maturities have different underlying futures contracts. If that elision is important, it can be expected to appear as a deviation of the estimated factor loadings from the predictions of the approximations (4)–(6).

but that they have the same curvature, consistent with having the same second derivative, as in equation (5). They only begin to difer noticeably as the returns get extreme. So straddles and strangles have equal *local* exposures to the underlying, but in the tails, e.g. in response to jumps, strangles become more sensitive. This shows why strangle returns help isolate the extra premium earned for exposure to tail risk.

4.2 Replicating portfolios

Cremers, Halling, and Weinbaum (2015) show that the implied sensitivities in (4)–(6) give a method for constructing portfolios that the Black–Scholes model says should give exposures *only* to realized volatility – $(f_{n,t}/\sigma_{t-1})^2$ – or implied volatility, measured by $\Delta\sigma_t/\sigma_{t-1}$. The method is to construct, for each market, two portfolios,

$$rv_{i,t} = \frac{5}{24}(r_{i,1,t} - r_{i,5,t}) \approx (f_t/\sigma_{t-1})^2, \quad (7)$$

$$iv_{i,t} = \frac{5}{4}r_{i,5,t} - \frac{1}{4}r_{i,1,t} \approx \Delta\sigma_t/\sigma_{t-1}. \quad (8)$$

where the approximations follow from equations (4)–(6).[21] Throughout this section, capitalized RV and IV refer to the levels of realized and implied volatility, while lower-case rv and iv refer to the associated portfolio returns. We use the one- and five-month options to construct the portfolios since it is exactly five-month implied volatility that is priced in the main analysis. The iv portfolio is dominated by an investment in the five-month options, with just a small short position in the one-month options. In that sense, the iv portfolio is a rather direct claim on exactly the implied volatility priced in the factor model.

The purpose of constructing these portfolios is to give a simple and direct method of measuring the premia associated with realized and implied volatility that does not require full estimation of the factor model. If the loadings used to construct the portfolios are correct, this method will also be more efficient. On the contrary, if the assumptions of the model are not correct, then the results will be biased (whereas the factor model will still be correct, as it *estimates* the risk exposures instead of using the ones implied by the model). There is thus a bias/variance trade-off between the factor model, which requires fewer assumptions but will have greater estimation error, and the replicating portfolios, which require stronger assumptions but will have less estimation error.

The key concern, then, is how accurate the Black–Scholes-implied loadings are. Figure

[21] Note that equation (5) gives the second derivative, which has weight 1/2 in the Taylor approximation. So the loading on the squared future return for a straddle of maturity n is $(2n)^{-1}$, which implies that the coefficient for (7) is 5/24.

A.2 and table A.2 in the appendix show that the theoretical predictions for the loadings are fairly accurate (though not perfect) empirically. Appendix A.4 also examines the accuracy of the Black–Scholes approximation for returns in a simulated setting.

Table A.2 shows that the biggest deviations from the model-implied loadings are for the S&P 500 *iv* portfolio. In that case, there is a large positive loading on realized volatility – a GARCH effect – and a large negative loading on the underlying futures return – the leverage effect. Both should be expected to bias the return on the *iv* portfolio down relative to the estimated implied volatility factor loading from above. The effects are three times larger for the S&P 500 than for any other market. That suggests that for measuring pricing of S&P 500 uncertainty, in particular, it is best to use the factor model, as in Constantinides, Jackwerth, and Savov (2013). For all other markets, instead, the Black-Scholes assumptions appear relatively accurate, so we would expect the results to line up well with those of the factor model.

Note that even though the *rv* and *iv* portfolios theoretically load on separate risk factors, they need not be uncorrelated. It is well known from the GARCH literature (e.g. Engle (1982) and Bollerslev (1986)) that in many markets, innovations to realized volatility are correlated with innovations to implied volatility. Table 4 reports the correlations between the *rv* and *iv* returns in the 19 markets. GARCH effects appear most strongly for the financial underlyings and precious metals, where the average correlation is 0.44. For the other nonfinancial underlyings, the effects are much smaller, and the correlation between the *rv* and *iv* returns is only 0.03 on average (it is 0.09 on average across all nonfinancials). So for the nonfinancials, innovations to realized and implied volatility returns are essentially independent on average. These weak correlations are valuable for the identification, since they show that surprises in realized and implied volatility are far from the same and can be hedged separately using the *rv* and *iv* portfolios.

4.3 Risk premia

4.3.1 Straddles

The solid blue series in the two panels of figure 3 report annualized Sharpe ratios for the *rv* and *iv* portfolios constructed from straddles in the 19 markets. As with the factor model, we begin by focusing on the straddle returns because they are most liquid and hence most conservative.

The results in figure 3 for the *rv* and *iv* portfolios are highly similar to those for the factor model. The *iv* portfolios earn returns close to zero on average for the financial underlyings and returns that are consistently positive for the nonfinancial underlyings. For the nonfinancials, the average Sharpe ratio for the *iv* portfolios is again statistically significantly positive. As expected, since the *iv* portfolios are formed using stronger assumptions, the standard errors for the risk premia are tighter than for the factor model.

The bottom panel of table 3 summarizes the estimates for the realized and implied volatility risk premia for financials and nonfinancials computed using the *rv* and *iv* portfolios, and also reports tests for whether the two are different. In all cases, the premia for the financials are insignificant while those for the nonfinancials are insignificant. However, note that there are fewer financial underlyings, limiting our statistical power. The difference between financials and nonfinancials itself is not significant – so we cannot actually say that there is strong evidence for a difference between the two in three out of four cases. The only case where the difference is statistically significant is for the Sharpe ratio on the *iv* portfolio.

That difference appears to be driven largely by the fact that the return on the S&P 500 *iv* portfolio is very different from the estimated risk premium for implied volatility from the factor model. In fact, the confidence bands do not even overlap. This result is driven by the fact that there are much stronger GARCH effects in the S&P 500 than the other underlyings that we study, creating a bias, as discussed above (see table A.1 showing that the S&P 500 *iv* portfolio actually loads strongly on realized volatility). We thus place relatively less trust in the results from the *rv* and *iv* portfolios (as opposed to the results from the factor model) for the S&P 500 than the other underlyings, for which there is very strong agreement between the factor model and the *iv* portfolio returns. Even in the case of the S&P 500, though, the premium for uncertainty shocks is not statistically significantly negative.

The Sharpe ratios for the *rv* portfolios are also highly similar to the estimated risk premia on realized volatility in the factor model (even for the S&P 500). The financial underlyings other than the S&P 500 again have premia generally close to zero, while the S&P 500 and the nonfinancials have consistently negative premia.

The returns on the *rv* and *iv* portfolios for the S&P 500 can be compared to those reported in table 1 of Cremers, Halling, and Weinbaum (2016). For their analog to our *rv* portfolio, they obtain a Sharpe ratio of -0.9, compared to -1.2 in our case, while for their analog to the *iv* portfolio, they report a Sharpe ratio of -0.5, compared to -0.2 here. In both cases, the confidence bands for our estimates easily contain theirs. We thus obtain substantial agreement with the findings of CHW for returns on option portfolios. Our results differ from theirs in two key ways. First, we focus on factor models using options as test assets, instead of equities. We choose to use options, similar to Constantinides, Jackwerth, and Savov (2013), because they have risk exposures very directly tied to uncertainty and volatility, whereas equity returns have many other risk exposures that have been explored in the literature. Second, obviously, we explore the pricing of options in a wide range of markets, not just the S&P 500.

4.3.2 Strangles

The results for strangles are again consistent with those for straddles, but more extreme. In figure 5, as in figure 3, the point estimates and confidence bands from the factor model (red) and the *rv* and *iv* portfolios (blue) are similar, with the model-based *rv* and *iv* portfolios again having narrower confidence intervals, showing that the results are robust to the estimation method.

We again find that the strangles have much more negative jump/gamma premia than the straddles. Since we showed above that the exposures of the strangles and straddles are the same up to second order, this section clearly indicates that it is the difference in higher order exposures of the different strategies that drives the larger premia for strangles.

4.4 Summary

The results in this section are useful for three reasons. First, they show that our results are not driven by some hidden detail of the factor model estimation. The *rv* and *iv* portfolios are simple to construct and yield highly similar results to the factor model, both for straddles and strangles. So the three key findings, zero or positive premia for uncertainty, substantially negative premia for realized volatility, and even larger premia for realized volatility for strangles, appear to be robust.

Second, the replicating portfolios help clarify exactly what the source of identification is in the factor model. The options have exposures to implied and realized volatility that differ across maturities, so including a panel of multiple maturities allows us to separately measure their premia.

Finally, by analyzing the risk exposures of the options, we can link the factor model estimates back to widely studied and applied features of options – their greeks. The estimate of the price of shocks to implied volatility from the factor model is essentially identical to the Sharpe ratio on a portfolio with positive vega and zero gamma, while the estimate of the price of shocks to realized volatility is almost the same as the Sharpe ratio on a portfolio with positive gamma and zero vega.

4.5 Combined portfolios

As we discussed in Section 2.3, the uncertainty in our 19 markets is related to various measures of aggregate uncertainty. It is then natural to ask what the cost of hedging is for aggregate uncertainty. A simple way to do that is to buy all the *iv* or *rv* portfolios simultaneously. We focus on just the straddles here since they are most liquid and thus most feasible for an investor to hold. Since tables 1 and 2 show that realized and implied volatility are imperfectly correlated across markets, even larger Sharpe ratios can be earned by holding portfolios that diversify across the various underlyings. Table 5 reports results of various implementations of such a strategy. Looking first at the top panel, the first row reports results for portfolios that put equal weight on every available underlying in each period, the second row uses only nonfinancial underlyings, and the third row only financial underlyings. The columns report Sharpe ratios for various combinations of the *rv* and *iv* portfolios. The first two columns report Sharpe ratios for strategies that hold only the *rv* or only the *iv* portfolios, the third column uses a strategy that is short *rv* and long *iv* portfolios in equal weights, while the final column is short *rv* and long *iv*, but with weights inversely proportional to their variances (i.e. a simple risk parity strategy).

The Sharpe ratios reported in table 5 are generally larger than those in figure 3. The portfolios that are short *rv* and long *iv* are able to attain Sharpe ratios above 1. The largest Sharpe ratios come in the portfolios that combine *rv* and *iv*, which follows from the fact that they are positively correlated, so going short *rv* and long *iv* leads to internal hedging. All of that said, these Sharpe ratios remain generally plausible. Values near 1 are observed in other contexts (e.g. Broadie, Chernov, and Johannes (2009) for put option returns, Asness and Moskowitz (2013) for global value and momentum strategies, and Dew-Becker et al. (2017) for variance swaps).

The portfolios that take advantage of all underlyings simultaneously seem to perform best, presumably because they are the most diversified. While holding exposure to implied volatility among the financials earns effectively a zero risk premium, it is still generally worthwhile to include financials for the sake of hedging.

Finally, the bottom panel of table 5 reports the skewness of the various strategies from above. One might think that the negative returns on the *rv* portfolio are driven by its positive skewness, but the *iv* portfolio also is positively skewed and has positive average returns. So the degree of skewness does not seem to explain differences in average returns in this setting.

5 Hedging uncertainty indexes

The results so far give the cost of directly hedging shocks in commodity markets. This section examines how options can be used to hedge shocks to macro uncertainty indexes. Section 2.3 showed that the commodity IVs do a good job of spanning the macro uncertainty indexes. We now discuss those indexes in more detail and examine the cost of hedging both the implied and realized parts of macro volatility.

The JLN index is developed in a pair of papers by Jurado, Ludvigson, and Ng (JLN; 2015) and Ludvigson, Ma, and Ng (2017). We follow their construction methodology and further extend it to yield separate measures of uncertainty that pertain to financial markets, real activity, and goods prices, with the latter two also being combined into an overall macroeconomic uncertainty index.[22] The goal of the JLN framework is to estimate uncertainty on each date, σ_t^2. The method can also be extended to create a realized volatility index.[23] We refer to the JLN uncertainty indexes by $JLNU$ and realized volatility indexes by $JLNRV$.

The Economic Policy Uncertainty (EPU) index of Baker, Bloom, and Davis (2015) is constructed based on media discussion of uncertainty, the number of federal tax provisions changing in the near future, and forecaster disagreement. Unlike JLN, there is no distinction in this case between volatility and uncertainty, so we treat EPU as measuring only uncertainty.

Figure 2 shows that the 19 IVs span most of the variation in the JLN and EPU uncertainty indexes. We can then measure risk premia associated with those indexes by constructing hedging portfolios using our straddles. For each index, we obtain the weights for the hedging portfolio from the coefficients of the projection we presented in section 2.3. Specifically, for each uncertainty index j, we estimate the regression

$$JLNU_t^j = a + \sum_i b_i^j IV_{i,t} + \varepsilon_{j,t} \tag{9}$$

[22] The construction involves two basic steps. First, realized squared forecast errors are constructed for 280 macroeconomic and financial time series. 134 macro series are from McCracken and Ng (2016), while the remaining financial indicators are from Ludvigson and Ng (2007). Our analysis uses code from the replication files of JLN. The macro price series are defined as those referring to price indexes, and the real series are the remainder of the macro time series. Denoting the error for series i as $\varepsilon_{i,t}$, there is a variance process, $\sigma_{i,t}^2 = E\left[\varepsilon_{i,t}^2\right]$. So $\varepsilon_{i,t}^2$ constitutes a noisy signal about $\sigma_{i,t}^2$. JLN then estimate $\sigma_{i,t}^2$ from the history of $\varepsilon_{i,t}^2$ using a two-sided smoother and create an uncertainty index as the first principal component of the estimated $\sigma_{i,t}^2$. For the component indexes, we take the first principal component of the $\sigma_{i,t}^2$ corresponding to the relevant group of indicators.

[23] This is done by taking the first principal component from the cross-section of the $\varepsilon_{i,t}^2$ in a given month, instead of the $\sigma_{i,t}^2$.

We then use the risk premia estimated in the factor model to calculate a premium for hedging the JLN indexes. In particular, we construct a hypothetical portfolio that has exposure b_i^j to $\Delta IV_{i,t}/IV_{i,t-1}$. The mean return on that portfolio can be calculated from equation (3), while the standard deviation is obtained from the covariance matrix of $\Delta IV_{i,t}/IV_{i,t-1}$ across i (again weighting by b_i^j). The same method also yields a risk premium for the EPU and $JLNRV$ indexes (see appendix figure A.1 for the analogous of figure 2 for realized volatilities).

The right-hand section of figure 3 (red lines) reports the Sharpe ratios for straddle portfolios hedging the EPU and JLN indexes, computed using the estimates from the factor models. Since those hedging premia are constructed combining the individual factor premia, it is not surprising that they are similar. In all three cases, the risk premium for JLN indexes – financial, macro, and price uncertainty – is positive, in one case statistically significantly. Furthermore, the confidence bands rule out economically large negative premia – the lowest confidence band only runs to -0.32. For EPU we find a point estimate of approximately zero (-0.03), though a confidence band that runs to -0.49.

The right-hand section of the bottom panel of figure 3 reports the returns from the JLN realized volatility hedging portfolios (again, the red lines use the risk premia estimates from the factor model). Again, consistent with the fact that the RV risk premia themselves are consistently negative, hedging the JLN indexes for realized volatility historically has a positive cost. For all three subindexes, the risk premia are very negative, with the Sharpe ratios for financial, real, and price volatility at -1.15, -0.62, and -0.65, all three of which are statistically significant. So the conclusions from hedging the JLN and EPU indexes are highly similar to those in the main analysis, providing further evidence that in the macroeconomy, it is realized volatility that is priced, rather than uncertainty about the future. The blue lines in the figure, that use the estimates from the rv and iv portfolio, show similar results, with the uncertainty Sharpe ratios slightly lower but still statistically indistinguishable from zero, and the realized volatility premia strongly negative. Figure 5 shows that the results for straddles are again similar, with hedging realized volatility in this case again carrying a more negative premium.

6 Robustness

This section examines some potential concerns about the robustness of the results.

6.1 One-week holding period returns

Our main analysis is based on two-week holding period returns for straddles, which strike a balance between having more precise estimates of risk premia and reducing the impact of measurement error in prices. We have repeated all of our analysis using one-week holding period returns, and find very similar results. Appendix figure A.6 is the analog of figure 3, but constructed using one-week returns. The results are qualitatively and quantitatively similar to the baseline.

6.2 Split sample and rolling window results

To address the concern that the results could be driven by outliers (though note that there would need to be outliers in all 19 markets), figures A.7 and A.8 replicate the main results in figure 3, but splitting the sample in half (before and after June 2000). The confidence bands are naturally wider, and the point estimates vary more from market to market in the two figures. Nevertheless, the qualitative results are the same as in the full-sample case, showing that realized volatility earns a negative premium while the premium on implied volatility is positive.

To further evaluate the possibility that the results are driven by a small number of observations, figure A.9 plots Sharpe ratios for the rv and iv portfolios in five-year rolling windows for each of the 19 markets, as well as for the equal-weighted portfolios of all 19 markets. The sample Sharpe ratios are reasonably stable over time. In no case do the results appear to be driven by a single outlying period or episode. Note that these results are not informative about variation in the conditional risk premium; with a five-year window, the standard error for the Sharpe ratios is 0.45, so even if the true conditional Sharpe ratios are constant, the five-year rolling estimates should display large amounts of variation over time.

6.3 Alternative maturities

Our main results use the five-month maturity for implied volatility, both in the factor model and as the second leg in the rv and iv portfolios. Figure A.10 in the appendix replicates the analysis using two-month implied volatility instead in both cases. The results are qualitatively and quantitatively similar to the main specification. Note that the GARCH effects – that bias the estimates for the iv portfolio risk premium (blue) in the top panel downward relative to the estimates from the factor model (red) – are stronger when using 2-month IV instead of 5-month IV (see the loadings of the iv portfolio on realized volatility in table A.4).

To help understand why the maturity choice does not have strong effects, the top panel of table A.3 in the appendix reports loadings of the *rv* portfolio on changes in implied volatility at maturities of one to five months. In all cases, the coefficients are close to zero – no larger than 0.1 – indicating that the exposures to implied volatility at any maturity are economically small (especially in comparison to the loading on realized volatility, which can be seen from table A.2 to be closer to 1). The bottom panel shows the same loadings, but for the RV-hedging portfolio built using the factor model (a portfolio that by construction has loading 1 on RV and 0 on 5-month IV, as the last column of the table highlights – see Appendix A.3 for more details).

6.4 Weighted least squares

Johnson (2019) argues that there can be efficiency gains from weighting by implied volatility in estimating risk premia. We explore that in figure A.11 in the appendix, which reports the risk premia (computed with the factor model) with and without weighting by implied volatility. Weighting drives most of the risk premia to be less negative or more positive, but the patterns all remain qualitatively and quantitatively similar. The premium for implied volatility shocks becomes even more strikingly positive.

6.5 Pricing the independent parts of realized and implied volatility

The main results above report returns associated with assets that hedge innovations to realized and implied volatility. Table 4 shows that those returns are positively correlated: months with increases in realized volatility also tend to have increases in implied volatility. A natural question is what would happen if we were to construct a portfolio that loaded on the independent part of those returns, e.g. an increase in implied volatility holding realized volatility fixed. Section A.8 in the appendix reports an SDF-based analysis that prices the independent components and shows that the results are similar to the main specification (see figure A.12).

6.6 Liquidity

If the options used here are highly illiquid, the analysis will be substantially complicated for three reasons. First, to the extent that illiquidity represents a real cost faced by investors – e.g. a bid/ask spread – then returns calculated from settlement prices do not represent returns earned by investors. Second, illiquidity itself could carry a risk premium that the options might be exposed to. Third, bid/ask spreads represent an added layer of noise in prices. The identification of the premia for realized volatility and uncertainty depends on differences in returns on options across maturities, so what is most important for our purposes is how liquidity varies across maturities. This section shows that the liquidity of the straddles studied here is generally highly similar to that of the widely studied S&P 500 contracts traded on the CBOE, and the liquidity does not appear to substantially deteriorate across maturities.

While a long history of bid/ask spreads is not available to us, we obtained posted bid/ask spreads for the options closest to the money on Friday, 8/4/2017 for our 19 contracts plus the CBOE S&P 500 options at maturities of 1, 4, and 7 months. Those spreads are plotted in figure A.13. For the majority of the options, the spreads are less than 3 percent, consistent with the 4.1-percent bid/ask spread for one-month S&P 500 options at the CBOE. Across nearly all the contracts, the posted spreads again decline with maturity, and for 10 of the 19 contracts the one-month posted spreads are nearly indistinguishable from that for the S&P 500, which is typically viewed as a highly liquid market and where incorporating bid-ask spreads generally has minimal effects on return calculations (Bondarenko (2014)).

Figure A.13 yields two important results. First, it shows that the liquidity of the straddles is reasonably high, in the sense that posted spreads are currently relatively narrow in absolute terms for most of the contracts and that they compare favorably with spreads for the more widely studied S&P 500 options traded at the CBOE. Second, liquidity does not appear to deteriorate as the maturity of the options grows, and in fact in many cases there are improvements with increasing maturities, again consistent with CBOE data.

Section A.4.5 in the appendix reports statistics for volume across maturities, showing that the markets are generally fairly similar. Section A.4.6 reports an additional robustness test that measures returns using a method that is robust to certain types of measurement errors in prices, showing that the main results are essentially identical.

Finally, it is useful to note that while the liquidity of option markets changed significantly in the last 30 years, the patterns in risk premia for the rv and iv portfolios appear stable over time (see, for example, the rolling Sharpe ratios of figure A.9), suggesting that liquidity is not the main driver of our results.

Even though the liquidity is similar across many of the markets, one might still ask how trading costs affect the returns we have been studying. Any trading cost will lower the return of a portfolio, regardless of whether an investor is long or short. By studying returns based on quoted prices, we are essentially looking at the return averaged across what the buyer and seller receive. For example, if the return on a portfolio based on quoted prices is 10 percent and there are total trading costs to each side of 1 percent, then the buyer earns a return of 9 percent while the seller has a loss of 11 percent. Looking at quotes is therefore natural for illustrating the return that the average investor sees.

7 Conclusion

This paper studies the pricing of uncertainty and realized volatility across a broad array of options on financial and commodity futures. Uncertainty is proxied by implied volatility – which theoretically measures investors' conditional variances for future returns – and a number of uncertainty indexes developed in the literature. Realized volatility, on the other hand, measures how large *realized* shocks have been. In modeling terms, if $\varepsilon_{t+1} \sim N(0, \sigma_t^2)$, uncertainty is σ_t^2, while volatility is the realization of ε_t^2.

A large literature in macroeconomics and finance has focused on the effects of uncertainty on the economy. This paper explores empirically how investors perceive uncertainty shocks. If uncertainty shocks have major contractionary effects so that they are associated with high marginal utility for the average investor, then assets that hedge uncertainty should earn negative average returns. On the other hand, the finance literature has recently argued that in many cases uncertainty can be good. For example, during the late 1990's, it may have been the case that investors were not sure about how *good* new technologies would turn out to be.

The contribution of this paper is to construct hedging portfolios for a range of types of macro uncertainty, including interest rates, energy prices, and uncertainty indexes. The cost of hedging uncertainty shocks reveals the relative importance of good and bad types of uncertainty. Furthermore, using a wide range of options is important for capturing uncertainty about the real economy and inflation, as opposed to just about financial markets. The empirical results imply that uncertainty shocks, no matter what type of uncertainty we look at, are not viewed as being negative by investors, or at least not sufficiently negative that it is costly to hedge them. Financial uncertainty appears to be roughly equally split between the good and bad types, while nonfinancial uncertainty is relatively more strongly driven by good uncertainty – the cost of hedging nonfinancial uncertainty shocks is negative.

What is highly costly to hedge is realized volatility. Portfolios that hedge extreme returns in futures markets – and hence large innovations in macroeconomic time series – earn strongly negative returns, with premia that are in many cases one to two times as large as the premium on the aggregate stock market over the same period. So what is consistently high in bad times is not uncertainty, but realized volatility. Periods in which futures markets and the macroeconomy are highly volatile and display large movements appear to be periods of high marginal utility, in the sense that their associated state prices are high. This is consistent with (and complementary to) the findings in Berger, Dew-Becker, and Giglio (2019), who provide VAR evidence that shocks to volatility predict declines in real activity in the future, while shocks to uncertainty do not.

Berger, Dew-Becker, and Giglio (2019) show that the VAR evidence and pricing results for realized volatility are consistent with the view that it is downward jumps in the economy that investors are most averse to. They show that a simple model in which fundamental shocks are both stochastically volatile and negatively skewed can quantitatively match the pricing of uncertainty and realized volatility, along with the VAR evidence. Similarly, Seo and Wachter (2018a,b) show that negative skewness can explain the pricing of credit default swaps and put options. This paper thus also contributes to the growing literature studying the effects of skewness. In a world where fundamental shocks are negatively skewed, the most extreme shocks – those that generate realized volatility – tend to be negative, which can explain why realized volatility would be so costly to hedge.

References

Ait-Sahalia, Yacine, Mustafa Karaman, and Loriano Mancini, "The Term Structure of Variance Swaps, Risk Premia and the Expectations Hypothesis," 2019. Forthcoming, Journal of Econometrics.

Alexopoulos, Michelle and Jon Cohen, "Uncertain Times, Uncertain Measures," 2009. Working paper.

Andersen, Torben G, Nicola Fusari, and Viktor Todorov, "The risk premia embedded in index options," *Journal of Financial Economics*, 2015, *117* (3), 558–584.

___ , ___ , and ___ , "Short-term market risks implied by weekly options," *The Journal of Finance*, 2017, *72* (3), 1335–1386.

Andries, Marianne, Thomas M Eisenbach, Martin C Schmalz, and Yichuan Wang, "The term structure of the price of variance risk," 2015.

Asness, Clifford S, Tobias J Moskowitz, and Lasse Heje Pedersen, "Value and momentum everywhere," *The Journal of Finance*, 2013, *68* (3), 929–985.

Bachmann, Rudiger and Christian Bayer, ""Wait-and-See" business cycles?," *Journal of Monetary Economics*, 2013, *60* (6), 704–719.

___ and Giuseppe Moscarini, "Business Cycles and Endogenous Uncertainty," 2012. Working paper.

Baker, Scott R, Nicholas Bloom, and Steven J Davis, "Measuring economic policy uncertainty," *The Quarterly Journal of Economics*, 2016, *131* (4), 1593–1636.

Bakshi, Gurdip and Nikunj Kapadia, "Delta-Hedge Gains and the Negative Market Volatility Risk Premium," *The Review of Financial Studies*, 2003, *16(2)*, 527–566.

——, ——, and Dilip Madan, "Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options," *Review of Financial Studies*, 2003, *16* (1), 101–143.

Barro, Robert J., "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 2006, *121(3)*, 823–866.

Basu, Susanto and Brent Bundick, "Uncertainty Shocks in a Model of Effective Demand," 2015. Working paper.

Bekaert, Geert, Eric Engstrom, and Andrey Ermolov, "Bad environments, good environments: A non-Gaussian asymmetric volatility model," *Journal of Econometrics*, 2015, *186* (1), 258–275.

——, Marie Hoerova, and Marco Lo Duca, "Risk, uncertainty and monetary policy," *Journal of Monetary Economics*, 2013, *60* (7), 771–788.

Berger, David and Joseph Vavra, "Volatility and Pass-through," 2013. Working paper.

Binsbergen, Jules H. Van and Ralph S.J. Koijen, "The Term Structure of Returns: Facts and Theory," *Journal of Financial Economics*, 2017, *124* (1), 1–21.

Black, Fischer, "The pricing of commodity contracts," *Journal of Financial Economics*, 1976, *3* (1-2), 167–179.

—— and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *The Journal of Political Economy*, 1973, *81* (3), 637–654.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77(3)*, 623–685.

——, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry, "Really Uncertain Business Cycles," 2014. Working paper.

Bollerslev, Tim, "Generalized autoregressive conditional heteroskedasticity," *Journal of econometrics*, 1986, *31* (3), 307–327.

—— and Viktor Todorov, "Tails, fears, and risk premia," *Journal of Finance*, 2011, *66(6)*, 2165–2211.

Bondarenko, Oleg, "Why Are Put Options So Expensive?," *Quarterly Journal of Finance*, 2014, *04* (03), 1450015.

Bretscher, Leonid, Alex Hsu, and Andrea Tamoni, "The Real Response to Uncertainty Shocks: the Risk Premium Channel," 2019. Working paper.

Bretscher, Lorenzo, Lukas Schmid, and Andrea Vedolin, "Interest rate risk management in uncertain times," *The Review of Financial Studies*, 2018, *31* (8), 3019–3060.

Broadie, Mark, Mikhail Chernov, and Michael Johannes, "Understanding Index Option Returns," *The Review of Financial Studies*, 2009, *22(11)*, 4493–4529.

Caballero, Ricardo J, "Aggregate investment," *Handbook of macroeconomics*, 1999, *1*, 813–862.

Cesa-Bianchi, Ambrogio, M. Hashem Pesaran, and Alessandro Rebucci, "Uncertainty and Economic Activity: A Multi-Country Perspective," 2018. Working paper.

Choi, Hoyong, Philippe Mueller, and Andrea Vedolin, "Bond Variance Risk Premiums," *Review of Finance*, 2017, *21* (3), 987–1022.

Cochrane, John H., *Asset Pricing*, Princeton University Press, 2005.

Constantinides, George M, Jens Carsten Jackwerth, and Alexi Savov, "The puzzle of index option returns," *Review of Asset Pricing Studies*, 2013, *3* (2), 229–257.

Cremers, Martijn, Michael Halling, and David Weinbaum, "Aggregate Jump and Volatility Risk in the Cross-Section of Stock Returns," *The Journal of Finance*, 2015, *70* (2), 577–614.

Darby, Julia, Andrew Hughes Hallett, Jonathan Ireland, and Laura Piscitelli, "The impact of exchange rate uncertainty on the level of investment," *The Economic Journal*, 1999, *109* (454), 55–67.

Decker, Ryan, Pablo N. D'Erasmo, and Hernan Moscoso Boedo, "Market exposure and endogenous firm volatility over the business cycle," *American Economic Journal: Macroeconomics*, 2016, *8* (1), 148–198.

Dew-Becker, Ian, Alireza Tahbaz-Salehi, and Andrea Vedolin, "Skewness and stochastic volatility in a network production model," 2019. Working paper.

___ **, Stefano Giglio, Anh Le, and Marius Rodriguez**, "The price of variance risk," *Journal of Financial Economics*, 2017, *123* (2), 225 – 250.

Diercks, Anthony, Alex Hsu, and Andrea Tamoni, "When It Rains It Pours: Cascading Uncertainty Shocks," 2019. Working paper.

Drechsler, Itamar and Amir Yaron, "What's Vol Got to Do with it?," *The Review of Financial Studies*, 2011, *24(1)*, 1–45.

Egloff, Daniel, Markus Leippold, and Liuren Wu, "The Term Structure of Variance Swap Rates and Optimal Variance Swap Investments," *Journal of Financial and Quantitative Analysis*, 2010, *45(5)*, 1279–1310.

Elder, John, "Another perspective on the effects of inflation uncertainty," *Journal of Money, Credit, and Banking*, 2004, *36* (5), 911–928.

___ **and Apostolos Serletis**, "Oil price uncertainty," *Journal of Money, Credit and Banking*, 2010, *42* (6), 1137–1159.

Engle, Robert F, "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econometrica: Journal of the Econometric Society*, 1982, pp. 987–1007.

Farago, Adam and Roméo Tédongap, "Downside risks and the cross-section of asset returns," *Journal of Financial Economics*, 2018, *129* (1), 69–86.

Gilchrist, Simon and John C Williams, "Investment, capacity, and uncertainty: a putty–clay approach," *Review of Economic Dynamics*, 2005, *8* (1), 1–27.

Gourio, Francois, "Credit Risk and Disaster Risk," *American Economic Journal: Macroeconomics*, 2013, *5(3)*, 1–34. Working paper.

Herskovic, Bernard, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh, "The common factor in idiosyncratic volatility: Quantitative asset pricing implications," *Journal of Financial Economics*, 2016, *119* (2), 249–283.

Huizinga, John, "Inflation uncertainty, relative price uncertainty, and investment in US manufacturing," *Journal of Money, Credit and Banking*, 1993, *25* (3), 521–549.

Ilut, Cosmin, Matthias Kehrig, and Martin Schneider, "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News," *NBER Working Paper Series*, 2015.

Jones, Christopher S, "A nonlinear factor analysis of S&P 500 index option returns," *The Journal of Finance*, 2006, *61* (5), 2325–2363.

Jurado, Kyle, Sydney Ludvigson, and Serena Ng, "Measuring Uncertainty," *American Economic Review*, 2015, *105* (3), 1177–1216.

Kozlowski, Julian, Laura Veldkamp, and Venky Venkateswaran, "The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation," 2016. Working paper.

Leduc, Sylvain and Zheng Liu, "Uncertainty shocks are aggregate demand shocks," *Journal of Monetary Economics*, 2016, *82*, 20–35.

Ludvigson, Sydney C, Sai Ma, and Serena Ng, "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?," Technical Report, National Bureau of Economic Research 2015.

McCracken, Michael W and Serena Ng, "FRED-MD: A monthly database for macroeconomic research," *Journal of Business & Economic Statistics*, 2016, *34* (4), 574–589.

Pástor, L'uboš and Pietro Veronesi, "Technological Revolutions and Stock Prices," *The American Economic Review*, 2009, pp. 1451–1483.

Prokopczuk, Marcel, Lazaros Symeonidis, and Chardin Wese Simen, "Variance risk in commodity markets," *Journal of Banking & Finance*, 2017, *81*, 136–149.

Salgado, Sergio, Fatih Guvenen, and Nicholas Bloom, "Skewed Business Cycles," 2016. Working paper.

Schwert, G William, "Business cycles, financial crises, and stock volatility," in "Carnegie-Rochester Conference series on public policy," Vol. 31 Elsevier 1989, pp. 83–125.

—, "Stock volatility during the recent financial crisis," *European Financial Management*, 2011, *17* (5), 789–805.

Segal, Gill, Ivan Shaliastovich, and Amir Yaron, "Good and bad uncertainty: Macroeconomic and financial market implications," *Journal of Financial Economics*, 2015, *117* (2), 369–397.

Seo, Sang Byung and Jessica A. Wacheter, "Do Rare Events Explain CDX Tranche Spreads?," *The Journal of Finance*, *0* (ja).

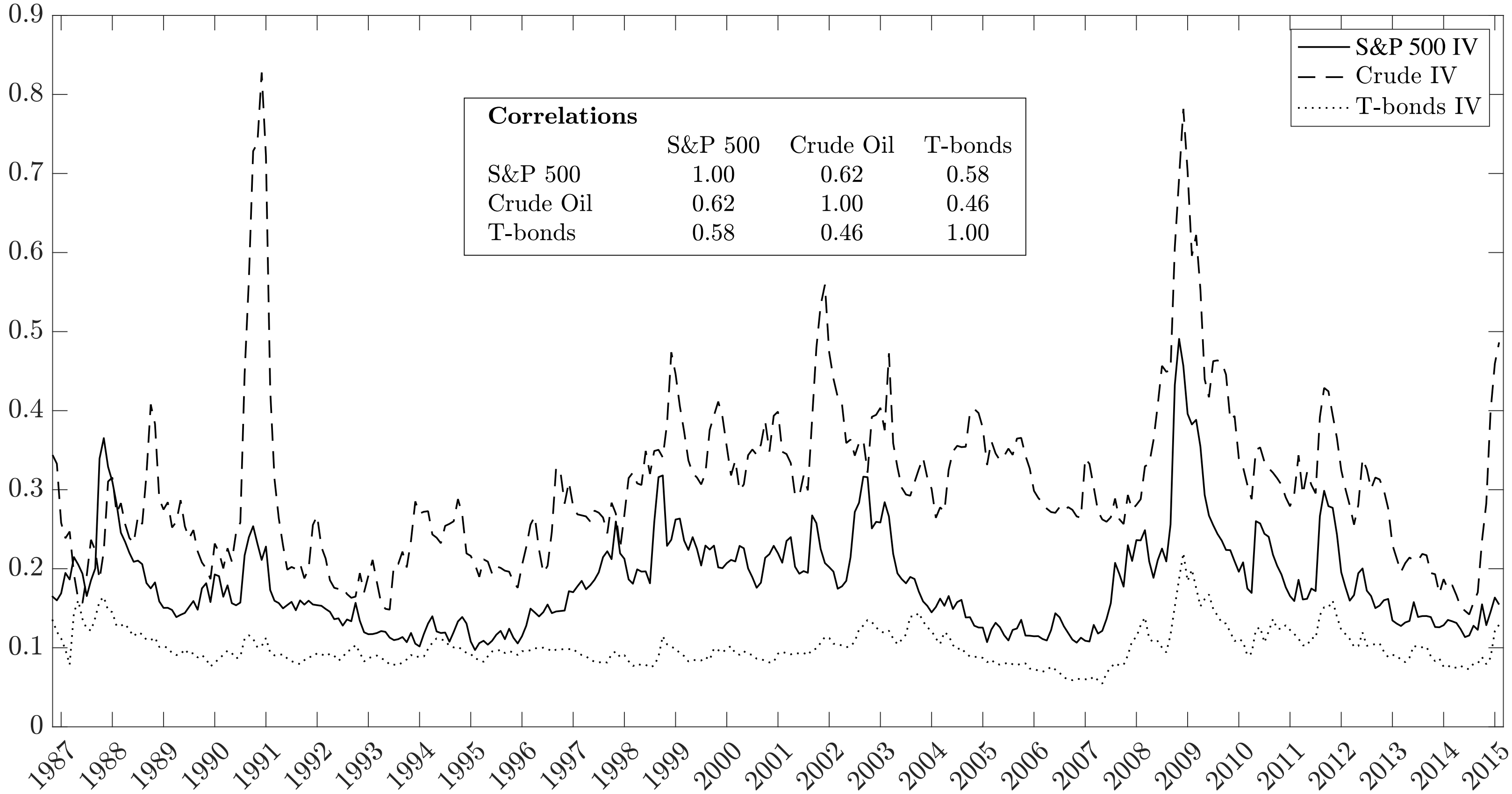
——— **and Jessica A Wachter**, "Option prices in a model with stochastic disaster risk," *Management Science*, 2018.

Siriwardane, Emil, "The probability of rare disasters: Estimation and implications," *Harvard Business School Finance Working Paper*, 2015, (16-061).

Trolle, Anders B and Eduardo S Schwartz, "Variance risk premia in energy commodities," *The Journal of Derivatives*, 2010, *17* (3), 15–32.

Wachter, Jessica A., "Can time-varying risk of rare disasters explain aggregate stock market volatility?," *Journal of Finance*, 2013, *68(3)*, 987–1035.

Figure 1: Sample implied volatilities

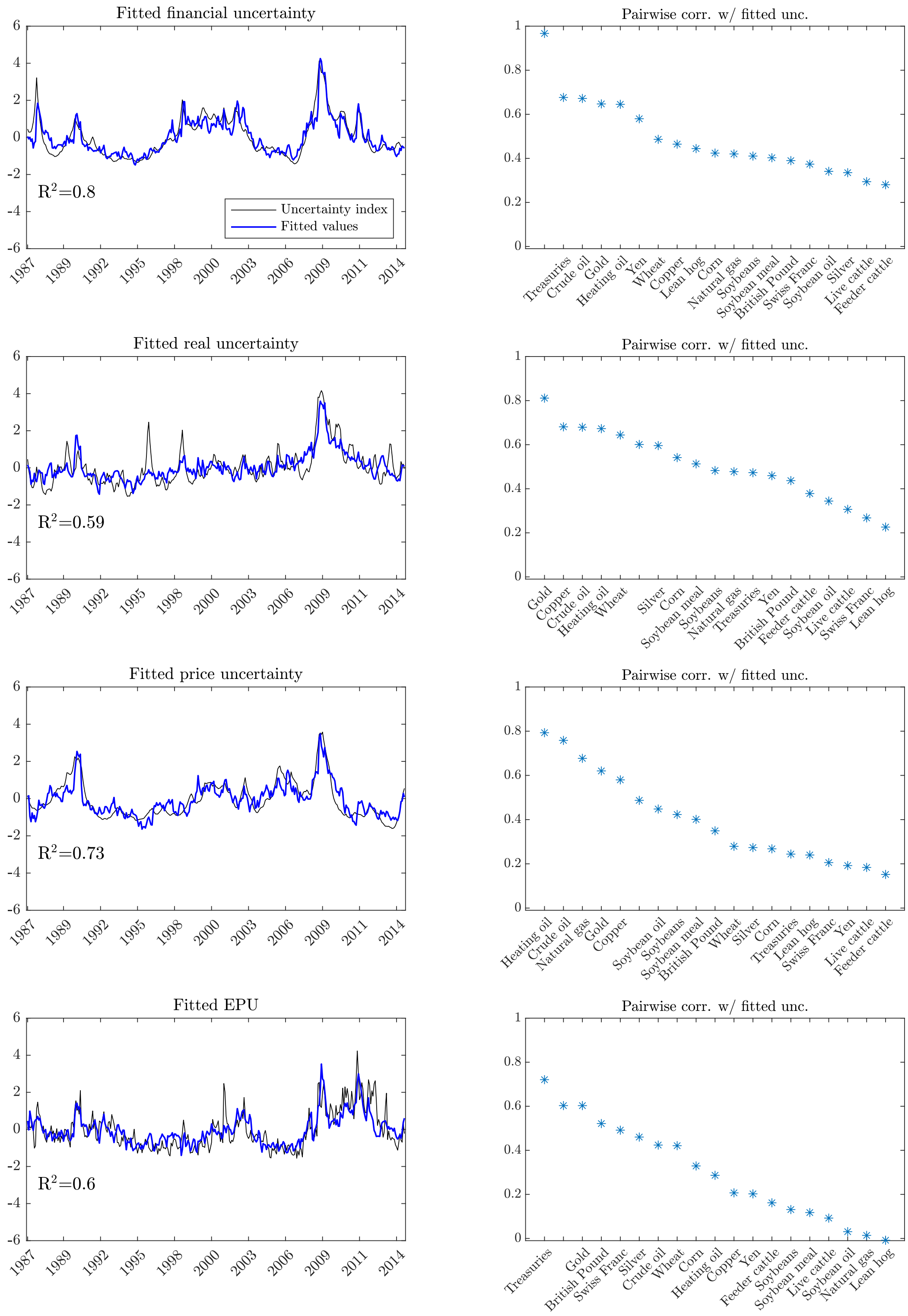


Correlations

	S&P 500	Crude Oil	T-bonds
S&P 500	1.00	0.62	0.58
Crude Oil	0.62	1.00	0.46
T-bonds	0.58	0.46	1.00

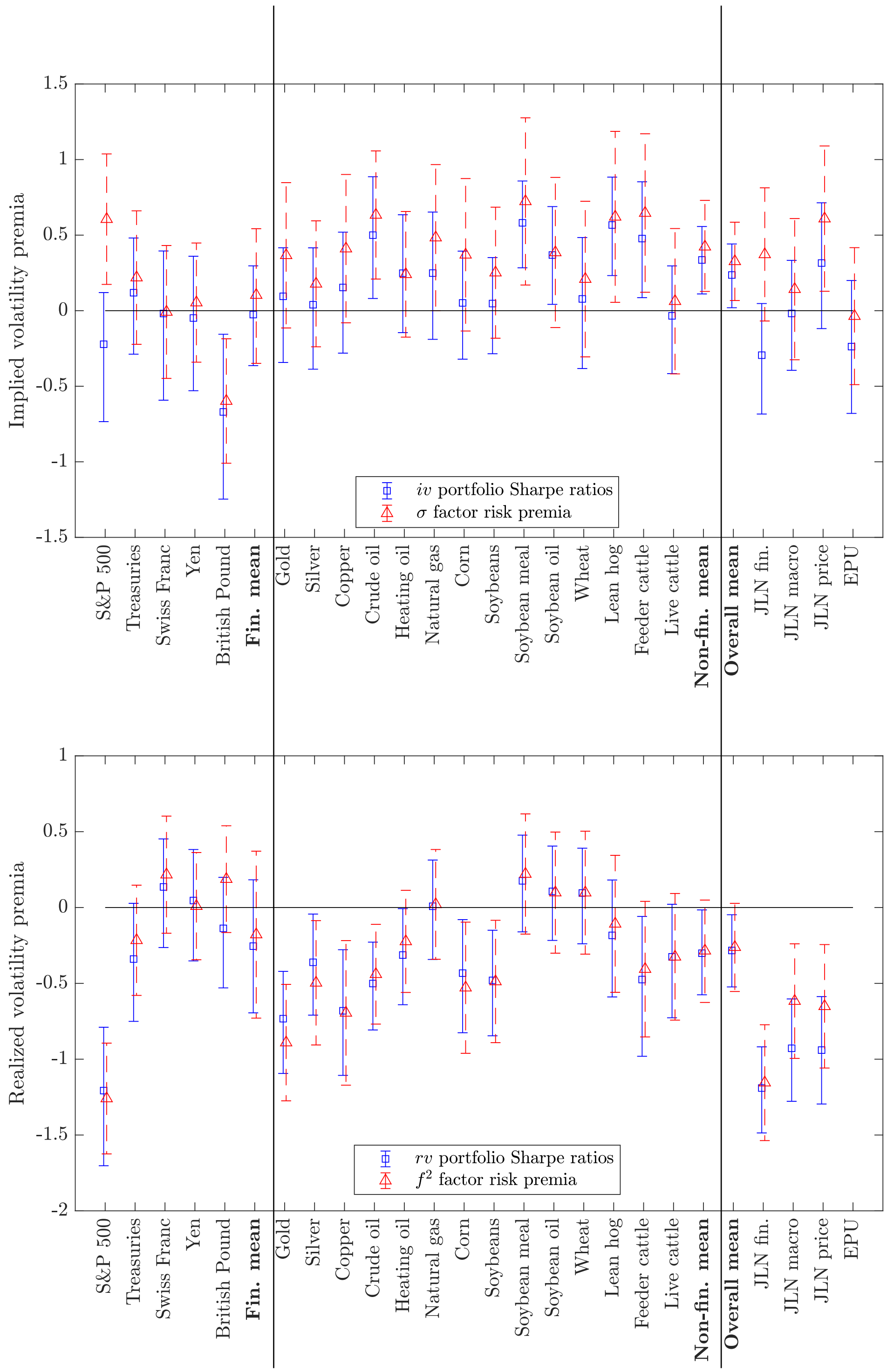
Note: Monthly implied volatilities calculated from three-month options using the Black–Scholes model.

Figure 2: Fit to uncertainty indexes



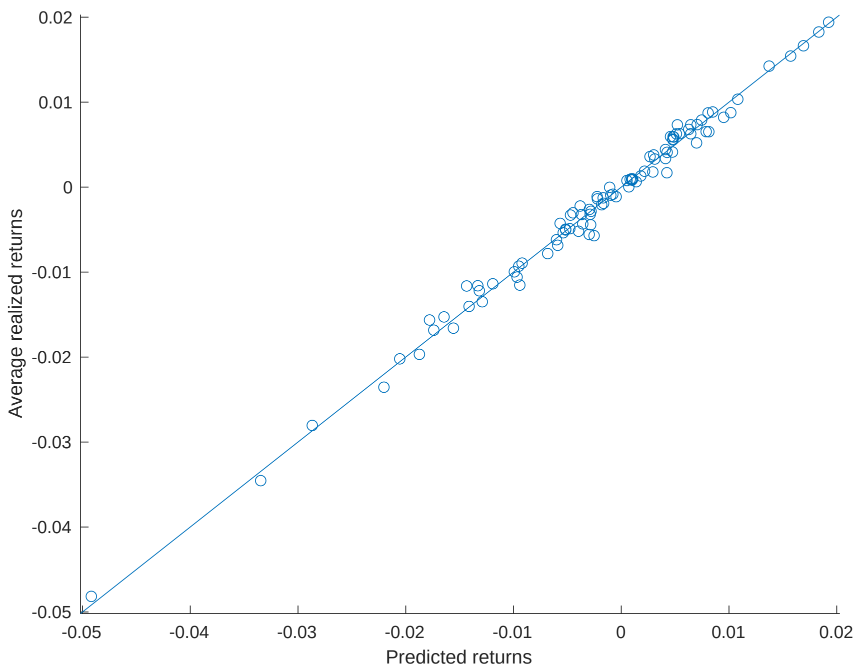
Note: The left-hand panels plot the fitted values from the regressions of the EPU and JLN indexes on three-month implied volatility in the 19 markets. The right-hand panels plot pairwise correlations between the individual implied volatility series and the fitted values from the regressions.

Figure 3: RV and IV portfolio Sharpe ratios and factor risk premia: straddles



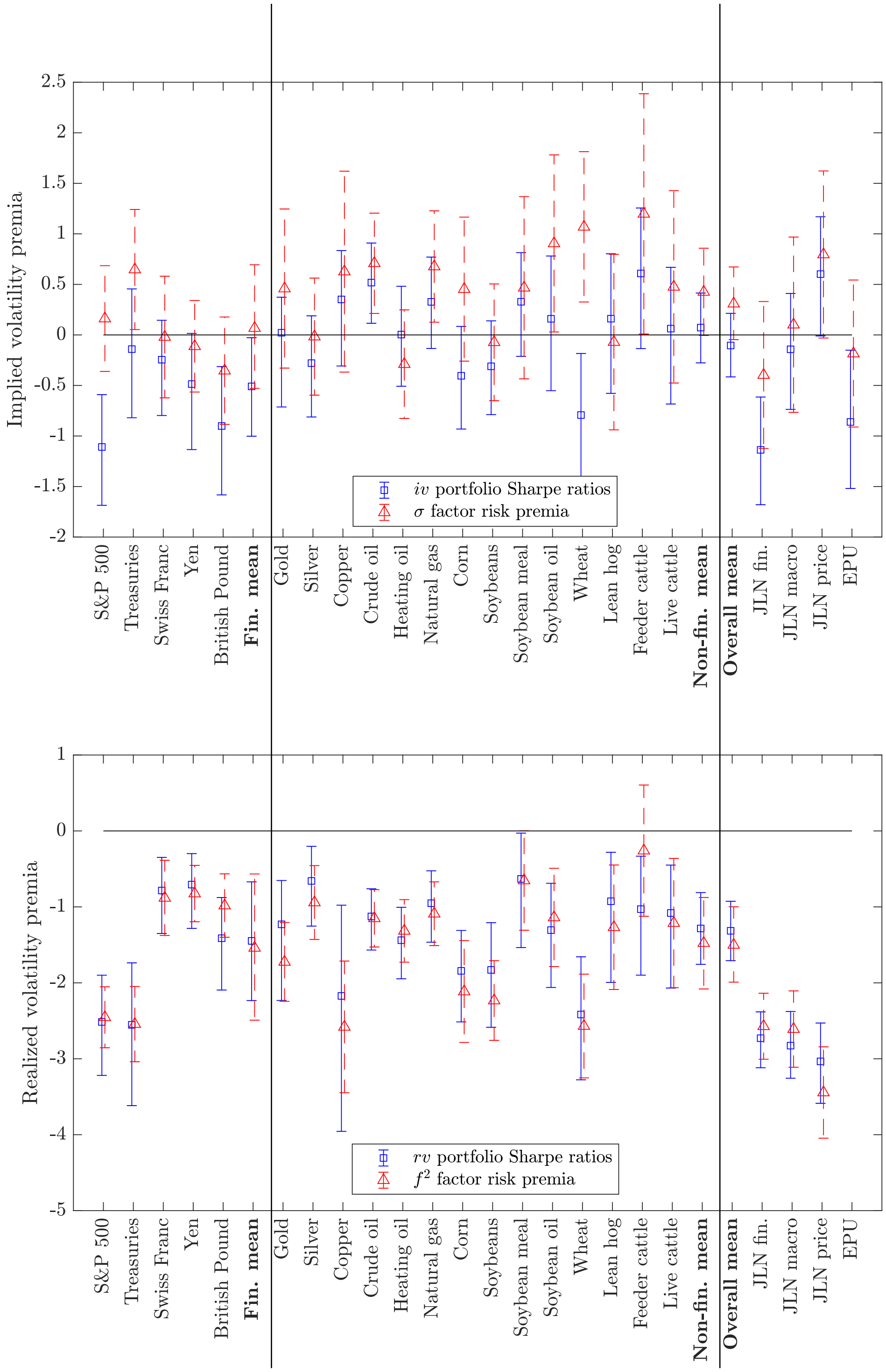
Note: Squares are point estimates and vertical lines represent 95-percent confidence intervals. The solid series plots the Sharpe ratios for the *rv* and *iv* portfolios. The dotted series plots the estimated risk premia from the factor model. In both cases, all estimation uses straddles. The confidence bands for the *rv* and *iv* Sharpe ratios are calculated through a 50-day block bootstrap, while those for the factor model use GMM standard errors with the Hansen–Hodrick (1980) method used to calculate the long-run variance. The "Fin. mean", "Non-fin. mean", and "Overall mean" points represent random effects estimates of group-level and overall means. The "JLN" and "EPU" points are for the portfolios that hedge those indexes.

Figure 4: Cross-sectional fit of factor models



Note: For each straddle of maturity 1 to 5 months, and for each of the 19 markets, the figure reports the predicted risk premium against the realized average excess return. Predicted risk premia are obtained estimating a linear factor model separately in each market.

Figure 5: RV and IV portfolio Sharpe ratios and factor risk premia: strangles



Note: See figure 3. This figure differs only in replacing the straddles with 1-sigma strangles.

Table 1: Pairwise correlations of implied volatility across markets

IV	Treasuries	S&P 500	Swiss Franc	Yen	British Pound	Gold	Silver	Copper	Crude oil	Heating oil	Natural gas	Corn	Soybeans	Soybean meal	Soybean oil	Wheat	Lean hog	Feeder cattle
S&P 500	0.56																	
Swiss Franc	0.53	0.29																
Yen	0.40	0.56	0.48															
British Pound	0.45	0.40	0.75	0.45														
Gold	0.52	0.57	0.21	0.28	0.37													
Silver	0.42	0.34	0.19	0.29	0.34	0.78												
Copper	0.39	0.49	0.15	0.35	0.36	0.74	0.77											
Crude oil	0.42	0.63	0.25	0.39	0.27	0.54	0.31	0.48										
Heating oil	0.41	0.64	0.23	0.36	0.23	0.51	0.28	0.51	0.95									
Natural gas	0.11	0.44	-0.03	0.04	0.03	0.33	0.06	0.44	0.49	0.63								
Corn	0.25	0.37	-0.11	0.14	0.11	0.50	0.56	0.58	0.22	0.18	0.11							
Soybeans	0.22	0.35	-0.05	0.17	0.17	0.47	0.48	0.57	0.29	0.29	0.21	0.85						
Soybean meal	0.28	0.33	-0.08	0.16	0.06	0.53	0.50	0.57	0.30	0.27	0.23	0.81	0.94					
Soybean oil	0.31	0.30	0.10	0.12	0.23	0.48	0.49	0.56	0.26	0.29	0.23	0.73	0.89	0.83				
Wheat	0.38	0.42	0.01	0.19	0.10	0.62	0.62	0.60	0.34	0.31	0.17	0.84	0.77	0.75	0.64			
Lean hog	0.29	0.42	-0.03	0.28	-0.10	0.27	0.16	0.35	0.40	0.47	0.40	0.29	0.37	0.39	0.38	0.36		
Feeder cattle	0.45	0.35	0.11	0.16	0.07	0.40	0.51	0.50	0.31	0.34	0.13	0.48	0.47	0.50	0.48	0.52	0.43	
Live cattle	0.51	0.28	0.24	0.18	0.07	0.38	0.41	0.45	0.32	0.39	0.26	0.32	0.33	0.43	0.49	0.43	0.47	0.84

Note: Pairwise correlations of three-month option-implied volatility across markets. The darkness of the shading represents the degree of correlation.

Table 2: Pairwise correlations of realized volatility across markets

RV	Treasuries	S&P 500	Swiss Franc	Yen	British Pound	Gold	Silver	Copper	Crude oil	Heating oil	Natural gas	Corn	Soybeans	Soybean meal	Soybean oil	Wheat	Lean hog	Feeder cattle
S&P 500	0.63																	
Swiss Franc	0.17	0.12																
Yen	0.31	0.32	0.15															
British Pound	0.43	0.36	0.24	0.31														
Gold	0.44	0.47	0.15	0.24	0.31													
Silver	0.42	0.43	0.15	0.22	0.27	0.65												
Copper	0.52	0.51	0.11	0.24	0.43	0.50	0.53											
Crude oil	0.24	0.24	0.13	0.20	0.20	0.32	0.14	0.24										
Heating oil	0.20	0.22	0.04	0.14	0.15	0.30	0.11	0.15	0.91									
Natural gas	0.03	0.08	0.04	-0.04	0.00	0.05	-0.06	0.00	0.08	0.18								
Corn	0.33	0.35	0.04	0.09	0.27	0.37	0.40	0.50	0.12	0.03	-0.04							
Soybeans	0.33	0.30	0.03	0.16	0.30	0.33	0.35	0.40	0.11	0.05	-0.07	0.74						
Soybean meal	0.33	0.25	0.03	0.19	0.19	0.31	0.32	0.30	0.08	0.02	-0.06	0.68	0.94					
Soybean oil	0.48	0.43	0.11	0.21	0.42	0.40	0.41	0.51	0.17	0.12	-0.04	0.67	0.88	0.72				
Wheat	0.30	0.24	0.02	0.08	0.11	0.31	0.34	0.33	0.11	0.04	-0.08	0.63	0.51	0.47	0.47			
Lean hog	0.12	0.12	0.08	0.20	-0.03	0.00	0.00	0.05	0.10	0.09	0.11	0.07	0.11	0.12	0.11	0.12		
Feeder cattle	0.22	0.20	0.03	0.04	0.07	0.10	0.16	0.30	0.10	0.07	0.12	0.35	0.32	0.32	0.27	0.22	0.26	
Live cattle	0.41	0.24	0.13	0.11	0.11	0.17	0.24	0.28	0.07	0.07	0.09	0.22	0.22	0.27	0.30	0.23	0.28	0.63

Note: Pairwise correlations of monthly realized volatility across markets. The darkness of the shading represents the degree of correlation.

Table 3: Risk premia for financials and nonfinancials, and their difference

		Financials	Nonfinancials	Difference
Factor model	RV	-0.18	-0.29	0.11
		[-0.63]	[-1.62]	[0.34]
	IV	0.10	0.43	-0.32
		[0.47]	[2.82]	[-1.28]
Replicating port.	rv	-0.25	-0.30	0.05
		[-1.13]	[-2.14]	[0.18]
	iv	-0.02	0.34	-0.36
		[-0.14]	[2.95]	[-2.02]

Note: The table reports the average risk premia for RV and IV risks, across financials (first column), across nonfinancials (second column) and for the difference between the two groups (third column), with corresponding t-statistics in square brackets. The top panel estimates the risk premia using the linear factor model; the bottom panel estimates the risk premia as the average excess returns of the rv and iv portfolios.

Table 4: Correlations between rv and iv portfolio returns in each market

	Std(rv)	Std(iv)	Corr(rv,iv)
S&P 500	0.03	0.08	0.48
T-bonds	0.03	0.08	0.01
CHF	0.04	0.08	0.63
JPY	0.04	0.08	0.61
GBP	0.04	0.07	0.41
Gold	0.04	0.12	0.48
Silver	0.04	0.08	0.45
Copper	0.03	0.10	0.03
Crude Oil	0.04	0.09	0.05
Heating oil	0.04	0.08	0.01
Natural gas	0.04	0.08	-0.17
Corn	0.04	0.08	0.06
Soybeans	0.04	0.09	0.17
Soybean meal	0.04	0.11	0.20
Soybean oil	0.04	0.09	0.21
Wheat	0.04	0.08	0.08
Lean hog	0.05	0.10	-0.24
Feeder cattle	0.05	0.10	0.03
Live cattle	0.04	0.08	-0.12

Note: The table reports, for each underlying, the standard deviation of the two-week returns to the rv and iv portfolios, and their correlation.

Table 5: Portfolios of rv and iv across markets

Panel A: Sharpe ratios

	rv		iv		rv+iv Equal weight		rv+iv Risk-parity	
All underlyings	-0.74	***	0.49	**	1.05	***	0.90	***
Nonfinancials	-0.63	***	0.62	***	0.91	***	0.90	***
Financials	-0.37	**	-0.04		0.42	***	0.13	

Panel B: Skewness

	rv		iv		rv+iv Equal weight		rv+iv Risk-parity	
All underlyings	1.23	***	1.82	***	-0.79	***	1.05	***
Nonfinancials	2.11	***	1.55	***	-2.00	***	0.75	***
Financials	2.01	***	2.91	***	-1.40	***	2.19	***

Note: Sharpe ratios and skewness of portfolios combining rv and iv portfolios across markets. For each panel, the first row reports a portfolio constructed using straddles from all available markets on each date, the second row using only nonfinancial underlyings, the third row only financial underlyings. Each column corresponds to a different portfolio. The first column is an equal-weighted RV portfolio, the second is an equal-weighted IV portfolio, the third is an equal-weighted long-short IV minus RV portfolio, and the last is the same long/short portfolio but weighted by the inverse of the variance (risk-parity). *** indicates significance at the 1-percent level, ** the 5-percent level, and * the 10-percent level.

A.1 Data filters and transformations

The observed option prices very often appear to have nontrivial measurement errors. This section describes the various filters we use and then proceeds to provide more information about the specifics of the data transformations we apply. Code is available on request.

First, we note that the price formats for futures and strike prices for many of the commodities change over time. That is, they will move between, say, 1/8ths, 1/16ths, and pennies. We make the prices into a consistent decimal time series for each commodity by inspecting the prices directly and then coding by hand the change dates.

We then remove all options with the following properties

1. Strikes greater than 5 times the futures price
2. Options with open interest below the 5th percentile across all contracts in the sample
3. Price less then 5 ticks above zero
4. Maturity less than 9 days
5. Maturity greater than 8 months.
6. Options with prices below their intrinsic value (the value if exercised immediately)

Note that in our baseline results, we do not remove options for which we have no volume information, or for which volume is zero. However, we have reproduced our main analysis (figure 3) including that filter and find, if anything, stronger results. We report them in Appendix figure A.5.

We then calculate implied volatilities using the Black–Scholes formula, treating the options as though they are European. We have also replicated the analysis using American implied volatilities and find nearly identical results (the reason is that in most cases we ultimately end up converting the IVs back into prices, meaning that any errors in the pricing formula are largely irrelevant – it is just a temporary data transformation, rather than actually representing a volatility calculation).

The data are then further filtered based on the IVs:

1. Eliminate all zero or negative IVs
2. All options with IV more than 50 percent (in proportional terms) different from the average for the same underlying, date, and maturity
3. We then filter outliers along all three dimensions, strike, date, and maturity, removing the following:
    (a) If the IV changes for a contract by 15 percent or more on a given day then moves by 15 percent or more in the opposite direction in a single day within the next week, and if it moves by less than 3 percent on average over that window, for options with maturity greater than 90 days (this eliminates temporary large changes in IVs that are reversed that tend to be observed early in the life of the options).

(b) If the IV doubles or falls by half in either the first or last observation for a contract

(c) If, looking across maturities at a given strike on a given date, the IV changes by 20 percent or more and then reverses by that amount at the next maturity (i.e. spikes at one maturity). This is restricted to maturities within 90 days of each other.

(d) If the last, second to last, or third to last IV is 40 percent different from the previous maturity.

(e) If, looking across strikes at a given maturity on a given date, the IV changes by 20 percent and reverses at the next strike (for strikes within 10 percent of each other).

(f) If the change in IV at the first or last strike is greater than 20 percent, or the change at the second or second to last option is greater than 30 percent.

At-the-money (ATM) IVs are constructed by averaging the IVs of the options with the first strike below and above the futures price. The ATM IV is not calculated for any observation where we do not have at least one observation (a put or a call) on both sides of the futures price.

To calculate ATM straddle returns for each maturity, we interpolate linearly between the IVs of the two closest out of the money options on either side of the spot, and use this to compute the implied price of the ATM straddle at the beginning of the holding period; similarly, we interpolate linearly the IVs of those options at the end of the holding period, and obtain the corresponding price of the straddle at the end of the holding period. These prices are then used to compute the holding period return. Finally, to calculate returns of straddles at standardized maturities, we interpolate linearly the returns across maturities (which corresponds to a feasible portfolio). If options are not available on the maturities on both sides of the target one, then we use a single straddle if it has a maturity within 35 days of the target maturity.

A.2 Implied volatility and regression forecasts

Implied volatilities are, under certain assumptions, expectations of future realized volatility under the risk-neutral measure. If there is a time-varying volatility risk premium, then implied volatilities will be imperfectly correlated with physical expectations of future realized volatility, which constitutes actual uncertainty. This section compares implied volatilities to regression-based forecasts of future volatility to evaluate the quantitative magnitude of that deviation.

For each market, we estimate the regression

$$RV_{i,t} = a_i + b_i(L)\, RV_{i,t-1} + c_i IV_{i,t-1} + \varepsilon_{i,t} \tag{A.1}$$

where $b_i(L)$ is a polynomial in the lag operator, L, and a_i and c_i are coefficients. $RV_{i,t}$ is realized volatility in month t for market i – the sum of squared daily futures returns during the month. $IV_{i,t}$ is the (at-the-money) implied volatility at the end of month t in market i.

The table below reports the correlation between the fitted values from that regression – which represent physical uncertainty – and implied volatility. That is, it reports $corr\left(b_i\left(L\right)RV_{i,t-1}+c_iIV_{i,t-1},IV_{i,t-1}\right)$. Ideally, we would like that correlation to be 1, so that implied volatility is perfectly correlated with physical uncertainty, and hedging implied volatility hedges uncertainty. Note that this does not require that risk premia are constant. If $b_i\left(L\right)=0$ but $c_i\neq 1$, risk premia are time-varying, but the physical uncertainty is still perfectly correlated with implied volatility. It is only deviations of $b_i\left(L\right)$ from zero that reduce the correlation. To the extent that the implied volatility summarizes all available information, we would expect $b_i=0$.

Correlations of implied volatility with fitted uncertainty

S&P 500	0.966	Crude oil	0.998	Silver	0.984
Treasuries	0.940	Feeder cattle	0.951	Soybeans	0.970
British Pound	0.987	Gold	0.994	Soybean meal	0.974
Swiss Franc	0.994	Heating oil	0.992	Soybean oil	0.946
Yen	0.976	Lean hogs	0.937	Wheat	0.998
Copper	0.963	Live cattle	0.919		
Corn	0.994	Natural gas	0.949		

The table shows that across the various markets, the correlations are all high, with a minimum of 91.1 percent and a mean of 97.0 percent. So while implied volatility is not literally the same as physical uncertainty, it appears to be fairly close. In the baseline results, we allow for two lags in the polynomial b, but we have experimented with alternative specifications and obtain similar results.

A.3 Factor models and factor-hedging portfolios

In this section we review a useful result from the algebra of cross-sectional regressions: given a set of K nontradable factors F_t, the cross-sectional estimates of the K risk premia, λ, are the average excess returns of K portfolios, each of which has betas of exactly 1 with respect to one factor, and 0 with respect to the other $K-1$ factors: we refer to these as *factor-hedging portfolios* for the K factors in F_t. The time series of returns for the factor-hedging portfolios are the slopes of period-by-period cross-sectional regressions. These results hold in population.

Consider K nontradable factors F_t, and a vector of N excess returns r_t of test assets. Nontradable factors have a risk premium of λ (a $K\times 1$ vector), so the factor model can be written as:

$$\underbrace{r_t}_{N\times 1}=\underbrace{\beta}_{N\times K}\underbrace{\lambda}_{K\times 1}+\underbrace{\beta}_{N\times K}\underbrace{(F_t-E[F_t])}_{K\times 1}+\underbrace{e_t}_{K\times 1} \tag{A.2}$$

Cross-sectional regressions operate in two stages. First, they estimate the $N\times K$ matrix β from time series regressions of the form:

$$r_t=k+\beta F_t+e_t$$

where the constant k also depends on the means of the factors $E[F_t]$, which is not interpretable in general when factors are nontradable, and is irrelevant for computing β. The second step of the cross-sectional regression could either be estimated using *average* returns (in one cross-sectional regression), or as a sequence of period-by-period cross-sectional regressions. The latter approach is often used in practice (as in the Fama-MacBeth version of the two-step regressions) because it makes standard errors calculation easier, but either method yields the same point estimates for risk premia λ. Here, we also follow the second method, but for a different reason: because it generates a time-series of factor-hedging portfolios.

We therefore run, for each period t, cross-sectional regressions of r_t on the estimated β:

$$r_t = a_t + \beta g_t + u_t$$

obtaining a time-series of $K \times 1$ slope vectors g_t. Risk premia λ are then estimated as the time-series average of the slopes: $\lambda = E[g_t]$.

The time-series slopes g_t have a useful interpretation. They are calculated in each period as:

$$g_t = (\beta' \beta)^{-1} \beta' r_t \tag{A.3}$$

This equation clarifies that g_t are themselves excess returns (they are the returns of portfolios of the underlying N assets, with weights $w = (\beta' \beta)^{-1} \beta'$); the risk premia λ are the (risk premia) average excess returns of these K portfolios g_t. We can now explore the properties of these portfolios. Substituting r_t out from (A.2) we have:

$$g_t = (\beta' \beta)^{-1} \beta' (\beta\lambda + \beta(F_t - E[F_t]) + e_t) = \lambda + (F_t - E[F_t]) + (\beta' \beta)^{-1} \beta' e_t$$

Under suitable assumptions on the cross-sectional dispersion in the β (see Giglio and Xiu (2019) for a formal analysis) the last term is close to zero for large N (intuitively, the idiosyncratic errors are diversified away, and the g_t are well-diversified portfolios). We therefore can write:

$$g_t \simeq \lambda + (F_t - E[F_t])$$

From this equation, it is clear that, as expected, $E[g_t] = \lambda$. In addition, these K portfolios have the special property of being exposed to exactly one of the underlying factor F_t each: the matrix of exposures of g_t to factor innovations $F_t - E[F_t]$ is simply the identity matrix. So the first portfolio has betas [1, 0, 0 , 0, ...], the second portfolio has betas [0, 1, 0 , 0, ...], and so on. This is why we refer to these portfolios as *factor-hedging portfolios.*

Finally, it is worth pointing out that the latter property also holds in *any* sample: the estimated betas of the factor-hedging portfolios with respect to the nontradable factors will be the vectors [1, 0, 0 , 0, ...], [0, 1, 0 , 0, ...] and so on in every sample.

A.4 Approximating return sensitivities

This section describes the approximation of option returns used to obtain the rv and iv portfolios. P denotes the price of an at-the-money straddle or strangle. σ is the Black–

Scholes volatility, n is the time to maturity, F is the forward price, and K is the strike. N denotes the standard Normal cumulative distribution function.

For the calls and puts, respectively, we set

$$K_{call} = F \exp\left(b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) \tag{A.4}$$

$$K_{put} = F \exp\left(-b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) \tag{A.5}$$

We calculate everything for arbitrary b. A straddle is the special case where $b = 0$, while a strangle has positive b, so that both the put and call are out of the money.

A.4.1 Prices

We first calculate the price of a strangle. The Black–Scholes formula gives

$$P_{call} = FN(-b) - F \exp\left(b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) N\left(-b - \sigma\sqrt{n}\right) \tag{A.6}$$

$$P_{put} = -FN(-b) + F \exp\left(-b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) N\left(-b + \sigma\sqrt{n}\right) \tag{A.7}$$

So the total price is

$$P = P_{call} + P_{put} = F(N(-b) - N(-b)) \tag{A.8}$$

$$-F\left(\exp\left(b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) N\left(-b - \sigma\sqrt{n}\right) - \exp\left(-b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) N\left(-b + \sigma\sqrt{n}\right)\right) \tag{A.9}$$

$$\approx FN'(-b)\, 2\sigma\sqrt{n} \tag{A.10}$$

where the second line uses a first order approximation to $N(x)$ around $-b$ and $\exp\left(b\sigma\sqrt{n} + \frac{\sigma^2}{2}n\right) \approx 1$.

A.4.2 Return derivatives

The local approximation for returns that we use is

$$\frac{\partial r_{t+1}}{\partial x_{t+1}} = \frac{\partial}{\partial x_{t+1}} \frac{P(F_{t+1}, \sigma_{t+1})}{P(F_t, \sigma_t)} \tag{A.11}$$

and we evaluate the derivatives at the point $F_{t+1} = F_t$, $\sigma_{t+1} = \sigma_t$.

We have

$$\frac{\partial r_{t+1}}{\partial \sigma_{t+1}} = \frac{P_{\sigma,t+1}}{P_t} \tag{A.12}$$

$$= \frac{N'(-b) + N'(b)}{N'(-b)\, 2\sigma_t} \tag{A.13}$$

$$\approx \frac{1}{\sigma_t} \tag{A.14}$$

where $P_{\sigma,t}$ denotes $\partial P\left(F_{t+1}, \sigma_{t+1}\right)/\partial \sigma_{t+1}$ (evaulated at $\sigma_{t+1}^2 = \sigma_t^2$), and using the approximation that $N'(b) \approx N'(-b)$. We then have

$$\frac{\partial r_{t+1}}{\partial\left(\Delta\sigma_{t+1}/\sigma_t\right)} \approx 1 \tag{A.15}$$

Next, for squared returns, we have

$$\frac{\partial r_{t+1}}{\partial F_{t+1}^2} = \frac{P_{FF,t}}{P_t} \tag{A.16}$$

$$= \frac{1}{F_t N'(-b)\, 2\sigma\sqrt{n}} \frac{N'(-b) + N(b)}{F_t \sigma_t \sqrt{n}} \tag{A.17}$$

$$\approx \frac{1}{F_t^2 \sigma_t^2 n} \tag{A.18}$$

Again using $N'(b) \approx N'(-b)$. Finally, note that $\partial f_{t+1} = \partial F_{t+1}/F_{t+1}$, so that

$$\frac{\partial^2 r_{t+1}}{\partial\left(f_{t+1}/\sigma_t\right)^2} = \frac{\partial r_{t+1}}{\partial F_{t+1}^2} F_t^2 \sigma_t^2 \tag{A.19}$$

$$\approx \frac{1}{n} \tag{A.20}$$

A.4.3 Accuracy

To study how effective the above approximation is, we examine a simple simulation. We assume that options are priced according to the Black–Scholes model. We set the initial futures price to 1 and the initial volatility to 30 percent per year. We then examine instantaneous returns (i.e. through shifts in σ and S) on the *iv* and *rv* portfolios for straddles defined exactly as in the main text, allowing the futures return to vary between between $+/-23.53$ percent, which corresponds to variation out to four two-week standard deviations. We allow volatility to move between 15 and 60 percent – falling by half or doubling.

The top two panels of figure A.3 plot contours of returns on the *rv* and *iv* portfolios defined in the main text, while the middle panels plot the contours predicted by the approximations for the partial derivatives. For the *iv* portfolio, except for very large instantaneous returns – 15–20 percent – the approximation lies very close to the truth. The bottom-right panel plots the error – the middle panel minus the top panel – and except for cases where the underlying has an extreme movement and the implied volatility falls – the exact opposite of typical behavior – the errors are all quantitatively small, especially compared to the overall return.

For the *rv* portfolio, the errors are somewhat larger. This is due to the fact that we approximate the *rv* portfolio using a quadratic function, but its payoff has a shape closer to a hyperbola. Again, for underlying futures returns within two standard deviations (where the two-week standard deviation here is 5.88 percent), the errors are relatively small quantitatively, especially when σ does not move far. Towards the corners of the figure, though, the errors grow somewhat large.

These results therefore underscore the discussion in the text. The approximations used to construct the *iv* and *rv* portfolios are qualitatively accurate, and except in more extreme cases also hold reasonably well quantitatively. But they are obviously not fully robust to all events, so the factor model estimation, which does not rely on any approximations, should be used in situations where the nonlinearities are a concern.

A.4.4 Empirical return exposures

To check empirically the accuracy of the expressions for the risk exposures of the straddles, appendix figure A.2 plots estimated factor loadings for straddles at maturities from one to five months for each market from time series regressions of the form

$$r_{i,n,t} = a_{i,n} + \beta_{i,n}^{f} \frac{f_{i,t}}{IV_{i,t-1}} + \beta_{i,n}^{f^2} \frac{1}{2} \left(\frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \beta_{i,n}^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}} + \varepsilon_{i,n,t} \tag{A.21}$$

The prediction of the analysis above is that $\beta_{i,n}^{f} = 0$, $\beta_{i,n}^{f^2} = 1/n$, and $\beta_{i,n}^{\Delta IV} = 1$.

Across the panels, the predictions hold surprisingly accurately. The loadings on $f_{i,t}$ are all near zero, if also generally slightly positive. The loadings on the change in implied volatility are all close to 1, with little systematic variation across maturities. And the loadings on the squared futures return tend to begin near 1 (though sometimes biased down somewhat) and then decline monotonically, consistent with the predicted n^{-1} scaling.

Table A.2 reports results of similar regressions for each underlying of the returns on the *rv* and *iv* portfolios on the underlying futures return, the squared futures return, and the change in implied volatility. The table shows that while the Black–Scholes predictions do not hold perfectly, it is true that the *rv* portfolio is much more strongly exposed to realized than implied volatility, and the opposite holds for the *iv* portfolio. The coefficients on $(f_t/\sigma_{t-1})^2$ average 0.78 for the *rv* portfolio and 0.12 for the *iv* portfolio (though that average masks some variation across markets). Conversely, the coefficients on $\Delta\sigma_t/\sigma_{t-1}$ average 0.03 for the *rv* portfolio and 0.81 for the *iv* portfolio Furthermore, the R^2s are large, averaging 70 percent across the various portfolios, implying that their returns are well described by the approximation (4).

A.4.5 Volume

Figure A.14 reports the average daily volume of all of the option contracts across maturities 1 to 6 months. For crude oil, which we use here as a reference contract, the figure reports average daily volume in dollars; for all other contracts, it reports the average daily volume relative to crude oil. Empirically, crude oil options have volume numbers of the same order of magnitude as the S&P 500, while there is more heterogeneity across the other markets. Looking across maturities, the general pattern is that dollar volume declines by about a factor of three in almost all the markets between the 1- and 6-month maturities – so the 6-month maturity has less volume, but far from zero.

A.4.6 Alternative scaling for returns

Because returns have a price in the denominator, if that price is measured with error, returns can be biased upwards. The *iv* portfolio is net long the straddles, while the *rv* portfolio has a total weight of zero, so measurement error in prices would bias *iv* returns up but not *rv* returns. To account for that possibility, this section examines results when all the straddle returns are scaled by the price of the one-month straddle, instead of the price of a straddle with the same maturity.

Specifically, denoting $P_{n,t}$ the price of a straddle of maturity n on date t, the return on an n-month straddle used in the main results is

$$R_{n,t} = \frac{P_{n-1,t+1} - P_{n,t}}{P_{n,t}} \tag{A.22}$$

We consider returns on a portfolio that puts weight $\frac{P_{n,t}}{P_{1,t}}$ on the n-month straddle and weight $1 - \frac{P_{n,t}}{P_{1,t}}$ on the risk-free asset (which is a tradable portfolio), which is

$$r_{n,t+1}^{rescaled} = \frac{P_{n-1,t+1} - P_{n,t}}{P_{n,t}} \frac{P_{n,t}}{P_{1,t}} + \left(1 - \frac{P_{n,t}}{P_{1,t}}\right) r_{f,t} \tag{A.23}$$

$$= \frac{P_{n-1,t+1} - P_{n,t}}{P_{1,t}} + \left(1 - \frac{P_{n,t}}{P_{1,t}}\right) r_{f,t} \tag{A.24}$$

This portfolio is useful for two reasons. First, the one-month maturity has the highest volume in most markets we study, and it is typically considered to be the most accurate. Second, this eliminates differences in bias across maturities since in this specification, the denominator is the same for all n.

For $r_{n,t+1}^{rescaled}$, similar calculations to those above yield the results that

$$\frac{\partial^2 r_{n,t+1}^{rescaled}}{\partial \left(f_{t+1}/\sigma_t\right)^2} \approx \frac{1}{\sqrt{n}} \tag{A.25}$$

$$\frac{\partial r_{n,t+1}^{rescaled}}{\partial \left(\Delta\sigma_{t+1}/\sigma_t\right)} \approx \sqrt{n} \tag{A.26}$$

We then calculate alternative rv and iv portfolios as

$$iv_t^{rescaled} = \frac{3}{\sqrt{12}}\left(\sqrt{5/12}r_{5,t}^{rescaled} - \sqrt{1/12}r_{1,t}^{rescaled}\right) \tag{A.27}$$

$$rv_t^{rescaled} = \frac{5/48}{\sqrt{12}}\left(\sqrt{12}r_{1,t}^{rescaled} - \sqrt{12/5}r_{5,t}^{rescaled}\right) \tag{A.28}$$

Figure A.15 replicates figure 3 with the rescaled returns. The results are nearly identical to the baseline for both the Sharpe ratios on the iv and rv portfolios and the estimated factor risk premia. These results show that when we correct for the potential bias induced by low liquidity and measurement error at longer maturities, the estimates are essentially unchanged.

A.5 Random effects models

Denote the vector of true Sharpe ratios for the straddles in market i as sr_i. Our goal is to estimate the distribution of sr_i across the various underlyings. A natural benchmark distribution for the means is the normal distribution,

$$sr_i \sim N\left(\mu_{sr}, \Sigma_{sr}\right) \tag{A.29}$$

This section estimates the parameters μ_{sr} and Σ_{sr}. μ_{sr} represents the high-level mean of Sharpe ratios across all the markets, and Σ_{sr} describes how the market-specific means vary. The estimates of the market-specific Sharpe ratios differ noticeably across markets, but much of that is variation is likely driven by sampling error. Σ_{sr} is an estimate of how much the *true* Sharpe ratios vary, as opposed to the sample estimates.

Denote the sample estimate of the Sharpe ratio in each market as $\widehat{sr}_i$, and the stacked vector of sample Sharpe ratios as $\widehat{\mathbf{sr}} \equiv \left[\widehat{sr}_1', \widehat{sr}_2', ...\right]'$. Similarly, denote the vector of true Sharpe ratios as $\mathbf{sr} \equiv \left[sr_1', sr_2', ...\right]'$. Under the central limit theorem,

$$\widehat{\mathbf{sr}} \Rightarrow N\left(\mathbf{sr}, \Sigma_{\widehat{\mathbf{sr}}}\right) \tag{A.30}$$

where $\Rightarrow$ denotes convergence in distribution and the covariance matrix $\Sigma_{\widehat{\mathbf{sr}}}$ depends on the covariance between all the returns, across both maturities and underlyings, along with the lengths of the various samples.[1] Appendix A.6 describes how we construct $\Sigma_{\widehat{\mathbf{sr}}}$.

The combination of (A.29) and (A.30) represents a fully specified distribution for the data as a function of μ_{sr} and Σ_{sr}. It is then straightforward to construct point estimates and confidence intervals for μ_{sr} and Σ_{sr} with standard methods.

To allow for the possibility that average returns differ between the financial and nonfinancial underlyings, the mean in the likelihood can be replaced by $\mu_{sr} + \mu_D I_F$, where μ_D

[1] More formally, we would say that $\widehat{\mathbf{sr}}$ properly scaled by the square root of the sample size converges to a normal distribution. The expression (A.30) implicitly puts the sample size in $\Sigma_{\widehat{\mathbf{sr}}}$. The derivation of this result is a straightforward application of the continuous mapping theorem, nearly identical to the proof that a sample t-statistic is asymptotically Normally distributed.

is the difference in Sharpe ratios and I_F is a 0/1 indicator for whether the associated underlying is financial. We calculate the sampling distribution for the estimated parameters through Bayesian methods, treating the parameters as though they are drawn from a uniform prior. The point estimates are therefore identical to MLE, and the confidence bands represent samples from the likelihood.[2]

A.6 Calculating the covariance of the sample mean returns

There are two features of our data that make calculating covariance matrix of sample means difficult: we have an unbalanced panel and the covariance matrix is either singular or nearly so. We deal with those issues through the following steps.

1. For each market, we estimate the two largest principal components, therefore modeling straddle returns for underlying i and maturity n on date t as

$$r_{i,n,t} = \lambda_{1,i,n} f_{1,i,t} + \lambda_{2,i,n} f_{2,i,t} + \theta_{i,n,t} \tag{A.31}$$

where the λ are factor loadings, the f are estimated factors, and θ is a residual that we take to be uncorrelated across maturities and markets (it is also in general extremely small).

2. We calculate the long-run covariance matrix of all $J \times 2$ estimated factors. The covariance matrix is calculated using the Hansen–Hodrick method to account for the fact that the returns are overlapping (we use daily observations of 2-week returns). The elements of the covariance matrix are estimated based on the available nonmissing data for the associated pair of factors. That means that the covariance matrix need not be positive semidefinite. To account for that fact, we set all negative eigenvalues of the estimated covariance matrix to zero.

Given the estimated long-run covariance matrix of the factors, denoted Σ_f, and given the (diagonal) long-run variance matrix of the residuals θ, denoted Σ_θ, the long-run covariance matrix of the returns is then

$$\Sigma_r \equiv \Lambda \Sigma_f \Lambda' + \Sigma_\theta \tag{A.32}$$

where Λ is a matrix containing the factor loadings λ.

3. Finally, it is straightforward to show that the covariance matrix of the sample mean returns is

$$\Sigma_{\hat{r}} = M \odot \Sigma_r \tag{A.33}$$

where $\odot$ denotes the elementwise product and M is a matrix where the element for a given return pair is equal to the ratio of the number of observations in which both returns are available to the product of the number of observations in which each return is available individually (if all returns had the same number of observations T, then we would obtain

[2] We use Bayesian methods to calculate the sampling intervals because likelihood-based methods require inverting large second derivative matrices, which can be numerically unstable. The estimation in this section is performed using the Bayesian computation engine Stan, which provides functions that both maximize the likelihood and rapidly sample from the posterior distribution. Code is available on request.

the usual T^{-1} scaling). We then have the asymptotic approximation that

$$\hat{r} \Rightarrow N\left(\bar{r}, \Sigma_{\hat{r}}\right) \tag{A.34}$$

where $\hat{r}$ is a vector that stacks the $\hat{r}_i$ and $\bar{r}$ stacks the $\bar{r}_i$ and $\Rightarrow$ denotes convergence in distribution.

To construct $\Sigma_{\widehat{\mathbf{sr}}}$, we simply divide the i, j element of $\Sigma_{\hat{r}}$ by the product of the sample standard deviations of r_i and r_j.

A.7 Calculating risk prices with unbalanced panels and correlations across markets

In estimating the factor models, we have two complications to deal with: the sample length for each underlying is different, and returns are correlated across underlyings. This section discusses how we deal with those issues.

We have the model

$$E_{T_i}\left[R_i\right] = \lambda_i \beta_i + \alpha_i \tag{A.35}$$

where E_{T_i} denotes the sample mean in the set of dates for which we have data for underlying i, R_i is the vector of returns of the straddles, λ_i is a vector of risk prices, β_i is a vector of risk prices, and α_i is a vector of pricing errors. Note that these objects are all population values, rather than estimates. In order to calculate the sampling distribution for the estimated counterparts, we need to know the covariance of the pricing errors. Note that there is also a population cross-sectional regression with

$$E_{T_i}\left[R_i\right] = a_i + \beta_i E_{T_i}\left[f_i\right] + E_{T_i}\left[\varepsilon_i\right] \tag{A.36}$$

where ε_i is a vector of residuals and f_i is a vector of pricing factors. That formula can be used to substitute out returns and obtain

$$\alpha_i = a_i + \beta_i E_{T_i}\left[f_i\right] + E_{T_i}\left[\varepsilon_i\right] - \lambda_i \beta_i \tag{A.37}$$

Since a_i, λ_i, and β_i are fixed in the true model, the distribution of α_i depends only on the distributions of the sample means $E_{T_i}\left[f_i\right]$ and $E_{T_i}\left[\varepsilon_i\right]$. Denoting the long-run (i.e. Hansen–Hodrick) covariance matrix of f_i as Σ_{f_i} and that of ε_i as Σ_{ε_i}, we have

$$var\left(\alpha_i\right) = \beta_i T_i^{-1} \Sigma_f \beta_i' + T_i^{-1} \Sigma_{\varepsilon_i} \tag{A.38}$$

Since the λ_i are estimated from a regression, if we denote their estimates as $\hat{\lambda}_i$, we obtain the usual formula for the variance of $\hat{\lambda}_i - \lambda_i$

$$var\left(\hat{\lambda}_i - \lambda_i\right) = \left(\beta_i'\beta_i\right)^{-1} \beta_i' var\left(\alpha_i\right) \beta_i \left(\beta_i'\beta_i\right)^{-1} \tag{A.39}$$

$$= \Sigma_f + \left(\beta_i'\beta_i\right)^{-1} \beta_i' \Sigma_{\varepsilon_i} \beta_i \left(\beta_i'\beta_i\right)^{-1} \tag{A.40}$$

Beyond the variance of $\hat{\lambda}_i$, we also need to know the covariance of any pair of estimates, $\hat{\lambda}_i$ and $\hat{\lambda}_j$. Using standard OLS formulas, we have

$$\begin{bmatrix} \hat{\lambda}_i - \lambda_i \\ \hat{\lambda}_j - \lambda_j \end{bmatrix} = \begin{bmatrix} (\beta_i'\beta_i)^{-1} \beta_i'\alpha_i \\ (\beta_j'\beta_j)^{-1} \beta_j'\alpha_j \end{bmatrix} \tag{A.41}$$

$$= \begin{bmatrix} (\beta_i'\beta_i)^{-1} \beta_i' \left(\beta_i E_{T_i}[f_t] + E_{T_i}[\varepsilon_{j,t}]\right) \\ (\beta_j'\beta_j)^{-1} \beta_j' \left(\beta_2 E_{T_j}[f_t] + E_{T_j}[\varepsilon_{j,t}]\right) \end{bmatrix} \tag{A.42}$$

The covariance between $\hat{\lambda}_i$ and $\hat{\lambda}_j$ is then

$$\frac{T_{12}}{T_1 T_2} \left(\Sigma_{f,i,j} + (\beta_1'\beta_1)^{-1} \beta_1' \Sigma_{\varepsilon,i,j} \beta_2 (\beta_2'\beta_2)^{-1}\right) \tag{A.43}$$

where $\Sigma_{f,i,j}$ and $\Sigma_{\varepsilon,i,j}$ are now long-run covariance matrices (again from the Hansen–Hodrick method). Using these formulas, we then have estimates of risk prices in each market individually along with a full covariance matrix of all the estimates.

A.8 SDF-based analysis

The marginal effects of realized and implied volatility can be estimated using the stochastic discount factor representation of the factor model estimated in the previous section. Specifically, given the set of straddle returns in each market, one can construct a pricing kernel M_t of the form

$$M_t = \bar{M} - m_i^f \frac{f_{i,t}}{IV_{i,t-1}} - m_i^{f^2} \left(\frac{f_{i,t}}{IV_{i,t-1}}\right)^2 - m_i^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}} \tag{A.44}$$

where M_t represents state prices (or marginal utility) and $1 = E_{t-1} M_t R_t$ for any return priced by M. The difference between this specification and that in the previous section is that the coefficients $m^{\cdots}$ represent the marginal impact of each term on marginal utility, whereas the $\gamma^{\cdots}$ coefficients represent the premium for total exposure to the factors. Cochrane (2001) discusses the distinction extensively.

Denoting the covariance matrix of the factors in market i by Σ_i, the m coefficients can be recovered as

$$\left[m_i^f, m_i^{f^2}, m_i^{\Delta IV}\right]' = \Sigma_i^{-1} \left[\gamma_i^f, \gamma_i^{f^2}, \gamma_i^{\Delta IV}\right]' \tag{A.45}$$

The m's now represent Sharpe ratios on portfolios with exposure to each of the individual factors, orthogonalized to the other two. That is, $m_i^{\Delta IV}$ is the Sharpe ratio for a portfolio exposed to the part of $\frac{\Delta IV_{i,t}}{IV_{i,t-1}}$ that is orthogonal to $\frac{f_{i,t}}{IV_{i,t-1}}$ and $\left(\frac{f_{i,t}}{IV_{i,t-1}}\right)^2$.

Figure A.12 reports the results of this exercise. The findings are qualitatively consistent with the main results in figure 3 and in fact even stronger quantitatively. The marginal effect of an increase in uncertainty on marginal utility, holding realized volatility fixed, is consistently negative, while an increase in realized volatility increases marginal utility. The fact that these results are close to the benchmark case is a consequence of the weak correlation between innovations in realized and implied volatility, so that the rotation by Σ_i^{-1} has small effects.

Figure A.12 also reports premia on orthogonalized versions of the rv and iv portfolios.[3] Again, the results are similar to the main analysis.

A.9 Robustness: ETF options

This section provides an alternative check on the results for crude oil options by examining returns on straddles for options on two exchange traded funds. The first is the United States Oil Fund (USO), which invests in short-term oil futures. USO has existed since 2006, and Optionmetrics reports quotes for options beginning in May, 2007. The second fund is the Energy Select Sector SPDR fund (XLE), which tracks the energy sector of the S&P 500. XLE has existed since 1998 and Optionmetrics reports data since December, 1998.

We eliminate observations using the following filters:

1. Volume less than 10 contracts
2. Time to maturity less than 15 days
3. Bid-ask spread greater than 20 percent of bid/ask midpoint
4. Initial log moneyness – log strike divided by the futures price – greater than 0.75 implied volatility units in absolute value (where implied volatility is scaled by he square root of time to maturity).

We then calculate straddle returns as in the main text over two-week periods and average across the two straddles nearest to the money for each maturity, weighting them by the inverse of their absolute moneyness.

The top section of table A.9.1 reports the number of (potentially overlapping) two-week straddle return observations across maturities for USO, XLE, and the CME Group futures options used in the main analysis. Since the CME data goes back to 1983, there are far more observations for that series than the other two. More interestingly, though, the number of observations only declines by about 10 percent between the 1- and 6-month maturities, while it falls by more than 2/3 for the XLE and USO samples. The CME data therefore has superior coverage at longer horizons, which justifies its use in our main analysis.

The bottom section of table A.9.1 reports the correlations of the USO and XLE straddle returns with those for the CME on the days where they overlap. The correlations are approximately 90 percent at all maturities for USO and 50 percent for XLE. The 90-percent correlations for USO and the CME sample provide a general confirmation of the accuracy of the CME straddle returns, since we would expect the USO and CME options to be highly similar as USO literally holds futures. The lower correlation for XLE is not surprising given that it holds energy sector stocks rather than crude oil futures.

Table A.9.1.

[3] These are constructed simply through a rotation. The $rv_\perp$ portfolio has a positive correlation with rv and zero correlation with iv, whie the $iv_\perp$ portfolio has zero correlation with rv and a positive correlation with iv.

	Maturity:	1	2	3	4	5	6
# obs.	USO	1640	1616	1721	1679	1118	525
	XLE	2612	2545	2454	1928	1134	369
	CME	6762	6645	6817	6801	6606	5998
Corr. w/	USO	0.93	0.96	0.95	0.92	0.89	0.83
CME	XLE	0.43	0.48	0.50	0.49	0.50	0.53

In the main text, the RV and IV portfolio returns are calculated using 5- and 1-month straddles. Since the number of observations drops off substantially between 4 and 5 months for both XLE and USO, though, here we examine returns on RV and IV portfolios using both 5- and 4-month straddles for the long-maturity side.

Figure A.16 plots estimated annualized Sharpe ratios along with 95-percent confidence bands for the RV and IV portfolios using 4- and 5-month straddles for the three sets of options. In all four cases, the three confidence intervals always overlap substantially. The fact that the sample for the CME options is far larger is evident in its confidence bands being much narrower than those for the other two sources. For the IV portfolios, USO has returns that are close to zero, but its confidence bands range from -1 to greater than 0.5, indicating that it is not particularly informative about the Sharpe ratio.

Table A.9.2 reports confidence bands for the *difference* between the IV and RV average returns constructed with the CME data and the same portfolios constructed using USO and XLE. The top panel shows that the differences for the IV portfolios are negative for USO and positive for XLE, but only the difference for USO constructed with the 4-month straddle is statistically significant. The bottom panel similarly shows mixed results for the point estimates for the differences for the RV portfolios, with none of the differences being statistically significant.

Table A.9.2. Differences between CME and USO, XLE mean returns

	USO - CME, 4mo.	USO - CME, 5mo.	XLE - CME, 4mo.	XLE - CME, 5mo.
IV return	-2.2	-2.2	-0.8	-1.4
	[-3.9,-0.2]	[-4.8,0.4]	[-2.5,4.1]	[-4.1,6.3]
RV return	0.43	0.47	-0.27	0.67
	[-0.6,1.4]	[-0.6,1.4]	[-1.8,1.3]	[-1.5,2.6]

Notes: the table reports percentage (two-week) returns on USO and XLE minus returns on CME RV and IV portfolios. 95-percent confidence intervals are reported in brackets.

The fact that the USO and CME straddle returns are highly correlated does not necessarily mean that the CME data is accurate for the mean return on the straddles. To check whether the difference in the means observed in the USO and XLE data would affect out main results, we ask how the Sharpe ratios of the RV and IV portfolios in the CME data would change if we shifted their means by the average differences reported in table A.9.2. The bars labeled "CME, USO adj." and CME, XLE adj." show how the confidence bands would change if we shifted them by exactly the point estimates from table A.9.2. Note that this is not the same as shifting the Sharpe ratio for the CME data to match that for the XLE or USO data. The reason is that the difference in table A.9.2 is calculated only for the returns on matching dates, whereas the Sharpe ratio calculated in figure A.16 is calculated using the full sample for the CME data. So the two adjusted bands take the full-sample band and then shift it by the mean difference calculated on the dates that overlap between the CME data and XLE or USO.

Figure A.16 shows that the economic conclusions drawn for the crude oil straddles are not changed if the mean returns are shifted by the differences observed in table A.9.1. The RV portfolio returns remain statistically significantly negative in all four cases, the changes in the point estimates are well inside the original confidence intervals. The top panel shows that the IV returns using 5-month straddles are similarly unaffected. For the 4-month straddles, the only difference is that with the USO options, the estimated Sharpe ratio falls by about half and is no longer statistically significantly greater than zero. So, again, out of eight cases – IV and RV with 4- and 5-month straddles – in only one is there a nontrivial change in the conclusions, and even there the Sharpe ratio on the IV portfolio does not become negative, it is simply less positive.

Overall, the period in which the USO and XLE options are traded is too short to use them for our main analysis. This section shows that the USO straddle returns are highly correlated with the CME returns. The mean returns on the XLE and CME straddles are highly similar, while they differ somewhat more for CME and USO. However, shifting the means used for the CME options in the main analysis by the observed difference between the CME and USO options does not substantially change any of the conclusions.

A.10 Robustness: Oil and gas equity options

As a further extension of the results for ETS above, we also analyse the returns on options on oil and gas companies. Specifically, we obtain data from Optionmetrics on firms with an Optionmetrics industry code between 120 and 125. We then constructed rv and iv portfolios for those firms using the same methods as for the main analysis, again with maturities of one and five months. We construct two-week returns and sum them across whatever firms are available on each date, weighting by market capitalization. The Optionmetrics data covers the period 1996–2018.

	Sharpe ratio
rv	-0.56
95% CI	[-1.02,-0.10]
iv	0.05
95% CI	[-0.42,0.52]

The Sharpe ratios for the rv and iv portfolios for oil and gas companies are below. Similar to the main results, we obtain a significantly negative premium on realized volatility and a marginally positive premium on implied volatility. The premium for the iv portfolio for oil and gas companies is less positive than for crude oil futures options, but more positive than for S&P 500 index options. In other words, the results imply that options on oil and gas companies behave as though they are a mixture of options on the S&P 500 and on crude oil, which is not an unrealistic desciption of oil and gas companies.

Because of the relatively short sample compared to the main results, similar to the previous section, this analysis has relatively low power. The point estiamte for rv is outside the confidence band for iv and vice versa, but their confidence bands do overlap and the Sharpe ratios are not statistically significantly different from each other. That also illustrates the benefit in the main analysis of using information from many different markets to help increase estimation power. Nevertheless, the results in this section are consistent with our main findings, if statistically weaker.

A.11 Model

To help provide some context for the empirical results and fit them into a standard framework, this section describes results from a simple extension of the standard long-run risk model of Bansal and Yaron (2004). The technical analysis is in section A.12; here we report the specification and key results.

Agents have Epstein–Zin preferences over consumption, C_t, with a unit elasticity of substitution, where the lifetime utility function, v_t, satisfies

$$v_t = (1-\beta)\log C_t + \frac{\beta}{1-\alpha}\log E_t \exp\left((1-\alpha)\, v_{t+1}\right) \tag{A.46}$$

where α is the coefficient of relative risk aversion. Consumption growth follows the process

$$\begin{aligned}
\Delta c_t &= x_{t-1} + \sqrt{\sigma^2_{B,t-1} + \sigma^2_{G,t-1}}\varepsilon_t + Jb_t && \text{(A.47)}\\
x_t &= \phi_x x_{t-1} + \omega_x \eta_{x,t} + \omega_{x,G}\eta_{\sigma,G,t} - \omega_{x,B}\eta_{\sigma,B,t} && \text{(A.48)}\\
\sigma^2_{j,t} &= (1-\phi_\sigma)\,\bar{\sigma}^2_j + \phi_\sigma \sigma^2_{j,t-1} + \omega_j \eta_{\sigma,j,t}, \text{ for } j \in \{B, G\} && \text{(A.49)}
\end{aligned}$$

where ε_t and the $\eta_{\cdot,t}$ are independent standard normal random variables. x_t represents the consumption trend. We have two deviations from the usual setup. First, we include jump shocks, Jb_t, where b_t is a Poisson distributed random variable with intensity λ and J is the magnitude of the jump. This addition allows for random variation in realized volatility and is drawn from Drechsler and Yaron (2011). Second, there are two components to volatility, which we refer to as bad and good. Bad volatility, σ^2_B, is associated with low future consumption growth, while good volatility, σ^2_G, is associated with high future growth (where all of the $\omega_\cdot$ coefficients are nonnegative).

Define realized volatility to be the realized quadratic variation in consumption growth, while implied volatility is the conditional variance of consumption growth (these are formalized in the appendix).

Proposition 1 *The average excess returns on forward claims to realized and implied volatility for consumption growth in this model are,*

$$\begin{aligned}
E\left[RV_{t+1} - P_{RV,t}\right] &= J^2\lambda\left(1-\exp(-\alpha J)\right) && \text{(A.50)}\\
E\left[IV_{t+1} - P_{IV,t}\right] &= (\alpha-1)\left(v_{Y,x}\left(\omega_{x,G}\omega_G - \omega_{x,B}\omega_B\right) + v_{Y,\sigma}\left(\omega_G^2 + \omega_B^2\right)\right) && \text{(A.51)}
\end{aligned}$$

where $P_{x,t}$ is the forward price for x. $E\left[IV_{t+1} - P_{IV,t}\right] > 0$ for $\omega_{x,G}$ sufficiently larger than $\omega_{x,B}$. Furthermore, the sign of $E\left[RV_{t+1} - P_{RV,t}\right]$ is the same as the sign of J and of the conditional skewness of consumption growth (i.e. the skewness of Δc_{t+1} conditional on date-t information).

Proposition 1 contains our key analytic results. We analyze premia for realized and implied volatility on consumption – real activity – consistent with the focus in the empirical analysis on macro volatility and uncertainty. The negative premium on realized volatility is driven by downward jumps, similar to the literature on the volatility risk premium in equities (Drechsler and Yaron (2011), Wachter (2013)). The sign of the premium on implied volatility depends on the contribution of good versus bad volatility. When good volatility shocks, where high volatility is associated with high future growth (e.g. due to learning about new technologies), are relatively larger than bad volatility shocks ($\omega_{x,G}\omega_G > \omega_{x,B}\omega_B$) the premium on implied volatility can be positive.

Section A.12 provides a numerical calibration of the model using values close to those in Bansal and Yaron's (2004) original choices. It shows that the model generates quantitatively realistic Sharpe ratios for implied and realized volatility in addition to a reasonable equity premium.

The key economic mechanism for the positive pricing of uncertainty shocks is that high volatility is sometimes associated with higher long-term growth. Intuitively, that mechanism contributes positive skewness to consumption growth, while the jumps contribute negative skewness. The appendix provides novel evidence on the skewness of consumption growth consistent with the model. In particular, conditional skewness in the model, which depends only on the jumps, is more negative than the skewness of expected consumption growth, which depends on the relationship of volatility and long-run growth (x). We show that consumption growth displays exactly the same pattern in US data.

So a simple version of the long-run risk model with good and bad volatility shocks and jumps in consumption can match our key empirical facts. Furthermore, the empirical results are sharp, in the sense that the sign of the premium on implied volatility identifies the relative importance of the bad and good volatility shocks, while the sign of the premium on realized volatility identifies the sign of consumption jumps.

A.12 Model details

A.12.1 Dynamics

Consumption growth follows

$$\begin{aligned} \Delta c_t &= x_{t-1} + \sqrt{\sigma_{B,t-1}^2 + \sigma_{G,t-1}^2}\varepsilon_t + Jb_t && \text{(A.52)} \\ x_t &= \phi_x x_{t-1} + \omega_x \eta_{x,t} + \omega_{x,G}\eta_{\sigma,G,t} - \omega_{x,B}\eta_{\sigma,B,t} && \text{(A.53)} \\ \sigma_{j,t}^2 &= (1-\phi_\sigma)\,\bar{\sigma}_j^2 + \phi_\sigma \sigma_{j,t-1}^2 + \omega_j \eta_{\sigma,j,t} && \text{(A.54)} \end{aligned}$$

for $j \in \{G, B\}$. The shocks ε, η_x, η_G, η_B are independent and Gaussian with unit variances. The ω coefficients are all assumed to be positive. b_t is a Poisson random variable with intensity λ.

The dynamics can also be written as

$$\begin{bmatrix} x_t \\ \sigma_t^2-\bar{\sigma}^2 \end{bmatrix} = \begin{bmatrix} \phi_x & 0 \\ 0 & \phi_\sigma \end{bmatrix}\begin{bmatrix} x_{t-1} \\ \sigma_{t-1}^2-\bar{\sigma}^2 \end{bmatrix} + \begin{bmatrix} \omega_x & \omega_{x,G} & 0 \\ 0 & \omega_G & \omega_B \end{bmatrix}\begin{bmatrix} \eta_{x,t} \\ \eta_{G,t} \\ \eta_{B,t} \end{bmatrix} \quad \text{(A.55)}$$

$$\Delta c_t = x_{t-1}+\sigma_{t-1}^2\varepsilon_t+Jb_t \quad \text{(A.56)}$$

$$Y_t = FY_{t-1}+G\eta_t \quad \text{(A.57)}$$

where $Y_t=[x_t,\sigma_t^2-\bar{\sigma}^2]'$, etc. The fact that the model can be rewritten with only a single variance process follows from the linearity of the two processes, the fact that they have the same rate of mean reversion, and the fact that they appear additively. We can then write consumption and dividend growth as

$$\Delta c_t = c_Y'Y_{t-1}+\sqrt{\bar{\sigma}^2+g_Y'Y_{t-1}}\varepsilon_t+Jb_t \quad \text{(A.58)}$$

$$\Delta d_t = \gamma\left(c_Y'Y_{t-1}+\sqrt{\bar{\sigma}^2+g_Y'Y_{t-1}}\varepsilon_t+Jb_t\right)+\omega_d\varepsilon_{d,t} \quad \text{(A.59)}$$

for vectors c_Y and g_Y. Δd_t is log dividend growth, which we will use for modeling equities. It satisfies $\Delta d_t=\gamma\Delta c_t+\omega_d\varepsilon_{d,t}$ ($\varepsilon_{d,t}\sim N(0,1)$), where γ determines the leverage of equities.

A.12.2 Preferences

We assume agents have Epstein–Zin preferences with a unit IES,

$$v_t = (1-\beta)c_t+\frac{\beta}{1-\alpha}\log E_t\exp((1-\alpha)v_{t+1}) \quad \text{(A.60)}$$

$$vc_t = \frac{\beta}{1-\alpha}\log E_t\exp((1-\alpha)(vc_{t+1}+\Delta c_{t+1})) \quad \text{(A.61)}$$

where vc_t is the log utility/consumption ratio, $vc_t=v_t-c_t$. We look for a solution to the model of the form

$$vc_t=\bar{v}+v_Y'Y_t \quad \text{(A.62)}$$

Inserting into the recursion for vc,

$$vc_t = \frac{\beta}{1-\alpha}\log E_t\exp\left((1-\alpha)\left(\bar{v}+v_Y'Y_{t+1}+c_Y'Y_t+\sqrt{g_Y'Y_t}\varepsilon_{t+1}+Jb_{t+1}\right)\right) \quad \text{(A.63)}$$

$$= \frac{\beta}{1-\alpha}\log E_t\exp\left((1-\alpha)\left(\bar{v}+v_Y'(FY_t+G\eta_{t+1})+c_Y'Y_t+\sqrt{\bar{\sigma}^2+g_Y'Y_t}\varepsilon_{t+1}+Jb_{t+1}\right)\right) \quad \text{(A.64)}$$

$$= \beta(\bar{v}+(v_Y'F+c_Y')Y_t)+\beta\frac{1-\alpha}{2}\left(v_Y'GG'v_Y+\bar{\sigma}^2+g_Y'Y_t\right)+\frac{\beta}{1-\alpha}\lambda(\exp((1-\alpha)J)-1) \quad \text{(A.65)}$$

Matching coefficients,

$$v_Y' = \beta\left(v_Y'F + c_Y'\right) + \beta\frac{1-\alpha}{2}g_Y' \tag{A.66}$$

$$v_Y' = \beta\left(c_Y' + \frac{1-\alpha}{2}g_Y'\right)(I-\beta F)^{-1} \tag{A.67}$$

$$\bar{v} = \frac{\beta}{1-\beta}\left(\frac{1-\alpha}{2}\left(v_Y'GG'v_Y + \bar{\sigma}^2\right) + \frac{1}{1-\alpha}\lambda\left(\exp\left((1-\alpha)J\right)-1\right)\right) \tag{A.68}$$

The pricing kernel is then

$$M_{t+1} = \beta\frac{\exp\left((1-\alpha)\left(vc_{t+1}\right)\right)}{E_t\exp\left((1-\alpha)\left(vc_{t+1}+\Delta c_{t+1}\right)\right)}\exp\left(-\alpha\Delta c_{t+1}\right) \tag{A.69}$$

$$m_{t+1} = -\log\beta + (1-\alpha)\,vc_{t+1} - \alpha\Delta c_{t+1} - \log E_t\exp\left((1-\alpha)\left(vc_{t+1}+\Delta c_{t+1}\right)\right) \tag{A.70}$$

Or, equivalently,

$$m_{t+1} = m_0 + m_Y'Y_t + m_\eta\eta_{t+1} - \alpha\sqrt{\bar{\sigma}^2 + g_Y'Y_t}\varepsilon_{t+1} - \alpha Jb_{t+1} \tag{A.71}$$

$$m_0 = -\log\beta - \frac{(1-\alpha)^2}{2}\left(v_Y'GG'v_Y + \bar{\sigma}^2\right) - \lambda\left(\exp\left((1-\alpha)J\right)-1\right) \tag{A.72}$$

$$m_Y' = -c_Y - \frac{(1-\alpha)^2}{2}g_Y \tag{A.73}$$

$$m_\eta = (1-\alpha)\,v_Y'G \tag{A.74}$$

A.12.3 Pricing equities

We have the usual Campbell–Shiller approximation for the return on equities, r_{t+1}, with

$$r_{t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1} \tag{A.75}$$

where z_t is the log price/dividend ratio of equities. We look for a solution of the form $z_t = z_0 + z_Y'Y_t$, which leads to the pricing equation

$$0 = \log E_t\exp\left(\begin{array}{c} m_0 + m_Y'Y_t + m_\eta\eta_{t+1} - \alpha\sqrt{\bar{\sigma}^2 + g_Y'Y_t}\varepsilon_{t+1} - \alpha Jb_{t+1} \\ +\kappa_0 + (\kappa_1 - 1)\,z_0 + \kappa_1 z_Y'\left(FY_t + G\eta_{t+1}\right) - z_Y'Y_t \\ +\gamma\left(c_Y'Y_t + \sqrt{\bar{\sigma}^2 + g_Y'Y_t}\varepsilon_{t+1} + Jb_{t+1}\right) + \omega_d\varepsilon_{d,t+1} \end{array}\right) \tag{A.76}$$

The solution satisfies

$$z_0 = (1-\kappa_1)^{-1}\left(\begin{array}{c} m_0 + \kappa_0 + \lambda\left(\exp\left((\gamma-\alpha)J\right)-1\right) \\ +\frac{1}{2}\left(\left(m_\eta + \kappa_1 z_Y'G\right)\left(m_\eta + \kappa_1 z_Y'G\right)' + (\gamma-\alpha)^2\bar{\sigma}^2 + \omega_d^2\right) \end{array}\right) \tag{A.77}$$

$$z_Y' = \left(m_Y' + \gamma c_Y' + \frac{1}{2}(\gamma-\alpha)^2 g_Y'\right)(I-\kappa_1 F)^{-1} \tag{A.78}$$

A.12.3.1 Average excess returns

To get average returns, on equities, first note that

$$\log E_t\left[\exp\left(r_{t+1}-r_{f,t}\right)\right] = \log E_t\left[\exp\left(\begin{array}{c}\kappa_0+(\kappa_1-1)z_0+\kappa_1 z_Y'\left(FY_t+G\eta_{t+1}\right)-z_Y'Y_t\\ +\gamma\left(c_Y'Y_t+\sqrt{\bar{\sigma}^2+g_Y'Y_t}\varepsilon_{t+1}+Jb_{t+1}\right)+\omega_d\varepsilon_{d,t+1}\\ -r_{f,0}-r_{f,1}'Y_t\end{array}\right)\right] \tag{A.79}$$

$$= \kappa_0+(\kappa_1-1)z_0-r_{f,0}+\left(\kappa_1 z_Y'F-z_Y'+\gamma c_Y'-r_{f,1}'\right)Y_t \tag{A.80}$$

$$+\frac{1}{2}\left(\kappa_1^2 z_Y'GG'z_Y+\gamma^2\left(\bar{\sigma}^2+g_Y'Y_t\right)\right)+\frac{1}{2}\omega_d^2+\lambda\left(\exp(\gamma J)-1\right) \tag{A.81}$$

The risk-free rate is of the form $r_{f,t}=r_{f,0}+r_{f,1}'Y_t$, with

$$r_{f,0} = \log\beta+\frac{(1-2\alpha)}{2}\bar{\sigma}^2+\lambda\left(\exp\left((1-\alpha)J\right)-\exp(-\alpha J)\right) \tag{A.82}$$

$$r_{f,1}' = c_Y'-\frac{1}{2}\alpha^2 g_Y' \tag{A.83}$$

which allows for the calculation of the average excess return on equities. The conditional standard deviation of equity returns is

$$\sqrt{\kappa_1^2 z_Y'GG'z_Y+\gamma^2\bar{\sigma}^2+\gamma^2J^2\lambda} \tag{A.84}$$

A.12.4 Pricing realized volatility

Since our empirical work estimates premia for realized and implied volatility for macro variables, we examine here the pricing of realized and implied volatility for Δc_{t+1}. The cumulative innovation in consumption between dates t and $t+1$ is

$$\Delta c_{t+1}-E_t\Delta c_{t+1}=\sigma_t^2\varepsilon_{t+1}+J\left(b_{t+1}-\lambda\right)$$

The first part is typically thought of as a diffusive component. That is, we can think of $\varepsilon_{t+1}=B_{t+1}-B_t$, for a standard (continuous-time) Brownian motion B_t. Similarly, b_{t+1} is the innovation in a pure jump process, $b_{t+1}=N_{t+1}-N_t$, where N_t is a (continuous-time) Poisson counting process. Now consider measuring the total quadratic variation in those two processes (i.e. as though we were measuring realized volatility from daily futures returns, as in our empirical analysis). The quadratic variation in B between dates t and $t+1$ is exactly 1, while the quadratic variation in N is exactly $N_{t+1}-N_t=b_{t+1}$. We then say that the realized volatility in consumption growth between period t and $t+1$ is

$$RV_{t+1}=\sigma_t^2+J^2b_{t+1} \tag{A.85}$$

In this case, the diffusive part of the realized volatility is entirely predetermined. This is a typical result. It is only the jumps that contribute an unexpected component to realized volatility. The pricing of realized volatility will therefore depend on the pricing of jumps.

The price of a forward claim on RV_{t+1} is

$$\begin{aligned} P_{RV,t} &= E_t\left[\frac{\exp(m_{t+1})}{E_t\exp(m_{t+1})}RV_{t+1}\right] \\ &= E_t\left[\exp\left(\begin{array}{c} (1-\alpha)v_Y'G\eta_{t+1} - \alpha\left(\sqrt{\bar{\sigma}^2+g_Y'Y_t}\varepsilon_{t+1}+Jb_{t+1}\right) \\ -\left[\frac{1}{2}\left((1-\alpha)^2 v_Y'GG'v_Y' + \alpha^2\left(\bar{\sigma}^2+g_Y'Y_t\right)\right)+\lambda\left(\exp(-\alpha J)-1\right)\right] \end{array}\right)\left(\sigma_t^2+J^2 b_{t+1}\right)\right] \\ &= \sigma_t^2 + J^2\lambda\exp(-\alpha J) \end{aligned}$$

The average excess return on that forward is then

$$\begin{aligned} E_t\left[RV_{t+1}-P_{RV,t}\right] &= \sigma_t^2+J^2\lambda-\sigma_t^2-J^2\lambda\exp(-\alpha J) && \text{(A.86)} \\ &= J^2\lambda\left(1-\exp(-\alpha J)\right) && \text{(A.87)} \end{aligned}$$

The sign of this object is equal to the sign of J. Note also that this is the sign of the conditional skewness of consumption growth.

A.12.5 Pricing uncertainty

We define uncertainty on date t as expected realized volatility on date $t+1$. That is, it is the conditional variance for Δc_{t+1}. So we say

$$IV_t \equiv \sigma_t^2 + J^2\lambda \qquad \text{(A.88)}$$

We now consider the price and excess return for a forward claim to IV_{t+1}.

$$\begin{aligned} P_{IV,t} &= E_t\left[\frac{\exp(m_{t+1})}{E_t\exp(m_{t+1})}IV_{t+1}\right] \\ &= J^2\lambda+\bar{\sigma}^2+\phi_\sigma\hat{\sigma}_t^2+E_t\left[\exp\left(\begin{array}{c}(1-\alpha)v_Y'G\eta_{t+1} \\ -\frac{1}{2}\left((1-\alpha)^2 v_Y'GG'v_Y'\right)\end{array}\right)g_Y'\eta_{t+1}\right] \\ &= J^2\lambda+\bar{\sigma}^2+\phi_\sigma\hat{\sigma}_t^2+\frac{E_t\left[\exp\left((1-\alpha)v_Y'G\eta_{t+1}\right)g_Y'G\eta_{t+1}\right]}{\exp\left(\frac{1}{2}\left((1-\alpha)^2 v_Y'GG'v_Y'\right)\right)} \\ &= J^2\lambda+\bar{\sigma}^2+\phi_\sigma\hat{\sigma}_t^2+(1-\alpha)\left(\begin{array}{c}\omega_G\left(v_{Y,x}\omega_{x,G}+v_{Y,\sigma}\omega_G\right) \\ +\omega_B\left(v_{Y,\sigma}\omega_B-v_{Y,x}\omega_{x,B}\right)\end{array}\right) \end{aligned}$$

where the last line follows from straightforward but tedious algebra. The average return on the claim on uncertainty is then

$$\begin{aligned} E\left[IV_{t+1}\right]-P_{IV,t} &= J^2\lambda+\bar{\sigma}^2+\phi_\sigma\hat{\sigma}_t^2-\left(J^2\lambda+\bar{\sigma}^2+\phi_\sigma\hat{\sigma}_t^2+(1-\alpha)\left(\begin{array}{c}\omega_G\left(v_{Y,x}\omega_{x,G}+v_{Y,\sigma}\omega_G\right) \\ +\omega_B\left(v_{Y,\sigma}\omega_B-v_{Y,x}\omega_{x,B}\right)\end{array}\right)\right) && \text{(A.89)} \\ &= -(1-\alpha)\left(\begin{array}{c}\omega_G\left(v_{Y,x}\omega_{x,G}+v_{Y,\sigma}\omega_G\right) \\ +\omega_B\left(v_{Y,\sigma}\omega_B-v_{Y,x}\omega_{x,B}\right)\end{array}\right) && \text{(A.90)} \end{aligned}$$

In the standard case from Bansal and Yaron (2004), we would have $\omega_{x,G} = \omega_{x,B} = 0$, so this would be

$$E\left[IV_{t+1}\right] - P_{IV,t} = (\alpha - 1)\, \upsilon_{Y,\sigma} \left(\omega_G^2 + \omega_B^2\right) \tag{A.91}$$

Since $\upsilon_{Y,\sigma} < 0$, the premium for IV will be negative in that case. Now when $\omega_{x,G}$ can be positive, we have

$$E\left[IV_{t+1}\right] - P_{IV,t} = (\alpha - 1) \left(\upsilon_{Y,x} \left(\omega_{x,G}\omega_G - \omega_{x,B}\omega_B\right) + \upsilon_{Y,\sigma} \left(\omega_G^2 + \omega_B^2\right)\right) \tag{A.92}$$

Since $\upsilon_{Y,x} > 0$, if $\omega_{x,G}$ is sufficiently large relatively to $\omega_{x,B}$, the premium can be positive.

The Sharpe ratio on this object depends on the standard deviation of $IV_{t+1} - P_{IV,t}$, which is exactly $\sqrt{\omega_G^2 + \omega_B^2}$.

A.12.6 Calibration

The calibration is relatively close to Bansal and Yaron's (BY; 2004) choices, with a few changes. For the preferences, we set $\beta = 0.998$ and $\alpha = 15$. β is as in BY, while α is set somewhat higher to help match the equity premium. We study post-war data here, in which the volatility of consumption growth is lower, thus necessitating higher risk aversion to match the equity premium. Leverage, γ, is set to 3.5, on the upper end of the range of values studied by BY.

The jump intensity is 1/18, implying jumps occur on average once every 18 months, while the jump size $J = -0.015$.

The persistence of x and σ^2 are 0.979 and 0.987, as in BY.

$\bar{\sigma} = 0.0039$, which is half the value used in BY in order to match the lower consumption volatility noted above. The standard deviation of innovations to x is set to $0.06{\times}\sigma$, which is somewhat higher than the value of 0.044 in BY. Of that, $\omega_x = \omega_{x,G} = 0.0129$ and $\omega_{x,B} = 0$. Similarly, $\omega_G = \omega_B = 1.62 \times 10^{-6}$, so that the standard deviation of innovations to σ^2 is $0.23{\times}10^{-5}$, as in BY. Finally, $\omega_d = 0.01$.

A.12.7 Results

The table below lists key moments from the model along with analogs from the data. The model moments are based on a monthly simulation of the model that is aggregated to the quarterly frequency to match quarterly data observed empirically (see also BY).

The first three rows on the left show that the model is able to generate realistic values for mean, standard deviation, and Sharpe ratio for equity returns. The top row on the right shows that the volatility of consumption growth is somewhat higher than in the data. However, this value is still smaller than that used by Bansal and Yaron (2004) by 40 percent. Our calibration of 0.87 percent is the midpoint between Bansal and Yaron's (2004) original value and the value in the post-war data. Using a smaller volatility would require either increasing some other form of risk (e.g. long-run risk or stochastic volatility) or risk aversion in order to generate a realistic equity premium.

Next, the table shows that the Sharpe ratios for claims on RV and IV are approximately -0.21 and 0.19, respectively, which agree well with the empirical values (which are calculated as the overall means across all 19 markets we study; see figure 3). These are the key moments that the model was designed to match. They show that it is able to generate quantitatively realistic premia for uncertainty and realized volatility shocks.

As discussed in the main text, the economic mechanism behind the negative premium on RV is negative conditional skewness in consumption growth, while the mechanism behind the positive premium for IV – the good volatility shocks that raise future consumption growth – pushes in the direction of positive skewness. That implies that the skewness of the conditional expectation of consumption growth should be less negative than conditional skewness. To test that idea, we examine skewness in the model and data. The information set used for conditioning here is lagged consumption growth. That is, we look at results involving regressions of consumption growth on three of its own lags in both the model and the data.

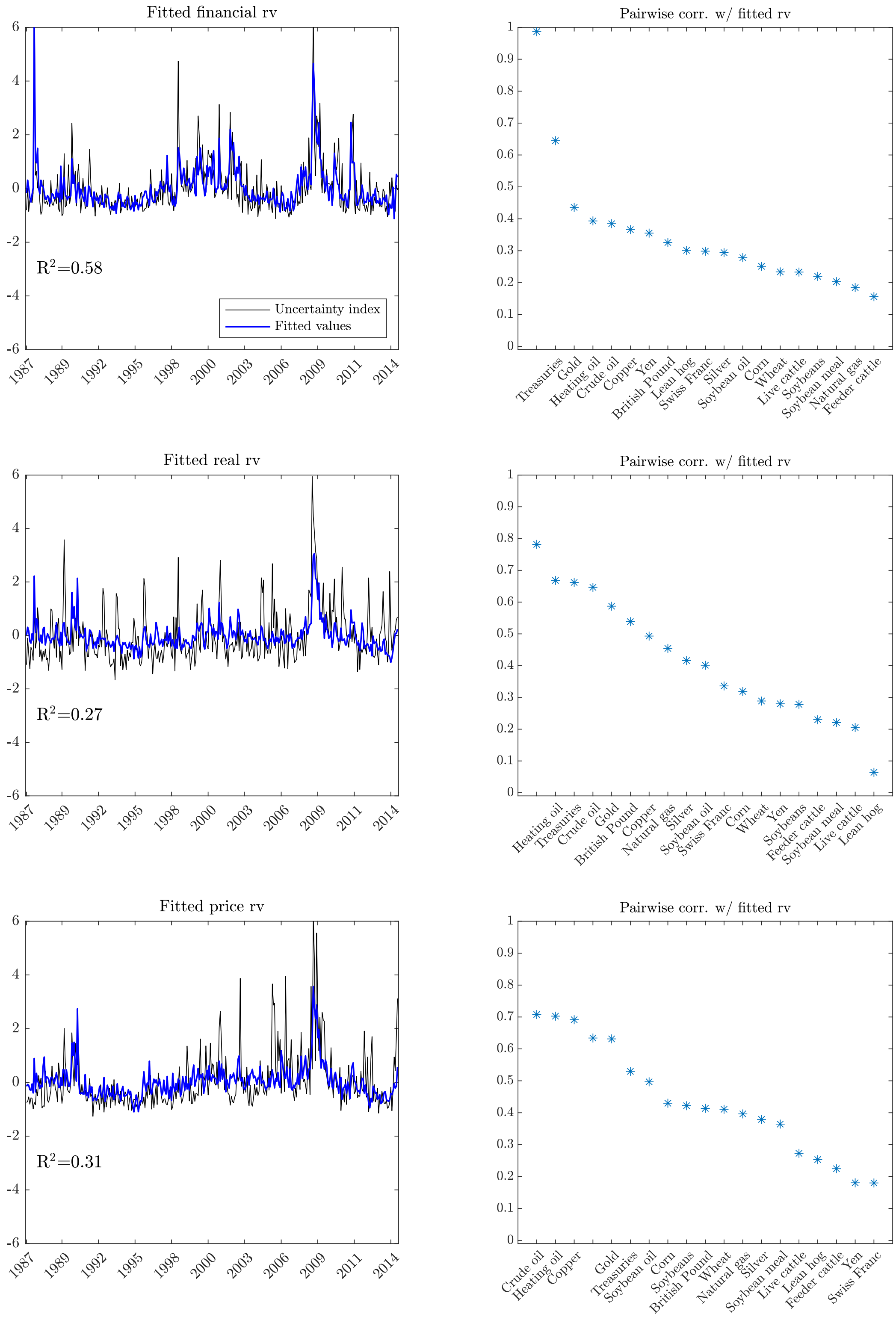
The table shows that the data and model both share the feature that the conditional expectation of consumption growth is much less negatively skewed than the surprise in consumption growth, consistent with the main mechanism in the model. This is not a moment that the model was explicitly designed to match. The model was meant to match the premia on RV and IV, so this represents an additional test of the proposed mechanism.

To be clear, the main contribution of the paper is not meant to be this model, but nevertheless this section shows that the empirical results can be rationalized in a standard structural asset pricing model.

Summary statistics from the model and empirical data, 1947–2018

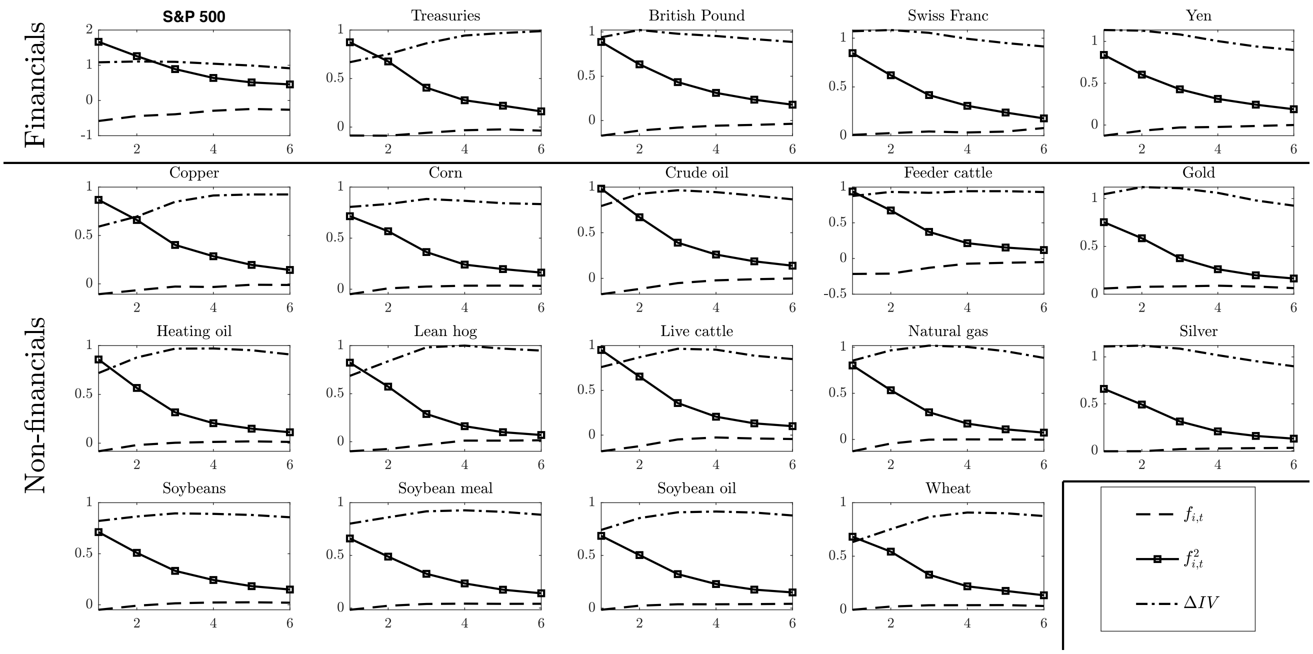
	Model	Data		Model	Data
$E\left[r_m - r_f\right]$	0.077	0.056	$std\left(\Delta c\right)$	0.0087	0.0052
$std\left(r_m - r_f\right)$	0.14	0.11	$skew_t\left(\Delta c_{t+1}\right)$	-0.32	-0.15
$\frac{E\left[r_m - r_f\right]}{std\left(r_m - r_f\right)}$	0.53	0.52	$skew\left(E_t \Delta c_{t+1}\right)$	-0.10	-0.07
$\frac{E\left[RV_{t+1} - P_{RV,t}\right]}{std\left[RV_{t+1} - P_{RV,t}\right]}$	-0.21	-0.32			
$\frac{E\left[RV_{t+1} - P_{RV,t}\right]}{std\left[RV_{t+1} - P_{RV,t}\right]}$	0.19	0.26			

Figure A.1: Fit to realized volatility indexes



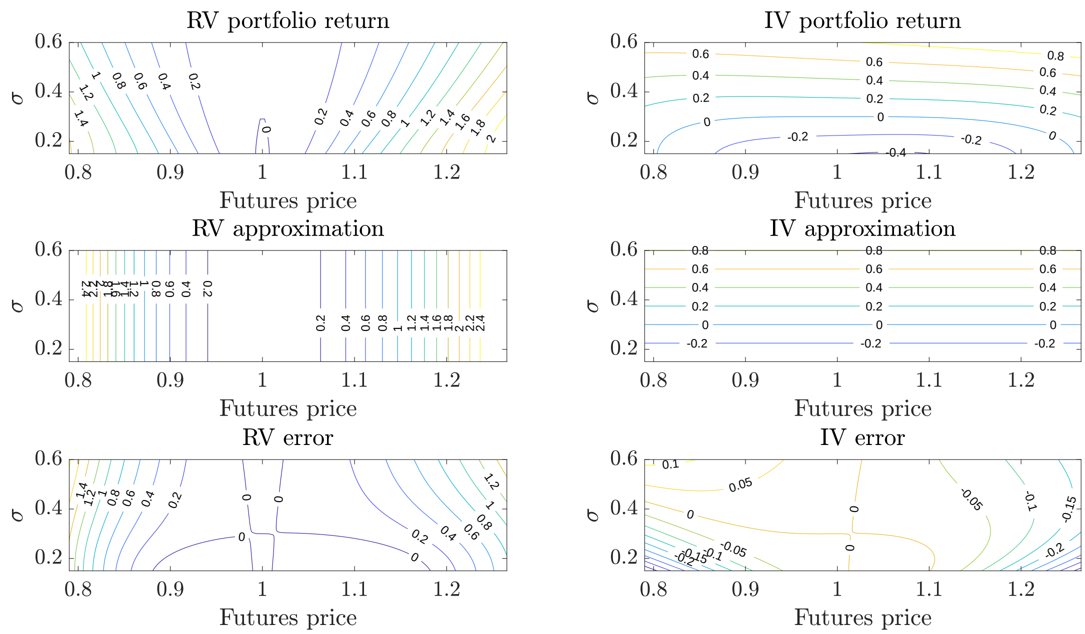
Note: See figure 2. This figure uses the JLN realized volatility series instead of uncertainty.

Figure A.2: Factor loadings



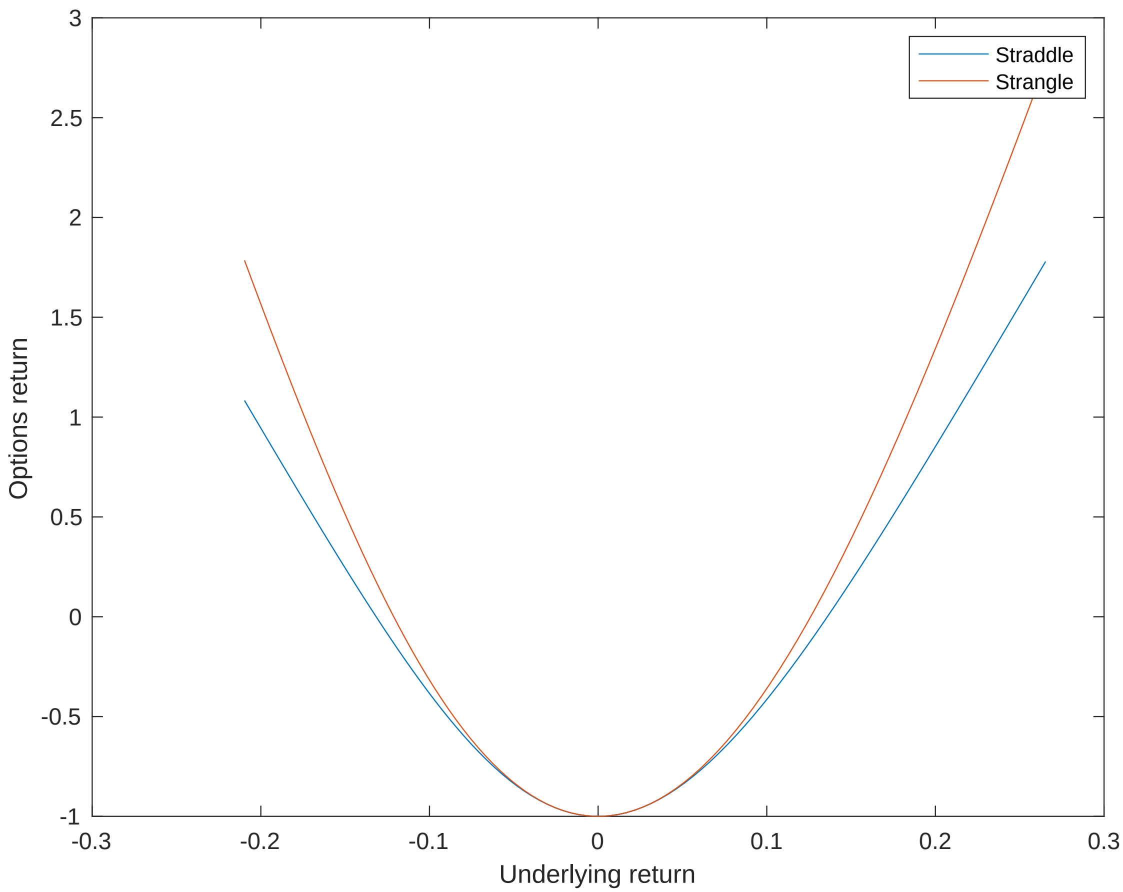
Note: Loadings of two-week straddle returns on the three risk factors. The factors are all scaled by current IV, as in equation 2. The loadings are scaled so that if the Black-Scholes approximation was exact, the loading on ΔIV would be 1 at all maturities, the loading on $f_{i,t}$ would be 0 at all maturities, and the loading on $f_{i,t}^2$ would be $1/n$ where n is the maturity in months.

Figure A.3: rv and iv portfolio approximation errors



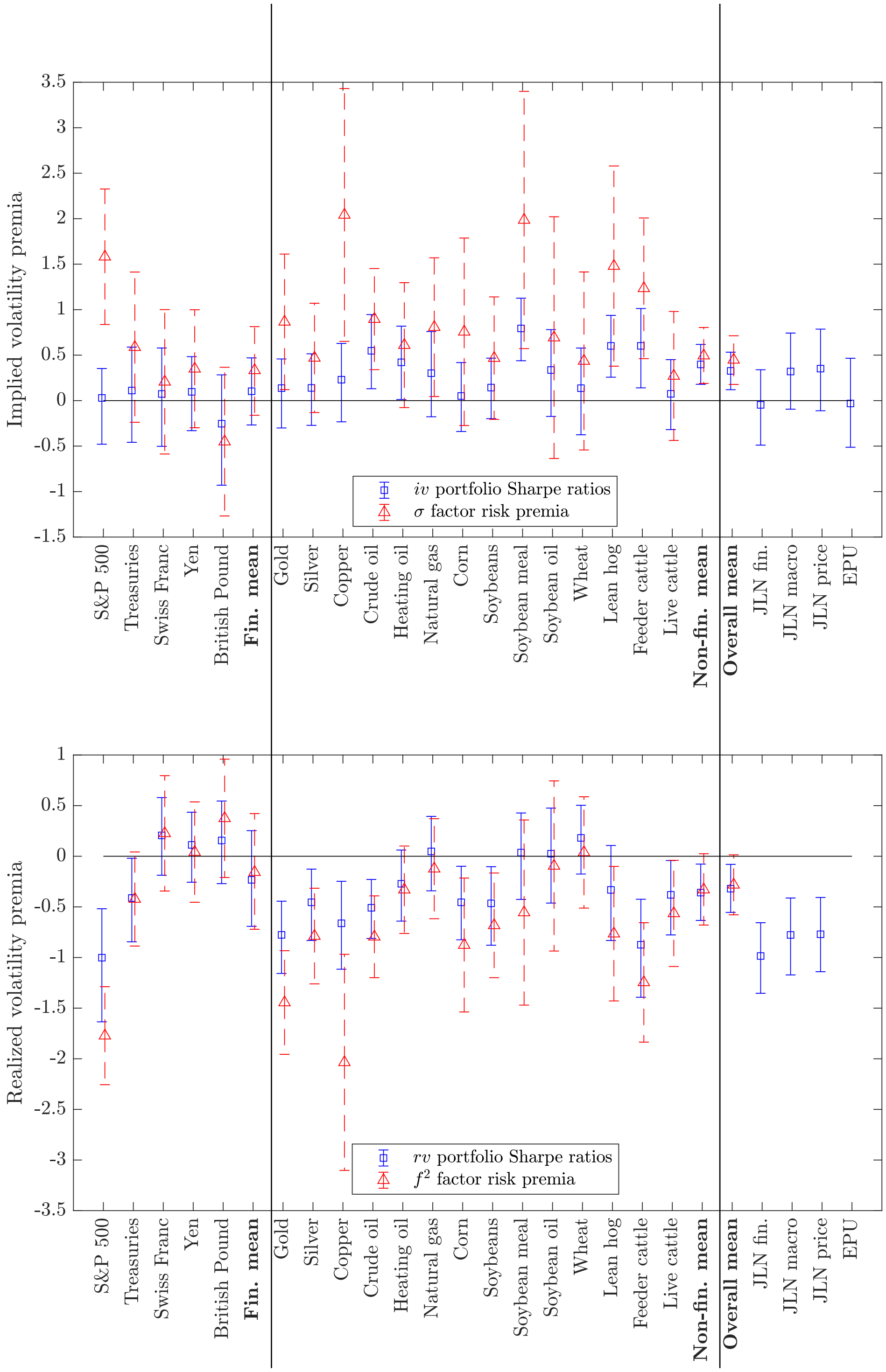
Note: The initial futures price is 1 and the initial volatility, σ, is 0.3. The top panels calculate the return on the rv and iv portfolios given an instantaneous shift in the futures price and volatility to the values reported on the axes under the assumption that the Black–Scholes formula holds. The middle panels plot returns under the approximations used in the text. The bottom panels are equal to the middle minus the top panels. All returns and errors are reported as decimals.

Figure A.4: Straddle and strangle returns



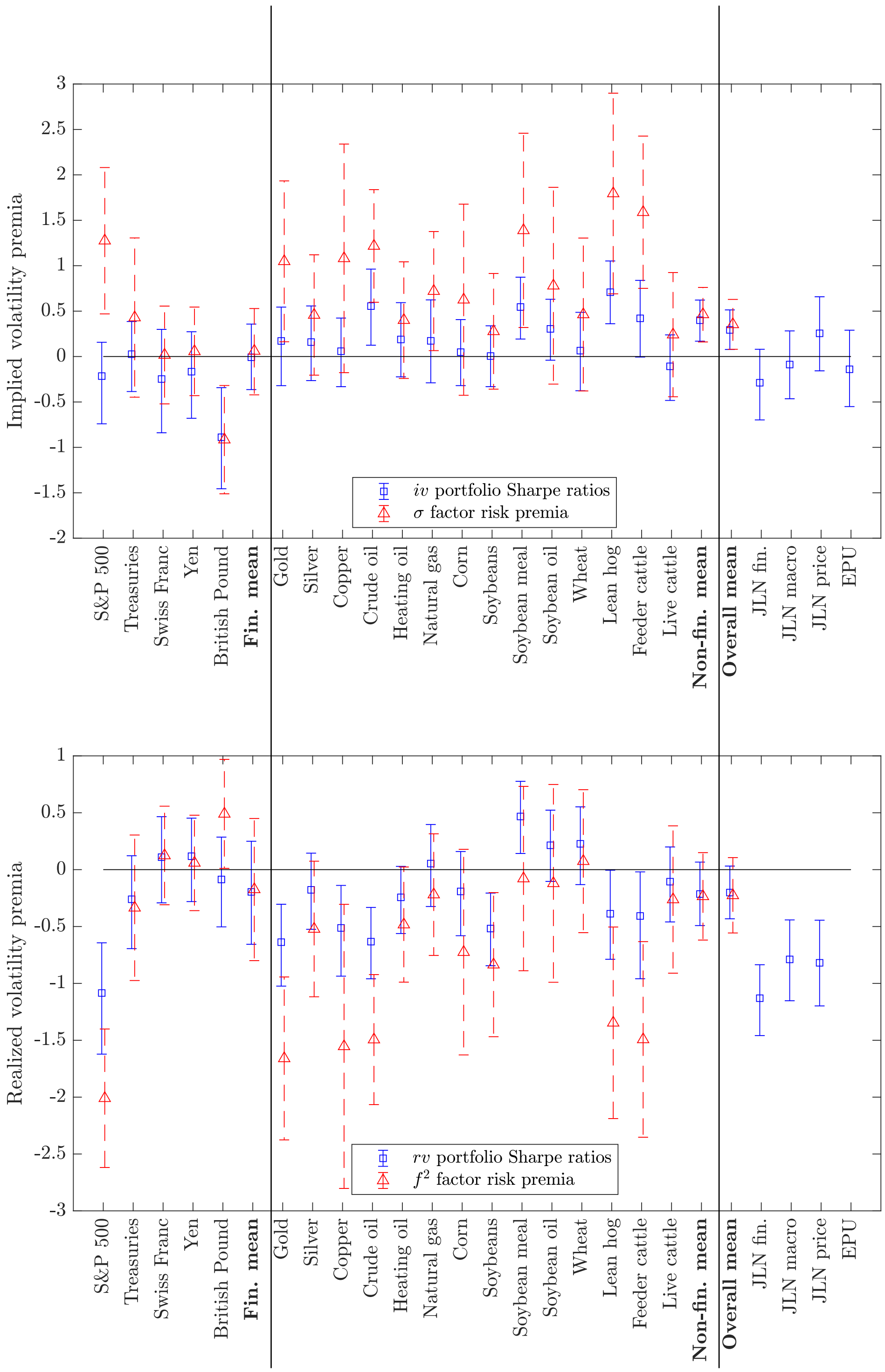
Note: Returns of 1-standard deviation strangles and straddles as function of the underlying's return.

Figure A.5: Imposing a filter on volume



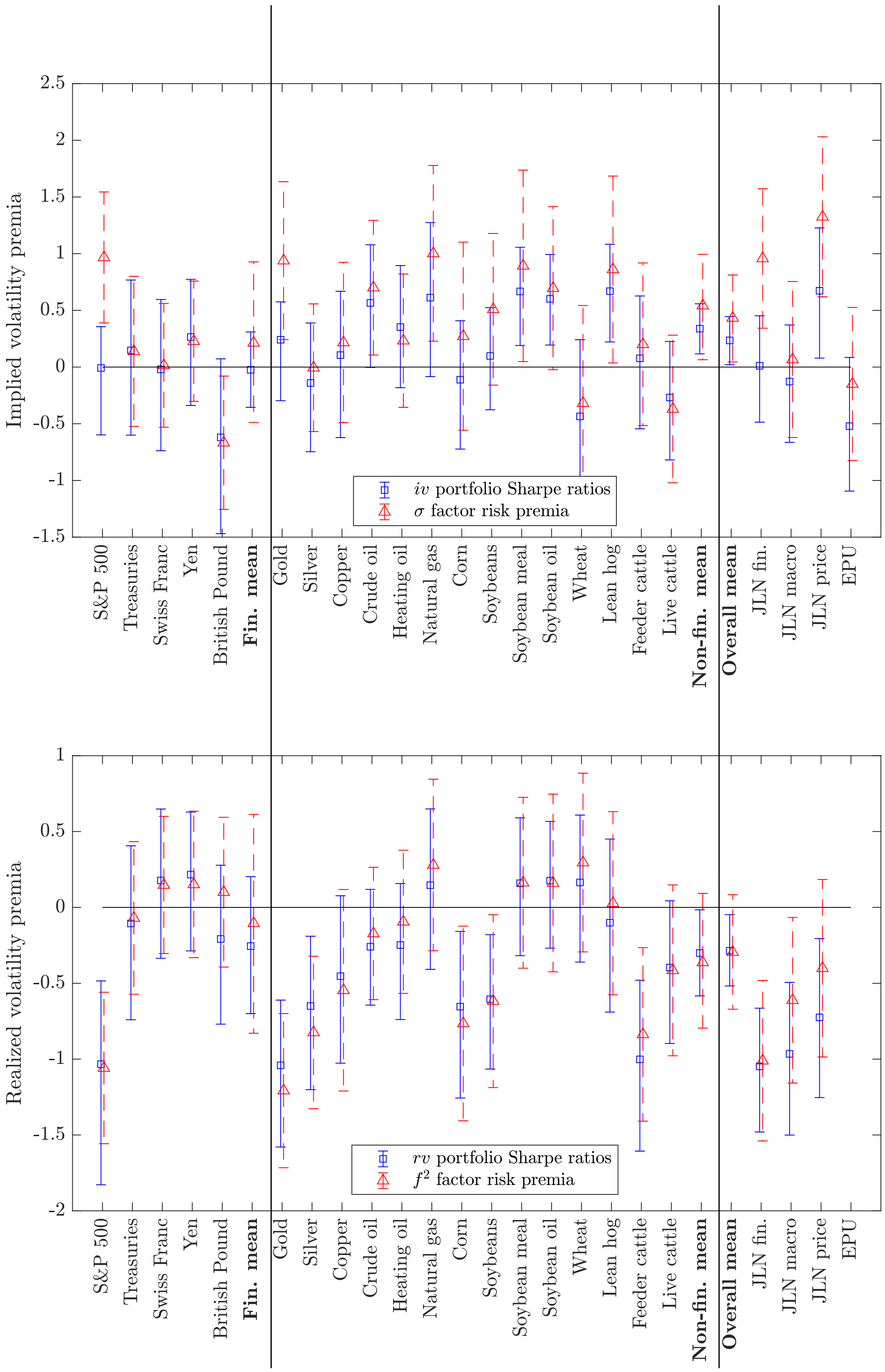
Note: Same as figure 3, but using only options for which volume is neither zero nor missing.

Figure A.6: RV and IV portfolio Sharpe ratios and factor risk premia, one-week holding period



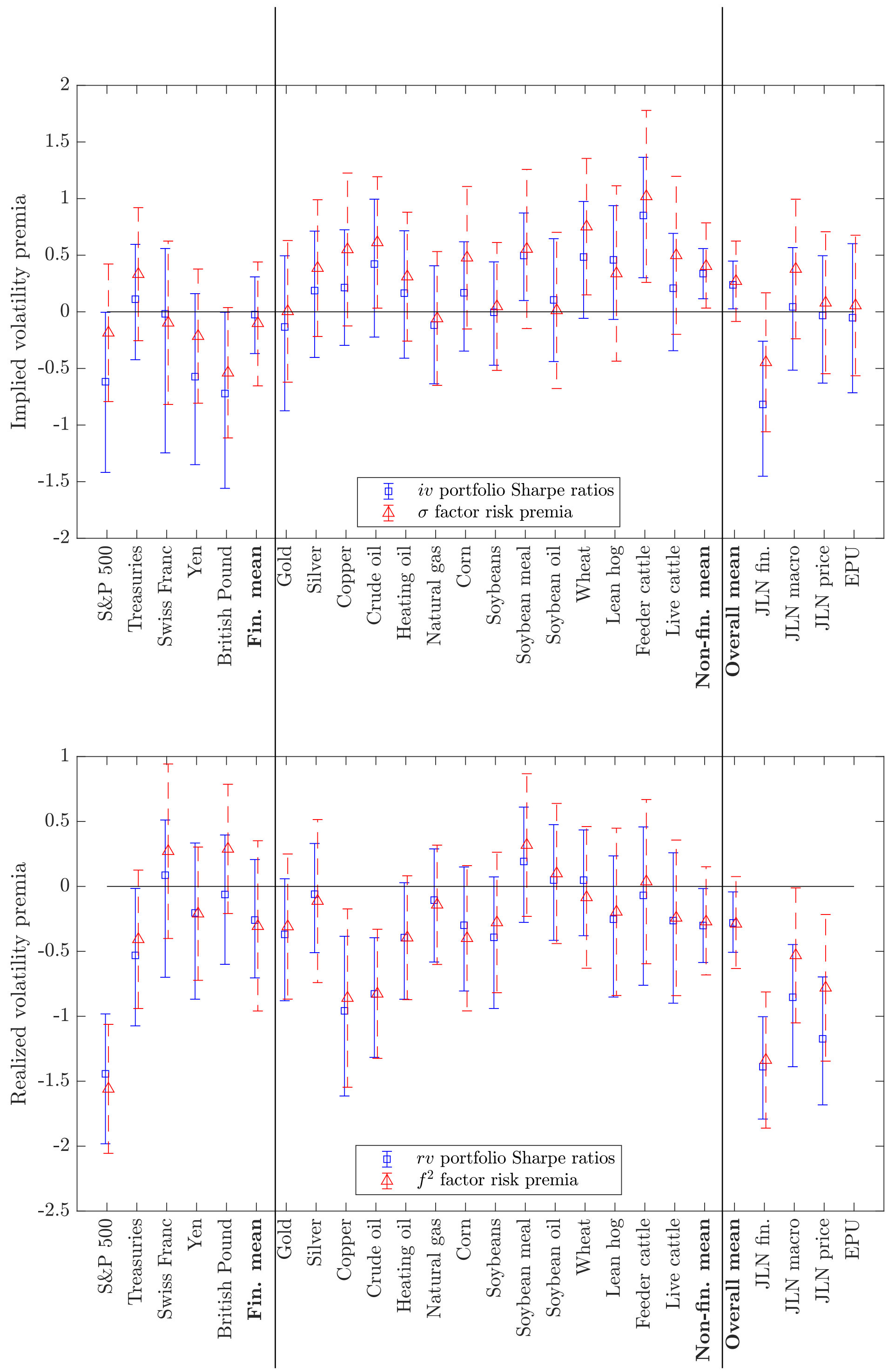
Note: Same as figure 3, but using one-week holding periods.

Figure A.7: RV and IV portfolio Sharpe ratios and factor risk premia (first half of the sample)



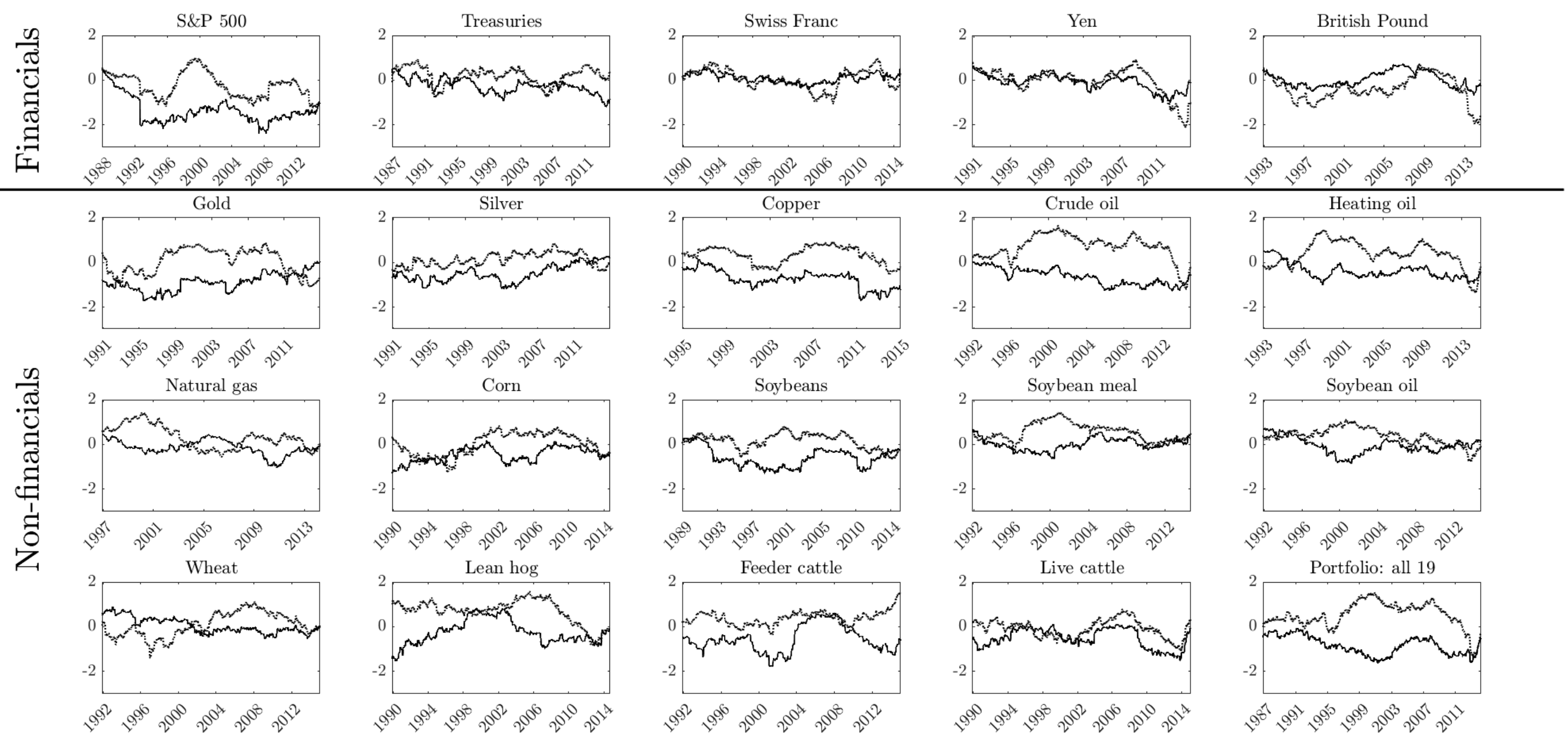
Note: Same as Figure 3, but only using the first half of the sample (up to June 2000).

Figure A.8: RV and IV portfolio Sharpe ratios and factor risk premia (second half of the sample)



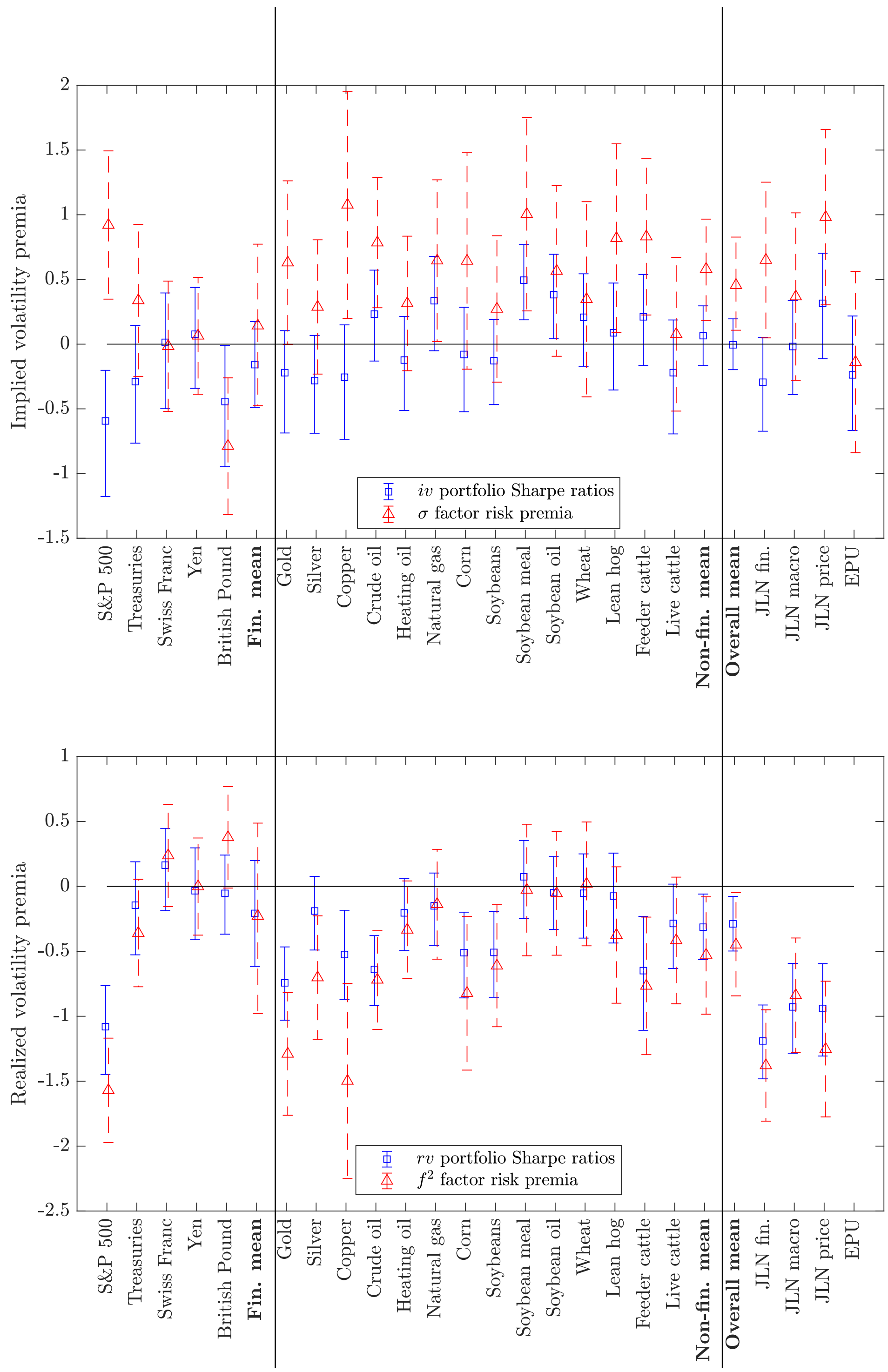
Note: Same as Figure 3, but only using the second half of the sample (after June 2000).

Figure A.9: Rolling Sharpe ratios of RV and IV portfolios



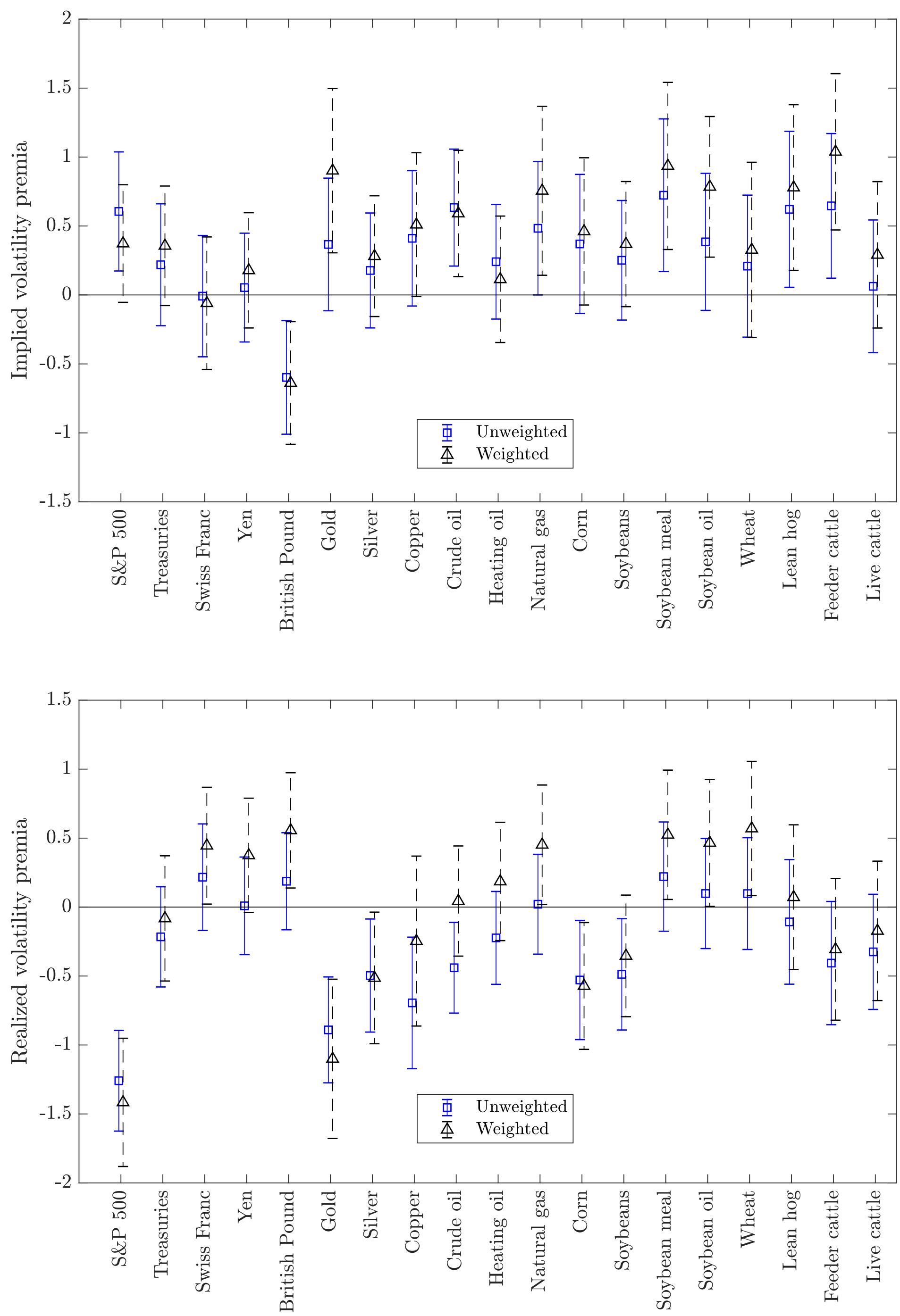
Note: 5-year rolling Sharpe ratios for RV portfolios (solid line) and IV portfolios (dotted lines). The bottom-right panel reports the rolling Sharpe ratio for RV and IV portfolio of all available markets.

Figure A.10: RV and IV portfolio Sharpe ratios and factor risk premia (using 2-month IV)



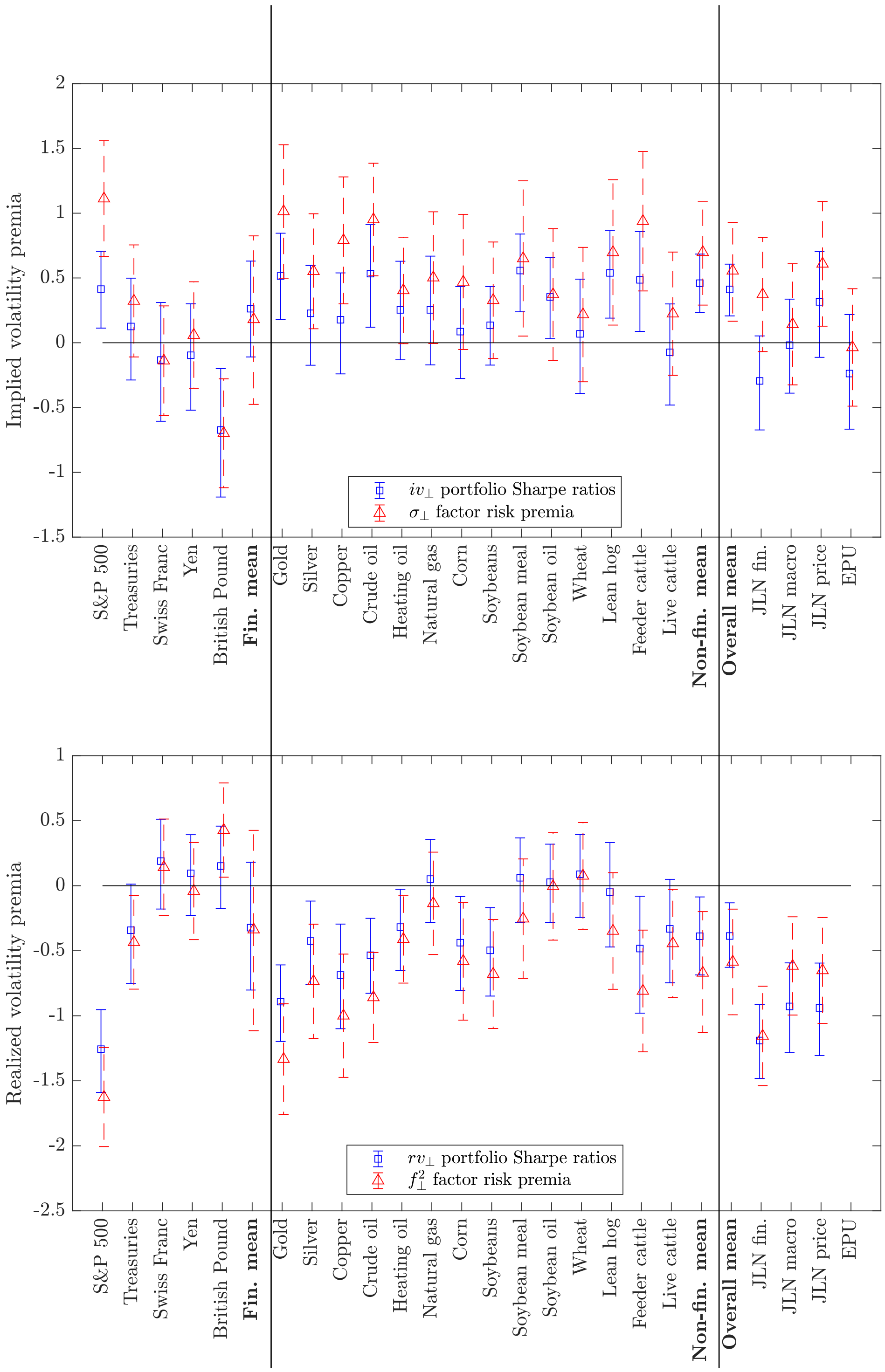
Note: Same as Figure 3, but using 2-month instead of 5-month IV.

Figure A.11: RV and IV risk premia estimates with and without weighting



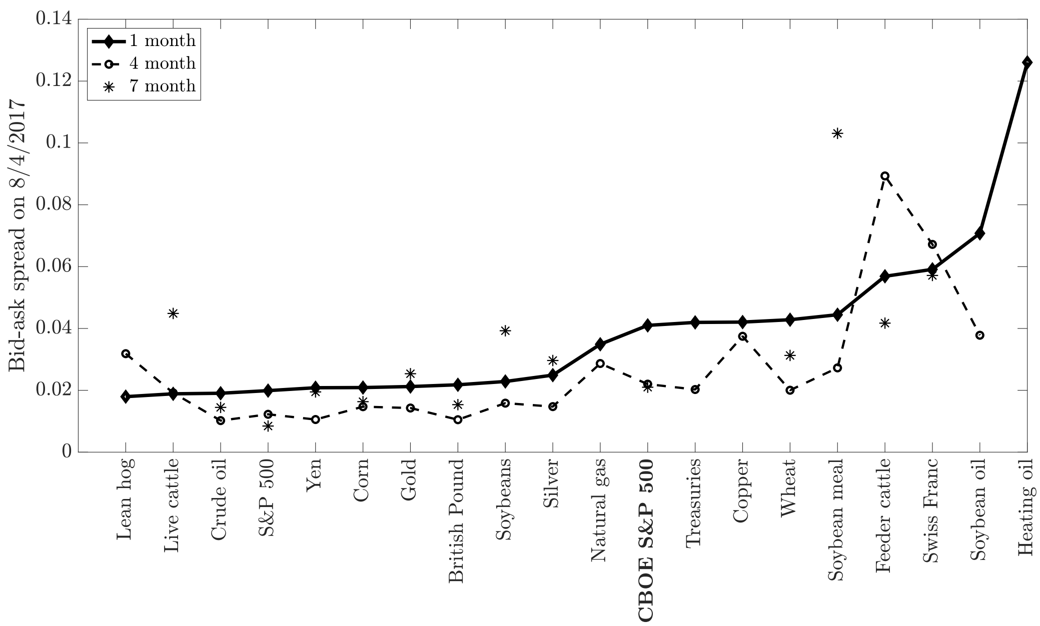
Note: The figure reports risk premia for the factor model, unweighted (as in figure 3) or weighting each observation by the implied volatility.

Figure A.12: SDF loadings on RV and IV (Sharpe ratios)



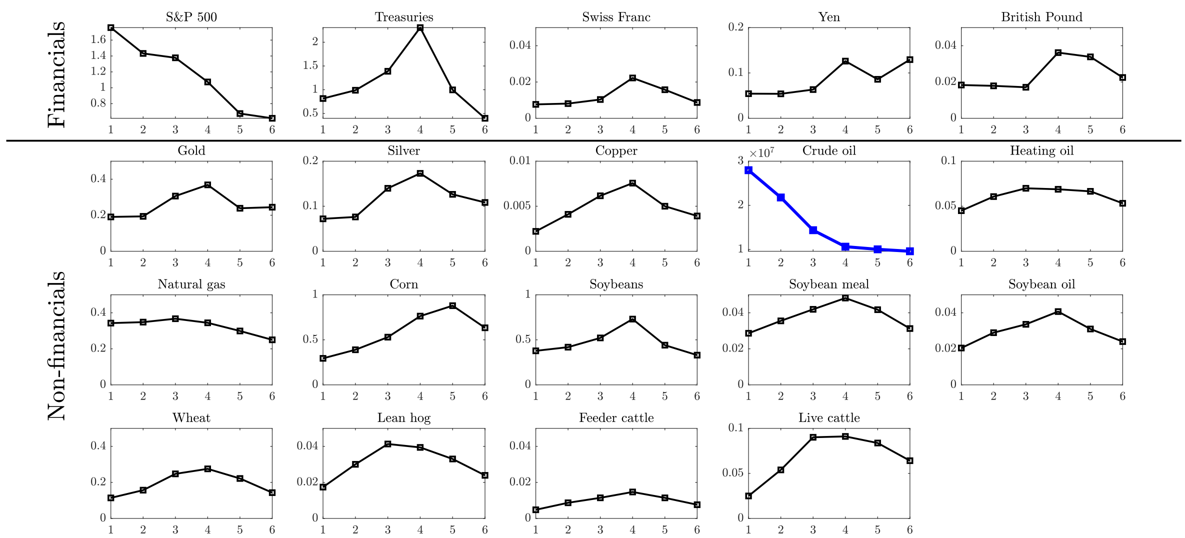
Note: The figure reports the stochastic discount factor (SDF) loadings on IV and RV. The loadings are scaled to correspond to Sharpe ratios of orthogonalized RV and IV portfolios, whose risk premia is equal to the corresponding SDF loading.

Figure A.13: Bid-ask spreads on 8/4/2017



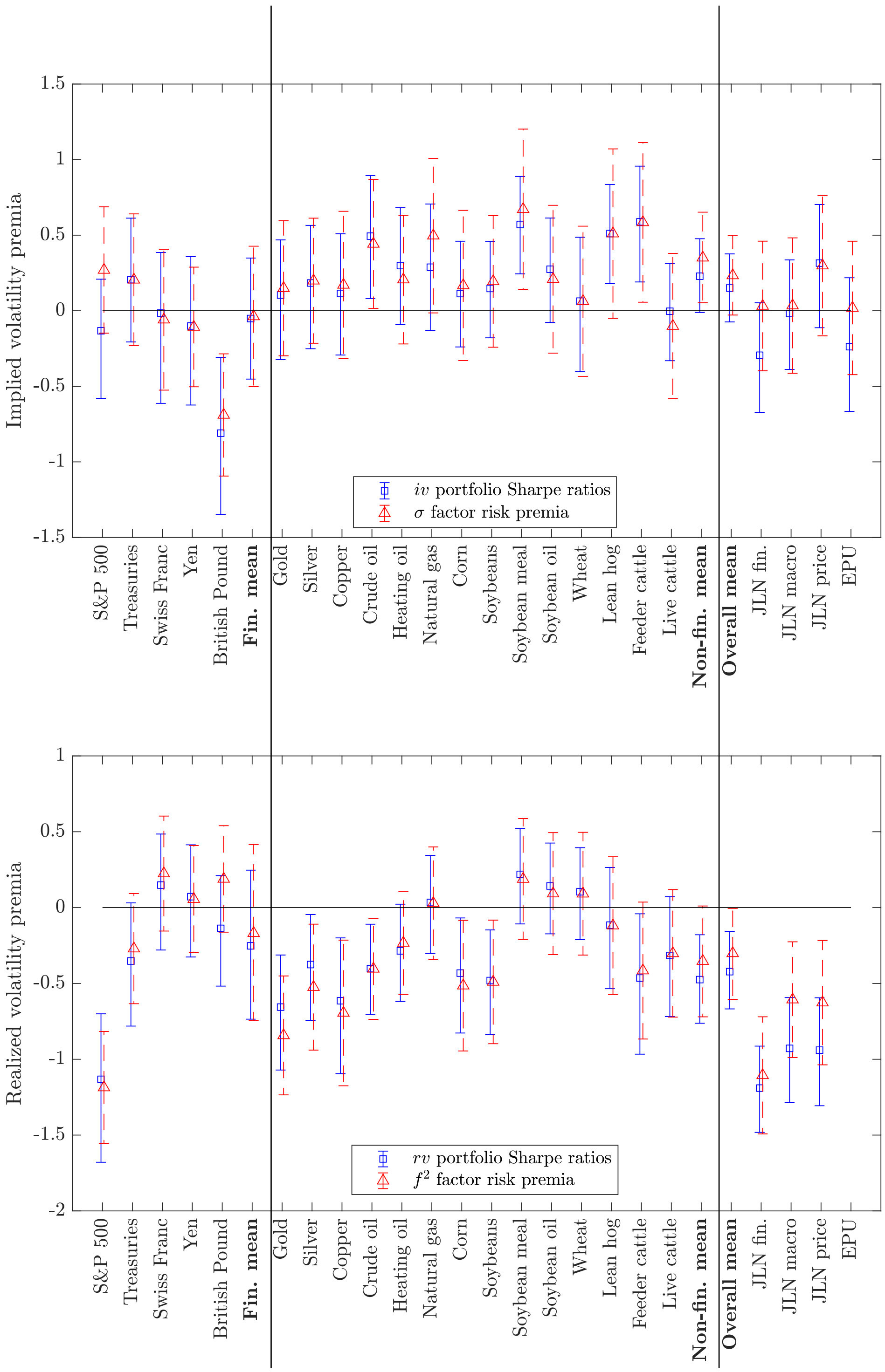
Note: The figure reports posted bid-ask spreads for at-the-money straddles obtained from Bloomberg on of August 4, 2017 (the CBOE S&P 500 spreads on that date are also obtained from Optionmetrics).

Figure A.14: Volume across markets and maturities



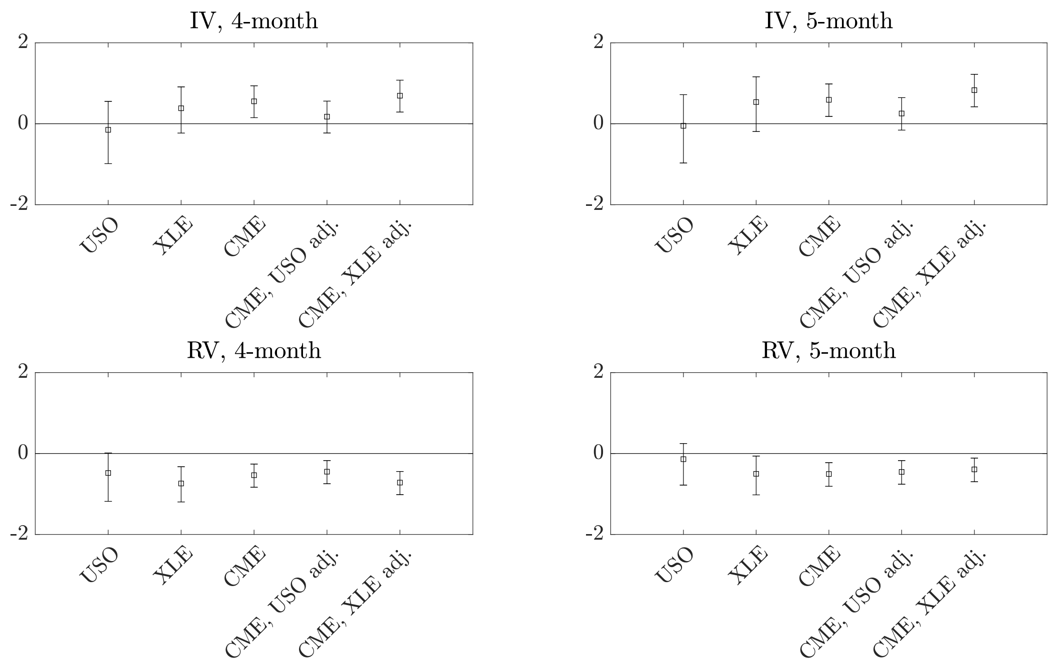
Note: Average daily volume of options in different markets. The panel corresponding to crude oil reports values in dollars. All other panels show values relative to the volume in the crude oil market, matched by maturity.

Figure A.15: RV and IV portfolio Sharpe ratios and factor risk premia (robust to measurement error)



Note: Same as Figure 3, but returns are computed using the same denominator at all maturities, to provide robustness with respect to measurement error in the prices (see section A.4.6).

Figure A.16: Options on crude futures vs ETFs



Note: Sharpe ratios on *rv* and *iv* portfolios using straddles for CME crude oil futures and the XLE and USO exchange traded funds. "4-month" and "5-month" refers to the longer of the two maturities used to construct each portfolio (the short maturity is always one month). The squares are point estimates based on the full sample available for each series. The lines are 95-percent confidence bands constructed with a 50-day block bootstrap. "CME, USO adj." and "CME, XLE adj." are identical to the "CME" numbers but with the mean return in the denominator of the Sharpe ratio shifted by the point estimate for the mean difference from table A.6.2.

Table A.1: χ^2 test of the factor model

	p-value
S&P 500	0.22
T-bonds	0.02
GBP	0.01
CHF	0.38
JPY	0.75
Copper	0.75
Corn	0.00
Crude oil	0.08
Feeder cattle	0.25
Gold	0.44
Heating oil	0.14
Lean hog	0.19
Live cattle	0.80
Natural gas	0.30
Silver	0.68
Soybeans	0.21
Soybean meal	0.41
Soybean oil	0.11
Wheat	0.29

Note: For each market, the table reports bootstrapped p-values for the χ^2 of on the squared fitting errors of the factor model (bootstrapped following Constantinides, Jackwerth, and Savov (2013).

Table A.2: Risk exposures of *rv* and *iv* portfolios

rv portfolio	f	f^2	ΔIV	R^2	**iv portfolio**	f	f^2	ΔIV	R^2	Corr(rv,iv)
S&P 500	-0.07	1.44	0.02	0.68	S&P 500	-0.16	1.37	0.96	0.75	0.48
T-bonds	-0.01	0.81	-0.06	0.75	T-bonds	-0.01	0.35	1.05	0.78	0.13
GBP	-0.03	0.81	0.00	0.82	GBP	-0.02	0.44	0.91	0.86	0.47
CHF	0.00	0.75	0.03	0.73	CHF	0.05	0.52	0.91	0.72	0.64
JPY	-0.02	0.74	0.04	0.80	JPY	0.02	0.57	0.89	0.87	0.63
Copper	-0.01	0.79	-0.06	0.62	Copper	0.01	0.23	1.00	0.85	0.07
Corn	-0.02	0.65	-0.01	0.69	Corn	0.06	0.41	0.85	0.75	0.08
Crude oil	-0.03	1.00	-0.02	0.75	Crude oil	0.03	-0.07	0.93	0.77	0.06
Feeder cattle	-0.03	0.98	-0.01	0.66	Feeder cattle	-0.02	-0.25	0.96	0.78	0.02
Gold	0.00	0.70	0.01	0.68	Gold	0.08	0.35	0.97	0.68	0.48
Heating oil	-0.02	0.88	-0.04	0.76	Heating oil	0.04	-0.17	1.00	0.77	-0.02
Lean hog	-0.02	0.90	-0.06	0.75	Lean hog	0.04	-0.49	1.03	0.64	-0.24
Live cattle	-0.03	1.03	-0.03	0.72	Live cattle	0.00	-0.44	0.92	0.78	-0.12
Natural gas	-0.03	0.87	-0.02	0.80	Natural gas	0.03	-0.38	0.98	0.64	-0.17
Silver	-0.01	0.63	0.03	0.71	Silver	0.04	0.20	0.92	0.85	0.45
Soybeans	-0.02	0.66	-0.01	0.71	Soybeans	0.04	0.30	0.89	0.80	0.18
Soybean meal	-0.01	0.61	-0.02	0.74	Soybean meal	0.05	0.31	0.93	0.69	0.19
Soybean oil	-0.01	0.64	-0.02	0.73	Soybean oil	0.05	0.29	0.94	0.77	0.20
Wheat	-0.01	0.63	-0.05	0.78	Wheat	0.05	0.30	0.97	0.78	0.16
Average	-0.02	0.82	-0.01	0.73	Average	0.02	0.20	0.95	0.76	

Note: The table reports regression coefficients of the *rv* and *iv* portfolios for each market onto three market-specific factors: the futures return, the squared futures return, and the change in IV. The column on the right reports the correlation between the rv and iv portfolio returns.

Table A.3: Risk exposures of rv portfolio to IV innovations at different maturity

	Maturity of IV shock				
rv portfolio	1	2	3	4	5
S&P 500	0.08	0.08	0.07	0.05	0.02
T-bonds	0.07	0.06	0.03	0.00	-0.06
GBP	0.07	0.07	0.06	0.04	0.00
CHF	0.07	0.07	0.07	0.06	0.03
JPY	0.07	0.07	0.07	0.06	0.04
Copper	0.08	0.08	0.05	0.00	-0.06
Corn	0.08	0.08	0.07	0.05	-0.01
Crude oil	0.06	0.06	0.04	0.01	-0.02
Feeder cattle	0.09	0.08	0.06	0.03	-0.01
Gold	0.07	0.07	0.07	0.05	0.01
Heating oil	0.07	0.07	0.05	0.02	-0.04
Lean hog	0.09	0.08	0.06	0.01	-0.06
Live cattle	0.08	0.08	0.06	0.02	-0.03
Natural gas	0.08	0.08	0.07	0.03	-0.02
Silver	0.09	0.09	0.09	0.07	0.03
Soybeans	0.07	0.07	0.06	0.03	-0.01
Soybean meal	0.07	0.07	0.05	0.03	-0.02
Soybean oil	0.07	0.07	0.05	0.02	-0.02
Wheat	0.05	0.05	0.03	-0.01	-0.05

	Maturity of IV shock				
RV-hedging	1	2	3	4	5
S&P 500	0.05	0.04	0.04	0.02	0.00
T-bonds	0.11	0.11	0.10	0.07	0.00
GBP	0.08	0.08	0.07	0.05	0.00
CHF	0.07	0.07	0.06	0.04	0.00
JPY	0.07	0.07	0.06	0.04	0.00
Copper	0.13	0.13	0.12	0.07	0.00
Corn	0.12	0.12	0.12	0.08	0.00
Crude oil	0.07	0.07	0.06	0.03	0.00
Feeder cattle	0.09	0.09	0.07	0.04	0.00
Gold	0.10	0.10	0.08	0.05	0.00
Heating oil	0.09	0.09	0.08	0.06	0.00
Lean hog	0.11	0.11	0.10	0.06	0.00
Live cattle	0.09	0.09	0.07	0.04	0.00
Natural gas	0.10	0.10	0.09	0.06	0.00
Silver	0.12	0.12	0.11	0.07	0.00
Soybeans	0.11	0.11	0.09	0.06	0.00
Soybean meal	0.12	0.12	0.10	0.07	0.00
Soybean oil	0.12	0.12	0.11	0.07	0.00
Wheat	0.11	0.11	0.10	0.06	0.00

Note: The table reports the loading of the rv portfolio (top panel) and of the RV-hedging portfolio built using the factor model (bottom panel) on shocks to IV of different maturity, from 1 to 5 months.

Table A.4: Risk exposures of *rv* and *iv* portfolios, 2-month IV

rv portfolio	f	f^2	ΔIV	R^2	**iv portfolio**	f	f^2	ΔIV	R^2	Corr(rv,iv)
S&P 500	-0.04	0.74	0.04	0.38	S&P 500	-0.33	4.79	0.78	0.66	0.27
T-bonds	0.00	0.37	0.00	0.39	T-bonds	-0.08	2.44	0.84	0.72	0.14
GBP	-0.02	0.45	0.03	0.50	GBP	-0.05	2.04	0.70	0.73	0.27
CHF	0.00	0.40	0.03	0.44	CHF	0.07	2.16	0.74	0.69	0.40
JPY	-0.02	0.42	0.04	0.54	JPY	-0.01	2.04	0.72	0.82	0.46
Copper	-0.01	0.33	0.02	0.25	Copper	0.00	2.18	0.78	0.68	-0.04
Corn	-0.02	0.27	0.03	0.32	Corn	0.10	2.17	0.64	0.72	0.07
Crude oil	-0.01	0.58	0.01	0.50	Crude oil	-0.06	1.72	0.77	0.71	0.19
Feeder cattle	0.00	0.45	0.04	0.36	Feeder cattle	-0.21	2.07	0.77	0.58	0.05
Gold	-0.01	0.28	0.02	0.35	Gold	0.09	2.21	0.84	0.66	0.23
Heating oil	-0.02	0.54	0.01	0.49	Heating oil	0.04	1.31	0.80	0.62	0.09
Lean hog	-0.01	0.45	0.03	0.45	Lean hog	-0.01	1.59	0.74	0.59	0.08
Live cattle	-0.02	0.53	0.02	0.47	Live cattle	-0.06	1.75	0.75	0.68	0.18
Natural gas	-0.03	0.50	0.02	0.55	Natural gas	0.03	1.28	0.77	0.67	0.18
Silver	0.00	0.28	0.04	0.41	Silver	-0.01	1.70	0.79	0.76	0.30
Soybeans	-0.02	0.38	0.03	0.50	Soybeans	0.05	1.52	0.69	0.77	0.29
Soybean meal	-0.01	0.31	0.03	0.47	Soybean meal	0.07	1.60	0.66	0.75	0.26
Soybean oil	-0.01	0.33	0.02	0.43	Soybean oil	0.07	1.63	0.73	0.72	0.20
Wheat	-0.01	0.26	0.00	0.33	Wheat	0.07	2.16	0.70	0.79	0.14
Average	-0.01	0.41	0.02	0.43	Average	-0.01	2.02	0.75	0.70	

Note: Same as table A.2, but 2-month IV is used as one of the factors (as opposed to 5-month IV) and in the construction of the *rv* and *iv* portfolios.