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Abstract

This paper explores how insurance companies can coordinate to extend their joint capacity for the coverage of new and undiversifiable risks. The undiversifiable nature of such risks causes a shortage of insurance capacity and their limited knowledge makes learning and information sharing necessary. We develop a unified theoretical model to analyse co-insurance agreements. We show that organizing this insurance supply amounts to sharing a common value divisible good between capacity constrained and privately informed insurers with a reserve price. Coinsurance via the creation of an insurance pool turns out to operate as a uniform price auction with an ``exit/re-entry'' option. We compare it to a discriminatory auction for which no specific agreements are needed. Both auction formats lead to different coverage/premium tradeoffs. If at least one insurer provides an optimistic expertise about the risk, the pool offers higher coverage. This result is reversed when all insurers are pessimistic about the risk. Static comparative results with respect to the severity of the capacity constraints and the reserve price are provided. In the case of completely new risks, a regulator aiming at maximizing the expected coverage should promote the pool when the reserve price is low enough or when competition is high enough.

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Organizing insurance supply for new and undiversifiable risks *

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April 17, 2023

Abstract

This paper explores how insurance companies can coordinate to extend their joint capacity for the coverage of new and undiversifiable risks. The undiversifiable nature of such risks causes a shortage of insurance capacity and their limited knowledge makes learning and information sharing necessary. In practice, organizing such insurance supply amounts to sharing a common value divisible good between capacity constrained and privately informed insurers with a reserve price. Widely used ad-hoc co-insurance agreements out to operate as a uniform price auction with an "exit/re-entry" option. We compare it to a discriminatory auction, another auction present in the insurance industry. Both auction formats lead to different coverage/premium tradeoffs. If at least one insurer provides an optimistic expertise about the risk, ad-hoc co-insurance agreements offer higher coverage. This result is reversed when all insurers are pessimistic about the risk. Static comparative results with respect to the severity of the capacity constraints and the reserve price are provided. In the case of completely new risks, a regulator aiming at maximizing the expected coverage should promote ad-hoc co-insurance agreements when the reserve price is low enough or when capacity constraints are large enough.

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1 Introduction

This paper develops an explanatory model for the insurance of undiversifiable and new risks. Certain large unconventional risks such as terrorism, nuclear power production, environmental protection or pandemic risks cannot be born by a single insurance company. Indeed, when the amounts of claims are very high, they exceed the financial capacity of insurers, equity and reinsurance included. In addition, the regulation of insurance companies requires a minimum level of capital and a target capital to absorb such risks (Solvability II in the European Union for instance): a single insurer can hardly fulfill this condition. Finally, these so-called unconventional risks are often poorly understood, the claim history being limited or even non-existent. All these characteristics explain the difficulty of their coverage by a standard insurance mechanism. A widespread practice in the insurance industry to achieve co-insurance is to set up ”ad-hoc co-insurance agreements”. Ad-hoc co-insurance agreements are developed in the market, facilitated and negotiated by a broker. Such cooperations not only directly and significantly increases the financial capacity of participating insurers, but also allow to organize information sharing on the nature and intensity of the risks insured. The latter characteristic is crucial for the insurance of unknown and new risks since the sources of information are often dispersed, moving and heterogeneous.

We propose to develop a unified theoretical model using auction theory to analyze these ad-hoc co-insurance agreements. Our theoretical model builds on a simplified representation of insurers' interactions based on empirical findings of a report commissioned by the European Commission [7].[1] Even if some country-specific differences exist, they share some common features. The prevalent procedure for the conclusion of an ad-hoc co-insurance agreement is broker-led within a two-stage auction, defining a leading insurer and following ones. The leader's selection process may combine the following factors: capacity, premium, insurer's expertise or reputation, terms and conditions of the offer. As European Commission [7] note, “*the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer*”. Based on this, we propose to study a simplified scenario where ad-hoc agreements operate as a uniform price auction with an “exit/re-entry” option and in which a leading insurer is selected on the basis of the more competitive bid premium.

Following the observations drawn up, we incorporate several key ingredients to our modelling strategy. First, the undiversifiable nature of the risks causes a shortage of insurance capacity so that a single insurance company cannot insure such risks. These capacity constraints may come either from legal solvency regulation constraints and capital requisites that are imposed to prevent from insurers' bankruptcy or from the characteristics of the risk itself. Second, when contemplating insuring new risks whose knowledge is limited, insurers use their own expertise to evaluate them. Each insurer has its own learning on the occurrence probability of the risk. At the industry level, this creates informational asymmetries not only between insurers and insureds, but also between insurers.

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[1] The European Commission [7] provides a detailed description of the procedures leading to the ad-hoc agreements in several European countries.

The contracts' terms will not only depend on insurers' beliefs on the probability of the risk, but also on the functioning rules of the coinsurance agreement. Third, we will deal with the most general case in which the insurance of such risk is not mandatory. Policyholders may remain uninsured or partially insured if insurance is too costly (see Kousky and Cooke [14] for instance). In terms of modelling, we will assume that insureds have a reserve price.

The game we consider is then a particular auction of a common value divisible good between capacity constrained agents who have private information in presence of a reserve price. We refer this particular auction as the "Flexible Uniform auction" (FU auction). We characterize the equilibrium risk premium of this auction and the resulting insurance capacity offered when two identical insurers compete for the risk. We then compare the outcome of the FU auction to a discriminatory auction (in which each insurer offers its own conditions (capacity and premium)), an auction format also prevalent in the insurance sector.[2] This leads us to compare different auction pricing rules and to understand the role of the exit/re-entry option. These auction formats are compared with respect to premiums and coverage, taking into account the impact of different characteristics (severity of capacity constraints and risk aversion via the reserve price). We then provide two kinds of results: some results directly help the insurance industry and other complete the auction literature.

Let us first discuss the outcome of the FU auction. We determine the unique equilibrium in symmetric and strictly increasing bidding strategies. Conditional on bidding, the equilibrium strategy completely reveals the signal an insurer observed. The reserve price implies the existence of a maximum signal determining the participation to the first stage of the auction. The equilibrium then exhibits both a complete market failure (no insurance) when both insurers have private pessimistic evaluations (above the threshold) and a partial market failure (partial insurance) when only one insurer is pessimistic about the risk. The re-entry option impacts these market failures in two ways: insurers refrain from bidding ex-ante (increasing the no-insurance region) but an insurer always has the possibility to re-enter the auction ex-post if he discovers that his opponent received a good signal (increasing the full insurance region). All these market failure regions are affected by the parameters of the model: intensity of the capacity constraints and reserve price. Relaxing capacity constraints increases the opportunity from being the follower. As a result, insurers bid less aggressively. This has two opposite effects on coverage: partial coverage is more likely but the proportion of uninsured risks decreased. A larger reserve price unambiguously increases insurance coverage: full coverage is more likely, and in case this latter is not achieved, partial coverage occurs more often when the reserve price increases.

Let us now turn to the discriminatory auction. We show that the equilibrium in symmetric and strictly increasing bidding strategies is semi-separating or separating and also involves some complete and partial market failure regions. The nature of the equilibrium

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[2] As European Commission [7] note, "*Co-insurance may include cases where proportions of a risk are placed separately, perhaps in different markets, using separate documentation, and with no insurer assuming the role of leader*".

and the maximum signal determining the participation depend on the parameters: capacity contraints and reserve price. The weaker the capacity constraints, the smaller the bidding regions. Similarly, the smaller the reserve price, the smaller the bidding region. Also, the bidding regions in the discriminatory auctions are always larger than in the FU one. However, we may observe full insurance with the FU auction and partial insurance in the discriminatory auction when the leader is optimistic enough about the risk occurrence and re-enters. The follower's position is essential to understand the efficiency of a given auction format. In the FU auction, the follower does not take any risk, but only enjoys relatively low profits (because of uniform pricing). On the contrary, in the discriminatory auction, the potential negative profit of the follower is counterbalanced by higher premiums. It must be noted that the comparison of the equilibrium bidding strategies differ from the comparison of the premiums. If pricing is uniform in the FU auction, the leader and the follower offers different premiums in the discriminatory auction, the follower's premium being larger. The difficult comparison of the equilibrium bidding strategies makes the comparison between the premiums quite involved.

Both auction formats then lead to different coverage/premium tradeoffs and the analysis shows that ex-ante there is no clear dominance of one auction format. When we compare these two auctions ex-post (for any possible realization of the two insurers' signals), we show that if at least one insurer provides an optimistic expertise about the risk, the FU auction offers higher coverage. At the opposite, if all insurers receive pessimistic information, the discriminatory auction offers a better coverage. If insurers agree about the ex-post evaluation of risk, the FU option has less scope so that the size of the bidding region is key to determine the insurance coverage. The discriminatory auction therefore offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows to increase insurance coverage.

We provide some results about the ex-ante comparison of auctions for the case of completely new risks (insurers' signals are therefore independent). We show that a regulator aiming at maximizing the expected coverage should promote the FU auction when the reserve price is low enough or when capacity constraints are large. Indeed in these two cases, the bidding regions of the two auction formats converge to the same region. As a result, the FU auction allows to increase insurance coverage.

Contribution to the Literature. This paper is related to two different parts of the literature: insurance of catastrophic and undiversifible risks and auction theory. On important concern of the literature on catastrophic risk consists in explaining the failure of the purchase of disaster insurance. Many reasons are evoked: behavioral biases (Kunreuther [15]), the role of solvency constraints (Kousky and Cooke [14]), the presence of default risk (Charpentier and Le Maux [5]). Louaas and Picard [17] highlight the intrinsic determinants of demand and supply in the insurance market for catastrophic risks. A model of the supply side is proposed that takes into account the default risk by introducing collateral. However, capacity constraints are not taken into account.

The insurance of large risks thanks to a co-insurance mechanism is relatively limited. The literature on undiversifiable risks has mainly focused on the risk sharing problem between insurers and policyholders. This risk sharing problem is analyzed for instance in Doherty and Dionne [6] who introduce a new form of insurance contract called Decomposed Risk Transfer contract (DRT contract) defined by an insurance policy packaged with a residual claim on an insurance pool. They show that this contract increases policyholders welfare. They characterize the optimal coverage and the risk premium as a function of the cost of risk bearing derived from asset pricing models. Our setting builds on such a two dimensional contract (a risk premium and a coverage) but we do not discuss any risk sharing issue associated with the undiversifiable risk. We consider instead that because of the particular competition emerging from the auction, the risk premium (paid by the policyholders) may differ from the actuarial rate (paid by the insurer). Mahul and Write [18] examine catastrophic risk sharing arrangements in presence of default risk from the consumers' perspective. Inderst [10] proposes a detailed description of co-insurance pools and ad-hoc co-insurance agreements. However, the microeconomic analysis of the supply side of co-insurance mechanisms under solvency constraints is not treated to our knowledge.

The organization of insurance supply amounts to sharing a common value divisible good between capacity constrained agents who have private information in presence of a reserve price. The auction literature is abundant in this topic and incorporates some of these characteristics. In presence of reserve price and the existence of secondary markets for the goods being sold, Haile [9] analyzes a second-price auction between two bidders with imperfect information about their valuations. A symmetric equilibrium exhibiting some pooling exists when the reserve price is sufficiently far below the maximum valuation. Jehiel and Moldovanu [11] analyze a more general setting by introducing positive or negative externalities in a standard second-price auction in presence of reserve price. They show that there must be some pooling at the reserve price in presence of positive externalities (as a resale opportunity for instance). Lizzeri and Persico [16] prove existence and uniqueness of equilibrium for a general class of two player bidding games in presence of a reserve price and interdependent values. In our paper, the divisibility of the good (the risk) makes the analysis quite different from these three papers, but we also prove the existence of a separating or semi-separating equilibrium in the discriminatory auction depending on reserve price. Moreover, we provide some comparative statics with respect to the strength of capacity constraints on the equilibrium outcome. This allows to extend part of the results of Lizzeri and Persico [16] to the multi-unit auction setting in case of the discriminatory auction.

The auction format used to reach ad-hod co-insurance agreements introduces the possibility for the follower to exit or to re-enter ex-post, after the first bidding stage occurred. By analyzing such two-stage auction setting, we are part of the study of sequential auctions that has developed very quickly since the 2000's. Caillaud and Mezzetti [4] analyze sequential auctions in which bidders have correlated valuations for multiple units and where the seller can freely set the reserve price at the beginning of each auction. Haile [9] consider a two-stage model in which an auction in the first stage is followed by a resale auction, held by the first-stage winner. Our work is between these two settings because we consider a two-stage auction of a multi unit good. Moreover, the second stage only concerns the follower. The originality of our approach is to provide a theoretical model of a practice used in the insurance industry and to compare it to a more standard auction setting.

The question of agreeing on a common coverage of a risk is akin to the one of exchanging Treasury debt and other divisible securities (where bonds are usually exchanged through a uniform auction or through a discriminatory auction). However, there is a need to develop a theoretical model since the auction rules are indeed specific to the insurance industry. Also, the nature of the good that is exchanged (reserve price and capacity constraints) differs from the nature of Treasury bonds. Finally, our objective slightly differs from the literature on Treasury auctions : if existing studies mainly compare the auctions with respect to revenues (as Back and Zender [2], Ausubel and Cramton [1] or Klemperer [13] for instance), we emphasize the ability of each auction to provide full insurance coverage.

The paper is organized as follows. We present the model in section 2. We then solve the equilibrium of the FU auction in section 3. In section 4, we introduce the discriminatory auction and determine the equilibrium. Section 5 is devoted to the comparison between the two auction formats. All proofs are relegated to the appendix.

2 The model

2.1 Risk, insurers and contract

There exists a *new* and *undiversifiable* risk in the economy characterized by its occurrence probability p and its loss size L.

In this paper, we analyze insurance supply given an exogenous demand. Therefore, on the demand side, we assume policyholders ask for the full coverage of this risk. Insurance is not assumed to be mandatory. Policyholders may prefer to remain uninsured or they have access to other means of sharing risk. This is captured by the presence of a reserve premium rate $\overline{P}$.

On the supply side, two identical risk neutral insurers, a and b, compete for the coverage of this risk: $i \in \{a, b\}$ will refer to an insurer and $-i$ to the other. The insurers propose a linear contract characterized by the premium rate P_i at which they provide insurance and by the quantity they insure q_i. Because of the idiosyncratic characteristics of both this *undiversifiable* risk and the insurance companies and because of solvency regulation and capital requisites applying to them (see for instance Kousky and Cooke (2012)), insurance companies are assumed to be capacity constrained so that $q_i \leq \kappa_i \in (1/2, 1)$, with $\kappa_a = \kappa_b = \kappa$. The lower and upper bounds on κ reflect the strength of regulation. When $\kappa = 1/2$, regulation is strong so that the quantity a unique insurer can cover only equals half of the demand; on the contrary, when $\kappa = 1$, regulation is milder and a unique insurer can cover the whole demand. Therefore, in some sense, the value of κ echoes competition between insurers: there is no compaetition between in surers when $\kappa = 1/2$, whereas competition is intense when κ equals 1. The minimum risk premium rate that insurers are willing to accept for this coverage is the actuarial premium rate p, and they do not ask more than the reserve premium rate $\overline{P}$. Insurer i' net expected benefit reads $q_i(P_i - p)L$ with $p \leq P_i \leq \overline{P}$. As a consequence, insurer i offers the maximum capacity, $q_i = \kappa$.

2.2 Insurers' expertise

As the risk is *new*, its occurrence probability p is not perfectly known by the insureds, nor by the insurers. However, the latter can use their expertise to infer p. We assume that insurer i privately observes a free signal S_i related to the true occurrence probability. Signals S_a and S_b are distributed according to the same continuous distribution on the interval $[0,1]$ and are assumed to be affiliated. As a consequence, if $g(.|s)$ denote the (symmetric) probability distribution function of an insurer's signal conditional on the other insurer having observed signal s, the following condition holds.

Assumption 1

$$\forall s'_i > s_i \textit{ and } s'_{-i} > s_{-i}, \frac{g(s'_i|s'_{-i})}{g(s'_i|s_{-i})} \geq \frac{g(s_i|s'_{-i})}{g(s_i|s_{-i})}. \tag{1}$$

The actuarial premium rate is updated according to all events conveying valuable information about the signals. We assume that it can be expressed as a function of insurers' private information. It is identical for the two insurers and is a symmetric function of all insurers' signals.

$$p(s_i, s_{-i}) = p(s_{-i}, s_i) \equiv \mathbb{E}\left[p | S_i = s_i, S_{-i} = s_{-i}\right]. \tag{2}$$

We impose the following regularity assumptions on the actuarial premium rate.

Assumption 2 *The actuarial premium rate p satisfies the following properties.*

(i) *Function p is twice continuously differentiable and strictly increasing in the two variables;*

(ii) $\mathbb{E}\left[p(S_i, 0)\right] < \overline{P} < \mathbb{E}\left[p(S_i, 1)\right], \forall S_i$.

A high value of s signals a risk that is assumed to be more costly to insure and some risks cannot be insured. Indeed, Assumption 2(ii) means that if insurer $-i$ observes the best (resp. worst) possible signal, covering the risk is always (resp. never) profitable for insurer i.

Let us also define signal $\widetilde{\sigma}$ as the maximal signal for which the two insurance companies accept to cover the risk in case they observe the same signal and function α that can be interpreted as an isocost curve evaluated at the maximal premium rate $\overline{P}$.[3]

Definition 1

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[3] According to Assumption 2(ii), α is a decreasing function. Moreover, the symmetry of α with respect to its arguments implies that $\alpha^{-1} = \alpha$.

(i) $\widetilde{\sigma}$ *is implicitly defined by*

$$p\left(\widetilde{\sigma}, \widetilde{\sigma}\right) = \overline{P}. \tag{3}$$

(ii) α *is implicitly defined by*

$$p(\alpha(s), s) = \overline{P} \ \forall s \in [0,1]. \tag{4}$$

2.3 Insurers' syndication

The organization of insurance supply amounts to the problem of sharing a common value divisible good between capacity constrained agents with a reserve price. The objective of this paper is to analyze different auction rules to constitute this syndicate, namely the Flexible Uniform Auction and the Discriminatory Auction. Each auction determines a game of incomplete information among the insurers: we look for a symmetric Bayesian Nash equilibrium that is increasing in the bidding strategies of each resulting game.

3 Analysis of the Flexible Uniform Auction

3.1 The Flexible Uniform Auction rules

In this section, we model ad-hoc co-insurance agreements, a representative organization of the insurance sector. The European Commission [7] provides a detailed description of the procedures leading to these agreements in several European countries. Even if country-specific differences exist, they share common features that we decide to highlight. The premium rate is unique and equals the lowest bid among insurers. The European Commission [7] also notes that "*the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer*".

We label this particular auction the *"Flexible Uniform Auction"* (FUA) whose following rules incorporate these features. After insurers receive their private signals s_a and s_b, a first price auction determines the risk premium rate. If at least one insurer submits a bid $P_i \leq \overline{P}$, the insurer that submitted the smallest premium rate is the leader and sells κ at rate P^{FU}; the other insurer is the follower and observes P^{FU}. He decides whether he sells $1-\kappa$ at rate P^{FU} or not. In this particular auction, the follower has both an exit and a re-entry option since he can join or exit the agreement whatever his initial choice to submit a bid ex-ante. Indeed, the follower has the possibility to exit the syndicate after having observed P^{FU}: in this case, the leader is still committed to serve its announced capacity and there is partial insurance. Moreover, if an insurer is too pessimistic to submit a bid ex-ante, he may still re-enter and participate ex-post if the leader's bid reveals a good risk.

3.2 Separating equilibrium

We look for a separating equilibrium characterized by a threshold $\widehat{\sigma}^{FU}$. When $s_i \leq \widehat{\sigma}^{FU}$, insurer i bids according to a strictly increasing bidding strategy $P^{FU}(s_i)$ with $P^{FU}(\widehat{\sigma}^{FU}) =$

$\overline{P}$. When $s_i > \widehat{\sigma}^{FU}$, insurer i is willing to participate ex-post only. The bid unambiguously reveals the private signal the bidding insurer observed. The profit of insurer i who observed a signal s_i and bids a premium rate $P^{FU}(s_i)$ reads

$$\Pi^{FU}(s_i) = \begin{cases} \kappa\left(1 - G\left(s_i|s_i\right)\right)\mathbb{E}\left[P^{FU}(s_i) - p(s_i, S_{-i})|S_{-i} > s_i\right] & \\ \quad +(1-\kappa)G\left(s_i|s_i\right)\mathbb{E}\left[\left(P^{FU}(S_{-i}) - p(s_i, S_{-i})\right)_+|S_{-i} < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^{FU} \quad (5a) \\ (1-\kappa)G\left(\widehat{\sigma}^{FU}|s_i\right)\mathbb{E}\left[\left(P^{FU}(S_{-i}) - p(s_i, S_{-i})\right)_+|S_{-i} < \widehat{\sigma}^{FU}\right] & \text{for } s_i > \widehat{\sigma}^{FU} \quad (5b) \end{cases}$$

When $s_i \leq \widehat{\sigma}^{FU}$, insurer i submits a bid ex-ante. The first term of equation (5a), *the leader's value*, corresponds to the case where insurer i observes the lowest signal. This happens when $S_{-i} > s_i$, an event of probability $1 - G(s_i|s_i)$. In such case, insurer i proposes the lowest premium rate and becomes the leader, serving the quantity κ at his bid $P^{FU}(s_i)$. The second term of equation (5a) corresponds to the case where insurer i observes the highest signal (this happens with probability $G(s_i|s_i)$). He proposes the highest risk premium rate and becomes the follower, serving the residual demand, $1-\kappa$, at insurer j's rate $P^{FU}(s_{-i})$. Note that if his payoff turns out to be negative, insurer i can withdraw from the auction (hence the subscript "$_+$"). Therefore, this term is named *the exit option value.* Equation (5b) corresponds to the case where insurer i observes a signal greater than $\widehat{\sigma}^{FU}$ and therefore does not want to participate ex-ante whereas his opponent submits a bid smaller than $\overline{P}$. Insurer i, as the follower, agrees to re-enter in case it is profitable. In such case, he serves the residual demand $1-\kappa$ at insurer $-i$'s proposed rate $P^{FU}(s_{-i})$. We refer to this term as *the re-entry option value.* Submitting a bid in this auction does not commit to stay in the auction for the highest bidder. This feature is unusual in the auction literature and implies a strong asymmetry between the leader and the follower.

The participation constraint requires that bidders with signals greater than $\widehat{\sigma}^{FU}$ prefer not to bid to submitting the bid $\overline{P}$. Next Lemma characterizes $\widehat{\sigma}^{FU}$.

Lemma 1 *The threshold $\widehat{\sigma}^{FU}$ exists and is uniquely defined by*

$$\mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_{-i})|S_{-i} > \widehat{\sigma}^{FU}\right] = 0 \tag{6}$$

Moreover, $\widehat{\sigma}^{FU} < \widetilde{\sigma}$.

Given the specific rules of the FUA (uniform pricing, options to exit and to re-enter), the follower profit is the same whatever the choice to participate to the auction ex-ante. At $s_i = \widehat{\sigma}^{FU}$, the exit option value exactly compensates the re-entry option value. Therefore, the threshold $\widehat{\sigma}^{FU}$ only matters for the leader's strategy and is determined to guarantee that the maximum net expected benefit is equal to zero which refrains from bidding when signals are too high. The tradeoff for an insurer is between insuring a large capacity with the risk of ex-post negative profit (leader) and insuring a smaller capacity at no risk of loss (follower).

The equilibrium bidding strategy is described in Proposition 1.

Proposition 1 *There exists a unique symmetric Nash equilibrium in strictly increasing equilibrium bidding strategies where*

$$P^{FU}(s) = \frac{2\kappa - 1}{\kappa}\left(1 - L(\widehat{\sigma}^{FU}|s)\right)\overline{P} + \int_{s}^{\widehat{\sigma}^{FU}} p(x,x)dL(x|s) \;\; \forall \;\; s \leq \widehat{\sigma}^{FU} \tag{7}$$

with

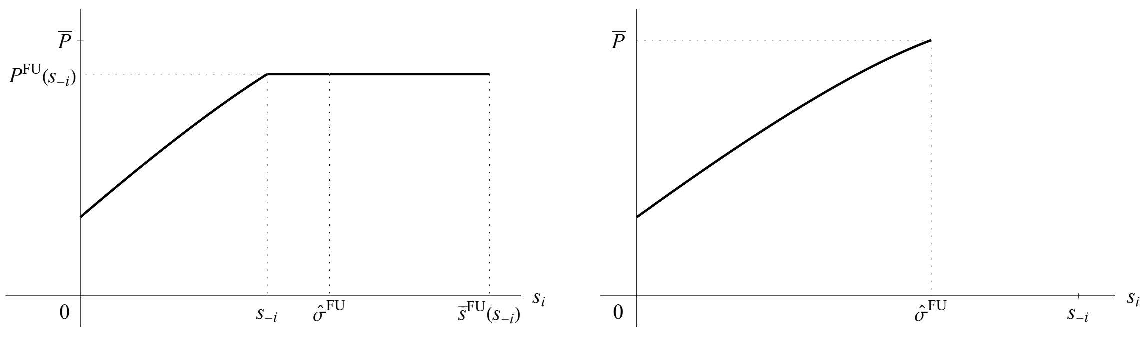
$$L(x|s) = 1 - \exp\left(-\frac{2\kappa - 1}{\kappa}\int_{s}^{x} \frac{g(\tau|\tau)}{1 - G(\tau|\tau)}d\tau\right) \tag{8}$$

and $\widehat{\sigma}^{FU}$ *defined by equation (6).*

The expression of the equilibrium bidding strategy is standard in the auction literature: the first term takes into account the reserve rate whereas the upper bound in the integral of the second term reflects the exit/re-entry option.[4] As for the premium rate that is indeed paid by the policyholders, it depends on the signal of the two insurers. In Figure 1(a), insurer i turns out to be the leader when $s_i \leq s_{-i}$ in which case the premium rate corresponds to its bid. But when $s_i > s_{-i}$, the premium rate corresponds to insurer $-i$'s bid. When $s_i > \widehat{\sigma}^{FU}$, observe that insurer i decides to re-enter ex-post when $P^{FU}(s_{-i}) > p(s_i, s_{-i})$. Let us define function $\overline{s}^{FU}(s_i)$ for $s_i \in [0, \widehat{\sigma}^{FU}]$ that takes value on $[\widehat{\sigma}^{FU}, 1]$.[5] It is implicitly defined by

$$P^{FU}(s_i) = p(s_i, \overline{s}^{FU}(s_i)). \tag{9}$$

Therefore, insurer i decides to re-enter ex-post if $s_i < \overline{s}^{FU}(s_{-i})$. In Figure 1(b), insurer i's opponent observes a signal higher than $\widehat{\sigma}^{FU}$, so that he does not bid and insurer i is always the leader.



(a) $s_{-i} < \widehat{\sigma}^{FU}$.

(b) $s_{-i} > \widehat{\sigma}^{FU}$.

Figure 1: Insurer i's premium for different insurer $-i$'s signal values.

One of the specificities of this FUA is that insurance coverage can be full even if only one insurer submits a bid ex-ante (when the re-entry option value is exerted). Similarly, when the exit option value is exerted, insurance coverage can be partial even if the two

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[4] Observe that $x \mapsto L(x|s)$ is an increasing function with $L(s|s) = 0$.

[5] This defines a function since p is increasing in each of its argument.

insurers submit a bid ex-ante. The following lemma tells us that, under an additional assumption, an insurer that submits a bid ex-ante never wants to withdraw ex-post.

Assumption 3 *Function p is supermodular,*

$$\frac{\partial^2 p(s_i, s_{-i})}{\partial s_i \partial s_{-i}} \geq 0, \forall (s_i, s_{-i}) \in [0,1]^2; \tag{10}$$

Supermodularity implies that some form of positive correlation exists between the two signals values.[6]

Lemma 2 *Under Assumption 3, an insurer that submits a bid ex-ante never wants to withdraw ex-post.*

$$P^{FU}(s_{-i}) - p(s_i, s_{-i}) > 0 \ \ \forall \ s_{-i} < s_i \leq \widehat{\sigma}^{FU}. \tag{11}$$

Lemma 2 implies that there will always be full coverage when the two insurers submit a bid ex-ante. Indeed, in this case, both insurers are optimistic enough on the risk to submit a bid. If Assumption 3 is satisfied, the least optimistic insurer is reassured by the leader's bid and remains in the agreement.

Figure 2 describes insurance coverage for all signals' values. The diagram is symmetric with respect to the 45 degree line and three possible cases appear: full coverage, partial coverage and no coverage. Assume insurer i observes the smallest signal. There is full coverage when both insurers initially receive a signal smaller than $\widehat{\sigma}^{FU}$, but also when insurer $-i$ who decided not to submit a bid ex-ante decides to re-enter ex-post ($s_i \leq \widehat{\sigma}^{FU}$ and $\widehat{\sigma}^{FU} \leq s_{-i} \leq \overline{s}^{FU}(s_i)$, so that $P^{FU}(s_i) > p(s_i, s_{-i})$). There is partial coverage when insurer i bids ex-ante and insurer $-i$ does not participate to the auction, so that only capacity κ is provided. The boundary between the full coverage and the partial coverage regions is $\left\{(s_i, s_{-i}) \in [0,1]^2 | P^{FU}(s_i) = p(s_i, s_{-i})\right\}$ and corresponds to $\overline{s}^{FU}(s_i)$: insurer $-i$ is indifferent between entering ex-post and never participating to the FUA since its payoff is zero. Therefore, when there is partial coverage ($s_{-i} > \overline{s}^{FU}$ and $s_i \leq \widehat{\sigma}^{FU}$), it holds that $P^{FU}(s_i) < p(s_i, s_{-i})$. The leader's payoff is negative and there is a winner's curse. According to Lemma 2, the boundary $\overline{s}^{FU}$ is greater than $\widehat{\sigma}^{FU}$. Note that $\overline{s}^{FU}(s_i)$ might be non-monotonic with respect to s_i. In particular, as $\frac{\partial^2 P^{FU}(s_i)}{\partial s_i \partial \kappa} \geq 0$, the higher κ, the steeper $P^{FU}(s_i)$. Therefore, if $s_i \mapsto P^{FU}(s_i) - p(s_i, s_{-i})$ is a decreasing function of s_i when $\kappa = \frac{1}{2}$, it might be a non monotonic function of s_i when κ is close to 1 as the Figure 2 illustrates. In the no coverage region, the two insurers observe a signal greater than $\widehat{\sigma}^{FU}$, none of them submits a bid.

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[6] Note that this additional assumption is compatible with MLRP. Assume for instance that the true probability is a function of the two signals and a third continuous random variable Y with a pdf f_Y distributed on an interval $[a, b]$ and independent of the signals such that $p = \psi(S_i, S_{-i}, Y)$. In this case $p(s_i, s_{-i}) = p(s_{-i}, s_i) = \int_a^b \psi(s_i, s_{-i}, y) f_Y(y) dy$. Assumption 3 is satisfied if $\int_a^b \frac{\partial^2}{\partial s_i \partial s_{-i}} \psi(s_i, s_{-i}, y) f_Y(y) dy \geq 0$.

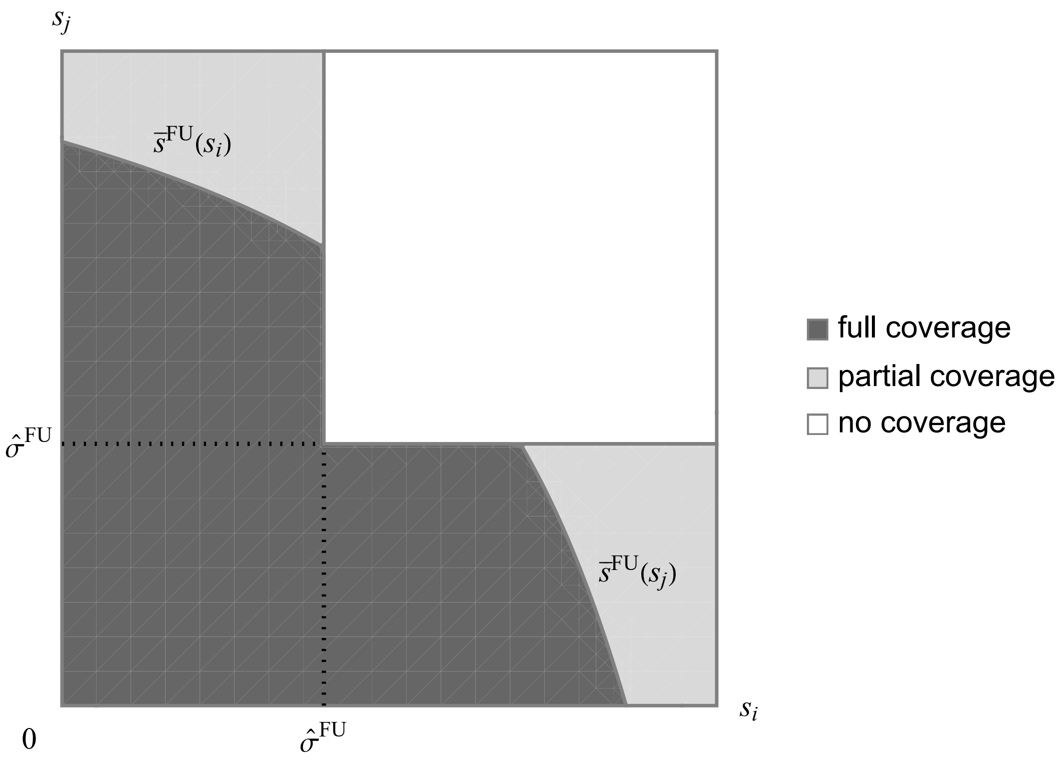

Figure 2: The different insurance coverages for all signals' values.

3.3 Equilibrium properties

Strengthening capacity constraints. The value of κ determines the strength of the capacity constraints that depends on regulation. Interval $[0, \widehat{\sigma}^{FU}]$, that represents the region of the signal values for which insurers decide to submit a bid ex-ante, is independent of κ (see equation (6) defining $\widehat{\sigma}^{FU}$). However, the value of κ modifies the equilibrium bid P^{FU} which in turn affects the follower's decision to enter or not ex-post.

Proposition 2 *When capacity constraints strengthen (κ decreases), the equilibrium bidding strategy P^{FU} increases and the full coverage region decreases.*

The equilibrium bidding strategy is represented in Figure 3 for two values of κ. Larger capacity constraints unambiguously lowers bids. Indeed, an increase in κ decreases the opportunity from being the follower. As a result, insurers bid more aggressively. Correspondingly, an increase in κ has two opposite effects on coverage: partial coverage is more likely but the quantity of uninsured risk decreases.

Our analysis highlights that a regulatory cap for the market share of the co-insurance syndicate members (low κ) should generate high commercial premiums and low full coverage. Remember that the value κ determines the intensity of competition. A low κ induces less competition between insurers so that bids are larger.

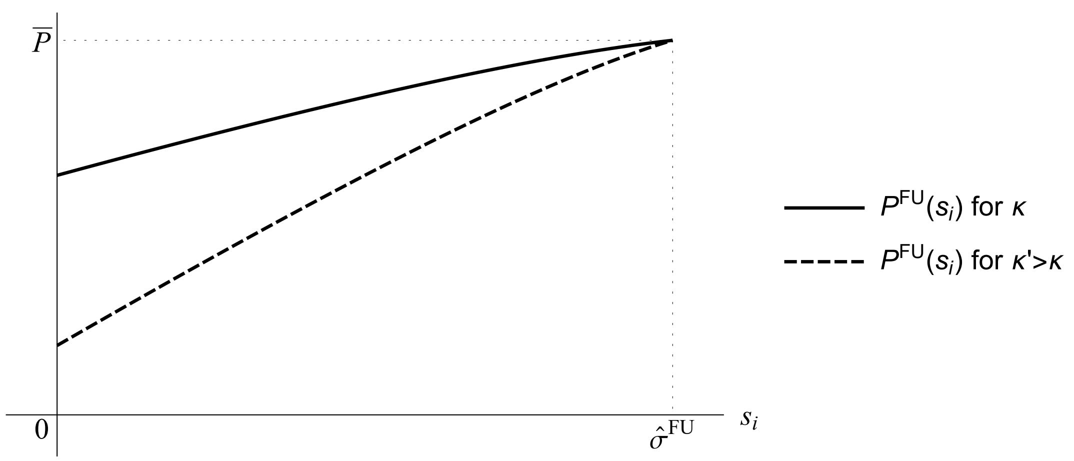

Figure 3: The equilibrium bidding strategy $P^{FU}(s)$ for two values of κ.

Modifying the reserve premium rate. An increase in the reserve premium rate can result from a lower access to outside risk sharing possibilities.

Proposition 3 *When the reserve premium rate increases, $\hat{\sigma}^{FU}$ increases and the equilibrium bidding strategy P^{FU} increases.*

If a higher reserve premium rate unambiguously increases the bidding regions, it has an ambiguous effect on the equilibrium bidding strategy. Indeed, on the one hand, for a given bidding region, a greater reserve premium rate tends to increase the bid (direct effect). But on the other hand, as bidding regions increase ($\hat{\sigma}^{FU}$ increases with $\overline{P}$) and because the equilibrium is in strictly increasing strategies, this tends to lowers the bids for a given signal value (indirect effect). However, Proposition 3 shows that the direct effect always dominates so that the higher the reserve premium rate, the higher the equilibrium bidding strategy as Figure 4 illustrates. As a result, a larger reserve premium rate unambiguously increases insurance coverage.

An increase in the reserve premium rate can also be viewed as a strengthening of the participation constraint for the insureds. In this case, making insurance mandatory (high reserve premium rate) trivially increases insurance coverage, but increases bids too.

4 An alternative syndication: the Discriminatory Auction

We now consider the Discriminatory Auction (DA) that is also widespread in the insurance industry. In this bidding process, each insurer proposes a risk premium ex-ante according to the private signal he received. Unlike the FUA, insurers sell insurance coverage at their announced premium so that bids equal premium rates.

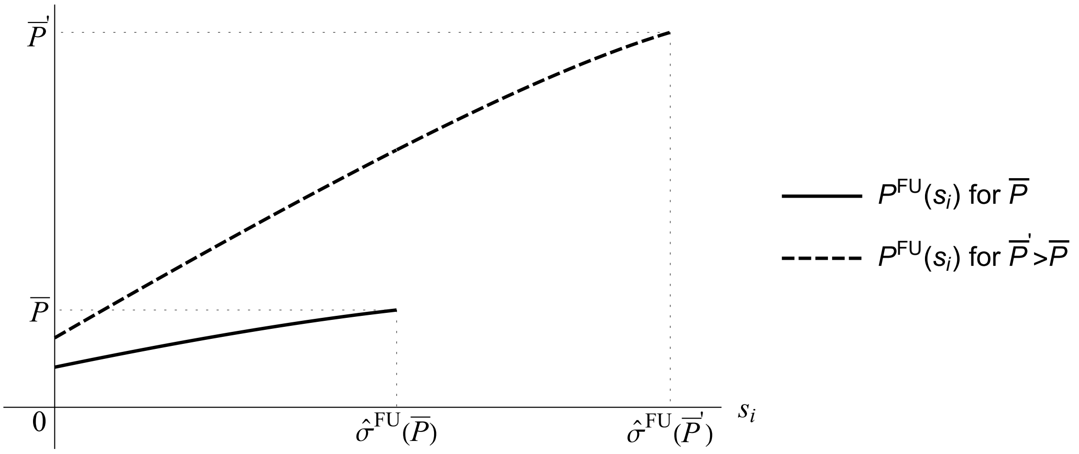

Figure 4: The equilibrium bidding strategy $P^{FU}(s)$ for two values of $\overline{P}$.

4.1 Equilibrium analysis

Separating equilibrium We first look for a separating equilibrium characterized by a threshold $\widehat{\sigma}^D$ such that when $s_i \leq \widehat{\sigma}^D$, insurer i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\widehat{\sigma}^D) = \overline{P}$ and when $s_i > \widehat{\sigma}^D$, insurer i does not participate anymore. The profit of insurer i that observed a signal s_i and bids a risk premium rate $P^D(s_i)$ reads

$$\Pi^D(s_i) = \begin{cases} \kappa\,(1 - G(s_i|s_i))\,\mathbb{E}\left[P^D(s_i) - p(s_i, S_{-i})|S_{-i} > s_i\right] & \\ \quad +(1-\kappa)G(s_i|s_i)\,\mathbb{E}\left[P^D(s_i) - p(s_i, S_{-i})|S_{-i} < s_i\right] & \text{for } s_i \leq \widehat{\sigma}^D \quad (12a) \\ 0 & \text{for } s_i > \widehat{\sigma}^D. \quad (12b) \end{cases}$$

As for the FUA, the *leader's value* (first term of equation (12a)) correspond to the case where insurer i proposes the smallest risk premium rate. Unlike the FUA, the *follower's value* (second term of equation (12a)) now depends on the follower's premium rate and not on the leader's premium rate. It tends therefore to be greater than the follower's value in the FUA. Note also that in the follower's value, the subscript "+" does not appear since the follower cannot withdraw the DA.

Incentive compatibility requires that bidders with signals greater than $\widehat{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}$. A new variable κ^* is needed to characterize threshold $\widehat{\sigma}^D$.

Definition 2

$$\kappa^*\left(\overline{P}\right) = \max\left(\frac{G(\widetilde{\sigma}|\widetilde{\sigma})\,\mathbb{E}\left[\overline{P} - p(\widetilde{\sigma}, S_{-i})|S_{-i} < \widetilde{\sigma}\right]}{G(\widetilde{\sigma}|\widetilde{\sigma})\,\mathbb{E}\left[\overline{P} - p(\widetilde{\sigma}, S_{-i})|S_{-i} < \widetilde{\sigma}\right] - (1 - G(\widetilde{\sigma}|\widetilde{\sigma}))\mathbb{E}\left[\overline{P} - p(\widetilde{\sigma}, S_{-i})|S_{-i} > \widetilde{\sigma}\right]}, \frac{1}{2}\right). \quad (13)$$

Lemma 3 *The threshold $\widehat{\sigma}^D$ exists and is uniquely defined on $[0, \widetilde{\sigma}]$ by*

$$\begin{aligned} & \kappa \left(1 - G\left(\widehat{\sigma}^D | \widehat{\sigma}^D\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^D, S_{-i}) | S_{-i} > \widehat{\sigma}^D\right] \\ + \quad & (1-\kappa)\, G\left(\widehat{\sigma}^D | \widehat{\sigma}^D\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^D, S_{-i}) | S_{-i} < \widehat{\sigma}^D\right] = 0 \end{aligned} \tag{14}$$

if and only if $\kappa \geq \kappa^* \left(\overline{P}\right)$.

Contrary to equation (6) that defined the FUA's threshold, the follower's payoff (the second term of equation (14)) matters in the definition of $\widehat{\sigma}^D$. The leader's expected payoff (first terme of Equation 14) is negative at the threshold making the winner's curse more intense.[7] Indeed, an insurer bids until $\widehat{\sigma}^D$ in the expectation of being the follower rather than the leader. As a consequence, $\widehat{\sigma}^D$ may be larger than $\widetilde{\sigma}$ depending on the parameters of the model as the lemma explains.

Semi-separating equilibrium. If $\kappa < \kappa^* \left(\overline{P}\right)$, there does not exist a threshold $\widehat{\sigma}^D \leq \widetilde{\sigma}$. Therefore, we must look for another equilibrium strategy that involves pooling for some values of the signal. More precisely, we look for a semi-separating equilibrium characterized by two thresholds $\underline{\sigma}^D$ and $\overline{\sigma}^D > \underline{\sigma}^D$ such that: *(i)* when $s_i \in [0, \underline{\sigma}^D]$, insurer i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\underline{\sigma}^D) = \overline{P}$; *(ii)* when $s_i \in [\underline{\sigma}^D, \overline{\sigma}^D]$, insurer i bids $\overline{P}$; and *(iii)* when $s_i > \overline{\sigma}^D$, insurer i does not participate anymore. The profit of insurer i that received a signal s_i and proposes a risk premium rate $P^D(s_i)$ reads

$$\Pi^D(s_i) = \begin{cases} \kappa \left(1 - G(s_i|s_i)\right) \mathbb{E}\left[P^D(s_i) - p(s_i, S_{-i}) | S_{-i} > s_i\right] & \\ \quad + (1-\kappa) G(s_i|s_i)\, \mathbb{E}\left[P^D(s_i) - p(s_i, S_{-i}) | S_{-i} < s_i\right] & \text{for } s_i \leq \underline{\sigma}^D \quad \text{(15a)} \\ \kappa \left(1 - G\left(\overline{\sigma}^D | s_i\right)\right) \mathbb{E}\left[\overline{P} - p(s_i, S_{-i}) | S_{-i} > \overline{\sigma}^D\right] & \\ \quad + \frac{1}{2}\left(G\left(\overline{\sigma}^D|s_i\right) - G\left(\underline{\sigma}^D|s_i\right)\right) \mathbb{E}\left[\overline{P} - p(s_i, S_{-i}) | \underline{\sigma}^D < S_{-i} < \overline{\sigma}^D\right] & \text{for } \underline{\sigma}^D < s_i \leq \overline{\sigma}^D \\ \quad + (1-\kappa) G\left(\underline{\sigma}^D | s_i\right) \mathbb{E}\left[\overline{P} - p(s_i, S_{-i}) | S_{-i} < \underline{\sigma}^D\right] & \text{(15b)} \\ 0 & \text{for } s_i > \overline{\sigma}^D. \quad \text{(15c)} \end{cases}$$

The first term corresponds to the *leader's value*, the last to the *follower's value*. As for the second term, it corresponds to the case where the two insurers bid $\overline{P}$ so that they equally share the market. Incentive compatibility conditions imply that the two thresholds are thus defined by the following system.[8]

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[7] In the DA, $\left(1 - G\left(\widehat{\sigma}^D|\widehat{\sigma}^D\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^D, S_{-i}) | S_{-i} > \widehat{\sigma}^D\right] < 0$, whereas, in the FAU, it holds that $\left(1 - G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_{-i}) | S_{-i} > \widehat{\sigma}^{FU}\right] = 0$.

[8] Incentive compatibility requires that insurers with signal in $[\underline{\sigma}^D, \overline{\sigma}^D]$ prefer submitting $\overline{P}$ to not participating and to submitting any lower bid. Moreover, insurers with signals greater than $\overline{\sigma}^D$ prefer not to bid to submitting the bid $\overline{P}$.

$$\begin{cases} \left(G\left(\overline{\sigma}^D|\underline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\underline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\underline{\sigma}^D, S_{-i})|\underline{\sigma}^D < S_{-i} < \overline{\sigma}^D\right] = 0 & \text{(16a)} \\ \kappa\left(1 - G\left(\overline{\sigma}^D|\overline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_{-i})|S_{-i} > \overline{\sigma}^D\right] & \\ +\frac{1}{2}\left(G\left(\overline{\sigma}^D|\overline{\sigma}^D\right) - G\left(\underline{\sigma}^D|\overline{\sigma}^D\right)\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_{-i})|\underline{\sigma}^D < S_{-i} < \overline{\sigma}^D\right] & \\ +(1-\kappa)\,G\left(\underline{\sigma}^D|\overline{\sigma}^D\right)\mathbb{E}\left[\overline{P} - p(\overline{\sigma}^D, S_{-i})|S_{-i} < \underline{\sigma}^D\right] = 0. & \text{(16b)} \end{cases}$$

It must also be checked that an insurer bidding $\overline{P}$ when it observes a signal comprised between $\underline{\sigma}^D$ and $\overline{\sigma}^D$ does not have an incentive to underprice.[9]

When the separating equilibrium exists ($\kappa \geq \kappa^*\left(\overline{P}\right)$), the three thresholds $\underline{\sigma}^D$, $\overline{\sigma}^D$ and $\widehat{\sigma}^D$ vanish into the same value eliminating any pooling. We can then state the following proposition that characterizes the equilibrium strategy.[10]

Proposition 4 *If $\kappa \geq \kappa^*\left(\overline{P}\right)$, the unique equilibrium in increasing strategy is the separating equilibrium.*

If $\kappa < \kappa^*\left(\overline{P}\right)$, the unique equilibrium in increasing strategy is the semi-separating equilibrium. In this case, the following ranking holds

$$\alpha(\overline{\sigma}^D) \leq \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) \leq \overline{\sigma}^D \leq \widehat{\sigma}^D.$$

Corollary 1 *In the separating equilibrium, the strictly increasing bidding equilibrium strategies read*

$$P^D(s) = \overline{P}(1 - K(\widehat{\sigma}^D|s)) + \int_s^{\widehat{\sigma}^D} p(x,x)dK(x|s) \;\; \forall s \leq \widehat{\sigma}^D. \tag{17}$$

with $\widehat{\sigma}^D$ defined by equation (14).

In the semi-separating equilibrium, the increasing bidding equilibrium strategies read

$$P^D(s) = \begin{cases} \overline{P}(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x,x)dK(x|s) & \text{for } s \leq \underline{\sigma}^D & \text{(18a)} \\ \overline{P} & \text{for } \underline{\sigma}^D < s \leq \overline{\sigma}^D. & \text{(18b)} \end{cases}$$

with $\underline{\sigma}^D$ and $\overline{\sigma}^D$ defined by equations (16a) and (16b).

Function K equals

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{(2\kappa - 1)\,g(\tau|\tau)}{\kappa - (2\kappa - 1)\,G(\tau|\tau)}d\tau\right). \tag{19}$$

---

[9] This comes down to checking that

$$\left(G\left(\overline{\sigma}^D|s_i\right) - G\left(\underline{\sigma}^D|s_i\right)\right)\mathbb{E}\left[\overline{P} - p(s_i, S_{-i})|\underline{\sigma}^D < S_{-i} < \overline{\sigma}^D\right] \leq 0 \;\; \forall s_i \in [\underline{\sigma}^D, \overline{\sigma}^D].$$

This is done in the proof of Lemma 4.

[10] The complete characterization of the equilibrium is presented in Appendix A.2.

Note that these results complement the literature on auction. We prove the existence of a separating or a semi-separating equilibrium depending on the reserve price or the capacity constraint level in a common value multi-auction with affiliated values.

4.2 Equilibrium properties

This section provides a comparative static analysis of the DA equilibrium where we emphasize the role of capacity constraints and of the reserve premium rate.

Strengthening capacity. The characterization of the equilibrium already highlighted the role of κ on the nature of the equilibrium that emerges. Therefore, contrary to the FUA, the region in which insurance companies submit bids now depends on the capacity constraints strength.

Lemma 4 *The following comparative static results hold for the different thresholds:*

$$\frac{\partial \hat{\sigma}^D}{\partial \kappa} \leq 0, \ \frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0 \ \textit{and} \ \frac{\partial \overline{\sigma}^D}{\partial \kappa} \leq 0.$$

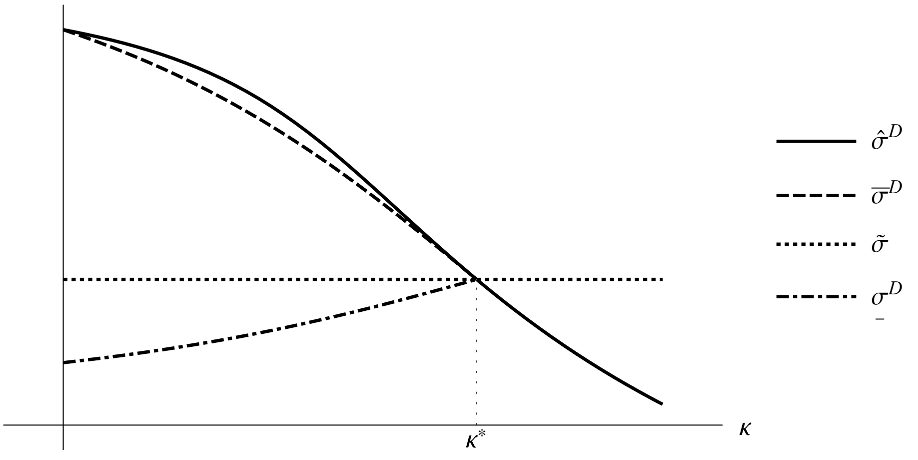

Figure 5: The different thresholds as a function of κ.

In Figure 5, we retrieve the results of Proposition 4 that the equilibrium is separating when $\kappa \geq \kappa^* \left(\overline{P}\right)$ and semi-separating when $\kappa < \kappa^* \left(\overline{P}\right)$. The weaker the capacity constraint (large κ), the smaller the bidding regions. The perspective of weak capacity constraints makes insurers more prudent when bidding because of the resulting competition between insurance companies. Note that when the equilibrium is semi-separating, $\overline{\sigma}^D$ decreases with κ and $\underline{\sigma}^D$ increases with κ: the region for which insurers bid the highest premium rate $\overline{P}$ shrinks.

The results on the bidding regions are closely related to the way the bidding strategies evolve with respect to κ. Holding the bidding regions constant, an increase in κ has the direct effect of reducing the bidding strategies. However, as we just saw, an increase in

κ also affects the bidding regions introducing an indirect effect.[11] When $\kappa < \kappa^*\left(\overline{P}\right)$, as $\underline{\sigma}^D$ increases, $\overline{P}$ is reached for higher signals' values so that the direct and the indirect effect go in the same direction, implying that bidding strategies decrease with κ. On the contrary, when $\kappa \geq \kappa\left(\overline{P}\right)^*$, $\widehat{\sigma}^D$ decreases: the indirect effect therefore tends to increase bidding strategy and the total effect is ambiguous.

Modifying the reserve premium rate. Observe first that, in the general case, it is not straightforward to determine whether the equilibrium is separating or semi-separating when the reserve premium rate increases. Indeed, the dependance of $\kappa^*(\overline{P})$ with respect to $\overline{P}$ is difficult to analyze. This is in part due to the dependence of $\widetilde{\sigma}$ with respect to $\overline{P}$ that intervenes both directly (through $p(S, \widetilde{\sigma})$) and indirectly (through affiliation and the distribution function) in the definition of $\kappa^*(\overline{P})$.

As for the comparative static of the thresholds with respect to the reserve premium rate, the following result holds.

Lemma 5 *It holds that*

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} \geq 0 \text{ and } \frac{\partial \overline{\sigma}^D}{\partial \overline{P}} \geq 0.$$

As for the FUA, a higher reserve premium rate unambiguously increases the bidding regions. There is more surplus to extract from the insureds so that insurance companies continue to bid even if they are less optimistic about the risk occurrence. However, the comparative statics of the bidding strategy does not lead to direct results.

An illustration is provided in Figure 6. In this example, the equilibrium is separating if and only if the reserve premium rate is smaller than some threshold $\overline{P}^*(\kappa)$. For a large reserve premium rate, there is more surplus to extract from the insureds so that insurance companies can afford bidding less aggressively. Therefore, the equilibrium is semi-separating.

5 Which auction for the coverage of these risks?

In order to compare auctions, several perspectives can be adopted. From the insurers viewpoint, the expected profit of the two auctions could be compared. From the insureds view point, a possibility would be to determine their expected utility. An social planner would focus on both insurers and insureds. Our model is quite rich and depend on several parameters : the distribution of signals and the fact that they are affiliated, the actuarial premium rate p and the functioning rules of each syndicate. Taking into account consumers preferences would make our model more complex. Therefore, we choose to focus on the insurers side and to analyze their bids. This will allow us to compare premium rates

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[11] This indirect effect writes:

$$-\frac{\partial \min(\widehat{\sigma}^D, \underline{\sigma}^D)}{\partial \kappa} \left(\overline{P} - p(\min(\widehat{\sigma}^D, \underline{\sigma}^D), \min(\widehat{\sigma}^D, \underline{\sigma}^D))\right) \left.\frac{dK(x|s)}{dx}\right|_{x=\min(\widehat{\sigma}^D, \underline{\sigma}^D)}. \quad (20)$$

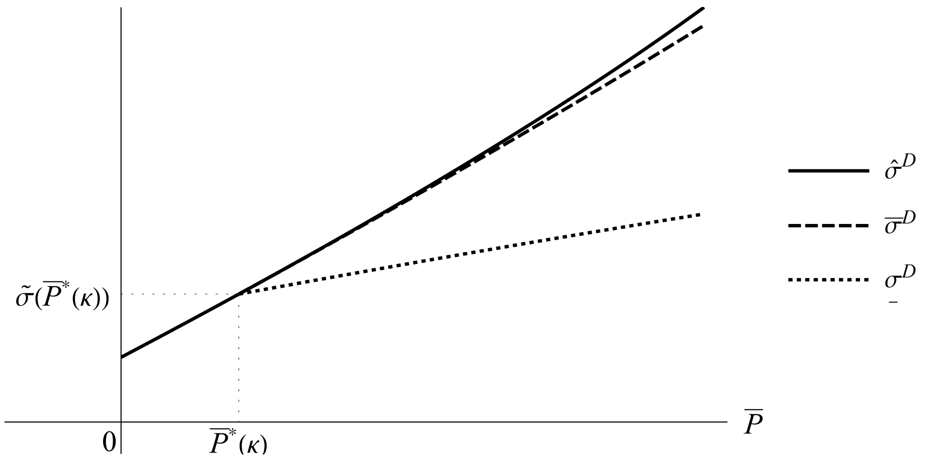

Figure 6: The different thresholds as a function of $\overline{P}$.

and resulting coverage. In order to compute and compare expected profits and expected coverage, we will specify both the signals' distribution and the premium rate p. This will allow us to highlight the role of the reserve premium rate $\overline{P}$ as well as the strength of capacity constraints that depend on the strength of regulation.

5.1 FUA vs DA

Both auctions deal with different risk/return tradeoff for the insurers and in particular for the one turning out to be follower. The follower's position is essential to understand the efficiency of a given auction format. In the FUA, the follower does not take any risk (because of the re-entry option), but only enjoys relatively low profits (because of uniform pricing), whereas in the DA the potential negative profit of the follower is counterbalanced by higher bids.

Lemma 6 *It holds that*

$$\widehat{\sigma}^{FU} \leq \min\left(\widehat{\sigma}^{D}, \underline{\sigma}^{D}\right).$$

A direct consequence of this lemma is that the DA offers a positive coverage (partial or full) of the risk whereas the FUA offers no coverage when the leader signal is between $\widehat{\sigma}^{FU}$ and $\widehat{\sigma}^{D}$. However, in order to be able to compare coverage when the leader's signal is smaller than $\widehat{\sigma}^{FU}$, it is necessary to compare $\overline{s}^{FU}(s)$ to $\widehat{\sigma}^{D}$. To do that, a comparison of the bids in the two settings is necessary. Unfortunately, the very general setting of the model does not allow us to have a clear result.

Lemma 7 *The bidding strategies in the FUA and in the DA cross at most one.*

- *Either* $P^D(s) < P^{FU}(s)$ *for all* $s \in [0, \widehat{\sigma}^{FU}]$,
- *Or,* $P^D(s) > P^{FU}(s)$ *and then* $P^D(s) > P^{FU}(s)$, *when* s *increases from 0 to* $\widehat{\sigma}^{FU}$.

When the leader's signal $s \leq \widehat{\sigma}^{FU}$ *is such that* $P^{D}(s) < P^{FU}(s)$*, the FUA always offer more coverage than the DA.*

Note that since we show in the proof of Lemma 7 that the two bids cross at most once, the key comparison is between $P^{FU}(0)$ and $P^{D}(0)$.

The classic tradeoff between quantity and price is highlighted as being the one driving the differences between the two syndications. Observe that there are only two regions of the signals values for which the coverage in the two organizations differs.

- When the leader's signal belongs to $[\widehat{\sigma}^{FU}, \min\left(\widehat{\sigma}^{D}, \underline{\sigma}^{D}\right)]$, no coverage is offered in the FUA whereas either full of partial coverage is offered in the DA (depending on the follower's signal);
- When the leader's signal is smaller than $\widehat{\sigma}^{FU}$, the follower's signal determines whether coverage is partial or full. When the boundary $\overline{s}^{FU}$ is larger than $\widehat{\sigma}^{D}$, the FUA offers full coverage whereas only partial coverage is offered in the DA. The reverse holds when $\overline{s}^{FU}$ is smaller than $\widehat{\sigma}^{D}$. We show in the proof of Lemma 7, that when $P^{D}(s) < P^{FU}(s)$ then $\overline{s}^{FU}$ is larger than $\widehat{\sigma}^{D}$. However, in the general case, comparing the boundary $\overline{s}^{FU}$ to $\widehat{\sigma}^{D}$ is difficult and $\overline{s}^{FU}$ may cross $\widehat{\sigma}^{D}$.

As a consequence, when one of the insurance companies is optimistic about the risk (the leader's signal is smaller than $\widehat{\sigma}^{FU}$) and $P^{D}(s) < P^{FU}(s)$, the FUA tends to offer more coverage at a larger rate. However, if the forecasts about the risk occurrence are rather pessimistic (when the most optimistic insurance company is already quite pessimistic, that is when the leader's signal belongs to $[\widehat{\sigma}^{FU}, \min\left(\widehat{\sigma}^{D}, \underline{\sigma}^{D}\right))$, only the DA allows to offer a positive coverage (partial or full coverage).

As it has already been highlighted, the comparison of the equilibrium bidding strategies differ from the comparison of the premium rates. If pricing is uniform in the FUA, the leader and the follower offer different premiums in the DA, the follower's premium being larger. The difficult comparison of the equilibrium bidding strategies makes the ranking of the premiums even more tricky.

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This analysis shows that ex-ante there is no clear dominance of one auction format. However, we can highlight two messages depending on the realization of the signals. If insurers agree about the ex-post evaluation of risk (s_i close to s_{-i}), the FUA has less scope so that the size of the bidding region is key to determine the insurance coverage. The DA therefore offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows to increase insurance coverage.

As Lemma 7 underlines, the generality of our model makes the comparison of the two organizations quite involved (the fact that the bidding strategies cross or not depends on the comparison of $P^{FU}(0)$ with $P^{D}(0)$ which involves all the parameters of the model).[12] Therefore, as an illustration, we propose to solve explicitly an example in the next subsection.

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[12] Observe that for parameters' values such that $P^{D}(0) < P^{FU}(0)$, bids are larger in the FUA for all

5.2 The case of independent signals and linear premium rate

We consider the case in which the two insurance companies receive independent signals that are distributed according to a uniform distribution on $[0,1]$. This corresponds to the case where the risk is new so that each insurer gathers together its own information on the occurrence probability. The cost function is moreover assumed to be linear in the two signals

$$p(s_i, s_{-i}) = \frac{s_i + s_{-i}}{2}.$$

In this case, Assumption 2(ii) implies that $\overline{P} \in [1/4, 3/4]$. Note also that $\widetilde{\sigma} = \overline{P}$. All the computations are detailed in the Appendix C. In particular, we do not detail here the equilibrium strategies that are explicitly computed in the Appendix. In this illustration, we restrict ourselves to the case where the separating equilibrium exists in the DA. This implies that we restrict κ to be greater than $\kappa^*(\overline{P}) = \max\left(\frac{2\overline{P}-1}{\overline{P}^2}, 0\right)$. Observe that when $\overline{P} \in [1/4, 1/2]$, the equilibrium is separating for any value of κ ($\kappa^*(\overline{P}) = 0$).

Proposition 5 *If $\kappa \geq \kappa^*(\overline{P})$, there exists a unique $\widehat{\overline{P}}$ such that*

- *if $\overline{P} < \widehat{\overline{P}}$, $P^{FU}(s) > P^D(s)$, $\forall s \in [0, \widehat{\sigma}^{FU}]$*
- *if $\overline{P} \geq \widehat{\overline{P}}$, $P^{FU}(s) < P^D(s)$ and then $P^{FU}(s) > P^D(s)$, when s increases from 0 to $\widehat{\sigma}^{FU}$.*

When the reserve price is low, the FUA offers higher premiums and higher coverage than the DA. All these results are ex-post as they depend on the signals' realization. This example allows us to have a look at an ex-ante analysis. To do that, we compute both the expected profit and the expected coverage in the two organizations for all parameters' values (whether the equilibrium in the discriminatory auction is separating or semi-separating). While their expression is given is the Appendix, we provide a comparison of both of them in Figures 7 and 8.

Remember the basic tradeoff highlighted in the previous subsection. Depending on the parameters' values (when the reserve price is low enough), the FUA provides more coverage at a higher premium when at least one insurer is optimistic about the risk occurrence probability. But it does not offer any coverage when the two insurers provide a pessimistic estimation of the risk occurrence probability. According to Figure 7, for a given κ, the FUA generates a larger expected profit if and only if the reserve price is smaller than a threshold. Indeed, when the reserve price is large, insurers taking part to the DA can extract higher rents from the insureds, in part because the follower has a larger risk premium than the leader.

Another criteria that might be of interest for a regulator concerns the expected coverage generated by each of the organization.

As expected, we observe in Figure 8 that the expected coverage is larger in the FUA when the reserve price is low. However, for a given value of κ, when the reserve price

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realizations of the signals. As a result, in expectation, the FUA offers ore coverage and generates larger expected profit than the DA.

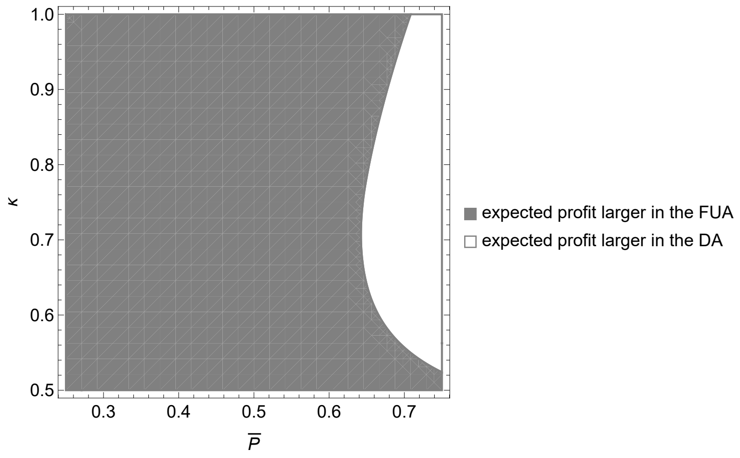

Figure 7: Comparison of the expected profits.

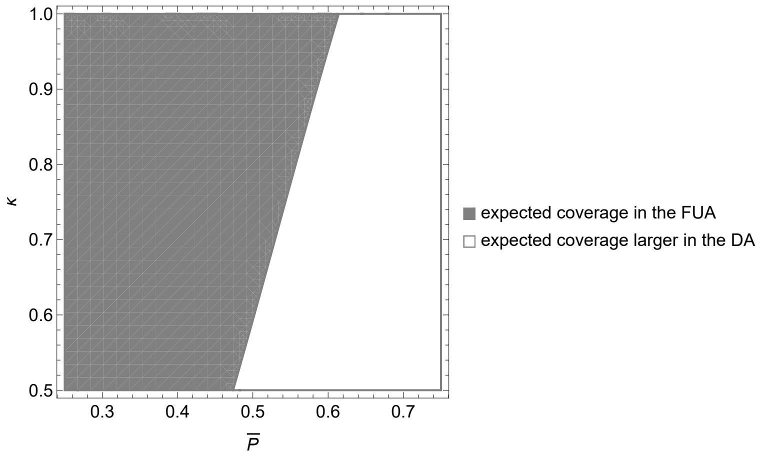

Figure 8: Comparison of the expected coverage.

increases, observe that the expected coverage becomes larger in the DA. As a matter of fact, the region of signals values such that insurance coverage is only provided with the DA $(\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^{FU})$ increases when $\overline{P}$ increases. On the contrary, for a given $\overline{P}$ high enough, when κ increases, the discriminatory auction offers a smaller expected coverage. Indeed, as κ increases, $\widehat{\sigma}^D$ and $\overline{\sigma}^D$ decreases so that the region in which only the discriminatory auction provides coverage $(\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^{FU})$ decreases.

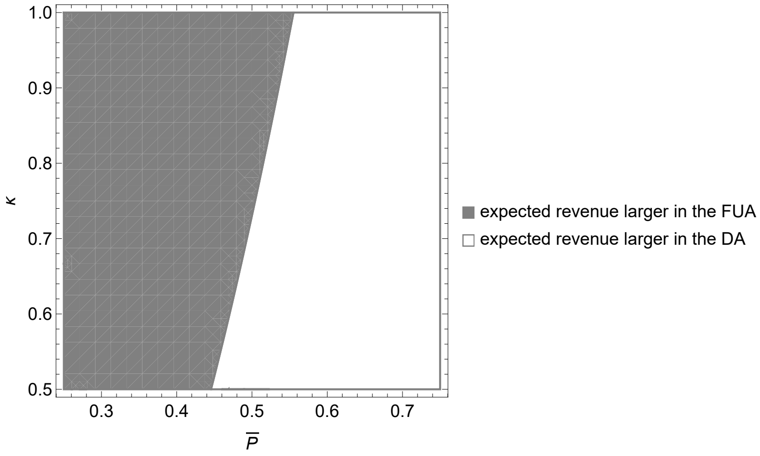

Figure 9: Comparison of the expected revenue.

5.3 Conclusion

In this paper, we investigate how insurance companies can coordinate to extend their joint capacity for the coverage of large new risks. Organizing such insurance supply amounts to auction a common value divisible good between capacity constrained insurers where insurers have private information. Two auctions formats, a flexible uniform auction (FUA) and a discriminatory auction (DA), are compared with respect to premiums and their ability to offer full and partial coverage.

We show that both auction formats lead to different coverage/premium tradeoffs. Our findings may not entirely negate the existence of ad-hoc co-insurance agreements to cover large unconventional risk categories in the insurance sector. If insurers agree about the ex-post evaluation of risk, the discriminatory auction offers more coverage. On the contrary, if insurers receive opposite evaluations, the exercise of the re-entry option allows to increase insurance coverage. We also provide some results about the ex-ante comparison of the two auctions for the case of new risks (independent insurers' signals). We show that a regulator aiming at maximizing the expected coverage should promote the FUA when the reserve price is low enough or when capacity constraints are large enough.

Two major differences exist between the two settings. The follower pricing rule (uniform versus heterogenous pricing) and the re-entry option. When re-entry is possible, an insurer takes less risks in its bidding strategy since submitting a bid is not a necessary condition to participate to the auction anymore. Therefore, an insurer reduces the risk of being a leader despite a high signal by reducing the bidding region. Introducing the re-entry option in the DA then leads to more conservative strategies. Also, being a follower in the first stage leads to higher premiums than re-entering in the second stage and being a follower at the leader's premium. Therefore, the re-entry option is less valuable under heterogenous pricing. As a consequence, insurers take advantage of the first stage by enlarging the bidding region, compared to the FUA. The presence of re-entry option in an auction then yields to more complete market failure but reduces the partial market failure. In the DA, re-entry raises premium since re-entering followers benefit from the leader's revenue. When re-entry is possible, the comparison between heterogenous and uniform pricing relies on the same forces than the ones described in Lemma 7. Re-entry also increases premium in the uniform auction. The presence of re-entry option yields to higher premiums both with heterogenous and uniform pricing.

A Equilibrium analysis

A.1 Equilibrium analysis in the FUA

We look for an equilibrium strategy such that insurer i has an incentive to submit a bid according to its true signal. The profit of insurer i who observed a signal s_i and bids a risk premium $P^{FU}(b)$ reads

$$\Pi^{FU}(b,s_i)=\begin{cases}\kappa\left(1-G\left(b|s_i\right)\right)\mathbb{E}\left[P^{FU}(b)-p(s_i,S_{-i})|S_{-i}>b\right] & \\ \quad +(1-\kappa)G\left(b|s_i\right)\mathbb{E}\left[\left(P^{FU}(S_{-i})-p(s_i,S_{-i})\right)_+|S_{-i}<b\right] & \text{for } b\leq\widehat{\sigma}^{FU}\\ (1-\kappa)G\left(\widehat{\sigma}^{FU}|s_i\right)\mathbb{E}\left[\left(P^{FU}(S_{-i})-p(s_i,S_{-i})\right)_+|S_{-i}<\widehat{\sigma}^{FU}\right] & \text{for } b>\widehat{\sigma}^{FU}.\end{cases}$$

Therefore, in order the incentive compatibility constraint to be satisfied, the risk premium satisfies $\forall s_i\leq\widehat{\sigma}^{FU}$

$$\frac{\partial\Pi^{FU}\left(b,s_i\right)}{\partial b}|_{b=s_i}=0.$$

As a consequence, the equilibrium bid $P^{FU}(s_i)$ satisfies the following differential equation

$$P^{FU\prime}(s_i)=\frac{2\kappa-1}{\kappa}\frac{g(s_i|s_i)}{1-G(s_i|s_i)}\left(P^{FU}(s_i)-p(s_i,s_i)\right),\forall s_i\leq\widehat{\sigma}^{FU}.$$

It is solved with the boundary condition that $P^{FU}(\widehat{\sigma}^{FU})=\overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^{FU}(s)=\overline{P}(1-L(\widehat{\sigma}^{FU}|s))+\int_s^{\widehat{\sigma}^{FU}}p(x,x)dL(x|s)\ \ \forall\ \ s\leq\widehat{\sigma}^{FU}$$

with

$$L(x|s)=1-\exp\left(-\frac{2\kappa-1}{\kappa}\int_s^x\frac{g(\tau|\tau)}{1-G(\tau|\tau)}d\tau\right).$$

A.2 Equilibrium analysis in the DA

First case: $\kappa\geq\kappa^*$. We look for an equilibrium strategy such that insurer i has an incentive to submit a bid according to his true signal. Therefore, at equilibrium, the risk premium satisfies $\forall s_i\leq\widehat{\sigma}^D$

$$\frac{\partial\Pi^{D}\left(b,s_i\right)}{\partial b}|_{b=s_i}=0$$

so that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation

$$P^{D\prime}(s_i)=\frac{\left(2\kappa-1\right)g(s_i|s_i)}{\kappa-\left(2\kappa-1\right)G(s_i|s_i)}\left(P^D(s_i)-p(s_i,s_i)\right),\forall s_i\leq\widehat{\sigma}^D.$$

It is solved with the boundary condition that $P^D(\widehat{\sigma}^D) = \overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^D(s) = \overline{P}(1 - K(\widehat{\sigma}^D|s)) + \int_s^{\widehat{\sigma}^D} p(x,x)dK(x|s) \ \ \forall \ \ s \leq \widehat{\sigma}^D$$

with

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{(2\kappa - 1)\, g(\tau|\tau)}{\kappa - (2\kappa - 1)\, G(\tau|\tau)} d\tau\right).$$

Second case: $\kappa < \kappa^*$. We look for an equilibrium strategy such that insurer i has an incentive to submit a bid according to his true signal. Therefore, at equilibrium, the risk premium satisfies $\forall s_i \leq \underline{\sigma}^D$

$$\frac{\partial \Pi^D\left(b, s_i\right)}{\partial b}|_{b=s_i} = 0$$

so that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation

$$P^{D\prime}(s_i) = \frac{(2\kappa - 1)\, g(s_i|s_i)}{\kappa - (2\kappa - 1)\, G(s_i|s_i)} \left(P^D(s_i) - p(s_i, s_i)\right), \forall s_i \leq \underline{\sigma}^{FU}.$$

It is solved with the boundary condition that $P^D(\underline{\sigma}^D) = \overline{P}$. Using the method of the parameters' variation, we obtain that

$$P^D(s) = \overline{P}(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x,x)dK(x|s) \ \ \forall \ \ s \leq \underline{\sigma}^D$$

with

$$K(x|s) = 1 - \exp\left(-\int_s^x \frac{(2\kappa - 1)\, g(\tau|\tau)}{\kappa - (2\kappa - 1)\, G(\tau|\tau)} d\tau\right).$$

When $s \in [\underline{\sigma}^D, \overline{\sigma}^D]$, $P^D(s) = \overline{P}$.

B Proofs

B.1 Proof of Lemma 1

The participation constraint requires that bidders with signals greater than $\widehat{\sigma}^{FU}$ prefer not to bid to submitting the bid $\overline{P}$:

$$\widehat{\sigma}^{FU} = \inf\{\sigma \in [0,1] : (1 - G\left(\sigma|\sigma\right))\, \mathbb{E}\left[\overline{P} - p(\sigma, S_{-i})|S_{-i} > \sigma\right] \leq 0\}. \tag{22}$$

Since p in continuous and because of Assumption 2, this implies that

$$\left(1 - G\left((\widehat{\sigma}^{FU}|(\widehat{\sigma}^{FU}\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_j)|S_j > \widehat{\sigma}^{FU}\right] = 0. \tag{23}$$

We first prove that $\widehat{\sigma}^{FU} < \widetilde{\sigma}$. Assume by contradiction that $\widehat{\sigma}^{FU} \geq \widetilde{\sigma}$. Then,

$$\begin{aligned}
\left(1 - G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_{-i})|S_{-i} > \widehat{\sigma}^{FU}\right] &= \int_{\widehat{\sigma}^{FU}}^{1} \left(\overline{P} - p(\widehat{\sigma}^{FU}, s_{-i})\right) g\left(s_{-i}|\widehat{\sigma}^{FU}\right) ds_{-i} \\
&< \left(\overline{P} - p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU})\right) \left(1 - G(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU})\right) \\
&\leq 0
\end{aligned}$$

where the first inequality comes from the fact that p is strictly increasing in each of its argument, and the second from the fact that $\widehat{\sigma}^{FU}$ is assumed to be greater than or equal to $\widetilde{\sigma}$. This contradicts the definition of $\widehat{\sigma}^{FU}$ (see equation (23)). A direct consequence is that $\left(1 - G\left((\widehat{\sigma}^{FU}|(\widehat{\sigma}^{FU}\right)\right) > 0$ since $\widehat{\sigma}^{FU} < \widetilde{\sigma}$ so that $\widehat{\sigma}^{FU}$ can be defined as

$$\mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_j)|S_j > \widehat{\sigma}^{FU}\right] = 0. \tag{24}$$

We introduce function

$$\Psi(x) \equiv \mathbb{E}\left[\overline{P} - p(x, S)|S > x\right]$$

that is decreasing (Milgrom and Weber (1982)). As $\Psi(0) > 0$ and $\Psi(1) < 0$ (Assumption 2), we conclude that $\widehat{\sigma}^{FU}$ is unique. □

B.2 Proof of Proposition 1

See Subsection A.1.

B.3 Proof of Lemma 2

We have that $s_i \mapsto P^{FU}(s_{-i}) - p(s_i, s_{-i})$ is a decreasing function. We prove the result by showing that $P^{FU}(s_{-i}) - p(\widehat{\sigma}^{FU}, s_{-i}) > 0$, $\forall s_{-i} \leq \widehat{\sigma}^{FU}$.

$$\begin{aligned}
& P^{FU}(s_{-i}) - p(\widehat{\sigma}^{FU}, s_{-i}) \\
={}& \overline{P}\left(1 - L\left(\widehat{\sigma}^{FU}|s_{-i}\right)\right) + \int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x, x) dL(x|s_{-i}) - p\left(\widehat{\sigma}^{FU}, s_{-i}\right) \\
={}& \overline{P}\left(1 - L\left(\widehat{\sigma}^{FU}|s_{-i}\right)\right) + L\left(\widehat{\sigma}^{FU}|s_{-i}\right) \int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x, x) \frac{dL(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} - p\left(\widehat{\sigma}^{FU}, s_{-i}\right) \\
\geq{}& \overline{P}\left(1 - L\left(\widehat{\sigma}^{FU}|s_{-i}\right)\right) + L\left(\widehat{\sigma}^{FU}|s_{-i}\right) p\left(\widehat{\sigma}^{FU}, s_{-i}\right) - p\left(\widehat{\sigma}^{FU}, s_{-i}\right) \\
={}& \left(\overline{P} - p\left(\widehat{\sigma}^{FU}, s_{-i}\right)\right)\left(1 - L\left(\widehat{\sigma}^{FU}|s_{-i}\right)\right) \\
>{}& 0
\end{aligned}$$

where the first inequality holds if $\int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x, x) \frac{dL(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} \geq p\left(\widehat{\sigma}^{FU}, s_{-i}\right)$ and the second inequality holds since $s_{-i} \leq \widehat{\sigma}^{FU} < \widetilde{\sigma}$.

The rest of the proof consists in proving that $\int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x, x) \frac{dL(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} \geq p\left(\widehat{\sigma}^{FU}, s_{-i}\right)$, $\forall s_{-i} \leq \widehat{\sigma}^{FU}$. As p is symmetric and supermodular (Assumption 3), it holds that $p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU})+$

$p(s_{-i}, s_{-i}) \geq 2p(\widehat{\sigma}^{FU}, s_{-i})$ so that it is sufficient to prove that

$$\int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x,x) \frac{dL(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} \geq \frac{p(s_{-i}, s_{-i}) + p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU})}{2} \forall s_{-i} \leq \widehat{\sigma}^{FU}. \tag{25}$$

An integration by part implies that

$$\int_{s_{-i}}^{\widehat{\sigma}^{FU}} p(x,x) \frac{dL(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} = p\left(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU}\right) - \int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} dx.$$

Therefore, inequality (25) reads

$$p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU}) - p(s_{-i}, s_{-i}) - 2\int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} dx \geq 0 \ \forall s_{-i} \leq \widehat{\sigma}^{FU}.$$

The derivative of function

$$s_{-i} \mapsto p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU}) - p(s_{-i}, s_{-i}) - 2\int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} dx$$

equals

$$-\frac{d}{dx} p(x,x)|_{x=s_{-i}} - 2\int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L_2(x|s_{-i}) L\left(\widehat{\sigma}^{FU}|s_{-i}\right) - L(x|s_{-i}) L_2\left(\widehat{\sigma}^{FU}|s_{-i}\right)}{\left(L\left(\widehat{\sigma}^{FU}|s_{-i}\right)\right)^2} dx < 0$$

since $L(s_{-i}|s_{-i}) = 0$ and since $x \mapsto \frac{L_2(x|s)}{L(x|s)}$ is decreasing. Moreover, as

$$p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU}) - p(s_{-i}, s_{-i}) - 2\int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} dx = 0$$

when $s_{-i} = \widehat{\sigma}^{FU}$ it follows that

$$p(\widehat{\sigma}^{FU}, \widehat{\sigma}^{FU}) - p(s_{-i}, s_{-i}) - 2\int_{s_{-i}}^{\widehat{\sigma}^{FU}} \frac{d}{dx} p(x,x) \frac{L(x|s_{-i})}{L\left(\widehat{\sigma}^{FU}|s_{-i}\right)} dx \geq 0 \ \forall s_{-i} \leq \widehat{\sigma}^{FU}$$

and the result is proved. □

B.4 Proof of Proposition 2

Observe first that $L_\kappa(x|s)$ is increasing in κ.[13] Remember that

$$P^{FU}(s) = \overline{P}(1 - L_\kappa(\widehat{\sigma}^{FU}|s)) + \int_s^{\widehat{\sigma}^{FU}} p(x,x) dL_\kappa(x|s).$$

As $\widehat{\sigma}^{FU}$ is independent from κ, $L_\kappa(\widehat{\sigma}^{FU}|s)$ is increasing in κ as we just underlined. This also implies that $\int_s^{\widehat{\sigma}^{FU}} p(x,x) dL_\kappa(x|s)$ is increasing in κ because, for $\kappa' > \kappa$ function L_κ first

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[13] The subscript "κ" indicates the parameter value, κ.

order stochastic dominates $L_{\kappa'}$. The boundary between regions the full coverage region and the partial coverage region is defined by $\kappa\left\{\{(s_i,s_{-i})\in[0,1]^2|P^{FU}(s_i)=p(s_i,s_{-i})\right\}$. For a given s_{-i}, if κ increases, in order the equality $P^{FU}(s_i)=p(s_i,s_{-i})$ to hold it must be the case that s_i decreases, so that the region where full coverage happens shrinks. As a consequence, the region in which partial coverage holds expands. □

B.5 Proof of Proposition 3

We know that $\widehat{\sigma}^{FU}$ is such that $\Psi(\widehat{\sigma}^{FU})=0$. Differiating this expression with respect to $\overline{P}$ implies that

$$\frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}}=-\frac{\frac{\partial\Psi(\widehat{\sigma}^{FU})}{\partial\overline{P}}}{\Psi'(\widehat{\sigma}^{FU})}=-\frac{1}{\Psi'(\widehat{\sigma}^{FU})}>0$$

since the denominator is negative (see the proof of Lemma 1).

As for the equilibrium bidding strategy, observe that

$$\frac{\partial P^{FU}(s)}{\partial\overline{P}}=1-L\left(\widehat{\sigma}^{FU}|s\right)-L_1\left(\widehat{\sigma}^{FU}|s\right)\frac{\widehat{\sigma}^{FU}}{\partial\overline{P}}\left(\overline{P}-p\left(\widehat{\sigma}^{FU},\widehat{\sigma}^{FU}\right)\right).$$

The first term is positive and corresponds to the direct effect, whereas the second term is negative and corresponds to the indirect effect. It is therefore necessary to compute all the terms together. Observing that

$$L_1\left(\widehat{\sigma}^{FU}|s\right)=\frac{2\kappa-1}{\kappa}\frac{g\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)}{1-G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)}\left(1-L\left(\widehat{\sigma}^{FU}|s\right)\right),$$

we have that

$$\begin{aligned}\frac{\partial P^{FU}(s)}{\partial\overline{P}} &= \left(1-L\left(\widehat{\sigma}^{FU}|s\right)\right)\left(1-\frac{\widehat{\sigma}^{FU}}{\partial\overline{P}}\left(\overline{P}-p\left(\widehat{\sigma}^{FU},\widehat{\sigma}^{FU}\right)\right)\frac{2\kappa-1}{\kappa}\frac{g\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)}{1-G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)}\right)\\ &= \frac{\left(1-L\left(\widehat{\sigma}^{FU}|s\right)\right)\left(\left(1-G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)\right)\Psi'(\widehat{\sigma}^{FU})-\frac{1-\kappa}{\kappa}\left(\overline{P}-p\left(\widehat{\sigma}^{FU},\widehat{\sigma}^{FU}\right)\right)g\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)\right)}{\left(1-G\left(\widehat{\sigma}^{FU}|\widehat{\sigma}^{FU}\right)\right)\Psi'(\widehat{\sigma}^{FU})}\\ &> 0.\end{aligned}$$

It follows that $P^{FU}(s)$ is an increasing function of $\overline{P}$. □

B.6 Proof of Lemma 3

Introduce functions ϕ, ψ and θ defined by

$$\begin{aligned}\phi(x) &\equiv G\left(x|x\right)\mathbb{E}\left[\overline{P}-p(x,S)|S<x\right] && (26)\\ \psi(x) &\equiv \left(1-G\left(x|x\right)\right)\mathbb{E}\left[\overline{P}-p(x,S)|S>x\right] && (27)\\ \theta(x) &\equiv \kappa\psi(x)+(1-\kappa)\phi(x) && (28)\end{aligned}$$

$\widehat{\sigma}^D$ (Equation (14)) is defined by $\theta(\widehat{\sigma}^D) = 0$. Observe that the only possibility for Equation (14) to be satisfied is that $\phi(\widehat{\sigma}^D) > 0$ and $\psi(\widehat{\sigma}^D) < 0$.
Introduce in addition functions Φ defined by

$$\Phi(x) \quad \equiv \quad \mathbb{E}\left[\overline{P} - p(x,S)|S < x\right] \tag{29}$$

and remember that we defined in Lemma 1

$$\Phi(x) \quad \equiv \quad \mathbb{E}\left[\overline{P} - p(x,S)|S < x\right]. \tag{30}$$

It follows that $\theta\left(x\right) = \kappa\left(1 - G\left(x|x\right)\right)\Psi\left(x\right) + \left(1-\kappa\right)G\left(x|x\right)\Phi\left(x\right)$. As we already noted, Φ and Ψ are decreasing functions (Milgrom and Weber (1982)).

Step 1. Assume first that $\kappa \geq \kappa^*$. We show that a unique threshold $\widehat{\sigma}^D \in [0, \widetilde{\sigma}]$ exists. Observe that $\theta(0) > 0$ and that $\theta(\widetilde{\sigma}) \leq 0$ when $\kappa \geq \kappa^*$. Therefore, function θ has at least one zero. Therefore, $\widehat{\sigma}^D$ exists. We are going to prove that the derivative of θ is negative at $\widehat{\sigma}^D$.

$$\theta'(\widehat{\sigma}^D) = (1-\kappa)G(\widehat{\sigma}^D|\widehat{\sigma}^D)\Phi'(\widehat{\sigma}^D) + \kappa(1-G(x|x))\Psi'(\widehat{\sigma}^D) + \left.\frac{dG\left(x|x\right)}{dx}\right|_{x=\widehat{\sigma}^D}\Big((1-\kappa)\Phi(\widehat{\sigma}^D) - \kappa\Psi(\widehat{\sigma}^D)\Big)$$

The first two terms are negative. Since $\phi(\widehat{\sigma}^D) > 0$ and $\psi(\widehat{\sigma}^D) < 0$, it follows that $(1-\kappa)\Phi(\widehat{\sigma}^D) - \kappa\Psi(\widehat{\sigma}^D) \geq 0$. Therefore, if $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} < 0$, $\theta'(\widehat{\sigma}^D) \leq 0$. To complete this part of the proof, we must treat the case where $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$. To do that, we have to detail the expressions of $\phi'(x)$ and $\psi'(x)$.

$$\begin{aligned}
\theta'(x) \quad &= \quad \kappa\psi'(x) + (1-\kappa)\phi'(x) \\
&= \quad -\kappa\left(\overline{P} - p(x,x)\right)g\left(x|x\right) - \kappa\int_x^1 p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} \\
&\qquad +\kappa\int_x^1\left(\overline{P} - p(x,s_{-i})\right)g_2\left(s_{-i}|x\right)ds_{-i} + (1-\kappa)\left(\overline{P} - p(x,x)\right)g\left(x|x\right) \\
&\qquad -(1-\kappa)\int_0^x p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} + (1-\kappa)\int_0^x\left(\overline{P} - p(x,s_{-i})\right)g_2\left(s_{-i}|x\right)ds_{-i} \\
&= \quad -\kappa\int_x^1 p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} - (1-\kappa)\int_0^x p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} \\
&\qquad -(2\kappa-1)\left(\overline{P} - p(x,x)\right)g\left(x|x\right) + \kappa\int_x^1\left(\overline{P} - p(x,s_{-i})\right)g_2\left(s_{-i}|x\right)ds_{-i} \\
&\qquad +(1-\kappa)\int_0^x\left(\overline{P} - p(x,s_{-i})\right)g_2\left(s_{-i}|x\right)ds_{-i}.
\end{aligned}$$

An integration by part of the last two terms imply that

$$\begin{aligned}\theta'(x) &= -\kappa \int_x^1 p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} - (1-\kappa)\int_0^x p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} \\ &\quad -(2\kappa-1)\left(\overline{P}-p(x,x)\right)g\left(x|x\right) + \kappa\int_x^1 p_2(x,s_{-i})G_2\left(s_{-i}|x\right)ds_{-i} \\ &\quad +(1-\kappa)\int_0^x p_2(x,s_{-i})G_2\left(s_{-i}|x\right)ds_{-i} - (2\kappa-1)\left(\overline{P}-p(x,x)\right)G_2\left(x|x\right) \\ &= -\kappa \int_x^1 p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} - (1-\kappa)\int_0^x p_1(x,s_{-i})g\left(s_{-i}|x\right)ds_{-i} \\ &\quad +\kappa\int_x^1 p_2(x,s_{-i})G_2\left(s_{-i}|x\right)ds_{-i} + (1-\kappa)\int_0^x p_2(x,s_{-i})G_2\left(s_{-i}|x\right)ds_{-i} \\ &\quad -(2\kappa-1)\left(\overline{P}-p(x,x)\right)\frac{dG\left(x|x\right)}{dx}.\end{aligned}$$

The first four terms are negative (remember that affiliation implies that $G_2 < 0$). As for the last term, it is negative when evaluated at $x = \widehat{\sigma}^D$ since $\widehat{\sigma}^D < \widetilde{\sigma}$ and $\frac{dG(x|x)}{dx}|_{x=\widehat{\sigma}^D} > 0$. When θ equals zero, its derivative is negative meaning that $\widehat{\sigma}_D$ is unique.

Step 2. Assume now that $\kappa < \kappa^*$. We show that function θ does not cancel on $[0,\widetilde{\sigma}]$ so that there does not exist $\widehat{\sigma}^D$.
When $\kappa < \kappa^*$, $\theta(\widetilde{\sigma}) > 0$, so that either θ never cancels on $[0,\widetilde{\sigma}]$, or it cancels an even number of times. However, we have shown that when θ cancels on $[0,\widetilde{\sigma}]$, its derivative is negative. Therefore, when $\kappa < \kappa^*$, θ is positive and $\widehat{\theta}^D$ does not exist. ☐

B.7 Proof of Proposition 4

We prove first that if the separating equilibrium does not exist ($\kappa < \kappa^*$), then the semi-separating equilibrium exists and is unique.

The first part of this proof goes through a series of steps. Let us first introduce function I, J and K

$$I(x,y) \equiv \left(G\left(y|x\right) - G\left(x|x\right)\right)\mathbb{E}\left[\overline{P} - p(x,S)|x < S < y\right] \tag{31}$$

$$J(x,y) \equiv \left(G\left(y|y\right) - G\left(x|y\right)\right)\mathbb{E}\left[\overline{P} - p(y,S)|x < S < y\right] \tag{32}$$

$$H(x,y) \equiv \theta(y) + \frac{1}{2}J(x,y) \tag{33}$$

where θ is defined by equation (28). Observe that $\psi(x) = I(x,1)$, $\phi(x) = J(0,x)$ and $\theta(x) = \kappa I(x,1) + (1-\kappa)J(0,x)$. Let us also introduce

$$\mathcal{J}(x,y) \equiv \mathbb{E}\left[\overline{P} - p(y,S)|x < S < y\right] \tag{34}$$

$\mathcal{J}$ is decreasing in each of its argument (Milgrom and Weber [19]).
Step 1.

We show that $\alpha(\overline{\sigma}^D) < \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) < \overline{\sigma}^D < \widehat{\sigma}^D$.
To prove this step, assume that $(\underline{\sigma}^D, \overline{\sigma}^D)$, with $\underline{\sigma}^D, \overline{\sigma}^D$, is a solution meaning that $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$. $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ implies that

$$\begin{aligned}\overline{P}\left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} p(\underline{\sigma}^D, t) g(t|\underline{\sigma}^D) dt \\ &> p(\underline{\sigma}^D, \underline{\sigma}^D)\left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right).\end{aligned}$$

Therefore, $\overline{P} > p(\underline{\sigma}^D, \underline{\sigma}^D)$ implying that $\underline{\sigma}^D < \widetilde{\sigma}$.

Remember that $t \mapsto \overline{P} - p(\underline{\sigma}^D, t)$ is a decreasing function. In order to have $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$, it must be the case that $\overline{P} - p(\underline{\sigma}^D, t)$ is first positive and then negative as t increases from $\underline{\sigma}^D$ to $\overline{\sigma}^D$. In particular, we must have that $\overline{P} - p(\underline{\sigma}^D, \overline{\sigma}^D) < 0$. This implies that $\overline{\sigma}^D > \alpha(\underline{\sigma}^D)$ and $\underline{\sigma}^D > \alpha(\overline{\sigma}^D)$ where function α is defined by equation (4).[14]

To prove that $\overline{\sigma}^D < \widehat{\sigma}^D$, we show that $J(\underline{\sigma}^D, \overline{\sigma}^D) < 0$ so that $\theta(\overline{\sigma}^D) > 0$.

$$\begin{aligned}J(\underline{\sigma}^D, \overline{\sigma}^D) &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \overline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &< \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P} - p(t, \underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|\overline{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\ &\leq \frac{g(\alpha(\underline{\sigma}^D)|\overline{\sigma}^D)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\ &= 0.\end{aligned}$$

The second inequality holds because $t \mapsto \frac{g(t|y)}{g(t|x)}$ is an increasing function $\forall x \leq y$.

It therefore holds that $\alpha(\overline{\sigma}^D) < \underline{\sigma}^D < \widetilde{\sigma} < \alpha(\underline{\sigma}^D) < \overline{\sigma}^D < \widehat{\sigma}^D$. This allows us to define the region $\mathcal{D} \equiv \left\{(x, y) \in [0,1]^2 | \alpha(y) < x < \widetilde{\sigma} < \alpha(x) < y < \widehat{\sigma}^D\right\}$ to which the solution to the following system should belong to

$$\begin{cases} I(x, y) &= 0 \\ H(x, y) &= 0. \end{cases}$$

Step 2.
We show that $x_I(y)$ defined by $I(x_I(y), y) = 0$ on $\mathcal{D}_y = \left\{\widetilde{\sigma} < y < \widehat{\sigma}^D | \alpha(y) < x_I(y) < \widetilde{\sigma}\right\}$ is a decreasing function.
The implicit function theorem implies that

$$\frac{dx_I(y)}{dy} = -\frac{I_2(x_I(y), y)}{I_1(x_I(y), y)}.$$

$I_2(x_I(y), y) = (\overline{P} - p(x_I(y), y)) g(y|x_I(y)) \leq 0$ since $\alpha(y) < x_I(y)$ (or equivalently $y >$

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[14] Note that the symmetry of p and the fact that it is increasing with respect to each of its argument imply that $\alpha = \alpha^{-1}$.

$\alpha(x_I(y)))$.

$$\begin{aligned} I_1(x_I(y),y) &= -\left(\overline{P}-p(x_I(y),x_I(y))\right)g(x_I(y)|x_I(y)) - \int_{x_I(y)}^{y} p_1(x_I(y),t)g(t|x_I(y)) \\ &\quad + \int_{x_I(y)}^{y} (\overline{P}-p(x_I(y),t))\mathcal{L}\left(t|x_I(y)\right)g(t|x_I(y))dt \end{aligned}$$

The first two terms are negative (the first because $x_I(y) < \widetilde{\sigma}$). The third term is negative, too. To see that, let us introduce

$$\mathcal{L}\left(s|x\right) \equiv \frac{1}{g\left(s|x\right)}\frac{dg\left(s|x\right)}{dx}. \tag{35}$$

Function $s \mapsto \mathcal{L}(s|x)$ is increasing according to Assumption 1. As a consequence,

$$\begin{aligned} & \int_{x_I(y)}^{y} (\overline{P}-p(x_I(y),t))\mathcal{L}\left(t|x_I(y)\right)g(t|x_I(y))dt \\ \leq\ & \mathcal{L}\left(\alpha(x_I(y))|x_I(y)\right)\int_{x_I(y)}^{y} (\overline{P}-p(x_I(y),t))g(t|x_I(y))dt \\ =\ & 0. \end{aligned}$$

This implies that $y \mapsto x_I(y)$ is a decreasing function.

Note moreover that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$ and that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$. To prove this last inequality, observe that

$$\begin{aligned} \int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P}-p(\alpha(\widehat{\sigma}^D),t)\right)g(t|x_I(\widehat{\sigma}^D))dt &> \int_{x_I(\widehat{\sigma}^D)}^{\widehat{\sigma}^D} \left(\overline{P}-p(\alpha(\widehat{\sigma}^D),\widehat{\sigma}^D)\right)g(t|x_I(\widehat{\sigma}^D))dt \\ &= 0, \end{aligned}$$

implying that $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$.

The last property that remains to be shown for this function x_I is that $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross once when $y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. This is not a priori obvious since the two functions are decreasing. We know that $x_I(\widetilde{\sigma}) = \alpha(\widetilde{\sigma}) = \widetilde{\sigma}$. Assume that there exists $\overline{y} \in (\widetilde{\sigma}, \widehat{\sigma}^D]$ such that $x_I(\overline{y}) = \alpha(\overline{y})$. By definition of x_I, this implies that

$$\int_{\alpha(\overline{y})}^{\overline{y}} \left(\overline{P} - p\left(\alpha(\overline{y}),t\right)g\left(t|\alpha(\overline{y})\right)\right)dt = 0.$$

However, $p\left(\alpha(\overline{y}),t\right) < p\left(\alpha(\overline{y}),\overline{y}\right) = \overline{P}$, $\forall t \in [\alpha(\overline{y}),\overline{y}]$, so that it is not possible that the integral equals 0. Therefore such an $\overline{y}$ does not exist. As a consequence, $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross for $y = \widetilde{\sigma}$, and $\forall y \in [\widetilde{\sigma}, \widehat{\sigma}^D]$, $x_I(y) > \alpha(y)$.

Step 3.
We show that $y_H(x)$ defined by $H(x, y_H(x)) = 0$ on $\mathcal{D}_x = \left\{\alpha\left(\widehat{\sigma}^D\right) < x < \widetilde{\sigma} | \alpha(x) < y_H(x) < \widehat{\sigma}^D\right\}$ is an increasing function.

The implicit function theorem implies that

$$\frac{dy_H(x)}{dx} = -\frac{H_1(x, y_H(x))}{H_2(x, y_H(x))}.$$

Remembering that $H(x, y) = \theta(y) + (1/2)J(x, y)$ (where θ is defined by equation (28)), it follows that

$$\begin{aligned}
H_1(x, y_H(x)) &= \frac{1}{2}J_1(x, y_H(x)) \\
&= -\frac{1}{2}(\overline{P} - p(x, y_H(x)))g(x|y_H(x))) \\
&> -\frac{1}{2}(\overline{P} - p(x, \alpha(x)))g(x|y_H(x))) \\
&= 0.
\end{aligned}$$

In order to prove that $H_2(x, y_H(x))$ is negative, let us first write

$$\begin{aligned}
H(x, y) &= \kappa\left(1 - G\left(y|y\right)\right)\Psi\left(y\right) + \left(1 - \kappa\right)G\left(y|y\right)\Phi\left(y\right) + \frac{1}{2}\left(G\left(y|y\right) - G\left(x|y\right)\right)\mathcal{J}\left(x, y\right) \\
&= \kappa\int_y^1 \left(\overline{P} - p(y, t)\right)G_1\left(t|y\right)dt + \left(1 - \kappa\right)\int_0^y \left(\overline{P} - p(y, t)\right)G_1\left(t|y\right)dt \\
&\quad + \frac{1}{2}\int_x^y \left(\overline{P} - p(y, t)\right)G_1\left(t|y\right)dt
\end{aligned}$$

Differentiating H with respect to y implies that

$$\begin{aligned}
H_2(x, y) &= -\kappa\int_y^1 p_1(y, t)G_1(t|y)dt - (1 - \kappa)\int_0^y p_1(y, t)G_1(t|y)dt - \frac{1}{2}\int_x^y p_1(y, t)G_1(t|y)dt \\
&\quad - \left(\frac{3}{2} - 2\kappa\right)\left(\overline{P} - p(y, y)\right)G_1\left(y|y\right) + \kappa\int_y^1 \left(\overline{P} - p(y, t)\right)G_{12}\left(t|y\right)dt \\
&\quad + (1 - \kappa)\int_0^y \left(\overline{P} - p(y, t)\right)G_{12}\left(t|y\right)dt + \frac{1}{2}\int_x^y \left(\overline{P} - p(y, t)\right)G_{12}\left(t|y\right)dt
\end{aligned}$$

The first four terms are negative (remember that $\kappa < \kappa^* \leq 1/2$). Let us analyze the last three. The objective is to evaluate them at $(x, y_H(x))$.

$$\begin{aligned}
& \kappa \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{12}\left(t|y_H(x)\right) dt + (1-\kappa) \int_{0}^{y_H(x)} \left(\overline{P} - p(y_H(x),t)\right) G_{12}\left(t|y_H(x)\right) dt \\
& + \frac{1}{2} \int_{x}^{y_H(x)} \left(\overline{P} - p(y_H(x),t)\right) G_{12}\left(t|y_H(x)\right) dt \\
= \; & \kappa \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x),t)\right) \mathcal{L}\left(t|y_H(x)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + (1-\kappa) \int_{0}^{\alpha(y_H(x))} \left(\overline{P} - p(y_H(x),t)\right) \mathcal{L}\left(t|y_H(x)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + (1-\kappa) \int_{\alpha(y_H(x))}^{1} \left(\overline{P} - p(y_H(x),t)\right) \mathcal{L}\left(t|y_H(x)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + \frac{1}{2} \int_{x}^{y_H(x)} \left(\overline{P} - p(y_H(x),t)\right) \mathcal{L}\left(t|y_H(x)\right) G_{1}\left(t|y_H(x)\right) dt \\
\leq \; & \kappa \mathcal{L}\left(1|y_H(x)\right) \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + (1-\kappa) \mathcal{L}\left(0|y_H(x)\right) \int_{0}^{\alpha(y_H(x))} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + (1-\kappa) \mathcal{L}\left(y|y_H(x)\right) \int_{\alpha(y_H(x))}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
& + \frac{1}{2} \mathcal{L}\left(y|y_H(x)\right) \int_{x}^{y_H(x)} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
= \; & \kappa \left(\mathcal{L}\left(1|y_H(x)\right) - \mathcal{L}\left(y_H(x)|y_H(x)\right)\right) \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
& - (1-\kappa) \left(\mathcal{L}\left(y_H(x)|y_H(x)\right) - \mathcal{L}\left(0|y_H(x)\right) - \right) \int_{0}^{\alpha(y_H(x))} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
\leq \; & 0
\end{aligned}$$

where the first and the last inequalities come from the fact that $t \mapsto \mathcal{L}(t|y)$ is an increasing function and the second equality holds because $H_2(x, y_H(x)) = 0$ so that

$$\begin{aligned}
& \frac{1}{2} \int_{x}^{y_H(x)} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt + (1-\kappa) \int_{\alpha(y_H(x))}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt \\
= \; & -(1-\kappa) \int_{0}^{\alpha(y_H(x))} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt - \kappa \int_{y_H(x)}^{1} \left(\overline{P} - p(y_H(x),t)\right) G_{1}\left(t|y_H(x)\right) dt
\end{aligned}$$

As a result, $H_2(x, y_H(x)) \leq 0$ and y_H is an increasing function.

Moreover note that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. Suppose by contradiction that $y_H(\widetilde{\sigma}) > \widehat{\sigma}^D$. In this case, $\theta(y_H(\widetilde{\sigma})) < 0$ and $J(\widetilde{\sigma}), y_H(\widetilde{\sigma}) > 0$. This implies that

$$\begin{aligned} 0 &< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P} - p\left(y_H(\widetilde{\sigma}), t\right)\right) g\left(t | y_H(\widetilde{\sigma})\right) dt \\ &< \int_{\widetilde{\sigma}}^{y_H(\widetilde{\sigma})} \left(\overline{P} - p\left(\widetilde{\sigma}, t\right)\right) g\left(t | y_H(\widetilde{\sigma})\right) dt \\ &< 0, \end{aligned}$$

leading to a contradiction. Therefore $y_H(\widetilde{\sigma}) \leq \widehat{\sigma}^D$. The same reasoning implies that $y_H(\widetilde{\sigma}) \geq \widetilde{\sigma}$.

Step 4.

We show that the solution $(\underline{\sigma}^D, \overline{\sigma}^D)$ is unique.

x_I is a decreasing function such that $x_I(\widetilde{\sigma}) = \widetilde{\sigma}$, $x_I(\widehat{\sigma}^D) > \alpha(\widehat{\sigma}^D)$ and y_H is an increasing function such that $y_H(\widetilde{\sigma}) \in [\widetilde{\sigma}, \widehat{\sigma}^D]$. As $x_I(y) > \alpha(y)$, $\forall y \in (\widetilde{\sigma}, \widehat{\sigma}^D]$, the two function cross only once on $\mathcal{D}$. This intersection point that is unique corresponds to the unique solution of the system that is $(\underline{\sigma}^D, \overline{\sigma}^D)$.

The last part of the proof consists in showing that if the semi-separating equilibrium exists, then the separating equilibrium does not exist. We are going to prove that if the separating equilibrium exists then the semi-separating equilibrium does not exist. Assume therefore that $\widehat{\sigma}^{FU} < \widetilde{\sigma}$. We have proven in Step 1 that $\overline{\sigma}^{FU} \leq \widehat{\sigma}^{FU}$, this implies that $\underline{\sigma}^D < \overline{\sigma}^D < \widetilde{\sigma}$. But in this case, $I(\underline{\sigma}^D, \overline{\sigma}^D) > 0$. Therefore, if $\widehat{\sigma}^{FU} < \widetilde{\sigma}$, there do not exist $(\underline{\sigma}^D, \overline{\sigma}^D)$ satisfying $I(\underline{\sigma}^D, \overline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \overline{\sigma}^D) = 0$.

It remains to prove that

$$\int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(s,t)\right) g\left(t|s\right) dt \leq 0 \quad \forall s \in [\underline{\sigma}^D \overline{\sigma}^D].$$

$$\begin{aligned} \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(s,t)\right) g\left(t|s\right) dt &= \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t,s)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\leq \int_{\underline{\sigma}^D}^{\overline{\sigma}^D} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\quad + \int_{\alpha(\underline{\sigma}^D)}^{\overline{\sigma}^D} \left(\overline{P} - p(t,\underline{\sigma}^D)\right) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\leq \frac{g(\alpha(\underline{\sigma}^D)|s)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \overline{\sigma}^D) \\ &= 0. \end{aligned}$$

B.8 Proof of Corollary 1

See Subsection A.2.

B.9 Proof of Lemma 4

$\theta(\widehat{\sigma}^D) = \kappa\psi(\widehat{\sigma}^D) + (1-\kappa)\,\phi(\widehat{\sigma}^D) = 0$. As we noted in the proof of Lemma 3, $\phi(\widehat{\sigma}^D) > 0$.

The implicit function theorem implies that

$$\frac{\partial \widehat{\sigma}^D}{\partial \kappa} = -\frac{\psi\left(\widehat{\sigma}^D\right) - \phi\left(\widehat{\sigma}^D\right)}{\kappa\psi'\left(\widehat{\sigma}^D\right) + (1-\kappa)\,\phi'\left(\widehat{\sigma}^D\right)} = \frac{\phi\left(\widehat{\sigma}^D\right) - \psi\left(\widehat{\sigma}^D\right)}{\theta'\left(\widehat{\sigma}^D\right)}.$$

In Lemma 3, we have proved that $\theta'(\widehat{\sigma}^D) \leq 0$ so that $\frac{\partial \widehat{\sigma}^D}{\partial \kappa} \leq 0$.

If $\widehat{\sigma}^D > \tilde{\sigma}$, $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are such that

$$\begin{cases} I(\underline{\sigma}^D, \overline{\sigma}^D) & = & 0 \\ H(\underline{\sigma}^D, \overline{\sigma}^D) & = & 0. \end{cases}$$

The implicit function theorem implies that

$$\begin{aligned} \frac{\partial \overline{\sigma}^D}{\partial \kappa} &= \frac{I(\overline{\sigma}^D, 1) - J(0, \overline{\sigma}^D)}{\frac{I_2(\underline{\sigma}^D, \overline{\sigma}^D)}{I_1(\underline{\sigma}^D, \overline{\sigma}^D)} H_1(\underline{\sigma}^D, \overline{\sigma}^D) - H_2(\underline{\sigma}^D, \overline{\sigma}^D)} \\ \frac{\partial \underline{\sigma}^D}{\partial \kappa} &= -\frac{\partial \overline{\sigma}^D}{\partial \kappa} \frac{I_2(\underline{\sigma}^D, \overline{\sigma}^D)}{I_1(\underline{\sigma}^D, \overline{\sigma}^D)}. \end{aligned}$$

We have proven in the proof of Proposition 4 that $H_2(\underline{\sigma}^D, \overline{\sigma}^D) \leq 0$ and that $H_1(\underline{\sigma}^D, \overline{\sigma}^D) \geq 0$ (in Step 3) and that $\frac{I_1(\underline{\sigma}^D, \overline{\sigma}^D)}{I_2(\underline{\sigma}^D, \overline{\sigma}^D)} \geq 0$ (Step 2). This implies that the denominator of $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ is negative and that $\frac{\partial \overline{\sigma}^D}{\partial \kappa}$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa}$ have opposite signs. Moreover, we know that $I(\overline{\sigma}^D, 1) < 0$ and $J(0, \overline{\sigma}^D) > 0$. As a consequence, $\frac{\partial \overline{\sigma}^D}{\partial \kappa} \leq 0$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0$.

B.10 Proof of Lemma 5

We proceed as in the proof of Lemma 4. Therefore, the implicit function theorem applied to the definition of $\widehat{\sigma}^D$ implies that

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} = -\frac{\kappa\left(1 - G(\widehat{\sigma}^D | \widehat{\sigma}^D)\right) + (1-\kappa) G(\widehat{\sigma}^D | \widehat{\sigma}^D)}{\theta'(\widehat{\sigma}^D)}.$$

The numerator is positive and we already proved that the denominator is negative. It follows that

$$\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} \geq 0.$$

The implicit function theorem applied to the definition of $(\underline{\sigma}^D, \overline{\sigma}^D)$ implies that

$$\begin{aligned} \frac{\partial \overline{\sigma}^D}{\partial \overline{P}} &= -\frac{-\frac{H_1(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}\left(G(\overline{\sigma}^D|\underline{\sigma}^D) - G(\underline{\sigma}^D|\underline{\sigma}^D)\right) + \kappa + \left(\frac{3}{2} - 2\kappa\right)G(\overline{\sigma}^D|\overline{\sigma}^D) - \frac{1}{2}G(\underline{\sigma}^D|\overline{\sigma}^D)}{\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}H_1(\underline{\sigma}^D,\overline{\sigma}^D) - H_2(\underline{\sigma}^D,\overline{\sigma}^D)} \\ \frac{\partial \underline{\sigma}^D}{\partial \overline{P}} &= -\frac{G(\overline{\sigma}^D|\overline{\sigma}^D) - G(\underline{\sigma}^D|\overline{\sigma}^D)}{I_1\left(\underline{\sigma}^D,\overline{\sigma}^D\right)} - \frac{\partial \overline{\sigma}^D}{\partial \overline{P}}\frac{I_2(\underline{\sigma}^D,\overline{\sigma}^D)}{I_1(\underline{\sigma}^D,\overline{\sigma}^D)}. \end{aligned}$$

The denominator of $\frac{\partial \overline{\sigma}^D}{\partial \overline{P}}$ is negative as we proved in Lemma 4. The numerator is negative since $I_1(\underline{\sigma}^D, \overline{\sigma}^D) \leq 0$, $H_1(\underline{\sigma}^D, \overline{\sigma}^D) \geq 0$ and $\kappa \leq 1/2$, implying that $(3/2) - 2\kappa \geq 1/2$. It follows that

$$\frac{\partial \overline{\sigma}^D}{\partial \overline{P}} \geq 0.$$

However, $\frac{\partial \underline{\sigma}^D}{\partial \overline{P}}$ equals the difference between two negative terms so that its sign is ambiguous. □

B.11 Proof of Lemma 6

Remember that

- $\widehat{\sigma}^{FU}$ is such that $\psi(\widehat{\sigma}^{FU}) = 0$ where ψ is defined in Equation (27),
- $\widehat{\sigma}^D$ is such that $\theta(\widehat{\sigma}^D) = \kappa\psi(\widehat{\sigma}^D) + (1-\kappa)\phi(\widehat{\sigma}^D) = 0$ where ϕ and θ are defined in Equations (26) and (28),
- $\underline{\sigma}^D$ and $\overline{\sigma}^D$ are the solution of the system $I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ where I and J are defined in Equations (32) and (33).

Assume first that $\widehat{\sigma}^D \leq \widetilde{\sigma}$.
We already noted (see the proof of Lemma 3) that $\psi(\widehat{\sigma}^D) < 0$. In addition, we also proved in Lemma 1 that $\psi(x) > 0 \Leftrightarrow x < \widehat{\sigma}^{FU}$. As $\psi(\widehat{\sigma}^{FU}) = 0$, this implies that $\widehat{\sigma}^{FU} \leq \widehat{\sigma}^D$.

Assume now that $\widehat{\sigma}^D > \widetilde{\sigma}$.
$\psi(\underline{\sigma}^D) = I(\underline{\sigma}^D, 1) < I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$. The same reasoning implies that $\underline{\sigma}^{FU} \leq \widehat{\sigma}^D$.

B.12 Proof of Proposition 7

Note first that $P^{FU}(\widehat{\sigma}^{FU}) = \overline{P} > P^D(\widehat{\sigma}^{FU})$. Second, imagine that the two functions P^{FU} and P^D cross at some point such that $P^{FU}(s) = P^D(s)$ at this point. Using the differential equations satisfied by the two premiums (Equations (**??**) and (**??**)), we have that $P^{P'}(s) > P^{D'}(s)$. Therefore, if the two functions cross, it happens only once. If $P^D(0) \leq P^{FU}(0)$, the two curves never crossed.

To prove that the boundary s^{FU} is larger than $\widehat{\sigma}^D$, let us first remark that Lemma 2 could easily apply to bidding strategy of the discriminatory auction. This implies that $P^D(s) - p(s,t) > 0$, $\forall s < t < \widehat{\sigma}^D$, so that if $\overline{s}^D$ is defined by $P^D(s) = p(s, \overline{s}^D)$, it holds that $\overline{s}^D > \widehat{\sigma}^D$. Therefore, if $P^{FU}(s) > P^D(s)$, then $\overline{s}^{FU}(s) > \overline{s}^D(s) > \widehat{\sigma}^D > \widehat{\sigma}^{FU}$. As a result, when $s \leq \widehat{\sigma}^{FU}$, the FU auction offers full coverage for a larger set of the follower's signals than the discriminatory auction. □

C Analysis of the example

As an illustration and to obtain explicit results, we consider the case in which the two insurance companies receive independent signals that are distributed according to a uniform distribution on $[0,1]$. The cost function is moreover assumed to be linear in the two signals

$$p(s_i, s_{-i}) = \frac{s_i + s_{-i}}{2}.$$

In this case, Assumption 1(ii) implies that $\overline{P} \in [1/4, 3/4]$. Note also that $\widetilde{\sigma} = \overline{P}$.

With this specification, the FU auction is characterized by a threshold $\sigma^{FU} = \frac{4\overline{P}-1}{3}$. The bidding strategy equals

$$P^{FU}(s) = \frac{\kappa + (2\kappa - 1)\, s}{3\kappa - 1} - \frac{(\kappa+1)(1-\overline{P})}{3(3\kappa-1)} \left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}} \left(\frac{1-\overline{P}}{1-s}\right)^{\frac{2\kappa-1}{\kappa}}, \forall s \in [0, \sigma^{FU}].$$

Proof. Remember that

$$\left(1 - G\left((\widehat{\sigma}^{FU} | (\widehat{\sigma}^{FU}\right)\right) \mathbb{E}\left[\overline{P} - p(\widehat{\sigma}^{FU}, S_{-i}) | S_{-i} > \widehat{\sigma}^{FU}\right] = 0.$$

With the specification, this reads

$$\int_{\widehat{\sigma}^{FU}}^{1} \left(\overline{P} - \frac{\widehat{\sigma}^{FU} + s}{2}\right) ds = 0$$

implying that

$$\widehat{\sigma}^{FU} = \frac{4\overline{P} - 1}{3}.$$

As for the equilibrium bidding strategy, Proposition 1 tells us that

$$P^{FU}(s) = \overline{P}(1 - L(\widehat{\sigma}^{FU}|s)) + \int_s^{\widehat{\sigma}^{FU}} p(x,x) dL(x|s) \ \forall \ s \leq \widehat{\sigma}^{FU}$$

with

$$L(x|s) = 1 - \exp\left(-\frac{2\kappa-1}{\kappa} \int_s^x \frac{g(\tau|\tau)}{1 - G(\tau|\tau)} d\tau\right)$$

In this example,

$$L(x|s) = 1 - \exp\left(-\frac{2\kappa-1}{\kappa} \int_s^x \frac{1}{1-\tau} d\tau\right) = 1 - \left(\frac{1-x}{1-s}\right)^{\frac{2\kappa-1}{\kappa}},$$

so that

$$\begin{aligned} P^{FU}(s) &= \overline{P}\left(\frac{1-\widehat{\sigma}^{FU}}{1-s}\right)^{\frac{2\kappa-1}{\kappa}} + \frac{\frac{2\kappa-1}{\kappa}}{(1-s)^{\frac{2\kappa-1}{\kappa}}}\int_s^{\widehat{\sigma}^{FU}} \tau(1-\tau)^{\frac{2\kappa-1}{\kappa}-1}d\tau \\ &= \frac{\kappa+(2\kappa-1)\,s}{3\kappa-1} - \frac{\kappa+1}{3(3\kappa-1)}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}\left(\frac{1}{1-s}\right)^{\frac{2\kappa-1}{\kappa}}(1-\overline{P})^{\frac{2\kappa-1}{\kappa}+1}. \end{aligned}$$

□

As for the discriminatory auction,

$$\kappa^*(\overline{P}) = \max\left(\frac{\overline{P}^2}{2\overline{P}^2-2\overline{P}+1}, \frac{1}{2}\right).$$

In the case where the equilibrium is separating ($\kappa \geq \kappa^*(\overline{P})$),

$$\widehat{\sigma}^D = \frac{\kappa+2\,(2\kappa-1)\,\overline{P} - \sqrt{\left(\kappa+2\,(2\kappa-1)\,\overline{P}\right)^2 - 3\kappa\,(2\kappa-1)\,(4\overline{P}-1)}}{3^{\frac{2\kappa-1}{\kappa}}}.$$

The bidding strategy equals

$$P^D(s) = \frac{\overline{P}(\kappa-(2\kappa-1)\,\widehat{\sigma}^D)}{\kappa-(2\kappa-1)\,s} + \frac{(2\kappa-1)\,(\widehat{\sigma}^D-s)(\widehat{\sigma}^D+s)}{2(\kappa-(2\kappa-1)\,s)}, \forall s \in [0, \sigma^D].$$

In the case where the equilibrium is semi-separating ($\kappa < \kappa^*(\overline{P})$),

$$\begin{cases} \overline{\sigma}^D &= \dfrac{9\kappa+2\,(2\kappa-1)\,\overline{P} - 3\sqrt{9\kappa^2+11\kappa\,(2\kappa-1) - 20\,(2\kappa-1)\,\overline{P}(2\kappa-(2\kappa-1)\,\overline{P})}}{11\,(2\kappa-1)} \\ \underline{\sigma}^D &= \dfrac{4\overline{P}-\overline{\sigma}^D}{3} \end{cases}$$

$$P^D\,(s) = \begin{cases} \dfrac{\overline{P}(\kappa-(2\kappa-1)\,\underline{\sigma}^D)}{\kappa-(2\kappa-1)\,s} + \dfrac{(2\kappa-1)\,(\underline{\sigma}^D-s)(\underline{\sigma}^D+s)}{2(\kappa-(2\kappa-1)\,s)} & \text{for } s \leq \underline{\sigma}^D \\ \overline{P} & \text{for } \underline{\sigma}^D \leq s \leq \overline{\sigma}^D \end{cases}$$

Proof. From the definition of $\kappa^*(P)$, we immediately get that

$$\kappa^*(P) = \max\left(\frac{\overline{P}^2}{2\overline{P}^2-2\overline{P}+1}, \frac{1}{2}\right).$$

Let us first focus on the case where $\kappa \geq \kappa^*(\overline{P})$. Observe that the constraint is always satisfied when $\overline{P} \in [1/4, 1/2]$. $\widehat{\sigma}^D$ is the solution smaller than $\overline{P}$ such that

$$(1-G\,(x|x))\,\mathbb{E}\left[\overline{P}-p(x,S_{-i})|S_{-i}>x\right] + \left(1-\frac{2\kappa-1}{\kappa}\right)G\,(x|x)\,\mathbb{E}\left[\overline{P}-p(x,S_{-i})|S_{-i}<x\right] = 0.$$

This implies that $\widehat{\sigma}^D$ is the solution smaller than $\overline{P}$ of

$$\begin{aligned}\int_0^1 \left(\overline{P}-\frac{x+s}{2}\right) ds-\frac{2\kappa-1}{\kappa}\int_0^x \left(\overline{P}-\frac{x+s}{2}\right) ds &= 0\\ \Leftrightarrow \frac{3}{4}\frac{2\kappa-1}{\kappa}x^2-\left(\frac{1}{2}+\frac{2\kappa-1}{\kappa}\overline{P}\right)x+\overline{P}-\frac{1}{4} &= 0. \qquad (38)\end{aligned}$$

This quadratic equation has two solutions, one of them larger than 1, the other belonging to $[0,\overline{P}]$.[15] It follows that

$$\widehat{\sigma}^D=\frac{\kappa+2\left(2\kappa-1\right)\overline{P}-\sqrt{(\kappa+2\left(2\kappa-1\right)\overline{P})^2-3\kappa\left(2\kappa-1\right)(4\overline{P}-1)}}{3\left(2\kappa-1\right)}.$$

In the analysis of the general case, we proved that, in the separating equilibrium,

$$P^D(s)=\overline{P}(1-K(\widehat{\sigma}^D|s))+\int_s^{\widehat{\sigma}^D} p(x,x)dK(x|s) \ \ \forall s\leq \widehat{\sigma}^D,$$

where

$$K(x|s)=1-\exp\left(-\int_s^x \frac{\left(2\kappa-1\right)g(\tau|\tau)}{\kappa-\left(2\kappa-1\right)G(\tau|\tau)}d\tau\right).$$

With our specification,

$$\begin{aligned}K(x|s) &= 1-\exp\int_s^x \frac{-\left(2\kappa-1\right)}{\kappa-\left(2\kappa-1\right)\tau}d\tau\\ &= 1-\frac{\kappa-\left(2\kappa-1\right)x}{\kappa-\left(2\kappa-1\right)s},\end{aligned}$$

so that

$$\begin{aligned}P^D(s) &= \overline{P}\left(\frac{\kappa-\left(2\kappa-1\right)\widehat{\sigma}^D}{\kappa-\left(2\kappa-1\right)s}\right)+\int_s^{\widehat{\sigma}^D}\frac{\left(2\kappa-1\right)x}{\kappa-\left(2\kappa-1\right)s}dx\\ &= \overline{P}\left(\frac{\kappa-\left(2\kappa-1\right)\widehat{\sigma}^D}{\kappa-\left(2\kappa-1\right)s}\right)+\frac{\left(2\kappa-1\right)(\widehat{\sigma}^D-s)(\widehat{\sigma}^D+s)}{2(\kappa-\left(2\kappa-1\right)s)}.\end{aligned}$$

Second, let us analyze the case of the semi-separating equilibrium. As we underlined in the beginning of the proof, the equilibrium is semi-separating iff $\overline{P}\in[1/2,3/4]$ and $\kappa\leq\kappa^*(\overline{P})$. $\underline{\sigma}^{FU}$ and $\overline{\sigma}^{FU}$ are the roots (smaller than $\overline{P}$ for $\underline{\sigma}^{FU}$ and greater than $\overline{P}$ for

---

[15] The discriminant $\Delta^x=\left(\frac{1}{2}+\frac{2\kappa-1}{\kappa}\overline{P}\right)^2-4\left(\overline{P}-\frac{1}{4}\right)\frac{3}{4}\frac{2\kappa-1}{\kappa}$ is a deceasing function of $\overline{P}$. When $\overline{P}=\frac{3}{4}$, it is positive so that two real solutions exist. As the highest root is greater than 1 (this is straightforward to prove), it follows that $\widehat{\sigma}^D$ is the smallest root.

$\overline{\sigma}^{FU}$) of the following system of equations

$$\begin{cases} \left(G\left(z|y\right)-G\left(y|y\right)\right)\mathbb{E}\left[\overline{P}-p(y,S_{-i})|y<S_{-i}<z\right]=0 \\ \left(1-G\left(z|z\right)\right)\mathbb{E}\left[\overline{P}-p(z,S_{-i})|S_{-i}>z\right]+\frac{1}{2}\left(G\left(z|z\right)-G\left(y|z\right)\right)\mathbb{E}\left[\overline{P}-p(z,S_{-i})|y<S_{-i}<z\right] \\ +\left(\frac{1-\kappa}{\kappa}\right)G\left(y|z\right)\mathbb{E}\left[\overline{P}-p(z,S_{-i})|S_{-i}<y\right]=0. \end{cases}$$

In our illustration, this reads

$$\begin{cases} \int_y^z\left(\overline{P}-\frac{y+s}{2}\right)ds=0 \\ \int_z^1\left(\overline{P}-\frac{z+s}{2}\right)ds+\left(1-\frac{\frac{2\kappa-1}{\kappa}}{2}\right)\int_y^z\left(\overline{P}-\frac{z+s}{2}\right)ds+(1-\frac{2\kappa-1}{\kappa})\int_0^y\left(\overline{P}-\frac{z+s}{2}\right)ds=0. \end{cases}$$

implying that

$$\overline{P}-\frac{y}{2}-\frac{1}{4}(z+y)=0 \tag{41a}$$

$$\overline{P}-\frac{z}{2}-\frac{1}{4}-\frac{2\kappa-1}{2\kappa}\left(\left(\overline{P}-\frac{z}{2}\right)(z-y)-\frac{1}{4}(z-y)(z+y)\right)-\frac{2\kappa-1}{\kappa}\left(\left(\overline{P}-\frac{z}{2}\right)-\frac{y^2}{4}\right) \tag{41b}$$

By eliminating y thanks to (41a), one gets that

$$\frac{11}{4}\frac{2\kappa-1}{\kappa}z^2-\left(\frac{9}{2}+\overline{P}\frac{2\kappa-1}{\kappa}\right)z+9\left(\overline{P}-\frac{1}{4}\right)-4\frac{2\kappa-1}{\kappa}\overline{P}=0.$$

This quadratic equation has two solutions, one greater than 1, the other belonging to $[\overline{P},1]$.[16] It follows that

$$\overline{\sigma}^D=\frac{9\kappa+2\overline{P}\left(2\kappa-1\right)-3\sqrt{9\kappa^2+11\kappa\left(2\kappa-1\right)-20\kappa\left(2\kappa-1\right)\overline{P}(2\kappa-\left(2\kappa-1\right)P)}}{11\left(2\kappa-1\right)}.$$

Using (41a),

$$\underline{\sigma}^D=\frac{4\overline{P}-\overline{\sigma}^D}{3}.$$

Using the computations done for the separating equilibrium, it follows that

$$P^D\left(s\right)=\begin{cases} \frac{\overline{P}(\kappa-\left(2\kappa-1\right)\underline{\sigma}^D)}{\kappa-\left(2\kappa-1\right)s}+\frac{\left(2\kappa-1\right)(\underline{\sigma}^D-s)(\underline{\sigma}^D+s)}{2(\kappa-\left(2\kappa-1\right)s)} & \text{for } s\leq\underline{\sigma}^D \\ \overline{P} & \text{for } \underline{\sigma}^D\leq s\leq\overline{\sigma}^D \end{cases}$$

□

---

[16] The discriminant equals $\Delta^z=45\frac{2\kappa-1}{\kappa}^2\overline{P}^2-90\frac{2\kappa-1}{\kappa}\overline{P}+\frac{81+99\frac{2\kappa-1}{\kappa}}{4}$. It is a decreasing function of $\overline{P}$ and is positive for $\overline{P}=\frac{3}{4}$ so that it is positive for all values of $\overline{P}$. To check that the smallest root is smaller than 1, it is necessary to study the function $\overline{P}\mapsto16\frac{2\kappa-1}{\kappa}^2\overline{P}^2-4\frac{2\kappa-1}{\kappa}(9-\frac{2\kappa-1}{\kappa})\overline{P}+\frac{2\kappa-1}{\kappa}(27-11\frac{2\kappa-1}{\kappa})$. This is a decreasing function, taking positive values for $\overline{P}=3/4$. It is straightforward to prove that the smallest root is greater than $\overline{P}$.

In order to continue the analysis of this example, let us focus on the parameters' values such that the separating equilibrium exist in the discriminatory auction. This implies that we restrict κ to be greater than $\kappa^*(\overline{P})$. Observe that when $\overline{P} \in [1/4, 1/2]$, the equilibrium is separating for any value of κ ($\kappa^*(\overline{P}) = 1/2$).

Proposition 5 *If $\kappa \geq \kappa^*(\overline{P})$, there exists a unique $\widehat{\overline{P}}$ such that*

- *if $\overline{P} < \widehat{\overline{P}}$, $P^{FU}(s) > P^{D}(s)$, $\forall s \in [0,1]$*
- *if $\overline{P} \geq \widehat{\overline{P}}$, $P^{FU}(s) < P^{D}(s)$ and then $P^{FU}(s) > P^{D}(s)$, when s increases from 0 to 1.*

Proof. We have proven in the general case that P^{FU} and P^{D} cross at most once and this happens when $P^{D}(0) > P^{FU}(0)$. The objective of this proof is therefore to determine the sign of $P^{FU}(0) - P^{D}(0)$. We decide to analyze this difference as a function of $\overline{P}$. Highlighting the dependence with respect to $\overline{P}$, we compute the following

$$
\begin{aligned}
\frac{\partial P^{FU}(0;\overline{P})}{\partial \overline{P}} &= \frac{(\kappa+1)(3\kappa-1)}{3\kappa(3\kappa-1)}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}(1-\overline{P})^{\frac{2\kappa-1}{\kappa}} > 0\\
\frac{\partial^2 P^{FU}(0;\overline{P})}{\partial \overline{P}^2} &= -\frac{(\kappa+1)\,(2\kappa-1)}{3\kappa^2}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}(1-\overline{P})^{\frac{2\kappa-1}{\kappa}-1} < 0\\
\frac{\partial^3 P^{FU}(0;\overline{P})}{\partial \overline{P}^3} &= -\frac{(\kappa+1)\,(2\kappa-1)\,(1-\kappa)}{3\kappa^3}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}(1-\overline{P})^{\frac{2\kappa-1}{\kappa}-2} < 0
\end{aligned}
$$

$\overline{P} \mapsto \frac{\partial^2 P^{FU}(0;\overline{P})}{\partial \overline{P}^2}$ is therefore a decreasing function. As

$$
\frac{\partial^2 P^{FU}(0;\frac{1}{4})}{\partial \overline{P}^2} = -\frac{(\kappa+1)\,(2\kappa-1)}{3\kappa^2}\frac{4}{3} < 0,
$$

this implies that $\frac{\partial^2 P^{FU}(0;\overline{P})}{\partial \overline{P}^2}$ is negative $\forall \overline{P}$. The same analysis is conducted for P^D

$$
\begin{aligned}
\frac{\partial P^{D}(0;\overline{P})}{\partial \overline{P}} &= 1 - \frac{2\kappa-1}{\kappa}\widehat{\sigma}^D - \frac{2\kappa-1}{\kappa}\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\left(\overline{P} - \widehat{\sigma}^D\right)\\
\frac{\partial^2 P^{D}(0;\overline{P})}{\partial \overline{P}^2} &= -2\frac{2\kappa-1}{\kappa}\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} + \frac{2\kappa-1}{\kappa}\left(\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}}\right)^2 - \frac{2\kappa-1}{\kappa}\left(\overline{P} - \widehat{\sigma}^D\right)\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\\
\frac{\partial^3 P^{D}(0;\overline{P})}{\partial \overline{P}^3} &= 3\frac{2\kappa-1}{\kappa}\frac{\partial^2 \widehat{\sigma}^D}{\partial \overline{P}^2}\left(\frac{\partial \widehat{\sigma}^D}{\partial \overline{P}} - 1\right) - \frac{2\kappa-1}{\kappa}\left(\overline{P} - \widehat{\sigma}^D\right)\frac{\partial^3 \widehat{\sigma}^D}{\partial \overline{P}^3}
\end{aligned}
$$

In order to compute the partial derivatives of $\widehat{\sigma}^D$, we apply the implicit function theorem to (38).

$$\begin{aligned}
\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}} &= \frac{2\left(\kappa-(2\kappa-1)\,\widehat{\sigma}^D\right)}{\kappa+2\,(2\kappa-1)\,\overline{P}-3\,(2\kappa-1)\,\widehat{\sigma}^D}>0\\
\frac{\partial^2\widehat{\sigma}^D}{\partial\overline{P}^2} &= \frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}\frac{2\kappa-1}{\kappa}\frac{\left(3\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}-4\right)\kappa}{\kappa+2\,(2\kappa-1)\,\overline{P}-3\,(2\kappa-1)\,\widehat{\sigma}^D}\\
&= \frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}2\,(2\kappa-1)\,\frac{\kappa+3\,(2\kappa-1)\,\widehat{\sigma}^D-4\,(2\kappa-1)\,\overline{P}}{\left(\kappa+2\,(2\kappa-1)\,\overline{P}-3\,(2\kappa-1)\,\widehat{\sigma}^D\right)^2}>0\\
\frac{\partial^3\widehat{\sigma}^D}{\partial\overline{P}^3} &= \frac{\partial^2\widehat{\sigma}^D}{\partial\overline{P}^2}\,(2\kappa-1)\,\frac{-6+9\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}}{\kappa+2\,(2\kappa-1)\,\overline{P}-3\,(2\kappa-1)\,\widehat{\sigma}^D}\\
&= \frac{\partial^2\widehat{\sigma}^D}{\partial\overline{P}^2}\frac{2\kappa-1}{\kappa}\frac{12\left(1-\frac{2\kappa-1}{\kappa}\overline{P}\right)}{\left(1+2\frac{2\kappa-1}{\kappa}\overline{P}-3\frac{2\kappa-1}{\kappa}\widehat{\sigma}^D\right)^2}>0.
\end{aligned}$$

It follows that

$$\begin{aligned}
&\frac{\partial^3 P^D(0;\overline{P})}{\partial\overline{P}^3}\\
=\ &\frac{\partial^2\widehat{\sigma}^D}{\partial\overline{P}^2}\frac{3\,(2\kappa-1)\left(\kappa^2-4\kappa\,(2\kappa-1)\,\overline{P}+4\,(2\kappa-1)^2\,\overline{P}\widehat{\sigma}^D+2\kappa\,(2\kappa-1)\,\widehat{\sigma}^D-3\,(2\kappa-1)^2\left(\widehat{\sigma}^D\right)^2\right)}{\kappa\left(\kappa+2\,(2\kappa-1)\,\overline{P}-3\,(2\kappa-1)\,\widehat{\sigma}^D\right)^2}\\
>\ &0.
\end{aligned}$$

$\overline{P}\mapsto\frac{\partial^2 P^D(0;\overline{P})}{\partial\overline{P}^2}$ is therefore an increasing function. As

$$\frac{\partial^2 P^D(0;\frac{1}{4})}{\partial\overline{P}^2}=\frac{(2\kappa-1)\left((2\,(2\kappa-1)-4\kappa)^2+8\kappa\,(2\kappa-1)\right)}{(4\kappa-1)^3}>0,$$

this implies that $\frac{\partial^2 P^D(0;\overline{P})}{\partial\overline{P}^2}$ is positive $\forall\overline{P}$.

This analysis allows us to conclude that $\overline{P}\mapsto\frac{\partial P^{FU}(0;\overline{P})}{\partial\overline{P}}-\frac{\partial P^D(0;\overline{P})}{\partial\overline{P}}$ is a decreasing function.

$$\begin{aligned}
\frac{\partial P^{FU}(0;\frac{1}{4})}{\partial\overline{P}}-\frac{\partial P^D(0;\frac{1}{4})}{\partial\overline{P}} &= \frac{(2\kappa-1)\,(1-\kappa)}{3\kappa(4\kappa-1)}\\
\frac{\partial P^{FU}(0;\frac{3}{4})}{\partial\overline{P}}-\frac{\partial P^D(0;\frac{3}{4})}{\partial\overline{P}} &= \frac{3(4\kappa-3)(\kappa+1)-2\kappa^2\times 4^{\frac{2\kappa-1}{\kappa}}}{9\kappa\times 4^{\frac{2\kappa-1}{\kappa}}(4\kappa-3)}<0.
\end{aligned}$$

This concludes the proof. □

It follows that $\overline{P}\mapsto P^{FU}(0;\overline{P})-P^D(0;\overline{P})$ is increasing and then decreasing as $\overline{P}$

increases.

$$\begin{aligned}
P^{FU}(0;\frac{1}{4}) - P^{D}(0;\frac{1}{4}) &= 0\\
P^{FU}(0;\frac{1}{4}) - P^{D}(0;\frac{3}{4}) &= \frac{3^{\frac{2\kappa-1}{\kappa}}\left(19\kappa\left(2\kappa-1\right)-9\left(2\kappa-1\right)^{2}-8\kappa^{2}-3\left(2\kappa-1\right)\left(\kappa+1\right)\right)}{(3\kappa-1)36\left(2\kappa-1\right)3^{\frac{2\kappa-1}{\kappa}}} < 0
\end{aligned}$$

Therefore there exists a unique $\widehat{\overline{P}}$ such that

- for all $\overline{P} < \widehat{\overline{P}}$, $P^{FU}(0;\overline{P}) > P^{D}(0;\overline{P})$ and

- for all $\overline{P} > \widehat{\overline{P}}$, $P^{FU}(0;\overline{P}) < P^{D}(0;\overline{P})$.

Let us now compute the expected profit in both organizations using expressions (5), (12) and (15). In the FU auction, computations imply that

$$\begin{aligned}
\Pi^{FU} &= \int_{0}^{\widehat{\sigma}^{FU}}\pi^{FU}(s)ds+\int_{\widehat{\sigma}^{FU}}^{\overline{s}^{FU}(\widehat{\sigma}^{FU})}\pi^{FU}(s)ds+\int_{\overline{s}^{FU}(\widehat{\sigma}^{FU})}^{1}\pi^{FU}(s)ds\\
&= \int_{0}^{\widehat{\sigma}^{FU}}(1-s)\kappa\left(\frac{\kappa+(2\kappa-1)s}{3\kappa-1}-\frac{\kappa(\kappa+1)(1-\overline{P})^{\frac{2\kappa-1}{\kappa}+1}}{3(3\kappa-1)(1-\kappa)}\left(\frac{4}{3(1-s)}\right)^{\frac{2\kappa-1}{\kappa}}-\frac{1+3s}{4}\right)ds\\
&+ \int_{0}^{\widehat{\sigma}^{FU}}(1-\kappa)\left(\frac{s(4\kappa-s(5\kappa-1))}{4(3\kappa-1)}-\frac{\kappa(\kappa+1)(1-\overline{P})^{\frac{2\kappa-1}{\kappa}+1}}{3(3\kappa-1)(1-\kappa)}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}\left(1-(1-s)^{1-\frac{2\kappa-1}{\kappa}}\right)\right)ds\\
&+ \int_{\widehat{\sigma}^{FU}}^{\overline{s}^{FU}(\widehat{\sigma}^{FU})}(1-\kappa)\frac{\widehat{\sigma}^{FU}\left(4\kappa-(1-\kappa)\widehat{\sigma}^{FU}-2s(3\kappa-1)\right)}{4(3\kappa-1))}ds\\
&- \int_{\widehat{\sigma}^{FU}}^{\overline{s}^{FU}(\widehat{\sigma}^{FU})}(1-\kappa)\frac{\kappa(\kappa+1)(1-\overline{P})^{\frac{2\kappa-1}{\kappa}+1}}{3(3\kappa-1)(1-\kappa)}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}\left(1-(1-\widehat{\sigma}^{FU})^{1-\frac{2\kappa-1}{\kappa}}\right)ds\\
&+ \int_{\overline{s}^{FU}(\widehat{\sigma}^{FU})}^{1}(1-\kappa)\frac{(\overline{s}^{FU})^{-1}(s)\left(4\kappa-(1-\kappa)(\overline{s}^{FU})^{-1}(s)-2s(3\kappa-1)\right)}{4(3\kappa-1)}ds\\
&- \int_{\overline{s}^{FU}(\widehat{\sigma}^{FU})}^{1}(1-\kappa)\frac{\kappa(\kappa+1)(1-\overline{P})^{\frac{2\kappa-1}{\kappa}+1}}{3(3\kappa-1)(1-\kappa)}\left(\frac{4}{3}\right)^{\frac{2\kappa-1}{\kappa}}\left(1-(1-(\overline{s}^{FU})^{-1}(s))^{1-\frac{2\kappa-1}{\kappa}}\right)ds.
\end{aligned}$$

As for the discriminatory auction, we have to distinguish the case where $\kappa \geq \kappa^{*}(\overline{P})$ or $\kappa < \kappa^{*}(\overline{P})$. In case the separating equilibrium exists ($\kappa \geq \kappa^{*}(\overline{P})$),

$$\Pi^{D} = \int_{0}^{\widehat{\sigma}^{D}}\left(\frac{2\overline{P}(\kappa-(2\kappa-1)\widehat{\sigma}^{D})+(2\kappa-1)(\widehat{\sigma}^{D}-s)(\widehat{\sigma}^{D}+s)-s(\kappa-(2\kappa-1)s)}{2}-\frac{\kappa-(2\kappa-1)s^{2}}{4}\right)ds.$$

When $\kappa < \kappa^{*}(\overline{P})$,

$$\begin{aligned}
\Pi^{D} &= \int_{0}^{\underline{\sigma}^{D}}\left(\frac{2\overline{P}(\kappa-(2\kappa-1)\underline{\sigma}^{D})+(2\kappa-1)(\widehat{\sigma}^{D}-s)(\underline{\sigma}^{D}+s)-s(\kappa-(2\kappa-1)s)}{2}-\frac{\kappa-(2\kappa-1)s^{2}}{4}\right)ds\\
&\quad+\int_{\underline{\sigma}^{D}}^{\overline{\sigma}^{D}}\left(\frac{2\overline{P}-s}{4}\left(2\kappa-(2\kappa-1)\overline{\sigma}^{D}-(2\kappa-1)\underline{\sigma}^{D}\right)-\frac{1}{8}\left(2\kappa-(2\kappa-1)(\overline{\sigma}^{D})^{2}-(2\kappa-1)(\underline{\sigma}^{D})^{2}\right)\right)ds.
\end{aligned}$$

As for the expected coverage denoted $\mathcal{C}$, it holds that

$$\mathcal{C}^{FU} = 2\kappa\widehat{\sigma}^{FU} + (1-\kappa)\int_0^{\widehat{\sigma}^{FU}} \overline{s}^{FU}(x)dx - (\widehat{\sigma}^{FU})^2.$$

As for the discriminatory auction, if $\kappa \geq \kappa^*(\overline{P})$, then

$$\mathcal{C}^{D} = (\widehat{\sigma}^{D})^2 - 2\kappa(1-\widehat{\sigma}^{D})\widehat{\sigma}^{D}.$$

When $\kappa < \kappa^*(\overline{P})$, then

$$\mathcal{C}^{D} = (\overline{\sigma}^{D})^2 - 2\kappa(1-\overline{\sigma}^{D})\overline{\sigma}^{D}.$$

To finish the analysis this example, let us analyze how $\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^{FU}$ evolves with respect to $\overline{P}$ and κ. Let us first focus on the comparative statics with respect to κ. As $\widehat{\sigma}^{FU}$ is independent of κ and as we proved in Lemma 4 that both $\widehat{\sigma}^D$ and $\overline{\sigma}^D$ are decreasing with κ, this implies that $\min(\widehat{\sigma}^D, \overline{\sigma}^D) - \widehat{\sigma}^{FU}$ decreases when κ increases.

To determine the comparative statics with respect to $\overline{P}$, observe that $\widehat{\sigma}^{FU}$ is linear with respect to $\overline{P}$ and that we proved that $\frac{\partial^2\widehat{\sigma}^D}{\partial\overline{P}^2} > 0$. Therefore, $\overline{P} \mapsto \frac{\partial\widehat{\sigma}^D}{\partial\overline{P}} - \frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}}$ is increasing. Moreover,

$$\begin{aligned} \left.\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}\right|_{\overline{P}=\frac{1}{4}} &= 4\kappa \\ \left.\frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}}\right|_{\overline{P}=\frac{1}{4}} &= \frac{3}{4}. \end{aligned}$$

It follows that $\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}} - \frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}}$ is positive for all values of $\overline{P}$ and that $\overline{P} \mapsto \widehat{\sigma}^D - \widehat{\sigma}^{FU}$ is an increasing function. Let us now focus on $\overline{\sigma}^D - \widehat{\sigma}^{FU}$.

$$\begin{aligned} \frac{\partial\overline{\sigma}^D}{\partial\overline{P}} &= \frac{2}{11} + \frac{60}{11}\frac{\kappa - (2\kappa-1)\overline{P}}{\sqrt{9\kappa^2 + 11\kappa(2\kappa-1) - 20(2\kappa-1)\overline{P}(2\kappa - (2\kappa-1)\overline{P})}} \\ \frac{\partial^2\overline{\sigma}^D}{\partial\overline{P}^2} &= \frac{11(2\kappa-1)(1-\kappa)\kappa^2}{(\sqrt{31\kappa^2 - 11\kappa + -20(2\kappa-1)\overline{P}(2\kappa - (2\kappa-1)\overline{P})})^3} > 0. \end{aligned}$$

Therefore, $\overline{P} \mapsto \frac{\partial\overline{\sigma}^D}{\partial\overline{P}} - \frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}}$ is increasing. Moreover,

$$\left.\frac{\partial\widehat{\sigma}^D}{\partial\overline{P}}\right|_{\overline{P}=\frac{1}{2}} = 2.$$

As $\frac{\partial\widehat{\sigma}^{FU}}{\partial\overline{P}} = \frac{3}{4}$, it follows that $\frac{\partial\overline{\sigma}^D}{\partial\overline{P}} - \frac{\partial\overline{\sigma}^{FU}}{\partial\overline{P}}$ is positive for all values of $\overline{P}$ and that $\overline{P} \mapsto \overline{\sigma}^D - \widehat{\sigma}^{FU}$ is an increasing function.

References

[1] Ausubel L.M. and P. Cramton (2004), “Auctioning many divisible goods”, *Journal of the European Economic Association*, 2(2–3) pp 480-493.

[2] Back, K. and J. Zender (1993), “Auction of divisible goods : on the rationale for the treasury experiment”, *Review of Financial Studies*, 6 pp 733-764.

[3] Berger, L.A., Cummins, J.D. and S. Tennyson (1992), “Reinsurance and the liability insurance crisis”, *Journal of Risk and Uncertainty,* 5 pp 253-272.

[4] Caillaud, B.and C. Mezzetti, (2004), “Equilibrium reserve prices in sequential ascending auctions”, *Journal of Economic Theory*, 117(1), 78-95.

[5] Charpentier, A. and B. Le Maux, (2014), “Natural catastrophe insurance: How should the government intervene?”, *Journal of Public Economics*, 115, 1-17.

[6] Doherty, N., A. and G. Dionne (1993), “Insurance with undiversifiable risk: Contract structure and organizational form of insurance insurer”, *Journal of Risk and Rncertainty*, 6 pp 187-203.

[7] European Commission, (2014), “Study on co(re)insurance pools and on ad-hoc co(re)insurance agreements on the subscription market”.

[8] Gollier, C (2004), The Economics of Risk and Time, *MIT Press.*

[9] Haile, P. A. (2000), “Partial pooling at the reserve price in auctions with resale opportunities”, *Games and Economic Behavior*, 33(2), 231-248.

[10] Inderst, R. (2016), “Efficiencies of coinsurance pools”, Association of the German Insurance Industry.

[11] Jehiel, P. and B. Moldovanu, B(2000), “Auctions with downstream interaction among buyers”, *Rand Journal of Economics*, 768-791.

[12] Klemperer, P. (1999), “Auction theory: A guide to the literature”, *Journal of economic surveys*, 13(3), 227-286.

[13] Klemperer P. (2002), “What Really Matters in Auction Design”, *Journal of Economic Perspectives*, 16(1), 169-189.

[14] Kousky, C. and R. Cooke, (2012), “Explaining the failure to insure catastrophic risks”, *Geneva Papers on Risk and Insurance: Issues and Practice*, 37(2), 206–227.

[15] Kunreuther, H. (1996), “Mitigating disaster losses through insurance”, *Journal of risk and Uncertainty*, 12(2-3), 171-187.

[16] Lizzeri, A. and N. Persico, (2000), “Uniqueness and existence of equilibrium in auctions with a reserve price”, *Games and Economic Behavior*, 30(1), 83-114.

[17] Louaas, A. and P. Picard, (2020), “Optimal insurance coverage of low-probability catastrophic risks”, *The Geneva Risk and Insurance Review*, 1-28.

[18] Mahul, O. and B. Wright (2004), “Efficient risk sharing within a catastrophe insurance pool”, mimeo.

[19] Milgrom, P. R. and R. J. Weber, (1982), “A theory of auctions and competitive bidding”, *Econometrica*; 50(5), 1089-1122.

[20] Wilson, R. (1979), “Auctions of shares”, *Quarterly Journal of Economics*, 93 pp 675-689.