DISCUSSION PAPER SERIES

DP15136

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PUBLIC ECONOMICS

ISSN 0265-8003

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Discussion Paper DP15136
Published 06 August 2020
Submitted 05 August 2020

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Decision makers called to evaluate and approve a reform, proposed by an interest group, a politician, or a bureaucracy, suffer from a double asymmetric information problem: about the competence of the proposer and the consequences of the proposal. Moreover, the ability of decision makers to evaluate proposals depends on the complexity of the legislative environment, itself a product of past reforms. We model the strategic interaction between reformers and decision makers as a function of legislative complexity, and study the dynamics of endogenous complexity and stability of reforms. Complexi cation-simpli cation cycles can occur on the equilibrium path, and expected long-run complexity may be higher when competence of reform proposers is lower. The results apply to regulatory reforms, legislative politics, and institutional design.

JEL Classification: D73, G28, H83, L51

Keywords: Information, Regulatory Complexity, competence, interest groups, Politicians, bureaucracy, Checks and balances, Incremental Reforms

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Acknowledgements
We wish to thank Avidit Acharya, David Baron, Pierpaolo Battigalli, Jon Bendor, Steve Callander, Gabriele Gratton, Niall Hughes, Matthew O. Jackson, Julien Manili, Federico Masera, Marco Ottaviani, Vincent Pons, Ken Shotts, Fernando Vega-Redondo, Richard vanWeelden, and (virtual) seminar participants at Bocconi, Stanford GSB, UNSW, and the Virtual Formal Theory Seminar for useful suggestions. We acknowledge support from the European Research Council grant 694583. The usual disclaimer applies.

Equilibrium Reforms and Endogenous Complexity*

Dana Foarta†and Massimo Morelli‡

August 5, 2020

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1 Introduction

The increasing production of laws, regulations and rules over the past decades has been extensively documented in both the United States and Europe.[1] More often than not, new laws or regulations have built upon and gradually modified existing statutes. Provisions were added or subtracted in order to respond to technological, economic, or ideological changes.[2] In some domains, for instance in tax policy, this process has led over time to high regulatory complexity.[3] In other domains, for instance the regulation of autonomous vehicles, more regulatory detail is necessary in order to keep up with the changing technologies.[4] Yet in other domains, like financial regulation, successive regulatory changes have historically resulted in cycles of regulation and deregulation (Dagher, 2018). The policy making process has led to different reform paths and resulting levels of legislative and regulatory complexity across policy domains.

In this paper, we model the reform process as a function of the complexity of the environment, and we trace the endogenous evolution of complexity as a function of equilibrium reform incentives. Most legislative and regulatory reform processes involve an offer by a *proposer* (an interest group, a specialized agency, or a politician, depending on the context) and approval by a *decision maker* (politician, regulator, or institution, depending on the context). Two aspects fundamental to most reform processes will be the main tenets of our model: the first tenet is that there is typically asymmetric information between the decision maker and the proposer. Second, there is history dependence, as reforms are evaluated within the regulatory environment created by past legislative activity. We show how the resulting reform process given these central features may lead to different patterns in the

[1] For instance, in the U.S., the total pages published in the code of Federal Regulations increased from 120.000 in 2000 to over 180.000 in 2019 ("Pages in the Code of Federal Regulations," Regulatory Studies Center, George Washington University); similarly, Gratton et al. (2020) document a significant increase in legislative production in Italy since the early 90s.

[2] See for instance the discussions in Teles (2013); Kawai et al. (2018), or the process of policy making described by Lindblom (1959).

[3] For the U.S. case, see Joint Committee on Taxation (JCX-49-15), "Complexity in the Federal Tax System," March 2015.

[4] Congressional Research Service (R45985), "Issues in Autonomous Vehicle Testing and Deployment," February 2020.

adoption of reforms and in the complexity of the legislative environment.

The asymmetric information between the decision maker and the proposer may manifest along two main dimensions. It may concern the proposer's ability in drafting or implementing a reform (private type asymmetric information) or the consequences of the reform itself, as a reform's benefit depends on the realization of some state of the world (common state asymmetric information). Thus, in general, a decision maker's relevant beliefs concern both the proposer's ability and the state of the world.

As an example of the simultaneous relevance of these two types of asymmetric information, consider two possible policy reforms that could be adopted by a government in the wake of an economic crisis: one is a *simple* blanket stimulus policy, for instance a universal tax cut or universal government guarantees for struggling firms; the other is a set of *complex* targeted tax credits or guarantees for specific sectors. Typically a government agency or dedicated task force proposes one of the two, and the executive (or the parliamentary majority) faces the double information problem we postulate: first, the executive is uncertain about the agency's ability to distinguish which firms benefit the most from the stimulus policy, in order to implement targeted policies (private type);[5] second, the executive faces uncertainty about the nature of the crisis, e.g., whether it is concentrated primarily in a few sectors or generalized (state of the world). A complex reform is effective if the crisis is concentrated and the implementing agency is capable of carrying it out. If these conditions are likely to be met, the executive is expected to adopt a complex proposal. If the crisis is likely to be generalized, a blanket policy is likely to be effective, and hence the executive is expected to approve such a proposal. The worst situation for the executive is when the crisis is likely confined to specific sectors, but the agency is believed to lack the ability to implement targeted transfers. In this situation, inaction by the executive could be expected.

The second tenet of our proposed framework is that the decision maker's understanding of a reform's desirability depends on the complexity of the policy domain. The decision

[5] The decision maker's belief about the agency's ability should matter more when the reform is a complex one, like going for selective tax credits. When instead a blanket policy is proposed, there is less room for discretion, and the agency's drafting or implementation ability matters much less for the final outcome.

maker can extract less information about the proposal's effects when the domain is more complex, e.g., a reform would interact with many more existing provisions. This complexity is the consequence of past policy choices, which have resulted in the current legislative environment. Thus, past reforms affect the evaluation of new reform proposals. To follow on the above example, implementing selective tax credits means that future responses to crises have to take into account their differential effects on sectors with and without tax credits. This complicates the evaluation problem for future executives responding to new crises.

In the model, an informed proposer, with information advantage both about his ability and about the state of the world, makes a reform proposal to an uninformed decision maker. Both agents live only one period and are followed in the next period by a new set of agents. The decision maker would like to adopt a reform that matches the state of the world and that is implemented by a high ability proposer. The proposer's objective is to convince the decision maker to adopt his reform proposal. For example, the proposer can be an interest group who drafts a reform, and the decision maker is a politician who may adopt or reject that reform to the legislation. Alternatively, the proposer may be a politician who makes a reform bill proposal, and the decision maker is the legislature's majority leader, who can support or reject the bill. The proposer can be thought of as offering one of two possible reforms, one that is simpler or one that is more complex: a blanket policy versus a tailored policy, repealing an existing regulatory provision or writing in a new one, etc. The outcome of a simple reform depends on the state of the world, while the outcome of a complex reform depends both on the state of the world and on the ability of the proposer. The decision maker in turn is endowed with the decision to adopt or reject the reform after evaluating it. The evaluation produces a noisy public signal about the state of the world. The decision maker's approval or rejection has dynamic consequences: if a simpler reform is adopted, then the legislation also becomes simpler. This means that next period's policy problem becomes easier for a future decision maker: evaluating a new reform will deliver a more precise signal. Similarly, if a more complex reform is adopted, then the legislation becomes more complex. This makes next period's evaluation problem more difficult: evaluating a new reform will deliver a noisier signal to a future decision maker. If the decision maker rejects the reform, then the status quo is maintained, and the next period's evaluation problem will be identical.

We obtain a full characterization of the best perfect Bayesian equilibrium for the decision maker of each period for every pair of parameters defining the asymmetric information and for any given level of complexity of the environment. When the proposer is more likely to be high-ability and the state of the world is more likely to require simplifications, complex reforms are proposed only by the high-ability proposer and only when the state requires them. The decision maker therefore receives complex reforms only when they are beneficial. When the proposer is less likely to be high-ability and the state of the world is expected to require complexity, a complex reform may be proposed even when the realized state of the world calls for simplification. This happens because the proposer strategically chooses the reform that is more likely to be adopted. A decision-maker who expects the state of the world to require complexity is more likely to adopt a reform in line with her prior. This leads to increasing legislative complexity even when this is not desirable. This strategy is contingent, however, on some expected competence on the part of the proposer. If the likelihood of competence is low, then the decision maker rejects reforms, halting reformism.

Our characterization allows us to compare the equilibrium proposals and adoption decisions under different levels of complexity of the environment. The probability of simple proposals being offered increases as the environment becomes more complex. With more complexity, the decision maker's evaluation of the reform is less precise. The decision maker therefore relies more on her prior about the desirability of a reform and may favor a simple reform, the outcome of which does not depend on the proposer's type. In turn, the proposer might prefer to offer a simple reform over a complex reform, to increase his chance of approval. While proposals become simpler, the probability that a reform is adopted over the status quo may increase or decrease as the environment becomes more complex: More complex reforms generate more(less) reformism when the prior of the decision maker on the importance of adopting a complex reform is low(high).

Dynamically, there is the possibility of complexity cycles: faced with high complexity, the decision maker receives simpler reform proposals that she adopts, and the environment becomes less complex. But once the decision maker has better information about the state of the world, more complex reform proposals materialize on the equilibrium path, as the proposer offers such policies in the state in which they are beneficial. This makes the system fluctuate around an intermediate level of complexity. The endogenous cycling is driven by the change in the proposer's equilibrium strategy as the environment's complexity changes. We show that this dynamic has some important features that are unique to the "checks and balances" nature of our model: in our setting the level of complexity around which cycling happens *decreases* in the probability that the proposer is high ability, while in a situation where a decision maker can freely pick reforms it would *increase* in the probability of high proposer ability. In the former case, the proposer responds to the decision maker's expectation of higher ability by strategically offering a simpler reform. In the latter case, the decision maker simply responds to less uncertainty about the state of the world.

Finally, we examine how the two dimensions of asymmetric information affect the expected long run endogenous complexity of the environment. Expected long run complexity is path dependent when there is a higher probability that the state of the world favors a complex reform and that the proposer is low-ability. Otherwise, it is path independent, and we show when it fluctuates around an intermediate level versus reaching extreme values.

The predictions of the model help to reconcile many different findings in the recent literature on the relationship between legislative complexity and efficiency: the negative effects of excessive legislative activism by politicians described in Gratton et al. (2020) are rationalized in our model by the combination of intermediate valence (mapped to ability in the model) of politicians in Italy and a large demand for reforms (mapped to high uncertainty about the state of the world) at the beginning of the Second Republic, leading to unwarranted complexification; on the other hand, when the state of the world is more likely to require complex reforms, for instance due to technological changes, or when the environment's complexity decreases exogenously due to information about similar reforms already adopted elsewhere, complexification improves efficiency, as shown in Ash et al. (2019).

The model also adds to the literature of regulatory cycles, where the role of interest groups as proposers has been widely documented empirically. In this application, we show how proposer ability can be mapped to alignment of an interest group with the public interest. Our mechanism points to a novel interpretation for endogenous cycles of regulation and deregulation, emerging due to the strategic incentives of interest groups that propose regulatory changes. We show how these endogenous cycles may amplify cycles driven by exogenous changes in the economic environment.

Finally, the model also has implications for understanding the role played by checks and balances in democracies: in the absence of vetoing by a decision maker separate from the proposer, there could be excessive reforms. Moreover, complexity may endogenously evolve to one extreme or to the other. Separation between decision maker and proposer limits reformism and can also maintain an intermediate level of complexity.

The paper is organized as follows. Section 2 discusses the related literature, Section 3 presents the model, Section 4 characterizes the equilibrium for any given complexity of the environment, and Section 5 derives the dynamics of complexity. Section 6 analyzes a number of applications and Section 7 concludes. All proofs are in the Appendix.

2 Related Literature

Our paper contributes to the literature on reform processes and legislative and regulatory complexity. The reforms in our model have the general feature of being incremental (Dewatripont and Roland, 1992, 1995; Callander, 2011), in that policy change happens gradually – a proposer cannot propose something that massively increases or decreases complexity in one step. Incrementalism emerges endogenously in Kawai et al. (2018), in an evolutionary model where entanglements and interdependencies among policies make it very difficult to make grand reforms. Beside justifying incrementalism, entanglements and interdependencies also create a bias in favor of policy complexity, and in their framework policies that start complex tend to become ever more complex, whereas simple policies stay simple forever.

In contrast, our framework does not consider entanglements and each policy domain could cycle endogenously between simplification and complexification equilibria. Taking reforms as incremental, we show how the complexity of the status quo, the need for reform, and the ability of policymakers affect reform adoption and we trace the resulting dynamics of complexity.

Central to our model is the view that complexity of the environment refers to the difficulty for the decision maker to discern the consequences of a proposed reform. This notion of complexity is introduced and analyzed in a general model in Asriyan et al. (2020). Here, the policymaking environment requires adjustment along two dimensions: First, we allow the consequences of a policy to depend on the state of the world, so that a more complex reform is not always costlier than a simpler reform; second, the complexity of the environment is history dependent, determined by the reforms adopted up to the current date. Implicitly, a more complex reform affects the future complexity of the legislative environment. The evolution of complexity is not an explicit choice of the proposer, and by not making the players long-lived, we abstract away from learning motives.[6] By assuming short-lived agents we also ignore the additional causes and consequences of complexity due to electoral concerns.[7]

We focus on the feedback between reform proposals and the complexity of the policymaking environment for future regulators or legislators. This is in line with the type of questions studied in Gratton et al. (2020), who study the consequences of greater uncertainty (in the form of political instability) on quantity and quality of legislation and rules. The evolution of the quantity and quality of rules in turn affects the functioning of the bureaucracy and politicians' incentives. Like in their model, we study the dynamic consequences of regulation decisions, but without focusing on politicians' signaling incentives vis--vis voters: in our paper the dynamics of complexity depends on the noise in understanding the consequences of policies for a decision-maker, without signaling distortions.

Another literature to which our paper contributes is the one on regulatory complexity and

[6]See Callander (2011) as part of the literature that has focused on learning.

[7]See Levy et al. (2019) and Morelli et al. (2020) for some insights on the connection between the demand of populism and the strategic supply of simplistic policy platforms even when perhaps the state of the world would require a complex strategy.

lobbying. Our framework maps to a model of informational lobbying, where the proposer is an industry-level interest group. This approach complements the literature focused on self-regulation (McCarty, 2017) or on quid-pro-quo lobbying conducted by individual companies.

Finally, the paper relates to the formal literature on checks and balances. Beside the classic Barro (1973), the closest related articles are Rogers (2003), Tsebelis (1999), and Gratton and Morelli (2018), who study the optimality of checks and balances under policy uncertainty when the veto player has the same signaling incentives as the proposer. To the best of our knowledge our model is the first to focus on the endogenous complexity consequences for the comparison between systems with and without checks and balances.

3 Model

Consider an environment in which a proposer can propose a reform to a status quo regulation. The proposal is adopted or rejected by a decision-maker (DM). A reform may either remove or add a contingency to the regulation. We denote a (simplifying) reform that removes a contingency by y^S, and a (complexifying) reform that adds a contingency by y^C.

The benefit to the DM from a reform depends on the state of the world (θ) and on the type of proposer who drafts it. There are two possible states of the world, θ^S and θ^C, the latter occurring with known probability κ. There are also two proposer types, $P \in \{A, B\}$, where type A is high-ability and type B is low-ability. The probability of a proposer being of type A is π. The realized state of the world and proposer type are known to the proposer, but they are not observable to the DM.

The status quo regulation delivers a benefit normalized to $\omega = 0$ to the DM. Adopting reform y^S adds a positive net benefit $v > 0$ if the state is θ^S and a net loss $-l < 0$ if the state is θ^C. The benefit from a proposal y^C depends on both the state of the world and on the type of proposer. If the proposer is of type A, then the DM receives the net benefit v in state θ^C, and $v - a \geq 0$ in state θ^S. If the proposal comes from proposer B, the DM receives the net loss $-l$ in state θ^C, and $-l - a$ in state θ^S. Below, we summarize the net gain to the

DM from each reform proposal, if it is adopted:

$\mathbf{y^S}$:

	A	**B**
$\theta^{\mathbf{S}}$	v	v
$\theta^{\mathbf{C}}$	$-l$	$-l$

$\mathbf{y^C}$:

	A	**B**
$\theta^{\mathbf{S}}$	$v-a$	$-l-a$
$\theta^{\mathbf{C}}$	v	$-l$

Intuitively, a simplifying proposal is good if the state of the world is one where simplification is needed, and the proposer's ability shouldn't matter much for simple blanket policies; on the other hand, the welfare consequences of more complex policies depend greatly on the proposer's ability. Moreover, adopting a complex reform when it is not needed adds unnecessary compliance costs (a).

The proposer derives a benefit normalized to 1 if his proposal is approved, and 0 otherwise.

Reform Process. We consider the following reform process.

1. Nature chooses a state of the world, $\theta \in \left\{\theta^S, \theta^C\right\}$.

2. After observing θ and his type P, the proposer offers a reform proposal, either y^S (simplify) or y^C (complexify).

3. The DM receives a private signal $\rho \in \{s, c\}$ about θ, with $\Pr(\rho = \theta) = 1 - z$, where $z \in \left[z^{\min}, \frac{1}{2}\right]$, and $z^{\min} > 0$.[8] The value z is public information.

4. The DM makes decision d to approve ($d = 1$) or deny ($d = 0$) the reform.

We assume that both the DM and the proposer live for only one period.[9] In the next period, nature independently draws another DM and another proposer from a pool in which type A exists with probability π, and the above reform process is repeated each period $t \in \{1, 2, ...\}$.

[8] We assume strict inequality because at $z = 0$ there is no imperfect information about the state of the world.

[9] Allowing the DM to live for longer than one period does not fundamentally change the analysis or the insights we will present below. If the DM takes into account the expected consequences of changing complexity, we can show that this leads to less complex proposals and more rejections. Details available upon request.

Decision-Maker's Information. In period t, the DM's signal $\rho_t \in \{s, c\}$ comes with noise z_t. A proposal that is approved in period t changes the status quo, and the noise in the following period is given by

$$z_{t+1} = \begin{cases} \min\{z_t + \Delta, \frac{1}{2}\} & \text{after } y^C \text{ is approved,} \\ \max\{z_t - \Delta, z^{\min}\} & \text{after } y^S \text{ is approved,} \end{cases} \tag{1}$$

where $\Delta > 0$. The noise therefore increases after y^C is approved, until the upper bound of $\frac{1}{2}$ is reached, at which point the noise is maximal, rendering the signal fully uninformative. Similarly, the noise decreases after y^S is approved, until the lower bound of $z^{\min}$, at which the noise is minimal.

Complexity. Complexity in our model has two aspects. First, reform y^C is more complex in that its payoff depends both sources of asymmetric information, the state of the world and the proposer's ability, whereas y^S only depends on the state of the world. Second, adopting y^C increases noise z next period. Intuitively, a higher z means higher complexity of the environment, in that understanding the effects of a reform requires more expertise or a costlier analysis. The proposer can observe θ directly, the underlying assumption being that he has more expertise on the exact effects of the proposed reform.

Equilibrium Concept

Fixing any initial condition with a given triplet (κ, π, z_0), we select the *best* Pure Strategy Perfect Bayesian Equilibrium for the DM.[10]

Definition 1 *A pure strategy Perfect Bayesian Equilibrium of the game is defined as a profile of strategies* $d : \{s, c\} \times \{y^S, y^C\} \times [z^{\min}, \frac{1}{2}] \to \{0, 1\}$ *for the DM and* $m_P : \{\theta^S, \theta^C\} \times [z^{\min}, \frac{1}{2}] \to \{0, 1\}$ *for the proposer, and a system of beliefs* $\mu : \{y^S, y^C\} \times [z^{\min}, \frac{1}{2}] \to [0, 1]$

[10] As typical with Perfect Bayesian Equilibria, we obtain multiplicity due to the freedom to set off-path beliefs. As such, there exist equilibria in which the DM does not approve any proposal, and the status quo is unchanged. The selection of best PBE for any set of initial conditions is standard.

for the DM such that (i) the proposer's strategy $m_P(\theta, z) \equiv \Pr(y^C|\theta, z)$ is optimal given DM's strategy, (ii) the DM's approval strategy $d(\rho, y, z)$ is optimal given her belief, and (iii) the DM's belief μ must be consistent with Bayes' Rule whenever possible.

Call our selected equilibrium $BPBE(\kappa, \pi, z)$. Given our payoff assumptions, it is possible to obtain multiple equilibria that deliver the same maximal expected welfare for the DM. In that case, we select among those equilibria the one that minimizes the future noise z, i.e., between two payoff equivalent actions, one where y^S is proposed and one where y^C is proposed, we select the equilibrium with y^S, stacking the deck against complexification.

Stable equilibrium Starting from noise z_0 in period $t = 0$, an initial $BPBE(\kappa, \pi, z_0)$ is *stable* if the endogenous evolution of z_t for $t = \{1, 2, ..., \infty\}$ never challenges the existence conditions for that initial $BPBE$. If $BPBE(\kappa, \pi, z_0)$ is not stable it means that for some z_t potentially reached at some time t on the equilibrium path dictated by the initial equilibrium strategies, $BPBE(\kappa, \pi, z_t)$ does not call for the same best equilibrium strategy profile as $BPBE(\kappa, \pi, z_0)$.

4 Equilibrium

To solve for the equilibrium, we first examine the DM's approval decision given a proposer's strategy. Afterwards, we derive the proposer choice given the DM's beliefs. Finally, we impose consistency between the proposer's strategies and the DM's beliefs.

Benchmark with Observable Types

An essential part of the model is that a reform may be proposed strategically even when the state of the world does not call for it, leading to changes in complexity driven by strategic reasons. Hence, the most relevant benchmark is the one where such strategic incentives are removed, that is, the one where the proposer's type is observable. In other words, this benchmark can be thought of as the baseline checks and balances model without asymmetric information on ability.

Proposition 1 *When the proposer's type P is observable, in the BPBE:*

1. *If $P = A$, the proposer offers y^S after θ^S, and y^C after θ^C; the DM approves with probability one.*

2. *If $P = B$, the proposer offers y^S in both states; the DM's approval strategy is*

$$d = \begin{cases} 1 & \text{if } \kappa \leq v/(v+l) \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

The DM derives a net benefit from any proposal made by proposer A. Therefore, any proposal from A is approved. Proposer B only offers a net benefit when he proposes y^S and the state is θ^S. Thus, the DM approves only if the state is sufficiently likely to be θ^S.

Proposal y^C is adopted and noise increases when $P = A$ and $\theta = \theta^C$. In the long-run, the expected noise increases to the maximum if this case is more likely than the others, i.e., $\pi \cdot \kappa \geq 1/2$. Otherwise, in the long-run, the expected noise decreases to the minimum, or, if $\pi = 0$ and $\kappa > v/(v+l)$ the status quo does not change. Thus, the dynamics are trivial, and fluctuations in noise are driven only by occasional realizations of $P = A$ and $\theta = \theta^C$.

Equilibrium Characterization

Let us now consider the general case in which both information asymmetries exist. We consider the $BPBE$ given any initial conditions (π, κ, z_0). We know that the DM obtains the highest payoff when she adopts policy y^S in state θ^S and policy y^C proposed by A in state θ^C. The proposer's goal is to have his proposal approved, regardless of the state of the world. Therefore, proposer B has an incentive to choose the reform that will induce the DM to believe that she is facing proposer A or state θ^S. Thus, the best outcome for the DM can be sustained in equilibrium only if the $DM's$ belief is that she is likely to face proposer

A and state θ^S. Otherwise, the proposer's equilibrium strategy will differ. The following Proposition describes the $BPBE$ in each region of the (π, κ) space, for a given z.[11]

Proposition 2 *Given any $z \in \left[z^{\min}, 1/2\right]$, there exist thresholds $\pi_1(\kappa, z), \pi_2(\kappa, z), \pi_3(\kappa, z)$ and $\bar{\kappa}$ such that the pure strategy $BPBE(\kappa, \pi, z)$ has the following form:*

1. *(**Simplification**) If $\pi \geq \pi_3$, proposer A offers y^S after θ^S, and y^C after θ^C; proposer B offers y^S in both states, and the DM approves the proposal:*

$$m_A(\theta, z) = \mathbb{1}_{\theta=\theta^C}; m_B(\theta, z) = 0; d(\rho, y, z) = 1. \tag{3}$$

2. *(**Matching**) If $\pi \in [\pi_2, \pi_3)$, both proposer types offer y^S after θ^S, and y^C after θ^C, and the DM approves the proposal:*

$$m_A(\theta, z) = m_B(\theta, z) = \mathbb{1}_{\theta=\theta^C}; d(\rho, y, z) = 1. \tag{4}$$

3. *(**Complexification**) If $\pi < \pi_3$ and $\pi \in [\pi_1, \pi_2)$, proposer A offers y^C in both states, proposer B offers y^S after θ^S and y^C after θ^C, and the DM approves the proposal:*

$$m_A(\theta, z) = 1; m_B(\theta, z) = \mathbb{1}_{\theta=\theta^C}; d(\rho, y, z) = 1. \tag{5}$$

4. *(**Pooling**) If $\pi < \min\{\pi_1, \pi_3\}$, both proposer types offer y^S in both states, and the DM approves conditional on $\rho = s$ and $\kappa \leq \bar{\kappa}$:*

$$m_A(\theta, z) = m_B(\theta, z) = 0; d(\rho, y, z) = \mathbb{1}_{\{\rho=s \text{ and } \kappa\leq\bar{\kappa}\}}. \tag{6}$$

Proposition 2 shows that for any noise z, we can map four distinct regions (with two sub-regions for Pooling) in the (κ, π) space. Figure 1 illustrates these equilibria in the (κ, π) space for low z (Panel a), medium z (Panel b) and high z (Panel c).

[11] In a very small space of parameters the BPBE is in mixed strategies, but we ignore them for simplicity. See appendix if interested also in the mixed strategy equilibria, which do not add anything substantive.

In the Simplification equilibrium, both proposer types use the same strategies as in the benchmark, and the DM approves all proposals. Thus, the DM suffers a loss only if the proposer is B and the state is θ^C. The threshold π_3 reflects the maximum loss the DM can tolerate: the value at which the DM is indifferent between approving and rejecting the proposal when her evaluation indicates state θ^C is more likely, i.e., her signal is $\rho = c$:

$$\pi_3(\kappa, z) = \max\{0, 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}\}. \tag{7}$$

The threshold π_3 increases in κ, because in that case the DM has a higher belief that the state is θ^C, in which she may suffer a loss. The threshold also decreases in z. Higher z gives a signal $\rho = c$ less weight in the DM's posterior belief about θ. Thus, the DM places lower weight on being in the state y^C and is willing to tolerate a higher probability that $P = B$.

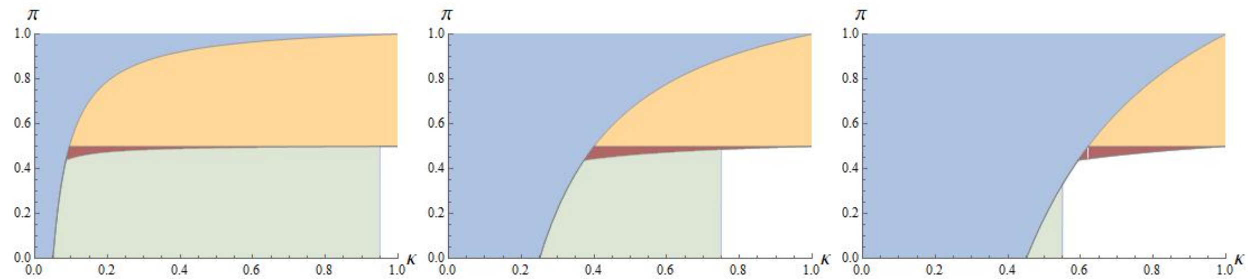
The Matching equilibrium is the one where proposer B offers y^C after θ^C. In this equilibrium, y^S is guaranteed to produce a gain over the status quo. After y^C, however, the DM knows that the state must be θ^C, and that she registers a loss only if the proposer is a B type. Thus, π_2 captures the minimum probability that $P = A$ needed to sustain this equilibrium:

$$\pi_2 = \frac{l}{v+l}. \tag{8}$$

This threshold does not depend on κ or z, as the equilibrium play reveals the state θ to the DM.

In the Complexification equilibrium, proposer A offers y^C in all states. The event in which the state is θ^C and the proposer is B, in which case the DM makes a loss, is less likely after observing y^C than in the previous equilibria, since now y^C is offered by A in all states. Given these proposer strategies, threshold π_1 is the value at which the DM is indifferent between approving and rejecting the proposal after a signal $\rho = c$:

$$\pi_1(\kappa, z) = \frac{l \cdot (1-z) \cdot \kappa}{(v+l) \cdot (1-z) \cdot \kappa + (v-a) \cdot z \cdot (1-\kappa)}. \tag{9}$$



(a) Low noise: $z = 0.1$ (b) Medium noise: $z = 0.25$ (c) High noise: $z = 0.45$

Figure 1: Illustrates the $BPBE$ in the parameter space (κ, π): in the blue area it is the Simplification equilibrium, in the yellow area it is the Matching equilibrium, in the red area it is the Complexification equilibrium, in the green area it is the Pooling equilibrium with $\kappa \leq \bar{\kappa}$, and in the white area it is the Pooling equilibrium with $\bar{\kappa} > \bar{\kappa}$. This and all subsequent figures take $l = v = 1$ and $a = 0.5$.

As with π_3, the threshold π_1 is increasing in κ and decreasing in z. The drivers are the same: the DM's probability of suffering a loss is higher when θ^C is more likely and when the signal $\rho = c$ is more precise.

In the Pooling Equilibrium, y^S is offered in both states by both proposer types. Thus, the proposer's type is irrelevant for the DM's payoff. What matters is whether y^S is adopted in the state of the world in which it brings a benefit. The DM approves the proposal as long as state θ^S is sufficiently likely, i.e., as long as κ is sufficiently low and the signal is $\rho = s$:

$$\kappa \leq \bar{\kappa}(z) = \frac{(1-z) \cdot v}{(1-z) \cdot v + z \cdot l}. \tag{10}$$

If $\kappa > \bar{\kappa}$, the DM expects a loss compared to the status quo and thus rejects the proposal.

Given the above description, we note that each boundary between regions changes monotonically with noise z.

Corollary 1 *Bounds $\pi_1(\kappa, z)$, $\pi_3(\kappa, z)$, and $\bar{\kappa}(z)$ decrease as z increases. The bound π_2 is independent of z.*

The three panels of Figure 1 indicate how the $BPBE$ changes with z. The Simplification and the Complexification equilibria exist for more parameter values (κ, π) at higher values of z. At a low z, a signal $\rho = c$ would deter the DM from approving, due to the expectation of a loss in state θ^C. As the noise increases, the DM receives less precise information and thus she places less weight on the signal.

The Matching and Pooling equilibria are the $BPBE$ for a smaller range of parameter values as z increases. For the Matching equilibrium, this is due to the Simplification equilibrium being sustainable where it was not before. For the Pooling equilibrium, this happens because the DM responds to less precise information from the signal by demanding a higher probability of θ^S, i.e., lower κ, in order to approve. These observations lead to the following insight.

Corollary 2 *Reformism (the probability that a reform proposal is adopted) increases in the complexity of the environment z when $\kappa \leq \frac{v}{l+v}$. It decreases in the complexity of the environment when $\kappa > \frac{v}{l+v}$.*

A more complex environment makes it more difficult for the DM to gather information about the implications of a reform proposal. With less precise information, the DM essentially has a more porous sifter through which to filter reform proposals. This results in more proposals getting approved when the DM attaches a prior probability that the state is θ^S greater than a threshold. With a high prior that the state is θ^S, an increase in z can make the DM switch from a signal-dependent approval strategy to accepting all proposals. Conversely, when the DM attaches a higher probability to state θ^C, an increase in z increases the likelihood of rejection.

5 The Dynamics of Complexity

In this Section, we consider the dynamics in the complexity of the environment, induced by reform decisions taken each period. As described in the model setup, adopting a reform in period t impacts the environment's complexity in period $t+1$, z_{t+1}. Adopting policy y^S reduces z_{t+1}, while adopting policy y^C increases z_{t+1}. We examine what this implies for the evolution and stability of the $BPBE$ given κ and π.

Proposition 3 *Let $z^{\min} \to 0$. Then, the parameter space (κ, π) can be divided in the following regions, which characterize the evolution of the initial BPBE given thresholds $\pi_1(\kappa, \frac{1}{2})$, π_2, $\pi_3(\kappa, \frac{1}{2})$, and $\pi_4(\kappa, \frac{1}{2}) \in [0, \pi_2)$:*

1. *A **Stable region** $\mathcal{S}$, where $\pi < \pi_3$ and $\pi \notin [\pi_1, \pi_2]$. In this region, the $BPBE(\kappa, \pi, z_0)$ is stable $\forall z_0 \in \left[z^{\min}, \frac{1}{2}\right]$.*
2. *An **Unstable region** $\mathcal{U}$, where $\pi \geq \max\{\pi_4, \pi_3\}$. In this region, there the $BPBE(\kappa, \pi, z_0)$ is not stable $\forall z_0 \in \left[z^{\min}, \frac{1}{2}\right]$.*
3. *A **Complexity Dependent region** $\mathcal{D}$, where $(\kappa, \pi) \notin \{\mathcal{S}, \mathcal{U}\}$. In this region, there exists $z^D(\kappa, \pi) \in (z^{\min}, \frac{1}{2})$ such that the $BPBE(\kappa, \pi, z_0)$ is stable(unstable) if $z_0 \leq (>)z^D$.*

The regions described in Proposition 3 are illustrated in Figure 2. Region $\mathcal{S}$ contains the locations (κ, π) where the $BPBE(\kappa, \pi, z_0)$ is stable. To arrive at this region, we use the monotonicity of boundary π_3, shown in Corollary 1. Then, the stable region is obtained by examining the case $z = \frac{1}{2}$. At that extreme, the threshold $\pi_3(\kappa, \frac{1}{2})$ gives the minimum $\pi(\kappa)$ above which the Simplification equilibrium may exist. Thus, any point $\pi(\kappa) \in [\pi_2, \pi_3]$ is in the Matching equilibrium for all z. Any point $\pi(\kappa) < \min\{\pi_1, \pi_3\}$ is in the Pooling equilibrium for all z. In that equilibrium, if z_0 is high, the signal is not sufficiently informative, and the DM rejects any proposal going forward; if z_0 is low, the signal is sufficiently informative, and the DM approves each proposal conditional on $\rho = s$.

Region $\mathcal{D}$ contains the locations (κ, π) where the $BPBE$ is stable if z_0 is sufficiently small, so that the location starts in the Pooling Equilibrium. From Pooling, there cannot be transitions into a different $BPBE$, as z weakly decreases in all subsequent periods. Thus, we have a stable $BPBE$ moving forward. If z_0 is sufficiently large, then we are in the Simplification or in the Complexification equilibrium. From either of these equilibria, if y^S is adopted repeatedly such that z decreases sufficiently, the $BPBE$ switches to Pooling. Thus, the $BPBE$ is not stable.

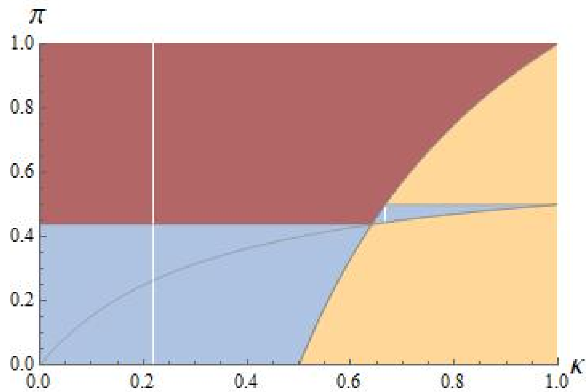


Figure 2: Illustrates the regions described in Proposition 3. Region $\mathcal{S}$ is in yellow, region $\mathcal{U}$ is in red, region $\mathcal{D}$ is in blue. The boundaries are given by curves $\pi_1 - \pi_4$.

In region $\mathcal{U}$, the equilibrium can change along the endogenous path of z, starting from any z_0. Consider, for instance, a path of repeated realizations of θ^C and $P = B$. Then, starting in the Simplification equilibrium, y^S is proposed, and z endogenously decreases each period. As z reaches a sufficiently low value, the $BPBE$ changes to Matching or Complexification. In this case, y^C is proposed and z endogenously increases. This raises z until the $BPBE$ switches to Simplification. As this example suggests, we are in region $\mathcal{U}$ when, for different values of z, either the Simplification equilibrium or the Matching (Complexification) equilibrium can be the $BPBE$ at a given (κ, π). The boundary between the Simplification and the Matching regions is given by π_3. The boundary between the Simplification to Complexification regions is given by the value π_4 at which both these equilibria may exist for a given κ and z, i.e. $\pi_3(\kappa, z) = \pi_1(\kappa, z) = \pi_4$.

Throughout region $\mathcal{U}$, switches between equilibria are possible. Next, we show when this is expected to lead to cycling between equilibria and fluctuations between higher and lower complexity:

Proposition 4 *In a subset of region $\mathcal{U}$, cycles between the Simplification equilibrium and the Matching or Complexification equilibrium occur in expectation in equilibrium. As z evolves endogenously,*

1. *if $\frac{1}{2} < \kappa < \frac{1}{2\pi}$ and $\max\{\pi_2, \pi_3(\kappa, \frac{1}{2})\} < \pi$, then the BPBE cycles between the Simpli-*

fication and the Matching equilibria. The expected frequency of fluctuations increases in κ*.*

2. *if* $\max\{\frac{1-2\kappa}{2(1-\kappa)}, \pi_4, \pi_3(\kappa, \frac{1}{2})\} < \pi < \pi_2$*, then the BPBE cycles between the Simplification and the Complexification equilibria. The expected frequency of fluctuations increases (decreases) in* κ *if* $\pi < (>)\frac{1}{2}$*.*

Cycling is expected in equilibrium in a non-trivial subset of region $\mathcal{U}$, illustrated in Figure 3. The expectation of cycling between equilibria is a stronger result than the condition that characterizes the broader region $\mathcal{U}$, which is that there exists at least one possible sequence $\{(\theta_t, P_t)\}_{t=0}^{\infty}$ such that the $BPBE$ switches between two equilibria.

At a location (κ^*, π^*), cycling emerges whenever the equilibrium play in one region leads, on average, to the region boundary changing such that we cross into a new region. In the new region, the equilibrium play leads, on average, to the boundary changing such that we cross back into the original region. For instance, if our location is in the Simplification $BPBE$, and y^S is proposed and adopted, the noise z decreases. This increases the boundary $\pi_3(\kappa, z)$. After sufficiently many proposals y^S, the boundary crosses location (κ^*, π^*), such that our location switches to a Matching $BPBE$. In this region, each proposer offers y^S after θ^S and y^C after θ^C. After proposal y^C, the noise z increases. If state θ^C is sufficiently frequent, then z increases, lowering the boundary π_3. Our location crosses back into the Simplification $BPBE$, and the cycle continues. This dynamic hinges on y^S being adopted sufficiently often in the Simplification $BPBE$, and y^C being adopted sufficiently often in the Matching $BPBE$, given the equilibrium strategies. The former requirement places an upper bound on the probability κ, while the latter places a lower bound on κ. The same dynamic and intuition applies to cycling between the Simplification and the Complexification equilibria. We illustrate this result in Figure 4.

The frequency of these cycles is higher when the region boundaries move on average faster in the direction that generates cycles. In our case, this means that y^S is proposed on average more often in the Simplification region and y^C is proposed on average more often

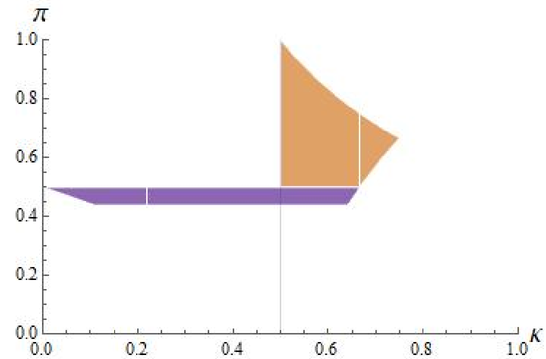


Figure 3: Illustrates the regions where cycling occurs (in expectation) between the Simplification and the Matching equilibria (orange) and between the Simplification and Complexification equilibria (violet).

in the Matching / Complexification region. The frequency of y^S in the Simplification region decreases in $\kappa \cdot \pi$; the frequency of y^C in the Matching and the Complexification regions increases in κ and $\kappa \cdot (1-\pi)$, respectively. Thus, there is a trade-off between increasing the average transition duration from the Simplification equilibrium to the Matching (or Complexification) equilibrium and increasing the reverse average transition duration.

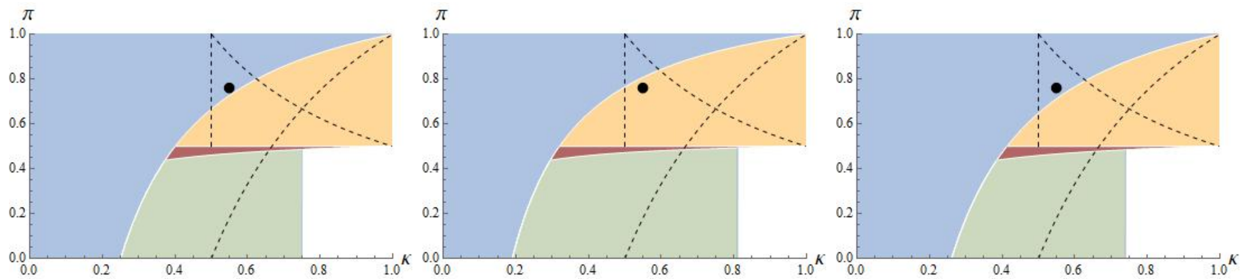
The cycling described in Proposition 4 happens around the boundary $\pi_3(\kappa, z)$, which separates the Simplification $BPBE$ from the Matching (or Complexification) $BPBE$. Thus, for any location (κ, π), the cycling happens where $\pi_3(\kappa, z^*) = \pi$, which implies that the environment's expected complexity must be

$$z^*(\kappa, \pi) = \frac{l \cdot \kappa \cdot (1-\pi)}{l \cdot \kappa \cdot (1-\pi) + v \cdot (1-\kappa)}. \tag{11}$$

We summarize this insight below and describe how z^* changes with κ and π.

Proposition 5 *Cycling between a Simplification equilibrium and a Matching or Complexification equilibrium happens around complexity $z^*(\kappa, \pi)$. The value z^* decreases in π and increases in κ.*

When the proposer is more likely to be low-ability, or the state is more likely to be θ^C, the cycling happens around a higher complexity level, that is, when the DM has less precise information. This comes in contrast with the standard intuition that a DM would be more



(a) Cycling, $z = 0.25$ (b) Cycling, $z = 0.19$ (c) Cycling, $z = 0.26$

Figure 4: Panels (a)-(c) illustrate the cycling between the Simplification and the Matching equilibria for the location $\kappa = 0.55$ and $\pi = 0.76$. The dashed lines represent the lower and upper limits on κ from Proposition 4 and the boundary of the Matching region, $\pi_3(\kappa, \frac{1}{2})$.

likely to adopt a potentially costlier policy y^C when she has more precise information. The result emerges because cycling here happens due to the change in the proposer's strategies, while the DM approves the proposal non-contingently on her signal. Thus, cycling happens because the proposer adapts his strategy to ensure approval. For cycling to exist, the DM must choose to not make use of her signal. If the DM is more likely to suffer a loss, either due to lower π or higher κ, the information provided by the signal is more valuable. For her to not use this information, the signal must be less precise. This insight highlights that the cycling here is driven by the strategic interaction between the DM and the proposer. As we show in Section 6.3, any cycling that could emerge with a singular uninformed proposer-decider relies on the signal being used. Here, cycling relies on the signal *not* being used, and this leads to distinct comparative statics. Specifically, the complexity level around which there is cycling (z^*) increases as the proposer is more likely to be low-ability (lower π). In a model with a single proposer-decider, complexity would decrease, as shown in Section 6.3.

We next consider the long-run implications of the endogenous evolution of z.

Proposition 6 *In the long-run (as $t \to \infty$), the environment's complexity z_t is expected to:*

1. *converge to $\frac{1}{2}$ in the Stable region $\mathcal{S}$ if $\pi > \pi_2$, and in the Unstable region $\mathcal{U}$ if $\kappa \cdot \pi > \frac{1}{2}$.*

2. *oscillate around $z^*(\kappa, \pi)$ in the cycling regions described in Proposition 4.*

3. *converge to* $z^{\min}$ *in the Complexity Dependent region* $\mathcal{D}$ *if* $\pi < \pi_4$*, and in region* $\mathcal{U}$ *if* $\kappa < \frac{1}{2}$ *and* $\pi > \pi_2$ *or* $\pi < \frac{1-2\kappa}{2(1-\kappa)}$.

4. *reach an expected long-run value that depends on the starting* z_0 *in region* $\mathcal{S}$ *if* $\pi < \pi_1(\kappa, \frac{1}{2})$*, and in region* $\mathcal{D}$ *if* $\pi > \pi_1(\kappa, \frac{1}{2})$*. This value is* $z^{\min}$ *if* $BPBE(\kappa, \pi, z_0)$ *is the Pooling equilibrium and* $\kappa \leq \bar{\kappa}$*,* z_0 *if the* $BPBE(\kappa, \pi, z_0)$ *is the Pooling equilibrium and* $\kappa > \bar{\kappa}$*, and* $\frac{1}{2}$ *if the* $BPBE(\kappa, \pi, z_0)$ *is the Complexification equilibrium.*

We illustrate these regions in Figure 5 in the (κ, π) space, and in Figure 6 in a simulation of the model which shows the endogenous evolution of z given different starting locations (κ, π). In each region, we compute which policy is expected to be adopted more often, given the equilibrium strategies. These depend on the proposer's identity and the state of the world. If the proposer is more likely to be high-ability (high π) or the state of the world is more likely to be θ^C (high κ), then proposal y^C is more likely to be offered and adopted.

In a subset of region $\mathcal{U}$, we have cycling, as described in Proposition 5. In the cycling regions, an intermediate average complexity is maintained in the long-run, whereas in our benchmark, intermediate levels of complexity cannot be expected in the long-run. Another substantive difference from the benchmark is that in a non-trivial region, the expected evolution of complexity depends on the initial complexity. The initial complexity determines the path along which z evolves, whether in the direction of simplification (if in the Pooling Equilibrium with low z), complexification (if in the Complexification Equilibrium) or maintaining the status quo (if in the Pooling equilibrium with high z). In this region, there is a high probability of a loss from the reform: a high likelihood of state θ^C and proposer B. Thus, starting in an environment with low complexity, the DM can rely on the signal and accept proposals y^S when $\rho = s$. This further simplifies the environment. Starting with high complexity, the DM does not follow the noisy signal. If π is sufficiently high, she approves any proposal, including a complex one, which then creates even more complexity. If π is sufficiently low, she rejects any proposal, and the status quo remains in place.

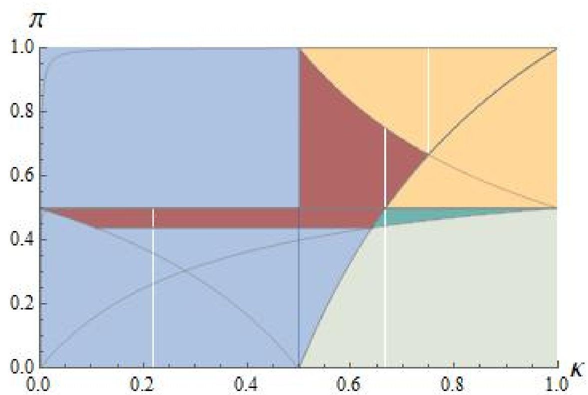
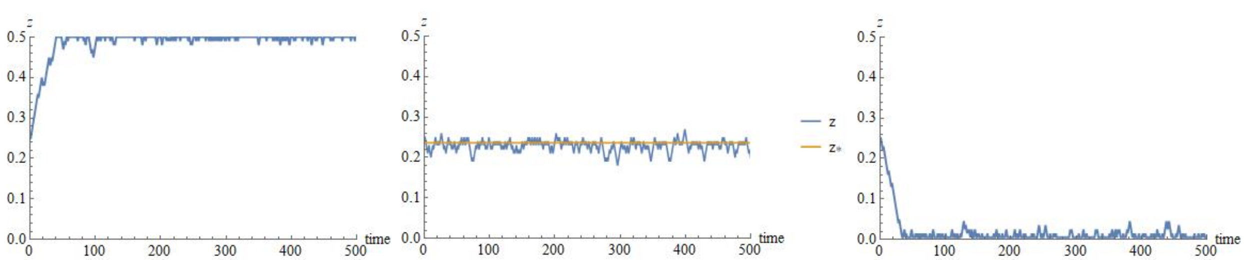


Figure 5: Illustrates the long-run expected convergence regions for z. In the blue region, z is expected to converge to $z^{\min}$. In the yellow region, z is expected to converge to $\frac{1}{2}$. In the red region, long-run z oscillates around z^*. In the dark green and the light green regions, expected convergence depends on the initial starting z_0: if at z_0 the location (κ, π) is in the Complexification equilibrium (the dark green region), then z is expected to converge to $\frac{1}{2}$, if the location is in the Pooling equilibrium and $z \leq z^D$, then z converges to $z^{\min}$, and if $z > z^D$, then z stays at z_0.



(a) Stable Region at location $(\kappa, \pi) = (0.8, 0.6)$

(b) Cycling Region at location $(\kappa, \pi) = (0.38, 0.495)$

(c) Unstable Region at location $(\kappa, \pi) = (0.25, 0.8)$

Figure 6: Illustrates the endogenous z resulting from a simulation of the model over $T = 500$ periods, with $l = v = 1$ and $a = 0.5$, and (κ, π) indicated for each panel, starting from $z_0 = 0.25$.

We have shown that each type of policy may be proposed in equilibrium by either type of proposer, and that the complexity of the environment, as captured by the noise z, may endogenously evolve to maximally informative signals, to fully uninformative signals, or it may cycle around intermediate values. Next, we move to discussing these results in light of several applications, and we show how this framework relates to empirical studies.

6 Applications

In this section, we use our model to shed light on several empirical puzzles regarding the evolution of complexity and its relationship to regulatory and bureaucratic outcomes.

6.1 Legislative Complexity, Bureaucratic Efficiency and Growth

We start by mapping our model to the production of legislation. It is common in the production of legislation for a better-informed agent to propose reforms, which must be approved by a less-informed decision maker.

In the U.S. legislative context, both at the state and federal levels, the proposer is oftentimes a bureaucrat, who has expertise on the topic (Bendor and Meirowitz, 2004), i.e, better information on the relevant state of the world θ.[12] Moreover, the bureaucrat is tasked with the implementation of any adopted reforms, and therefore his ability P is consequential for the reform's outcome. The decision-maker is a politician, who can vote to approve or reject the reform. The politician is electorally accountable for the reform's effects, while the bureaucrat is not, which maps into different objectives. The politician's electoral benefit depends on the reform's outcome, so both on the economic value of the reform and the private value of the bureaucrat's competence. A bureaucrat who is motivated by career concerns (Alesina and Tabellini, 2007) may find implementing the reform valuable, and he may not

[12] For instance, Cates (1983), as quoted in Ting (2009), provides the following anecdote: faced with a proposal to reform Social Security in 1950, Senator Eugene Millikin (R,CO) complained that [t]he cold fact of the matter is that the basic information is alone in possession of the Social Security Agency. There is no private actuary...that can give you the complete picture ...I know what I am talking about because I tried to get that.

be directly impacted by the outcome of that reform.

In the context of parliamentary systems, like in many European countries, the proposer is usually a politician, in the legislature or in the executive (Laver et al., 1996). The decision-maker is the relevant majority leader in the parliament, who controls the vote over its approval. The proposer politician may have the sole interest of getting a bill passed if he is strongly office-motivated, and showing legislative activity signals competence to voters or furthers his career prospects (Canes-Wrone et al., 2001; Gratton and Morelli, 2018; Gratton et al., 2020). The majority party leadership instead may be evaluated by voters based on the reform's outcome. The outcome depends both on the economic state (θ) and the competence of the proposing politician (P).

Finally, a reform may reduce legislative contingencies, i.e., it may cut a provision, or it may add contingencies. As in our model, a reform that cuts a provision has the same effect regardless of who proposes it, because it does not allow the proposer the freedom to add text to the law. A reform that adds a provision depends on the competence of the politician, as new text is added to the legislation.

A growing number of empirical studies have examined the effect of increasing legislative complexity on the quality of regulatory outcomes, and by extension, on growth. Studies from different institutional contexts and time periods show potentially opposing effects of increasing complexity. On the one hand, higher legislative complexity has been shown to accompany lower quality legislation, worse bureaucratic efficiency and growth (Giommoni et al., 2020; Gratton et al., 2020). Gratton et al. (2020) examine the production of legislation in Italy during the First Republic (1948-1992) and the Second Republic (1992-2017). They show that higher political instability in the Second Republic is associated with lower quality and more complex legislation compared to the First Republic. They rationalize these findings by noting that higher political instability shortens the expected political horizon of legislators. This means that voters are called to evaluate the performance of legislators before their legislative proposals are fully implemented. This in turn incentivizes incompetent politicians to propose bad quality legislation, in order to appear hard-working and competent to voters. The increase in the production of low quality laws is then shown to have increased the complexity of the legislation and decreased bureaucratic efficiency.[13].

On the other hand, higher legislative complexity has been shown to accompany higher efficiency and economic growth in different contexts. Ash et al. (2019) examine legislative complexity in U.S. states over the period 1965-1998. They find that more legislative complexity, in the form of more contingencies and legislative detail, leads to higher economic growth. The effect is larger when there is higher economic uncertainty, i.e., the state of the world in which adding more provisions or contingencies is socially beneficial (higher κ).

At first glance, the above results present a puzzle as to when reforms that increase legislative complexity are desirable. Our model sheds light on this puzzle. Consider starting from values π and κ at which we are in the Simplification region described in Proposition 2. We examine the effect of decreasing the expected quality of politicians (lower π) and of a technological shock that increases the benefit of more legislative detail (higher κ).

Remark 1 *There exists $\bar{\pi}, \bar{\kappa} \in (0,1)$ such that starting from any (κ, π) with $\pi \geq \bar{\pi}, \kappa \geq \bar{\kappa}$, a one-time shock that decreases π or a one-time shock that increases κ may lead to an increase in the frequency of proposals y^C, and higher average complexity z over time. However, the decrease in π lowers expected payoffs more than the increase in κ, for any given local increase in complexity caused by such shocks.*

A large political instability shock can trigger a shift from the Simplification equilibrium to the Complexification equilibrium, where all politicians are more likely to propose reforms that complexify. The increase in ill-fitting reforms reduces expected benefit for voters, and it leads to increasing complexity of the legislative environment, z (as per Proposition 6).[14]

On the other hand, considering the effect of a technological shock that makes it more likely for new legislative provisions to improve private contracting, as in Ash et al. (2019), the

[13] This Kafkaesque loop also determines endogenously a reduction of the expected quality of politicians through selection (may endogenously lower π, an element outside our model)

[14] The Complexification equilibrium in a legislative setting can also be related to kludged politics equilibrium of Kawai et al. (2018). In their setting, complexity begets complexity through the cost of disentangling new provisions from old ones, whereas for us complexity increases due to the strategic choices made by proposers when the decision maker has high information costs.

model predicts a shift from the Simplification equilibrium to the Matching equilibrium (or the Simplification equilibrium if π is sufficiently high). In this case, the frequency of reform proposals that add provisions (y^C) increases, because κ increases, as does the complexity of the legislative environment, z (as per Proposition 6). An alternative mechanism in the Ash et al. (2019) study is that the results are driven by competition between states. Under this mechanism, the complexity z suffers an exogenous downward shock, due to information arriving from other states that adopted similar reforms. This exogenous shock leads to a shift from the Simplification equilibrium to the Matching equilibrium.

Intuitively, the instability shock documented in Gratton et al. (2020) lands the system in the dark green (Kafkaesque) zone of Figure 5; the shock to κ that maps to the likely motivations for increases in complexification reforms in Ash et al. (2019) lands the system in the yellow region of Figure 5, which does not entail a significant reduction in payoff for the DM. In the first case, more proposals y^C are made when they are not beneficial for the decision-maker: the expected need for reforms that add provisions does not change (κ stays the same); however, in the Complexification region, proposers switch to offering these reforms more often, both when they are not needed (in the case of competent proposers) and when they cannot be aptly drafted (in the case of incompetent proposers). In the second case, more proposals y^C are made because the need for such reforms increases (κ increases). Translating the change in the decision-maker's expected payoff to growth / efficiency, we obtain the prediction that growth / efficiency would be higher in the second case compared to the first case.

6.2 Regulation and Deregulation Cycles

Another natural application of our model is to lobbying and regulation. The central role played by special interest and lobby groups in regulatory policy making is well documented in the literature (Grossman and Helpman, 2001). These groups explicitly write bills to be introduced in legislatures (Levy and Razin, 2013) or rules to added during the rulemaking process (Bertrand et al., 2018). In terms of our model, the proposer maps to an interest group. This group's goal is to have its proposal adopted by the decision maker, who is either the relevant legislator or regulator. Legislators possess less information about the effects of the proposal compared to the interest groups, who are experts on their industry. Similarly, in many industries, the high degree of specialization and higher complexity of the environment (higher z) makes industry insiders also better informed than regulators on the effects of their proposal (McCarty, 2017). Moreover, an interest group's current stance on an issue may be aligned or misaligned with the public interest, and this positioning is many times not transparent. For instance, Bertrand et al. (2018) show that non-profit groups are tied to firms through difficult to trace links (donations by charitable arms of U.S. corporations), and these non-profits submit proposals for rules that favor their donors. Then, a misaligned interest group maps in our model to proposer B, whose drafting of the reform is detrimental to welfare. An aligned interest group maps to proposer A, whose drafting of the reform is beneficial to welfare.

A simplifying reform (y^S) can be mapped to removing a regulatory contingency (deregulation), while a complexifying reform (y^C) can be mapped to adding a regulatory contingency (more regulation). Our model delivers several insights regarding the evolution of regulation and the complexity of the regulatory environment. Interestingly, the model shows that cycles of regulation and deregulation may be obtained both through exogenous changes in economic conditions and endogenously due to the incentives of interest groups.

Remark 2 *Cycles between increasing and decreasing regulatory complexity may emerge:*

1. *exogenously, due to shocks to κ, the likelihood of being in the state of the world in which additional contingencies are beneficial.*

2. *endogenously, due to the evolution in the complexity of the regulatory environment, z, that changes the incentives of interest groups to propose or remove contingencies.*

First, our model allows us to consider shocks to economic conditions that change the likelihood that more regulatory contingencies are beneficial (changes in κ). Such shocks may shift the equilibrium play from one in which reform proposals more often reduce regulatory contingencies (the Simplification region) to one in which reform proposals increase regulatory contingencies (the Complexification region). Such variation is consistent with the empirical evidence that periods of financial innovation are accompanied by simplification of regulation, while periods of crisis are accompanied by increases in regulatory contingencies. This pattern has been documented empirically in both the United States and in Europe, since the South Sea Bubble and up to the 2008 financial crisis (Almasi et al., 2018; Barth et al., 2012; Dagher, 2018). It is worth mentioning that the shift described above cannot happen in the model if the proposer is highly likely to be aligned with the regulator (high π). Empirically, Dagher (2018) finds that the aforementioned pattern is not encountered around the Swedish banking crisis of the 1990s. Consistent with our model, he points to political institutions which grant less access to private interests (which corresponds to high π in our model).

Second, cycles of regulation and deregulation are not necessarily driven by shocks to economic conditions or technology. In fact, they can emerge endogenously due to the incentives of interest groups and regulators, as derived in Proposition 4. The complexity of the regulatory environment, z, evolves endogenously, and it changes the interest groups' incentives. When z is small, both aligned and misaligned interest groups are more likely to propose adding contingencies. As these contingencies are adopted, the regulatory environment becomes more complex, and interest groups switch to proposing simpler rules. These proposals are adopted, they simplify the regulatory environment, thus generating a cycle of regulation and deregulation. This dynamic complements the result in Asriyan et al. (2020) that aligned proposers may complexify regulation, while misaligned proposers may simplify regulation.

Exogenous changes in κ or π do, however, affect the endogenous regulatory cycles. Proposition 6 implies that the long-run complexity around which regulatory cycles occur, z^*, increases in κ and decreases in π. Thus, technological shocks that increase the likelihood that more regulation is beneficial also increase the long-run complexity around which we may observe cycling. Similarly, if proposers are less aligned, long-run complexity may actually increase, even if there are episodes of deregulation (simplification) as part of the cycle.

Finally, Proposition 4 shows that cycling happens for moderate values of both π and κ.

These are the situations in which there is higher uncertainty about the alignment of interest groups and about the appropriateness of making regulatory changes. In industries in which these conditions are met, the lobbying process and the incentives of interest groups to have their proposals approved lead to a cycle of complexification and simplification. This cycle emerges due to the rule-making process in regulatory agencies, complementing or amplifying similar dynamics that emerge due to the electoral motivations (McCarty et al., 2013) or due to exogenous shocks (Fernández-Villaverde et al., 2013).

6.3 Checks and Balances

Our model can contribute to the discussion on the desirability of checks and balances. The proposer P, whether a politician or an interest group, cannot have his policy implemented unless it is approved by a gatekeeper, DM. This gatekeeper represents a system of checks and balances. We assume that the DM in this case seeks to maximize the expected social welfare. We can compare this to having a singular proposer-decider.

The case in which there is a single decision-maker who is uncertain about her competence at implementing a reform, $\pi \in (0, 1)$, is equivalent to assuming that the decision whether to undertake a reform is taken before the identity of the reformer is revealed. Moreover, as in our main model, the decision maker cannot observe θ. She only receives signal ρ, with noise z. Thus, with a single decision maker, we no longer have a proposer with an informational advantage over the regulator. After observing signal ρ, the decision maker chooses the policy that maximizes her expected utility (y^S, y^C, or the status quo).

The decision-maker's optimal choice at each (κ, π) is illustrated in Figure 7. If the probability of a loss from reform (state θ^C and B competence) is low, then the decision-maker adopts reform y^S regardless of signal, as this reform is most likely to match the state and deliver a benefit. The region of the parameter space where this decision is optimal is illustrated in dark blue. If the probability of state θ^C is high, and competence is expected to be high, the decision-maker chooses y^C regardless of signal, as she expects this reform to match the state and produce a gain. This region of the parameter space is illustrated in

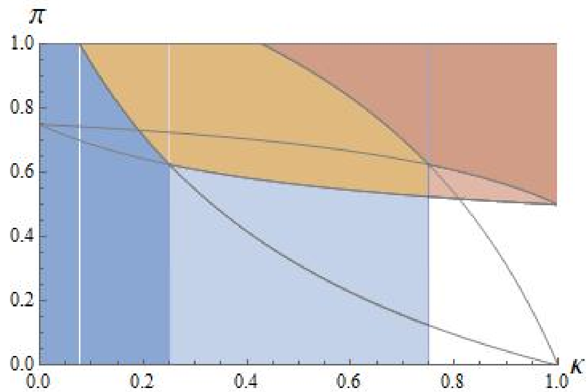


Figure 7: Illustrates the single decision-maker's policy choice given $z = 0.25$. In the dark blue region, y^S is chosen regardless of signal, in the light blue region, y^S is chosen after $\rho = s$ and the status quo is maintained after $\rho = c$. In the dark orange region, y^C is chosen regardless of signal, in the light orange region, y^C is chosen after $\rho = c$ and the status quo is maintained after $\rho = s$. In the yellow region, y^S is chosen after $\rho = s$ and y^C is chosen after $\rho = c$. In the white region, the status quo is maintained regardless of signal.

dark orange. For intermediate values of κ, there is high uncertainty about the state of the world, and the decision maker relies on her signal to choose policy. She chooses policy y^S after signal $\rho = s$. After $\rho = c$, she expects state θ^C to be more likely. In that case, she wants to implement policy y^C only if she is likely to be competent, i.e., π is sufficiently high (the yellow region in the figure). Otherwise, maintaining the status quo is preferable (the light blue region in the figure). Finally, if κ is very high, the regulator expects state θ^C. If there is high uncertainty about her competence (intermediate π), then she uses the signal to choose y^C after $\rho = c$ and maintain the status quo otherwise (the light orange region in the figure). If she expects to not be competent, then a reform is expected to produce a loss, and she therefore maintains the status quo regardless of signal (the white region in the figure).

Cycling can emerge with a single decision-maker, but in a different form and for different reasons than in the main model. There is cycling between implementing a reform and implementing no reform, and the cycling is contingent on the signal $\rho = c$.[15] This result

[15] When z is high, the signal $\rho = c$ is less informative, and the decision-maker is not willing to take a risk of implementing policy y^C, as it only delivers a benefit if the state is θ^C. She then maintains the status quo after $\rho = c$. After signal $\rho = s$, the decision-maker still implements y^S, as state θ^S is sufficiently likely. Implementing y^S decreases the average noise z, which in turn makes the signal more informative. This induces the decision-maker to implement y^C after $\rho = c$.

has a straightforward intuition: as the decision maker receives more precise information, she acts on that information to implement a more risky, complex reform. As the information becomes less precise, the decision-maker maintains the safe status quo. In contrast to the cycling obtained in the main model, this cycling is driven by the decision maker conditioning reform y^C on the precision of her information. This also implies that the $z^{*S}(\kappa, \pi)$ around which cycling happens in this case increases in its arguments. As π decreases, the decision-maker faces a higher likelihood of a loss from reform y^C, and therefore requires a more precise signal in order to adopt this reform. In our main model the cycling emerges when the decision maker is not making decisions contingent on her signal, while here the cycling is driven by the decision maker acting contingent on her signal. This leads to the contrasting result: For a (κ, π) in the intersection of region $\mathcal{U}$ and this cycling region, cycling that happens at high complexity in the main model happens at low complexity here, and vice-versa.

Given our assumption that the regulator is benevolent, and her payoff equals the social welfare, we can evaluate checks and balances in terms of their welfare implications.

Remark 3 *Checks and balances deliver higher expected social welfare than a single uninformed decision maker.*

Compared to the single decision maker, checks and balances allow for more information to be accessed before a decision is made. This is because the proposer has private information, which is reflected in the equilibrium strategies, and the decision maker forms beliefs that are consistent with those strategies. In sum, checks and balances allow for some access to information (because of the informed proposer) and some rejection of proposals which are expected to bring a loss. They limit the downsides of a single uninformed decision maker. In fact, the solution of the single decision maker could be implemented in the system with checks and balances, as it is in the Pooling region. Yet, strategies with different proposer types proposing different policies yield higher expected welfare.

Another dimension along which we can evaluate checks and balances is the resulting complexity of the regulatory environment.

Remark 4 *A system of checks and balances does not lead to everywhere higher or to everywhere lower expected complexity of the environment (z) in the long-run compared to a single uninformed decision-maker.*

Intuitively, on the one hand, checks and balances reduce long-run complexity because policy y^C is implemented less often for most parameter values. With an informed proposer, policy y^C may be offered only after θ^C, and the decision maker may reject proposals for high values of κ and low values of π. On the other hand, checks and balances increase long-run complexity because they support equilibria where y^C is offered more often, i.e., the Complexification equilibrium. The informed proposer chooses y^C more often in order to get approval from the decision maker given her beliefs. This insight complements that of Gratton and Morelli (2018). In their model, checks and balances reduce the frequency with which bad reforms are approved (type I errors), but they also increase the frequency with which good reforms are rejected (type II errors). We also obtain the result that checks and balances help decrease the frequency of type I errors, outside the Complexification region, where she approves y^C less often. Yet, checks and balances increase the frequency of type I errors inside the Complexification region, where the decision maker approves y^C when it follows state θ^S or proposer B. Finally, Remark 4 relates to the debate on whether shifting the authority over approving the details of reforms from legislators to regulators will result in more simplification (as argued by Teles, 2013) or whether removing checks and balances increases instability (as argued by Besley and Mueller, 2018), and by extension complexity, as the environment becomes more uncertain. Our results bring a note of caution to both these theses. As shown above, long-run complexity comparisons depend on the fundamentals.

7 Concluding Remarks and Future Directions

In this paper we have analyzed a large class of situations where decision making typically involves an informed but potentially low-ability proposer and a decision maker, typically less informed. In particular, we have endowed the proposer with a choice between proposing new details (in a law or regulation) or proposing elimination of details or contingencies. When the proposer cares about passing her proposal no matter what, her proposal can sometimes be a more detailed law even though the situation is such that this complexification is not beneficial for the general public.

We have first characterized the equilibrium between proposer and decision maker for every pair of common state and private type asymmetric information parameters, and then we have studied the implications of such equilibrium reforms for the endogenous evolution of complexity. In the long run, for a large set of intermediate parameter values, the endogenous level of complexity fluctuates around an intermediate level. This level is increasing in the probability that the proposer is low-ability. This finding highlights that even if complexity relates to the common state information asymmetry, the additional fear of captured or incompetent proposers actually has negative spillovers on complexity itself.

We showed that the proposed model allows us to nest and reconcile a number of recent findings in legislative politics and regulation studies, and could be useful for institutional design. We closed the analysis by showing that indeed with our assumptions the presence of a veto player (checks and balances) is justified, because welfare is higher than in the absence of such a player. The comparison in terms of long run complexity is instead ambiguous.

In future research, the model may be connected to the literature on endogenous incompleteness of contracts (Tirole, 1999). Adopting a complexification reform can be mapped to making a contract more complete. This may be beneficial or it may be detrimental, depending on the alignment of interests between proposer and decision maker. A regulator may decide not to introduce a proposed completion of a contract for lack of trust that the benefits from the additional contingencies will truly outweigh the costs of writing them and the costs of enforcing them. For instance, the regulator may think that the costs may be on everybody whereas the benefits might be concentrated in the proposing interest group only. Endogenous incompleteness is more likely to emerge when there is a higher likelihood that additional contingencies are beneficial (high κ), the regulator faces high uncertainty (π is not too high), and there is high noise (z is higher). This insight is distinct from that obtained when endogenous incompleteness is due to writing costs alone. In Battigalli and Maggi (2002), where the focus is on endogenous incompleteness due to writing costs, greater uncertainty could cause an increase in the likelihood that more contingencies are beneficial, thus decreasing endogenous incompleteness. Making the link between contract incompleteness and our framework leads to the conjecture that higher uncertainty in the form of higher complexity of the environment could make contracts more incomplete. This in turn may have the collateral effect of transferring more power to some institutions over others, for instance giving more power and discretion to the bureaucracy.

Another important direction for future research is the comparative politics or comparative institutions direction. In the US, policy reforms are delegated to a bureaucratic agency. In Europe, the European Commission or other bureaucratic agencies decide on regulation. But in many other countries the legislators delegate less, and the decision maker is a member of parliament. For this case, we must consider also accountability, coalition formation, etc. Relatedly, we could extend the analysis to reform complementarities. A complexification reform could induce positive returns when combined with other changes proposed by other proposers on connected issues. This complementarity would reduce the incentive of the regulator to adopt a complex policy, given the uncertainty about the possibility that the complementary reform will also occur.

Some research could also be dedicated to the connection between complexity cycles and the business cycle. At a time in which complexity of the environment is very high it may be good to simplify decision making. Especially in the case in which complementarities exist or coordination is necessary, like in the case of the Covid-19 crisis, it would seem that complex designs could lose all their potential strength if adopted in isolation.

Lastly, our framework has focused on incremental reforms only. A next step would be to also consider the alternative of radical reforms or revolutions. The incremental dynamics of reforms could then be contrasted to the dynamics of major policy changes. In line with the view of Acemoglu and Robinson (2019), our model suggests that the continuous process of incremental reforms may stop when the perception is that more elaborate reforms would be necessary but the elite of proposers is perceived to be bad or captured (low trust in institutions and low confidence in expertise in their terminology). Hence, such situations are exactly those where the world of incremental reforms stops and the chapter of institutional regime change begins.

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A Proofs

A.1 Proof of Proposition 1

If the proposer is A, the DM's payoff is maximized for y^S after θ^S and y^C after θ^C. If the proposer is B, the DM's payoff is positive only after θ^S and y^S. Thus, in an equilibrium where A offers y^S after θ^S and y^C after θ^C, the DM's best response is to approve with with probability 1. This proposer strategy gives the DM the highest payoff. If the proposer is B, the DM's best response is to approve only if she receives proposal y^S and she expects θ^S is sufficiently likely such that $\kappa \cdot (-l) + (1-\kappa) \cdot v \geq 0$. Thus, she approves if $\kappa \geq v/(v+l)$. Proposing y^S regardless of state is a dominating strategy for B, since y^C would be rejected with probability one.

A.2 Proof of Proposition 2

The decision-maker's approval strategy :

Given a proposal y, signal ρ and noise z, the DM approves it if

$$\mathbb{E}\left[v\left(\theta,y\right)|y,\rho\right] - w \geq 0. \tag{12}$$

We examine this approval condition after each possible combination of y and ρ:

1. After y^S and $\rho = s$:

$$\frac{\left[\Pr\left(y^S|\theta^H,A\right)\cdot\pi + \Pr\left(y^S|\theta^H,B\right)\cdot(1-\pi)\right]\cdot\kappa}{\left[\Pr\left(y^S|\theta^L,A\right)\cdot\pi + \Pr\left(y^S|\theta^L,B\right)\cdot(1-\pi)\right]\cdot(1-\kappa)} \leq \frac{v}{l}\cdot\frac{1-z}{z} \tag{13}$$

2. After y^S and $\rho = c$:

$$\frac{\left[\Pr\left(y^S|\theta^H,A\right)\cdot\pi + \Pr\left(y^S|\theta^H,B\right)\cdot(1-\pi)\right]\cdot\kappa}{\left[\Pr\left(y^S|\theta^L,A\right)\cdot\pi + \Pr\left(y^S|\theta^L,B\right)\cdot(1-\pi)\right]\cdot(1-\kappa)} \leq \frac{v}{l}\cdot\frac{z}{1-z} \tag{14}$$

3. After y^C and $\rho = s$:

$$\frac{1}{1+\Psi\left(1-z\right)} - \frac{c}{v+l}\cdot\frac{1}{1+\Upsilon\left(1-z\right)} \geq \frac{l}{v+l}, \tag{15}$$

where

$$\Psi\left(1-z\right) \equiv \frac{z\cdot\kappa\cdot\Pr\left(y^C|\theta^H,B\right) + (1-z)\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L,B\right)}{z\cdot\kappa\cdot\Pr\left(y^C|\theta^H,A\right) + (1-z)\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L,A\right)}\frac{1-\pi}{\pi}, \tag{16}$$

$$\Upsilon\left(1-z\right) \equiv \frac{z\cdot\kappa\cdot\left[\Pr\left(y^C|\theta^H,A\right)\cdot\pi + \Pr\left(y^C|\theta^H,B\right)\cdot(1-\pi)\right]}{(1-z)\cdot(1-\kappa)\cdot\left[\Pr\left(y^C|\theta^L,A\right)\cdot\pi + \Pr\left(y^C|\theta^L,B\right)\cdot(1-\pi)\right]}. \tag{17}$$

4. After y^C and $\rho = c$:

$$\frac{1}{1+\Psi(z)} - \frac{c}{v+l} \cdot \frac{1}{1+\Upsilon(z)} \geq \frac{l}{v+l}, \tag{18}$$

where

$$\Psi(z) \equiv \frac{(1-z)\cdot\kappa\cdot\Pr\left(y^C|\theta^H,B\right)+z\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L,B\right)}{(1-z)\cdot\kappa\cdot\Pr\left(y^C|\theta^H,A\right)+z\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L,A\right)}\frac{1-\pi}{\pi}, \tag{19}$$

$$\Upsilon(z) \equiv \frac{(1-z)\cdot\kappa\cdot\left[\Pr\left(y^C|\theta^H,A\right)\cdot\pi+\Pr\left(y^C|\theta^H,B\right)\cdot(1-\pi)\right]}{z\cdot(1-\kappa)\cdot\left[\Pr\left(y^C|\theta^L,A\right)\cdot\pi+\Pr\left(y^C|\theta^L,B\right)\cdot(1-\pi)\right]}. \tag{20}$$

Pure Strategy Equilibria We consider all the possible pure strategy equilibria where there is a positive probability of acceptance:

1. Equilibrium where B proposes y^S, and A proposes y^S after θ^S and y^C after θ^C

The DM approves with probability 1 after observing y^C. After observing y^S, the DM's response is

- after signal $\rho = s$, approve if

$$1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{1-z}{z} \leq \pi, \tag{21}$$

- after $\rho = c$, approve if

$$1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z} \leq \pi. \tag{22}$$

Thus, the DM approves regardless of signal (and this equilibrium exists) if

$$\pi \geq 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}. \tag{23}$$

The DM's expected payoff given this equilibrium play is

$$U^{(1)} - \omega = \kappa\cdot\pi\cdot v - \kappa\cdot(1-\pi)\cdot(l-a) + (1-\kappa)\cdot v. \tag{24}$$

2. Equilibrium where all proposers chose y^S after θ^S and y^C after θ^C After observing y^S, the DM approves with probability one. After observing y^C, the DM approves if

$$\pi \geq \frac{l}{v+l}. \tag{25}$$

The DM's expected payoff given this equilibrium play is

$$U^{(2)} - \omega = \kappa\cdot\pi\cdot v - \kappa\cdot(1-\pi)\cdot l + (1-\kappa)\cdot v. \tag{26}$$

3. Equilibrium where proposer A chooses y^C for all θ and proposer B chooses y^S if θ^L and y^C if θ^H.

After observing y^S, the DM approves with probability 1. After observing y^C, the DM approves in the following cases:

- after $\rho = c$, if

$$\pi \geq \pi^{mch} = l \cdot \frac{(1-z)\cdot\kappa}{(1-z)\cdot\kappa\cdot(v+l)+(v-a)\cdot z\cdot(1-\kappa)}. \tag{27}$$

- after $\rho = s$, if

$$\pi \geq \pi^{mcl} = l \cdot \frac{z\cdot\kappa}{z\cdot\kappa\cdot(v+l)+(v-a)\cdot(1-z)\cdot(1-\kappa)}. \tag{28}$$

Since $\frac{1-z}{z} \geq \frac{z}{1-z}$, we have

$$\pi^{mch} \geq \pi^{mcl}. \tag{29}$$

Thus, this equilibrium exists if $\pi \geq \pi^{mch}$. The DM's expected payoff given this equilibrium play is

$$U^{(3)} - \omega = \kappa\cdot\pi\cdot v - \kappa\cdot(1-\pi)\cdot l + (1-\kappa)\cdot\pi\cdot(v-a) + +(1-\kappa)\cdot(1-\pi)\cdot v. \tag{30}$$

4. Pooling on y^S for all θ:

The DM's approval condition reduces to

1. If $\rho = c$:

$$(1-z)\cdot(1-k)\cdot v - z\cdot k\cdot l \geq 0. \tag{31}$$

2. If $\rho = s$:

$$z\cdot(1-k)\cdot v - (1-z)\cdot k\cdot l \geq 0. \tag{32}$$

Thus, an equilibrium with pooling on y^S (regardless of θ) and probability one of approval exists if

$$k \leq k^{pool} \equiv \frac{z\cdot v}{z\cdot v+(1-z)\cdot l} \tag{33}$$

An equilibrium with pooling on y^S and approval conditional on $\rho = s$ exists if

$$k \leq k^{cp} \equiv \frac{(1-z)\cdot v}{(1-z)\cdot v + z\cdot l}, \tag{34}$$

and the DM's off path belief is that a deviation has come from proposer B.

The DM's expected payoff given the equilibrium play with conditional approval is

$$U^{(4)} - \omega = -z\cdot\kappa\cdot l + (1-z)\cdot(1-\kappa)\cdot v. \tag{35}$$

5. Pooling on y^C regardless of θ
If the proposer only offered y^C the DM's approval decision reduces to:

- After $\rho = c$:
$$\pi \geq \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1+\frac{(1-z)\cdot\kappa}{z\cdot(1-\kappa)}}, \tag{36}$$

- After $\rho = s$:
$$\pi \geq \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1+\frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)}}. \tag{37}$$

If y^S were proposed (off-equilibrium), then the DM's expected payoff from accepting, given her belief about who deviated is

$$-\widehat{\kappa} \cdot l + (1-\widehat{\kappa}) \cdot v, \tag{38}$$

so a deviation is not profitable as long as

$$\widehat{\kappa} > \frac{v}{v+l},$$

where $\widehat{\kappa} = \Pr(\theta^C | y^S)$ is the belief that a deviation is undertaken in state θ^C. Then, an equilibrium with pooling on y^C and positive probability of acceptance exists and

- If
$$\pi \in \left[\frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1+\frac{(1-z)\cdot\kappa}{z\cdot(1-\kappa)}}, \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1+\frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)}}\right], \tag{39}$$
the DM accepts contingent on $\rho = c$;

- If
$$\pi \geq \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1+\frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)}}, \tag{40}$$
the DM accepts regardless of signal.

The DM's expected payoff given this equilibrium play is

$$U^{(5)} - \omega = \pi \cdot v - (1-\pi) \cdot l - (1-\kappa) \cdot c. \tag{41}$$

6. Equilibrium where proposer A offers y^S after θ^S and y^C after θ^C, and proposer B offers y^C in all states
After y^S, the DM accepts with probability 1. After, y^C the DM approves if

- after signal $\rho = c$:

$$\pi \geq \frac{l \cdot (1-z) \cdot \kappa + (l+c) \cdot z \cdot (1-\kappa)}{(v+l) \cdot (1-z) \cdot \kappa + (l+c) \cdot z \cdot (1-\kappa)}. \tag{42}$$

- after signal $\rho = s$:

$$\pi \geq \frac{l \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}{(v+l) \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}. \tag{43}$$

Thus, the DM approves regardless of signal if

$$\pi \geq \frac{l \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}{(v+l) \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}. \tag{44}$$

The DM's expected payoff given this equilibrium play is

$$U^{(6)} - \omega = \kappa \cdot \pi \cdot v - \kappa \cdot (1-\pi) \cdot l + (1-\kappa) \cdot \pi \cdot v + -(1-\kappa) \cdot (1-\pi) \cdot (l+c). \tag{45}$$

Ranking on Equilibria Notice then that $U^{(1)} \geq U^{(2)} > U^{(3)} > U^{(6)} > U^{(4)}$. Also, $U^{(6)} > U^{(5)}$, and Then,

$$\pi_3 \equiv 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}. \tag{46}$$

$$\pi_2 \equiv \frac{l}{v+l}. \tag{47}$$

$$\pi_1 \equiv \frac{l \cdot (1-z) \cdot \kappa}{(1-z) \cdot \kappa \cdot (v+l) + (v-a) \cdot z \cdot (1-\kappa)}. \tag{48}$$

Also, let

$$\pi^{pool} \equiv \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1 + \frac{z \cdot \kappa}{(1-z) \cdot (1-\kappa)}} \tag{49}$$

$$\pi^{compl} \equiv \frac{l \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}{(v+l) \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)} \tag{50}$$

Notice that $\pi_2 \leq \pi^{pool}$ and $\pi_2 \leq \pi^{compl}$ for any κ and z. Thus, the parameter values for which equilibria 5 and 6 exist are also the parameter values for which equilibrium 2 exists. Notice also that at k^{pool}, we have $\pi_3(k^{pool}) = 0$, and π_3 increases in k. Thus the equilibrium with pooling at y^S at approval probability one exists only when equilibrium 1 also exists. However, equilibrium 4 with approval conditional on $\rho = s$ exists since $\pi(k^{cp}) > 0$.

Thus, the Best Perfect Bayesian equilibrium may take forms 1-4, with the boundaries between these regions given by $\pi_1 - \pi_3$.

Other proposer strategies. Notice that the other possible pure strategies are not part of an equilibrium in which there is a positive probability of acceptance. Specifically, if all proposers offer y^S after θ^H and y^C after θ^L, the DM would surely reject after y^S. Thus, this cannot be an equilibrium. Similarly, consider the case where proposer A chooses y^S in all states and proposer B chooses y^S after θ^S and y^C after θ^C. After y^C, the DM rejects with probability 1. Thus, this cannot be an equilibrium.

A.3 Proof of Corollary 1

From (46) - (48), it follows that

$$\frac{\partial \pi_3}{\partial z} \leq 0; \frac{\partial \pi_2}{\partial z} = 0; \frac{\partial \pi_1}{\partial z} \leq 0. \tag{51}$$

Also, the upper bound $\bar{\kappa}$ for the Pooling region changes as follows:

$$\frac{\partial \bar{\kappa}}{\partial z} = \frac{-v \cdot l}{((1-z) \cdot v + z \cdot l)^2} < 0. \tag{52}$$

A.4 Proof of Corollary 2

The boundary of the region where proposals are accepted with probability one is given by

$$\pi^B \equiv \min\{\pi_1, \pi_2, \pi_3\}. \tag{53}$$

From Corollary 1, it follows that π^B weakly decreases in z at each κ. Thus, the boundaries of the Simplification and the Complexification equilibria expand. At $z = \frac{1}{2}$, $\pi_3 \geq 0$ for $\kappa \leq \frac{v}{l+v}$. Thus, in the region where $\kappa \leq \frac{v}{l+v}$, $\pi^B \geq 0$ and it increases. The $BPBE$ becomes ones in which the proposal is accepted with probability one, instead of the Pooling equilibrium, where the proposal is accepted with probability $z \cdot \kappa + (1-z) \cdot (1-\kappa)$.

If $\kappa > \frac{v}{l+v}$, the bound $\bar{\kappa}$ decreases as z increases:

$$\frac{\partial \bar{\kappa}}{\partial z} = \bar{\kappa} \cdot \frac{-v}{1-z}. \tag{54}$$

The bound π_1 decreases as z increases:

$$\frac{\partial \pi_1}{\partial z} = \pi_1 \cdot \frac{-(v-a) \cdot (1-\kappa)}{1-z}. \tag{55}$$

Thus, the region where the proposal is rejected changes approximately by $-\frac{\partial \bar{\kappa}}{\partial z} \cdot \pi_1(\bar{\kappa}, z) + \frac{\partial \pi_1}{\partial z} \cdot \bar{\kappa}$. Notice that

$$\frac{\frac{\partial \bar{\kappa}}{\partial z} \cdot \pi_1(\bar{\kappa}, z)}{\frac{\partial \pi_1}{\partial z} \cdot \bar{\kappa}} = \frac{v}{(v-a) \cdot (1-\kappa)} > 1. \tag{56}$$

Thus, the region of the parameter space (κ, π) where the proposal is rejected expands as z increases. At the maximum z, $\bar{\kappa} = \frac{v}{l+v}$. Thus, the region where the proposal is rejected expands as z increases, from a lower bound $\bar{\kappa} \to 1$ as $z^{\min} \to 0$ to $\frac{v}{/}v + l$ when $z = \frac{1}{2}$.

A.5 Proof of Proposition 3

In the Simplification, Matching and Complexification equilibria, both y^S and y^C may be proposed with positive probability in equilibrium. Thus, the noise z may either increase or decrease. In each of these regions, there are realizations of θ and P such that z increases in Δ increments until the maximum noise of $\frac{1}{2}$ is reached. Similarly, there exist paths with realizations of θ and P such that z decreases by Δ each period, until the minimum noise $z^{\min}$ is reached.

The Stable Region. Consider the Simplification $BPBE$. By Corollary 1, the boundary value π_3 weakly decreases in z. Thus, the $BPBE$ is stable and the same for any $z_0 \in [z^{\min}, 0.5]$ if $\pi \geq \pi_3(\kappa, z^{\min})$. As $z^{\min} \to 0$, $\pi_3(\kappa, z^{\min}) \to 1$, meaning that this region contracts to 0.

Consider the Matching $BPBE$. The lower bound $\pi_2 = \frac{l}{l+v}$ does not change with z. However, the upper bound of this region, given by π_3 decreases in z. Thus, the region where $\pi \in \left[\pi_2, \pi_3(\kappa, \frac{1}{2})\right)$ is a Matching $BPBE$ for all $z \in [z^{\min}, 0.5]$. It is therefore stable and the same for all $z \in [z^{\min}, 0.5]$.

Consider the Complexification $BPBE$. The upper bound $\pi_2 = \frac{l}{l+v}$ does not change with z. The lower bound for this region is given by $\pi_1(\kappa, z)$, which decreases in z. Thus, the $BPBE$ is the Complexification equilibrium for all z if $\pi \in \left[\pi_1(\kappa, z^{\min}), \pi_2\right)$. As $z^{\min} \to 0$, the interval for π contracts to 0.

Consider the Pooling $BPBE$. The boundary of the region where Pooling is the $BPBE$ expands as z decreases: π_1, π_3, and $\bar{\kappa}$ all increase. The value of $\bar{\kappa}$ is $\frac{v}{l+v}$ at $z = 0.5$ and converges to 0 as $z^{\min} \to 0$. Thus, for any (κ, π), there exists a value z' at which the BPBE is Pooling with approval conditional on signal for $z < z'$ and Pooling with rejection for $z > z'$. Hence, the $BPBE$ is stable.

The Unstable and Complexity Dependent Regions. Consider the regions where the $BPBE$ is not stable. This implies that for each (κ, π) in these regions, there exists $z'(\kappa, \pi) \in (z^{\min}, 0.5)$ at which (κ, π) is on the boundary between two different $BPBEs$.

Consider first the boundary $\pi_3(\kappa, z)$, with $z'(\kappa, \pi)$ defined as $\pi = \pi_3(\kappa, z')$. For $\pi_3(\kappa, z) \geq \pi_2$, the boundary is between the Simplification $BPBE$, when $z \geq z'$ and the Matching $BPBE$, when $z < z'$. In both of these equilibria, either y^S or y^C may be proposed along the equilibrium path. Thus, we can construct an path of possible realizations of θ and P such that y^C is proposed when $z < z'$ and y^S when $z > z'$. Along such a path, the $BPBE$ switches between Simplification and Matching, and it is hence not stable. Each location (κ, π) at which $\pi = \pi_3(\kappa, z') > \pi_2$ is therefore not stable. If the location is outside region $\mathcal{C}$, then there In the Transitional region.

Let κ^M be defined as the value at which $\pi_3(\kappa^M, \frac{1}{2}) = \pi_1(\kappa^M, \frac{1}{2})$. For any $\kappa \leq \kappa^M$, let π_4 be defined as the value at which the following equality is satisfies for some z in $\left[z^{\min}, \frac{1}{2}\right]$:

$\pi_3(\kappa, z) = \pi_1(\kappa, z)$. Given the expressions for $\pi_3(\kappa, z)$ and $\pi_1(\kappa, z)$, we obtain

$$\pi_4 = 1 - \frac{\sqrt{((v)^2 - l \cdot c)^2 + 4 \cdot l \cdot (v)^3} - ((v)^2 + l \cdot c)}{2 \cdot l \cdot (v - a)}. \tag{57}$$

Hence, for $\pi \geq \pi_4$ and z, the $BPBE$ is Simplification if $z \geq z'$ and it is Complexification if $z < z'$. Thus, if $\pi_3(\kappa, z) \in [\pi_4, \pi_2)$, then the boundary is between the Simplification $BPBE$, when $z \geq z'$ and the Complexification $BPBE$, when $z < z'$. In both of these equilibria, both y^S and y^C may be proposed along the equilibrium path. Thus, as above, we can construct an path of possible realizations of θ and P such that y^C is proposed when $z < z'$ and y^S when $z > z'$. Along such a path, the $BPBE$ switches between Simplification and Complexification, and it is hence not stable. Each location (κ, π) at which $\pi = \pi_3(\kappa, z') \in [\pi_4, \pi_2)$ is therefore in the Transitional region.

For $\pi < \pi_4$, the $BPBE$ is Pooling if $z \leq z'$, and it is Simplification if $z > z'$. In the Pooling equilibrium, proposals y^C are not approved. If $z \leq z'$, we are in the Pooling equilibrium. Once there, z may only further decrease. Hence, the $BPBE$ is stable for $z < z'$. If $z_0 > z'$, then any realized sequence of θ and P results in either (i) $z_t \leq z'$ in some period t, in which case the $BPBE(\kappa, \pi, z_t)$ is stable, or (ii) $z_t > z'$ for all $t > 0$, in which case the $BPBE(\kappa, \pi, z_t)$ is stable as well (Simplification along the entire path). Thus, if $\pi = \pi_3(\kappa, z') < \pi_4$, then we are in the Complexity Dependent region.

Consider next the boundary $\pi_1(\kappa, z)$, with $z'(\kappa, \pi)$ now defined as $\pi = \pi_1(\kappa, z')$. The case where $\pi_1(\kappa, z) = \pi_3(\kappa, z)$ was discussed above. For $\kappa > \kappa^M$, the $BPBE$ is Pooling if $z \leq z'$, and it is Complexification if $z > z'$ and $\pi < \pi_2$. In the Pooling equilibrium, proposals y^C are not approved and thus z may only decrease (or stay the same). Hence, the $BPBE$ is stable for $z < z'$. If $z_0 > z'$, then any realized sequence of θ and P results in either (i) $z_t \leq z'$ in some period t, in which case the $BPBE(\kappa, \pi, z_t)$ is stable, or (ii) $z_t > z'$ for all $t > 0$, in which case the $BPBE(\kappa, \pi, z_t)$ is stable as well (Complexification along the entire path). Thus, if $\pi > \pi_1(\kappa, z')$ and $\pi < \pi_2$, then we are in the Complexity Dependent region.

A.6 Proof of Proposition 4

Consider $z_0 \in \left(z^{min}, \frac{1}{2}\right)$ and a location (κ, π) in the Transitional region such that $\pi_3(\kappa, z_0) \leq \pi$. Then, z_t, for $t = 1, 2, ...$ is expected to evolve as follows:

$$z_t = z_{t-1} + \Pr(y^C, a = 1) \cdot \Delta - \Pr(y^S, a = 1) \cdot \Delta. \tag{58}$$

Thus, z_t is expected to decrease if $\Pr(y^C, a = 1) < \Pr(y^S, a = 1)$, and it is expected to increase if $\Pr(y^C, a = 1) > \Pr(y^S, a = 1)$. For the Simplification $BPBE$, given the equilibrium strategies, $\Pr(y^C, a = 1) < \Pr(y^S, a = 1)$ reduces to

$$\kappa \cdot \pi < \kappa \cdot (1 - \pi) + (1 - \kappa), \tag{59}$$

or

$$\kappa < \frac{1}{2\pi}. \tag{60}$$

If at period t the noise z_t decreases to z^* such that $\pi \leq \pi_3(\kappa, z^*)$, the Simplification BPBE is not longer achievable. The equilibrium play switches to another $BPBE$. If this $BPBE$ is Matching, then given the equilibrium strategies, $\Pr(y^C, a = 1) > \Pr(y^S, a = 1)$ reduces to

$$\kappa > 1 - \kappa. \tag{61}$$

Thus,

$$\kappa > \frac{1}{2}. \tag{62}$$

If condition (62) is satisfied, z_t is expected to increase, which lowers $\pi_3(\kappa, z_t)$. Then, for $z_t \geq z^*$, $\pi \geq \pi_3(\kappa, z^*)$, and the BPBE switches to Simplification. Thus, under conditions (60) and (62), we obtain cycling between Simplification and Matching in region $\mathcal{U}$.

In the Complexification $BPBE$, given the equilibrium strategies, $\Pr(y^C, a = 1) > \Pr(y^S, a = 1)$ reduces to

$$\kappa + (1 - \kappa) \cdot \pi > (1 - \kappa) \cdot (1 - \pi). \tag{63}$$

Thus,

$$\kappa > \frac{1 - 2\pi}{2(1 - \pi)}. \tag{64}$$

Then, under conditions (60) and (64), we obtain cycling in region $\mathcal{U}$, along the Simplification / Complexification boundary, i.e., when $\pi > \max\{\pi_4, \pi_3\}$.

Frequency of Fluctuations The frequency of fluctuations is highest when the number of periods needed to cross from one region to the other and back is minimized, i.e., when $\Pr(y^S|\text{Simplification})$ and $\Pr(y^C|\text{Matching/Complexification})$ are maximized. For cycling between Simplification and Matching, $\Pr(y^S|\text{Simplification}) = 1 - \kappa \cdot \pi$ and $\Pr(y^C|\text{Matching}) = \kappa$. Then, $1 - \kappa \cdot \pi + \kappa$ decreases in π and it increases in κ.

For cycling between Simplification and Complexification, $\{\Pr(y^S|\text{Simplification}) = 1 - \kappa \cdot \pi$ and $\Pr(y^C|\text{Complexification}) = \kappa + \pi \cdot (1 - \kappa)$. The frequency of fluctuations is then increasing (decreasing) in κ when $\pi < (>) \frac{1}{2}$, and it is increasing (decreasing) in π when $\kappa < (>) \frac{1}{2}$.

A.7 Proof of Proposition 5

From (11), it follows immediately that $\frac{\partial z^*}{\partial \pi} < 0$ and $\frac{\partial z^*}{\partial \kappa} > 0$.

A.8 Proof of Proposition 6

As shown in the proof to Proposition 4, in the Simplification $BPBE$, z decreases on average if condition (60) is satisfied, i.e., $\kappa \cdot \pi < \frac{1}{2}$. Under this condition, z_t is expected to decrease, and as $t \to \infty$, it is expected to go to $z_t = z^{\min}$. The noise increases on average if $\kappa \cdot \pi > \frac{1}{2}$, so, as $t \to \infty$, the expected $z_t = \frac{1}{2}$. Finally, the noise is expected to stay at the status quo value if $\kappa \cdot \pi = \frac{1}{2}$. So, as $t \to \infty$, expected $z_t = z_0$.

In the Matching BPE, from (62) as $t \to \infty$, expected $z_t = \frac{1}{2}$. Proposition 3 says that region $\mathcal{I}$ exists if $\pi_2 = \frac{l}{l+v} < 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} = \pi_3$. This can be re-written as

$$\kappa > \frac{l+v}{l+v+l} > \frac{1}{2}. \tag{65}$$

Therefore, condition (62) is satisfied everywhere in region $\mathcal{I}$.

In the Complexification BPE, Condition (64) must be satisfied in order for average z to increase. Otherwise, average z decreases. Thus, if Condition (64) is satisfied, as $t \to \infty$, z_t is expected to converge to $\frac{1}{2}$. Otherwise, if $\kappa < \frac{1-2\pi}{2(1-\pi)}$, then as $t \to \infty$, expected z_t converges to $z^{\min}$.

In the Pooling BPE, only proposals y^S are made, and they are approved conditional on $\rho = s$. Every period, z_t is thus expected to decrease with probability $(1-z) \cdot (1-\kappa) + z \cdot \kappa$. Therefore, $\lim_{t\to\infty} \mathbb{E}\left[z_t\right] = z^{\min}$.

In the Rejection $BPBE$, no proposal is approved, thus $z_t = z_0$, for all $t > 0$.

Finally, in the regions where there is cycling, the cycling happens around the boundary between regions, i.e., around the value of z at which the location (κ, π) is on the boundary. Since cycling only happens between the Simplification region and another region (either Matching or Complexificaton), the boundary is given by $\pi = \pi_3(\kappa, z)$. Thus, the location (κ, π) is on the boundary at noise z^* defined implicitly by $\pi = \pi_3(\kappa, z^*)$.

A.9 Proof of Remark 1

For any $\kappa \in (0, 1)$ and $z \in \left[z^{\min}, 0.5\right]$, let $\bar{\pi} \equiv \max\{\pi_1(\kappa, z), \pi_3(\kappa, z)\}$. Let $\bar{\kappa}$ be the value of κ at which $\pi_1(\kappa, z) = \pi_3(\kappa, z)$.

Consider any $\pi \geq \bar{\pi}$ and $\kappa \geq \bar{\kappa}$. By Proposition 2, this location is in the Simplification region. Then, for $\Delta\pi \in (\pi - \pi_2(\kappa, z), \pi - \pi_1(\kappa, z))$, a shock which decreases π by $\Delta\pi$ is a move to location $(\pi - \Delta\pi)$ in the Complexification region described in Proposition 2. Then, the equilibrium play switches from y^S being proposed by A after θ^S to y^C being proposed by A in that state. Also, proposer B switches to from proposing y^S after θ^C to proposing y^C. Hence, the probability of y^C being proposed increases. This also increases the probability of z going up.

Similarly, let κ_2 be given by the value at which $\pi_1(\kappa_2, z) = \pi$, and let $\kappa_l = \min\{\kappa_2, \frac{1}{2\pi}\}$. Then, for $\Delta\kappa \in (\kappa_l - \kappa, 1 - \kappa)$, a shock which increases κ by $\Delta\kappa$ is a move to location $(\pi, \kappa + \Delta\kappa)$. This new locations in the Matching region if $\kappa + \Delta\kappa > \kappa_2$, or otherwise in the part of the Simplification region where the long-run z converges to $\frac{1}{2}$ (Proposition 6). In either case, κ increases, which means a higher probability of state θ^C, in which proposal y^C is made (by A in the Simplification region and by both types in the Matching region). Moreover, in both of these regions, z increases on average and converges to $\frac{1}{2}$, as shown in Proposition 6.

A.10 Proof of Remark 2

Follows from Remark 1 and Corollary 4.

A.11 The Single-Decision Marker's Policy Choice

After signal $\rho = s$, the decision-maker gets the following gain over the status quo:

- if she implements y^S:

$$v \cdot \frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa} - l \cdot \frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{66}$$

- if she implements y^C:

$$v\cdot\pi - l\cdot(1-\pi) - a\cdot\frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{67}$$

After signal $\rho = c$, the decision-maker gets the following gain over the status quo:

- if she implements y^S:

$$v \cdot \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa} - l \cdot \frac{(1-z)\cdot\kappa}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{68}$$

- if she implements y^C:

$$v\cdot\pi - l\cdot(1-\pi) - a\cdot\frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{69}$$

Since $z \leq \frac{1}{2}$, notice that

$$\frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa} \geq \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{70}$$

Thus, the decision-maker:

1. implements y^S regardless of signal if

$$\pi \leq \pi^{sb}(\kappa, z) \equiv \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}\cdot\frac{v+l+c}{v+l} \text{ and} \tag{71}$$

$$\kappa \leq \kappa^{rbh} \equiv \frac{z\cdot v}{z\cdot v+(1-z)\cdot l}. \tag{72}$$

2. implements y^C regardless of signal if

$$\pi \geq \pi^{cb}(\kappa, z) \equiv \frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa} \cdot \frac{v+l+c}{v+l} \text{ and} \tag{73}$$

$$\pi \geq \pi^{rbl}(\kappa, z) \equiv \frac{l}{l+v} + \frac{c}{l+v} \cdot \frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{74}$$

3. implements y^S after $\rho = s$ and y^C after $\rho = c$ if

$$\pi \in \left(\pi^{sb}(\kappa, z), \pi^{cb}(\kappa, z)\right) \text{ and} \tag{75}$$

$$\pi \geq \pi^{rbh}(\kappa, z) \equiv \frac{l}{l+v} + \frac{c}{l+v} \cdot \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa} \text{ and} \tag{76}$$

$$\kappa \leq \kappa^{rbl} \equiv \frac{(1-z)\cdot v}{(1-z)\cdot v + z\cdot l}. \tag{77}$$

4. implements y^S after $\rho = s$ and maintains status quo after $\rho = c$ if

$$\kappa \in \left(\kappa^{rbh}, \kappa^{rbl}\right) \text{ and } \pi \leq \pi^{rbh}. \tag{78}$$

5. implements y^C after $\rho = c$ and maintains status quo after $\rho = s$ if

$$\pi \in \left(\pi^{rbh}(\kappa, z), \pi^{rbl}(\kappa, z)\right), \text{ and } \kappa > \kappa^{rbl}. \tag{79}$$

6. maintains status quo after any signal if

$$\pi < \pi^{rbh}(\kappa, z) \text{ and } \kappa > \kappa^{rbl}. \tag{80}$$

Cycling between regions happens if endogenous changes in z move a location (κ, π) between two regions. Notice that π^{cb}, π^{rbh}, and κ^{rbh} decrease in z, while π^{sb}, π^{rbh}, and κ^{rbl} increase in z. Moreover, notice that $\kappa^{rbh} \leq \frac{1}{2} \leq \kappa^{rbl}$ and $\pi^{rbl} \geq \pi^{rbh} \geq \frac{l}{l+v}$. These properties imply that cycling between regions can occur only starting from the region $\kappa \in \left(\frac{1}{2}, \kappa^{rbl}\right)$ and $\pi \in \left(\pi^{rbh}(\kappa, z^{\min}), \pi^{rbh}\right)$, at some $z \in (z^{\min}, \frac{1}{2})$. Then, starting from such a point (κ, π), the decision-maker implements y^S after $\rho = s$ and maintains the status quo otherwise. Thus, expected z falls. This in turn reduces π^{rbh}. Then, at some z^*, $\pi^{rbh}(\kappa, z^*) \leq \pi$, i.e., the location crosses into the region where the decision-maker implements y^S after $\rho = s$ and y^C after $\rho = c$. The average z is expected to increase if $\rho = c$ is more likely than $\rho = s$, i.e., if $z\cdot(1-\kappa)+(1-z)\cdot\kappa > (1-z)\cdot(1-\kappa)+z\cdot\kappa$. This reduces to the condition that $\kappa > \frac{1}{2}$.

In the region where there is cycling at location (κ, π), it happens around z^{**} where

$$\pi = \pi^{rbh}(\kappa, z^{**}). \tag{81}$$

Given (77),

$$z^{**}(\kappa,\pi)=\frac{(\pi\cdot(v+l)-l)\cdot\kappa}{(l+c-\pi\cdot(l+v))\cdot(1-\kappa)+(\pi\cdot(l+v)-l)\cdot\kappa} \tag{82}$$

Then, $\frac{\partial z^{**}(\kappa,\pi)}{\partial\pi}>0$ and $\frac{\partial z^{**}(\kappa,\pi)}{\partial\kappa}>0$. Moreover, as $\pi\to\frac{l}{l+v}$, $z^{**}\to 0$. As $\pi\to\frac{l+c}{l+v}$, $z^{**}\to\frac{1}{2}$.

Consider now comparing z^{**} to the z^{*} from the main model. Let (κ,π) be a location in the parameter space that satisfies the conditions for cycling both in the main model and in the model with a single decision maker. Then, from (11), $z^{*}(\kappa,\pi)$ decreases in π. As $\pi\to\frac{l}{l+v}$, $z^{*}>z^{**}\to 0$. As $\pi\to\frac{l+c}{l+v}$, $z^{*}<z^{**}\to\frac{1}{2}$. Hence, there exists $\pi^{*}\in\left[\frac{l}{l+v},\frac{l+c}{l+v}\right]$,

$$\pi^{*}=\frac{2\cdot l\cdot(v+l)+(v)^{2}+l\cdot c-\sqrt{\gamma}}{l\cdot(v+l)}, \tag{83}$$

where $\gamma=(2\cdot l\cdot(v+l)+(v)^{2}+l\cdot c)^{2}-4\cdot l\cdot(v+l)\cdot(l\cdot a+(l)^{2}+l\cdot v)$. For $\pi<\pi^{*}$, we have $z^{*}>z^{**}$. For $\pi>\pi^{*}$, we have $z^{*}<z^{**}$.

A.12 Proof of Remark 3

Notice that the self-regulation maps into the main model framework in the following way: it is equivalent to the Simplification equilibrium play for the proposers, where the DM is forced to play $a=1$. Allowing the DM to choose $a\in\{0,1\}$ can only improve the DM's welfare. A proposed Simplification equilibrium is not sustainable when the DM would reject a proposal y^{S}. The DM would reject if she expects the outside option (status quo) to yield higher welfare.

The problem with a single decision-maker has policy y^{C} as part of the solution only for $\pi\geq\pi^{rbh}\geq\frac{l}{l+v}$. For those values of π, in the main model, y^{C} is proposed only after θ^{C}. Policy y^{C} delivers a higher expected payoff when used only after θ^{C} than when used after any θ or after $\rho=c$. Thus, in the region in which policy choice is contingent on signal, the Simplification / Matching equilibrium yields higher welfare, as it offers y^{C} only in the state θ^{C}. For $\pi<\pi^{rbh}$, the outcome with a single-decision maker can be achieved in an equilibrium of our main model (the Pooling equilibrium or the Rejection equilibrium). Yet, the main model allows for the Complexification equilibrium in a region which the decision-maker would implement the play from the Pooling or the Rejection regions. Since these equilibria are possible the main model for those parameter values, it must be the case that the DM expects higher welfare under the Complexification equilibrium.

A.13 Proof of Remark 4

In the self-regulation case, average z increases if

$$\kappa\cdot\pi>(1-\kappa)+\kappa\cdot(1-\pi), \tag{84}$$

So if $\kappa \cdot \pi \geq \frac{1}{2}$. Thus, in this case, z increases on average until it reaches the upper bound $\frac{1}{2}$. If Otherwise, $\kappa \cdot \pi < \frac{1}{2}$, z decreases on average, until it reaches the lower bound $z^{\min}$.

In the case of a single decision-maker, we consider each of the regions:

1. where she implements y^S regardless of signal, z decreases on average until it reaches the lower bound $z^{\min}$.

2. where she implements y^C regardless of signal z decreases on average until it reaches the lower bound $\frac{1}{2}$.

3. where she implements y^S after $\rho = s$ and y^C after $\rho = c$, the average z increases if

$$z \cdot (1 - \kappa) + (1 - z) \cdot \kappa \geq (1 - z) \cdot (1 - \kappa) + z \cdot \kappa, \tag{85}$$

   i.e., if $\kappa \geq \frac{1}{2}$, and average z decreases otherwise.

4. where she implements y^S after $\rho = s$ and maintains status quo after $\rho = c$, average z decreases.

5. where she implements y^C after $\rho = c$ and maintains status quo after $\rho = s$, average z increases.

6. where maintains status quo after any signal, z remains at its initial value z_0.

Outside the cycling region described in the single decision-maker's problem, if average z decreases, then it decreases until it reaches the lower bound $z^{\min}$. If average z increases, then it increases until it reaches the upper bound $\frac{1}{2}$. In the cycling region, the cycling happens around the bound π^{rbh}, so at the z^* at which $\pi^{rbh}(\kappa, z^*) = \pi$.

For self-regulation versus checks and balances and for single decision-maker versus checks and balances, we show that there exist regions where complexity is higher in one case compared to the other, and regions where the opposite is true.

Comparing the main model to the self-regulation outcome, consider the case in which $\frac{l}{l+v} > 1/2$. In that case, for $\kappa \to 1$ and $\pi \in \left(\frac{1}{2}, \frac{l}{l+v}\right)$, we have $z_\infty = \frac{1}{2}$ in the case of self-regulation, and $z_\infty = z_0$ in the case of checks and balances. However, for the cycling regions of Proposition 4, we have we have $z_\infty = z^{\min}$ in the case of self-regulation (since the cycling requires $\kappa < \frac{1}{2\pi}$), and $z_\infty = z^* > z^{\min}$ in the case of checks and balances.

Comparing the main model to the single decision-maker, the Complexification region in the main model has $z_\infty > z^{\min}$, while in the case of a single decision-maker, the same region (with $\pi < \frac{l}{l+v}$ has $z_\infty = z^{\min}$. Then, at $\kappa = 1/2 + \epsilon$, with $\epsilon \to 0$, and $\pi \in \left(\frac{l}{l+v}, \pi^{rbh}\right)$, the main model is in the Simplification region with $z_\infty = z^{\min}$, while with a single decision-maker we are in the cycling region with $z_\infty > 0$.