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Dirk Bergemann, Stephen Morris and Tibor Heumann

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
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Dirk Bergemann - dirk.bergemann@yale.edu
Yale University and CEPR

Stephen Morris - semorris@mit.edu
M.I.T. and CEPR

Tibor Heumann - tibor.heumann@gmail.com
HEC Montreal

Information, Market Power and Price Volatility*

Dirk Bergemann† Tibor Heumann‡ Stephen Morris§

July 23, 2020

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†Department of Economics, Yale University, New Haven, CT 06520, U.S.A., dirk.bergemann@yale.edu.

‡Instituto de Economía, Pontificia Universidad Católica de Chile, Santiago, Chile; tibor.heumann@uc.cl.

§Department of Economics, MIT, Cambridge, MA 02139, USA, semorris@mit.edu

1 Introduction

1.1 Motivation and Results

Models of demand function competition (or equivalently, supply function competition) are a cornerstone of the analysis of markets in industrial organization and finance. Economic agents submit demand functions and an auctioneer chooses a price that clears the market. Demand function competition is an accurate description of many important economic markets, such as treasury auctions or electricity markets. In addition, it can be seen as a stylized representation of many other markets in which there may not be an actual auctioneer, but agents can condition their bids on market prices and markets clear at equilibrium prices.

Under complete information, there is a well-known multiplicity of equilibria under demand function competition (see Grossman (1981), Klemperer and Meyer (1989)). In particular, under demand function competition, the degree of market power – which measures the distortion of the allocation as a result of strategic withholding of demand – is indeterminate. This indeterminacy arises because, under complete information, an agent is indifferent about what demand to submit at prices that do not arise in equilibrium. Making the realistic assumption that there is incomplete information removes this indeterminacy because every price can arise with positive probability in equilibrium. We therefore analyze demand function competition under incomplete information (Vives (2011)). We consider a setting where a finite number of agents have linear-quadratic preferences over their holdings of a divisible good, and the marginal utility of an agent is determined by a payoff shock; we restrict attention to symmetric environments (in terms of payoff shocks and information structures) and symmetric linear Nash equilibria.

The outcome of demand function competition under incomplete information will depend not only on the fundamentals of the economic environment - the number of agents and the distribution of payoff shocks - but also on which information structure is assumed. However, it will rarely

be clear what would be reasonable assumptions to make about the information structure. We therefore examine whether it is possible to make predictions about outcomes under demand function competition in a given economic environment that are robust to the exact modeling of the information structure.

Our first main result establishes the impossibility of robust predictions about market power. We show that any degree of market power can arise in the unique equilibrium under an information structure that is arbitrarily close to complete information. In particular, regardless of the number of agents and the correlation of payoff shocks, market power may be arbitrarily close to zero (so we obtain the competitive outcome) or arbitrarily large (so there is no trade in equilibrium). The reason is that under incomplete information prices convey information to agents. The slope of the demand function that an agent submits will then depend on what information is being revealed, and this will pin down market power in equilibrium.

Given the sharp indeterminacy in the level of market power induced by the information structure, it is natural to ask what predictions— if any— hold across *all* information structures.

Our second main result shows that – for any level of market power – price volatility is always (that is, regardless of the information structure) less than the price volatility that is achieved by an equilibrium under complete information. A direct corollary of our result is that price volatility is lower than the variance of the average shock across agents independent of the information structure. Hence, we show that it is possible to provide sharp bounds on some equilibrium statistics, which hold across all information structures.

We can always decompose agents' payoff shocks into idiosyncratic and common components. If the common component were common knowledge, but agents observed noisy signals of their idiosyncratic components, there would be a unique equilibrium and we can identify the market power as noise goes to zero. If instead the idiosyncratic components were common knowledge, but each agent observed a different noisy signal of the common component, there will be a different unique equilibrium and a different market power in the limit as the noise goes to zero.

In the latter case, unlike in the former case, higher prices will reveal positive information about the value of the good to agents, and as a result, agents will submit less price-elastic demand functions, and there will be high market power. More generally, if agents have distinct noisy but accurate signals of the idiosyncratic and common components of payoff shocks, then market power will be determined by the relative accuracy of the signals, even when all signals are very accurate.

The information structures giving rise to extremal outcomes are special because they are constructed to simplify the Bayesian updating when solving for the Nash equilibrium, so they do not necessarily have an immediate interpretation. Thus, one could have expected that it is possible to sharpen the predictions about market power and price volatility if one restricts attention to some class of parametrized information structures. To address this conjecture, we study three classes of information structures that are natural in the sense that each signal an agent observes is only about one payoff shock, and the noise terms are independently distributed. We do find tighter bounds on market power in these classes: in particular, market power is bounded below by the market power arising in Klemperer and Meyer (1989) and bounded above by one. However, we also show that market power can take any value in this reduced range for any degree of interdependence in the payoff environment (that is, the correlation of the payoff shocks), even with arbitrarily small amounts of asymmetric information.

We interpret our first main result as establishing that the indeterminacy of market power is not an artifact of particular modeling choices, such as complete information, but rather an intrinsic feature of the game. If economic agents interact in a market where demands can be conditioned on prices, then there can be extreme sensitivity to the inferences that market participants draw from prices, meaning that it will not be possible to make ex ante predictions about market power. Even restricting attention to information structures close to complete information does not allow us to provide sharper predictions about market, unless one is able to make additional restrictive assumptions regarding the nature of the incomplete information. By contrast, it is possible to

provide sharp predictions regarding price volatility with demand function competition.

On the other hand, we interpret our second main result as showing that the same economic feature that gives rise to the indeterminacy of market power – conditioning demand on market prices – imposes tight bounds on price volatility that do not hold in other economic environments.

The tight bounds on price volatility and indeterminacy of market power are important features of demand function competition. Our methodology allows us to make an exact comparison of outcomes under demand function competition (under any information structure) with what could have arisen under alternative trading mechanisms. We illustrate this by showing that under Cournot competition, our qualitative results are reversed: market power is now completely determined by the number of firms (and independent of the information structure), while the bounds on price volatility are now very weak.

1.2 Related Literature

The multiplicity of equilibria in demand function competition under complete information was identified by Wilson (1979), Grossman (1981) and Hart (1985) – see also Vives (1999) for a more detailed account. Klemperer and Meyer (1989) emphasized that the multiplicity under complete information was driven by the fact that agents' demand at non-equilibrium prices was indeterminate. They showed that introducing noise that pinned down best responses led to a unique equilibrium and thus determinate market power. They also showed that the equilibrium selected was independent of the shape of the noise, as the noise became small. They were thus able to offer a compelling prediction about market power. Our results show that their results rely on maintaining the private value assumption, which implies that agents do not learn from prices. We replicate the Klemperer and Meyer (1989) finding that small perturbations select a unique equilibrium but – by allowing for the possibility of a common value component of values – we can say nothing about market power in the perturbed equilibria.

Vives (2011) pioneered the study of asymmetric information under demand function competition, and we work in his setting of linear-quadratic payoffs and interdependent values. He studied a particular class of information structures where each trader observes a *one-dimensional normal* noisy signal of his own payoff type. The noise is represented by an idiosyncratic error term around his payoff type. We study what happens for *all multidimensional normal* information structures. In particular, we allow each trader to observe signals about the other traders' payoff types. Moreover, each multivariate signal can be either noise-free or noisy, and the noise term can have idiosyncratic or common components.

We show that the impact of asymmetric information on the equilibrium market power can even be larger than those derived from the one-dimensional signals studied in Vives (2011). Our results reverse some of the comparative statics and bounds that are found when using the specific class of one-dimensional signal structures. In particular, in this paper but not in Vives (2011) market power can be large even when the amount of asymmetric information is small; this holds regardless of the number of players, or the correlation of the payoff shocks. Rostek and Weretka (2012) and Rostek and Weretka (2015) relaxed the symmetry in the correlation of payoff shocks across agents, while maintaining the one-dimensional signal model of Vives (2011). This allows for a rich structure in the induced correlation of signals and large variation in market power. In our setting, the variation in market power arises through multidimensional signals despite maintaining symmetry in the correlation of payoff shocks across agents.

Our result demonstrating the extreme sensitivity of market power to the information structure has the same flavor as abstract game theory results establishing that fine details of the information structure can be chosen to select among multiple rationalizable or equilibrium outcomes of complete information games (Rubinstein (1989) and Weinstein and Yildiz (2007)). However this work relies on extremal information structures and, in particular, a "richness" assumption in Weinstein and Yildiz (2007), which in our context would require the strong assumption that

there exist "types" with a dominant strategy to submit particular demand functions.[1] Our results do not require richness and do exploit the structure of the demand function competition game. Moreover, we show that although it is impossible to make sharp predictions about market power, the complete information equilibria generate the maximum price volatility, thus allowing sharp predictions for the second moment.

2 Model

2.1 Payoff Environment

There are N agents who have demand for a divisible good. The utility of agent $i \in \{1, ..., N\}$ who buys $q_i \in \mathbb{R}$ units of the good at price $p \in \mathbb{R}$ is given by:

$$u_i(\theta_i, q_i, p) \triangleq \theta_i q_i - pq_i - \frac{1}{2}q_i^2,$$

where $\theta_i \in \mathbb{R}$ is the payoff shock of agent i. The payoff shock θ_i describes the marginal willingness to pay of agent i for the good at $q_i = 0$. The payoff shocks are symmetrically and normally distributed across the agents, and for any i, j:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta}\sigma_\theta^2 \\ \rho_{\theta\theta}\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}\right),$$

where $\rho_{\theta\theta}$ is the correlation coefficient between the payoff shocks θ_i and θ_j.

The *realized average* payoff shock among all the agents is denoted by:

$$\overline{\theta} \triangleq \frac{1}{N}\sum_{i \in N} \theta_i,$$

[1] Weinstein and Yildiz (2011) provide a similar result without requiring a richness condition. However, their results apply only for games with one-dimensional strategies and continuous and concave payoffs.

and the corresponding joint distribution of θ_i and $\overline{\theta}$ is given by

$$\begin{pmatrix} \theta_i \\ \overline{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \frac{1+(N-1)\rho_{\theta\theta}}{N}\sigma_\theta^2 \\ \frac{1+(N-1)\rho_{\theta\theta}}{N}\sigma_\theta^2 & \frac{1+(N-1)\rho_{\theta\theta}}{N}\sigma_\theta^2 \end{pmatrix} \right).$$

The supply of the good is given by an exogenous supply function $S(p)$ as represented by a linear inverse supply function with $\alpha, \beta \in \mathbb{R}_+$:

$$p(q) = \alpha + \beta q.$$

For notational simplicity, we normalize the intercept α of the affine supply function to zero.[2]

2.2 Information Structure

Each agent i observes a multidimensional signal $s_i \in \mathbb{R}^J$ about the payoff shocks:

$$s_i \triangleq (s_{i1}, ..., s_{ij}, ..., s_{iJ}).$$

The joint distribution of signals and payoff shocks $(s_1, ..., s_N, \theta_1, ..., \theta_N)$ is symmetrically and normally distributed. We discuss specific examples of information structures in the following sections.

2.3 Demand Function Competition

The agents compete via demand functions. Each agent i submits a demand function $x_i : \mathbb{R} \times \mathbb{R}^J \to \mathbb{R}$ that specifies the demanded quantity as a function of the market price $p \in \mathbb{R}$ and the private signal $s_i \in \mathbb{R}^J$, denoted by $x_i(p, s_i)$. The Walrasian auctioneer sets a price p^* such that the market clears for every realization of signal profiles s:

$$p^* = \beta \sum_{i \in N} x_i(p^*, s_i). \tag{1}$$

[2] The general affine case with $\alpha \neq 0$ is equivalent to a market with a different mean payoff shock μ_θ. Specifically, considering $\alpha \neq 0$ is mathematically equivalent to considering a model in which $\tilde{\alpha} = 0$ and $\tilde{\theta}_i = \theta_i - \alpha$.

If a market clearing price satisfying (1) does not exist, then we assume that there is a market shut-down; thus, $q_1 = \cdots = q_N = 0$. We note that in the class of linear equilibria we study, a market clearing price will always exist, so the aforementioned rule is mentioned only for completeness.

We study the Nash equilibrium of the demand function competition game. The strategy profile $(x_1^*, ..., x_N^*)$ forms a Nash equilibrium if:

$$x_i^* \in \underset{\{x_i:\mathbb{R}^{J+1}\to\mathbb{R}\}}{\arg\max}\ \mathbb{E}\left[\theta_i x_i(p^*, s_i) - p^* x_i(p^*, s_i) - \frac{x_i(p^*, s_i)^2}{2}\right], \tag{2}$$

where

$$p^* = \beta(x_i(p^*, s_i) + \sum_{j\neq i} x_j^*(p^*, s_{_j})).$$

We say that a Nash equilibrium $(x_1^*, ..., x_N^*)$ is linear and symmetric if there exists a vector of coefficients $(c_0, ..., c_J, m) \in \mathbb{R}^{J+2}$ such that for all $i \in N$:

$$x_i\,(p, s_i) = c_0 + \sum_{j\in J} c_j s_{ij} - mp.$$

Thus, the private information s_i of agent i determines the intercept of the demand curve, whereas the slope m of the demand curve–determined in equilibrium–is invariant with respect to the signal s_i. Throughout the paper we focus on symmetric linear Nash equilibria and so hereafter we drop the qualifications "symmetric" and "linear". When we say that an equilibrium is unique, we refer to uniqueness within this class of equilibria.

2.4 Equilibrium Statistics: Market Power and Price Volatility

We analyze the set of equilibrium outcomes in demand function competition under incomplete information. We frequently describe the equilibrium outcome through two central statistics of the equilibrium: *market power* and *price volatility*.

The marginal utility of agent i from consuming the q_i-th unit of the good is $\theta_i - q_i$. We define the market power of agent i as the agent's gross marginal utility minus the price divided by the

equilibrium price:

$$l_i \triangleq \frac{\theta_i - q_i - p}{p}.$$

This is the natural demand-side analogue of the supply-side price markup defined by Lerner (1934), commonly referred to as the "Lerner index." We define the (expected) *equilibrium market power* by:

$$l \triangleq \mathbb{E}\left[\frac{1}{N}\sum_{i\in N} l_i\right] = \frac{1}{N}\mathbb{E}\left[\frac{\sum_{i\in N}(\theta_i - q_i - p)}{p}\right]. \tag{3}$$

The market power l is defined as the expected average of the Lerner index across all agents. If the agents were price takers, then the marginal utility would be equal to the equilibrium price, $\theta_i - q_i = p$, and the market power would be $l = 0$.

A second equilibrium statistic of interest is *price volatility*, the variance of the equilibrium price:

$$\sigma_p^2 \triangleq \mathrm{var}(p).$$

Price volatility measures the ex ante uncertainty about the equilibrium price.

These two statistics of the equilibrium outcome, market power and price volatility, will completely describe the first and second moments of aggregate market outcomes. In more detail, the equilibrium market power l will determine the expected equilibrium price and the expected aggregate demand. Similarly, the variance of the equilibrium price σ_p^2 will determine the variance of the aggregate demand. Thus, within the linear-quadratic normal environment, these two statistics completely describe the aggregate equilibrium outcomes.

While most of our paper focuses on price volatility and market power, we will explain how our results extend to other statistics of an equilibrium outcome, such as the mean and variance of individual demand.

3 The Case of Complete Information

As previously discussed, the existence of multiple equilibria in demand function competition has long been established. Here, we focus on the implications that such multiplicity has for the induced market power and price volatility. With complete information every agent i observes the entire vector of payoff shocks $\theta = (\theta_1, ..., \theta_N)$ before submitting his demand $x_i(p, \theta)$. We use a brief review of the complete information setting to introduce some key ideas.

The residual supply faced by agent i, denoted by $r_i(p, \theta)$, is determined by the demand functions of all the agents other than i:

$$r_i(p, \theta) \triangleq S(p) - \sum_{j \neq i} x_j(p, \theta).$$

Agent i is a monopsonist over his residual supply. That is, if agent i submits demand $x_i(p, \theta)$, then the equilibrium price p^* satisfies $x_i(p^*, \theta) = r_i(p^*, \theta)$ for every i. Hence, agent i only needs to determine the optimal point along the curve $r_i(p, \theta)$; this will determine the quantity that agent i purchases and the equilibrium price.

To compute the first-order condition for agent i's demand, we define the *price impact* λ_i of agent i as the reciprocal of the residual demand derivative:

$$\frac{1}{\lambda_i} \triangleq \frac{\partial r_i(p, \theta)}{\partial p}. \tag{4}$$

This parameter is frequently referred to as "Kyle's lambda" following the seminal contribution of Kyle (1985). The price impact determines the rate at which the price increases when the quantity bought by agent i increases:

$$\lambda_i = \frac{\partial p}{\partial r_i(p, \theta)}.$$

The first-order condition of agent i determines the equilibrium demand of agent i:

$$x_i(p^*) = \frac{\theta_i - p^*}{1 + \lambda_i}.$$

Thus λ_i determines how much demand agent i withholds to decrease the price at which he purchases the good. For example, if $\lambda_i = 0$, then agent i behaves as a price taker. As λ_i increases, agent i withholds more demand to decrease the equilibrium price. Hence, λ_i determines the propensity of agent i to withhold demand to decrease the price. The following proposition establishes what the equilibrium outcomes with complete information in the linear-quadratic setting.

Proposition 1 (Equilibrium Statistics with Complete Information)
Market power and price volatility are induced by some complete information Nash equilibrium if and only if $\left(l, \sigma_p^2\right)$ *satisfy:*

$$l \geq -\frac{1}{2\beta N} \quad \textit{and} \quad \sigma_p^2 = \frac{(\beta N)^2}{(1+\beta N+\beta N l)^2}\sigma_{\tilde{\theta}}^2. \tag{5}$$

Proposition 1 can be established using the general arguments provided in Theorem 1 and the proof is relegated to the Appendix. In the complete information equilibrium, market clearing implies that market power l and price impact λ are related as follows:

$$l = \frac{\lambda}{\beta N}. \tag{6}$$

In the special case, when there is only one agent, then the market power is equal to 1 as the price impact is then given by the supply function itself, and $\lambda = \beta$. In this case, the agent fully internalizes the impact of increasing his demand on the total expenditure. The lower bound on market power in Proposition 1 indicates that a small amount of negative market power can occur. This happens when an agent faces a downward-sloping residual supply. However, the slope of the residual supply cannot be too inelastic because otherwise the agent would be able to achieve infinite utility by buying an arbitrarily large quantity at an arbitrarily low price. The relation between price volatility and market power is intuitive. As market power increases, every agent withholds more demand to lower the price. This leads to a smaller response to the payoff shocks and consequently, lower price volatility.

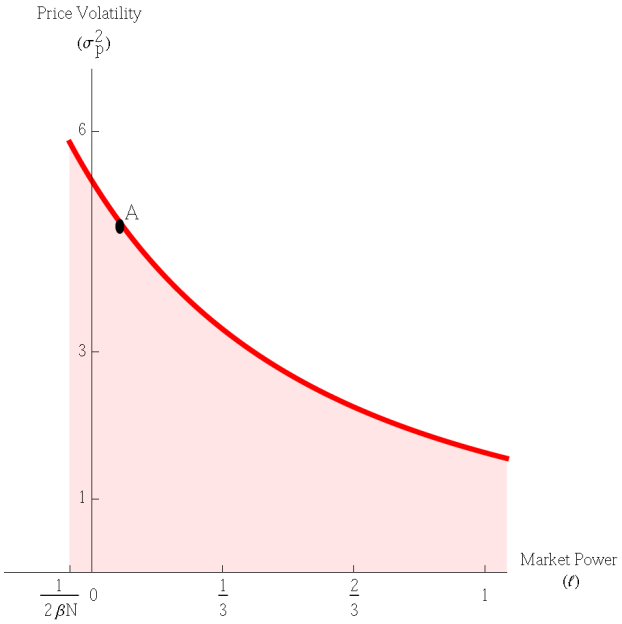

Figure 1: Set of equilibrium pairs $\left(l, \sigma_p^2\right)$ of market power and price volatility with complete information ($\beta = 3, N = 3$).

The reason for multiple equilibria has been thoroughly investigated in the literature (see Grossman (1981), Klemperer and Meyer (1989)). The key source of equilibrium multiplicity is that each agent has multiple best responses. In particular, there are multiple affine demand functions that agent i can submit that would intercept his residual supply at the same point (hence, inducing the same equilibrium price and quantities). Thus, agent i is indifferent between the multiple demand functions that intercept with his residual supply at the same point. Importantly, the slope of the *demand function of agent* i determines the slope of the *residual supply of agent* j.

In Figure 1 the bold red curve plots all feasible equilibrium pairs of market power and price volatility that can be attained under complete information. The point labeled A depicts the equilibrium outcome that would be attained under complete information if we selected the outcome using the equilibrium selection proposed by Klemperer and Meyer (1989).

The results in the next section will establish that the set of all possible pairs of market power

and price volatility is the set of pairs under this red curve established by the complete information equilibria, thus the area in light red under the boundary curve in bold red.

4 Robust Prediction of Market Power and Price Volatility

With incomplete information, market power and price volatility will be uniquely pinned down given a specific information structure. What robust predictions can then be made that do not depend on the fine details of the information structure? We will show that we cannot make any robust predictions about market power: any positive level of market power can arise as the unique equilibrium even when we restrict attention to arbitrarily small amounts of incomplete information. However, we can offer a sharp prediction about price volatility regardless of the amount of incomplete information, price volatility cannot be higher than that under complete information.

We say that an information structure is $\varepsilon-$close to complete information if the conditional variance of the estimate of each payoff shock θ_j is small given the signal s_i received by agent i:

$$\forall i, j \in N, \qquad \text{var}(\theta_j | s_i) < \varepsilon. \tag{7}$$

In an information structure that is ε-close to complete information an agent can observe his own payoff shock and the payoff shock of the other agents with a residual uncertainty of at most ε. If an information structure is $\varepsilon-$close to complete information for a sufficiently small ε, then the information structure will effectively be a perturbation of complete information. We now show that any equilibrium under complete information can be selected as the unique equilibrium in a perturbation of complete information.

We prove the result by decomposing the payoff shock θ_i into two independent payoff shocks:

$$\theta_i \triangleq \eta_i + \phi_i, \tag{8}$$

where the sets of payoff shocks $\{\eta_i\}_{i\in N}$ are independent of the shocks $\{\phi_i\}_{i\in N}$. The shocks are jointly normally distributed:

$$\mu_\eta = \mu_\phi = \mu_\theta/2 \quad \text{and} \quad \text{corr}(\eta_i, \eta_j) = \text{corr}(\phi_i, \phi_j) = \text{corr}(\theta_i, \theta_j), \tag{9}$$

and the variance of the shocks are:

$$\text{var}(\phi_i) = \varepsilon \quad \text{and} \quad \text{var}(\eta_i) = \sigma_\theta^2 - \varepsilon. \tag{10}$$

It follows from (9) and (10) that:

$$\text{var}(\phi_i + \eta_i) = \sigma_\theta^2 \quad \text{and} \quad \text{cov}(\phi_i + \eta_i, \phi_j + \eta_j) = \text{cov}(\theta_i, \theta_j).$$

Thus, the joint distribution of the random variables $\{\eta_i + \phi_i\}_{i\in N}$ is equal to the joint distribution of the original payoff shocks $\{\theta_i\}_{i\in N}$.

With respect to the information structure, we assume that every agent observes the realization of all shocks $\{\eta_i\}_{i\in N}$. Additionally, agent i observes a signal t_i that is equal to a weighted difference between his shock ϕ_i and the average of the shocks $\{\phi_j\}_{j\in N}$:

$$t_i = \phi_i - (1-\gamma)\frac{1}{N}\sum_{j\in N}\phi_j. \tag{11}$$

Thus, the weight $\gamma \in \mathbb{R}\backslash\{0\}$ serves to *confound* the payoff shocks ϕ_i with ϕ_j for all $j \neq i$. The signal vector s_i that is observed by agent i is then given by:

$$s_i = (t_i, \eta_1, ..., \eta_N). \tag{12}$$

We remark that under this information structure:

$$\forall i, j \in N, \quad \text{var}(\theta_i|\eta_1, ..., \eta_N, s_j) = \text{var}(\phi_i|s_j) \leq \text{var}(\phi_i) = \varepsilon.$$

It follows that under this information structure, (7) is satisfied. We refer to the information structure given by (12) as *noise-free signals* because the signals represent the payoff shocks (in a linear combination) without adding any extraneous noise.We discuss the role of the confounding parameter γ following the statement of our first main result.

Theorem 1 (Equilibrium Selection)

For every $\varepsilon > 0$ and every complete information equilibrium market power and price volatility (l, σ_p^2), there exists an information structure that is $\varepsilon-$close to complete information and induces (l, σ_p^2) as the unique equilibrium.

Proof. We now construct a symmetric linear Nash equilibrium using a "guess-and-verify" method. A linear demand function x^* is a symmetric Nash equilibrium if and only if it solves (1) and (2) for all i. In a linear Nash equilibrium $x^*(p^*, s_i)$ is linear in p^*, and so the first-order condition of (2) is given by:

$$x^*(p^*, s_i) = \frac{\mathbb{E}[\theta_i | p^*, s_i] - p^*}{1 + \lambda}, \tag{13}$$

where λ is the derivative of the inverse residual supply defined above in (4). We now write λ explicitly:

$$\lambda \triangleq \left(\frac{\partial r_i(p, s_i)}{\partial p} \right)^{-1} = \frac{\beta}{1 + \beta m(N-1)}. \tag{14}$$

The objective function of (2) is a quadratic function of $x(p^*, s_i)$ and the coefficient on the quadratic component is equal to $-(\lambda + 1/2)$. Thus, the second-order condition is satisfied if and only if $\lambda \geq -1/2$. It is clear that if $\lambda < -1/2$, then the objective function is strictly convex and hence (2) does not have a solution. Therefore, there is no equilibrium satisfying $\lambda < -1/2$.

We first establish the argument for $l \in \mathbb{R} \backslash \{0, 1\}$; we address the special cases of $l = 0$ and $l = 1$ at the end of the proof. In any linear Nash equilibrium, the equilibrium price must be a linear function of the shocks $\{\eta_i\}_{i \in N}$ and the signals $\{t_i\}_{i \in N}$. The symmetry of the conjectured equilibrium implies that there exist constants $\hat{c}_0, \hat{c}_1, \hat{c}_2$ such that the equilibrium price satisfies:

$$p^* = \hat{c}_0 + \hat{c}_1 \bar{\phi} + \hat{c}_2 \bar{\eta}.^3$$

Regardless of the values of $\hat{c}_0, \hat{c}_1, \hat{c}_2$, as long as $\hat{c}_1 \neq 0$, the following equation is satisfied:

$$\mathbb{E}[\theta_i | \{\eta_i\}_{i \in N}, t_i, p^*] = \theta_i. \tag{15}$$

[3]Recall that according to the notation already introduced $\bar{\eta} = \sum_{i \in N} \eta_i / N$ and $\bar{\phi} = \sum_{i \in N} \phi_i / N$.

That is, agent i can perfectly infer θ_i using the realization of the shocks $\{\eta_i\}_{i\in N}$, the signal t_i and the equilibrium price. This is because agent i can infer $\bar{\phi}$ from p^*, which in addition to t_i, allows agent i to perfectly infer ϕ_i (note that $\bar{\eta}$ is common knowledge). We now verify that there is no equilibrium in which $\hat{c}_1 = 0$. If $\hat{c}_1 = 0$, then $\mathbb{E}[\theta_i|\{\eta_i\}_{i\in N}, t_i, p^*] = \eta_i + \mathbb{E}[\phi_i|t_i]$, and so each agent will submit a demand function:

$$x^*(p^*, s_i) = \frac{\eta_i + \mathbb{E}[\phi_i|t_i] - p^*}{1+\lambda},$$

for some λ. Therefore, market clearing implies that:

$$\beta \sum_{i\in N} \frac{\eta_i + \mathbb{E}[\phi_i|t_i] - p^*}{1+\lambda} = p^*.$$

Thus,

$$p^* = \frac{1}{1+\beta N+\lambda}\beta \sum_{i\in N} \eta_i + \mathbb{E}[\phi_i|t_i].$$

However, $\sum_{i\in N} \mathbb{E}[\phi_i|t_i] \propto \bar{\phi}$ (recall that $\gamma \neq 0$, so $\sum_{i\in N} t_i \neq 0$). Thus, the market clearing price p^* must depend on $\bar{\phi}$, which contradicts $\hat{c}_1 = 0$.

Using (13) and (15), we conclude that, in equilibrium, agent i buys a quantity equal to:

$$x^*(p^*, s_i) = q_i^* = \frac{\theta_i - p^*}{1+\lambda}, \tag{16}$$

for some $\lambda \geq -1/2$. The market clearing condition implies that $p^* = \beta \sum q_i^*$, and so the equilibrium price is given by:

$$p^* = \frac{\beta N \bar{\theta}}{1+\lambda+\beta N}, \tag{17}$$

for some $\lambda \geq -1/2$.

Given the expression for the equilibrium price in (17), we note that:

$$\mathbb{E}[\theta_i|p^*, t_i, \{\eta_i\}_{i\in N}] = t_i + \eta_i + (1-\gamma)\left(\frac{p^*}{\beta N}(1+\lambda+\beta N) - \bar{\eta}\right) = \theta_i.$$

Recall that in equilibrium, agent i submits demand function (13), so the slope of the demand submitted by agent i is given by:

$$m = -\frac{\partial x_i(p)}{\partial p} = \frac{-1}{1+\lambda}\left(\frac{\partial \mathbb{E}[\theta_i|p^*, t_i, \{\eta_i\}_{i\in N}]}{\partial p^*} - 1\right) = \frac{1-(1-\gamma)\frac{1}{\beta N}(1+\lambda+\beta N)}{1+\lambda}.$$

This gives a relation between agent i's price impact (i.e., λ) and the slope of the demand function submitted by agent i (i.e., m). Using these equations and (14), we find λ in terms of the confounding parameter γ:

$$\lambda = \frac{1}{2}\left(-1 - N\beta\frac{\gamma(N-1)-1}{\gamma(N-1)+1} \pm \sqrt{\left(N\beta\frac{\gamma(N-1)-1}{\gamma(N-1)+1}\right)^2 + 2N\beta + 1}\right). \tag{18}$$

Only the positive root is a valid solution because the negative root yields λ less than $-1/2$. Hence, for every γ, there is a unique linear Nash equilibrium. In this equilibrium, the price impact is equal to the positive root of (18), and each agent i submits a demand function:

$$x(p, s_i) = \frac{1}{1+\lambda}\left(t_i + \eta_i + (1-\gamma)\left(\frac{p}{\beta N}(1+\lambda+\beta N) - \bar{\eta}\right) - p\right).$$

We note that this demand function is equal to (13), so the demand of every agent satisfies the first-order condition by construction and $\lambda \geq -1/2$, so the second-order condition is also satisfied. This shows that this is a Nash equilibrium.

We note that for all $\lambda \geq -1/2$, there exists γ that satisfies (18) (as a positive root). To verify this, note that (18) is continuous as a function of γ except at $\gamma = -1/(N-1)$. Moreover, at $\gamma = -1/(N-1)$, the right limit is $+\infty$ while the left limit is $-1/2$. Since (18) is equal to 0 in the limits $\gamma \to \pm\infty$, we have that every $\lambda \in [-1/2, \infty)$ is achieved by some γ.

From (17), it follows directly that in equilibrium, the price volatility is:

$$\sigma_p^2 = \left(\frac{\beta N}{1+\lambda+\beta N}\right)^2 \sigma_{\bar{\theta}}^2.$$

Using the definition of market power (3), the expression for price (17) and the expression for q_i (16) we obtain that:

$$l = \frac{1}{N}\mathbb{E}\left[\frac{\sum_{i\in N}\left(\theta_i - \frac{\theta_i - \frac{\beta N\bar{\theta}}{1+\lambda+\beta N}}{1+\lambda} - \frac{\beta N\bar{\theta}}{1+\lambda+\beta N}\right)}{\frac{\beta N\bar{\theta}}{1+\lambda+\beta N}}\right] = \mathbb{E}\left[\frac{\left(\bar{\theta} - \frac{\bar{\theta} - \frac{\beta N\bar{\theta}}{1+\lambda+\beta N}}{1+\lambda} - \frac{\beta N\bar{\theta}}{1+\lambda+\beta N}\right)}{\frac{\beta N\bar{\theta}}{1+\lambda+\beta N}}\right].$$

Simplifying terms, we obtain the following:

$$l = \frac{\lambda}{\beta N}. \tag{19}$$

Thus, we have that $l \geq -1/(2\beta N)$ and price volatility can be written as a function of market power. Recall that we have in fact constructed a linear Nash equilibrium in which every agent i submits a demand function given by (13), which by construction satisfied the agent's first-order condition, and we have also shown that the second-order condition is satisfied.

Finally, we address the cases of $l = 0$ and $l = 1$, which correspond to the cases in which the confounding parameter is either $\gamma = \infty$ or $\gamma = 0$, respectively. An equilibrium with market power $l = 1$ and price volatility given by (5) can be attained by the following information structure. Every agent observes $\{\eta_i\}_{i\in N}$ and $\{(\theta_i - \bar{\phi})\}_{i\in N}$, and additionally agent i privately observes signal $t_i = \bar{\phi} + \varepsilon_i$, where ε_i is an error term normally distributed with variance one and correlation across agents equal to $-1/(N-1)$. Since the errors are perfectly negatively correlated, this implies that $\sum_{i\in N} \varepsilon_i = 0$. Note that agent i knows η_i and $(\phi_i - \bar{\phi})$, so by virtue of observing only his own signals, he does not know $\bar{\phi}$.[4] We also note the following:

$$\mathbb{E}[\bar{\phi} \mid t_i] = \frac{1}{N}\sum_{j\in N} t_j, \ \forall t_i, \forall i \in N. \tag{20}$$

That is, $\bar{\phi}$ can be inferred perfectly by averaging over signals $\{t_j\}_{j\in N}$.

An equilibrium with $l = 0$ and price volatility given by (5) is attained by considering the following information structure. Every agent knows $\{\eta_i\}_{i\in N}$ and $\bar{\phi}$, and additionally agent i privately observes a signal $t_i = \phi_i - \bar{\phi} + \varepsilon$, where ε is a common error term that is normally distributed with variance one. Note that agent i knows η_i and $\bar{\phi}$, so by virtue of observing only his own signals, he cannot know $\phi_i - \bar{\phi}$. We also note the following:

$$\mathbb{E}[\phi_i - \bar{\phi} \mid t_i] = t_i - \frac{1}{N}\sum_{j\in N} t_j, \quad \forall t_i, \forall i \in N. \tag{21}$$

That is, $\phi_i - \bar{\phi}$ can be inferred perfectly by subtracting from i the average of the signals $\{t_i\}_{i\in N}$.

Using the two aforementioned information structures, one can construct a unique linear equilibrium in which $l = 1$ and $l = 0$. The equilibrium construction is the same as before, so we will

[4] Agent i cannot infer $\bar{\phi}$ by observing $\{(\phi_i - \bar{\phi})\}_{i\in N}$, as by construction the sum of all signals $\{(\phi_i - \bar{\phi})\}_{i\in N}$ is equal to zero. Thus, knowing $\{(\phi_i - \bar{\phi})\}_{i\in N}$ is equivalent to knowing only $(\phi_i - \bar{\phi})$.

not repeat the steps. However, we note that for the noise-free signals we constructed in (11), we have that:

$$\mathbb{E}[\phi_i \mid \{t_i\}_{i\in N}] = t_i + \frac{1-\gamma}{\gamma}\frac{1}{N}\sum_{j\in N} t_j. \tag{22}$$

The two special cases of $\gamma = 0$ and $\gamma = \infty$ correspond intuitively to the cases in which the expectations are given by (20) and (21), respectively.[5] ■

Theorem 1 shows that all combinations of market power and price volatility that can be achieved as an equilibrium under complete information can also be achieved as a unique equilibrium in an information structure that is close to complete information. In fact, the result is stronger: every equilibrium outcome under complete information is the unique equilibrium outcome of an information structure that is close to complete information.

To provide intuition for Theorem 1 it is useful to consider a *one-dimensional* version of the noise-free signals described above in (12). Consider a situation in which every agent i observes a one-dimensional signal:

$$s_i = \theta_i - (1-\gamma)\frac{1}{N}\sum_{j\in N}\theta_j. \tag{23}$$

This one-dimensional information structure is a particular instance of the earlier information structure (12). Now, we remove the restriction on the variance of ϕ_i, which we required to satisfy $\text{var}(\phi_i) \leq \varepsilon$. In fact, the one-dimensional signal (23) sets $\phi_i = \theta_i$, and so $\text{var}(\phi_i) = \sigma_\theta^2$ and $\text{var}(\eta_i) = 0$. While this one-dimensional information structure clearly cannot approximate complete information, we now establish that it *can* attain the complete information equilibrium outcome.

Corollary 1 (Equilibrium Selection with One-Dimensional Signals)

For every complete information equilibrium market power and price volatility (l, σ_p^2) there exists a one-dimensional information structure as in (23) *that induces (l, σ_p^2) as the unique equilibrium.*

[5] The right-hand side of (22) diverges in the limit as $\gamma \to 0$. However, what is relevant for the analysis is that in this limit $\mathbb{E}[\phi_i \mid \{t_i\}_{i\in N}] \propto \frac{1}{N}\sum_{j\in N} t_j$, which is satisfied in (20).

By changing the weight γ that signal s_i places on an agent's own payoff shock θ_i relative to the other agents' payoff shocks, one can affect the (perceived) degree of payoff interdependence between the agents. To see this, note that the expected payoff of agent i conditional on all signals $\{s_i\}_{i\in N}$ is given by:

$$\mathbb{E}[\theta_i \mid \{s_i\}_{i\in N}] = s_i + \frac{1-\gamma}{\gamma}\frac{1}{N}\sum_{j\in N} s_j. \tag{24}$$

The conditional expectation of θ_i now places weights on both the signal s_i observed by agent i and the signals observed by other agents, $\{s_j\}_{j\neq i}$. By changing γ, the relative weight that the conditional expectation places on agent's i signal relative to the signals of others changes.

Of course, agent i only observes his own private signal s_i and not the private signals of other agents. However, the equilibrium price allows agent i to deduce the average of the signals observed by the other agents. Hence, each agent uses the equilibrium price as a signal about his own payoff shock. With a change in the confounding parameter γ, the weight that agent i places on the price to predict his own payoff shock changes, which in turn changes the equilibrium degree of market power. We illustrate the relation between γ and the equilibrium degree of market power in Figure 2.[6]

Our construction uses an $(N+1)$-dimensional signal. Given that there are N different shocks in our model, in any information structure that is close to complete information, agents must observe at least N different signals. Therefore, any information structure that is close to complete information must consist of at least N signals.[7]

When agents observe one-dimensional signals as in (23), agents will not be close to complete information. However, an agent's private information plus the information that can be deduced

[6] The idea that confounding informational shocks can strongly influence equilibrium behavior goes back at least to Lucas (1972). In a seminal contribution, he shows how monetary shocks can have a real effect in the economy, even when under complete information monetary shocks would have no real effect.

[7] The information structure that we use to prove Theorem 1 generates an $N+1$-dimensional signal. However, it is possible to prove this theorem using an N-dimensional signal. Consider an information structure in which agent i observes a signal as in (23) and, additionally, $N-1$ signals given by $s_{ij} = \theta_j + \varepsilon_{ij}$, for every $j \neq i$, where

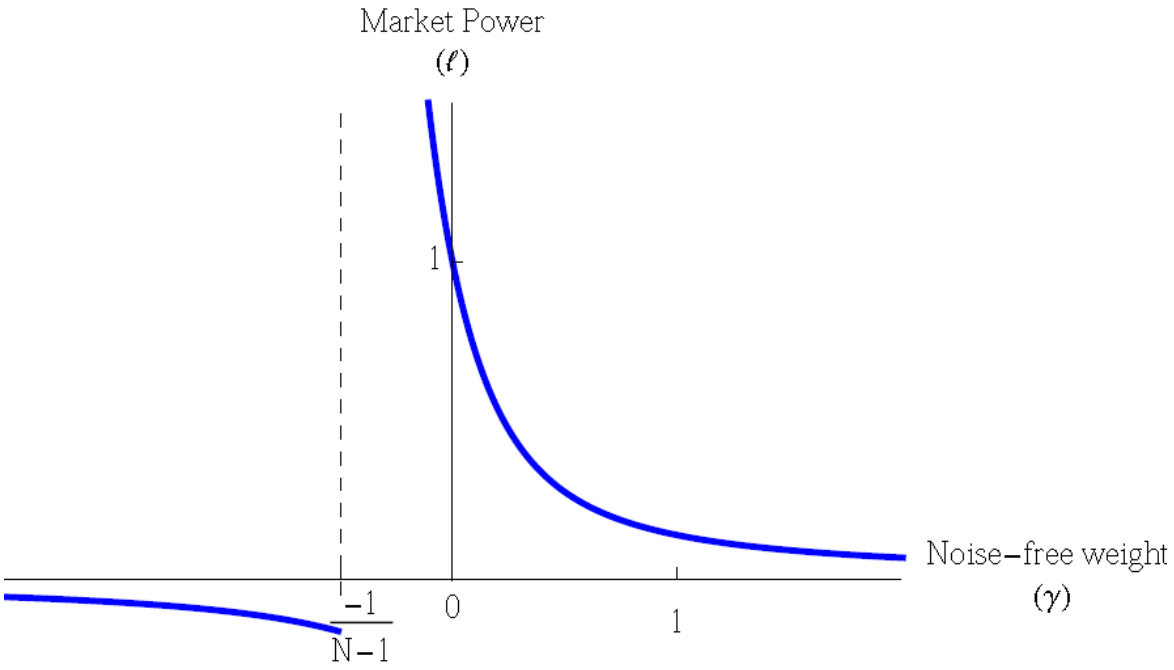

Figure 2: Set of equilibrium market power l with noise-free signal parametrized by γ.

from knowing the equilibrium price is sufficient to deduce this agent's payoff shock and the average payoff shock across all agents. Hence, despite the fact that agents have non-negligible uncertainty about their own payoff shock and the average payoff shock, every agent perfectly knows his own payoff shock and the average shock of all agents ex post (i.e., after learning the equilibrium price).

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all noise terms are independently normally distributed. First, note the following:

$$\tilde{s}_i \triangleq \frac{1}{1-(\gamma-1)/N} s_i - \frac{(\gamma-1)/N}{(1-(\gamma-1)/N)} \sum_{j \neq i} s_{ij} = \theta_i + \frac{(\gamma-1)/N}{(1-(\gamma-1)/N)} \sum_{j \neq i} \varepsilon_{ij}.$$

Agent i can infer $\tilde{s}_i$ by taking a linear combination of the N signals he observes. Therefore, if the noise terms $\{\varepsilon_{ij}\}_{j \neq i}$ are small enough, each agent will be ε-close to complete information. With this N-dimensional signal, there is an equilibrium that is equivalent to that in which agents observe only signal (23); that is, there is an equilibrium in which the $N-1$ additional signals are ignored. Therefore, this N-dimensional signal supports every market power and price volatility (l, σ_p^2) satisfying (5). Additionally, since this is an ex post equilibrium in which each agent can perfectly infer θ_i ex post, the results in Heumann (2018) imply that this will be the unique equilibrium. Hence, this $N-$dimensional signal would suffice to prove Theorem 1.

Theorem 1 shows that (i) all equilibrium outcomes under complete information can turn into unique equilibrium outcomes under incomplete information, and (ii) restricting attention to information structures close to complete information does not allow us to provide sharper predictions about market power and price volatility. The large indeterminacy in the set of possible outcomes suggests that it is difficult to offer robust predictions of market power under demand function competition. By contrast, it is possible to provide sharp predictions regarding price volatility with demand function competition.

We consider the earlier decomposition of the payoff shocks as given by (8) and (9). By contrast to the earlier construction, we now allow the variance of the decomposed payoff shocks ϕ_i and η_i to vary. In particular, the variance $\text{var}(\phi_i)$ is no longer restricted anymore to be less than ε:

$$\text{var}(\phi_i) = \sigma_p^2 \frac{N}{\rho_{\theta\theta}(N-1)+1} \frac{(1+\beta N(1+l))^2}{(\beta N)^2} \quad \text{and} \quad \text{var}(\eta_i) = \sigma_\theta^2 - \text{var}(\phi_i). \tag{25}$$

We assume that each agent receives no information about the realization of the shocks $\{\eta_i\}_{i\in N}$ and that each agent simply observes the following *one-dimensional* signal about the payoff shocks $\{\phi_i\}_{i\in N}$:

$$t_i = \phi_i - (1 - \frac{(l\beta N+1)(\beta N - l\beta N)}{\beta N l(N-1)(\beta N + l\beta N + 1)}) \frac{1}{N} \sum_{j\in N} \phi_j.$$

Note that the signal t_i here has the same structure as the signal t_i that we used in the proof of Theorem 1, and we simply replace the weight γ with the fraction; thus:

$$\gamma = \frac{(l\beta N+1)(\beta N - l\beta N)}{\lambda(N-1)(\beta N + l\beta N + 1)}.$$

Theorem 2 (Equilibria Under All Information Structures)

There exists an information structure that induces a pair of market power and price volatility (l, σ_p^2) *if and only if:*

$$l \geq -\frac{1}{2}\frac{1}{\beta N} \quad \textit{and } \sigma_p^2 \leq \frac{(\beta N)^2}{(1+\beta N(1+l))^2}\sigma_{\bar{\theta}}^2. \tag{26}$$

Moreover, all feasible pairs (l, σ_p^2) *are induced by a unique equilibrium for some information structure.*

Proof. We prove necessity and sufficiency separately and start with necessity.

We established in Theorem 1 that in any linear Nash equilibrium, an agent's demand is given by (13). Adding (13) over all agents and multiplying by β, we obtain:

$$\beta \sum_{i \in N} x^*(s_i, p^*) = \beta \sum_{i \in N} \frac{\mathbb{E}[\theta_i | s_i, p^*] - p^*}{1+\lambda}.$$

Market clearing implies that $\beta \sum_{i \in N} x^*(s_i, p^*) = p^*$. It follows that

$$p^* = \beta \sum_{i \in N} \frac{\mathbb{E}[\theta_i | s_i, p^*] - p^*}{1+\lambda}.$$

Rearranging terms, we obtain:

$$p^* = \frac{\beta N}{1+\lambda+\beta N}\frac{1}{N}\sum_{i \in N} \mathbb{E}[\theta_i | s_i, p^*]. \tag{27}$$

Taking the expectation of the previous equation conditional on p^* (i.e., taking the expectation $\mathbb{E}[\cdot|p^*]$) and using the law of iterated expectations, we obtain:

$$p^* = \frac{\beta N}{1+\lambda+\beta N}\frac{1}{N}\sum_{i \in N} \mathbb{E}[\theta_i | p^*] = \frac{\beta N}{1+\lambda+\beta N}\mathbb{E}[\frac{1}{N}\sum_{i \in N} \theta_i | p^*].$$

It follows that:

$$\sigma_p^2 = \left(\frac{\beta N}{1+\lambda+\beta N}\right)\text{cov}(p, \bar{\theta}) = \left(\frac{\beta N}{1+\lambda+\beta N}\right)\rho_{p\bar{\theta}}\sigma_p\sigma_{\bar{\theta}}.$$

Thus, we have that:

$$\sigma_p^2 = \left(\frac{\beta N}{1+\lambda+\beta N}\right)^2 \rho_{p\bar{\theta}}^2\sigma_{\bar{\theta}}^2. \tag{28}$$

We now prove that $\lambda = l\beta N$ in every linear equilibrium. We write market power (3) as follows:

$$l = \frac{1}{N}\mathbb{E}\left[\frac{1}{p}\mathbb{E}\left[\sum_{i\in N}(\theta_i - q_i - p) \mid p\right]\right], \tag{29}$$

where we used the fact that the law of iterated expectations implies that $\mathbb{E}[\cdot] = \mathbb{E}[\mathbb{E}[\cdot \mid p]]$. The first-order condition (13) implies that:

$$q_i = \frac{\mathbb{E}[\theta_i|p, s_i] - p}{1+\lambda}.$$

Substituting q_i back into equation (29) for market power, we obtain:

$$l = \frac{1}{N}\mathbb{E}\left[\frac{1}{p}\mathbb{E}\left[\sum_{i\in N}\left(\theta_i - \frac{\mathbb{E}[\theta_i|p, s_i] - p}{1+\lambda} - p\right) \mid p\right]\right].$$

Using the law of iterated expectations, we obtain that $\mathbb{E}[\mathbb{E}[\theta_i|p, s_i] \mid p] = \mathbb{E}[\theta_i|p]$. Simplifying terms, we obtain:

$$l = \frac{1}{N}\mathbb{E}\left[\frac{1}{p}\sum_{i\in N}\frac{\lambda}{1+\lambda}\left(\mathbb{E}\left[\theta_i \mid p\right] - p\right)\right]. \tag{30}$$

Using (27), we obtain the following:

$$Np\frac{1+\lambda+\beta N}{\beta N} = \mathbb{E}[\sum_{i\in N}\theta_i|p].$$

Substituting the conditional expectation of the payoff shock back into (30), we have that:

$$l = \mathbb{E}\left[\frac{1}{p}\frac{\lambda}{1+\lambda}\left(\frac{1+\lambda+\beta N}{\beta N}p - p\right)\right].$$

Here, p cancels out, so we can omit the expectation on the right-hand side of this equation. Simplifying terms, we obtain that $\lambda = l\beta N$.

Since $\rho^2_{p\bar{\theta}} \leq 1$, we have that (28) implies that:

$$\sigma_p^2 \leq \frac{(\beta N)^2}{(1+\beta N(1+l))^2}\sigma^2_{\bar{\theta}}.$$

Moreover, in the proof of Theorem 1, we also established that in any linear Nash equilibrium $\lambda \geq -1/2$ and thus that $l \geq -1/(2\beta N)$.

Sufficiency. Let (l, σ_p^2) be such that (26) is satisfied. We show that there exists an information structure that induces this market power and price volatility as a unique equilibrium. We can now construct a linear equilibrium in the same way as in the proof of Theorem 1, so we obtain that the price volatility is:

$$\frac{(\beta N)^2}{(1+\beta N+\beta N l)^2}\text{var}(\frac{1}{N}\sum_{i\in N}\phi_i) = \frac{(\beta N)^2}{(1+\beta N+\beta N l)^2}\frac{\rho_{\phi\phi}(N-1)+1}{N}\text{var}(\phi_i).$$

Since $\text{var}(\phi_i)$ is defined as in (25) and $\rho_{\phi\phi} = \rho_{\theta\theta}$, the previous equation implies that the price volatility is given by σ_p^2. Moreover, market power is l. Thus, (l, σ_p^2) is induced as an equilibrium outcome, and it is the unique equilibrium, as we already established in Theorem 1. ■

Theorem 2 provides a sharp bound on all possible equilibrium outcomes. It shows that the equilibrium outcome is bounded by the outcomes that are achieved under complete information. Thus, the outcomes that arise under complete information can be seen as the "upper boundary" of the set of outcomes that can arise under all information structures, as illustrated above in Figure 1.

The "if" part of the statement closely resembles the proof of Theorem 1. In particular, the set of market power and price volatility that satisfy (26) would be achieved under complete information if one could reduce the variance of the aggregate shocks (i.e., by making $\text{var}(\overline{\theta})$ smaller). By decomposing the payoff shocks into an observable and an unobservable component, we can effectively achieve the same outcomes as if there were complete information but the variance of the shocks were smaller. We note that $\text{var}(\eta_i)$ (as defined in (25)) is always positive because Theorem 2 states that:

$$\sigma_p^2 \leq \frac{(\beta N)^2}{(1+\beta N(1+l))^2}\sigma_{\overline{\theta}}^2,$$

and so $\text{var}(\phi_i)$ (as defined in (25)) is less than or equal to $\text{var}(\theta_i)$.

The "only if" part of the statement is economically more interesting because it uses the restrictions that arise from the agents' first-order condition. By aggregating the agents' demands

and using the market clearing condition, we can establish that the equilibrium price satisfies:

$$p^* = \frac{\beta}{1+\beta N(1+l)} \sum_{i \in N} \mathbb{E}[\theta_i | s_i, p^*]. \tag{31}$$

That is, the equilibrium price is proportional to the average of the agents' expected payoff shocks. Note that the expectation includes the information that can be deduced from observing the equilibrium price. Hence, the price reflects the agents' valuation, and the price is consistent in the sense that the expectations also include the information that can be deduced from observing the equilibrium price.

Taking expectations of (31) conditional on p^* and using the law of iterated expectations, we can write the equilibrium price as follows:

$$p^* = \frac{\beta N}{1+\beta N(1+l)} \mathbb{E}[\bar{\theta} | p^*]. \tag{32}$$

Since (32) relates p^* to the expectation of $\bar{\theta}$ conditional on p^*, it follows that the variance of p^* is directly related to the variance of $\bar{\theta}$ and the correlation between p^* and $\bar{\theta}$.[8]

It is crucial for the argument that the expected payoff shock of agent i is computed conditional on the equilibrium price — this is an implication of the fact that agents compete on demand functions and hence agent i can condition the quantity he buys on the equilibrium price. The fact that an agent can condition on the equilibrium price disciplines beliefs, which ultimately allows us to bound price volatility. This allows us to relate p^* to the average payoff shock $\bar{\theta}$ (as in (32)), instead of p^* being related only to the average of the agents' expected payoff shocks (as in (31)).

An interesting conclusion that one can derive from the proof of Theorem 2 is that the expected price is uniquely determined by the equilibrium degree of market power. Specifically, (27) implies that:

$$\mu_p = \frac{\beta N}{1+\lambda+\beta N} \mu_\theta.$$

[8] For any two random variables (y, z), if $y = \mathbb{E}[z|y]$, then $\sigma_y = \rho_{yz}\sigma_z$.

Thus, the equilibrium degree of market power l and the price volatility describe the first and second moments of the equilibrium price. By extension, they also describe the first and second moment of aggregate demand. This in turn allows us to describe other statistics of the equilibrium outcome. For example, the expected revenue is given by:

$$\mathbb{E}[p\sum_{i\in N} q_i] = \mathbb{E}[\frac{p^2}{\beta}] = \frac{1}{\beta}(\mu_p^2 + \sigma_p^2).$$

Here, we used the fact that $\sum_{i\in N} q_i = p/\beta$ by the market clearing condition. Since Theorem 2 provides bounds on price volatility and market power and market power determines the mean price, Theorem 2 also provides bounds on the equilibrium revenue of the seller.

5 Parametrized Information Structures

We now study parametrized classes of information structures. We first consider two different forms of one-dimensional signals about the payoff shock. These information structures impose tighter lower and upper bounds on market power and lead to qualitatively very different relationships between market power and volatility. We then consider a class of information structures with multidimensional signals and show that while it maintains the tighter restriction on market power, it also replicates in a parameterized class the feature that we can approximate extremal complete information market power and price volatility pairs with arbitrarily small incomplete information and thus hints that our main results do not rely on outlandish information structures.

One-Dimensional Signal of Payoff Shock The first information structure consists of a *one-dimensional signal* about the payoff shock θ_i, which was the case studied by Vives (2011). For completeness, we show how the model fits into our framework. Each agent i observes his payoff shock with conditionally independent noise; thus, the *noisy one-dimensional* signal is:

$$s_i = \theta_i + \varepsilon_i, \tag{33}$$

where the noise terms $\{\varepsilon_i\}_{i\in N}$ are independent normal with variance σ_ε^2. This class of information structure is therefore parametrized by the variance $\sigma_{\varepsilon_\omega}^2$ of the noise term $\varepsilon_{i\omega}$.

The equilibrium strategies can be computed using a guess-and-verify approach (as we did in the proof of Theorem 1), and for every σ_ε, there is a unique equilibrium. Given the equilibrium strategies, we can then compute equilibrium market power and price volatility as in Theorem 1.

To gain some intuition for how the noise term changes market power, we consider the conditional expectation of θ_i given the realization of *all* signals:

$$\mathbb{E}[\theta_i \mid \{s_i\}_{i\in N}] = \frac{(1-\rho_{\theta\theta})\sigma_\theta^2}{(1-\rho_{\theta\theta})\sigma_\theta^2+\sigma_\varepsilon^2}\left(s_i + \frac{1-\gamma}{\gamma}\frac{1}{N}\sum_{j\in N} s_j\right), \tag{34}$$

where

$$\gamma = \frac{(1-\rho_{\theta\theta})\sigma_\theta^2}{(\rho_{\theta\theta}N+(1-\rho_{\theta\theta}))\sigma_\theta^2}\frac{(\rho_{\theta\theta}N+(1-\rho_{\theta\theta}))\sigma_\theta^2+\sigma_\varepsilon^2}{(1-\rho_{\theta\theta})\sigma_\theta^2+\sigma_\varepsilon^2}. \tag{35}$$

We obtain an expression that is identical to (24), except that the expectation is multiplied by a constant. While agent i only observes s_i, the conditional expectation (34) represents the equilibrium inference of agent i because he can deduce the average signal of other agents from observing the equilibrium price.

We can then compute the equilibrium market power using the calculations developed in the proof of Theorem 1:

$$\hat{l}(\gamma) \triangleq \frac{1}{2\beta N}\left(-N\beta\frac{(N-1)\gamma-1}{(N-1)\gamma+1} - 1 + \sqrt{\left(N\beta\frac{(N-1)\gamma-1}{(N-1)\gamma+1}\right)^2 + 2\beta N + 1}\right). \tag{36}$$

We note that when agents observe signals about payoff shocks (23), then the equilibrium market power is given by $l = \hat{l}(\gamma)$ (this can be deduced from (18) and (19)). We obtain an expression for price volatility similar to (28) but in which we replace $\lambda = \beta N l$:

$$\sigma_p^2 = \left(\frac{\beta N}{1+\beta N(1+\hat{l}(\gamma))}\right)^2 \rho_{p\bar{\theta}}^2\sigma_{\bar{\theta}}^2. \tag{37}$$

Here, $\rho_{p\bar{\theta}}^2$ is the correlation between the equilibrium price and the average payoff shock:

$$\rho_{p\bar{\theta}}^2 = \frac{(\rho_{\theta\theta}N+(1-\rho_{\theta\theta}))\sigma_\theta^2}{(\rho_{\theta\theta}N+(1-\rho_{\theta\theta}))\sigma_\theta^2+\sigma_\varepsilon^2}. \tag{38}$$

We can summarize our discussion in the following corollary, which establishes a tighter range of possible market power levels.

Corollary 2 (One-Dimensional Signal About Payoff Shock)

There exists an information structure σ_ε^2 that induces market power and price volatility (l, σ_p^2) as a unique equilibrium if and only if market power l is given by (34)-(36) *and price volatility is given by* (37)-(38)*. In particular, market power $l \in \left(\hat{l}(1), 1\right) \subset (0, 1)$.*

The noise-free signals and the class of noisy one-dimensional signals (33) have the common feature that in equilibrium prices are privately revealing. This means that the information contained in the price plus the private signals observed by agent i are a sufficient statistic for all the signals observed by every agent to predict θ_i.

In Figure 3, we illustrate how market power and price volatility change with the variance σ_ε^2 of the noise term. We present the equilibrium behavior for different correlation coefficients $\rho_{\theta\theta}$ of the payoff shocks θ_i and θ_j.

Point A corresponds to the outcome when $\sigma_\varepsilon^2 = 0$: every agent knows his own payoff shock but remains uncertain about the payoff shocks of other agents. Market power is increasing in σ_ε^2 and price volatility is decreasing in σ_ε^2. Price volatility decreases at a faster rate (as a function of market power) than under complete information. Therefore, the relation between market power and price volatility is different from the one that arises under complete information.

As the payoff shocks become more correlated (i.e., as $\rho_{\theta\theta}$ increases), market power increases. One easy way to verify this is to note that the weight γ given by (35) is decreasing in $\rho_{\theta\theta}$. In the limit in which the payoff shocks are perfectly correlated γ converges to zero, and consequently market power converges to 1. This is the maximum market power that can be attained with a one-dimensional noisy signal about payoff shocks.

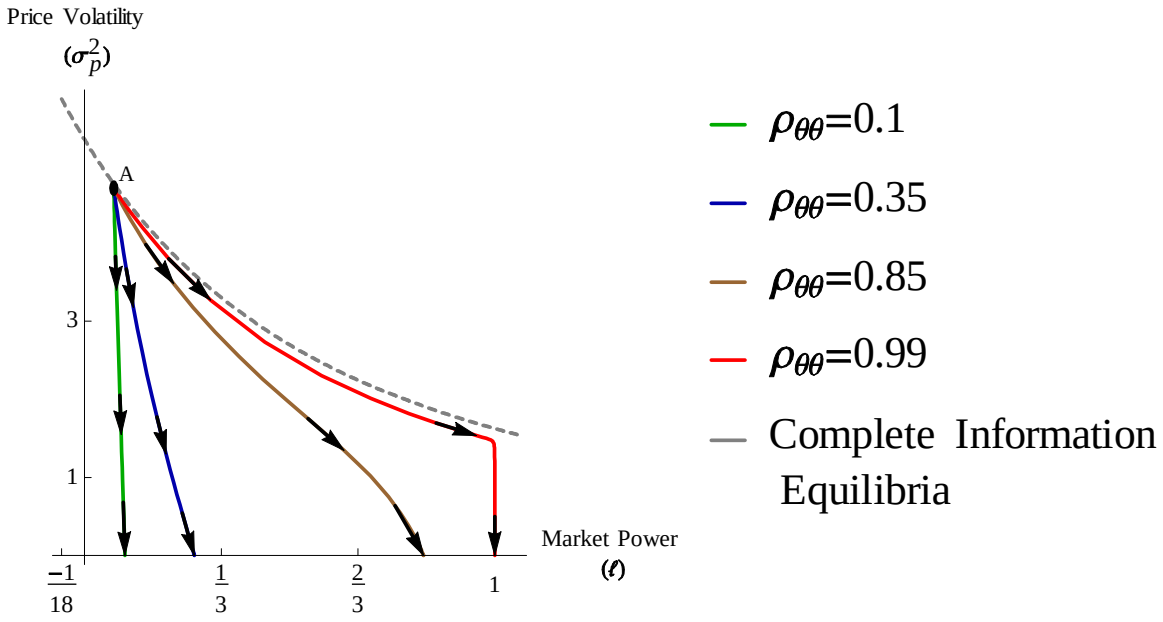

Figure 3: Set of equilibrium pairs $\left(l, \sigma_p^2\right)$ of market power and price volatility under signals about payoff shocks.

One-Dimensional Signal About Common Payoff Shock Our next example highlights that different one-dimensional signals will give rise to different implications for market power and price volatility.

We allowed the individual payoff shocks θ_i and θ_j to be positively correlated. One plausible reason for this is that the individual payoff shocks reflect common and idiosyncratic components. This suggests that we decompose the individual payoff shocks into a common and an idiosyncratic component, ω and τ_i, respectively:

$$\theta_i = \omega + \tau_i,$$

where ω and $\{\tau_i\}_{i \in N}$ are normally distributed and independent of each other.[9] It is now natural

[9] Given that θ_i are normally distributed with mean 0, standard deviation σ_θ and correlation $\rho_{\theta\theta}$, the decomposition leads ω and τ_i to be independently normally distributed with mean 0 and standard deviations $\sigma_\omega^2 = \rho_{\theta\theta}\sigma_\theta^2$ and $\sigma_\tau^2 = (1 - \rho_{\theta\theta})\sigma_\theta^2$, respectively. Observe that the variance of the realized average payoff shock $\overline{\theta}$ can now be written as $\sigma_{\overline{\theta}}^2 = \text{var}(\omega + \frac{1}{N}\sum \tau_i) = \sigma_\omega^2 + \sigma_\tau^2/N$.

to allow information to reflect common and idiosyncratic components in different ways.

In the second class of information structure, each agent i now receives a *noisy one-dimensional signal* about the common component ω:

$$s_{i\omega} = \omega + \varepsilon_{i\omega},$$

where the noise terms are again independent normal with variance $\sigma^2_{\varepsilon_{i\omega}}$. By contrast, the idiosyncratic components $\{\tau_i\}_{i\in N}$ are assumed to be public information. This class of information structure is therefore parametrized by the variance $\sigma^2_{\varepsilon_{i\omega}}$ of the noise term $\varepsilon_{i\omega}$.

The expectation of agent i's payoff shock conditional on all the signals is given by:

$$\mathbb{E}[\theta_i \mid \{(\tau_j, s_{j\omega})\}_{j\in N}] = \tau_i + \frac{\sigma^2_\omega}{\sigma^2_\omega + \frac{\sigma^2_{\varepsilon_{i\omega}}}{N}} \frac{1}{N} \sum_{j\in N} s_{j\omega}.$$

We can see that agent i places the same weight on all private signals $\{s_{j\omega}\}_{j\in N}$. Since the shocks $\{\tau_j\}_{j\in N}$ are public information, we can construct a linear equilibrium in the same way as we constructed an equilibrium for noise-free signals in the proof of Theorem 1.

When $\sigma^2_{\varepsilon_{i\omega}} = 0$, we have the same market power as with noise-free signals when the confounding parameter is equal to zero. To see why this is true, note that when the confounding parameter is zero, an agent places the same weight on his own private signal as on the private signals of other agents. For this reason, market power l is equal to 1. As an agent increases the quantity he demands, in the symmetric equilibrium all agents increase the quantity demanded by the same amount. Hence, while each agent only spends a fraction $1/N$ of the total expenditure, increasing the quantity demanded leads to N times higher price increase (since all other firms follow suit). Hence, when agents only observe signals about a common shock, in equilibrium there is complete internalization of the impact that demand has on the equilibrium price. On the other hand, to compute the price volatility, we can use (37) and compute the correlation between the price and the aggregate payoff shock. It is easy to verify that the correlation is

bounded from below by:

$$\rho^2_{p\bar{\theta}} \geq \frac{\frac{\sigma^2_\tau}{N}}{\frac{\sigma^2_\tau}{N} + \sigma^2_\omega}.$$

The lower bound is attained when $\sigma^2_{\varepsilon_{i\omega}} = \infty$. We summarize these findings in the following corollary.

Corollary 3 (One-Dimensional Signal About Common Payoff Shock)

There exists an information structure $\sigma^2_{\varepsilon_{i\omega}}$ that induces market power and price volatility (l, σ^2_p) as a unique equilibrium if and only if market power l is equal to 1 and price volatility σ^2_p is in the interval:

$$\left[\left(\frac{\beta N}{1+2\beta N}\right)^2 \frac{\sigma^2_\tau}{N},\ \left(\frac{\beta N}{1+2\beta N}\right)^2 (\frac{\sigma^2_\tau}{N} + \sigma^2_\omega)\right].$$

When the only source of uncertainty is the common-valued shock, market power is always equal to 1. In this case, the noise term in the signal changes the price volatility because it changes how much the price co-moves with the shocks. We illustrate the set of market power and price volatility that is attained by signals about common component in Figure 4.

Multidimensional Signals Our final information structure consists of *multidimensional signals*. We enrich the preceding information structure by relaxing the assumption that there is common knowledge of the idiosyncratic components. Instead, each agent observes a noisy signal of the idiosyncratic component of the other agents' idiosyncratic payoff shocks. Thus, each agent i observes $N+1$ private signals:

$$s_{ii} = \tau_i, \quad s_{ij} = \tau_j + \varepsilon_{ij}, \quad s_{i\omega} = \omega + \varepsilon_{i\omega}, \quad \forall i \in N, \forall j \neq i \in N;$$

where all noise terms are again independent normal. Thus, each agent knows his own idiosyncratic component. In addition, each agent has noisy signals of the idiosyncratic components of the others, which are very accurate (i.e., $0 < \sigma_{\varepsilon_{ij}} \ll 1$). The *multidimensional signals* are parametrized by the standard deviation of noise on the common component $\sigma_{\varepsilon_{i\omega}} \in [0, \infty)$.

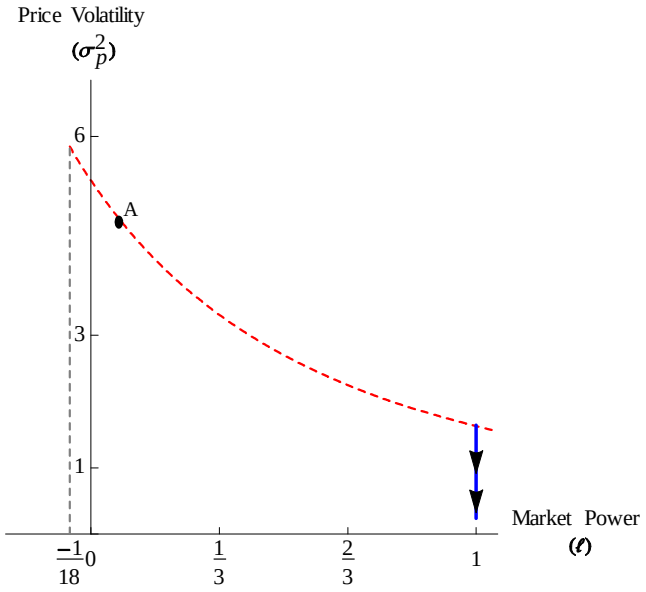

Figure 4: Set of equilibrium pairs $\left(l, \sigma_p^2\right)$ of market power and price volatility under signals about common component.

Computing the equilibrium strategies is analytically much more intricate than in the previous cases, and explicit solutions are only available in special cases.[10] Formally, we can state the following result for information structures near complete information.

Corollary 4 (Multidimensional Signals near Complete Information)

There exists a multidimensional signal that induces a pair of market power and price volatility (l, σ_p^2) *in the limit as* $\sigma^2_{\varepsilon_{ij}}, \sigma_{\sigma_{\varepsilon_{i\omega}}} \to 0$ *if and only if* $l \in [\hat{l}(1), 1]$ *and price volatility* σ_p^2 *satisfies* (5).

The detailed analysis is in the Appendix. In Figure 5, we plot the set of market power and price volatility that is achieved by multidimensional signals for all $\sigma_{\varepsilon_{i\omega}} \in \mathbb{R}_+$ (the red dashed curve is the set of outcomes under complete information). As before, point A corresponds to the outcome when $\sigma_{\varepsilon_{i\omega}} = 0$: an agent knows his own payoff shock but remains uncertain about the payoff shocks of the other agents. Initially, as $\sigma_{\varepsilon_{i\omega}}$ increases, market power increases. The

[10] Heumann (2018) shows that computing the coefficients of a linear equilibrium amounts to finding the roots of a polynomial of degree $2 \times$ (number of signals) $- 1$.

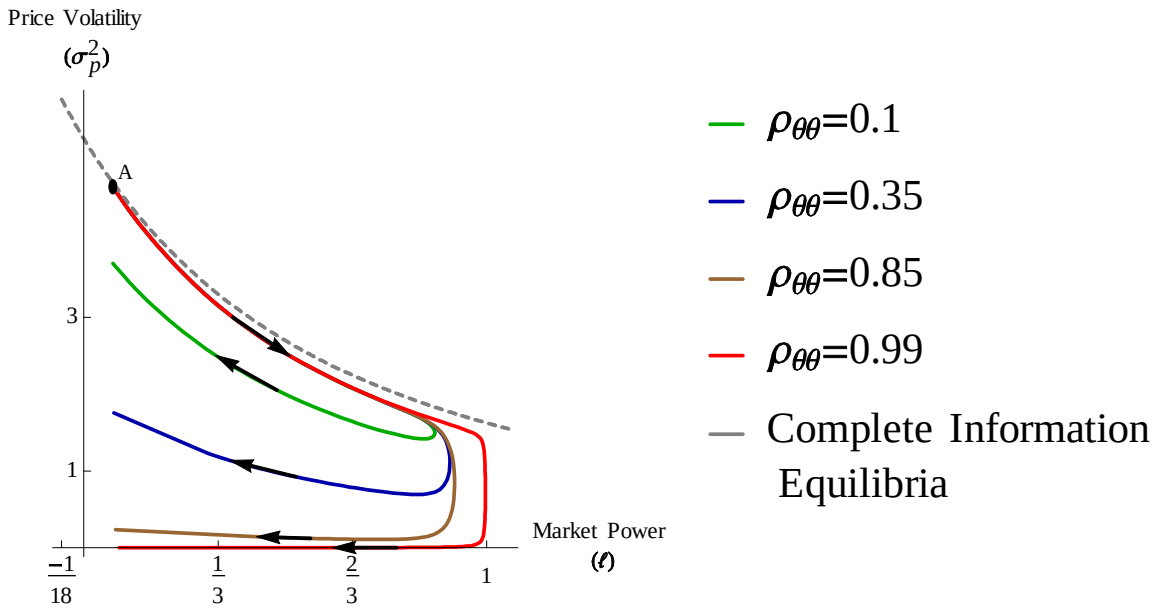

Figure 5: Set of equilibrium pairs $\left(l, \sigma_p^2\right)$ of market power and price volatility under noisy multidimensional signals.

intuition is similar to the case of a one-dimensional signal about the payoff shocks. Because the payoff shocks θ_i of the agents are correlated, every agent wants to increase the correlation between the quantity he buys and the quantity bought by the other agents. Figure 5 illustrates the equilibrium outcomes as we vary the correlation $\rho_{\theta\theta}$ in the payoff shocks. As $\sigma_{\varepsilon_{i\omega}} \to \infty$ the signals about the common shock become irrelevant, and so we are back to the case in which all the relevant sources of uncertainty are idiosyncratic shocks. Therefore, as $\sigma_{\varepsilon_{i\omega}} \to \infty$, market power is reduced back to the same level as $\sigma_{\varepsilon_{i\omega}} = 0$, but price volatility is lower because the price does not reflect the common component. Market power and price volatility under multidimensional signals track very closely the set of outcomes under complete information. The agents are effectively close to complete information as each agent i observes precise signals about $\{\tau_j\}_{j\in N}$ and ω. We conclude that a wide range of outcomes that are close to the complete information outcomes can be attained by parametrized information structures that have a natural statistical decomposition into idiosyncratic and common shocks.

Thus, when each agent observes noisy one-dimensional signals about his payoff shocks, market power always increases with the amount of incomplete information, and high market power can only be supported by a large amount of incomplete information. If each agent observes a noisy signal about the common payoff shock, then market power is always equal to 1 and price volatility is determined by the variance of the noise term. For the noisy multidimensional signals, the equilibrium outcomes closely track —within some range— the set of outcomes under complete information. Although a small amount of incomplete information can support high market power, in our parametrized classes of information structure market power is never above 1. However, Theorem 1 establishes that there is no upper bound on market power across all information structures.

6 Comparing Market Mechanisms

Our paper focuses on characterizing the set of possible pairs of market power and price volatility that can be attained by a Nash equilibrium of a particular mechanism, the demand function competition game. A methodological contribution of the paper is that we provide a characterization of key statistics of the equilibrium outcome independent of the specific equilibrium strategies that generate these outcomes. This approach has the advantage that it permits an easy and insightful comparison of *different* mechanisms or game forms.

We now illustrate this by means of Cournot competition. Thus, we consider the outcome of the economic environment as described in Section 2 in terms of payoff functions and payoff shocks but where agents now compete by submitting quantities (i.e., Cournot competition) instead of submitting demand schedules. Bergemann, Heumann, and Morris (2015) characterizes all the equilibrium outcomes of the quantity competition game. We can use their results (and the results we have given here) to compare the two forms of market competition.[11]

[11] The explicit calculations and comparison can be found in an earlier version of this paper (see Bergemann,

We first characterize market power and price volatility in quantity competition, which is analogous to Theorem 2.

Corollary 5 (Cournot Equilibria Under All Information Structures)

There exists an information structure that induces a pair of market power and price volatility, (l, σ_p^2)*, under quantity competition if and only if:*

$$l = \frac{1}{N} \quad \textit{and} \quad \sigma_p^2 \leq \frac{1}{4}\left(\frac{\sqrt{1+\beta}\sigma_{\bar{\theta}} + \sqrt{(\beta+\beta N+1)\sigma_\theta^2 + \beta N \sigma_{\bar{\theta}}^2}}{\sqrt{1+\beta}(\beta+\beta N+1)}\right)^2.$$

The restrictions that quantity competition imposes on market power and price volatility are strikingly different from those imposed under demand function competition. We illustrate the possible market power and price volatility values that can be attained by some information structure under the two forms of market competition in Figure 6; this figure illustrates the discussion that we present next.

Under quantity competition, market power is constant instead of being (almost) completely indeterminate. The expression for price volatility is slightly more cumbersome, but there is one important feature that is worth highlighting. Even if the aggregate shock is close to zero (i.e., $\sigma_{\bar{\theta}}^2 \approx 0$), there may be non-negligible price volatility. In contrast, under demand function competition, price volatility is always negligible if the variance of the aggregate shock is negligible (see Theorem 2). The difference between quantity competition and demand function competition is explained as follows.

Under quantity competition, an agent's price impact is equal to the slope of the exogenous supply; this explains why market power is constant across all information structures. In contrast, when agents submit price-contingent demands, an increase in the demand of an agent affects the price, which in turn affects the demand of other agents. Thus, an agent's price impact is not determined solely by the exogenous supply.

Heumann, and Morris (2018)), and the analysis of Cournot competition with a continuum of firms appears in Bergemann and Morris (2013).

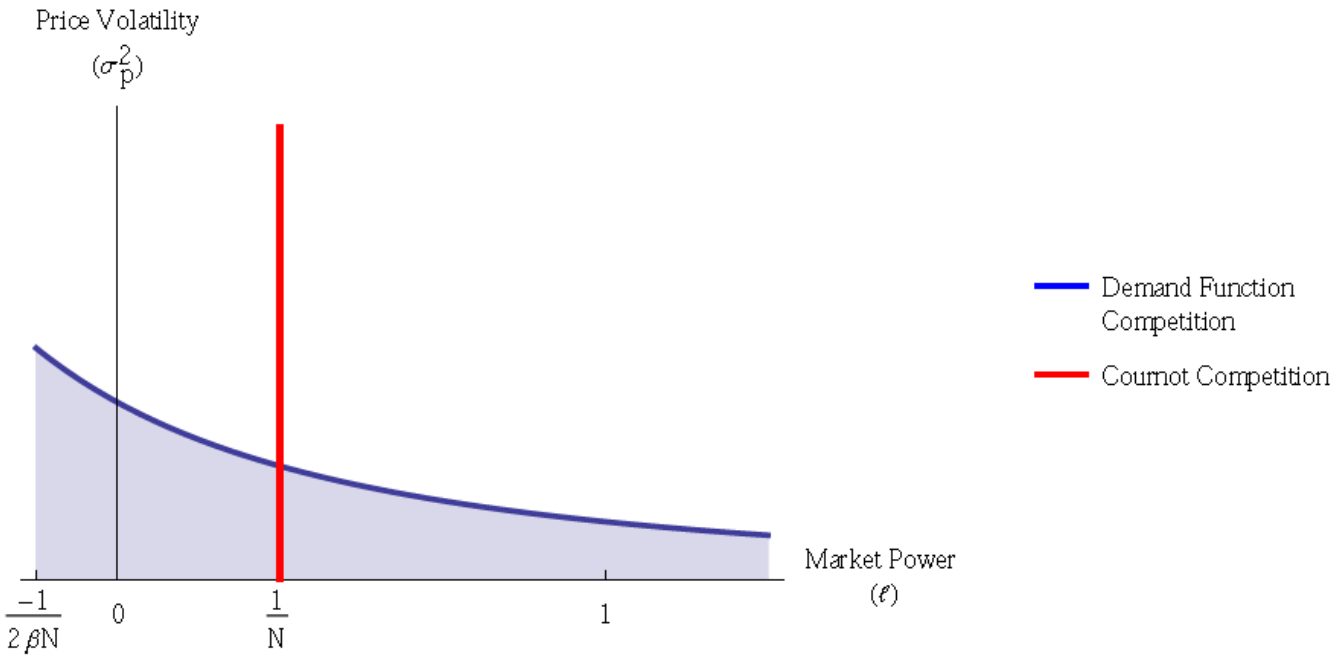

Figure 6: Comparison of the first and second equilibrium moments under demand function competition and Cournot competition.

For price volatility, the intuition is as follows. In quantity competition the equilibrium quantities may be correlated because signals are correlated, which may lead to aggregate volatility even in the absence of aggregate shocks. In contrast, when agents submit price-contingent demands, their beliefs are disciplined by the equilibrium price. Specifically, because agents choose their demand contingent on the price, the price serves as a public signal about the average quantity purchased by all agents. This additional signal disciplines beliefs in such a way that the quantities they purchase cannot be correlated more than the correlation of the payoff shocks, which ultimately disciplines price volatility.

7 Discussion

In this paper, we study demand function competition. Our findings provide positive and negative results regarding our ability to make predictions in this empirically important market microstruc-

ture. We showed that any market power is possible— from $-1/(2\beta N)$ to infinity. Considering small amounts of incomplete information does not allow us to provide any sharper predictions, unless one is able to make additional restrictive assumptions regarding the nature of the incomplete information. However, we showed that we can provide many substantive predictions regarding demand function competition that are robust to weak informational assumptions.

While our analysis focused on studying market power and price volatility, the conclusions can be extended to other equilibrium statistics. For example, an analyst may be interested in the variance of the quantities bought by each agent, that is, $\text{var}(q_i)$. In an earlier version of this paper (see Bergemann, Heumann, and Morris (2018)), we explored more broadly how information may determine *any* given equilibrium statistic. Our conclusions there extend the current results in the sense that the equilibria under complete information are "extremal" equilibria. For example, the variance of the quantities is bounded by the variance attained under a complete information equilibrium. Thus, while it is difficult to rule out any of the equilibria that arise under complete information, these equilibria can be used to provide bounds of what can happen across all information structures.

Our analysis relies on several simplifying assumptions, most notably the symmetry of agents and the normality of signals. The symmetry assumption facilitates the analysis by allowing us to describe the equilibrium outcome using a smaller set of equilibrium variables. If agents were heterogeneous, then describing an equilibrium outcome would amount to describing each agent's market power, the price volatility, and the variance of each agent's demand. We believe that this would be an algebraically more cumbersome exercise, but the tools and ideas we have developed could be applied to such a problem. The most important role that the normality assumption plays in our analysis is that it leads to constant market power (meaning that it does not depend on the realization of signals). If market power is not constant, then we may have an additional source of price volatility that does not come from the realization of the average payoff shock, but from market power volatility. Hence, we may attain higher price volatility than when we focus on

Gaussian signals. Overall, we believe that allowing for richer settings in terms of heterogeneous agents and non-Gaussian signals would not overturn the main message of Theorem 1: that is, it is difficult to offer predictions even when close to complete information. On the other hand, allowing for non-Gaussian signals may lead to higher price volatility, and hence, the bounds on price volatility that would allow us to extend Theorem 2 may show more slackness.

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8 Appendix

Proof of Proposition 1. Using the equilibrium construction in the proof of Theorem 1, in particular (13) we find that in any symmetric Nash equilibrium x^*:

$$x^*(s_i, p) = \frac{\theta_i - p}{1 + \lambda}, \tag{39}$$

where λ is the reciprocal of the derivative of an agent's residual supply. This equation is the complete information counterpart to (13). The market clearing condition implies that the equilibrium price is given by:

$$p^* = \frac{\beta N \bar{\theta}}{1 + \lambda + \beta N}. \tag{40}$$

The arguments established in the proof of Theorem 1 imply that market power satisfies $l = \lambda/\beta N$ and that the second-order condition of an agent's maximization problem implies that $\lambda \geq -1/2$. Thus, (40) implies that the equilibrium relation between price volatility and market power is as in (5).

To check that the conditions are sufficient, we consider the following family of demand functions parametrized by $\gamma \in \mathbb{R}$:

$$x_i(p) = \frac{1}{1 + \lambda}\left(\theta_i - (1 - \gamma)\bar{\theta}\right) - \frac{1}{N - 1}(\frac{1}{\lambda} - \frac{1}{\beta})p, \tag{41}$$

where λ is determined as a function of γ by the positive root of (18). We first observe that this is the same demand function as the one that the agents submit in the Nash equilibrium when they observe the information structure constructed in Theorem 1 (there γ parametrizes the one-dimensional signal (11)). Thus, if (41) constitutes a Nash equilibrium, then the equilibrium market power and price volatility are given by:

$$l = \lambda/\beta N \quad \text{and} \quad \sigma_p^2 = \frac{(\beta N)^2}{(1 + \beta N + \lambda)^2}\sigma_{\bar{\theta}}^2,$$

where $\lambda \in [-1/2, \infty)$ is the positive root of (18). To check that (41) is an equilibrium, we note

that when λ is determined by the positive root of (18) we have that:

$$x_i(p) = \frac{1}{1+\lambda}\left(\theta_i - (1-\gamma)\bar{\theta}\right) - \frac{1}{N-1}(\frac{1}{\lambda} - \frac{1}{\beta})p = \frac{\theta_i - p}{1+\lambda}.$$

Thus, demand satisfies the first-order condition (39), and it also satisfies the second-order condition because $\lambda \geq -1/2$. Therefore, (41) satisfies the optimality conditions and thus constitutes a symmetric Nash equilibrium. ■

Proof of Corollary 4. We recall that the first-order condition is given by (13). In the limit $\sigma^2_{\varepsilon_{ij}}, \sigma^2_{\varepsilon_{i\omega}} \to 0$ we have that:[12]

$$\mathbb{E}[\theta_i | p^*, s_i] \to \theta_i.$$

This limit is satisfied regardless of the rate at which we take the limits. Following the same steps as in the proof of Theorem 1 we conclude that the equilibrium price converges to:

$$p \to \frac{\beta N \bar{\theta}}{1 + \beta N(1+l)}. \tag{42}$$

This immediately implies that price volatility converges to the formula in (5). We are thus left with proving that, by taking the limits $\sigma^2_{\varepsilon_{ij}}, \sigma^2_{\varepsilon_{i\omega}} \to 0$ at the appropriate rate, every market power $l \in [\hat{l}(1), 1]$ can be attained in the limit.

We now use the fact that the price converges to (42), and we recall that $\bar{\theta} = \omega + \bar{\tau}$. We first note that, if $\sigma^2_{\varepsilon_{ij}}, \sigma^2_{\varepsilon_{i\omega}} \to 0$ and $\sigma^2_{\varepsilon_{ij}}/\sigma^2_{\varepsilon_{i\omega}} \to \infty$, then:

$$\mathbb{E}[\theta_i \mid p, s_i] \to s_{ii} + s_{i\omega}.$$

In other words, if $\sigma^2_{\varepsilon_{i\omega}}$ converges to zero at a faster rate than $\sigma_{\varepsilon_{ij}}$, then s_{iw} becomes arbitrarily more informative (about ω) than p. Hence, to predict ω, agents simply use their private signals s_{iw}. Following the same steps as in the proof of Theorem 1 to find the equilibrium market power, we conclude that the equilibrium market power will be $\hat{l}(1)$.

[12] All convergences of expectations are convergences in probability.

We now note that if $\sigma^2_{\varepsilon_{ij}}, \sigma^2_{\varepsilon_{i\omega}} \to 0$ and $\sigma^2_{\varepsilon_{ij}}/\sigma^2_{\varepsilon_{i\omega}} \to 0$, then:

$$\mathbb{E}[\theta_i \mid p, s_i] \to s_{ii} + (\frac{1 + N\beta(1+l)}{\beta N} p - \frac{1}{N}\sum_{j \in N} s_{ij}).$$

In other words, if $\sigma^2_{\varepsilon_{ij}}$ converges to zero at a faster rate than $\sigma_{\varepsilon_{i\omega}}$, then agents can subtract $\bar{\tau}$ from the equilibrium price, which allows them to deduce $\sum_{j \in N} s_{jw}/N$ by observing the equilibrium price. Hence, to predict ω, agents simply use the equilibrium price. Following the same steps as in the proof of Theorem 1 to find the equilibrium market power, we conclude that the equilibrium market power will be 1.

We now observe that for every $\kappa \in [0, 1]$ we can take the limits $\sigma^2_{\varepsilon_{ij}}, \sigma^2_{\varepsilon_{i\omega}} \to 0$ at an appropriate rate such that:

$$\mathbb{E}[\theta_i \mid p, s_i] \to s_{ii} + \kappa(\frac{1 + N\beta(1+l)}{\beta N} p - \frac{1}{N}\sum_{j \in N} s_{ij}) + (1 - \kappa) s_{i\omega}.$$

Therefore, changing the rate at which $\sigma^2_{\varepsilon_{ij}}$ and $\sigma^2_{\varepsilon_{i\omega}}$ converge to zero changes κ, which has the same effect on the equilibrium market power as changing γ under noise-free signals (23). The set of equilibrium market power values that can be attained by the set $\kappa \in [0, 1]$ corresponds to the set of market power values that can be attained by noise-free signals with parameter $\gamma \in [0, 1]$. Hence, every market power in $l \in [\hat{l}(1), 0]$ can be attained. ■