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Abstract

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JEL Classification: E4, D8, G0

Keywords: forecast dispersion, uncertainty, Variability, private noisy information, public information

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Uncertainty and dispersion in professional interest rate forecasts: International evidence and theory

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July 11, 2020

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1 Introduction

The future course of interest rates is key for current decisions. When deciding how much to borrow and for how long, information about future interest rates is useful to credit demanders. For similar reasons, information about future interest rates is beneficial to credit suppliers. Accurate forecasts of future interest rates are important for financial institutions, and in particular for banks, that derive a large part of their revenues from interest rate differentials between borrowing and lending interest rates. Positive differentials are usually achieved by longer maturities on the asset than on the liability side of banks' balance sheets. Achieving an optimal balance between high interest rate differentials and maintenance of adequate liquidity fundamentally depends on accurate forecasts of interest rates.[1]

Furthermore, forecasting short-term interest rates that are related to central bank policy is essential for evaluating the stance of monetary policy. Beliefs about the future course of long-term yields constitute an important link in the transmission of monetary policy to economic activity and inflation.

This paper documents cross-country differences in the variability of interest rates and yields, as well as in the magnitudes of aggregate forecast uncertainties and forecast dispersion. The bulk of the paper documents systematic cross-country relations between those three aggregate variability measures and presents a theoretical model with Bayesian expectations that accounts for the findings.

The relation between forecast dispersion and uncertainty has been investigated in the past mostly in the context of other variables such as inflation and economic growth. Using forecasts on GNP growth and inflation in the US Zarnowitz & Lambros (1987) examine the extent to which forecast dispersion can be used as a proxy for objective individual forecast uncertainty and report a positive association between those concepts. Ottaviani & Sorensen (2006) report a similar regularity for GDP growth in the US. Using data from the US survey of professional forecasters Lahiri & Sheng (2010) find that disagreement is a reliable measure for individual subjective uncertainty in stable periods. Cukierman & Wachtel (1979) report positive and significant correlations between the cross-sectional variance of inflation expectations and the variance of nominal income change in the US. A positive correlation between inflation's forecast uncertainty and the variance of nominal income changes in the US is also documented by Cukierman & Wachtel (1982).

Barron & Struerke (1998) argue that dispersion in analysts' earnings forecasts is a good measure of earning uncertainty. Using a panel of forecasts from the US Survey of Profes-

[1] Using a sample of nine developed economies Istrefi & Mouabbi (2018) find that, in many cases shocks to subjective interest rate uncertainty have large and persistent negative effects on economic activity.

sional Forecasters Cicarrelli & Hubrich (2010) conclude that in stable periods disagreement is a reliable measure of uncertainty. Combining data on inflation expectations from the Livingston survey and the Survey Research Center with an ARCH model to proxy for subjective uncertainty Rich, Raymond & Butler (1992) find a positive association between measures of forecast uncertainty and forecast dispersion. But this finding is sensitive to the choice of survey series. In general dispersion and uncertainty are obviously distinct concepts that might be positively related under some conditions. The theory section of this paper provides such conditions for a cross-country positive association between those two measures.

Using professional forecasting data on short-term interest rates and long-term yields for up to 33 countries this paper investigates cross country relations between proxies of uncertainty, dispersion and variability. Forecast uncertainty is characterized by the average (over forecasters and time) root mean square forecast error in a country (FU). Forecast dispersion is measured as the, over time, average forecast dispersion across forecasters in a country (FD) and variability ($Var[r_t]$) is the variance of each original interest rate series and is measured over time within a given country. The associations between those three proxies are generally positive. In particular, the average forecast uncertainty measure, FU, and the average cross-sectional forecast dispersion, FD, are positively and significantly related across countries for both short-term interest rate and long-term yield forecasts. For short-term interest rates, those two variables are also positively related to the standard deviations of interest rates and yields ($\sqrt{Var[r_t]}$).[2] Similar, but albeit not always significant, regularities are found for long-term yields and their forecasts.

The remainder of the paper describes the data base, compares the three proxy measures with each other and across countries and embeds the empirical findings into a theoretical framework.

2 Data

The data consists of short-term interest rate forecasts and long-term yield forecasts collected and maintained by Consensus Economics. It consists of monthly forecasts of short-term interest rates for 33 countries (mostly with maturity of three months) and forecasts of 10 years government bond yields for 23 countries. Table 1 in the Appendix lists the countries and their country codes. Professional forecasters such as financial institutions and other forecasting agencies report their forecasts to Consensus Economics, starting at the earliest in October 1989 and ending in June 2017. Two forecast horizons are provided: three- and twelve-months.

[2]Cukierman (1984) documents similar regularities for inflationary expectations.

We use realized interest rates and yields from Refinitiv Datastream to derive their variability and forecast errors.

3 Evidence on cross-country relations between forecast uncertainty, forecast dispersion and variability

This section presents cross-country evidence on the relation between forecast uncertainty, forecast dispersion and variability. Operational proxies are required to examine the relation between these three variables. The over time average root mean square forecast error of the cross-sectional mean forecasts is the proxy for forecast uncertainty in a given country ($\sqrt{FU}$). We proxy forecast dispersion ($\sqrt{FD}$) as the overtime average of the cross-sectional standard deviations of forecasts in a given country. Variability is measured as the overtime standard deviation of the variable in a given country (interest rate or yield; $\sqrt{Var[r_t]}$). We calculate all three proxy measures for each country in the sample. The first three columns in Table 2 show those aggregate measures for each country.

In the following, we examine the cross-country relations between the three measures by plotting them against each other. In addition, we estimate the cross-country correlations R of the three measures and report their t-Values. We also show the corresponding regression lines in the figures.

3.1 Forecast dispersion versus forecast uncertainty

Figure 1 plots least squares regression lines between forecast dispersion and forecast uncertainty along with the individual countries' observations. The four panels of the figure correspond respectively to short-term interest rate forecasts at the three- and twelve-months forecast horizons and to three- and twelve-months forecasts of long-term yields. The coefficient R shown in each panel denotes the corresponding correlation coefficient and the value in parenthesis reports the t-Value of R. Outlier countries have been excluded in those figures.[3] Inclusion of outliers would lead to even stronger correlations than reported in the figures below.

[3]For interest rates the outlier countries are ARG, TUR, IDN, and VEN. For yields, the outlier countries are ITA and IDN.

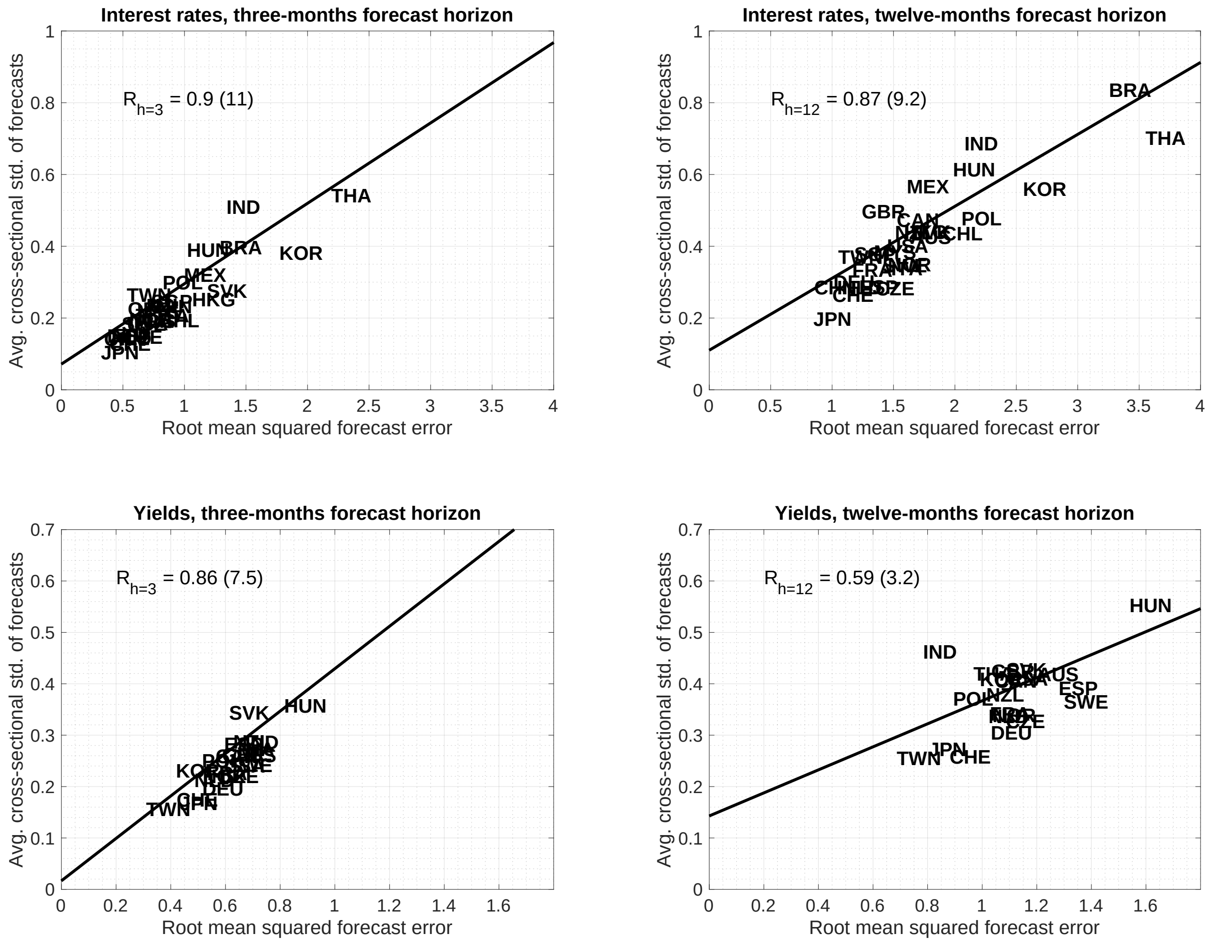

Figure 1: Forecast dispersion versus forecast uncertainty

All four panels in Figure 1 strongly support the conclusion that there is a positive and significant cross-country relation between forecast uncertainty and the dispersion of individual interest rate and yield forecasts.[4] This evidence is consistent with the view that, as uncertainty about interest rates and yields increases, consensus about the future course of those variables diminishes. In other words, in countries characterized by more uncertain short-term rates and long-term yields there will be less consensus about the future course of those variables.

3.2 Forecast uncertainty versus variability

Although uncertainty and variability are not identical, it is likely that they have common elements. The reason is that part, but not all, of the variability is predictable (Cukierman

[4]Istrefi & Mouabbi (2018) document a positive relation between individual uncertainty and dispersion within each of 9 advanced economies.

& Wachtel (1982)). The four panels of Figure 2 show the relation between our measure of forecast uncertainty for the three- and twelve-months forecast horizons and variability for both short-term interest rates and long-term yields using a figure format identical to Figure 1.

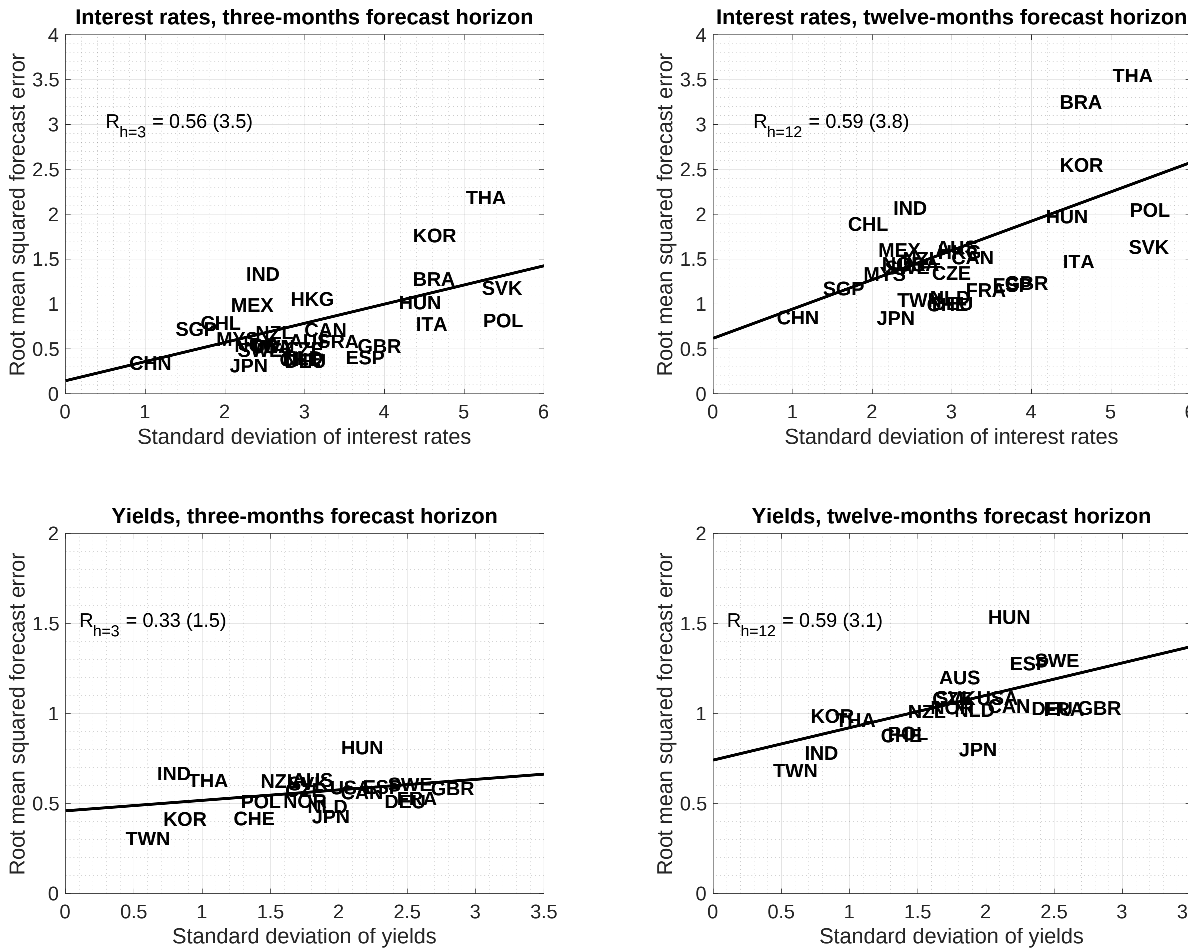

Figure 2: Forecast uncertainty versus variability

In all four cases the relation is positive. Yet, this relation is not significant for yield forecasts at the three-months horizon (Figure 2, quadrant 3). For the other three sets, the relation is statistically significant. But the significance is not as strong as in the case of the relation between forecast dispersion and forecast uncertainty shown in Figure 1. Generally, Figure 2 supports the view that parts of the variability in interest rates and yields are predictable, but not all of it is predictable a priori.

3.3 Forecast dispersion versus variability

Figure 1 supports a relatively strong positive relation between forecast dispersion and forecast uncertainty. Figure 2 reports a, somewhat weaker, positive relation between forecast uncertainty and variability. A natural third step is to examine the relation between forecast dispersion and variability. We use the same empirical proxies for those two concepts as in the previous two figures. Figure 3 presents the results using the same figure formats as before.

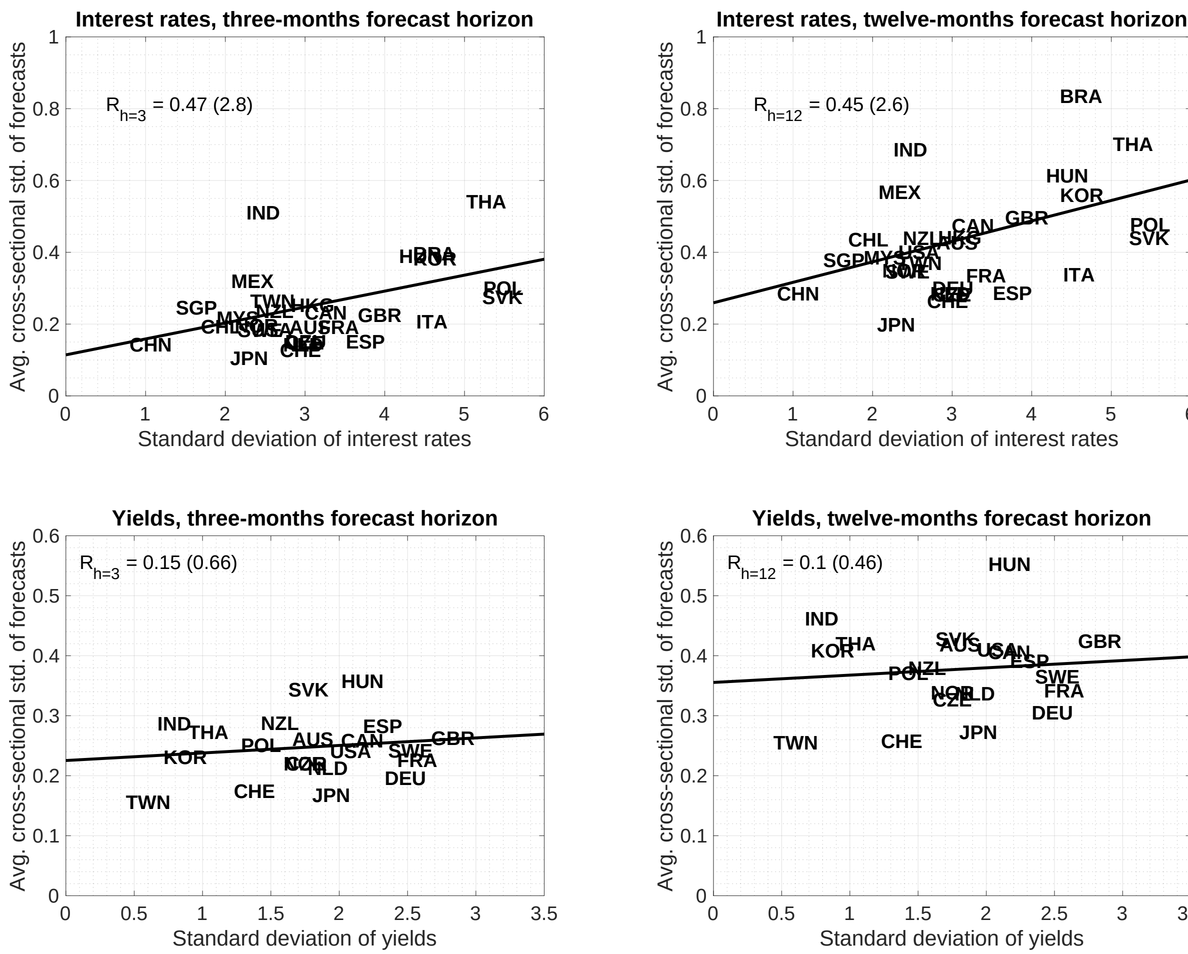

Figure 3: Forecast dispersion versus variability

Figure 3 shows that there is a small and significant positive relation between variability and forecast dispersion for short-term interest rates at both forecast horizons, but no meaningful relation between those variables for long-term yields. Of all the three relationships investigated in the paper, forecast dispersion versus variability is the weakest.

In summary, the results reported in this section are consistent with the view that the most meaningful cross-country relation is the positive relation between forecast dispersion and forecast uncertainty.

4 Theoretical foundations

We embed our empirical results in a theoretical model with Bayesian expectations.[5] We postulate that the interest rate (yield) is given by the following stationary first order Markov process

$$
\begin{aligned}
r_t &= r + \theta_t \\
\theta_t &= \rho \cdot \theta_{t-1} + v_t, 0 < \rho < 1.
\end{aligned}
\tag{1}
$$

with innovation v_t. There is a large number of forecasters forecasting the future path of the interest rate r_{t+h} at time t with forecast horizon h and the fundamental parameters of the interest rate process (ρ, σ_v^2, r) are common knowledge. For forecast horizon h equation (1) implies

$$\theta_{t+h} - \rho^h \cdot \theta_t = v_{t+h} + \rho \cdot v_{t+h-1} + ... + \rho^{h-1} \cdot v_{t+1} \equiv \delta_{t+h} \tag{2}$$

where δ_{t+h} is the cumulative innovation to the stochastic part of the interest rate process over the $[t+1, t+h]$ period. In addition to the current state, r_t, forecaster i also observes a private unbiased noisy signal (s_{it}) about the future change of the interest rate (δ_{t+h}) between the present (period t) and period $t+h$. The signal is unbiased but subject to noise, ε_{it+h}, and thus of precision $1/\sigma_\varepsilon^2$. Since the signal is private information predictions about the future generally differ across forecasters.[6] Each individual combines public information about the current state with his private information to produce the following optimal Bayesian predictor.

$$
\begin{aligned}
f_{it} &\equiv E\left[r_{t+h} | r_t, s_{it}\right] = r + \rho^h \cdot \theta_t + \frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\varepsilon^2} \cdot s_{it} \\
&\equiv r + \rho^h \cdot \theta_t + w_s \cdot (\delta_{t+h} + \varepsilon_{it+h}).
\end{aligned}
\tag{3}
$$

[5] The full model and detailed derivations are presented in the theoretical Appendix.

[6] Manski (2018) suggests that macroeconomists should give up the simplicity of homogeneous expectations and strive to develop tractable models of economies in which forecasters have flexibly heterogeneous expectations.

The model implies that the three aggregate measures variability ($Var[r_t]$), forecast uncertainty (FU), and forecast dispersion (FD) are given by:

$$Var[r_t] \equiv \frac{\sigma_\delta^2}{1-\rho^{2h}} \tag{4}$$

$$FD \equiv \left(\frac{\sigma_\delta^2}{\sigma_\delta^2+\sigma_\varepsilon^2}\right)^2 \cdot \sigma_\varepsilon^2 \tag{5}$$

$$FU \equiv \left(\frac{\sigma_\varepsilon^2}{\sigma_\delta^2+\sigma_\varepsilon^2}\right)^2 \cdot \sigma_\delta^2. \tag{6}$$

where $(\sigma_\delta^2, \sigma_\varepsilon^2)$ are the basic underlying variances. They are shown for each country in the fourth and fifth column of Table 2. From equation (4), we observe that $Var[r_t]$ increases in σ_δ^2, but is unaffected by σ_ε^2. To establish the impacts of σ_δ^2 and σ_ε^2 on FD and FU further analysis is needed. Evaluation of the partial derivatives of FD and FU with respect to σ_δ^2 and σ_ε^2 yields

$$\begin{aligned} \frac{\partial FD}{\partial \sigma_\delta^2} &= \frac{2\sigma_\delta^2\sigma_\varepsilon^4}{(\sigma_\delta^2+\sigma_\varepsilon^2)^3} & \frac{\partial FU}{\partial \sigma_\delta^2} &= \frac{\sigma_\varepsilon^4}{(\sigma_\delta^2+\sigma_\varepsilon^2)^3}(\sigma_\varepsilon^2-\sigma_\delta^2) \\ \frac{\partial FD}{\partial \sigma_\varepsilon^2} &= \frac{\sigma_\delta^4}{(\sigma_\delta^2+\sigma_\varepsilon^2)^3}(\sigma_\delta^2-\sigma_\varepsilon^2) & \frac{\partial FU}{\partial \sigma_\varepsilon^2} &= \frac{2\sigma_\delta^4\sigma_\varepsilon^2}{(\sigma_\delta^2+\sigma_\varepsilon^2)^3}. \end{aligned} \tag{7}$$

Proposition 1 *Given $\sigma_\varepsilon^2 - \sigma_\delta^2 > 0$ cross country variations in σ_δ^2 induce a positive cross country relation between FD and FU and cross country variations in σ_ε^2 induce a negative cross country relation between FD and FU.*

Proof. Immediate from inspection of the first and second rows in equation (7). ■

Note that, given $\sigma_\varepsilon^2 - \sigma_\delta^2 > 0$, all derivatives in equation (7) except $\partial FD/\partial\sigma_\varepsilon^2$ are positive. The values of the derivatives in Proposition 1 are shown for each country in the last four columns of Table 2.

Proposition 2 *Given $\sigma_\varepsilon^2 - \sigma_\delta^2 > 0$ and provided $\partial FD/\partial\sigma_\delta^2$ is sufficiently larger than the absolute value of $\partial FD/\partial\sigma_\varepsilon^2$ the positive cross country relation between FD and FU induced by variations in σ_δ^2 dominates the negative relation induced by variations in σ_ε^2.*

Proof. Immediate by inspection of the first column in equation (7). ■

The intuition underlying the signs of the derivatives in equation (7) follows

(i) Sign of $\partial FD/\partial\sigma_\delta^2$: Equation (3) implies that when uncertainty, σ_δ^2, about the future increases forecasters increase the weight on private information about the future. As a consequence, a given realization of the vector of individual noises, ε_{it+h}, raises the dispersion of forecasts.

(ii) Sign of $\partial FU/\partial\sigma_\delta^2$: Inspection of equation (6) reveals that an increase in σ_δ^2 triggers two opposing effects. One is due to the positive direct impact of σ_δ^2 on FU and the other is the increase in the optimal weight given to private information about the future that moderates this uncertainty. When $\sigma_\varepsilon^2 - \sigma_\delta^2 > 0$ the direct positive effect dominates.

(iii) Sign of $\partial FD/\partial\sigma_\varepsilon^2$: Inspection of equation (5) shows that an increase in the noise, σ_ε^2, of private information triggers two opposing effects on forecast dispersion. One is the positive direct impact of an increase in forecast inaccuracy (σ_ε^2 increases) on forecasts' dispersion. The other is the negative impact due to a decrease in the optimal weight on private information. When $\sigma_\varepsilon^2 - \sigma_\delta^2 > 0$ the negative impact on dispersion dominates.

(iv) Sign of $\partial FU/\partial\sigma_\varepsilon^2$: When the private signals' precision decreases (σ_ε^2 increases) forecasters rely less on their private information and this raises FU (equation (6)).

Empirical evidence presented in Table 2 of the appendix shows that the conditions in propositions 1 and 2 are satisfied for all countries in the sample. In particular:[7]

(i) σ_ε^2 is uniformly larger across countries than σ_δ^2 and usually by a substantial margin. For example , for three-months interest rate forecasts the cross country mean value of σ_ε^2 is almost 12 times larger than σ_δ^2.

(ii) The positive impact of σ_δ^2 on FD is uniformly larger than the absolute value of the negative impact on FD due to changes in σ_ε^2. Thus, for three-months interest rate forecasts the cross country mean value of $\partial FD/\partial\sigma_\delta^2$ is almost 18 times larger than the absolute value of $\partial FD/\partial\sigma_\varepsilon^2$

In summary the two propositions along with the estimates in Table 2 in the Appendix imply that there should be a positive cross country association between forecast uncertainty FU, forecast dispersion FD and variability $Var[r_t]$.

5 Conclusions

We presented a set of results which take their roots from literature on measures of forecast uncertainty, forecast dispersion and variability. Variability is often taken as a measure of uncertainty in finance and economics. However, variability and uncertainty measures generally differ when some of the variability is predictable. Recognizing this difference, existing literature has used two alternative measures of uncertainty. One is the root mean square

[7]Details appear in Table 2 of the Appendix.

forecast error and the other is a measure of dispersion of individual forecasts from the mean forecast. The first measure is the most direct measure of forecast uncertainty while the second is a direct measure of disagreement among forecasters rather than a direct measure of uncertainty. Nonetheless, the literature has occasionally used it as a measure of subjective individual uncertainty (Barron & Stuerke (1998)).

Since our main objective is to examine the **cross-country** relations between uncertainty, dispersion and variability our work focuses on aggregate versions of those measures in which variations across individuals and time have been replaced by their appropriate country averages. Correspondingly the theoretical framework highlights the relations between uncertainty, dispersion and variability that are caused by cross-country variations in the variability of innovations to the interest rate process (σ_v^2) and in the precision of private information ($1/\sigma_\varepsilon^2$). As one might expect the measure of forecast uncertainty is directly related to σ_v^2 and the measure of dispersion is directly related to σ_ε^2. But in addition cross country differences also induce different individual weights on private information across countries that may reinforce or offset the direct effects depending on parameter values.

Availability of forecasting data for a sample of up to 33 countries from consensus economics makes it possible to examine the relation between these three measures across countries. The data generally supports the conclusion that all three measures are positively related across countries for both short-term interest rates and long-term bond yields at the three- and twelve-months forecast horizons and is consistent with the conceptual framework.[8] The strongest relations are found between forecast dispersion and the root mean square forecast error. The weakest positive association is between forecast dispersion and variability.

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[8] One exception to this statement concerns the correlation between forecasts' dispersion and the variability of long-term yields which is not significantly different from zero.

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Appendix: Tables

Table 1: Consensus Economics data sets and country codes

Consensus Forecasts (advanced economies)		Asia Pacific Consensus Forecasts	
USA	United States of America	AUS	Australia
JPN	Japan	CHN	China
DEU	Germany	HKG	Hong Kong
FRA	France	IND	India
GBR	United Kingdom	IDN	Indonesia
ITA	Italy	MYS	Malaysia
CAN	Canada	NZL	New Zealand
NLD	Netherlands	SGP	Singapore
NOR	Norway	KOR	South Korea
ESP	Spain	TWN	Taiwan
SWE	Sweden	THA	Thailand
CHE	Switzerland		
Eastern Europe Consensus Forecasts		**Latin American Consensus Forecasts**	
CZE	Czech Republic	ARG	Argentina
HUN	Hungary	BRA	Brazil
POL	Poland	CHL	Chile
TUR	Turkey	MEX	Mexico
SVK	Slovakia	VEN	Venezuela

Table 2: Estimates of basic underlying variances, aggregate variability measures and their derivatives across countries

Country	$\sqrt{\widehat{FU}}$	$\sqrt{\widehat{FD}}$	$\sqrt{\widehat{Var[r_t]}}$	$\widehat{\sigma}_{\delta}^2$	$\widehat{\sigma}_{\varepsilon}^2$	$\frac{\partial FU}{\partial \sigma_{\delta}^2}$	$\frac{\partial FU}{\partial \sigma_{\varepsilon}^2}$	$\frac{\partial FD}{\partial \sigma_{\delta}^2}$	$\frac{\partial FD}{\partial \sigma_{\varepsilon}^2}$
Interest rates, three-months forecast horizon (33)									
Data set Consensus Forecasts (advanced economies)									
USA	0.533	0.186	2.325	0.36	2.93	0.621	0.021	0.173	-0.009
JPN	0.321	0.106	2.060	0.13	1.17	0.654	0.017	0.159	-0.008
DEU	0.372	0.152	2.749	0.19	1.12	0.523	0.035	0.210	-0.015
FRA	0.589	0.193	3.175	0.43	3.94	0.656	0.017	0.159	-0.008
GBR	0.540	0.226	3.664	0.40	2.31	0.509	0.038	0.216	-0.016
ITA	0.783	0.209	4.395	0.70	9.89	0.756	0.008	0.116	-0.004
CAN	0.719	0.233	2.996	0.63	5.99	0.662	0.016	0.156	-0.007
NLD	0.406	0.144	2.725	0.21	1.66	0.612	0.022	0.176	-0.010
NOR	0.550	0.197	2.119	0.38	3.01	0.608	0.023	0.178	-0.010
ESP	0.410	0.152	3.512	0.22	1.58	0.585	0.026	0.187	-0.011
SWE	0.491	0.184	2.158	0.31	2.23	0.578	0.027	0.190	-0.011
CHE	0.391	0.129	2.684	0.19	1.73	0.655	0.017	0.159	-0.008
Data set Asia Pacific Consensus Forecasts									
AUS	0.592	0.193	2.802	0.43	4.03	0.659	0.017	0.157	-0.007
CHN	0.347	0.144	0.799	0.17	0.97	0.516	0.036	0.213	-0.015
HKG	1.062	0.253	2.823	1.26	22.12	0.799	0.005	0.096	-0.003
IND	1.342	0.511	2.264	2.36	16.29	0.570	0.028	0.193	-0.012
IDN	3.760	1.109	8.093	16.71	191.97	0.711	0.012	0.136	-0.005
MYS	0.614	0.218	1.889	0.48	3.80	0.613	0.022	0.176	-0.010
NZL	0.691	0.237	2.381	0.60	5.06	0.632	0.020	0.169	-0.009
SGP	0.729	0.247	1.380	0.66	5.76	0.639	0.019	0.166	-0.008
KOR	1.771	0.383	4.357	3.44	73.46	0.831	0.004	0.082	-0.002
TWN	0.532	0.265	2.313	0.44	1.77	0.385	0.064	0.256	-0.024
THA	2.193	0.542	5.021	5.42	88.58	0.786	0.006	0.102	-0.003
Data set Eastern Europe Consensus Forecasts									
CZE	0.505	0.149	2.749	0.30	3.45	0.710	0.012	0.136	-0.005
HUN	1.021	0.391	4.179	1.37	9.34	0.566	0.028	0.194	-0.012
POL	0.823	0.301	5.236	0.87	6.50	0.594	0.025	0.184	-0.011
TUR	2.182	1.717	9.737	12.49	20.17	0.090	0.181	0.292	-0.034
SVK	1.184	0.277	5.223	1.56	28.50	0.806	0.005	0.093	-0.002
Data set Latin American Consensus Forecasts									
ARG	9.532	2.523	10.185	104.04	1485.19	0.759	0.008	0.114	-0.004
BRA	1.286	0.398	4.355	1.99	20.78	0.688	0.014	0.145	-0.006
CHL	0.795	0.194	1.694	0.71	11.90	0.791	0.006	0.100	-0.003
MEX	0.996	0.321	2.075	1.21	11.62	0.666	0.016	0.155	-0.007
VEN	4.070	1.874	4.782	24.33	114.76	0.443	0.050	0.238	-0.020
Std.	1.701	0.548	2.145	18.16	254.08	0.140	0.030	0.047	0.007
Min	0.321	0.106	0.799	0.13	0.97	0.090	0.004	0.082	-0.034
Max	9.532	2.523	10.185	104.04	1485.19	0.831	0.181	0.292	-0.002
Average	1.277	0.435	3.603	5.61	65.56	0.626	0.026	0.166	-0.010

Table 2: Continued

Country	$\sqrt{\widehat{FU}}$	$\sqrt{\widehat{FD}}$	$\sqrt{\widehat{Var[r_t]}}$	$\widehat{\sigma}^2_\delta$	$\widehat{\sigma}^2_\varepsilon$	$\frac{\partial FU}{\partial \sigma^2_\delta}$	$\frac{\partial FU}{\partial \sigma^2_\varepsilon}$	$\frac{\partial FD}{\partial \sigma^2_\delta}$	$\frac{\partial FD}{\partial \sigma^2_\varepsilon}$
Interest rates, twelve-months forecast horizon (33)									
Data set Consensus Forecasts (advanced economies)									
USA	1.449	0.401	2.325	2.43	31.68	0.739	0.009	0.123	-0.004
JPN	0.849	0.199	2.057	0.80	14.53	0.804	0.005	0.094	-0.002
DEU	1.011	0.301	2.749	1.21	13.65	0.706	0.012	0.138	-0.006
FRA	1.164	0.336	3.175	1.59	19.13	0.722	0.011	0.131	-0.005
GBR	1.241	0.498	3.664	2.08	12.91	0.537	0.033	0.206	-0.014
ITA	1.481	0.339	4.397	2.43	46.29	0.813	0.005	0.090	-0.002
CAN	1.527	0.475	2.996	2.81	29.00	0.685	0.014	0.147	-0.006
NLD	1.079	0.286	2.725	1.33	18.97	0.758	0.008	0.115	-0.004
NOR	1.454	0.350	2.122	2.37	40.73	0.795	0.006	0.098	-0.003
ESP	1.221	0.288	3.512	1.66	29.92	0.803	0.005	0.094	-0.002
SWE	1.411	0.348	2.159	2.24	36.87	0.787	0.006	0.102	-0.003
CHE	1.002	0.266	2.684	1.15	16.40	0.759	0.008	0.114	-0.004
Data set Asia Pacific Consensus Forecasts									
AUS	1.635	0.428	2.802	3.05	44.61	0.764	0.008	0.112	-0.004
CHN	0.855	0.287	0.799	0.90	8.05	0.645	0.018	0.163	-0.008
HKG	1.587	0.441	2.823	2.92	37.81	0.738	0.010	0.124	-0.004
IND	2.077	0.687	2.264	5.31	48.57	0.652	0.018	0.160	-0.008
IDN	7.517	1.437	8.093	60.72	1660.47	0.865	0.002	0.066	-0.001
MYS	1.341	0.386	1.889	2.11	25.44	0.722	0.011	0.131	-0.005
NZL	1.512	0.440	2.381	2.69	31.78	0.717	0.011	0.133	-0.005
SGP	1.181	0.379	1.380	1.70	16.43	0.668	0.016	0.154	-0.007
KOR	2.554	0.560	4.357	7.17	148.79	0.827	0.004	0.084	-0.002
TWN	1.050	0.371	2.313	1.40	11.19	0.615	0.022	0.175	-0.010
THA	3.552	0.702	5.021	13.62	348.10	0.856	0.003	0.070	-0.001
Data set Eastern Europe Consensus Forecasts									
CZE	1.353	0.284	2.749	2.00	45.33	0.840	0.003	0.077	-0.002
HUN	1.984	0.615	4.179	4.73	49.23	0.687	0.014	0.146	-0.006
POL	2.054	0.478	5.236	4.69	86.58	0.807	0.005	0.092	-0.002
TUR	3.770	2.564	9.772	30.40	65.73	0.172	0.137	0.296	-0.037
SVK	1.639	0.441	5.223	3.09	42.68	0.752	0.008	0.117	-0.004
Data set Latin American Consensus Forecasts									
ARG	12.308	3.114	10.185	171.49	2678.76	0.777	0.007	0.106	-0.003
BRA	3.255	0.836	4.355	12.04	182.49	0.771	0.007	0.109	-0.003
CHL	1.899	0.437	1.694	4.00	75.51	0.811	0.005	0.091	-0.002
MEX	1.608	0.568	2.075	3.27	26.19	0.615	0.022	0.176	-0.010
VEN	5.813	3.243	4.782	58.10	186.71	0.306	0.086	0.276	-0.030
Std.	2.256	0.760	2.148	31.47	523.60	0.142	0.026	0.051	0.007
Min	0.849	0.199	0.799	0.80	8.05	0.172	0.002	0.066	-0.037
Max	12.308	3.243	10.185	171.49	2678.76	0.865	0.137	0.296	-0.001
Average	2.286	0.690	3.604	12.65	185.77	0.713	0.016	0.131	-0.006

Table 2: Continued

Country	$\sqrt{\widehat{FU}}$	$\sqrt{\widehat{FD}}$	$\sqrt{\widehat{Var[r_t]}}$	$\widehat{\sigma}_\delta^2$	$\widehat{\sigma}_\varepsilon^2$	$\frac{\partial FU}{\partial \sigma_\delta^2}$	$\frac{\partial FU}{\partial \sigma_\varepsilon^2}$	$\frac{\partial FD}{\partial \sigma_\delta^2}$	$\frac{\partial FD}{\partial \sigma_\varepsilon^2}$
Yields, three-months forecast horizon (23)									
Data set Consensus Forecasts (advanced economies)									
USA	0.594	0.242	1.933	0.48	2.89	0.527	0.035	0.209	-0.014
JPN	0.433	0.169	1.802	0.25	1.63	0.554	0.030	0.199	-0.013
DEU	0.516	0.197	2.334	0.35	2.40	0.568	0.028	0.194	-0.012
FRA	0.533	0.227	2.424	0.40	2.19	0.497	0.040	0.220	-0.016
GBR	0.587	0.263	2.673	0.50	2.47	0.461	0.047	0.232	-0.019
ITA	0.743	0.300	3.339	0.75	4.59	0.533	0.034	0.207	-0.014
CAN	0.564	0.260	2.014	0.47	2.21	0.443	0.050	0.238	-0.020
NLD	0.486	0.214	1.769	0.34	1.74	0.475	0.044	0.227	-0.018
NOR	0.516	0.221	1.593	0.37	2.04	0.493	0.040	0.221	-0.017
ESP	0.595	0.283	2.174	0.53	2.35	0.419	0.056	0.246	-0.022
SWE	0.609	0.243	2.358	0.50	3.14	0.540	0.032	0.204	-0.014
CHE	0.421	0.175	1.229	0.24	1.41	0.512	0.037	0.215	-0.015
Data set Asia Pacific Consensus Forecasts									
AUS	0.635	0.262	1.656	0.55	3.24	0.518	0.036	0.213	-0.015
IND	0.672	0.288	0.670	0.63	3.45	0.493	0.041	0.221	-0.017
IDN	1.091	0.551	2.155	1.88	7.35	0.376	0.066	0.258	-0.025
NZL	0.629	0.288	1.427	0.58	2.76	0.446	0.050	0.237	-0.020
KOR	0.419	0.232	0.714	0.30	0.98	0.310	0.084	0.275	-0.029
TWN	0.310	0.157	0.440	0.15	0.59	0.377	0.066	0.258	-0.025
THA	0.633	0.273	0.895	0.56	3.02	0.487	0.042	0.223	-0.017
Data set Eastern Europe Consensus Forecasts									
CZE	0.579	0.221	1.608	0.44	3.03	0.568	0.028	0.194	-0.012
HUN	0.815	0.358	2.016	0.95	4.89	0.474	0.044	0.228	-0.018
POL	0.514	0.252	1.281	0.41	1.70	0.399	0.060	0.252	-0.023
SVK	0.614	0.344	1.629	0.65	2.07	0.302	0.087	0.277	-0.030
Std.	0.152	0.079	0.673	0.33	1.40	0.074	0.016	0.023	0.005
Min	0.310	0.157	0.440	0.15	0.59	0.302	0.028	0.194	-0.030
Max	1.091	0.551	3.339	1.88	7.35	0.568	0.087	0.277	-0.012
Average	0.587	0.262	1.745	0.53	2.70	0.468	0.047	0.228	-0.018

Table 2: Continued

Country	$\sqrt{\widehat{FU}}$	$\sqrt{\widehat{FD}}$	$\sqrt{\widehat{Var[r_t]}}$	$\widehat{\sigma}_\delta^2$	$\widehat{\sigma}_\varepsilon^2$	$\frac{\partial FU}{\partial \sigma_\delta^2}$	$\frac{\partial FU}{\partial \sigma_\varepsilon^2}$	$\frac{\partial FD}{\partial \sigma_\delta^2}$	$\frac{\partial FD}{\partial \sigma_\varepsilon^2}$
Yields, twelve-months forecast horizon (23)									
Data set Consensus Forecasts (advanced economies)									
USA	1.090	0.411	1.933	1.55	10.93	0.576	0.027	0.191	-0.012
JPN	0.804	0.274	1.802	0.80	6.93	0.636	0.019	0.167	-0.009
DEU	1.031	0.306	2.334	1.26	14.31	0.708	0.012	0.137	-0.005
FRA	1.028	0.343	2.424	1.31	11.73	0.648	0.018	0.162	-0.008
GBR	1.033	0.425	2.673	1.46	8.64	0.520	0.036	0.212	-0.015
ITA	1.469	0.400	3.339	2.49	33.61	0.747	0.009	0.120	-0.004
CAN	1.046	0.406	2.014	1.45	9.60	0.557	0.030	0.198	-0.013
NLD	1.023	0.338	1.769	1.29	11.79	0.653	0.017	0.160	-0.008
NOR	1.038	0.339	1.593	1.32	12.34	0.659	0.017	0.158	-0.008
ESP	1.280	0.392	2.174	1.96	20.93	0.693	0.013	0.143	-0.006
SWE	1.299	0.366	2.358	1.97	24.78	0.732	0.010	0.126	-0.005
CHE	0.881	0.259	1.229	0.92	10.59	0.712	0.012	0.135	-0.005
Data set Asia Pacific Consensus Forecasts									
AUS	1.203	0.419	1.656	1.82	14.96	0.623	0.021	0.172	-0.009
IND	0.784	0.463	0.670	1.12	3.20	0.265	0.099	0.284	-0.032
IDN	1.999	0.829	2.155	5.49	31.92	0.514	0.037	0.214	-0.015
NZL	1.015	0.380	1.427	1.34	9.56	0.580	0.026	0.189	-0.011
KOR	0.991	0.410	0.714	1.35	7.89	0.517	0.036	0.213	-0.015
TWN	0.687	0.256	0.440	0.61	4.40	0.583	0.026	0.188	-0.011
THA	0.968	0.421	0.895	1.33	7.02	0.483	0.042	0.225	-0.017
Data set Eastern Europe Consensus Forecasts									
CZE	1.087	0.328	1.608	1.41	15.50	0.701	0.013	0.140	-0.006
HUN	1.540	0.553	2.016	3.02	23.40	0.605	0.023	0.179	-0.010
POL	0.894	0.372	1.281	1.10	6.35	0.513	0.037	0.214	-0.015
SVK	1.089	0.429	1.629	1.58	10.22	0.549	0.031	0.201	-0.013
Std.	0.275	0.114	0.673	0.97	8.01	0.105	0.018	0.038	0.006
Min	0.687	0.256	0.440	0.61	3.20	0.265	0.009	0.120	-0.032
Max	1.999	0.829	3.339	5.49	33.61	0.747	0.099	0.284	-0.004
Average	1.099	0.396	1.745	1.65	13.50	0.599	0.027	0.179	-0.011

The table shows estimates for the square roots of forecast uncertainty FU, forecast dispersion FD and variability $Var[r_t]$ for interest rate and yield forecasts at the three-months and twelve-months forecast horizon (four sets). Moreover, it shows estimates for σ_δ^2 and σ_ε^2 and the derivatives of FU and FD. Each set is summarized with its cross-country standard deviation, minimum and maximum value as well as the cross-country average. For derivations of the theoretical values see Appendix: Theory.

Appendix: Theory

A stationary interest rate model

The stationary interest rate model (equation (1)) in the text implies

$$\begin{aligned} \theta_t &= r_t - r = \rho \cdot \theta_{t-1} + v_t \\ \Leftrightarrow \theta_{t+1} &= \rho \cdot \theta_t + v_{t+1}. \end{aligned} \tag{8}$$

Recursive use of equation (8) yields an expression for the cumulative innovation, δ_{t+h}, to the stochastic part of the interest rate process over the $[t+1, t+h]$ forecast horizon (equation (2) in the text). Equation (2) implies that the variance of δ_{t+h} is given by

$$\sigma_\delta^2 = E\left[v_{t+h} + \rho \cdot v_{t+h-1} + ... + \rho^{h-1} \cdot v_{t+1}\right]^2 = \frac{1-\rho^{2h}}{1-\rho^2} \cdot \sigma_v^2 \tag{9}$$

Structure of information and the forecasting model

In period t forecaster i possesses observations on all past values of θ up to and including period t. The long run value, r, of the interest rate and ρ are known to all forecasters in all periods. Each forecaster also observes an individual noisy signal s_{it} on the stochastic component of the interest rate δ_{t+h}. The noisy signal is given by

$$s_{it} \equiv \delta_{t+h} + \varepsilon_{it+h} \tag{10}$$

where ε_{it+h} is a white noise process with a constant variance σ_ε^2 across forecasters, and has no contemporaneous correlation with the noises of the other forecasters. The variances σ_δ^2 and σ_ε^2 are common knowledge. Since all forecasters try to forecast the same stochastic variable, δ_{t+h}, there will be contemporaneous correlation between the signals of any two different forecasters. In addition, since the forecast horizon, h, is larger than one there will be serial correlation in each individual forecast. But none of these facts interferes with the derivation of the optimal predictor that follows.

Since the current value of the state is known by all forecasters the common prior of θ_{t+h} is $\rho^h \cdot \theta_t$ and the prior of δ_{t+h} is zero. The joint distribution of θ_{t+h} and s_{it} is given by

$$\begin{bmatrix} \theta_{t+h} \\ s_{it} \end{bmatrix} \sim N\left(\begin{bmatrix} \rho^h \cdot \theta_t \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\delta^2, & \sigma_\delta^2 \\ \sigma_\delta^2, & \sigma_\delta^2 + \sigma_\varepsilon^2 \end{bmatrix} \right)$$

We use this joint distribution to derive the conditional forecast. For that purpose, we make use of the general formula for normally distributed conditional expectations (Bayesian expectations).[9]

$$E\left[x_1|x_2\right] = \mu_1 + \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot (x_2 - \mu_2)$$

with $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$.

Here x_2 is the observed signal about x_1, μ_1 is its prior, and Σ is the covariance matrix. In our case the relevant conditional expectation is

$$f_{it} \equiv E\left[r_{t+h}|r_t, s_{it}\right] = r + E\left[\theta_{t+h}|\theta_t, s_{it}\right]$$

and the general matrices above specialize to $x_2 = [s_{it}]$, $\mu_1 = [\rho^h \cdot \theta_t]$, $\mu_2 = [0]$, $\Sigma_{11} = [\sigma_\delta^2]$, $\Sigma_{12} = [\sigma_\delta^2]$, and $\Sigma_{22} = [\sigma_\delta^2 + \sigma_\varepsilon^2]$. Applying the general formula to our framework yields

$$\begin{aligned} f_{it} &\equiv r + E\left[\theta_{t+h}|\theta_t, s_{it}\right] = r + \rho^h \cdot \theta_t + w_s \cdot s_{it} \\ &= r + \rho^h \cdot \theta_t + \frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\varepsilon^2} \cdot (\delta_{t+h} + \varepsilon_{it+h}) \end{aligned} \tag{11}$$

which is equation (3) in the text. Here $w_s \equiv \sigma_\delta^2/(\sigma_\delta^2 + \sigma_\varepsilon^2)$.

[9]See, for example, theorem B.7 in Greene (2012), p. 1081 ff.

Variability of rates ($Var[r_t]$)

By using equation (1) repeatedly it is possible to express θ_t as the following infinite weighted sum of past values of the innovations, v_t.

$$\theta_t = \sum_{i=0}^{\infty} \rho^i v_{t-i}$$

Equation (1) therefore implies

$$E[r_t] = r + E[\theta_t] = r + E\left[\sum_{i=0}^{\infty} \rho^i v_{t-i}\right] = r$$

Here E stands for the unconditional expected value and the last equality follows from the fact that the unconditional expected value of the v's is zero. Hence

$$\begin{aligned} Var[r_t] &= E\left[r + \theta_t - r\right]^2 = E\left[\theta_t\right]^2 = E\left[\sum_{i=0}^{\infty} \rho^i v_{t-i}\right]^2 \\ &= \sum_{i=0}^{\infty} \rho^{2i} \cdot \sigma_v^2 = \frac{\sigma_v^2}{1-\rho^2} = \frac{\sigma_\delta^2}{1-\rho^{2h}} \end{aligned} \tag{12}$$

where the last equality folows from equation (9). It is easy to see that the variability of interest rates is increasing in the variance of the cummulative innovation, σ_δ^2, and in the persistence, ρ, of the innovation, v. The positive impact of σ_δ^2 on $Var[r_t]$ is not surprising since it reflects the variability in v which is at the origin of variability in the interest rate.

Forecasts' dispersion (FD)

From equation (11) we have

$$f_{it} = r + \rho^h \cdot \theta_t + w_s \cdot (\delta_{t+h} + \varepsilon_{it+h}) \tag{13}$$

An approximate expression for the mean forecast across forecasters in a given period is:[10]

$$\bar{f}_t = r + \rho^h \cdot \theta_t + w_s \cdot \delta_{t+h} \tag{14}$$

Subtracting equation (14) from equation (13) yields

$$f_{it} - \bar{f}_t = w_s \cdot \varepsilon_{it+h} \tag{15}$$

FD in a given sample/period is defined as the unconditional expected value of the square of the expression in equation (15)

$$\begin{aligned} FD &\equiv E\left[f_{it} - \bar{f}_t\right]^2 = w_s^2 \cdot E\left[\varepsilon_{it+h}\right]^2 = w_s^2 \cdot \sigma_\varepsilon^2 \\ &= \left(\frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\varepsilon^2}\right)^2 \cdot \sigma_\varepsilon^2. \end{aligned} \tag{16}$$

Substituting equation (9) into equation (16) yields

$$FD = \left(\frac{\frac{1-\rho^{2h}}{1-\rho^2} \cdot \sigma_v^2}{\frac{1-\rho^{2h}}{1-\rho^2} \cdot \sigma_v^2 + \sigma_\varepsilon^2}\right)^2 \cdot \sigma_\varepsilon^2 \tag{17}$$

Obviously, the mean, over all periods of FD in a given country is also equal to the expressions in the last two equations. Inspection reveals that it is increasing in both σ_δ^2 and σ_v^2. This is intuitive: When either σ_δ^2 or σ_v^2 goes up each individual forecaster gives more weight to his private information and less to the uniform public information. Since the private information is noisy this raises the dispersion of forecasts across forecasters.

Forecasts' uncertainty (FU)

The measure of forecast uncertainty is based on the difference between r_{t+h} and its period t forecast. Combining equations (1) and (2) in the text yields the following expression for

[10]The expression is approximate because, for convenience, we approximate the sample mean of ε_{it+h} by its population mean which is zero. The accuracy of the approximation increases with the number of forecasters.

r_{t+h}

$$r_{t+h} = r \cdot (1-\rho^h) + \rho^h \cdot r_t + \delta_{t+h} \tag{18}$$

Substracting the mean forecast in equation (14) from equation (18)

$$\begin{aligned} r_{t+h} - \bar{f}_t &= (1-w_s) \cdot \delta_{t+h} = \left(1 - \frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_\varepsilon^2}\right) \cdot \delta_{t+h} \\ &= \left(\frac{\sigma_\varepsilon^2}{\sigma_\delta^2 + \sigma_\varepsilon^2}\right) \cdot \delta_{t+h}. \end{aligned} \tag{19}$$

The difference in equation (19) is the mean forecast error. The corresponding measure of forecast uncertainty for a particular sample in period t is

$$FU \equiv E\left[\frac{\sigma_\varepsilon^2}{\sigma_\delta^2 + \sigma_\varepsilon^2} \cdot \delta_{t+h}\right]^2 = (1-w_s)^2 \cdot \sigma_\delta^2 = \left(\frac{\sigma_\varepsilon^2}{\sigma_\delta^2 + \sigma_\varepsilon^2}\right)^2 \cdot \sigma_\delta^2. \tag{20}$$

Obviously, the unconditional expected value of the mean over all periods in a particular country is identical to the expression in equation (20).

Derivation of σ_ε^2 and σ_δ^2 from FU and FD

In order to evaluate whether the conditions in Propsitions 1 and 2 are satisfied in the data empirical proxies for σ_ε^2 and σ_δ^2 are needed. Since we have empirical measures of FD and FU the proxies for σ_ε^2 and σ_δ^2 can be obtained by expressing those basic variances in terms of FD and FU.

Proposition 3

$$\sigma_\varepsilon^2 = \frac{(FD+FU)^2}{FD} \tag{21}$$

$$\sigma_\delta^2 = \frac{(FD+FU)^2}{FU} \tag{22}$$

Proof. Dividing equation (6) by equation (5)

$$\frac{FU}{FD} = \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}. \tag{23}$$

Rearranging equation (5)

$$FD = \frac{1}{\left(1 + \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\right)^2} \cdot \sigma_\varepsilon^2. \tag{24}$$

Substituting equation (23) into equation (24), rearranging and solving for σ_ε^2 establishes equation (21) of the proposition. Inserting equation (21) into equation (23)

$$\sigma_\delta^2 = \sigma_\varepsilon^2 \frac{FD}{FU} = \frac{(FD + FU)^2}{FD} \frac{FD}{FU} = \frac{(FD + FU)^2}{FU}$$

which establishes equation (22). ■