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**BUSINESS CYCLE DURING
STRUCTURAL CHANGE: ARTHUR
LEWIS' THEORY FROM A
NEOCLASSICAL PERSPECTIVE**

Kjetil Storesletten, Bo Zhao and Fabrizio Zilibotti

MACROECONOMICS AND GROWTH



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Abstract

We document that business cycles change during the process of development. In countries with large declining agricultural sectors, aggregate employment is uncorrelated with GDP. During booms, agricultural employment declines even though agricultural labor productivity increases relative to other sectors. We construct a unified theory of business cycles and structural change consistent with the stylized facts. The theory focuses on the simultaneous decline and modernization of agriculture. As capital accumulates, agriculture becomes increasingly capital intensive as traditional agriculture is crowded out. We estimate the model and show that it accounts well for both structural transformation and business cycle fluctuations in China.

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Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective.*

Kjetil Storesletten

Bo Zhao

Fabrizio Zilibotti

March 25, 2020

Abstract

We document that the nature of business cycles evolves over the process of development and structural change. In countries with large declining agricultural sectors, aggregate employment is uncorrelated with GDP. During booms, employment in agriculture declines while labor productivity increases in agriculture more than in other sectors. We construct a unified theory of business cycles and structural change consistent with the stylized facts. The focal point of the theory is the simultaneous decline and modernization of agriculture. As capital accumulates, agriculture becomes increasingly capital intensive as modern agriculture crowds out traditional agriculture. Structural change accelerates in booms and slows down in recessions. We estimate the model and show that it accounts well for both the structural transformation and the business cycle fluctuations of China.

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1 Introduction

The nature of economic fluctuations differs systematically at different stages of economic development (see, e.g., Acemoglu and Zilibotti (1997), Aguiar and Gopinath (2007)). China provides a good example. The country has experienced a profound economic transformation with the share of employment in agriculture falling from about 2/3 in 1980 to about 1/3 today. During this time, structural change has systematically accelerated during economic booms and stagnated during recessions, implying procyclical employment in nonagriculture and countercyclical employment in agriculture. Aggregate employment has instead been uncorrelated with GDP, while nonagricultural employment has been strongly procyclical. The cyclicity of labor productivity in agriculture is also noteworthy: relative labor productivity in agriculture has increased in booms as workers have been leaving the rural sector.

These features are shared by the majority of countries at a comparable stage of development. Extending the work of Da Rocha and Restuccia (2006) beyond OECD countries, we show that the correlation between agricultural employment and aggregate GDP varies systematically with the relative size of the agricultural sector. While employment is procyclical in industrialized countries, it is acyclical in economies with a large agricultural sector. In these poorer economies, employment in agriculture is negatively correlated with employment in the rest of economy, while such a correlation is zero or even positive in the industrial world. Finally, downswings in agricultural employment are associated with upswings in the relative productivity and capital intensity of the agricultural sector in developing countries. No such pattern is observed in mature economies where the agricultural sector is typically very small and its behavior shows no significant correlation with aggregate GDP. At the aggregate level, employment is on average more procyclical and volatile in fully industrialized countries than in developing and emerging countries.

To explain these observations, we propose a neoclassical theory of growth and structural change where the economy is subject to stochastic productivity shocks. In our theory, the same technological forces drive the structural transformation and business cycle fluctuations at different stages of development. The key drivers of growth and structural change are sector-specific TFP growth (as in Ngai and Pissarides (2007)) and endogenous capital accumulation (as in Acemoglu and Guerrieri (2008)). Investments, capital deepening, and TFP growth cause both reallocation from agriculture to nonagriculture and *modernization* of agriculture. To capture modernization, we depart from standard theories of structural change and assume that the rural sector comprises two subsectors, modern and traditional agriculture, producing imperfect substitutes with different technologies. In particular, modern agriculture uses capital which is instead not an input of the traditional sector. Under empirically plausible assumptions about parameters, agriculture shrinks as a share of total GDP and becomes more productive – not only in an absolute sense but also relative to the rest of the economy. In the long run, the equilibrium converges to an asymptotic balanced growth path where the agricultural sector is small, modernized, and highly productive.

The theory embeds a single exogenous frictions, namely, a time-invariant wedge between agriculture and nonagriculture that prevents the equalization of wages and keeps agriculture inefficiently too large. This friction has no bearing on the qualitative predictions but affects the quantitative results for reasons we explain below. Capital can move freely across sectors, although in an extension we consider sectoral capital adjustment costs.

The mechanism in our theory is reminiscent of Lewis (1954), where the existence of a labor-intensive sector makes the supply of labor to nonagriculture very elastic. In Lewis' theory, labor supply is infinitely elastic for as long as the traditional sector exists. In such a model, the elasticity would fall discretely as soon as the last worker moves out of the traditional sector. In our model, instead, the elasticity of labor supply declines gradually during the process of modernization of agriculture. A poor economy behaves like a Lewis economy; then, throughout the development process, it becomes more and more similar to a standard

neoclassical economy.

Structural change has implications for business cycles. When the agricultural sector is large and predominantly traditional, the economy responds to sector-specific shocks by reallocating workers towards the more productive sectors with limited effects on wages and relative prices. As the agricultural sector declines, this margin of adjustment is muted. Wages and prices respond more to productivity shocks leading to larger swings in labor supply. We show that these theoretical predictions are in line with stylized facts about both structural change and business cycles across countries.

We estimate the growth model using data from China. The key parameters are the elasticities of substitution between the output of the agricultural and nonagricultural sector and, within agriculture, between the two agricultural subsectors. We estimate both elasticities to be significantly larger than unity. Because a declining agricultural sector is also consistent with nonhomothetic preferences, we allow preferences to be nonhomothetic using a generalized Stone-Geary specification as in Herrendorf et al. (2013). The estimated deviation from homothetic preferences is small. While nonhomothetic preferences are known to be important at the micro level, they do not seem to carry an important role in explaining macroeconomic dynamics. Our findings are in line with the recent evidence in Alvarez-Cuadrado and Poschke (2011).¹

Having estimated the deterministic model, we introduce stochastic shocks. We show that our model fits quantitatively well salient features of the business of industrializing economies, most notably China. This step is technically challenging. Because structural change is still ongoing we cannot rely on the standard approach of approximating the model in the neighborhood of a steady state. The moments from the model must then be estimated by simulating a large number of trajectories approaching the steady state, and calculating moments from that.

Among other things, the model explains why positive TFP shocks in nonagriculture cause a temporary acceleration of the process of structural change. Such shocks trigger an increase in total investments and a reallocation of capital and labor out of agriculture. Labor productivity grows, both because of the increase in average TFP and because the resource reallocation reduces misallocation. Interestingly, labor productivity increases more in agriculture than in the rest of the economy, as most of the temporary reallocation of labor away of the agriculture is drawn from the traditional sector. Therefore, agriculture experiences a sharp increase in capital intensity and average labor productivity. Out-of-sample predictions also confirm that as structural change progresses the business cycle properties of the model become increasingly similar to those of advanced economies.

Our research contributes to the existing literature on structural change pioneered by Baumol (1967) which includes, among others, Kongsamut et al. (2001), Gollin et al. (2002), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Buera and Kaboski (2009), Alvarez-Cuadrado and Poschke (2011), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2015), and Alder et al. (2018). In our model, exogenous TFP growth and capital accumulation induce transition away from agriculture. The properties of the transition are consistent with Acemoglu and Guerrieri (2008), although in their model there is no traditional sector. The closest theoretical contribution in the literature is the recent paper by Alvarez-Cuadrado and Poschke (2011). They study the properties of two-sector models where both sectors use capital and labor as inputs. The elasticity of substitution between capital and labor is assumed to be constant within each sector but can differ across sectors. In contrast, in our model this elasticity changes over the process of development due to

¹Note that our estimates is different from that obtained by Herrendorf et al. (2013). They estimate a production function with three sectors, agriculture, manufacturing, and services, and find a low elasticity of substitution close to Leontief when using value-added data. Their estimate hinges on an assumption of symmetry, namely that the same elasticity of substitution is imposed across the three sectors. We show that if one relaxes the symmetry assumption the estimated elasticity of substitution between manufacturing and services is indeed close to zero, whereas the elasticity of substitution between agricultural and nonagricultural goods is larger than unity.

the reallocation between traditional and modern agriculture.² While our main focus is on the business cycle implications of the theory, Alvarez-Cuadrado and Poschke (2011) do not consider economic fluctuations.

Our paper is also related to the debate on the effects of technological progress and demand factors on structural change and industrialization. The traditional development literature suggests that productivity in growth agriculture combined with Engel’s law released resources from agriculture and generated industrialization (*labor push*), cf. Nurkse (1953) and Rostow (1960) (see Gollin et al. (2002) for a recent formalization of these ideas). An alternative view – the *labor pull* theory – is that fast productivity growth in manufacturing attracted workers from agriculture to manufacturing (cf. Mokyr (1976)). Our model can be consistent with either of these two views depending on parameters. Under the assumption that agricultural and industrial production are gross substitutes, which is the outcome of our structural estimates for China, it is in line with the labor pull theory. We document that our results are in line with the common narrative about episodes of business cycles in recent Chinese history. The findings are also consistent with the recent empirical literature on the Green Revolution that we review in more detail in Section 6.3 (see, e.g., Foster and Rosenzweig (2004), Bustos et al. (2016), Jayachandran (2006), and Moscona (2018)). These studies point at the fact that both across countries and within developing countries, exogenous positive productivity shocks in agriculture (such as rainfall or the introduction of new technologies) slow down industrialization and structural change and halts the demise of the agricultural sector.

Our work also adds to the existing business-cycle literature by incorporating endogenous structural change. The classical multi-sector model focus on the stable economic structures or abstract from growth, for example, Benhabib et al. (1991), Horvath (2000), Boldrin et al. (2001), etc. We extend the standard business cycle model to account for the business cycles properties at different stages of development. Related researches on cross-country business cycles differences including Da Rocha and Restuccia (2006) and Aguiar and Gopinath (2007).³ Recent studies on the properties of the Chinese business cycle include Yao and Zhu (2018) and Chang et al. (2016). Yao and Zhu (2018) construct a two-sector model of the Chinese economy in which consumers have non-homothetic preferences and show that in such a model the size of the agricultural sector affects the nature of business cycles. Different from our theory, their model abstracts from capital and investments nor does it focus on modernization in agriculture. Our structural estimation allows for nonhomothetic preferences but these appear to have a modest quantitative effect relative to technological factors. The two approaches are complementary. The process of modernization of agriculture is instead at the core of the recent papers by Chen (2018) and Boppart et al. (2019), although the focus of their papers is very different from ours.

The remainder of the paper is organized as follows. Section 2 presents a set of stylized facts about business cycles and structural change across countries, zooming on China and the US. Section 3 lays out the model. Section 4 describes the estimation of the growth model. Section 5 performs a quantitative analysis of the business cycle properties of the model. Section 6 compares the results with additional international evidence. Section 7 concludes. An appendix includes proofs and additional material referred in the text.

²Another difference is that in our specification the labor income share in agriculture remains bounded away from zero, while in the CES technology proposed by Alvarez-Cuadrado and Poschke (2011) it would fall to zero with development. The empirical labor income share in the U.S. farm sector has remained above 44%.

³In particular, Da Rocha and Restuccia (2006) documents that among OECD countries, economies with larger agricultural sectors have smoother aggregate employment fluctuations and that agricultural employment is less correlated with aggregate GDP. Relative to their paper we show that this pattern holds up when extending the sample of countries to include a large number of countries, including very poor countries that are predominantly agrarian. Moreover, we document several additional salient differences in business cycle properties between poor and rich countries and show that business cycle properties of China are representative of countries with a similar share of agriculture.

2 Evidence on Modernization of Agriculture and Business Cycles

The process of economic development is associated with a significant downsizing of the agricultural sector. In the US, a third of the workforce was employed at farms in 1900. This employment share fell below 2% by 2000. Today, the average employment in agriculture is 4.6% in OECD countries, which compares with 31.6% in non-OECD countries (World Development Indicators 2017). The cross-country correlation between the employment share of agriculture and log GDP per capita is -0.84.

2.1 Modernization of Agriculture

The relative decline of agriculture is accompanied by a number of economic transformations. First, as employment in agriculture declines, the capital intensity of this sector rises faster than in the rest of the economy. Second, labor productivity grows faster in agriculture than in the rest of the economy.

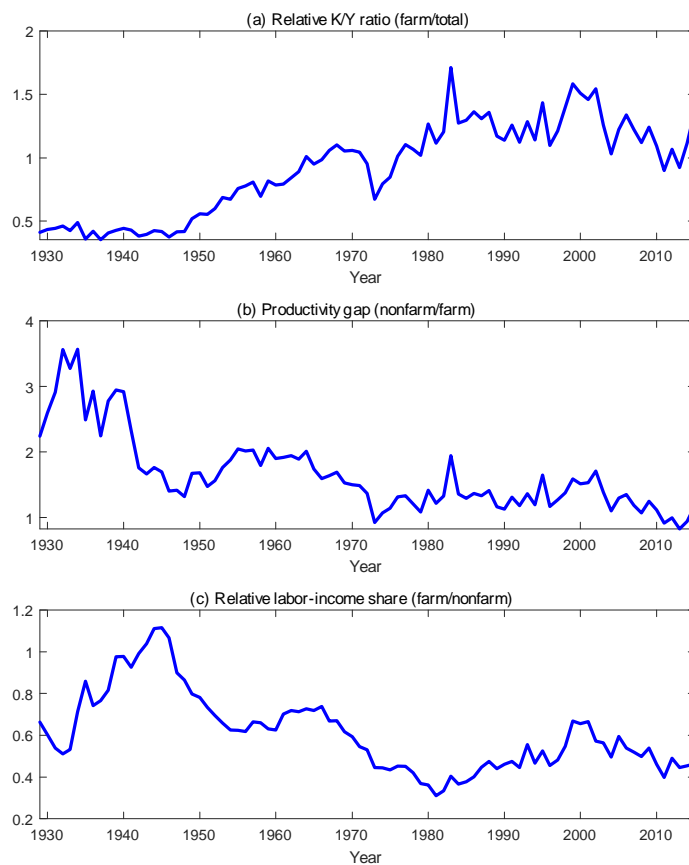


Figure 1: Panel a plots the farm capital-output ratio divided by the total capital-output ratio in the US. Panel b plots the Labor productivity gap over time in the US. The labor productivity gap is measured as the nonfarm value added per worker divided by the farm value added per worker. Panel c plots the ratio of the labor income share in the farm sector over the labor-income share in the non-farm sector in the US. These labor-income shares are measured as the compensation of employees divided by the value added excluding proprietors' income. Sources: Capital stocks by sectors 1929-2015 are from the U.S. Bureau of Economic Analysis (BEA) Fixed Asset Tables 6.1 "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods". The value added by sector come from the National Income and Product Accounts (NIPA) Table 1.3.5. Employment by sector is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. Proprietors' income by sectors come from the NIPA Table 1.12. Compensation of employees by sectors come NIPA Table 6.2A, 6.2B, 6.2C, and 6.2D.

Relative Capital Deepening. While capital deepening is pervasive over the development process, it is especially pronounced in agriculture: both the capital-output and the capital-labor ratio grow faster in agriculture than in the nonfarm sector. Figure 1 (panel a) shows the time series of capital-output (K-Y) ratio in agriculture relative to the aggregate K-Y ratio (henceforth, the *relative K-Y ratio*) for the US. The relative K-Y ratio increased from about 40% in the pre-war period, to about 120% since the 1980s. Figure 2

(panel a) plots the relative K/Y ratio against the employment share of agriculture for the period 1995-2016 across countries. The relative K-Y ratio is significantly lower in developing than in industrialized countries. For example, it is very low in Sub-Saharan African countries, where agriculture is very labor intensive and employs a large proportion of the labor force. Since there are concerns about data quality of agricultural capital in very poor countries, in panel (b) we drop African countries from the sample. This strengthens the results: the regression coefficient becomes more negative and significant.

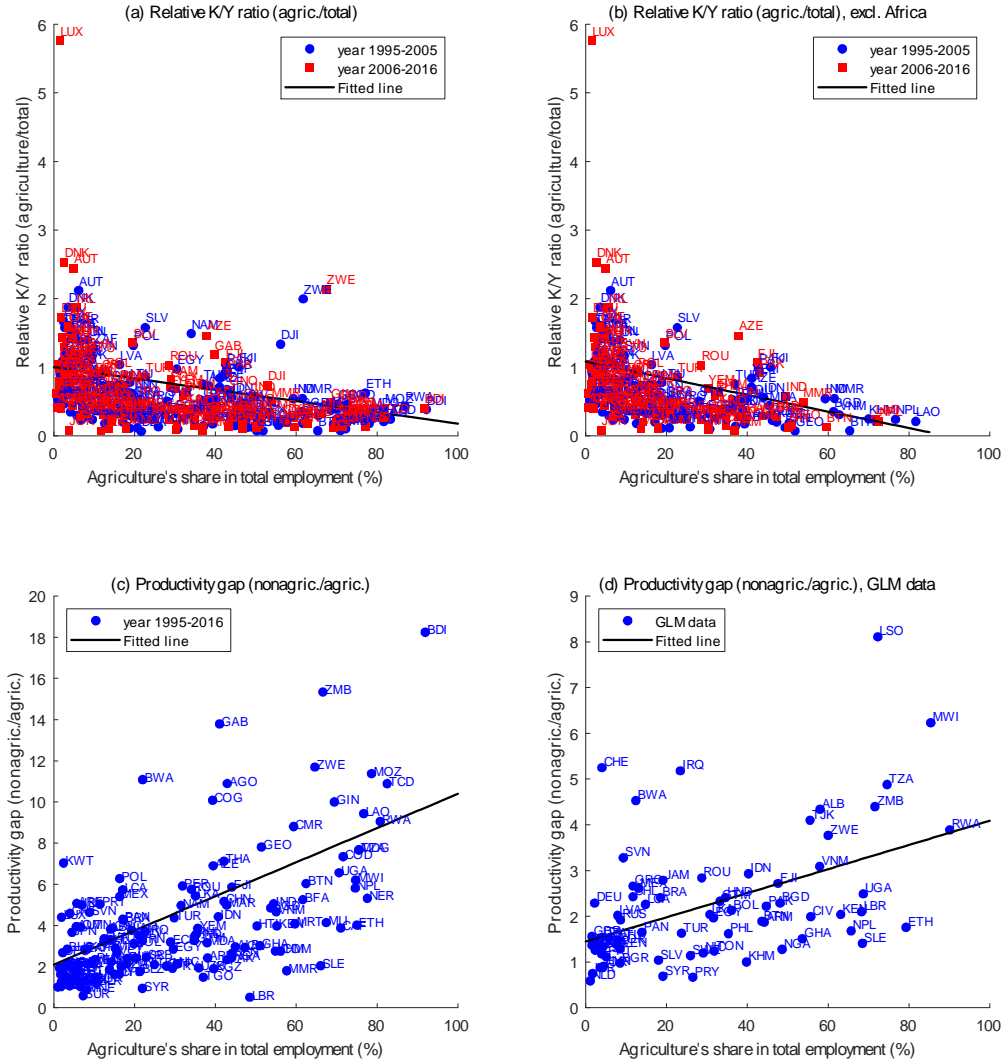


Figure 2: Panel a and b plot the relative capital-output ratio (agriculture vs. total) $(K^G/Y^G)/(K/Y)$ against the average agricultural employment share across countries. Each country has two observations: the average for the period 1995-2005 and the average for the period 2006-2016. Panel b excludes African countries. Panel c plots labor productivity gap across countries. The labor productivity gap is measured as the nonagriculture value added per worker divided by the agriculture value added per worker. The horizontal axis shows the average employment share of agriculture over the sample period for each country. Panel d shows the same relationship using the data in Gollin et al. (2014) who control for sectoral differences in hours worked and human capital per worker. Source: FAO (K^G is measured by the net capital stock; Y^G is value added, both at current prices) and Penn World Table (capital stock and real GDP at current national prices from detailed Penn World Table 9.1). Agriculture's employment share comes from ILO modeled estimates. Agriculture's value added output share comes from FAO extracted UNSD AMA data.

Since data are available over a 22 year panel, we can also study the within-country coevolution of employment shares in agriculture and relative K-Y ratios. To this aim, we split the sample for each country into two observations, the average for the period 1995-2005 and the average for the period 2006-2016, and regress the relative K-Y ratio on the employment share of agriculture and a full set of country dummies. Panel (a) in appendix Table 7 shows a set of regression results. Since the relationship is nonlinear, we regress the logarithm of the K-Y ratio on the logarithm of the employment share. We run both pooled

regressions and regressions with country fixed effects. The results confirm a negative (in most cases, highly significant) relationship between employment in agriculture and relative K-Y ratios. The coefficients of the different regressions are similar in magnitude, indicating a strong consistency in the patterns across and within countries.⁴

Productivity Gap. Capital deepening and technical change bring about higher labor productivity in all sectors of the economy. However, productivity grows faster in agriculture than in the rest of the economy. To show this, define the *productivity gap* to be the average labor productivity in nonagriculture relative to agriculture. Figure 1 (panel b) shows that the productivity gap declines over time in the United States.⁵ Figure 2 (panel c) shows that a similar pattern holds true across countries: the productivity gap is especially high in poor countries with a large agricultural sector (a fact that has been documented by, among others, Restuccia et al. (2008) and Gollin et al. (2014)). The same pattern emerges from fixed effect regressions exploiting the within-country variation over time (see panel b in appendix Table 7.) Panel d shows the same relationship using the cross-country data in Gollin et al. (2014) which controls for sectoral differences in hours worked and human capital per worker. The regression coefficient is in both cases significantly positive.

The behavior of the labor income share in agriculture relative to the aggregate labor-income share mirrors that of the productivity gap. Figure 1 (panel c) shows that the labor-income share in the farm sector relative to that of the nonfarm sector fell over time in the US since the end of World War II. Unfortunately, it is difficult to find comparable data across a large number of countries since the labor income share in agriculture is often poorly measured.

2.2 Business Cycles

Consider, next, economic fluctuations. Panels a and b in Figure 3 compare the business cycle of China with that of the US. While aggregate employment is volatile and highly procyclical in the US, aggregate employment is acyclical and relatively smooth in China.⁶⁷ The data are annual and are filtered using an HP smoothing parameter of 6.25 ((Ravn and Uhlig 2002)). These patterns are robust to using alternative filters such as first differencing the data.

Interestingly, aggregate fluctuations in employment are closely associated with the process of structural change. Consider panel c in Figure 3. Until 1960, NBER recessions in the US were associated with a slowdown and at occasion even a reversal of the industrialization process. Namely, the employment share of agriculture used to fall in booms and increase in downturns. The cyclicity of employment in agriculture declines over time and ceases to be noticeable after 1960. China today looks similar to the US of the earlier days. Panel d in Figure 3 shows that structural change – measured by the decline in agricultural employment – accelerates during periods of high growth and slows down or halts during periods of low growth in China.

Panel b of Figure 4 documents that agricultural employment is volatile and countercyclical in China.

⁴The regressions in column (3) and (4) do not include time dummies and identify the effect of interest out of both global trends and cross-country deviations. The regressions in columns (5) and (6) include time dummies, filtering out common trends as in standard panel regressions. The magnitude of the coefficients is similar in all specifications, although the statistical significance drops in panels (5)-(6).

⁵Labor productivity is measured as value added per worker in current prices.

⁶Appendix Figure 14 shows that the volatility of employment relative to the volatility of output in the US has increased over time. Before 1980, employment was significantly less volatile than output, while after 2000 employment has been more volatile than output.

⁷Sectoral employment and aggregate employment in China are from the Statistic Year Book by the China National Bureau of Statistics. From 1990 and onward NBS estimates aggregate employment based on a labour force survey, while employment before 1990 is based on annual administrative data. To make the series consistent, we apply the correction proposed by Holz (2006) revising the official employment data before 1990 using the 1982 and 1990 population census. Sectoral employment after 1990 is based on working status one week before the survey is conducted. (Brandt and Zhu 2010) estimate an alternative series of agricultural employment based on the rural employment statistics. Their series have very similar cyclical fluctuations as the data we use, albeit a steeper trend (see the appendix for further details).

There is no such pattern in the contemporary data for the US, where the correlation is positive rather than negative. In contrast, the cyclical pattern of nonagricultural employment is similar in the US and in China: Panels c and d of Figure 4 shows that nonagricultural employment is highly procyclical and roughly as volatile as GDP in both China and the US. It follows that agricultural and nonagricultural employment are strongly negatively correlated in China.

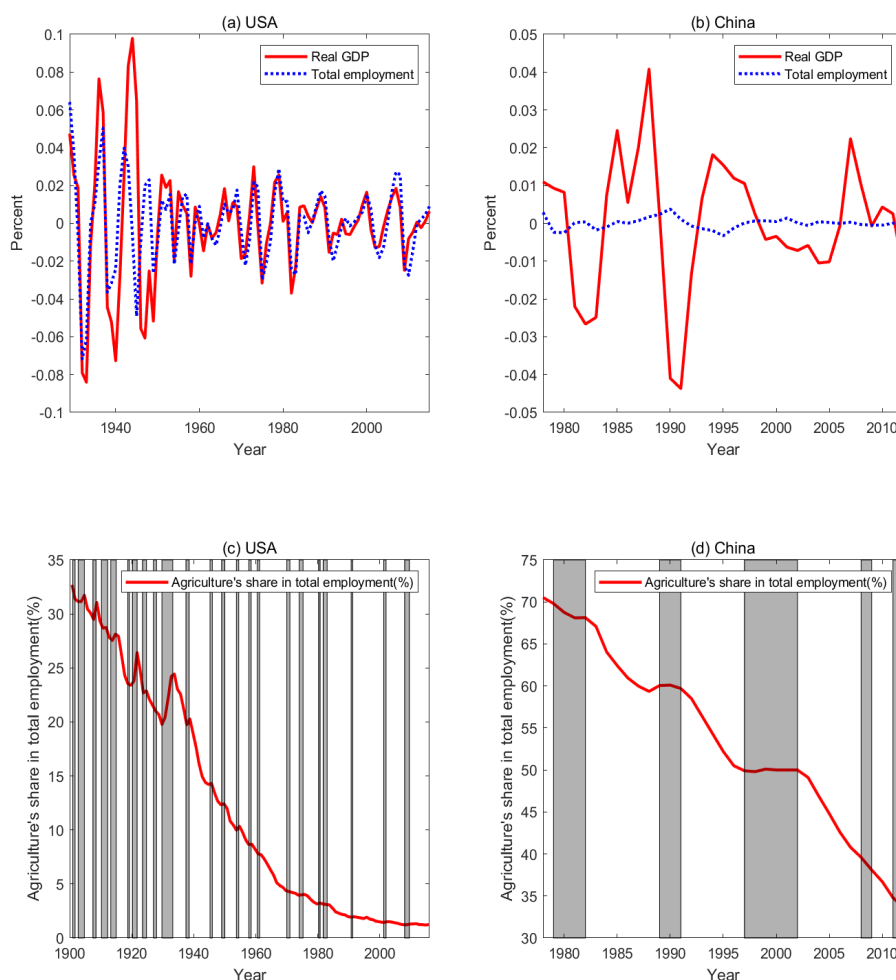


Figure 3: Panel a and b plot the volatility of HP-filtered total employment and real GDP in the US (left panel) and China (right panel). Source: NBS (see also footnote 7). Panel c and d plot the agriculture’s share in total employment over the business cycles. The left panel plots the farm employment share in the US. Grey ranges denote recessions as classified by the NBER. The right panel plots the agriculture employment share over time in China. Grey ranges denote recessions, defined as years when the real GDP growth rate was below the average growth rate (9.7 percent during 1978-2012).

The stylized facts documented above are consistent with international data.⁸ Panel (a) in Figure 5 shows that the time-series correlation of the HP-filtered employment in agriculture and nonagriculture is positive (on average) for countries with a small agricultural sector like the US and negative for countries with a large agricultural sector like China. The volatility of employment relative to GDP is also negatively correlated with the employment share of agriculture (see appendix Figure 13), consistent with the time series evidence for the US – see appendix Figure 14.⁹ Panel (c) in Figure 5 shows that the correlation between aggregate employment and GDP declines strongly with the employment share of agriculture, being large and positive

⁸We use sector-level employment data from the International Labor Organization (ILO) (see Appendix A).

⁹This relationship is not statistically significant. If we weight countries by size, the relationship is negative and significant.

for industrialized countries like the US and negative for countries with a large agricultural sector. This is in line with the US-China contrast discussed above.

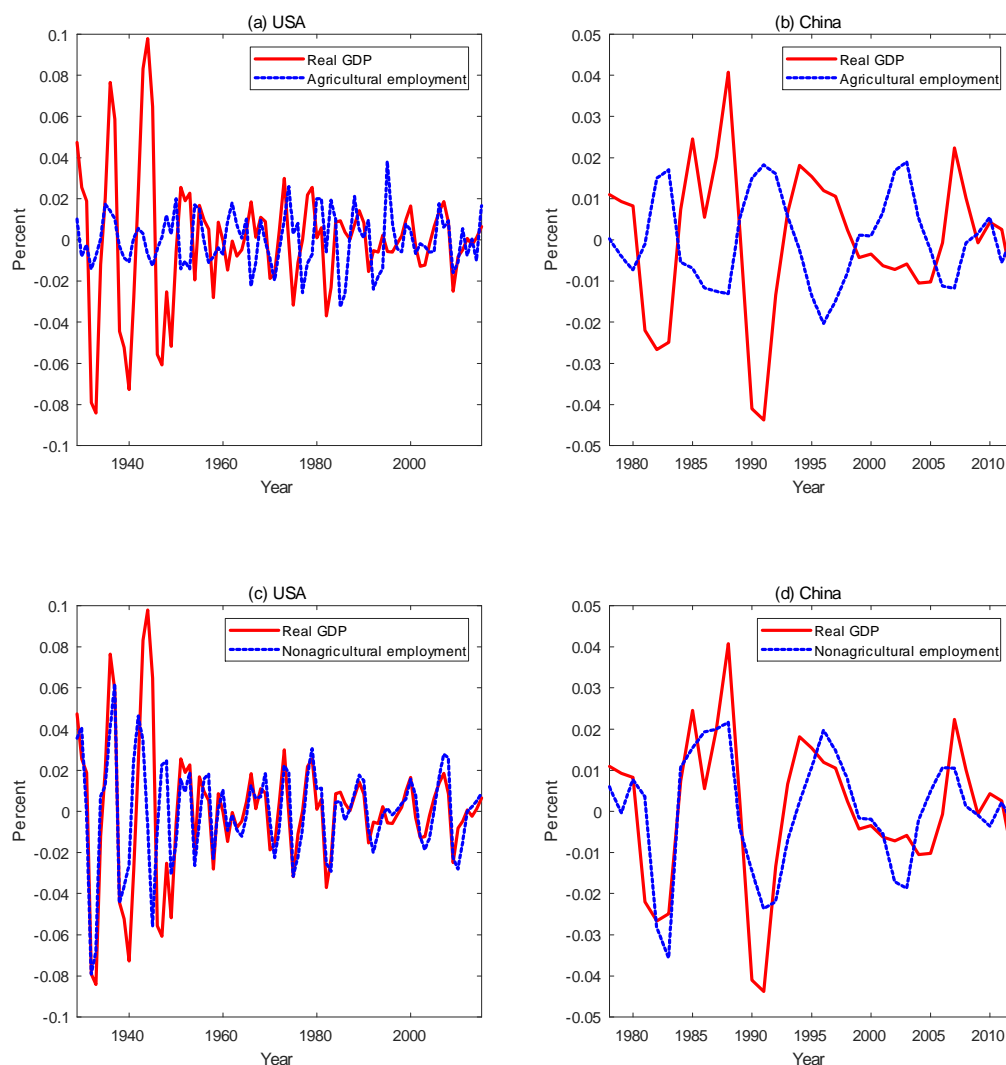


Figure 4: Panel a and b plot the HP-filtered log agricultural employment vs HP-filtered log real GDP in the US 1929-2015 and in China 1978-2012. Panel c and d plot the HP-filtered log nonagricultural employment vs HP-filtered log real GDP in the US and in China. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). Source: The US employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The sectoral employment data in China is from the Statistic Year Book by the China National Bureau of Statistics, Table "Number of employed persons at year-end by three strata of industry". The number is calculated based on the households survey on both urban and rural households in China. The nonagriculture employment is the sum of both employment in the secondary industry and the employment in the tertiary industry. We incorporate a correction proposed by (Holz 2006). The correction takes care of the reclassification of employed workers that was made by the NBS in 1990.

Next, we highlight the dynamics of the productivity gap. We have already shown that the productivity gap falls throughout structural change, (see panel b of Figures 1-2). Panel c in Figure 5 shows a similar pattern at business-cycle frequencies. Namely, in countries with large agricultural sectors, the productivity gap is negatively correlated with employment in nonagriculture, while this correlation is close to zero in countries with a small agricultural sector. For instance the correlation is -0.54 for China. We conclude that relative productivity and relative employment (in agriculture) move in opposite directions. This happens both during the process of structural change and over the business cycle in countries that are undergoing structural change away from agriculture.

Finally, panel d shows that consumption volatility relative to GDP volatility declines over the process of development, consistent with the evidence in Aguiar and Gopinath (2007).

In summary, the characteristics of the business cycles change systematically across different stages of the

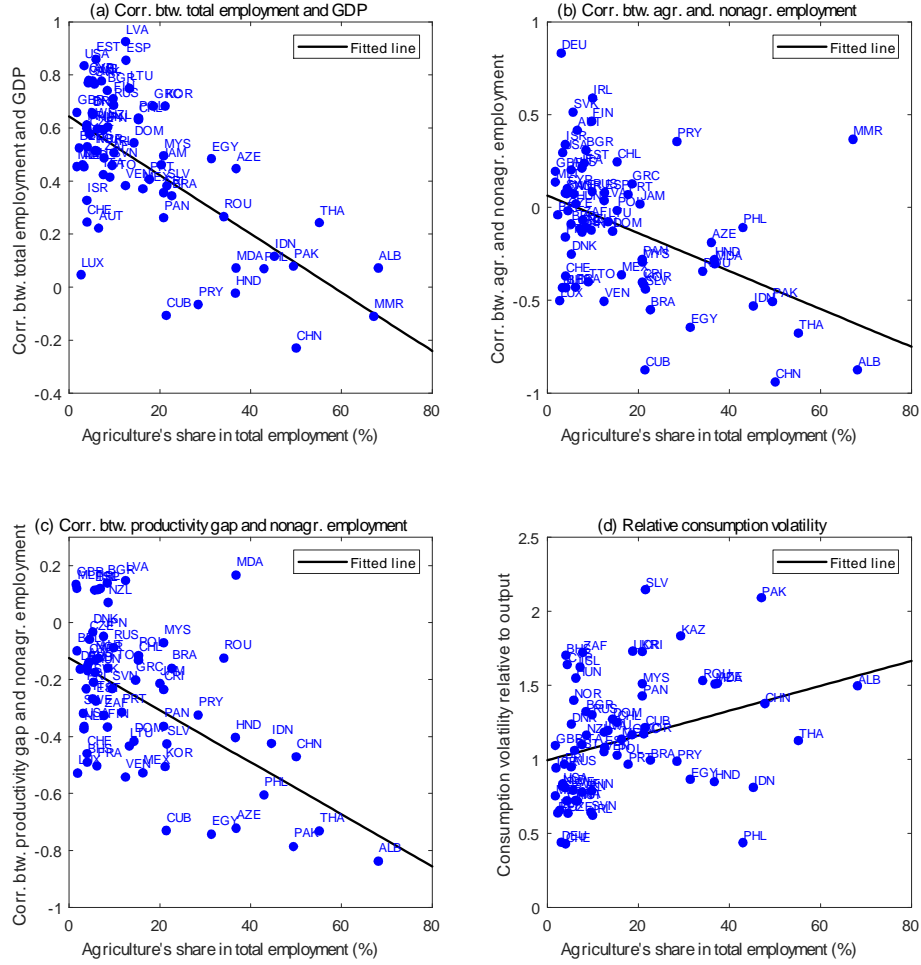


Figure 5: The figure is a cross-country scatter plot of several business cycle statistics. The sample period is 1970-2015 and some countries have fewer observations. We keep the countries that have at least more than 15 years of consecutive observations in order to calculate the business cycle statistics. Panel a plots the time series correlation of HP-filtered log nonagricultural employment and HP-filtered log agricultural employment in a sample of 66 countries. Panel b shows the correlation between the HP-filtered log total employment and HP-filtered log real GDP in a sample of 66 countries. Panel c plots the time series correlation of the HP-filtered log productivity gap and the HP-filtered log nonagricultural employment in a sample of 63 countries. Panel d shows the volatility of consumption relative to the volatility of GDP in a sample of 64 countries. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). The x-axis denotes the mean agriculture's share in total employment over the sample period for each country. Appendix A describe how we constructed the data.

process of structural change. In countries with a large agricultural sector, we observe:

1. Aggregate employment is less correlated with GDP and less volatile;
2. Employment in agriculture is countercyclical;
3. The labor productivity gap is negatively correlated with employment in nonagriculture;
4. Consumption is highly volatile relative to GDP.

These observations (especially, point 3) suggest a systematic relationship between business cycles and the drivers of structural change. In developing and emerging economies, recessions are times of slowdown and even reversal of structural change: People stop leaving farms or even move back to rural areas, and recessions have a sullyng effect on the productivity of agriculture. This pattern has been documents in a recent empirical study for China. Using a unique panel data set, Zhang et al. (2001) document that farm employment increases during recessions and increases in booms. They conclude the agricultural sector contributed to stabilizing total employment in China.

3 A Model of Business Cycle with Structural Change

In this section, we present a dynamic general equilibrium model close in spirit to Acemoglu and Guerrieri (2008) which describes the process of growth and structural change in an economy with a declining agriculture. We first derive an analytical characterization of the equilibrium.¹⁰ Then, in the following section, we estimate the key structural parameters of the deterministic version of the model using data for China. Finally, we introduce uncertainty and productivity shocks, and show that the stochastic version of the estimated model is consistent with salient business cycle features of China.

3.1 Environment

3.1.1 Production

The consumption good, assumed to be the numeraire, is a CES aggregate of two goods,

$$Y = F(Y^G, Y^M) = \left[\gamma (Y^G)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) (Y^M)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (1)$$

We label the sectors producing the two goods *agriculture* (superscript G) and *nonagriculture* (superscript M, as in "manufacturing"), respectively. We denote by $\varepsilon > 0$ the elasticity of substitution between the two goods.¹¹

The technology of nonagriculture is described by the following Cobb Douglas production function

$$Y^M = (K^M)^{1-\alpha} \times (Z^M H^M)^\alpha, \quad (2)$$

where $H^M = hN^M$ is the labor input. N^M denotes the number of workers and h denotes the number of hours worked by each of them. K^M denotes capital and Z^M is a productivity parameter (henceforth, TFP).

Agriculture is a CES aggregate of two subsectors producing imperfect substitutes with an elasticity of substitution $\omega > 0$. More formally,

$$Y^G = \left[\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}} + (1-\varsigma) (Y^S)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (3)$$

where $\varsigma \in (0, 1)$. Modern agriculture (superscript AM) uses a Cobb-Douglas technology with capital and labor (in the quantitative section we also introduce land in agriculture):

$$Y^{AM} = (K^{AM})^{1-\beta} (Z^{AM} H^{AM})^\beta. \quad (4)$$

Traditional agriculture (superscript S, as in "subsistence") does not use any capital:

$$Y^S = Z^S H^S. \quad (5)$$

Note that the presence of a traditional sector implies a variable elasticity of substitution between capital and labor in agriculture. When $\omega > 1$, the elasticity of substitution is larger than unity and declines as the economy develops. For large K^{AM} , the elasticity of substitution approaches unity.

We assume the TFPs Z^M , Z^{AM} , and Z^S to grow at the exponential rates g^M , g^{AM} , and g^S , respectively (we later introduce stochastic disturbances). All goods are produced competitively. Both capital and labor

¹⁰In the text, we summarize the main findings. The technical analysis including proofs of all propositions and lemmas are provided in Appendix B.

¹¹The technology parameters γ and ε can alternatively be interpreted as preference parameters, reflecting the relative weight and the elasticity of substitution between goods produced by the agricultural and nonagricultural sector.

are perfectly mobile across sectors. Capital depreciates at the rate δ .

3.1.2 Households

Agent's preferences are described by a logarithmic utility function.

$$U = \int_0^\infty (\theta \log c + (1 - \theta) \log(1 - h)) \times e^{-(\rho-n)t} dt. \quad (6)$$

Here, $c \equiv C/N$ denotes the consumption per capita, $1 - h$ is leisure, and ρ is the time discount rate. Population grows at the exogenous rate $n < \rho$. In the analytical section, for simplicity, we assume an inelastic labor supply by setting $\theta = 1$ (hence, $H^i = N^i$). We introduce an endogenous labor-leisure choice below where we estimate the model and study economic fluctuations. When we introduce uncertainty, (6) is replaced by an expected utility function with unit relative risk aversion. We suppress time indexes when it is not a source of confusion.

The representative household maximizes expected utility subject to a set of period budget constraints $Nc + \dot{K} = WN + RK + Tr$, where $K = K^M + K^{AM}$ and $N = N^M + N^{AM} + N^S$ denote the aggregate capital stock and number of workers, respectively. W denotes the after-tax wage that is equalized across sectors in equilibrium; R denotes the equilibrium (gross) interest rate.

Since in the data we see persistent labor wage differences across agriculture and nonagriculture (see, e.g., Gollin et al. (2014)), we introduce an exogenous wedge by assuming that the government taxes wages in nonagriculture at the rate τ . The government runs a balanced budget each period and rebates the tax proceeds to the representative household as lump-sum transfers, denoted by Tr . Thus, $Tr = \tau W^M H^M$, where W^M denotes the nonagriculture pre-tax wage. In equilibrium, $W^{AM} = W^S = (1 - \tau) W^M = W$. Note that the wedge distorts resource allocation preventing the equalization of the marginal product of labor across sectors. Misallocation increases employment in agriculture and reduces the equilibrium after-tax wage W . The wedge τ is a stand-in for a variety of frictions leading to rural overpopulation.

3.2 Competitive Equilibrium

Since the wedge τ is the only distortion, we can characterize the recursive competitive equilibrium as the solution to a distorted planning problem. More formally, the planner maximizes (6) subject to the resource constraint

$$\dot{K} = F(Y^G, Y^M) - \delta K - C - \tau \bar{W} N^M + Tr, \quad (7)$$

and to the technological constraints implied by eq. (1)–(5). Here, \bar{W} denotes the marginal product of labor in manufacturing, i.e., $\bar{W} = W^M$. The distortion arises from the planner taking \bar{W} as parametric when she solves the maximization problem.

It is useful to introduce some normalizations.

Notation 1 *Define:*

$$\begin{aligned} c &\equiv \frac{C}{N}, \quad \chi \equiv \frac{K}{N}, \\ \kappa &\equiv \frac{K^M}{K}, \quad \nu^M \equiv \frac{N^M}{N}, \quad \nu^{AM} \equiv \frac{N^{AM}}{N}, \quad \nu^S \equiv \frac{N^S}{N} \\ v &\equiv \frac{\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}}}{\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}} + (1 - \varsigma) (Y^S)^{\frac{\omega-1}{\omega}}}. \end{aligned}$$

χ is the aggregate capital-labor ratio, the key endogenous state variable of the economy. κ is the share

of the aggregate capital stock used in nonagriculture. Since the traditional sector does not use capital, $1 - \kappa$ is the corresponding share in modern agriculture. $\nu^i \equiv N^i/N$ denotes the employment share in sector $i \in \{AM, M, S\}$. v measures the GDP share of modern agriculture in total agriculture.

We characterize equilibrium in two stages. First, we solve the static problem defined the maximized current output per capita subject to the wedge τ and a given aggregate stock of capital per worker χ and the TFP levels. Then, we solve the dynamic equilibrium involving capital accumulation and technical progress.

3.2.1 Static Equilibrium

Let $\mathbf{x} \equiv (\kappa, \nu^S, \nu^{AM}, \nu^M)$. Then, the competitive equilibrium maximizes the distorted output per worker y

$$y(\chi) = \max_{\mathbf{x}} f(y^G(\mathbf{x}, \chi), y^M(\mathbf{x}, \chi)) - \tau \bar{W} \nu^M + Tr. \quad (8)$$

subject to the technology constraint

$$f(y^G(\mathbf{x}, \chi), y^M(\mathbf{x}, \chi)) = \left(\gamma (y^G(\mathbf{x}, \chi))^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) (y^M(\mathbf{x}, \chi))^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (9)$$

where

$$\begin{aligned} y^G(\mathbf{x}, \chi) &= \left(\varsigma \left((Z^{AM})^\beta \times ((1-\kappa)\chi)^{1-\beta} \times (\nu^{AM})^\beta \right)^{\frac{\omega-1}{\omega}} + (1-\varsigma) (Z^S \nu^S)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}, \\ y^M(\mathbf{x}, \chi) &= \left((Z^M)^\alpha \times (\kappa\chi)^{1-\alpha} \times (\nu^M)^\alpha \right), \end{aligned}$$

and to the resource constraints $\kappa \in [0, 1]$ and $\nu^M + \nu^{AM} + \nu^S = 1$.

The static allocation attains constrained production efficiency, conditional on the wedge τ . This requires the equalization of the marginal product capital across nonagriculture and modern agriculture and of the marginal product of labor across the three sectors.

Proposition 1 *Given χ and $\mathbf{Z} = (Z^M, Z^{AM}, Z^S)$, a static competitive equilibrium is characterized by a set of policy functions $\kappa = \kappa(\chi, \mathbf{Z})$, $v = v(\chi, \mathbf{Z})$, $\nu^M(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$, $\nu^{AM}(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$, and $\nu^S(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$ satisfying the First Order Conditions of the program (8) – see equations (22), (23), (24), (25) Appendix B – and the technology constraint (9).*

Next, we characterize the properties of the equilibrium functions $\kappa(\chi, \mathbf{Z})$ and $v(\chi, \mathbf{Z})$. Once these functions are known, it is straightforward to determine the allocation of labor across the three sectors using the FOCs. Here, we focus on the case $\varepsilon > 1$ (which will be consistent with our estimates) but return to the general case in the long-run convergence analysis. We study first the role of capital deepening and then the role of TFP growth.

Capital deepening: In the general case, the comparative statics of κ and v with respect to changes in χ are involved and possibly nonmonotonic. However, we can establish sharp results for the special case in which the two elasticities, ω and ε , are close to each other. In this case, capital deepening triggers a relative decline of agriculture (i.e., increasing κ) and modernization (i.e., increasing v) at all stages of economic transition. The analysis of the first-order conditions of the program (8) (see Appendix B) leads to the result that

$$\lim_{\omega \rightarrow \varepsilon} \frac{\partial \ln \kappa(\chi, \mathbf{Z})}{\partial \ln \chi} = \frac{(\varepsilon - 1)(\beta - \alpha)(1 - \kappa)}{1 + (\varepsilon - 1)((\beta - \alpha)(\kappa - \nu^M) + \nu^S(1 - \beta))}, \quad (10)$$

which is a generalization of a result in Acemoglu and Guerrieri (2008). If $\varepsilon > 1$ and $\beta > \alpha$, this derivative is

necessarily positive (note, in particular that $\beta > \alpha \Rightarrow \kappa - \nu^M > 0$, since nonagriculture is the capital-intensive sector.) Building on this result, we establish the following lemma.

Lemma 1 *Suppose $\beta > \alpha$, and $\varepsilon > 1$. Then, there exists $\bar{x} > 0$ such that, if $\|\omega - \varepsilon\| < \bar{x}$, then, both $\kappa(\chi, \mathbf{Z})$ and $v(\chi, \mathbf{Z})$ are monotone increasing in χ . Moreover, ν^M , ν^M/ν^{AM} , and ν^{AM}/ν^S are monotone increasing in χ while ν^S is monotone decreasing in χ .*

After deriving eq. (10), the lemma is easily established by exploiting the fact that the capital labor ratio in modern agriculture is proportional to that in nonagriculture. More formally, the First Order Conditions imply that

$$\frac{\kappa}{\nu^M} = \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \frac{1 - \kappa}{\nu^{AM}}, \quad (11)$$

which in turn implies that capital accumulation triggers capital deepening in both sectors. Since, by eq. (10), $\partial\kappa/\partial\chi > 0$, (11) implies that $\partial(\nu^M/\nu^{AM})/\partial\chi > 0$. Moreover, capital deepening in agriculture implies that modern agriculture accounts for a growing share of the total agricultural production, i.e., $\partial v/\partial\chi > 0$. Finally, employment in the traditional sector must fall ($\partial\nu_S/\partial\chi > 0$), since, on the one hand, capital deepening increases the productivity of modern agriculture and, on the other hand, modern and traditional agriculture are substitutes ($\omega > 1$).

TFP differences: Next, we study the comparative statics of the sectoral TFPs. First, we note that a proportional increase in all TFPs leaves κ and v unaffected, i.e., $\kappa(\chi, \mathbf{Z}) = \kappa(\chi, \lambda\mathbf{Z})$ and $v(\chi, \mathbf{Z}) = v(\chi, \lambda\mathbf{Z})$ for any $\lambda > 0$. Thus, what matters are the relative TFPs in the three sectors. With some abuse of notation, we express the problem in terms of relative TFPs: $\kappa(\chi, \mathbf{Z}) = \kappa(\chi, z^{AM}, z^S)$ and $v(\chi, \mathbf{Z}) = v(\chi, z^{AM}, z^S)$ where $z^{AM} \equiv (Z^{AM})^\beta / (Z^M)^\alpha$ and $z^S \equiv Z^S / (Z^M)^\alpha$. The following lemma provides a complete characterization.

Lemma 2 *Suppose $\beta > \alpha$ and $\varepsilon > 1$. Then, $\kappa(\chi, z^{AM}, z^S)$ is decreasing in z^{AM} and increasing in z^S , whereas $v(\chi, z^{AM}, z^S)$ is increasing in z^{AM} and decreasing in z^S . Moreover, ν^{AM} , ν^{AM}/ν^S and ν^{AM}/ν^M are increasing in z^{AM} and decreasing in z^S .*

Lemma 2 establishes two intuitive results. First, an increase in TFP in nonagriculture relative to modern agriculture (with constant Z^S) induces a reallocation of both capital and labor towards nonagriculture. Second, an increase in TFP in modern relative to traditional agriculture (with constant Z^M) triggers more modernization of agriculture.

3.2.2 Dynamic Equilibrium

In this section, we characterize the dynamic equilibrium. We continue to exploit the equivalence between the distorted planning solution and the competitive equilibrium. Thus, we can write:

$$\max_{\{c_t, \mathbf{x}_t, \chi_t\}_{t=0}^{\infty}} U = \int_0^{\infty} \log(c) \times e^{-(\rho-n)t} dt$$

subject to the resource constraint

$$\dot{\chi} = f(y^G(\mathbf{x}, \chi), y^M(\mathbf{x}, \chi)) - (\delta + n + g^M)\chi - c,$$

where $f(y^G, y^M)$ is given by (9), to the exogenous law of motion of TFPs, $\dot{Z}_t^M/Z_t^M = g^M$, $\dot{Z}_t^{AM}/Z_t^{AM} = g^{AM}$, and $\dot{Z}_t^S/Z_t^S = g^S$. The problem is subject to a vector of initial conditions $(\chi_0, \mathbf{Z}_0) = (\bar{\chi}_0, \bar{\mathbf{Z}}_0)$.

Using the set of static equilibrium conditions allows us to write the present-value Hamiltonian

$$H(c_t, \chi_t, \mathbf{Z}_t, \xi_t) = e^{-(\rho-n)t} \log(c_t) + \xi_t (y(\chi_t, \mathbf{Z}_t) - (\delta + n)\chi_t - c_t), \quad (12)$$

where c_t is a control variable, (χ_t, \mathbf{Z}_t) is the state vector, and ξ_t is a dynamic Lagrange multiplier. The production function $y(\chi_t, \mathbf{Z}_t)$ subject to productive efficiency (static equilibrium) can be expressed as

$$y(\chi, \mathbf{Z}) = \eta(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z})) \times (\chi \kappa(\chi, \mathbf{Z}))^{1-\alpha} (\nu^M(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z})))^\alpha, \quad (13)$$

where

$$\eta(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z})) = (1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}} \left(1 + \frac{1 - \alpha}{1 - \beta} \frac{1 - \kappa(\chi, \mathbf{Z})}{\kappa(\chi, \mathbf{Z})} \frac{1}{v(\chi, \mathbf{Z})} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (14)$$

Note that $\kappa(\chi, \mathbf{Z})$, $\nu^M(\chi, \mathbf{Z})$, and $v(\chi, \mathbf{Z})$ are the policy function consistent with the static equilibrium. Thus, eq. (13) yields the technology constraint subject to the static equilibrium conditions.

Appendix B (Proposition 3 and Corollary 1) provides a complete characterization of the dynamic equilibrium in terms of a system of ordinary differential equations, a set of initial conditions, and a terminal (transversality) condition. We prove that the equilibrium can be represented by an autonomous system of ordinary differential equations (eq. (36)–(39) in the appendix) in the endogenous variables c_t, κ_t, v_t , and χ_t , and the exogenous law of motion $\dot{Z}_t^M/Z_t^M = g^M$, where the initial conditions are given by χ_0 (i.e., the initial capital stock), the initial state of technology, and κ_0 and v_0 such that they satisfies the static equilibrium conditions at time zero, i.e., $\kappa_0 = \kappa(\chi_0, \mathbf{Z}_0)$ and $v_0 = v(\chi_0, \mathbf{Z}_0)$. A standard transversality condition pins down the (unique) optimal choice of the control variable c_0 . This dynamic system fully characterizes the unique equilibrium trajectories of all variables in the economy, thereby providing a full description of the process of structural change. Note, in particular, that the static equilibrium conditions will be satisfied for all $t \geq 0$.

Finally, we discuss the long-term properties of the model. We provide conditions under which the economy converges to an asymptotic balanced growth path (ABGP) where the agriculture is fully modernized and the share of nonagriculture in total GDP is unity.

Proposition 2 *Let $k^M = \kappa\chi$ and $k^{AM} = (1 - \kappa)\chi$. Then, there exists an Asymptotic Balanced Growth Path (ABGP) such that*

$$\begin{aligned} \frac{\dot{c}_t}{c_t} &= \frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{k}_t^M}{k_t^M} = g^M; \\ \kappa &= \nu^M = 1; \quad \frac{\dot{\kappa}_t}{\kappa_t} = \frac{\dot{v}_t}{v_t} = 0; \\ \frac{\dot{k}_t^M}{k_t^M} &= g^M, \quad \frac{\dot{k}_t^{AM}}{k_t^{AM}} = g^M - (\varepsilon - 1)\beta(g^M - g^{AM}); \\ \frac{\dot{N}_t^M}{N_t^M} &= n, \quad \frac{\dot{N}_t^{AM}}{N_t^{AM}} = n - (\varepsilon - 1)\beta(g^M - g^{AM}); \\ \frac{\dot{N}_t^S}{N_t^S} &= \frac{\dot{N}_t^{AM}}{N_t^{AM}} - (\omega - 1)[(g^{AM} - g^S) + (1 - \beta)(g^M - g^{AM})]. \end{aligned}$$

Along the ABGP,

$$\left(\frac{c}{\chi}\right)^* = \left(\frac{g^M + \delta + \rho}{1 - \alpha}\right) - (g^M + \delta + n), \quad (15)$$

$$\left(\frac{\chi}{Z^M}\right)^* = \left(\frac{(1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1 - \alpha)}{g^M + \delta + \rho}\right)^{\frac{1}{\alpha}}. \quad (16)$$

If $\varepsilon > 1$, $\omega > 1$, and $g^M \geq g^{AM} \geq g^S$, then, the ABGP is asymptotically stable, i.e., given a vector of initial conditions $(\chi_0/Z_0^M, \kappa_0, v_0)$ close to the ABGP, the economy converges to the ABGP. The same is true if, instead, $\varepsilon < 1$, $\omega > 1$, and $g^{AM} \geq g^M \geq g^S$.

The ABGP features a vanishing agriculture. Sufficient conditions for this to happen are that either the elasticity of substitution between nonagriculture and agriculture be larger than unity and technical progress be faster in nonagriculture than in agriculture or that, alternatively, $\varepsilon < 1$ and $g^{AM} \geq g^M$. Note that as long as ε is not too large, β is not too large and the TFP growth gap between nonagriculture and modern agriculture is not too high, capital accumulation in modern agriculture remains positive in the ABGP (i.e., k^{AM} grows at a positive rate). In addition, the ABGP features modernization of agriculture: the traditional sector vanishes both as a share of value added and as a share of total agricultural employment. This is due to the combination of a high elasticity of substitution ($\omega > 1$) and technical progress being faster in modern than in traditional agriculture.

Our theory bears predictions about the labor income shares and the productivity gap. To highlight them, we move from the planner's allocation to its decentralized counterpart. Denote by $LIS^j \equiv W^j H^j / P^j Y^j$ for $j \in \{G, M\}$ the labor income share in sector j . The labor share in nonagriculture is constant, owing to the Cobb-Douglas production function. The labor share in agriculture declines as long as the (labor-intensive) traditional sector shrinks, consistent with Figure 1 above. More precisely, $LIS^G = \beta v + 1 - v$, implying that the labor share in agriculture declines from unity – when capital is very low and the agriculture is dominated by the traditional sector – to β when agriculture is fully modernized.¹² The theory also predicts a declining productivity gap in line with the data in Figures 1 and 2. The two predictions are two sides of the same coin. More formally, $APL^M/APL^A = ((1 - \tau) \times LIS^M/LIS^A)^{-1}$.

3.3 Equilibrium in the Lewis Model

A particular case for which a global characterization of the equilibrium dynamics is available is when the output of traditional and modern agriculture are perfect substitutes ($\omega \rightarrow \infty$). This model is interesting for its close resemblance with the seminal contribution of Lewis (1954). In addition, it illustrates why the dynamics of κ and v may be nonmonotone. Intuitively, when ω is large, labor in the traditional sector is a close substitute of capital in modern agriculture. When capital is scarce, it is then efficient to allocate its entire stock to nonagriculture and defer the modernization of agriculture to a later stage in which capital is more abundant.

The technical analysis is deferred to Appendix B. Here, we summarize the main qualitative findings in the case where $\varepsilon > 1$. The dynamic equilibrium evolves through three stages. In the first stage, (*Early Lewis* stage), capital is very scarce, all agricultural production takes place in the traditional sector ($v = 0$) and all capital is allocated to the manufacturing sector ($\kappa = 1$). Capital accumulation brings about a steady increase in the relative price of agricultural goods and a growth in the real wage. The interest rate decreases over time.

At some point, the increase in the capital stock and the growing price of the agricultural good make it efficient to activate modern agriculture allocating part of the capital stock to this subsector. We enter then the *Advanced Lewis* stage, when the share of capital in modern agriculture (κ) increases in χ . Employment increases in both nonagriculture and modern agriculture and declines in traditional sector (thus, v increases). The sectoral capital-labor ratios are constant over time, and so are factor prices.

¹²One could obtain a declining labor share by assuming an aggregate CES production function in agriculture with a high elasticity of substitution between capital and labor, like in Alvarez-Cuadrado et al. (2017). However, such alternative model would feature, counterfactually, an ever declining labor share that would converge to zero in the long run. In our model, like in the data, the labor share in agriculture declines but remains bounded away of zero.

Finally, when the labor force reserve in traditional agriculture is exhausted, the economy enters the third stage (*Neoclassical* stage). In this stage, all the agricultural production takes place in modern farms using capital ($v = 1$). Since $\varepsilon > 1$ and nonagriculture is more capital intensive than agriculture, the output share of nonagriculture keeps growing. There is positive wage growth and a declining interest rate.

Figure 6 displays the equilibrium dynamics during economic transition, assuming no technical progress, for simplicity. Each panel has the aggregate capital labor ratio χ on the horizontal axis. Panel *a* plots the share of labor in each sector. The labor share in the traditional sector (ν^S) starts high and declines with χ in both the Early and Advanced Lewis stages. The labor share in nonagriculture increases throughout the entire transition. The labor share in modern agriculture is nonmonotone: it is zero in the Early Lewis stage, takes off in the Advanced Lewis stage, and declines again in the neoclassical stage.

Panel *b* plots the factor price dynamics. During the Early Lewis stage, the interest rate falls and wages increase with capital accumulation. Wages and interest rates are flat during the Advanced Lewis stage. Eventually, the interest rate resumes its fall and wages resume their increase during the Neoclassical stage.

Panel *c* plots the productivity gap, which tracks the dynamics of v . The gap is constant during the Early Lewis stage, when $v = 0$. It falls in χ during the Advanced Lewis stage, when the traditional sector declines. It is constant again in the Neoclassical stage, when $v = 1$.

Finally, panel *d* plots the relative capital-output ratio $\frac{K^G}{P^G Y^G} / \frac{K^M}{P^M Y^M}$. The ratio stays at zero during the Early Lewis stage. It increases during the Advanced Lewis stage. Eventually, it becomes constant during the Neoclassical stage.

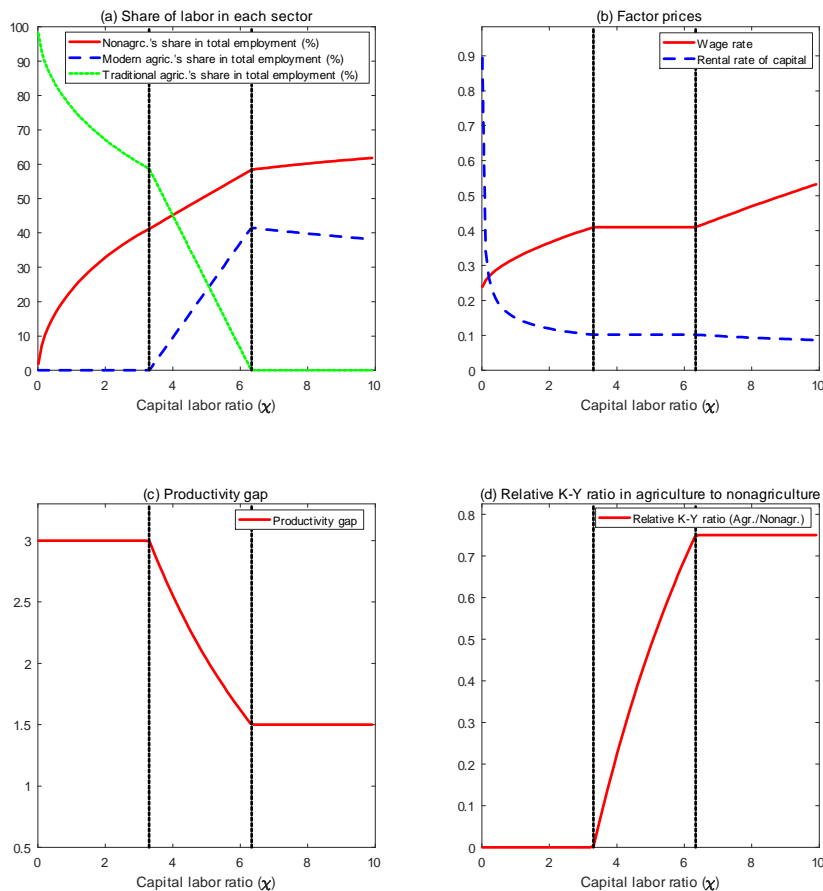


Figure 6: The figure illustrates the allocations and prices as a function of capital per worker, χ , in a version of the Lewis model ($\omega \rightarrow \infty$ and no technical change).

4 Estimating the Model

In this section, we estimate the model. We generalize the theory in three dimensions. First, we use discrete time.¹³ Second, we introduce an endogenous labor supply choice as in eq. (6). Third, we introduce land as a fixed factor in modern agriculture.¹⁴ We could add land also in the traditional sector. However, in the spirit of Lewis (1954), we retain the property of a traditional sector working as a labor force reserve at a constant marginal cost. We then pursue the following two steps:

1. We estimate the deterministic model with constant productivity growth in each sector. We first calibrate some parameters externally. We then estimate the structural parameters and initial conditions to match moments of the structural change of China between 1985 and 2012.
2. We introduce productivity shocks. We estimate stochastic processes for the three TFP shocks and simulate the model. Then, we evaluate the ability of the stochastic model to account for the business cycle properties of China. Finally, we use the estimated model to forecast how the Chinese business cycle will evolve as economic development progresses further.¹⁵

Parameters calibrated externally: We assume a 4% annual time discount rate. Note that log preferences in Equation (6) ensure that there is no trend in labor supply in the ABGP. θ is chosen so that in the long run agents work one third of their time. We set the annual population growth rate (n) to 1.5% following Acemoglu and Guerrieri (2008). This parameter has no significant impact on the results. Capital is assumed to depreciate at a 5% annual rate.

Estimated parameters based on nonagricultural production data: We set $\alpha = 0.5$ to match the labor-income share in the nonagricultural sector in China (see Bai and Qian (2010)). Then, we estimate g^M using standard growth accounting based on a Cobb Douglas production function – as in the model – to match the trend of real GDP in the nonagricultural sector of China between 1985 to 2012. This yields $g^M = 6.5\%$.

Parameters estimated based on endogenous moments: We estimate the remaining parameters using Simulated Method of Moments. The parameters we estimate are

$$\{\varepsilon, \omega, \tau, \gamma, \varsigma, \beta, \beta_T, g^{AM}, g^S, Z_{1985}^M, Z_{1985}^{AM}, Z_{1985}^S\}.$$

We normalize $Y_{1985} = 1$ and target (the natural logarithms of) the following empirical annual observations from 1985 to 2012 in China:¹⁶ (i) the share of agricultural employment in total employment; (ii) the share of capital in agriculture relative to the total capital stock; (iii) the ratio of real output in agriculture to total GDP; (iv) the relative value added share of agriculture, evaluated at current prices (i.e., the expenditure share of agricultural goods); (v) the aggregate GDP growth; (vi) the initial and final aggregate capital-output ratios; and (vii) the 1985-2012 change in the productivity gap between agriculture and nonagriculture, adjusted for rural-urban wage differences. We calculate this gap as the ratio of labor productivity in

¹³We provide a formal description of the discrete-time model in the appendix.

¹⁴We assume $Y^{AM} = (K^{AM})^{1-\beta-\beta_T} (Z^{AM} H^{AM})^\beta T^{\beta_T}$, where T is land and β_T is the output elasticity of land.

¹⁵We decided to use China instead of a fully industrialized economy because it is difficult to find data for periods in which on such economy had a large agricultural sector. For instance, the available time-series for the US cover a period when the employment in agriculture is already quite low (20.3% of the total US employment in 1929) and the activities captured by our traditional agricultural sector are arguably negligible. In contrast, China has a large and declining share of employment in agriculture ranging from 62.4% in 1985 to 33.6% in 2012. As late as in 2004 more than 10% of China's agricultural output was produced without machine equipment (source: Fixed Point Survey of Research Center for Rural Economy (RCRE)).

¹⁶We choose the year 1985 as our initial period because it was a turning point in internal migration policy. In earlier years restrictions on labor mobility between rural and urban areas were very severe. These restrictions were relaxed in January 1985, following the issuance of the "Ten Policies on Further Active Rural Economy" from the CPC Central Committee and the State Council.

nonagriculture to agriculture times the ratio of wages in agriculture to nonagriculture. Recall that in the model this gap equals $(1 - v(1 - \beta))/\alpha$, i.e., the ratio of the labor-income share in agriculture to the labor-income share in nonagriculture.¹⁷

This procedure yields 143 moment conditions – 28 for each of the annual moments (i)-(v) plus moments (vi)-(vii). The estimation is based on equal weights on the annual moments (i)-(v) and corresponding weights for moments (vi) and (vii) (i.e., 14 times the weight on the annual moments for each of the two K/Y ratios, and 28 times for the relative deviation of the output gap).

We measure real output in agriculture and nonagriculture following the same approach as China’s National Bureau of Statistics (NBS).¹⁸ To be consistent with the empirical data, we start the model in 1980 and base the 1985-1990 growth rates using 1980 as the base year, then update and chain the output levels using 1990 relative prices, etc.

Nonhomothetic preferences: We also estimate a model with non-homothetic Stone-Geary preferences, where the agricultural good is a necessity. To this end, we reinterpret the goods M and G as final goods that are allocated to consumption (denoted C^M and C^G) and investment (denoted X^M and X^G), where $Y^G = C^G + X^G = Nc^G + X^G$ and $Y^M = C^M + X^M = Nc^M + X^M$. More formally, we replace the per-capita utility function in Equation (6) by

$$u(c^G, c^M, h) = \theta \log \left(\left[\gamma (c^G - \bar{c})^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \gamma) (c^M)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) + (1 - \theta) \log(1 - h),$$

while the investment good continues to be a CES aggregate of the agricultural and nonagricultural goods,

$$\left[\gamma (X^G)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \gamma) (X^M)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} = I.$$

The aggregate resource constraint (7) can be written as

$$\Delta K = I - \delta K - \tau \bar{W} H^M + Tr. \quad (17)$$

We denote the total expenditure by $Y = P^G Y^G + P^M Y^M$, the total consumption expenditure as $C = P^G C^G + P^M C^M$, and the total investment expenditure as $I = P^G X^G + P^M X^M$. The constant \bar{c} is added to the list of parameters we estimate. The baseline model is a restricted version with $\bar{c} = 0$.

Estimation results:

Table 1 summarizes the results of the estimation. The benchmark model with homothetic utility is labeled CRRA.

The estimated elasticities of substitution ε and ω are significantly larger than unity, both with and without a subsistence level in food consumption. This is in line with our conclusion from Section 6 and

¹⁷We could alternatively have targeted the empirical labor-income shares directly. However, such data are available only up until 2003. From 2004 onwards, the labor income shares are not comparable to their counterparts in the pre-2004 period. See Bai and Qian (2010) for details. As it turns out, the overall change in the ratio of labor income shares is comparable to the change in the labor productivity gap: over the 1985-2003 period these ratios fall by 13 and 14 log points, respectively.

¹⁸The NBS measures real growth in agricultural and nonagricultural output using prices in a base year. The base year was updated in 1980, 1990, 2000, 2005, and 2010. The levels of real sectoral output are chained when a new base year is adopted. Therefore, in the year of change in base year prices the real levels are by construction invariant to using the new or the old set of prices. This approach is similar to that pursued in the U.S. National Income and Product Accounts before 1996.

As is well known, this approach introduces a bias when the relative prices change substantially over time. In Figure 17 in the appendix we quantify the magnitude of this bias in China by plotting the series measured in the same way as does the NBS against the exact measures of real sectoral output in the model. The average bias is 0.2 percentage points: the real GDP growth is overstated by 0.5 percentage points over the sample period 1985-1990. Then the bias decreases over time. In contrast, the bias would be negligible if real growth were calculated using chain weighting.

Parameters Set Exogenously		CRRA	Stone Geary		NoPrGap
			ε unrestricted	$\varepsilon = 0.5$	
n	popul. growth rate (15+	1.5%	1.5%	1.5%	1.5%
δ	capital deprec. rate	5%	5%	5%	5%
$\frac{1}{1+\rho}$	discount factor	0.96	0.96	0.96	0.96
θ	pref. weight on consumption	0.73	0.73	0.64	0.71
Parameters Estimated Within Model					
Targeting Data on Nonagr. Prod.		CRRA	StGeary	$\varepsilon = 0.5$	NoPrGap
α	labor share in nonagr.	0.50	0.50	0.50	0.5
g^M	nonagr. TFP growth rate	6.5%	6.5%	6.5%	6.5%
Z_{1985}^M	initial TFP level in nonagr.	4.33	4.45	1.69	3.42
Targeting Empirical Moments		CRRA	StGeary	$\varepsilon = 0.5$	NoPrGap
\bar{c}	Subsist. level in food cons.	—	0.05	0.69	0.05
ε	ES btw agric. and nonagric. cons.	3.60	3.36	0.5	4.00
ω	ES btw modern and trad. agr.	9.00	9.00	9.0	8.22
γ	weight on agric. goods	0.61	0.60	0.000074	0.54
ς	weight on modern-agr. output	0.40	0.39	0.41	0.50
$1 - \beta - \beta_T$	capital's income share in modern agr.	0.14	0.13	0.20	0.11
β	labor's income share in modern-agric.	0.61	0.60	0.67	0.68
τ	labor wedge	0.76	0.75	0.63	0.73
g^{AM}	TFP growth rate in modern-agr.	6.1%	6.1%	6.8%	5.9%
g^S	TFP growth rate in trad. sector	0.9%	0.9%	0.0%	1.0%
Z_{1985}^S	initial TFP level in trad. agr.	1.23	1.18	2.17	1.35
Z_{1985}^{AM}	initial TFP level in modern-agr.	2.26	2.25	3.28	2.42

Table 1: Estimated parameters.

$\varepsilon = 3.6$ is within the range of the estimates we found for China in Section 6.¹⁹ Moreover, the finding that $\varepsilon > 1$ and $g^M \geq g^{AM} \geq g^S$ is consistent with the assumption of the first case of Proposition 2 (convergence to an ABGP when $\varepsilon > 1$).

With Stone-Geary preferences (second column), the estimated ε falls slightly. This is not surprising. With homothetic preferences there is only a substitution effect: nonagricultural products turn relatively cheaper due to capital accumulation and faster technical progress causing a decline in the relative demand of agricultural goods. To match the declining expenditure share on agricultural good observed in the data, the price elasticity of the relative demand must be sufficiently high (recall that if $\varepsilon = 1$ the expenditure shares would not change at all). In contrast, with Stone-Geary preferences, the expenditure share in agriculture falls also because of an income effect. Therefore, the Stone Geary model has a potential of lowering the estimate of ε (even, possibly, turning it smaller than unity). However, the estimated subsistence level \bar{c} turns out to be very small, corresponding to 11% of agricultural production in 1985. Therefore, the estimated ε falls only marginally, and the remaining parameters are also very similar to the case with homothetic preferences.

In column 3 we restrict ε to be equal to $\varepsilon = 0.5$. Namely, we impose complementarity between agricultural and nonagricultural goods. This case yields very large estimates for the subsistence level in agriculture, which now amounts to 98% of agricultural production in 1985. Moreover, in this case the TFP growth in modern agriculture exceeds the TFP growth in nonagriculture, in line with the conditions for the second case of Proposition 2 (convergence to an ABGP when $\varepsilon < 1$).

¹⁹Recall that when estimating equation (20) on Chinese data we obtain $\varepsilon \approx 4$ (using 5-year differences), while we obtained $\varepsilon = 1.7$ when estimating a model based on equation (19).

Finally, in the fourth column, we show the estimation results when we do not impose the productivity gap as a target moment. Since the model without a traditional agricultural sector cannot generate a declining productivity gap by construction, one might worry that the estimation results hinge on the inclusion of the productivity gap as a targeted moment. However, the results are highly robust, being quantitatively very similar to those in the second column. In particular, the data demand a large traditional sector ($\zeta = 0.5$) and estimated elasticities that are very similar to the baseline case. The nonhomotheticity continues to be small. In other words, the estimated model predicts a declining productivity gap over the process of structural change even though if this moment were not targeted.

The estimated elasticity of substitution between modern and traditional agriculture ω is in all cases very large. The productivity growth rate is very high in both manufacturing and modern agriculture, reflecting the high growth rate of the Chinese economy. To avoid the unrealistic implication of a long run annual growth rate in excess of 6%, we assume that the productivity growth rate in both nonagriculture and modern agriculture gradually (linearly) declines to 1.8% over the 2012-2085 period. Thereafter, the economy grows at an annual 1.8% growth rate. This assumption is for the sake of realism and has negligible effects on the quantitative results. Finally, we note that the estimated productivity growth in modern and traditional agriculture satisfy the (sufficient) conditions for convergence to the ABGP set forth in Proposition 2.

4.1 Accounting for Structural Change in China

In this section, we show that the benchmark model fits well the data along salient dimensions of the process of structural change.²⁰ Figures 7 and 8 plot the time series for the seven empirical targets against the implications of the estimated model – with and without homothetic preferences. Figure 7 shows that, in line with the data, the benchmark model predicts that the shares of employment, capital, value added, and expenditure in the agricultural sector relative to total should be falling over time. In contrast, the $\varepsilon = 0.5$ economy cannot generate a monotone decline in the share of aggregate capital allocated to agriculture. That economy also generates too large a decline in the employment share, expenditure share, and output share of agriculture.

Figure 8 displays an index of the log GDP per capita (index), the capital-output ratio, and the productivity gap measured by the output per worker in agriculture relative to total output per worker. The estimated models capture well the trend in GDP and capital-output ratio. They can also capture the falling productivity gap, although the data feature large swings.

Finally, Figure 9 shows the demise of the traditional agriculture. The employment share of the modern sector in total agriculture increases from about 25% in 1985 to almost one in 2012. The output share (corresponding to the variable v in the model) exhibits a similar behavior. Note that in the data we do not observe a distinction between traditional and modern agriculture, and thus the transition from traditional to modern agriculture is identified from the change in the productivity gap (the bottom panel of Figure 8). The decline of the traditional sector is due to both relative TFP growth and fast capital accumulation.

In conclusion, the unrestricted model does a good job at matching the data, irrespective of whether or not the productivity gap is targeted. The model with $\varepsilon = 0.5$ captures some salient features of the data but does less well on others, as discussed above.

5 Business Cycle Analysis

In this section, we introduce uncertainty in the form of shocks to the three sectoral TFPs.

²⁰Using a somewhat larger value for ε , our model would also be consistent with structural change in the U.S. between 1800 and 1945, as documented by Alvarez-Cuadrado and Poschke (2010). See Appendix C.5 for details.

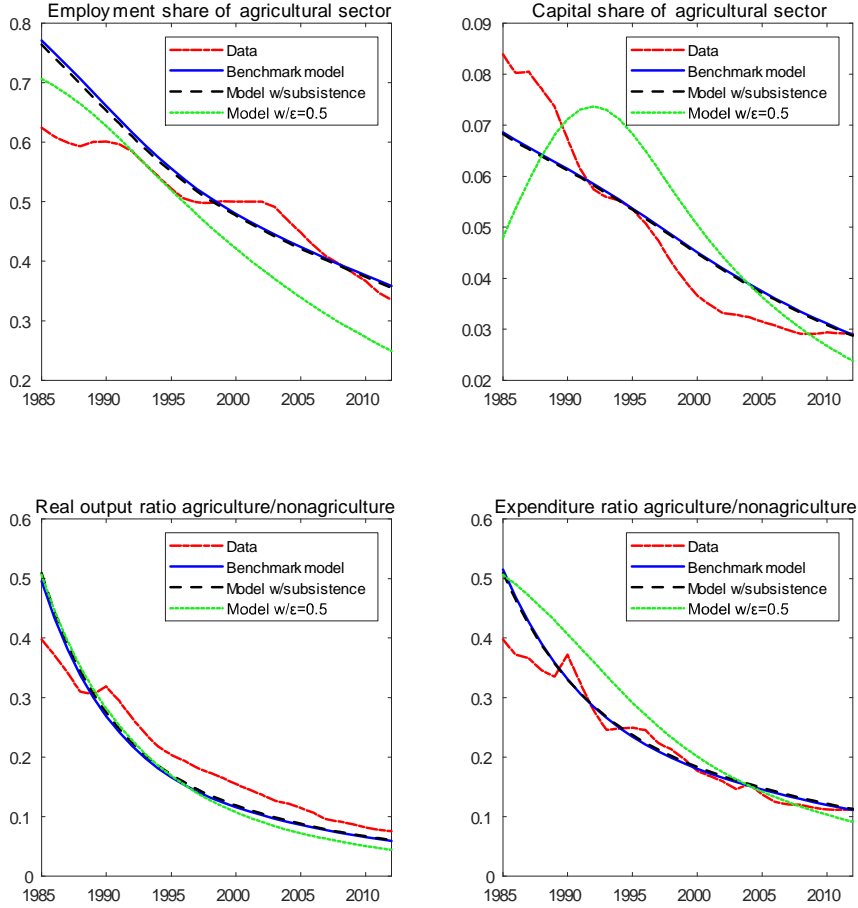


Figure 7: Structural change in model versus targeted empirical moments. Solid blue lines: homothetic model. Dashed lines: the nonhomothetic models with unrestricted ε (black) and restricted $\varepsilon = 0.5$ (green). The top left panel displays agricultural employment as a share of total employment. The top right panel displays the share of aggregate capital invested in agriculture. The bottom left panel displays the agricultural value added as a share of aggregate GDP at current prices. The bottom right displays the expenditure on agricultural goods as a share of aggregate GDP.

5.1 Estimating the Stochastic TFP Process

The estimation of the process for technology shocks presents a measurement problem because, as mentioned above, we do not have direct separate measures of traditional and modern agriculture. To overcome this problem we exploit two equilibrium conditions in the theory, namely that the marginal product of capital is equated across manufacturing and modern agriculture, and that the marginal product of labor is the same in traditional and modern agriculture. Combining these conditions with direct annual observations of capital, labor, and value added in agriculture and non-agriculture, yields a uniquely identified sequence $\{Z_t^M, Z_t^{AM}, Z_t^S\}_{t=1985}^{2012}$.

We decompose each TFP level into a trend and a cyclical part, assuming that the trend is deterministic. To this aim, define $z_t^j \equiv \log(Z_t^j) - \log(\bar{Z}_t^j)$, where $\bar{Z}_t^j = \bar{Z}_0^j (1 + g^j)^t$ is the deterministic trend for $j \in \{M, AM, S\}$. We assume that the stochastic process is VAR(1), such that

$$\begin{bmatrix} z_{t+1}^M \\ z_{t+1}^{AM} \\ z_{t+1}^S \end{bmatrix} = \begin{bmatrix} \phi^M & 0 & 0 \\ 0 & \phi^{AM} & 0 \\ 0 & 0 & \phi^S \end{bmatrix} \cdot \begin{bmatrix} z_t^M \\ z_t^{AM} \\ z_t^S \end{bmatrix} + \epsilon_t, \quad (18)$$

where $\epsilon_t = A \cdot \tilde{\epsilon}_t$, A is a 3×3 matrix and $\tilde{\epsilon}_t$ denotes a vector of orthogonal i.i.d. shocks. Off-diagonal elements in the matrix A will capture correlation between the three TFP innovations ϵ_t^j .

The estimated parameters are: $\hat{\phi}^M = 0.63$, $\hat{\phi}^{AM} = 0.9$, and $\hat{\phi}^S = 0.42$, all significant at 1% level.²¹ The

²¹Note that the autocorrelation matrix has no off-diagonal elements. If we allow for off-diagonal elements in this matrix, we

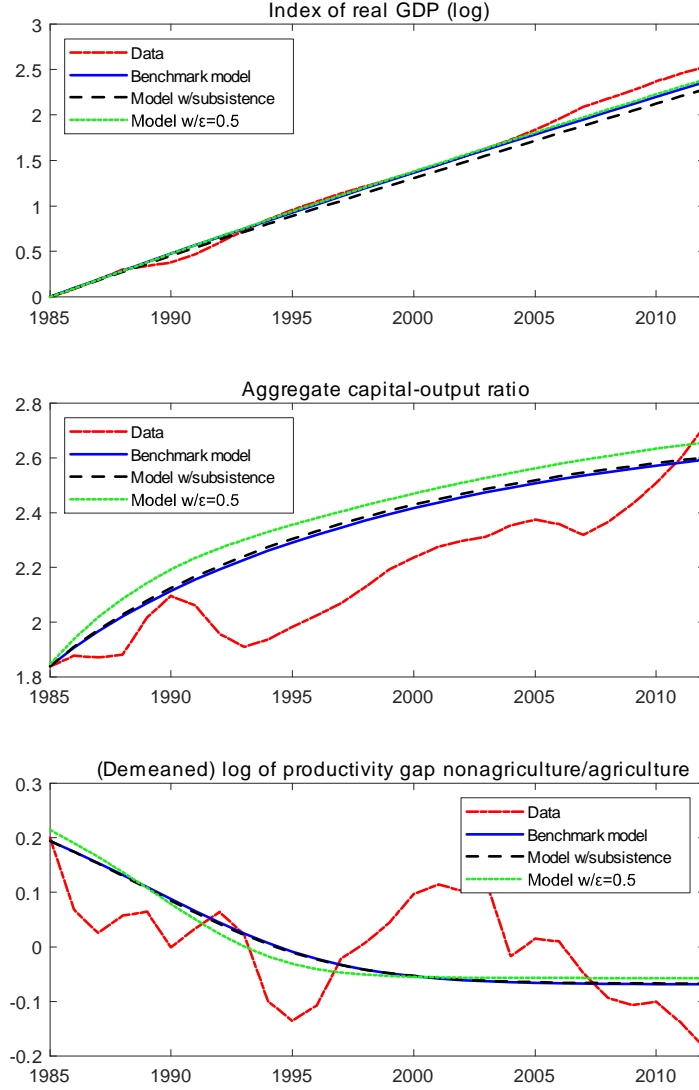


Figure 8: Structural change in model versus targeted empirical moments. Solid blue lines: homothetic model. Dashed lines: the nonhomothetic models with unrestricted ε (black) and restricted $\varepsilon = 0.5$ (green). The top panel displays an index of real GDP (logarithm). The middle panel displays the capital-output ratio. The bottom panel displays changes the labor productivity gap.

persistence of the three shocks on a quarterly basis, is 0.89, 0.97, and 0.81, respectively. The estimated correlation matrix is

$$Corr(A) = \begin{bmatrix} 1 & 0.42^{**} & -0.48^{**} \\ 0.42^{**} & 1 & -0.15 \\ -0.48^{**} & -0.15 & 1 \end{bmatrix}.$$

Note that innovations to nonagriculture and modern agriculture are positively correlated, whereas innovations to nonagriculture and traditional agriculture are negatively correlated. Below we perform a sensitivity analysis where we impose that the three shocks in ϵ_t are orthogonal. Finally, the implied standard deviations of $(\epsilon_t^M, \epsilon_t^{AM}, \epsilon_t^S)$ are given by $\sigma(\epsilon_t^M) = 0.042$, $\sigma(\epsilon_t^{AM}) = 0.036$, and $\sigma(\epsilon_t^S) = 0.053$.

We assume that the realization of the stochastic productivity shock is observed after capital is installed in each sector. Therefore, capital can only adjust in the following period.

obtain:

$$\begin{aligned} \log z_{t+1}^M &= 0.72^{***} \log z_t^M - 0.07 \log z_t^A + 0.12 \log z_t^S + \varepsilon_{t+1}^M \\ \log z_{t+1}^A &= -0.03 \log z_t^M + 0.906^{***} \log z_t^A - 0.05 \log z_t^S + \varepsilon_{t+1}^A \\ \log z_{t+1}^S &= 0.10 \log z_t^M + 0.02 \log z_t^A + 0.438^{***} \log z_t^S + \varepsilon_{t+1}^S \end{aligned}$$

Since all off-diagonal coefficients are insignificant, we set them to zero and estimate a restricted VAR.

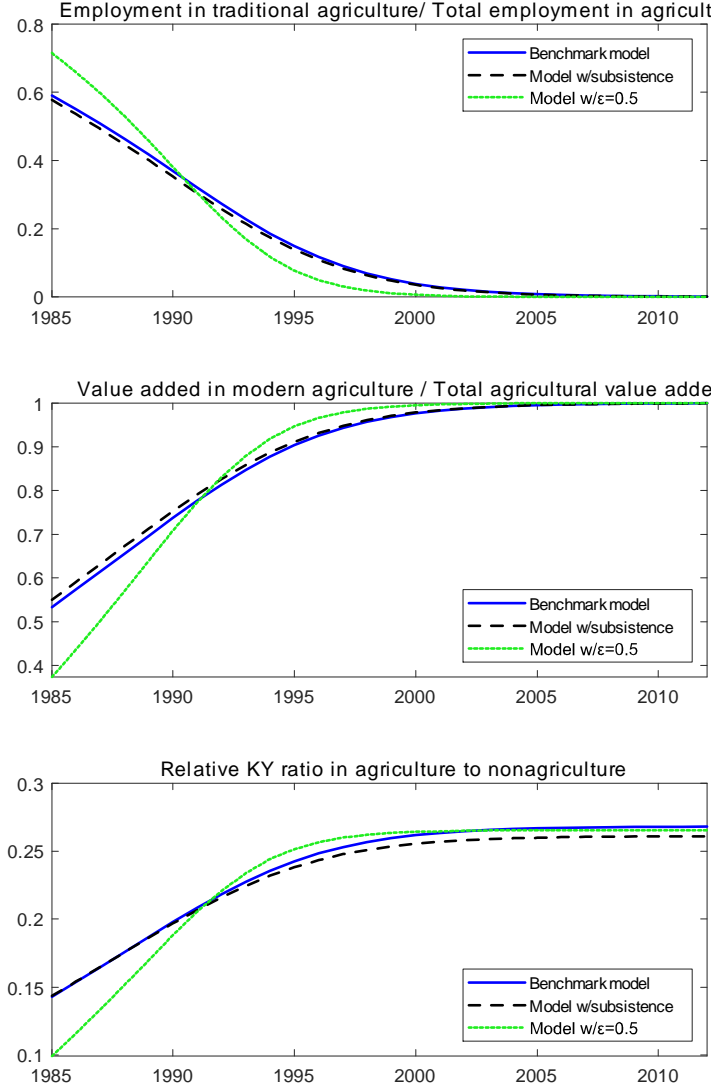


Figure 9: Structural change, according to the models. Solid blue lines: homothetic model. Dashed lines: the nonhomothetic models with unrestricted ε (black) and restricted $\varepsilon = 0.5$ (green). Panel A displays the share of agricultural employment working in traditional sector. Panel B displays the evolution of v ; value added in modern agriculture as a share of total agricultural value added, in current prices.

5.2 Simulating the Stochastic Economy

We simulate the model using the estimates in Table 1 that were obtained to match the structural change of China. We augment the model with the stochastic process for TFPs estimated above.

5.2.1 Methodology

Solving the model is a nontrivial task. We cannot approximate the economy around a balanced growth path. Instead, we proceed as follows. We first solve for a stochastic one-sector version of our model without agriculture, using standard methods. We assume that our benchmark three-sector model converges to such one-sector model after 250 periods. Proposition 2 shows that the ABGP of our benchmark economy indeed converges to the balanced growth path of this one-sector model. We then solve the stochastic economy recursively for each time period, back to period $t = 0$ using global methods.

The stochastic process for $z_t \equiv [z_t^M, z_t^{AM}, z_t^S]'$ is approximated by a 27-state Markov chain with three realizations for each shock, using a standard Tauchen method (Tauchen 1986). There are two continuous state variables, κ and χ . We approximate the next-period decision rules for (c_{t+1}, h_{t+1}) with piecewise linear functions over the state variable $(\kappa_{t+1}, \chi_{t+1}, z_{t+1})$. We solve for the optimal decisions on a grid with

75 points for κ and χ . The location of this grid is adjusted over time.²² We verify that realized optimal (κ_t, χ_t) never exceeds this range. Given decision rules for (c_{t+1}, h_{t+1}) , the optimal control variables follow from the state and the optimality conditions. In particular, we solve for current-period optimal choices for $(\chi_{t+1}, \kappa_{t+1}, h_t, c_t, v_t, \nu_t^M, \nu_t^{AM}, \nu_t^S)$. The decision rules for $\chi_{t+1}, \kappa_{t+1}, h_t, c_t, \nu_t^M, v_t$ are approximated by piecewise linear functions over (κ_t, χ_t, z_t) and the decision rules for ν_t^{AM} and ν_t^S follow directly from the optimality conditions once the values for κ_t and ν_t^M are determined.

We start each sample economy off with an initial value for $\hat{\chi}_{1980}$ such that the deterministic model reaches the empirical $\hat{\chi}_{1985}$ in 1985. Moreover, κ_{1985} is set to the 1985 value on the ABGP. We then simulate 1,000 versions of the economy starting in 1980 and calculate statistics from 1985 to 2185.

5.2.2 Results

Both the empirical data and the simulated data are filtered to remove trends. We report results for the benchmark economy based on HP-filtered data and data in First Differences. The upper panel of each table presents the business cycle statistics for China 1985-2012. The lower panel reports the same statistics for the simulated economies. Table 2 reports results based on HP-filter (with an HP parameter of 6.25), whereas appendix Table 8 reports results based on first differences. In the discussion we focus on the HP-filtered data. As an inspection of the tables shows, the choice of filter is not important.

We complement the discussion of Table 2 with an analysis of the impulse-response functions since these are useful to explain our findings. Figure 10 shows the impulse response functions for employment, value added, and the productivity gap to sectoral TFP shocks. The analysis assumes that the economy is at the ABGP in 1985 and that one cyclical TFP level z_{1985}^j experiences a positive shock while the other cyclical TFP levels remain at zero.²³

The figure illustrates the response to each shock individually, holding the other TFP shocks at the deterministic equilibrium level.

Productivity gap. The correlations between the productivity gap and sectoral labor supplies have the same sign as their empirical counterparts, although the model predicts a lower volatility of the fluctuations in productivity gap than is observed in the data. The productivity gap is countercyclical; it decreases when the employment in nonagriculture is high and increases when the employment in modern agriculture is high. Intuitively, a positive productivity shock in nonagriculture attracts workers from agriculture and induces modernization in agriculture. Interestingly, a temporary boom is associated with a temporary acceleration of the process of structural change (cf. Lemma 2).

The impulse response functions in Figure 10 illustrate the mechanism. An increase in z^M (upper panels) triggers a shift of labor and capital to nonagriculture. In the period when the shock occurs, only labor adjusts (recall that capital is set one period in advance). Subsequently, capital goes up in the more productive sector, sourced from both modern agriculture and net capital accumulation. Labor comes from traditional agriculture and, to a lesser extent, from modern agriculture. Thus, as labor supply in agriculture falls, this sector becomes modernized: both the average capital intensity and the average labor productivity increase in agriculture. Modernization reduces the productivity gap (upper right panel of Figure 10). A shock to z^S (bottom panels) has a similar but opposite effect on the employment and output dynamics: a higher z^S causes labor to be reallocated to traditional agriculture sourced from modern agriculture and nonagriculture. This in turn increases the relative size of traditional agriculture and the productivity gap. Thus, shocks to

²²In period t the grid for χ_t is distributed from $0.90\bar{\chi}_t$ to $1.1\bar{\chi}_t$ where $\bar{\chi}_t$ denotes the deterministic trend. Similarly, the grid for κ_t distributed from $\bar{\kappa}_t - 0.025$ to $\bar{\kappa}_t + 0.025$.

²³The impulse response functions change with structural change. In the appendix we illustrate the corresponding impulse response functions for year 2000. The main quantitative difference relative to 1985 is that the productivity gap changes less in response to shocks. The reason is that traditional agriculture is smaller in 2000.

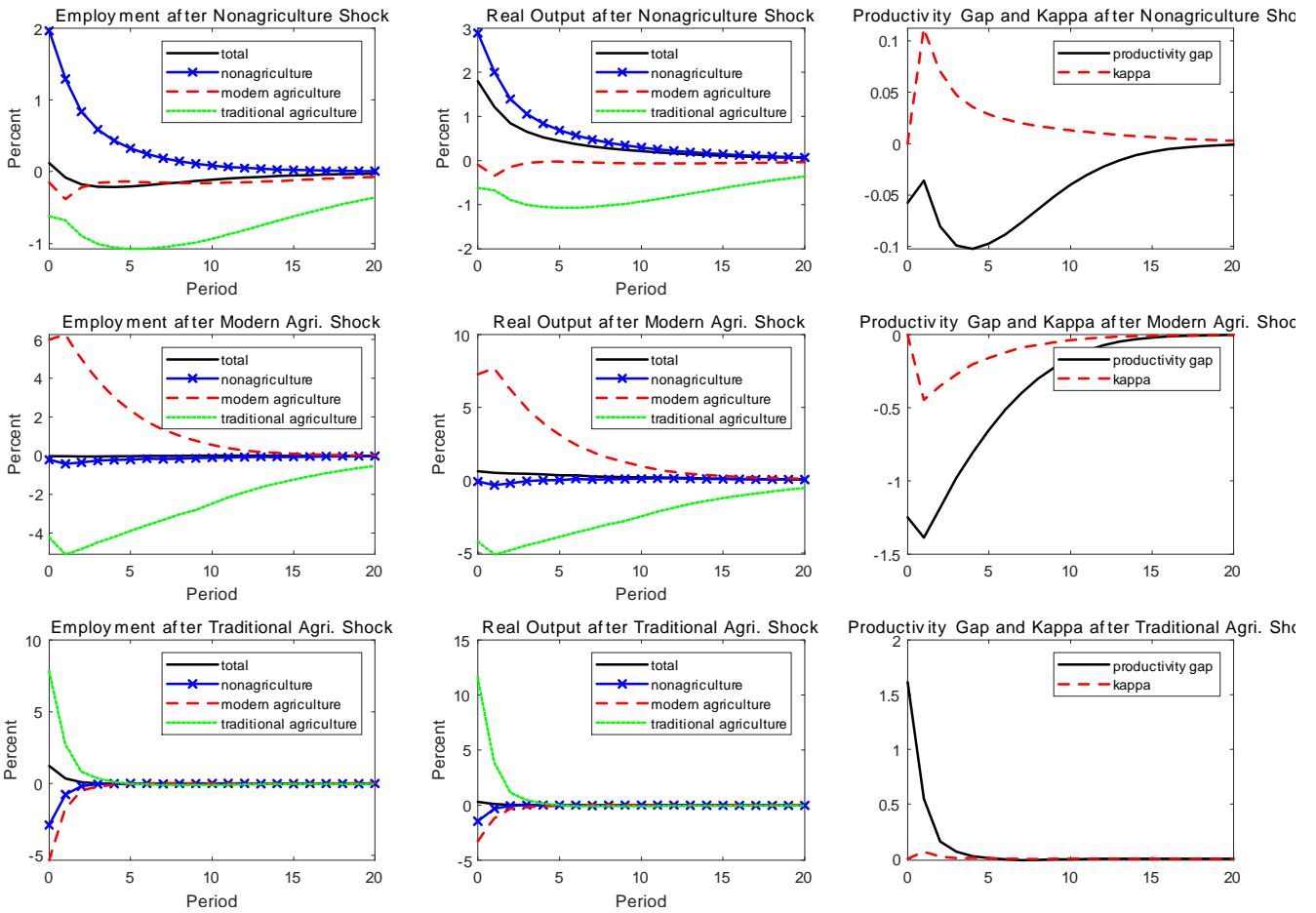


Figure 10: **Impulse Response Functions.** All graphs show impulse response as percentage deviation from the deterministic path in 1985. The three top panels show responses to a one-standard deviation change in nonagricultural TFP (Z^M). The three middle panels show responses to modern agricultural TFP (Z^{AM}). The three bottom panels show responses to traditional agricultural TFP (Z^S).

z^M and z^S induce a positive comovement in the productivity gap and agricultural employment. In contrast, a positive shock to modern agriculture induces both an increase in total employment and modernization of agriculture. Overall, shocks to modern agriculture mitigate the countercyclicality of the productivity gap.

Structural change and sectoral productivity growth. Figure 11 shows the impulse responses for the agricultural employment share in response to TFP shocks. As mentioned above, structural change accelerates in response to positive TFP shocks to z^M and slows down in response to TFP shocks in agriculture. This prediction is in line with the causal evidence for employment dynamics associated with the Green Revolution discussed in the introduction and reviewed in more detail in Section 6 below.

Our impulse-response functions also conform with the common narrative about business-cycle fluctuations in contemporary China. For example, during the early 1980s China implemented large agricultural reforms through the so-called Household Responsibility System. This reform induced substantial productivity growth in agriculture (Lin (1988)). According to Figure 3, this period is associated with a slowdown in structural change.²⁴ Conversely, the period 2002–08 witnessed a remarkable TFP growth in manufacturing, associated with WTO accession and important market reforms (Song et al. (2011)), while TFP in agriculture stagnated (Wang et al. (2013)). In Figure 3, this period is characterized by an acceleration of structural change, which is in line with the prediction of our model after a positive TFP shock in manufacturing.

Sectoral labor supply and GDP. Both in the model and in the data (see Table 2) employment in nonagriculture ν^M is positively correlated with GDP, consumption, and investment whereas employment in agriculture ν^G is negatively correlated with GDP, consumption, and investment. The reason for these asymmetries lies in the presence of misallocation. Because of the wedge τ , the agricultural sector is suboptimally too large. Our estimate is in line with a number of studies which document the presence of a significant wedge between urban and rural sector in China, see e.g., Brandt and Zhu (2010), Cheremukhin et al. (2015). Thus, resource reallocation away from agriculture has in general a positive efficiency effect.

With this in mind, consider the contrasting effect of a TFP shock in nonagriculture and in agriculture. When TFP in nonagriculture increases, both capital and labor move towards nonagriculture. Therefore, GDP goes up both by the direct effect of the productivity increase and because misallocation falls. In contrast, when TFP increases in agriculture the positive direct effect of a higher average TFP is dampened (and possibly offset) by the increase in misallocation. This dichotomy is clear in the second column of Figure 10: GDP increases sharply after a positive TFP shock in nonagriculture, while GDP hardly moves after TFP shocks to agriculture.

These predictions are again consistent with the recent history of China – see Figure 3. The early 1980s (positive TFP shock in agriculture) is associated with below-trend GDP growth whereas the period 2002–08 (positive TFP shock in nonagriculture) is associated with boom in GDP.

Note that the existence of a labor reserve in the traditional sector speeds up reallocation since many workers can leave agriculture without much effect on the marginal product of labor. This effect is reminiscent of Lewis (1954). Both the reduction in misallocation and the low opportunity cost of sourcing workers from agriculture result in a strong correlation between employment in nonagriculture and GDP.

Expenditure and value added. Expenditure and value added exhibit a cyclical pattern similar to that of sectoral labor: expenditure on nonagricultural good is strongly procyclical, while expenditure on agricultural good is acyclical in the data and only weakly procyclical in the model. However, the predictions of the model are inconsistent with the empirical observation that agricultural value added is positively

²⁴Clearly, we do not mean to diminish the value of the agriculture reforms in the 1980s on the development of China. The Household Responsibility System was important in highlighting the salience of individual incentives, with crucial lessons for the subsequent reforms that increased productivity in the industrial sector. It also had dramatic effects on the living conditions of millions of people. These and other aspects are abstracted from in our stylized theory whose focus is positive rather than normative.

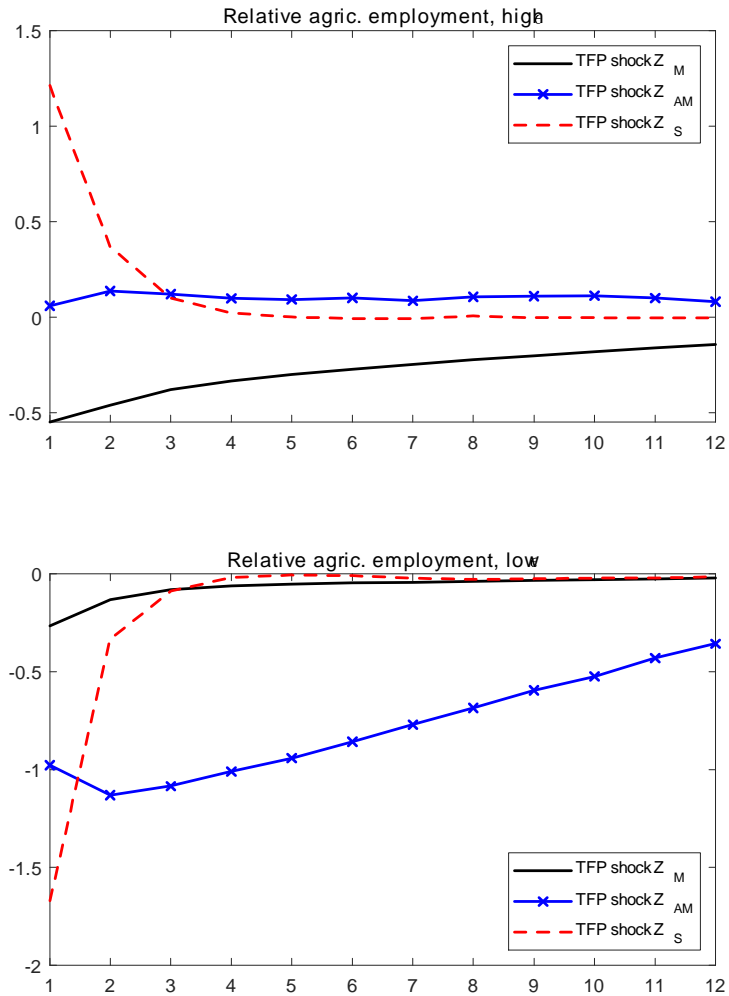


Figure 11: Impulse Response Functions. The graphs show impulse responses for the agricultural employment share, i.e., $(N^S + N^{AM})/N$, as percentage deviation from the deterministic path. The top panel shows the employment dynamics in the estimated case ($\varepsilon = 3.6$) for each of the TFP shocks (Z^S , Z^{AM} , and Z^M). The bottom panel shows the corresponding responses in the case with a low elasticity ($\varepsilon = 0.5$).

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
A. HP Filtered China Data, 1985-2012						$std(y) = 1.7\%$		
$\frac{std(x)}{std(y)}$	0.99	3.53	1.63	1.34	2.04	0.64	0.73	0.10
$corr(x, y)$	0.70	0.65	0.06	0.95	-0.17	-0.69	0.73	-0.23
$corr(x, n^G)$	-0.60	-0.31	-0.37	-0.55	0.48	1.00	-0.94	0.48
$corr(x, n^M)$	0.60	0.37	0.41	0.57	-0.54	-0.94	1.00	0.04
B. HP Filtered Model, Homothetic model						$std(y) = 1.7\%$		
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr(x, n^G)$	-0.08	-0.25	0.78	-0.43	0.73	1	-0.75	0.69
$corr(x, n^M)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21

Table 2: Summary Statistics for China data and Model: HP-filtered

(negatively) correlated with nonagricultural labor (agricultural labor) in China.

GDP and aggregate labor supply. The model generates the same volatility of GDP as in the data. It also predicts a low volatility of employment and a low correlation with GDP, although both are larger than in the Chinese data. Moreover, the model is consistent with the observation that aggregate labor supply is highly correlated with employment in agriculture and approximately uncorrelated with employment in nonagriculture. The low volatility and low correlation of employment with total GDP stem from the availability of a large labor reserve in agriculture that can be reallocated across sectors without generating large wage fluctuations. Therefore, total labor supply is less volatile (and less correlated with GDP) than in a one-sector model. Note that TFP shocks to nonagriculture are responsible for the lion's share of GDP fluctuations: a variance decomposition exercise shows that they account for 95 percent of GDP fluctuations over the 1985-2012 period.

Finally, the model shares many of the features of a standard one-sector RBC model concerning consumption and investment: investment is more volatile and consumption is less volatile than output.

5.3 Business Cycle and Economic Development

In this section, we simulate the model beyond the stage of structural change reached by China in 2012. This allows us to forecast the future evolution of business cycles and to compare the predictions of the model for an economy at the stage of development of China with that of a fully industrialized economy. We focus on four statistics: (i) the correlation between agricultural employment and productivity gap; (ii) the correlation between agricultural and nonagricultural employment, (iii) the correlation between total employment and GDP, and (iv) the volatility of labor supply and consumption relative to GDP.

The results are shown in Figure 12. Each dot in the figure represents a statistic covering a 28-year rolling window. The first dot on the right of each figure (i.e., that corresponding to the largest agricultural share) corresponds to simulations over the period 1985-2012 which are those reported in Table 2); the second dot corresponds to 1986-2013, and so on. In each figure, the process of economic development is a shift from left to right. A fully industrialized economy is an economy with an employment share of agriculture lower than 10%.

The top left panel shows that the correlation between the size of the agricultural sector and the productivity gap decreases as the agriculture shrinks. This is in line with the empirical cross-country evidence documented in Section 2. The reason for this behavior in the model is that as the agricultural sector modernizes and the traditional sector shrinks, the economy converges to Hansen and Prescott (2002) economy with a constant productivity gap.

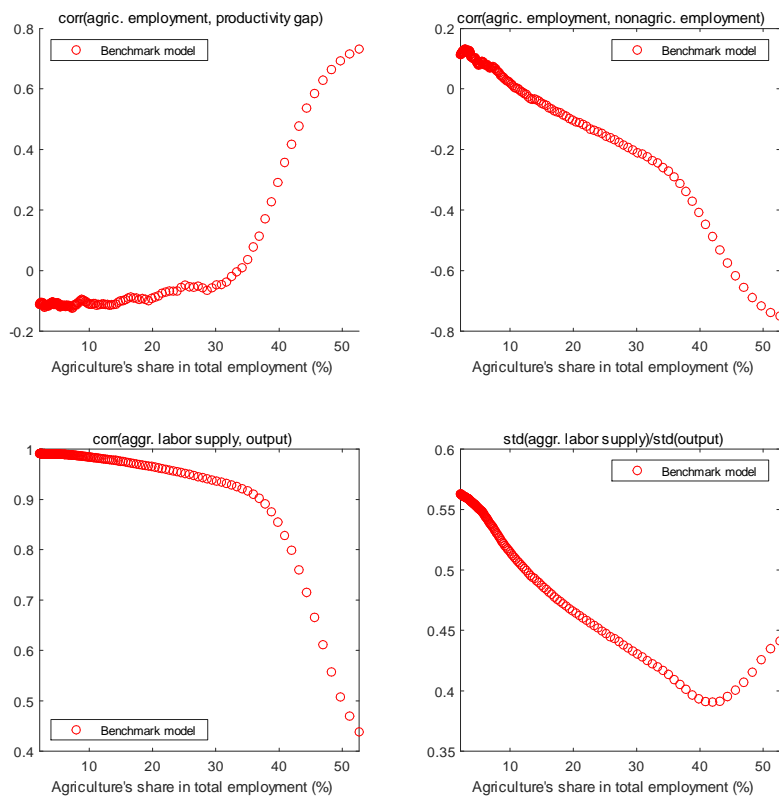


Figure 12: Business Cycle during Structural Change. The graphs show the evolution of business cycle statistics as a function of the employment share in agriculture. Each dot shows a statistic covering a 28-year rolling window. Simulated data are HP-filtered. The upper left panel shows the correlation between employment in agriculture and the productivity gap. The upper right panel shows the correlation between employment in agriculture and employment in nonagriculture. The lower left panel shows the correlation between total employment and GDP. The lower right panel shows the volatility of aggregate employment relative to GDP.

The upper right panel shows that the correlation between employment in agriculture and nonagriculture increases as the share of agriculture falls. In particular, the correlation is negative and large in absolute value as long as the employment share in agriculture is between 40%-50% (close to the value for today's China), falling to zero in fully modernized economies. This is again in line with the empirical cross-country evidence (cf. Panel b of Figure 5). The reason is twofold. First, a large agricultural sector works as a buffer: when there are sector-specific shocks, it is possible to move labor in and out of agriculture. Second, the labor reserve in the traditional sector offers a high elasticity of substitution between sectoral employments. As this sector shrinks, the effective elasticity of substitution across sectors falls.

The bottom left panel shows that the correlation between total employment and GDP increases from 40% to 100% as the share of agriculture falls. The increase in the correlation along the process of structural change, again in line with the empirical evidence in Figure 5.

The bottom right panel also shows that the volatility of employment relative to GDP is lower for an economy like China than in a fully industrialized economy.²⁵ Finally, our model generates a decline in the volatility of consumption relative to GDP as the economy develops, in line with the empirical evidence in Aguiar and Gopinath (2007). The reason is that in an economy where agriculture has a large share of production, the aggregate capital-output ratio is low. Therefore, a fixed absolute change in aggregate capital causes large swings in the marginal product of capital. This makes consumption more volatile and more positively correlated with GDP through a standard Euler Equation mechanism. In turn, this lowers the fluctuations in labor supply due to income effects.

5.4 Robustness Analysis

This section explores four robustness analysis exercises: (1) a low elasticity $\varepsilon = 0.5$ combined with a large food subsistence level, (2) introducing capital adjustment costs; (3) assuming shocks to Z^S have the same persistence of as shocks to Z^{AM} ; and (4) uncorrelated shocks.

5.4.1 Low Elasticity of Substitution

Our estimation identified a high elasticity of substitution ε and small subsistence component in agriculture \bar{c} . It is interesting to contrast the results of the estimated benchmark model with those of a model with a low ε and a large \bar{c} . Section 4.1 estimates a version of the economy when ε is restricted to $\varepsilon = 0.5$. In this case the estimation yields a large subsistence level \bar{c} . We have already documented that the success of this model at accounting for the structural change in China is mixed. Here we document the business-cycle properties of such a model.

Consider first the response of relative sectoral employment to TFP shocks to agriculture. The lower panel of Figure 11 shows that the $\varepsilon = 0.5$ economy implies a *decline* in agricultural employment when TFP increases in agriculture. The reason is twofold. On the one hand, the relative price of agricultural goods falls sharply when ε is low, causing an increase in nonagriculture. On the other hand, when TFP in agriculture increases, the demand for nonagricultural goods increases more, due to a stronger income effect. This implication runs counter to the causal evidence from the Green Revolution and from rainfall patterns reviewed in Section 6: positive technology and weather shocks slow down the structural change

²⁵Note that in our calibration, employment volatility decreases with development at earlier stages of the process of structural change (i.e., when agriculture employs more than 45% of the total hours worked). The reason for this nonmonotone behavior is that when traditional agriculture is large, fluctuations are largely driven by TFP shocks to this sector (Z^S). The estimated persistence of the TFP shock in traditional agriculture happen to be low which implies a large response in labor supply. We believe that this result may partly arise from measurement error, which is particularly important in the traditional sector (where, recall, we retrieve TFP shocks indirectly rather than through standard growth accounting, because of lack of data). Measurement error is likely to exaggerate the volatility and underestimate the persistence of shocks to Z^S . We return to this in the robustness section.

from agriculture to nonagriculture in the data. It also collides with the common narrative about the Chinese business cycle discussed above.

The effect of a TFP shock to nonagriculture is qualitatively similar to that of the benchmark economy: it increases relative employment in nonagriculture. Although the relative price of nonagricultural goods falls, the income effect comes to rescue. However, the overall quantitative effect is significantly smaller than in the benchmark economy. For this reason, TFP shocks to agriculture play a significantly more important role as drivers of business cycles. A variance decomposition analysis shows that agricultural TFP shocks account for about half of the GDP fluctuations in the $\varepsilon = 0.5$ economy whereas in the benchmark economy GDP fluctuations are almost entirely driven by TFP shocks to nonagriculture.

The business-cycle properties of the $\varepsilon = 0.5$ economy are presented in panel B of Table 3. For convenience, the statistics for the benchmark economy are restated as panel A of this table. Many of the qualitative properties are similar to those of the benchmark economy. However, one important failure is that the productivity gap is uncorrelated with sectoral employment in the $\varepsilon = 0.5$ economy. The reason is that TFP shocks to traditional sector increases relative employment in nonagriculture (cf. Figure 11) and at the same time increases the productivity gap (through labor reallocation within agriculture).

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
A. Benchmark Economy								$std(y) = 1.7\%$
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr(x, n^G)$	-0.08	-0.25	0.78	-0.43	0.73	1	-0.75	0.69
$corr(x, n^M)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21
B. $\varepsilon = 0.5$								$std(y) = 1.6\%$
$\frac{std(x)}{std(y)}$	0.38	2.46	1.72	1.20	0.40	1.27	0.93	0.41
$corr(x, y)$	0.81	0.99	-0.002	0.99	-0.26	-0.42	0.80	0.39
$corr(x, n^G)$	-0.37	-0.41	0.84	-0.43	-0.001	1	-0.76	0.58
$corr(x, n^M)$	0.60	0.82	-0.47	0.82	-0.12	-0.76	1	-0.13
C. Capital Adjustment Cost								$std(y) = 1.7\%$
$\frac{std(x)}{std(y)}$	0.71	1.47	1.22	1.22	0.71	1.35	1.05	0.45
$corr(x, y)$	0.99	0.99	0.14	0.96	-0.41	-0.49	0.58	-0.27
$corr(x, n^G)$	-0.45	-0.49	0.74	-0.68	0.78	1	-0.92	0.93
$corr(x, n^M)$	0.53	0.77	-0.09	0.88	-0.50	-0.56	1	0.14
D. Same autocorrelation for Z^S and Z^{AM}								$std(y) = 1.6\%$
$\frac{std(x)}{std(y)}$	0.27	2.40	0.97	1.18	0.38	0.82	0.97	0.32
$corr(x, y)$	0.80	0.99	0.33	0.98	-0.68	-0.34	0.83	0.54
$corr(x, n^G)$	-0.15	-0.35	0.72	-0.48	0.51	1	-0.70	0.50
$corr(x, n^M)$	0.49	0.86	-0.15	0.93	-0.77	-0.70	1	0.08
E. Orthogonal Shocks								$std(y) = 1.5\%$
$\frac{std(x)}{std(y)}$	0.25	2.43	1.25	1.19	0.67	1.20	1.08	0.48
$corr(x, y)$	0.79	0.99	0.16	0.96	-0.06	-0.19	0.68	0.49
$corr(x, n^G)$	-0.06	-0.18	0.87	-0.40	0.66	1	-0.76	0.69
$corr(x, n^M)$	0.40	0.68	-0.48	0.84	-0.53	-0.76	1	-0.22

Table 3: Robustness analysis, benchmark model versus alternative model. All statistics refer to HP-filtered simulated data for 1985-2012.

5.4.2 Capital Adjustment Costs

In the benchmark model, capital in each sector is set one period in advance, and after one period reallocation of capital between sectors can occur without cost. In this section, we investigate the effect of introducing additional costs of reallocating capital between agriculture and nonagriculture. Capital adjustment costs are commonly assumed in the quantitative DSGE literature (cf. Christiano et al. (2005); Smets and Wouters (2007)), including papers studying business cycles in models with multiple sectors (see e.g. Horvath (2000); Bouakez et al. (2009); Iacoviello and Neri (2010)).

Following Bouakez et al. (2009) and Iacoviello and Neri (2010), we consider a canonical model where it is costly to change the investment rate. Recall that for each sector j , the law of motion of capital is given by $K_{t+1}^j = (1 - \delta) K_t^j + I_t^j$, where $j \in \{M, G\}$ and I_t^j is the effective investment in sector j . The investment cost is reflected in an aggregate resource constraint for investment goods,

$$\left[\gamma (X_t^G)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \gamma) (X_t^M)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} = \Psi_t^G \left(\frac{I_t^G}{K_t^G}, K_t^G \right) + \Psi_t^M \left(\frac{I_t^M}{K_t^M}, K_t^M \right) + I_t^G + I_t^M,$$

where the terms Ψ_t^{AG} and Ψ_t^M capture the adjustment cost. Recall also that X_t^i is the quantity of good $i \in \{M, G\}$ allocated to investment. We assume that the adjustment cost function Ψ has a standard quadratic form,

$$\Psi_t^j \left(\frac{I_t^j}{K_t^j}, K_t^j \right) = \frac{\xi}{2} \left(\frac{I_t^j}{K_t^j} - \delta - g_t^j \right)^2 K_t^j,$$

where the parameter g_t^j is the growth rate of capital K_t^j in period t in the deterministic structural transition and ξ is a nonnegative adjustment cost parameter. It follows that as long as the capital stock K_t^j grows at the same rate as in the deterministic transition, g_t^j , the adjustment costs are zero. However, when the investment rate deviates from this level then quadratic costs are incurred.

We set the adjustment cost parameter to $\xi = 2.5$, which is slightly lower than the annual equivalent to the value of ξ estimated by Iacoviello and Neri (2010) based on quarterly data for the US ($\xi = 11$).²⁶ Since the deterministic model of structural change is not affected by the capital adjustment cost, we keep all other parameters the same as in the benchmark economy.

Because adjustment costs make investments more sluggish, consumption must respond more to TFP shocks, which in turn increases its volatility. Adjustment costs also affect sectoral employment because sluggish capital makes less advantageous moving labor across sectors in the short run.

Panel C of Table 3 shows the results. The main effect of introducing adjustment costs is that the cyclical behavior of aggregate labor supply and consumption is more in line with the data: n becomes negatively correlated with GDP and positively correlated with nonagricultural labor supply, as it is for China (see panel A of Table 2). Moreover, consumption becomes substantially more volatile, more negatively correlated with agricultural labor supply, and get a more reasonable correlation with GDP. However, aggregate investment turns too smooth.

5.4.3 More Persistent Shocks in the Traditional Sector

In the benchmark model the persistence of shocks to z^S is significantly lower than that of shocks to z^{AM} ($\hat{\phi}^S = 0.42$ versus $\hat{\phi}^{AM} = 0.90$). We argued above that the transitory nature of shocks to traditional sector

²⁶Recall that in an annual model it is impossible to change the capital stock more often than annually, so our benchmark economy already embeds some frictions since sectoral capital is set one period in advance.

z^S was the cause behind the falling volatility of aggregate employment in the initial phase of the transition (see the lower right panel of Figure 12).

Since neither traditional nor modern agricultural production are directly observed, measurement error in agricultural capital and employment will show up as movements in the two TFP levels. This will affect the estimated TFP processes for z^S and z^M . For this reason, we consider a sensitivity analysis where TFP shocks to traditional agriculture have the same persistence as shocks to modern agriculture ($\hat{\phi}^S = \hat{\phi}^{AM} = 0.9$). Moreover, we adjust the volatility of the innovations to z^S , $\sigma(\epsilon_t^S)$, so that the stationary variance of z^S is kept constant. The results are shown in panel C of Table 3. The main effect of a higher persistence of shocks to z^S is that the aggregate volatility of labor supply n falls and the correlation between n and GDP increase somewhat. Moreover, the relative volatility of employment is predicted to increase monotonically during structural change (see panel D of appendix Figure 15). Lowering the standard deviation of innovations to z^S , $\sigma(\epsilon_t^S)$, has similar effects.

5.4.4 Orthogonal Shocks

The benchmark model estimated the correlation between TFP shocks based on the empirical time series for China. In order to quantify the role of this correlation structure for the results, we investigate the effect of abstracting from this correlation by assuming orthogonal shocks. The results are presented in panel E of Table 3. The differences relative to the benchmark model are negligible.

6 International Evidence

The elasticity of substitution ε plays an important role in our analysis. When estimating the model for China we found $\varepsilon > 1$. Using data from other countries, other studies have instead estimated $\varepsilon < 1$. For example, Herrendorf et al. (2013) shows that a model with $\varepsilon < 1$ and higher TFP growth in agriculture than in manufacturing accounts well for structural change in the US after 1950. In this section we use data from other countries to evaluate the robustness of our estimate of ε outside of China.

We proceed in two steps. First, estimate aggregate multi-sector production functions based on time-series evidence on structural change in the world's largest economies. Rather than committing to the structure of our model, we use a production function approach that is comparable to previous studies in the literature. We also consider some correlation analysis using cross-country data. Then, we review causal evidence of how TFP shocks in agriculture affect structural change, and discuss its implications for ε .

6.1 Estimating Aggregate Production Functions

Herrendorf et al. (2013) estimate a three-sector CES production function where the elasticity of substitution between agricultural goods, manufacturing goods, and services is assumed to be identical across sectors. Using a consumption value added approach, they estimate an elasticity of substitution between manufacturing, services and agriculture close to zero for the US. Following their lead, we now extend our analysis to a class of models where the nonagricultural sector comprises two sectors: a service sector and a manufacturing sector. More formally, we assume that

$$Y^M = \left[\hat{\gamma} \left(Y^{Manuf} \right)^{\frac{\varepsilon_{ms}-1}{\varepsilon_{ms}}} + (1 - \hat{\gamma}) \left(Y^{Serv} \right)^{\frac{\varepsilon_{ms}-1}{\varepsilon_{ms}}} \right]^{\frac{\varepsilon_{ms}}{\varepsilon_{ms}-1}}$$

where the superscripts *Manuf* and *Serv* denote manufacturing and services, respectively, and the parameter ε_{ms} represents the elasticity of substitution between services and manufacturing in production of the nonagricultural good. Moreover, following the approach proposed by Herrendorf et al. (2013), we construct

Table 4: IFGNLS Estimates of Nested CES Elasticities					
	Elasticity	Cons. value added		GGDC	
		3-sector	2-sector	3-sector	2-sector
USA	ε	2.49*** (0.28)	2.32*** (0.32)	1.36*** (0.13)	1.53*** (0.48)
	ϵ_{ms}	0	-	0.48*** (0.005)	-
Japan	ε	1.58*** (0.053)	2.24*** (0.36)	5.46*** (0.80)	6.06*** (0.91)
	ϵ_{ms}	0.79*** (0.003)	-	1.66*** (0.13)	-
China	ε	1.70*** (0.22)	1.34*** (0.25)	1.30*** (0.13)	1.75*** (0.08)
	ϵ_{ms}	0.007 (0.05)	-	0.65*** (0.04)	-

Table 4: Estimates of elasticities under the nested CES preference specification. ε is the elasticity between agriculture and nonagriculture and ϵ_{ms} is the elasticity between manufacturing and services. Table 9 in Appendix D reports the full results.

consumption value-added data consistent with national accounts and with input-output tables. We also estimate the model using standard production value-added data from the Gröningen data (GGDC) 10-Sector Database. We estimate the model for the world’s three largest countries – China, USA, and Japan.

The estimation procedure follows Herrendorf et al. (2013). More formally, we estimate the following Stone-Geary demand system,

$$Y = \left[\gamma \left(\left[\hat{\gamma} (Y^{Manuf})^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1 - \hat{\gamma}) (Y^{Serv} + \bar{s})^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} \right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} + (1 - \gamma) (Y^G + \bar{c})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}. \quad (19)$$

This specification follows that used in a different context by (Krusell, Ohanian, Rios-Rull, and Violante 2000). In particular, we allow ε to differ from ϵ_{ms} . Moreover, as in Herrendorf et al. (2013) we allow for a subsistence level in agriculture ($\bar{c} \leq 0$) and home production in services ($\bar{s} \geq 0$). Note that the generalized model nests the specification of Herrendorf et al. (2013) as the particular case in which $\epsilon_{ms} = \varepsilon$. The estimated nonlinear equations are stated in Appendix C.

Table 4 summarizes the main results (the full estimation results are reported in appendix Table 9). We present results for both the three-sector model and a restricted version where we do not distinguish between manufacturing and services (this is closer to the specification we use in our model).

We obtain estimates of the agriculture-nonagriculture elasticity above unity for all countries and with both consumption value added and production value added. The benchmark estimation – the three-sector model using consumption value added data – imply $\varepsilon_{US} = 2.5$, $\varepsilon_{JP} = 1.6$, and $\varepsilon_{CN} = 1.7$, and the estimates are significantly larger than unity at a 1% level for all countries. The production-based estimate of ε is higher for Japan and slightly lower for the US and China. Moreover, in all cases the estimates originating from the three-sector model are very similar to those of the two-sector model.

We estimate ϵ_{ms} to be close to zero for the US and China, and 0.8 for Japan. Thus, our findings strongly reject the null hypothesis that $\epsilon_{ms} = \varepsilon$, and suggests that the elasticity of substitution between agricultural and nonagricultural goods is substantially higher than the elasticity between manufacturing and services.²⁷

²⁷Note that when we impose the restriction $\epsilon_{ms} = \varepsilon$ and reestimate the model, we replicate the finding of as Herrendorf et al. (2013) that the elasticity is close to zero for the US. Similarly, when this restriction is imposed for Japan and China, we estimate the elasticity to be smaller in unity for both countries, 0.65 and 0.58, respectively.

6.2 Correlation between Expenditure and Output (or Prices)

Cross-country data could also be used to shed light on the elasticity ε . In particular, the value of ε has implications about the correlation between relative expenditure and relative production in agriculture. To this aim, consider an aggregate CES production function like in Equation (1), which implies a standard isoelastic demand condition $P_t^G/P_t^M = \frac{\gamma}{1-\gamma} (Y_t^G/Y_t^M)^{-1/\varepsilon}$. Taking log on both sides and considering the first difference yields

$$\Delta \ln \left(\frac{Y_t^G P_t^G}{Y_t^M P_t^M} \right) = \frac{\varepsilon - 1}{\varepsilon} \Delta \ln \left(\frac{Y_t^G}{Y_t^M} \right) \quad (20)$$

or, equivalently,

$$\Delta \ln \left(\frac{Y_t^G P_t^G}{Y_t^M P_t^M} \right) = (1 - \varepsilon) \cdot \Delta \ln \left(\frac{P_t^G}{P_t^M} \right) \quad (21)$$

We take these two versions of the demand equation to the data. When doing so, one should bear in mind that this exercise is potentially problematic since both sides of Equations (20)-(21) are endogenous. We return to these endogeneity issues below.

When $\varepsilon > 1$, the model predicts the expenditure ratio growth will be positively correlated with the real output ratio growth (eq. (20)) and negatively correlated with the relative price growth (eq. (21)). Conversely, these correlations will be reversed if $\varepsilon < 1$.

We use value added for 10 sectors from the Gröningen Growth and Development Centre, and use the UN Analysis of Main Aggregates dataset (1970-2017) for robustness.

Appendix Table 5 documents the results from estimating equation (20) with OLS across a number of specifications. All specifications yield regression coefficients in the range 0.46-0.84, which implies an elasticity ε in the range between 2 and 6. In all cases the estimates are significantly positive at a 1 percent level. Conversely, appendix Table 6 documents the OLS estimates implied by Equation (21). This specification yields regression coefficients in the range -0.44 to 0.78, which implies estimates of ε in the range 0.2 to 1.4. The bulk of the estimates implies $\varepsilon < 1$.

The overall evidence is mixed. In Appendix C.2 we show that (classical) measurement error is not a likely cause of the differences across specifications. Measurement error should, if anything, mitigate the differences between the estimators. In Appendix C.3 we show that these inconclusive results could be caused by an identification issue. Namely, the comovements could be driven by both productivity and demand (preference) shocks, which we show have opposite implications for the sign of the correlations. This problem applies to the estimation of both Equation (20) and (21). With this in mind, we now turn our attention to studies that consider exogenous and well identified shocks to TFP in agriculture.

6.3 Causal Effects of TFP Shocks in Agriculture

Empirical analyses of demand systems using data on prices and quantities, as in Equations (20)-(21) and (6.1), suffer from a potential omitted variable bias, since their comovements may be affected by factors other than productivity shocks that influence relative prices and output simultaneously. To address this identification problem, we turn to event studies that focus on plausibly exogenous productivity shocks.

Exogenous shocks to agriculture are especially informative about ε . In the model with $\varepsilon < 1$, a positive TFP shock to agriculture will lead to structural change from agriculture to industry. In contrast, when $\varepsilon > 1$, such shocks slow down the process of structural change. This was evident in the discussion of the impulse response functions of TFP shocks in our model, see Section 5.

One prominent event that offers sharp identification is the Green Revolution: the introduction of high-yield crop varieties caused a sharp increase in agricultural TFP in developing countries, and the impact

was particularly pronounced in the 1970s. Foster and Rosenzweig (2004) and Moscona (2018) exploit the fact that, due to differences in ecological and geographic characteristics, different regions were able to adopt and reap the benefits of high-yield crops to different degrees. The differences generated by this exogenous variation provide a valid instrument for agricultural TFP growth. Foster and Rosenzweig (2004) shows that the Green Revolution slowed down non-farm growth across rural Indian villages. In another study covering the period 1970-2000, Foster and Rosenzweig (2004) use time series for 240 Indian villages and document that the growth of rural industry was systematically slower in areas where crop yields grew faster. They conclude that the evidence is consistent with a model in which industrial capital is mobile and industrialization and technological development in agriculture are substitutes. Note that this causal pattern holds true not only for small local areas, but also at the country level. Moscona (2018) finds that countries that for exogenous reasons were better placed to benefit from the Green Revolution witnessed a larger increase in agricultural TFP together with an expansion of employment in agriculture. However, this also caused a slower growth of manufacturing and non-farm labor as well a slower process of urbanization. His conclusions again suggest substitutability rather than complementarity between agriculture and industrial technology.

Similarly, in a study of the Green Revolution in Brazil, Bustos et al. (2016) find that the introduction of high-yield maize, which increased the marginal product of land, resulted in a reduction in industrial employment.²⁸ Finally, Jayachandran (2006) estimates the response of agricultural wages and labor supply to positive TFP shocks in agriculture. He uses rainfall as an instrument for TFP shocks and shows that higher crop yield is strongly associated with higher wages and higher labor supply in agriculture across districts in India over the 1956-1987 period.

In conclusion, the evidence from causal studies estimating the effects of technological progress in agriculture are consistent with the predictions of our structural model when the output of agriculture and nonagriculture are substitutes ($\varepsilon > 1$).²⁹

7 Conclusion

Business cycle fluctuations in countries undergoing structural transformation differ systematically from business cycles in mature economies. We document that countries with large declining agricultural sectors – including China – have aggregate employment fluctuations that are smooth and acyclical, while these countries experience volatile and procyclical reallocation of labor between agriculture and nonagriculture. We provide a unified theoretical framework for studying business cycle during structural change. The central aspect of the theory is the modernization of agriculture that occurs during the structural change: agriculture is becoming increasingly capital intensive and less labor intensive as a large traditional sector is crowded out. With a large traditional sector the expansion of manufacturing draws workers from traditional agriculture, triggering modernization in agriculture and sustaining aggregate productivity. This process is driven by capital accumulation and differential productivity growth between agriculture and nonagriculture. At business cycle frequencies, positive TFP shocks in nonagriculture accelerate this process while TFP shocks in agriculture slowdown structural change in line with the evidence from the Green Revolution discussed in the paper.

We estimate the model using data for China and show that its quantitative predictions are consistent both with China’s structural transformation and China’s business cycles. Moreover, the model is consistent with

²⁸However, they also find that the introduction of genetically modified soy, which does not require tilling of land and hence was labor saving, tended to reduce employment in agriculture.

²⁹We also note that the elasticity of substitution between agriculture and nonagriculture is potentially different at the household preferences (consumption) level versus at the aggregate country level (production). There are many reasons for that. For example, if goods could be stored or traded across countries, the relative production would differ from the relative consumption and the effective production elasticity would be larger than the consumption elasticity.

the changing business cycle properties as a country evolves from a poor economy with a large agricultural sector to a modern industrialized economy with negligible agricultural employment.

Our business cycle analysis only focuses on productivity shocks. Future research will extend it to a broader set of disturbances including demand shocks. Another limitation that we plan to address in future work is the closed-economy environment. In spite of these and other shortcomings, we believe the theory casts new light on the relationship between business cycles and economic development.

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Online Appendix for the paper “Business Cycle during Structural Change: Arthur Lewis’ Theory from a Neoclassical Perspective”

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A Appendix: Data Description in Section 2

In this appendix, we describe how we constructed the data to document the evidence on the structural change and business-cycle facts across countries. In all cases, we exclude countries. We exclude the following city states: Bahrein, Hong Kong, Macao, Qatar, Singapore. Agricultural sector’s shares of employment and GDP are negligible in these countries.

When computing the K/Y ratio, we exclude countries that have K/Y ratio higher than 100. When calculating the productivity gap, we exclude countries that have productivity gap higher than 20.

The data for aggregate GDP, capital stocks, investment, and consumption are from the World Development Indicators. The data for value added in agriculture and capital stocks in agriculture is from the FAO. The data for sectoral employment comes from the International Labor Organization (ILO). The real consumption is constructed by dividing households consumption by CPI index. The real output is constructed by dividing the GDP in current price by GDP deflator.

The data set is constructed as follows. First, we use data from the labor force surveys, households surveys, official statistics, and population censuses. We exclude data from firm surveys. Second, we exclude data that are not representative of the whole country. In particular, we exclude data from some countries which report data that only cover the urban population. Third, if multiple sources exist for the same country and these data cover overlapping time periods, we merge (by chaining) the different sources provided that data in the overlapping time periods are small. However, if the differences are large across different sources, we only retain the most recent data source, provided that the sample period is at least 15 years. If the most recent data cover less than 15 years, we retain the less recent data series (provided the sample covers at least 15 years).

Fourth, if multiple sources exist for the same country and these data do not cover overlapping time periods, then we do not merge the data. Instead we retain only the most recent data, provided that the sample period is at least 15 years. Again, if the most recent data covers less than 15 years, we use less recent data, provided that the data cover at least 15 years. The country is dropped if there are no data series longer than 15 years.

The sample of countries plotted in Figure 3 ranges from 63 to 66 across panels (a)-(c). We further exclude countries with relative consumption volatility higher than 3 in panel (d). The detailed description of which countries appear in each panel and for which sample period is provided in Appendix E.2.

B Appendix: Formal Analysis in Section 3

This appendix contains proofs and analytical derivations.

B.1 Proof of Proposition 1

We take FOCs of the program 8 using standard methods. After rearranging terms, the equalization of the marginal product of capital in modern agriculture and nonagriculture yields:

$$\frac{1 - \kappa}{\kappa} = \frac{1 - \beta}{1 - \alpha} \frac{\gamma}{1 - \gamma} \left(\frac{y^G}{y^M} \right)^{\frac{\varepsilon - 1}{\varepsilon}} v. \quad (22)$$

The equalization of the marginal product of labor in modern agriculture and nonagriculture, and in traditional agriculture and nonagriculture yield, respectively,

$$\nu^{AM} = \frac{1}{1 - \tau} \frac{1 - \alpha}{1 - \beta} \frac{\beta}{\alpha} \frac{1 - \kappa}{\kappa} \nu^M, \quad (23)$$

$$\nu^S = \frac{1}{\beta} \frac{1 - v}{v} \nu^{AM}. \quad (24)$$

Combining (22), (23), and (24) with the resource constraint on labor, and solving for ν^M , yields

$$\nu^M = \left(1 + \frac{1}{1 - \tau} \frac{1 - \kappa}{\kappa} \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \left(1 + \frac{1}{\beta} \frac{1 - v}{v} \right) \right)^{-1}. \quad (25)$$

These four equations, together with the definitions of y^G and y^M provided in the text, implicitly define the unique set of equilibrium policy functions $\kappa = \kappa(\chi, \mathbf{Z})$, $v = v(\chi, \mathbf{Z})$, $\nu^M(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$, $\nu^{AM}(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$, and $\nu^S(\kappa(\chi, \mathbf{Z}), v(\chi, \mathbf{Z}))$.

B.2 Derivation of Equation 10

Proof. The FOC (22) that equates the marginal product of capital can be rewritten as

$$\frac{1 - \kappa}{\kappa} = \varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \left(\frac{Y^G}{Y^M} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \left(\frac{Y^{AM}}{Y^G} \right)^{\frac{\omega - 1}{\omega}}.$$

Taking logarithms, and letting $\omega \rightarrow \varepsilon$ yields

$$\ln(1 - \kappa) - \ln \kappa = \ln \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right) + \frac{\varepsilon - 1}{\varepsilon} \ln \left(\frac{y^{AM}}{y^M} \right)$$

Substituting in the expressions for y^{AM} and y^G , and differentiating with respect to $\ln \chi$ yields

$$\left(\frac{\varepsilon - 1}{\varepsilon} (\beta - \alpha) - \frac{1}{\varepsilon} \frac{1}{1 - \kappa} \right) \frac{\partial \ln \kappa}{\partial \ln \chi} = -\frac{\varepsilon - 1}{\varepsilon} (\beta - \alpha) \times \left(1 - \frac{\partial \ln \nu^M}{\partial \ln \chi} \right) \quad (26)$$

Next, consider (25). Differentiating with respect to $\ln \chi$ yields

$$\begin{aligned} \frac{\partial \ln(\nu^M)}{\partial \ln \chi} &= \frac{\nu^{AM} + (1 + (1 - \kappa)(\varepsilon - 1)(1 - \beta)) \nu^S}{(1 + \nu^S(1 - \beta)(\omega - 1))} \frac{1}{1 - \kappa} \frac{\partial \ln \kappa}{\partial \ln \chi} \\ &\quad + \frac{\nu^S(\omega - 1)(1 - \beta)}{1 + \nu^S(1 - \beta)(\varepsilon - 1)} \end{aligned} \quad (27)$$

Plugging in (27) into (26) and letting $\omega \rightarrow \varepsilon$ yields

$$\lim_{\omega \rightarrow \varepsilon} \frac{\partial \ln \kappa}{\partial \ln \chi} = \frac{(\varepsilon - 1)(\beta - \alpha)(1 - \kappa)}{1 + (\varepsilon - 1)((\beta - \alpha)(\kappa - \nu^M) + \nu^S(1 - \beta))} > 0. \quad (28)$$

■

B.3 Proof of Lemma 2

Proof. Having defined $z_S \equiv Z^S / (Z^M)^\alpha$ and $z_A \equiv (Z^{AM})^\beta / (Z^M)^\alpha$, the four static equilibrium conditions and the definition of Ξ can be expressed as

$$\frac{\nu^S}{\nu^{AM}} = \left(\frac{1 - \varsigma}{\beta} \frac{1 - \alpha}{\kappa} \right)^\varepsilon \left(\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \Xi \right)^{-(1-\beta)(\varepsilon-1)} \left(\frac{z_S}{z_A} \right)^{\varepsilon-1} \quad (29)$$

$$\frac{\nu^{AM}}{\nu^M} = \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \frac{1 - \kappa}{\kappa} \quad (30)$$

$$\left(\frac{\nu^{AM}}{\nu^M} + \frac{\nu^S}{\nu^M} \right) \kappa \chi = \left(\frac{1}{\nu^M} - 1 \right) \kappa \chi = \Xi - \kappa \chi \quad (31)$$

$$\left(\frac{1 - \kappa}{\kappa} \right) (\Xi)^{(\beta-\alpha)(\varepsilon-1)} = \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^\varepsilon \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{\beta(\varepsilon-1)} (z_A)^{(\varepsilon-1)} \quad (32)$$

$$\Xi \equiv \frac{\kappa \chi}{\nu^M} = \left(\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \right)^{-1} \frac{(1 - \kappa) \chi}{\nu^{AM}} \quad (33)$$

We start with the comparative statics for z_A . Rewrite eq. (32) as

$$\Xi = \left(\left(\frac{1 - \kappa}{\kappa} \right)^{-1} \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^\varepsilon \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{\beta(\varepsilon-1)} (z_A)^{(\varepsilon-1)} \right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}} \quad (34)$$

Substitute (29)-(30) into (31) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in κ and Ξ ;

$$\frac{\Xi}{\kappa \chi} - 1 = \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \frac{1 - \kappa}{\kappa} + \left(\frac{1 - \varsigma}{\alpha} \frac{\gamma}{1 - \gamma} \right)^\varepsilon \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{(\varepsilon-1)(\beta-\alpha)} (\Xi)^{-(\varepsilon-1)(1-\alpha)} (z_S)^{\varepsilon-1}$$

Substitute in eq. (34), and simplify

$$\begin{aligned} & \frac{1}{\chi} \left(\frac{\kappa}{1 - \kappa} \right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}} \left(\left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^\varepsilon \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{\beta(\varepsilon-1)} \right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}} (z_A)^{\frac{1}{\beta-\alpha}} \\ &= \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} - \frac{\beta - \alpha}{\alpha(1 - \beta)} \kappa + \left(\frac{1 - \varsigma}{\alpha} \frac{\gamma}{1 - \gamma} \right)^\varepsilon \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{\beta}{\alpha} \right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}\varepsilon} \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}(\beta(\varepsilon-1)-\varepsilon)+(\varepsilon-1)(\beta-\alpha)} \\ & \times (1 - \kappa)^{\frac{(1-\alpha)}{(\beta-\alpha)}} (\kappa)^{-\frac{1-\beta}{\beta-\alpha}} (z_A)^{-(\varepsilon-1)\frac{(1-\alpha)}{(\beta-\alpha)}} (z_S)^{\varepsilon-1} \end{aligned}$$

The LHS is increasing in z_A and in κ . The RHS is decreasing in z_A and in κ . It follows that an increasing in z_A must be associated with a decline in κ . Equation (30) then implies that ν^{AM}/ν^M increases. Suppose $\partial \ln \Xi / \partial \ln z_A \geq 0$. Then the RHS of eq. (29) unambiguously falls, implying that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also

fall. Suppose instead that $\partial \ln \Xi / \partial \ln z_A < 0$. Combine eq. (30) with the identity $1 - \nu^S = \nu^M + \nu^{AM}$,

$$\begin{aligned} 1 - \nu^S &= \left(1 + \frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \frac{\kappa}{1 - \kappa} \right) \nu^{AM} \\ &= \chi \left(\frac{1}{\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha}} - \frac{\beta - \alpha}{\alpha(1 - \beta)} \kappa \right) \frac{1}{\Xi} \end{aligned}$$

Since $\partial \ln \kappa / \partial \ln z_A < 0$ and since $\partial \ln \Xi / \partial \ln z_A < 0$ by assumption, the RHS must increase. It follows that ν^S must fall, implying that the ratio ν^S / ν^{AM} must also fall. This proves that $\partial \ln (\nu^S / \nu^{AM}) / \partial \ln z_A < 0$. It follows immediately that both v and ν^{AM} must increase.

Consider now the comparative statics for z_A . Substitute (29)-(30) into (31) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in κ and Ξ ;

$$\frac{1}{1 - \kappa} = \frac{\chi}{\Xi - \chi} \left[\left(1 + \left(\frac{1 - \beta}{\beta} \frac{1 - \alpha}{\kappa} \right)^\varepsilon \left(\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \frac{\Xi}{\kappa} \right)^{-(1-\beta)(\varepsilon-1)} \left(\frac{z_S}{z_A} \right)^{\varepsilon-1} \right) \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} - 1 \right]$$

Rewrite (32) as,

$$\frac{1}{1 - \kappa} = \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^{-\varepsilon} \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{-\beta(\varepsilon-1)} (z_A)^{-(\varepsilon-1)} (\Xi)^{(\beta-\alpha)(\varepsilon-1)} + 1.$$

Equate these expressions and rearrange to get one equation in Ξ ;

$$\begin{aligned} &\ln \left(\left(\frac{1 - \beta}{\beta} \frac{1 - \alpha}{\kappa} \right)^\varepsilon \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{(1-\beta)(\varepsilon-1)+1} (\Xi)^{-(1-\beta)(\varepsilon-1)} (z_S)^{\varepsilon-1} + \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} - \frac{\Xi}{\chi} \right) (z_A)^{\varepsilon-1} \right) \\ &= \ln \left(\frac{1}{\chi} \left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^{-\varepsilon} \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{-\beta(\varepsilon-1)} \right) + (\beta - \alpha)(\varepsilon - 1) \ln(\Xi) + \ln(\Xi - \chi) \end{aligned} \quad (35)$$

Differentiate Equation (35) w.r.t. z_S , and rearranging terms yields

$$\begin{aligned} &\frac{(\varepsilon - 1) \left(\frac{1 - \beta}{\beta} \frac{1 - \alpha}{\kappa} \right)^\varepsilon \left(\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \right)^{-(1-\beta)(\varepsilon-1)-1} (\Xi)^{-(1-\beta)(\varepsilon-1)} \left(\frac{z_S}{z_A} \right)^{\varepsilon-1}}{\left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^{-\varepsilon} \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{-\beta(\varepsilon-1)} (z_A)^{-(\varepsilon-1)} (\Xi)^{(\beta-\alpha)(\varepsilon-1)} \left(\frac{\Xi - \chi}{\chi} \right)} \\ &= \left[\frac{(\beta - \alpha)(\varepsilon - 1) + \frac{\Xi}{\Xi - \chi}}{\left(\varsigma \frac{\gamma}{1 - \gamma} \frac{1 - \beta}{1 - \alpha} \right)^{-\varepsilon} \left(\frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} \right)^{-\beta(\varepsilon-1)} (z_A)^{-(\varepsilon-1)} (\Xi)^{(\beta-\alpha)(\varepsilon-1)} \left(\frac{\Xi - \chi}{\chi} \right)} + \frac{\frac{\Xi}{\Xi - \chi}}{\left(\frac{1 - \beta}{\beta} \frac{1 - \alpha}{\kappa} \right)^\varepsilon \left(\frac{\alpha}{\beta} \frac{1 - \beta}{1 - \alpha} \right)^{-(1-\beta)(\varepsilon-1)-1} (\Xi)^{-(1-\beta)(\varepsilon-1)} + \frac{\Xi}{\chi}} \right] \frac{\partial \ln \Xi}{\partial \ln z_S} \end{aligned}$$

Recall that $\Xi = \frac{\kappa \chi}{\nu^{AM}} > \chi$ due to $\kappa > \nu^M$. Therefore both coefficient on $\partial \ln \Xi / \partial \ln z_S$ is positive. It follows that $\partial \ln \Xi / \partial \ln z_S > 0$. Now take log on both sides of Equation (32) and differentiate w.r.t. $\ln z_S$; $0 = \partial / \partial \ln z_S \ln [(1 - \kappa) / \kappa] + (\beta - \alpha)(\varepsilon - 1) \partial \ln \Xi / \partial \ln z_S$. Since $(\beta - \alpha)(\varepsilon - 1) \partial \ln \Xi / \partial \ln z_S > 0$, it must be that $\partial / \partial \ln z_S \ln [(1 - \kappa) / \kappa] < 0$, which in turn implies that $\partial / \partial \ln z_S \ln \kappa > 0$. Recall that $\nu^{AM} = \chi(1 - \kappa) / \Xi$. Since both Ξ and κ are increasing in z_S , ν^{AM} must be falling in z_S . Now substitute (30) into (31) to obtain an equation in $\frac{\nu^S}{\nu^{AM}}$, κ and Ξ ;

$$\left(\frac{\Xi}{\chi} - 1 \right) \frac{1}{1 - \kappa} = \left(1 + \frac{\nu^S}{\nu^{AM}} \right) \frac{\beta}{\alpha} \frac{1 - \alpha}{1 - \beta} - 1$$

Since both κ and Ξ are increasing in z_S , it follows that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also be increasing in z_S . Since $\nu^S / \nu^{AM} = \beta^{-1} (1 - v) / v$, it follows immediately that v must fall in z_S . ■

B.4 Analysis of the Dynamic Equilibrium

The dynamic equilibrium can be characterized by solving the Hamiltonian (12). The following proposition summarizes the results equilibrium:

Proposition 3 *The dynamic competitive equilibrium is characterized by the following system of ordinary differential equations:*

$$\frac{\dot{c}_t}{c_t} = (\eta(\kappa(\chi_t, \mathbf{Z}_t), v(\chi_t, \mathbf{Z}_t)))^{\frac{1}{\varepsilon}} (1 - \gamma) (1 - \alpha) \times \quad (36)$$

$$\begin{aligned} & \left(\frac{\kappa(\chi_t, \mathbf{Z}_t) \chi_t}{Z_t^M \nu^M(\kappa(\chi_t, \mathbf{Z}_t), v(\chi_t, \mathbf{Z}_t))} \right)^{-\alpha} - \delta - \rho \\ \dot{\chi}_t &= \eta(\kappa(\chi_t, \mathbf{Z}_t), v(\chi_t, \mathbf{Z}_t)) \times (\chi_t \kappa(\chi_t, \mathbf{Z}_t))^{1-\alpha} \times \\ & (Z_t^M \nu^M(\kappa(\chi_t, \mathbf{Z}_t), v(\chi_t, \mathbf{Z}_t)))^\alpha - (\delta + n) \chi_t - c_t \end{aligned} \quad (37)$$

$$\frac{Z_t^M}{Z_t^M} = g^M, \quad \frac{Z_t^{AM}}{Z_t^{AM}} = g^{AM}, \quad \frac{Z_t^S}{Z_t^S} = g^S$$

where $\eta(\kappa, v)$ is given by (14), $\nu^M(\kappa, v)$ satisfies (25), and $\kappa(\chi_t, \mathbf{Z}_t)$ and $v(\chi_t, \mathbf{Z}_t)$ are the static equilibrium policy functions. The solution is subject to a vector of initial conditions $(\chi_0, \mathbf{Z}_0) = (\bar{\chi}_0, \bar{\mathbf{Z}}_0)$ and a transversality condition.

Equation (36) is a standard Euler Equation for consumption. For constant TFPs, the growth rate of consumption is decreasing in χ because the aggregate production function exhibit decreasing returns to capital. Equation (37) is a resource constraint.

It is useful to rewrite the equilibrium conditions in Proposition 3 in terms of an autonomous system of differential equations. To this aim, we differentiate with respect to time the set of static equilibrium (22), (23), (24), and (25). After rearranging terms, we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = (1 - \kappa_t) \frac{\left(\left(\alpha g^M - \beta g^{AM} + (\beta - \alpha) \frac{\dot{\chi}_t}{\chi_t} \right) + \left(\frac{1}{\omega - 1} - \frac{(\beta - \alpha)(1 - \nu^M(\kappa_t, v_t))}{1 - v_t(1 - \beta)} \right) \frac{\dot{v}_t}{v_t} \right)}{\frac{1}{\varepsilon - 1} + (\beta - \alpha)(\kappa_t - \nu^M(\kappa_t, v_t))}, \quad (38)$$

$$\frac{\dot{v}_t}{v_t} = \frac{(1 - v_t) \left(\beta g^{AM} - g^S + (1 - \beta) \left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t} \frac{\kappa_t - \nu^M(\kappa_t, v_t)}{1 - \kappa_t} \right) \right)}{\frac{1}{\omega - 1} + \frac{(1 - v_t)(1 - \beta)(1 - \nu^M(\kappa_t, v_t))}{1 - v_t(1 - \beta)}}. \quad (39)$$

This dynamic system is defined up to a pair of initial conditions: $\kappa_0 = \kappa(\chi_0, \mathbf{Z}_0)$ and $v_0 = v(\chi_0, \mathbf{Z}_0)$ consistent with the static equilibrium conditions at time zero.

Corollary 1 *The dynamic competitive equilibrium is fully characterized by the solution to the autonomous system of ordinary differential Equations (36)-(37)-(38)-(39) and the exogenous law of motion $\dot{Z}_t^M/Z_t^M = g^M$, after setting $\kappa(\chi_t, \mathbf{Z}_t) = \kappa_t$ and $v(\chi_t, \mathbf{Z}_t) = v_t$ for $t > 0$, with initial conditions $\kappa(\chi_0, \mathbf{Z}_0) = \kappa(\bar{\chi}_0, \bar{\mathbf{Z}}_0) \equiv \kappa_0$ and $(\chi_0, \mathbf{Z}_0) = v(\bar{\chi}_0, \bar{\mathbf{Z}}_0) \equiv v_0$.*

Equations (38)-(39) allow us to eliminate Z_t^{AM} and Z_t^S from the dynamic system, while only retaining their initial levels and their growth rates. In other words, $\kappa_0 = \kappa(\bar{\chi}_0, \bar{\mathbf{Z}}_0)$ and $v_0 = v(\bar{\chi}_0, \bar{\mathbf{Z}}_0)$ are sufficient statistics. If κ_0 and v_0 are set at the static equilibrium level at time zero, Equations (38)-(39) guarantee that κ_t and v_t will also be consistent with the static equilibrium in all future periods.

B.5 Proof of Proposition 2

Proof. We start by evaluating Equations (36)-(37) under the ABGP conditions. Note that (14) implies that $\eta(1, 1) = (1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}}$. Thus,

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= g_M = (1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}} (1 - \alpha) \left(\frac{\chi}{Z^M} \right)^{-\alpha} - \delta - \rho, \\ \frac{\dot{\chi}_t}{\chi_t} &= g_M = (1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}} \times \left(\frac{\chi}{Z^M} \right)^{-\alpha} - (\delta + n) - \frac{c}{\chi}.\end{aligned}$$

Solving for c/χ and χ/Z^M yields the expressions in (15) and (16). Therefore, (36)-(37) hold true under the ABGP conditions. It is straightforward to see that under the ABGP conditions (in particular, when $\kappa = v = 1$) (38)-(39) yields $\frac{\dot{\kappa}}{\kappa} = \frac{\dot{v}}{v} = 0$. Likewise, (22) holds true when $\kappa = v = 1$.

Next, consider the asymptotic growth rates of the sectoral capital. Taking logarithms and differentiating with respect to time the definitions of k^M and k^{AM} yields $\dot{k}^M/k^M = \dot{\kappa}/\kappa + \dot{\chi}/\chi = g^M$ and $\dot{k}^{AM}/k^{AM} = -(1 - \kappa)^{-1} \times \dot{\kappa}/\kappa + \dot{\chi}/\chi = g^M - (\varepsilon - 1) \beta (g^M - g^{AM})$.

Next, consider the asymptotic growth rates of the sectoral employments of labor. First, observe that Equation (25) yields $\nu^M = 1$ at the ABGP conditions $\kappa = v = 1$. Second, note that $\nu^M = 1$ implies that $\dot{N}^M/N^M = \dot{N}/N = n$. In order to establish the growth rate of N^{AM} , observe that taking logarithms on both side of Equation (23), differentiating with respect to time, and using the ABGP conditions and Equation (38) yields

$$\frac{\dot{N}^{AM}}{N^{AM}} = -\frac{1}{1 - \kappa} \frac{\dot{\kappa}}{\kappa} + \frac{\dot{N}^{AM}}{N^{AM}} = n - (\varepsilon - 1) \beta (g^M - g^{AM}).$$

Finally, to establish the growth rate of N^S , observe that taking logarithms on both side of Equation (24), differentiating with respect to time, and using the ABGP conditions and Equation (39) yields

$$\frac{\dot{N}^S}{N^S} = -\frac{1}{1 - v} \frac{\dot{v}}{v} + \frac{\dot{N}^{AM}}{N^{AM}} = \frac{\dot{N}^{AM}}{N^{AM}} - (\omega - 1) [(g^{AM} - g^S) + (1 - \beta) (g^M - g^{AM})].$$

To establish convergence, we linearize the dynamic system in a neighborhood of the ABGP. The system has three predetermined variables (χ, κ, v) and one jump variable (c) . Therefore, we must prove that the linear system has three negative eigenvalues and one positive eigenvalue. The rest of the proof is devoted to establish that this is the case.

Let $\tilde{\chi} = \frac{\chi}{Z^M}$ and $\tilde{c} = \frac{c}{Z^M}$, implying that $\frac{d\tilde{\chi}/dt}{\tilde{\chi}} = \frac{\dot{\chi}}{\chi} - g^M$. Then, we can write the dynamic system (36)-(37)-(38)-(39). We can rewrite the system as

$$\begin{aligned}\frac{d\tilde{c}/dt}{\tilde{c}} &= \eta(\kappa_t, v_t)^{\frac{1}{\varepsilon}} (1 - \gamma) (1 - \alpha) \left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M(\kappa_t, v_t)} \right)^{-\alpha} - \delta - \rho - g^M \\ \frac{d\tilde{\chi}/dt}{\tilde{\chi}} &= \eta(\kappa_t, v_t)^{1-\alpha} \left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M(\kappa_t, v_t)} \right)^{-\alpha} \kappa_t - \delta - \tilde{c}_t/\tilde{\chi}_t - n - g^M \\ \frac{\dot{\kappa}}{\kappa} &= (1 - \kappa) \frac{\left(\left(\alpha g^M - \beta g^{AM} + (\beta - \alpha) \left(\frac{d\tilde{\chi}/dt}{\tilde{\chi}} + g^M \right) \right) + \left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)(1-\nu^M)}{1-v(1-\beta)} \right) \frac{\dot{v}}{v} \right)}{\frac{1}{\varepsilon-1} + (\beta - \alpha) (\kappa - \nu^M)} \\ \frac{\dot{v}}{v} &= (1 - v) \frac{\left(\beta g^{AM} - g^S + (1 - \beta) \left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t} \frac{\kappa_t - \nu^M(\kappa_t, v_t)}{1 - \kappa_t} \right) \right)}{\frac{1}{\omega-1} + \frac{(1-v)(1-\beta)(1-\nu^M)}{1-v(1-\beta)}}\end{aligned}$$

where we use Equation (25) implying that

$$\frac{1 - \nu^M}{\nu^M} = \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{1 - \beta} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1 - v}{v} \right).$$

The transversality condition (TVC) becomes

$$\lim_{t \rightarrow \infty} \xi e^{-(\rho-n)t} K = 0$$

Substitute the condition that

$$\lim_{t \rightarrow \infty} \frac{\dot{K}}{K} = n + g^M$$

The TVC becomes

$$\lim_{t \rightarrow \infty} \frac{\dot{\xi}}{\xi} + g^M + n < \rho - n$$

Because

$$-\frac{\dot{c}}{c} = (1 - \theta)(1 - \sigma) \frac{\dot{h}}{h} \frac{h}{1 - h} + \frac{\dot{\xi}}{\xi} + n$$

Then we have

$$\lim_{t \rightarrow \infty} \frac{\dot{c}}{c} = - \lim_{t \rightarrow \infty} \frac{\dot{\xi}}{\xi} - n$$

where we use $\lim_{t \rightarrow \infty} \frac{\dot{h}}{h} = 0$. Plug $\lim_{t \rightarrow \infty} \frac{\dot{\xi}}{\xi} = -g^M - n$ into the TVC to get

$$-g^M - n + g^M + n < \rho - n$$

In the end, the TVC can be rewritten as

$$\rho - n > 0$$

Letting $\Psi = (\tilde{c}, \tilde{\chi}, \kappa, v)'$, we can write the system of differential equations as

$$A\dot{\Psi} = f(\Psi)$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & a & 1 & b \\ 0 & c & d & 1 \end{bmatrix}$$

and, after

$$\begin{aligned} a &= -\frac{(1 - \kappa)(\beta - \alpha)}{\frac{1}{\varepsilon - 1} + (\beta - \alpha)(\kappa - \nu^M)} \\ b &= -(1 - \kappa) \frac{\left(\frac{1}{\omega - 1} - \frac{(\beta - \alpha)(1 - \nu^M)}{1 - v(1 - \beta)} \right)}{\frac{1}{\varepsilon - 1} + (\beta - \alpha)(\kappa - \nu^M)} \\ c &= -\frac{(1 - v)(\omega - 1)(1 - \beta)}{\left(1 + \frac{1 - v}{v} \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{\alpha} (\omega - 1) \nu^M \right)} \\ d &= -\frac{(1 - v)(\omega - 1)(1 - \beta) \left(1 - \frac{1 - \alpha}{(1 - \beta)\alpha} \frac{\nu^M}{\kappa} \left(\beta + \frac{1 - v}{v} \right) \right)}{1 + (\omega - 1) \frac{1 - v}{v} \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{\alpha} \nu^M} \end{aligned}$$

$$f = \begin{bmatrix} \eta (\kappa_t, v_t)^{\frac{1}{\varepsilon}} (1 - \gamma) (1 - \alpha) \left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M (\kappa_t, v_t)} \right)^{-\alpha} - \delta - \rho - g^M \\ \eta (\kappa_t, v_t) \left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M (\kappa_t, v_t)} \right)^{-\alpha} \kappa_t - \delta - \tilde{c}_t / \tilde{\chi}_t - n - g^M \\ (1 - \kappa) \frac{\beta (g^M - g^{AM})}{\frac{1}{\varepsilon - 1} + (\beta - \alpha) (\kappa - \nu^M)} \\ \frac{(1 - v)(\omega - 1) (g^{AM} - g^S + (1 - \beta) (g^M - g^{AM}))}{1 + \frac{1 - v}{v} \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{\alpha} \nu^M (\omega - 1)} \end{bmatrix}$$

where we use the

$$\begin{aligned} \frac{1 - \nu^M}{\nu^M} &= \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{(1 - \beta)} \left(\frac{\beta}{\alpha} + \frac{1 - v}{\alpha v} \right) \\ \frac{1 - \nu^M}{1 - v (1 - \beta)} &= \nu^M \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{(1 - \beta)} \frac{1}{\alpha v} \\ \frac{\kappa - \nu^M}{1 - \kappa} &= -1 + \frac{\nu^M}{\kappa} \frac{1 - \alpha}{(1 - \beta)} \frac{1}{\alpha} \left(\beta + \frac{1 - v}{v} \right) \end{aligned}$$

The inverse of matrix A is given by

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{bd-1} (a - bc) & -\frac{1}{bd-1} & \frac{b}{bd-1} \\ 0 & \frac{1}{bd-1} (c - ad) & \frac{d}{bd-1} & -\frac{1}{bd-1} \end{bmatrix}$$

Along the approximate balanced growth path,

$$a^* = 0, b^* = 0, c^* = 0, d^* = 0$$

Now we compute

$$J(\Psi) = \begin{pmatrix} J_1(\Psi) \\ J_2(\Psi) \\ J_3(\Psi) \\ J_4(\Psi) \end{pmatrix} = A^{-1}(\Psi) f(\Psi)$$

where

$$\begin{aligned} J_1(\Psi) &= \eta^{\frac{1}{\varepsilon}} (1 - \gamma) (1 - \alpha) \left(\frac{\kappa \tilde{\chi}}{\nu^M (\kappa, v)} \right)^{-\alpha} - \delta - \rho - g^M \\ J_2(\Psi) &= \eta \left(\frac{\kappa \tilde{\chi}}{\nu^M (\kappa, v)} \right)^{-\alpha} \kappa - \delta - \tilde{c} (\tilde{\chi})^{-1} - n - g^M \\ J_3(\Psi) &= \frac{1}{bd - 1} (a - bc) \left(\left(\frac{\kappa \tilde{\chi}}{\nu^M (\kappa, v)} \right)^{-\alpha} \kappa - \delta - \tilde{c} / \tilde{\chi} - n - g^M \right) \\ &\quad - \frac{1}{bd - 1} (1 - \kappa) \frac{\beta (g^M - g^{AM})}{\frac{1}{\varepsilon - 1} + (\beta - \alpha) (\kappa - \nu^M)} \\ &\quad + \frac{b}{bd - 1} \frac{(1 - v)(\omega - 1) (g^{AM} - g^S + (g^M - g^{AM}) (1 - \beta))}{1 + \frac{1 - v}{v} \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{\alpha} (\omega - 1) \nu^M} \\ J_4(\Psi) &= \frac{1}{bd - 1} (c - ad) \left(\left(\frac{\kappa \tilde{\chi}}{\nu^M (\kappa, v)} \right)^{-\alpha} \kappa - \delta - \tilde{c} / \tilde{\chi} - n - g^M \right) \\ &\quad + \frac{d}{bd - 1} (1 - \kappa) \frac{\beta (g^M - g^{AM})}{\frac{1}{\varepsilon - 1} + (\beta - \alpha) (\kappa - \nu^M)} \\ &\quad - \frac{1}{bd - 1} \frac{(1 - v)(\omega - 1) (g^{AM} - g^S + (g^M - g^{AM}) (1 - \beta))}{1 + \frac{1 - v}{v} \frac{1 - \kappa}{\kappa} \frac{1 - \alpha}{\alpha} (\omega - 1) \nu^M} \end{aligned}$$

From (25) it follows that

$$\begin{aligned}\dot{\nu}^M &= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{1 - \nu^M}{1 - \kappa} + \frac{\dot{v}}{v} \frac{1 - \nu^M}{1 - v(1 - \beta)} \right) \\ &= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{\nu^M}{\kappa} \frac{1 - \alpha}{(1 - \beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1 - v}{v} \right) + \frac{\dot{v}}{v} \frac{1 - \nu^M}{1 - v(1 - \beta)} \right)\end{aligned}$$

Hence

$$\begin{aligned}\frac{\partial \nu^M}{\partial \kappa} &= \left(\frac{\nu^M}{\kappa} \right)^2 \frac{1 - \alpha}{(1 - \beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1 - v}{v} \right) \\ \frac{\partial \nu^M}{\partial v} &= \frac{\nu^M}{v} \frac{1 - \nu^M}{1 - v(1 - \beta)}.\end{aligned}$$

Computing the Jacobian evaluated at the balanced growth path $(\tilde{c}, \tilde{\chi}, \kappa, v)'$, we obtain

$$J = \begin{bmatrix} 0 & J_{12}^* & J_{13}^* & 0 \\ J_{21}^* & J_{22}^* & J_{23}^* & 0 \\ 0 & 0 & J_{33}^* & 0 \\ 0 & 0 & J_{43}^* & J_{44}^* \end{bmatrix}$$

with determinant given by $-J_{12}^* J_{21}^* J_{33}^* J_{44}^*$ and four eigenvalues given by

$$\begin{bmatrix} \frac{1}{2} J_{22}^* + \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4J_{12}^* J_{21}^*} \\ \frac{1}{2} J_{22}^* - \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4J_{12}^* J_{21}^*} \\ J_{33}^* \\ J_{44}^* \end{bmatrix},$$

where

$$\begin{aligned}J_{12}^* &= -\alpha(1 - \gamma)^{\frac{\varepsilon}{\varepsilon - 1}} (1 - \alpha) (\tilde{\chi}^*)^{\alpha - 1} < 0 \\ J_{21}^* &= -(\tilde{\chi}^*)^{-1} < 0 \\ J_{22}^* &= (\tilde{\chi}^*)^{-1} (\rho - n) > 0 \\ J_{33}^* &= -(\varepsilon - 1) \beta (g^M - g^{AM}) < 0 \\ J_{44}^* &= -(\omega - 1) (g^{AM} - g^S + (g^M - g^{AM}) (1 - \beta)) < 0\end{aligned}$$

Thus, three eigenvalues are negative, while one is positive ($\frac{1}{2} J_{22}^* + \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4J_{12}^* J_{21}^*} > 0$), establishing the result. ■

B.6 The Lewis Model

In this section we provide the details of the analysis in Section 3.3. We abstract from technical progress and set $Z^M = Z^{AM} = Z^S = 1$. Moreover, we set $\tau = 0$. Endogenous capital accumulation is then the only source of transition. We continue to assume that $\varepsilon > 1$ and $\beta > \alpha$.

Stage 1 (Early Lewis). Consider an economy in which capital is very scarce. When $\chi < \underline{\chi}$, then, $\nu^{AM} = 0$, $\nu^M > 0$, $\nu^S > 0$, and $\kappa = 1$.³⁰ Intuitively, because capital is scarce, it is optimal to use it only in nonagriculture, where it is an essential factor, to take advantage of the high relative price of the

³⁰In particular, $\underline{\chi} = \frac{\beta(1 - \alpha)}{\alpha(1 - \beta)} \left(\frac{(1 - \zeta)}{\beta \zeta} \right)^{\frac{1}{1 - \beta}} \frac{\Xi}{1 + \Xi}$, where $\Xi \equiv \left(\frac{\alpha(1 - \gamma)}{\gamma(1 - \zeta)^{\frac{\varepsilon - 1}{\varepsilon}}} \right)^\varepsilon \left(\frac{\beta(1 - \alpha)}{\alpha(1 - \beta)} \left(\frac{(1 - \zeta)}{\beta \zeta} \right)^{\frac{1}{1 - \beta}} \right)^{(\varepsilon - 1)(1 - \alpha)}$.

nonagricultural good. Over time, employment grows in nonagriculture and falls in agriculture.³¹

The average labor productivity is higher in nonagriculture than in agriculture, reflecting the nonagriculture uses capital. More formally, the productivity gap is given by the inverse ratio of the labor-income shares in the two sectors, which equals $1/\alpha$.

Consider, next, the evolution of the aggregate capital-output ratio and factor prices in the Early Lewis stage. If the agricultural and nonagricultural goods were perfect substitutes, both the wage and the interest rate would stay constant as capital accumulates. However, for $\varepsilon < \infty$ capital accumulation triggers an increase in the relative price of agricultural goods and real wage growth. Wage growth in turn causes capital deepening in the nonagricultural sector and a declining interest rate.

As capital accumulation progresses, the relative price of the agricultural good increases. Once capital is sufficiently abundant, (i.e., as $\chi \geq \underline{\chi}$), the relative price of agriculture is so high that it becomes optimal to put some capital in the modern agricultural sector. At this point the modernization process of agriculture starts and the economy enters the Advanced Lewis stage.

Stage 2 (Advanced Lewis). In this stage the equalization of factor returns across the two sectors implies that they have a constant capital-labor ratios. These are given by

$$k^{AM} \equiv \frac{(1-\kappa)\chi}{\nu^{AM}} = \left(\frac{1-\zeta}{\beta\zeta} \right)^{\frac{1}{1-\beta}}, \quad k^M \equiv \frac{\kappa\chi}{\nu} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} k^{AM}. \quad (41)$$

The share of capital that goes to nonagriculture declines over the process of development in Stage 2:

$$\kappa = \frac{K^M}{L^M} \frac{L^M}{L} \frac{1}{\chi} = \frac{1 + \left(\frac{1-\zeta}{\beta\zeta} \right)^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta} \frac{1}{\chi}}{1 + \frac{\alpha}{1-\alpha} \left(\frac{1+\Xi}{\Xi} \right)} \quad (42)$$

The optimal allocation of labor in manufacturing and modern agriculture yields

$$\nu^M = \frac{\chi + k^{AM} \frac{\beta}{1-\beta}}{k^{AM} \frac{\beta}{1-\beta} \left(\frac{1+\Xi}{\Xi} \right) + k^M}, \quad \nu^{AM} = \frac{\beta}{1-\beta} \frac{1+\Xi}{\Xi} \nu^M - \frac{\beta}{1-\beta}. \quad (43)$$

These expressions shows that employment in both manufacturing and modern agriculture increase as χ grows. Since the sectoral capital-labor ratios are constant, this also implies that capital and output in these sectors are increasing at the expense of a falling production of the traditional agriculture. Since factor prices are constant while the aggregate capital intensity in the economy is increasing, then the aggregate share of GDP accruing to capital grows while the labor share falls.

An interesting observation is that throughout this stage the expenditure share on agriculture and nonagriculture remain constant, even though $\varepsilon \neq 1$. To understand why, consider an economy without a Lewis sector. In this case, when $\varepsilon > 1$ and $\beta > \alpha$, capital accumulation would imply that the capital-intensive sector (in our case, nonagriculture) would grow faster over time. Although this implies an increase in the relative price of the agricultural product, the expenditure share on nonagricultural goods would increase over time. However, reallocation within agriculture with the decline of the Lewis sector offsets this force by increasing labor productivity in agriculture.

³¹The key equilibrium condition is the equalization of the marginal product of labor in nonagriculture and traditional agriculture. Using the implicit function theorem, we can show that ν^M is an increasing function of χ . More formally,

$$\frac{\nu^M}{1-\nu^M} = \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}} \right)^{\varepsilon} \left(\frac{\chi}{\nu^M} \right)^{(\varepsilon-1)(1-\alpha)}, \quad (40)$$

where the LHS is increasing in ν^M and the RHS is increasing in χ and decreasing in ν^M . Thus, standard differentiation implies that $\partial \nu^M / \partial \chi > 0$.

More formally, we can show that $\frac{P^M Y^M}{P^G Y^G} = \frac{1-\gamma}{\gamma} \left(\frac{Y^M}{Y^G}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \Psi$, where $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)^\varepsilon \left(\frac{\alpha}{1-\zeta}\right)^{\varepsilon-1} (k^M)^{(1-\alpha)(\varepsilon-1)}$. This implies that the productivity gap between agriculture and nonagriculture is shrinking, since

$$\frac{\frac{P^M Y^M}{L^M}}{\frac{P^G Y^G}{L^G}} = \Psi \frac{1 - \nu^M}{\nu^M},$$

and, recall, ν^M is increasing in the Advanced Lewis stage. In particular, the productivity gap (which is the inverse of the ration between the labor income share in the two sectors) declines from $1/\alpha$ to β/α in this stage, where, recall, β is the labor income share in modern agriculture. Finally, in the Advanced Lewis stage, the capital-output ratio in agriculture increases relative to the capital-output ratio in nonagriculture. More formally,

$$\frac{\frac{K^G}{P^G Y^G}}{\frac{K^M}{P^M Y^M}} = \Psi \frac{\alpha}{1-\alpha} \left(\frac{1+\Xi}{\Xi} - \frac{\frac{\alpha}{1-\alpha} \left(\frac{1+\Xi}{\Xi}\right) + 1}{\frac{1}{k^M} \chi + \frac{\alpha}{1-\alpha}} \right),$$

which is increasing in χ .

Stage 3 (Neoclassical). As the process of capital accumulation proceeds, the labor reserve in traditional agriculture becomes eventually exhausted. This happens when

$$\bar{\chi} = \frac{\beta}{\beta + \Xi} \left(\frac{(1-\zeta)}{\beta\zeta} \right)^{\frac{1}{1-\beta}} \left(1 + \frac{(1-\alpha)\Xi}{\alpha(1-\beta)} \right) > \underline{\chi}.$$

For any $\chi > \bar{\chi}$, $\nu^S = 0$ and $v = 1$. Henceforth, the economy exhibit standard properties of neoclassical models. In particular, if $\varepsilon > 1$ and $\beta > \alpha$, the nonagriculture sector grows in relative size, capital share (i.e., κ increases) and expenditure share. The productivity gap remains constant at β/α and the relative (agriculture vs. nonagriculture) capital-output ratio is also constant. During this stage, the interest rate falls and the real wage increases as capital accumulates.

Proposition 4 *Suppose $\varepsilon > 1$, $\beta > \alpha$ and $\omega \rightarrow \infty$. Then, as χ grows, economic development progresses through three stages:*

1. **Early Lewis:** *If $\chi \leq \underline{\chi}$, then, $\nu^{AM} = v = 0$, $\kappa = 1$. Moreover, ν^M is increasing and ν^S is decreasing in χ . The interest rate is decreasing and the wage rate is increasing in χ . The (average labor) productivity gap is constant and equal to $1/\alpha$.*
2. **Advanced Lewis:** *If $\chi \in [\underline{\chi}, \bar{\chi}]$ then, ν^M and ν^{AM} are increasing linearly in χ while ν^S is falling linearly in χ (cf. Equation (43)). Therefore, v is increasing in χ . Moreover, κ is decreasing in χ (cf. Equation (42)). The capital-labor ratio in nonagriculture and modern agriculture is constant (cf. Equation (41)), but the relative nonagriculture-to-agriculture capital-output ratio is falling in χ . The interest rate and the wage rate are constant, implying that the aggregate labor income share is falling. The (average labor) productivity gap is monotonically decreasing.*
3. **Neoclassical:** *If $\chi \geq \bar{\chi}$, then, $\nu^S = 0$ and $v = 1$. ν^M is increasing and ν^{AM} is decreasing in χ . Moreover, κ is increasing in χ . The capital-labor ratio is increasing in χ in both nonagriculture and modern agriculture, but the relative nonagriculture-to-agriculture capital-output ratio is constant. The interest rate is decreasing in χ and the wage rate is increasing in χ , while the aggregate labor income share is falling. The (average labor) productivity gap is constant. As χ becomes arbitrarily large, $\kappa \rightarrow 1$, $\nu^M \rightarrow 1$ and the expenditure share of agriculture tends to zero.*

C Appendix: Details of the Analysis of Section 6

C.1 CES estimation with cross-sectional data

We now estimate the elasticities implied by a CES model based on Equations (20)-(21) and cross-country data.

We use the 10-sector data from the Gröningen Growth and Development Centre as our benchmark data set, henceforth GGDC. These data provide long time series on sectoral output for 41 countries. We also use the UN Analysis of Main Aggregates dataset which covers more countries (220 countries) albeit over a shorter time period (1970-2017), henceforth UN AMA. Using value-added sectoral output and real output data (in 2005 prices) from GGDC, we can derive the implied price deflator for each sector. We then aggregate the sectoral price indices and real quantities into agriculture, manufacturing, and nonagriculture following the cyclical expansion procedure used in Herrendorf et al. (2013). We define manufacturing sector as the union of “Mining”, “Manufacturing”, “Utilities”, “Construction”. Similarly, we aggregate all sectors except agriculture sector and government sector into the nonagriculture sector. Following Comin et al. (2015), our benchmark specification focuses on ratios of agriculture to manufacturing. We show that all our findings are robust to using a broader data set (UN data) and a broader definitions of nonagriculture.

It is appropriate to estimate the model in first differences since one cannot reject the hypothesis that the logarithm of the empirical relative value added-, price-, and output ratios have unit roots. For robustness we also estimate the models in levels, allowing country-specific fixed effects.

We start by estimating Equation (20). Table 5 documents the results. Specification (1) is based on calculating average ratios over non-overlapping 10-year periods and taking the first difference across these observations (the results are robust to instead taking simple 10-year differences of annual data). Specification (2), which we consider our benchmark specification, adds time fixed effects to the regression. The estimated coefficient of interest is significantly positive (0.838 and 0.746). According to Equation (20), a coefficient of 0.746 implies a high elasticity; $\varepsilon \approx 4$. This finding is robust to different data and specifications. Specification (3) adds log of aggregate real consumption as a control. Specification (4) estimates the model in levels, allowing country and time fixed effects. Motivated by our focus on China, specification (5) estimates the model on Asian countries. Specification (6) estimates the model using a higher frequency (annual). Finally, specification (7) considers ratios of agriculture to nonagriculture (as opposed to manufacturing).³² The coefficient of interest is significantly positive in all specifications and imply an elasticity ε between 2 and 6. These results are robust to using UN AMA data.

We now estimate equation (21). Table 6 documents the results. The specifications (1) and (2) yield positive coefficients which are all significant at a 10% level. According to Equation (21), the benchmark estimate in column (2) implies a low elasticity; $\varepsilon \approx 0.6$. This finding is robust to adding aggregate consumption as a control (3), to estimating the model in levels (4), at a higher frequency (6), and using ratios of agriculture to nonagriculture (7). These regressions suggest that ε is between 0.2 and 0.6. However, when considering only Asian countries, the estimated coefficient implies an elasticity above unity, $\varepsilon \approx 1.4$.

In conclusion, the simple regressions based on expenditure, prices, and quantities, summarized in Tables 5 and 6, give mixed evidence on ε . These inconclusive results could be due to the fact that both sides of Equations (20)-(21) are endogenous. In Appendix C.3 we present a structural model with two goods where relative prices, quantities, and expenditures are endogenous and where the economy is subject to both supply shocks and demand shocks, captured by TFP shocks and preference shocks. We show that demand shocks and supply shocks have opposite implications for the sign of the correlations between prices and quantities.

³²The coefficient in column (7) of Table 5 is somewhat lower than the other estimates. However, when estimating this specification with UN AMA data, the coefficient is higher, 0.862.

EXPENDITURE SHARE OF AGRICULTURE, regressed on QUANTITY RATIO							
Data	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	GGDC	GGDC	GGDC	GGDC	GGDC Asia	GGDC	GGDC
	Agriculture/Manufacturing				Agri./Nonagr.		
	10yr FD	10yr FD	10yr FD	10yr level	10yr FD	1yr FD	10yr FD
$\Delta\logrel_quantity$	0.838*** (0.0893)	0.746*** (0.100)	0.664*** (0.141)		0.685*** (0.0625)	0.681*** (0.0591)	0.459*** (0.119)
$\Delta\logconsumption$			-0.204 (0.142)				
$\logrel_quantity$				0.787*** (0.0928)			
Year Fixed Effect	N	Y	Y	Y	Y	Y	Y
Observations	139	139	139	179	33	1,923	139
R-squared	0.684	0.756	0.763	0.830	0.961	0.361	0.541

Standard errors (clustered at country level) in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 5: The table shows regressions based on equation 20. Specification (1) is based on differences over averages for 10-year periods for all countries in the GGDC data. Specification (2) adds time fixed effects as additional control. Specification (3) adds log real consumption from PWT as additional control. Specification (4) is based on 10-year average levels. Specification (5) is based on GGDC Asia sample, excluding HKG and SGP. (6) uses 1-year difference instead, and specification (7) estimates the model based on ratios of agriculture to nonagriculture.

EXPENDITURE SHARE OF AGRICULTURE, regressed on PRICE RATIO							
Data	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	GGDC	GGDC	GGDC	GGDC	GGDC Asia	GGDC	GGDC
	Agriculture/Manufacturing				Agri./Nonagr.		
	10yr FD	10yr FD	10yr FD	10yr level	10yr FD	1yr FD	10yr FD
$\Delta\logrel_price$	0.539** (0.233)	0.356* (0.203)	0.501** (0.200)		-0.445 (0.285)	0.775*** (0.0497)	0.518*** (0.103)
$\Delta\logconsumption$			-0.983*** (0.125)				
\logrel_price				0.451*** (0.112)			
Year Fixed Effect	N	Y	Y	Y	Y	Y	Y
Observations	139	139	139	179	33	1,923	139
R-squared	0.099	0.381	0.648	0.671	0.820	0.548	0.591

Standard errors (clustered at country level) in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 6: The table shows regressions based on equation 21. Specification (1) is based on differences over averages for 10-year periods for all countries in the GGDC data. Specification (2) adds time fixed effects as additional control. Specification (3) adds log real consumption from PWT as additional control. Specification (4) is based on 10-year average levels. Specification (5) is based on GGDC Asia sample, excluding HKG and SGP. (6) uses 1-year difference instead, and specification (7) estimates the model based on ratios of agriculture to nonagriculture.

This problem applies to the estimation of both Equation (20) and (21).

Finally, note that (classical) measurement error is not a likely cause of the differences across specifications. To see this, note that in the presence of classical measurement error, potential biases should be smaller at lower frequencies, as the signal to noise ratio is larger. Consider first Table 5. The estimated coefficients are virtually unchanged when going from differences over 10-year periods to annual differences (specification (6) versus (2)) in Table 5). This suggests that potential biases due to measurement error must be negligible. Consider, next, Table 6. The coefficient of interest in Table 6 is higher at an annual frequencies (see specification (6) relative to (2)). We show below that measurement error in this regression gives rise to an attenuation bias, which should bias the estimated coefficient toward zero. The fact that this coefficient is larger at higher frequencies suggests that classical measurement error is not plausible explanation for the differences in results across specifications.

C.2 Classical measurement error in the price-quantity regression

This section examines how the estimates of ε from regressions in (20)-(21) would be influenced by classical measurement error.

Assume that there is classical measurement error in both log expenditure and prices, and that quantities are defined as expenditures over prices, i.e.,

$$\begin{aligned}\ln \frac{\widehat{P_t^G Y_t^G}}{\widehat{P_t^M Y_t^M}} &= \ln \frac{P_t^G Y_t^G}{P_t^M Y_t^M} + \eta_e \\ \ln \frac{\widehat{P_t^G}}{\widehat{P_t^M}} &= \ln \frac{P_t^G}{P_t^M} + \eta_p \\ \ln \frac{\widehat{Y_t^G}}{\widehat{Y_t^M}} &\equiv \ln \frac{\widehat{P_t^G Y_t^G}}{\widehat{P_t^M Y_t^M}} - \ln \frac{\widehat{P_t^G}}{\widehat{P_t^M}} = \ln \frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p\end{aligned}$$

Note first that $\hat{\beta}_p$ in the price regression is biased toward zero (relative to the true coefficient) through an attenuation bias;

$$\begin{aligned}\ln \frac{\widehat{P_t^G Y_t^G}}{\widehat{P_t^M Y_t^M}} &= \beta_{0p} + \hat{\beta}_p \ln \frac{\widehat{P_t^G}}{\widehat{P_t^M}} + \delta_t \\ \Rightarrow \hat{\beta}_p &= \beta_p \cdot \frac{\text{var} \left(\frac{P_t^G}{P_t^M} \right)}{\text{var} \left(\frac{P_t^G}{P_t^M} \right) + \text{var} (\eta_p)} = \begin{cases} < \beta_p & \text{if } \beta_p > 0 \\ \geq \beta_p & \text{if } \beta_p \leq 0 \end{cases},\end{aligned}$$

where β_p is the true coefficient. It follows that since $\beta_p = 1 - \varepsilon$ and $\hat{\beta}_p > 0$, the estimated $\hat{\varepsilon}$ in the price regression is biased upward.

Second, note that $\hat{\beta}_y$ in the quantity regression is also biased toward zero (relative to the true coefficient) provided that measurement error in expenditures is small relative to measurement error in prices. This is

again driven by through an attenuation bias;

$$\begin{aligned}
\ln \frac{\widehat{P_t^G Y_t^G}}{\widehat{P_t^M Y_t^M}} &= \beta_0 + \hat{\beta}_y \ln \frac{\widehat{Y_t^G}}{\widehat{Y_t^M}} + \delta_t \\
\Rightarrow \hat{\beta}_y &= \frac{\text{cov} \left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M} + \eta_e, \frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p \right)}{\text{var} \left(\frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p \right)} \\
&= \beta_y \frac{\text{var} \left(\frac{Y_t^G}{Y_t^M} \right)}{\text{var} \left(\frac{Y_t^G}{Y_t^M} \right) + \text{var} (\eta_e) + \text{var} (\eta_p)} + \frac{\text{var} (\eta_e)}{\text{var} \left(\frac{Y_t^G}{Y_t^M} \right) + \text{var} (\eta_e) + \text{var} (\eta_p)}.
\end{aligned}$$

Assume that $\text{var} (\eta_e)$ is small. This seems reasonable given that expenditure ratios are calculated from national accounts. It follows that if $\beta_p > 0$, then the attenuation bias will dominate and $\hat{\beta}_y < \beta_y$. Since $\beta_y = (\varepsilon - 1)/\varepsilon$ and the estimated coefficient $\hat{\beta}_y \gg 0$, the estimated $\hat{\varepsilon}$ in the quantity regression must be biased downward.

C.3 Estimation bias in the presence of demand shocks

This appendix argues that in the presence of demand shocks affecting the relative demand for agricultural goods, the estimate of ε from Equation (21) is biased toward zero while that from Equation (20) is biased away from zero.

Consider a dynamic production economy where a representative household has preferences over the goods Y_g and Y_m given by Equation (1). There is no capital and no storage, so equilibrium consumption equals production of each good.

The goods are traded at prices P_g and P_m . Utility maximization implies the demand equation

$$\frac{Y_g}{Y_m} = \frac{\gamma}{1 - \gamma} \left(\frac{P_g}{P_m} \right)^{-\varepsilon}. \quad (44)$$

A continuum of firms, owned by the household, produces Y_g and Y_m via the function

$$Y_i = A_i L_i^\varphi, \quad (45)$$

where $\varphi \in [0, 1)$ and $i \in \{g, m\}$. The household has one unit of labor that is split between sectors. Firm optimization implies that the marginal product of labor is equalized across sectors, i.e., $P_g \varphi A_g (L_g)^{\varphi-1} = P_m \varphi A_m (L_m)^{\varphi-1}$.

Combine this equation with Equations (44)-(45) to substitute out the labor ratio L_g/L_m and the price ratio P_g/P_m . Rearranging terms yields an expression for Y_g/Y_m in terms of $\gamma/(1 - \gamma)$ and A_g/A_m ,

$$\ln \left(\frac{Y_g}{Y_m} \right) = \frac{\varphi}{\varphi + \varepsilon(1 - \varphi)} \ln \left(\frac{\gamma}{1 - \gamma} \right) + \frac{\varepsilon}{\varphi + \varepsilon(1 - \varphi)} \ln \left(\frac{A_g}{A_m} \right). \quad (46)$$

Combining Equations (44) and (46) yields an expression for P_g/P_m in terms of $\gamma/(1 - \gamma)$ and A_g/A_m ,

$$\ln \left(\frac{P_g}{P_m} \right) = \frac{1 - \varphi}{\varphi + \varepsilon(1 - \varphi)} \ln \left(\frac{\gamma}{1 - \gamma} \right) - \frac{1}{\varphi + \varepsilon(1 - \varphi)} \ln \left(\frac{A_g}{A_m} \right). \quad (47)$$

The economy is subject to supply and demand shocks, interpreted as shocks to growth in relative TFP and relative preference weights. For notational convenience, define the variables $p_t = \ln(P_{g,t}/P_{m,t}) - \ln(P_{g,t-1}/P_{m,t-1})$, $y_t = \ln(Y_{g,t}/Y_{m,t}) - \ln(Y_{g,t-1}/Y_{m,t-1})$, $\xi_t = \ln[\gamma_t/(1 - \gamma_t)] - \ln[\gamma_{t-1}/(1 - \gamma_{t-1})]$, and $a_t = \ln(A_{g,t}/A_{m,t}) - \ln(A_{g,t-1}/A_{m,t-1})$. The growth in relative expenditure, $x_t \equiv \ln[P_{gt}Y_{gt}/(P_{mt}Y_{mt})] -$

$\ln [P_{g,t-1}Y_{g,t-1}/(P_{m,t-1}Y_{m,t-1})]$, is then $x_t = p_t + y_t$. For simplicity, assume that $\text{var}(\xi_t)$ and $\text{var}(a_t)$ are constant and that $\text{corr}(\xi, a) = 0$.

Consider now simulating time-series data from this model and running OLS regressions on Equations (20)-(21). The following proposition derives the implied coefficients from these regressions.

Proposition 5 *The regression coefficient on the relative price growth p_t in the expenditure-price regression $x_t = \beta_0 + \beta_p p_t + \eta_t$ is given by*

$$\beta_p = 1 - \varepsilon + \left(\varepsilon + \frac{\varphi}{1 - \varphi} \right) \frac{(1 - \varphi)^2 \text{var}(\xi)}{(1 - \varphi)^2 \text{var}(\xi) + \text{var}(a)} \geq 1 - \varepsilon \quad (48)$$

The regression coefficient on the relative output growth y_t in the expenditure-output regression $x_t = \beta_0 + \beta_y y_t + \delta_t$ is given by

$$\beta_y = \frac{\varepsilon - 1}{\varepsilon} + \frac{\varphi + \varepsilon(1 - \varphi)}{\varphi\varepsilon} \frac{\varphi^2 \cdot \text{var}(\xi)}{\varphi^2 \cdot \text{var}(\xi) + \varepsilon^2 \cdot \text{var}(a)} \geq \frac{\varepsilon - 1}{\varepsilon} \quad (49)$$

Proof. The regression coefficients are defined as $\beta_p = \text{cov}(p + y, p) / \text{var}(p)$ and $\beta_y = \text{cov}(p + y, y) / \text{var}(y)$. The expressions in (48)-(49) follow from the equilibrium relationships (46)-(47) and the assumption $\text{corr}(\xi, a) = 0$. The inequalities in Equations (48)-(49) follow from $\varphi \in [0, 1)$ so the last term in each equation is non-negative. ■

Proposition 5 shows that when there are no demand shocks ($\text{var}(\xi) = 0$) then β_p and β_y are unbiased estimates of $1 - \varepsilon$ and $(\varepsilon - 1) / \varepsilon$, respectively. However, when $\text{var}(\xi) > 0$, then $\hat{\varepsilon} \equiv 1 - \beta_p < \varepsilon$ and $\tilde{\varepsilon} \equiv 1 / (1 - \beta_y) > \varepsilon$. It follows that the estimate from Equation (21) yields a downward bias in ε and that from Equation (20) yields an upward bias in ε .

C.4 Estimated equations in Section 6.1

The production function (19) in Section 6.1 gives rise to the following equilibrium conditions, based on first-order conditions,

$$\begin{aligned} \frac{p^{Serv} Y^{Serv}}{p^{Serv} Y^{Serv} + p^{Manuf} Y^{Manuf}} &= \frac{(1 - \hat{\gamma}) (p^{Serv})^{1 - \varepsilon_{ms}}}{\hat{\gamma} (p^M)^{1 - \varepsilon_{ms}} + (1 - \hat{\gamma}) (p^S)^{1 - \varepsilon_{ms}}} \\ &\times \frac{p^{Serv} Y^{Serv} + p^{Manuf} Y^{Manuf} + p^{Serv} \bar{s}}{p^{Serv} Y^{Serv} + p^{Manuf} Y^{Manuf}} - \frac{p^{Serv} \bar{s}}{p^{Serv} Y^{Serv} + p^{Manuf} Y^{Manuf}} \\ \frac{p^G Y^G}{p^G Y^G + p^M Y^M} &= \frac{\gamma (p^G)^{1 - \varepsilon}}{\gamma (p^G)^{1 - \varepsilon} + (1 - \gamma) (p^M)^{1 - \varepsilon}} \\ &\times \frac{p^G Y^G + p^M Y^M + p^G \bar{c} + p^S \bar{s}}{p^G Y^G + p^M Y^M} - \frac{p^G \bar{c}}{p^G Y^G + p^M Y^M} \end{aligned}$$

The nonlinear least square regressions in Section 6.1 are based on these conditions.

C.5 Calibrating ε using time-series data for the U.S.

Combining equations (22) and (25) with the demand relationship $\Delta \ln(Y_t^G / Y_t^M) = -\varepsilon \cdot \Delta \ln(P_t^G / P_t^M)$ yields

$$\varepsilon = 1 - \left[\Delta \ln \left(\frac{1 - \nu^M}{\nu^M} \right) - \Delta \ln \left(v + \frac{1}{\beta} (1 - v) \right) \right] / \Delta \ln \left(\frac{P_t^G}{P_t^M} \right).$$

Alvarez-Cuadrado and Poschke (2010) report that the employment share of U.S. agriculture – $1 - \nu^M$ in our model – went from 73% in 1800 to 16% in 1945. Over the same time period the price ratio of agricultural goods to nonagricultural goods increased by 50%. Our model is consistent with these changes for $\varepsilon \approx 7.5$. This calculation assumes the factor shares estimated for China, and any $\nu \geq 0$. A shorter time span, for example 1800-1920, yields an even larger estimate for ε .

D Appendix: Tables and Figures

(a) Relative K-Y ratio in agriculture						
	(1)	(2)	(3)	(4)	(5)	(6)
log(agr. empl. share)	-0.296*** (0.0186)	-0.297*** (0.0241)	-0.262*** (0.0964)	-0.249** (0.105)	-0.281* (0.150)	-0.219 (0.223)
Sample	FULL	NO AFRICA	FULL	NO AFRICA	FULL	NO AFRICA
Time dummy	YES	YES	NO	NO	YES	YES
Country FE	NO	NO	YES	YES	YES	YES
Observations	292	206	292	206	292	206
R-squared	0.638	0.586	0.047	0.052	0.047	0.052

(b) Productivity Gap						
	(1)	(2)	(3)	(4)	(5)	(6)
log(agr. empl. share)	0.370*** (0.0165)	0.348*** (0.0176)	0.300*** (0.0979)	0.191** (0.0863)	0.901*** (0.139)	0.965*** (0.163)
Sample	FULL	NO AFRICA	FULL	NO AFRICA	FULL	NO AFRICA
Time dummy	YES	YES	NO	NO	YES	YES
Country FE	NO	NO	YES	YES	YES	YES
Observations	290	208	290	208	290	208
R-squared	0.797	0.771	0.066	0.036	0.261	0.281

Table 7: Panel (a) reports the results of a set of regressions whose dependent variable is the logarithm of the K-Y ratio. Panel (b) reports the results of a set of regressions whose dependent variable is the logarithm of the productivity gap. Columns (1) and (2) are pooled regressions with time effects. Columns (3) and (4) are within regressions with country fixed effects. Robust standard errors clustered at country level are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
A. FD- Filtered China Data, 1985-2012: $std(y) = 2.4\%$								
$\frac{std(x)}{std(y)}$	1.27	3.34	1.82	1.31	2.39	1.00	0.76	0.33
$corr(x, y)$	0.57	0.63	0.12	0.93	-0.09	-0.57	0.66	-0.25
$corr(x, n^G)$	-0.74	-0.34	-0.38	-0.38	0.35	1.00	-0.50	0.71
$corr(x, n^M)$	0.32	0.37	0.40	0.53	-0.52	-0.50	1	0.19
B. FD- Filtered Model, Homothetic model $std(y) = 2.7\%$								
$\frac{std(x)}{std(y)}$	0.30	2.36	1.11	1.25	0.72	1.10	1.27	0.49
$corr(x, y)$	0.80	0.99	0.24	0.95	-0.42	-0.30	0.69	0.18
$corr(x, n^G)$	-0.22	-0.27	0.80	-0.51	0.79	1	-0.78	0.75
$corr(x, n^M)$	0.55	0.66	-0.40	0.88	-0.81	-0.78	1	-0.52

Table 8: Summary Statistics for China data and Model: First-differenced

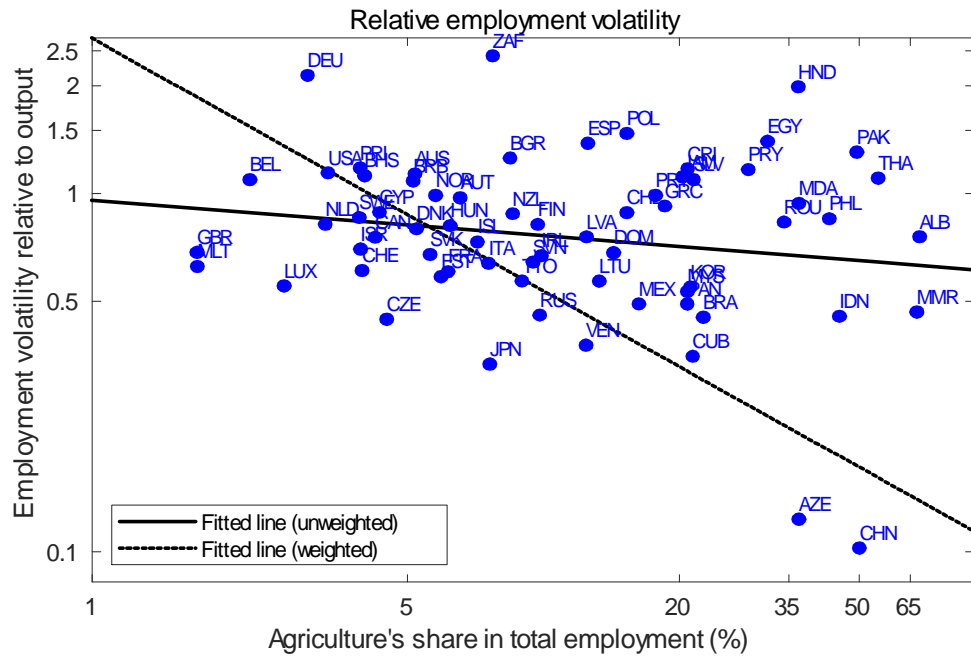


Figure 13: The cross country panel on the relative employment volatility. The figure has log-log scale. The two regression lines stand for unweighted OLS regression (solid line) and the regression weighted by the size of total employment (dotted line).



Figure 14: The figure shows the time evolution of the volatility of total employment in private sector (excluding government) relative to the GDP in the US from 1955 to 2015. The relative volatility is measured by the standard deviation of the HP-filtered log total employment divided by the HP-filtered log real output, both of which are computed on a 28-year rolling window. The x-axis denotes the end year of the sample window. The HP-filter use the smoothing parameter 6.25 (Ravn and Uhlig 2002). Source: Employment in private sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The GDP in current price is deflated by the implicit price deflators from NIPA Table 1.1.9.

Param.	USA						Japan						China					
	Cons. value added		GGDC		Cons. value added		GGDC		Cons. value added		GGDC		Cons. value added		GGDC			
	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector		
ε	2.49*** (0.28)	2.32*** (0.32)	1.36*** (0.13)	1.53*** (0.48)	1.58*** (0.05)	2.24*** (0.36)	5.46*** (0.80)	6.06*** (0.91)	1.70*** (0.22)	1.34*** (0.25)	1.30*** (0.13)	1.75*** (0.08)						
ϵ_{ms}	0	-	0.48*** (0.005)	-	0.79*** (0.003)	-	1.66*** (0.13)	-	0.007 (0.05)	-	0.65*** (0.04)	-						
\bar{c}	-160.6*** (3.8)	-160.6*** (3.8)	-215.0*** (6.0)	-224.0*** (5.7)	-669.6*** (43.1)	-828.2*** (46.1)	-58334*** (5417)	-59631*** (5417)	-9.0*** (0.4)	-9.2*** (0.4)	-654.2*** (41.6)	-614.9*** (41.4)						
\bar{s}	4299.4*** (444.0)	-	12897*** (431.8)	-	3234.0*** (92.9)	-	36056*** (6269)	-	0.01 (0.008)	-	139.8*** (31.1)	-						
γ	0.99*** (0.0005)	0.99*** (0.005)	0.997*** (0.0004)	0.994*** (0.0005)	0.97*** (0.03)	0.97*** (0.005)	0.994*** (0.003)	0.995*** (0.003)	0.89*** (0.02)	0.92*** (0.02)	0.90*** (0.005)	0.90*** (0.006)						
$\hat{\gamma}$	0.15*** (0.004)	-	0.20*** (0.003)	-	0.21*** (0.002)	-	0.44*** (0.004)	-	0.63*** (0.008)	-	0.61*** (0.004)	-						
Sample	1947-2010	1947-2010	1947-2010	1947-2010	1955-2017	1955-2017	1953-2010	1953-2010	1985-2012	1985-2012	1980-2010	1980-2010						
AIC	-909.3	-525.0	-840.0	-506.0	-596.6	-259.6	-663.8	-392.2	-270.2	-9.2	-302.1	-158.8						
RMSE _G	0.004	0.004	0.003	0.003	0.022	0.022	0.008	0.008	0.015	0.015	0.018	0.017						
RMSE _M	0.011	-	0.020	-	0.016	-	0.021	-	0.016	-	0.025	-						
RMSE _S	0.012	-	0.018	-	0.037	-	0.025	-	0.020	-	0.016	-						

Table 9: Estimation results for the models in Section 6.1. “3-sector” refers to agriculture, manufacturing, and services. “2-sector” refers to agriculture and nonagriculture. Consumption value added data for USA are from Herrendorf et al. (2013). Consumption value added data for China and Japan are constructed following the methodology in Herrendorf et al. (2013) using official input-output tables from NBS and Japan’s Ministry of Internal Affairs and Communications, respectively. Production value added data are from GGDC.

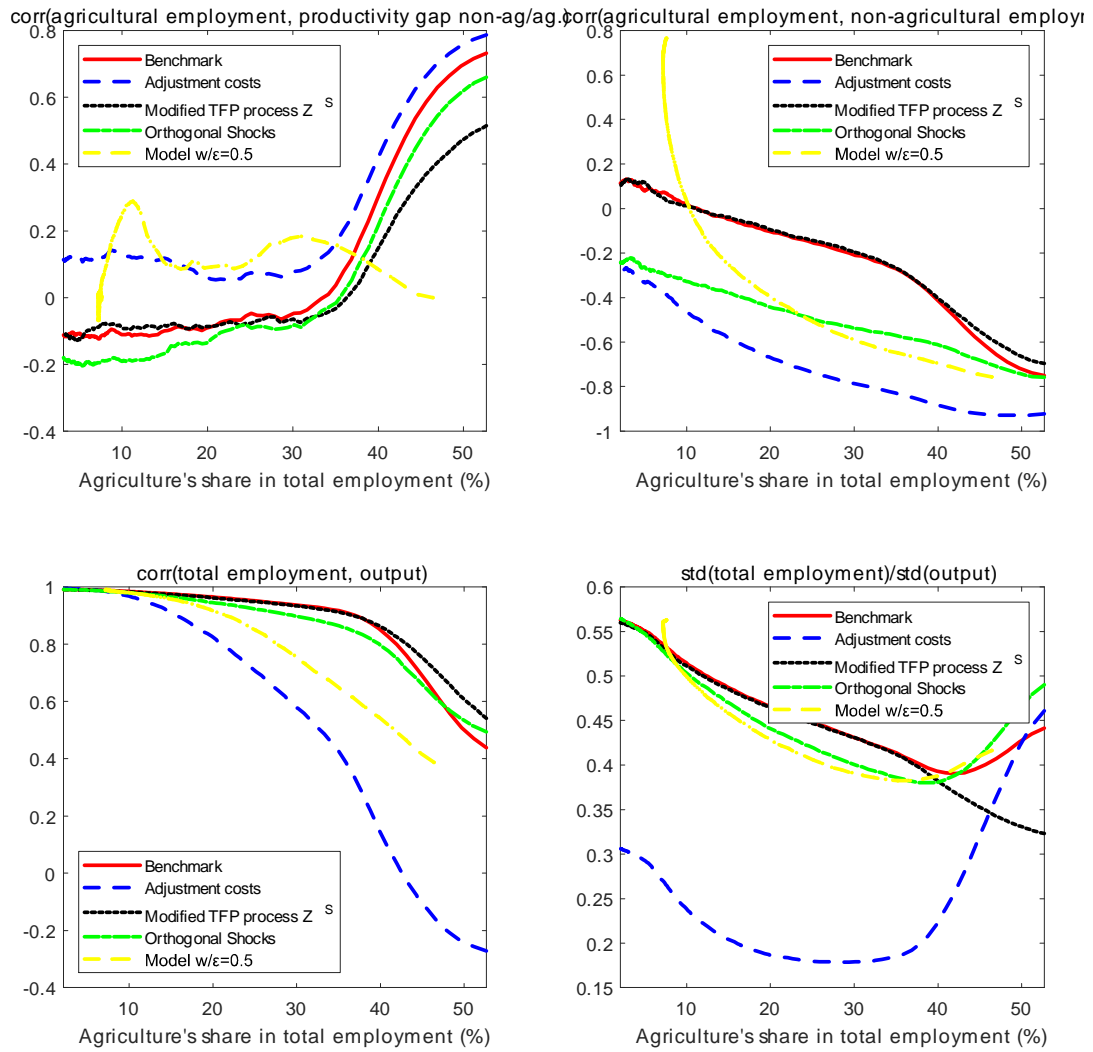


Figure 15: The figure reports the result of the robustness analysis discussed in Section 5.4.

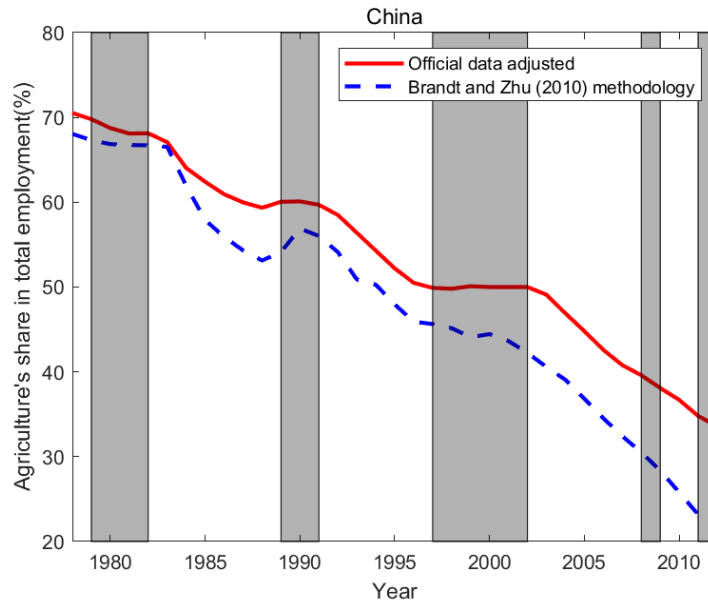


Figure 16: Share of agricultural employment in China. Official data versus data series using methodology from Brandt and Zhu (2010). Shaded areas are times with GDP growth rate below 9.7%.

E Additional Material

E.1 Details on employment data for China

We rely on official Chinese data on aggregate and sectoral employment. The official series has a structural break in 1990. After 1990, the data are based on the labour force survey and the population census. Before 1990 the official data comes from the annual administrative data on registered workers from the Administration for Industry and Commerce. To make the series consistent, we apply the correction proposed by Holz (2006), which revises the official employment data before 1990 using the 1982 and 1990 population census. See appendix 13 in Holz (2006) for details. The resulting revised data series has the same agricultural employment share as in Young (2003) between 1978-1990. See column "labor force of society" in table 8 in Young (2003).

The survey data are preferable to the administrative data because the survey captures also informal employment outside cities and towns, where the registered employees reside. We acknowledge that the survey data also have potential drawbacks. For example, the question about working status is based on the reference period, which is usually one week before the survey starts. Therefore, migrants might be counted as agricultural workers or nonagricultural workers depending on which week they are surveyed (cf. (Brandt and Zhu 2010)). Brandt and Zhu (2010) provides an alternative measure of agricultural employment based on rural employment statistics. Their methodology is to subtract the rural employment in the township and village enterprises and in rural private firms from the rural employment statistics. We are particularly interested in the cyclical fluctuations in labor supply, including agricultural labor. Figure 16 plots the series of Brandt and Zhu (2010) against the official data we use. The figure shows that the data of Brandt and Zhu (2010) has a steeper downward trend in agricultural employment. However, the cyclical fluctuations in agricultural employment share are similar in the two data series.

E.2 Details of Samples used in Figure 3

For panels a in Figure 3 (aggregate employment and GDP), the sample comprises 66 countries.

Countrycode	Start	End	Countrycode	Start	End
ALB	1996	2014	ITA	1977	2015
AUS	1970	2015	JPN	1970	2015
AUT	1983	2015	KAZ	2001	2015
AZE	1992	2012	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	TTO	1977	2010
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1970	2015	ZAF	2000	2015

For panel b in Figure 3 (agricultural versus nonagricultural employment), the sample comprises 66 countries. The sample time periods for each country are the following,

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1983	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1989	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1993	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	TTO	1977	2010
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1970	2015	ZAF	2000	2015

For panel c in Figure 3 (productivity gap versus nonagricultural employment), the sample comprises 63 countries. The sample time periods for each country are the following

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1990	2015
AUS	1990	2015	JAM	1993	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1995	2015	LTU	1998	2015
BGR	2000	2015	LUX	1995	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1990	2014	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2014
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1991	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1971	2011
EGY	1989	2015	PRT	1995	2015
ESP	1995	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1975	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1990	2015	SVK	1995	2015
GRC	1995	2015	SVN	1995	2015
HND	1990	2015	SWE	1980	2015
HUN	1995	2015	THA	1971	2015
IDN	1987	2015	TTO	1984	2010
IRL	1995	2015	USA	1970	2015
ISL	1997	2015	VEN	1975	2013
			ZAF	2000	2015

For panel d in Figure 3 (relative consumption to output volatility versus nonagricultural employment), the sample comprises 64 countries. The sample time periods for each country are the following,

Countrycode	Start	End	Countrycode	Start	End
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
CAN	1970	2012	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1990	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1986	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1972	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	UKR	2001	2015
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1995	2015	ZAF	2000	2015

E.3 Discrete Time Model

In this section we provide a complete description of the discrete time model with endogenous labor supply estimated in Section 4. Our baseline discrete time model adds the following model features to the continuous-time model: (1) endogenous labor supply (2) land as a factor of production in modern-agriculture sector (3) TFP shocks (4) capital stocks in each sector are predetermined.

Time t is discrete, indexed by $0, 1, 2, \dots$. Given the initial capital stock in each sector, i.e., $\bar{K}_0 \kappa_0$ and $\bar{K}_0 (1 - \kappa_0)$, and initial TFP levels, $Z_0^i, i = AM, M, S$, the representative household maximizes expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t (\theta \log c_t + (1 - \theta) \log (1 - h_t))$$

subject to the budget constraint

$$N_t c_t + K_{t+1} = W_t N_t + (R_t - \delta) K_t + Tr_t$$

where $K_t = K_t^M + K_t^{AM}$, $N_t = N_t^M + N_t^{AM} + N_t^S$. W_t denotes the after-tax equilibrium wage. $Tr_t = \tau W_t^M h_t N_t^M$ denotes the lump-sum transfer from the government to the representative household. Note that μ denotes the discount factor.

The production side is identical to the model in the text, except that the production of modern agriculture has been modified to include land

$$Y_t^{AM} = (K_t^{AM})^{1-\beta-\beta_T} (Z_t^{AM} H_t^{AM})^\beta$$

where the land income share is denoted by $\beta_T \geq 0$. We assume $\beta + \beta_T < 1$.

As explained in Section 3.2, we can exploit the equivalence between the competitive equilibrium and the distorted social planner problem, and write the Lagrangian as:

$$L = E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \begin{array}{l} \theta \log c_t + (1 - \theta) \log (1 - h_t) + \\ \xi_t \left[\begin{array}{l} Y_t + (1 - \delta) K_t - c_t N_t - K_{t+1} \\ -\tau W_t^M H_t^M + Tr_t \end{array} \right] \end{array} \right\}$$

where we use the notation $\chi_t, \kappa_t, \nu_t^M, \nu_t^{AM}, \nu_t^S, v_t$ introduced in the main text, but we modify the notations of η_t and introduce $\tilde{\eta}_t$

$$\begin{aligned} \eta_t &\equiv \left[\gamma \left(\frac{Y_t^G}{Y_t^M} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \gamma) \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \tilde{\eta}_t &\equiv \left[\gamma + (1 - \gamma) \left(\frac{Y_t^M}{Y_t^G} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Therefore, by definition $Y_t = Y_t^M \eta_t$ and $Y_t = Y_t^G \tilde{\eta}_t$. Recall that $H_t^i \equiv h_t N_t^i$.

The FOC with respect to ν_t^{AM} and ν_t^S are as in the continuous time problem.

$$\begin{aligned} \gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} v_t \beta \frac{1}{\nu_t^A} &= (1 - \tau) (1 - \gamma) (Y_t^M)^{1-\frac{1}{\varepsilon}} \alpha \frac{1}{\nu_t^M} \\ \gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} (1 - v_t) \frac{1}{\nu_t^S} &= (1 - \tau) (1 - \gamma) (Y_t^M)^{1-\frac{1}{\varepsilon}} \alpha \frac{1}{\nu_t^M} \end{aligned}$$

Therefore, we have

$$\begin{aligned}\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} v_t \beta}{(1-\tau)(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}} \alpha} \\ \frac{\nu_t^S}{\nu_t^M} &= \frac{\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} (1-v_t)}{(1-\tau)(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}} \alpha}\end{aligned}$$

sum up together to have the expenditure ratio agr./non-agr. as

$$\frac{\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}}} = \frac{1-\nu_t^M}{\nu_t^M} \frac{(1-\tau)\alpha}{(1-v_t)+v_t\beta}$$

and express ν_t^M as

$$\nu_t^M = \left(1 + \frac{\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} (1-v_t) + v_t\beta}{(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}} (1-\tau)\alpha} \right)^{-1}$$

Those with respect to c_t and h_t yield, respectively:

$$\theta \frac{1}{c_t} = \xi_t N_t, \quad (50)$$

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}} \left[\begin{aligned} &\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} \left(v_t \frac{\beta}{h_t} + (1-v_t) \frac{1}{h_t} \right) \\ &+ (1-\tau)(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}} \frac{\alpha}{h_t} \end{aligned} \right]. \quad (51)$$

Substituting the FOCs with respect to ν_t^{AM} and ν_t^S into (51) yields

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}} (1-\tau)\alpha(1-\gamma)(Y_t^M)^{1-\frac{1}{\varepsilon}} \frac{1}{h_t \nu_t^M}. \quad (52)$$

Combining (52) with (50) yields

$$\frac{1-\theta}{\theta} \frac{c_t}{1-h_t} = (1-\tau) Y_t^{\frac{1}{\varepsilon}} \alpha (1-\gamma) (Y_t^M)^{1-\frac{1}{\varepsilon}} \frac{1}{\nu_t^M h_t N_t}.$$

The FOC w.r.t. κ_{t+1} and K_{t+1} yield, respectively (after combining the two equations and rearranging terms):

$$\xi_t = E_t \left[\mu \xi_{t+1} \left(Y_{t+1}^{\frac{1}{\varepsilon}} \gamma (Y_{t+1}^G)^{-\frac{1}{\varepsilon}} \varsigma \left(\frac{v_{t+1}}{\varsigma} \right)^{-\frac{1}{\omega-1}} (1-\beta-\beta_T) \frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} + 1 - \delta \right) \right] \quad (53)$$

$$\xi_t = \mu E_t \left[\xi_{t+1} \left(Y_{t+1}^{\frac{1}{\varepsilon}} \frac{1}{K_{t+1}} \frac{1}{\kappa_{t+1}} (1-\gamma) (Y_{t+1}^M)^{1-\frac{1}{\varepsilon}} (1-\alpha) + 1 - \delta \right) \right]. \quad (54)$$

Substitute $Y_t = \eta_t Y_t^M$ and $Y_t = Y_t^G \tilde{\eta}_t$ Substituting in this into (53) and (54) respectively to eliminate Y yields

$$\xi_t = \mu E_t \left[\xi_{t+1} \left(\gamma \tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}} \varsigma \left(\frac{v_{t+1}}{\varsigma} \right)^{-\frac{1}{\omega-1}} (1-\beta-\beta_T) \frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} + 1 - \delta \right) \right] \quad (55)$$

$$\xi_t = \mu E_t \left[\xi_{t+1} \left(\eta_{t+1}^{\frac{1}{\varepsilon}} \frac{Y_{t+1}^M}{K_{t+1}^M} (1-\gamma) (1-\alpha) + 1 - \delta \right) \right]. \quad (56)$$

Equation (50), 55), and (56) yields the standard Euler equations for consumption:

$$\begin{aligned} 1 &= \frac{\mu}{1+n} E_t \left\{ \frac{c_t}{c_{t+1}} \left[\gamma \tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}} \varsigma \left(\frac{v_{t+1}}{\varsigma} \right)^{-\frac{1}{\omega-1}} (1-\beta-\beta_T) \left((1-\kappa_{t+1}) \chi_{t+1} \right)^{-\beta-\beta_T} \right. \right. \\ &\quad \left. \left. \times \left(Z_{t+1}^{AM} h_{t+1} \nu_{t+1}^{AM} \right)^\beta N_{t+1}^{-\beta_T} + 1 - \delta \right] \right\} \\ 1 &= \frac{\mu}{1+n} E_t \left\{ \frac{c_t}{c_{t+1}} \left[1 + \eta_{t+1}^{\frac{1}{\varepsilon}} (1-\gamma) (1-\alpha) \left(\frac{Z_{t+1}^M h_{t+1} \nu_{t+1}^M}{\kappa_{t+1} \chi_{t+1}} \right)^\alpha - \delta \right] \right\} \end{aligned}$$

We can simplify the intertemporal condition:

$$\frac{1-\theta}{\theta} \frac{c_t}{1-h_t} = (1-\tau) \eta_t^{\frac{1}{\varepsilon}} \alpha (1-\gamma) \left(\frac{Z_t^M h_t \nu_t^M}{\kappa_t \chi_t} \right)^{\alpha-1}$$

The resource constraint becomes

$$\chi_{t+1} (1+n) = \eta_t (\kappa_t)^{1-\alpha} (Z_t^M \nu_t^M h_t)^\alpha \chi_t^{1-\alpha} + (1-\delta) \chi_t - c_t$$

E.4 Algorithm to Solve the Rational Expectation Equilibrium

We can rewrite the model in discrete time recursively. Denote the state space as $\Theta_t \equiv (\hat{\chi}_t, \kappa_t, z_t^M, z_t^{AM}, z_t^S, t)$. The Bellman Equation (during the structural change transition) is given by

$$V(\Theta_t) = \max_{\hat{c}_t, h_t, \kappa_{t+1}, \hat{\chi}_{t+1}, v_t, \nu_t^M} \{ u(\hat{c}_t, h_t) + \mu E_t V'(\Theta_{t+1}) \}$$

where use the following notations to detrend the variables

$$\hat{\chi}_t = \frac{\chi_t}{\Lambda_t^M}, \hat{c}_t = \frac{c_t}{\Lambda_t^M}, z_t^i = \frac{Z_t^i}{\Lambda_t^i}, i = AM, M, S$$

and

$$\Lambda_t^i = Z_0^i \prod_{k=1}^t (1+g_k^i), i = AM, M, S$$

We solve the Bellman Equation using its detrended first order conditions and budget constraint.

In each period, we have 6 unknown policy function to solve

$$\hat{c}_t(\Theta_t), h_t(\Theta_t), \kappa_{t+1}(\Theta_t), \hat{\chi}_{t+1}(\Theta_t), v_t(\Theta_t), \nu_t^M(\Theta_t).$$

We can reduce the number of unknown policy function further by expressing \hat{c}_t and $\hat{\chi}_{t+1}$ in terms of other state variables and decision variables.

$$\begin{aligned} \hat{c}_t &= (1-\tau) \eta_t^{\frac{1}{\varepsilon}} (1-\gamma) \alpha (\kappa_t \hat{\chi}_t)^{1-\alpha} (z_t^M h_t \nu_t^M)^{\alpha-1} (1-h_t) \frac{\theta}{1-\theta} \\ \hat{\chi}_{t+1} &= \frac{\eta_t (\kappa_t)^{1-\alpha} (z_t^M \nu_t^M h_t)^\alpha (\hat{\chi}_t)^{1-\alpha} + (1-\delta) \hat{\chi}_t - \hat{c}_t}{(1+n) (1+g_{t+1}^M)} \end{aligned}$$

where

$$\begin{aligned}\eta_t &= \left[\gamma \left(\frac{Y_t^G}{Y_t^M} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \frac{Y_t^G}{Y_t^M} &= \frac{\left(z_t^{AM} \Lambda_t^{AM} h_t \frac{\nu_t^{AM}}{\nu_t^M} \nu_t^M N_t \right)^\beta \left((1-\kappa_t) \hat{\chi}_t \Lambda_t^M N_t \right)^{1-\beta-\beta_T}}{\left(z_t^M \Lambda_t^{AM} h_t \nu_t^M N_t \right)^\alpha \left(\kappa_t \hat{\chi}_t \Lambda_t^M N_t \right)^{1-\alpha}} \left(\frac{v_t}{\varsigma} \right)^{-\frac{\omega}{\omega-1}} \\ \frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M} \frac{(1-\tau)\alpha}{(1-v_t)+v_t\beta} \frac{v_t\beta}{(1-\tau)\alpha}\end{aligned}$$

Therefore, we are left with four unknowns $h_t(\Theta_t), \kappa_{t+1}(\Theta_t), \nu_t^M(\Theta_t), v_t(\Theta_t)$. We need to solve four non-linear equations as well.

The first two are the detrended the Euler Equations

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)(1+g_{t+1}^M)} E_t \left\{ \hat{c}_{t+1}^{-1} \times \left[\eta_{t+1}^{\frac{1}{\varepsilon}} \left(\frac{z_{t+1}^M \nu_{t+1}^M h_{t+1}}{\kappa_{t+1} \hat{\chi}_{t+1}} \right)^\alpha (1-\gamma)(1-\alpha) + (1-\delta) \right] \right\} \quad (57)$$

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)(1+g_{t+1}^M)} E_t \left[\hat{c}_{t+1}^{-1} \left(\eta_{t+1}^{\frac{1}{\varepsilon}} \varsigma \left(\frac{v_{t+1}}{\varsigma} \right)^{-\frac{1}{\omega-1}} (1-\beta-\beta_T) \frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} + 1 - \delta \right) \right] \quad (58)$$

where

$$\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} = \left((1-\kappa_{t+1}) \hat{\chi}_{t+1} \Lambda_{t+1}^M \right)^{-\beta-\beta_T} \left(z_{t+1}^{AM} \Lambda_{t+1}^{AM} \lambda_{t+1}^{AM} h_{t+1} \right)^{\alpha_A} N_{t+1}^{-\beta_T}$$

and

$$\frac{Y_{t+1}^G}{Y_{t+1}^M} = \frac{\left(z_{t+1}^{AM} \Lambda_{t+1}^{AM} h_{t+1} \nu_{t+1}^{AM} N_{t+1} \right)^\beta \left((1-\kappa_{t+1}) \hat{\chi}_{t+1} \Lambda_{t+1}^M N_{t+1} \right)^{1-\beta-\beta_T}}{\left(z_{t+1}^M \Lambda_{t+1}^{AM} h_{t+1} \nu_{t+1}^M N_{t+1} \right)^\alpha \left(\kappa_{t+1} \hat{\chi}_{t+1} \Lambda_{t+1}^M N_{t+1} \right)^{1-\alpha}} \left(\frac{v_{t+1}}{\varsigma} \right)^{-\frac{\omega}{\omega-1}}$$

The other two are the equations about v_t and ν_t^M .

$$v_t = \frac{\varsigma (Y_t^{AM})^{\frac{\omega-1}{\omega}}}{\varsigma (Y_t^{AM})^{\frac{\omega-1}{\omega}} + (1-\varsigma) (Y_t^S)^{\frac{\omega-1}{\omega}}} \quad (59)$$

$$\nu_t^M = \left(1 + \frac{\gamma (Y_t^G)^{1-\frac{1}{\varepsilon}} (1-v_t) + v_t\beta}{(1-\gamma) (Y_t^M)^{1-\frac{1}{\varepsilon}} (1-\tau)\alpha} \right)^{-1} \quad (60)$$

where

$$\begin{aligned}Y_t^{AM} &= (1-\kappa_t)^{1-\beta-\beta_T} (\hat{\chi}_t \Lambda_t^M)^{1-\beta-\beta_T} \left(\frac{\nu_t^{AM}}{\nu_t^M} \right)^\beta \left(z_t^{AM} \Lambda_t^{AM} \nu_t^M \right)^\beta h_t^\beta N_t^{-\beta_T} \\ Y_t^S &= z_t^S \nu_t^M \frac{\nu_t^S}{\nu_t^M} h_t \Lambda_t^S N_t \\ \frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M} \frac{(1-\tau)\alpha}{(1-v_t)+v_t\beta} \frac{v_t\beta}{(1-\tau)\alpha} \\ \frac{\nu_t^S}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M} \frac{(1-\tau)\alpha}{(1-v_t)+v_t\beta} \frac{(1-v_t)}{(1-\tau)\alpha}\end{aligned}$$

Summary 1 *We solve the Bellman Equation backwards from the last period, which corresponds to the long-run approximate balanced growth path with (approximately) only one sector, that is, the non-agr. sector.*

1. We choose to number of transition period to be a large number ($T = 250$). In practice, we can increase the number until the beginning transition period we are interested in are no longer affected by the choice of T . We can also check whether the economy will converge to the long-run ABGP within the period T .
2. We discretize the state space. In the deterministic case, we choose the state space for $\hat{\chi}$ as $[0.5\hat{\chi}_0, 1.5\hat{\chi}^*]$ and choose the state space for κ as $[0.5, 1]$. We discretize both $\hat{\chi}$ and κ using 250 equally spaced grid points; In the stochastic case, we choose the state space for $\hat{\chi}$ as $[0.9\hat{\chi}_t^*, 1.1\hat{\chi}_t^*]$ and choose state space for κ as $[\kappa_t^* - 0.025, \kappa_t^* + 0.025]$. We discretize both $\hat{\chi}$ and κ using 75 equally spaced grid points, where $\hat{\chi}_t^*$ and κ_t^* are the realized path in the deterministic model. We further discretize the joint process for the three types of shocks using 27 grid points using Tauchen's method (Tauchen 1986).
3. We solve the transitional path backwards.

(a) In the last period, the economy is almost identical to a one-sector RBC model. Therefore, we set

$$\begin{aligned}\kappa_{T+1}(\Theta_T) &= 1, v_T(\Theta_T) = 1 \\ \nu_T^M(\Theta_T) &= 1, \nu_T^{AM}(\Theta_T) = 0, \nu_T^S(\Theta_T) = 0\end{aligned}$$

We solve the Bellman Equation using value function iteration, with linear interpolation between grid points. We can solve for the rest of the policy functions

$$c_T(\Theta_T), h_T(\Theta_T), \hat{\chi}_{T+1}(\Theta_T)$$

(b) From period $t = T - 1$ to 1, we solve the nonlinear system of Equations (57), (58), (59), and (60) for the policy functions $\kappa_{t+1}(\Theta_t), v_t(\Theta_t), \nu_t^M(\Theta_t), h_t(\Theta_t)$. In each period, we first express $\hat{c}_t(\Theta_t), \hat{\chi}_{t+1}(\Theta_t)$ in terms of other state variables and decision variables

E.5 Measuring sector-specific TFP levels: theory

We observe $\{Y, Y^M, K^M, N^M, Y^G, K^G, N^G\}$ but we do not have direct observations of allocations of labor and output across the two agricultural technologies (we also presume that we have already estimated all relevant parameters). We describe the procedure to estimate the three-shock process

To measure the sector-specific TFP levels we impose three equilibrium conditions: (i) marginal return to capital are equated across manufacturing and modern agriculture, and (ii) marginal return to labor is equated across the two agricultural technologies; and (iii) hours per worker h is equalized across sectors. In addition we assume that h is constant over time so that employment is a sufficient statistic for measuring labor input.

Recall the following definitions of sectoral outputs;

$$\begin{aligned}Y^G &= \left[\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}} + (1-\varsigma) (Y^S)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \\ Y^M &= (K^M)^{1-\alpha_M} \times (Z^M H^M)^{\alpha_M} \\ Y^{AM} &= (K^{AM})^{\beta_{AM}} \times (Z^{AM} H^{AM})^{\alpha_{AM}} \\ Y^S &= Z^S H^S\end{aligned}$$

We express aggregate output using current prices,³³ $Y = P^G Y^G + P^M Y^M$.

³³The advantage of focusing on aggregate output in terms of current prices instead of specifying the production function is

The marginal products of sector-specific capital and labor are

$$\begin{aligned}
\frac{\partial Y}{\partial K_M} &= P^M \frac{\partial Y_M}{\partial K_M} = P^M (1 - \alpha_M) \left(\frac{K^M}{H^M} \right)^{-\alpha_M} \times (Z^M)^{\alpha_M} \\
\frac{\partial Y}{\partial K_G} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial K_G} = P^G \varsigma \left(\frac{Y^{AM}}{Y} \right)^{-\frac{1}{\omega}} \beta_{AM} (K^G)^{\beta_{AM}-1} \times (Z^{AM} H^{AM})^{\alpha_{AM}} \\
\frac{\partial Y}{\partial N_S} &= P^G \frac{\partial Y_G}{\partial Y_S} \frac{\partial Y_S}{\partial N_S} = P^G (1 - \varsigma) \left(\frac{Y^S}{Y^G} \right)^{-\frac{1}{\omega}} Z^S h \\
\frac{\partial Y}{\partial N_{AM}} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial N_{AM}} = P^G \varsigma \left(\frac{Y^{AM}}{Y^G} \right)^{-\frac{1}{\omega}} \alpha_{AM} (K^G)^{\beta_{AM}} \times (H^{AM})^{\alpha_{AM}-1} (Z^{AM})^{\alpha_{AM}} h
\end{aligned}$$

We now proceed to measuring the TFP levels in five steps:

1. The marginal product of capital is the same in manufacturing and modern agriculture,

$$\begin{aligned}
P^M (1 - \alpha_M) (K^M)^{-\alpha_M} \times (Z^M H^M)^{\alpha_M} &= \varsigma \left(\frac{Y^{AM}}{Y^G} \right)^{-\frac{1}{\omega}} P^G \beta_{AM} (K^G)^{\beta_{AM}-1} \times (Z^{AM} H^{AM})^{\alpha_{AM}} \\
&\Rightarrow \\
\left(\frac{Y^{AM}}{Y^G} \right)^{1-\frac{1}{\omega}} &= \frac{K^G P^M Y^M}{K^M P^G Y^G} \frac{1 - \alpha_M}{\beta_{AM}}
\end{aligned} \tag{61}$$

The variables on the right-hand side of Equation (61) are observable. This identifies the ratio Y^{AM}/Y^G .

2. Use the agricultural production function (3) to derive a relationship between the ratios Y^S/Y^G and Y^{AM}/Y^G ,

$$1 = \varsigma \left(\frac{Y^{AM}}{Y^G} \right)^{\frac{\omega-1}{\omega}} + (1 - \varsigma) \left(\frac{Y^S}{Y^G} \right)^{\frac{\omega-1}{\omega}} \tag{62}$$

Given the imputed ratio Y^{AM}/Y^G , this equation identifies the ratio Y^S/Y^G .

3. The marginal product of labor is equated across the two agricultural sectors. This implies

$$\begin{aligned}
P^G (1 - \varsigma) \left(\frac{Y^S}{Y^G} \right)^{-\frac{1}{\omega}} Z^S h &= P^G \varsigma \left(\frac{Y^{AM}}{Y^G} \right)^{-\frac{1}{\omega}} \alpha_{AM} (K^G)^{\beta_{AM}} \times (H^{AM})^{\alpha_{AM}-1} (Z^{AM})^{\alpha_{AM}} h \\
&\Rightarrow \\
\frac{H^{AM}}{H^S} &= \alpha_{AM} \frac{\varsigma}{1 - \varsigma} \left(\frac{Y^{AM} Y^G}{Y^G Y^S} \right)^{1-\frac{1}{\omega}}
\end{aligned} \tag{63}$$

4. Use the accounting identity $N^G = N^{AM} + N^S$ to identify N^{AM} and N^S ;

$$\frac{H^{AM}}{H^S} = \frac{N^{AM}}{N^S} = \frac{N^{AM}}{N^G - N^{AM}} \Rightarrow$$

$$N^{AM} = \frac{N^G}{1 + \left(\frac{H^{AM}}{H^S} \right)^{-1}} \tag{64}$$

$$N^S = N^G - N^{AM} \tag{65}$$

that while a subsistence level in agricultural consumption will be subsumed in the relative prices, it will not affect the equations determining how TFP levels are measured.

5. Normalize hours per worker to $h = 1$ for all periods (so aggregate hours equals employment). Equations (61)-(65) then allows us to identify Z^M , Z^S , and Z^{AM} ;

$$\ln(Z^S) = \ln(Y^S) - \ln(N^S) \quad (66)$$

$$\ln(Z^{AM}) = \frac{1}{\alpha_{AM}} \ln(Y^{AM}) - \frac{\beta_{AM}}{\alpha_{AM}} \ln(K^{AM}) - \ln(N^{AM}) \quad (67)$$

and finally TFP in manufacturing;

$$\ln(Z^M) = \frac{1}{\alpha_M} \ln(Y^M) - \frac{1 - \alpha_M}{\alpha_M} \ln(K^M) - \ln(N^M) \quad (68)$$

E.6 Additional Figures

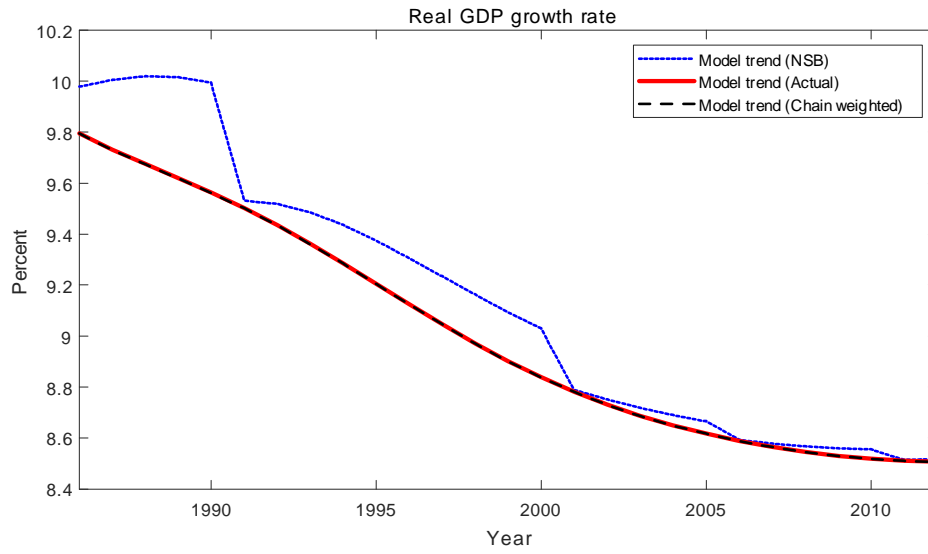


Figure 17: This figure compare different ways of calculating the real GDP growth rate with data.

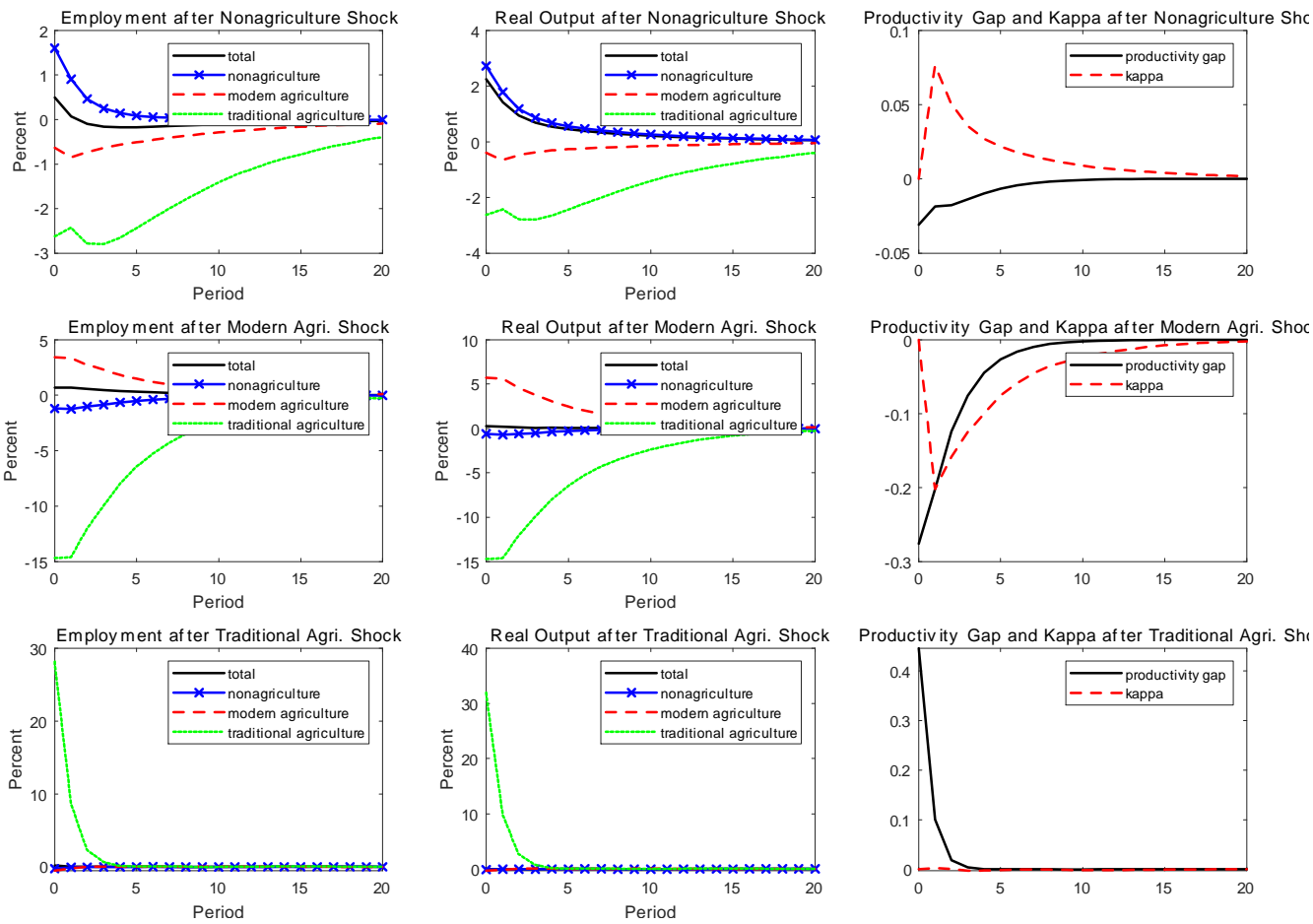


Figure 18: **Impulse Response Functions.** All graphs show impulse response as percentage deviation from the deterministic path in year 2000. The three top panels show responses to a one-standard deviation change in nonagricultural TFP (Z^M). The three middle panels show responses to modern agricultural TFP (Z^{AM}). The three bottom panels show responses to traditional agricultural TFP (Z^S).