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When is Debt Odious? A Theory of Repression and Growth Traps[†]

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Abstract

How is a developing country affected by its odious government's ability to borrow in international markets? We examine the dynamics of a country's growth, consumption, and sovereign debt, assuming that the government is myopic and wants to maximize short-term, socially unproductive, spending. Interestingly, access to external borrowing can extend the government's effective horizon; the government's ability to borrow hinges on its convincing investors they will be repaid, which gives it a stake in the future. The lengthening of the government's effective horizon can incentivize it to tax less, resulting in higher steady-state household consumption than if it could not borrow. However, in a developing country that saves little, the government may engage in more repressive policies to enhance its debt capacity, which only ensures that successor governments repress as well. This leads to a "growth trap" where household steady-state consumption is lower than if the government had no access to debt. We characterize circumstances in which odious government leads to odious debt and those in which it does not, and discuss policies that might ameliorate the welfare of the citizenry.

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1 Introduction

Is the ability to borrow in international markets good for a country, especially for a developing one? Many theories of international borrowing emphasize the better risk-sharing a country can achieve – in case of an economic or natural calamity, it can borrow to smooth consumption – as well as its ability to draw on international savings to finance domestic growth (see, for example, Kletzer and Wright [2000]). Yet it is hard empirically to see a positive correlation between a developing country's use of foreign financing and good outcomes such as stronger economic growth (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]). What might explain the divergence between theory and evidence?

One weakness with many existing models is that they tend to assume that the government of the country in question maximizes the utility of its citizenry over the long run. Yet an important reality in many developing countries is that their governments are often myopic, wasteful, and even rapacious. Whether poverty reduces the quality of government or the poor quality of government entrenches poverty is unclear.

A second weakness is related to the first. Once the government is assumed to maximize the welfare of its citizenry, often the best thing it can do is to default on its foreign debt (Bulow and Rogoff [1989*a*], Bulow and Rogoff [1989*b*], and Tomz [2012]). To explain the existence of sovereign debt, researchers then have to appeal to a variety of mechanisms that enforce sovereign repayment such as a government's concern for its reputation or punishment strategies by other countries. Unfortunately, there is little empirical evidence for these mechanisms (Eichengreen [1987], Özler [1993], Flandreau and Zumer [2004], Sandleris et al. [2004] and Arellano [2008]).

In this paper, we start with an extreme view of a goverment, that it is myopic and self-interested so all of its spending does little for the welfare of its citizenry. It turns out that in this setting it is relatively easy to explain the enforceability of sizeable amounts of sovereign borrowing even with small costs of default. We then ask whether access to sovereign borrowing, taking into account the need for international investors to be confident that the borrowing will be repaid, is welfare-improving for the country's citizens.

A literature on "odious" debt takes the view that it is not (see Buchheit, Gulati and Thompson [2006] and Jayachandran and Kremer [2006]). The ability to borrow essentially gives the government more resources to waste or steal, with the repayment eventually extracted by international lenders from the citizens. Therefore, some commentators advocate declaring debt issued by such governments odious, and recommend limiting the enforcement of such debt in international courts. Other commentators (for example, Choi and Posner [2007]) have argued that the value of such proposals should turn on whether a country's citizens will be better off when it no longer has the ability to borrow. The answer, it turns out, is not straightforward.

Let us explain. Consider an overlapping generations model of a country with a representative young citizen each period – the citizen is a composite of the households and the productive private sector, and we will use these terms interchangeably. The other agents in the model are the government and international investors.

The representative citizen has an initial endowment (smaller if a poorer country)[1] that she can either consume, save in domestic government bonds, or invest in private enterprise. She maximizes the sum of her consumption this period and the discounted endowment left behind for the next generation, a proxy for the future stream of her descendants' consumption.

The myopic government rules only one period, and thus has a short horizon. It is assumed to spend in ways that do not enhance citizen welfare, including wasteful populist spending (such as election propaganda), white elephant projects (such as gigantic power plants that are not economic to run), or plain theft (luxury flats in Miami or London or Cayman Island bank accounts). The government maximizes the resources it can raise for spending, which consist of the sum of the taxes it levies on private sector output and the amount it can raise through debt issuance (net of repayment of past debt).

Government debt is issued to both domestic investors and foreign investors in the form of bearer bonds, and we assume the government cannot tell who holds its debt.[2] Successor governments inherit the obligation to repay sovereign debt, though they can default. If the government defaults on past debt, it pays the default cost (we elaborate shortly) and cannot issue new debt for the rest of the period. In that situation, which we term "debt autarky", it will set the tax rate on private output at the level that trades off the disincentivizing effect of higher taxes on private investment against their impact on government revenues (the "Laffer curve" maximizing level).

International investors do not care about the quality of government spending, but will lend only if they expect to get their money back with interest. Therefore, given the model has no uncertainty, there will be no over-lending, and no default. This allows us to highlight the central tradeoffs.

The assumption of government myopia and self interest, consistent with odious governments, is one difference in our assumptions from the traditional ones. Another, following a recent set of papers, is the source of the incentive for the government to repay. The government cannot default selectively on foreign investors alone since it cannot tell domestic holders

[1] Throughout the paper, we refer to "poorer" country as one that has a smaller initial endowment, *ceteris paribus*, that is, holding constant all other parameters of the economy.

[2] See, for example, Broner, Martin and Ventura [2010]. This assumption ensures the government cannot default selectively on foreigners.

of government bonds apart from foreign holders.[3] So if it defaults, domestic investors experience significant losses. For instance, if these are banks, the government will have to bail them out to have any reasonable economic output. We assume the default costs rise in the size of sovereign bonds held by domestic investors. So the government does not default on the debt for two reasons. First, it will incur the deadweight cost immediately. Second, it has a short horizon, so it does not trade off the deadweight cost of default against the present value of the outstanding debt, but instead only against the net debt repayments it has to make in its period in power. This implies that a sizeable amount of debt issuance can be supported with modest deadweight costs.

The government's ability to borrow alters the tax it will impose on the real sector or the extent to which it will repress it financially. The higher the tax it imposes, the lower the amount that the private sector allocates to real investment, leaving more of its endowment to consumption and financial savings in government bonds. So when a government has access to sovereign debt issuance, its taxation is driven by two sets of opposing concerns. A higher tax rate will curb real investment, resulting in lower future revenues to be taxed. It will also reduce the surplus available to future governments to repay debt, thus lowering how much can be borrowed today. A higher tax rate therefore reduces the future government's *ability to pay*. But it will also raise the domestic private sector's financial savings in government debt, increasing the *willingness to pay* of a future government, and thus increasing how much debt can be issued today.

So whether the country's ability to issue debt raises or lowers the tax that the myopic rapacious government imposes on the private sector depends on which of these incentives predominates. For a country that starts at low endowments (a developing country) and a high propensity to save among the citizenry, the government may have little need to channel more into financial savings, and it will lower tax rates relative to autarky in our model. The government's ability to issue debt here tends to be beneficial for the citizen over the long run because the need to convince debt holders of repayment limits the government's rapacity and enhances steady-state consumption relative to autarky, *i.e.*, there is a "growth boost."

Conversely, for a country with low starting endowment and a low propensity to save among the citizenry, the government may set higher-than-autarky tax rates. This could push the country into a lower consumption "growth trap," precisely because each rapacious government represses in order to enhance its debt issuance, in the process leaving the next period government also with a low-endowment economy that is heavily indebted so that the repression gets entrenched *ad infinitum*. For the citizens of such countries, sovereign debt is truly odious.

To summarize, whether traversing the *extensive* margin (autarky versus access to debt) im-

[3] See, for example, Bolton and Jeanne [2011], Acharya and Rajan [2013], Acharya, Drechsler and Schnabl [2014], Gennaioli, Martin and Rossi [2014], Andrade and Chhaochharia [2018], and Farhi and Tirole [2018].

proves or hurts citizen welfare for a developing country depends on its specific situation, not just on the odious nature of its government. The debate on whether a country's external sovereign debt should be declared odious has centered on the misuse of such borrowing by the odious government, not on the possibility that access to borrowing will affect the government's incentives and behavior. We examine both, as well as the costs to citizens of defaulting on existing sovereign external debt, in evaluating such proposals.

A related but different question is on the *intensive* margin: How does the variation in reliance on foreign borrowing across developing countries that have the ability to borrow internationally affect growth (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]). This literature has documented a puzzling weak or negative correlation between developing country growth and its use of foreign borrowing. It turns out that our model can also provide a potential explanation for this phenomenon.

Specifically, suppose the differential reliance on foreign borrowing across countries arises due to cross-country differences in the citizen's propensity to save, keeping the nature of the government the same. Our model implies that governments of countries that have a high domestic propensity to save will be more growth-friendly in their policies; under certain conditions, these countries will rely less on foreign borrowing than countries with a low domestic propensity to save, with the latter experiencing more repressive policies from their government as it tries to boost its capacity to borrow. As a result, the absence of a positive correlation between foreign borrowing and economic growth for developing countries documented in the literature may stem from the *endogenous* selection of which countries rely more on foreign borrowing, rather than some direct adverse effect of foreign borrowing on country growth and development.

Typically, debt relief in our model will do little for a country's citizens, even when it is in a growth trap. The government will simply use the expanded space to borrow, and spend it quickly. It will soon be back to pre-relief levels of debt – this was a common concern with the debt relief measures undertaken in developing countries in the late 1990s and early 2000s. Indeed, in a number of cases, debt relief has had little long-term beneficial effect as governments continue to misspend.[4] Many countries whose debt was written off have returned to situations of debt stress.

In contrast to the ineffectiveness of debt relief in the model, debt ceilings that also bind future governments can be desirable. In particular, countries could decide to limit their own ability to borrow through a constitutional debt ceiling. Alternatively, well-intentioned international lenders could set informal debt limits for countries – though this requires a collective

[4] See https://www.worldbank.org/en/topic/debt-sustainability#2 for a list of countries and the risk of debt distress prepared by the World Bank.

agreement since the country is capable of servicing more debt at the ceiling. We examine the consequences of such debt limits, and discuss why they might be more effective in developing countries than in rich countries, even if the quality of their governments is similar.

Of course, our assumption that the government is both myopic and unconcerned about citizen welfare is a caricature, perhaps on par with the more traditional assumption that the government cares only about its citizenry and is benevolent. We trace the consequences of moving away from this assumption, including what happens when governments have very productive public goods they can invest in.

Despite the fact that government defaults are costly by design in our model, we observe also that countries in a growth trap can benefit from default – potentially caused by unanticipated shocks to endowment or changes in parameters such as the interest rate. Because growth is suppressed by the government's repressive policies, a significant one-period boost to growth can arise from the economy entering debt autarky post-default (see Levy-Yeyati and Panizza [2011] for empirical evidence on the positive effects of sovereign default on growth). In some cases, the boost can be even larger than the cost of default such that in the medium run the economies outgrow their original endowment levels. They may even emerge out of the trap.

It is also possible that shocks to endowment can trap a high-flying country in a low-endowment equilibrium. The effects of such defaults assume importance with the ongoing Coronavirus pandemic, which we will argue is akin to a negative endowment shock in our model, coupled with the possibility of investment in a high return public good (investing in medical resources to contain the virus' spread). We will describe why a policy of debt relief with targeted new lending might work well in such cases.

The rest of the paper is as follows. In Section 2, we discuss the baseline model and the main Bellman equation capturing the model dynamics. In Section 3, we present an in-depth analysis of the properties of the solution and explain how a growth trap or growth boost arises. In Section 4, we discuss the merits of the odious debt proposal, the link between foreign financing and growth, and the relationship of our model to the literature on sovereign debt. In Section 5, we analyze various policy instruments that can help the economy escape the growth trap. In Section 6, we discuss the impact of unanticipated shocks to the economy in the steady state and derive further policy implications. In Section 7, we examine what happens when government spending is not entirely wasteful, and offer concluding remarks and possible future extensions in Section 8.

2 Baseline Model

We consider an overlapping generations model. Time is discrete and the horizon is infinite. The world consists of a single country and the rest of the world. The country is a small open economy with two agents, the private sector and the government. Foreign investors invest in the country's sovereign debt as well as its private sector's debt. We assume all debt issued matures in one period and pays the required world interest rate of $r > 0$.

Consider period i. The private sector is a representative household that maximizes the sum of the log of current period consumption c_i and the log of next period endowment e_{i+1} (which is the endowment it leaves for the next generation) times a parameter ρ, where $\rho \in (0, \frac{1}{r})$ captures the overall preference for savings (bequest) of the household. At the beginning of the period i, the household inherits an endowment e_i, which it allocates to financial savings s_i and physical investment k_i so as to maximize utility. The household has a mild home bias so financial savings are invested in domestic government bonds at the rate r (rather than internationally) whenever the government borrows. Physical investment produces $f(k_i)$ at the end of the period, where $f' > 0$ and $f'' < 0$. The government can potentially tax the production at a rate t_i, in which case the net proceeds for the household from production is $(1-t_i)f(k_i)$.

We assume the private household's financial savings into government debt are not taxed (equivalently, it bears a relatively lower tax than household investment in real assets). This is a key assumption. Consider three justifications. First, fixed hard assets are easier to tax than fungible financial savings. Since financial savings are more mobile and also easily converted to concealable assets like gold, the government typically keeps taxes on financial savings relatively low. Second, we have in mind here both actual taxes as well as the implicit taxes the government collects through corruption, which usually falls more heavily on business enterprise. Third and most important, governments that need resources tend to direct flows toward themselves through *financial repression*. For instance, capital controls are deployed to ensure that domestic savings do not leave the economy, financial institutions are required to allocate a significant part of their assets to government debt, and tax breaks are provided to domestic investors for the return earned on government bond holdings, potentially crowding out the private sector's access to finance (effectively a tax).[5] For simplicity, we do not model any of these effects, assuming they are fully captured by the tax falling only on real investment. It should be kept in mind, though, that real repression (high taxes on private sector real investment) and financial repression (guiding financial savings into government instruments) are two sides of the same coin.

[5] In this vein, Gennaioli, Martin and Rossi [2018] find that there is a negative and statistically significant correlation between a bank's holding of domestic government bonds and its ratio of loans to assets, especially in developing countries.

The government in our model is incumbent for only one period and its sole objective is to maximize its wasteful spending, wasteful in that it does not directly augment the economy's endowment or private consumption. The spending could be on itself (high government salaries or corruption), on grandiose white elephant projects, or on political propaganda. It finances the spending by imposing a tax on the private sector, as well as issuing debt which is sold to both domestic and foreign investors. We assume that debt is short-term, *i.e.*, it matures next period.[6] We assume the government cannot default selectively on foreign debt holders, which would be true if it issued bearer bonds. All we really need, however, is that a default on external sovereign debt spills over to domestic debt. This is hardwired in the model by assuming the two forms of debt are indistinguishable, but there are a variety of other sources of spillover that could be invoked.[7]

The government can decide whether to default or to repay the maturing debt that the previous government issued. If it defaults, the economy's infrastructure incurs direct damage – for instance, banks holding government debt are "run" upon, the payment system freezes, and repo markets collateralized by government debt are disrupted. To ensure the private sector produces this period (and can be taxed) the government has to commit a part of its spending on cleaning up the disruption. We model this cost as $C + zD^{Dom}$, where $C > 0$, $z > 1$ are constant parameters and D^{dom} is the face value of government debt held by the domestic residents at the time of default; C captures a fixed cost of default, whereas zD^{Dom} captures the idea that the default cost is increasing in the face value amount that the domestic private sector has invested in the government debt.[8] In addition to this cost, the government is excluded post default from debt markets for that period – this could be thought of as the period the debt is being renegotiated (down to zero for simplicity in our model); the government thus experiences "debt autarky" with no access to the sovereign debt market. We assume that the investors – both domestic and foreign – are fully rational and are therefore willing to lend to the government only to the extent that the debt will be fully repaid in the next period.

The timeline of the model is shown in Fig. 1.

[6] Our results are robust to allowing for the issuance of long-term debt, as explained in the concluding remarks of the paper.

[7] There is evidence consistent with such spillovers. Borensztein and Panizza [2009] show that public defaults are associated with banking crises; Brutti [2011] finds more financially dependent sectors tend to grow relatively less after sovereign default; De Paoli, Hoggarth and Saporta [2009] show that sovereign default is associated with substantial output costs for the domestic economy; Arteta and Hale [2008] use firm-level data to show that syndicated lending by foreign banks to domestic firms declines after default; Ağca and Celasun [2012] also use firm-level data to show the corporate borrowing costs increase after default.

[8] Because household savings s can be negative in our model, we need a high enough C to ensure that the default cost itself never becomes negative.

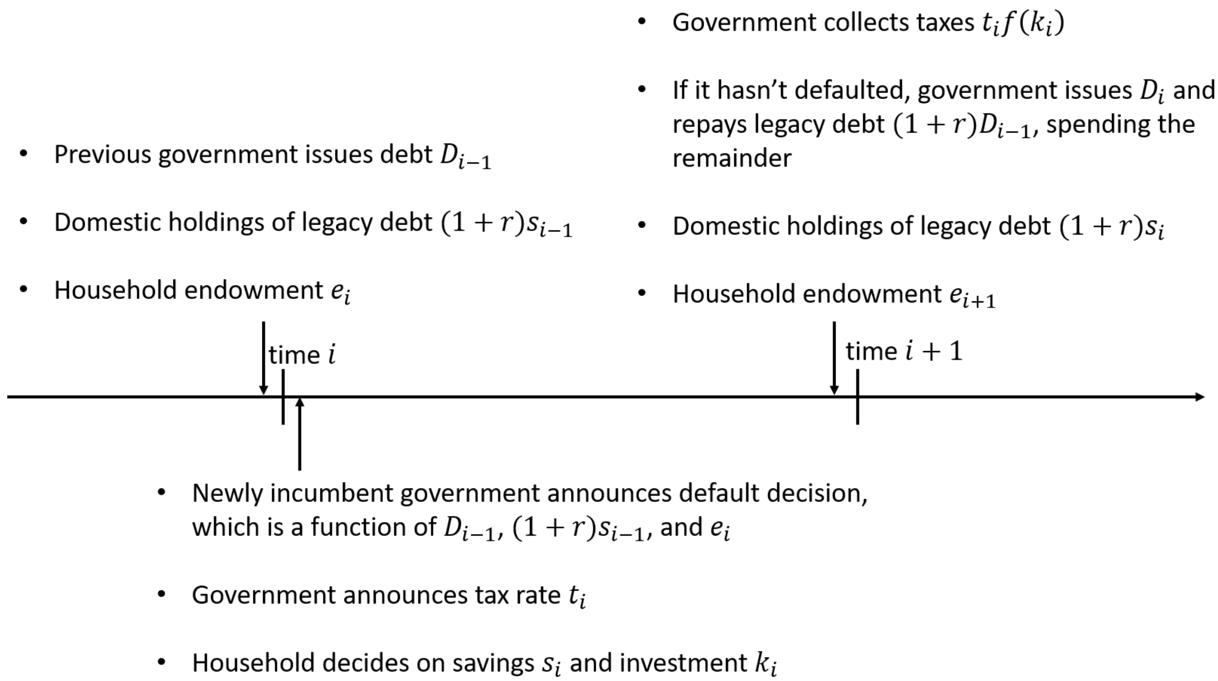

Figure 1: Timeline of the model.

The private sector's problem can be summarized by the following constrained optimization problem:

$$\max_{c_i, e_{i+1}, k_i, s_i} \quad \ln c_i + \rho \ln e_{i+1} \tag{2.1}$$

$$s.t. \quad c_i + s_i + k_i \leq e_i, \text{ and} \tag{2.2}$$

$$e_{i+1} \leq (1+r)s_i + (1-t_i)f(k_i). \tag{2.3}$$

2.1 Household problem

The government decides whether to service legacy debt, sets the tax rate, and issues the maximum new debt consistent with these decisions, while expecting the household to maximize its utility in reaction to government policy. Start with the household's problem in period i. The representative household receives an endowment e_i from the past generation, and takes the tax rate t_i as given.[9] It solves the maximization problem in (2.1). Let us set λ and μ as the Lagrangian multipliers for constraints in (2.2) and (2.3), respectively. The corresponding

[9] To keep the timing straight, we assume only financial investment held between periods accrues interest, real investment takes place at the beginning of the period, real production pays taxes to the government at the end of the period, and the after-tax production is returned to households at the beginning of the next period as part of their endowment.

Lagrangian is the following:

$$\mathscr{L} = \ln c_i + \rho \ln e_{i+1} - \lambda(e_i - c_i - s_i - k_i) - \mu[(1+r)s_i + (1-t)f(k_i) - e_{i+1}].$$

Obtaining the first order conditions (FOC's) for our four choice variables yields:

$$c_i : \quad 0 = \frac{1}{c_i} + \lambda; \tag{2.4}$$

$$s_i : \quad 0 = \lambda - (1+r)\mu; \tag{2.5}$$

$$k_i : \quad 0 = \lambda - (1-t_i)f'(k_i)\mu; \text{ and} \tag{2.6}$$

$$e_{i+1} : \quad 0 = \frac{\rho}{e_{i+1}} + \mu. \tag{2.7}$$

It is easily seen (see) Lemma C.1 in the appendix) that FOC's (2.4) - (2.7) lead to the following set of decision functions for the households:

$$k_i = f'^{-1}\Big(\frac{1+r}{1-t_i}\Big), \tag{2.8}$$

$$c_i = \kappa_0[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \tag{2.9}$$

$$e_{i+1} = \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \tag{2.10}$$

$$s_i = \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i); \text{ where} \tag{2.11}$$

$$\kappa_0 := \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}.$$

The government chooses the tax rate and debt issuance knowing the household will react according to (2.8), (2.9), (2.10), and (2.11).

Remark 2.1. We discuss some properties of the solutions (2.8) - (2.11).

(1) The household's physical investment is a function of the exogenous interest rate and the government-set tax rate only (see (2.8)). So the total amount of tax collected by the government is $tf(k(t))$, a function of t. We denote this function as $\tau(t)$.

(2) Note from (2.9) and (2.10) that $\forall i$, $c_i = \frac{1}{\rho(1+r)}e_{i+1}$. This implies that there is a one-to-one relationship between the level of endowment and consumption in the model.

(3) Note from (2.10) that the next-period endowment depends on the current-period endowment linearly with a coefficient $\kappa_1(1+r)$. In order to rule out exploding economies, we impose a condition that $\kappa_1(1+r) < 1 \Leftrightarrow \rho < 1/r$.

(4) Note in (2.11) that household financial savings is increasing in the tax rate t (because

investment is decreasing in the tax rate from (2.8)).[10]

2.2 Government Problem: Debt autarky

Let us turn now to the government's problem. The benchmark case is one where the government cannot issue any debt. Since this government can only spend what it raises from tax, it will simply choose a tax rate that maximizes tax revenues $\tau(t)$. Let $**$ denote this benchmark "debt autarky" case:

$$
\begin{aligned}
t^{**} &:= \text{benchmark tax rate} = \underset{t}{\operatorname{argmax}}\, \tau(t), \\
k^{**} &:= \text{benchmark investment} = k(t^{**}), \text{ and} \\
\tau^{**} &:= \text{benchmark tax revenue} = \tau(t^{**}).
\end{aligned}
$$

For instance, in the case of a power production function $f(k) = Ak^{\gamma}$, $t^{**} = 1 - \gamma$.

2.3 Optimization problem of myopic government with debt

Consider now the government's problem when it can borrow. It has legacy debt $(1+r)D_{i-1}$ due, of which $(1+r)D_{i-1}^{Dom}$ is held domestically. Suppose for now that the government finds default suboptimal and decides to pay back the legacy debt. It finances its spending by issuing debt D_i and collecting taxes from the private sector at rate t_i. It expects the household to react as in (2.8), (2.10), and (2.11). Suppose that the maximum resource that next period's government can raise – through taxation and borrowing – is S_{i+1}. Debt issuance D_i today is then constrained by the next-period government's *ability to pay*:

$$D_i(1+r) \leq S_{i+1}. \tag{2.12}$$

Consider now the next-period government's *willingness to pay*. In the event that the next-period government defaults, its tax revenues are at the autarky level τ^{**}. It follows that in order for the next-period government to be willing to pay, the amount it can spend if it doesn't default

[10] Under the log-utility assumption for households, investment declines and savings increase with the tax rate t; in other words, economic and financial repression map one-for-one in this case. With a more general utility function for households, the impact of the tax rate on savings would depend on the elasticity of inter-temporal substitution (EIS); what is crucial to note is that the government's incentive in the willingness-to-pay region is to channel domestic savings to its bonds, for which it may in general have to employ financial repression explicitly, beyond economic repression.

should be more than τ^{**} minus the spending to clean up the post-default financial disruption:

$$\begin{aligned}
\underbrace{S_{i+1} - D_i(1+r)}_{\text{net spending on no default}} \geq & \underbrace{\tau^{**}}_{\text{revenues in autarky}} - \underbrace{\left(C + zD_i^{Dom}(1+r)\right)}_{\text{spending to clean up default}} \qquad (2.13)\\
\Rightarrow & D_i(1+r) \leq S_{i+1} + zD_i^{Dom}(1+r) + C - \tau^{**}\\
\Rightarrow & D_i(1+r) \leq S_{i+1} + zs_i(1+r) + C - \tau^{**}. \qquad \text{(in equlibrium)}
\end{aligned}$$

Since both the ability-to-pay constraint as well as the willingness-to-pay constraint must be met, the effective constraint on current-period debt is

$$\begin{aligned}
& D_i(1+r) \leq \min\{S_{i+1}, S_{i+1} + zs_i(1+r) + C - \tau^{**}\}\\
\Rightarrow & D_i(1+r) \leq S_{i+1} - \max\{0, \tau^{**} - C - zs_i(1+r)\}. \qquad (2.14)
\end{aligned}$$

It can be seen that $\tau^{**} - C - zs_i(1+r) = 0$ traces the threshold between willingness-to-pay and ability-to-pay constraint; when $\tau^{**} - C - zs_i(1+r)$ is positive, the willingness-to-pay constraint is binding, whereas when it is negative, the ability-to-pay constraint is binding.

Notice from (2.11) that s_i is increasing linearly in e_i. This implies that for sufficienty high endowments, $\tau^{**} - C - zs_i(1+r) < 0$, implying that the ability-to-pay constraint is binding. Conversely, for sufficiently low levels of endowment, the willingness-to-pay constraint is binding. This will be important in what follows.

Constraint (2.14) highlights the double-edged nature of sovereign debt that is at the heart of our model. On the one hand, the willingness-to-pay constraint implies D_i increases in s_i, which incentivizes the myopic government to repress investment with higher taxation in order to boost financial savings in government debt. On the other hand, when focusing on the next-period government's available resources to pay debt (its ability to pay), it turns out that D_i increases in S_{i+1}, which increases in e_{i+1}. In this case, the current-period government has an incentive to increase e_{i+1} by lowering taxation and boosting growth. As we show in the following sections, the government can under-tax or over-tax – relative to our benchmark case, which is the debt autarky optimum ($\text{argmax}_t \, tf(k(t))$) – depending on which term is more sensitive. If S_{i+1} is more sensitive to current-period taxation than the penalty term $\max\{0, \tau^{**} - C - zs_i(1+r)\}$, then the myopic government will choose a lower-than-benchmark tax rate, otherwise it will choose a higher-than-benchmark tax rate. Since for the current-period government,

$$\text{spending} = S_i - \text{legacy debt} = \max_t [D_i(t) + \tau(t)] - D_{i-1}(1+r), \qquad (2.15)$$

and the debt capacity $D_i(t)$ depends on S_{i+1}, the problem is inherently infite-horizon, even though the myopic government only optimizes a one-period problem. This is why debt is potentially a horizon-lengthening device.

2.4 Recursive Formulation of the Government's Problem

Let us formulate this problem recursively. Note that a myopic government i takes e_i, D_{i-1}^{Dom}, and D_{i-1} as given, and maximizes (2.15). This implies that the natural set of state variables is $(e_i, D_{i-1}^{Dom}, D_{i-1})$; however, since D_{i-1} enters (2.15) only additively, the maximization problem is independent of D_{i-1}. Moreover, D_{i-1}^{Dom} only governs the government's decision to default or not. Therefore, conditional on the government finding default suboptimal, the only state variable is e_i. Furthermore, since a myopic government will always choose D_i at the maximum, we can replace D_i with the expression in (2.14). Note that since the maximum is derived from no-default condition for the next government, there will be no government defaults in our model on the equilibrium path. Therefore, we have:

Lemma 2.1. *(Main Bellman equation)*

$$S(e) = \max_t \left[\frac{1}{1+r} \left[S(e') - \max\{0, \tau^{**} - C - zs(1+r)\} \right] + \tau(t) \right] \tag{2.16}$$

$$s.t. \quad e' = \kappa_1 \left[(1+r)(e - k(t)) + (1-t) f(k(t)) \right], \tag{2.17}$$

$$s = \kappa_1 (e - k(t)) - \kappa_0 (1-t) f(k(t)), \text{ and} \tag{2.18}$$

$$k(t) = f'^{-1}\left(\frac{1+r}{1-t} \right). \tag{2.19}$$

The value function $S(e)$ as well as the policy function $t(e)$, i.e., the decision rule conditional on the myopic government finding default suboptimal, constitute the complete solution for (2.16), which is sufficient for the no-default equilibrium path.

The decision rule encompassing (off-equilibrium) default can be obtained by revisiting the two constraints, (2.12) and (2.13); for given endowment e, legacy domestic debt D_{-1}^{Dom} (the face value of which is $(1+r)D_{-1}^{Dom}$), and legacy total debt D_{-1} (the face value of which is $(1+r)D_{-1}$),

- *(i) If $S(e) - (1+r)D_{-1} < 0$, the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges the autarkic tax rate t^{**}.*
- *(ii) If $S(e) - (1+r)D_{-1} < \tau^{**} - C - z(1+r)D_{-1}^{Dom}$, the government potentially can pay back the legacy debt, but finds defaulting more advantageous. In other words it defaults strategically, enters autarky, and charges the autarkic tax rate t^{**}.*
- *(iii) If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax $t(e)$ and issues $D(e) := S(e) - \tau(t(e))$ amount of debt. Government spending is $S(e) - (1+r)D_{-1}$.*

Finally, note that the debt issuance $D(e)$ can be further decomposed into domestic and foreign debt:

$$D^{Dom} := \text{Domestic debt} = s(e,t(e)), \text{ and} \tag{2.20}$$

$$D^{For} := \text{Foreign debt} = \text{Total debt} - \text{Domestic debt} = D(e) - s(e,t(e)). \tag{2.21}$$

2.5 A Numerical Example

Before we go into the details of the solution, a numerical example can help fix ideas. Fig. 2 shows a solution from the model specialized to $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$, and $C = 1$. We have

$$e_+(e,t) := \kappa_1[(1+r)(e-k(t)) + (1-t)f(k(t))]; \tag{2.22}$$

$$s(e,t) := \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)); \tag{2.23}$$

$$\pi(e,t) := (1-t)f(k(t)) - (1+r)k(t). \tag{2.24}$$

as the next-period endowment, financial savings, and private profit from investment, respectively, set based on the current-period endowment e and tax rate t. We prove formally in Proposition 2.2 that the solution possesses the following properties which are illustrated in Fig. 2:

- There exists a low-e region (see Fig. 2, regions annotated "WTP") where only the willingness-to-pay constraint is binding. In this region, the future government's ability to pay exceeds its willingness to pay. The government gains debt capacity by pushing default costs up, that is, with high repressive taxes that channel incremental household endowments entirely into savings in government bonds. Depending on the parameter set, there can be a steady state below a threshold endowment $\bar{e}^1$ such that for $\forall e < \bar{e}^1$, $t(e) = t^W > t^{**}$. The government represses investment so much with high taxes that the economy never escapes the WTP region, and the ability-to-pay constraint is rendered irrelevant.
- There exists a middle-e region (see Fig. 2, regions annotated "WTP & ATP") where the optimal solution for the government is to "slide" between the two constraints, *i.e.*, setting $\tau^{**} - z(1+r)s = 0$. In this region, the policy tax rate $t(e)$ is always strictly decreasing in e (see Fig. 2(b)). Essentially, the government channels incremental endowment into household investment (see Fig. 2(d)) by lowering taxes, which increases the household's future endowment and the future government's ability to pay. Marginal household productivity is high enough that the current government's borrowing capacity increases more than the foregone taxes. Household financial savings (see Fig. 2(c)) are constant so the

incremental borrowing is all foreign. The limit of this process is reached when household productivity falls enough at high enough investment that incremental reductions in the tax rate do not incentivize enough production and borrowing capacity to offset the loss in tax revenues. The limiting lower bound for the tax rate turns out to be the autarkic tax rate.

- There exists a high-e region (see Fig. 2, regions annotated "ATP") where only the ability-to-pay constraint is binding. Depending on the parameter set, there can be a steady state after a threshold endowment $\bar{e}^2$ such that for $\forall e > \bar{e}^2$, $t(e) = t^{**} := \text{argmax}_t\, \tau(t)$. Large-endowment economies have so much domestic savings that default is ruled out. However, when the willingness-to-pay constraint is not binding, the size of the government's surplus and its ability to borrow does not vary with the private sector endowment (see Fig. 2(e)). In this region, government debt capacity rises by less than the loss of tax revenues when taxes are lowered below the autarkic rate. So the government fixes taxes at the autarkic rate, and this does not vary with endowment. Household investment is commensurately fixed, and all incremental endowment goes into financial savings. In sum, a myopic government with a wealthy household sector taxes as if it has no access to debt, *i.e.*, our benchmark autarkic case.

[Fig. 2 about here]

We formalize the intuition from the example in Proposition 2.2. The proposition requires a set of regularity conditions set out in Definition 2.1, imposed mainly to ensure convexity and single-crossing properties of the derived functions. Any power production function of the form $f(k) = Ak^{\gamma}$ automatically meets regularity conditions A and B, and therefore will be used in all our numerical exercises throughout (as in Fig. 2). All proofs are in appendix C.

Definition 2.1. We assume that the following regularity conditions are met:

A. (Convexity of investment in t) $k(t)$ is decreasing and convex in t, from which it follows that private profit $\pi(t)$ is also decreasing and convex in t.

B. (Single-crossing properties) $\frac{k'(t)}{\pi'(t)}$ is decreasing in t, and $\frac{\tau'(t)}{\pi'(t)}$ is strictly increasing in t.

C. (Minimal government feasibility in autarky) $\tau^{**} > C$.

Proposition 2.2. *There is a unique bounded and weakly monotonic value function $S(e)$, and a corresponding policy function $t(e)$, that solve (2.16). Suppose that model's specifications satify the regularity conditions in Definition 2.1. Then, the solution has the following properties:*

(i) $S(e)$ is weakly concave, and $S'(e) \to 0$ as $e \to \infty$.

(ii) $\exists \hat{e}^1 \leq \hat{e}^2$ *such that for* $e < \hat{e}^1$*, only the willingness-to-pay constraint binds; for* $e > \hat{e}^2$*, only the ability-to-pay constraint binds; and, for* $e \in [\hat{e}^1, \hat{e}^2]$*, both constraints bind.*

(iii) $t(e)$ *is continuous, (weakly) increasing in the region* $e \in [0, \hat{e}^1]$*, (weakly) decreasing in the region* $[\hat{e}^1, \hat{e}^2]$*, and (weakly) increasing in the region* $[\hat{e}^2, \infty)$*. Also,* $t(e) \to t^{**}$ *as* $e \to \infty$*.*

3 Steady States and their Properties

Let us now characterize steady states and the path towards them. We first need some definitions regarding the growth path:

Definition 3.1. Given the solution program $t(e)$ from the Bellman equation (2.16) and the private sector reaction function (2.17)–(2.19), we define

- An endowment path $\{e_i\}_{i=0}^{\infty}$ as $e_{i+1} := e_+(e_i, t(e_i))$ starting at e_0. In addition, we define $e_\infty(e_0)$ as the limit (if it exists) of this endowment path: $e_\infty(e_0) := \lim_{i\to\infty} e_i$.
- Steady state (e^{ss}, t^{ss}) as a pair satisfying[11]

$$t^{ss} = t(e^{ss}), \text{ and} \tag{3.1}$$

$$e^{ss} = e \text{ such that } e = e_+(e, t^{ss}). \tag{3.2}$$

- As discussed earlier, consumption at the steady state $c^{ss} = \frac{1}{\rho(1+r)} e^{ss}$.

From Proposition 2.2, it must be the case that e^{ss} is in (i) the willingness-to-pay constraint region; or, (ii) the ability-to-pay constraint region; or, (iii) the "sliding" region. We derive the necessary conditions for the steady states should it exist in each of the three regions.

Suppose first that e^{ss} exists in the willingness-to-pay constraint region (region (i)). We note first that using the envelope condition as well as the definition $e^{ss} = e_+(e^{ss}, t^{ss})$, we can get the exact $\frac{dS}{de}$ at this point:

$$\begin{aligned} &\frac{dS}{de} = \kappa_1 \frac{dS}{de} + z\kappa_1 \\ \Rightarrow &\frac{dS}{de} = z\frac{\kappa_1}{1-\kappa_1} = \rho z. \end{aligned} \tag{3.3}$$

[11] In addition, a no-saddle-point condition is imposed as follows: $\exists\ \epsilon > 0$ such that for all $e \in (e^{ss} - \epsilon, e^{ss} + \epsilon)$, $e_\infty(e) = e^{ss}$. This excludes the measure-zero set of fixed-point endowments on which a small shock can push the endowment path away from the fixed point in the long run.

Also, the optimal t should also satisfy the FOC:

$$\frac{1}{1+r}\left[\frac{de_+}{dt}\frac{dS}{de}+z(1+r)\frac{ds}{dt}\right]+\tau'=0. \tag{3.4}$$

Plugging (3.3) into (3.4), we get the following characteristic equation:

$$\frac{de_+}{dt}\underbrace{\frac{dS}{de}}_{=\rho z}+z(1+r)\frac{ds}{dt}+(1+r)\tau'=0. \tag{3.5}$$

It is straightforward to see that the equation above is independent of e. Therefore, it follows that if such a steady state were to exist, the tax rate t^{ss} can be completely characterized from the model primitives, which we define as t^W. Then, the corresponding endowment e^{ss} can be derived simply by solving $e^W = e_+(e^W, t^W)$. We denote this as **steady state W**. Interestingly, t^W can be both greater or smaller than the autarkic tax rate t^{**}. We offer an in-depth discussion of this in Proposition 3.2.

Next, suppose that e^{ss} exists in region (ii), the ability-to-pay constraint region. The corresponding envelope condition and the FOC yield respectively

$$\frac{dS}{de}=\kappa_1\frac{dS}{de}\Rightarrow\frac{dS}{de}=0,\ \text{and} \tag{3.6}$$

$$\frac{de_+}{dt}\underbrace{\frac{dS}{de}}_{=0}+(1+r)\tau'=0. \tag{3.7}$$

Following the same logic as for case (i), it follows that, if such a steady state were to exist, the tax rate t^{ss} must be equal to $t^A = \text{argmax}_t\,\tau = t^{**}$. Again, e^{ss} in this region can be derived by solving $e^A = e_+(e^A, t^{**})$. Note that the steady-state taxation will be set at the debt autarky level, even though the government will be borrowing. We denote this as **steady state A**, which achieves the same endowment as the benchmark autarky case.

Finally, suppose that e^{ss} exists in region (iii). Since it is sliding between the constraints, and because it is a steady state, the following must be simultaneously met:

$$e=e_+(e,t),\ \text{and} \tag{3.8}$$

$$0=\tau^{**}-C-z(1+r)s(e,t). \tag{3.9}$$

We refer to the solution (e^S, t^S) for (3.8)-(3.9) as **steady state S**. The endowment in this steady state is higher than the benchmark autarky case.

In Appendix B, we formally characterize the three steady states A, W, and S, and argue why the limit of any endowment path must be one of them. We also discuss the conditions under which each of the steady state can exist. Importantly, when multiple steady states exist, the limit of an endowment path depends on the initial endowment; in particular, endowment paths starting from lower endowments converge to a lower steady state than those starting from higher endowments. This is the core reason why growth traps exist in our model.

We now turn to the central result of the paper, *i.e.*, whether access to international borrowing helps or hurts a country. For the benchmark case, we use the notation $\{e_n^{**}\}_{n=0}^{\infty}$ where $e_{n+1}^{**} = e_+(e_n^{**}, t^{**})$ and the corresponding steady state as e_∞^{**}.[12]

Proposition 3.1. *Access to sovereign borrowing can lead the government to set steady-state taxation at levels that are below or above the benchmark. Steady-state endowments and consumption vary correspondingly. Specifically :*

- *Suppose that $t^{**} < t^W$. Then, $e_\infty(e_0)$ is in general not independent of e_0, and $e_\infty(e_0) \leq e_\infty^{**}$ always. In particular, for a set of parameters of strictly positive measure, $\exists \bar{\bar{e}}$ such that*
  - *$\forall e_0 < \bar{\bar{e}}$, $e_\infty(e_0) < e_\infty^{**}$ **(Growth Trap)**, and*
  - *$\forall e_0 \geq \bar{\bar{e}}$, $e_\infty(e_0) = e_\infty^{**}$ **(Benchmark)**.*
- *Suppose instead that $t^{**} \geq t^W$. Then, $e_\infty(e_0)$ is independent of e_0 and $e_\infty(e_0) \geq e_\infty^{**}$ always. Depending on the parameter set,*
  - *e_∞ is either equal to e_∞^{**} **(Benchmark)**, or*
  - *e_∞ is strictly greater than e_∞^{**} **(Growth Boost)**.*

In order to graphically illustrate the growth dynamics for a myopic government that can borrow internationally, we show in Fig. 3 the simulated endowment paths. In Fig. 3(a), both steady states A and W exist. Therefore, the long-run endowments depend on the initial endowment. Indeed, it can be observed that economies starting at sufficiently low endowments may never escape the lower endowment region. The willingness-to-pay constraint will always be binding. The government is highly repressive, which leads the economy to a growth trap (in fact, the growth in endowment can be negative as seen in Fig. 3(a) for some starting endowments). The economy never converges to the benchmark steady state. However, if it were to start at a higher endowment, then the willingness-to-pay constraint is never binding, and the economy converges to the "better" steady state.

In the case of Fig. 3(b), only steady state A exists and there is no growth trap. Therefore, all economies eventually converge to the benchmark steady state. Obviously, poorer economies take longer to reach there.

[12]We exclude measure zero events as even a small perturbation would remove the possibility of their existence.

Finally, in Fig. 3(c), only steady state W exists, and the equilibrium tax rate is smaller than that of the benchmark case ($t^W < t^{**}$). Access to borrowing acts as a growth boost, and all economies converge to a better-than-benchmark equilibrium, no matter what endowment they start with. While not shown in the figure, steady state S behaves similarly to this case of a growth boost.

[Fig. 3 about here]

Proposition 3.1 begs the question which model parameters determine the steady state outcomes and how. We identify the household savings parameter ρ as the most critical parameter in determining the existence of growth traps; in particular, growth traps exist only for low-saving economies. Consider the following intuition. As mentioned before, the government in willingness-to-pay region trades off both the incentive to boost growth (horizon-lengthening effect of debt) as well as the repression incentive. Evidently, the boost incentive is greater for the governments of higher-saving economies because their endowment growths are more sensitive to taxation. Conversely, the repression incentive is larger for governments of economies that save sufficiently little, since more domestic financial savings are necessary for the government to borrow internationally.

We detail the preceding argument in Section B.1, which leads to the following result:

Proposition 3.2. *A necessary and sufficient condition for $t^{**} < t^W$, which is a necessary condition for the growth trap to exist, is an upper bound on the savings parameter ρ:*

$$t^{**} < t^W \Leftrightarrow \rho < \frac{1}{t^{**}}. \tag{3.10}$$

Low endowment countries with low propensities to save are particularly likely to have governments who repress in order to boost external borrowing, and thus push their countries into a growth trap. We can also show the following:

Proposition 3.3. *A sufficient condition for the economy to converge to the benchmark steady state is a lower bound on the propensity to save parameter ρ:*

$$\rho \in \left(\bar{\rho}, \frac{1}{r}\right), \;\; where\ \bar{\rho} < \frac{1}{r}. \tag{3.11}$$

The intuition is that with a high savings parameter, household endowments grow quickly, enabling the economy to escape from the willingness-to-pay region to the ability-to-pay region swiftly, and in turn, leading to convergence to the benchmark case. Combining the two results above (Propositions 3.2 and 3.3), we conclude that when households have a high propensity to save, sovereign debt can be (weakly) beneficial to growth even in the presence of a myopic and wasteful government.

Whether growth is strictly boosted by access to borrowing depends on whether the default cost parameter z, the importance of government bonds to the domestic financial sector, is sufficiently small. Here is why: Recall that the growth boost in our model occurs only when the economy's steady state remains in the willingness-to-pay region, which is when $\tau^{**} - C - zs(1+r) \geq 0$. Therefore, when z is low, $\tau^{**} - C - zs(1+r)$ stays positive and the willingness-to-pay constraint can remain binding for a longer duration; conversely, when z is high, the willingness-to-pay region is small and the steady state moves quickly to the benchmark steady state which is in the ability-to-pay region. These results on how the savings parameter ρ and the default cost parameter z affect the nature of the steady state (growth trap, benchmark or growth boost) are illustrated in Fig. 4. In sum, this suggests that developing countries with low z (recall that z in our model is related to the importance of government bonds in the financial sector's transactions, and is a measure of the sophistication or development of the country's financial system) and high propensities to save ρ will tend to benefit from access to foreign borrowing, even though their governments are self interested and myopic.

[Fig. 4 about here]

When the "growth boost" steady state exists, it is the unique steady state. When the "growth trap" steady state exists in the WTP region, it is also possible to have the benchmark steady state in the ATP region. Let us characterize these steady states more fully when they both exist before turning to implications.

Lemma 3.4. *Suppose that the model parameters admit two steady states, depending on the starting endowment e_0. Consider a steady state where all subsequent governments choose the same policies (t, D) with none defaulting. Then, equilibrium quantities chosen at the two steady states can be derived as the following, where* PV *stands for the "present value of":*

Steady state A. *In the ability-to-pay region steady state, the tax rate is t^{**} and the corresponding endowment is $e^A = e^{sat}(t^{**})$. The debt D^A, its domestic and foreign components, and government spending are:*

- $D^A = \frac{\tau^{**}}{r} = PV(\textit{future period tax revenue})$,
- $D^{Dom} = s(e^A, t^{**})$,
- $D^{For} = \frac{\tau^{**}}{r} - s(e^A, t^{**})$, *and*
- *Government spending* $= 0$.

Steady state W. *In the willingness-to-pay region steady state W, the tax rate is chosen at* $t^W > t^{**}$ *and the corresponding endowment is* $e^W = e^{sat}(t^W) < e^{**}$*. The debt* D^W*, its domestic and foreign components, and government spending are:*

- $D^W = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} = NPV$ *(future period tax revenue - spending),*
- $D^{Dom} = s(e^W, t^W)$*,*
- $D^{For} = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} - s(e^W, t^W)$*, and*
- *Government spending* $= \tau^{**} - C - z(1+r)s(e^W, t^W)$*.*

Interestingly, in the ability-to-pay region, the borrowing by the previous government leaves the current government with no room to spend. In contrast, the government in the willingness-to pay-region can spend $\tau^{**} - C - z(1+r)s(e^W, t^W)$. In steady state, all future governments will act in the exact same way, collecting taxes $\tau(t^W)$ and spending $\tau^{**} - C - z(1+r)s(e^W, t^W)$. It follows that the debt capacity of the government in this steady state equals to the present value of tax revenues, net of spending.

4 Discussion and Related Literature

4.1 Odious Debt

Our results are relevant to the debate on whether countries should have access to external debt or not. In particular, it might seem natural to declare the debt issued by myopic rapacious government "odious" and non-enforceable going forward, as suggested by some (see, for example, Buchheit, Gulati and Thompson [2006] and Jayachandran and Kremer [2006]). Yet, as we have just shown, it is possible that an "odious" government's incentives could be improved by access to borrowing. The key to the change in its behavior on gaining access to debt may not be the nature of the government (they are uniformly odious in our model thus far) but the nature of the country's environment – for instance, the propensity to save of households, the size of their endowment, or the centrality of government debt to the private sector's functioning (as captured in the default cost parameter). Governments may choose growth-enhancing policies relative to the autarky benchmark in order to boost their successor government's willingness to repay, and in turn, borrow more today; this dynamic enables the economy to experience a growth boost in the form of a steady-state endowment that is above the autarkic one. Odious government, therefore, does not always imply that access to borrowing has odious consequences.

Of course, we also show the converse possibility: access to borrowing can lead the government to repress its country into a poverty trap (Kharas and Kohli [2011]), especially if the country is poor (small endowments) and has a low propensity to save. Even so, because the country does not start with a blank slate, a declaration that the debt issued by the government is odious and unenforceable is not necessarily beneficial to its citizens. Such a declaration will immediately trigger default (since the government cannot borrow to repay legacy debt), which may be costlier to the country's citizens than keeping access open. It may be better, as we will see shortly, for the country to be eased into a better equilibrium through a combination of debt relief and debt ceilings. The odious debt declaration, while benefiting from being simple, may have unintended consequences.

One of them is for a country that does not currently have an odious government. The increased possibility that one of its successors could be deemed "odious" could reduce prospects for rolling over debt, and thus constrict the market for new debt issuance today. This too could precipitate costly default.

The broader point is that proposals to declare newly issued debt odious should take into account not just the use of the debt, but its effect on government incentives, its effect on the repayment of past debt, as well as the uncertainty they may create for regimes that are perfectly reasonable today, but could be followed at a future date by odious regimes. Since few countries can guarantee the quality of successor governments, the unintended consequences of proposals to declare debt "odious" on curtailing country access to borrowing and precipitating default could be quite substantial.

Note that our model focuses on the economic consequences of access to debt for self-interested governments, governments that hurt their citizenry only through oppressive taxation and wasteful spending. We do not explore the consequences of access to debt for governments that actually imprison, maim, and murder their citizens (or those of neighboring countries) freely, as in the characterization of "odious" governments in Bolton and Skeel [2007]. While some of the issues pertaining to murderous myopic governments are different, the incentive effects we have alluded to from access to international borrowing will not be entirely absent so long as the myopic governments rationally want to increase their resource base. Of course, in such situations, we will also have to model the negative utility to citizens from the government spending more on rifles and flame-throwers. Finally, our model considers international debt provided by investors who are at arm's length rather than strategic in their extension of credit (for instance, governments and public institutions who often provide "official debt" to odious governments to gain political leverage, as considered by Gelpern [2007]).

4.2 Why the Weak or Negative Correlation between Foreign Finance and Growth

A number of studies (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]) have explored whether countries that borrow more internationally do better (focusing on the *intensive* margin, while the "odious" debt literature focuses on the *extensive* margin). The surprising finding is a weak or even negative correlation between developing country growth and its use of foreign borrowing, within the set of countries that all have the ability to borrow internationally. In particular, Prasad et al. [2006] find that over the period 1970-2004, there is no positive correlation for nonindustrial countries between current account balances and growth, or equivalently, that developing countries that have relied more on foreign finance have not grown faster in the long run, and have typically grown more slowly. They conclude this runs counter to the predictions of standard theoretical models. Similarly, Aizenman, Pinto and Radziwill [2004] construct a “self-financing” ratio for countries in the 1990s and find that countries with higher ratios grew faster than countries with lower ratios.

Our model can shed light on this so-called “allocation puzzle". Suppose the differential reliance on foreign borrowing across countries arises due to differences across countries in the citizen’s propensity to save (ρ), keeping the nature of the government the same. We focus on the willingness-to-pay region or the sufficiently low endowment region which typically represents developing countries and emerging markets.

From Lemma 3.4, we can decompose $\frac{D^{For}}{e^W}$ as the following:

$$\frac{D^{For}}{e^W} = \underbrace{\frac{\tau(t^W)/r}{e^W}}_{\sum \text{tax revenues}} - \underbrace{\frac{(\tau^{**} - C - z(1+r)s(e^W,t^W))/r}{e^W}}_{\text{willingness-to-pay wedge}} - \underbrace{\frac{s(e^W,t^W)}{e^W}}_{\text{domestic debt}} . \tag{4.1}$$

As ρ increases, the steady-state endowment is higher mechanically as households prefer endowment over consumption, but the repressive tax rate t^W decreases (see Figures 5(a) and (b)). As a result, the first term on the right hand side in (4.1), which is proportional to tax revenues and inversely proportional to endowment, is decreasing.

[Fig. 5 about here]

However, rearranging slightly, the other terms on the right hand side are increasing in ρ. Since e^W increases with ρ, $-\frac{(\tau^{**}-C)}{e^W}$ is increasing in ρ. Furthermore, $\frac{s(e^W,t^W)}{e^W}$ is multiplied by a positive coefficient ($z > 1$ which implies that $z\frac{(1+r)}{r} - 1 > 0$). This term is increasing in ρ since savings increase at a faster rate than the endowment as ρ increases.

When z is low, the first term in (4.1) can dominate and $\frac{D^{For}}{e^W}$ may be decreasing in ρ, as shown in Figure 5(e), whereas e^W is increasing in ρ regardless of z (Figures 5(c) and (d)). This gives rise to a *negative* relation between the foreign debt to endowment ratio and the steady-state endowment.

In contrast, when z is high, the term containing $\frac{s(e^W,t^W)}{e^W}$ dominates the decrease in repression so that the foreign debt normalized by endowment is increasing in ρ, giving rise to a *positive* relation between the foreign debt to endowment ratio and steady-state endowment.

Recall that z in our model is related to the importance of government bonds in the financial sector's transactions, and is a measure of the sophistication or development of the country's financial system. Hence in developing countries with low financial development, a higher propensity to save can drive the steady-state endowment up and the extent of foreign borrowing down, and conversely a lower propensity to save can drive the steady-state endowment down and the extent of foreign borrowing up. To the extent that the steady-state endowment proxies for measures of well-being such as consumption and growth, we generate the negative relationship between foreign borrowing and these measures documented in the literature. Interestingly, the relationship for countries with more sophisticated financial systems may be different, an implication for which there is some evidence (see, for example, Prasad et al. [2006]).

Our model clarifies the broader point that *ceteris* is not *paribus* across countries, so the relationship between foreign borrowing and economic growth may be confounded by the endogenous selection of which countries rely more on foreign borrowing. This can be driven by variation in other factors that affect both foreign borrowing and steady-state endowments – in our case, by the country's propensity to save. Put differently, it is not that foreign financing is necessarily bad for developing country growth, but that some of the countries that are seen to have more foreign financing (because of low endowments and low propensities to save) may also have greater repression.

There are, of course, other explanations. Gourinchas and Jeanne [2013] conclude that poorer countries are poor because they have lower productivity or more distortions than richer countries, not because capital is scarce in them – the implication being that access to foreign capital by itself would not generate much additional growth in these countries. In contrast, we argue that distortions are lower in countries that have substantial domestic savings.

Aguiar and Amador [2011] argue that high borrowed foreign debt can lead to an underinvestment problem for myopic governments that also have the ability to expropriate capital in future, giving rise in their model to a reduction in the accumulation of capital stock and in the speed of convergence to the steady state. The steady state, however, remains unaffected by these government distortions. In contrast, our model's implication is that when government myopia is combined with wasteful expenditures, there can in fact be a *permanent* impact on endowments for developing economies: The steady-state endowment can be trapped below the debt autarky levels, as the government taxes heavily and discourages private investment.

4.3 Relationship to the Literature

There is a vast literature on sovereign debt that we have benefited from but cannot do justice to, including, but not limited to, Eaton and Gersovitz [1981], Bulow and Rogoff [1989*a*], Bulow and Rogoff [1989*b*], Fernandez and Rosenthal [1990], Eaton and Fernandez [1995], Cole and Kehoe [1998], Guembel and Sussman [2009], Reinhart and Rogoff [2010], Amador [2012], and Tomz [2012]. Our paper is most related to an emerging literature that embeds a cost of sovereign default that is tied to the extent to which the economy's private sector is entangled with sovereign debt. Specifically, we build on Acharya and Rajan [2013], who present a two-period (three-date) model of sovereign debt with a myopic wasteful government. Given their model, they cannot examine long-run or steady-state equilbria, nor do they address the choice between consumption, investment, and savings by the household sector. Our model enables us to examine dynamics, wherein lie the key results of our paper.

Basu [2009], Bolton and Jeanne [2011] and Gennaioli, Martin and Rossi [2014] relate the costs of sovereign default to the amount of debt held by domestic banks. They examine the trade-offs between more credible sovereign borrowing (when domestic banks hold more sovereign bonds) against the greater costs when the sovereign defaults. A version of this trade-off is also in our model, but our focus is on how access to sovereign borrowing can alter long-run growth. Moreover, our fundamental assumption – of myopic wasteful governments – is different from these papers.

We have examined repression from the perspective of developing countries attempting to grow. Our model allows for both real and financial represssion, but has little to say on the relative magnitude of each. Reinhart, Kirkegaard and Sbrancia [2011], Reinhart [2012], Reinhart and Sbrancia [2015], and Chari, Dovis and Kehoe [2020] look at financial repression as a way to ease the debt repayment burden for a rich country that has suddenly experienced a large accumulation of debt (due to crisis or war). Roubini and Sala-i Martin [1992] model financial repression as a way for governments to raise "easy" resources for the public budget when tax evasion by the private sector is high, with consequent effects on efficiency of the financial sector and long-run growth. An interesting avenue for research is to compare the nature of repression in industrial countries with repression in developing countries, and to compare their relative deadweight costs in terms of effects on long-run growth.

5 Policy Instruments for Escaping the Trap

We now discuss possible policy instruments to help economies escape from, or remove, the poverty or growth trap we have identified earlier.

5.1 Debt ceiling

The primary reason economies are trapped is because their governments adopt repressive policies in order to enhance borrowing. Therefore, a natural policy instrument would be to cap the government's ability to borrow with a constitutional debt ceiling (as, for example, in Germany) or through a common understanding imposed by external lenders (as, for instance, in the call for multilateral agencies like the IMF to monitor and limit debt build up in poor countries). An extreme version would be to declare all new debt "odious" and set the debt ceiling at zero.

Suppose that debt ceiling takes the general form $\{\bar{D}_i\}_{i=0}^{\infty}$ where each government i faces the debt ceiling $\bar{D}_i$. Conditional on not defaulting, government's actions are independent of past government debt ceilings and legacy debt, but not of future debt ceilings. Let us denote the current government's spendable surplus as $S(e;\bar{D}_0,\bar{D}_1,\ldots)$. We can show $S(e;\bar{D}_0,\bar{D}_1,\ldots)$ exhibits the following intuitive property:

Proposition 5.1. *$S(e;\bar{D}_0,\bar{D}_1,\ldots)$ is weakly decreasing in all debt ceilings, $\bar{D}_i$, current ($i=0$) and future ($i>0$). It follows that lowering the debt ceiling – whether for the government itself or future governments – weakly decreases the current government's ability to spend.*

We now consider a special form of debt ceiling where $D_i = \bar{D}\ \forall i$ (flat debt ceiling). Let us define $e_\infty(e_0;\bar{D})$ as the limit of the endowment sequence under debt ceiling $\bar{D}$. We first prove that

Proposition 5.2. *(Optimal debt ceiling). Suppose that $t^{**} < t^W$ (corresponding to the trap case). Then, in general $e_\infty(e_0) \le e_\infty(e_0;\bar{D})$. In particular, there exists a threshold debt ceiling $\bar{\bar{D}} = D^W$ such that for all $\bar{D} < \bar{\bar{D}}$, $e_\infty(e_0;\bar{D}) = e_\infty^{**}$ for all e_0, completely removing the trap. Recall, from Proposition 3.1, that $e_\infty(e_0) \le e_\infty^{**}$ without the debt ceiling.*

Suppose instead that $t^{**} > t^W$. Then, in general $e_\infty(e_0) \ge e_\infty(e_0;\bar{D})$. Similarly, $\exists\bar{\bar{D}}$ such that for all $\bar{D} < \bar{\bar{D}}$, $e_\infty(e_0;\bar{D}) = e_\infty^{**}$ for all e_0. Recall, from Proposition 3.1, that $e_\infty(e_0) \ge e_\infty^{**}$ in the original problem without debt ceiling.

In summary, the best that the debt ceiling can achieve when there is a growth trap is the benchmark steady-state endowment e_∞^{**}. In this case, it can help enhance long-run growth; conversely, when debt in the presence of government myopia boosts growth, a debt ceiling can hurt long-run growth.

One way to see this intuitively is to analyze the marginal incentives for a myopic government in the short run. Recall the original Bellman equation and suppose for simplicity that e is in the willingness-to-pay region:

$$t(e) = \underset{t}{\operatorname{argmax}} \frac{1}{1+r}\Big[S(e') - \tau^{**} + C + z(1+r)s\Big] + \tau(t).$$

Recall that the myopic governments' optimal taxation was chosen by trading off the incentive to boost ($\frac{de'}{dt}\frac{dS}{de} < 0$) and to repress ($\frac{ds}{dt} > 0$). We consider two cases:

- The debt ceiling is imposed only on the current government. In this case, the problem is changed to

  $$t(e) = \underset{t}{\operatorname{argmax}} \frac{1}{1+r}\Big[\min\{S(e') - \tau^{**} + C + z(1+r)s, \bar{D}\}\Big] + \tau(t).$$

  If $\bar{D}$ is low enough so that $S(e') - \tau^{**} + C + z(1+r)s$ is greater than or equal to $\bar{D}$, then the government's marginal incentives to both boost or repress disappear. Therefore, the government would simply choose $t = t^{**}$ that maximizes $\tau(t)$.

- The debt ceiling is imposed on all future governments but not on the current government. In this case, the problem is changed to

  $$t(e) = \underset{t}{\operatorname{argmax}} \frac{1}{1+r}\Big[S(e';\bar{D}) - \tau^{**} + C + z(1+r)s\Big] + \tau(t).$$

  The incentive to repress remains unchanged; however, because $S(e')$ is constrained by $\bar{D}$ in some states of the world, the incentive to boost is lower. Therefore, the government engages in even higher repression than without debt ceiling.

Given that a flat ceiling is a combination of the debt ceiling now and a debt ceiling starting tomorrow for ever, it follows that a debt ceiling either moves the tax rate to the benchmark tax rate t^{**}, or induces the government to repress even more. It follows that if $t^{**} < t^W$, then the debt ceiling could improve the steady state by achieving the benchmark steady state instead. On the other hand if $t^{**} > t^W$, then the debt ceiling always hurts when it is binding. Fig. 6 offers an illustration; in Fig. 6(a), the debt ceiling is placed on the parameter case where the growth trap exists ($t^{**} < t^W$). In can be observed that the debt ceiling generally reduces the tax rate for most values of endowment; in Fig. 6(b), the debt ceiling is placed on the parameter case where the growth boost exists ($t^{**} > t^W$). In this case, the debt ceiling increases the tax rate everywhere.

[Fig. 6 about here]

Finally, we should note that a debt ceiling is a less abrupt way of nudging an irresponsible borrowing government into responsibility than simply declaring its debt odious. It is likely to embed a lower expected cost of default. Indeed, when combined with debt relief which we explore next, the default costs can be avoided entirely.

5.2 Debt relief

Consider now debt relief, that is, forgiveness of a certain amount of the face value of debt. Debt relief alone is inconsequential in our model. It simply allows the current-period government to increase spending by the amount of the relief.

Lemma 5.3. *In an equilibrium path, any debt relief in a period is transfered one-to-one to government spending in that period. The ensuing tax rates and endowment paths remain unchanged.*

This is not very far from reality. Of the 36 countries that received significant official debt relief under the Highly Indebted Poor Country (HIPC) Initiative and Multilateral Debt Relief Initiative (MDRI) in the early 2000s, 15 were either back in debt distress or had a high risk of debt distress by 2019. Another 13 had a moderate risk of debt distress.[13] Even the remaining did not all have a low risk of debt distress – some simply did not produce the data to compute debt sustainability.

However, when coupled with a debt ceiling, debt relief can be beneficial in moving a country to a better equilibrium. Suppose, that the debt ceiling was not initially in place and governments are trapped in steady-state W equilibrium (*i.e.*, the scenario analyzed in Lemma 3.4). Only a debt ceiling below the steady-state level of debt will have effect, but imposing it will cause the country to default, thus causing it to incur the deadweight costs. Therefore, if default is a dominated option,[14] any attempt to impose a debt ceiling should first be preceded by debt relief so as to avoid immediate default.

Formally, let the debt amount be reduced by fraction λ. Our debt restructuring scheme then can be summarized by a pair $(\lambda, \bar{D})$. We analyze how various restructuring schemes $(\lambda, \bar{D})$ can affect the utilities of different interested parties.

We first take the perspective of external creditors. Clearly, creditors want no debt relief since their claims are being serviced, and their utility is decreasing in the amount of debt relief. Therefore, assuming a debt reduction has to be undertaken, they would want to minimize λ

[13] See https://www.worldbank.org/en/topic/debt-sustainability#2 for a list of countries and the risk of debt distress prepared by the World Bank.

[14] Note that default followed by debt autarky can ameliorate repressive taxation and potentially help the economy move from a growth trap to a higher steady state, as we will see later in Section 6.1. Here, we focus on the case where default is not welfare-improving in the long run.

given $\bar{D}$, such that relief is enough to prevent default. Intuitively, λ required to prevent default is a decreasing function of the debt ceiling $\bar{D}$, as a lower ceiling constrains the government's resources more. By Proposition 5.2, lowering $\bar{D}$ eventually gets the economy out of the trap. It follows, then, that finding an efficient scheme can be reduced to finding the threshold debt ceiling $\bar{\bar{D}}$ at or below which the economy escapes the trap. It is intuitive to conjecture that the threshold $\bar{\bar{D}}$ is smaller than the debt issued in steady state W, as anything higher is not going to change the current and subsequent government's behavior.

We formalize this argument in Proposition 5.4.

Proposition 5.4. *For any debt ceiling $\bar{D}$, debt relief λ prevents government default if and only if*

$$\lambda \geq \lambda^{min}(\bar{D}) := 1 - \frac{S(e^W;\bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D^W_{-1}}.$$

Since $S(e^W;\bar{D})$ is increasing and continuous in $\bar{D}$, $\lambda^{min}(\bar{D})$ is decreasing and continuous in $\bar{D}$.

A debt restructuring scheme that minimizes λ while ensuring no default as well as no growth trap $(e_\infty = e^{**})$ can be characterized as choosing the debt ceiling $\bar{\bar{D}}$ that is arbitarily smaller than the current level of debt

$$\bar{\bar{D}} := D^W = \frac{\tau^W - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r},$$

and choosing a λ arbitarily close to 0. At this debt ceiling, the tax rate is initially arbitarily close to t^W as well.

Fig. 7(a) illustrates the patterns exhibited by $\lambda^{min}(\bar{D})$ and $e_\infty(\bar{D})$. Note first a sharp discontinuity of $e_\infty(\bar{D})$; for $\bar{D}$ higher than the steady-state level D^W, the trap is unchanged. For $\bar{D}$ slightly lower than D^W, the trap is suddenly removed. However, $\lambda^{min}(\bar{D})$ is continuous in $\bar{D}$, and need only be vanishingly small. Essentially, the debt ceiling dislodges the country from the trap steady state, and the ensuing dynamics take it to the ability to pay region. In sum, while some debt relief is required, when coupled with a debt ceiling just below D^W, the debt relief can be an arbitarily small amount to get the economy out of the growth trap without default.

Next, we take the perspective of the long-run interest of the private sector. It cares about the discounted sum of consumption by the households. This depends on how fast the economy converges to the ability-to-pay steady state A after the debt ceiling has been placed. Interestingly, while the levels of debt ceilings do not affect the level of long-run endowment once the debt ceiling is below the threshold $\bar{\bar{D}}$ – as stated in Proposition 5.2 – lower debt ceilings induce faster convergence to the long-run endowment. Fig. 7(b) illustrates this point. At a debt ceiling just below the threshold (99.95% of level of debt in steady state W (D^W) in this parameter set), it takes about 100 periods for the economy to reach the benchmark steady state, whereas a lower debt ceiling (80% in the figure) achieves it in 40 periods. Intuitively, governments do not start charging the autarkic tax rate right away; if the debt ceiling is just below D^W, they will set the tax rate just below τ^W and only slowly will it decline to the autarkic tax rate. Convergence is faster when the debt ceiling is set lower and debt relief is set accordingly higher, as can be seen in Fig 7(c).

[Fig. 7 about here]

Formalizing the preceding argument:

Proposition 5.5. *Suppose that model parameters admit a trap equilibrium, and that the economy initially is trapped at endowment e^W. Suppose now that a permanent debt ceiling $\bar{D}$ is placed at $t = 0$, along with adequate levels of debt relief such that the debt ceiling does not trigger default. Let $\{t_i^{\bar{D}}\} := \{t_0^{\bar{D}}, t_1^{\bar{D}}, \ldots\}$ denote the collection of tax rates that the governments in periods $i = 0, 1, 2, \ldots$ charge, and similarly, let $\{e_i^{\bar{D}}\} := \{e_0^{\bar{D}}, e_1^{\bar{D}}, \ldots\}$ be the corresponding endowments. Then, for two debt ceilings $\bar{D}^1 < \bar{D}^2$, $t_i^{\bar{D}^1} \leq t_i^{\bar{D}^2}$ holds for all $i \in \mathbb{Z}_+$. This immediately implies that $e_i^{\bar{D}^1} \geq e_i^{\bar{D}^2}$ for all i as well.*

Propositions 5.4 and 5.5 show that there is an understandable conflict of interest between creditors and the domestic private sector on the extent of government debt haircuts. While creditors would prefer the minimum debt relief that allows the country to escape the growth trap, the domestic private sector would prefer higher levels of debt relief for faster convergence to the steady state. In reality, debt renegotiation will be a bargaining process, taking these and other factors into account.

It should also be noted that debt ceilings are inherently time-inconsistent. While suitable debt relief combined with a debt ceiling is in the present government's incentive, it is not in the future governments' incentive; future governments benefit, if possible, from removing or relaxing the debt ceilings and increasing their spending by borrowing more. And future creditors have an incentive to lend. Therefore, the bargaining between creditors and the present government may potentially break down should the creditors anticipate that there is a lack of commitment on future governments' or creditors' behavior in complying with the debt ceilings.

Finally, the knife-edged nature of debt ceilings and debt relief (no effect above a threshold ceiling, large effects below so minor debt relief is enough) are largely driven by the fact that in the model there is no uncertainty and all parameters are exactly known. In the presence of various forms of uncertainties, the minimum debt relief would likely be higher.

6 Policy Response to Unexpected Shocks

Let us now analyze the effects of unexpected shocks to model parameters. We focus on our benchmark case where the model exhibits both steady states A and W, as defined in Lemma B.1. We again assume that the model economy has stayed at either of the steady states for a long enough time, such that the endowment, taxes, and debt issuances all follow quantities defined in Lemma 3.4. The shock occurs shortly before the end of the period.

Specifically, we consider a shock to the current endowment e; a permanent shock to the propensity to save ρ; a permanent shock to private sector productivity ϕ which level-shifts the production function $f(k) \to \phi \times f(k)$; and a permanent shock to the interest rate r. We analyze the effects of these shocks on (i) the current government's decision to default, and (ii) the steady states. We first consider the impact of small shocks, and next, that of large shocks.

6.1 Small shocks

Proposition 6.1. *Consider the spendables function $S(e;\rho,\phi,r)$ where ρ, ϕ, and r are savings parameter, productivity parameter, and interest rate, respectively. Partial derivatives of the spendables function with respect to e, ρ, ϕ, and r (for sufficiently low r), at steady state A and steady state W, are as follows:*

$$\frac{\partial S}{\partial e}\Big|_{e^W} > 0, \quad \frac{\partial S}{\partial \rho}\Big|_{e^W} > 0, \quad \frac{\partial S}{\partial \phi}\Big|_{e^W} < 0, \quad \frac{\partial S}{\partial r}\Big|_{e^W} < 0; \text{ and}$$
$$\frac{\partial S}{\partial e}\Big|_{e^A} = 0, \quad \frac{\partial S}{\partial \rho}\Big|_{e^A} = 0, \quad \frac{\partial S}{\partial \phi}\Big|_{e^A} > 0, \quad \frac{\partial S}{\partial r}\Big|_{e^A} < 0.$$

At steady states, a shock triggers default if and only if it decreases current spendables S. It follows then that

1. *In steady state W, a negative shock to endowment e, a negative shock to savings ρ, and a positive shock to productivity ϕ, all trigger default.*
2. *In steady state A, a negative shock to productivity ϕ triggers default.*
3. *A positive shock to interest rate r triggers default in both steady states.*
4. *Endowments in both steady states are positively related to savings ρ and productivity ϕ:*

$$\frac{\partial e^W}{\partial \rho}, \quad \frac{\partial e^W}{\partial \phi} > 0; \text{ and}$$
$$\frac{\partial e^A}{\partial \rho}, \quad \frac{\partial e^A}{\partial \phi} > 0.$$

Perhaps the most intriguing part in Proposition 6.1 is the fact that government spendable in steady state W is negatively related to the productivity parameter. This is driven by two forces: (i) An increase in productivity induces a decrease in financial savings by the private sector; in steady state W, this drives down the government debt capacity. (ii) An increase in productivity also increases tax revenue in case of default, which weakens the government's commitment to not default, thereby further reducing the debt capacity. Lower debt capacity will in turn trigger default if the government had previously maximized borrowing.

In the willingness-to-pay steady state, even a small negative endowment shock causes default. However, somewhat counter-intuitively the shock may be beneficial in the long run: Because the next-period government is in autarky, it charges the autarkic tax rate, which is lower than the original repressive tax rate; as a result, the economy gets a large push to growth in the following period. In some cases, this boost in growth can be large enough to eventually get the economy out of the growth trap it was originally in.

[Fig. 8 about here]

Panel (a) of Fig. 8 illustrates this result; the economy initially in steady state W is given a small shock (5% of original endowment) in period 10, causing a sovereign default. However, it can be noticed in the following period, the government charges the autakic tax rate which boosts growth significantly. This boost is large enough to counter the effects of the initial contraction so that in the long run the economy converges to the higher steady state A.

In contrast, an economy initially in steady state A is impervious to small endowment shocks. In panel (b), such an economy is given a small shock (5% of original endowment). This does not trigger government default; in this sense, government debt of this economy is a "safe haven." The economy goes through a minor contraction but bounces back to its original path.

6.2 Large shocks

To show that large shocks can lead to significantly different implications compared to small shocks, we focus on shocks to endowment.

In panel (c) of Fig. 8, the economy initially in steady state W is given a large shock (50% of original endowment). In this case too, the government defaults; however, unlike the case of a small shock (panel (a)), the economy is unable to recover from the initial shock in spite of the short-term boost to growth and converges back to steady state W.

In fact, panel (d) shows that a large shock can cause even the government of the economy initially in steady state A to default, unlike the case of a small shock (panel (b)). With a large shock, the economy is pushed into a growth trap and the endowment only converges to the lower steady state W.

6.3 Policy implications

Our analysis of the impact of small shocks shows that policy intervention might be unnecessary in response, even when such shocks lead to sovereign defaults, as in the case of low-endowment economies. However, this is not the case while considering the impact of large shocks.

We write this paper when the world is enveloped by the Covid pandemic, arguably a large endowment shock both to developing countries and developed ones.

First, consider the implications for developing countries, especially those with myopic self-interested governments. The pandemic clearly reduces production, taxes, future endowments, and the government's ability to service debt, possibly pushing these countries into growth traps. Furthermore, the nature of the shock is such that the government must undertake socially useful healthcare expenditures and also boost fiscal transfers to boost household endowments. Our model suggests that an efficient mechanism to help the developing economy recover well from such a shock could be "targeted relief," *i.e.*, a combination of (i) debt relief to avoid the default costs which can be a significant shock to government resources; and, (ii) continued access to debt markets, with the utilization of proceeds from debt issuance monitored (perhaps by a multilateral agency) for specific deployment toward containing the pandemic and its economic fallout. Within the context of our model, even myopic self-interested governments will have some interest in containing the pandemic and helping households survive – the fruits of that spending will be reaped within their horizon. However, they have little interest in spending that has benefits outside that horizon, so they will underspend relative to the socially desirable level, and access to borrowing will not help them spend better. Therefore, some amount of monitoring of the targeted relief is warranted.

Second, it may be argued that the lack of commitment to repay is a problem irrelevant to the governments of developed countries. However, with a large negative shock in household endowment and the ensuing rise of public debt relative to GDP, as witnessed during the Global Financial Crisis and the recent COVID-19 outbreak, this argument has perhaps weakened. In particular, the lack of commitment to repay, the associated incentive to repress the economy – economically and financially, and the resulting vulnerability to growth traps, may be very pertinent in the not-distant future, even for some rich industrial countries that have experienced large shocks. Therefore, while the debate on the sustainability of external sovereign debt has primarily focused on developing countries, our model's policy implications may have some bearing on sovereign debt more generally, including in industrial countries.

7 Extensions

Thus far, we have assumed the government spends wastefully. Let us now consider two ways the government can behave better without going all the way back to a benevolent long-horizon government.

7.1 Productive Government Investment

Assume the government has access to a productive technology which yields a cash flow of $g(I)$ for the government in the next period, in return for today's investment I. This is best thought of as investment in a state-owned steel plant or a toll road. We assume that the investment is made at the end of current period, when the government undertakes other spending, and the return of the investment is at the end of the next period. We assume that the government technology g satisfies Inada conditions, *i.e.*, $g'(0) \to \infty$, $g' > 0$, $g'' < 0$.

Since $g(I)$ is created only in the next period, the myopic current government does not enjoy the future cash flow *per se*. However, non-zero investment may still be in the government's incentive if it increases its debt capacity. Importantly, the government will invest if it is in the ability-to-pay region, but not if it is in the willingness-to-pay region.

To see this, suppose for simplicity that the next period government's total surplus is fixed at S and the option to invest in technology g is only available to the current government. Note that the next period government's ability-to-pay constraint, with respect to the current government's debt issuance D and investment I is now :

$$D(1+r) \leq S + g(I) \Rightarrow D \leq \frac{1}{1+r}\left(S + g(I)\right) . \tag{7.1}$$

Clearly, if the next period government is constrained by the ability to pay, an investment in government technology I increases the debt capacity of the current government by $\frac{1}{1+r}g(I)$. In contrast, the next government's willingness-to-pay constraint is:

$$S + g(I) - D(1+r) \geq \tau^{**} - \text{default cost} + g(I) \tag{7.2}$$

$$\Rightarrow D \leq \frac{1}{1+r}\left[S - \tau^{**} + \text{default cost}\right] . \tag{7.3}$$

Interestingly, if the next period government is constrained by the willingness to pay, investment does not help the current government's debt capacity at all. Although the incremental cash flow $g(I)$ increases the net spending by the future government in case it honors the legacy debt, it also increases its net spending in the default state by exactly the same amount. The two effects offset each other so that the debt capacity is left unchanged in the willingness-to-pay region.

We illustrate it in Fig. 9. The corresponding formal results are summarized in Lemma C.9.

[Fig. 9 about here]

As we have noted earlier, countries with low endowments (developing countries) are likely to be in the willingness to pay region. The government of the developing country cannot take advantage of public investment opportunities, not because it is less capable or more corrupt than a rich-country government, but because the willingness-to-pay constraint binds more strongly. Effectively, public investment does nothing to alleviate this constraint, so the government sees no value in such investments. Developing country governments, according to the model, are not intrinsically bad, their circumstances give them less incentive to be good.

7.2 Fiscal transfer

If private endowments matter, can the government transfer some of its funds to households to get the economy out of a growth trap? Assume at the end of the period, the government simultaneously engages in three actions we have already considered so far, as well as a new one: (a) raises debt by selling bonds; (b) raises taxes; (c) pays back its legacy debt; and, in addition, (d) shares some of the surplus with the households, spending the rest. We assume the sharing is not foreseen in prior periods and one-off, meant to dislodge the economy from the repressive steady state. We also assume that the present government is perfectly committed to the announced transfer at the end of the period, and this is understood by households at the beginning of the period when they choose investment.

The myopic government may have a private incentive to engage in the fiscal transfer, because the anticipated increase in the household endowment increases the government's debt capacity, which ultimately increases its spending today.

Recall that a government with endowment e has the objective function to maximize:

$$\begin{aligned}\text{spending} &= S(e) - D^{legacy}(1+r)\\ &= \max_t \quad \left[\frac{1}{1+r}\left[S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}\right] + \tau(t)\right] - D^{legacy}(1+r).\end{aligned}$$

Suppose that the government can take out $\Delta e \geq 0$ from its spending and transfer it to households at the end of the period. Under the assumption that $e = e^W$, we know that (i) the next period endowment is also e^W, and (ii) from (3.3), the marginal sensitivity of optimal t to endowment is zero, so that $\frac{dS}{de} = \rho z$. Therefore, collecting only the terms dependent on Δe, we have that

$$\text{spending} = \frac{1}{1+r} S(e^W + \Delta e) - \Delta e.$$

This immediately implies that there is a positive Δe that increases the objective function if and only if $\frac{\rho z}{1+r} > 1$. In Fig. 10(a), we plot spending as a function of Δe, alongside Fig. 10(b) which plots the endowment path after the transfer privately optimal for the government. Clearly, for some parameters, there is a non-zero fiscal transfer that increases the government's spending.Therefore:

Proposition 7.1. *Let the model parameters admit a trap equilibrium. There is a non-zero fiscal transfer to the households that increases the government's spending if and only if* $\rho > \frac{1+r}{z}$*. The fiscal transfer that maximizes the government's spending can be large enough that the economy escapes the growth trap.*

[Fig. 10 about here]

Notice again the importance of household savings. We have established in Proposition 3.2 that the trap occurs only if $\rho < \frac{1}{t^{**}}$. Proposition 7.1 shows that as the savings parameter falls even further, such that $\rho < \frac{1+r}{z} < \frac{1}{t^{**}}$, the myopic government will not engage in growth-friendly fiscal transfer, even given a chance; it will do so only for $\rho \in \left(\frac{1+r}{z}, \frac{1}{t^{**}}\right)$.

Note also that a substantial degree of commitment is required for the government to find these fiscal transfers worthwhile. For after announcing the transfer and affecting household investment, the government has an incentive to renege on the transfer. As such, this exercise suggests the very high degree of commitment required to get away from the growth trap in the baseline model. Implicitly, it also suggests some robustness to it and the results in Section 2.

8 Conclusion

We analyzed the effects of access to debt under the assumption that the government is myopic and spends wastefully. The key takeaway is that sovereign debt is a double-edged sword. When the economy is poor or has a low propensity to save, access to debt can lead to a growth trap where the economy's steady state is worse than under debt autarky (without access to debt) as the government adopts repressive policies to channel domestic savings to government bonds; in other cases, however, access to debt can extend a myopic government's horizon, resulting in steady states that are the same as or even better than autarky. When debt induces a growth trap, policy instruments such as debt ceilings and fiscal transfers can be effective, provided there is adequate commitment to enforce them. Some of these implications are worthy of further empirical investigation.

There are a number of extensions that are possible to our model. Our model considered sovereign debt only in the form of short-term or one-period contract. It turns out that long-term debt does not lead to any different outcomes under the assumptions that (i) any default by the government on any portion of the debt that is due in a period triggers cross-default clauses on all other debt; and (ii) the resulting default costs are therefore linked to the domestic portion of *all* outstanding debt. Since governments are myopic and care only about the current-period spending, it is immaterial to outcomes whether their ability to spend is reduced by their having to repay all legacy short-term debt, or whether their ability to issue debt is lowered by the stock of legacy long-term debt. In either case, the government can tap all debt capacity into the indefinite future regardless of the maturity of debt issued.

Another extension would be to allow the government to have a longer horizon. It can be shown that if the government is sufficiently far-sighted in nature (as characterized by its discount rate on future spending), then its capacity to borrow can collapse leading to autarky. The collapse in access to borrowing naturally improves economic outcomes when access to debt leads to a growth trap and worsens them when such access leads instead to a growth boost.

An interesting extension could be to allow uncertainty in the model. The key difference in this extension would be the optimal choice of the myopic government between issuing large quantities of risky debt or smaller quantities of riskless debt. We conjecture that similar trade-offs would arise in the choice between risky debt and safe debt. When the government issues risky debt, the level of endowment in the future high-endowment states matters for the government, and therefore the government will have an extra incentive to boost growth by lowering tax rates. This effect will be much attenuated if the government issues smaller quantities of safe debt. However, risky debt exposes the economy to the costs of government default in low-endowment states, as well as other adverse spillovers such as the reduced ability of real and financial sectors to use government bonds as safe collateral in borrowing contracts. There is clearly scope for more research.

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A Figures

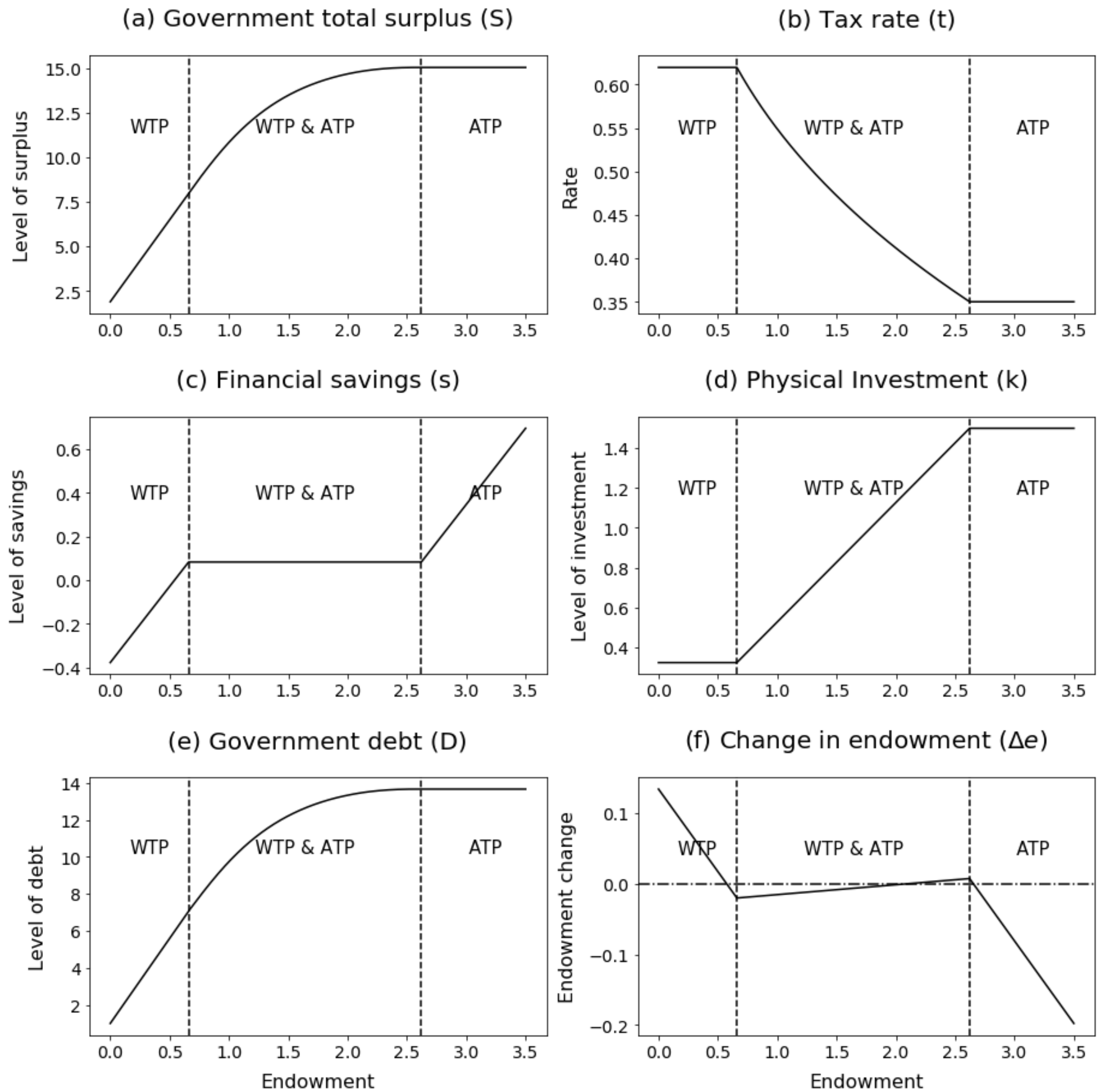

Figure 2: Solution from the baseline model, with parameters $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$. "WTP" stands for willingness-to-pay region; "ATP" for the ability-to-pay region; and "WTP & ATP" for the sliding region where both willingness-to-pay and ability-to-pay constraints bind.

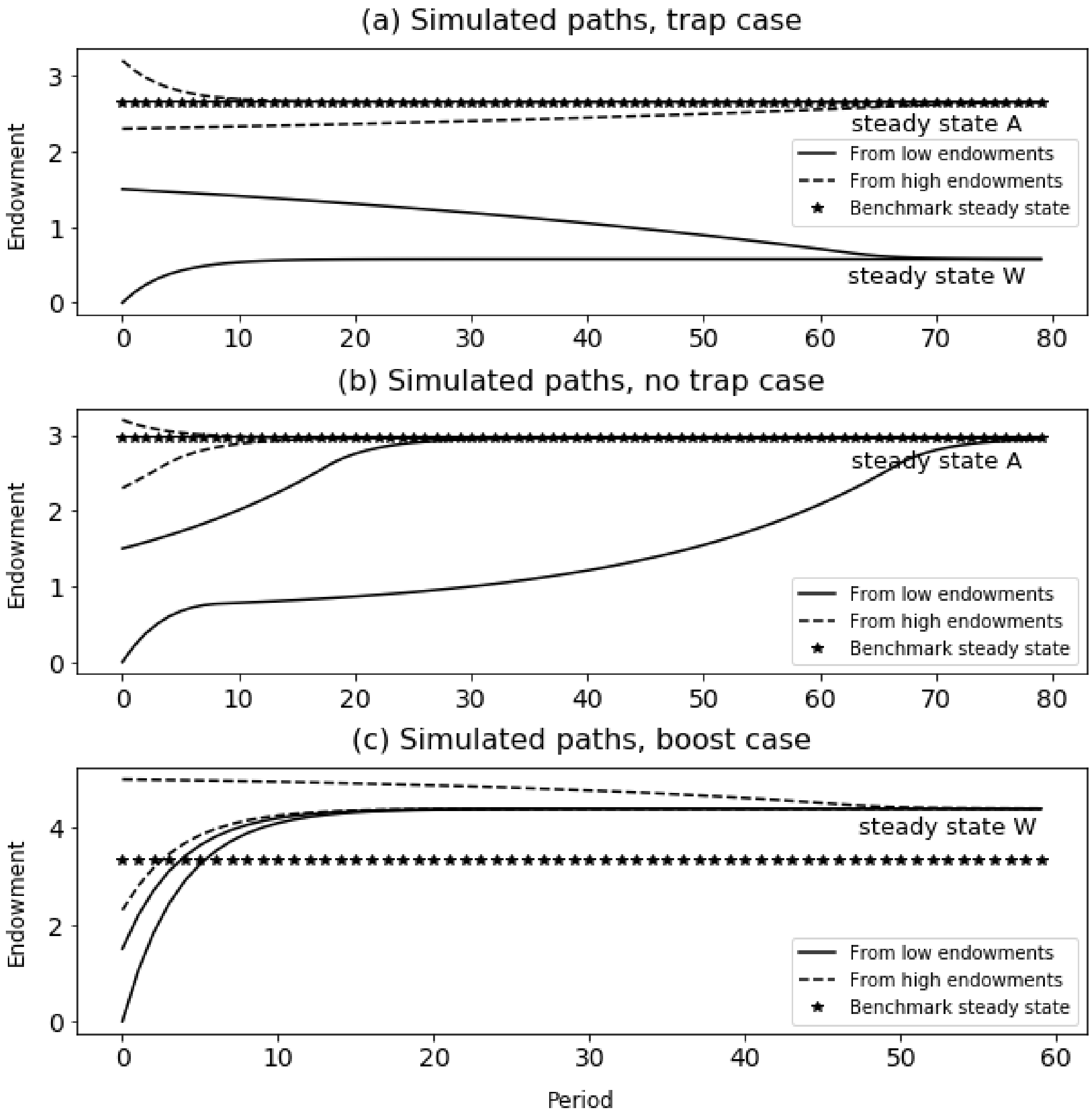

Figure 3: Simulated endowment paths for three different parameter sets. The model in panel (a) exhibits two steady states, W and A. Endowment paths starting from low endowments (solid lines) converge to steady state W (lower), whereas those starting from high endowments (dashed lines) converge to steady state A (higher). The model in panel (b) exhibits only one steady state (steady state A). All endowment paths converge to the same endowment regardless of the starting endowment. The model in panel (c) exhibits only steady state W. Contrary to other parameter configurations, steady state W in this case is at a higher endowment level than the benchmark autarky case. All endowment paths converge to the same endowment regardless of the starting endowment. Parameters used: $f = 3k^{.65}$, $C = 1$, (a) $r = 10\%$, $\rho = 2.3$, and $z = 4$. (b) $r = 10\%$, $\rho = 2.5$, and $z = 4$. (c) $r = 1\%$, $\rho = 3.1$, and $z = 1.1$.

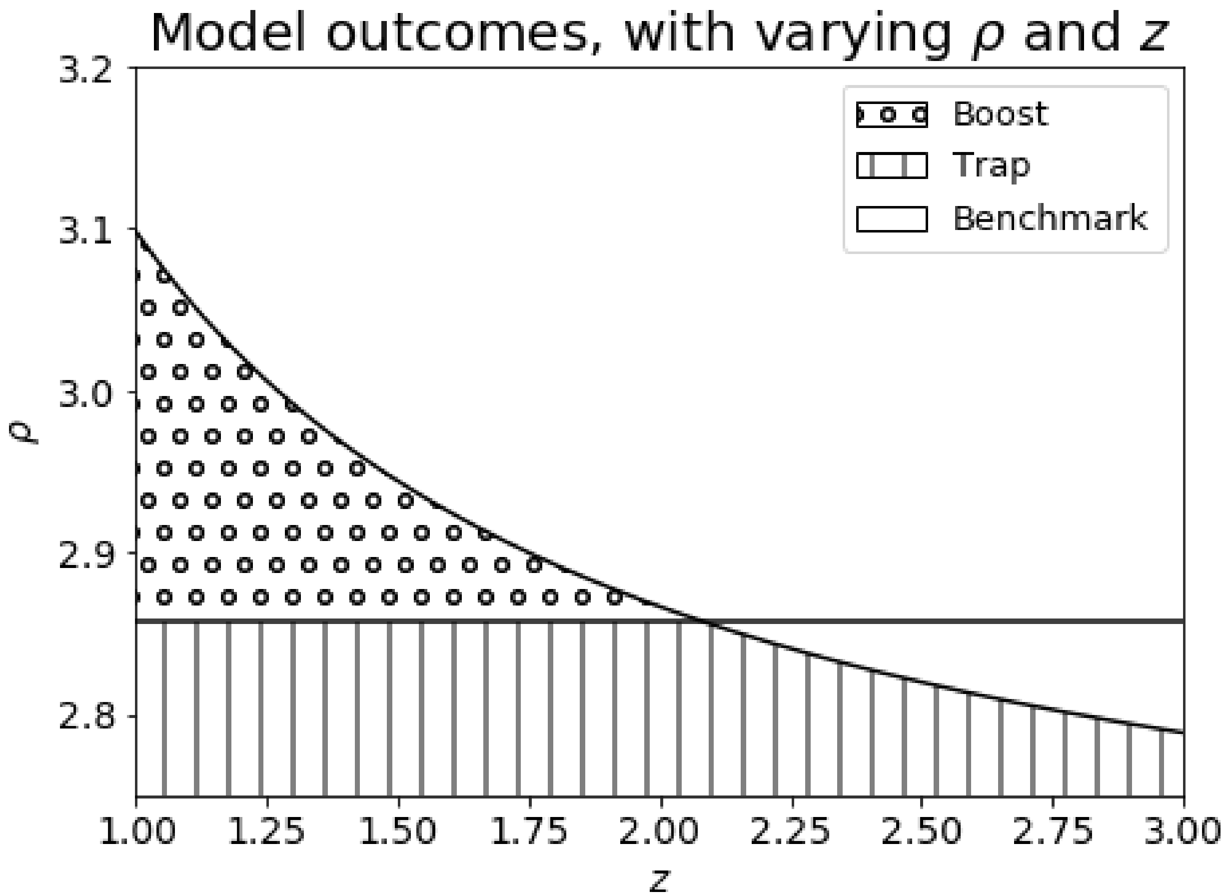

Figure 4: Model outcomes in terms of steady states. ρ and z are varied, while the following parameters have been used: $f = 3k^{.65}$, $r = 3\%$, and $C = 1.0$. The straight horizontal line is at $\rho = \frac{1}{t^{**}}$, markedly separating the boost and trap cases.

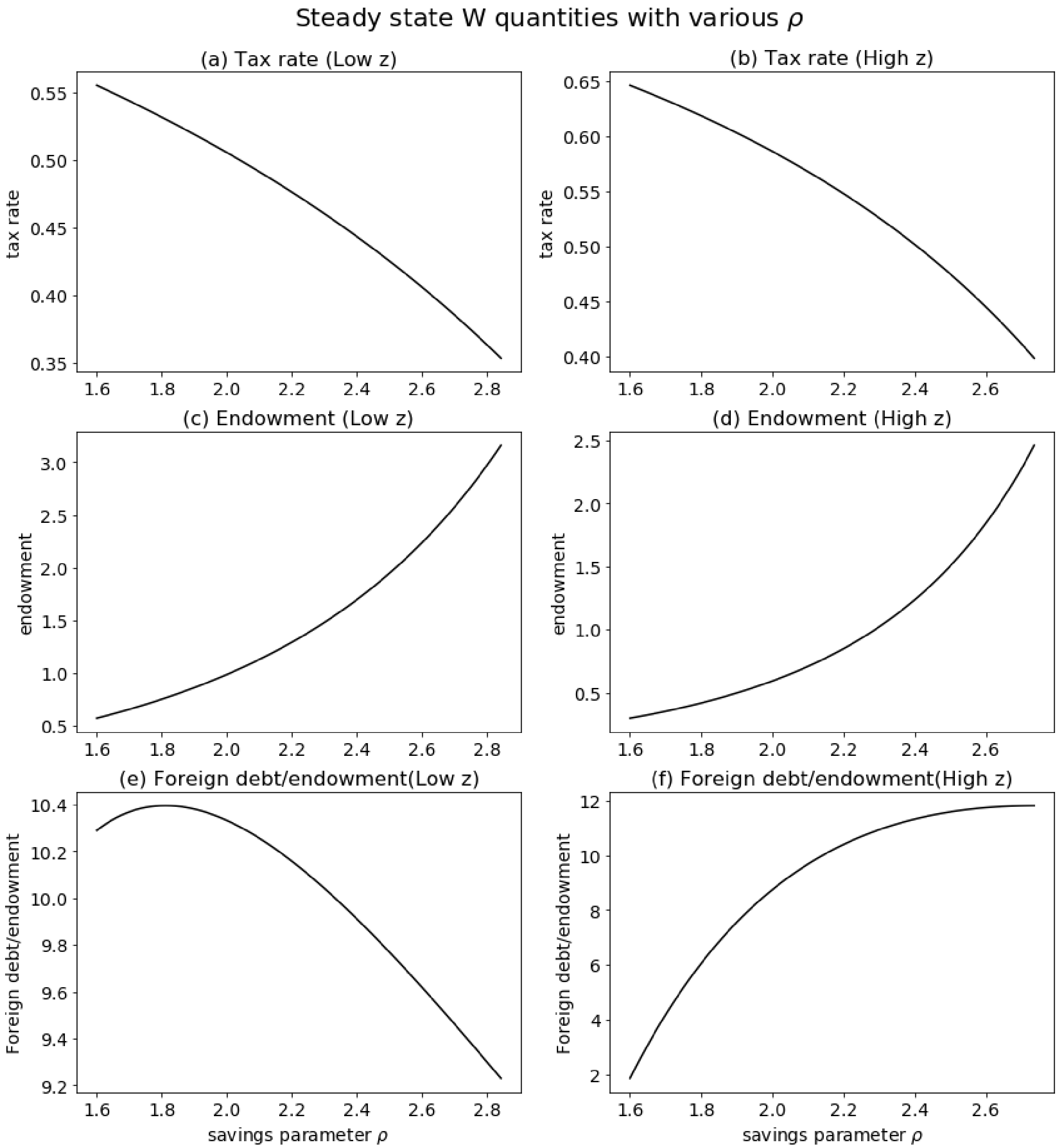

Figure 5: Comparative statics on ρ – households' propensity to save – to tax rates, endowments, and foreign debt normalized by endowment, in the willingness-to-pay steady state. The following parameters are used: $f = 3k^{.65}$, $r = 10\%$, $C = 1.0$, low $z = 1.1$, high $z = 2$.

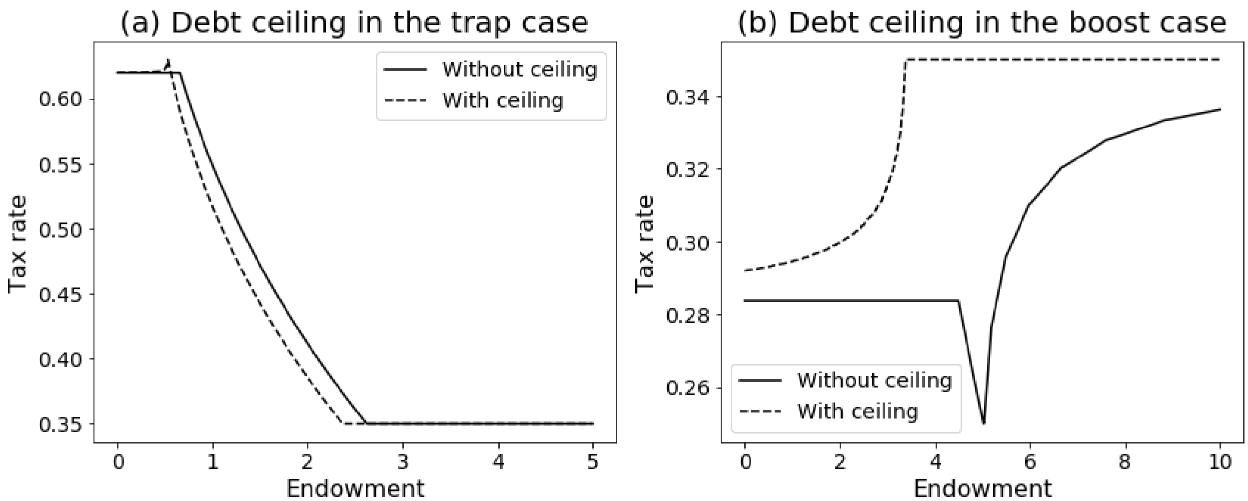

Figure 6: Tax policy of a myopic government facing a debt ceiling equal to 95% of the debt amount taken at steady state W, D^W. In panel (a), the debt ceiling is placed on a model which originally exhibited a growth trap. It can be seen that the debt ceiling lowers the tax rate for the most part. In panel (b), the debt ceiling is placed on a model which originally exhibited a growth boost. In this case, the debt ceiling raises the tax rate uniformly. Parameters used: (a) $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$. (b) $f = 3k^{.65}$, $r = 1\%$, $z = 1.1$, $\rho = 3.1$ and $C = 1.0$

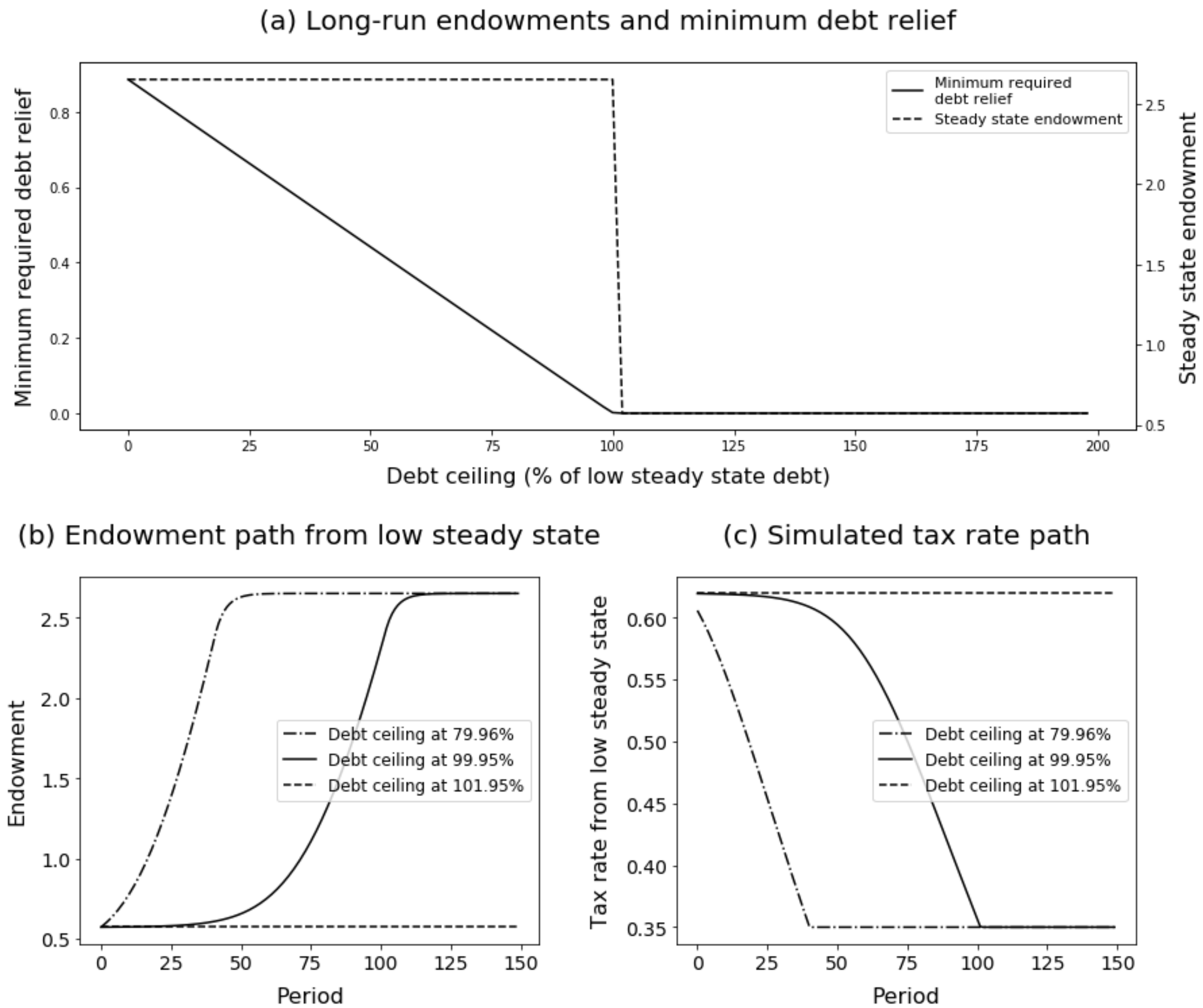

Figure 7: (a) Minimum required relief (left scale) and steady-state endowment (right scale), as functions of debt ceiling. Simulated endowment (b) and tax rate (c) paths after different levels of debt ceilings are placed on a trapped economy. In all figures, The debt ceilings are expressed as % of the level of debt in steady state W, D^W. Parameters used: $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$.

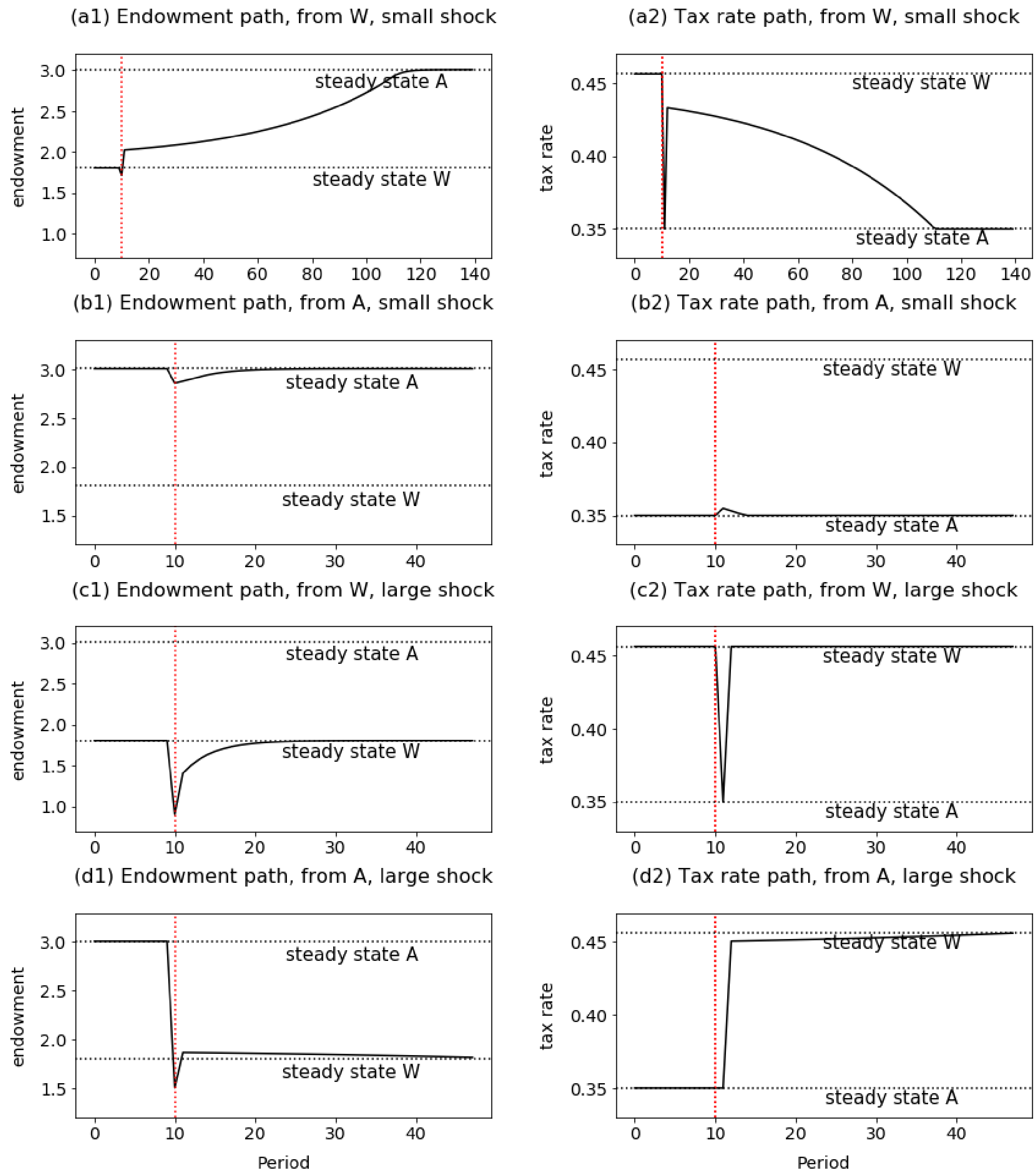

Figure 8: Short- and long-run results of small (5% of original) and large (50% of original) negative endowment shock , for economies in steady states W and A. The shock is experienced shortly before the end of period 10. Panels (a) and (c) pertain to economies initially in steady state W, whereas panels (b) and (d) pertain to those initially in steady state A. All economies except the one initially in steady state A and experiencing a small shock (panel (b)) go through a default in period 10. The following parameters are used: $f = 3k^{.65}$, $r = 4\%$, $z = 4.24$, $\rho = 2.72$ and $C = 1.0$.

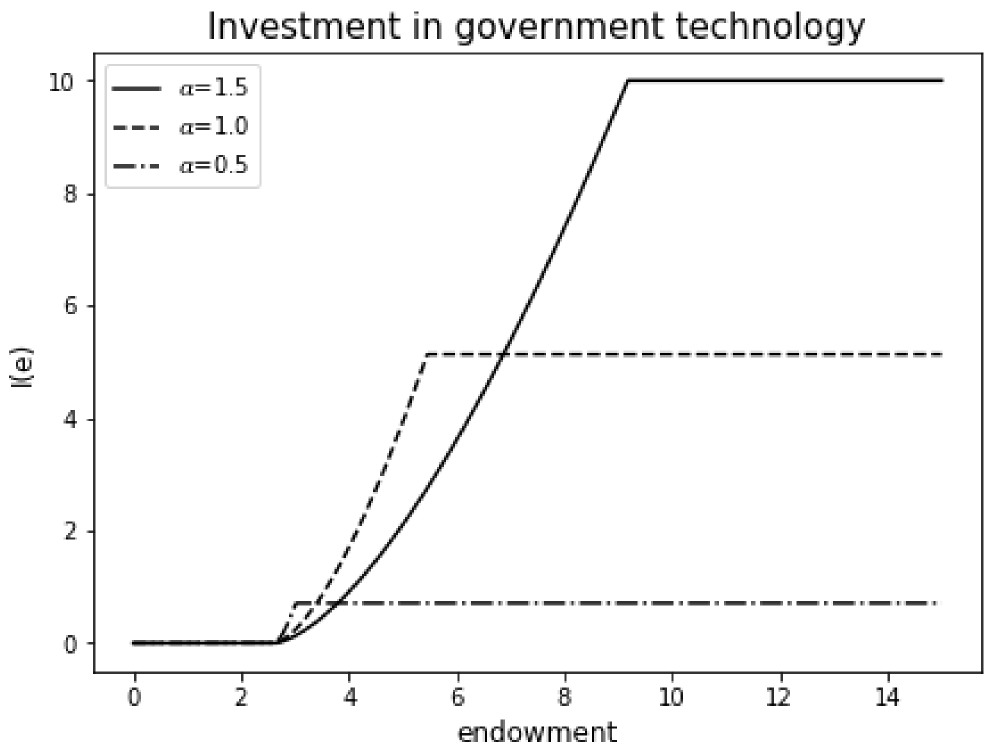

Figure 9: Numerical solution for the extension with government technology. α is the varied parameter, where $g(\cdot) = \alpha \times f(\cdot)$. All other parameters are the same as in Fig. 2; $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$.

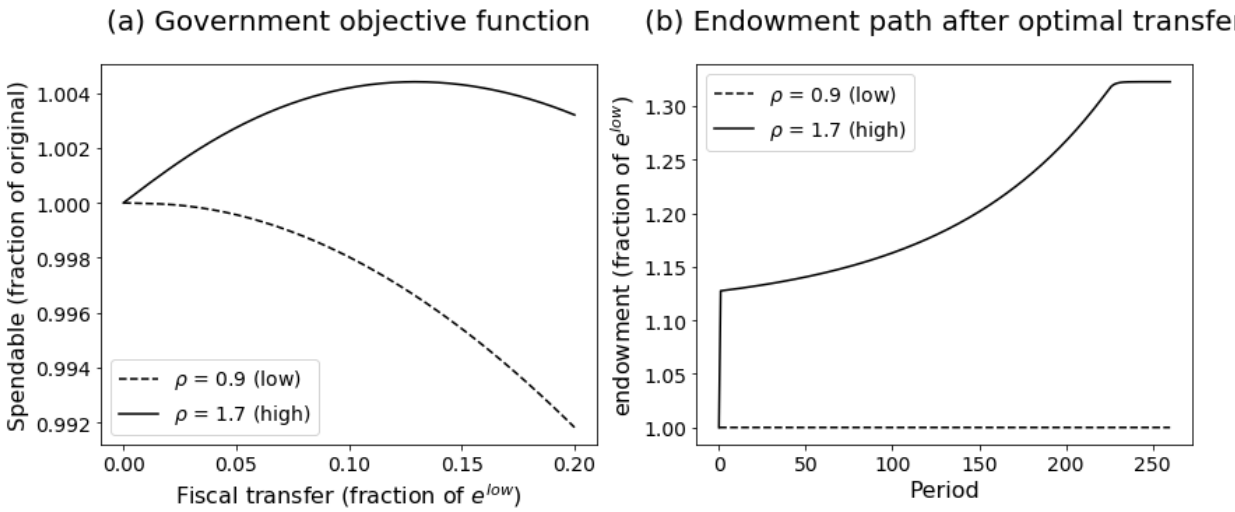

Figure 10: Panel (a) plots the government objective functions against fiscal transfers, for low and high savings parameters $\rho^{low} = 0.9$, $\rho^{high} = 1.7$. They are low and high in a relevative sense to z, *i.e.*, in the "low" parameter configuration, $\rho^{low} < \frac{1}{z^{low}}$, and in the "high" parameter configuration, $\rho^{high} > \frac{1}{z^{high}}$. Both parameter configurations admit a trap equilibrium. It can be seen that a non-zero fiscal transfer can increase the objective function for the model with high savings parameter, whereas it does not for the one with low savings parameter. Panel (b) plots endowments paths after optimal transfers for the two models. Notice that the fiscal transfer at $t = 1$ by the government with high savings parameter leads an eventual escape of the trap, whereas it does not happen for the government with low savings parameter. Other parameters used: $f = 3k^{.5}$, $r = 10\%$, $z = 1.1$, and $C = 1.0$.

B Characterization of the Steady States

In Section 3, we have stated that the steady state has to fall in one of (i) ability-to-pay region, (ii) willingness-to-pay region, and (iii) sliding region. We then derived necessary conditions for a steady state in each of the three regions:

$$e^A = e_+(e^A, t^A); \text{ and } \tau'(t^A) = 0. \qquad \text{(steady state A)}$$

$$e^W = e_+(e^W, t^W); \text{ and } \frac{\rho z}{1+r}\frac{de_+}{dt}(e^W, t^W) + z\frac{ds}{dt}(e^W, t^W) + \tau'(t^W) = 0. \quad \text{(steady state W)}$$

$$e^S = e_+(e^S, t^S); \text{ and } \tau^{**} - C - z(1+r)s(e^S, t^S). \qquad \text{(steady state S)}$$

In addition, for steady states A and W, the other necessary condition is that they indeed fall under the correct regions. That is,

$$\text{Steady state A exists only if } \tau^{**} - C - zs(e^A, t^{**}) \leq 0, \text{ and} \tag{B.1}$$

$$\text{Steady state W exists only if } \tau^{**} - C - zs(e^W, t^W) > 0. \tag{B.2}$$

We show in Lemma C.6, via an application of the contraction-mapping theorem, that conditions in (B.1) and (B.2) are not only necessary, but also sufficient for the existence of each of the steady states, respectively.

Finally, we prove in Lemma B.1 that because any endowment path $\{e_i\}_{i=0}^{\infty}$ (see Definition 3.1) is a monotonic sequence, it must have a limit. Moreover, the limit must be one of the steady states characterized above. In Lemma B.1 as well as Appendix C, we make use of the intermediate function e^{sat}:

Definition B.1. Define the following function:

$$\begin{aligned} & e^{sat}(t) := e \quad s.t. \quad e_+(e,t) = e \\ \Rightarrow & e^{sat}(t) = \frac{(1-t)f(k(t)) - (1+r)k(t)}{1/\kappa_1 - (1+r)}. \end{aligned} \tag{B.3}$$

In intuitive terms, $e^{sat}(t)$ is the point towards which the economy "saturates" under the given t: $\left[\lim e_n = e_+(e_+(\cdots(e_+(e,t),\cdots),t),t) = e^{sat}(t)\right]$. It also follows that for a given t, at $e > e^{sat}(t)$ the economy is "contracting" $(e_+(e,t) < e)$, and at $e < e^{sat}(t)$, the economy is "growing" $(e_+(e,t) > e)$.

Summarizing all arguments above, we have the following formal result:

Lemma B.1. *Any endowment path $\{e_i\}_{i=0}^{\infty}$ is a monotone sequence (increasing or decreasing) and has a limit. It follows that $e_\infty(e_0)$ is always well-defined. Furthermore, $e_\infty(e_0)$ is always one of* *three possible steady states:*

- ***(steady state A)*** *Steady state is in the ability-to-pay constraint region* $(\hat{e}^2, \infty)$, *and* $e^{ss} = e^A := e^{sat}(t^{**})$.
- ***(steady state W)*** *Steady state is in the willingness-to-pay constraint region* $[0, \hat{e}^1)$, *and* $e^{ss} = e^W := e^{sat}(t^W)$ *where*

$$t^W = t \quad \textit{such that} \quad \rho z \frac{de_+}{dt} + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0.$$

- ***(steady state S)*** *The sliding region is a singleton set* ($\hat{e}^1 = \hat{e}^2$)*, and the steady state is in this set. In this case, the pair* (e^S, t^S) *simultaneously solve*

$$\begin{aligned} &e = e_+(e,t), \textit{ and} \\ &0 = \tau^{**} - C - z(1+r)s(e,t). \end{aligned}$$

In general, in the case where there are multiple steady states in the model, $e_\infty(e_0)$ *is not independent of* e_0*. In particular,* $e_\infty(e_1) \le e_\infty(e_2)$ *if* $e_1 < e_2$.

The proof is in the appendix. Notably, steady state S exists only when the sliding region is a singleton set; this is because when it is of positive measure, the steady state within the region is bound to be a saddle point.

Finally, in Lemma C.7 we discuss how six different parameter cases yield distinct combinations of the above three steady states, which provide the basis for Proposition 3.1.

B.1 Savings parameter and growth traps

We showed in Proposition 3.1 that $t^W > t^{**}$ is a necessary condition for a growth trap to exists for lower endowments. In this section, we analyze the government incentives in the willingness-to-pay region to show how ρ emerges as a critical parameter.

First, suppose that the economy is in the willingness-to-pay region. Government's optimal tax rate is chosen as the following:

$$t^W := \underset{t}{\operatorname{argmax}} \quad \Big[\frac{1}{1+r}\big[S(e') - \tau^{**} + C + zs(1+r)\big] + \tau(t)\Big].$$

Note that $e' = \kappa_1\big[\pi(t) + (1+r)e\big]$. Differentiating, and collecting all terms except the last, we get

$$\underbrace{\frac{dS}{de}}_{=\rho z}\frac{\rho}{1+r}\pi'(t)-z\left[\rho k'(t)+\frac{1}{1+r}\frac{d}{dt}(1-t)f(k(t))\right].$$

Whether t^W is lower or higher than $t^{**}=\text{argmax}_t\, tf(k(t))$ depends on whether this expression, evaluated at $t=t^{**}$, is positive or not. The two conflicting incentives for the myopic government follow:

$$\underbrace{\underbrace{\frac{dS}{de}}_{=\rho z}\frac{\rho}{1+r}\pi'(t)}_{\text{Incentive to lower taxes to boost growth to increase next-period government's spendable}} - \underbrace{z\left[\rho k'(t)+\frac{1}{1+r}\frac{d}{dt}(1-t)f(k(t))\right]}_{\text{Incentive to repress investment with higher taxes to increase next-period government's willingness-to-pay}}.$$

In the equation above, we observe that (i) z enters linearly in both terms, so that when determining the sign of the expression, z is irrelevant; (ii) ρ enters as a quadratic term in the first term (+ incentive to grow), and as a linear term in the second term (− incentive to grow). This is because the savings parameter ρ influences both the marginal sensitivity of the future endowment to current tax rate ($\frac{de_+}{dt}$) and the marginal sensitivity of next period government's repayment capacity to endowment ($\frac{dS}{de}$). For high enough ρ, the first term dominates and the myopic government chooses an even lower tax rate than benchmark. For low enough ρ, the second term dominates and the opposite occurs. In the proof of Proposition 3.2, we show that the threshold savings parameter is equal to $\frac{1}{t^{**}}$.

C Mathematical Appendix

Lemma C.1. *Household's optimization problem in (2.1) - (2.3) and the associated FOC's (2.4) - (2.7) is solved by the following set of decision functions:*

$$\begin{aligned} k_i &= f'^{-1}\Big(\frac{1+r}{1-t_i}\Big), \\ c_i &= \kappa_0[(1+r)(e_i-k_i)+(1-t_i)f(k_i)], \\ e_{i+1} &= \kappa_1[(1+r)(e_i-k_i)+(1-t_i)f(k_i)], \text{ and} \\ s_i &= \kappa_1(e_i-k_i)-\kappa_0(1-t_i)f(k_i); \text{ where} \\ \kappa_0 &:= \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}. \end{aligned}$$

Proof: *Combining (2.5) and (2.6), we get the investment decision as a function of tax rate t_i only:*

$$k_i = f'^{-1}\Big(\frac{1+r}{1-t_i}\Big). \tag{C.1}$$

Combining (2.4), (2.5), and (2.7), we obtain the following marginal condition between the next-period endowment e_{i+1} and the current-period consumption c_i:

$$\frac{1}{c_i} - (1+r)\frac{\rho}{e_{i+1}} \Rightarrow e_{i+1} = \rho(1+r)c_i. \tag{C.2}$$

Given our four equations (two each from resource constraints and FOC's), we solve for the four unknowns. c_i can be solved by adding (2.3) to $(1+r)\times$ (2.2) and plugging in (C.2):

$$\begin{aligned} &(1+r)c_i + \cancel{(1+r)s_i} + (1+r)k_i + \underbrace{e_{i+1}}_{=\rho(1+r)c_i} = (1+r)e_i + \cancel{(1+r)s_i} + (1-t_i)f(k_i) \\ \Rightarrow &(1+r)(1+\rho)c_i = (1+r)(e_i-k_i)+(1-t_i)f(k_i) \\ \Rightarrow &c_i = \underbrace{\frac{1}{(1+\rho)(1+r)}}_{=\kappa_0}[(1+r)(e_i-k_i)+(1-t_i)f(k_i)]. \end{aligned}$$

and k_i is determined in (C.1). Similarly, we can derive conditions for e_{i+1} and s_i:

$$\begin{aligned} e_{i+1} &= \kappa_1[(1+r)(e_i-k_i)+(1-t_i)f(k_i)], \text{ and} \\ s_i &= \kappa_1(e_i-k_i)-\kappa_0(1-t_i)f(k_i). \end{aligned}$$

■

Proof of Proposition 2.2: It suffices to show that the mapping T implied by the Bellman equation preserves monotonicity and concavity. In what follows, we denote $F : \mathbb{R}_+ \to \mathbb{R}$ as a generic weakly increasing and concave function. In addition, we let e_1 and e_2 denote generic real values of endowments where $e_1 < e_2$, and t_1, t_2 the respective optimal tax rates.

Monotonicity. Observe first that both $e_+(e,t)$ and $s(e,t)$, defined respectively in (2.22) and (2.23), are increasing in e. Next, note that

$$\begin{aligned}
TF(e_2) &= \max_t \frac{1}{1+r}[F(e_+(e_2,t)) - \max\{0, \tau^{**} - C - z(1+r)s(e_2,t)\}] + \tau(t) \\
&\geq \frac{1}{1+r}[F(e_+(e_2,t_1)) - \max\{0, \tau^{**} - C - z(1+r)s(e_2,t_1)\}] + \tau(t_1) \\
&\geq \frac{1}{1+r}[F(e_+(e_1,t_1)) - \max\{0, \tau^{**} - C - z(1+r)s(e_1,t_1)\}] + \tau(t_1) \\
&= TF(e_1).
\end{aligned}$$

This proves the preservation of monotonicity under the mapping T. ■

(i) Concavity. Take some (e_1,t_1), (e_2,t_2) and $\alpha \in (0,1)$. Let

$$\begin{aligned}
&e_\alpha := (1-\alpha)e_1 + \alpha e_2; \\
&t_\alpha : e_+(e_\alpha,t_\alpha) = (1-\alpha)e_+(e_1,t_1) + \alpha e_+(e_2,t_2).
\end{aligned}$$

It is immediate that such a t_α always exists. We prove the following lemma first:

Lemma C.2. *For (e_1,t_1), (e_2,t_2), and (e_α,t_α) defined as above,*

$$\begin{aligned}
\tau(t_\alpha) &\geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2); \\
s(e_\alpha,t_\alpha) &\geq (1-\alpha)s(e_1,t_1) + \alpha s(e_2,t_2).
\end{aligned}$$

Proof: From the definition of t_α, denoting $k_\alpha := k(t_\alpha)$, $f_\alpha := f(k(t_\alpha))$, $s_\alpha := s(e_\alpha,t_\alpha)$, and $\pi_\alpha := \pi(t_\alpha)$, and recognizing that by definition $e_\alpha = (1-\alpha)e_1 + \alpha e_2$, it follows that

$$\begin{aligned}
&e_+(e_\alpha,t_\alpha) = (1-\alpha)e_+(e_1,t_1) + \alpha e_+(e_2,t_2) \\
\Rightarrow &(1-t_\alpha)f_\alpha - (1+r)k_\alpha = (1-\alpha)[(1-t_1)f_1 - (1+r)k_1] + \alpha[(1-t_2)f_2 - (1+r)k_2] \\
\Rightarrow &\pi(t_\alpha) = (1-\alpha)\pi(t_1) + \alpha\pi(t_2),
\end{aligned}$$

where π is defined in (2.24). From Lemma 1.1 in the Online Appendix, assumptions stated in Definition 2.1 imply that

$$k(t_\alpha) \leq (1-\alpha)k(t_1) + \alpha k(t_2); \tag{C.3}$$

$$\tau(t_\alpha) \geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2). \tag{C.4}$$

In addition, from the definition of π in (2.24), we also have that

$$\begin{aligned}
&\pi_\alpha = (1-\alpha)\pi_1 + \alpha\pi_2 \\
\Rightarrow &(1-t_\alpha)f_\alpha - (1+r)k_\alpha = (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2 - (1+r)((1-\alpha)k_1 + \alpha k_2) \\
\Rightarrow &(1-t_\alpha)f_\alpha = (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2 - \underbrace{(1+r)((1-\alpha)k_1 + \alpha k_2 - k_\alpha)}_{\geq 0} \\
\Rightarrow &(1-t_\alpha)f_\alpha \leq (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2,
\end{aligned}$$

which leads to

$$\begin{aligned}
s_\alpha &= \kappa_1(e_\alpha - k_\alpha) - \kappa_0(1-t_\alpha)f_\alpha \\
&\geq (1-\alpha)s_1 + \alpha s_2.
\end{aligned}$$

■

To show that concavity is preserved under T, we need to show that

$$TF(e_\alpha) \geq (1-\alpha)TF(e_1) + \alpha TF(e_2).$$

First, by the definition of t_α and the concavity of F,

$$\begin{aligned}
&e_+(e_\alpha, t_\alpha) = (1-\alpha)e_+(e_1, t_1) + \alpha e_+(e_2, t_2) \qquad (\because \text{Construction of } t_\alpha) \\
\Rightarrow &F(e_+(e_\alpha, t_\alpha)) \geq (1-\alpha)F(e_+(e_1, t_1)) + \alpha F(e_+(e_2, t_2)).
\end{aligned} \tag{C.5}$$

Second, since $\max(x, y) + \max(a, b) \geq \max(x + a, x + b)$, we have

$$\begin{aligned}
&(1-\alpha)\max\{0, \tau^{**} - Cz(1+r)s_1\} + \alpha \max\{0, \tau^{**} - Cz(1+r)s_2\} \\
\geq &\max\{0, \tau^{**} - C - z(1+r)[(1-\alpha)s_1 + \alpha s_2]\} \\
\geq &\max\{0, \tau^{**} - C - z(1+r)s_\alpha\}.
\end{aligned} \tag{C.6}$$

Then,

$$\begin{aligned}
TF(e_\alpha) &= \max_t \frac{1}{1+r}[F(e_+(e_\alpha, t)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t)\}] + \tau(t) \\
&\geq \frac{1}{1+r}[F(e_+(e_\alpha, t_\alpha)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t_\alpha)\}] + \tau(t_\alpha) \\
&\geq (1-\alpha)TF(e_1) + \alpha TF(e_2),
\end{aligned}$$

where the last step comes from the combination of (C.5), (C.6), and (C.4). ■

(ii) (Binding constraints). We prove the following logically equivalent statement: let $e_1 < e_2$. If at e_1 the ability-to-pay constraint is binding, that so it must at e_2 also. If instead at e_2 the willingness-to-pay binds, then so it must at e_1 also.

Proof: First let us set forth the associated first-order conditions (FOC's). If at e the ability-to-pay constraint is binding, then the following FOC is satisfied:

$$\underbrace{\frac{de_+}{dt}}_{=\pi'(t)} \frac{dS}{de} + (1+r)\tau'(t) = 0$$

$$\Rightarrow \underbrace{\frac{dS}{de} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{ability}(e,t)} = 0.$$

If instead at e the willingness constraint is binding, then the following FOC is satisfied:

$$\underbrace{\frac{de_+}{dt}}_{=\pi'(t)} \frac{dS}{de} + z(1+r)\frac{ds}{dt} + (1+r)\tau'(t) = 0$$

$$\Rightarrow \underbrace{\frac{dS}{de} + z(1+r)\frac{s'(t)}{\pi'(t)} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{willingness}(e,t)} = 0.$$

Since $s' > 0$ and $\pi' < 0$, it follows that $FOC^{willingness}(e,t) < FOC^{ability}(e,t)$ always.

If both are binding, then it must be that $\tau^{**} - C - z(1+r)s = 0$ and

$$FOC^{ability}(e,t) > 0, \text{ and}$$
$$FOC^{willingness}(e,t) < 0.$$

as increasing t by dt would enter the region where only the ability-to-pay constraint is binding ($\tau^{**} - C - z(1+r)s < 0$) and increase the objective function by $\pi' FOC^{ability}(e,t)dt$. Since $\pi' < 0$ and $dt > 0$, $FOC^{ability}$ must be greater than 0 for this not to be a perturbation that increases the objective function. Similar argument applies in the opposite direction ($dt < 0$) for $FOC^{willingness}$.

We then prove the following lemma:

Lemma C.3. *Both $FOC^{ability}(e,t)$ and $FOC^{willingness}(e,t)$ are (weakly) decreasing in e and (strictly) increasing in t.*

Proof: For $FOC^{ability}(e,t)$, observe that $e_+(e,t)$ is increasing in e and decreasing in t. Combined with the fact that S is concave, it follows that dS/de is decreasing in e and increasing in t. From the assumptions stated in Definition 2.1, $\frac{\tau'}{\pi'}$ is increasing in t. This proves the properties for $FOC^{ability}(e,t)$.

For $FOC^{willingness}(e,t)$, it only remains to be proved that $\frac{s'}{\pi'}$ is increasing in t as the function is independent of e. Notice that since $\pi = (1-t)f - (1+r)k$ and $s = \kappa_1(e-k) - \kappa_0(1-t)f$,

$$\begin{aligned}\frac{s'}{\pi'} &= \frac{-\kappa_1 k' - \kappa_0(\pi' + (1+r)k')}{\pi'} \\ &= -[\kappa_1 + \kappa_0(1+r)]\frac{k'}{\pi'} - \kappa_0.\end{aligned}$$

Since $\frac{k'}{\pi'}$ is assumed to be decreasing in t in Definition 2.1, this proves the properties for $FOC^{willingness}(e,t)$. ■

Now, consider the first case where at e_1 the ability-to-pay constraint is binding and suppose per contra that at e_2 the ability-to-pay constraint is non-binding. This implies that

$$\begin{aligned}\tau^{**} - C - z(1+r)s(e_1,t_1) \le 0, \text{ and} \\ \tau^{**} - C - z(1+r)s(e_2,t_2) > 0.\end{aligned}$$

Observe that since s is increasing in both e and t,

$$\begin{aligned}&\tau^{**} - C - z(1+r)s(e_2,t_2) > 0 \ge \tau^{**} - C - z(1+r)s(e_1,t_1) \\ \Rightarrow & z(1+r)s(e_2,t_2) < z(1+r)s(e_1,t_1) \\ \Rightarrow & z(1+r)s(e_1,t_2) < z(1+r)s(e_1,t_1) \\ \Rightarrow & t_1 > t_2.\end{aligned}$$

At e_1, the FOC should be met, which implies that $FOC^{ability}(e_1,t_1) = 0$ and accordingly $FOC^{willingness}(e_1,t_1) < 0$. At e_2, $FOC^{willingness}(e_2,t_2) = 0$ and accordingly $FOC^{ability}(e_2,t_2) > 0$. Comparing $FOC^{ability}$ evaluated at different parameters,

$$FOC^{ability}(e_2,t_2) > 0 = FOC^{ability}(e_1,t_1) > FOC^{ability}(e_2,t_1) \Rightarrow t_2 > t_1,$$

leading to a contradiction. The proof of the second case is a mirror image. ■

(iii) (Continuity). By the theorem of the maximum, we only have to prove that for each e, there is a unique t that maximizes the objective function. First observe that, since $s(e,t)$ is concave in t, the penalty function $-\max\{0,\cdot\}$ is concave in t. Next, Let e be an arbitrary number and consider $t_1 < t_2$ and suppose per contra that t_1 and t_2 both achieve the maximum. Consider an arbitrary $\alpha \in (0,1)$ and pick t_α as in Lemma C.2. By the stated lemma and the fact that S is concave, we know respectively that

$$\tau(t_\alpha) \geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2), \text{ and}$$
$$S(e'(t_\alpha, e)) \geq (1-\alpha)S(e'(t_1, e)) + \alpha S(e'(t_2, e)).$$

Since this holds true for any arbitrary α, by picking t_α we should achieve a larger objective function. The claim is then proved by contradiction. ■

$t(e)$ **increasing in** $[0, \bar{e}^1]$**:** Suppose not, and suppose that $e_1 < e_2$ and $t_1 > t_2$. This creates the following contradiction:

$$0 = FOC^{willingness}(t_1, e_1) \geq FOC^{willingness}(t_1, e_2) > FOC^{willingness}(t_2, e_2) = 0.$$

$t(e)$ **decreasing in** $[\bar{e}^1, \bar{e}^2]$**:** In this region, the optimal t is such that $\tau^{**} - C - z(1+r)s = 0$. The proof follows from the fact that s is increasing in both e and t.
$t(e)$ **increasing in** $[\bar{e}^2, \infty]$**:** Suppose not, and suppose that $e_1 < e_2$ and $t_1 > t_2$. This creates the following contradiction:

$$0 = FOC^{ability}(t_1, e_1) \geq FOC^{ability}(t_1, e_2) > FOC^{ability}(t_2, e_2) = 0.$$

(iv) (Asymptotics). We first prove that $S(e)$ is bounded. First observe that, since max$\{0, \tau^{**} - C - zs(1+r) \geq 0$, $S(e)$ is bounded from above by an alternative value function $\tilde{S}(e)$

$$\tilde{S}(e) := \max_t \frac{1}{1+r}\tilde{S}(e') + \tau(t)$$

for which the solution is simply $\tilde{S} = \frac{\tau^{**}}{r}$. Therefore, we conclude that $S(e) \leq \frac{\tau^{**}}{r}$ $\forall e$. Combined with the fact that $S(e)$ is weakly increasing and concave in e, we have that $S'(e) \to 0$ as $e \to \infty$. Note, then, at sufficiently high e, the optimal $t = \text{argmax}_t \frac{1}{1+r}S(e') + \tau(t) = t^{**}$. ■

Proof of Lemma B.1: In order to prove this lemma, we prove Lemmas C.4 - C.6 first.

Lemma C.4. *Any endowment path $\{e_i\}_{i=0}^{\infty}$ is a monotone sequence (increasing or decreasing). This immediately implies that any growth path has a limit, and it must be a fixed point of the policy function $h(e) := e_+(e, t(e))$.*

Proof: It suffices to prove that $h(e)$ is a monotonic increasing function, because $e_i < e_{i+1} = h(e_i)$ would imply that $e_{i+2} = h(e_{i+1}) > h(e_i) = e_{i+1}$, which leads by induction that $e_{j+1} > e_j$

for $\forall j \geq i$. We have proved in Proposition 2.2 that there are three regions to consider: $[0, \hat{e}^1]$, $[\hat{e}^1, \hat{e}^2]$, and $[\hat{e}^2, \infty]$. We prove piecewise monotonicity in each of these regions, which suffices for overall monotonicity given the continuity of $t(e)$ proved in Proposition 2.2. Recall from (2.22) that $e_+(e,t)$ is increasing in e and decreasing in t.

- (Region 1) Take $e_1 < e_2$, $e_1, e_2 \in [0, \hat{e}^1]$ and suppose per contra $h(e_1) > h(e_2)$. This must imply that $t_1 < t_2$. Note that $FOC^{willingness}$ must be met at both points and recall that both $\frac{s'}{\pi'}$ and $\frac{\tau'}{\pi'}$ are strictly increasing in t (Lemma C.3). This leads to

$$\begin{aligned}
0 &= \frac{dS}{de}\Big|_{h(e_1)} + z(1+r)\frac{s'(t_1)}{\pi'(t_1)} + (1+r)\frac{\tau'(t_1)}{\pi'(t_1)} \\
&< \frac{dS}{de}\Big|_{h(e_1)} + z(1+r)\frac{s'(t_2)}{\pi'(t_2)} + (1+r)\frac{\tau'(t_2)}{\pi'(t_2)} && (\because t_1 < t_2) \\
&\leq \frac{dS}{de}\Big|_{h(e_2)} + z(1+r)\frac{s'(t_2)}{\pi'(t_2)} + (1+r)\frac{\tau'(t_2)}{\pi'(t_2)} && (\because h(e_1) > h(e_2) \text{ and concavity of } S) \\
&= 0.
\end{aligned}$$

  which is a contradiction.

- (Region 2) Take $e_1 < e_2$, $e_1, e_2 \in [\hat{e}^1, \hat{e}^2]$. We have proved in Proposition 2.2 that $t_1 > t_2$ in this region. Therefore $h(e_1) < h(e_2)$ immediately follows.

- (Region 3) This part is similar to region 1. ■

Lemma C.4 allows us limit the analysis of only the fixed points of the policy function $h(e)$. Essentially, these are steady states defined in Definition 3.1 plus the saddle fixed points. Saddle fixed points are limiting endowments of a measure zero starting endowment - only if it starts at that exact point - and therefore we exclude them from our analysis.

Next we characterize all possible steady states. Recall that $e^{sat}(t)$ is defined in (B.3). In addition, we define an additional auxiliary function $e^{abil}(t)$ in (C.7):

Definition C.1. Define the following function:

$$\begin{aligned}
&e^{abil}(t) := e \quad s.t. \quad \tau^{**} - C - z(1+r)s(e,t) = 0 \\
\Rightarrow &e^{abil}(t) = k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\kappa_1(1+r)}; \text{ and,}
\end{aligned} \tag{C.7}$$

In intuitive terms, for any given t, $e^{abil}(t)$ is the boundary endowment at which both constraints are binding ($\tau^{**} - C - z(1+r)s(e,t) = 0$).

Lemma C.5. *e^{ss} must satisfy one of the following:*

- *(Steady state W) $e^{ss} \in [0, \hat{e}^1)$ and is characterized by*

$$t^W := t \quad \text{such that} \quad \rho \frac{de_+}{dt} z + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0; \tag{C.8}$$
$$e^{ss} = e^{sat}(t^W).$$

- *(Steady state A) $e^{ss} \in (\hat{e}^2, \infty)$ and is characterized by $e^{ss} = e^{sat}(t^{**})$.*
- *(Steady state S) $e^{ss} = \hat{e}^1 = \hat{e}^2$, and is characterized by t^{ss} such that $e^{ss} = e^{abil}(t^{ss}) = e^{sat}(t^{ss})$.*

Proof: It is straightforward to see that e^{ss} must belong in one of the three regions $[0, \hat{e}^1)$, $[\hat{e}^1, \hat{e}^2]$, $(\hat{e}^2, \infty)$. We first prove that in the interior in the region ($[0, \hat{e}^1)$) and region ($(\hat{e}^2, \infty)$), the fixed points must take the aforementioned form. Suppose that $e^{ss} \in [0, \hat{e}^1)$. Then, in the neighborhood of e^{ss}, the Bellman equation is

$$S = \max_t \frac{1}{1+r}\Big[S(e') - \tau^{**} + C + zs(1+r)\Big] + \tau(t).$$

From the envelope condition, we get that $\frac{dS}{de} = \rho z$. Then, the optimal t can be derived by solving the following isolated equation:

$$\rho \frac{de_+}{dt} z + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0. \tag{C.9}$$

Finally, since e^{ss} must be a fixed point, it follows that $e^{ss} = e^{sat}(t^W)$ where t^W is the solution to (C.9). The steady-state endowment in the region $\big((\hat{e}^2, \infty)\big)$ can be obtained similarly.

Next, we prove that if $\hat{e}^1 < \hat{e}^2$, then e^{ss} cannot belong to the middle region ($[\hat{e}^1, \hat{e}^2]$). We prove that in order for a fixed point $t^{ss} : e^{abil}(t^{ss}) - e^{sat}(t^{ss}) = 0$ to be a stable point, $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t)$ must be non-positive at t^{ss}. Suppose per contra that $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) > 0$. Note that in a small neighborhood of e^{ss}, the two functions can be approximated as

$$e^{abil}(t) = e^{ss} + \frac{d}{dt}e^{abil}(t)(t - t^{ss}) \Rightarrow {e_{abil}}^{-1}(e) = t^{ss} + \left(\frac{d}{dt}e^{abil}(t)\right)^{-1}(e - e^{ss});$$
$$e^{sat}(t) = e^{ss} + \frac{d}{dt}e^{sat}(t)(t - t^{ss}) \Rightarrow {e_{sat}}^{-1}(e) = t^{ss} + \left(\frac{d}{dt}e^{sat}(t)\right)^{-1}(e - e^{ss}).$$

Note that in this neighborhood $e < e^{ss} \Rightarrow e^{-1}_{abil}(e) > e^{-1}_{sat}(e)$.

Suppose now WLOG[15] that in the left neighborhood of e^{ss}, the optimal policy is sliding

[15] without loss of generality

between the two constraints, *i.e.*, $t(e) = {e_{abil}}^{-1}(e)$. Consider e in this neighborhood $e \in (e^{ss}-\epsilon, e^{ss})$ and consider $e_+(e, t(e))$. By definition of e^{sat}, $e_+(e,t) < e$ if and only if $t > e_{sat}^{-1}(e)$. Therefore, it follows that $e_+(e,t(e)) = e_+(e, {e_{abil}}^{-1}(e)) < e$. Since this applies to all elements of the left neighborhood of e^{ss}, combined with the fact from Lemma C.4 $h(e)$ is a monotonic increasing function, that endowment paths are it follows that e can never converge to e^{ss}. Therefore, $e^{ss} \notin [\hat{e}^1, \hat{e}^2]$ if $\hat{e}^1 < \hat{e}^2$.

We next prove that the derivative condition $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) \leq 0$ is impossible. Recall that $e^{abil}(t) - e^{sat}(t) = \psi_1\pi(t) + \psi_2 k(t) + \psi_3$ where ψ_2 and ψ_3 are positive. By the definition of t^{ss},

$$\psi_1\pi(t^{ss}) + \psi_2 k(t^{ss}) + D = 0 \Rightarrow \psi_1\pi(t^{ss}) + \psi_2 k(t^{ss}) < 0 \Rightarrow \psi_1 < -\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}, \text{ so that}$$

$$\begin{aligned}\frac{d}{dt}e^{abil}(t^{ss}) - \frac{d}{dt}e^{sat}(t^{ss}) &= \psi_1\pi'(t^{ss}) + \psi_2 k'(t^{ss}) \\ &> -\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}\pi'(t^{ss}) + \psi_2 k'(t^{ss}). \quad (\because \pi' < 0)\end{aligned}$$

Note that

$$\begin{aligned}-\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}\pi'(t^{ss}) + \psi_2 k'(t^{ss}) \geq 0 &\Leftrightarrow -\frac{\pi'(t^{ss})}{\pi(t^{ss})} + \frac{k'(t^{ss})}{k(t^{ss})} \geq 0 \qquad (\because \psi_2, k > 0) \\ &\Leftrightarrow -\frac{d}{dt}\log(\pi(t^{ss})) + \frac{d}{dt}\log(k(t^{ss})) \geq 0 \\ &\Leftrightarrow \frac{d}{dt}\log\left(\frac{k(t^{ss})}{\pi(t^{ss})}\right) \geq 0 \\ &\Leftrightarrow \frac{d}{dt}\frac{k(t^{ss})}{\pi(t^{ss})} \geq 0 \\ &\Leftarrow \frac{k(t)}{\pi(t)} \text{ is weakly increasing.}\end{aligned}$$

Therefore, the assumption that $\frac{k(t)}{\pi(t)}$ is weakly increasing (it is constant for power production function) is a sufficient condition for any fixed point in $[\hat{e}^1, \hat{e}^2]$ not to be a stable fixed point. ■

Lemma C.6. *The following facts are true:*

A. *Steady state W* $\left(e^{ss} \in [0, \hat{e}^1)\right)$ *exists if and only if* $e^{abil}(t^W) \geq e^{sat}(t^W)$.

B. *Steady state A* $\left(e^{ss} \in (\hat{e}^2, \infty)\right)$ *exists if and only if* $e^{abil}(t^{**}) \leq e^{sat}(t^{**})$.

C. *If either of conditions A and B are met, then* $\hat{e}^1 < \hat{e}^2$ *almost always, implying that steady state S cannot exist.*

D. *If neither of conditions A and B are met, then* $\hat{e}^1 = \hat{e}^2$ *and the only steady state is steady state S:* $e^{ss} = \hat{e}^1 = \hat{e}^2$.

Proof: The proof follows four steps A-D below.

A. The "only if" part is proved in Lemma C.5. To show the "if" part, recall the Bellman equation

$$\begin{aligned} S(e) = \max_t \quad & \Big[\frac{1}{1+r}\big[S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}\big] + \tau(t)\Big] \qquad \text{(C.10)} \\ s.t. \quad & e' = \kappa_1\big[(1+r)(e - k(t)) + (1-t)f(k(t))\big], \\ & s = \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big). \end{aligned}$$

Now conjecture that $S(e) = \alpha + \beta e$ and $t(e) = t^W$ $\forall e \le e^{abil}(t^W)$. It can be verified that the conjecture is correct if

$$\begin{aligned} \alpha &= \frac{1+r}{r} - r(\tau^{**} - C), \text{ and} \\ \beta &= \rho z. \end{aligned}$$

owing to the fact that $e'(e, t^W) < e^{abil}(t^W)$ if $e < e^{abil}(t^W)$ and thus the ability-to-pay constraint is never binding in this region.

B. Similar to A., we can verify a conjectured partial solution $S(e) = \frac{1+r}{r}\tau^{**}$ and $t(e) = t^{**}$ $\forall e \ge e^{abil}(t^{**})$, owing to the fact that $e'(e, t^{**}) > e^{abil}(t^{**})$ if $e < e^{abil}(t^{**})$ and thus the willingness-to-pay constraint is never binding in this region.

C. Suppose per contra that steady state A exists, and that $\hat{e}^1 = \hat{e}^2$. Note that steady state W cannot exist as it would directly violate the continuity of $t(e)$ proved in Proposition 2.2. Now suppose that it does not, and consider an endowment e arbitarily lower than $\hat{e}^1$. Because steady state W does not exist, the next-period endowment must be over $\hat{e}^2$, at which point the spendables function S is a constant value. Note that this would imply the optimal tax rate t to be the solution of:

$$t = \operatorname*{argmax}_t \Big[\frac{1}{1+r}\big[S(e') - \tau^{**} + C + zs(1+r)\}\big] + \tau(t)\Big] \qquad (\because e < \hat{e}^1))$$

$$= \underset{t}{\operatorname{argmax}}\big[zs(e,t) + \tau(t)\big] \qquad (\because S(e') \text{ is constant})$$

which is almost surely different from $t^{**} := \operatorname{argmax} \tau(t)$. This violates the continuity of $t(e)$. The proof of the case where steady state W exists is a mirror image. ■

D. This immediately follows from Lemma C.5. ■

Proof of Proposition 3.1: The following corollary of Lemma C.6 is a sufficient condition for the proposition:

Lemma C.7. *We analyze six different parameter cases, which span all possible cases due to the fact that $e^{abil}(1) > e^{sat}(1)$ always, and the single-crossing properties implied by the assumptions in Definition 2.1. [Refer to Figs. 1–4 of the Online Appendix for the solution characteristics for each of the six cases.]*

- **Case A.** *$t^{**} < t^W$, and*
  - **A1.** ***(Benchmark)*** *$e^{sat}(t) \geq e^{abil}(t)$ for both t^{**} and t^W: Regardless of the starting endowment e_0, the economy converges to e^{**}_∞ $\big(\forall e_0,\ e_\infty(e_0) = e^{**}_\infty\big)$.*
  - **A2.** ***(Trap)*** *$e^{sat}(t) \leq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to the same point* **lower** *than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = e^{sat}(t^W) < e^{**}_\infty\big)$.*
  - **A3.** ***(Trap or Benchmark)*** *$e^{sat}(t^{**}) > e^{abil}(t^{**})$ and $e^{sat}(t^W) < e^{abil}(t^W)$: There is a unique crossing point for the two functions e^{sat} and e^{abil}, say $\bar{\bar{e}}_A$. Then,*

$$e_\infty(e_0) = \begin{cases} e^{sat}(t^W) & \text{if } e_0 < \bar{\bar{e}}_A;\ \text{and} \\ e^{**}_\infty & \text{if } e_0 \geq \bar{\bar{e}}_A. \end{cases}$$

- **Case B.** *$t^{**} \geq t^W$, and*
  - **B1.** ***(Benchmark)*** *$e^{sat}(t) \geq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to e^{**}_∞ $\big(\forall e_0,\ e_\infty(e_0) = e^{**}_\infty\big)$.*
  - **B2.** ***(Boost)*** *$e^{sat}(t) \leq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to the same point* **higher** *than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = e^{sat}(t^W) > e^{**}_\infty\big)$.*
  - **B3.** ***(Boost)*** *$e^{sat}(t^{**}) < e^{abil}(t^{**})$ and $e^{sat}(t^W) > e^{abil}(t^W)$: There is a unique crossing point for the two functions e^{sat} and e^{abil}, say $\bar{\bar{e}}_B$. Then, regardless of e_0, the economy converges to $\bar{\bar{e}}_B$ which is* **higher** *than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = \bar{\bar{e}}_B > e^{**}_\infty\big)$. Also, it is only at this singleton point that both constraints are binding.*

Proof of Proposition 3.2: Note that t^W maximizes

$$t^W = \underset{t}{\operatorname{argmax}}\, \rho z \frac{\kappa_1}{1+r}\pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t)f(k(t))] + \tau(t). \tag{C.11}$$

Note that

$$\rho z \frac{\kappa_1}{1+r}\pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t)f(k(t))] + \tau(t) = \tau(t) - z(\frac{1-\rho}{1+r}\pi(t) + k(t))$$

Since by assumption π and k are convex, and τ is concave, expression in (C.11) is concave. This implies that $t^W > t^{**}$ if and only if the FOC at t^{**} is positive. This translates to

$$\frac{\rho z \kappa_1}{1+r}\pi'(t^{**}) - z\kappa_1 k'(t^{**}) + z\kappa_0 f(k(t^{**})) - z\kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0, \tag{C.12}$$

It is sufficient to derive conditions for (C.12) to hold. Using $\pi'(t) = -f(k)$ as well as

$$\begin{aligned}
&\big(tf(k(t))\big)'|_{t^{**}} = 0 \\
\Rightarrow &f(k(t^{**})) + tf'(k)k'(t^{**}) = 0 \\
\Rightarrow &f(k(t^{**})) = -t\frac{1+r}{1-t}k'(t^{**}),
\end{aligned}$$

we can simplify the expression in (C.12) as the following:

$$\begin{aligned}
&\frac{\rho z \kappa_1}{1+r}\pi'(t^{**}) - z\kappa_1 k'(t^{**}) + z\kappa_0 f(k(t^{**})) - z\kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0 \\
\Rightarrow &z\kappa_0\Big[\rho^2 t^{**}\frac{1+r}{1-t^{**}} - \rho(1+r) - t^{**}\frac{1+r}{1-t^{**}} - (1+r)\Big]k'(t^{**}) > 0 \\
\Rightarrow &z\kappa_0\frac{1+r}{1-t^{**}}\big[t^{**}\rho^2 - (1-t^{**})\rho - 1\big] < 0.
\end{aligned}$$

The characteristic quadratic equation has two roots:

$$\frac{(1-t^{**}) \pm \sqrt{((1-t^{**})^2 + 4t^{**})}}{2t^{**}} = \Big\{\frac{1}{t^{**}}, -1\Big\}.$$

Since $\rho > 0$, the second root is economically irrelevant and therefore we get that

$$t^W > t^{**} \Leftrightarrow \rho < \frac{1}{t^{**}}. \qquad \blacksquare$$

Proof of Proposition 3.3: First, we prove that $t^{**} < 1$. Recall that $t^{**} = \operatorname{argmax}_t \tau(t)$ and $\tau(t) \geq 0$. Since $\tau(1) = 0$ always, it cannot be the case that $1 = \operatorname{argmax}_t \tau(t)$. Therefore, $t^{**} < 1$. Further, t^{**} does not vary with ρ.

Next, we prove that for any $t < 1$, $\exists \hat{\rho}$ such that $e^{abil}(t) < e^{sat}(t)$. Recall that

$$e^{abil}(t) = k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\left(\frac{\rho}{1+\rho}\right)(1+r)}, \text{ and}$$
$$e^{sat}(t) = \frac{(1-t)f(k(t)) - (1+r)k(t)}{\frac{1}{\rho} - r}.$$

Note that for $t < 1$, $(1-t)f(k(t)) - (1+r)k(t) > 0$, and that keeping all else equal, $e^{sat}(t)$ is monotonically increasing in ρ, reaching infinity as $\rho \to \frac{1}{r}$, whereas e^{abil} is monotonically decreasing in ρ. It follows that for any given $t < 1$, there must exist a threshold $\hat{\rho}(t) < \frac{1}{r}$ such that $e^{sat}(t) > e^{abil}(t)$.

Finally, it suffices to consider the case where $\rho > \frac{1}{t^{**}}$, under which case $t^W < t^{**}$. Notice that due to the single-crossing properties of e^{abil} and e^{sat}, $e^{sat}(t^{**}) > e^{abil}(t^{**}) \Rightarrow e^{sat}(t^W) > e^{abil}(t^W)$ in this case. Given that t^{**} does not vary with ρ, it follows that for $\rho > \bar{\rho} = \hat{\rho}(t^{**})$, $e^{sat}(t) > e^{abil}(t)$ for both t^{**} and t^W. From Lemma C.7, this implies that model outcomes are either A1 or B1, where endowments always converge to the benchmark steady state. ■

Proof of Proposition 5.1: The formal problem is stated for the general case in Lemma C.8:

Lemma C.8. *Conditional on not defaulting, government's actions are independent of past government debt ceilings and legacy debt. Suppose that the debt ceiling that the government in period i faces is $\bar{D}_i$, $\forall i \in \mathbb{Z}_+$. Then, the current government's problem can be summarized as solving the following Bellman equation:*

$$S(e;\bar{D}_0,\bar{D}_1,\dots) = \max_t \left[\min\left[\frac{1}{1+r}(S(e';\bar{D}_1,\bar{D}_2,\dots) - \max\{0, \tau^{**} - C - zs(1+r)\}), \bar{D}_0\right] + \tau(t) \right] \tag{C.13}$$

$$\begin{aligned} s.t. \quad & e' = \kappa_1\left[(1+r)(e-k(t)) + (1-t)f(k(t))\right], \\ & s = \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\left(\frac{1+r}{1-t}\right). \end{aligned}$$

Then, similarly to Lemma 2.1, the decision rule encompassing default for government i which has inherited an economy with endowment e_i, legacy debt D_{i-1}, and legacy domestic debt D_{i-1}^{Dom} can be characterized as the following. For the sake of brevity, we use the notation $S_i(\cdot) := S(\cdot\,;\bar{D}_i,\bar{D}_{i+1},\dots)$ and $t_i(\cdot) := t(\cdot\,;\bar{D}_i,\bar{D}_{i+1},\dots)$.

(i) If $S_i(e_i) - (1+r)D_{i-1} < 0$, the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges autarkic tax rate t^{**}.

(ii) *If $S_i(e_i) - (1+r)D_{i-1} < \tau^{**} - C - z(1+r)D_{i-1}^{Dom}$, the government potentially can pay back the legacy debt, but finds defaulting more advantageous. In other words it strategically defaults, enters autarky, and charges the autarkic tax rate t^{**}.*

(iii) *If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax $t_i(e_i)$ and issues $S_i(e_i) - \tau(t_i(e_i))$ amount of debt. Total spending of the government is $S_i(e_i) - (1+r)D_{i-1}$.*

The flat debt ceiling case corresponds to setting $D_i = \bar{D}\ \forall i$. Let us first prove that the mapping $T(\bar{D})$:

$$F \to T(\bar{D})F = \max_t \frac{1}{1+r} \min\Big[F(e') - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D}\Big] + \tau(t),$$

is monotonic:

$$F \le G \quad \forall e \Rightarrow TF \le TG \quad \forall e;\ \text{and} \tag{C.14}$$

$$\bar{D}^1 \le \bar{D}^2 \Rightarrow T(\bar{D}^1)F \le T(\bar{D}^2)F \quad \forall e. \tag{C.15}$$

In the interest of brevity, let us define:

$$T^t(\bar{D})F := \frac{1}{1+r} \min\Big[F(e') - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D}\Big] + \tau(t),$$

so that $T(\bar{D}) = \max_t T^t(\bar{D})$. Note that fixing t, T^t is a monotonic transformation: $F \ge G \Rightarrow T^t F \ge T^t G$, $\bar{D}^1 \le \bar{D}^2 \Rightarrow T(\bar{D}^1)F \le T(\bar{D}^2)F$. Next, we prove (C.14) and (C.15).

Proof of (C.14). Suppose per contra that for some e, $TF > TG$. Let the associated tax rates be t_F and t_G. This leads to the following contradiction:

$$\begin{aligned} T^{t_F}F(e) &> T^{t_G}G(e) && \text{(by assumption)} \\ &\ge T^{t_F}G(e) && (\because \text{optimality of } t_G) \\ &\ge T^{t_F}F(e). && (\text{monotonicity of } T^t) \end{aligned}$$

Proof of (C.15). Similarly, suppose per contra that $T(\bar{D}^1)F > T(\bar{D}^2)F$ for some e. Let the associated tax rates be t_1 and t_1. This leads to the following contradiction:

$$\begin{aligned} T^{t_1}(\bar{D}^1)F(e) &> T^{t_2}(\bar{D}^2)F(e) && \text{(by assumption)} \\ &\ge T^{t_1}(\bar{D}^2)F(e) && (\because \text{optimality of } t_G) \\ &\ge T^{t_1}(\bar{D}^1)F(e). && (\text{monotonicity of } T^t) \end{aligned}$$

Now consider two generic value functions $S^1 := S(\cdot;\bar{D}_1,\ldots,\bar{D}_n^1,\ldots)$ and $S^2 := S(\cdot;\bar{D}_1,\ldots,\bar{D}_n^2,\ldots)$ where the debt ceiling is different for only one period $i = n$, and suppose WLOG that $\bar{D}_n^1 < \bar{D}_n^2$. Note that

$$S^1 = \Big(\prod_{i=1}^{n-1} T(\bar{D}_i)\Big) T(\bar{D}_n^1) S^{n+1}, \text{ and}$$
$$S^2 = \Big(\prod_{i=1}^{n-1} T(\bar{D}_i)\Big) T(\bar{D}_n^2) S^{n+1};$$

where $S^{n+1} := S(\cdot;\bar{D}_{n+1},\bar{D}_{n+2},\ldots)$. Note that from (C.15),

$$S_n^1 := T(\bar{D}_n^1)S^{n+1} \leq T(\bar{D}_n^2)S^{n+1} =: S_n^2.$$

Then, by successive application of (C.14) for $i = 1,\ldots,n-1$, we derive that $S^1 \leq S^2$. ■

Proof of Proposition 5.2: First note that in this special case the Bellman equation takes the following form:

$$\begin{aligned} S(e;\bar{D}) = \max_t \quad & \Big[\frac{1}{1+r}\min\big[S(e';\bar{D}) - \max\{0,\tau^{**} - C - zs(1+r)\},\bar{D}\big] + \tau(t)\Big] \\ s.t. \quad & e' = \kappa_1\big[(1+r)(e-k(t)) + (1-t)f(k(t))\big], \\ & s = \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big). \end{aligned} \tag{C.16}$$

It follows similarly to Lemma C.6 that there are only two possible steady states, A and W, which must satisfy conditions specified in Lemma C.5. What remains to be proved is that the necessary and sufficient condition for the willingness-to-pay region steady state W to exist is that $\bar{D} \geq \bar{\bar{D}}$ for some $\bar{\bar{D}}$.

Let us conjecture that $\bar{\bar{D}} = D^W$ defined in Lemma 3.4, and suppose first that $\bar{D} > D^W$. Note that in steady state W, the current and all future governments on the equilibrium path take on the debt of amount D^W which is below the debt ceiling. Using this logic, we can verify that a conjectured partial solution $S(e;\bar{D}) = S(e)\ \forall e \leq \hat{e}^1$ solves the Bellman equation in (C.16), similarly to Lemma C.6. By the uniqueness of the solution, this proves that $\bar{D} > D^W$ does not alter the behavior of the model economy for $e < \hat{e}^1$.

Now suppose instead that $\bar{D} < D^W$. We know that if the steady state were to exist, the tax rate must satisfy (C.8), and that $e^{ss} = e^{sat}(t^W)$. We then verify the impossibility of the existence by observing the fact that at (e^{ss}, t^W), the optimality condition is violated because of the debt ceiling binding.

It can be seen that once the debt ceiling starts binding, the marginal sensitivity of the first term (min$\{\cdot, \bar{D}\}$) to the tax rate is zero. Therefore, the government's choice of tax rate in this case would be t^{**}. Therefore, if steady state W is removed, the only steady state that can survive is $e^A = e^{sat}(t^{**})$. ■

Proof of Proposition 5.4: In a steady state, the government defaults if and only if the new government spendings under the debt restructuring scheme, $\big(S(e^W;\bar{D}) - (1+r)(1-\lambda)D^W_{-1}\big)$, is lower than the original spending $\big(\tau^{**} - C - z(1+r)s(e^W, t^W)\big)$, the expression for which is derived in Lemma 3.4. Observe that

$$\begin{aligned} & S(e^W;\bar{D}) - (1+r)(1-\lambda)D^W_{-1} \geq \tau^{**} - C - z(1+r)s(e^W, t^W) \\ \Rightarrow & (1-\lambda) \leq \frac{S(e^W;\bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D^W_{-1}} \\ \Rightarrow & \lambda \geq 1 - \frac{S(e^W;\bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D^W_{-1}}. \end{aligned}$$ ■

Proof of Proposition 5.5: First observe that for all endowment paths starting from the trap endowment, the debt ceiling is binding. Therefore, there are only three possible choices of tax rate: choose tax rate such that either (i) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) = \bar{D}$, (ii) $S(e';\bar{D}) = \bar{D}$ or (iii) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$ and $\tau'(t) = 0$.

We show that in all possible cases, $t(e;\bar{D})$ is weakly decreasing in $\bar{D}$, having e fixed. Observe that using the envelope theorem – given that the debt ceiling is binding – yields $\frac{\partial S(e;\bar{D})}{\partial \bar{D}} < 1$. Using this, and supposing $\bar{D}^1 < \bar{D}^2$, we assess the property in each case:

(i) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) - \bar{D} = 0$. Note that the LHS is decreasing in $\bar{D}$, and therefore t has to increase the LHS to counteract. The LHS is decreasing in t implying that t should be decreasing as $\bar{D}$ is decreasing.

(ii) $S(e';\bar{D}) - \bar{D} = 0$. This case is similar to case (i) above.

(iii) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$ and $\tau'(t) = 0$. In this case $t = t^{**}$ and therefore the stated condition that $t(e;\bar{D})$ is weakly decreasing condition in $\bar{D}$ is preserved. ■

Proof of Proposition 6.1: The partial derivatives of S and e in the two steady states were proved in Lemma C.6. For the savings parameter ρ, notice first that an application of envelope theorem on the Bellman equation in (2.16) yields, in steady state W:

$$\frac{\partial S}{\partial \rho} = \frac{1}{1+r}\Big[\frac{\partial S}{\partial \rho} - \frac{\partial \tau^{**}}{\partial \rho} + z\frac{\partial s}{\partial \rho}(1+r)\}\Big] + \frac{\partial \tau(t)}{\partial \rho}$$

$$\Rightarrow \frac{\partial S}{\partial \rho} = \frac{1+r}{r} z \frac{\partial s}{\partial \rho}(1+r) > 0 \qquad (\because \tfrac{\partial \tau}{\partial \rho} = 0)$$

It follows similarly that at steady state A, $\frac{\partial S}{\partial \rho} = 0$. For the productivity parameter ϕ, an application of envelope theorem yields, in steady state W:

$$\begin{aligned}
\frac{\partial S}{\partial \phi} &= \frac{1}{1+r}\Big[\frac{\partial S}{\partial \phi} - \frac{\partial \tau^{**}}{\partial \phi} + z\frac{\partial s}{\partial \phi}(1+r)\}\Big] + \frac{\partial \tau(t)}{\partial \phi} \\
\Rightarrow \frac{r}{1+r}\frac{\partial S}{\partial \phi} &= z\frac{\partial s}{\partial \phi}(1+r) - \frac{1}{1+r}\frac{\partial \tau^{**}}{\partial \phi} + \frac{\partial \tau(t)}{\partial \phi} \\
\Rightarrow \frac{r}{1+r}\frac{\partial S}{\partial \phi} &= \underbrace{z\frac{\partial s}{\partial \phi}(1+r)}_{<0} - \Big[\frac{1}{1+r}\tau^{**} - \tau(t)\Big] \qquad (\because \tfrac{\partial \tau(t)}{\partial \phi} = \tau(t))
\end{aligned}$$

Now notice that since $\tau^{**} = \max_s \tau(s) \geq \tau(t)$, the second term $\Big[\frac{1}{1+r}\tau^{**} - \tau(t)\Big] > 0$ for sufficiently low r. The partial derivative in steady state A $\frac{\partial S}{\partial \phi} > 0$ follows similarly. ■

Lemma C.9. *The government's problem, with access to a technology that for investment I generates cash flow $g(I)$ acrruing to the next-period government, is characterized by the following Bellman equation:*

$$S(e) = \max_{t,I} \quad \Big[\frac{1}{1+r}\big[S(e') + \min\{g(I), C + zs(1+r) - \tau^{**}\}\big] + \tau(t)\Big] - I.$$

The optimal investment function $I(e)$ has the following property: $\exists \bar{e}^1_{gcf} < \bar{e}^2_{gcf}$ such that $\forall e < \bar{e}^1_{gcf}$, $I(e) = 0$, and $\forall e > \bar{e}^2_{gcf}$, $I(e) = I^{**} := \text{argmax}_i \big[\frac{1}{1+r}g(i) - i\big]$. In other words, governments in economies with low endowments may not see any value in spending productively, even if the technology exists.

Proof: First, note that since $g(I)$ is concave, the optimal I is always smaller or equal to I^{**}. We then consider the two limits of the endowment.

Consider $e \to 0$. For sufficiently small e, $C + zs(1+r) - \tau^{**} < 0$, implying that $\min\{g(I), C + zs(1+r) - \tau^{**}\} = C + zs(1+r) - \tau^{**}$ $\forall I \geq 0$. In this case, the dependence of the objective function on I only comes from the $-I$ term. Therefore, the maximum is achieved at $I = 0$, regardless of other values.

Then consider $e \to \infty$. For sufficiently large e, $C + zs(1+r) - \tau^{**} > g(I^{**})$, implying that $\min\{g(I), C + zs(1+r) - \tau^{**}\} = g(I)$ $\forall I \in [0, I^{**}]$. In this case, the optimization problem is separable for I, *i.e.*, $I(e) = \text{argmax}_i \big[\frac{1}{1+r}g(i) - i\big] = I^{**}$.

In the interim region, the optimal I is such that it slides between the two constraints, i.e., $g(I) = C + zs(1+r) - \tau^{**}$. ■

Proof of Proposition 7.1: The optimality condition for the fiscal transfer can be expressed as

$$\max_{\Delta e} \quad \frac{1}{1+r} S(e^W + \Delta e) - \Delta e \quad s.t. \quad \Delta e \geq 0$$

Notice that due to the concavity of S, the optimal $\Delta e > 0$ if and only if $\frac{dS}{de}(e^W) > 1 + r$. Therefore, we conclude that

$$\Delta e > 0 \Leftrightarrow 1 + r < \frac{dS}{de}(e^W) = \rho z \Leftrightarrow \rho > \frac{1+r}{z} \ . \qquad \blacksquare$$