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THE RISE AND FALL OF ELITES: A THEORY OF ECONOMIC DEVELOPMENT AND SOCIAL POLARIZATION IN RENT-SEEKING SOCIETIES

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ABSTRACT

The Rise and Fall of Elites: A Theory of Economic Development and Social Polarization in Rent-seeking Societies*

This paper analyses how political institutions, wealth distribution and economic activities affect each other during the process of development. A simple general equilibrium model of rent-seeking political elites with two productive sectors (modern and traditional) is presented. Political participation is viewed as a costly activity. We show what drives entry into politics and how the size of the elite affects the level of distortions. The model also highlights the role played by the initial distribution of wealth in determining the long-run pattern of political participation and economic performance. We show why one society may converge to an equilibrium with low distortions and social equality, while others may end up with an institutional framework that brings about high distortions and social polarization. The model is then extended to account for the provision of public goods, to analyse the effects of rent-seeking on technological change, and to allow for random shocks in intergenerational transfers.

JEL Classification: D31, D72, O10
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NON-TECHNICAL SUMMARY

This paper analyses the role political institutions play during the process of development, and how they are in turn influenced by it. It emphasizes *inter*-group redistributions (redistributions between the politically powerful and the politically weak) at the expense of the more standard notion of *intra*-group redistributions (those between rich and poor voters), and examines their implications for economic performance.

In the model developed here, political participation is endogenous. This allows us not only to examine how politics affects economic performance, but also how political institutions evolve in response to changes in economic performance.

The structure is simple. We integrate rent-seeking political *elites* into a two-sector model of production. Unhindered by rent-seeking, this economy would achieve the efficient outcome, i.e. productive inputs would be allocated to their best use and the economy would reach its production possibility frontier. But the existence of a *res publica* provides incentives for political *elites* to organize and develop. We then show how the optimizing behaviour of these political *elites* distorts the allocation of productive resources across sectors in the economy and the extent of political redistributions in society.

The existence of political rents begs the question of entry into the *elite*, however. Two variables play a key role in shaping the dynamics of political participation in our model: the technology of political participation and the initial distribution of wealth.

As to the first, we assume that belonging to the ruling class is costly. The second key variable is the initial level and distribution of wealth. Our model emphasizes credit constraints as a crucial determinant of political outcomes. When costly political participation is combined with imperfect capital markets, the distribution of wealth plays a crucial role: *how* wealth is distributed in society will determine *who* will be able to belong to the *elite*.

In our model, the size and ultimate fate of political *elites* is, to a large extent, explained by distributional considerations. But the distribution of wealth is in turn determined by the size and fate of political *elites*. By imposing distortionary taxes, the ruling class can affect the allocation of resources in the economy and, ultimately, the patterns of wealth accumulation and distribution in society.

We show how the initial distribution of wealth and the technology of political participation affect entry into politics, and how the size of the resulting *elite* affects distortions, resource allocation and political redistributions. We also study the main determinants of a society's long-run institutional equilibrium. In particular, we show why one society converges to an equilibrium with low distortions and social equality while others end up with an institutional framework that brings about high distortions and social polarization.

Then we discuss the comparative statics and comparative dynamics of the system. We show, in particular, that temporary changes in the technologies of production or political participation might have permanent effects on the nature of the institutional equilibrium to which society converges in the long run.

Lastly, we show how the model can be extended to account for public goods, growth and uncertainty. In particular, cycles with endogenous rise and fall of elites are shown to exist under some conditions.

Until two or three hundred years ago, it was characteristic almost everywhere - and to this day, it is characteristic in the majority of countries and in countries containing the majority of the world's population - that the primary government activity was and is extraction of surpluses (...) and use of such surpluses to benefit tiny groups of people in and near the government - Mills (1986, p. 134).

Why do some countries industrialize, while others remain agricultural or stagnate for decades under the weight of distortions that inhibit productive investments and growth? Why should apparently similar countries enjoy radically different forms of political organization, ranging from regimes dominated by small and violently autocratic *elites* to institutions that grant access to political participation for a large fraction of the population? What role does income distribution play in the process of economic development and political change?

These very old questions are the starting point of our paper. We analyze the role that political institutions play during the process of development, and how they are in turn influenced by it.

By looking at how political institutions affect economic performance, we contribute to the recent (and not so recent) literature on the political economy of growth.[1] This literature notes that the incentives for accumulation of physical or human capital are largely determined by the ability of individuals to appropriate the fruits of their investments. According to this view, societies where distributional conflicts are important will tend to adopt policies that allow less private appropriation and more redistribution in favor of the poor. Building on median-voter type of results, these models predict higher taxes and higher distortions for countries where income disparities are more acute.

In this tradition, political institutions play a crucial role. Indeed, the extent of these distortionary redistributions will depend on the ability of the poor to express their demands through political channels. A standard empirical prediction of these models is that (holding measures of income or wealth distribution constant), democracies should have more redistribution, higher distortions and lower growth of per capita GDP than dictatorships.

But the empirical evidence on this point is far from incontrovertible. Przeworski and Limongi (1993) report that out of 18 studies that have looked at the effects of democracy on growth since 1966, 8 find

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[1] See, for example, Helliwell (1992), Alesina and Rodrik (1991), Persson and Tabellini (1991), Saint-Paul and Verdier (1992a and 1992b), Ades (1992), and Murphy, Shleifer and Vishny (1991), North (1981 and 1990), and Olson (1982).

a positive effect of democracy on growth, 8 a negative effect, and 2 find no significant relationship. Using cross-sectional and pooled data for up to 125 countries over the period 1960 to 1985, Helliwell (1992) fails to identify a robust effect from democracy to growth.[2]

To some people, these findings come as no surprise. While there are good theoretical reasons for believing that more representative institutions might hinder growth by allowing more redistributions *within* the group of political participants (voters), there are also arguments to the contrary. In regimes where political power is unevenly distributed, rulers might find it easier to extract rents from weaker groups in their favor.[3] Alternatively, dictators might find it easier to benefit from the spoils of distortions than democratic assemblies, increasing their incentives for redistributing *between* politically active and inactive groups (as argued by Eckelund and Tollison, 1981). Olson (1991) has also argued for the inability of autocrats to credibly commit themselves to growth enhancing policies.

All these ideas lead, therefore, to the opposite empirical prediction: holding income or wealth distribution constant, political redistributions, high distortions and low growth will be less likely in democratic regimes. De long and Shleifer (1992) have provided some empirical support for these views. In their analysis of city growth before the Industrial Revolution, they find that cities grew rapidly and commerce flourished where princes were weak or absent and political power was either held by merchant oligarchies ruling self-governing cities or checked by representative assemblies.[4]

Our paper incorporates these ideas into a simple model in which rulers device policies in their favor. We emphasize *inter*-group redistributions (redistributions between the politically powerful and the politically weak) at the expense of the more standard notion of *intra*-group redistributions (those between rich and poor voters), and examine their implications for economic performance.

Any model in which (in equilibrium) political *elites* are able to capture positive rents from the least powerful must somehow address the issue of entry into the *elite*. If political activity is profitable, why

[2] A related line of research does not look *directly* at the effects of democracy on growth. Instead, it focuses on how income distribution variables affect growth through political channels. See for example Alesina and Rodrik (1991), Persson and Tabellini (1991), and Perotti (1992). These studies find that income inequality affects growth *only in democracies*.

[3] This is a standard prediction in models like North (1982), where rulers devise property rights in their own interests.

[4] One such example is given by the contrast between the booming commercial economies of republican Holland and constitutional England in the eighteenth century and the stagnant economy of absolutist France.

is it that we do not observe more of it? Wouldn't competition for these rents drive them down to zero? To address these issues, we develop a model where political participation is endogenous. We thus examine not only how politics affects economic performance, but also how political institutions evolve in response to changes in economic performance.

The structure of our model is simple. We integrate rent-seeking political *elites* into a two sector model of production. Unhindered by rent-seeking, this economy would achieve the efficient outcome, i.e., productive inputs would be allocated to their best use and the economy would reach its production possibility frontier. But the existence of a *res publica* provides incentives for political *elites* to organize and develop. We then show how the optimizing behavior of these political *elites* distorts the allocation of productive resources across sectors in the economy and the extent of political redistributions in society.[5]

The existence of political rents begs, however, the question of entry into the *elite*. Two variables play a key role in shaping the dynamics of political participation in our model: the technology of political participation and the initial distribution of wealth.

As to the first, we assume that belonging to the ruling class is costly. This assumption has already a long tradition in the literature.[6] These costs can be thought off as capturing how easy it is for political *elites* to get organized and redistribute from the *masses*. Small urbanized regions make political organization easier than societies where a large segment of the population is dispersed and unorganized in large rural areas. Other things equal, the first type of regions are more likely to encourage the formation of large ruling *elites*.

The second key variable is the initial level and distribution of wealth. Borrowing from another tradition in the literature on income distribution and growth (Galor and Zeira (1993), Banerjee and Newman (1993), and Aghion and Bolton (1991)), our model emphasizes credit constraints as a crucial determinant

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[5] The literature has emphasized several damaging effects of rent-seeking on development and growth. First, as the rent-seeking sector expands, it draws productive resources into it and so reduces income. The size of government bureaucracies and armies in some developing countries illustrates this effect. Second, if the rent-seeking sector attracts the most talented people, the rate of technological progress and growth are likely to be lower. Murphy, Shleifer and Vishny (1991) discuss the importance of this effect, and present it as a possible explanation for the recent U.S. productivity growth slowdown. Finally, the tax imposed by the rent-seeking sector on the productive sector reduces incentives to produce and to invest, and so reduces income and possibly growth. Our paper focuses on this last channel, and shows the damaging effects that rent-seeking has on efficiency.

[6] See, for example, Downs (1957), Huntington and Nelson (1976), Mayer (1984).

of political outcomes.[7] When costly political participation is combined with imperfect capital markets, the distribution of wealth plays a crucial role: *how* wealth is distributed in society will determine *who* will be able to belong to the *elite*.[8]

In our model, the size and ultimate fate of political *elites* is, to a large extent, explained by distributional considerations. But the distribution of wealth is in turn determined by the size and fate of political *elites*. By imposing distortionary taxes, the ruling class can affect the allocation of resources in the economy and, ultimately, the patterns of wealth accumulation and distribution in society. Our paper aims at studying the behavior of this dynamic process.

Section I presents a simple model of rent-seeking political *elites* with two productive sectors: an untaxed traditional sector, and a taxable modern sector.[9] We show how the initial distribution of wealth and the technology of political participation affect entry into politics, and how the size of the resulting *elite* affects distortions, resource allocation and political redistributions. We also study what are the main determinants of a society's long-run institutional equilibrium. In particular, we show why one society converges to an equilibrium with low distortions and social equality while others end up with an institutional framework that brings about high distortions and social polarization. In section II, we discuss the comparative statics and comparative dynamics of the system. We show, in particular, that temporary changes in the technologies of production or political participation might have permanent effects on the nature of the institutional equilibrium to which society converges in the long run.

In section III we show how the model can be extended to account for public goods, growth and uncertainty. In section 3.1., we relax the assumption of purely *parasitic elites* and consider the case in which the ruling group can decide on the allocation of revenues between transfers for themselves and the provision of public goods beneficial to production. We then show how *elites* decide on the shares allocated to those different uses, and how in turn this choice determines their evolution. The results from

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[7] In these models, the poor are credit constrained and cannot borrow to invest in education. In turn, this results in lower rates of human capital accumulation and (in models with endogenous growth) lower steady state growth.

[8] To the extent that bequests make a sizeable fraction of total wealth, participation costs can alternatively be seen as capturing the role of hereditary considerations in the formation of *elites*, or equivalently, the degree of social mobility in society. In this interpretation, the existence of well developed credit markets is seen as an important determinant in the path of institutional development (as suggested by Putnam, 1993).

[9] The reason for having two sectors will soon become clearer.

this section explain why governments in developed countries tend to allocate larger shares of expenditures to the provision of services with a public good nature.

Section 3.2. allows for learning effects arising in the modern sector of the economy. We then consider how political participation and growth performance are connected. Section 3.3. extends the basic model of section I to allow for randomness in the technology of intergenerational transmission of wealth. Section IV suggests some avenues for future research and concludes.

I.- A Model of Political Rent-seeking and Economic Development

1.1. The Game

This section presents a model of production and political participation. The model is based on Ades (1992), but it is enriched to allow for dynamic considerations in a way similar to Saint Paul and Verdier (1992b). We consider an economy with a continuum of consumers-citizens that are alive for one period. Population size is normalized to 1. Each individual has one offspring and generations are altruistically linked by a "joy of giving" motive for bequests. Preferences are described by a utility function $U(c_t,b_{t+1})$ where c_t is consumption at time t and b_{t+1} is the bequest left for a child born at $t+1$.[10] We assume that $U(.,.)$ is twice continuously differentiable, increasing in each argument c_t and b_{t+1}, strictly concave and homothetic. The marginal rate of substitution U'_c/U'_b between c_t and b_{t+1} is therefore an increasing function $\varphi(b_{t+1}/c_t)$ of the ratio b_{t+1}/c_t. In what follows, it is useful to define $\rho=\varphi^{-1}(1)$. ρ has then the natural interpretation of being the relative weight of the "joy of giving" component in the utility function.

Consumers are endowed with one unit of labor, which they can allocate to any one of the two productive sectors in the economy. The first sector has a decreasing returns to scale production technology. In this sector, l_t units of labor produce $\phi(l_t)$ units of output, with $\phi'>0$, $\phi''<0$, $\phi'(0)=\infty$ and $\phi'(1)=0$. This activity is not taxable, and can be thought of as agriculture, though informal, household production or underground sectors in many developing countries also fit the general characteristics described above. The other productive sector of the economy has a constant returns to scale production function, and its technology is such that l_t units of labor produce l_t units of output. This

[10] This utility function has a long tradition in the development/political-economy tradition. See, for example, Banerjee and Newman (1992), Galor and Zeira (1988), Ekstein and Zilcha (1991) and Saint Paul and Verdier (1992b).

activity is taxable, and it can be thought of as the "modern" sector of the economy or as manufacturing.[11]

The reason for introducing two sectors should now be clear. In order for taxes to have distortionary effects, agents must be able to substitute away from the taxed activity. In our model, agents optimize at the margin given by modern/manufacturing vs. traditional/agriculture. Of course, in real world situations, agents will most likely find themselves optimizing at many margins at the same time (*e.g.*, labor vs. leisure, exports vs. import substitution, domestic investment vs. capital flight, etc). Our choice of manufacturing vs. agriculture is not important for our results and must be taken either as an example or as a simplifying assumption.[12]

Consumers are also endowed with a bequest b_t left over by their parents in period $t-1$. This bequest can be consumed, or it can be used to enter into political activity. If a consumer decides to enter politics at time t, he will belong to the ruling *elite*. This enables him to decide on the level of taxes in the economy and to get a share of those tax revenues, unlike those consumers that decide not to participate in politics and stay in the *masses*. If a consumer decides to enter the ruling *elite*, he has to bear a fixed cost π.[13] Because of capital market imperfections, agents are not able to borrow to enter the *elite* using their future income as collateral. Thus, in order to belong in the *elite* at time t, an agent must have a starting level of wealth $b_t \geq \pi$.[14]

Each agent's life can be subdivided into three stages. In the first stage, (i) they receive their

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[11] Reynolds (1985) notes that at early stages of development, the economy is dominated by the *household production* sector. These activities are typically less easy to detect and measure and, thus, more to tax than activities in the monetary sector of the economy.

[12] Ades (1992) presents a case in which the relevant margin is labor vs. leisure.

[13] We thank Andrei Shleifer for pointing to us that the assumption of a fixed cost of entry derives naturally from the standard notion that the State is an institution with monopoly powers to tax.

[14] The importance that liquidity has always had for gaining access to positions of political power can hardly be emphasized. Schama (1989) provides one of many historical examples: that of the Farmer-General in 18th century France. Overburdened by the huge debts that resulted from its participation in the American war of independence and desperate for cash, the French monarchy contracted every six years "with a syndicate of these men for a *bail*, or lease, by which they agreed to advance a specific sum to the Treasury in return for the right to "farm" certain indirect taxes." Brutally efficient, they were characterized by Darigrand's *L'Anti Financier* as "blood-suckers fattening themselves off the substance of the people". In order to collect their dues they were entitled with the right to enter, search and seize any property or household they deemed suspicious.

endowments, and (ii) decide whether they will enter politics in period 2. Let z be the fraction of individuals that do not enter politics. Therefore, $(1-z)$ will be the fraction of agents that belong to the *elite*.

In stage 2, (iii) those individuals that have decided to enter politics "vote" for a proportional tax τ on the income of the modern sector. In deciding, they consider the way in which other agents and themselves will react to τ.[15]

In the final third stage, (iv) the *masses* and the *elite* make their choice of how to allocate their labor among the two productive activities, (v) production and consumption take place, and (vi) each agent decides how much bequest he will leave to his descendant.

The timing of events is, thus, as follows

t			$t+1$
	* receives bequest b_t	* elite votes on τ_t	* decision on labor supply
	* decision to enter politics		* decision on bequest for $t+1$
			* production and consumption take place

There are three features of our political mechanism that should not be taken literally. The first is the assumption of proportional taxes.[16] This assumption is less restrictive than what it might seem at a first glance. In fact, the results of our model are quite general and are robust to most non-linear tax schedules, as long as members of the *elite* are unable to avoid *all* the distortions imposed by the tax system.

The second feature that should not be taken literally is the voting mechanism itself, which should be best seen as the reduced form of a more complex mechanism in which government activity is influenced through various channels, voting being only one of them. In fact, we think of the *elite* as all those agents who can exert, through different channels, a noticeable influence on the choice of policies by the government.

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[15] In order to illustrate in the simplest possible way the effects of *inter*-group redistributions on inequality and economic performance, we have completely abstracted from issues concerning *intra*-group redistributions. This has the important implication that differences among members of the *elite* will not play a role in political decisions. In the conclusion, we suggest a way in which the model can be extended to account for both types of distributional conflict.

[16] Reliance on proportional taxes follows a well-established tradition. Barro (1990), Alesina and Rodrik (1991), Murphy, Shleifer and Vishny (1991) and others work within that framework.

Finally, our use of an income tax is only intended to capture the effects of rent-seeking activities by one group on the welfare of others. As such, it can be used to describe similar behavior in related contexts (*e.g.*, trade unions, regulation, foreign trade, exchange rate policies, etc). For instance, our model can be easily reinterpreted as one with rent-seeking and trade taxes.[17] The only important feature about τ is that it illustrates a rent extracting instrument imposing distortions in a less than perfect discriminatory way in the economy. On the contrary, the rents generated by this instrument are assumed to be perfectly targeted to the people that are politically influential.

The problem can be solved by backward induction.

(i) Stage 3

Consumers decide on how to allocate their labor and how much bequest to leave to maximize utility. Then, the problem to be solved is

$$\max_{<l_t, b_{t+1}>} U_t = U(c_t, b_{t+1}) \tag{2}$$

subject to

$$c_t + b_{t+1} = (1-\tau_t) l_t + \phi(1-l_t) + b_t + R_t \tag{2}$$

and

$$0 \leq l_t \leq 1 \tag{3}$$

where R_t are the rents collected from political participation. R_t is zero if an agent does not belong to the *elite*. If an agent participates in politics, he shares equally with other members of the *elite* in the rents collected. Thus, for an agent who participates in politics, R_t is equal to T_t-π, where

[17] Consider a small open economy that produces two goods: an exportable good with a technology that exhibits constant returns to scale, and an import competing good manufactured domestically with a decreasing returns to scale technology *($\phi(l_t)$)*. Take the importable good as the numeraire, (by an appropriate choice of units) fix the terms of trade of the country to be equal to 1, and reinterpret c_t as a composite index of consumption of both goods. Then, one can think of τ as a tax on the exportable sector, with revenues being only redistributed among the people that enter successfully in politics (what we call the *elite*).

$$T_t = \frac{\tau_t l_t}{1-z_t} \tag{4}$$

In deciding how to allocate their labor and how much bequest to leave, agents take τ_t and the size of the *elite* as given. The solution to this problem yields the following first order condition

$$(1-\tau_t) = \phi'(1-l^*(\tau_t)) \tag{5}$$

from where optimal labor supply $l^*(\tau)$ is just a function of the tax rate, with $dl_t^*/d\tau<0$, $0<l^*(0)<1$ and $l^*(1)=0$. Given our assumption of homotheticity, it is easy to see that consumption and bequests are given, respectively, by

$$c_t = \frac{1}{1+\rho}[(1-\tau_t)l_t^* + \phi(1-l_t^*) + b_t + R_t] \tag{6}$$

$$b_{t+1} = \frac{\rho}{1+\rho}[(1-\tau_t)l_t^* + \phi(1-l_t^*) + b_t + R_t] \tag{7}$$

Thus, both consumption and the level of bequests left over for the generation born at time $t+1$ are a fixed proportion of disposable income.

(ii) Stage 2

Those agents that belong to the *elite* choose their preferred τ to maximize their own utility. In doing so, they consider the effect that τ will have on revenues through its effect on the allocation of labor by every member of society, including themselves.

Using (4), (5), (6) and (7) to substitute in the utility function $U(c_t, b_{t+1})$, we can compute indirect utilities both for members of the *elite* and of the *masses*. Because of homothetic preferences, these functions take respectively the following form

$$V_t^E = V(k(\rho)[(1-\tau_t)l_t^* + \phi(1-l_t^*) + T_t - \pi + b_t]) \tag{8}$$

and

$$V_t^M = V(k(\rho)[(1-\tau_t)l_t^* + \phi(1-l_t^*) + b_t]) \tag{9}$$

where $k(\rho)$ is a constant depending only on ρ and $V(.)$ is a monotonically increasing function.

Maximization of V_t^E with respect to τ yields the following F.O.C. condition

$$z_t l_t^*(\tau_t) + \tau_t \frac{dl_t^*(\tau_t)}{d\tau_t} = 0 \tag{10}$$

which defines implicitly the optimal τ_t^* as a function of z_t. For a well defined concave problem, τ_t^* is increasing in z_t, the share of total population that does not participate in politics.[18]

Moreover, equation (10) shows that τ must balance two effects. The first is the maximization of surplus extracted from the *masses*. This is given by the first term on the LHS. As mentioned before, revenues have Laffer-type properties. Therefore, there exists a τ^* that maximizes surplus extraction. The second effect that will be taken into account by the representative member of the *elite* when choosing τ is the deadweight loss that he has to bear in order to extract surplus from the *masses*. This is shown by the second term on the LHS. As we can see, τ approaches the revenue maximizing level as z approaches 1 and the size of the *elite* converges to zero. Conversely, τ tends to zero as z converges to 0 and the size of the *elite* converges to 1. We thus have that $\tau_t^*(1) = \bar{\tau} = Argmax\ \tau_t l_t^*(\tau_t)$ and $\tau_t^*(0) = 0$. *In other words, the smaller the ruling class, the more it will be concerned with revenue maximization, and the less it will be concerned with efficiency.*

(iii) Stage 1

Two conditions determine the decision to belong in the *elite*. The first is that $b_t \geq \pi$. In other words, the agent must not be liquidity constrained and is thus able to pay the entry cost into the *elite*. The second condition is that it is profitable to enter. This last condition is given by $T_t \geq \pi$, where

[18] A sufficient condition for the second order condition to be globally satisfied is that ϕ is of class C^3 and $1 + \phi'''(1-l)/[\phi''(1-l)]^2 \leq 0,\ \forall\ l \in (0,1)$.

$$T_t = T_t(z_t) = \frac{\tau_t^*(z_t)I_t^*(\tau_t^*(z_t))}{1-z_t} \tag{11}$$

Because $\tau_t^*(z_t) \leq \hat{\tau}$ (*i.e.*, we are on the "good" side of the Laffer curve), it is clear that $T_t(z_t)$ is increasing in z_t, $T_t(0)=0$ and, moreover, $T_t(z_t) \to \infty$ as $z_t \to 1$.

If liquidity constraints were not binding, entry would drive political rents to zero. Let $(1-\bar{z})$ be the equilibrium size of the *elite* under such conditions. This size would be defined by the equality of T_t and π. In other words, $(1-\bar{z})$ would be such that

$$\frac{\tau_t^*(\bar{z}_t)I_t^*(\tau_t^*(\bar{z}_t))}{1-\bar{z}_t} = \pi \tag{12}$$

and it corresponds to the level of political participation that makes the *elite* members indifferent between belonging to the *elite* or to the *masses*.

However, as long as liquidity constraints *do* matter for political participation, $(1-\bar{z})$ will not be attained. At any time t, we can then define the share of total population that is liquidity constrained. This is given by

$$q_t = \int_0^{\pi} f_t(b_t)\,db_t = F_t(\pi) \tag{13}$$

where $f_t(.)$ and $F_t(.)$ are, respectively, the density and cumulative functions of the distribution of bequests b_t at time t. Thus, q_t shows the fraction of people that have received a bequest less than π at time t. Then, the path of z_t, the fraction of total population that belongs to the *masses* at time t, is given by

$$z_t = Max\{q_t, \bar{z}\} \tag{14}$$

1.2. Equilibrium Dynamics

Substituting with equation (10) in (7), it is easy to see that the bequest left by an agent that belongs to the *elite* at time t will be given by

$$b_{t+1} = \frac{\rho}{\rho+1}[(1-\tau^*(z_t))l^*(z_t) + \phi(1-l^*(z_t)) + T(z_t) - \pi + b_t] \tag{15}$$

where

$$l^*(z_t) = l^*(\tau^*(z_t)) \tag{16}$$

Similarly, an agent that is not in the *elite*, will leave a bequest

$$b_{t+1} = \frac{\rho}{\rho+1}[(1-\tau_t^*(z_t))\,l^*(z_t) + \phi(1-l^*(z_t)) + b_t] \tag{17}$$

The two difference equations shown by (15) and (17) describe the dynamics of wealth accumulation for both the *elite* and the *masses* as a function of the level of political participation.

For notational convenience, we define

$$A(z) = (1-\tau^*(z))l^*(z) + \phi(1-l^*(z)) + T(z) - \pi \tag{18}$$

and

$$B(z) = (1-\tau^*(z))l^*(z) + \phi(1-l^*(z)) \tag{19}$$

where *A(z)* and *B(z)* are total after-tax incomes (net of political participation costs) for the *elite* and the *masses*, respectively. Then, (15) and (17) can be rewritten as

$$b_{t+1} = \frac{\rho}{1+\rho}[A(z_t) + b_t] \tag{20}$$

for an agent that belongs to the *elite*, and

$$b_{t+1} = \frac{\rho}{1+\rho}[B(z_t) + b_t] \tag{21}$$

if he belongs to the *masses*. Notice that *A(z)* and *B(z)* are such that

$$A(\bar{z}) = B(\bar{z}) \tag{22}$$

which implies that political rents are driven down to zero if liquidity constraints are not binding in equilibrium. Also, *A(z)* and *B(z)* are, respectively, increasing and decreasing in z as

$$A'(z) = \frac{\tau^*(z) l^*(z)}{(1-z)^2} > 0 \tag{23}$$

and

$$B'(z) = -l^*(z)\frac{\partial \tau^*}{\partial z} < 0 \tag{24}$$

Thus, after-tax income net of participation costs is decreasing in the size of the *elite* for the *elite*, and increasing for the *masses*.

Equations (14), (18), (19), (20), and (21) describe the whole dynamics of the system. Due to its nonlinear nature, a complete formal analysis is not easy. However, one can get some interesting insights into the dynamics of taxes, income, wealth and political participation by using a simple diagrammatic analysis.

Figure 1 shows difference equations (20) and (21). These two equations describe the dynamics of wealth accumulation for the *elite* and the *masses* in our two sector economy. As it can be seen, these two equations are linear in b_t. The vertical distance between the two schedules is given by $\rho/(1+\rho)[A(z)-B(z)]$ and is, thus, proportional to the amount of political rents collected by each member of the *elite*. At any time t, these rents are given by $T(\tau^*(z_t))-\pi$. Therefore, the vertical distance between the two schedules is a function of z. This diagram is similar to the one described in the basic model of Galor and Zeira (1993) except for the important fact that here the position of the two schedules may shift with time as the size of the *masses* z_t changes. In particular, an increase in political participation (a fall in z) reduces political rents and brings the two schedules closer to each other. Notice, finally, that at any time t, the values of E and F are given by $\rho B(z_t)$ and $\rho A(z_t)$, respectively.

Another variable that plays an important dynamic role is π. At any time t (provided that some cohorts are liquidity constrained), this variable determines which of them belong to the *elite* and which to the *masses*. Only those cohorts that at time t receive an endowment b_t larger than π will belong to the *elite* and evolve according to equation (20). All the others will do it according to (21), as they will belong to the *masses*.

The initial distribution of wealth at time 0 is also very important as it determines the initial fraction of agents in the *masses*. Let this initial distribution be described by a continuous density function $f_0(.)$ and an associated cumulative function $F_0(.)$ on the closed interval $[b_0^-, b_0^+]$ with $0 \leq b_0^-$ small enough and $\pi < b_0^+$. Then, at time zero, z_0 will be given by $F_0(\pi)$, with $F'(.) > 0$.

In what follows, we assume that initial conditions are such that liquidity constraints are binding for some cohorts, *i.e.*, $z_0=q_0=F_0(\pi)>\hat{z}(\pi)$.[19] Thus, the *elite* enjoys strictly positive rents. This seems to be a reasonable departing assumption to analyze the social dynamics of political *elites*. Then, both π and the initial distribution of wealth play a crucial role in shaping the institutional equilibrium to which society converges in the long run. There are three cases to consider depending on the initial conditions of the system.

Case 1

Suppose that at some initial time *0*, the cost of political participation π and the fraction of people in the *masses* $z_0=F_0(\pi)$ are such that the dynamics of bequest accumulation can be represented as in figure 2. Formally, we require that $\rho A(z_0)<\pi<b^+_0$. This case is most likely to occur when the cost of political participation π is relatively high.

We have then the following proposition:

PROPOSITION 1: Suppose that the political technology and the initial distribution of wealth $F_0(b)$ defined on the support $[b_0^-,b_0^+]$ are such that $\rho A(F_0(\pi))<\pi<b^+_0$: then there exists a finite time S such that:
i) For all $t \leq S$, $z_t=q_t<q_{t+1}=z_{t+1}$. ii) For all $t > S$, $z_t=z_{t+1}=q_{S+1}$.
iii) The wealth distribution $F_t(.)$ converges to the degenerate long run distribution
F_∞ : $\rho B(q_{S+1})$ with mass q_{S+1}
$\rho A(q_{S+1})$ with mass $1-q_{S+1}$.

Proof: See Appendix 1.

Proposition 1 basically says that in this case q_t, the number of people that are liquidity constrained and cannot therefore participate in the political decision-making process rises through time until a finite time *S* after which the system stabilizes with a stationary size of the *masses*. The intuition is simple: as figure 2 shows, all those agents born at time *t* with an endowment b_t smaller that π will also leave to their children an endowment b_{t+1} smaller than π. Thus, neither themselves or their lineage will belong to the

[19] This will be satisfied provided that the initial distribution of bequests is sufficiently skewed to the left.

elite. Conversely, *some* agents that were in the *elite* at time t will not be able to preserve the same situation for their children. As z_t increases, $A(z_t)$ also increases and the schedule that describes the dynamics of wealth accumulation for the *elite* shifts upwards. At the same time, the schedule that describes the bequest function for the *masses* moves downwards. There is then a finite time S such that $\rho A(q_S) < \pi \leq \rho A(q_{S+1})$. At that point, the *elite* stops shrinking and the fraction of the population in the *masses* stays constant at q_{S+1}. Graphically, at S, the schedule that describes the bequest function for the *elite* hits (or crosses) the $45°$ degree line. From then onwards, the size of the *elite* remains fixed as all those agents who received endowments larger than π at S will also leave to their children an endowment larger than π.

Notice that convergence towards the steady state involves a process of impoverishment for those dynasties that already belonged to the *elite* at time 0, as their level of wealth converges towards $\rho A(q_{S+1})$ (point F) which is not significantly different from π. At the same time, an equalizing process of wealth takes place amongst the cohorts that belong to the *masses*, as their level of wealth converges to $\rho B(q_{S+1})$ (point E). This equilibrium path is one in which there is a long run polarization of society into two homogenous groups: an *elite*, that enjoys a level of wealth given by F, and the *masses*, whose long-run level of wealth is given by E.

The path of distortions can also be easily described. Before time S, as the *elite* shrinks, taxes rise monotonically. This is a situation in which the *elite* struggles to maintain its social and political position by raising taxes. During this phase, distortions increase and output falls. The economy moves inward, away from its production possibility frontier. After time S, taxes are sufficiently high so that the dynasties that belong to the *elite* can remain in the *elite*. The tax rate stays constant from that time onwards and output does not fall any further. The long run equilibrium is one in which members of the *elite* make positive political rents at the expense of high distortionary taxes. They are better off than members of the *masses*. These would rather belong to the *elite*, but they cannot as they are liquidity constrained.

Case 2

This case will occur if the cost of political participation π is initially at an intermediate level, higher than E but lower than F. Formally, what we need is that $\rho B(z_0) \leq \pi \leq \rho A(z_0)$. In this case, we have the simple proposition:

PROPOSITION 2: Suppose that the political technology and the initial distribution of wealth $F_0(b)$ are such that $\rho B(F_0(\pi)) < \pi < \rho A(F_0(\pi))$, then i) for all $t \geq 0$, $z_t = F_0(\pi)$; ii) the wealth distribution $F_t(.)$ converges to the degenerate long run distribution F_∞: $\rho B(F_0(\pi))$ with mass $F_0(\pi)$
$\rho A(F_0(\pi))$ with mass $1-F_0(\pi)$.

Proof: See appendix 1.

Agents will converge to different steady state levels of wealth depending on their initial endowments. As we know, all those agents with endowments smaller than π at time t did not belong to the *elite*. As figure 3 shows, these cohorts will leave to their children a bequest that will also be smaller than π, and thus their children will not belong to the *elite* either. These cohorts will converge towards a stationary state level of bequests given by E. Conversely, all those agents who received an endowment higher than π at time t already belonged to the *elite* at that time and will leave to their children a bequest that will also be higher than π at $t+1$. Thus, their children will also belong to the *elite*. All these cohorts will converge towards a long run stationary state level of bequests given by F. Therefore, this case is one in which no entry takes place and the level of political participation stays constant.

As taxes are just a function of the level of participation, they do not change with time. Consequently, income does not change either. Taxes are just given by

$$\tau_t = \tau^*(z_0) \tag{25}$$

where

$$z_0 = F_0(\pi) \tag{26}$$

In the long run, while society polarizes around E and F, a process of equalization takes place within the *masses* and the *elite*. Unlike case 1, the *elite* does not undergo a generalized process of impoverishment. Those dynasties within the *elite* that were poorer than $\rho A(z_0)$ at time 0 get richer with time, while those dynasties that were initially richer than $\rho A(z_0)$ experience a decline in their wealth.

Again, the long run equilibrium is one in which members of the *elite* make strictly positive political rents.

Case 3

This last case is given by an initial situation as the one described in figure 4, where the cost of entry into politics π is relatively low. The initial configuration of parameters must be such that $\pi < \rho B(z_0)$. This case might be seen as a stage of historical development characterized by a relatively large share of urbanized population that makes it easier for *elites* to get organized. Alternatively, a low π would characterize societies in which hierarchical considerations play a minor role in the formation of *elites*, or where credit markets are relatively well developed.

Under such conditions, we have the following proposition:

PROPOSITION 3: *Suppose that the political technology and the initial distribution of wealth $F_0(b)$ are such that $\rho B(F_0(\pi)) \geq \pi$, then there exists a finite time S' such that:*
i) for all $t \leq S'$, $z_t = q_t > q_{t+1} = z_{t+1}$; ii) for all $t > S'$, $z_t = \hat{z}(\pi)$. iii) The wealth distribution $F_t(.)$ converges towards the degenerate long run distribution $F_\infty(.)$: $\rho A(\hat{z}(\pi)) = \rho B(\hat{z}(\pi))$ with mass 1.

Proof: see appendix 1.

Proposition 3 says that until a finite time S', the fraction of people that are liquidity constrained in period $t+1$ is smaller than that of period t, q_t. The reason is simple: on the one hand, all those agents that at time t received an endowment b_t larger than π will leave a bequest b_{t+1} larger than π to their children. On the other hand, some of the cohorts that received at time t a bequest b_t smaller than π, are nevertheless able to leave to their children a bequest b_{t+1} larger than π. Thus, q_{t+1} must be smaller than q_t. Now, it is easy to describe the dynamics of wealth accumulation and taxation in this economy. By (14), z_t falls as q_t falls.[20] At some finite time S', q_t becomes smaller than $\hat{z}(\pi)$ and z_t remains constant at $\hat{z}(\pi)$. After that time, those agents that are not liquidity constrained are indifferent between belonging to the *elite* or to the *masses*. In fact, members of the *elite* do not enjoy positive political rents any longer. Graphically, since $A(z)$ and $B(z)$ are respectively increasing and decreasing in z, the schedule that describes the bequest function for the *elite* shifts downwards while the one describing the bequest function

[20] As $B(z)$ is decreasing in z, if $\pi < \rho B(z_t)$ is satisfied at time t, it will also be satisfied at time $t+1$. Therefore q_t is a decreasing sequence and as far as z_t is larger than $z(\pi) = \hat{z}$, z_t is also a decreasing sequence.

for the *masses* moves upwards. Thus, while point F moves to the left, point E moves to the right. We can easily see that at time $t+1$, we are in a situation identical to that of time t except that E and F are closer to each other. This process continues until at some finite time S', q_t is smaller or equal to $\bar{z}$. At that point, both schedules become identical. A process of social homogenization thus takes place. The fraction of people in the *masses* remains constant and equal to $\bar{z}$. People that are not liquidity constrained are completely indifferent between being in the *elite* or remaining in the *masses*.

The path of taxes can also be easily described. Before S', as the fraction of people that are liquidity constrained falls, the *elite* expands. By (10), the tax rate falls as the size of the *elite* increases and output increases as the economy gets closer to its production possibility frontier. After time S', when q_t becomes equal to $\bar{z}$, the size of the *elite* and, consequently, the level of distortions remain constant. Notice that the long run equilibrium is just the equilibrium found by Ades (1992) in a static model of *elite* formation. In particular, no dynasty is liquidity constrained in equilibrium and they are all indifferent between being in the *elite* or remaining in the *masses*. The static model from Ades (1992) can be interpreted, therefore, as the steady state equilibrium of this dynamic model under a configuration of parameters where liquidity constraints do not matter in the long run.

This pattern of increased political competition leading to falling distortions and improved welfare fits well the demise of mercantilism and the rise of *laissez-faire* in England. Indeed, Ekelund and Tollison (1981) argue that the debate over monopolies was not a debate over free trade versus crown grants of patents but, rather, a competition between Parliament and the Crown for the rents implied by the power to supply those regulations. This process and Parliament's ultimate victory unintentionally helped bring about institutional changes that made special interest rent-seeking less feasible. Liberalism and free trade became the only viable alternatives under the modified institutional structure.

II. Discussion

2.1. Hierarchies, Credit Markets and Social Polarization

The model presented above shows that the politico-economic equilibrium to which different societies will converge in the long run depends on the initial pattern of wealth distribution and on the technology of access to the political decision making process. It suggests that societies with unequal distributions of wealth and relatively high costs of entry into politics will experience growing distortions, stagnation, social polarization and declining elites. Putnam (1992) argues that such conditions characterized the powerful feudal monarchy that Norman mercenaries built at the beginning of the 12th century in Southern Italy. This kingdom only experienced a short flourishing period, which reached its zenith by the end of the 12th century under Frederick II, *stupor mundi* ("the wonder of the world"). But it soon became clear that the structure of incentives made the success of this form of social organization depend too much on how enlightened the incumbent prince was. And, as Putnam points out, enlightenment would turn out to be much rarer than rapacity. Indeed, these strictly autocratic societies with steep vertical social structures soon came to be dominated by feudal barons "while at the bottom masses of peasants struggled wretchedly close to the limits of physical survival."

An the other extreme, the model also suggests that societies in which access to the political decision making process is relatively easy or where the distribution of wealth is not very unequal will enjoy lower distortions, and experience economic development and a process of wealth homogenization as time goes by. The communes that emerged in the North of Italy during the 12th century illustrate this case. They sprang out of voluntary associations and they were organized around ideas of horizontal cooperation and social equality. The civic republicanism of these northern cities laid the foundations for a growing prosperity that was rooted in the rapid growth of finance and commerce. Interestingly enough, the invention of credit was at the heart of these long-lasting economic developments.

2.2. *Improvements in Political Technology and Stages of Development*

Another interesting feature of our model is that it shows how any given society will go through successive stages of economic development as its technology for political participation becomes more efficient. We have already stressed that the parameter π can be seen as a measure of the cost of organizing *elites* and setting up the apparatus of political rent-seeking. It is reasonable to believe that these costs will change as societies increase their shares of population living in cities, and the means of communication and transportation develop. In our analysis, changes in π bring about changes in efficiency and in the level of political participation.[21]

In this section, we consider how improvements in the technology of political participation affect the long run steady state of an economy, *once society has reached that steady state*. Section 2.3. considers

[21] See Bates (1981) for an analysis of how in West Africa, agricultural policies usually involve a redistribution from a productive segment of the population that is vulnerable because it is geographically dispersed and unorganized in the rural areas to a small and concentrated urban elites.

the case in which these changes take place while the economy is on its transition path.

Suppose that society has converged to a steady state as that described under case 1, with relatively high costs of participation and small *elites*. This stage of development is characterized by high distortions, and large wealth differences between members of the *elite* that are clustered around F, and members of the *masses* at E.

Consider a reduction in the cost of political participation from π_0 to π_1, with $E < \pi_1 < F$. For those that belong to the *elite*, things get still better. Because the cost of political participation is higher than E, this reduction in π does not increase the level of political participation. Thus, it has no effects on taxes or revenues. Political rents, given by T_t-π_1, increase as $\pi_1 < \pi_0$. An increase in political rents shifts the schedule that corresponds to the *elite* upwards and point F moves to the right. However, it leaves unchanged the schedule that corresponds to the *masses*. Thus, the new level of steady state wealth to which the *elite* converges is further apart from that of the *masses*, and inequality increases.

A further reduction in the cost of participation to π_2, with π_2 lower than E allows all members of the *masses* to enter the *elite*. The size of the *elite* jumps to $\bar{z}$, taxes fall and political rents vanish completely. The new steady state implies a lower level of distortions, a higher level of income, and an elimination of social polarization.

2.3. *The Importance of Timing for Political Reform*

In this section, we consider the effects of sudden exogenous changes in the technology of political participation that take place *before the economy has reached its steady state*. A thoughtful analysis of the comparative dynamics along the transition path is not easy. However, our diagrammatic presentation can provide some insights. Our analysis shows that the extent to which a change in π affects the long run equilibrium of the economy depends very much on *when* this change takes place. Stated differently, the size in the changes required to achieve a certain institutional equilibrium in the long run depends on *when* these changes take place.

Consider, for example, the case in which society is converging towards a long run steady state as that described under case 1. During the transition, distortions grow, the economy moves away from its PPF, and the ruling *elite* declines. Suppose that at some time t before the steady state is reached, an exogenous shock reduces the cost of participation from π_0 to π_1, with $\pi_1 > E$. The immediate effect is to increase the number of dynasties that will be able to stay in the *elite*. In fact, all those dynasties which at time t owned a level of wealth b_t such that $\pi_1 < b_t < \pi_0$, and belonged to the *masses* will now belong to the *elite*. Hence, the fraction of people in the *masses* falls immediately, distortions decrease and output jumps up.

Notice that there is a sense in which reforms can occur too late. Indeed, if reform takes place after most cohorts have already converged to the steady state described under case 1, its effects on the size of the *elite* and the level of distortions will not be sizeable. A larger fall in π will be required to achieve the effects that reform would have had if it had taken place earlier.

An interesting implication of this analysis is that temporary shifts in the technology of political participation can have long run effects. If after the economy has converged to the new steady state implied by a cost of political participation given by π_1 we revert to π_0, this change will not necessarily bring back the initial social dynamics associated with π_0. Hence, the timing of historical political accidents may importantly affect the nature of economic and political dynamics in a society.[22]

2.4. *Productivity Shocks*

It is relatively straightforward to incorporate economy-wide productivity shocks into our model. Just imagine that one unit of labor at time t now produces a_t and $a_t\phi(l)$ units of output in each sector, where a_t is the state of technology at time t.

Assume that the economy was converging on a path such as that of case 3, with increasing participation and equality, and falling distortions. Consider a negative shock to productivity that lowers the state of technology from a_0 *to* a_1. The effects of such a shock would be to shift both schedules that describe the dynamics for bequest accumulation downwards. This would have the effect of moving both E and F to the left. If the change in a_t is not too small, it might bring F to the left of π, and put the economy on a path of development such as that described under case 1, with growing distortions and social polarization. Notice that the extent of the shifts is given by the values of $A(z)$ and $B(z)$ at the time of the shock. Even if the state of technology goes later back to a_0, the economy might not be able to settle in its older development path. As we saw in section 2.3., temporary shocks can have permanent effects.

III. Extensions

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[22] These results are consistent with the patterns of path-dependence in institutional change described in North (1991). See Saint Paul and Verdier (1992) for an analogous statement in a two class society model with human capital accumulation.

3.1. Public Goods

In section I, we considered the State's single role to be an instrument of predation for the ruling class. Historically, this has been an important role played by the State. Indeed, one only has to look at the way in which public finances were administered in Frederick II's thirteenth-century Kingdom of Naples, or in the seventeenth-century France of Louis XIV. Schama (1989) notes that "the most chronically ancient feature of the *ancien régime* seems to be that it was unable to distinguish adequately between the public and the private realms in matters as vital as its own finances." A similar impression can be obtained today by looking at some developing countries, where the notion of government revenues is indistinguishable from that of the ruler's pocket.

However, ruling *elites* have traditionally fulfilled another important function: to provide for public goods essential for economic activity. Obvious examples are infrastructure and communications, contract enforcement laws, defense and public education. In fact, this second function of providing essential public goods plays a much larger role in rich western democracies than in underdeveloped countries with autocratic regimes.[23]

In this section, we extend the model of section I to account for the case in which the State can be used by the *elite* not only as an instrument of predation but also for the provision of public goods beneficial to economic activity. One interesting issue in this context is the way in which State resources are allocated by the *elite* between direct transfers to themselves and production of the public good, as well as how this allocation process evolves through time as the size of the *elite* changes.

We modify the model of section I to incorporate public goods as an input to production. Now, technology in the manufacturing or modern sector is such that l_t units of labor produce $h(G_t)l_t$ units of output; in the traditional sector l_t units of labor produce $h(G_t)\phi(l_t)$ units of output. G_t and l_t are, respectively, the amount of public goods provided and the amount of labor allocated to the modern sector. The function $h(.)$ is assumed to be strictly increasing and concave in G_t, *i.e.*, $h'(.)>0$ and $h''(.)<0$. We thus assume that the provision of the public good G_t affects positively and equally the productivity of the taxable and the nontaxable activities.

[23] In a cross-section of countries, the average share from 1970 to 1985 of public education expenditures in total government consumption is correlated at the 65 percent with the initial level of GDP. The same share is correlated at the 45 percent with the Gastil index of political rights. Thus, richer countries and more representative forms of political organization are correlated with larger shares of government consumption allocated to public education.

The structure of the game is the same as that of section I, except that the *elite* can now choose both the tax rate τ_t and the way in which the resulting tax revenues are allocated between redistribution and the provision of public goods.

Solving the game backwards, it is easy to see that labor supply in the taxable activity still depends only on the tax rate τ_t, and is such that

$$(1-\tau_t) = \phi'(1-l^*(\tau_t)) \tag{27}$$

The dynamics of wealth accumulation follow a similar pattern as that of the previous section. If we define earned income as

$$Y(G_t,\tau_t) = h(G_t)\,x(\tau_t) \tag{28}$$

with

$$x(\tau_t) = [(1-\tau_t)l^*(\tau_t) + \phi(1-l^*(\tau_t))] \tag{29}$$

the dynamics of bequests for the *elite* are given

$$b_{t+1} = \frac{\rho}{1+\rho}\,(Y(G_t,\tau_t) + T_t - \pi + b_t) \tag{30}$$

and for the *masses*

$$b_{t+1} = \frac{\rho}{1+\rho}(Y(G_t,\tau_t) + b_t) \tag{31}$$

where T_t is the amount of transfers that each member of the *elite* captures.

The levels of utility for members of the *elite* and for the *masses* are given by

$$V^E(\tau_t,G_t,T_t) = V\,[k(\rho)\,(Y(G_t,\tau_t) + T_t - \pi + b_t)] \tag{32}$$

and

$$V^M(\tau_t,G_t) = V[\,k(\rho)\,(\,Y(G_t,\tau_t) + b_t)\,] \tag{33}$$

The representative *elite* member thus maximizes $V^E_t(\tau_t,G_t,T_t)$ subject to the following budget and nonnegativity constraints

$$(1-z_t)T_t+G_t=\tau_t h(G_t)l^*(\tau_t) \tag{34}$$

$$T_t\geq 0 \tag{35}$$

Equations (34) and (35) can be summarized as

$$\tau_t h(G_t)l^*(\tau_t)-G_t\geq 0 \tag{36}$$

Substituting T_t from the budget constraint, we get the following first order conditions

$$-l^*(\tau_t)+(\tau_t\frac{dl^*(\tau_t)}{d\tau_t}+l^*(\tau_t))[\frac{1}{1-z_t}+\lambda]=0 \tag{37}$$

$$\frac{d\,h(G_t)}{dG}(x(\tau_t))+[\frac{dh(G_t)}{dG}\tau_t l^*(\tau_t)-1][\frac{1}{1-z_t}+\lambda]=0 \tag{38}$$

and exclusion conditions $\lambda>0$ if $T_t=0$ and $\lambda=0$ if $T_t>0$, where λ is the Lagrange multiplier in (36).

In equilibrium, transfers will always be larger than π so that λ is equal to zero. Assuming that second order conditions are satisfied, (37) and (38) define both τ^*_t *and* G^*_t as functions of the size of the *elite* $(1-z_t)$ only. They can be rewritten as

$$-z_t l^*(\tau_t)+\tau_t\frac{dl^*(\tau_t)}{d\tau_t}=0 \tag{39}$$

$$\frac{d\,h(G_t)}{dG}(x(\tau_t))+[\frac{dh(G_t)}{dG}\tau_t l^*(\tau_t)-1][\frac{1}{1-z_t}]=0 \tag{40}$$

Inspection of (39) reveals that the optimal tax rate $\tau^*_t(z_t)$ is exactly the same as that of the section without public goods.[24] Hence the tax rate is determined independently from the public good. Also, as before, τ^*_t is decreasing in the size of the *elite*.

Equation (40) shows the FOC for the optimal allocation of revenues between public goods and transfers for the *elite*. Each extra dollar of revenues allocated to the provision of the public good has positive effects on productivity. This increases income for the *elite* through two channels: first, through its effect

[24] This comes from our assumption that the public good affects the two sectors in the same way.

on increased earned income; and, secondly, through an increase in the size of the tax base. These positive effects the extra dollar allocated to the provision of the public good will be balanced, at the margin, against the direct loss in transfers, which is given by $1/(1-z_t)$.

The interesting novelty comes from the optimal public good provision $G^*_t(z_t)$. Differentiating totally equation (40), making use of the envelope theorem and of equation (39), we can show that

$$h''(G_t^*)\ x(\tau_t^*)\ dG_t^* = \frac{dz_t}{(1-z_t)^2}\ (1-\tau_t^*)\ h'(G_t^*)\ l^*(\tau_t^*) \tag{41}$$

Notice that the right hand side of (41) is always positive. Therefore, as $h(.)$ is concave, $G^*_t(z_t)$ is a decreasing function of z_t. Stated differently, as the size of the *elite* $(1-z_t)$ increases, the amount of State revenues devoted to the provision of the public good increases. The intuition behind this result is simple. As the *elite* broadens, transfers have to be shared among more people. Thus, the returns to allocating tax revenues to transfers decreases relative to the return to allocating them to providing public goods. Hence, as the size of the *elite* increases, each individual in the *elite* finds it optimal to vote for a reduction in the share of revenues allocated to redistribution and an increase in the share of expenditures allocated to productive public goods.

As in section I, the two conditions that determine entry into the *elite* are that $b_t \geq \pi$, and that

$$T(z_t) = \frac{h(G^*(z_t))\,\tau^*(z_t)\,l^*(\tau^*(z_t)) - G^*(z_t)}{1-z_t} \geq \pi \tag{42}$$

It can be easily seen that this function $T(z_t)$ is increasing in z_t, and that $T(0)=0$ and $T(1)=+\infty$. Hence there exists a unique $\bar{z}$ such that $T(\bar{z})=\pi$.

The equilibrium dynamics of wealth accumulation are essentially the same as in the case without public goods, and are described by (13), (14), (20) and (21) with $A(z_t)$ and $B(z_t)$ now defined as

$$A(z_t) = Y(G^*(z_t), \tau^*(z_t)) + T(z_t) - \pi \tag{43}$$

and

$$B(z_t) = Y(G^*(z_t), \tau^*(z_t)) \tag{44}$$

Using the envelope theorem, it is easy to see $A(z_t)$ and $B(z_t)$ are again respectively increasing and decreasing in z_t.

Depending on the initial distribution of wealth $f_0(b)$ and the cost of entry into the *elite* π, one has still the three cases described in the previous section. The interesting new issue concerns the dynamics of the allocation of State revenues between transfers and the public good.

When the cost of political participation is relatively high, we are again in case 1. During the transition, the size of the *elite* falls. This decline of the *elite* brings about an increase in taxes and distortions. Simultaneously, the share of tax revenues allocated to the provision of the public good falls. The State becomes larger in terms of fiscal burden, less productive and more redistributive. Because of this, stagnation and a decrease of output ensue. After the size of the *elite* stabilizes, taxes, public good and output remain constant.

When the cost of political participation lies between E and F, no entry takes place and the size of the *elite* remains fixed at its initial value $(1-z_0)$. As we know, this a case in which the tax rate, the amount of public good and the output level remain constant through time. In the long run, society polarizes around the two levels of wealth shown by E and F.

The last case to consider is that in which the cost of political participation is relatively low. This is a case in which the size of the *elite* tends to increase as less people are liquidity constrained. As more people get involved in the political decision making process, taxes and distortions fall. Simultaneously, during the transition, equation (41) tells us that the amount of resources devoted to the provision of the public good increases. The State becomes more productive and less redistributive. Because of a smaller fiscal burden and a better allocation of public funds for market activities, output increases until we reach a times S' after which it remains constant.

Summarizing the previous discussion, we see that allowing the State to provide public goods reinforces our findings of section I: namely, the politico-economic system allocates resources in a more efficient way as access to the political decision making-process becomes widespread. When the *elite* can provide public goods, a reduction in the cost of political participation π has two effects: first, it reduces allocative distortions by lowering the tax rate imposed on manufacturing activities; second, it improves the allocation of tax revenues, increasing the share devoted to the provision of productive public goods.

3.2. Growth

In the models presented in previous sections, all economies experienced zero growth in steady state. Growth was only a temporary phenomenon, with income rising or falling as the economy got closer or further away from its production possibility frontier.

In that setup, the allocation of resources between the modern or traditional sectors only mattered for static efficiency. In particular, it had no effects on society's state of technology. However, there is ample evidence, both anecdotal and statistical, that innovation, learning and spillovers are very much tied to the manufacturing or modern sector of the economy.[25]

In this section, we extend the model of section I to allow for changes in the state of technology arising from learning effects in the modern sector of the economy. The model allows for the possibility of non-zero growth in steady state. Of particular interest is the fact that a small change in the cost of political participation might have strong long-run effects on output, shifting the economy from a low to a high growth equilibrium.

In order to analyze steady state dynamics, we modify the model of section I in three ways. First, we assume that in the modern sector, one unit of labor yields $l_t a_t$ units of output, where a_t is the state of technology at time t. In the DRS sector, one unit of labor yields $\phi(l_t)a_t$ of output, with $\phi'>0$, $\phi''<0$ and $\phi'(0)=\infty$. Second, we assume that the path of the state of technology a is such that its growth is proportional to the share of labor allocated to the constant returns to scale modern sector. Thus, the level of technology at $t+1$ is given by

$$a_{t+1}=(1-\delta)a_t+\alpha a_t l_t \tag{45}$$

where δ is the rate at which technology depreciates, and $\alpha>1$ is just a constant. Finally, we assume that at any one time the cost of political participation π is a function of the state of technology a at that time, *i.e.*, $\pi=\pi(a)$ with $\pi'(.)>0$. This could be rationalized in several ways. For example as the level of technology improves, more sophisticated political networks have to be built in order to get access to the political decision making process and it costs more to get into those network. Alternatively this cost of political participation could be considered as the opportunity cost of the time allocated to political activity, which increases as agents become more productive.

The problem to be solved is now given by

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[25] De Long and Summers (1991) note that "the history of economic growth is often written as if nations and industries seized the opportunity to intensify their specialization in manufactures and grew rapidly, or failed to seize such opportunities and stagnated." They also provide evidence showing a strong association between machinery investment and productivity growth for a broad cross-section of nations since World War II. See also De Long (1991), who provides similar evidence for a sample of six currently-industrialized nations (Canada, Germany, Italy, Japan, the United Kingdom and the United States) over the past century.

$$\max_{<l_t, b_{t+1}>} U_t = U(c_t ; b_{t+1}) \tag{46}$$

subject to

$$c_t + b_{t+1} = (1-\tau_t) l_t a_t + \phi(1-l_t) a_t + T_t - \pi(a_t) + b_t \tag{47}$$

$$0 \leq l_t \leq 1 \tag{48}$$

and

$$a_{t+1} = (1-\delta) a_t + \alpha a_t l_t \tag{49}$$

The solution to this problem is almost identical to that of section I. The solution to the third stage of the game implies that optimal labor supply is again a function of τ only, with $dl_t^*/d\tau < 0$. In particular, it is not a function of the state of technology. As in equations (6) and (7), $c_t = (1/\rho) b_{t+1}$.

The solution to the second stage of the game yields the same first order condition for the choice of the tax rate. This is again given by

$$z_t l_t^*(\tau_t) + \tau_t \frac{d l_t^*(\tau_t)}{dt} = 0 \tag{50}$$

which defines implicitly the optimal τ_t^* as a function of z_t. For a well defined concave problem, τ_t^* is again increasing in z_t.[26]

Again, the two conditions that determine the decision to belong to the *elite* in the first stage of the game are $b_t \geq \pi(a_t)$ and $T_t \geq \pi(a_t)$, where now

$$T_t = \frac{\tau_t^*(z_t) l_t^*(\tau_t^*(z_t)) a_t}{1 - z_t} \tag{51}$$

The bequest left by an agent in the *elite* at time t is now given by

[26] Notice, also, that τ^* is independent of the level of technology.

$$b_{t+1} = \frac{\rho}{1+\rho}[(1-\tau^*(z_t))\,l^*(z_t)a_t + \phi(1-l^*(z_t))a_t + T(z_t,a_t) - \pi(a_t) + b_t] \qquad (52)$$

and that left by someone in the *masses* by

$$b_{t+1} = \frac{\rho}{1+\rho}[(1-\tau^*(z_t))\,l^*(z_t)a_t + \phi(1-l^*(z_t))a_t + b_t] \qquad (53)$$

Equations (52) and (53) describe the dynamics of b for any given z and a. It is convenient for the analysis to "detrend" the level of wealth at time t by dividing it by the state of technology at t. Thus, using (45), we can rewrite (52) and (53) as

$$\frac{b_{t+1}}{a_{t+1}} = \frac{1}{1-\delta+\alpha l^*(z_t)}\left[\frac{\rho}{1+\rho}[(1-\tau^*(z_t))\,l^*(z_t) + \phi(1-l^*(z_t)) + T(z_t) - \frac{\pi(a_t)}{a_t}] + \frac{\rho}{1+\rho}\frac{b_t}{a_t}\right] \qquad (54)$$

and

$$\frac{b_{t+1}}{a_{t+1}} = \frac{1}{1-\delta+\alpha l^*(z_t)}\left[\frac{\rho}{1+\rho}[(1-\tau^*(z_t))\,l^*(z_t) + \phi(1-l^*(z_t))] + \frac{\rho}{1+\rho}\frac{b_t}{a_t}\right] \qquad (55)$$

We can analyze the dynamics of this economy as we did in section I. Figure 5 shows the dynamics of wealth accumulation in this economy, where $g_t = g(z_t) = 1-\delta+\alpha l^*(z_t)$. In order to analyze the case of stationary long-run growth, we consider the case in which $\pi(a_t)$ is just proportional to a_t, so that $\pi(a_t)/a_t = \pi$ stays constant with a_t. Also, to ensure the convergence of detrended wealth levels, we need to assume that *for all* z, $\rho/(1+\rho) < 1+g(z)$. This is summarized in the following sufficient condition:

Assumption G1: $\rho/(1+\rho) < 1-\delta$

Equations (55) and (56) may now be rewritten in terms of $A(z)$ and $B(z)$ as

$$\frac{b_{t+1}}{a_{t+1}} = \frac{1}{1+g(z_t)}\frac{\rho}{1+\rho}\left[A(z_t) + \frac{b_t}{a_t}\right] \qquad (56)$$

and

$$\frac{b_{t+1}}{a_{t+1}} = \frac{1}{1+g(z_t)}\frac{\rho}{1+\rho}\left[B(z_t) + \frac{b_t}{a_t}\right] \qquad (57)$$

As shown in Figure 5, the similarity of the analysis with that of section I is apparent. Again, the position of the schedules that describe the dynamics of wealth accumulation for the *masses* and the *elite* shifts as z changes. As in section I, a change in z_t affects directly the bequest schedules by changing $A(z_t)$ and $B(z_t)$. However, there is also an additional growth effect operating through $g(z_t)$. Graphically, this additional effect changes both the slope and the vertical intercept of the bequest schedules.

To be more precise, consider an increase in z_t. This fall in political participation reduces the growth rate $g(z_t)$ and consequently affects positively the slope and intercept of (56) and (57). Through this channel, the bequest schedules tend to shift upwards. However, these effects must be combined with the direct effect of z_t on $A(z_t)$ and $B(z_t)$ to deduce the overall effect of a change in the size of the *masses* on (56) and (57).

It is easy to see that the direct effect and the growth effect go in the same direction for (56). An increase in z_t thus implies an upward shift of the *elite's* schedule. On the contrary, for equation (57) (the bequest schedule of the *masses*), the two effects act in opposite directions. On the one hand, an increase in z_t reduces $B(z_t)$ and therefore affects negatively b_{t+1}/a_{t+1}. On the other hand, an increase in z_t affects negatively the growth rate and has therefore a positive effect on $b_{t+1}\backslash a_{t+1}$. The total effect of a change in z_t for (57) is therefore a priori ambiguous.

For consistency with section I, we assume that the direct effect (operating through $B(z_t)$) outweighs the growth effect (operating through $g(z_t)$). More precisely, we need to assume that for the bequest schedule that corresponds to the *masses*, the intersection with the 45 degree line remains to the left of π as z increases. This is summarized by the following assumption:

Assumption G2: $\forall z \geq 0$: $B_z - \pi(1+\rho)/\rho \; g_z < 0$

Finally, we define for convenience:

(i) $\Omega_A(z)=A(z)-\pi[(1+g(z))(1+\rho)/\rho-1]$, and

(ii) $\Omega_B(z)=B(z)-\pi[(1+g(z))(1+\rho)/\rho-1]$.[27]

As we did in section I, we also assume that initial conditions are such that liquidity constraints are binding for some cohorts, *i.e.*, $z_0=q_0=F_0(\pi a_0)>\hat{z}(\pi)$.

Propositions 4, 5 and 6 describe the three possible cases.

[27] These two functions are useful for examining the position of the intersection of the bequest schedules with the 45 degree line relative to π. In fact, it is easy to see that in Figure 5, E (resp. F) would be to the left (resp. right) of π if $\Omega_B(z)<0$ (resp. $\Omega_A(z)>0$). Assumption $G2$ simply ensures that $\Omega_B(z)$ is decreasing in z, so that E always remains to the left of π as z increases.

PROPOSITION 4: Suppose that the political technology and the initial distribution of wealth $F_0(b)$ defined on the support $[b_0^-, b_0^+]$ are such that $\Omega_A(F_0(\pi a_0)) < 0$ and $\pi a_0 < b^+_0$; then there exists a finite time S such that:

(i) For all $t \leq S$, $z_t = q_t < q_{t+1} = z_{t+1}$. (ii) For all $t > S$, $z_t = z_{t+1} = q_{S+1}$. (iii) the growth rate $g(z_t)$ decreases for $t \leq S$; then remains constant at $g(q_{S+1})$ for $t > S$. (iv) In the long run, all wealth levels grow at the same rate $g(q_{S+1})$ and the distribution of b/a_t (detrended wealth levels) converges to the degenerate long run distribution:

F_∞ : $\{\rho/[(1+g(q_{S+1}))(1+\rho)-\rho]\}$ $B(q_{S+1})$ *with mass* q_{S+1}

$\{\rho/[(1+g(q_{S+1}))(1+\rho)-\rho]\}$ $A(q_{S+1})$ *with mass* $1-q_{S+1}$.

Proof: See Appendix 2.

The general features of this case are quite similar to those of section I. Convergence towards the steady state involves a process of reduction in the size of the *elite*. As the *elite* shrinks, taxes increase monotonically. A process of *de-industrialization* sets in, whereby labor is increasingly allocated to the informal sector. As this happens, growth falls and the economy stagnates.[28] Once time S is reached, the economy settles along a stationary path with a constant growth rate $g(q_{S+1})$. A process of social polarization takes place towards points E and F in figure 5. The main difference with section I is the fact that in the long run, the levels of wealth grow at a constant common rate of $g(q_{S+1})$.

PROPOSITION 5: Suppose that the political technology and the initial distribution of wealth $F_0(b)$ defined on the support $[b_0^-, b_0^+]$ are such that $\Omega_B(F_0(\pi a_0)) < 0 \leq \Omega_A(F_0(\pi a_0))$ and $\pi a_0 < b^-_0$; then (i) for all $t \geq 0$ $z_t = z_0 = F_0(\pi a_0)$; (ii) the growth rate of the economy is constant and equal to $g(z_0)$; (iii) the distribution of detrended wealth levels b/a_t converges to the degenerate long run distribution

F_∞: $\{\rho/[(1+g(z_0))(1+\rho)-\rho]\}.B(z_0)$ *with mass* z_0 *and*

$\{\rho/[(1+g(z_0))(1+\rho)-\rho]\}A(z_0)$ *with mass* $1-z_0$.

[28] Reynolds (1985) notes that such a process characterized the experience of Uganda under the Amin regime of 1971-79: "people moved back from the monetary economy to the subsistence economy, monetized output dropped sharply, investment and exports almost disappeared."

Proof: See appendix 2.

In figure 5, this case corresponds to the cost of political participation $\pi(a_t)/a$, being initially higher than E but lower than F. Agents converge to different steady state levels of wealth depending on their initial endowments. As we saw in section I, this case is one in which no entry takes place so that the level of political participation remains constant. Since τ is just a function of the level of participation, taxes do not change with time nor does the allocation of labor across sectors in the economy. Thus, the rate of growth stays constant throughout. In the long run, the economy is characterized by a relatively large nontaxable sector. While some growth is achieved, the level of distortions imposed on the modern sector does not allow a full scale industrialization process to take-off. At a social level, while a process of equalization takes place within the *masses* and the *elite*, society polarizes around two different levels of wealth, E and F.

Finally, there is the case with relatively low costs of political participation.

PROPOSITION 6: Suppose that the political technology and the initial distribution of wealth $F_0(b)$ defined on the support $[b_0^-, b_0^+]$ are such that $\Omega_B\ (F_0(\pi a_0)) \geq 0$ and $\pi a_0 < b^+_0$. Then there exists a finite time S' such that: (i) for all $t \leq S'$, $z_t = q_t > q_{t-1} = z_{t-1}$; (ii) for all $t > S'$, $z_t = \bar{z}(\pi)$; (iii) The growth rate is increasing for $t \leq S'$ and remains constant at $g(\bar{z}(\pi))$ for $t > S'$; (iv) In the long run, the levels of wealth grow at the same constant rate $g(\bar{z}(\pi))$ and the distribution of detrended wealth converges towards the degenerate long run distribution $F_\infty(.)$: $\{\rho/[(1+g(\bar{z}(\pi)))(1+\rho)-\rho]\}\ B(\bar{z}(\pi))$ with mass 1.

Proof: see Appendix 2.

Under such conditions, the dynamics of *elite* formation and wealth accumulation can be easily followed. Since the fraction of agents that are liquidity constrained at time $t+1$ is smaller than that of time t, z_t falls with time and the size of the *elite* grows. In turn, this leads to a fall in taxes and a surge in the modern sector. The economy "takes-off", and there is a massive shift of labor out of the traditional sector and into the modern sector of the economy. As more labor is allocated to the modern sector, the growth rate of technology increases. The economy turns from a low growth-large traditional type to a fast growing industrialized one. After time S', there is free entry in politics and the growth rate remains constant . As in section I, a process of wealth equalization takes place in the meantime. The economy reaches a new steady state with higher growth, higher political participation and more equality.

3.3. Uncertainty

One feature that is common to all the versions of the model that we have presented up to now is that each cohort's fate is completely determined from the beginning of times. With no uncertainty, a first generation's endowment determines the whole future of the dynasty.

Our model can be extended to allow for some randomness. We consider the possibility of idiosyncratic shocks that affect the value of the endowment left over for the following generation. Some examples might be technological innovations, changes in tastes, floods, marriage or invasions by bordering countries. This modification will imply an increase in social mobility.

One simple way of introducing uncertainty is to imagine that at the beginning of each period t an idiosyncratic shock ϵ_t affects the actual value of the endowment received at that time by a family's generation. We assume that such shock is additive, and that it affects the value of the bequest by an amount equal to $\rho/(1+\rho)\epsilon_t$.[29]

To keep things simple, assume that this shock can take only two values $(-\epsilon_b, \epsilon_g)$ (bad and good) with probabilities 1-p and p, respectively. Assume that (ϵ_b, ϵ_g) are both positive and $E(\epsilon)=-(1-p)\epsilon_b+p\epsilon_g=0$. Moreover, assume that shocks are *iid* and independent across agents in the population. These assumptions and our convenient normalization imply that the dynamics of bequest accumulation for a dynasty in the *elite* can be described by the following equation

$$b_{t+1}=\frac{\rho}{1+\rho}(A(z_t)+b_t+\epsilon_t) \quad if \quad b_t \geq \pi \tag{58}$$

and for a dynasty in the *masses* by

$$b_{t+1}=\frac{\rho}{1+\rho}(B(z_t)+b_t+\epsilon_t) \quad if \quad b_t < \pi \tag{59}$$

In particular, notice that (58) and (59) are essentially identical to (20) and (21), their deterministic counterparts.

We may then describe the dynamics of wealth distribution in the following way. Starting from a wealth distribution F_t in period t, one may compute the wealth dstribution in period $t+1$, as follows:

Consider an individual with an inherited wealth *level* b_{t+1}. Four cases are possible. 1) It may come

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29 [29] The factor $\rho/(1+\rho)$ in front of ϵ is just a useful normalization.

from a parent in the *elite*, who experienced a good shock, in which case the parent's initial endowment b_t is such that

$$\pi \leq b_t = R_A(b_{t+1}, z_t, \epsilon_g) = \frac{1+\rho}{\rho} b_{t+1} - A(z_t) - \epsilon_g \tag{60}$$

2) Or it may come from a parent in the *elite*, who experienced a bad shock, in which case the parent's initial endowment b_t is such that

$$\pi \leq b_t = R_A(b_{t+1}, z_t, \epsilon_b) = \frac{1+\rho}{\rho} b_{t+1} - A(z_t) - \epsilon_b \tag{61}$$

3) Or it may come from a parent in the *masses*, who experienced a good shock, in which case the parent's initial endowment b_t is such that

$$\pi \geq b_t = R_B(b_{t+1}, z_t, \epsilon_g) = \frac{1+\rho}{\rho} b_{t+1} - B(z_t) - \epsilon_g \tag{62}$$

4) Or finally it may come from a parent in the *masses*, who experienced a bad shock, in which case the parent's initial endowment b_t is such that

$$\pi \geq b_t = R_B(b_{t+1}, z_t, \epsilon_b) = \frac{1+\rho}{\rho} b_{t+1} - B(z_t) - \epsilon_b \tag{63}$$

The wealth distribution at $t+1$ is obtained by adding up over the total mass of lineages who give rise to a level of wealth smaller than b_{t+1}. One obtains:

$$\begin{aligned} F_{t+1}(b) = [T_{z_t} F_t](b) = p \int_{Min(\pi, R_A(b,z_t,\epsilon_g))}^{R_A(b,z_t,\epsilon_g)} dF_t(x) + (1-p) \int_{Min(\pi, R_A(b,z_t,\epsilon_b))}^{R_A(b,z_t,\epsilon_b)} dF_t(x) \\ + p \int_0^{Min(\pi, R_B(b,z_t,\epsilon_g))} dF_t(x) + (1-p) \int_0^{Min(\pi, R_B(b,z_t,\epsilon_b))} dF_t(x) \end{aligned} \tag{64}$$

The dynamics of the system can be then described as:

$$\begin{aligned} F_{t+1} &= T_{z_t} F_t \\ z_{t+1} &= Max\,[F_{t+1}(\pi), z(\pi)] \end{aligned} \tag{65}$$

Of particular interest is the question of the existence and convergence of the system to a steady state (ie.

an invariant distribution F^* such that:

$$F^* = T_\epsilon . F^* \\ z^* = Max\,[F^*(\pi), z(\pi)] \tag{66}$$

and how does this invariant distribution (when it exists) depend on the initial distribution F_0. Because the dynamic system is a non linear markovian system, unfortunately it is quite difficult to fully characterize its dynamics. However, for some particular configurations of parameters, one may get some insights with our simple diagrammatic presentation. Consider Figure 6 which plots equations (58) and (59). To each family now correspond two bequest schedules depending on the value taken by the shock $-\epsilon_b$ *or* ϵ_g. If it belongs to the *elite*, these schedules are A_b and A_g; B_b and B_g if it belongs to the *masses*. Several cases are possible depending on initial conditions and the size of the shocks. The discussion can be then organized by considering the different conditions under which the "bad shock" schedule (B_b, A_b) and the "good shock" schedule (B_g, A_g) look like those of cases 1, 2 or 3 discussed in the deterministic model. Given that the schedule (B_b, A_b) always has to lie under schedule (B_g, A_g), this leaves us with 6 possibilities:

1. (B_b, A_b) and (B_g, A_g) look both like in case 1.
2. (B_b, A_b) is like in case 1 and (B_g, A_g) is like case 2.
3. (B_b, A_b) is like case 1 and (B_g, A_g) is like case 3.
4. (B_b, A_b) and (B_g, A_g) are both like case 2.
5. (B_b, A_b) is like case 2 and (B_g, A_g) is like case 3
6. (B_b, A_b) and (B_g, A_g) are both like case 3

We restrict the discussion here to three illlustrative examples. The first two describe situations in which the *elite* is stable but they differ in terms of the nature of this *elite*. In the first example, there is social polarization with the same dynasties staying indefinitely in power. On the contrary, the second case illustrates the case of a stable *elite* with subtancial social and political mobility, each dynasty having always a positive probability of ending up among the politically powerful. Finally, the third example suggests that one may get cycles in policies and political participation with an endogenous rise and fall of *elites*.

Example 1: Stable Elite with Social Polarization.

This situation is illustrated by Figure 7. It is the stochastic equivalent of case 2 in the deterministic model.[30] This is the picture of a very static society. Dynasties that belong to the *elite* (resp. the *masses*) at time t remain in the *elite* (resp. the *masses*) at time $t+1$ no matter what. Shocks are too small to make a difference.

The value of endowments depend mainly on what was left over by the previous generation. Hence, there is no way for someone in the *masses* to see his son enter the *elite* because of good luck; nor is there any possibility for someone in the *elite* to have his descendant belong to the *masses* because of bad luck. Therefore, there is no entry into politics. The share of total population in the *masses* and the level of distortions stay constant at z_0 and $\tau(z_0)$, respectively.

The steady-state distribution of wealth is no longer degenerate, and we will no longer find perfect homogenization within the *elite* and the *masses*. Instead, there are now two ergodic sets supporting a long run distribution of wealth. These are included in segments *EE'* and *FF'* for the *masses* and the *elite*, respectively.[31]

Example 2: A Stable Elite with Social Mobility

This case is illustrated in figure 8, and characterizes a society with a high degree of social mobility, and is thus the extreme opposite to the case presented above.[32] In this case, shocks are large enough to make unlucky dynasties in the *elite* fall into the *masses*, while lucky dynasties in the *masses* have a chance of ending up in the *elite*. There is, therefore, a substantial amount of social mobility.

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[30] Formally, the initial parameters of the model should satisfy the following conditions:

$$\rho(B(z_0)-\epsilon_b) < \rho(B(z_0)+\epsilon_h) < \pi < \rho(A(z_0)-\epsilon_b) < \rho(A(z_0)+\epsilon_h) \quad (1)$$

[31] In this example, as z_t remains constant, the system given by (65) can be analysed using techniques on convergence of stationary Markov Processes (Futia 1982). In particular, using theorems of existence of an invariant distribution for stable quasi compact Markov operators, one may show that a slightly continuously pertubed version of system (65) has, for any initial distribution, an invariant distribution that adequately describes the time average pattern of wealth lineages. When the pertubation tends to zero, the pertubed system is arbitrarily close to the unpertubed system (65) and an invariant distribution of the former system is asymptotically an invariant distribution of the latter. As there is more than one ergodic set, the invariant distribution is not unique and depends on the initial distribution from which the system starts.

[32] Formally, the initial configuration of parameters should satisfy the following conditions:

$$\rho(B(z_0)-\epsilon_b) < \pi < \rho(B(z_0)+\epsilon_h) \quad and \quad \rho(A(z_0)-\epsilon_b) < \pi < \rho(A(z_0)+\epsilon_h) \quad (1)$$

In order to get some insight, let us consider the extreme case in which the support of the initial distribution of wealth $[b_0^-, b_0^+]$ is such that all dynasties in the *masses* may end up in the *elite* after a positive shock, and all dynasties in the *elite* may end up in the *masses* after a negative shock. Formally, we need that

$$\frac{\rho}{1+\rho}(B(z_0)+b_0^- +\epsilon_g) \geq \pi \geq \frac{\rho}{1+\rho}(A(z_0)+b_0^+ -\epsilon_b) \tag{67}$$

Under such conditions (and assuming that political rents are initially strictly positive, *i.e.*, $z_t > \bar{z}$) the number of dynasties in the *masses* at time $t+1$ will be given by

$$z_{t+1} = z_t - p\ z_t + (1-p)(1-z_t) \tag{68}$$

where the second term in the right hand side of (62) shows the average number of "lucky" dynasties moving up from the *masses* into the *elite*, and the third term is the average number of "unlucky" dynasties moving down from the *elite* into the *masses*. It is quite straightforward to see that $z_{t+1}=(1-p)$ for any value of z_t.

Immediately after the first period, the fraction of people in the *masses* jumps to $1-p$ and stays there forever. Depending on whether z_0 is larger or smaller than $1-p$, the tax rate will decrease or increase from $\tau(z_0)$ to $\tau(1-p)$. The long run distribution has his support in the interval $[E_p, F_p]$. If $1-p$ is larger than $\bar{z}$, then in the long run, we have that the *elite* enjoys positive rents during the time they stay in power. However, because of the importance of the shocks, there is a substantial degree of social mobility and renewal of the ranks in the *elite*. The same families do not stay indefinitely in power. Instead, a constant fraction moves up and down the social ladder. Everybody has its chance to enjoy his share of social rents. If $1-p$ is smaller than $\bar{z}$, the *elite* does not enjoy positive rents, and z_t remains constant at $\bar{z}$. The pattern of social mobility is the same as above.

Example 3: Cycles or The Rise and Fall of Elites.

So far the dynamics of the system led to a stationary size of the *elite*. However due to its non-linearity, one may also expect to have richer social dynamics. The following example decribes the case of a cycle of period 2. This is described in figure 8.

i) We suppose that the two bequest schedules associated to an initial size of the *masses* z_0, B_gA_g and B_bA_b, have the position described as 5 in this section, namely under a good shock, the size of the *elite* increases while under a bad shock, one has a stable *elite*.

ii) The picture also shows the bequest schedules C_g and C_b corresponding to free entry in politics ($z = \hat{z}$) respectively under the good and the bad shock. These curves C_g and C_b should be respectively between A_g and B_g, and A_b and B_b. The position of C_b is such that it intersects the 45_o line at a point smaller than π.

iii) We also assume that with free entry in politics, all dynasties with an endowment smaller than π will end up with an endowment larger than π after a positive shock, and similarly that all dynasties with a bequest larger than π will end up with a bequest smaller than π after a negative shock. In order to ensure this, it is sufficient (a) to have that someone with a wealth level corresponding to point A ends up leaving a bequest smaller than π in the case of a bad shock, while someone with a level of wealth that corresponds to point B ends up leaving a bequest larger than π in the case of a good shock, and (b) that the support of the initial distribution $[b_0^-, b_0^+]$ is in between points B and A.[33]

Finally, assume that $(1-p)^2 < \hat{z} < z_0 = (1-p)$. Then starting from $z_0 > \hat{z}$, the dynamics of the system are represented by curves B_gA_g and B_bA_b. All dynasties in the *elite* remain with an endowment larger than π while a fraction p of those in the *masses* are not liquidity constrained in the next period. It results from this that the number of liquidity constrained dynasties in period 1 is smaller and given by $q_1 = (1-p)z_0 = (1-p)^2 < \hat{z}$. Hence in period 1, we are in the free entry politics regime and the bequest dynamics are described by schedules C_g and C_b. Because of assumptions ii) and iii) however, all dynasties now that experience a bad shock end up with a period 2 endowment smaller than π, while on the contrary all dynasties with a wealth smaller than π, end up, under a positive shock, with an endowment larger than π. It results that the number of liquidity constrained dynasties in period 2 is given by: $q_2 = q_1 - pq_1 + (1-p)(1-q_1) = (1-p) = z_0$ which is larger than $\hat{z}$. The bequest schedules in period 2 are then again

[33] Formally conditions i), ii) and iii) may be stated as:

$$\begin{aligned} \rho[B(z_0)+\epsilon_b] &< \pi < \rho[A(z_0)+\epsilon_b] \\ \pi &< \rho[B(z_0)+\epsilon_g] < \rho[A(z_0)+\epsilon_g] \end{aligned} \tag{1}$$

$$\rho[B(z(\pi))+\epsilon_b] < \pi < \rho[B(z(\pi))+\epsilon_g] \tag{2}$$

$$\frac{\rho}{1+\rho}[\rho(A(z_0)+\epsilon_g)+B(z(\pi))+\epsilon_b] < \pi < \frac{\rho}{1+\rho}[\rho(B(z_0)+\epsilon_b)+B(z(\pi))+\epsilon_g] \tag{3}$$

$$\rho B(z_0)+\epsilon_b < b^-_0 < \pi < b^+_0 < \rho A(z_0)+\epsilon_g \tag{4}$$

represented by B_gA_g and B_bA_b and the dynamics of the *elite* starting in period 2, is exactly the same as the one starting in period 0. Hence a cycle of period 2. The size of the *masses* oscillates between the two values $(1-p)$ and z and one gets endogenously a rise and fall of *elites*. A period of good times characterized by free entry in politics, low distortions and a relatively efficient productive system is followed by a period of bad times with restricted entry in politics, a smaller *elite* and higher distortions, after which comes again a period of good times.

While certainly very specific, this example suggests how redistributive conflict considerations connected with individual randomness (here on bequest transmission) may lead to aggregate endogenous cycles in policies, politics and wealth distribution.

IV.- Conclusion

There are several possible explanations for why countries develop or fail to do so. Many of these explanations have to do with the sort of policies that their governments adopt. In general, those policies are taken as exogenous or, at most, they are seen as the result of the will of a benevolent social planner.

Following a reemerging tradition in positive political economy, this paper makes an attempt to endogenize those policy decisions. In particular, we show how these policies can be the result of the optimizing behavior of rent-seeking political *elites*. We show how they affect the allocation of resources across sectors and the distribution of wealth, and how they can lead society to prosperity or stagnation. We also endogenize the decision to belong to these *elites* and present a first analysis of how these *elites* are formed, what benefits they obtain, and what determines their fate in the long run. The paper illustrates, thus, a world in which political and economic dynamics are inextricably linked, and both are endogenously determined.

Of course, several aspects have been left out of our analysis and should merit further attention in future research. In what follows we discuss some of them.

4.1 Heterogenous Elites and intra-group redistribution effects:

One of the main purposes of this paper was to emphasize how *inter*-group redistribution effects and endogenous political participation might help explain some of the ambiguous empirical results about the relationship between economic development and democracies. In order to do this in the neatest way, we abstracted from *intra*-group redistribution effects and heterogeneity among the *elite*.

But, historically, *elites* have often been far from homogenous. *Intra-elite* redistributive conflicts have played an important role in economic and political development. In the model presented here, one may add a dimension of *intra-elite* redistribution by assuming that agents are endowed with different levels of ability. In this case, the equilibrium level of taxes will be the one preferred by the median agent in the *elite*. Enlarging the size $(1-z_t)$ of the *elite* has then two opposite effects on the tax rate τ_t. On the one hand, a larger *elite* reduces the incentives for *inter*-group redistribution as emphasized in this paper. On the other hand, *intra*-group redistributions become more important when the poor enter the *elite*, as emphasized so often by the median voter literature. The overall effect of an increase in participation on τ_t is a priori ambiguous and depends on the relative strength of the two aforementioned effects.[34]

While this could complicate substancially the model, an interesting area for future research would be to analyse the dynamic implications of the interaction of these effects on growth and distribution. In particular, the relative strength of the inter-group and the intra-group effects need not remain constant through time. For the same economy, an increase in political participation could thus be good for property rights and economic performance at certain points in time, and disastrous at others.

4.2 Endogenous political technology and strategic behavior

In our model the technology of political participation is exogenous and given by the cost of participation π. It would be interesting to have it endogenized through rational actions of political and economic agents. In that way, we would be able to study how institutional change and political modernization are related to economic growth and development.

Another extension would be to introduce some strategic behavior, and allow agents in the *elite* to preempt entry into the political *elite* as the economy develops. This could potentially introduce some interesting trade-offs for the *elite*, faced with a choice between raising the costs of participation or lowering the pace of development.

4.3. Increasing returns of rent-seeking

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[34] See Ades (1992) for an analysis of these effects in a static setting.

Finally, it would be interesting to examine the issue of specialization in rent-seeking versus productive activities, and the respective roles played by political and economic entrepreneurs during the development process. In our model, political rent-seeking is always a part-time job. Allowing for specialization would enable a better understanding of the roles that economies of scale and learning-by-doing effects play in the process of political institutionalization and economic growth, and their influence on the dynamics of income distribution.

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Appendix 1

* Proof of Proposition 1:

The proof is based on the following lemmata 1 and 2:

Lemma 1: (i) If q_t is such that $q_t > \hat{z}(\pi)$ and $\rho A(q_t) < \pi$, then: $z_{t+1} = q_{t+1} > q_t = z_t$. *(ii) If q_t is such that $q_t > \hat{z}(\pi)$ and $\rho A(q_t) \geq \pi$, then for all $k > 0$, $z_{t+k} = z_t = q_t$.*

Proof: (i) As $q_t > \hat{z}(\pi)$, it is clear that $z_t = q_t$. Moreover $\rho A(q_t) < \pi$ implies that there exist dynasties that at time t had an endowment b_t such that

$$\pi < b_t < \frac{1+\rho}{\rho}\pi - A(q_t) \tag{69}$$

Inspection of the dynamic bequest equation (20) reveals that those dynasties will leave a bequest b_{t+1} smaller than π. Hence, their descendants will not stay in the *elite* and will increase the size of the *masses* in the following period. It is also clear from equation (21) and $\rho B(q_t) < \rho A(q_t) < \pi$ that all dynasties with an endowment b_t smaller than π will also leave a bequest smaller than π. Hence By defining:

$$\bar{b}(q_t, \pi) = \frac{1+\rho}{\rho}\pi - A(q_t) \tag{70}$$

we can trace the path of q_t. This is given by

$$q_{t+1} = q_t + \int_{\pi}^{\bar{b}(q_t,\pi)} f_t(b_t)\,db_t \tag{71}$$

From this it follows that $q_{t+1} > q_t > \hat{z}(\pi)$. Hence $z_{t+1} = q_{t+1}$ will also be larger than $z_t = q_t$.
(ii) If $\rho A(q_t) \geq \pi$, then $([1+\rho]/\rho)\pi - A(q_t) < \pi$. Therefore all dynasties having an endowment of b_t larger than π will also leave a bequest b_{t+1} larger than π. Consequently, after time t the size of the *masses* remains stationary and equal to its value at t which, given that $q_t > \hat{z}(\pi)$, is equal to q_t. In other words for *all* $k > 0$, $z_{t+k} = z_t = q_t$. QED. ■

Lemma 2: There exists a finite time $S > 0$ such that $\rho A(q_S) < \pi \leq \rho A(q_{S+1})$.

Proof: The proof is by contradiction. Suppose that for all $t > 0$, we have $\rho A(q_t) < \pi$. Then by lemma 1(i), we know that for all t, $z_{t+1} = q_{t+1} > q_t = z_t$. The sequence q_t (or z_t) is strictly monotonically increasing and bounded from above by 1. Hence, it has a limit q^* which is also the limit size of the *masses* z^*. $A(z)$ being increasing in z, one gets that for all $t > 0$, $\rho A(q_t) < \rho A(q^*)$. Also by continuity of $A(z)$ one should have that $\rho A(q^*) = \rho A(z^*) \leq \pi$. From this and the fact that $A(z)$ tends to ∞ when z tends to 1, one concludes that $z^* < 1$. Hence the dynasty with the largest initial endowment b_0^+ should indefinitely remain in the *elite*. Using recurrently the dynamic bequest equation (20), one gets that the level of bequest left at time t by that dynasty is such that:

$$
\begin{aligned}
b_t &= \sum_0^{t-1} [\frac{\rho}{1+\rho}]^{t-i} A(z_i) &+ [\frac{\rho}{1+\rho}]^t b^*_0 \\
&< A(q^*) \sum_0^{t-1} [\frac{\rho}{1+\rho}]^{t-i} &+ [\frac{\rho}{1+\rho}]^t b^*_0 \\
&\leq \quad \pi &+ [\frac{\rho}{1+\rho}]^t b^*_0
\end{aligned}
$$

which shows that for t large enough the richest dynasty cannot remain strictly in the *elite*. This implies that the long run size of the *masses* z^* should be equal to 1 and consequently $A(z^*)$ is equal to ∞. This contradicts the fact that $A(z^*)$ remains smaller than π/ρ. QED. ■

$q_0=z_0>\hat{z}(\pi)$,$\rho A(q_0)<\pi$, lemma 1(i) and lemma 2, give immediately part (i) of proposition 1). lemma 1(ii) and lemma 2 give us part (ii) of proposition 1. Finally part (iii) of proposition 1) follows directly from inspection of the dynamic bequest equations (20) and (21) and the fact that after time S, z_t remains constant and equal to q_{S+1}. QED. ■

* Proof of Proposition 2:

We have $\rho B(F_0(\pi))<\pi<\rho A(F_0(\pi))$ and $z_0=q_0=F_0(\pi)$. Simple inspection of (20) and (21) gives us that all dynasties with an endowment smaller (larger) than π will leave a bequest smaller (larger) than π. Hence the size of the *masses* does not change and remains stationary at z_0. Convergence of wealth to $\rho B(F_0(\pi))$ and $\rho A(F_0(\pi))$ follows directly from (20) and (21). QED. ■

* Proof of Proposition 3:

The proof is based on the following lemmata 3 and 4:

Lemma 3: (i) If q_t *is such that* $q_t>\hat{z}(\pi)$ *and* $\rho B(q_t)>\pi$, *then:* $q_{t+1}<q_t=z_t$ *and* $z_{t+1}<z_t$; *(ii) If* q_t *is such that* $q_t\leq\hat{z}(\pi)$ *and* $\rho B(q_t)>\pi$, *then for all* $k>0$, $z_{t+k}=z_t=\hat{z}(\pi)$.

Proof: (i) If q_t is such that $q_t>\hat{z}(\pi)$, then $z_t=q_t$. Moreover as $\rho B(q_t)>\pi$, simple inspection of (21) reveals that all those agents that enjoy an initial level of wealth b such that:

$$\frac{(1+\rho)}{\rho}\pi - B(z_t) < b_t < \pi \tag{73}$$

will leave a bequest b_{t+1} larger than π. Also, because $\rho A(q_t)>\rho B(q_t)>\pi$, equation (20) shows that all agents with endowments larger than π, leave a bequest larger than π. Hence if we define $\underline{b}$ as

$$\underline{b}(\pi,z_t) \equiv \frac{(1+\rho)}{\rho}\pi - B(z_t) \quad (74)$$

the number of people that are liquidity constrained in period $t+1$ is just given by

$$q_{t+1} = q_t - \int_{\underline{b}(\pi,q_t)}^{\pi} f_t(b_t)db_t \quad (75)$$

Hence $q_{t+1}<q_t$. Also $z_{t+1}=Max(\hat{z}(\pi),q_{t+1})<q_t=z_t$.

(ii) If q_t is such that $q_t\leq\hat{z}(\pi)$ and $\rho B(q_t)>\pi$, then $z_t=\hat{z}(\pi)$, also as $B(z)$ is decreasing in z, one has that $\rho B(z_t)>\pi$. By an argument similar to the one of (i) (with z_t instead of q_t in $\underline{b}(\pi,q_t)$, one can see again that $q_{t+1}<q_t$. Hence $z_{t+1}=Max(\hat{z}(\pi),q_{t+1})=\hat{z}(\pi)=z_t$. By recurrence, one also gets that for all $k>0$, $z_{t+k}=z_t=\hat{z}(\pi)$. QED. ■

Lemma 4: Suppose $\rho B(q_0)>\pi$, *Then there exists a finite time S' such that* $q_{S'}>\hat{z}(\pi)\geq q_{S'+1}$.

Proof: The proof is by contradiction. Suppose that for all $t\geq 0$, one has $q_t>\hat{z}(\pi)$. Because $\rho B(q_0)>\pi$, one knows from lemma 3(i) that $q_1<q_0$ and because $B(z)$ is decreasing in z, $\rho B(q_1)>\rho B(q_0)>\pi$. Hence by recurrence, one gets from lemma 3(i) that for all t, $q_{t+1}<q_t$ and $\rho B(q_t)>\rho B(q_0)>\pi$. The sequence q_t is monotonically decreasing and bounded from below by $\hat{z}(\pi)$. It has a limit $q^*\geq\hat{z}(\pi)$. From $q_t>\hat{z}(\pi)$, we have also that for all t, $z_t=q_t$. Hence the long run value of the size of the *masses* will be $z^*=q^*\geq\hat{z}(\pi)$. Also from $\rho B(q_t)>\rho B(q_0)>\pi$, one gets $\rho B(z^*)=\rho B(q^*)\geq\rho B(q_0)>\pi$. Inspection of (21) reveals that the poorest dynasty starting with b^-_0 has then a bequest level that converges towards $\rho B(z^*)$. Thus there is a finite large enough time t' such that this dynasty will have a bequest level b' larger that π. But this implies that after time t' nobody is liquidity constrained contradicting the fact that for all time t, $z_t=q_t>\hat{z}(\pi)$.

Proposition 3 follows then directly from lemmata 3 and 4 and the fact that initially $z_0=q_0=F_0(\pi)>\hat{z}(\pi)$ and $\rho B(F_0(\pi))\geq\pi$. Lemma 4 ensures that there exists a finite time S' after which political rents are driven to zero. Lemma 3 describes the transitionary path before time S' and after time S'. Finally inspection of equation (21) with $z_t=\hat{z}(\pi)$ gives immediately the result of convergence of the wealth distribution. QED.■

Appendix 2: Proofs of propositions 4,5,6.

* Proof of proposition 4.

Lemma 5: (i) If q_t *is such that* $q_t>\hat{z}(\pi)$ *and* $\Omega_A(q_t)<0$, *then:*

$z_{t+1}=q_{t+1}>q_t=z_t$. *(ii) If q_t is such that $q_t>\hat{z}(\pi)$ and $\Omega_A(q_t)\geq 0>\Omega_B(q_t)$, then for all $k>0$, $z_{t+k}=z_t=q_t$.*

Proof: (i) As $q_t>\hat{z}(\pi)$, it is clear that $z_t=q_t$. Moreover, $\Omega_A(q_t)<0$ implies that there exist dynasties that at time t had an "detrended" endowment b_t/a_t such that:

$$\pi<\frac{b_t}{a_t}<\pi\frac{1+\rho}{\rho}[1+g(q_t)]-A(q_t) \tag{76}$$

Inspection of the dynamic bequest equation (56) reveals that those dynasties will leave a detrended bequest b_{t+1}/a_{t+1} smaller than π. Hence, their descendants will not stay in the *elite* and will increase the size of the *masses* in the following period. It is also clear from equation (57) and $\Omega_B(q_t)<\Omega_A(q_t)<0$ that all dynasties with an detrended endowment b_t/a_t smaller than π will also leave a detrended bequest smaller than π. Hence the size of the *masses* increases and one gets $z_{t+1}=q_{t+1}>q_t=z_t$.

(ii) If $\Omega_A(q_t)\geq 0$, then $\pi[1+g(q_t)][1+\rho]/\rho-A(q_t)<\pi$. Therefore all dynasties having a detrended endowment b_t/a_t larger than π will also leave a detrended bequest b_{t+1}/a_{t+1} larger than π. Similarly $\Omega_B(q_t)<0$, then $\pi[1+g(q_t)][1+\rho]/\rho-B(q_t)<\pi$. Therefore, all dynasties having a detrended endowment b_t/a_t smaller than π will also leave a detrended bequest b_{t+1}/a_{t+1} smaller than π. Consequently, after time t the size of the *masses* remains stationary and equal to its value at t which, given that $q_t>\hat{z}(\pi)$, is equal to q_t. In other words for *all* $k>0$, $z_{t+k}=z_t=q_t$. QED ■

Lemma 6: There exists a finite time $S>0$ such that $\Omega_A(q_S)<0\leq\Omega_A(q_{S+1})$.

Proof: The proof is by contradiction. Suppose that for all $t>0$, we have $\Omega_A(q_t)<0$. Then by lemma 5(i), we know that for all t, $z_{t+1}=q_{t+1}>q_t=z_t$. The sequence q_t (or z_t) is strictly monotonically increasing and bounded from above by 1, Hence it has a limit q^* which is also the limit size of the *masses* z^*. It is immediate to check that $\Omega_A(z)$ is increasing in z. Hence for all $t>0$, $\Omega_A(q_t)<\Omega_A(q^*)$. Also by continuity of $\Omega_A(z)$ one should have that $\Omega_A(q^*)=\Omega_A(z^*)\leq 0$. From this and the fact that $A(z)$ and therefore $\Omega_A(z)$ tends to ∞ when z tends to 1, one concludes that $z^*<1$. Using the dynamic bequest equation (56) and the fact that $\Omega_A(z_t)<0$, one gets the following relation for all t:

$$\frac{b_{t+1}}{a_{t+1}}-\pi\leq[\frac{b_t}{a_t}-\pi]\frac{\rho}{1+\rho}\frac{1}{1+g(z_t)} \tag{77}$$

Define $u_t=(b^+_0)/a_t-\pi$ *where* b^+_0 is the initial endowment of the richest dynasty. Then $z^*<1$ implies that there exists a positive number ϵ such that for all $t>0$, $u_t>\epsilon$.(Otherwise at most the dynasty with the largest initial endowment b^+_0 remains indefinitely in the *elite* and $z^*=1$). Using equation (56) one finds that:

$$u_{t+1}\leq u_0[\frac{\rho}{(1+\rho)(1+g(z_t))}]^t<u_0[\frac{\rho}{(1+\rho)(1-\delta)}]^t \tag{78}$$

Hence because of (78) and assumption GI, u_t tends to 0. This contradicts the fact that for all $t>0$, $u_t>\epsilon$ and $z^*<1$. QED■

The proof of proposition 4 follows immediately from lemma 5 and 6. Lemma 6 gives us the existence of a finite time S such that $\Omega_A(q_S)<0\leq\Omega_A(q_{S+1})$. From $\Omega_A(z_0)<0$ and lemma 5(i) we conclude that for all time $t\leq S$, the sequence $q_t=z_t$ is increasing and the growth rate $g(z_t)$ is decreasing. It is easy to check that assumption G2 in the text implies that the function $\Omega_B(z)$ is decreasing in z. Hence as $z_{S+1}=q_{S+1}>q_0=z_0$, and from $\Omega_B(z_0)<\Omega_A(z_0)<0$, one concludes that: $\Omega_B(z_{S+1})<\Omega_B(z_0)<0\leq\Omega_A(z_{S+1})$. From this and lemma 5(ii) one obtains immediately that for all $t>S$, the size of the *masses* z_t and consequently the growth rate remain constant at z_{S+1} and $g(z_{S+1})$. Inspection of (56) and (57) with z_t equals to z_{S+1} gives us immediately Part (iii) of proposition 4. QED ■

* Proof of Proposition 5:

We have $\Omega_B(F_0(a_0\pi))<0<\Omega_A(F_0(a_0\pi))$ and $z_0=q_0=F_0(a_0\pi)$. Simple inspection of (56) and (57) gives us that all dynasties with a detrended endowment smaller (larger) than π will leave a bequest smaller (larger) than π. Hence the size of the *masses* (as well as the growth rate $g(z_t)$) does not change and remains stationary at z_0 $(g(z_0))$. Convergence of detrended wealth levels follows directly from (56) and (57). QED. ■

* Proof of proposition 6.

Lemma 7: i) If q_t is such that $q_t>\hat{z}(\pi)$ and $\Omega_B(q_t)>0$, then: $q_{t+1}<q_t=z_t$ and $z_{t+1}<z_t$. ii) If $\Omega_B(z_0)>0$ and q_t is such that $q_t\leq\hat{z}(\pi)$, then for all $k>0$, $z_{t+k}=z_t=\hat{z}(\pi)$.

<u>Proof:</u> (i) If q_t is such that $q_t>\hat{z}(\pi)$, then $z_t=q_t$. Moreover, as $\Omega_B(q_t)>0$, simple inspection of (57) reveals that all those agents that enjoy an initial level of detrended wealth b_t/a_t such that:

$$\pi[\frac{(1+g(z_t))(1+\rho)}{\rho}]-B(z_t)\leq\frac{b_t}{a_t}<\pi \tag{79}$$

will leave a detrended bequest b_{t+1}/a_{t+1} larger than π. Also, because $\Omega_A(q_t)>\Omega_B(q_t)>0$, equation (56) shows that all agents with detrended endowments larger than π, leave a detrended bequest larger than π. Hence the number of liquidity constrained dynasties decreases as well as the size of the *masses* z_t. In other words $q_{t+1}<q_t$. Also $z_{t+1}=Max(\hat{z}(\pi),q_{t+1})<q_t=z_t$.

(ii) If q_t is such that $q_t\leq\hat{z}(\pi)$, then $z_t=\hat{z}(\pi)$, also as $\Omega_B(z)$ is decreasing in z, $\hat{z}(\pi)<z_0$ and by assumption $\Omega_B(z_0)>0$, one has that $\rho B(z_t)>\pi$. By an argument similar to the one of (i), one can see again that $q_{t+1}<q_t$. Hence $z_{t+1}=Max(\hat{z}(\pi),q_{t+1})=\hat{z}(\pi)=z_t$. By recurrence, one also get that for all $k>0$, $z_{t+k}=z_t=\hat{z}(\pi)$. QED. ■

Lemma 8: Suppose $\Omega_B(z_0)>0$, Then there exists a finite time S' such that $q_{S'}>\hat{z}(\pi)\geq q_{S'+1}$.

<u>Proof:</u> The proof is by contradiction. Suppose that for all $t\geq 0$, one has $q_t>\hat{z}(\pi)$. Because $\Omega_B(z_0)>0$, one knows from lemma 7(i) that $q_1<q_0$ and because $\Omega_B(z)$ is decreasing in z, $\Omega_B(q_1)>\Omega_B(q_0)>0$. Hence, by recurrence, one gets from lemma 7(i) that for all t $q_{t+1}<q_t$ and $\Omega_B(q_t)>\Omega_B(q_0)>0$. The sequence q_t is monotonically decreasing and bounded from below by $\hat{z}(\pi)$. It has a limit $q^*\geq\hat{z}(\pi)$. From $q_t>\hat{z}(\pi)$, we have also that for all t, $z_t=q_t$. Hence, the long run value of the size of the *masses* will be $z^*=q^*\geq\hat{z}(\pi)$. Also from $\Omega_B(q_t)>\Omega_B(q_0)>0$, one gets $\Omega_B(z^*)=\Omega_B(q^*)\geq\Omega_B(q_0)>0$. Inspection of (57) and

Figure 1

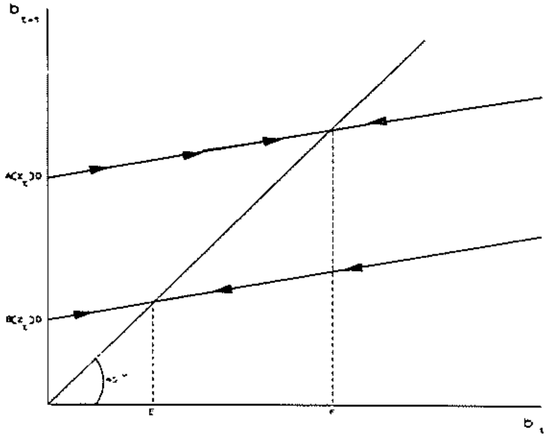

Figure 2

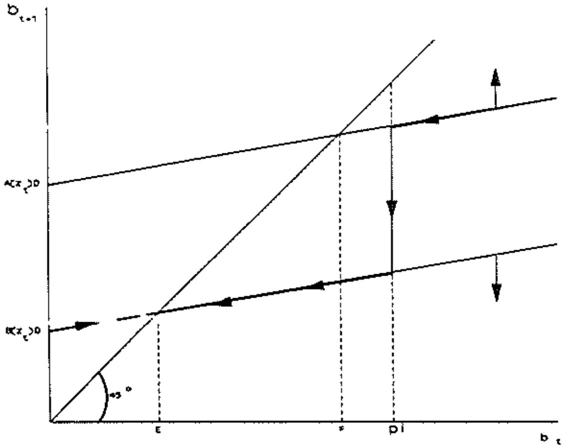

Figure 3

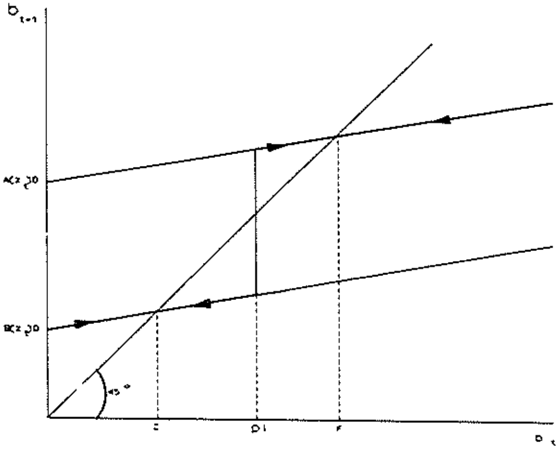

Figure 4

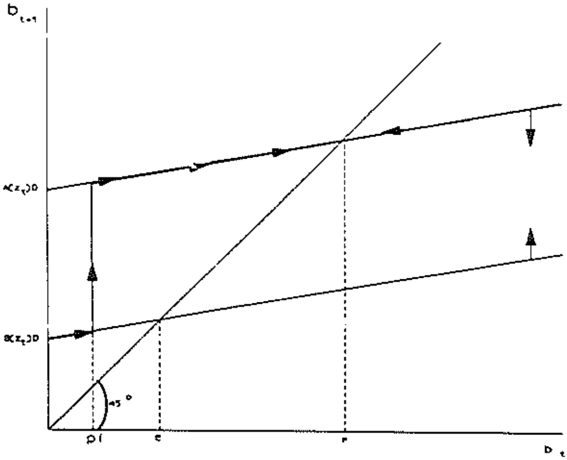

Figure 5

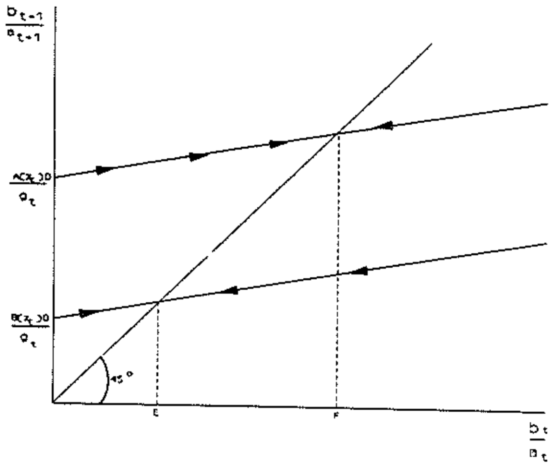

Figure 5

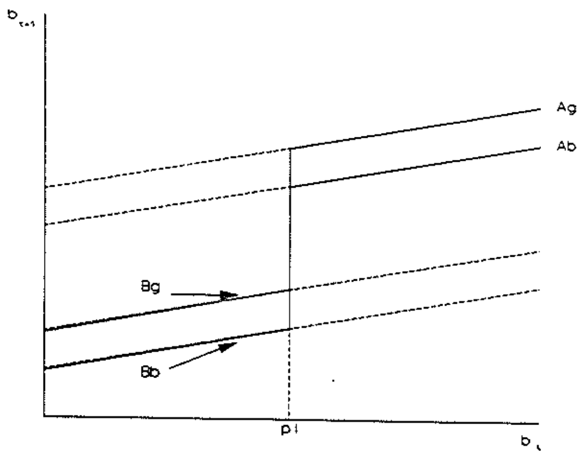

Figure 7

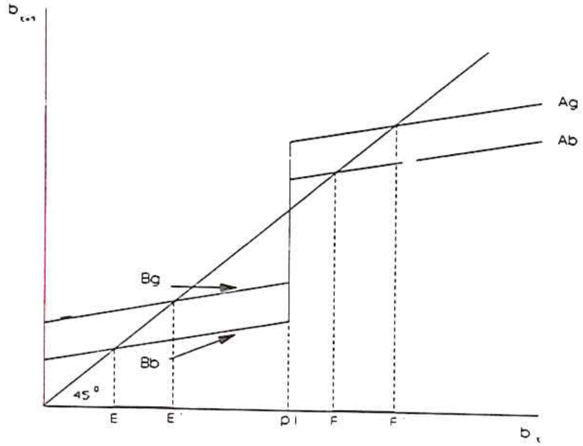

Figure 8

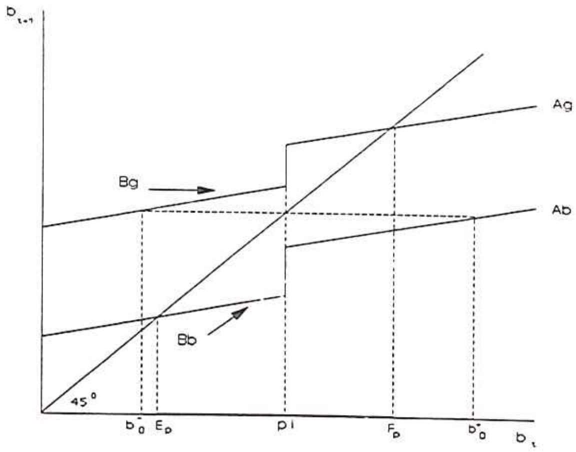

Figure 9