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JEL Classification: C72, C73, D72, D78

Keywords: democracy, dynamic elections, Political Polarization, costs of change, Markov perfect equilibrium

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Electoral Competition with Costly Policy Changes: A Dynamic Perspective*

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**Matthew Jackson is an external faculty member of the Santa Fe Institute.

†Philippe Muller was funded by SNF (Project no. 100010_153482/1).

1 Introduction

Motivation

Policy changes are costly, and the costs increase with the extent of the policy shift. This is graphically illustrated, for example, by the negotiations between the EU and UK about the implementation of Brexit. In general, policy shifts may render original investments in human and physical capital by the private and public sectors obsolete, or may require effort from the government to design new policies and overcome resistance from groups wanting to preserve the status quo and even to create new institutions and/or dismantle old ones.[1] These costs of change are borne by the entire citizenry, including party members and politicians.

In the presence of costs of change, office-holders face a trade-off. Changing the policy towards their own bliss point is desirable, but the benefits associated with the new policy may be outweighed by the costs of carrying out the policy change. Depending on the level of these costs, politicians may thus engineer major or minor policy changes, or none at all. How do these costs influence policies and elections in the short and the long term?

A preview of the model

We develop an infinite-horizon election model with a continuum of citizens and two political parties, in which policy changes are costly. We consider a one-dimensional policy space over which citizens have standard quadratic utility. In every election, one candidate from each party competes for office.[2] The winning candidate chooses a policy for that period. Following an electoral defeat, the losing candidate is replaced in the next election by a new candidate from the same party. Neither candidates nor citizens can commit to particular policies before election. Political parties also have quadratic utility over the one-dimensional policy space, which represents the interests of the median party member, a citizen. A candidate's bliss point is that of the party s/he runs for, so the candidate's and the party's objectives are perfectly aligned.

Once in office, each candidate is characterized by his/her valence or *capacity* to efficiently carry out the usual governmental tasks. The capacity of an elected politician can be interpreted in a narrow sense as pure ability to perform such tasks or, in a broader sense, as the ability and propensity of key employees and political partners to perform the tasks involved in governance and administration. During tenure, exogenous events may negatively affect an incumbent's capacity as perceived by the

[1] The phasing-out of nuclear energy production in Germany and the repeal of the Patient Protection and Affordable Care Act in the US are further examples of costly policy changes.

[2] In the baseline case, candidates are policy-motivated. Yet the results extend readily to the case where candidates are also office-motivated.

voters. For instance, a corruption scandal involving the incumbent or some of his/her governmental partners may be understood as a proxy for a bad politician; a failed public project may be considered a signal of the incumbent's inability as a policy-maker; a worldwide crisis may affect the incumbent's perceived ability—and that of his/her administration—, and so forth. We assume that in every period there is a probability that the current incumbent's capacity declines significantly enough for a majority of voters to vote for the challenger in the next election, regardless of any other consideration.[3] Although low capacity is a *sufficient* condition for an incumbent to be ousted, it is not clear a priori whether, and if so under what circumstances, it is also *necessary*.

Differently from most models of electoral competition, we make the assumption that the further the policy choice in the current period deviates from the status quo, the higher the costs are. These *costs of change* establish a dynamic link between policies across periods.[4] Although in our baseline model costs are linear in the extent of the policy shift, this assumption is not crucial for our results. It is only necessary for such costs to be moderately convex in the extent of policy changes. Assuming linear costs of change conveys the main mechanisms underlying our dynamic election model in a transparent way. Later, we examine (moderately) convex costs of change and identify how different degrees of convexity affect the results.

Besides costs of change, other parameters are important in our model. *Party polarization* captures the distance between the two parties' preferred policies. Whenever costs of change are not negligible the extent of party polarization can have a large impact on policies. For instance, high party polarization could induce large and costly policy switches after an incumbent has been ousted from power. Throughout the paper, we take party polarization as given and study how it influences policy choices. Due to the existence of costs of change, the policies implemented need not coincide with the preferred policies of the political parties. Another crucial parameter is the initial level of *policy polarization*, which corresponds to the distance between the past policy and the median voter's current bliss point. Because the initial level of policy polarization is exogenous, our model embeds the possibility that preferences of parties and citizens have changed. Lastly, we also take into account how *capacity-shock probability*—i.e. the probability that the incumbent receives a capacity shock in any given period—influences turnover and policy choices.

[3] As we will see, our model features a strong incumbency advantage. Then it suffices to focus on negative shocks, since positive capacity shocks would only reinforce the incumbency advantage. Assuming constant probability facilitates the analysis but does not affect the results qualitatively. Finally, the assumption that any shock is sufficiently large in extent for the incumbent to be ousted facilitates the analysis but can also be dispensed with. With capacity shocks that individually have a lower extent, there is always a number of them such that the incumbent is ousted if and only s/he has suffered at least such a number of shocks.

[4] Costs of change are borne, no matter whether the office-holder has held office in the previous period or not.

Our goal

The main object of the paper is to study the short- and long-term impact of costs of change on policy choices and office-holder turnover. We proceed as follows: To disentangle the impact of the costs of change from voter and party farsightedness, we first examine what happens when voters and office-holders are myopic and choose strategies that maximize their current utility. This corresponds to an extreme case of present-biased preferences. An alternative interpretation is that of a non-overlapping generation framework in which voters and policy-makers live for one period and where, after each period, power can shift to the other party. This is the case if election cycles are long, in which case the relevant discount factors are small. For the game-theoretic analysis of the baseline model, we introduce the concept of a *Myopic Stationary Markov Perfect Equilibrium (MSMPE)*. Subsequently, we modify the notion of MSMPE and let either voters or parties—and hence office-holders—be forward-looking.

Results

We show that the game described above has a unique MSMPE and characterize this equilibrium. Four findings stand out. First, costs of change generate a strong electoral advantage for incumbents that can only be offset by a negative shock to their capacity: office-holders are re-elected if and only if they have normal capacity. This means that costly policy changes are a potential source of the commonly observed incumbency advantage. In the long run, such an advantage is maximal for moderate marginal costs of change. The electoral advantage for incumbents is *not* a general property of costs of change, but it holds only if these costs are moderately convex.

Second, also in the long run, policies converge to a stochastic alternation between two states or (regions of) policies, with the transition probability between the two states being equal to the probability of a negative capacity shock. For low costs of change, these states are independent of the initial policy and are located on either side of the median voter's bliss point. Thus, no convergence to the median voter position takes place. A consequence is that the steady state of policy alternation is *robust*, in the sense that it is always restored after some periods if a policy perturbation occurs.

Third, policy paths display strong history-dependence in general, even if costs of change are low. Specifically, the transition phase from the initial policy to the long-term sequence of moderate policies crucially depends on the level of initial policy polarization. If initial policy polarization is high, the equilibrium policy path starts with a short sequence of extreme policies followed by an infinite sequence of less extreme policies. For low initial policy polarization, on the other hand, the moderate policy stage is reached in the first period.

Fourth, the candidates' long-term equilibrium policy choices are more moderate (i.e., closer to the median) than their bliss points. In other words, *policy polarization*—i.e. the distance between the policies implemented by candidates of different party affiliation—is lower than party polarization (unless costs of change are so high that they prevent policy change altogether). This means that from a long-term perspective costs of change can have positive consequences for welfare.

These findings offer clear-cut predictions about party and citizen behavior. Accordingly, they can be formulated as a series of hypotheses with regard to policies and elections that can then be tested empirically for particular political systems. While this is beyond the scope of the paper, we argue in Section 4.4 that they can be related to some observations in real-world political environments.

Finally, the equilibrium characterization is robust for varying degrees of convexity in costs of change and varying levels of voter and party farsightedness. This enables us to formulate further testable predictions (see Section 5.3). First, convex costs may lead to gradual reforms and may guarantee that turnover is welfare-improving. Second, extreme policies are carried out only when voters are (fully) myopic. Third, high political instability and moderate costs of change lead to a negative relation between the office-holders' discount factor and short- and long-term policy polarization.

Organization of the paper

The rest of the paper is organized as follows: In Section 2 we review the strands of literature related to our paper. In Section 3 we present our baseline model and introduce the corresponding equilibrium concept. Section 4 contains the equilibrium analysis of our baseline set-up. In Section 5 we study three generalizations of the model introduced in Section 3. These generalizations enable us to relax the assumption on voters' and parties' shortsightedness and to study the case of non-linear costs of change. We show that our results remain valid in all generalizations. Section 6 concludes. The proofs can be found in Appendices A and B.

2 Related Literature

The paper is related to several strands in the literature.

Dynamic electoral competition

Our paper contributes to the literature on dynamic elections with endogenous state variables (see e.g. Battaglini et al., 2012) by showing that costs of change offer incumbents the possibility of choosing policies that create an electoral advantage. In contrast to many papers in this literature, the state variable—viz. the previous policy choice—is not necessarily economic in nature in our model. The first papers to highlight strategic incentives for office-holders to manipulate economic variables—mainly, the debt level—for electoral gains are Persson and Svensson (1989) and Alesina and Tabellini

(1990) (see also the more recent Bouton et al., 2016). These papers find that political competition results in higher debt accumulation, which can negatively affect welfare. This contrasts with our paper, in which the distortion created by costs of change—which induce burning utility—can have positive welfare effects, at least in the long run.

Two closely related papers are Forand (2014) and Nunnari and Zápal (2017). Both study dynamic models of electoral competition based on a static model of partisan competition in which policy-makers are committed to implementing the same policy in all future periods in office. A rationale for such behavior is that it precludes reputation losses associated with flip-flop policies (see e.g. Alesina, 1988; Miller and Schofield, 2003; Tavits, 2007). We differ by considering positive costs of change that reduce but do not eliminate the flexibility of *all* future office-holders, incumbents and challengers alike, in engineering policy changes.[5] As in our model, Forand (2014) shows that policy converges to an alternation between two limit points, but for different reasons. And, in contrast to our model, these points generally depend on the initial policy. Nunnari and Zápal (2017) also feature policy alternations, but they converge to the median voter's position. This follows because opposition parties are not constrained by the status quo, which allows them to cater sufficiently well to the median voter to win the election against the constrained incumbent. Unlike Forand (2014) and Nunnari and Zápal (2017), our model generates an electoral advantage for the incumbent (and not the challenger).

Costly policy changes

The literature examining the costs associated with policy changes is scant. A few papers have focused on costs of change arising only when a newly enacted policy—whose initial implementation involves no costs as there is no status quo—is reformed. Gersbach and Tejada (2018) show that to create an electoral advantage, the incumbent chooses extreme (moderate) policies when he or she is more (less) efficient in implementing policy changes than the challenger. Extreme policies creating an electoral incumbency also arise in Glazer et al. (1998), but they occur because the challenger is committed to his/her position, and the reform costs are disproportionately large. In Gersbach et al. (2019), policy moderation arises as a compromise between the incumbent's ideological position and the utility losses that result when s/he loses power. The more likely it is that power will shift, the higher the degree of moderation. Because we assume that there is an initial status quo, moderation in this policy dimension arises in our case also as an attempt to reduce the costs associated with policy changes in the *current* period. Moreover, we model elections explicitly and are hence able to endogenize re-election. This enables us to identify costs of change as a source of incumbency

[5]With its focus on shortsighted candidates, our baseline set-up is similar to Kramer (1977) and Wittman (1977), who consider candidates basing their policy choices on the expected outcome of the upcoming election.

advantage, as we shall see below. In a companion paper, see Gersbach et al. (2020), we focus on the socially optimal length of political terms in the presence of costs of change.

Our paper is also relevant from a technical viewpoint since we are the first to look at the effect of costs of change on elections and policy within an infinite-horizon framework. We add to the scant literature on such costs by proving *(a)* the existence of a Markovian equilibrium and *(b)* that such an equilibrium is unique if strategies are of a certain reasonable type.[6] This transcends the merely technical dimension because it enables us both to describe the long-term behavior of political competition with costs of change and to characterize the transition phase towards the steady state. Our analysis also yields interesting comparative statics results regarding the degree of convexity of costs of change and the discount factor of politicians and voters.

Further dynamic links in political competition

The implications of dynamic links across periods is the subject of other recent papers. In a framework of public-good provision with public transfers, Bowen et al. (2014) show that when there is some degree of persistence in political power, mandatory programs—programs that set the default level of the public good in the case of disagreement between parties—are more efficient than fully flexible programs. In Bowen et al. (2014), public spending only changes when a new party comes to office, as in our model. In a similar vein, Bowen et al. (2017) argue that efficiency can only be attained if institutions allow some degree of flexibility in spending, but not full flexibility. These results on the trade-off between flexibility and efficiency bear some resemblance to one insight of our paper, namely that some degree of costs of change—say, in the form of a system of checks and balances—can have positive effects for welfare in the steady state by moderating policy. Chen and Eraslan (2017) assume that a change in one policy dimension precludes the possibility of another change in the future. They show that a system of checks and balances enables strategic behavior of many types, as do costs of change in our model. Assuming that the status quo carries over to the next period if no unanimity is reached, Buisseret and Bernhardt (2017) analyze the dynamic consequences of yet another institutional feature: veto power. Buisseret and Bernhardt (2017) show that incumbents also moderate policy in the current period, but they do so because they do not know the veto player's preferences in the future. Callander and Raiha (2017) assume that investment decisions are durable and accumulate over time. They show that the possibility of exhausting the budget for future periods can be used to create an incumbency advantage. Unlike costs of change, this possibility unambiguously leads to inefficient outcomes. Finally, dynamic links can also occur by inducing changes in preferences (see e.g. Glaeser and Shleifer, 2005). In our paper, preferences are

[6]Gersbach et al. (2020) cannot guarantee uniqueness of equilibria in their model because they consider a type of strategies that lead to weaker off-equilibrium threats than the type of party strategies we consider.

publicly known, and they do *not* change over time.

Also worthy of mention are Baron (1996) and Zápal (2016). Focusing on Markovian equilibria as we do, these papers take the policy chosen in the previous period as the status quo for the current period, which is pitted against the policy proposed by some (exogenously-selected) agenda-setter.[7] This establishes a dynamic link across policies. The outcome in Baron (1996) and Zápal (2016) is that policies fully converge to the median. This contrasts with our model, where election forces are present and policy moderation is only partial. At the other extreme, we find Dziuda and Loeper (2016) and Dziuda and Loeper (2018).[8] Also focusing on Markovian dynamics with an endogenous status quo, their papers show that polarization obtains (more often) as a result of gridlock when preferences evolve over time. One conclusion is that checks and balances may generate welfare-reducing inertia. Our model features this property only when costs of change are very large. If this is not the case, a moderate limit state is restored some periods after a preference shock has occurred. Inefficiencies associated with status-quo inertia are also studied by Strulovici (2010) and others.

Incumbency advantage

There is a large literature on the existence and the causes of incumbency advantage. Gelman and King (1990) and Alford and Brady (1989) empirically measure incumbency advantages in congressional elections. Levitt and Wolfram (1997) and Cox and Katz (1996) go beyond measurement and decompose the sources of this advantage. We find that normal-capacity candidates are always reelected, which is in line with the existence of a substantial incumbency advantage. We add to the existing literature by identifying (linear or moderately convex) costs of change as a potential source of incumbency advantage and by showing that they can have positive implications for welfare.

Policy commitment

In electoral competition models, it is a divisive issue whether candidates can commit to policy positions before elections. In the classic formulation by Hotelling (1929) and Downs (1957), politicians can do so. Yet various other strands of literature, and most notably models of political accountability, assume that competitive elections are not enough to guarantee that politicians who break promises are ousted—see Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989), Persson et al. (1997), or Ashworth (2012) for a recent review of models of electoral accountability. Our model can be interpreted as a model of imperfect accountability in which the costs of policy change enable incumbents (but not challengers) to commit to a particular policy that yields an electoral advantage. This is because policy choices in the current period provide an anchor for future behavior, as they

[7] Cho (2014) and Baron (2018) are recent papers that belong to the endogenous status-quo literature considering elections in proportional systems.

[8] See also Austen-Smith et al. (2019).

influence the policy choices in all subsequent periods. The present paper thus attempts to bridge the aforementioned divide between full and no commitment for politicians, and in doing so proves that the existence of limited commitment—in the form of costs of change—can moderate policies without reducing welfare.

Behavioral political economy

There is a growing literature on behavioral political economy (see e.g. Ortoleva and Snowberg, 2015; Attanasi et al., 2016). The paper most related to ours is Alesina and Passarelli (2017), who investigate public-good provision when citizens are loss-averse with respect to changes in the status-quo policy. They find that this behavioral feature moderates policies, as do costs of change in our model. A bridge between our paper and this literature can be established if we consider costs of change to be psychological in nature.

Political polarization

Party and policy polarization are important variables in our model. A large literature has examined the causes of both political phenomena (see e.g. Roberts and Smith, 2003; Theriault, 2006; Heberlig et al., 2006) and consequences (see e.g. Jones, 2001; Binder, 2003; Fiorina et al., 2005; Testa, 2012; Hetherington, 2001). Our paper adds to this knowledge by investigating the way in which policy polarization is determined in the long run by varied levels of party polarization, magnitude and convexity of costs of change, and voter and party farsightedness.

Present-biased preferences and time inconsistency

In our baseline set-up, voters and politicians are myopic, which is an extreme case of present-biased preference. In the political economy literature, preferences of this sort can lead to time-inconsistent decisions (see e.g. Bisin et al., 2015; Jackson and Yariv, 2015; Lizzeri and Yariv, 2017; Piguillem and Riboni, 2015). This is not the case in our model with costs of change. In the long run, elections and policy-making are qualitatively equal regardless of whether politicians and/or voters are myopic or have standard forward-looking preferences. In the short term, the main difference is that forward-looking voters can avoid finite cycles of extreme policies when policy polarization is initially large.

Gradualism and status-quo bias in reforms

There is a vast literature on why reforms sometimes occur gradually (see e.g. Murphy et al., 1992; Roland, 2000) or do not occur at all (see e.g. Fernández and Rodrik, 1991; Miller and Schofield, 2003; Tavits, 2007).[9] We contribute to this literature by showing that *(i)* convex costs of reform can lead naturally to gradual reforms, and *(ii)* large costs of change can lead to gridlock.

[9] Hwang and Möllerström (2017) study how time-inconsistent preferences may yield to gradual reforms.

3 The (Simple) Model

3.1 General set-up

We examine an infinite-horizon model ($t = 1, 2, \ldots$) of electoral competition. In each period, there is an election by which a society elects an office-holder who is responsible for policy-making in that period. The society consists of a continuum of voters of mass 1, with each voter indexed by $i \in [0, 1]$. There are two political parties: Party L and party R. One candidate from each party competes in every election in representation of the respective party.[10] A party in $\{L, R\}$ is denoted by K, with $-K$ such that $\{K, -K\} = \{L, R\}$. Similarly, a candidate in $L \cup R$ is denoted by k, with $-k$ such that $k \in K$ and $-k \in -K$ when parties are not specified. The candidate defeated in one election is replaced in the next election by another candidate from the same party.

Office-holder's tasks

Once in office, a politician k in a given period t can be of *normal capacity* (i.e., his/her capacity is $a_{kt} = 0$) or of *low capacity* (i.e. his/her capacity is $a_{kt} = -A$, with $A > 0$). As discussed in the Introduction, capacity is a broad term designed to capture all aspects of valence and of ability of an office-holder to perform well in policy-making. By default, an office-holder in his/her first term has normal capacity. At the end of each period t in which the incumbent is still in office, s/he suffers from a permanent negative capacity shock with probability $\lambda \in [0, 1]$. The capacity of a politician is common knowledge at all times, so we abstract from any signaling problem. For a given period t, office-holder k faces issues in two different dimensions:

- S/he undertakes the usual governmental tasks to provide basic public administration services. We assume that the output of all these tasks, denoted by g_{kt}, is directly proportional to the capacity of the office-holder k in period $t - 1$ before the capacity shock may occur, i.e. $g_{kt} = a_{k(t-1)}$, where $a_{k(t-1)}$ is either equal to zero or equal to $-A$. That is, the office-holder's capacity is all that matters, so g_{kt} is *not* a choice variable. Its only role is to reveal the office-holder's capacity. A low-capacity politician can be interpreted simply as incapable of performing basic governmental tasks correctly.

[10] We rule out entry of a third independent candidate locating at the median. One possibility is that parties have complete control over entry and only those who are part of one of the two parties can enter an election. Another possibility is that established parties choose their positions differently from the median—and hence induce a significant level of party polarization—precisely to prevent the entry of a third candidate (Palfrey, 1984). Such a candidate could be discouraged from entry if s/he anticipated a certain defeat. From an empirical perspective, it has been shown that institutional factors can also affect the chances of third candidates (see Dowling and Lem, 2009, for US gubernatorial elections between 1980 and 2005).

- S/he chooses a policy i_{kt}, with $i_{kt} \in [0,1]$. We use i_t instead of i_{kt} if the office-holder's identity does not matter. We interpret $[0,1]$ as the usual policy space that ranges from liberal ($i_t = 0$) to conservative ($i_t = 1$). We assume that candidates (and hence parties) cannot commit before the election to carrying out a particular policy if they are elected.

Elections

In each election, every citizen casts a vote for one of the two candidates running for office. Citizens cannot commit their vote before election. To break ties when voters are indifferent between voting for either candidate, we distinguish two cases. In the t-th election, with $t \geq 2$, ties are broken in favor of the incumbent. The first election, by contrast, is an open race as there is no incumbent. Then, we assume that citizens who are indifferent between the two candidates vote for the one whose bliss point is closest to the status-quo policy (if such a policy is different from the median voter's bliss point) or they simply decide according to the fair toss of a coin (if the status quo is the median voter's bliss point).[11] This reflects the idea that if policies that are to the right (to the left) of the median voter's bliss point, they are likely to have been put in place by a candidate from party R (from party L).

The sequence of events in period t, for $t \in \{1, 2, \ldots\}$, is summarized in Figure 1.

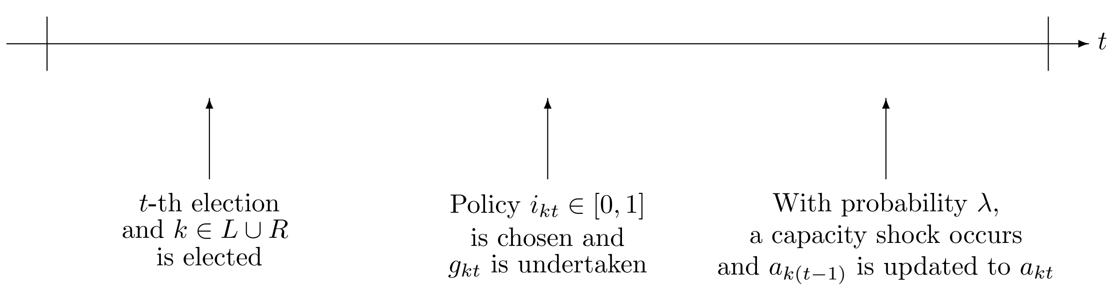


Figure 1: Sequence of events in period t, for $t \in \{1, 2, \ldots\}$.

Instantaneous utilities

First, the utility from the business-as-usual tasks in period t is the same for all voters and for both parties:

$$U^{\mathcal{B}}(g_t) = g_t.$$

[11] Qualitatively our results also hold if ties are broken in favor of the challenger or we assume other tie-breaking rules for the first election.

Second, citizens have differing preferences over $[0,1]$. More specifically, voter $i \in [0,1]$ has bliss point i and derives utility

$$U_i^{\mathcal{I}}(i_t) = -(i_t - i)^2$$

from the policy choice $i_t \in [0,1]$ made in period t. Thus, i refers to both the voter i and his/her ideal point. We assume for simplicity that the median voter is $m = 1/2$.[12] The exact distribution of preferences does not matter for equilibrium behavior, since voting outcomes are determined by the median voter. In turn, in period t, party $K \in \{R, L\}$ derives utility

$$U_K^{\mathcal{I}}(i_t) = -(i_t - \mu_K)^2$$

from the policy choice $i_t \in [0,1]$, where μ_R and μ_L are the parties' ideal points regarding $[0,1]$. Party R is the *right-wing* and party L is the *left-wing* party. Throughout the paper, $\Pi = \frac{1}{2} \cdot (\mu_R - \mu_L)$ denotes the level of *party polarization*. Accordingly, the higher the value of Π, the more opposed the two parties' interests are. Any party's candidate inherits the instantaneous utility of the party s/he represents, so their interests are fully aligned at any given period. In the rest of the paper we often slightly abuse language and refer indistinguishably to a party's and a politician's utility. We assume that

$$\frac{1}{2} < \mu_R \leq 1 \quad \text{and} \quad \mu_L = 1 - \mu_R.$$

The latter assumptions facilitate the presentation of the results but could be easily dispensed with. Imposing that both parties have bliss points that are located on different sides of the political spectrum relative to the median voter's bliss point is standard. If this were not the case, the party with a bliss point that is closer to that of the median voter would always be elected, all else being equal. For its part, the symmetry assumption enables us to abstract from exogenous differences between parties. Yet is not crucial for the dynamics of the model.[13] If voter preferences are biased towards one party, this party simply enjoys a *weakly* greater incumbency advantage than the other party. If the bias is large, the party whose bliss point is farther away from that of the median voter will lose elections even if its candidate has normal capacity. If the bias is small, by contrast, the dynamics are the same as in the case of no bias. These properties hold because of the incumbency advantage generated by (linear or, more generally, moderately convex) costs of change—see next.

Third, we assume that policy changes are costly for all voters and parties. More precisely, given policy choice $i_{t-1} \in [0,1]$ in period $t-1$, we assume that policy choice $i_t \in [0,1]$ in period t imposes

[12] For instance, this is the case if citizens' preferences are uniformly distributed on $[0,1]$.

[13] If parties' bliss points are not located symmetrically with respect to $1/2$, the median voter's bliss point, then the party whose bliss point is closest to that of the median voter would always be elected in the first election.

a utility loss in period t on voters and parties alike, given by[14]

$$U^c(i_{t-1}, i_t) = -c \cdot |i_{t-1} - i_t|.$$

The parameter $c \geq 0$ corresponds to the *marginal cost of policy change*.[15] In the first period, costs of change are equal to $U^c(i_0, i_1) = -c \cdot |i_0 - i_1|$, where $i_0 \in [0, 1]$ is the status-quo policy in $t = 1$. Note that $\left|i_0 - \frac{1}{2}\right|$ captures the *initial (absolute) level of policy polarization*. Thus, the higher the value of $\left|i_0 - \frac{1}{2}\right|$, the more distant the initial policy is from the interests of the median voter.

Finally, for each voter $i \in [0, 1]$ and each period $t \in \{1, 2, \ldots\}$, we have

$$U_i(i_{t-1}, i_t, g_t) = U^{\mathcal{B}}(g_t) + U_i^{\mathcal{I}}(i_t) + U^c(i_{t-1}, i_t). \tag{1}$$

Hence, $U_i(i_{t-1}, i_t, g_t)$ is the instantaneous utility of voter i in period t. Similarly, for party $K \in \{L, R\}$ and each period $t \in \{1, 2, \ldots\}$, we have[16]

$$U_K(i_{t-1}, i_t, g_t) = U^{\mathcal{B}}(g_t) + U_K^{\mathcal{I}}(i_t) + U^c(i_{t-1}, i_t). \tag{2}$$

We use $\mathcal{G}^{i_0}$ to denote the game described above, with the set of players being made up of all voters and the two political parties and the initial status-quo policy being $i_0 \in [0, 1]$.

3.2 Equilibrium concept

We assume in our baseline model that voters and parties (candidates) are *myopic*, i.e. they only care about their own utility in the current period. More precisely, voters base their voting decisions on the utility they expect from both candidates in the current period, while the office-holder does not care about re-election.[17] These assumptions are restrictive, but they enable us to obtain a first general approximation of the long-term effects on policies of marginal costs of change, party polarization, initial policy polarization, and capacity-shock probability. They also lead to results whose underlying mechanisms are transparent and can be easily interpreted. In Section 5 we relax these assumptions and analyze their marginal impact in the results. For this baseline model, we consider stationary equilibria in pure Markov strategies, which is standard in the literature on dynamic political economy (see e.g. Duggan and Kalandrakis, 2012; Duggan and Martinelli, 2014). We also impose the standard

[14] As already mentioned, in Section 5.1 we analyze the robustness of our results to costs of change that are convex (instead of linear) in the extent to which policies change.

[15] Politicians and voters all have the same parameter c. This simplifies notation, but the main thrust of our results also holds if there are cost differences between politicians and voters—see Figure 5.

[16] To avoid cumbersome notation, we do not consider office-motivation in office-holders until Section 5.2.2, as this only plays a role when parties (and office-holders) are forward-looking.

[17] We also assume that in the case of indifference, parties prefer to win the election. This is the case if parties are office-oriented. We only explicitly incorporate these considerations in Section 5.2.2, where parties are not myopic.

refinement that citizens eliminate stage-dominated strategies and hence vote as if they are pivotal, which rules out implausible equilibria (Baron and Kalai, 1993). We introduce some definitions.

Definition 1

A Stationary Markov Strategy for voter $i \in [0,1]$ is a function $\sigma_i : \{\emptyset, L, R\} \times [0,1] \times \{0, -A\} \to \{L, R\}$ that maps the identity and capacity of the incumbent and the status-quo policy into the candidate to vote for. A Stationary Markov Strategy for party $K \in \{L, R\}$ is a function $\sigma_K : [0,1] \times \{0, -A\} \to [0,1]$ that maps the status-quo policy into the policy of the current period given the incumbent's capacity.

Throughout the paper, we write $\sigma_v = (\sigma_i)_{i \in [0,1]}$ to denote the voters' strategy profile. Next, we define the notion of equilibrium that we use in Section 4.

Definition 2

A Myopic Stationary Markov Perfect Equilibrium (MSMPE) of $\mathcal{G}^{i_0}$ is a profile of Stationary Markov Strategies $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ such that for each $t \in \{1, 2, \ldots\}$, each previous policy $i_{t-1} \in [0,1]$, and incumbent $k \in K$ in period t with initial capacity $a_{k(t-1)} \in \{0, -A\}$:

$$\sigma_K^*(i_{t-1}, a_{k(t-1)}) \in \underset{i_t \in [0,1]}{\operatorname{argmax}}\, U_K(i_{t-1}, i_t, a_{k(t-1)}) \tag{3}$$

and, for every citizen $i \in [0,1]$:

$$\sigma_i^*(K, i_{t-1}, a_{k(t-1)}) = K \Leftrightarrow U_i\left(i_{t-1}, \sigma_K^*(i_{t-1}), a_{k(t-1)}\right) \geq U_i\left(i_{t-1}, \sigma_{-K}^*(i_{t-1}), 0\right). \tag{4}$$

To facilitate the analysis, we assume henceforth that for all i_{t-1}, i_t, i'_{t-1}, and $i'_t \in [0,1]$

$$U_i\left(i_{t-1}, i_t, 0\right) > U_i\left(i'_{t-1}, i'_t, -A\right) \text{ for all } i \in [0,1]. \tag{5}$$

On the one hand, (3) guarantees that candidates choose the policy that maximizes their instantaneous utility and hence that of the party they represent. On the other hand, according to (4), citizens always vote—there is no abstention in our model—and they do so for the candidate/party from whom they expect the higher instantaneous utility from policy-making, provided that both candidates are of normal capacity. Otherwise, (5) implies that if one candidate—the challenger—has normal capacity and the other candidate—the incumbent—has already suffered a negative capacity shock, the former is always elected because s/he yields higher instantaneous utility.[18]

[18] It suffices to assume that Equation (5) holds for a subset of citizens of measure of at least 1/2. As will be made clear in our analysis, costs of change generate an incumbency advantage. This advantage can only be counteracted by an exogenous event such as a capacity shock. Assuming that individual shocks can have less extreme values which may accumulate over time and do not offset the incumbency advantage completely would be equivalent. There would exist a threshold such that the following applies: If the capacity of the incumbent is above this threshold, s/he is re-elected; if the capacity of the incumbent is below the threshold, s/he is not re-elected. In either case, the set of equilibrium policy choices would not be affected.

4 Analysis of the Game

In this section we prove that an MSMPE of the dynamic electoral competition game $\mathcal{G}^{i_0}$ exists and is unique. Since we require sequential rationality in each period, we start with the analysis of the policies chosen by the incumbents and then find the election outcomes. We show that, in the steady state, policies alternate between two points that are (symmetrically) located to the left and right of the median—but are closer to the median than the parties' bliss points—and that policy changes occur *if and only if* the incumbent suffers a capacity shock, in which case the challenger is elected.

4.1 Equilibrium policy choices

First we solve the problem for the incumbent who chooses a policy that maximizes his/her utility.

Proposition 1

Let $k \in K$ be the office-holder in period t, with $t \in \{1, 2, \ldots\}$ and $K \in \{R, L\}$. In any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$, k's policy choice in period t is given by

$$\sigma_K^*(i_{t-1}) = \min\left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}, \mu_K + \frac{c}{2}\right\}, \tag{6}$$

where $i_{t-1} \in [0, 1]$ is the policy implemented in period $t-1$.

Proof: See Appendix A.

□

Note that $\sigma_L^*(i_{t-1}) \leq \sigma_R^*(i_{t-1})$ for all $i_{t-1} \in [0, 1]$, i.e., a left-wing office-holder always chooses a more leftist policy than a right-wing one. The policy choice given in (6) is illustrated by Figure 2.

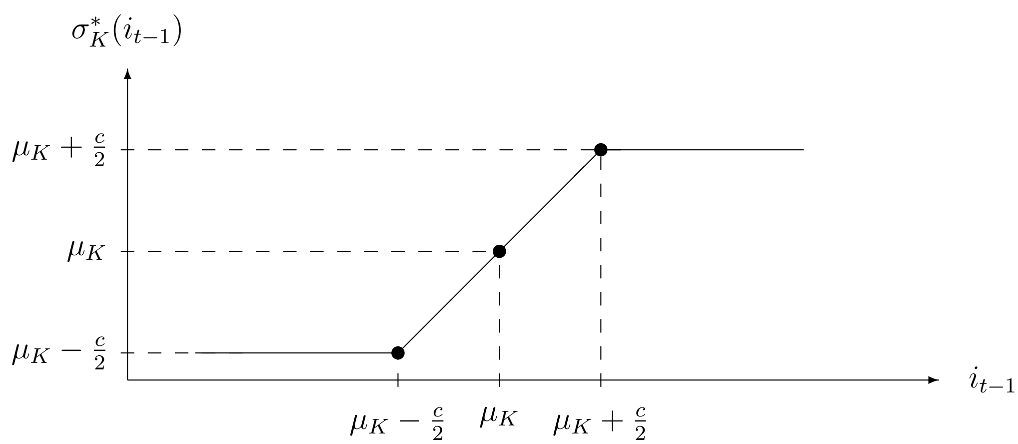


Figure 2: Best response of office-holder $k \in K$ to policy i_{t-1}.

We make two straightforward remarks: First, $\mu_L + \frac{c}{2} < \mu_R - \frac{c}{2}$ if and only if

$$c < 2\Pi. \tag{7}$$

If inequality (7) holds, then costs of change are sufficiently low so that, due to the existence of party polarization, office-holders of parties L and R implement different policies when the status quo is a moderate policy, i.e., when $i_{t-1} \in (\mu_L + c/2, \mu_R - c/2)$. It is straightforward to see that if inequality (7) holds, the median voter's position belongs to the latter set. Second, $0 < \mu_L - \frac{c}{2}$ and $\mu_R + \frac{c}{2} < 1$ holds if and only if

$$c < 1 - 2\Pi. \tag{8}$$

If inequality (8) holds, then costs of change are sufficiently low such that very extreme status-quo policies—i.e. policies that lie outside the interval $(\mu_L - c/2, \mu_R + c/2)$—are changed by office-holders of both parties.

Then it is convenient to introduce for each party $K \in \{L, R\}$, a two-dimensional measure $\mathcal{Y}_K = (\mathcal{Y}_K^-, \mathcal{Y}_K^+)$ of the *range of (endogenous) policy persistence*, which is defined by

$$\mathcal{Y}_K^- = \min\left\{\frac{c}{2}, \mu_K\right\} \quad \text{and} \quad \mathcal{Y}_K^+ = \min\left\{\frac{c}{2}, 1 - \mu_K\right\}.$$

Note that $\mathcal{Y}_R^- = \mathcal{Y}_L^+ = \min\left\{\frac{c}{2}, \frac{1}{2} + \Pi\right\}$ and $\mathcal{Y}_R^+ = \mathcal{Y}_L^- = \min\left\{\frac{c}{2}, \frac{1}{2} - \Pi\right\}$.[19] According to Proposition 1, the best response of office-holder $k \in K$ has the following properties: If the status-quo policy $i_{t-1} \in [0, 1]$ is sufficiently close to, but to the left of, his/her bliss point, i.e. $\mu_K - \mathcal{Y}_K^- < i_{t-1} < \mu_K$, then k's best response is to maintain the status-quo policy. By contrast, if the status-quo policy i_{t-1} is far away from its bliss point, i.e. $i_{t-1} \leq \mu_K - \mathcal{Y}_K^-$, then k's best response is to choose a more moderate policy than the status quo, which is exactly at a distance $\mathcal{Y}_K^-$ from the bliss point and is situated between μ_K and i_{t-1}. Analogous comments hold for $\mathcal{Y}_K^+$ and status-quo policies that are to the right of a party's bliss point. Hence, the bigger $\mathcal{Y}_K^-$ and $\mathcal{Y}_K^+$ are, the wider is the range of status-quo policies that persist when a candidate from party K is elected. Clearly, the interesting policy dynamics occur for low values of policy persistence. Specifically, if Conditions (7) and (8) hold, $\mathcal{Y}_L^- = \mathcal{Y}_L^+ = \mathcal{Y}_R^- = \mathcal{Y}_R^+$, so the persistence range in equilibrium is symmetric around the party's bliss point.

Finally, it is worth noting that office-holders' policy choice does not respond locally to changes in the status-quo policy. This follows from the assumption that costs associated with policy changes are linear in the extent of the change. As discussed in Section 5.1, this assumption is not knife-edged, and the validity of our results extends to convex specifications of such costs. With linear costs of change, a simpler analytical solution of the political game is possible, and costs of change also enable

[19] All candidates of one party have the same persistence, so we can refer to persistence as a feature at party level.

us to define the concept of a policy persistence range, which is particularly useful when parties are forward-looking—see Section 5.2.2.

4.2 Equilibrium voting decisions

We now characterize the election outcomes. Here, it suffices to focus on the case in which both candidates have the same capacity, since due to (5), a low-capacity incumbent is never re-elected in equilibrium.[20] Hence, let the status-quo policy be $i_{t-1} \in [0,1]$, the incumbent $K \in \{\emptyset, L, R\}$ have normal capacity, and the citizens vote according to the strategy profile $\sigma_v = (\sigma_i)_{i\in[0,1]}$. We denote the outcome of the t-th election in this case by

$$E(\sigma_v, K, i_{t-1}) \in \{L, R\}. \tag{9}$$

When the policy outcome is stochastic, we write $E(\sigma_v, K, i_{t-1}) = pL(1-p)R$ to denote the following outcome: party L's candidate is elected with probability p and party R's candidate is elected with probability $1-p$, with $p \in [0,1]$. Recall that the probability that a normal-capacity incumbent has low capacity at the end of his/her current term is λ.

We obtain the following result:

Proposition 2

Let $J \in \{L, R\}$ be such that $|\mu_J - i_{t-1}| \leq |\mu_{-J} - i_{t-1}|$. For any period $t \in \{1, 2, \ldots\}$, any status-quo policy $i_{t-1} \in [0,1]$, and any incumbent $K \in \{\emptyset, L, R\}$ with normal capacity, in any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$ the mapping $E(\sigma_v^*, K, i_{t-1})$ satisfies the following properties:

(i) If $t = 1$, then $E(\sigma_v^*, \emptyset, i_{t-1}) = \begin{cases} J & \textit{if } 0 < \left|i_0 - \frac{1}{2}\right| \leq \frac{1}{2}, \\ \frac{1}{2}L\frac{1}{2}R & \textit{if } \left|i_0 - \frac{1}{2}\right| = 0. \end{cases}$

(ii) If $t > 1$, then $E(\sigma_v^*, K, i_{t-1})$ is given by

$$\begin{cases} -K & \textit{if } \left|i_{t-1} - \frac{1}{2}\right| < \Pi + \frac{c}{2} \textit{ and } |\mu_{-K} - i_{t-1}| < |\mu_K - i_{t-1}|, \\ K & \textit{otherwise.} \end{cases}$$

Proof: See Appendix A.

□

[20] We build on the implicit assumption that the loss of support for a candidate affects the entire party, meaning that the latter cannot counteract the negative shock by simply replacing the bad candidate by another candidate of the same party in the first election after the shock. There are numerous instances in which the loss of a candidate's credibility has dragged the entire party down into certain electoral defeat. On some occasions, the candidate is very powerful within the party and is able to impose his/her own agenda, on other occasions the electorate likes to punish the entire party as also being accountable for bad policy-making. Formally, however, our model would not change if we were to assume that parties can replace bad candidates with a certain probability. In such a case, it would suffice to lower the exogenous probability with which a candidate is affected by a negative shock.

Together with the fact that low-capacity incumbents are always ousted, this proposition describes the optimal choices of the electorate. Part *(i)* refers to the first election, in which there is no incumbent. Since the candidates from the two parties both have normal capacity and their bliss points are symmetrically located with respect to the median voter, the electorate selects the politician who will effect the smaller policy shift. If the initial policy is biased towards the bliss point of one party, it is the candidate of that party who is elected. If there is no bias, the winner of the election is decided according to a fair coin toss. Part *(ii)* refers to any subsequent election. In this case, the challenger $-k$ wins the election if either the incumbent k has suffered a negative shock or if the challenger will carry out a smaller policy change than the incumbent while still choosing a moderate policy. This latter feature occurs when the status quo policy is closer to the challenger's bliss point than to the incumbent's bliss point, i.e., when $|\mu_{-K} - i_{t-1}| < |\mu_K - i_{t-1}|$, and when the status-quo policy is itself moderate enough, i.e., when $\left|i_{t-1} - \frac{1}{2}\right| < \Pi + \frac{c}{2}$. To understand the latter expression, assume that party R is in power and that $i_{t-1} < 1/2$. Then $\left|i_{t-1} - \frac{1}{2}\right| < \Pi + \frac{c}{2}$ is equivalent to

$$\mu_L - \frac{c}{2} < i_{t-1}.$$

Figure 3 depicts the regions of status-quo policies that guarantee re-election of the incumbent and those that guarantee the election of the challenger, assuming that $c < 2\Pi$.

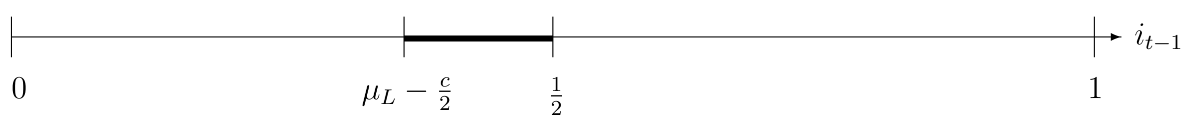


Figure 3: The winning candidate in some period $t > 1$ for different status-quo policies $i_{t-1} \in [0, 1]$ when $c < 2\Pi$. The incumbent of party R is re-elected if i_{t-1} belongs to the area with a thin line, while the challenger of party L is elected if i_{t-1} belongs to the area with a thick line.

That is, the incumbent is re-elected even when the status-quo policy is very extreme and located on the other side of the political spectrum from his/her perspective. In such a case, the costs associated with the larger policy shift effected by the incumbent exactly offset the benefits from his/her more moderate policy, and hence s/he is elected according to the tie-breaking rule that ensures re-election in the case of indifference. In equilibrium, the policy choice by party R is always to the right of $1/2$, while the policy choice by party L is to the left of $1/2$ but to the right of $\mu_L - c/2$, provided that $c < 2\Pi$. Hence, the leftmost thin segment of Figure 3 is never attained in equilibrium. This figure is nonetheless informative if we consider random shocks to the citizens' preferences, in which case

i_{t-1} could be any policy in $[0,1]$. Since the thin area is larger than the thick one, the incumbent can reasonably expect to be re-elected even if citizen preferences are unstable.

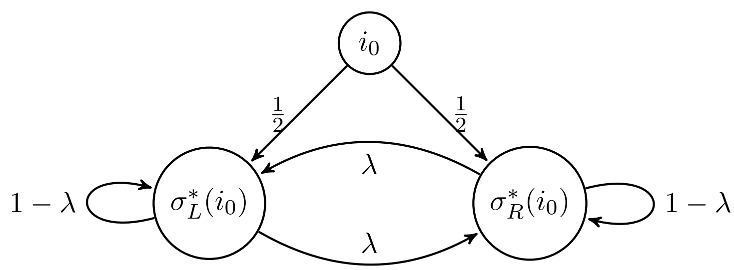
4.3 Unique MSMPE

From the combination of Proposition 1 and Proposition 2 it follows that there exists a unique MSMPE of $\mathcal{G}^{i_0}$. The characterization of this unique equilibrium is summarized in our first main result.

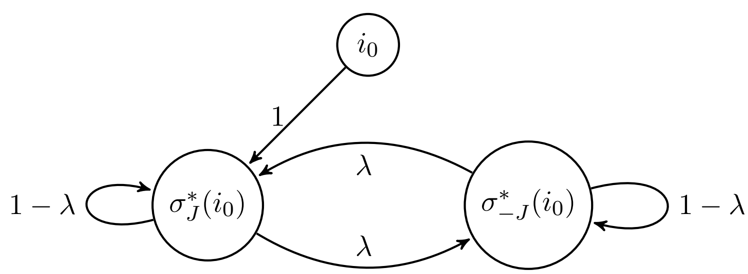
Theorem 1

Let $i_0 \in [0,1]$ be the status-quo policy in $t = 1$. There exists a unique MSMPE of $\mathcal{G}^{i_0}$, referred to as $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$, which is characterized by one of the following Markov transition diagrams, where the equilibrium policy choices are determined according to the best response given in Proposition 1 and $J \in \{L, R\}$ is a party that satisfies $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$:

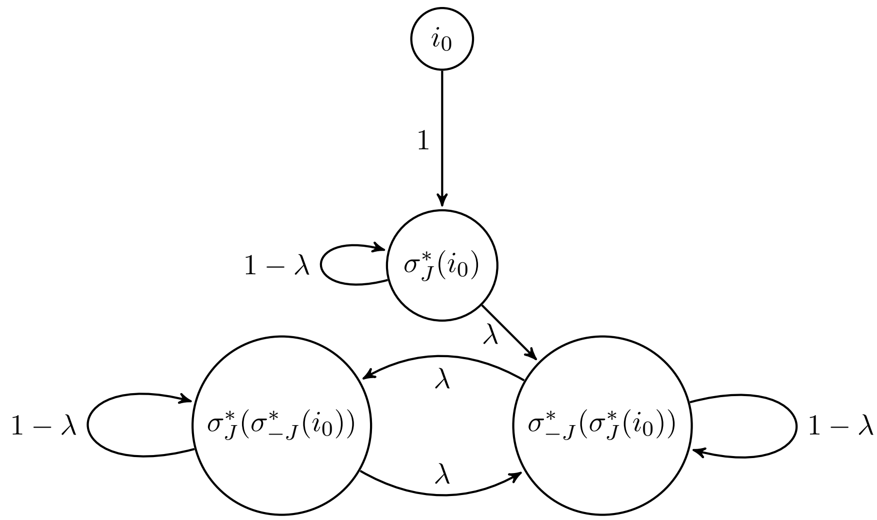
(i) If $\left|i_0 - \frac{1}{2}\right| = 0$, then σ^* is characterized by



(ii) If $0 < \left|i_0 - \frac{1}{2}\right| \leq \Pi - \frac{c}{2}$, then σ^* is characterized by



(iii) If $\Pi - \frac{c}{2} < \left|i_0 - \frac{1}{2}\right| \leq \frac{1}{2}$, then σ^* is characterized by



In all three cases described in Theorem 1, the infinite sequence of policies converges to a set of two (possibly identical) policies, each implemented by a candidate from one of the two parties. The outcome switches randomly from one policy to the other. More specifically, the shift from one long-term policy to the other only occurs when the respective office-holder is ousted from office, which happens in each period with probability λ. Whenever an office-holder is re-elected, s/he maintains the status-quo policy. That is to say, persistence in the incumbent's policy choices arises endogenously in our model when costs of change are linear. We summarize these insights in the following corollary:

Corollary 1

For any initial policy $i_0 \in \mathcal{I}$, the political outcomes converge to a stochastic fixed point with two policies and transition probability λ in each period.[21]

If $c \geq 2\Pi$, the policies chosen by the two parties are identical in the long run. That is, only the office-holder's party affiliation is stochastically alternating in this case, but not the actual policy choices. If $c < 2\Pi$, by contrast, the infinite sequence of policies consists of a stochastic alternation between $\mu_L + \frac{c}{2} = \frac{1}{2} - \Pi + \frac{c}{2}$ (implemented by a left-wing office-holder) and $\mu_R - \frac{c}{2} = \frac{1}{2} + \Pi - \frac{c}{2}$ (implemented by a right-wing office-holder). Importantly, these long-term policies are more moderate than the parties' bliss points. Theorem 1 thus implies that, in the long run, costs of change may have a moderating effect on policies. Theorem 1 also demonstrates that although the long-term sequence of policies is independent of the initial level of policy polarization, the transition path that describes how to get there crucially depends on i_0 (see the diagrams). More precisely, if $\left|i_0 - \frac{1}{2}\right|$ is large, the infinite sequence of alternating policies is only reached after an initial phase with more extreme policies. For low $\left|i_0 - \frac{1}{2}\right|$ such a sequence is immediately reached in $t = 1$. The long-term dynamics

[21] For the concept of stochastic fixed points, see e.g. Bharucha-Reid et al. (1976).

when costs of changes are small, i.e. when Condition (7) holds, is depicted in Figure 4.

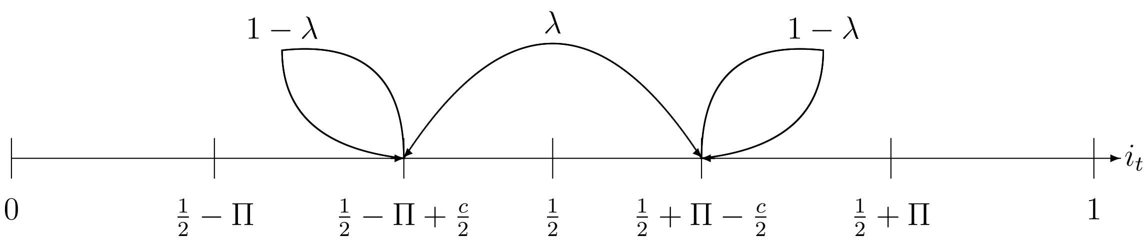


Figure 4: Long-term dynamics when $c < 2\Pi$—linear costs of change.

From Theorem 1 it also follows that, in each period, the incumbent is ousted with probability λ, which corresponds to the probability of normal-capacity office-holders being affected by a negative capacity shock in a given period. The reasoning, based on Proposition 2, is as follows: The best-response policy choices are always in the subset of $[0, 1]$ for which normal-capacity incumbents are re-elected. We summarize these insights as follows:

Corollary 2

In any period $t > 1$, the incumbent is always elected if s/he has normal capacity.

Corollary 2 highlights the fact that having normal capacity is not only *necessary* for the incumbent to be re-elected, it is also *sufficient*. This is *not* a general property of costs of change, even in our baseline set-up with myopic agents. For instance, if the disutility from policies were linear and costs of change were quadratic, an incumbent with normal capacity might not be re-elected.[22] This is because under this specification of utility and costs of change, neither the incumbent nor the challenger could credibly commit not to changing the policy further away from his/her bliss point. This contrasts with our baseline model, which specifies linear costs of change and quadratic utility. In Section 5.1, we show that considering that the disutility from policies is more convex than the disutility from costs of change suffices for incumbents to have an electoral advantage.

The fact that normal-capacity incumbents are always re-elected in equilibrium may be interpreted as a manifestation of the commonly observed incumbency advantage. Our paper identifies (linear or moderately convex) costs of change as one (of many) potential source for this type of electoral advantage. In our model, an office-holder with normal capacity can secure re-election by choosing

[22] Given the status-quo policy i_{t-1}, one can check that the best response of an office-holder with bliss point μ is to choose $\min\{i_{t-1}+1/2c, \max\{i_{t-1}-1/2c, \mu\}\}$. If $i_0 = 1/2$ and $c \geq 1/4\Pi$ and then party R is (without loss of generality) in power in period $t = 1$, it implements $1/2 + 1/2c \in [0, 1]$. Let us consider now period $t = 2$, and assume that both candidates have normal capacity. If the incumbent from party R is elected, s/he implements $1/2 + 1/c \in [0, 1]$ and carries out a shift equal to $1/2c$. If the challenger from party L is elected, s/he implements $1/2 \in [0, 1]$ and carries out a shift equal to $1/2c$. Clearly, the median voter prefers to elect the challenger. That is, there is no incumbency advantage.

a policy that makes it too costly for the median voter to switch to the equally capable and from an ideological viewpoint equally appealing challenger. As discussed below in Section 4.4, this property of elections in the presence of costs of change can have positive consequences for welfare. Figure 5 illustrates how costs of change generate this incumbency advantage when $c < 2\Pi$.

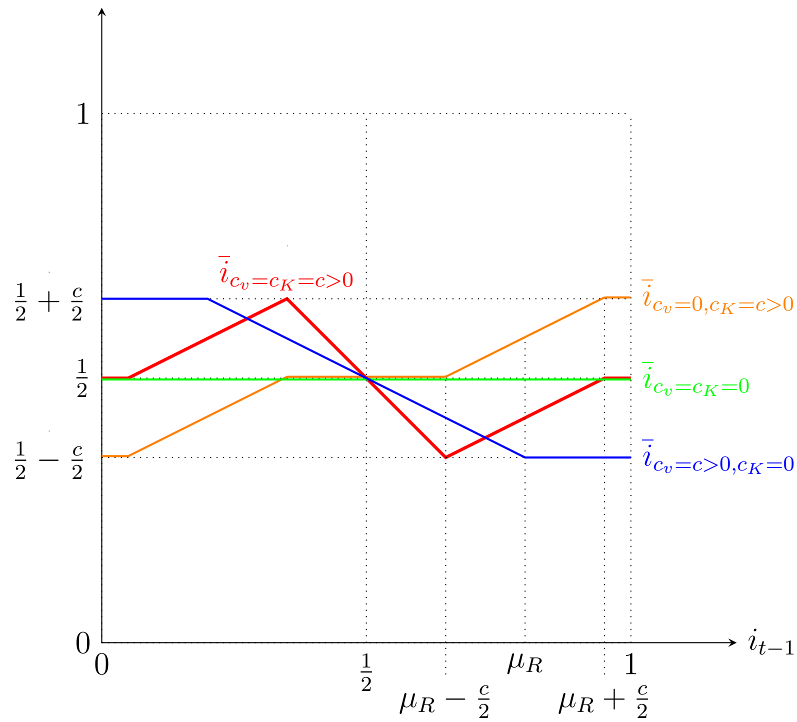


Figure 5: The indifference curves when $\Pi = c = 0.3$, assuming party R is in power and both candidates have normal capacity.

In Figure 5, the green line $\bar{i}_{c_v=c_K=0}$ plots for each status-quo policy $i_{t-1} \in \mathcal{I}$ the bliss point of a citizen who is indifferent between choosing either candidate in the absence of costs of change for voters ($c_v = 0$) and for parties ($c_K = 0$). The orange line plots the same indifference condition when there are costs of change for parties ($c_K = c > 0$) but not for voters ($c_v = 0$). The blue line plots the indifference condition when there are costs of change for voters ($c_v = c > 0$) but not for parties ($c_K = 0$). The red line also plots such an indifference condition, but considers costs of change both for voters and parties ($c_v = c_K = c > 0$) and hence describes the case analyzed in Section 4. The analysis of the other three cases depicted in Figure 5 follows the same logic.[23] One can easily see that the median voter always lies above the red line for status-quo policies i_{t-1} that are to the right of 1/2, and lie strictly above if in addition $i_{t-1} < \mu_R + c/2$. These policies guarantee that the candidate from party R wins the election. We stress that policies chosen by office-holders from party K belong to $[\mu_K - c/2, \mu_K + c/2]$ along the equilibrium path. This implies, in particular, that the

[23]Proofs can be provided upon request.

incumbency advantage is robust in general against small unexpected shocks to citizenry preferences that affect the median voter's bliss point—see below. Furthermore, although the orange line seems to indicate that for costs of change to create an incumbency advantage in all cases it must be the case that voters also incur them, this condition is only necessary off the equilibrium path.

For our analysis we have proceeded on the assumption that the median voter's preferred policy is $1/2$, which is equidistant from μ_L and μ_R. This may not always be the case. For one thing, parties may have difficulty adapting to a change in the electorate's preferences. Or there may exist a systemic bias in favor of or against the incumbent, e.g. due to differential abstention or redistricting. Although Corollary 2 may not hold in these circumstances for both parties (and their candidates), the equilibrium dynamics described in Theorem 1 remain valid, but with possibly different (and asymmetric) transition probabilities. If, in particular, a sudden, symmetric change in the parties' platforms brought about an increase (decrease) of party polarization, the above results could be immediately applied, notably if actual policies had already converged to the long-term stochastic alternation between two states.[24] Under this assumption, the new situation would be equivalent to Case *(ii)* or Case *(iii)* of Theorem 1, the only difference being that now the convergence would be towards the stochastic alternation between the two states calculated with the new policy platforms. Similarly, if costs of change are not large and an exogenous shock to the status-quo policy occurs while in the *steady state*—with preferences remaining the same—, this very same long-term dynamic will be restored after a few periods. This last observation is summarized in the next corollary.

Corollary 3

If Condition (7) holds, the stochastic fixed point described in Figure 4 is robust.

4.4 Model predictions

Over and above the theoretical insights, our results enable us to formulate a series of predictions regarding actual policy-making, all of which could be empirically tested. Specifically, our theory—both the baseline model of Section 3 and that of Section 5—provides support for hypothesizing that

1. All else being equal, incumbents have an advantage over challengers in elections, provided that costs of change are linear (or moderately convex, see Section 5.1). In the long run, such an advantage is maximal for moderate marginal costs of change.

2. In the long run and if the marginal cost of change is not too large, there is an alternation between two policies centered around the median voter position, the distance between which

[24] A similar reasoning can be applied if policies have not yet reached the long-term stochastic alternation.

is proportional to (but lower than) the difference in the parties' bliss points. Moreover, policy shifts occur only when a new party comes into office and the incumbents' policies are persistent during their tenure.[25]

3. The initial policy determines the long-term dynamics only if the marginal cost of change is sufficiently large. In this case, we should observe no policy changes, even after political turnover.

4. Moderate marginal costs of change induce more moderate policies in the long run, yielding in turn long-term welfare gains.

A full confirmation or rebuttal of the above hypotheses from an empirical perspective for particular political systems lies beyond the scope of the present paper. We limit ourselves to a brief discussion of each of the hypotheses in the light of other contributions in the literature and some empirical evidence.

First, in the last few decades, over 90% of the incumbents in the US House of Representatives have been successful when standing for re-election (Levitt and Wolfram, 1997). Erikson et al. (1993) find that US governors have comparable advantages when seeking re-election. Since World War II, the re-election rates of governors have been rising and reached nearly 90% in the period 2010–2013.[26] Many scholars argue that these high re-election rates are partially due to the so-called incumbency advantage. Being in office offers politicians a range of policy and non-policy tools that are not available to their challengers and can be used to create an electorate advantage. This is the case here. In our model, the incumbency advantage is a consequence of the possibility for office-holders to leverage on the partial commitment tool offered by costs of change when they choose a policy. In the absence of full policy commitment, linear (or moderately convex, see Section 5.1) costs of change enable incumbents to choose a policy that guarantees their re-election unless they experience a negative (exogenous) shock to their capacity. Of course, incumbency advantage does not hold universally (see e.g. Klašnja, 2015, 2016; Uppal, 2009), even in the US (Chatterjee and Eyigungor, 2019). In fact, we have argued that office-holders may suffer from an electoral disadvantage under specifications of costs of change that differ from ours—see Footnote 22. The link between the exact functional form of costs of change and incumbency (dis)advantage calls for in-depth empirical analysis. Back to our model, Figure 5 illustrates the incumbency advantage for a fixed marginal

[25] When costs of change are convex and not linear, the alternation occurs between two regions of policies and policies are only approximately persistent, though the degree to which policy changes decreases with the number of terms an incumbent has previously held power—see Section 5.1.

[26] The growing re-election rates can be found under `http://governors.rutgers.edu/on-governors/us-governors/when-governors-seek-re-election` (retrieved 18 May 2015).

cost $c \leq 2\Pi$. The incumbent can guarantee his/her re-election even if there is a majority of citizens whose bliss point is closer to the challenger's than to the incumbent's.

To see how the incumbency advantage changes with the marginal cost of change, let $c \leq 2\Pi$ and assume without loss of generality that the office-holder belongs to party R and has normal capacity. If s/he is re-elected, s/he maintains the status quo. If the challenger is elected, s/he instead implements a policy that is symmetrically located on the other side of the spectrum. This yields a utility difference to the median voter from voting for the incumbent that is equal to

$$c \cdot (2\Pi - c).$$

This (quadratic) term is maximized at $c = \Pi$. In particular, it increases with c if $0 < c < \Pi$, while it decreases if $\Pi < c < 2\Pi$. This implies that according to our model, we should observe higher re-election rates when policies that are salient in elections are associated with moderate marginal costs of change than when they are associated with either very large or very low marginal costs of change, all else being equal.

Second, the number of policy dimensions a government can influence is large. Policy alternation can thus be defined for a bundle of policies, or rather on a policy-by-policy basis. Focusing on the highest marginal tax rate in the US as a remarkable example of a one-dimensional policy, both the Democratic and Republican administrations have been reversing their opponent's decisions ever since Reagan's tax cuts.[27] Another example of policy reversals are the series of Education Laws approved in Spain whenever power shifted from the socialists to the conservatives and vice versa.[28,29]

Two features of policy alternation are worth discussing in more detail, namely the extent and the frequency of such alternation. With regard to extent, Wiesehomeier and Benoît (2009) have shown for the case of many Latin-American countries that polarization in parties' ideologies tends to be more pronounced than the differences in actual policies chosen by different parties.[30] In our model with moderate or low costs of change, policy alternation obtains because both parties are committed to their polarized positions, yet the degree to which policies alternate is determined by the degree to which policy shifts are costly. With regard to frequency, Budge et al. (2010) argue that parties become more partisan immediately after winning elections, less partisan right after losing elections, and that they do not tend to substantially change policies while they are in power (i.e., policies

[27]Source: Tax Foundation, see `http://taxfoundation.org/` (retrieved 18 November 2018).

[28]See `https://elpais.com/sociedad/2013/11/26/actualidad/1385489735_160991.html` (retrieved 9 September 2019).

[29]From a theoretical perspective, policy alternation is featured in some papers that have been discussed in Section 2 such as Forand (2014) and Nunnari and Zápal (2017).

[30]The property of policy polarization being lower than party polarization is also discussed in Alesina and Rosenthal (2000).

are persistent unless government switches from one party to the other). Budge et al. (2010) also show that, to some extent, this pattern is consistent with what can be observed in US politics. As we have seen, policy persistence for incumbents is a property documented in abundant theoretical and empirical research (see e.g. Miller and Schofield, 2003; Tavits, 2007). To mention only a few examples, voter behavior and internal politics often generate costs to policy flexibility. According to our analysis, costs of change are another factor that may explain persistence in policy choices, especially when such costs are linear or costs of change are convex and incumbents have served many terms. Our results can help to better understand how the frequency and extent of policy alternation is determined in real-world political systems.

Third, it seems intuitive that no reforms will occur if the costs of carrying them out are disproportionate even if they only involve small changes, in which cases societies will be stuck with the status quo. For instance, this suggests that an incumbent who wants to maintain the status quo may try to make salient as many policy dimensions as possible among those where s/he has made the last policy decision. This would create higher costs of change associated with electing the challenger. When the current policy is very unfavorable for the interests of the median voter and costs of change are very large, the (permanent) outcome is a "poverty trap". Our results suggest a threshold may exist such that if the frictions generated by the political system guarantee that policy reforms entail per-reform costs below this threshold, the corresponding society will always be able to escape any poverty trap. Above the threshold, by contrast, the political system is ill-defined and there is no escape from the status quo. If the latter policy is very harmful for society, a country could stagnate (Acemoglu, 2009).

Fourth, when the costs associated with policy changes are moderate, our model shows that there are positive long-term consequences for welfare that occur thanks to policy moderation in the steady state. Indeed, it follows from Theorem 1 that when $0 \leq c \leq 2\Pi$ and λ is strictly positive, the expected stage utility of the median voter in the long term is independent of the initial level of policy polarization and equal to

$$-\left(\Pi - \frac{c}{2}\right)^2 - \lambda \cdot (2\Pi - c)\,. \tag{10}$$

It is a matter of simple algebra to check that Equation (10) increases with c and is maximal at

$$c^* = 2\Pi. \tag{11}$$

That is, a certain positive level of costs of change has positive consequences for long-term welfare. When $c = c^*$, in particular, both parties choose the median voter position, which in addition eliminates all costs associated with policy changes.

Note that, in our model, there is ex-ante inefficiency if parties and voters are forward-looking and costs of change are not very large. The reason is that all participants—voters and parties alike—would be better off if they could simply fix policy at the median and never change it. This is clear for the median voter. As for parties, they know they will be losing office eventually, and hence they face a random walk between two policies over time. Given their concave utility over policies, they would prefer a sure outcome in the middle. When neither parties nor voters have commitment power—as is the case in our model—, this inefficiency provides some rationale for why one might see laws that make it hard to change policies or make them less discretionary. In actual democracies, this typically takes the form of a system of checks and balances. In our model, this can be captured by parameter c. As we have seen, Equation (11) calls in particular for a moderate system of checks and balances, while also allowing policy changes in the case of severe shocks to the citizens' preferences.

5 A More General Model

In the following we analyze the validity of Theorem 1, and hence that of Corollaries 1–3, when some features of the baseline model outlined in Section 3 are generalized. First, we consider the case where costs of change are not linear but moderately convex. Second, we assume that voters—but not parties—are forward-looking (or farsighted). Third, we consider that parties—but not voters—are forward-looking. For this last generalization of the baseline model, we restrict the parties' strategy space in a plausible way. Overall, we show that the main insights that derive from Theorem 1 are neither guided by the assumption of the linearity of costs of change nor by the assumption of myopia in both parties and voters, although the analysis is significantly more involved when neither simplifying assumption is imposed. The baseline set-up considered in the previous sections—namely, linear costs of change and myopic agents—is thus sufficient to capture the main channels by which costs associated with policy changes affect policy choices and re-election. Office-holders display an incumbency advantage (Corollary 2), and when costs of changes are not too large, long-term dynamics are robust (Corollary 3) and lead to a stochastic alternation between two (regions of) policies that are located on either side of the median voter's bliss point (Corollary 1). These three results remain valid in all generalizations. Disentangling the different modeling assumptions, as in the present section, provides us in addition with a better understanding of the marginal effect of each single assumption on equilibrium choices.

The analysis in the present section also reveals that the main thrust of dynamics described in Theorem 1 is preserved in the simultaneous combination of all generalizations of the model in Section 3. These dynamics are affected in the same qualitative way in all generalizations, the only exception concerning the first-period election when either initial policy polarization is very large ($\Pi + c/2 \le \left|i_0 - \frac{1}{2}\right|$) or initial polarization is large ($\Pi - c/2 \le \left|i_0 - \frac{1}{2}\right| < \Pi + c/2$) and the median voter's discount factor is sufficiently large. Our baseline model—linear costs and myopic agents—predicts that the candidate is elected whose bliss point is closest to the status-quo policy, and thus the policy remains extreme for a number of periods. The same considerations apply for strictly convex costs of change and forward-looking parties. When voters are forward-looking, by contrast, the candidate whose bliss point is farther away from the status-quo policy is elected whenever initial polarization is very large or if initial polarization is large and citizens care enough about future utility. In this case, convergence to the steady state where more moderate policies are in place already occurs in the first period, from which point the dynamics are qualitatively equal across all model generalizations. This is because the gains from policy moderation that will accrue in the future outweigh the costs of change needed to achieve policies of the kind needed in the present. At any rate, incumbents are re-elected if and only if they have normal capacity. The proofs of all the results of this section can be found in Appendix B.

5.1 Convex costs of change

In this first model generalization we consider a variation of game $\mathcal{G}^{i_0}$, denoted by $\mathcal{G}^{i_0}_{\eta}$, where

$$U^c(i_{t-1}, i_t) = -c \cdot |i_{t-1} - i_t|^{\eta}, \tag{12}$$

with $1 \le \eta \le 2$. In words, costs associated with policy changes are an increasing convex function of the magnitude of this change. Moreover, the degree of convexity captured by exponent η is lower than, or equal to, the one associated with the disutility derived from policies.[31] With convex costs, the results are quite similar to the linear case except for one difference: convex costs mean that parties move policies closer to a limit point—their own bliss point—in a series of moves that become more gradual over time. Rather than moving to the steady state points as in the linear case, incumbents now approach these limit points with large increments at first and then smaller ones over time. As with linear costs, incumbents are only replaced if and only if they have suffered a capacity shock.

We start by characterizing the office-holders' optimal policy choice in each period.

Proposition 3

Let $k \in K$ be the office-holder elected in period t, with $t \in \{1, 2, \ldots\}$ and $K \in \{R, L\}$. In any

[31] We assume for the sake of analysis that politicians and citizens obtain quadratic utilities from policies. Our analysis and results extend to the case in which $U_i^{\mathcal{I}}(i_t) = -(i_t - i)^{\eta'}$ and $U^c(i_{t-1}, i_t) = -c \cdot |i_{t-1} - i_t|^{\eta}$, with $1 \le \eta \le \eta'$. However, the analysis in this general case turns out to be much more cumbersome, without yielding any additional insight, so it is not included here.

MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ *of* $\mathcal{G}_\eta^{i_0}$*, k's policy choice in period* t *is given by* $\sigma_K^*(i_{t-1}) = x$*, with*

$$\mu_K - x = \frac{c\eta}{2}(x - i_{t-1})^{\eta-1} \ \text{ and } x \in (i_{t-1}, \mu_K) \qquad \text{if } i_{t-1} < \mu_K, \tag{13}$$

$$x = \mu_K \qquad \text{if } i_{t-1} = \mu_K, \tag{14}$$

$$x - \mu_K = \frac{c\eta}{2}(i_{t-1} - x)^{\eta-1} \ \text{ and } x \in (\mu_K, i_{t-1}) \qquad \text{if } i_{t-1} > \mu_K, \tag{15}$$

where $i_{t-1} \in [0, 1]$ *is the policy implemented in period* $t - 1$*.*

Proof: See Appendix B.

□

As a particular, very illustrative case, we note that if $\eta = 2$, these implicit equations can be solved explicitly to obtain

$$\sigma_k^*(i_{t-1}) = \frac{c}{c+1} \cdot \mu_K + \frac{1}{c+1} \cdot i_{t-1}.$$

The following corollary shows some further properties of the incumbents' policy choices in general.

Corollary 4

Let $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ *be any MSMPE of* $\mathcal{G}_\eta^{i_0}$ *and* $K \in \{L, R\}$*. Then,*

(i) $0 < \frac{d}{di_{t-1}}\sigma_K^*(i_{t-1}) < 1$,

(ii) $|\sigma_K^*(\sigma_K^*(i_{t-1})) - \sigma_K^*(i_{t-1})| < |\sigma_K^*(i_{t-1}) - i_{t-1}|$,

(iii) $\frac{d^2}{di_{t-1}^2}\sigma_K^*(i_{t-1}) > 0$ *if* $i_{t-1} < \mu_K$ and $\frac{d^2}{di_{t-1}^2}\sigma_K^*(i_{t-1}) < 0$ *if* $i_{t-1} > \mu_K$,

(iv) $\sigma_L^*(i_{t-1}) < \sigma_R^*(i_{t-1})$,

where $i_{t-1} \in [0, 1]$ *is the policy implemented in period* $t - 1$*.*

Proof: See Appendix B.

□

Part *(i)* of Corollary 4 implies that an incumbent makes policy choices that move monotonically with the status quo, while the slope of the choice function is lower than one. Hence, persistence in policy choices does not obtain in finite time when costs of change are convex, except when the status quo is the incumbent's bliss point—see (14). Nevertheless, the extent of the policy change decreases for every additional term of the incumbent's tenure and, moreover, it approaches zero change in the case where the status quo is the incumbent's bliss point. This is Part *(ii)* together with (14) and differentiability of $\sigma_K^*(\cdot)$. In other words, office-holders with long tenure choose policies that are approximately persistent. According to Part *(iii)*, the gradual policy movement taken by a given incumbent can be described by a convex function (if the status-quo policy is to the left of the incumbent's bliss point) or by a concave function (if the status-quo policy is to the right of the incumbent's bliss point). As in the linear case, right-wing office-holders choose policies to the right of left-wing office-holders' choices. This is Part *(iv)*. Figure 6 depicts the office-holders' best response for different values of c and thus illustrates the results of the above corollary.

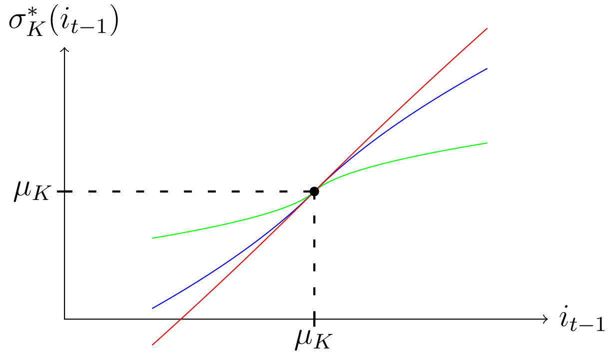


Figure 6: Best response of an office-holder with bliss point μ_K to policy i_{t-1} when $\eta = 3/2$ for $c = 1/3$ (green), $c = 4/3$ (blue), and $c = 4$ (red).

Because politicians cannot commit to policies before the election, citizens simply anticipate that, if elected, each candidate will behave in accordance with Proposition 3. Then they vote for the candidate from whom they expect the higher utility. The next corollary characterizes the electorate's decision.

Corollary 5

Let $k \in K$ be the office-holder elected in period t, with $t \in \{1, 2, \ldots\}$ and $K \in \{R, L\}$. In any MSMPE of $\mathcal{G}_\eta^{i_0}$, k is elected in period t if and only if s/he has normal capacity and

$$|i_{t-1} - \mu_K| \leq |i_{t-1} - \mu_{-K}|,$$

where $i_{t-1} \in [0, 1]$ is the policy that was implemented in period $t - 1$.

Proof: See Appendix B.

□

Corollary 5 establishes that if the status-quo policy is biased with respect to the median voter's ideal policy 1/2, then in equilibrium s/he prefers to elect the candidate whose preferred policy lies on the same side of the political spectrum as the status quo, provided that both candidates have the same capacity. Moreover, it turns out that if

$$c < \frac{2}{\eta} \cdot \Pi^{2-\eta}, \tag{16}$$

all office-holders of party R always choose a policy to the right of 1/2, while all office-holders of party L always choose a policy to the left of 1/2. This implies that as long as costs of change are moderate, Corollary 2 does not depend on the assumption that costs of change are linear. This is captured in the next corollary, which follows immediately from Corollary 5.[32]

Corollary 6

Assume Condition (16), and let $k \in K$ be the office-holder at the beginning of period t, with $t \in \{1, 2, \ldots\}$ and $K \in \{R, L\}$. In any MSMPE of $\mathcal{G}_{\eta}^{i_0}$, k is re-elected in period t if and only if s/he has normal capacity.

Proof: See Appendix B.

□

As with linear costs of change, the latter result implies that (moderate) costs of change generate an incumbency advantage. This property of elections stems from the politicians' lack of power to commit to arbitrary policies. One can draw a graph similar to Figure 5 and verify that with moderate convex costs, the corresponding red curve lies strictly below the median voter's bliss point for all status-quo policies that are to the right of such a bliss point. Corollary 6 also implies that because the challenger is only elected when the incumbent has suffered a capacity shock, the transition probability from one party being in power to the other party being in power is again pinned down by λ. This results in the uniqueness of MSMPE, analogously to the case when costs of change are linear—similar Markov transition diagrams can be drawn (see Theorem 4 in Appendix B). If we let Δ be implicitly defined as

$$\Delta := \frac{c\eta}{2} \left(2\Pi - \Delta\right)^{\eta-1}, \tag{17}$$

then Figure 7 depicts the long-term dynamics when costs are convex and costs of change are not very large—i.e., when Condition (16) holds. We stress that Condition (16) guarantees that $\Delta < \Pi$ and note that $\lim_{\eta \to 1^+} \Delta = \frac{c}{2}$.

Figure 7 illustrates that in steady state, policies alternate between two regions of policies marked in black following a change of party in power. When c is sufficiently small, neither of these regions

[32] When costs of change are linear and c is large, the median voter is indifferent between electing either normal-capacity candidate, since both of them will choose to maintain the status quo. Under strictly convex costs of change, this indifference is broken in favor of the normal-capacity candidate whose ideal policy lies on the same side of the political spectrum as the status quo, regardless of whether the latter candidate is the incumbent or the challenger.

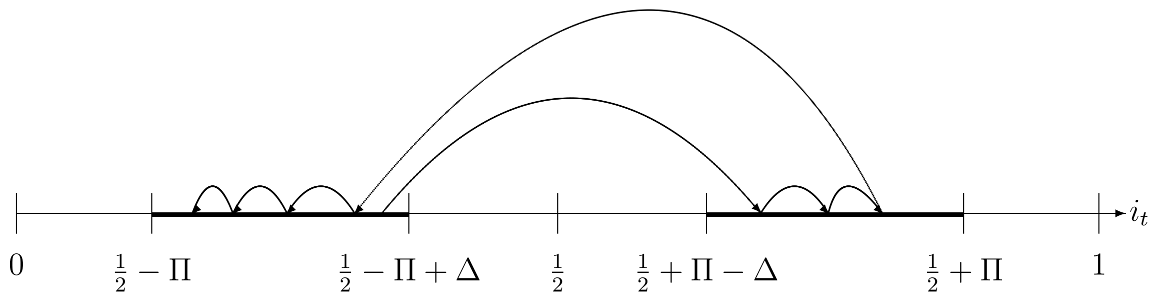


Figure 7: A long-term policy path when Condition (16) holds—convex costs of change.

contains the median voter's ideal policy, so convergence to this position does not obtain when costs of change are moderately convex and not linear, regardless of the exact value of turnover probability λ. This parameter only determines how likely it is to observe particular policies of these regions along the equilibrium path. In the polar case where $\lambda \to 1$, with probability one the alternation is between policies $1/2 - \Pi + \Delta$ and $1/2 + \Pi - \Delta$. This leads to an expected stage utility for the median voter equal to

$$W^1 := -(\Pi - \Delta) \cdot (\Pi - \Delta + 2c) .$$

In the polar case where $\lambda \to 0$, with probability one there is no alternation and the policy is either $1/2 - \Pi = \mu_L$ or $1/2 + \Pi = \mu_R$. This leads to an expected stage utility for the median voter equal to

$$W^0 := -\Pi^2.$$

Accordingly, the probability of capacity shocks can have major consequences for welfare when costs of change are moderately convex. With linear costs, higher instability in the political system (i.e., higher λ) unambiguously reduces welfare by making costs of change more frequent. This also occurs with convex costs. If $\eta > 1$, however, higher λ also reduces the incumbents' power to dictate policy for a series of consecutive periods, which leads to more moderate policies. Focusing on these two polar cases, it can be shown that $W^1 > W^0$ if and only if costs of change are sufficiently convex, provided that party polarization is not very small.[33] That is, with convex costs, political instability *can* be positive for welfare although it would lead to more political turnover, since it would moderate policies.

Finally, we show how Δ changes with some parameters. Note that Δ represents the distance between the most moderate and the most extreme policies a given party can implement along the equilibrium path.

[33]Trivially, W^1 is increasing in Δ. At the same time, Δ is increasing in η if $\Pi > 1/e^2$ (see Proposition 4, part *(ii).*). Finally, when $\eta = 2$, we have $\Delta = c/(c+1) \cdot 2\Pi$ and hence $W^1 > W^0$.

Proposition 4

Let $\Delta(c, \eta, \Pi)$ *as defined in (17), where* $\eta \in (1, 2)$, $\Pi \in \left(0, \frac{1}{2}\right)$ *and* $c \in \left(0, \frac{2}{\eta} \cdot \Pi^{2-\eta}\right)$. *Then,*

(i) $\lim_{c \to 0} \Delta(c, \eta, \Pi) = 0$ *and* $\frac{\partial \Delta(c,\eta,\Pi)}{\partial c} > 0$,

(ii) $\frac{\partial \Delta(c,\eta,\Pi)}{\partial \eta} > 0$ *if* $\Pi > e^{-\frac{1}{2}}$, *and*

(iii) $\frac{\partial \Delta(c,\eta,\Pi)}{\partial \Pi} > 0$.

Proof: See Appendix B.

□

First, Δ increases with the extent of the costs associated with policy changes, namely c, and in particular is maximal for $c \to \frac{2}{\eta} \cdot \Pi^{2-\eta}$. In this case, both parties always choose the median voter's bliss point. This is not surprising, since *ceteris paribus*, higher costs of change simply decrease the utility of moving the policy chosen by the other party. Second, Δ also increases with the degree of convexity of such costs, i.e. η, provided that party polarization is large enough. This is because with more convex costs, office-holders make smaller policy adjustments to spread out the costs associated with such adjustments over time. Finally, Δ also increases with Π. This implies that higher party polarization leads to a higher range of policies that can be observed along the equilibrium path.

5.2 Forward-looking agents

Next, we investigate the validity of the findings of Section 4 when either voters or parties are not myopic. It is convenient to introduce some additional concepts.

First, we use $\mathcal{S} = (s_1, s_2, \ldots)$ to denote an arbitrary realization of the stochastic process containing the capacity shocks to incumbents throughout all periods, i.e., for each $t \in \{1, 2, \ldots\}$, $s_t = 1$ with probability λ and $s_t = 0$ with probability $1 - \lambda$. That is, $s_t = 1$ indicates that the incumbent in period t had a capacity shock at the end of his/her term, while $s_t = 0$ indicates that s/he did not. Since the probability of a negative shock is independent of all other variables of the model, the above stochastic process can be defined independently of policy choices and incumbents' identities. Second, given a strategy profile $\sigma = (\sigma_v, \sigma_L, \sigma_R)$ and $\mathcal{S}$, we use $\mathcal{P}(\mathcal{S}) = \mathcal{P}(\mathcal{S}, \sigma, i_0) = (i_0, i_1, i_2, \ldots)$ to denote a path of policies in $[0, 1]$ generated when voters and parties decide according to strategy profile σ, the initial status-quo policy is $i_0 \in [0, 1]$, and office-holders suffer capacity shocks in accordance with $\mathcal{S}$. Third, we use $\mathcal{A}(\mathcal{S}) = \mathcal{A}(\mathcal{S}, \sigma, i_0) = (g_1, g_2, \ldots)$ to denote a path of outputs in the business-as-usual dimension generated for given σ and $i_0 \in [0, 1]$, when office-holders suffer capacity shocks in accordance with $\mathcal{S}$. When there is no possible confusion, we write $(i_0, i_1, i_2, \ldots)$ and $(g_1, g_2, \ldots)$ without explicitly referring to $\mathcal{S}$, σ, and i_0.

5.2.1 Forward-looking voters

This section discusses the robustness to voter shortsightedness for the main results in the baseline model. For this purpose we assume that voters—but not parties—are forward-looking and we have them discount future payoffs with a common discount factor $\theta \in [0, 1)$. This parameter captures the extent to which voters care about future outcomes. Note that since parties are still assumed to be myopic, they do not care whether their candidates will be re-elected in the next election when they choose their policies. Given the status-quo policy $i_0 \in [0, 1]$, we use $\mathcal{G}_\theta^{i_0}$ to denote the modification of the game $\mathcal{G}^{i_0}$, where the voters' discount factor is θ. In the following we modify the notion of equilibrium that we have used for our baseline model.

Definition 3

A Party-Myopic Stationary Markov Perfect Equilibrium (P-MSMPE) of $\mathcal{G}_\theta^{i_0}$ is a profile of Stationary Markov Strategies $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$ such that, for each $t \in \{1, 2, \ldots\}$, each $i_{t-1} \in [0, 1]$, and $k \in K$ denoting the incumbent in period t with initial capacity $a_{k(t-1)} \in \{0, -A\}$, we have

$$\sigma_K^*(i_{t-1}, a_{k(t-1)}) \in \underset{i_t \in [0,1]}{\operatorname{argmax}}\, U_K(i_{t-1}, i_t, a_{k(t-1)})$$

and, for all citizens $i \in [0, 1]$,

$$\begin{aligned}\sigma_i^*(K, i_{t-1}, a_{k(t-1)}) = K \Leftrightarrow {} & U_i\left(i_{t-1}, \sigma_K^*(i_{t-1}), a_{k(t-1)}\right) + \mathbb{E}_{\mathcal{S}}\left[\sum_{t' \geq t+1} \theta^{t'-t} \cdot U_i\left(i_{t'-1}, i_{t'}, g_{t'}\right)\right] \\ & \geq U_i\left(i_{t-1}, \sigma_{-K}^*(i_{t-1}), 0\right) + \mathbb{E}_{\mathcal{S}}\left[\sum_{t' \geq t+1} \theta^{t'-t} \cdot U_i\left(i'_{t'-1}, i'_{t'}, g'_{t'}\right)\right],\end{aligned}$$

where

$$\begin{aligned}&\mathcal{P}(\mathcal{S}, \sigma^*, \sigma_K^*(i_{t-1})) = (\sigma_K^*(i_{t-1}), i_{t+1}, i_{t+2}, \ldots) = (i_t, i_{t+1}, i_{t+2}, \ldots) \quad \textit{resp.} \\ &\mathcal{A}(\mathcal{S}, \sigma^*, \sigma_K^*(i_{t-1})) = (g_{t+1}, g_{t+2}, \ldots)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{P}(\mathcal{S}, \sigma^*, \sigma_{-K}^*(i_{t-1})) = (\sigma_{-K}^*(i_{t-1}), i'_{t+1}, i'_{t+2}, \ldots) = (i'_t, i'_{t+1}, i'_{t+2}, \ldots) \quad \textit{resp.} \\ &\mathcal{A}(\mathcal{S}, \sigma^*, \sigma_{-K}^*(i_{t-1})) = (g'_{t+1}, g'_{t+2}, \ldots)\end{aligned}$$

are the paths of policies in $[0, 1]$ (outputs in business as usual) that follow the office-holders' decision when $\mathcal{S} = (s_{t+1}, s_{t+2}, \ldots)$ is the realization of the stochastic process containing the capacity shocks, and voters and parties decide in accordance with σ^*.

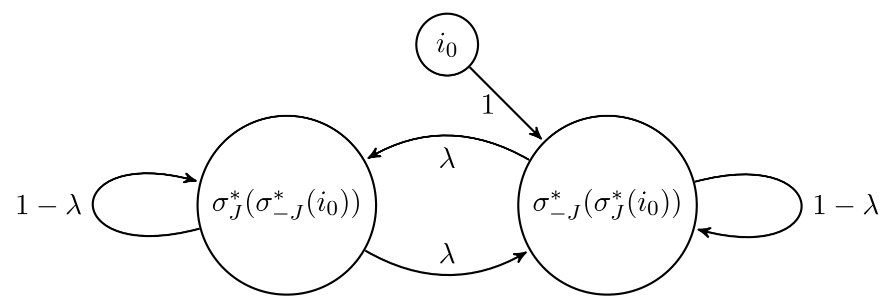
As in the baseline model with myopic voters, we assume that incumbents whose capacity is low are never re-elected. This greatly facilitates the analysis but does not affect the results qualitatively.[34] As in (9), we denote the outcome of an election when the incumbent has normal capacity by $E(\sigma_v, K, i_{t-1}) \in \{L, R\}$. Since candidates are myopic, Proposition 1 still holds in this modified setting. However, the behavior of forward-looking voters is no longer given by Proposition 2. Instead, it is the next result that describes the voters' behavior when they are forward-looking, i.e., when $\theta > 0$.

Theorem 2

Let $i_0 \in [0,1]$ be the status-quo policy in $t = 1$. Then there is $\theta^{i_0} \in (0,1]$ such that for all $\theta \in (0,1)$, $\mathcal{G}_\theta^{i_0}$ has a unique P-MSMPE, referred to as $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$, with the following properties:

(i) If $0 \leq \left|i_0 - \frac{1}{2}\right| \leq \Pi - \frac{c}{2}$ or $\Pi - \frac{c}{2} < \left|i_0 - \frac{1}{2}\right| < \Pi + \frac{c}{2}$ and $\theta \leq \theta^{i_0}$, then σ^* is characterized by the Markov transition diagrams given in (i)–(ii) of Theorem 1.

(ii) If either $\Pi + \frac{c}{2} \leq \left|i_0 - \frac{1}{2}\right|$ or $\Pi - \frac{c}{2} < \left|i_0 - \frac{1}{2}\right| < \Pi + \frac{c}{2}$ and $\theta > \theta^{i_0}$, then σ^* is characterized by the following Markov transition diagram:



where equilibrium policy choices are determined according to the best response given in Proposition 1, and $J \in \{L, R\}$ is a party such that $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$.

Proof: See Appendix B.

□

According to the above result, the long-term dynamics identified by Theorem 1 remain valid with forward-looking voters regardless of their discount factor. The short-term dynamics, in turn, also remain valid if either initial polarization is low (i.e., $\left|i_0 - \frac{1}{2}\right| \leq \Pi - \frac{c}{2}$) or initial polarization is large (i.e., $\Pi - \frac{c}{2} < \left|i_0 - \frac{1}{2}\right| < \Pi + \frac{c}{2}$) and the discount factor is sufficiently low (i.e., $\theta \leq \theta^{i_0}$). However, the picture changes when either initial polarization is large (i.e., $\Pi - \frac{c}{2} < \left|i_0 - \frac{1}{2}\right| < \Pi + \frac{c}{2}$) and the discount factor is sufficiently large (i.e., $\theta > \theta^{i_0}$) or when initial polarization is very large

[34]See Footnote 18.

(i.e., $\Pi + \frac{c}{2} \leq \left|i_0 - \frac{1}{2}\right|$). In either case, the median voter prefers to elect the candidate who will choose a more moderate policy in the short term and incur the associated costs of change as soon as possible. This candidate is always the one who has his/her bliss point located on the *other* side of the spectrum compared with the initial policy. This is in sharp contrast with the baseline case with $\theta = 0$, in which case the median voter disregards the higher utility that s/he could attain from more moderate policies in the future and elects the candidate who has his/her bliss point located on the *same* side of the spectrum as the initial policy. This candidate chooses a more extreme policy in the current period, but does not incur any costs of change. That is, with forward-looking voters no extreme policy is implemented along the equilibrium path, in which we observe an alternation between $\frac{1}{2} + \Pi - \frac{c}{2}$ (by party R) and $\frac{1}{2} - \Pi + \frac{c}{2}$ (by party L).

5.2.2 Forward-looking parties

Next, we analyze a case in which parties—but not voters—are forward-looking and investigate how the results in Theorem 1 are affected. Both parties discount future payoffs with a common discount factor $\psi \in [0, 1)$. Since voters are myopic, they only foresee (and care about) the policies that each of the candidates would implement in the first term after elections. Given the status-quo policy $i_0 \in [0, 1]$, we use $\mathcal{G}_{\psi}^{i_0}$ to denote the modification of the game $\mathcal{G}^{i_0}$ where the parties' discount factor is ψ. We start by assuming that office-holders and parties are solely policy-motivated, later we discuss the case where they also care about being in office. We show that while the main thrust of Theorem 1 remains valid, the parties' optimal policy locations change as they are now looking to a longer-term benefit from moving policy.

When costs of change are low and the initial status-quo policy is very extreme, we show that forward-looking candidates choose policies that are closer to their bliss point, and hence more moderate. This is because parties regard future policies as more valuable, and hence they trade off some costs associated with changing the policy in the current period against a policy that will be closer to their own bliss point in the future. When costs of change are low and the initial status-quo policy is moderate (relative to party polarization), a similar effect takes place with one major difference. Whether the policy choice is more moderate or closer to the parties' bliss point is mediated by how many periods they expect to be in office before they suffer a capacity shock. This expectation depends on the probability that such a shock occurs in any given period. If such a probability is low, parties choose more extreme policies that are closer to their bliss points. The opposite happens when the shock probability is large, in which case parties choose more moderate policies. We stress that the dynamics when the initial status-quo policy is moderate coincide with the long-term dynamics for any initial policy (provided that costs of change are low).

As the set of party strategies is now very large and strategies themselves can be very complicated, it is convenient for the analysis of forward-looking parties to adapt the notion of equilibrium used in our baseline model to this modified setting by considering a family of parametrized strategies for parties.[35] To do so, we first note that if an office-holder were certain to dictate policy for a finite number of consecutive periods (for all of which s/he would care from the perspective of the present time), s/he would choose the same policy in all periods. This is shown in the following result:

Lemma 1

Let $T \geq 1$ be a finite number of periods, $i_0 \in \mathcal{I}$, and $k \in K$ be the office-holder in periods $t = 1, \ldots, T$. Let also

$$\Gamma^T = \frac{c}{2} \cdot \frac{1}{\sum_{t=0}^{T} \psi^t}.$$

Then,

$$i_1 = \ldots = i_T = i^*(i_0) = \min\left\{\max\left\{\mu_K - \Gamma^T, i_{t-1}\right\}, \mu_K + \Gamma^T\right\}$$

maximizes

$$\sum_{t=1}^{T} \psi^{t-1} \cdot U_K\left(i_{t-1}, i_t, g_t\right)$$

over all possible $(i_1, \ldots, i_T) \in \prod_{t=1}^{T} \mathcal{I}$.

Proof: See Appendix B.

□

Hence, if an office-holder (mistakenly) believes that s/he will be able to dictate policy-making for a particular number T of periods, s/he will choose a certain policy and stick to it henceforth—and hence his/her policy choices are time-consistent as long as s/he stays in power believing that T periods are ahead of him/her in office. In particular, with linear costs of change and without capacity shocks that can be anticipated, policy persistence does not depend on the assumption that office-holders and parties are myopic. Note also that the persistence level Γ^T decreases with ψ, since utility losses from policies that differ from the party's ideal policy have more weight. Similarly, as T becomes larger, Γ^T shrinks, thereby approaching

$$\lim_{T\to\infty} \Gamma^T = \frac{c}{2} \cdot (1 - \psi). \tag{18}$$

[35]The set of parametrized strategies that we consider bears some resemblance to the one considered by Nunnari and Zápal (2017). Moreover, it shares the rationale behind the assumption shared by Forand (2014) that for exogenous reasons, incumbents are bound to choose the same policy throughout their tenure. Finally, we note that equilibria which are not Markov could also be considered, which might yield different predictions. The investigation of such equilibria lies beyond the scope of this paper.

Equation (18) represents the value of policy persistence when $\lambda = 0$. In this case, the office-holder (or his/her party) can, in fact, dictate policy forever.

We can now define a particular subset of Stationary Markov Strategies, for which Lemma 1 offers a micro-foundation and which are very helpful in describing equilibrium behavior when $\lambda > 0$.

Definition 4

A Simple Stationary Markov Strategy profile $\sigma = (\sigma_v, \sigma_L, \sigma_R)$ is a Stationary Markov Strategy profile where for each party $K \in \{L, R\}$ the best-response function $\sigma_K : [0,1] \times \{0, -A\} \to [0,1]$ that maps the status-quo policy into the policy of the current period given the incumbent's capacity can be written as

$$\sigma_K(i_{t-1}, a_{k(t-1)}) = \min\left\{\max\left\{\mu_K - \chi_K^-, i_{t-1}\right\}, \mu_K + \chi_K^+\right\} \tag{19}$$

for $\chi_K := (\chi_K^-, \chi_K^+) \in [0, \mu_K] \times [0, 1 - \mu_K]$.

A party strategy that is part of a Simple Stationary Markov Strategy profile is called *simple*. When parties play in accordance with a simple strategy, they choose the range of persistence in policy-making that they apply at any time when they are in power. It has been argued in the literature that incumbents always find it difficult to reverse the policies they have themselves chosen in the past—see the discussion in Section 4.4. Beyond the micro-foundation offered by Lemma 1, simple strategies also build on this assumption. Since a simple strategy for party K can be fully characterized by the pair $\chi_K = (\chi_K^-, \chi_K^+)$, we simplify (and slightly abuse) notation and write $\sigma_K = \chi_K$. Figure 8 is a generalization of Figure 2 and illustrates the shape of a party's simple strategy.

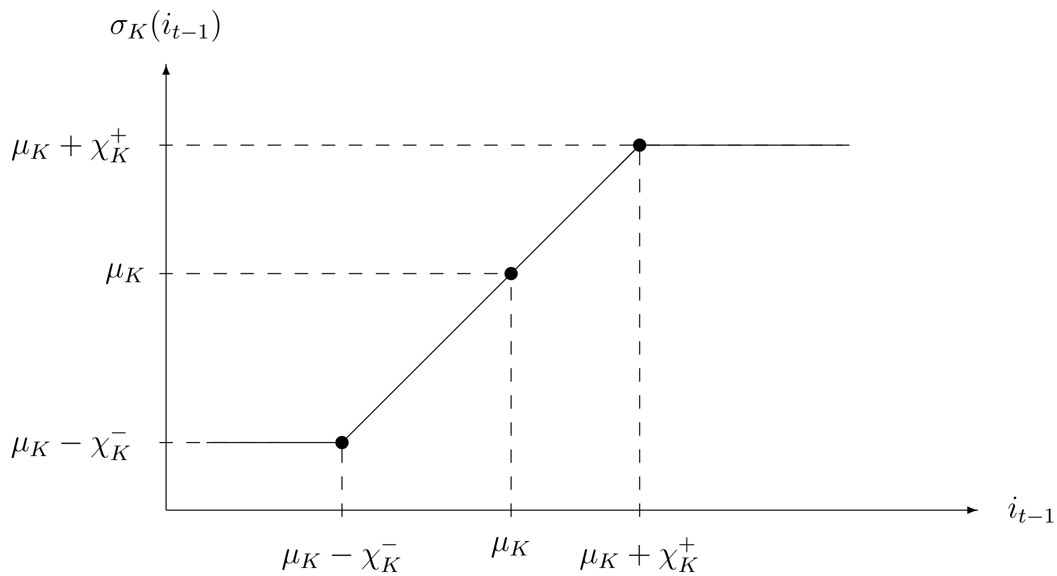


Figure 8: A Simple Strategy for Party K given the status-quo policy i_{t-1}.

We are now in a position to define the notion of equilibrium for $\mathcal{G}_\psi^{i_0}$.

Definition 5

A Voter-Myopic Stationary Markov Perfect Equilibrium (V-MSMPE) of $\mathcal{G}_{\psi}^{i_0}$ is a profile of Simple Stationary Markov Strategies $\sigma^* = (\sigma_v^*, \chi_L^*, \chi_R^*)$ such that for each $t \in \{1, 2, \ldots\}$, each $i_{t-1} \in [0,1]$, and $k \in K$ denoting the incumbent in period t with initial capacity $a_{k(t-1)} \in \{0, -A\}$, we have

$$\chi_K^*(i_{t-1}, a_{k(t-1)}) \in \operatorname*{argmax}_{i_t \in [0,1]} \left\{ U_K\left(i_{t-1}, i_t, a_k\right) + \mathbb{E}_{\mathcal{S}}\left[\sum_{t' \geq t+1} \psi^{t'-t} \cdot U_K\left(i_{t'-1}, i_{t'}, g_{t'}\right)\right]\right\}$$

and, for all citizens $i \in [0,1]$,

$$\sigma_i^*(K, i_{t-1}, a_{k(t-1)}) = K \Leftrightarrow U_i\left(i_{t-1}, \chi_K^*(i_{t-1}), a_{k(t-1)}\right) \geq U_i\left(i_{t-1}, \chi_{-K}^*(i_{t-1}), 0\right),$$

where $\mathcal{P}(\mathcal{S}, \sigma^*, i_t) = (i_t, i_{t+1}, i_{t+2}, \ldots)$ (or $\mathcal{A}(\mathcal{S}, \sigma^*, i_t) = (g_{t+1}, g_{t+2}, \ldots)$) is the path of policies in $[0,1]$ (or outputs in business as usual) that follows i_t, the decision of the office-holder in period t, when $\mathcal{S} = (s_{t+1}, s_{t+2}, \ldots)$ is the realization of the stochastic process containing the capacity shocks and voters and parties decide in accordance with σ^*.

We assume again that incumbents whose capacity is low are never re-elected, and once more use $E(\sigma_v, K, i_{t-1}) \in \{L, R\}$ to denote the outcome of an election where the incumbent has zero capacity, as in (9). To find the equilibria of the game, we first need to find the optimal range of policy persistence for parties and then characterize the optimal behavior of voters. This enables us to formulate the following result, which is the counterpart of Theorem 1 when parties are forward-looking:

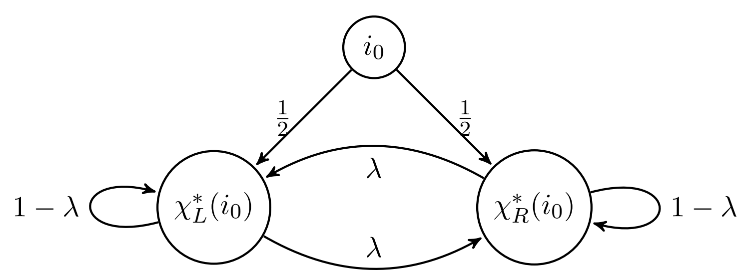
Theorem 3

Let $c \in \left(0, \frac{2\Pi}{1+\psi}\right) \cup \left[\frac{2\Pi+1}{1-\psi}, \infty\right)$ and let $i_0 \in [0,1]$ be the status-quo policy in $t = 1$. Then $\mathcal{G}_{\psi}^{i_0}$ has a unique V-MSMPE, referred to as $\sigma^* = (\sigma_v^*, \chi_L^*, \chi_R^*)$. Depending on $\left|i_0 - \frac{1}{2}\right|$, σ^* is characterized by one of the following Markov transition diagrams, where

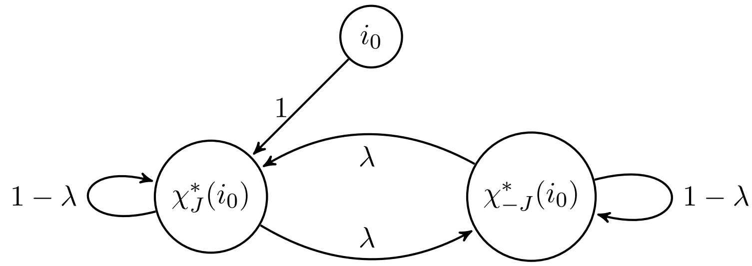
$$\chi_L^* = \left(\underline{\chi}^*, \overline{\chi}^*\right) = \left(\min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1-\psi)\right\}, \min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right\}\right),$$
$$\chi_R^* = \left(\overline{\chi}^*, \underline{\chi}^*\right) = \left(\min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right\}, \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1-\psi)\right\}\right),$$

and $J \in \{L, R\}$ is a party such that $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$:

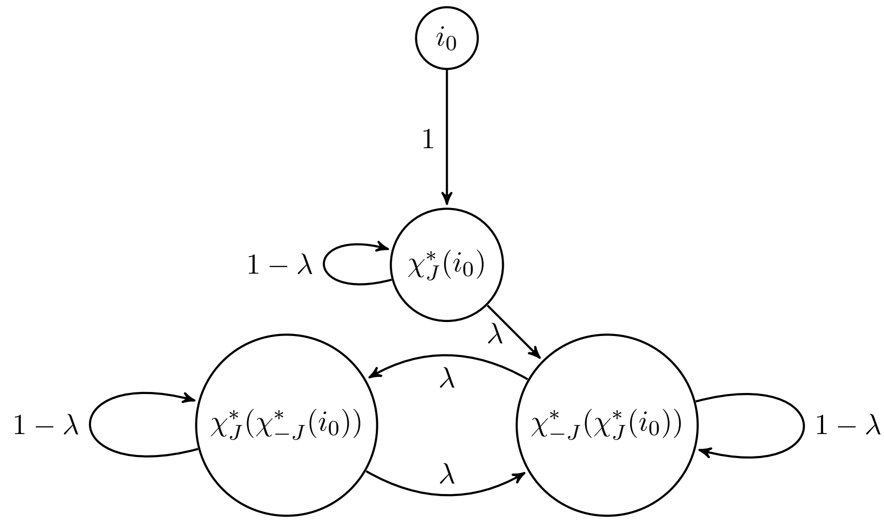
(i) If $\left|i_0 - \frac{1}{2}\right| = 0$, then σ^* is characterized by



(ii) If $0 < \left|i_0 - \frac{1}{2}\right| \leq \min\left\{\left|\Pi - \underline{\chi}^*\right|, \frac{1}{2}\right\}$, *then* σ^* *is characterized by*



(iii) If $\min\left\{\left|\Pi - \underline{\chi}^*\right|, \frac{1}{2}\right\} < \left|i_0 - \frac{1}{2}\right| \leq \frac{1}{2}$, *then* σ^* *is characterized by*



Proof: See Appendix B.

□

This result shows that, except for $c \in \left[\frac{2\Pi}{1+\psi}, \frac{2\Pi+1}{1-\psi}\right)$, the equilibrium dynamics when parties are forward-looking and use simple strategies are qualitatively identical to those described in Theorem 1.[36] In particular, if $c \in \left(\frac{2\Pi+1}{1-\psi}, \infty\right)$, costs are so large that the initial policy is maintained in all periods. When costs of change are low, i.e., $c \in \left(0, \frac{2\Pi}{1+\psi}\right)$, the policy path tends also towards a stochastic alternation between two policies equidistant to 1/2, regardless of the initial policy, as was the case with myopic parties. Since office-holders are forward-looking, however, the equilibrium policy choices now depend on ψ. If ψ tends to zero, the baseline model is recovered since

$$\lim_{\psi \to 0} \overline{\chi}^* = \lim_{\psi \to 0} \underline{\chi}^* = \frac{c}{2}.$$

[36] If $c \in [2\Pi/(1+\psi), 2\Pi/(1+\psi(2\lambda-1)))$, then *(i)* if there is a V-MSPME, it is the strategy profile described in Theorem 3 (see Remark 1 in Appendix B), and *(ii)* the strategy profile described in Theorem 3 remains a V-MSPME if parties—and office-holders—obtain a sufficiently large benefit from being in office (see Remark 3 in Appendix B). If $c \in [2\Pi/(1+\psi(2\lambda-1)), (2\Pi+1)/(1-\psi))$, a necessary condition for a V-MSPME to exist is that policy choices converge to one particular policy after some periods, and are not changed thereafter (see Remark 2 in Appendix B).

Additionally, if λ tends to zero, the level of policy persistence chosen by both parties coincides with the one described in Lemma 1.

Some further observations concerning policy choices are in order when costs of change are low. If $\left|i_0 - \frac{1}{2}\right| > \left|\Pi - \underline{\chi}^*\right|$ and thus initial polarization is large, the first-period office-holder implements an *extreme* policy during his/her tenure in office until s/he is ousted. The baseline model also features this property. However, because $\overline{\chi}^* = \frac{c}{2} \cdot (1-\psi) < \frac{c}{2}$, such an extreme policy is closer to the median voter's preferred policy than under the assumption that parties are myopic. The intuition behind this moderation effect is the following: While a myopic first-period office-holder merely weighs $U^c(i_1, i_0)$ against $U_K^{\mathcal{I}}(i_1)$, a forward-looking candidate anticipates (on behalf of his/her party) that accepting a larger policy shift in the first period (which brings about a higher cost) may increase the party's expected future utility. As a consequence, forward-looking candidates choose policies in the first period that are closer to their bliss point and thus more moderate, as the initial status-quo policy is very extreme. This moderating effect is stronger, the larger ψ is.

Theorem 3 implies in addition that if c is low, regardless of the initial policy the long-term policies are given by

$$\mu_L + \overline{\chi}^* = \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \quad \text{and} \quad \mu_R - \overline{\chi}^* = \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]. \tag{20}$$

Note that these policies are more moderate than their counterparts from Proposition 1, $\mu_L + \frac{c}{2}$ and $\mu_R - \frac{c}{2}$, if and only if $\lambda > \frac{1}{2}$. How the parties' farsightedness does precisely affect long-term policy polarization is then ambiguous and depends on λ, the probability that incumbents suffer a capacity shock in any term in office. In particular, the fact that parties are forward-looking may not be beneficial for society in the long run if capacity shocks are not frequent (i.e., if λ is low enough). Because office-holders expect to stay in office for many periods, they choose policies that are closer to their bliss points and, hence, more extreme.

Finally, if costs of change are low, we can fully address the case where office-holders and parties are not only policy-motivated but also office-motivated. Formally, we assume that $b \geq 0$ is added to the utility of parties for every period they are in office, and we denote the modified game by $\mathcal{G}^{i_0}_{\psi,b}$.[37] Thus far we have assumed that $b = 0$. We say that a simple stationary strategy profile $\sigma = (\sigma_v, \sigma_L, \sigma_R)$ is *symmetric* if $\underline{\chi}_R^+ = \underline{\chi}_L^-$ and $\underline{\chi}_L^+ = \underline{\chi}_R^-$, and is *regular* if all choices of party R are to the right of $1/2$ and all choices of party L are to the left of $1/2$. Assuming symmetry and regularity, we then obtain the following result, which follows from the property that normal-capacity incumbents enjoy a strong incumbency advantage generated by the existence of costs of change.

[37] Definition 5 should be altered accordingly to account for this modification.

Proposition 5

Let $c \in \left(0, \frac{2\Pi}{1+\psi}\right)$ *and* $i_0 \in [0,1]$ *be the status-quo policy at the beginning of period* $t = 1$. *Then,* σ^* *is a symmetric and regular V-MSMPE of* $\mathcal{G}^{i_0}_{\psi,b}$ *for* $b > 0$ *if and only if* σ^* *is V-MSMPE of* $\mathcal{G}^{i_0}_{\psi}$.

Proof: See Appendix B.

□

As a consequence, when costs of change are low, re-election concerns do not affect policy choices any differently from considerations about present and future policies.

5.3 More model predictions

The model and the results of this section give further support to the positing of the hypothesis of Section 4.4. Moreover, they enable us to formulate the following further testable hypotheses:

5. If political instability is high and costs of change are moderate, the more shortsighted office-holders are, the higher policy polarization is, both in the short and the long term.

6. If costs of change are moderate, welfare may increase with political instability only if costs are sufficiently convex. In such a case, reforms may occur gradually.

7. Extreme policies are effected only when voters are (fully) myopic.

First, there seems to be widespread agreement that policy polarization has increased in the recent past, especially in the US for both the House of Representatives and the Senate (see e.g. Poole and Rosenthal, 2001; Theriault, 2008) and for presidential platforms (see e.g. Budge et al., 2001). Simultaneously, some democratic political systems all over the world seem to be increasingly unstable, partly due to a rise in electoral support for outsiders. This phenomenon has eroded the control of party elites and has brought about a great deal of uncertainty, eventually leading to outcomes as unexpected as the election of Donald Trump as US President. In our model, this instability can be translated into a high value of λ, say above 1/2. Our theory suggests that the currently observable high levels of policy polarization may be due to the parties' lack of farsightedness in current politically unstable times or to the parties' loss of control over their candidates' agenda. According to Theorem 3, if $c \leq \frac{2\Pi}{1+\psi}$, the degree of policy polarization in the long run is proportional to $\Pi - \frac{c}{2}(1+\psi)$, and then increases if ψ becomes smaller. Policy polarization also increases in the short term if parties become more shortsighted.

Second, in Section 5.1 we have seen that it may be better for welfare for there to be turnover (say, $\lambda \approx 1$) rather than no turnover (say, $\lambda \approx 0$), but only if costs of change are sufficiently convex. That is, from a social perspective the exact structure of the costs incurred by the citizenry when policies change can be of paramount importance in our current agitated times. This can be particularly relevant for the design of checks and balances in the political system. From a general perspective, the degree of checks and balances can be parametrized not only by c (as we have already argued), but also by η. As we have seen, convex costs enable gradualism in reforms. This is a testable hypothesis in its own right.

Third and last, there have been many examples in the recent past where extreme policies have been adopted. Our theory offers one possible rationale: Voters have become increasingly unaware of the long term consequences of policy-making, and this has made it easier for policies to correspond to the transitional dynamics that precede the long-term alternation of more moderate policies. This has been formulated in part *(ii)* of Theorem 2.

6 Conclusion

We have developed an infinite-horizon model of electoral competition to analyze the long-term social consequences of costly policy changes. We find that if costs of change are not too large, the equilibrium policy path tends towards an infinite sequence of (regions of) policies that are equidistant to the median voter's preferred policy and more moderate than the office-holders' bliss points. The dynamics are fully determined by the incumbency advantage created by costs of change, on the one hand, and by the (exogenous) shocks that affect the incumbent's capacity on the other.

Our analysis provides a series of testable hypotheses that can serve as a basis for further inquiries and applications. For instance, the impact of costs of change in systems with more than two parties or the possibility of endogenizing party platforms (in particular, allowing entry at the median voter position) are possible avenues for further research. Our results also suggest that the major effects of costs of change on long-term policy outcomes already arise when voters and office-holders look only one term ahead. In turn, from a behavioral perspective, this may provide a rationale for issues such as why voters only concentrate on the consequences of their voting for the next term. This property of voter behavior calls for further investigation.

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Appendix A

Proof of Proposition 1

Let $k \in K$ be the office-holder in period t, with $t \geq 1$. For a given $(t-1)$-th period policy choice i_{t-1}, k chooses policy i_t so that his/her party's instantaneous utility in period t, as given in (2), is maximized. Recall that candidates are myopic and do not care about re-election. Hence, the office-holder maximizes

$$U_K(i_{t-1}, i_t, g_t) = a_{k(t-1)} - (i_t - \mu_K)^2 - c \cdot |i_{t-1} - i_t|.$$

We note that $U_K(i_{t-1}, i_t, g_t)$ is differentiable with respect to i_t on $(0,1)\backslash\{i_{t-1}\}$. We distinguish two cases, depending on the relative values of i_{t-1} and μ_K.

<u>*Case 1:*</u> $i_{t-1} < \mu_K$
In this case,

$$\frac{dU_K(i_{t-1}, i_t, g_t)}{di_t} = \begin{cases} -2(i_t - \mu_K) + c & \text{if } 0 < i_t < i_{t-1}, \\ -2(i_t - \mu_K) - c & \text{if } i_{t-1} < i_t < 1. \end{cases}$$

Therefore, $U_K(i_{t-1}, i_t, g_t)$ is strictly increasing if and only if

$$0 < i_t < \max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\},$$

which implies that

$$\operatorname*{argmax}_{i_t \in [0,1]} U_K(i_{t-1}, i_t, g_t) = \left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}\right\}.$$

<u>*Case 2:*</u> $i_{t-1} \geq \mu_K$
Analogous reasoning leads to

$$\operatorname*{argmax}_{i_t \in [0,1]} U_K(i_{t-1}, i_t, g_t) = \left\{\min\left\{\mu_K + \frac{c}{2}, i_{t-1}\right\}\right\}.$$

Finally, combining *Case 1* and *Case 2* yields

$$\operatorname*{argmax}_{i_t \in [0,1]} U_K(i_{t-1}, i_t, g_t) = \left\{\min\left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}, \mu_K + \frac{c}{2}\right\}\right\}.$$

This completes the proof.

□

Proof of Proposition 2

Without loss of generality, we can assume that $i_{t-1} \in \left[\frac{1}{2}, 1\right]$. The results for $i_{t-1} \in \left[0, \frac{1}{2}\right]$ follow by symmetry. Let $K \in \{\emptyset, L, R\}$ denote the incumbent's party at the moment of the t-th election. We now assume that $K \neq \emptyset$ and provide a detailed proof of Part *(ii)*, i.e., we assume that $t > 1$. Part *(i)* referring to the elections in period $t = 1$ can be proved using the same logic. The only difference is how ties are broken in the absence of an incumbent when the median voter is indifferent between electing either candidate. We recall that in this case, s/he chooses the candidate whose bliss point is closest to the status-quo policy i_0 when this policy is different from 1/2 or decides according to the fair toss of a coin if $i_0 = 1/2$.

When deciding whether to elect $k \in R$ or $k' \in L$ in the t-th election, any voter $i \in [0,1]$ compares the instantaneous utilities that s/he will receive from the two candidates being in office. We can assume that both candidates have normal capacity, because a low-capacity incumbent is never re-elected. Accordingly, voter i strictly prefers $k \in R$ to be in office if

$$\Delta U_i(i_{t-1}) := U_i(i_{t-1}, i_{kt}, g_{kt}) - U_i(i_{t-1}, i_{k't}, g_{k't}) > 0,$$

which, by (1), is equivalent to

$$\Delta U_i(i_{t-1}) = -(i_{kt} - i)^2 - c \cdot |i_{t-1} - i_{kt}| + (i_{k't} - i)^2 + c \cdot |i_{t-1} - i_{k't}| > 0.$$

From Proposition 1, we know that in any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$, the policy choices i_{kt} and $i_{k't}$ are given by $\sigma_R^*(i_{t-1})$ and $\sigma_L^*(i_{t-1})$, respectively. Thus

$$\Delta U_i(i_{t-1}) = -\left(\sigma_R^*(i_{t-1}) - i\right)^2 - c \cdot |i_{t-1} - \sigma_R^*(i_{t-1})| + \left(\sigma_L^*(i_{t-1}) - i\right)^2 + c \cdot |i_{t-1} - \sigma_L^*(i_{t-1})|.$$

We recall that an incumbent is re-elected if and only if s/he receives a vote-share of at least $\frac{1}{2}$. Thus, if k is the incumbent, s/he is re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \geq 0.$$

Indeed, we can focus on the median voter's decision because $\Delta U_i(i_{t-1})$ is increasing in i, and hence the median voter is decisive in the election. Analogously, if $k' \in L$ is the incumbent, s/he is re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \leq 0.$$

We now distinguish two different cases regarding the relative values of c and Π and analyze the sign of $\Delta U_{\frac{1}{2}}(i_{t-1})$ as a function of i_{t-1} in each case.

<u>*Case 1:*</u> $\frac{c}{2} < \Pi$

We distinguish three subcases.

<u>*Case 1a:*</u> $i_{t-1} \in \left[\frac{1}{2}, \mu_R - \frac{c}{2}\right]$

In this case, $\sigma_R^*(i_{t-1}) = \mu_R - \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, so it follows by simple algebra that

$$\Delta U_i(i_{t-1}) = 2i(2\mu_R - 1 - c) - (2\mu_R - 1 - 2ci_{t-1}).$$

As a consequence, $\Delta U_{\frac{1}{2}}(i_{t-1}) > 0$ for $i_{t-1} \in \left(\frac{1}{2}, \mu_R - \frac{c}{2}\right]$ and $\Delta U_{\frac{1}{2}}(i_{t-1}) = 0$ for $i_{t-1} = \frac{1}{2}$. On the one hand, if $k \in R$ is the incumbent, s/he is always re-elected (provided s/he has normal capacity). On the other hand, if $k' \in L$ is the incumbent, s/he is only re-elected for $i_{t-1} = \frac{1}{2}$.

<u>*Case 1b:*</u> $i_{t-1} \in \left(\mu_R - \frac{c}{2}, \mu_R + \frac{c}{2}\right)$

Since $\sigma_R^*(i_{t-1}) = i_{t-1}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, we obtain

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = \underbrace{\left(\mu_L + \frac{c}{2} - i_{t-1}\right)}_{<0} \cdot \underbrace{\left(\mu_L - \frac{c}{2} + i_{t-1} - 1\right)}_{<0} > 0.$$

Thus, the incumbent is always re-elected if s/he belongs to party R, whereas the incumbent is never re-elected if s/he belongs to party L.

<u>*Case 1c:*</u> $i_{t-1} \geq \mu_R + \frac{c}{2}$

Since $\sigma_R^*(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, it follows that $\Delta U_{\frac{1}{2}}(i_{t-1}) = 0$ for all $i_{t-1} \geq \mu_R + \frac{c}{2}$. Thus, the incumbent is always re-elected.

<u>*Case 2:*</u> $\frac{c}{2} \geq \Pi$

We distinguish three subcases.

<u>*Case 2a:*</u> $i_{t-1} \in \left[\frac{1}{2}, \mu_L + \frac{c}{2}\right]$

In this case, $\sigma_R^*(i_{t-1}) = \sigma_L^*(i_{t-1}) = i_{t-1}$, so that $\Delta U_i(i_{t-1}) = 0$ for all $i \in [0,1]$. Thus, incumbents are always re-elected.

Case 2b: $i_{t-1} \in \left(\mu_L + \frac{c}{2}, \mu_R + \frac{c}{2}\right)$

Now, $\sigma_R^*(i_{t-1}) = i_{t-1}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$. Accordingly, we obtain the same results as in *Case 1b*.

Case 2c: $i_{t-1} \geq \mu_R + \frac{c}{2}$

Since $\sigma_R^*(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, it follows that incumbents are always re-elected, as in *Case 1c*.

This completes the proof.

$\square$

Appendix B

Convex costs of change

Proof of Proposition 3

The best response of office-holder $k \in K$ is $\sigma_K^*(i_{t-1}) = x$, where x maximizes the following expression:

$$-(\mu_K - x)^2 - c \cdot |x - i_{t-1}|^\eta. \tag{21}$$

If $i_{t-1} = \mu_K$, we trivially have $\sigma_K^*(i_{t-1}) = i_{t-1}$. Accordingly, it suffices to distinguish the following two cases.

<u>*Case 1:*</u> $i_{t-1} < \mu_K$

By the symmetry of the term $|x - i_{t-1}|^\eta$ with respect to i_{t-1} and the symmetry of the term $(\mu_K - x)^2$ with respect to μ_K, we can focus on the case where $x \in (i_{t-1}, \mu_K)$. This enables us to write Equation (21) as

$$-(\mu_K - x)^2 - c \cdot (x - i_{t-1})^\eta. \tag{22}$$

Then, the first-order condition that results from maximizing Expression (22) with respect to x is

$$2(\mu_K - x) - c\eta \cdot (x - i_{t-1})^{\eta-1} = 0. \tag{23}$$

Next, consider the following function:

$$f(x) = 2(\mu_K - x) - c\eta \cdot (x - i_{t-1})^{\eta-1}.$$

Because

$$f(i_{t-1}) = 2(\mu_K - i_{t-1}) > 0 \quad \text{and} \quad f(\mu_K) = -c\eta \cdot (\mu_K - i_{t-1})^{\eta-1} < 0,$$

and, moreover, since $\eta > 1$,

$$f'(x) = -2 - c\eta(\eta - 1) \cdot (x - i_{t-1})^{\eta-2} < 0,$$

then Equation (23) has exactly one solution in (i_{t-1}, μ_K). This completes the proof of this case.

<u>*Case 2:*</u> $\mu_K < i_{t-1}$

Similarly to Case 1, we can focus on the case where $x \in (\mu_K, i_{t-1})$ and re-write Equation (21) as

$$-(x - \mu_K)^2 - c \cdot (i_{t-1} - x)^\eta. \tag{24}$$

The first-order condition that results from maximizing Expression (24) with respect to x is

$$-2(x - \mu_K) + c\eta \cdot (i_{t-1} - x)^{\eta-1} = 0. \tag{25}$$

Next, consider the following function:

$$f(x) = -2(x - \mu_K) + c\eta \cdot (i_{t-1} - x)^{\eta-1}.$$

Because

$$f(\mu_K) = c\eta \cdot (i_{t-1} - \mu_K)^{\eta-1} > 0 \quad \text{and} \quad f(i_{t-1}) = -2(i_{t-1} - \mu_K) < 0,$$

and, moreover, since $\eta > 1$,

$$f'(x) = -2 - c\eta(\eta - 1) \cdot (i_{t-1} - x)^{\eta-2} < 0,$$

then Equation (25) has exactly one solution in (μ_K, i_{t-1}). This completes the proof of this case, and hence the whole proof.

$\square$

Proof of Corollary 4

Throughout the proof, we let $K \in \{L, R\}$. First, we prove item (i). We distinguish two cases. On the one hand, assume that $i_{t-1} < \mu_K$. Then, by the Implicit Function Theorem, we obtain from Equation (13)

$$\frac{d}{di_{t-1}}\sigma_K^*(i_{t-1}) = \frac{1}{1 + \frac{1}{\eta-1}\frac{\sigma_K^*(i_{t-1})-i_{t-1}}{\mu_K-\sigma_K^*(i_{t-1})}} = \frac{1}{1 + \frac{1}{\frac{c\eta}{2}(\eta-1)(\sigma_K^*(i_{t-1})-i_{t-1})^{\eta-2}}} \in (0,1). \tag{26}$$

On the other hand, assume that $\mu_K < i_{t-1}$. Then, by the Implicit Function Theorem, we obtain from Equation (15)

$$\frac{d}{di_{t-1}}\sigma_K^*(i_{t-1}) = \frac{1}{1 + \frac{1}{\eta-1}\frac{i_{t-1}-\sigma_K^*(i_{t-1})}{\sigma_K^*(i_{t-1})-\mu_K}} = \frac{1}{1 + \frac{1}{\frac{c\eta}{2}(\eta-1)(i_{t-1}-\sigma_K^*(i_{t-1}))^{\eta-2}}} \in (0,1). \tag{27}$$

Second, we show item (ii). Note that if $i_{t-1} < \mu_K$, Equation (13) implies that

$$\frac{d}{di_{t-1}}[\sigma_K^*(i_{t-1}) - i_{t-1}] < 0 \Leftrightarrow \frac{d}{di_{t-1}}[\mu_K - \sigma_K^*(i_{t-1})] < 0.$$

The right-hand side of the above equivalence follows directly from (26). Similarly, Equation (15) implies that

$$\frac{d}{di_{t-1}}[i_{t-1} - \sigma_K^*(i_{t-1})] > 0 \Leftrightarrow \frac{d}{di_{t-1}}[\sigma_K^*(i_{t-1}) - \mu_K] > 0.$$

The right-hand side of the above equivalence follows directly from (27).

Third, we show item (iii). Again, we distinguish two cases, and in each we apply the Implicit Function Theorem. On the one hand, assume that $i_{t-1} < \mu_K$. Then,

$$\frac{d^2}{di_{t-1}^2}\sigma_K^*(i_{t-1}) > 0 \Leftrightarrow \frac{d}{di_{t-1}}\frac{\mu_K - \sigma_K^*(i_{t-1})}{\sigma_K^*(i_{t-1}) - i_{t-1}} > 0.$$

Moreover, since $\eta \leq 2$,

$$\begin{aligned}\frac{d}{di_{t-1}}\frac{\mu_K - \sigma_K^*(i_{t-1})}{\sigma_K^*(i_{t-1}) - i_{t-1}} &= \left(\frac{1}{\sigma_K^*(i_{t-1}) - i_{t-1}}\right)^2 \left(-\frac{d\sigma_K^*(i_{t-1})}{di_{t-1}}(\mu_K - \sigma_K^*(i_{t-1})) + \mu_K - \sigma_K^*(i_{t-1})\right) \\ &= \left(\frac{1}{\sigma_K^*(i_{t-1}) - i_{t-1}}\right)^2 (\mu_K - i_{t-1}) \left(-\frac{d\sigma_K^*(i_{t-1})}{di_{t-1}} + \frac{1}{1 + \frac{\sigma_K^*(i_{t-1})-i_{t-1}}{\mu_K-\sigma_K^*(i_{t-1})}}\right) > 0.\end{aligned}$$

On the other hand, assume that $\mu_K < i_{t-1}$. Then,

$$\frac{d^2}{di_{t-1}^2}\sigma_K^*(i_{t-1}) < 0 \Leftrightarrow \frac{d}{di_{t-1}}\frac{\sigma_K^*(i_{t-1}) - \mu_K}{i_{t-1} - \sigma_K^*(i_{t-1})} < 0.$$

Moreover, since $\eta \leq 2$,

$$\frac{d}{di_{t-1}}\frac{\sigma_K^*(i_{t-1}) - \mu_K}{i_{t-1} - \sigma_K^*(i_{t-1})} = \left(\frac{1}{i_{t-1} - \sigma_K^*(i_{t-1})}\right)^2 (i_{t-1} - \mu_K) \left(\frac{d\sigma_K^*(i_{t-1})}{di_{t-1}} - \frac{1}{1 + \frac{i_{t-1}-\sigma_K^*(i_{t-1})}{\sigma_K^*(i_{t-1})} - \mu_K}\right) < 0.$$

Fourth and last, we show item (iv). It is enough to prove that

$$\frac{\partial}{\partial \mu}\sigma_K^*(i_{t-1}, \mu) > 0.$$

On the one hand, if $i_{t-1} < \mu_K$, the Implicit Function Theorem applied to Equation (13) implies that

$$\frac{\partial}{\partial \mu}\sigma_K^*(i_{t-1}, \mu) = \frac{1}{1 + (\eta - 1)\frac{\mu_K - \sigma_K^*(i_{t-1})}{\sigma_K^*(i_{t-1}) - i_{t-1}}} > 0.$$

On the other hand, if $\mu_K < i_{t-1}$, the Implicit Function Theorem applied to Equation (15) implies that

$$\frac{\partial}{\partial \mu}\sigma_K^*(i_{t-1}, \mu) = \frac{1}{1 + (\eta - 1)\frac{i_{t-1} - \mu_K}{\sigma_K^*(i_{t-1}) - \sigma_K^*(i_{t-1})}} > 0.$$

This completes the proof.

□

Proof of Corollary 5

Given i_{t-1} the status-quo policy, we let $r = r(i_{t-1}) := \sigma_R^*(i_{t-1})$ denote the policy that candidate $k \in R$ implements if elected and $l = l(i_{t-1}) := \sigma_L^*(i_{t-1})$ denote the policy that candidate $k' \in L$ implements if elected. When deciding whether to elect k or k' in the t-th election, any voter $i \in [0, 1]$ compares the instantaneous utilities that s/he will receive from both candidates. As with linear costs, we can assume that both candidates have capacity $a_t = 0$, because a low-capacity incumbent is never re-elected. Accordingly, voter i strictly prefers k to be in office (and thus votes in his/her favor) if[38]

$$\Delta U_i(i_{t-1}) := U_i(i_{t-1}, i_{kt}, g_{kt})) - U_i(i_{t-1}, i_{k't}, g_{k't}) \geq 0,$$

which, by (1) and (12), is equivalent to

$$\Delta U_i(i_{t-1}) = -(r(i_{t-1}) - i)^2 - c \cdot |i_{t-1} - r(i_{t-1})|^\eta + (l(i_{t-1}) - i)^2 + c \cdot |i_{t-1} - l(i_{t-1})|^\eta \geq 0. \tag{28}$$

By symmetry, it trivially follows that $\Delta U_i\left(\frac{1}{2}\right) = 0$. Then, without loss of generality it suffices to consider that $i_{t-1} > \frac{1}{2}$. We distinguish two cases.

<u>*Case 1:*</u> $\frac{1}{2} < i_{t-1} < \mu_K$

From Corollary 4 we know that

$$\mu_L < l(i_{t-1}) < i_{t-1} < r(i_{t-1}) < \mu_R. \tag{29}$$

Accordingly, Condition (28) can be rewritten as

$$\Delta U_i(i_{t-1}) = -(r(i_{t-1}) - i)^2 - c \cdot (r(i_{t-1}) - i_{t-1})^\eta + (l(i_{t-1}) - i)^2 + c \cdot (i_{t-1} - l(i_{t-1}))^\eta \geq 0.$$

As with linear costs, we can focus on the decision of the median voter $i = \frac{1}{2}$, since s/he is the decisive voter in the election. Accordingly, k is elected if and only if[39]

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = -\left(r - \frac{1}{2}\right)^2 - c \cdot (r - i_{t-1})^\eta + \left(l - \frac{1}{2}\right)^2 + c \cdot (i_{t-1} - l)^\eta \geq 0, \tag{30}$$

where

$$\mu_R - r = \frac{c\eta}{2}(r - i_{t-1})^{\eta - 1} \tag{31}$$

[38]Recall that in the case of indifference, citizens vote in favor of the incumbent.
[39]For the sake of notation we do not make explicit the dependencies on i_{t-1}.

and

$$l-\mu_L=\frac{c\eta}{2}(i_{t-1}-l)^{\eta-1} \tag{32}$$

Conditions (31) and (32) follow from Proposition 3. It is also convenient to let $x:=r-i_{t-1}$ and $y:=i_{t-1}-l$. Note that due to (29), we have $x>0$ and $y>0$. Then, we can rewrite (30) as

$$\Delta U_{\frac{1}{2}}(i_{t-1})=f(x,y)=-\left(x+y\right)\left(x-y+2i_{t-1}-1\right)-cx^\eta+cy^\eta, \tag{33}$$

while (31) and (32) are now

$$x-\mu_R+i_{t-1}=-\frac{c\eta}{2}x^{\eta-1} \tag{34}$$

and

$$-y+i_{t-1}-1+\mu_R=\frac{c\eta}{2}y^{\eta-1}. \tag{35}$$

We emphasize that we have used $\mu_R=1-\mu_L$. Combining Equations (33), (34), and (35), we obtain

$$f(x,y)=-\frac{c\eta}{2}\left(x+y\right)\left(y^{\eta-1}-x^{\eta-1}\right)-cx^\eta+cy^\eta. \tag{36}$$

Now, using Equation (36), note that

$$\frac{\partial f}{\partial y}(x,y)=\frac{c\eta}{2}\left((2-\eta)y^{\eta-1}+x^{\eta-1}-(\eta-1)xy^{\eta-2}\right)$$

and hence

$$\frac{\partial f}{\partial y}(x,x)=c\eta x^{\eta-1}\left(2-\eta\right)\geq 0, \tag{37}$$

where the inequality holds because $\eta\leq 2$. Moreover,

$$\frac{\partial^2 f}{\partial y^2}(x,y)=\frac{c\eta}{2}(2-\eta)(\eta-1)y^{\eta-3}\left(x+y\right)\geq 0, \tag{38}$$

where the inequality holds because $1<\eta\leq 2$. Next, we claim that it must be the case that

$$x\leq y. \tag{39}$$

Suppose, on the contrary, that

$$x>y \tag{40}$$

which using (34) and (35) implies that

$$1-x>2i_{t-1}-y. \tag{41}$$

Adding inequalities in (40) and (41) yields

$$\frac{1}{2}>i_{t-1}.$$

Accordingly, we have arrived at a contradiction, so (39) must hold. Finally, Condition (30) follows from (37), (38) and (39).

<u>*Case 2:*</u> $\mu_K\leq i_{t-1}<1$

From Corollary 4 we know that

$$\mu_L<l(i_{t-1})<r(i_{t-1})<i_{t-1}. \tag{42}$$

Analogously to Case 1, it is easy to verify that k is now elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1})=-\left(r-\frac{1}{2}\right)^2-c\cdot(i_{t-1}-r)^\eta+\left(l-\frac{1}{2}\right)^2+c\cdot(i_{t-1}-l)^\eta\geq 0, \tag{43}$$

where

$$r - \mu_R = \frac{c\eta}{2}(i_{t-1} - r)^{\eta-1} \tag{44}$$

and

$$l - \mu_L = \frac{c\eta}{2}(i_{t-1} - l)^{\eta-1} \tag{45}$$

Conditions (44) and (45) follow again from Proposition 3. This time, it is further convenient to let $x := i_{t-1} - r$ and $y; = i_{t-1} - l$. Note that due to (42) we have $x > 0$ and $y > 0$. Then, we can rewrite (43) as[40]

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = f(x, y) = -(x - y)(x + y - 2i_{t-1} + 1) - cx^\eta + cy^\eta, \tag{46}$$

while (44) and (45) are now

$$-x - \mu_R + i_{t-1} = \frac{c\eta}{2}x^{\eta-1} \tag{47}$$

and

$$-y + i_{t-1} - 1 + \mu_R = \frac{c\eta}{2}y^{\eta-1}. \tag{48}$$

We emphasize that we have used $\mu_R = 1 - \mu_L$. Combining Equations (46), (47) and (48), we obtain

$$f(x, y) = \frac{c\eta}{2}(x - y)(y^{\eta-1} + x^{\eta-1}) - cx^\eta + cy^\eta. \tag{49}$$

Now, using Equation (49), note that

$$\frac{\partial f}{\partial y}(x, y) = \frac{c\eta}{2}\left((2 - \eta)y^{\eta-1} - x^{\eta-1} + (\eta - 1)xy^{\eta-2}\right)$$

and hence

$$\frac{\partial f}{\partial y}(x, x) = 0. \tag{50}$$

Moreover,

$$\frac{\partial^2 f}{\partial y^2} = \frac{c\eta}{2}(2 - \eta)(\eta - 1)y^{\eta-3}(y - x) \geq 0, \tag{51}$$

where the inequality holds because $1 < \eta \leq 2$ if we assume that

$$x \leq y. \tag{52}$$

Suppose, on the contrary, that

$$x > y \tag{53}$$

which using (47) and (48) implies that

$$1 - x > 2\mu_R - y. \tag{54}$$

Adding inequalities in (53) and (54) yields

$$\frac{1}{2} > \mu_R.$$

Accordingly, we have arrived at a contradiction, so (52) must hold. Finally, Condition (43) follows from (50), (51), and (52).

□

[40] For the sake of notation we again do not make explicit the dependencies on i_{t-1}.

Proof of Corollary 6

We emphasize that we already know that a necessary condition for re-election is that the incumbent should have normal capacity. To prove the corollary, we show that any incumbent always chooses a policy that is closer to his/her bliss point than to the bliss point of the other party's candidate. We assume without loss of generality that $k \in R$ is the incumbent in period t. Because of item *(i)* of Corollary 4 (see below), it suffices to analyze the case where the status quo is the leftmost possible policy, i.e., $i_{t-1} = 0$. In this case, $x := \sigma_R^*(0)$ satisfies

$$\mu_R - x = \frac{c\eta}{2} x^{\eta-1}.$$

Consider now the function

$$f(y) = \mu_R - y - \frac{c\eta}{2} y^{\eta-1}.$$

Clearly,

$$f(\mu_R) = -\frac{c\eta}{2} (\mu_R)^{\eta-1} < 0$$

and

$$f\left(\frac{1}{2}\right) = \mu_R - \frac{1}{2} - \frac{c\eta}{2} \left(\frac{1}{2}\right)^{\eta-1} > 0,$$

where the inequality holds by Condition (16). Moreover, because $\eta > 1$,

$$f'(y) < 0.$$

All the above implies that $x > \frac{1}{2}$. That is, incumbent k from party R always chooses a policy that is to the right of the median voter's preferred policy, i.e. to the right of 1/2. Item *(i)* of Corollary 4 and Corollary 5 then imply that incumbent k is re-elected in period t regardless of the status-quo policy, provided that s/he has normal capacity at the time of elections. This completes the result of the proof.

$\square$

Proof of Proposition 4

For the three items of the proposition, we apply the Implicit Function Theorem on $\Delta = \Delta(c, \eta, \Pi)$ and use the following inequalities:

$$2\Pi - \Delta > \Pi > 0.$$

For (i), we trivially have $\lim_{c \to 0} \Delta(c, \eta, \Pi) = 0$ and we obtain

$$\frac{\partial \Delta(c, \eta, \Pi)}{\partial c} = \frac{\frac{1}{c}}{\frac{1}{\Delta} + \frac{\eta-1}{2\Pi-\Delta}} > 0.$$

For (ii), we obtain

$$\frac{\partial \Delta(c, \eta, \Pi)}{\partial \eta} = \frac{\frac{1}{\eta} + \ln(2\Pi - \Delta)}{\frac{1}{\Delta} + \frac{\eta-1}{2\Pi-\Delta}} > 0,$$

where the inequality holds because

$$\frac{1}{\eta} + \ln(2\Pi - \Delta) > 0 \Leftrightarrow 2\Pi - \Delta > e^{-\frac{1}{\eta}}$$

and

$$\Pi > e^{-\frac{1}{2}} > e^{-\frac{1}{\eta}} \text{ and } \Pi - \Delta > 0.$$

For (iii), we obtain

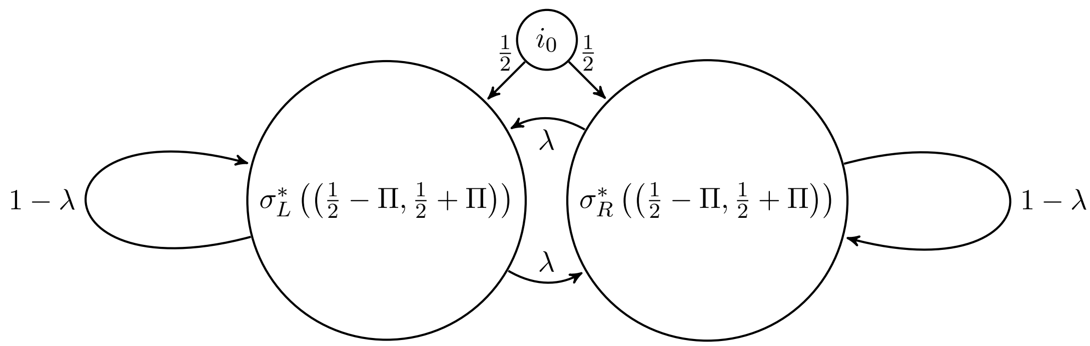
$$\frac{\partial \Delta(c,\eta,\Pi)}{\partial \Pi} = \frac{2\Delta(\eta-1)}{2\Pi + (\eta-2)\Delta} > 0.$$

The above inequality holds since $1 < \eta \leq 2$ and $2\Pi > \Delta$.

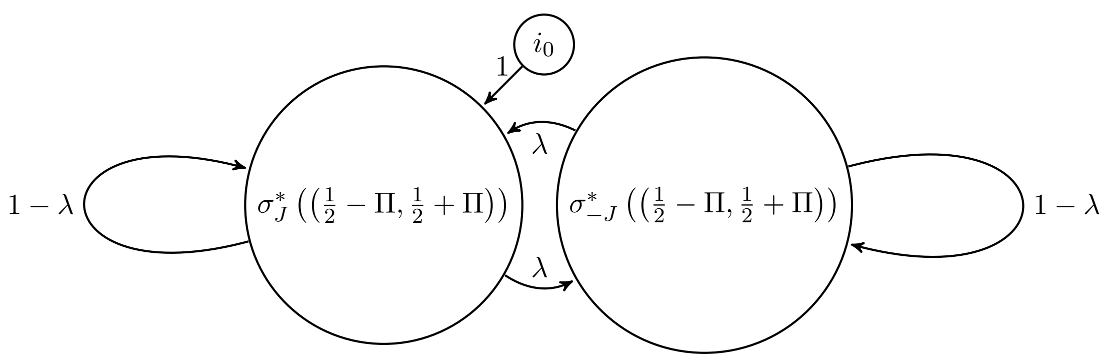
$\square$

For a given function $f : \mathbb{R} \to \mathbb{R}$, we let $f(A) := \{y : y = f(x), x \in A\}$ for any $A \subset \mathbb{R}$. All in all, we have proved the following result:

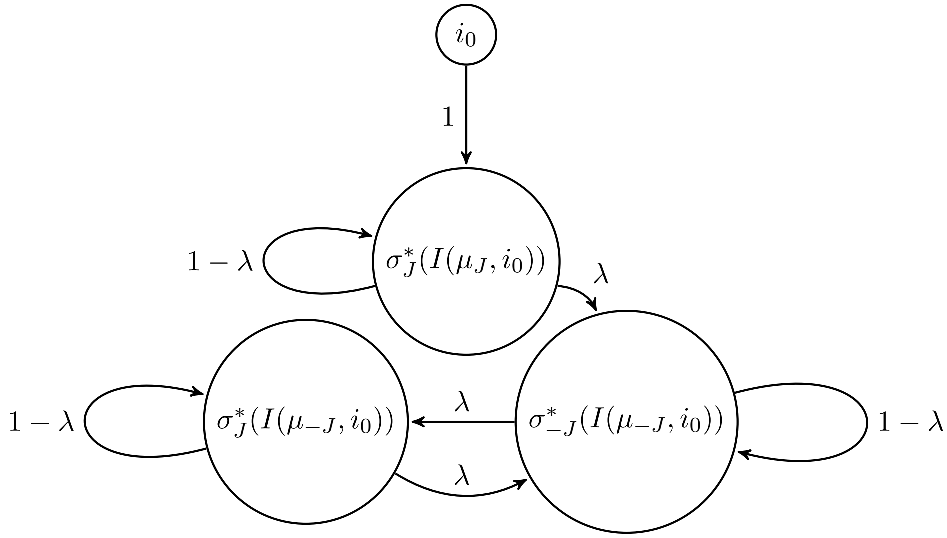
Theorem 4
Let $i_0 \in [0,1]$ be the status-quo policy in $t = 1$. Then, $\mathcal{G}^{i_0}$ has a unique MSMPE, referred to as $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$ and characterized as follows:
(i) If $\left|i_0 - \frac{1}{2}\right| = 0$, then σ^* is characterized by



(ii) If $0 < \left|i_0 - \frac{1}{2}\right| \leq \Pi$ and $J \in \{L, R\}$ satisfies $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$, then σ^* is characterized by



(iii) If $\Pi < \left|i_0 - \frac{1}{2}\right| \leq \frac{1}{2}$ and $J \in \{L, R\}$ is a party that satisfies $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$, with $I(\mu_J, i_0)$ denoting the interval between μ_J and i_0 and $I(\mu_{-J}, i_0)$ denoting the interval between μ_{-J} and i_0,



Forward-looking voters

Proof of Theorem 2

Since parties are still considered to be myopic, their policy choices are given by Proposition 1. In the present proof we thus focus on the election and not the policy choices. As in the proof of Proposition 2 where voters were also myopic, we can assume without loss of generality that $i_{t-1} \geq \frac{1}{2}$. The results for $i_{t-1} \leq \frac{1}{2}$ follow by symmetry. Let $K \in \{\emptyset, L, R\}$ denote the incumbent's party at the moment of the t-th election, with $t \geq 1$. The incumbent is denoted by k (if there is one).

When deciding whether to elect the candidate from party R or the candidate from party L in the t-th election, any voter $i \in [0,1]$ compares the utilities that s/he expects to receive if either of the two candidates are elected for office in period t. As in the proof of Proposition 2, we can focus on the case where $i = 1/2$, since the median voter is decisive in all elections. We can also focus on the case where both candidates have normal capacity since a low-capacity incumbent is never re-elected. Then, the median voter strictly prefers candidate k from party R (candidate k' from party L) to be in office if

$$\Delta U_{\frac{1}{2}} := U_{\frac{1}{2}}\left(i_{t-1}, i_{kt}, g_{kt}\right) - U_{\frac{1}{2}}(i_{t-1}, i_{k't}, g_{k't}) + \theta \cdot \Omega > (<) 0,$$

where Ω is a term capturing the expected difference in utilities—from the perspective of the median voter in period $t+1$—that will accrue in periods $t+1$, $t+2$, and so on from electing the two candidates. By (1), the above inequality can be written as

$$\Delta U_{\frac{1}{2}} = -\left(i_{kt} - \frac{1}{2}\right)^2 - c \cdot |i_{t-1} - i_{kt}| + \left(i_{k't} - \frac{1}{2}\right)^2 + c \cdot |i_{t-1} - i_{k't}| + \theta \cdot \Omega > (<) 0.$$

From Proposition 1 we know that in any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$, the policy choices i_{kt} and $i_{k't}$ are given by $\sigma_R^*(i_{t-1})$ and $\sigma_L^*(i_{t-1})$, respectively. Thus,

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = -\left(\sigma_R^*(i_{t-1}) - \frac{1}{2}\right)^2 - c \cdot |i_{t-1} - \sigma_R^*(i_{t-1})| + \left(\sigma_L^*(i_{t-1}) - \frac{1}{2}\right)^2 + c \cdot |i_{t-1} - \sigma_L^*(i_{t-1})| + \theta \cdot \Omega.$$

If $\Delta U_{1/2} = 0$, the incumbent is elected when there is one (if $t > 1$) or candidates are elected according to our tie-breaking rules when the election is an open race (if $t = 1$). In particular, if the incumbent belongs to party R, s/he is re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \geq 0.$$

If the incumbent belongs to party L, by contrast, s/he is re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \leq 0.$$

We now distinguish two different cases regarding the relative values of c and Π and analyze the sign of $\Delta U_{\frac{1}{2}}(i_{t-1})$ as a function of i_{t-1} in each case. We recall that $\Pi = \mu_R - 1/2$.

Case 1: $\frac{c}{2} < \Pi$

In this case, $\sigma_R^*(\cdot) \in [\mu_R - c/2, \mu_R + c/2]$ and $\sigma_L^*(\cdot) = \mu_L + c/2 < 1/2$. Since $i_{t-1} \geq 1/2$, it must be the case that the incumbent belongs to party R and that $i_{t-1} \in [\mu_R - c/2, \mu_R + c/2]$ or that there is no incumbent and $i_{t-1} = i_0 \geq 1/2$. We distinguish four subcases.

Case 1a: $i_{t-1} = \frac{1}{2}$

In equilibrium, this case can only happen when $t = 1$ and results in each candidate being elected with probability $1/2$.

Case 1b: $i_{t-1} \in (1/2, \mu_R - \frac{c}{2}]$

Since $\sigma_R^*(i_{t-1}) = \mu_R - \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, we obtain by simple algebra that

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = 2c \cdot \left(i_{t-1} - \frac{1}{2}\right) + \theta \cdot \Omega = 2c \cdot \left(i_{t-1} - \frac{1}{2}\right) > 0,$$

where the first equality can be obtained by algebraic manipulations and the second can be explained as follows: From the perspective of the median voter, it is equivalent from a (life-time) utility perspective for a normal-capacity candidate from party R to be elected who has chosen $\mu_R - c/2$ or for a normal-capacity candidate from party L to be elected who has chosen $\mu_L + c/2$. This holds due to the symmetry of the parties' bliss point with respect to the median voter's position—and hence of the policies that both parties choose—and because the probability that candidates from both parties suffer a capacity shock is the same in each period in which they hold power. Hence, $\Omega = 0$ and then

$$\Delta U_{\frac{1}{2}}(i_{t-1}) > 0.$$

To sum up, the candidate from party R is elected in period t (provided s/he has not received a capacity shock in the case where s/he has already held office).

Case 1c: $i_{t-1} \in \left(\mu_R - \frac{c}{2}, \mu_R + \frac{c}{2}\right)$

We start by noting that this case can only occur if $i_0 \in (\mu_R - c/2, \mu_R + c/2)$, no capacity shock has yet occurred, and the (same) candidate from party R has been elected in all prior elections (if there have been any). In particular, once the candidate from party L has been elected, Case 1c can no longer occur. We also know that in this case, $\sigma_R^*(i_{t-1}) = i_{t-1}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$. We now assume that whenever Case 1c happens for any period $t' > t$ the candidate from party R is elected, and then we check whether the median voter wants to deviate from such a decision in period t. Note that either this assumption holds in equilibrium or the candidate from party L is elected (in equilibrium), and hence the conditions that we find pin down the unique equilibrium. Let $U_R := U_R(i_{t-1})$ ($U_L := U_L(i_{t-1})$) denote the life-time utility that the median voter expects to obtain from the perspective of period t if s/he elects the candidate from party R (L), assuming that the latter has normal capacity. Then, we see that

$$U_R(i_{t-1}) = -\left(i_{t-1} - \frac{1}{2}\right)^2 + \theta\lambda \cdot U_L + \theta(1-\lambda) \cdot U_R \tag{55}$$

and

$$U_L(i_{t-1}) = -\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 - c \cdot \left(i_{t-1} - \mu_L - \frac{c}{2}\right) + \theta \cdot X, \tag{56}$$

where X is such that

$$X = -\left(\Pi - \frac{c}{2}\right)^2 - \lambda c \cdot (2\Pi - c) + \theta \cdot X. \tag{57}$$

In deriving these three equations we have assumed that candidates who have received a capacity shock are always ousted, and have then built on Case 1b. This guarantees that normal-capacity incumbents are always re-elected if their policy choices are on their side of the spectrum w.r.t. the median voter's position and at least at a distance of $c/2$ from their bliss points, and that, moreover, their choices are symmetrically located with respect to $1/2$. In particular, they are at distance $\Pi - c/2$ of $1/2$, which results in a policy shift equal to $2\Pi - c$ whenever a capacity shock occurs. We have also used the fact that the probability of receiving a capacity shock is independent of the identity of the incumbent. Equation (55) can be rewritten as

$$U_R(i_{t-1}) = -\frac{\left(i_{t-1} - \frac{1}{2}\right)^2}{1 - \theta(1-\lambda)} + \frac{\theta\lambda}{1 - \theta(1-\lambda)} \cdot U_L(i_{t-1}).$$

Then, we can define

$$\begin{aligned} \tilde{\Delta} U_{\frac{1}{2}}(i_{t-1}) &:= (1 - \theta(1-\lambda)) \cdot \Delta U_{\frac{1}{2}}(i_{t-1}) = (1 - \theta(1-\lambda)) \cdot (U_R(i_{t-1}) - U_L(i_{t-1})) \\ &= -\left(i_{t-1} - \frac{1}{2}\right)^2 - (1-\theta) \cdot U_L(i_{t-1}). \end{aligned} \tag{58}$$

Note that $\Delta U_{1/2}(i_{t-1})$ has the same sign as $\tilde{\Delta} U_{1/2}(i_{t-1})$, so it henceforth suffices to focus on the latter. In turn, Equation (57) can be written as

$$X = -\frac{1}{1-\theta} \cdot \left[\left(\Pi - \frac{c}{2}\right)^2 + \lambda c \cdot (2\Pi - c)\right].$$

Using $\mu_L + c/2 = c/2 + 1/2 - \Pi$, the above equation together with Equation (56) enables us to write

$$\begin{aligned} U_L(i_{t-1}) &= -\left(\frac{c}{2} - \Pi\right)^2 - c \cdot \left(i_{t-1} - \mu_L - \frac{c}{2}\right) - \frac{\theta}{1-\theta} \cdot \left[\left(\Pi - \frac{c}{2}\right)^2 + \lambda c \cdot (2\Pi - c)\right] \\ &= -\frac{1}{1-\theta} \cdot \left(\frac{c}{2} - \Pi\right)^2 - \frac{c}{1-\theta} \cdot \left[(1-\theta)\left(i_{t-1} - \mu_L - \frac{c}{2}\right) + \theta\lambda \cdot (2\Pi - c)\right]. \end{aligned} \tag{59}$$

Using Equations (58) and (59) then yields

$$\begin{aligned} &P(\theta) := \tilde{\Delta} U_{\frac{1}{2}}(i_{t-1}) \\ &= -\left(i_{t-1} - \frac{1}{2}\right)^2 + \left(\frac{c}{2} - \Pi\right)^2 + c \cdot \left[(1-\theta)\left(i_{t-1} - \mu_L - \frac{c}{2}\right) + \theta\lambda \cdot (2\Pi - c)\right] \\ &= -\left(i_{t-1} - \frac{1}{2}\right)^2 + \left(\frac{c}{2} - \Pi\right)^2 + c \cdot \left(i_{t-1} - \mu_L - \frac{c}{2}\right) + \theta c \cdot \left[\lambda \cdot (2\Pi - c) - \left(i_{t-1} - \mu_L - \frac{c}{2}\right)\right] \\ &= -\left(i_{t-1} - \frac{1}{2}\right)^2 + K_1 + \theta K_2 \cdot, \end{aligned}$$

where

$$K_1 := \left(\frac{c}{2} - \Pi\right)^2 + c \cdot \left(i_{t-1} - \mu_L - \frac{c}{2}\right)$$

and

$$K_2 := c \cdot \left[\lambda \cdot (2\Pi - c) - \left(i_{t-1} - \mu_L - \frac{c}{2}\right)\right].$$

Note that

$$K_2 \leq c \cdot \left[(2\Pi - c) - \left(i_{t-1} - \mu_L - \frac{c}{2}\right)\right] = c \cdot \left[\mu_R - \frac{c}{2} - i_{t-1}\right] < 0,$$

so

$$P'(\theta) = K_2 < 0. \tag{60}$$

From the proof of Case 1b in Proposition 2, we have

$$P(0) = -\left(i_{t-1} - \frac{1}{2}\right)^2 + K_1 > 0.$$

Then, there must be $\theta^{i_{t-1}} \in (0, 1]$ such that $P(\theta) \geq 0$ if and only if $\theta \leq \theta^{i_{t-1}}$. This means that the median voter is always content with his/her decision to vote for the candidate from party R (provided that s/he has normal capacity) if and only if $\theta \leq \theta^{i_{t-1}}$. For period $t = 1$, we can see that

$$\theta^{i_0} = \min\left\{1, \frac{(|i_0 - \frac{1}{2}|)^2 - K_1}{K_2}\right\} > 0.$$

<u>*Case 1d:*</u> $i_{t-1} \geq \mu_R + \frac{c}{2}$

In this case, $\sigma_R^*(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$. Using the same notation as in Case 1c, we can easily see that

$$\tilde{\Delta}U_{\frac{1}{2}}(i_{t-1}) = -\left(i_{t-1} - \frac{1}{2}\right)^2 + \left(\frac{c}{2} - \Pi\right)^2 + c \cdot \left(i_{t-1} - \mu_L - \frac{c}{2}\right) + \theta c \cdot \left[\lambda \cdot (2\Pi - c) - \left(i_{t-1} - \mu_L - \frac{c}{2}\right)\right]$$
$$-(1 - \theta(1 - \lambda)) \cdot \left[c \cdot \left(i_{t-1} - \mu_R - \frac{c}{2}\right) + \left(\mu_R + \frac{c}{2} - \frac{1}{2}\right)^2 - \left(i_{t-1} - \frac{1}{2}\right)^2\right].$$

The reason is that the only change over and against Case 1c is that if the candidate from party R is elected in period t, a policy change will occur from i_{t-1} to $\mu_R + \frac{c}{2}$ and that, as a consequence, the utility derived from policy in this same period t is equal to $-(\mu_R + c/2 - 1/2)^2$ instead of $-(i_{t-1} - 1/2)^2$. Then, by simple algebra (see also the proof of Case 1c in Proposition 2) we can verify that

$$T(\theta) := \Delta U_{\frac{1}{2}}(i_{t-1}) = \theta \cdot (K_2 + (1 - \lambda) \cdot K_3),$$

where

$$K_3 := c \cdot \left(i_{t-1} - \mu_R - \frac{c}{2}\right) + \left(\mu_R + \frac{c}{2} - \frac{1}{2}\right)^2 - \left(i_{t-1} - \frac{1}{2}\right)^2$$
$$= c\left(i_{t-1} - \frac{1}{2}\right) - \left(i_{t-1} - \frac{1}{2}\right)^2 + \left(\Pi + \frac{c}{2}\right) \cdot \left(\Pi - \frac{c}{2}\right).$$

Note that for all $i_{t-1} \geq \mu_R + \frac{c}{2}$,

$$\frac{\partial K_3(i_{t-1})}{\partial i_{t-1}} = c - 2\left(i_{t-1} - \frac{1}{2}\right) \leq -2\Pi < 0.$$

Together with the fact that

$$K_3(\mu_R + \frac{c}{2}) = 0,$$

this implies

$$K_3 \leq 0.$$

Hence, $T(\theta) < 0$ for all $\theta > 0$. That is, the candidate from party L is re-elected whenever $\theta > 0$ (provided that s/he has normal capacity).

<u>*Case 2:*</u> $\frac{c}{2} \geq \Pi$

We distinguish three subcases.

<u>*Case 2a:*</u> $i_{t-1} \in \left[\frac{1}{2}, \mu_L + \frac{c}{2}\right]$

In this case, $\sigma^*_R(i_{t-1}) = \sigma^*_L(i_{t-1}) = i_{t-1}$, so that

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = 0.$$

Thus, the incumbent is always re-elected (if $t > 1$) or candidates are elected according to our tie-breaking rules (if $t = 1$). We stress that the median voter derives the same utility regardless of who is in power and that policy i_0 is chosen in any period.

<u>*Case 2b:*</u> $i_{t-1} \in \left(\mu_L + \frac{c}{2}, \mu_R + \frac{c}{2}\right]$

Now, $\sigma^*_R(i_{t-1}) = i_{t-1}$ and $\sigma^*_L(i_{t-1}) = \mu_L + \frac{c}{2}$. Thus, we obtain the same results as in *Case 1c*.

<u>*Case 2c:*</u> $i_{t-1} > \mu_R + \frac{c}{2}$

Since $\sigma^*_R(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma^*_L(i_{t-1}) = \mu_L + \frac{c}{2}$, we now obtain the same results as in *Case 1d*.

□

Forward-looking parties

Proof of Lemma 1

We start from the last period. Clearly, given any choices $i_1, \ldots, i_{t-1}$, the optimal policy choice in period $t = T$ is given according to Proposition 1 by

$$i^*_t = \min\left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}, \mu_K + \frac{c}{2}\right\}. \tag{61}$$

Let $\Gamma^T = \frac{c}{2}$. Consider now that for any period $t = \tau + 1, \ldots, T$, with $1 \leq \tau < T$, the choice is

$$i^*_t = \min\left\{\max\left\{\mu_K - \Gamma^{\tau+1}, i_{t-1}\right\}, \mu_K + \Gamma^{\tau+1}\right\}, \tag{62}$$

with

$$\Gamma^{\tau+1} = \frac{c}{2} \cdot \frac{1}{\sum_{t=0}^{T-1-\tau} \psi^t}.$$

Then, i_τ should be chosen to maximize

$$G(i_\tau) := \sum_{t=\tau}^{T} \psi^{t-1} \cdot U_K\left(i_{t-1}, i_t, g_t\right), \tag{63}$$

where $i_{\tau+1}, \ldots, i_T$ are chosen in accordance with (62). On the one hand, if $\mu_K - \Gamma^{\tau+1} \leq i_t \leq \mu_K + \Gamma^{\tau+1}$, there is no policy shift in periods $\tau + 1, \ldots, T$, and thus we obtain

$$G(i_\tau) := -c \cdot |i_{\tau-1} - i_\tau| \cdot \psi^{t-1} - (i_\tau - \mu_K)^2 \cdot \sum_{t=\tau}^{T} \psi^{t-1} + M, \tag{64}$$

where constant M captures the utility term associated with public good provision. The same proof technique as with Proposition 1 can be directly applied here to obtain that

$$i^*_\tau = \min\left\{\max\left\{\mu_K - \Gamma^\tau, i_{t-1}\right\}, \mu_K + \Gamma^\tau\right\}$$

maximizes Expression (64), with

$$\Gamma^\tau = \frac{c}{2}\cdot\frac{1}{\sum_{s=0}^{T-\tau}\psi^s}. \tag{65}$$

On the other hand, if either $i_t < \mu_K - \Gamma^{\tau+1}$ or $i_t > \mu_K + \Gamma^{\tau+1}$, there are some costly policy changes in some period later than τ and the policy never gets closer to μ_K than $\Gamma^{\tau+1}$, hence implying

$$G(i_\tau) < \max\left\{G(\mu_K - \Gamma^{\tau+1}), G(\mu_K + \Gamma^{\tau+1})\right\}.$$

Finally, taking $\tau = 1$ in (65) yields

$$\Gamma^1 = \frac{c}{2}\cdot\frac{1}{\sum_{s=0}^{T-1}\psi^s},$$

which completes the proof.

□

Proof of Theorem 3

First of all, we analyze voter behavior for any strategy profile $\chi_K = (\chi_K^-, \chi_K^+)$ of party K, with $K \in \{L, R\}$, for some period $t \geq 1$. Because $A \gg 0$, we focus on the case where both candidates have normal capacity and for notational convenience we then drop any reference to the candidates' capacity. It seems intuitive that if this strategy profile is part of an equilibrium, there cannot exist $i_{t-1} \in [0,1]$ such that $\chi_R(i_{t-1}) < \chi_L(i_{t-1})$. The reason is that $\mu_L < \mu_R$ and costs of change are the same for both parties. Indeed, we claim—and prove below—that

$$\mu_L - \chi_L^- \leq \mu_R - \chi_R^- \tag{66}$$

and

$$\mu_L + \chi_L^+ \leq \mu_R + \chi_R^+. \tag{67}$$

Analogously to the myopic case, the median voter is the decisive voter in the election. We use $\Delta_{\frac{1}{2}}(i_{t-1})$ to denote the difference between the utility the median voter derives if candidate $k \in R$ is elected and the utility this same voter derives if candidate $-k \in L$ is elected. It follows that k is elected if $\Delta_{\frac{1}{2}}(i_{t-1}) > 0$ and $-k$ is elected if $\Delta_{\frac{1}{2}}(i_{t-1}) < 0$. By contrast, if $\Delta_{\frac{1}{2}}(i_{t-1}) = 0$, then the incumbent is re-elected if $t > 1$ and either candidate is elected with probability $\frac{1}{2}$ if $t = 1$. We next distinguish several cases.

<u>*Case 1:*</u> $i_{t-1} < \mu_L - \chi_L^-$

In this case, using the fact that $\mu_R + \mu_L = 1$, we obtain

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= c\cdot[(\mu_L - \chi_L^-) - i_{t-1}] + \left[(\mu_L - \chi_L^-) - \frac{1}{2}\right]^2 - c\cdot[(\mu_R - \chi_R^-) - i_{t-1}] - \left[(\mu_R - \chi_R^-) - \frac{1}{2}\right]^2 \\ &= [(\mu_L - \chi_L^-) - (\mu_R - \chi_R^-)]\cdot[c - \chi_L^- - \chi_R^-].\end{aligned} \tag{68}$$

<u>*Case 2:*</u> $\mu_L - \chi_L^- \leq i_{t-1} \leq \mu_L + \chi_L^+$ and $i_{t-1} < \mu_R - \chi_R^-$

In this case,

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= \left(i_{t-1} - \frac{1}{2}\right)^2 - c\cdot[(\mu_R - \chi_R^-) - i_{t-1}] - \left[(\mu_R - \chi_R^-) - \frac{1}{2}\right]^2 \\ &= [i_{t-1} - (\mu_R - \chi_R^-)]\cdot[c + i_{t-1} + (\mu_R - \chi_R^-) - 1].\end{aligned} \tag{69}$$

Case 3: $\mu_L - \chi_L^- \leq i_{t-1} \leq \mu_L + \chi_L^+$ and $\mu_R - \chi_R^- \leq i_{t-1} \leq \mu_R + \chi_R^+$

In this case,

$$\Delta_{\frac{1}{2}}(i_{t-1}) = \left(i_{t-1} - \frac{1}{2}\right)^2 - \left(i_{t-1} - \frac{1}{2}\right)^2 = 0. \tag{70}$$

Case 4: $\mu_L + \chi_L^+ \leq i_{t-1} \leq \mu_R - \chi_R^-$

In this case,

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= c \cdot [i_{t-1} - (\mu_L + \chi_L^+)] + \left[(\mu_L + \chi_L^+) - \frac{1}{2}\right]^2 - c \cdot [(\mu_R - \chi_R^-) - i_{t-1}] - \left[(\mu_R - \chi_R^-) - \frac{1}{2}\right]^2 \\ &= c \cdot [2i_{t-1} - 1] + [\chi_R^- - \chi_L^+] \cdot [c + (\mu_R - \chi_R^-) - (\mu_L + \chi_L^+)]. \end{aligned} \tag{71}$$

Case 5: $\mu_R - \chi_R^- \leq i_{t-1} \leq \mu_R + \chi_R^+$ and $\mu_L + \chi_L^+ < i_{t-1}$

In this case,

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= c \cdot [i_{t-1} - (\mu_L + \chi_L^+)] + \left[(\mu_L + \chi_L^+) - \frac{1}{2}\right]^2 - \left(i_{t-1} - \frac{1}{2}\right)^2 \\ &= [i_{t-1} - (\mu_L + \chi_L^+)] \cdot [c + 1 - i_{t-1} - (\mu_L + \chi_L^+)]. \end{aligned} \tag{72}$$

Case 6: $\mu_R + \chi_R^+ < i_{t-1}$

In this case, using the fact that $\mu_R + \mu_L = 1$, we obtain

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= c \cdot [i_{t-1} - (\mu_L + \chi_L^+)] + \left[(\mu_L + \chi_L^+) - \frac{1}{2}\right]^2 - c \cdot [i_{t-1} - (\mu_R + \chi_R^+)] - \left[(\mu_R + \chi_R^+) - \frac{1}{2}\right]^2 \\ &= [(\mu_R + \chi_R^+) - (\mu_L + \chi_L^+)] \cdot [c - \chi_L^+ - \chi_R^+]. \end{aligned} \tag{73}$$

After analyzing Cases 1–6, it remains to prove Conditions (66) and (67). By symmetry, it suffices to focus on Condition (66). Accordingly, assume that

$$\mu_R - \chi_R^- < \mu_L - \chi_L^-. \tag{74}$$

We start by analyzing voter behavior. If $i_{t-1} \in [\mu_R - \chi_R^-, \mu_L - \chi_L^-]$, then

$$\begin{aligned}\Delta_{\frac{1}{2}}(i_{t-1}) &= c \cdot [(\mu_L - \chi_L^-) - i_{t-1}] + \left[(\mu_L - \chi_L^-) - \frac{1}{2}\right]^2 - \left(i_{t-1} - \frac{1}{2}\right)^2 \\ &= [(\mu_L + \chi_L^-) - i_{t-1}] \cdot [c - 1 + i_{t-1} + (\mu_L - \chi_L^-)]. \end{aligned} \tag{75}$$

Now let x be such that $\mu_R - \chi_R^- = i_{t-1} \leq x \leq \mu_L - \chi_L^-$. On the one hand, assume that incumbent k belongs to party K. Hence, his/her lifetime utility at period t if s/he chooses policy x, given voter behavior as prescribed above, given the capacity shocks, and given the assumption that in future periods party choices will be given by χ_R and χ_L, is

$$\begin{aligned}U_K(x) = &- c \cdot (x - i_{t-1}) - (\mu_K - x)^2 \\ &+ \psi \cdot p_R(x) \cdot [U_K(x) + c \cdot (x - i_{t-1})] \\ &+ \psi \cdot p_L(x) \cdot \left[-c \cdot \left(\mu_L - \chi_L^- - x\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1 - \lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t + 1$, with $p_L(x) + p_R(x) = 1$, given that K is the incumbent's party. Accordingly,

$$U_K(x) = - c \cdot (x - i_{t-1}) - \frac{1}{1 - \psi \cdot p_R(x)} \left[(\mu_K - x)^2 + \psi \cdot p_L(x) \cdot c \cdot \left(\mu_L - \chi_L^- - x\right) - \psi \cdot p_L(x) \cdot C_L\right].$$

By (75), there exists $D \in [\mu_L + \chi_L^+, \mu_R - \chi_R^-]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \le D$ and $x \ge D$, respectively. Hence, for all $x \ne D$, we have

$$\frac{dU_K(x)}{dx} = -c + \frac{1}{1 - \psi \cdot p_R(x)} \left[2(\mu_K - x) + c \cdot \psi \cdot p_L(x)\right].$$

On the one hand, assume that k belongs to party R. Then,[41]

$$\frac{dU_R}{dx}(\mu_R - \chi_R^-) = -c + \frac{1}{1 - \psi \cdot p_R(\mu_R - \chi_R^-)} \left[2\chi_R^- + c \cdot \psi \cdot p_L(\mu_R - \chi_R^-)\right].$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_R - \chi_R^-$ than to choose $\mu_R - \chi_R^-$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_R^- \le \frac{c}{2}(1 - \psi). \tag{76}$$

On the other hand, assume that k belongs to party L. Then,

$$\frac{dU_L}{dx}(\mu_L - \chi_L^-) = -c + \frac{1}{1 - \psi \cdot p_R(\mu_L - \chi_L^-)} \left[2\chi_L^- + c \cdot \psi \cdot p_L(\mu_L - \chi_L^-)\right].$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_L - \chi_L^-$ than to choose $\mu_L - \chi_L^-$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_L^- \ge \frac{c}{2}(1 - \psi). \tag{77}$$

Finally, using (74), we deduce from (76) and (77) that

$$2\Pi < \chi_R^- - \chi_L^- \le \frac{c}{2}(1 - \psi) - \frac{c}{2}(1 - \psi) = 0,$$

a contradiction with $\Pi \ge 0$. That is, we have shown that Conditions (66) and (67) must hold.

In the following, we determine the optimal party choices in period $t \ge 1$, given voter behavior, the capacity shocks, and the assumption that in future periods party choices will be given by χ_R and χ_L. Again, we let $x \in [0, 1]$ denote the (possible) policy choice of the period-t office-holder, $k \in \{L, R\}$, and $U_K(x)$ denote his/her lifetime utility at period t if s/he chooses policy x. First, we show that only one equilibrium can exist at the most. Second, we show that there exists one equilibrium.

Uniqueness of equilibrium (for V-MSPME)

We proceed in two steps, Step 1 and Step 2. First, we consider different cases regarding χ_R and χ_L and derive certain conditions. Second, we apply such conditions to prove that for low and high values of c there exists one V-MSMPE at the most. Nevertheless, we discuss all values of c.

Step 1:

We distinguish some cases (which do not necessarily exclude each other).

Case 1.A: $0 < \mu_L - \chi_L^-$

We assume that incumbent k belongs to party L. Let x be such that $0 = i_{t-1} \le x \le \mu_L - \chi_L^-$. Then, we have

$$\begin{aligned} U_L(x) = &- c \cdot (x - i_{t-1}) - (\mu_L - x)^2 \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \chi_L^- - x\right) + C_L\right] \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

[41] If $D = \mu_R - \chi_R^-$, we could take the limit instead. Similar observations apply throughout the proof. They are omitted here.

where C_L and C_R are constants independent of x, and $p_L, p_R \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L + p_R = 1$, given that L is the incumbent's party. Note that by (68), p_L and p_R are also independent of x. It immediately follows that

$$\frac{dU_L(x)}{dx} = -c + 2(\mu_L - x) + c \cdot \psi,$$

and hence

$$\frac{dU_L}{dx}(\mu_L - \chi_L^-) = -c + 2(\mu_L - \mu_L + \chi_L^-) + c \cdot \psi = -c + 2\chi_L^- + c \cdot \psi.$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_L - \chi_L^-$ than to choose $\mu_L - \chi_L^-$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_L^- \geq \frac{c}{2}(1 - \psi). \tag{78}$$

Finally, because $\chi_L^- < \mu_L$, it must be that

$$c < \frac{1 - 2\Pi}{1 - \psi}.$$

Hence, we obtain

$$\mu_L - \chi_L^- > 0 \Rightarrow c < \frac{1 - 2\Pi}{1 - \psi}. \tag{79}$$

Case 1.B: $\mu_L + \chi_L^+ < \mu_R - \chi_R^-$

First of all, note that

$$\chi_L^+ + \chi_R^- < \mu_R - \mu_L = 2\Pi.$$

Hence,

$$\chi_L^+ < \Pi \quad \text{or} \quad \chi_R^- < \Pi. \tag{80}$$

On the one hand, assume that incumbent k belongs to party R. We focus first on the case where $i_{t-1} = \mu_L + \chi_L^+ \leq x \leq \mu_R - \chi_R^-$. Then, we have

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ \psi \cdot p_L(x) \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right] \\ &+ \psi \cdot p_R(x) \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_L and C_R are constants independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x) + p_R(x) = 1$, given that the incumbent's party is R. Note that by (71), there exists $D \in [\mu_L + \chi_L^+, \mu_R - \chi_R^-]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$ respectively. It then follows that for all $x \in [\mu_L + \chi_L^+, \mu_R - \chi_R^-]$ such that $x \neq D$, we have

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) + c \cdot \psi(2p_R(x) - 1).$$

Now, note that

$$\lim_{x \to \mu_R - \chi_R^-} \frac{dU_R}{dx}(x) = -c + 2\chi_R^- + c \cdot \psi \cdot (2p_R(\mu_R - \chi_R^-) - 1).$$

If the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_R - \chi_R^-$ than to choose $\mu_R - \chi_R^-$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_R^- \geq \frac{c}{2} \cdot [1 - \psi(2p_R(\mu_R - \chi_R^-) - 1)].$$

Note that because k is the incumbent and s/he will be ousted from office if s/he is affected by a capacity shock—which will happen with probability λ—, it must be that $p_R(\mu_R - \chi_R^-) \leq 1 - \lambda$. Hence,

$$\frac{c}{2} \cdot [1 - \psi(2p_R(\mu_R - \chi_R^-) - 1)] \geq \frac{c}{2} \cdot [1 - \psi(2(1-\lambda) - 1)].$$

Accordingly,

$$\chi_R^- \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]. \tag{81}$$

On the other hand, assume that incumbent k belongs to party L. We focus on the case where $\mu_L + \chi_L^+ \leq x \leq i_{t-1} = \mu_R - \chi_R^-$. Then, we have

$$\begin{aligned} U_L(x) = &- c \cdot (i_{t-1} - x) - (\mu_L - x)^2 \\ &+ \psi \cdot p_L(x) \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right] \\ &+ \psi \cdot p_R(x) \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_L and C_R are constants independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x) + p_R(x) = 1$, given that the incumbent's party is L. Note that by (71), there exists $D \in [\mu_L + \chi_L^+, \mu_R - \chi_R^-]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$ respectively. It then follows that for all $x \in [\mu_L + \chi_L^+, \mu_R - \chi_R^-]$ such that $x \neq D$, we have

$$\frac{dU_L(x)}{dx} = c + 2(\mu_L - x) + c \cdot \psi(1 - 2p_L(x)).$$

Now, note that

$$\frac{dU_L}{dx}(\mu_L + \chi_L^+) = c - 2\chi_L^+ + c \cdot \psi \cdot (1 - 2p_L(\mu_L + \chi_L^-)).$$

If the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_L + \chi_L^-$ than to choose $\mu_L + \chi_L^-$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned limit is non-positive or, equivalently, that

$$\chi_L^+ \geq \frac{c}{2} \cdot [1 - \psi(2p_L(\mu_L + \chi_L^+) - 1)].$$

Note that because k is the incumbent and s/he will be ousted from office if s/he is affected by a capacity shock—which will happen with probability λ—, it must be that $p_L(\mu_L + \chi_L^+) \leq 1 - \lambda$. Hence,

$$\frac{c}{2} \cdot [1 - \psi(2p_L(\mu_L + \chi_L^+) - 1)] \geq \frac{c}{2} \cdot [1 - \psi(2(1-\lambda) - 1)].$$

Accordingly,

$$\chi_L^+ \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]. \tag{82}$$

Finally, (80), (81), and (82) imply that

$$c < \frac{2\Pi}{1 + \psi(2\lambda - 1)}. \tag{83}$$

That is,

$$\mu_L + \chi_L^+ < \mu_R - \chi_R^- \Rightarrow c < \frac{2\Pi}{1 + \psi(2\lambda - 1)}. \tag{84}$$

Case 1.C: $\mu_R + \chi_R^+ < 1$

We assume that incumbent k belongs to party R. Let x be such that $\mu_R + \chi_R^+ \leq x \leq i_{t-1} = 1$. Then, we have

$$\begin{aligned} U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right] \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(x - \mu_R - \chi_R^+\right) + C_R\right], \end{aligned}$$

where C_L and C_R are constants independent of x, and $p_L, p_R \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L + p_R = 1$, given that R is the incumbent's party. Note that by (73), p_L and p_R are also independent of x. It immediately follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) - c \cdot \psi,$$

and hence

$$\frac{dU_R}{dx}(\mu_R + \chi_R^+) = c + 2(\mu_R - \mu_R - \chi_R^+) - c \cdot \psi = c - 2\chi_R^+ - c \cdot \psi.$$

If the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_R + \chi_R^+$ than to choose $\mu_R + \chi_R^+$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_R^+ \geq \frac{c}{2}(1-\psi). \tag{85}$$

Finally, because $\chi_R^+ < 1 - \mu_R$, it must be that

$$c < \frac{1 - 2\Pi}{1 - \psi}.$$

Hence, we obtain

$$\mu_R + \chi_R^+ < 1 \Rightarrow c < \frac{1-2\Pi}{1-\psi}. \tag{86}$$

Case 1.D: $\mu_L - \chi_L^- < \mu_L + \chi_L^+ \leq \mu_R - \chi_R^- < \mu_R + \chi_R^+$

On the one hand, we assume that incumbent k belongs to party L. We distinguish two cases. First, let x be such that

$$\mu_L - \chi_L^- = i_{t-1} \leq x \leq \mu_L + \chi_L^+.$$

Then, we have

$$\begin{aligned} U_L(x) = & - c \cdot (x - i_{t-1}) - (\mu_L - x)^2 \\ & + \psi \cdot p_L(x) \cdot [U_L(x) + c \cdot (x - i_{t-1}))] \\ & + \psi \cdot p_R(x) \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_R is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x) + p_R(x) = 1$, given that L is the incumbent's party. Note that by (69), there is $D \in [\mu_L - \chi_L^-, \mu_L + \chi_L^+]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. It immediately follows that

$$U_L(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1 - \psi p_L(x)} \cdot \left[(\mu_L - x)^2 + \psi p_R(x) c(\mu_R - \chi_R^- - x) - \psi p_R(x) C_R\right].$$

Accordingly, for all $x \neq D$, we have

$$\frac{dU_L(x)}{dx} = -c + \frac{1}{1 - \psi p_L(x)} \cdot [2(\mu_L - x) + c\psi p_R(x)],$$

and hence

$$\frac{dU_L}{dx}\left(\mu_L - \chi_L^-\right) = -c + \frac{1}{1 - \psi p_L(\mu_L - \chi_L^-)} \cdot \left[2\chi_L^- + c\psi p_R(\mu_L - \chi_L^-)\right].$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_L - \chi_L^-$ than to choose $\mu_L - \chi_L^-$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_L^- \leq \frac{c}{2}(1-\psi). \tag{87}$$

Second, let x be such that

$$\mu_L - \chi_L^- \leq x \leq i_{t-1} = \mu_L + \chi_L^+.$$

Then, we have

$$\begin{aligned} U_L(x) = &- c \cdot (i_{t-1} - x) - (\mu_L - x)^2 \\ &+ \psi \cdot p_L(x) \cdot [U_L(x) + c \cdot (i_{t-1} - x))] \\ &+ \psi \cdot p_R(x) \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_R is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x) + p_R(x) = 1$, given that L is the incumbent's party. Note that by (69), there is $D \in [\mu_L - \chi_L^-, \mu_L + \chi_L^+]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. It immediately follows that

$$U_L(x) = -c \cdot (i_{t-1} - x) - \frac{1}{1 - \psi p_L(x)} \cdot \left[(\mu_L - x)^2 + \psi p_R(x) c(\mu_R - \chi_R^- - x) - \psi p_R(x) C_R\right].$$

Accordingly, for all $x \neq D$, we have

$$\frac{dU_L(x)}{dx} = c + \frac{1}{1 - \psi p_L(x)} \cdot \left[2(\mu_L - x) + c\psi p_R(x)\right],$$

and hence

$$\frac{dU_L}{dx}\left(\mu_L + \chi_L^+\right) = c + \frac{1}{1 - \psi p_L(\mu_L + \chi_L^+)} \cdot \left[-2\chi_L^+ + c\psi p_R(\mu_L + \chi_L^+)\right].$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_L + \chi_L^+$ than to choose $\mu_L + \chi_L^+$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_L^+ \leq \frac{c}{2}[1 + \psi(1 - 2p_L(\mu_L + \chi_L^+))]. \tag{88}$$

In particular, it must be that

$$\chi_L^+ \leq \frac{c}{2}(1+\psi). \tag{89}$$

Using Equation (69), we have

$$\Delta_{\frac{1}{2}}(\mu_L + \chi_L^+) = [(\mu_L + \chi_L^+) - (\mu_R - \chi_R^-)] \cdot [c + \chi_L^+ - \chi_R^-] < 0$$

if and only if

$$\chi_R^- - \chi_L^+ < c. \tag{90}$$

We claim—and prove below—that Condition (90) holds. Accordingly, $p_L(\mu_L + \chi_L^+) = 1 - \lambda$ and

$$\chi_L^+ \leq \frac{c}{2}[1 + \psi(2\lambda - 1)]. \tag{91}$$

On the other hand, we assume that incumbent k belongs to party R. We distinguish two cases. First, let x be such that

$$\mu_R - \chi_R^- \leq x \leq i_{t-1} = \mu_R + \chi_R^+.$$

Then, we have

$$\begin{aligned}U_R(x) = & - c\cdot(i_{t-1}-x) - (\mu_R - x)^2 \\ & + \psi\cdot p_R(x)\cdot[U_R(x) + c\cdot(i_{t-1}-x))] \\ & + \psi\cdot p_L(x)\cdot\left[-c\cdot\left(x-\mu_L-\chi_L^+\right)+C_L\right],\end{aligned}$$

where C_L is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x)+p_R(x)=1$, given that R is the incumbent's party. Note that by (72), there is $D \in [\mu_R - \chi_R^-, \mu_R + \chi_R^+]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. It immediately follows that

$$U_R(x) = -c\cdot(i_{t-1}-x) - \frac{1}{1-\psi p_R(x)}\cdot\left[(\mu_R-x)^2 + \psi\cdot p_L(x)c\cdot(x-\mu_L-\chi_L^+) - \psi\cdot p_L(x)\cdot C_L\right].$$

Accordingly, for all $x \neq D$, we have

$$\frac{dU_R(x)}{dx} = c + \frac{1}{1-\psi p_R(x)}\cdot[2(\mu_R - x) - c\psi p_L(x)],$$

and hence

$$\frac{dU_R(x)}{dx}\left(\mu_R+\chi_R^+\right) = c - \frac{1}{1-\psi p_R(\mu_R+\chi_R^+)}\cdot\left[2\chi_R^+ + c\psi p_L(\mu_R+\chi_R^+)\right].$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_R+\chi_R^+$ than to choose $\mu_R+\chi_R^+$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_R^+ \leq \frac{c}{2}(1-\psi). \tag{92}$$

Second, let x be such that

$$\mu_R - \chi_R^- = i_{t-1} \leq x \leq \mu_R + \chi_R^+.$$

Then, we have

$$\begin{aligned}U_R(x) = & - c\cdot(x-i_{t-1}) - (\mu_R - x)^2 \\ & + \psi\cdot p_R(x)\cdot[U_R(x) + c\cdot(x-i_{t-1}))] \\ & + \psi\cdot p_L(x)\cdot\left[-c\cdot\left(x-\mu_L-\chi_L^+\right)+C_L\right],\end{aligned}$$

where C_L is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x)+p_R(x)=1$, given that R is the incumbent's party. Note that by (72), there is $D \in [\mu_R - \chi_R^-, \mu_R + \chi_R^+]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. It immediately follows that

$$U_R(x) = -c\cdot(x-i_{t-1}) - \frac{1}{1-\psi p_R(x)}\cdot\left[(\mu_R-x)^2 + \psi\cdot p_L(x)c\cdot(x-\mu_L-\chi_L^+) - \psi\cdot p_L(x)\cdot C_L\right].$$

Accordingly, for all $x \neq D$, we have

$$\frac{dU_R(x)}{dx} = -c + \frac{1}{1-\psi p_R(x)}\cdot[2(\mu_R - x) - c\psi p_L(x)],$$

and hence

$$\frac{dU_R}{dx}\left(\mu_R-\chi_R^-\right) = -c + \frac{1}{1-\psi p_R(\mu_R-\chi_R^-)}\cdot\left[2\chi_R^- - c\psi p_L(\mu_R-\chi_R^-)\right].$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_R-\chi_R^-$ than to choose $\mu_R-\chi_R^-$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_R^- \leq \frac{c}{2}[1 + \psi(1 - 2p_R(\mu_R - \chi_R^-))].$$

In particular, it must be that

$$\chi_R^- \leq \frac{c}{2}(1 + \psi). \tag{93}$$

Using Equation (72), we have

$$\Delta_{\frac{1}{2}}(\mu_R - \chi_R^-) = [(\mu_R - \chi_R^-) - (\mu_L + \chi_L^+)] \cdot [c + \chi_R^- - \chi_L^+] > 0$$

if and only if

$$\chi_L^+ - \chi_R^- < c. \tag{94}$$

We claim—and prove next—that Condition (94) holds. Accordingly, $p_R(\mu_R - \chi_R^-) = 1 - \lambda$ and

$$\chi_R^- \leq \frac{c}{2}[1 + \psi(2\lambda - 1)]. \tag{95}$$

Finally, it remains to prove the claims in Conditions (90) and (94). On the one hand, using (81) and (89), we obtain

$$\chi_L^+ - \chi_R^- < \frac{c}{2} \cdot (1 + \psi) - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] = \psi(1 - \lambda) \cdot c < c.$$

On the other hand, using (82) and (93), we obtain

$$\chi_R^- - \chi_L^+ \leq \frac{c}{2} \cdot (1 + \psi) - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] = \psi(1 - \lambda) \cdot c < c.$$

This completes the proof of this case.

Case 1.E: $0 = \mu_L - \chi_L^- < \mu_R - \chi_R^-$

We assume that incumbent k belongs to party L. Let $0 = i_{t-1} \leq x$ be such that

$$x \leq \mu_R - \chi_R^- \quad \text{and} \quad x \leq \mu_L + \chi_L^+.$$

Then, we have

$$\begin{aligned} U_L(x) = &- c \cdot (x - i_{t-1}) - (\mu_L - x)^2 \\ &+ \psi \cdot p_L \cdot [U_L(x) + c \cdot (x - i_{t-1}))] \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_R is a constant independent of x, and $p_L, p_R \in \{0, \lambda, 1 - \lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t + 1$, with $p_L + p_R = 1$, given that L is the incumbent's party. By (68), p_L and p_R are independent of x. It immediately follows that

$$U_L(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1 - \psi p_L} \cdot \left[(\mu_L - x)^2 + \psi p_R c(\mu_R - \chi_R^- - x) - \psi p_R C_R\right].$$

Hence,

$$\frac{dU_L}{dx}(0) = -c + \frac{1}{1 - \psi p_L} \cdot [2\mu_L + \psi p_R \cdot c].$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_L - \chi_L^- = 0$ than to choose 0 itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$c \geq \frac{1 - 2\Pi}{1 - \psi}. \tag{96}$$

Case 1.F: $0 = \mu_L - \chi_L^+ = \mu_R - \chi_R^-$

We assume that the incumbent belongs to party R. Let $0 = i_{t-1} \leq x$ be such that

$$x \leq \mu_R + \chi_R^+ \quad \text{and} \quad x \leq \mu_L + \chi_L^+.$$

Then, we trivially have

$$U_R(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1-\psi}(\mu_R - x)^2,$$

Hence,

$$\frac{dU_R}{dx}(0) = -c + \frac{2}{1-\psi} \cdot \mu_R.$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of 0 than to choose 0 itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$c \geq \frac{1+2\Pi}{1-\psi} \geq \frac{1-2\Pi}{1-\psi}. \tag{97}$$

Case 1.G: $\mu_L + \chi_L^+ < \mu_R + \chi_R^+ = 1$

We assume that incumbent k belongs to party R. Let $x \leq i_{t-1} = 1$ be such that

$$\mu_R - \chi_R^- \leq x \quad \text{and} \quad \mu_L + \chi_L^+ \leq x.$$

Then, we have

$$\begin{aligned} U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ &+ \psi \cdot p_R \cdot [U_R(x) + c \cdot (i_{t-1} - x))] \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x, and $p_L, p_R \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L + p_R = 1$, given that R is the incumbent's party. By (73), p_L and p_R are independent of x. It immediately follows that

$$U_R(x) = -c \cdot (i_{t-1} - x) - \frac{1}{1-\psi p_R} \cdot \left[(\mu_R - x)^2 + \psi p_L c(x - \mu_L - \chi_L^+) - \psi p_L C_L\right].$$

Hence,

$$\frac{dU_R}{dx}(1) = c + \frac{1}{1-\psi p_R} \cdot [2(\mu_R - 1) - \psi p_L \cdot c].$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_R + \chi_R^+ = 1$ than to choose 1 itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$c \geq \frac{1-2\Pi}{1-\psi}. \tag{98}$$

Case 1.H: $\mu_L + \chi_L^+ = \mu_R + \chi_R^+ = 1$

We assume that the incumbent belongs to party L. Let $x \leq i_{t-1} = 1$ be such that

$$x \geq \mu_L - \chi_L^- \quad \text{and} \quad x \geq \mu_R - \chi_R^-.$$

Then, we trivially have

$$U_L(x) = -\,c \cdot (i_{t-1} - x) - \frac{1}{1-\psi}(\mu_L - x)^2,$$

Hence,

$$\frac{dU_L}{dx}(1) = c + \frac{2}{1-\psi} \cdot (\mu_L - 1).$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of 1 than to choose 1 itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$c \geq \frac{1+2\Pi}{1-\psi} \geq \frac{1-2\Pi}{1-\psi}. \tag{99}$$

Case 1.I: $\mu_L - \chi_L^- = \mu_L + \chi_L^+$

Clearly, it follows that

$$\mu_L - \chi_L^- = \mu_L + \chi_L^+ = \mu_L \leq \mu_R - \chi_R^-.$$

We focus on the case where x is such that $0 = i_{t-1} \leq x \leq \mu_L$ and assume that incumbent k belongs to party L. Then,

$$\begin{aligned} U_L(x) = &-\,c \cdot (x - i_{t-1}) - (\mu_L - x)^2 \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \chi_L^- - x\right) + C_L\right] \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_L and C_R are constants independent of x, and $p_L, p_R \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L + p_R = 1$, given that L is the incumbent's party. Note that by (68), p_L and p_R are also independent of x. It immediately follows that if $c > 0$,

$$\frac{dU_L}{dx}(\mu_L) = -c \cdot (1-\psi) < 0,$$

and hence k would do better to choose a certain policy x to the left of μ_L than to choose μ_L itself, as prescribed by χ_L. However, this would contradict the fact that the latter strategy is part of an equilibrium.

Case 1.J: $\mu_R - \chi_R^- = \mu_R + \chi_R^+$

Clearly, it follows that

$$\mu_L + \chi_L^+ \leq \mu_R - \chi_R^- = \mu_R = \mu_R + \chi_R^+.$$

We focus on the case where x is such that $\mu_R \leq x \leq i_{t-1} = 1$ and assume that incumbent k belongs to party R. Then,

$$\begin{aligned} U_R(x) = &-\,c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right] \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(x - \mu_R - \chi_R^+\right) + C_R\right], \end{aligned}$$

where C_L and C_R are constants independent of x, and $p_L, p_R \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L + p_R = 1$, given that R is the incumbent's party. Note that by (73), p_L and p_R are also independent of x. It immediately follows that if $c > 0$,

$$\frac{dU_R}{dx}(\mu_R) = c \cdot (1-\psi) > 0,$$

and hence k would do better to choose a certain policy x to the right of μ_R than to choose μ_R itself, as prescribed by χ_R. However, this would contradict the fact that the latter strategy is part of an equilibrium.

Case 1.K: $\mu_R - \chi_R^- < \mu_L + \chi_L^+$

We start by noting that it must be that

$$2\Pi = \mu_R - \mu_L \leq \chi_R^- + \chi_L^+. \tag{100}$$

First, we focus on the case where $\mu_R - \chi_R^- \leq x \leq i_{t-1} = \mu_L + \chi_L^+$. Assume that incumbent k belongs to party L. It easily follows that

$$U_L(x) = -c \cdot (i_{t-1} - x) - \frac{1}{1-\psi}(\mu_L - x)^2.$$

Then,

$$\frac{dU_L}{dx}(\mu_L + \chi_L^+) = c - \frac{2}{1-\psi}\chi_L^+.$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of i_{t-1} than to choose i_{t-1} itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_L^+ \leq \frac{c}{2}(1-\psi). \tag{101}$$

Second, we focus on the case where $\mu_R - \chi_R^- = i_{t-1} \leq x \leq \mu_L + \chi_L^+$. Assume that incumbent k belongs to party R. It easily follows that

$$U_R(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1-\psi}(\mu_R - x)^2.$$

Then,

$$\frac{dU_R}{dx}(\mu_R - \chi_R^-) = -c + \frac{2}{1-\psi}\chi_R^-.$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of i_{t-1} than to choose i_{t-1} itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_R^- \leq \frac{c}{2}(1-\psi). \tag{102}$$

Adding (101) and (102), we obtain

$$c \geq \frac{\chi_L^+ + \chi_R^-}{1-\psi} \geq \frac{2\Pi}{1-\psi}, \tag{103}$$

where the inequality follows from (100).

Case 1.L: $0 \leq \mu_L - \chi_L^- < \mu_R - \chi_R^- \leq \mu_L + \chi_L^+$

We focus on the case where $\mu_L - \chi_L^- = i_{t-1} \leq x \leq \mu_R - \chi_R^-$. Then,

$$\begin{aligned} U_K(x) = &- c \cdot (x - i_{t-1}) - (\mu_K - x)^2 \\ &+ \psi \cdot p_L(x) \cdot [U_K(x) + c \cdot (x - i_{t-1})] \\ &+ \psi \cdot p_R(x) \cdot \left[-c \cdot \left(\mu_R - \chi_R^- - x\right) + C_R\right], \end{aligned}$$

where C_R is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1-\lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t+1$, with $p_L(x) + p_R(x) = 1$, given that K is the incumbent's party. Accordingly,

$$U_K(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1 - \psi \cdot p_L(x)} \left[(\mu_K - x)^2 + \psi \cdot p_R(x) \cdot c \cdot (\mu_R - \chi_R^- - x) - p_R(x) \cdot \psi \cdot C_R\right].$$

Note that by (69), there is $D \in [\mu_L - \chi_L^-, \mu_R - \chi_R^-]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. Then, for $x \neq D$,

$$\frac{dU_K(x)}{dx} = -c + \frac{1}{1 - \psi \cdot p_L(x)} \left[2(\mu_K - x) + c \cdot \psi \cdot p_R(x)\right]. \tag{104}$$

On the one hand, assume that incumbent k belongs to party R. Then,

$$\frac{dU_R}{dx}(\mu_R - \chi_R^-) = -c + \frac{1}{1 - \psi \cdot p_L(\mu_R - \chi_R^-)} \left[2\chi_R^- + c \cdot \psi \cdot p_R(\mu_R - \chi_R^-)\right]. \tag{105}$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $\mu_R - \chi_R^-$ than to choose $\mu_R - \chi_R^-$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-negative or, equivalently, that

$$\chi_R^- \geq \frac{c}{2} \cdot (1 - \psi). \tag{106}$$

Moreover, because $\chi_R^- < \mu_R$, it must be that

$$c < \frac{1 + 2\Pi}{1 - \psi}. \tag{107}$$

On the other hand, assume that incumbent k belongs to party L. Then,

$$\frac{dU_L}{dx}(\mu_L - \chi_L^-) = -c + \frac{1}{1 - \psi \cdot p_L(\mu_L - \chi_L^-)} \left[2\chi_L^- + c \cdot \psi \cdot p_R(\mu_L - \chi_L^-)\right]. \tag{108}$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $i_{t-1} = \mu_L - \chi_L^-$ than to choose $\mu_L - \chi_L^-$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_L^- \leq \frac{c}{2} \cdot (1 - \psi). \tag{109}$$

Case 1.M: $\mu_R - \chi_R^- \leq \mu_L + \chi_L^+ < \mu_R + \chi_R^+ \leq 1$

We focus on the case where $\mu_L + \chi_L^+ = x \leq i_{t-1} = \mu_R + \chi_R^+$. Then,

$$\begin{aligned} U_K(x) = &- c \cdot (i_{t-1} - x) - (\mu_K - x)^2 \\ &+ \psi \cdot p_R(x) \cdot [U_K(x) + c \cdot (i_{t-1} - x)] \\ &+ \psi \cdot p_L(x) \cdot \left[-c \cdot \left(x - \mu_L - \chi_L^+\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x, and $p_L(x), p_R(x) \in \{0, \lambda, 1 - \lambda, 1\}$ denote the probabilities that parties L and R respectively will be in power in period $t + 1$, with $p_L(x) + p_R(x) = 1$, given that K is the incumbent's party. Accordingly,

$$U_K(x) = -c \cdot (i_{t-1} - x) - \frac{1}{1 - \psi \cdot p_R(x)} \left[(\mu_K - x)^2 + \psi \cdot p_L(x) \cdot c \cdot (x - \mu_L - \chi_L^+) - p_L(x) \cdot \psi \cdot C_L\right].$$

Note that by (72), there is $D \in [\mu_L + \chi_L^+, \mu_R + \chi_R^+]$ such that $p_L(x)$ and $p_R(x)$ are constant for $x \leq D$ and $x \geq D$. Then, for $x \neq D$,

$$\frac{dU_K(x)}{dx} = c + \frac{1}{1 - \psi \cdot p_R(x)} \left[2(\mu_K - x) - c \cdot \psi \cdot p_L(x)\right]. \tag{110}$$

On the one hand, assume that incumbent k belongs to party L. Then,

$$\frac{dU_L}{dx}(\mu_L + \chi_L^-) = c - \frac{1}{1 - \psi \cdot p_R(\mu_L + \chi_L^+)} \left[2\chi_L^+ + c \cdot \psi \cdot p_L(\mu_L + \chi_L^+)\right]. \tag{111}$$

Now, if the above expression is positive, k would do better to choose a certain policy x to the right of $\mu_L + \chi_L^+$ than to choose $\mu_L + \chi_L^+$ itself, as prescribed by χ_L. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_L^+ \geq \frac{c}{2} \cdot (1-\psi). \tag{112}$$

Moreover, because $\chi_L^+ < 1 - \mu_L$, it must be that

$$c < \frac{1+2\Pi}{1-\psi}. \tag{113}$$

On the other hand, assume that incumbent k belongs to party R. Then,

$$\frac{dU_R}{dx}(\mu_R + \chi_R^-) = c - \frac{1}{1-\psi \cdot p_R(\mu_R + \chi_R^+)} \left[2\chi_R^+ + c \cdot \psi \cdot p_L(\mu_R + \chi_R^+)\right]. \tag{114}$$

Now, if the above expression is negative, k would do better to choose a certain policy x to the left of $i_{t-1} = \mu_R + \chi_R^+$ than to choose $\mu_R + \chi_R^+$ itself, as prescribed by χ_R. Because this would contradict the fact that the latter strategy is part of an equilibrium, it must be that the aforementioned expression is non-positive or, equivalently, that

$$\chi_R^+ \leq \frac{c}{2} \cdot (1-\psi). \tag{115}$$

Step 2:

In the following, we focus on the cases where either c is sufficiently small or it sufficiently large. More specifically, we assume that either

$$c < \frac{2\Pi}{1+\psi} \tag{116}$$

or

$$c \geq \frac{1+2\Pi}{1-\psi}. \tag{117}$$

Note that (116) implies

$$c < \frac{2\Pi}{1+\psi(2\lambda-1)}. \tag{118}$$

We prove that provided that either (116) or (118) hold, if there is a V-MSPME (χ_L, χ_R), it must be (outcome-equivalent to the V-MSPME such) that

$$\chi_L^- = \chi_R^+ = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1-\psi)\right\} \quad \text{and} \quad \chi_R^- = \chi_L^+ = \min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right\}.$$

We stress that

$$\frac{c}{2} \cdot (1-\psi) < \frac{1}{2} - \Pi \Longleftrightarrow c < \frac{1-2\Pi}{1-\psi} \tag{119}$$

and

$$\frac{c}{2} \cdot [1+\psi(2\lambda-1)] < \Pi \Longleftrightarrow c < \frac{2\Pi}{1+\psi(2\lambda-1)}. \tag{120}$$

Accordingly, we henceforth let (χ_L, χ_R) be an equilibrium. Because $c > 0$, we have $\chi_R^+, \chi_L^+, \chi_R^-, \chi_L^- > 0$—see Cases 1.I and 1.J. We stress that Definition 4 implies that

$$\mu_L^-, \mu_R^+ \geq \frac{1}{2} - \Pi \Longrightarrow \mu_L^-, \mu_R^+ = \frac{1}{2} - \Pi \tag{121}$$

and

$$\mu_L^+, \mu_R^- \geq \frac{1}{2} + \Pi \Longrightarrow \mu_L^+, \mu_R^- = \frac{1}{2} + \Pi. \tag{122}$$

In what follows, we distinguish three different cases from which we immediately derive the desired results.

Case 2.A: $c < \frac{2\Pi}{1+\psi}$ and $c < \frac{1-2\Pi}{1-\psi}$

First, note that it cannot be the case that $\mu_R - \chi_R^- < \mu_L + \chi_L^+$, since Case 1.K—see (103)—would then imply that

$$c \geq \frac{2\Pi}{1-\psi} > \frac{2\Pi}{1+\psi}.$$

Hence, $\mu_L + \chi_L^+ \leq \mu_R - \chi_R^-$. Then, Case 1.D—see (87), (91), (92), and (95)—implies that

$$\chi_L^- \leq \frac{c}{2} \cdot (1-\psi) \quad \text{and} \quad \chi_R^+ \leq \frac{c}{2} \cdot (1-\psi)$$

and

$$\chi_R^- \leq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \quad \text{and} \quad \chi_L^+ \leq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)].$$

Second, note that it must be that $0 < \mu_L - \chi_L^-$ and $\mu_R + \chi_R^+ < 1$, since otherwise Cases 1.E, 1.F, 1.G, and 1.H—see (96), (97), (98), and (99)—would then imply that

$$c \geq \frac{1-2\Pi}{1-\psi}.$$

Accordingly, Cases 1.A and 1.C—see (78) and (85)—imply respectively that

$$\chi_L^- \geq \frac{c}{2} \cdot (1-\psi) \quad \text{and} \quad \chi_R^+ \geq \frac{c}{2} \cdot (1-\psi).$$

Third, note that if $\mu_L + \chi_L^+ = \mu_R - \chi_R^-$, it must be the case that

$$2\Pi = \mu_R - \mu_L = \chi_L^+ + \chi_R^- \leq c \cdot [1 + \psi(2\lambda - 1)] \leq c(1+\psi) < 2\Pi,$$

a contradiction. This means that

$$\mu_L + \chi_L^+ < \mu_R - \chi_R^-.$$

Then Case 1.B—see (81) and (82)—implies that

$$\chi_R^- \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \quad \text{and} \quad \chi_L^+ \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)].$$

All in all,

$$\chi_L^- = \chi_R^+ = \frac{c}{2} \cdot (1-\psi) = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1-\psi)\right\}$$

and

$$\chi_R^- = \chi_L^+ = \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] = \min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right\},$$

where the last equalities in the above two chains of equalities can be easily seen using (119) and (120). This completes the proof of this case.

Case 2.B: $c < \frac{2\Pi}{1+\psi}$ and $c \geq \frac{1-2\Pi}{1-\psi}$

First, note that it cannot be the case that $\mu_R - \chi_R^- < \mu_L + \chi_L^+$, since Case 1.K—see (103)—would then imply that

$$c \geq \frac{2\Pi}{1-\psi} \geq \frac{2\Pi}{1+\psi}.$$

Hence, $\mu_L + \chi_L^+ \leq \mu_R - \chi_R^-$. Then, Case 1.D—see (91) and (95)—implies that

$$\chi_L^+ \leq \frac{c}{2} \cdot (1 + \psi(2\lambda - 1)) \quad \text{and} \quad \chi_R^- \leq \frac{c}{2} \cdot (1 + \psi(2\lambda - 1)).$$

Second, note that if $\mu_L + \chi_L^+ = \mu_R - \chi_R^-$, it must be the case that

$$2\Pi = \mu_R - \mu_L = \chi_L^+ + \chi_R^- \leq c \cdot [1 + \psi(2\lambda - 1)] \leq c(1 + \psi) < 2\Pi,$$

a contradiction. This means that

$$\mu_L + \chi_L^+ < \mu_R - \chi_R^-.$$

Accordingly, Case 1.B—see (81) and (82)—implies that

$$\chi_R^- \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \quad \text{and} \quad \chi_L^+ \geq \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)].$$

Third, together with (121), Cases 1.A and 1.C—see (78), (79), (85), and (86)—imply respectively that

$$\chi_L^- = \mu_L = \frac{1}{2} - \Pi \quad \text{and} \quad \chi_R^+ = 1 - \mu_R = \frac{1}{2} - \Pi.$$

All in all,

$$\chi_L^- = \chi_R^+ = \frac{1}{2} - \Pi = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1 - \psi)\right\}$$

and

$$\chi_R^- = \chi_L^+ = \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] = \min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right\},$$

where the last equalities in the above two chains of equalities can be easily seen using (119) and (120). This completes the proof of this case.

Case 2.C: $c \geq \frac{1+2\Pi}{1-\psi}$

First, together with (121), Cases 1.A and 1.C—see (78), (79), (85), and (86)—imply respectively that

$$\chi_L^- = \mu_L = \frac{1}{2} - \Pi \quad \text{and} \quad \chi_R^+ = 1 - \mu_R = \frac{1}{2} - \Pi.$$

Second, due to Case 1.B—see (84)—it must also be that

$$\mu_R - \chi_R^- \leq \mu_L + \chi_L^+.$$

Together with (122), from Cases 1.L and 1.M—see (107) and (113)—we then obtain

$$\chi_L^+ = 1 - \mu_L = \frac{1}{2} + \Pi \quad \text{and} \quad \chi_R^- = \mu_R = \frac{1}{2} + \Pi.$$

All in all,

$$\chi_L^- = \chi_R^+ = \frac{1}{2} - \Pi = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1 - \psi)\right\}$$

and

$$\chi_R^- = \chi_L^+ = \frac{1}{2} + \Pi = \min\left\{\frac{1}{2} + \Pi, \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right\}$$

where the last equalities in the above two chains of equalities can be easily seen using (119) and (120). This completes the proof of this case.

Two remarks are in order.

Remark 1

For cases 2.A and 2.B, one can assume the weaker condition (118) *instead of* (116).

Remark 2
Suppose that

$$\frac{2\Pi}{1+\psi(2\lambda-1)} \leq c < \frac{1+2\Pi}{1-\psi}.$$

Then, Case 1.B—see (84)—implies

$$\mu_R - \chi_R^- \leq \mu_L + \chi_L^+.$$

Existence of equilibrium:

To prove existence, we also consider that either c is sufficiently small or it is sufficiently large. First, we focus on the case where c is small. More specifically, we assume that

$$c < \frac{2\Pi}{1+\psi}, \tag{123}$$

which implies

$$c < \frac{2\Pi}{1+\psi(2\lambda-1)}. \tag{124}$$

Assume that parties $K \in \{R, L\}$ choose their policies according to χ_K^*, as given in Theorem 3. That is,

$$\sigma_L^*(i_{t-1}) = \min\left\{\max\left\{\mu_L - \min\left\{\frac{c}{2}\cdot(1-\psi), \frac{1}{2}-\Pi\right\}, i_{t-1}\right\}, \mu_L + \min\left\{\frac{1}{2}+\Pi, \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right\}\right\},$$
$$\sigma_R^*(i_{t-1}) = \min\left\{\max\left\{\mu_R - \min\left\{\frac{1}{2}+\Pi, \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right\}, i_{t-1}\right\}, \mu_R + \min\left\{\frac{c}{2}\cdot(1-\psi), \frac{1}{2}-\Pi\right\}\right\}.$$

Condition (123) guarantees that

$$\mu_L + \frac{c}{2}\cdot(1-\psi) < \frac{1}{2} < \mu_R - \frac{c}{2}\cdot(1-\psi) \tag{125}$$

and

$$\min\left\{\frac{1}{2}+\Pi, \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right\} = \frac{c}{2}\cdot[1+\psi(2\lambda-1)].$$

We therefore can write

$$\sigma_L^*(i_{t-1}) = \min\left\{\max\left\{\mu_L - \min\left\{\frac{c}{2}\cdot(1-\psi), \frac{1}{2}-\Pi\right\}, i_{t-1}\right\}, \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right\}, \tag{126}$$
$$\sigma_R^*(i_{t-1}) = \min\left\{\max\left\{\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], i_{t-1}\right\}, \mu_R + \min\left\{\frac{c}{2}\cdot(1-\psi), \frac{1}{2}-\Pi\right\}\right\}. \tag{127}$$

Let $J \in \{L, R\}$ be a party such that $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$. Then, from the analysis of voter behavior at the beginning of this proof, it follows that when the incumbent has normal capacity then σ_v^* is such that (or equivalently, voters' behavior is such that):

$$\text{For } t=1: \quad E(\sigma_v^*, \emptyset, i_{t-1}) = \begin{cases} J & \text{if } 0 < \left|i_0 - \frac{1}{2}\right| \leq \frac{1}{2} \\ \frac{1}{2}L\frac{1}{2}R & \text{if } \left|i_0 - \frac{1}{2}\right| = 0 \end{cases} \tag{128}$$

$$\text{For } t>1: \quad E(\sigma_v^*, \emptyset, i_{t-1}) = \begin{cases} K & \text{if } |\mu_K - i_{t-1}| \leq |\mu_{-K} - i_{t-1}| \\ -K & \text{otherwise.} \end{cases} \tag{129}$$

This shows that, if parties decide according to χ_K^*, then the transition probabilities between the different equilibrium policies are indeed given by the Markov transition diagrams from Theorem 3.

Next, let $t \geq 1$. We show that if *(i)* voters behave according to (128) and (129), *(ii)* all future office-holders behave according to χ_K^*, and *(iii)* the status quo is i_{t-1}, then it is optimal for the period-t office-holder $k \in \{R, L\}$, to choose his/her policy according to $\chi_K^*(i_{t-1})$ and no other policy choice is optimal for him/her. It is sufficient to consider that the office-holder in period t is $k \in R$, since the behavior of left-wing candidates follows by symmetry. Given $x \in [0,1]$, a policy choice of incumbent k, we let $U_R(x)$ be the lifetime utility that s/he derives from choosing policy x, from the perspective of period t. We distinguish the following cases—depending on the status-quo policy—and we build on the previous analysis of voter behavior:

Case 1: $i_{t-1} \leq \max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}$,
Case 2: $i_{t-1} \in \big(\max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}, \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\big)$,
Case 3: $i_{t-1} \in \big[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\big]$,
Case 4: $i_{t-1} \in \big(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}\big)$,
Case 5: $i_{t-1} \geq \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}$.

We start by analyzing Cases 1 and 5 and then discuss the remaining cases.

Case 1: $i_{t-1} \leq \max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}$

We show that $U_R(x)$ is uniquely maximized for $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$. We can focus on the case where $x \geq i_{t-1}$, since it is clear that it is never optimal for an office-holder from party R to choose a policy which is lower than $\mu_L - \frac{c}{2} \cdot (1-\psi)$. We distinguish several subcases.

Case 1.a: $x \in \big[i_{t-1}, \max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}\big]$

We assume that

$$\mu_L - \frac{c}{2} \cdot (1-\psi) \geq 0,$$

as otherwise this case is trivial to analyze. Then we have

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2}(1+\psi(2\lambda-1)) - x\right) + C_R\right] \\ &+ \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \frac{c}{2}(1-\psi) - x\right) + C_L\right], \end{aligned}$$

where C_R, C_L, p_R and p_L are constants independent of x, with $p_R + p_L = 1$—see Equation (68). It immediately follows that

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) + c \cdot \psi,$$

which is strictly positive if and only if

$$x < \mu_R - \frac{c}{2}(1-\psi).$$

The latter inequality follows from the following chain of inequalities:

$$x < \mu_L - \frac{c}{2}(1-\psi) < \mu_R - \frac{c}{2}(1-\psi).$$

To sum up,

$$x \in \left[i_{t-1}, \max\left\{0, \mu_L - \frac{c}{2}(1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{130}$$

Case 1.b: $x \in \big(\max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}, \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\big)$

In this case, Equation (69) implies that the challenger is elected—note that we have $\Delta_{\frac{1}{2}}(x) < 0$. Then, it follows that

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ \psi \cdot [U_R(x) + c \cdot (x - i_{t-1})], \end{aligned}$$

and thus

$$U_R(x) = -c \cdot (x - i_{t-1}) - \frac{1}{1-\psi}(\mu_R - x)^2.$$

We immediately obtain

$$\frac{dU_R(x)}{dx} = -c + \frac{2}{1-\psi}(\mu_R - x),$$

which is strictly positive if and only if

$$x < \mu_R - \frac{c}{2}(1-\psi).$$

The latter inequality follows again from the following chain of inequalities:

$$x < \mu_L + \frac{c}{2}(1 + \psi(2\lambda - 1)) < \mu_R - \frac{c}{2}(1-\psi). \tag{131}$$

The second inequality in (131) is equivalent to

$$c < \frac{2\Pi}{1 + \psi(2\lambda - 1) - \lambda\psi}.$$

The latter inequality is implied by Condition (124). Hence,

$$x \in \left(\max\left\{0, \mu_L - \frac{c}{2}(1-\psi)\right\}, \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{132}$$

Case 1.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)], \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right]$

In this case we have

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ p_R \cdot \psi \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] - x\right) + C_R\right] \\ &+ p_L \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right) + C_L\right], \end{aligned}$$

where C_R and C_L are two constants independent of x, $p_R = 1 - p_L$ and, by Equation (71),

$$p_L = \begin{cases} 1 & \text{if } x < \frac{1}{2}, \\ \lambda & \text{if } x \geq \frac{1}{2}. \end{cases}$$

It immediately follows that

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) + c \cdot p_R \cdot \psi - c \cdot p_L \cdot \psi,$$

which is strictly positive if and only if

$$x < \mu_R - \frac{c}{2} \cdot [1 + \psi(p_L - p_R)]. \tag{133}$$

Now, if $x \geq \frac{1}{2}$, Condition (133) is equivalent to

$$x < \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)],$$

which is trivially satisfied. If $x < \frac{1}{2}$, by contrast, Condition (133) reduces to

$$x < \mu_R - \frac{c}{2} \cdot [1 + \psi],$$

which trivially follows from Condition (123) (see also (125)). All in all,

$$x \in \Big[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\Big) \Longrightarrow \frac{dU_R(x)}{dx} > 0 \tag{134}$$

and, moreover,

$$\frac{dU_R}{dx}\Big(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\Big) = 0. \tag{135}$$

Case 1.d: $x \in \big(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}\big)$

In this case, by Equation (72) we obtain that the incumbent is re-elected unless s/he does not have normal capacity. This means that

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ (1-\lambda) \cdot \psi \cdot [U_R(x) + c \cdot (x - i_{t-1})] \\ &+ \lambda \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\Big) + C_L\Big], \end{aligned}$$

where C_L is a constant independent of x. Hence,

$$\begin{aligned} U_R(x) = &\frac{1}{1-(1-\lambda)\psi} \cdot \Big\{ - c \cdot (x - i_{t-1}) - (\mu_R - x)^2 + (1-\lambda) \cdot \psi \cdot c \cdot (x - i_{t-1}) \\ &+ \lambda \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\Big) + C_L\Big] \Big\}. \end{aligned}$$

This implies that

$$\frac{dU_R(x)}{dx} = \frac{-c + 2(\mu_R - x) + \psi \cdot c \cdot (1-2\lambda)}{1-(1-\lambda)\psi},$$

which is strictly negative if and only if

$$x > \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)],$$

and 0 otherwise. Hence,

$$x \in \Big(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \min\Big\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\Big\}\Big) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{136}$$

Case 1.e: $x \in \big[\min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi), 1\big]$

We assume that

$$\mu_R + \frac{c}{2} \cdot (1-\psi) < 1,$$

since otherwise this case is trivial to analyze. Then, using Equation (73) we obtain

$$\begin{aligned} U_R(x) = &- c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ &+ (1-\lambda) \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_R - \frac{c}{2} \cdot (1-\psi)\Big) + C_R\Big] \\ &+ \lambda \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\Big) + C_L\Big], \end{aligned}$$

where C_R and C_L are constants independent of x. It follows that

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) - c \cdot \psi,$$

which is strictly negative because

$$x \geq \mu_R + \frac{c}{2} \cdot (1-\psi) > \mu_R - \frac{c}{2} \cdot (1+\psi).$$

To sum up,

$$x \in \left(\min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}, 1\right] \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{137}$$

The combination of Conditions (130), (132), (134), (135), (136), and (137) together with the continuity of $U_R(x)$ in x, implies that $x^* = \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$ is the unique optimal policy choice of $k \in R$ when the status quo i_{t-1} satisfies $i_{t-1} \leq \max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}$. This completes *Case 1.*

<u>*Case 5:*</u> $i_{t-1} \geq \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}$.

We show that $U_R(x)$ is uniquely maximized for $x^* = \min\{1, \mu_R + \frac{c}{2} \cdot [1+\psi]\}$. We can focus on the case where $x \leq i_{t-1}$, since it is clear that it is never optimal for an office-holder from party R to choose a policy right to $\mu_R + \frac{c}{2} \cdot (1-\psi)$. We distinguish several subcases.

Case 5.a: $x \in \left[0, \max\{0, \mu_L - \frac{c}{2}(1-\psi)\}\right]$

We assume that

$$\mu_L - \frac{c}{2}(1-\psi) \geq 0,$$

since otherwise the analysis of this case is trivial. Then we have

$$\begin{aligned}
U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\
&+ \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2}(1 + \psi(2\lambda - 1)) - x\right) + C_R\right] \\
&+ \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \frac{c}{2}(1-\psi) - x\right) + C_L\right],
\end{aligned}$$

where C_R, C_L, p_R and p_L are constants independent of x, with $p_R + p_L = 1$—see Equation (68). It immediately follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) + c \cdot \psi,$$

which is strictly positive if and only if

$$x < \mu_R + \frac{c}{2}(1+\psi).$$

The latter inequality follows from the following chain of inequalities:

$$x < \mu_L - \frac{c}{2}(1-\psi) < \mu_R + \frac{c}{2}(1+\psi).$$

To sum up,

$$x \in \left[0, \max\left\{0, \mu_L - \frac{c}{2}(1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{138}$$

Case 5.b: $x \in \left(\max\{0, \mu_L - \frac{c}{2}(1-\psi)\}, \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right)$

In this case, Equation (69) implies that the challenger is elected—note that we have $\Delta_{\frac{1}{2}}(x) < 0$. Then, it follows that

$$\begin{aligned}
U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\
&+ \psi \cdot [U_R(x) + c \cdot (i_{t-1} - x)],
\end{aligned}$$

and thus

$$U_R(x) = -c \cdot (i_{t-1} - x) - \frac{1}{1-\psi}(\mu_R - x)^2.$$

We immediately obtain

$$\frac{dU_R(x)}{dx} = c + \frac{2}{1-\psi}(\mu_R - x),$$

and hence

$$\frac{dU_R(x)}{dx} \geq c + \frac{2}{1-\psi}\left[\mu_R - \left(\mu_L + \frac{c}{2}(1+\psi(2\lambda-1))\right)\right] > 0.$$

The latter inequality is equivalent to

$$c < \frac{2\Pi}{\lambda \cdot \psi}, \tag{139}$$

which is implied by Condition (124). Hence,

$$x \in \left(\max\left\{0, \mu_L - \frac{c}{2}(1-\psi)\right\}, \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \implies \frac{dU_R(x)}{dx} > 0. \tag{140}$$

Case 5.c: $x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right]$

In this case we have

$$\begin{aligned} U_R(x) = &- c\cdot(i_{t-1} - x) - (\mu_R - x)^2 \\ &+ p_R \cdot \psi \cdot \left[-c\cdot\left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)] - x\right) + C_R\right] \\ &+ p_L \cdot \psi \cdot \left[-c\cdot\left(x - \mu_L - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) + C_L\right], \end{aligned}$$

where C_R and C_L are two constants independent of x, $p_R = 1 - p_L$ and

$$p_L = \begin{cases} 1 & \text{if } x < \frac{1}{2}, \\ \lambda & \text{if } x \geq \frac{1}{2}. \end{cases}$$

It immediately follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) + c\cdot p_R \cdot \psi - c \cdot p_L \cdot \psi,$$

which is strictly positive if and only if

$$x < \mu_R + \frac{c}{2}\cdot[1 - \psi(p_L - p_R)]. \tag{141}$$

Now, if $x < \frac{1}{2}$, Condition (141) reduces to

$$x < \mu_R + \frac{c}{2}\cdot[1-\psi].$$

If $x \geq \frac{1}{2}$, by contrast, Condition (141) is equivalent to

$$x < \mu_R + \frac{c}{2}\cdot[1-\psi(2\lambda-1)].$$

The two latter conditions follow from Equation (71). All in all,

$$x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \implies \frac{dU_R(x)}{dx} > 0. \tag{142}$$

Case 5.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \min\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\}\right)$

In this case, using Equation (72), we have

$$\begin{aligned} U_R(x) = &- c\cdot(i_{t-1} - x) - (\mu_R - x)^2 \\ &+ (1-\lambda)\cdot\psi\cdot[U_R(x) + c\cdot(i_{t-1} - x)] \\ &+ \lambda\cdot\psi\cdot\left[-c\cdot\left(x - \mu_L - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x. Hence,

$$U_R(x) = \frac{1}{1-(1-\lambda)\psi} \cdot \Big\{ -c \cdot (i_{t-1} - x) - (\mu_R - x)^2 + (1-\lambda) \cdot \psi \cdot c \cdot (i_{t-1} - x) \\ + \lambda \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \Big) + C_L \Big] \Big\}.$$

This implies that

$$\frac{dU_R(x)}{dx} = \frac{c + 2(\mu_R - x) - \psi \cdot c \cdot}{1 - (1-\lambda)\psi},$$

which is strictly positive if and only if

$$x < \mu_R + \frac{c}{2} \cdot [1 - \psi].$$

Hence,

$$x \in \Big(\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)], \min\Big\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\Big\} \Big) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{143}$$

Case 5.e: $x \in \big[\min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}, i_{t-1} \big]$

We assume

$$\mu_R + \frac{c}{2} \cdot (1-\psi) \leq 1,$$

since otherwise this case is trivial to analyze. Then

$$U_R(x) = -c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ + (1-\lambda) \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_R - \frac{c}{2} \cdot (1-\psi) \Big) + C_R \Big] \\ + \lambda \cdot \psi \cdot \Big[-c \cdot \Big(x - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \Big) + C_L \Big],$$

where C_R and C_L are constants independent of x—see Equation (73). It follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) - c \cdot \psi,$$

which is strictly negative if and only if

$$x > \mu_R + \frac{c}{2} \cdot (1-\psi).$$

That is,

$$x \in \Big(\min\Big\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\Big\}, 1 \Big] \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{144}$$

The combination of Conditions (138), (140), (142), (143), and (144) together with the continuity of $U_R(x)$ in x, implies that $x^* = \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi\})$ is the unique optimal policy choice of $k \in R$ when the status quo i_{t-1} satisfies $i_{t-1} \geq \min\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\}$. This completes *Case 5.*

<u>*Case 2:*</u> $i_{t-1} \in \big(\max\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\}, \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \big)$,

We show that $U_R(x)$ is uniquely maximized for $x^* = \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$. We distinguish several subcases.

Case 2.a: $x \in \big[0, \max\big\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\big\}\big]$

This case is equivalent to Case 5.a. We therefore obtain

$$x \in \Big[0, \max\Big\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\Big\} \Big) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{145}$$

Case 2.b: $x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)]$

Applying Cases 1.b (if $i_{t-1} \leq x$) and 5.b (if $i_{t-1} \geq x$), we obtain

$$x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{146}$$

Case 2.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$

This case is equivalent to Case 1.c. Recall that we are assuming Condition (123) (see also (125)). We therefore obtain

$$x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0 \tag{147}$$

and, moreover,

$$\frac{dU_R}{dx}\left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) = 0. \tag{148}$$

Case 2.d: $x \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}\right)$

This case is equivalent to Case 1.d. We therefore obtain

$$x \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{149}$$

Case 2.e: $x \in \left[\min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}, 1\right]$

This case is equivalent to Case 1.e. We therefore obtain

$$x \in \left(\mu_R + \frac{c}{2} \cdot (1-\psi), 1\right] \frac{dU_R(x)}{dx} < 0. \tag{150}$$

The combination of Conditions (145), (146), (147), (148), (149), and (150) together with the continuity of $U_R(x)$ in x, implies that $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$ is the unique optimal policy choice of $k \in R$ when the status quo is i_{t-1} satisfies $\max\left\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\right\} < i_{t-1} < \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$. This completes *Case 2*.

<u>*Case 3:*</u> $i_{t-1} \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$,

We show that $U_R(x)$ is uniquely maximized for $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$. We distinguish several subcases.

Case 3.a: $x \in \left[0, \max\left\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\right\}\right]$

This case is equivalent to Case 5.a. We therefore obtain

$$x \in \left[0, \max\left\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{151}$$

Case 3.b: $x \in \left(\max\left\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\right\}, \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)]$

This case is equivalent to Case 5.b. We therefore obtain

$$x \in \left(\max\left\{0, \mu_L - \frac{c}{2} \cdot (1-\psi)\right\}, \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{152}$$

Case 3.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$

Recall that we are assuming Condition (123) (see also (125)). Then, applying Cases 1.c (if $i_{t-1} \leq x$) and 5.c (if $i_{t-1} \geq x$), we obtain

$$x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0 \tag{153}$$

and, moreover,

$$\frac{dU_R}{dx}\left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) = 0. \tag{154}$$

Case 3.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}\right)$

This case is equivalent to Case 1.d. We therefore obtain

$$x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{155}$$

Case 3.e: $x \in \left[\min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}, 1\right]$

This case is equivalent to Case 1.e. We therefore obtain

$$x \in \left(\min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}, 1\right] \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{156}$$

The combination of Conditions (151), (152), (153), (154), (155), and (156) together with the continuity of $U_R(x)$ in x, implies that $x^* = \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$ is the unique optimal policy choice of $k \in R$ when the status quo satisfies $\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)] \leq i_{t-1} \leq \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$. This completes *Case 3.*

Case 4: $i_{t-1} \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}\right)$,

We show that $U_R(x)$ is uniquely maximized for $x^* = i_{t-1}$. We distinguish several subcases.

Case 4.a: $x \in \left[0, \max\left\{0, \mu_L - \frac{c}{2}\cdot(1-\psi)\right\}\right]$

This case is equivalent to Case 5.a. We therefore obtain

$$x \in \left[0, \max\left\{0, \mu_L - \frac{c}{2}\cdot(1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{157}$$

Case 4.b: $x \in \left(\max\left\{0, \mu_L - \frac{c}{2}\cdot(1-\psi)\right\}, \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)]$

This case is equivalent to Case 5.b. We therefore obtain

$$x \in \left(\max\left\{0, \mu_L - \frac{c}{2}\cdot(1-\psi)\right\}, \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{158}$$

Case 4.c: $x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right]$

Recall that we are assuming Condition (123) (see also (125)). Then, this case is equivalent to Case 5.c, and thus we obtain

$$x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{159}$$

Case 4.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}\right)$

First, assume that $x < i_{t-1}$. Applying Case 5.d, we obtain

$$x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], i_{t-1}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{160}$$

Second, assume that $x < i_{t-1}$. Applying Case 1.d, we obtain

$$x \in \left(i_{t-1}, \min\left\{1, \mu_R + \frac{c}{2}\cdot(1-\psi)\right\}\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{161}$$

Case 4.e: $x \in \left[\mu_R + \frac{c}{2} \cdot (1-\psi), 1\right]$

This case is equivalent to Case 1.e. We therefore obtain

$$x \in \left(\min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}, 1\right] \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{162}$$

The combination of Conditions (157), (158), (159), (160), (161), and (162) together with the continuity of $U_R(x)$ in x, implies that $x^* = i_{t-1}$ is the unique optimal policy choice of $k \in R$ when the status quo i_{t-1} satisfies $\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] < i_{t-1} < \min\left\{1, \mu_R + \frac{c}{2} \cdot (1-\psi)\right\}$. This completes *Case 4*.

Note the following remark, which follows easily from analyzing Cases 1.c, 2.c, 3.c, and 4.c—all cases where Condition (123) is used because the weaker Condition (124) does not suffice.

Remark 3

Assume that a politician (or his/her party) obtains an additional utility $b > 0$ for every period s/he is in office. Then, if b is sufficiently large, existence of equilibrium is also guaranteed for $c \in \left[\frac{2\Pi}{1+\psi}, \frac{2\Pi}{1+\psi(2\lambda-1)}\right)$. The unique V-MSMPE is given by (126) and (127).

Finally, we focus on the case of large costs of change. That is, we assume that

$$c \geq \frac{2\Pi + 1}{1 - \psi}.$$

Then, it is straightforward to check that the proof of existence goes along the same lines as when Condition (123) holds. It suffices to consider Case 2 (or, equivalently, Case 4).

$\square$

Proof of Proposition 5

First, assume that σ^* is a V-MSMPE of $\mathcal{G}_{\psi}^{i_0}$. As we have seen, normal-capacity office-holders are always re-elected. Moreover, since $A \gg 0$, office-holders that have been affected by a capacity shock is never re-elected, so no deviation that improves on the expected rents from office is possible. Because σ^* is a V-MSMPE of $\mathcal{G}_{\psi}^{i_0}$, no deviation that improves on the expected utility from policies is possible either. Hence, σ^* must also be a V-MSMPE of $\mathcal{G}_{\psi,b}^{i_0}$ for any $b > 0$. Moreover, by Theorem 3 σ^* is also symmetric and regular.

Second, assume that σ^* is a symmetric and regular V-MSMPE of $\mathcal{G}_{\psi,b}^{i_0}$ for some $b > 0$. Let $\chi_R^* = (\underline{\chi}^*, \overline{\chi}^*)$ and $\chi_L^* = (\overline{\chi}^*, \underline{\chi}^*)$ be the party choices of persistence ranges in this equilibrium. Since all choices by party R are to the right of $\frac{1}{2}$ and all choices by party L are to the left of $\frac{1}{2}$, and both parties' strategies in σ^* are symmetrically defined with respect to 1/2, each party expects to have its office-holders re-elected *if and only if* they have normal capacity—we refer to the analysis of voter behavior in the proof of Theorem 3. Assume now that σ^* is not a V-MSMPE of $\mathcal{G}_{\psi}^{i_0}$. This means that there is a policy $i_{t-1} \in \mathcal{I}$ such that, w.l.o.g., incumbent $k \in R$ prefers to choose a policy $i_t \neq \chi_R^*(i_{t-1})$ that yields party R a higher utility from policies than $\chi_R^*(i_{t-1})$. By definition it must be that $i_t \geq \frac{1}{2}$, which guarantees that the probability of re-election is the same as if s/he chose $\chi_R^*(i_{t-1})$—we refer again to the analysis of voter behavior in the proof of Theorem 3. Accordingly, choosing i_t in this period must also be a profitable deviation for K in $\mathcal{G}_{\psi,b}^{i_0}$, a contradiction.

$\square$