DISCUSSION PAPER SERIES

DP14855

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LABOUR ECONOMICS

MACROECONOMICS AND GROWTH

ISSN 0265-8003

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Discussion Paper DP14855
Published 08 June 2020
Submitted 05 June 2020

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Abstract

Low-skill workers are concentrated in sectors that experience fast productivity growth and yet their wages have been stagnating. A multisector perspective is crucial to understand this stagnation as it is not due to an overall stagnation in the marginal product of low-skill workers but a labour reallocation into sectors with slower growth. We show this in a two-sector model where the faster productivity growth causes a fall in the relative price of the low-skill intensive output, which consists of capital and a consumption good that is a complement to the high-skill intensive output. When calibrated to the U.S., the model accounts for a substantial part of the low-skill wage stagnation and its divergence from aggregate productivity during 1980-2010.

JEL Classification: E24, J23, J31

Keywords: Wage stagnation, Wage-productivity divergence, Multisector model

Liwa Rachel Ngai - l.ngai@lse.ac.uk
london school of economics and CEPR

Orhun Sevinc - orhun.sevinc@tcmb.gov.tr
Central Bank of Turkey, Centre for Macroeconomics

Acknowledgements
We thank Mun Ho and Akos Valentinyi for the help with the US KLEMS data, Zso a Barany, Joseba Martinez, Ben Moll, Chris Pissarides for helpful discussion, and participants at London Business School, London School of Economics, London Macro workshop at UCL, Labour Market Adjustment Workshop at ECB, XIII. Koc University Winter Workshop in Economics, TOBB University, VIII Workshop on Structural Transformation and Macro Dynamics at Cagliari, and Uppsala University for comments. Hospitality from London Business School is gratefully acknowledged.

A Multisector Perspective on Wage Stagnation*

L. Rachel Ngai† Orhun Sevinc ‡

June 2020

Abstract

Low-skill workers are concentrated in sectors that experience fast productivity growth and yet their wages have been stagnating. A multisector perspective is crucial to understand this stagnation as it is not due to an overall stagnation in the marginal product of low-skill workers but a labour reallocation into sectors with slower growth. We show this in a two-sector model where the faster productivity growth causes a fall in the relative price of the low-skill intensive output, which consists of capital and a consumption good that is a complement to the high-skill intensive output. When calibrated to the U.S., the model accounts for a substantial part of the low-skill wage stagnation and its divergence from aggregate productivity during 1980-2010.

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†London School of Economics, CFM, and CEPR. Email: L.Ngai@lse.ac.uk

‡Central Bank of the Republic of Turkey, and CFM. Email: Orhun.Sevinc@tcmb.gov.tr

1 Introduction

Over the last four decades, the real wage of low-skill workers has experienced very little growth in the U.S. (Autor et al., 2008). The picture looks stark when put against growth in the aggregate labour productivity: the real wage of non-college workers increased by about 20% during the period 1980-2010, which is less than half of the increase in the aggregate labour productivity.[1] This stagnation is an important concern for the macro labour market as the hours worked by non-college workers represents two-third of the total hours worked, implying the average wage is also lagging behind the aggregate productivity. The stagnation persists even after controlling for age, race, gender, education and occupation, thus it is not due to compositional changes in the low-skill labour market.[2]

Our main objective is to understand the stagnation in the low-skill real wage and its divergence from the aggregate labour productivity, during a period of growing wage inequality between low-skill and high-skill workers. These three facts are interrelated but one does not necessarily imply the other.[3] This paper offers a novel perspective for understanding the low-skill wage stagnation through labour reallocation driven by uneven productivity growth across sectors. Using the U.S. data, it shows that this mechanism is quantitatively important in accounting for the three facts simultaneously.

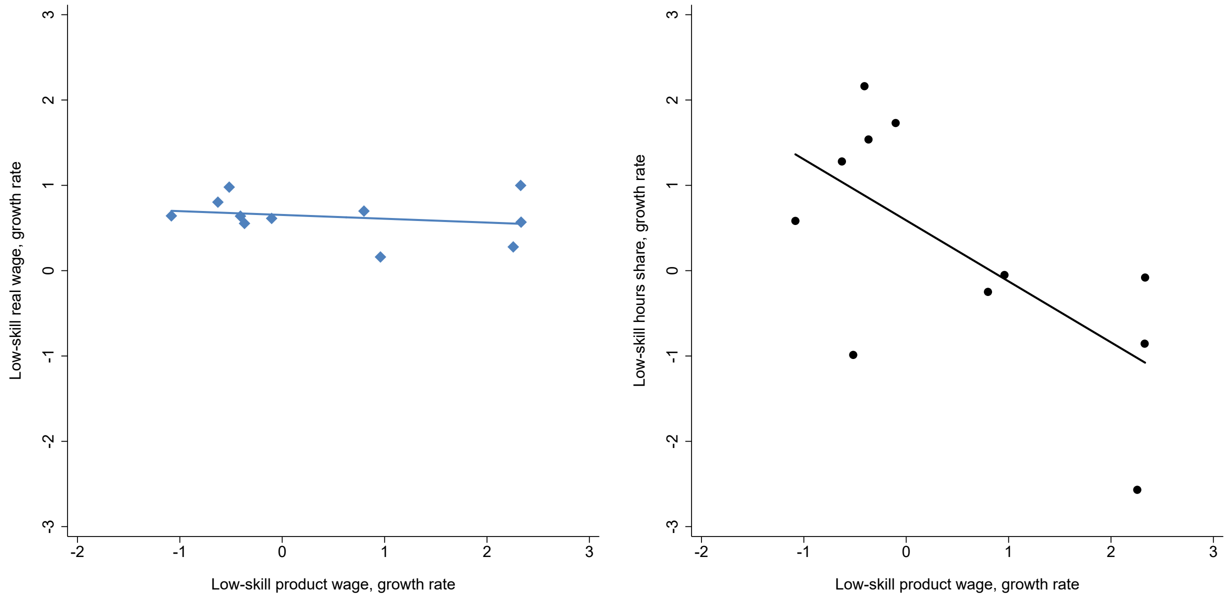
Real wage stagnation can reflect lack of growth in the productivity of low-skill workers. However, focusing on an aggregate production function masks the underlying driving forces at

[1]The precise increase in the non-college real wage ranges from 15% to 25% depending on choice of price deflators, composition adjustment, inclusion of non-wage compensation and self-employed, and whether it is only for nonfarm business sectors. For example, the literature on average wage and productivity divergence often focuses on nonfarm business sector. See Appendix A1.3. However, regardless of these choices, the findings that non-college real wage is stagnant and lags behind aggregate labour productivity growth are robust.

[2]The composition adjusted wages are calculated from CPS as the fixed-weighted mean of 216 cells based on 6 age, 2 gender, 2 race, 3 education categories (high school dropouts, high school, and some college) and 3 occupations (abstract, routine, manual), where the fixed weights are groups' long-run employment shares. It is important to note that, as documented in Acemoglu and Autor (2011), low-skill wage stagnation co-exists with occupational polarisation according to which the wages of low-wage occupations have been growing faster than the wages of middle-wage occupations. The low-skill wage stagnation is about a group of workers with given education qualifications whereas polarisation is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within occupations in understanding the different trends in wages by skill across workers and occupations.

[3]As will be shown later, economic forces that increase wage inequality can increase the divergence without causing low-skill stagnation. On the other hand, forces that contribute to low-skill wage stagnation may not contribute to wage inequality or the divergence.

Figure 1: Growth in Low-skill Wage, Product Wage and Hours Shares by Sector



Notes: Left panel plots the annual growth of low-skill real wage against the growth of sectoral low-skill product wage for the period 1980-2010. Right panel plots the annual growth of sectoral hour shares against the growth of sectoral product wage of the low-skill workers for the same period. Sectoral real wage is calculated as nominal wage divided by Personal Consumption Expenditure (PCE) price index. Sectoral product wage is calculated as nominal wage divided by sectoral value-added price. Sectoral wages and hours are from CPS and sectoral value-added prices are from WORLD KLEMS. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated from CPS as the fixed-weighted mean of 216 cells based on 6 age, 2 gender, 2 race, 3 education categories and 3 occupations (abstract, routine, manual), where the fixed weights are groups' long-run employment shares. See Data Appendix for the construction of variables and sectors.
Source: CPS, WORLD KLEMS, and authors' calculations.

the production floor, and the effects of technology on the real wage. In an economy with many sectors, stagnation in real wage does not necessarily imply stagnation in marginal product of low-skill worker in all sectors. Real wage (or sometimes referred as the consumption wage) is measured as the nominal wage deflated by an aggregate consumption price index and it is what workers care about as it measures what they can consume. However, what really matters for understanding marginal product of labour at the production floor is the product wage, which is measured as the nominal wage deflated by the sectoral value-added price. In a perfectly competitive labour market, as nominal wage of a sector equals the value of marginal product of labour, the sectoral product wage is an exact measure of the sectoral marginal product of labour.

Figure 1A plots the growth in the low-skill real wage against the growth in the low-skill product wage for the one-digit industries. It shows that there is very little variation in the growth of sectoral real wages compared to those of sectoral product wage. This is consistent with the intuition that workers move across sectors to seek better consumption wage so the growth in nominal wage is similar across sectors. The growth in product wage, however, varies substantially across sectors because of the changing relative prices. More specifically, given the similar growth in low-skill nominal wage, sectors with faster growing prices experienced stagnant (or even falling) low-skill product wages while sectors with slower growing prices experienced growing product wages. This variation suggests that while the marginal product of low-skill workers of some sectors are stagnant, others are growing. So how can this contribute to a stagnation in the aggregate marginal product of low-skill labour? The answer is given in Figure 1B which shows the growth of sectoral hour shares are negatively correlated with the growth of sectoral product wages, suggesting sectors with slower growing product wages are expanding at the expense of the faster growing ones. Thus stagnation in aggregate marginal product of low-skill labour can result from a reallocation from sectors with faster growing marginal product to the slower ones.

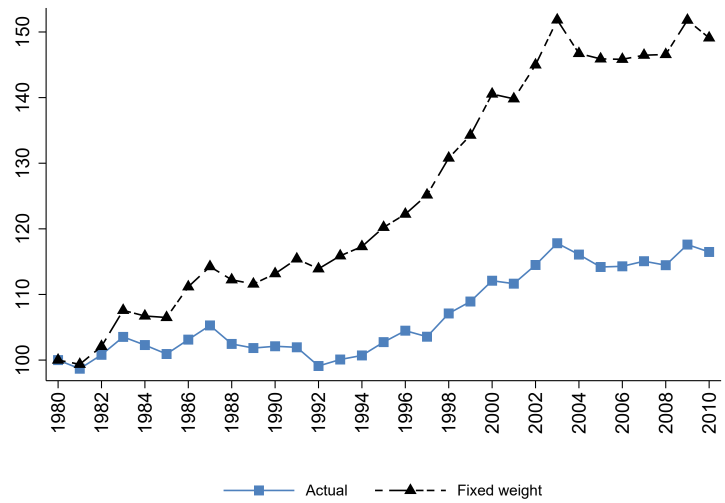
A simple counterfactual exercise can be used to illustrate the importance of this reallocation mechanism by expressing the average real wage as a weighted average of the sectoral product wages:

$$\frac{w_l}{P_C} = \sum_j \frac{w_{lj}}{p_j}\alpha_j; \quad \alpha_j \equiv \frac{p_j}{P_C}\frac{L_j}{L}; \tag{1}$$

where w_l is the average low-skill nominal wage and P_C is the consumption price deflator, so w_l/P_C is the average low-skill real wage. On the right-hand-side: w_{lj} and p_j are the low-skill nominal wage and value-added price in sector j, so w_{lj}/p_j is the low-skill product wage in sector j. The weight α_j is a product of the relative price p_j/P_C and the share of low-skill labour L_j/L in sector j.

The key observation of Figure 1A is that sectors with slower growth in prices experienced faster growth in low-skill product wage while Figure 1B shows that these are the declining sectors with falling low-skill hour shares. Putting together, Figure 1 shows that the weight α_j is falling for sector with faster growing product wage due to its slower growing prices p_j/P_C and declining labour share L_j/L. A natural question is what would happen to the low-skill real wage if the weight α_j were fixed. This is reported in Figure 2. The blue-square

Figure 2: Low-skill Real Wage and Sectoral Reallocation



Notes: The blue-square line is the CPS low-skill mean real wage, normalized to 100 in 1980. The black-triangle line holds the weights on the sectoral product wages fixed, i.e. α_j in equation (1) fixed. Real wage is equal to nominal wage deflated by PCE price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated from CPS as the fixed-weighted mean of 216 cells based on 6 age, 2 gender, 2 race, 3 education categories and 3 occupations (abstract, routine, manual), where the fixed weights are groups' long-run employment shares. See Data Appendix for the construction of variables and sectors.
Source: CPS, WORLD KLEMS and authors' calculations.

line is the data on low-skill real wage which grew by about 20% during the 30 years period. The black-triangle line reports the case when the weight α_j is hold fixed, i.e. when the relative prices p_j/P_C and hours shares L_j/L are fixed at the 1980 level. It shows that real wage would have increased by more than 40%. In other words, the reallocation mechanism highlighted in Figure 1 has reduced the growth of low-skill real wage by half.[4] The question is why does such reallocation happen and why only the low-skill wage is stagnant? The objective of the paper is to provide a mechanism that answers these questions.

The mechanism we propose is motivated by observations that consumers cannot easily substitute away from consuming services that use high-skill labour intensively (e.g. health care and education) yet these high-skill services are getting more expensive and gaining a bigger share of the economy. These observations form the basic mechanism of the paper:

[4]There are two other potential counterfactuals. If relative prices are fixed but hours shares vary, the average real wage would have increased by about 30%. Labour reallocation itself captures only part of the decline of the weight α_j. If hour shares are fixed but relative prices vary , the average real wage would behave like the data line because real wage growth are similar across sectors, as shown in Figure 1A. They confirm the key message of Figure 1B that labour reallocation matters for real wage stagnation because it is associated with changing sectoral relative prices.

low-skill workers are concentrated in sectors with faster productivity growth but they do not benefit as much since the output they produce is getting cheaper over time and is complementary to the high-skill labour.

The basic mechanism can be understood in an off-the-shelf two-sector and two-input model, with both sectors using high-skill and low-skill workers. The high-skill sector has a slower productivity growth and uses low-skill workers less intensively. Slower productivity growth in the high-skill sector implies a rising relative price of high-skill sector. Assuming the output from the two sectors are gross complements, the rise in the relative price of high-skill sector increases the relative expenditure of high-skill sector resulting in a labour reallocation into the high-skill sector. Given the expanding sector has a faster growth in price, this reallocation process reflects a shift of workers into the sector with a slower growing product wage, contributing to the stagnation in low-skill wage. But why does the stagnation only happen to low-skill wage? This is because the high-skill sector puts a lower input weight on low-skill workers, so the reallocation acts like a skill-biased demand shift which increases the wage inequality boosting the growth in the high-skill wage.[5]

The basic model delivers the key mechanism of how sectoral reallocation driven by uneven productivity growth can contribute to low-skill wage stagnation. It contributes to the divergence of low-skill wage from the aggregate productivity by predicting a rise in wage inequality. Using an accounting identity, which expresses total value-added of the economy as the sum of total factor payments, we show that there are two other potential drivers for the low-skill real wage and productivity divergence. They are the falling labour income share and the rising relative cost of living, measured by the ratio of the consumption deflator and the output deflator.

To quantify the contribution of our mechanism in accounting for the low-skill wage stagnation, capital is introduced to the basic model. The presence of capital is essential for understanding the full picture of the wage stagnation and its divergence all the three drivers of the wage and productivity divergence. The key assumption is that there is capital-skill

[5]In other words, specializing in sectors with faster productivity growth works against the low-skill workers as the output they produce are getting cheaper over time. This has a similar flavour, but the mechanism is different, to the early trade literature on immiserizing growth where faster productivity growth results in a country being worse off because of the deteriorating term of trade (Bhagwati, 1958).

complementarity. Thus, a falling relative price of capital implies a shift towards the high-skill labour in the production of both sectors. It acts as a within-sector skill-biased demand shift, reinforcing the rise in wage inequality implied by the between-sector skill-biased demand shift explained previously. In addition to predicting a rising wage inequality, the model can also contribute to the wage-productivity divergence through the other two channels in the accounting identity. First, it predicts a rise in the relative cost of living. Second, by generating endogenous skill-biased demand shifts, it predicts an increase in the income share of high-skill labour and a fall in the income share of low-skill labour, so it can potentially lower the aggregate labour income share if the fall in low-skill income share dominates. More importantly, the model demonstrates that factors that imply a rise in wage inequality always contribute to the divergence but they do not necessarily contribute to the low-skill wage stagnation.

The model is calibrated to match key features of the US labour market from 1980 to 2010. We group industries used in Figure 1 into high-skill and low-skill sector according to the importance of high-skill worker. The details of the data are provided in the Data Appendix A1. Consistent with the basic mechanism, the labour productivity growth is faster in the low-skill sector and both the relative price and the hour share of the high-skill services are increasing. The rise in the relative price of high-skill services imply that the low-skill product wage experienced very different trends in the two sectors: it grew in the low-skill sector but fell in the high-skill sector. This confirms the key message that low-skill wage stagnation is not due to an overall stagnation in marginal product of low-skill labour but it is due to reallocation of low-skill labour from the sector with faster growing one into the slower one.

The within- and between-sector skill-biased demand shifts are calibrated to the fall in the relative price of capital and the rise in the relative price of high-skill sector. The rest of the parameters are set to match the income shares of high- and low-skill workers in the two sectors and in the aggregate, and the aggregate labour productivity growth. Matching the aggregate income shares of the high-skill and low-skill labour implies a rise in the relative supply of high-skill labour. Matching the sectoral income shares, on the other hand, requires changes in the production weights of inputs which reflect other sources of skill-biased demand shift that are exogenous to our model. For instance, as a result of automation some tasks performed by low-skill are displaced by machines (Acemoglu and Autor, 2011), or skill-biased organizational change documented by Caroli and Van Reenen (2001) that increases the importance of human capital.

The predicted labour market changes from 1980 to 2010 are driven by the two sources of endogenous skill-biased demand shifts, increase in relative supply of high-skill labour and two sources of exogenous skill biased demand shifts. The endogenous skill-biased demand shifts can account for the divergence by predicting a rise in wage inequality and the relative cost of living, but they cannot generate a fall in labour income share. Among the two sources of endogenous skill-demand shifts, the between-sector (the basic mechanism) alone can contribute to 85% of the divergence by predicting 68% of the rise in wage inequality and all the rise in the relative cost of living. The exogenous skill-biased demand shifts are needed to account for the fall in the aggregate labour income share and the remaining rise in wage inequality, especially in the presence of rising relative supply of high-skill labour. The increase in relative supply of high-skill labour on its own implies convergence and a fall in the aggregate labour income share.

The quantitative exercise shows that all types of skill-biased demand shifts are important for understanding the rise in wage inequality and divergence. However, their effects on the growth of low-skill wage are very different. Among them, only the between-sector skill-biased demand shift induced by the faster productivity growth in the low-skill sector and the labour-displacing technical change that lowers the input weights of low-skill workers can generate low-skill real wage stagnation, but they work through different channels. The endogenous between-sector skill-biased demand shift delivers the result by predicting a rise in the relative price of high-skill services, resulting in a fall in the marginal product of low-skill labour in the high-skill sector and a rise in the low-skill sector. The exogenous labour-displacing technical change delivers the result by predicting low growth in the marginal product of low-skill labour in both sectors, which misses the differential trends observed in the data. This confirms the quantitative importance of the basic mechanism in accounting for the low-skill wage stagnation.

The role of different price deflators and falling labour income share have been empirically documented as the sources of the decoupling of the average wage and productivity (e.g. Lawrence and Slaughter, 1993; Stansbury and Summers, 2017). This paper shows that growing wage inequality is an important source of the divergence of the low-skill wage and aggregate productivity. There has been a large literature studying the effects of the skill-biased technical change on wage inequality (see Goldin and Katz, 2009, for a review). However, skill-biased technical change that simply improves the productivity of high-skill workers relative to the low-skill cannot explain wage stagnation for low-skill workers or for the middle-wage occupations (Acemoglu and Autor, 2011; Johnson, 1997). This has partly contributed to a growing literature on the effect of labour displacing technical change such as automation (see recent examples, Acemoglu and Restrepo, 2018; Martinez, 2019; Moll et al., 2019; Caselli and Manning, 2019; **?**, among others).[6]

In addition to low-skill labour displacing technical changes, there are other potential explanations for the low-skill wage stagnation, such as de-unionization and decline in minimum wage (Lee, 1999; Dustmann et al., 2009), increasing imports (Autor et al., 2013), and the decline in urban premium for non-college workers due to region-specific occupational changes (Autor, 2019).[7] Our paper offers a novel channel for the low-skill wage stagnation through the labour reallocation from sectors with growing marginal product of labour to the stagnant ones.[8] This mechanism shares some features of the Baumol's cost disease on the slowdown of aggregate growth in his seminal paper (Baumol, 1967). Yet, in its original form the cost disease would have applied to all workers and to the aggregate labour productivity. By including capital and allowing for heterogeneous workers, we show that the cost disease has a larger effect on the low-skill workers, resulting in low-skill wage stagnation, the decoupling of wage and labour productivity and growing wage inequality.

The mechanisms for the dynamics of relative wages across different types of workers are

[6]This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labour income shares (see e.g., Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2019, among others).

[7]The decline in manufacturing is an important part of our mechanism and it is modelled as a result of uneven productivity growth. Both Autor et al. (2013) and Kehoe et al. (2018) find that trade accounts for a quarter or less for the decline in U.S. manufacturing, and Kehoe et al. (2018) specifically shows that most of the decline is due to uneven productivity growth.

[8]To the extent that most of the expansion in high-skill services happens in urban areas, our mechanism is consistent with the finding of Autor (2019) on the decline of urban premium for the non-college workers.

related to Krusell et al. (2000), Ngai and Petrongolo (2017) and Buera et al. (2018). Our main objective is to understand low-skill wage stagnation and its divergence from aggregate labour productivity, which are not addressed in these papers. As in Ngai and Petrongolo (2017) and Buera et al. (2018), we show how factor-neutral productivity growth at the sector level can become factor-biased at the aggregate level, where the expansion of a sector can result in higher relative wage for the factor that is used more intensively in that sector. Both papers abstract from capital. Capital plays two important roles for our objective. First, it is needed for studying the decoupling of wage and aggregate labour productivity. Second, it provides an additional mechanism for the rise in the relative wage of high-skill workers through capital-skill complementarity and falling relative prices of capital as in Krusell et al. (2000).[9]

Section 2 uses the off-the-shelf two-sector and two-input model to show the basic mechanism of how sectoral reallocation can lead to low-skill wage stagnation. Section 3 presents the full model with capital to show how sectoral reallocation can imply low-skill wage stagnation and its divergence from productivity by predicting growing wage inequality, rising cost of living and potentially a falling aggregate labour income share. It shows how the model can generate a within-sector and a between-sector skill-biased demand shifts and disentangles factors that contribute to low-skill wage stagnation from those implying growing wage inequality and the divergence. The quantitative importance of the basic mechanism is presented in Section 4 when the model is calibrated to match key features of the U.S. labour market.

2 The Basic Mechanism

2.1 The Basic Model Setup

There is a measure H of high-skill household and a measure $L = 1 - H$ of low-skill households. Each household is endowed with one unit of time which they supply to the market inelastically. Household i maximizes utility defined over consumption of the output from the

[9] Autor and Dorn (2013) combines consumption complementarity with high substitutability between capital and routine tasks to study employment reallocation and relative wages across occupations.

two sectors c_{ij} $j = h, l$:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi)\, c_{ih}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

subject to the budget constraint :

$$p_h c_{ih} + p_l c_{il} = w_i, \tag{3}$$

where w_i is the wage of household i.

The economy consists of two sectors: the high-skill sector and the low-skill sector. The representative firm in sector $j = h, l$ uses low-skill labour and high-skill labour as input with a CES production function:

$$Y_j = A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j)\, H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4}$$

where parameter ξ_j captures the importance of low-skill labour in sector j. H_j and L_j are the high-skill and low-skill labour used in sector j.

There are two key assumptions in the model: (1) a sector uses different inputs of production with different intensities and (2) there is complementarity across output of different sectors. More specifically, we assume:

$$A1 \quad : \quad \xi_l > \xi_h \tag{5}$$

$$A2 \quad : \quad 1 > \varepsilon. \tag{6}$$

Assumption A1 implies the production in the low-skill sector puts a higher weight on low-skill worker than the production in the high-skill sector. Assumption A2 implies that output of the low-skill sector and the high-skill sector are gross complements in consumption.

The goods market clearing conditions are:

$$Y_j = C_j; \quad j = h, l \tag{7}$$

The labour market clearing conditions are

$$H_h + H_l = H; \quad L_h + L_l = L, \tag{8}$$

2.2 Household's optimization

Household $i = h, l$ maximizes utility taking prices p_h and services p_l as given. The optimal decision of household i implies the marginal rate of substitution across the two goods equal to their relative prices, which implies:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \tag{9}$$

thus relative expenditure is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}. \tag{10}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{11}$$

where x_j is the expenditure share of good j. These expenditure shares are identical across all household because of the homothetic preferences. Aggregation across households, the aggregate demand for good j is :

$$p_j C_j = x_j(Hw_h + Lw_l) \tag{12}$$

so the relative aggregate demand is the same as the relative individual demand:

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}; \qquad \frac{p_h C_h}{p_l C_l} = x, \tag{13}$$

and the aggregate relative expenditure is the same as individual relative expenditure.

Using the equilibrium condition from the household's optimization, Appendix A2.1 shows that the price index for consumption basket is the same across all household and derive the aggregate consumption price index as:

$$P_C = \left[\psi^{\varepsilon} p_g^{1-\varepsilon} + (1-\psi)^{\varepsilon} p_s^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{14}$$

and its growth rate as a weighted average of the prices of the two consumption goods where the weight is given by the expenditure share x_j:

$$\hat{P}_C \approx x_l \hat{p}_l + x_h \hat{p}_h \tag{15}$$

2.3 Firm's optimization

All sectors are perfectly competitive and the representative firm in each sector takes wages of high-skill and low-skill labour as given and maximize profit. The optimal decision of the firms implies the marginal rate of technical substitution across high-skill and low-skill labour is equal to the relative wages, thus the skill-intensity in each sector is:

$$\frac{H_j}{L_j} = \sigma_j^{\eta} q^{-\eta}; \quad q \equiv \frac{w_h}{w_l} \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}; \tag{16}$$

where q is the wage of high-skill labour relative to the low-skill labour. It follows directly from Assumption A1 that $\sigma_h > \sigma_l$, so the high-skill sector has a higher skill-intensity.

The income share of low-skill workers in sector j is

$$J_j(q) \equiv \frac{w_l L_j}{w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\left(\frac{1-\xi_j}{\xi_j}\right)^{\eta}\right]^{-1} \tag{17}$$

and the relative income share can be expressed as:

$$\frac{J_h(q)}{J_l(q)} = 1 - \frac{1}{1+\sigma_h^{-\eta} q^{\eta-1}}\left(1 - \left(\frac{\sigma_l}{\sigma_h}\right)^{\eta}\right) \tag{18}$$

Given Assumption A1 implies $\sigma_h > \sigma_l$, the low-skill income share is lower in the high-skill sector.

2.4 Equilibrium prices and allocation

The equilibrium low-skill wage is equal to the value of its marginal product, using the production function

$$w_l = p_j \frac{\partial Y_j}{\partial L_j}; \qquad \frac{\partial Y_j}{\partial L_j} = A_j \left[J_j(q) \xi_j^{-\eta} \right]^{\frac{1}{1-\eta}}, \tag{19}$$

where the marginal product of low-skill labour is decreasing in q as equation (17) implies $[J_j(q)]^{\frac{1}{1-\eta}}$ is decreasing in q. Thus an increase in the relative wage of high-skill will contribute to a fall in low-skill wage and a rise in high-skill wage $w_h = qw_l$.

The free mobility of labour implies the relative price of high-skill services:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h} \right) \left(\frac{\xi_l}{\xi_h} \right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h(q)}{J_l(q)} \right)^{\frac{1}{\eta-1}}. \tag{20}$$

It shows that an increase in the relative productivity of the low-skill sector (rising A_l/A_h) contributes to rising relative price of high-skill services. An increase in the relative wage of the high-skill also increases the relative price of high-skill services given (18) implies $[J_h(q)/J_l(q)]^{\frac{1}{\eta-1}}$ is increasing in q.

Appendix A2.4 show that the equilibrium of the model can be summarized as solving for relative wage q and the share of low-skill labour in the high-skill sector $l_h \equiv L_h/L$ using a supply condition and a demand condition. In a nut shell, the supply condition is derived using the labour market clearing conditions in equation (8) and the firm's optimal input usage in (16). The demand condition is derived using the goods market clearing conditions in equation (7) and the household's optimal consumption in (13).

The supply condition is derived as:

$$l_h = S(q; \zeta) = \frac{\zeta \sigma_l^{-\eta} q^{\eta} - 1}{(\sigma_h/\sigma_l)^{\eta} - 1}; \quad \zeta \equiv \frac{H}{L} \tag{21}$$

where ζ is the relative supply of high-skill labour. The supply $S(q; \zeta)$ is increasing in q given Assumption A1 implies $\sigma_h > \sigma_l$. In other words, when the low-skill sector uses the low-skill workers more intensively, the reallocation of low-skill labour from the low-skill sector to the high-skill sector (higher l_h) is associated with higher relative wage (higher q).

The demand condition is derived as:

$$l_h = D\left(q; \hat{A}_{lh}\right) = \left(1 + \frac{J_l(q)}{J_h(q)\, x\left(q; \hat{A}_{lh}\right)}\right)^{-1} \tag{22}$$

where the relative expenditure x is derived from (10) and (20) as:

$$x\left(q; \hat{A}_{lh}\right) = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{J_h(q)}{J_l(q)}\left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\varepsilon}{\eta-1}}; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{23}$$

The relative expenditure share $x\left(q; \hat{A}_{lh}\right)$ summarizes the effect of relative productivity on demand through its effect on relative prices. A rise in $\hat{A}_{lh}$ increases the relative price of high-skill services which increases the relative expenditure x, resulting in higher h_h for any given q.

The supply and demand conditions together solve for the equilibrium relative wage (q) and the equilibrium allocation of low-skill labour (l_h). The equilibrium relative prices (p_h/p_l) and relative expenditures (x) follow directly from (20) and (23). The equilibrium allocation of high-skill labour is derived using the optimal skill-intensity condition (16).

2.5 Low-skill wage stagnation

The low-skill real wage can be expressed as:

$$\frac{w_l}{P_C} = \left(\frac{p_l}{P_C}\right)\left(\frac{w_l}{p_l}\right); \quad \frac{p_l}{P_C} = \psi^{\frac{\varepsilon}{\varepsilon-1}} x_l^{\frac{1}{1-\varepsilon}}, \tag{24}$$

where the relative price (p_l/P_C) is derived from the consumption price index in (14), which is increasing in the expenditure share on low-skill goods $x_l = 1/(1+x)$ given $\varepsilon < 1$. The product wage w_l/p_l is the marginal product of low-skill workers in the low-skill sector in (19). A rise in the relative wage of the high-skill q implies a lower low-skill real wage by lowering both terms.

It is important to note that productivity growth itself has a positive effect on the level of low-skill real wage. This can be seen by substituting the marginal product of labour (19) and the equilibrium expenditure shares (23) into (24):

$$\frac{w_l}{P_C} = \left[\hat{A}_l^{\varepsilon-1}\left(\xi_l^{-\eta}J_l\right)^{\frac{1-\varepsilon}{\eta-1}} + \hat{A}_h^{\varepsilon-1}\left(\xi_h^{-\eta}J_h\right)^{\frac{1-\varepsilon}{\eta-1}}\right]^{\frac{1}{\varepsilon-1}}; \quad (25)$$

$$\hat{A}_l \equiv \psi^{\frac{\varepsilon}{\varepsilon-1}}A_l \quad \hat{A}_h \equiv (1-\psi)^{\frac{\varepsilon}{\varepsilon-1}}A_h \quad (26)$$

which is increasing in productivity parameters A_l and A_h. Clearly low-skill real wage will be stagnant if there is lack of growth in productivity (A_h, A_l). But the main issue in the data is that low-skill real wage is lagging behind productivity.

The mechanism proposed in the basic model is that: low-skill workers are concentrated in sectors with faster productivity growth but they do not benefit as much since the output they produce is getting cheaper and is complementary to the high-skill labour. This is stated in the following proposition:

Proposition 1 *When the low-skill sector has a higher production weight on low-skill worker (Assumption A1) and the output of the two sectors are complements (Assumption A2), a rise in the relative productivity of the low-skill sector contributes negatively to the change in the low-skill real wage, offsetting the positive effect from the rise in productivity.*

To understand Proposition 1, suppose A_h is fixed and there is an increase in A_l. The increase in the relative productivity in the low-skill sector (higher $\hat{A}_{lh}$) increases the relative price of high-skill services p_h/p_l in (20). Given the consumption complementarity ($\varepsilon < 1$), this leads to an increase in the relative expenditure on high-skill services x in (23), which shifts up the demand for labour in the high-skill sector as shown in (22). Given the supply is increasing, the increase in demand results in higher relative wage of the high-skill worker (q) and a labour reallocation into high-skill services (higher l_h). This process imposes two negative forces on the low-skill real wage, offsetting the positive effect from the rise in productivity A_l. They are shown in (24): the fall in the expenditure share on low-skill goods x_l implies a fall in p_l/P_C, and a higher relative wage of the high-skill worker q implies a fall in the product wage w_l/p_l as shown in (19).

The mechanism spelt out in Proposition 1 confirms the finding in the counterfactual Figure (2) that the rise in relative price of high-skill services (p_h/p_s) and the reallocation of low-skill labour into the high-skill services (higher l_h) are crucial for the stagnation in the low-skill.

Proposition 1 highlights the three key ingredients for the basic mechanism: (1) sector-specific productivity growth, (2) sector-specific input intensity and (3) consumption complementarity. Each of them is necessary for the low-skill real wage to fall behind productivity.

First, suppose the productivity growth was the same across sectors, i.e. $\hat{A}_{lh}$ does not change, there will be no change in the demand condition thus no change in inequality q, relative prices p_h/p_l or expenditure share x_l. Productivity growth will benefit all workers equally and the growth in the low-skill real wage will be the same as productivity growth as shown in equation (24).

It is important to note that a fall in ψ, e.g. due to a pure demand shift toward high-skill services, can also lead to an increase in $\hat{A}_{lh}$ and a rise in the relative expenditure x as shown in equation (23). Thus it can have a similar effect on the relative wage q and low-skill labour allocation l_h as an increase in the relative productivity A_l/A_g. But it will not have a direct effect on relative prices of service as shown in equation (20).[10] Most importantly, its effect on the relative price term p_l/P_C in the low-skill real wage equation (19) will be muted due to the offsetting effect of falling x_l and falling ψ. Thus a fall in ψ will not contribute much to the low-skill real wage stagnation even though its effect on relative wage is similar to the effect of an increase in the relative productivity A_l/A_g.

Second, suppose both sectors use inputs with the same weight ($\xi_s = \xi_g$,), equation (16) implies the factor intensity is identical across sectors and equal to the relative supply of labour, thus a rise in the relative productivity $\hat{A}_{lh}$ has no effect on inequality q. This implies the product wage w_l/p_l has the same growth as A_l. However, in the presence of consumption complementarity, higher relative prices of high-skill services (implied by higher relative productivity$\hat{A}_{lh}$) still leads to a fall in the expenditure share x_l, leading to a fall in p_l/P_C. Thus the Baumol's cost disease is present but applies to all workers.

Finally, the presence of consumption complementarity is also necessary. When $\varepsilon = 1$, (23) imply the relative expenditure x is independent of the relative productivity $\hat{A}_{lh}$, and so

[10] It will have an equilibrium effect on the relative prices through the rise in q that drives changes in the relative low-skill income share J_h/J_l as shown in equation(20), but the effect will be much smaller than the direct effect from the rise in relative productivity A_l/A_g.

is the demand. Thus a rise in relative productivity $\hat{A}_{lh}$ has no effect on inequality q or the expenditure share of low skill goods x_l, resulting in no change in the relative price p_l/P_C and the low-skill income share J_l. It follows from (19) and (24) that the growth in the low-skill real wage follows the growth in A_l.

To sum up, under assumptions A1-A2, the basic model shows that a rise in the relative productivity of low-skill sector can lead to a rise in the relative prices of high-skill services, a reallocation of low-skill workers into high-skill services, a higher wage inequality and a divergence of the low-skill real wage from aggregate productivity. It is shown that the rise in the relative prices play a crucial role in the basic mechanism to account for low-skill stagnation, a rise in wage inequality can only contribute to low-skill wage stagnation if it is associated with a rise in relative prices. This confirms the insight from the simple counter-factual exercise in Figure (2) for a two-sector setting: an increase in the relative price of the high-skill sector leads to both a slower growth in its low-skill product wage w_l/p_h and a rise in its weight α_h in the simple accounting equation (1).

3 Low-skill wage and productivity divergence

The main objective of this paper is to understand the stagnation in the low-skill real wage and its decoupling from labour productivity, which happened during a period of growing wage inequality between low-skill and high-skill workers. These three facts are interrelated but one does not necessarily imply the other.

The basic model delivers the key mechanism on how uneven productivity growth can contribute to low-skill wage stagnation by predicting a fall in the relative price of the sector that uses low-skill workers intensively. It generates a divergence in the low-skill wage and aggregate productivity by predicting a rise in wage inequality. This section first shows that there are two other potential drivers behind the divergence in the data that are missing from the basic model. It then presents a full model to incorporate all three drivers. Finally, in line with the basic model, the full model shows other factors that can imply a rise in wage inequality and contribute to the divergence, but they do not necessarily contribute to low-skill wage stagnation.

3.1 Accounting identity

An accounting relationship between low-skill wage and aggregate labour productivity exists given the sum of value-added must equal to sum of factor payment:

$$\beta \sum_j p_j Y_j = \sum_i w_i M_i, \tag{27}$$

where p_j and Y_j is the price and real value-added of sector j, w_i and M_i are the wage and market hours by labour input i, and β is the labour income share. Let P_Y be the aggregate output price index and M be the total market hours, the identity implies

$$\beta y = w, \quad y \equiv \frac{\sum_j p_j Y_j}{M}, \quad w \equiv \frac{\sum_i w_i M_i}{M}, \tag{28}$$

where y is the nominal aggregate labour productivity and w is the average nominal wage in the economy. So the ratio of real productivity relative to low-skill real wage is:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{\textit{Real}} = \underbrace{\left(\frac{y}{w_l}\right)}_{\textit{Nominal}} \underbrace{\left(\frac{P_C}{P_Y}\right)}_{\textit{Deflator}}, \qquad \frac{y}{w_l} = \underbrace{\left(\frac{w}{w_l}\right)}_{\textit{Wage Inequality}} \underbrace{\left(\frac{1}{\beta}\right)}_{\textit{Labour Share}} \tag{29}$$

It shows that the real divergence in the low-skill wage and productivity can be due to growth in the relative cost of living and a nominal divergence in low-skill wage and productivity, while the nominal divergence itself can be driven by the growth in wage inequality and a fall in labour income share. To put it differently, it also implies changes in low-skill real wages are driven by productivity, relative cost of living, wage inequality and labour income share.

Two of the drivers for the divergence, ratio of deflators and labour income share, are missing from the basic model. In the basic model, given both sectors only produce consumption goods, the value-added shares of the economy are the same as the expenditure shares. Thus it implies the consumption price defaltor and the output price deflators are the same. Second, in the absence of capital, the labour income share is equal to 1 in the basic model. The remaining parts of this section present a full model that incorporates all three drivers of the divergence.

3.2 The Model Economy

This section extends the basic model to include capital. To keep the framework simple, we assume the output of the low-skill sector can be converted into capital and there is full depreciation of capital. In the quantitative exercise, the objective will be to compare the labour market changes predicted by the model from 1980 to 2010 instead of studying the time path.

3.2.1 The model setup

The household problem is the same as the basic model but the firm's problem is different. The representative firm in sector $j = l, h$ uses low-skill labour, high-skill labour and capital as input with the following production function:

$$Y_j = A_j F_j \left(G_j \left(H_j, K_j\right), L_j\right) \tag{30}$$

$$F_j \left(G_j \left(H_j, K_j\right), L_j\right) = \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right)\left[G_j\left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{31}$$

$$G_j \left(H_j, K_j\right) = \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + \left(1-\kappa_j\right) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{32}$$

where parameter κ_j measures the importance of capital within the capital-skill composite. The key new assumption is that there is capital-skill complementarity, $\rho < 1$. Together with $\eta > 1$,, the nested CES structure implies that the elasticity of substitution across low-skill and capital are larger than the substitution across high-skill and capital, i.e. capital is a gross complement to high-skill labour but a gross substitute to low-skill labour.

The market clearing condition for high-skill services, and the labour market clear conditions are the same as before. The output of the low-skill sector can be used as consumption goods or converted into $1/\phi$ unit of capital, where ϕ can be interpreted as the price in capital relative to the low-skill goods.[11] As in (see Greenwood et al., 1997), an investment specific technical change can be implemented as a fall in ϕ.

[11] We show in the Appendix A2.3 formally how the low-skill sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production function except with a sector-specific TFP index. In this environment the relative price of capital is equal to ϕ which is the inverse of their TFP.

The low-skill goods market clearing conditions is

$$Y_l = C_l + \phi K, \tag{33}$$

and the capital market clearing condition is:

$$K = K_h + K_l. \tag{34}$$

3.2.2 Firm's optimal decision

All sectors are perfectly competitive and the representative firm in each sector takes price of capital q_k, high-skill labour w_h and low-skill labour w_l as given to maximize profit. The optimal decision of the firms implies the marginal rate of technical substitution across any two inputs is equal to its relative price. Across high-skill and capital input:

$$\frac{H_j}{K_j} = (\chi \delta_j)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \chi \equiv \frac{w_h}{q_k}. \tag{35}$$

Define $\tilde{I}_j$ as the high-skill income relative to total income that goes to high-skill and capital:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_k H_j} = \frac{1}{1 + \chi^{\rho-1} \delta_j{}^{\rho}}, \tag{36}$$

where the last equality follows from the condition (35).

Equalizing marginal rate of technical substitution to relative prices across high-skill and low-skill labour, Appendix A2.2.1 shows that relative skill-intensity in each sector is:

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta} (1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j{}^{\frac{\eta-\rho}{1-\rho}}, \tag{37}$$

Thus, the income share of low-skill in sector j is:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta} \sigma_j^{\eta} \left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{38}$$

The income share of high-skill in sector j is:

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = (1 - J_j)\,\tilde{I}_j. \tag{39}$$

Finally, the total labour income share in sector j is derived in Appendix A2.2.2 as

$$\beta_j = I_j + J_j = J_j \left[q^{1-\eta} \sigma_j^{\eta} \left[\tilde{I}_j \left(1 - \kappa_j\right)^{-\rho} \right]^{\frac{\eta-\rho}{1-\rho}} + 1 \right]. \tag{40}$$

3.2.3 Equilibrium prices and allocation

Using the production functions, Appendix A2.2.3 derive that the equilibrium low-skill wage to have the same expression as (19) with the income share J_l derived in (38). Thus labour mobility implies the relative price of high-skill services have the same expression as in (20).

The equilibrium conditions on input prices and output prices implies an equilibrium condition across the relative prices χ and q. It is shown in Appendix A2.3 that the two-sector model can be mapped into a three-sector economy where ϕ is the price of capital relative to low-skill goods, so $\phi = q_k/p_l$. Using the firm's optimal conditions, the equilibrium price of capital implies:

$$\chi = q \frac{A_l}{\phi} \left(J_l \xi_l^{-\eta} \right)^{\frac{1}{1-\eta}}. \tag{41}$$

Using the definition of income share, Appendix A2.4.1 shows that we can express q explicitly as a function of χ

$$q = \chi \left[\left(\frac{\phi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^{\eta} \left[\left(\chi^{1-\rho} + \delta_l^{\rho} \right) \left(1 - \kappa_l\right)^{\rho} \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}}, \tag{42}$$

where q is increasing in χ. Given that q is a function of χ, it follows that I_j, J_j and $\tilde{I}_j$ are functions of χ. Appendix A2.4 shows that the supply and demand conditions for the full model are

$$l_h = S\left(\chi; \zeta, \frac{\phi}{A_l}\right) \equiv \frac{\zeta q \left(\chi; \frac{\phi}{A_l}\right)^{\eta} \sigma_l^{-\eta} \left(1 - \kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l(\chi)^{\frac{\eta-\rho}{\rho-1}} - 1}{\left(\sigma_h/\sigma_l\right)^{\eta} \left(\frac{1-\kappa_l}{1-\kappa_h} \right)^{\frac{\rho(\eta-1)}{1-\rho}} \left(\frac{\tilde{I}_l(\chi)}{\tilde{I}_h(\chi)} \right)^{\frac{\eta-\rho}{\rho-1}} - 1}. \tag{43}$$

$$l_h = D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right) \equiv \left[1 + \frac{J_l\left(\chi; \frac{\phi}{A_l}\right)}{J_h\left(\chi; \frac{\phi}{A_l}\right)} \left(\frac{1}{x\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)\beta_l\left(\chi\right)} + \frac{1-\beta_h\left(\chi\right)}{\beta_l\left(\chi\right)}\right)\right]^{-1}, \quad (44)$$

where the relative expenditure share $x\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ has the same expression as (23). Note that when $\kappa_j \to 0$, $\beta_j \to 1$, the supply and demand conditions are the same as (21) and (22).

The supply (43) and the demand (44) together solve for (χ, l_h) which then imply value for equilibrium relative wage q from (42). The value-added shares of high-skill services is derived in the Appendix A2.5 as:

$$v_h \equiv \sum_j \frac{p_j Y_j}{\sum_j p_j Y_j} = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \quad (45)$$

3.3 Low-skill wage stagnation and wage inequality

This subsection uses the full model to show that factors that imply a rise in wage inequality does not always contributes to low-skill wage stagnation. Using the optimal capital-skill ratio in (35), the production function can be expressed as a function of high-skill and low-skill labour:

$$Y_j = \tilde{A}_j \left[\lambda_j H_j^{\frac{\eta-1}{\eta}} + (1-\lambda_j) L_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad (46)$$

$$\tilde{A}_j \equiv A_j \left(\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}\right)^{\frac{\eta}{\eta-1}}; \quad \lambda_j \equiv \frac{(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}{\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \quad (47)$$

which takes a similar form as the aggregate production used in the literature (see, for example, Heathcote et al., 2010). Changes in aggregate λ is often interpreted as skill-biased demand shift (SBDS). Our model provides two endogenous sources of SBDS. First, falling relative price of low-skill goods (implied by its faster productivity growth) induces a labour reallocation towards high-skill services due to consumption complementarity. This implies an increase in aggregate λ when $\lambda_h > \lambda_l$, contributing to a between-sector SBDS. Second, falling relative price of capital (driven by uneven productivity and investment specific technical change) implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies an increase in λ_j, contributing to a within-sector SBDS.

Both sources of endogenous SBDS imply a rise in wage inequality but they have different effects on the level of low-skill wage growth. The between-sector SBDS induces a shift from the low-skill sector with high $(1-\lambda_l)$ to the service sector with low $(1-\lambda_h)$, so it reduces the aggregate $(1-\lambda)$ contributing to a slow growth in low-skill wage. The within-sector SBDS, through rising $\tilde{I}_j$, reduces $(1-\lambda_j)$ in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$. This can be seen from equation (46) where the weight on low-skill worker is simply ξ_j. Thus though the within-sector SBDS can contribute to a rise in wage inequality, it does not contribute to the low-skill wage stagnation.

There are other sources of SBDS not captured by the model and can be incorporated as exogenous changes in κ_j and ξ_j which increase λ_j. For instance, as a result of automation some tasks performed by low-skill are displaced by machines (Acemoglu and Autor, 2011), or skill-biased organizational change documented by Caroli and Van Reenen (2001) increases the importance of human capital. Both changes in κ_j and ξ_j can contribute to a rise in wage inequality but as shown in equation (46) only the fall in ξ_j can contribute to the low-skill wage stagnation.

The skill-biased demand shifts discussed above can be put into perspective using the three classes of technical changes in Johnson (1997). The fall in κ_j is an *intensive skill-biased technical change* which raises the marginal product of high-skill workers without affecting those of low-skill labour directly, thus it contributes to wage inequality but not low-skill wage stagnation. The fall in ξ_j is an *extensive skill-biased technical change* which increases the marginal product of high-skill workers while lowers the the marginal product of low-skill workers, thus contributing to both wage inequality and low-skill wage stagnation. What is interesting is the rise in A_h and A_l, which are *skill-neutral technical change* at the sectoral level but becomes skill-biased at the aggregate level because of different factor intensities across sectors, contributing to both rising wage inequality and low-skill wage stagnation.

3.4 The decoupling of wage and productivity in the model

The accounting identity in Section 3.1 shows that the divergence of low-skill wage from the aggregate labour productivity can due to rising wage inequality, falling labour income shares and rising relative cost of living. We now study them through the lens of the model. As shown in equation (29), the divergence in real terms is a product of divergence in nominal terms and the price deflators. For this model with two labour input, the nominal divergence is equal to:

$$\frac{y}{w_l} = \frac{(w_h/w_l - 1)\,\mu_H + 1}{\beta}; \quad \beta = \beta_l v_l + \beta_h v_h, \quad \mu_H = \frac{M_h}{M_l + M_h} \tag{48}$$

Given the share of high-skill market hours μ_H, the model provides three sources for the divergence by predicting relative wage w_h/w_l, the aggregate labour income share β and the relative price indexes P_C/P_Y.

The model implies a rise in the relative wage due to the two sources of endogenous skill-biased demand shifts. It predicts a rise in the high-skill income share and a fall in the low-skill income share in both sectors, and a shift towards high-skill services, thus has an ambiguous prediction on labour income share β.

The growth of the relative price indexes P_C/P_Y is obtained from the difference in the growth of the two deflators. The growth of consumption price index is given in (15), and similarly, the growth rate of the aggregate output price index P_Y is computed as a weighted average of the price of each sector using value-added shares as weight (the Tornqvist formula). Thus the growth in the relative price of consumption price index relative to output price index is:

$$\hat{P}_C - \hat{P}_Y \approx (x_h - v_h)\,(\hat{p}_h - \hat{p}_l)\,. \tag{49}$$

The model predicts a rise in relative price of high-skill services which is transmitted into rising P_C/P_Y given the expenditure share on services exceeds its value-added share.

4 Quantitative Results

To quantify the role of our proposed mechanism in accounting for the low-skill wage stagnation and its divergence from the aggregate labour productivity, we calibrate the model to match key features of the US during 1980 to 2010. The forces that drive the mechanism of the model are $(A_{lT}/A_{l0}, A_{hT}/A_{h0}, \phi_T/\phi_0)$. They are calibrated to match the rise in the relative prices of high-skill services, the fall in the relative prices of capital and the aggregate labour productivity growth. The weights of each input in the production function $\{\xi_{gt}, \xi_{st}, \kappa_{gt}, \kappa_{st}\}_{t=0,T}$ are set to match the sectoral income shares while the relative supply of high-skill labour (ζ_0, ζ_T) are set to match the aggregate income shares of high-skill and low-skill labour.

In sum, the labour market changes are driven by changes in six parameters: $\hat{A}_{lh}$ in equation (23), $\hat{A}_l$ in equation (26), the relative price of capital ϕ, the production weights $\{\xi_l, \xi_h, \kappa_l, \kappa_h\}$ and the relative supply of high-skill labour ζ.[12]

4.1 Data Targets

The data targets used for the calibration are reported in Table 1. Data from the five-year average 1978-1982 is used for 1980 and 2006-2010 for 2008. The data Appendix A1 describes how the data targets are constructed using data from the WORLD KLEMS, the CPS and the BEA. In brief, WORLD KLEMS data is used to compute value-added, prices and labour income shares for each sector. We group sectors into low-skill and high-skill according to the importance of high-skill workers in each sector. The high-skill sector includes: finance, insurance, government, health and education services and the low-skill sector includes the remaining industries. As shown in Table 1 the high-skill income share (I_j) increases while the low-skill income shares (J_j) fall in both sectors. The total labour income share $(I_j + J_j)$ falls in the low-skill sector, rise in the high-skill sector and fall for the overall economy. The price of high-skill sector relative to the price of low-skill sector grows at 1.4% and the annual growth of the aggregate labour productivity deflated by the price of the low-skill sector was 2.1% during this period. Using the ratio of P_K/P_Y from the BEA and the ratio P_Y/P_l from the KLEMS, the implied price of capital relative to low-skill sector ϕ declines at 0.5% per year.[13]

To be consistent with the accounting equation (29), the aggregate wages are computed by merging the KLEMS data on total compensation and hours with the distribution of demographic subgroups in the CPS. It is important to note that labour compensation variable

[12]Given the definition of $\hat{A}_{lh}$ in equation (23) and $\hat{A}_l$ in equation (26), we do not need to separate the preference parameter ψ from A_l and A_h to solve for the model.

[13]It is worth noting that the growth of P_Y in KLEMS is growing at 2.94% which is almost identical to that of BEA at 2.86%.

Table 1: Calibration Data Summary

	Level							Growth (% p.a.)		
	J	J_h	J_l	I	I_h	I_l	q	$\frac{y}{p_l}$	ϕ	$\frac{p_h}{p_l}$
1980	0.41	0.23	0.46	0.17	0.33	0.12	1.44	-	-	-
2008	0.28	0.21	0.31	0.28	0.44	0.21	1.94	2.1	-0.5	1.4

of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labour input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. Wages by demographic groups are used to construct composition adjusted wages (w_{ht}, w_{lt}) for the two periods.[14] More specifically, we control for age, sex, race and education within high-skill and low-skill.[15] The relative wage q_t is obtained as w_{ht}/w_{lt}. Given the distribution of demographic subgroups is taken from the CPS, the implied relative wage is the same as the CPS.

4.2 Calibration

The elasticity of substitution across high-skill and low-skill labour $\eta = 1.4$ is taken from Katz and Murphy (1992) and the elasticity of substitution across capital and high-skill labour $\rho = 0.67$ is taken from Krusell et al. (2000). There is no direct estimate of elasticity of substitution across high-skill and low-skill goods ϵ. The literature on the structural transformation finds that the elasticity of substitution across agriculture, manufacturing and services is close to zero (Herrendorf et al., 2013). Given we re-group these three sectors into two sectors, this

[14] To compute (w_h, w_l), we allow the efficiency unit of labour to be different within subgroups (gender, age, education and race) of a skill-type, e.g. one hour of high-school graduate is not equal to that of high school dropout in efficiency units. The relative efficiency unit of an average high-skill relative to an average low-skill is assumed to be one, where the average worker in each skill-type is defined by long-run hours shares of subgroups. Instead of choosing the average worker as the reference group, we could make alternative assumption such as assuming the relative efficiency for a particular subgroup, e.g. a 18-25 years old white male, then compute the relative efficiency for an average high-skill relative to an average low-skill. As long as the relative efficiency does not change over time, the quantitative result on low-skill wage stagnation is robust to this alternative assumption given we match the initial w_l in the data.

[15] We do not control for occupation in constructing the composition adjusted wage for the quantitative exercise because unlike other controls, occupation is a choice variables for the worker. In contrast, Figure 1 and 2 control for changes in occupations given the objective there is to compare changes in low-skill wage across industries.

is likely to imply a higher degree of substitution. The equilibrium condition (9)), on the other hand, implies that the own price elasticity of the two goods is $-\varepsilon$. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services ranging from -0.3 to 0, this is informative but not an exact estimate for $-\varepsilon$ which is the price elasticity of high-skill services in our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value, which is also the benchmark value used in Buera et al. (2018) for the elasticity of substitution across high-skill and low-skill sector. We conduct sensitivity analysis in Appendix A3.3.

The relative wage q and incomes shares reported in Table 1 are used to determine the relative supply of high-skill efficiency labour ζ and the input weights $(\xi_l, \xi_h, \kappa_l, \kappa_h)$ in the two periods. In the aggregate economy, the income share of high-skill relative to the low-skill is:

$$\frac{I_t}{J_t} = \frac{w_{ht} H_t}{w_{lt} L_t} = q_t \zeta_t, \tag{50}$$

which implies a value for the relative supply of high-skill efficiency labour ζ_t given data on (q_t, I_t, J_t). [16]

Given a value for ϕ/A_l, equation (41) can be used together with the equations on income shares to set the input weights to match sectoral income shares in the data. To simplify the explanation, denote 1980 as period 0 and 2008 as period T. We normalized $\phi_0/A_{l0} = 1$, this pins down all input weights in period 0 (see Appendix A3.1 for details). Using these parameters the supply equation (43) implies a value of l_{h0}. The value of $\hat{A}_{lh0}$ is then set to match the relative wage q_0 using the demand equation (44).

For a given level of A_{lT}/A_{l0}, data on the fall in ϕ_t implies a value for ϕ_T/A_{lT}, which pins down all inputs weights in period T. We then set the change in relative productivity A_{lhT}/A_{lh0} so that the predicted relative price of high-skill services matches the data. Finally we adjust A_{lT}/A_{l0} so that the predicted changes in the aggregate labour productivity deflated by the price of the low-skill sector, y/p_l, matches the data. It is important to note that the model is not calibrated to match the relative wage in 2008.[17]

[16] Note that the H_j and L_j are not the raw market hours by the high-skill and low-skill workers in the data. The composition adjusted high-skill hours H_j in sector j is computed as high-skill income in sector j divided by the composition adjusted high-skill wage, similarly for L_j.

[17] The relative wage in 2008 is only used together with the income shares to calibrate $\{\xi_{gT}, \xi_{sT}, \kappa_{gT}, \kappa_{sT}\}$. As an alternative, we could choose growth in relative productivity, A_{lhT}/A_{lh0}, to match the rise in the

Table 2: Parameters of Calibration

A. Parameters from the literature		
Parameters	Values	Source
ε	0.2	Benchmark value, see main text
ρ	0.67	Krusell et al. (2000)
η	1.4	Katz and Murphy (1992)

B. Calibrated parameters				
Parameters	1980	2010	Growth (% p.a.)	Target
ϕ			-0.50	Price of capital relative to the low-skill sector
A_l			1.10	Labour productivity deflated by price of the low-skill sector
A_{lh}			1.82	Relative price of high-skill services
ξ_l	0.33	0.25	-0.93	Sectoral income share. See Appendix A3.1
ξ_h	0.20	0.19	-0.13	Sectoral income share. See Appendix A3.1
κ_l	0.74	0.69	-0.21	Sectoral income share. See Appendix A3.1
κ_h	0.41	0.33	-0.79	Sectoral income share. See Appendix A3.1
ζ	0.29	0.50	1.92	Relative aggregate labour income shares

Table 2 reports the calibrated parameters. The data implies faster productivity growth in the low-skill sector and higher input weights on low-skill worker in the low-skill sector $\xi_l > \xi_h$, confirming assumption A1 of the theory. The implied annual growth of ϕ, A_{lh}, A_l, ζ and input weights are reported in Panel B of Table 2.[18] Matching the aggregate income shares of the high-skill and low-skill labour implies a rise in the relative supply of high-skill efficiency labour. Matching the sectoral income shares, on the other hand, requires changes in the input weights reflecting other sources of skill-biased demand shifts that are exogenous to our model. In sum, the quantitative results are driven by the endogenous skill-biased demand shifts through rising relative TFP (A_l/A_h) and the investment specific technical change (ϕ), the increase in relative supply of high-skill (ζ), and the exogenous skill biased demand shifts (κ_j, ξ_j).

Using the calibrated parameters the model delivers predictions on wages, allocation of labour, relative prices and labour productivity for each sector. The baseline calibration

relative wage using the demand equation (44) for 2008. However, given the objective of the quantitative exercise is to examine the proposed mechanism in accounting for stagnant low-skill wage and its divergence, and the mechanism is governed by the changing relative prices, we choose to match the changes in relative prices instead of relative wage.

[18]Note that negative growth in κ_j is not necessarily a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

implies a rise of relative wage q from 1.44 to 1.92, accounting for 96% of the rise in the data (1.44 to 1.94). It predicts a rise in the share of low-skill efficiency labour in the high-skill services l_h from 0.14 to 0.20, accounting for 86% of the rise in the data (0.14 to 0.21). Consistent with the data, it predicts a fall in labour income share in the low-skill sector and a rise in labour income share in the high-skill sector, and a decline in aggregate labour income share. These results and the role played by endogenous skill-biased demand shifts $(A_l/A_h, \phi)$, the exogenous skill biased demand shifts (κ_j, ξ_j) and the increase in relative supply of high-skill (ζ) can be found in Appendix A3.2. Appendix Table A3 shows that both endogenous and exogenous skill-biased demand shifts are important for the rise in relative wage, but the basic mechanism through rising A_l/A_h is crucial for the observed labour reallocation while the low-skill labour displacing technical change through falling ξ_j is crucial for the fall in the labour income shares for the low-skill sector.

The sectoral real labour productivity growth in the model is

$$\frac{y_j}{p_j} \equiv \frac{Y_j}{L_j + H_j} = A_j \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{1 + H_j/L_j}\right), \tag{51}$$

which shows that in addition to TFP, other factors also contribute to the sectoral labour productivity growth. The calibrated model predicts the sectoral labour productivity growth is 2.2% for the low-skill sector and -0.2% for the high-skill sector, which match the 2.3% and 0% observed in the data almost perfectly.[19]

4.3 Predictions on Wage-Productivity Divergence

Table 3 reports the percentage change in the real divergence, decomposed into the changes in relative cost of living, wage inequality and the aggregate labour income share. Since KLEMS data does not contain information on consumption, we simply take P_C/P_Y as the ratio of PCE and GDP implicit deflators from the BEA. [20]

[19] The calibration implies that the TFP for high-skill services, A_h is falling. This decline can be rationalized through the findings of Aum et al. (2018) and Bárány and Siegel (2019). The former finds negative growth for high-skill occupations (Professional and Management) while the latter finds negative growth for abstract occupation. We do not model occupations, but their findings could be the sources of the falling A_h in our model given these occupations are concentrated in the high-skill sector.

[20] This implies P_C/P_Y increased by 2.8% as reported in Table 3. If we were to use CPI which grows faster than PCE, the increase in P_C/P_Y would be at 11.5%. This alternative value will imply a larger real

The data (row 1) provides an empirical decomposition for the accounting identity in equation (29). During this 30-year period, the negative forces imposed by rising relative cost of living, growing wage inequality and falling aggregate labour income share largely offset the impact of rising productivity on low-skill real wage. The rise in the relative cost of living contributes to 10%(=2.8/27) of the real divergence, the increase in the wage inequality contributes to 70% (=19/27) and the fall in the aggregate labour income share accounts for the remaining 20% of the real divergence.[21]

The baseline (row 2) can account for all the real and the nominal divergence. The remaining rows of Table 3 examine each of the five forces that drives these changes: the endogenous skill-biased demand shifts through rising relative productivity of the low-skill sector (higher A_l/A_h) and the investment specific technical change (lower ϕ), the exogenous skill-biased demand shifts through falling input weight on low-skill worker (lower ξ_j) and rising input weight on high-skill worker within the capital-skill composite (lower κ_j), and the increase in relative supply of high-skill (higher ζ).

Row 3 and 4 of Table 3 shows that both sources of endogenous skill-biased demand shifts contribute to the real divergence by predicting a rise in wage inequality and a rise in the relative cost of living. Among the two sources, the investment-specific technical change (Row 4) contributes more through the rise in wage inequality, while the uneven productivity growth (Row 3) contributes through both channels. More specifically, the uneven productivity growth alone (row 3) can account for 85% (=23/27) of the real divergence by predicting 68% (=13/19) of the rise in wage inequality and all the rise in the relative cost of living. This shows that the basic mechanism is quantitatively importance in accounting for the real divergence.

Row 5 and 6 of Table 3 shows that both sources of the exogenous skill-biased demand shifts contributes to the real divergence by predicting a rise in wage inequality but only the low-skill labour displacing technical change (Row 5) can generate a fall in labour income

divergence and a slower real wage growth in the data row in Table 3 and 4, but not the predictions of the model. Due to the concerns that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we follow the literature in using the PCE deflator.

[21] The literature on the average wage and productivity divergence often uses the nonfarm business sector. In Appendix A1.3 we conduct the empirical decomposition for the accounting identity in equation (29) using similar data.

Table 3: Real and Nominal Divergence, Cumulative Percentage Change, 1980-2008

		Real	Nominal			Deflator	
		$(y/w_l)(P_C/P_Y)$	y/w_l	w/w_l	β	P_C/P_Y	p_s/p_g
(1)	Data	27	24	19	-3.4	2.8	49
(2)	Model	34	23	19	-3.7	8.3	matched
		Counterfactual (keeping all else constant at 1980)					
Endogenous skill-biased demand shifts							
(3)	$A_l/A_h \uparrow$	23	9.3	13	3.5	12	79
(4)	$\phi \downarrow$	11	9.7	14	3.7	1.5	8.8
Exogenous skill-biased demand shifts							
(5)	$\xi_j \downarrow$	28	29	19	-7.5	-0.6	-3.3
(6)	$\kappa_j \downarrow$	6.3	5.9	14	7.6	0.3	2.1
Relative supply of high-skill labour							
(7)	$\zeta \uparrow$	-4.6	-3.4	-7.1	-3.8	-1.2	-6.6

Note: the combined effects are not the sum of the individual effects because the model is not linear.

share. Finally, the increase in relative supply of high-skill labour contributes negatively to the divergence as it reduces wage inequality but it contributes to a fall in labour income share (Row 7). This is because an increase in the supply of high-skill lowers its relative wage which contributes to a fall in wage inequality, but it induces an increase in capital income share due to capital-skill complementarity.

4.4 Predictions on Wage Stagnation

Table 3 shows that both the endogenous and exogenous skill-biased demand shifts are important in accounting for the decoupling of wage and productivity. The endogenous forces do the job by predicting the rise in the relative cost of living and the rise in wage inequality. The exogenous forces are needed for the remaining rise in wage inequality and the fall in the aggregate labour income share. We next turn to the their effects on the growth of low-skill real wage. As highlighted in equation (24), their effects depends crucially on how they affect the growth of the low-skill product wage in each sector (w_l/p_j), i.e. the marginal product of low-skill workers in each sector.

Table 4 shows that among the four types of skill-biased demand shifts, only the uneven

Table 4: Productivity and Wages, Cumulative Percentage Change, 1980-2008

		y/P_Y	w_l/P_C	y/p_l	w_l/p_l	w_l/p_h
(1)	Data	60	26	78	44	-3.4
(2)	Model	61	20	matched	44	-3.2
Counterfactual (keeping all else constant at 1980)						
Endogenous skill-biased demand shifts						
(3)	$\boldsymbol{A_l/A_h \uparrow}$	**43**	**17**	**68**	**54**	**-14**
(4)	$\phi \downarrow$	81	63	85	69	55
Exogenous skill-biased demand shifts						
(5)	$\boldsymbol{\xi_j \downarrow}$	**53**	**19**	**51**	**18**	**22**
(6)	$\kappa_j \downarrow$	61	51	62	53	50
Relative supply of high-skill labour						
(7)	$\zeta \uparrow$	80	89	77	83	96

productivity growth (Row 3) and the low-skill labour displacing technical change (Row 5) can contribute to stagnant low-skill real wages, but through different channels. The uneven productivity growth delivers the result by predicting rising relative price of high-skill services. The low-skill labour displacing technical change delivers the result by predicting low growth in marginal product of low-skill labour in both sectors.

In the data (Row 1) the marginal product of low-skill labour in the low-skill sector actually rose by 44% and fell in the high-skill sector due to the rise in the relative price of high-skill services. Consistent with the data, the uneven productivity growth (Row 3) implies a 54% rise in the low-skill sector and a fall in the high-skill sector by predicting the rising relative price of high-skill services. The uneven productivity growth implies a reallocation from the low-skill sector with high ξ_l to the high-skill sector with low ξ_h, contributing to a decline in the average ξ in the economy. The low-skill labour displacing technical change (Row 5) relies on lowering ξ_j directly in both sectors, thus predicts low growth in the marginal product of labour for both sectors, which misses the differential trends observed in the data. Finally, as discussed in Section 3.3, both the investment specific technical change (Row 4) and lower κ_j (Row 6) boost the growth in low-skill real wage as they increase the marginal product of low-skill labour in both sectors.

To sum up, the quantitative exercise shows that all four types of skill-biased demand shifts imply a rise in wage inequality and wage-productivity divergence, their effects on the growth of low-skill real wage are very different. Among them, only the between-sector skill-biased demand shift and the low-skill labour displacing technical change can generate stagnant low-skill wage. However, the between-sector skill-biased demand shift is essential for understanding the differential trends in the marginal product of low-skill labour observed in the two sectors. This mechanism is quantitative relevant for understanding the important features of low-skill wage stagnation as shown in Figure (1) and (2). It predicts a rise in the relative price of high-skill services, a reallocation of low-skill workers into high-skill services where the marginal product of low-skill worker is stagnant.

5 Conclusion

Despite strong growth in labour productivity, low-skill wage is stagnant because of the divergence in low-skill wage and productivity driven by rising wage inequality, falling labour share and rising relative cost of living. This paper develops a multi-sector model that uneven productivity growth across sectors implies a labour reallocation towards high-skill services which experience rising relative price and put a lower input weight on low-skill workers. This reallocation process can generate low-skill wage stagnation, rising wage inequality and the wage-productivity divergence.

Quantitatively, the model does a good job in accounting for these three facts. Other sources of skill-biased demand shift are needed to account for the full picture in the presence of rising relative supply of high-skill labour. The uneven productivity growth is essential in understanding the source of stagnant low-skill real wage in the data as it can replicate differential trends in the marginal product of low-skill labour across sectors by predicting a rise in the relative price of high-skill services. Finally, increasing relative supply of high-skill labour can reverse the divergence and boost the growth in low-skill wage.

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Appendix

A1 Data Appendix

A1.1 Industry Data

The main dataset at the industry level is the March 2017 Release of the Unites States data from the WORLD KLEMS database (Jorgenson et al., 2017), which reports industry value-added, price indexes, labour compensation and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S. This provides the data needed to compute value-added share, relative prices and labour income shares for all industries that are consistent with the official statistics.

To classify sectors into high-skill and low-skill sectors, we use April 2013 Release of the U.S. data from the WORLD KLEMS (Jorgenson et al., 2013) which provides a labour input file that allows computation of low- and high-skill workers' share in labour compensation and value-added. High-skill worker is defined as education greater than or equal to college degree. Table A1 reports the long-run (1980-2010) average high-skill share in total value-added and total labour income for 15 one-digit industries. For a sector to be classified as high-skill we require that the long-run high-skill labour income share out of total labour income and total value-added to be jointly above the total economy average. High-skill service sector includes finance, insurance, government, health and education services (code J,L,M,N), and the remaining industries are grouped into the low-skill sector.

Using this classification we map the 65 NAICS industries of the KLEMS 2017 Release and the three digit *ind1990* codes of the CPS into the two broad sectors for the quantitative analysis. Value-added and labour compensation for each broad sector are obtained by summing over industries in each broad sector. Sectoral value-added prices are calculated as Tornqvist indexes, where value-added shares are used as weights. For the ratio of aggregate consumption price deflator and output price deflator, we use the BEA's implicit price deflators of GDP and Personal Consumption Expenditures, respectively. The price of capital is calculated as the investment in total fixed assets divided by the chain type quantity index

Table A1: High-skill Income Shares by Industry, 1980-2010 average

		High-skill share in	
Industry	Code	Value-added	Labour income
Agriculture, Hunting, Forestry and Fishing	AtB	10	19
Mining and Quarrying	C	11	32
Total Manufacturing	D	20	31
Electricity, Gas and Water Supply	E	8.7	30
Construction	F	14	16
Wholesale and Retail Trade	G	22	30
Hotels and Restaurants	H	14	18
Transport and Storage and Communication	I	16	25
Financial Intermediation	**J**	**33**	**55**
Real Estate, Renting and Business Activity	K	21	55
Public Admin	**L**	**29**	**40**
Education	**M**	**58**	**77**
Health and Social Work	**N**	**39**	**49**
Other Community, Social and Personal Services	O	23	31
Private Households With Employed Persons	P	16	16
All Industries	TOT	25	40

Notes: The table reports the share of high-skill worker in total value-added and labour income by industry. High-skill is defined as education greater than or equal to college degree. Labour income reflects total labour costs and includes compensation of employees, compensation of self-employed, and taxes on labour.
Source: April 2013 Release of the WORLD KLEMS for the U.S.

for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts).

Industries in Figures 1 and 2 are the one-digit industries reported in Table A1 with some regrouping. Due to low number of observations in CPS we merge agriculture (AtB) with mining (C), and other services (O) with private households (P). We also regroup public administration (L), education (M), and health and social work (N) as a single industry to ensure consistency in industry definitions.[22] Our mapping across KLEMS 2013, KLEMS 2017 and CPS industries is provided in Table A2.

A1.2 Wages, Efficiency Hours, and Productivity

We use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012 (Ruggles et al., 2017). Our sample includes wage and salary workers with a job aged 16-64, who are not student, retired, or in the military. Hourly wage is calculated as annual wage income divided by annual hours worked, where the latter is the

[22]For instance, public education is included in the general government industry in KLEMS 2017, while it is part of education in KLEMS 2013.

product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars (based on PCE price index) are dropped.

The composition adjusted mean wages of low-skill workers for each of the sectors, used in Figures 1 and 2, are computed using the CPS data as follows. Within each sector, we calculate mean wages weighted by survey weights for each of 216 subgroups composed of two sexes, white and non-white categories, three education categories (high school dropout, high school graduate, some college), six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years), and three occupation categories (high-wage occupations including professionals, managers, technicians, and finance jobs, middle-wage occupations including clerical, sales, production, craft, and repair jobs, operators, fabricators, and labourers, and low-wage occupations including service jobs). Sector-level means by skill are calculated using the long-run average hours share of each subgroup in the labour market as weights. This way we obtain a measure of industry wage that only compares growth differences of subgroups across industries. However, applying long-run hours share by subgroup can still affect industry means through composition when for some subgroups there are missing observations in some of the industries. Cells containing missing wages are imputed for each year of the dataset using a regression of the log of hourly wages on industry dummies and dummies including the full set of interactions of subgroups. We assign predictions from this regression to the missing wage observations while keeping the observed wages. The growth rate of sector wages with and without imputation are very close. Finally, we deflate nominal wages by the PCE price index for real wages and by the value-added price index for product wages.

For the quantitative analysis, used in Table (3) and (4), the aggregate wage has to be consistent with the measure of aggregate productivity, so we use the aggregate labour compensation and aggregate hour from the KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skill and average low-skill workers, we merge KLEMS 2013 data on total labour compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sex, two race, five education, six age categories. Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours of each subgroup is calculated in a similar way. The wage for each of the subgroup is then calculated as total compensation divided by total hours. The aggregate wage for low-skill and high-skill are calculated as the average wage of the relevant subgroups using their long-run (1980-2010) hour shares as weights. It is important to note that labour compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labour input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescale the CPS total hours and total wage income to sum up to KLEMS total.

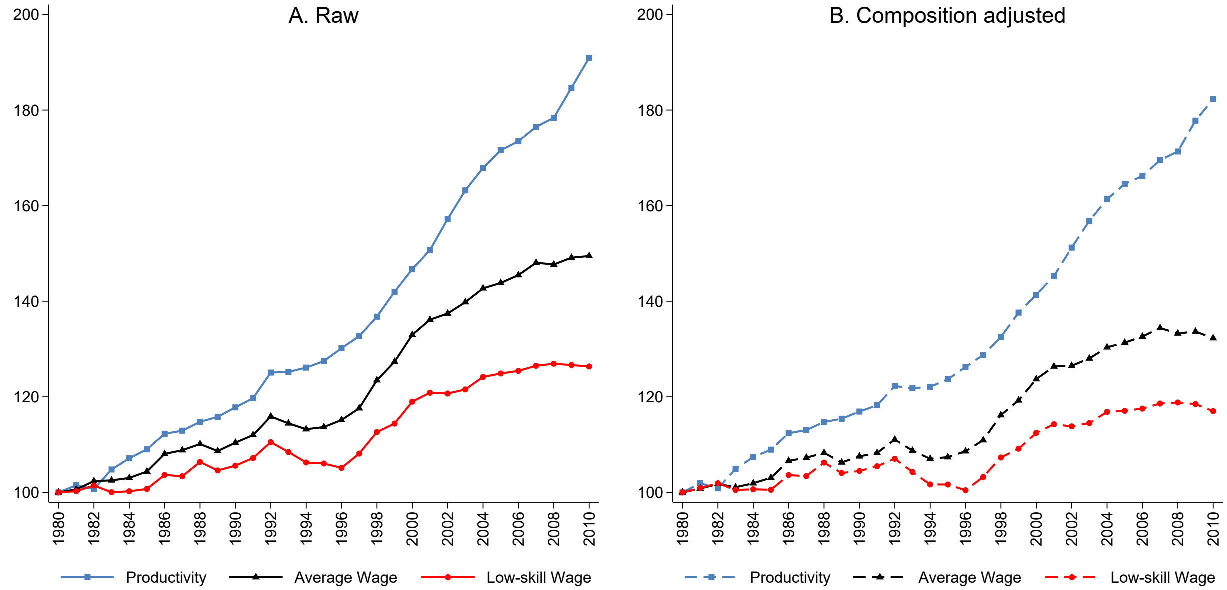
Efficiency hours, corresponding to (H, L) in the model, are computed as the labour compensation divided by composition-adjusted wage for high-skill and low-skill workers respectively. Total efficiency hours are the sum of low- and high-skill efficiency hours. We calculate real labour productivity as total value-added divided by total efficiency hours and deflate with the output price index.

Table A2: Industry Mapping

NACE (KLEMS 2013)	NAICS (KLEMS 2017)	IND1990 (CPS)
AtB & C	Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining	Agriculture, Forestry, and Fisheries, Mining
D	Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products	Manufacturing
E	Utilities	Utilities
F	Construction	Construction
G	Wholesale Trade, Retail Trade	Wholesale Trade, Retail Trade
H	Accodomation, Food Services and Drinking Places	Hotels and Lodging Places, Eating and Drinking Places
I	Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services	Transportation, Communications
J	Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles	Finance, Insurance
K	Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services	Real Estate, Business Services, Professional Services*
L & M & N	Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises	Public Administration, Education*, Health and Social Services*
O & P	Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government	Sanitary and Personal Services, Private Households, Entertainment and Recreation Services, Museums, Art Galleries, and Zoos, Labor Unions, Religious Organizations, Membership Organizations, n.e.c.

Notes: The table shows mapping of KLEMS 2013 industries to KLEMS 2017 and CPS industries. The description of KLEMS 2013 industries are provided in Table A1. Industries marked with * do not have separate sections in CPS industry classification. They are constructed as follows. Professional Services: Engineering, architectural, and surveying services, Accounting, auditing, and bookkeeping services, Research, development, and testing services, Management and public relations services, Miscellaneous professional and related services, Legal services. Education: Elementary and secondary schools, Colleges and universities, Vocational schools, Educational services, n.e.c. Health and Social Services Offices and clinics of physicians, Offices and clinics of dentists, Offices and clinics of chiropractors, Offices and clinics of optometrists, Offices and clinics of health practitioners, n.e.c., Hospitals, Nursing and personal care facilities, Health services, n.e.c., Job training and vocational rehabilitation services, Child day care services, Family child care homes, Residential care facilities, without nursing, Social services, n.e.c.

Figure A1: Divergence in the BLS Nonfarm Business Sector Data



Notes: The figure plots low-skill and average hourly real wage and average hourly real labour productivity in the U.S. economy, all normalized to 100 in 1980. Raw (composition adjusted) wage and hours are used in Panel A (B). Real labour productivity is from Bureau of Labor Statistics (BLS). Real hourly wages are calculated by merging hours and income shares in the Current Population Survey (CPS) with the total hours and labour income in BLS. Productivity is deflated by the output price index. Wages are deflated by Personal Consumption Expenditure (PCE) price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 120 demographic groups, where the fixed weights are groups' long-run employment shares. See the appendix subsection for the construction of variables.
Source: BLS nonfarm business sector multifactor productivity statistics, CPS, and authors' calculations.

A1.3 Divergence in the BLS Nonfarm Business Data

This subsection compares the wage growth and the decomposition of low-skill wage and productivity divergence by KLEMS, on which results in the main text are based, with Bureau of Labour Statistics (BLS) nonfarm business productivity data. BLS Nonfarm business data is typically used by the papers on U.S. wage-productivity divergence (e.g. Lawrence and Slaughter, 1993; Lawrence, 2016; Stansbury and Summers, 2017), and its labour share is a widely cited headline measure (Elsby et al., 2013).

In order to compute wages at skill-level that are consistent with the BLS productivity series' hourly compensation growth, the share of annual wage income and total hours of 120 demographic groups from March CPS are used. Demographic groups are based on six age, two gender, two race and five education categories. Compensation (hours) for each sub-group is calculated as compensation (hours) share times BLS total compensation (hours).

BLS-consistent wages for each subgroup is calculated as total compensation divided by total hours. Average and low-skill wages are then calculated as the mean hourly wages of relevant subgroups weighted by the their hours share. In the composition adjusted wages, long-run hours shares are used as weights. This is the same procedure as we followed for the quantitative analysis with two exceptions. First, we further exclude agriculture, private households, and public administration sectors to comply with nonfarm business sector. Second, aggregate labour income and hours are rescaled to those of nonfarm business sector. For real wages Personal Consumption Expenditure price index (PCE) is used as the wage deflator.

Real labour productivity is U.S. non farm business nominal output divided by nonfarm total composition adjusted hours and deflated by the output price deflator from BLS. Average wage for all workers is calculated as total compensation divided by total composition adjusted hours of the non farm business sector. Composition adjusted or efficiency hours are calculated for each skill as the total compensation divided by composition adjusted wages.

Figure A1 plots the raw and composition adjusted low-skill real wage, average real wage, and real labour productivity. From 1980 to 2010, the low-skill wage growth is around 25 percent which shrinks just below 20 percent when adjusted for compositional changes. These figures are slightly lower from those suggested by KLEMS (Table 4), and somewhat higher than those calculated directly from CPS (Figure 2). The former difference stems from the industry coverage that particularly affects growth rates in labour income, which is lower in the nonfarm business sector. Hours grow at the same rate in both. On the contrary, the latter difference, i.e. slower wage growth in CPS, is driven by the stronger growth in CPS hours compared to those in the macro sources, despite a bit higher growth in CPS wage income. [23]

As shown in Figure A1, low-skill real wage growth is less than a quarter of the labour productivity growth, suggesting, suggesting a higher real divergence than what is calculated from KLEMS. The reason for a higher divergence is partly greater decline in labour share of nonfarm business (7 percent as opposed to 3.4 in KLEMS), which is already hinted by the discussion above regarding the stronger labour income growth in KLEMS. A second but

[23] See Stewart and Frazis (2019) for an up-to-date discussion on the hours estimated by CPS and other BLS measures. Although total annual hours estimated from CPS is seen as problematic, authors recommend the use of CPS for comparing hours across demographic groups, which is consistent with our data approach.

more important reason is the large growth in the BLS nonfarm business output deflator compared to the BEA's output deflator. Accordingly, the relative cost of living increases by 13 percent compared to 2.8 in KLEMS. Not surprisingly, inequality growth is the same in the two sources given that they both employ hours and income distribution of CPS. Recall Table 3 implies increasing inequality, declining labour share and rising relative cost of living accounts for 70, 20 and 10 percent of the real divergence respectively. The corresponding decomposition based on nonfarm business sector are 48, 19 and 33 percent, implying a larger role for the rising relative cost of living for the real divergence, and a larger role of labour share relative to wage inequality for the nominal divergence.

A2 Theory Appendix

The proof here is for general case. It can be applied to the basic model with no capital by settting $\kappa_j = 0$.

A2.1 Deriving consumption price index

Define p_{ci} as household i' price index for the consumption basket. By definition:

$$p_{ci}c_i = p_l c_{ig} + p_h c_{is} = c_{ig} p_l \left(1 + x\right).$$

From the utility function,

$$\frac{c_i}{c_{ig}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{c_{is}}{c_{ig}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

substituting the optimal condition (9),

$$\frac{c_i}{c_{ig}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right)^{\varepsilon-1}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

simplify to

$$\frac{c_i}{c_{ig}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi}\right)^{\varepsilon} \left(\frac{p_l}{p_h}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\psi\left(1+x\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A1}$$

thus using the expression for c_i/c_{il} in (A1), the consumption price index becomes

$$p_{ci} = \left(\psi\left(1+x\right)\right)^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1+x\right),$$

which is identical across households due to the assumption of a homothetic preference with identical weight, so it is also the same as the aggregate price index for consumption P_C. Using the expression for x in (9),

$$P_C = p_{ci} = \psi^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1 + \left(\frac{p_h}{p_l}\right)^{1-\varepsilon} \left(\frac{1-\psi}{\psi}\right)^{\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$

which simplifies to

$$P_C = \left[\psi^{\varepsilon} p_l^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_h^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{A2}$$

Thus

$$\begin{aligned}
\frac{P_{Ct}}{P_{Ct-1}} &= \left[\frac{\psi^{\varepsilon} p_{gt}^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_{st}^{1-\varepsilon}}{\psi^{\varepsilon} p_{gt-1}^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_{st-1}^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\frac{\psi^{\varepsilon} p_{gt-1}^{1-\varepsilon}}{\psi^{\varepsilon} p_{gt-1}^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_{st-1}^{1-\varepsilon}} \left(\frac{p_{st}}{p_{st-1}}\right)^{1-\varepsilon} + \frac{\left(1-\psi\right)^{\varepsilon} p_{st-1}^{1-\varepsilon}}{\psi^{\varepsilon} p_{gt-1}^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_{st-1}^{1-\varepsilon}} \left(\frac{p_{st}}{p_{st-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[x_{gt} \left(\frac{p_{st}}{p_{st-1}}\right)^{1-\varepsilon} + x_{st} \left(\frac{p_{st}}{p_{st-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

In continuous time, the growth of P_C can be derived as follows. Taking log,

$$\ln P_C = \frac{1}{1-\varepsilon} \ln\left[\psi^{\varepsilon} p_l^{1-\varepsilon} + \left(1-\psi\right)^{\varepsilon} p_h^{1-\varepsilon}\right]$$

so differentiate w.r.t. time,

$$\frac{\dot{P}_C}{P_C} = \frac{\psi^{\varepsilon} p_l^{-\varepsilon} \dot{p}_l + \left(1-\psi\right)^{\varepsilon} p_h^{-\varepsilon} \dot{p}_h}{P_C^{1-\varepsilon}},$$

which can be rewritten as

$$\frac{\dot{P}_C}{P_C} = \left(\frac{\psi^{\varepsilon} p_l^{1-\varepsilon}}{P_C^{1-\varepsilon}}\right) \frac{\dot{p}_l}{p_l} + \left(\frac{(1-\psi)^{\varepsilon} p_h^{1-\varepsilon}}{P_C^{1-\varepsilon}}\right) \frac{\dot{p}_h}{p_h},$$

note from (A2) that

$$\left(\frac{P_C}{p_l}\right)^{1-\varepsilon} = \psi^{\varepsilon} + (1-\psi)^{\varepsilon} \left(\frac{p_h}{p_l}\right)^{1-\varepsilon} = \psi^{\varepsilon} (1+x),$$

so the growth rate of aggregate consumption price is a weighted average of the prices of the two consumption goods:

$$\frac{\dot{P}_C}{P_C} = x_l \frac{\dot{p}_l}{p_l} + x_h \frac{\dot{p}_h}{p_h}.$$

A2.2 Equilibrium prices

A2.2.1 Deriving the ratio H_j/L_j

Equating MRTS across high-skill and low-skill labour to relative wages:

$$q = \frac{1-\xi_j}{\xi_j} \left(\frac{L_j}{\tilde{H}_j}\right)^{\frac{1}{\eta}} (1-\kappa_j) \left(\frac{G_j(H_j, K_j)}{H_j}\right)^{\frac{1}{\rho}},$$

which can be re-written as

$$q = \sigma_j (1-\kappa_j) \left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}} \left(\frac{G_j(H_j, K_j)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}}; \qquad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}$$

where using equation (35), we can derive:

$$\begin{aligned}
\frac{G_j(H_j, K_j)}{H_j} &= \left[\kappa_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}} \left[\delta_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + 1\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}} \left[\delta_j (\chi\delta_j)^{\rho-1} + 1\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}} \left(\delta_j^{\rho} \chi^{\rho-1} + 1\right)^{\frac{\rho}{\rho-1}},
\end{aligned}$$

thus we have

$$\frac{G_j\left(H_j,K_j\right)}{H_j}=\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}} \tag{A3}$$

substituting (A3) into the MRTS condition across high-skill and low-skill:

$$q=\sigma_j\left(1-\kappa_j\right)\left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}}\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\eta-\rho}{(\rho-1)\eta}},$$

which implies

$$\frac{H_j}{L_j}=\left(\sigma_j/q\right)^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}.$$

A2.2.2 Labour income shares

The high-skill income share is

$$I_j=\left[1-J_j\right]\tilde{I}_j \tag{A4}$$

using (36) and (38),

$$I_j=\frac{\tilde{I}_j}{1+q^{\eta-1}\sigma_l^{-\eta}\left[\tilde{I}_j\left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-1}{\rho-1}}} \tag{A5}$$

The total labour income shares is

$$\begin{aligned}\beta_j &= I_j+J_j=\left(1-J_j\right)\tilde{I}_j+J_j\\ &= J_j\left[\frac{1-J_j}{J_j}\tilde{I}_j+1\right]\end{aligned}$$

substitute (36) and (38)

$$\beta_j=J_j\left[q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}}+1\right].$$

A2.2.3 Equilibrium low-skill wage w_l

The price for low-skill efficiency labour equals to its marginal product:

$$w_l=\xi_j p_j A_j\left(\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j}\right)^{\frac{1}{\eta}}$$

where using the production function

$$
\begin{aligned}
\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} &= \left[\left(1-\xi_j\right)\left[\frac{G_j\left(H_j,K_j\right)}{L_j}\right]^{\frac{\eta-1}{\eta}}+\xi_j\right]^{\frac{\eta}{\eta-1}} \\
&= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left[\frac{G_j\left(H_j,K_j\right)}{H_j}\right]^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}}
\end{aligned}
$$

substitute (A3) and (37) to obtain

$$
\begin{aligned}
\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} &= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}} \\
&= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j^{\eta}q^{1-\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-1}{1-\rho}}+1\right]^{\frac{\eta}{\eta-1}}
\end{aligned}
$$

Using the income shares (38)

$$
\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j}=\left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}}, \tag{A6}
$$

and low-skill wage is

$$
w_l=\xi_j^{\frac{\eta}{\eta-1}}p_jA_j\left[J_j\right]^{\frac{1}{1-\eta}}.
$$

A2.3 Mapping the two-sector model into a three-sector setting

Consider a three sector-economy where the service sector is as before, but in addition to the low-skill sector, there is a capital sector with the same production function as the low-skill sector in the baseline model. Assume the production function of the low-skill sector and the capital sector are identical except for their TFP index, equating the MRTS across the three inputs of production implies that the following two Lemmas.

Lemma A1 *Given the production functions for the low-skill sector and capital sector are identical except the TFP A_j, the relative inputs used in the low-skill sector is the same as that of the capital sector:*

$$
\frac{H_l}{K_l}=\frac{H_k}{K_k},\frac{H_l}{L_l}=\frac{H_k}{H_l}, \tag{A7}
$$

and the relative price of the two sectors is the inverse of their TFP:

$$\frac{p_l}{q_k} = \frac{A_k}{A_l}. \tag{A8}$$

Proof. Given $\kappa_l = \kappa_k$, it follows from (35) that $\frac{H_l}{K_l} = \frac{H_k}{K_k}$, thus (36) implies $\tilde{I}_l = \tilde{I}_k$, and together with $\xi_l = \xi_k$, optimal condition (37) implies $\frac{H_l}{L_l} = \frac{H_k}{L_k}$. It also follows from (38) and (A5) that $J_j = J_k$ and $I_j = I_k$, thus mobility of low-skill labour across the low-skill and capital sector implies the relative price is the inverse of the TFP from (20). ■

Lemma A2 *Given the production functions for the low-skill sector and capital sector are identical except their TFP, the low-skill sector and capital sectors can be aggregate into one sector with the following constraint:*

$$Y_l + \frac{q_k}{p_l} Y_k = A_l F_l \left(G_l \left(H_l + H_k, K_l + K_k \right), L_l + L_k \right) \tag{A9}$$

Proof. Given the production function is homogenous of degree 1,

$$\begin{aligned} & p_l Y_l + q_k Y_k \\ = \;& p_l A_l F_l \left(G_l \left(H_l, K_l \right), L_l \right) + q_k A_l F_l \left(G_l \left(H_k, K_k \right), L_k \right) \\ = \;& p_l A_l H_l F_l \left(G_l \left(1, \frac{K_l}{H_l} \right), \frac{L_l}{H_l} \right) + q_k A_l H_k F_l \left(G_l \left(1, \frac{K_k}{H_k} \right), \frac{L_k}{H_k} \right) \end{aligned}$$

Lemma A1 implies that

$$\frac{K_l + K_k}{H_l + H_k} = \frac{K_l}{H_l}; \qquad \frac{L_l + L_k}{H_l + H_k} = \frac{L_l}{L_k}$$

together with the result on relative price equation (A8),

$$p_l Y_l + q_k Y_k = p_l F_l \left(G_l \left(H_l + H_k, K_l + K_k \right), L_l + L_k \right),$$

thus result follows. ■

Lemma A2 implies that we can work with a two-sector economy where the final goods from the low-skill sector can be transformed into one unit of consumption goods and $1/\phi \equiv p_l/q_k = A_k/A_l$ unit of capital goods.

A2.4 Allocation of high-skill efficiency labour

A2.4.1 Expressing q as function of χ

Using (19), the equilibrium condition for price of capital is:

$$q_k = \frac{q}{\chi} p_l A_l \left[J_l \xi_l{}^{-\eta} \right]^{\frac{1}{1-\eta}}$$

Given $\phi = q_k/p_l$,

$$\chi = q \frac{A_l}{\phi} \left[J_l \xi_l{}^{-\eta} \right]^{\frac{1}{1-\eta}}.$$

Using the definition of income share $J_l(\chi, q)$ in (38),

$$\begin{aligned} \chi &= q \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[1 + q^{1-\eta} \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}} \\ &= \xi_l^{\frac{\eta}{\eta-1}} \frac{A_l}{\phi} \left[q^{\eta-1} + \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}} \end{aligned}$$

rearranging

$$q^{\eta-1} + \sigma_l^\eta \left[\tilde{I}_l (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} = \left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{\frac{\eta}{1-\eta}}$$

so

$$q = \left[\left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\tilde{I}_l(\chi) (1-\kappa_l)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{\frac{1}{\eta-1}},$$

Given the expression for $\tilde{I}_l$ in (36),

$$\begin{aligned} q &= \left[\left(\frac{\phi \chi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\left(1 + \chi^{\rho-1} \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}} \\ &= \chi \left[\left(\frac{\phi}{A_l} \right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^\eta \left[\left(\chi^{1-\rho} + \delta_l^\rho \right) (1-\kappa_l)^\rho \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta-1}}, \end{aligned}$$

so $q > 0$ requires

$$\left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} > \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}$$
$$\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{\eta-1}{1-\rho}} > \left(\frac{\phi}{A_l}\right)^{1-\eta}(1-\xi_l)^{\eta}$$

which requires

$$\chi > \chi_{\min} \equiv \left[\left(\frac{A_l}{\phi}\right)^{1-\rho}(1-\xi_l)^{\frac{\eta(1-\rho)}{\eta-1}}(1-\kappa_l)^{-\rho}-\delta_l^{\rho}\right]^{\frac{1}{1-\rho}}.$$

The supply condition The labour market clearing condition for high-skill worker can be rewritten as:

$$\frac{H_l+H_k}{L_l+L_k}(L_l+L_k)+\frac{H_h}{L_h}L_h = H,$$

which using Lemma 2 and high-skill labour market implies

$$\frac{H_l}{L_l}(L-L_h)+\frac{H_h}{L_h}L_h = H,$$

thus it follows that the share of low-skill efficiency labour in the high-skill services sector is:

$$l_h \equiv \frac{L_h}{L} = \frac{H/L - H_l/L_l}{H_h/L_h - H_l/L_l}, \tag{A10}$$

simplify to

$$l_h = \frac{\zeta/(H_l/L_l)-1}{(H_h/L_h)/(H_l/L_l)-1},$$

substitute MRTS condition (37)

$$l_h = \frac{\zeta\sigma_l^{-\eta}q^{\eta}(1-\kappa_l)^{\frac{\rho(\eta-1)}{1-\rho}}\tilde{I}_l^{\frac{\eta-\rho}{\rho-1}}-1}{(\sigma_h/\sigma_l)^{\eta}\left(\frac{1-\kappa_h}{1-\kappa_l}\right)^{\frac{\rho(\eta-1)}{\rho-1}}\left(\frac{\tilde{I}_h}{\tilde{I}_l}\right)^{\frac{\eta-\rho}{1-\rho}}-1}$$

for the special case $\kappa_j \to 0, \tilde{I}_l \to 1$

$$l_h = \frac{\zeta\sigma_l^{-\eta}q^{\eta}-1}{(\sigma_h/\sigma_l)^{\eta}-1}$$

The demand condition Next solve for the demand equation. Using the goods market clearing conditions, the relative demand of the two goods implies:

$$x = \frac{p_h C_h}{p_l C_l} = \frac{P_h Y_h}{P_l \left(Y_l - \phi K\right)}$$

which can be written as:

$$\frac{p_h Y_h}{p_l Y_l} = x\left(1 - \frac{\phi K}{Y_l}\right), \tag{A11}$$

where using relative price (20), x is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{\xi_h^{-\eta} J_h}{\xi_l^{-\eta} J_l}\right)^{\frac{1-\varepsilon}{\eta-1}}; \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, K is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)$$

so the relative demand equation (A11) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l}\left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right],$$

given $\phi \equiv q_k/p_l$, rewrite it in terms of low-skill income share J_j :

$$\begin{aligned}
\frac{J_l}{x J_h}\left(\frac{L_h}{L_l}\right) &= 1 - \frac{q_k J_l}{q_l L_l}\left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right] \\
&= 1 - \frac{J_l}{L_l}\left[\frac{q_k K_h}{q_l L_h} L_h + \frac{q_k K_l}{q_l L_l}\left(L - L_h\right)\right] \\
&= 1 - \frac{J_l}{L_l}\left[\frac{1-\beta_h}{J_h} L_h + \frac{1-\beta_l}{J_l}\left(L - L_h\right)\right],
\end{aligned}$$

where the last equality follows from the definition of labour income share β_j. Finally rewrite it in terms of l_h :

$$\frac{J_l}{x J_h}\left(\frac{l_h}{1-l_h}\right) = 1 - \frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h} l_h + \frac{1-\beta_l}{J_l}\left(1 - l_h\right)\right],$$

which implies

$$\frac{J_l}{xJ_h} l_h = 1 - l_h - J_l \left[\frac{1-\beta_h}{J_h} l_h + \frac{1-\beta_l}{J_l} (1-l_h) \right],$$

thus the demand for l_h is:

$$l_h = \frac{\beta_l}{\beta_l + \frac{J_l}{J_h}\left(\frac{1}{x} + 1 - \beta_h\right)}.$$

For the special case of no capital, $\beta_j \to 1$:

$$l_h = \left(1 + \frac{J_l}{xJ_h}\right)^{-1}.$$

A2.5 Value-added shares

The value-added shares of services is defined as:

$$v_h = \left[1 + \frac{p_l Y_l}{p_h Y_h}\right]^{-1}$$

So

$$v_h = \left[1 + \frac{p_l A_l F_l / L_l}{p_h F_h / L_h} \frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (20) and (A6),

$$v_h = \left[1 + \left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}} \left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}} \left(\frac{L_l}{L_h}\right)\right]^{-1}$$

simplify to

$$v_h = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1},$$

given l_h, v_h is determined.

A2.5.1 Endogenous skill-biased demand shift

The production function is

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j) H_j^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

Using the MRTS condition (35),

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\chi \frac{\kappa_j}{1-\kappa_j}\right)^{\rho-1} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\left(\chi^{\rho-1} \left(\frac{\kappa_j}{1-\kappa_j}\right)^{\rho} + 1\right) (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \end{aligned}$$

A3 Quantitative results

A3.1 Calibration

This section explains how the weights of each inputs are calibrated to match the sectoral income shares of high-skill and low-skill for 1980 and 2003.

A3.1.1 Normalization of ϕ/A_l

Next we show that the initial $\frac{\phi}{A_l}$ can be normalize to 1. Note that by definition of $\tilde{I}_j$

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j}\right]^{-1} \Longrightarrow \frac{K_j}{\chi H_j} = \frac{1-\tilde{I}_j}{\tilde{I}_j},$$

which is independent of ϕ/A_l. Also by definition of J

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q \frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of ϕ/A_l as well. Therefore it follows from the supply condition (A10) that l_h is independent of ϕ/A_l. So the allocation of low-skill labour is independent of ϕ/A_l. Given H_j/L_j and K_j/H_j are independent of ϕ/A_1, so the allocation of all inputs are independent of ϕ/A_1. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocations across the three sectors. The value of ϕ_T/A_{lT} is then determined by the growth in the relative price of capital ϕ_T/ϕ_0 and γ_l.

A3.1.2 Calibration of κ_l, ξ_l

Given ϕ/A_l, equation (41) express χ as a function of ξ_l given data on q and J_l :

$$\chi = qA_k \left[J_g \xi_g^{-\eta}\right]^{\frac{1}{1-\eta}} = qA_k J_l^{\frac{1}{1-\eta}} \xi_l^{\frac{\eta}{1-\eta}}.$$

Substitute this into the income share $\tilde{I}_l$ in (36) to solve out δ_l explicitly as function of ξ_l :

$$\delta_l = \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}}$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of ξ_l. Thus the income share equation (38) provides an implicit function to solve for ξ_g:

$$J_l = \left[1 + q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_{lg}\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1},$$

which can be used to solve for ξ_l given data on $\left(\tilde{I}_l, J_l\right)$. This procedure pins down χ, ξ_l and κ_l. Note that

$$\left(1-\kappa_l\right)^{-1} = 1 + \delta_l = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}} = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}\right)^{1-\rho}\right]^{\frac{1}{\rho}}$$

so

$$\sigma_l^\eta \left[(1-\kappa_l)^{-1}\right]^{\frac{\rho(\eta-1)}{1-\rho}} = \sigma_l^\eta \left[1 + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}} \left(qA_k J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}} \xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}$$

The implicit function is

$$f(\xi_l) = \left[1 + q^{1-\eta}\left[\left(\frac{1-\xi_l}{\xi_l}\right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}} \left(\frac{q\phi}{A_l} J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}} (1-\xi_l)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}\right]^{-1} - J_l,$$

thus we have

$$\begin{aligned} f'(\xi_l) &> 0 \\ \lim_{\xi_l \to 1} f(\xi_l) &= 1 - J_l > 0 \\ \lim_{\xi_l \to 0} f(\xi_l) &= -J_l < 0 \end{aligned}$$

so there is an unique solution for $\xi_l \in (0,1)$ for any given ϕ/A_l.

A3.1.3 Calibration of κ_h, ξ_h

Using income shares $\tilde{I}_h$ in ((36)):

$$\delta_h = \left[\frac{1-\tilde{I}_h}{\tilde{I}_h}\chi^{1-\rho}\right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}$$

given values $\tilde{I}_h$ and the implied value of χ from above, κ_h is obtained. Using income shares J_h in (38):

$$\sigma_h = \left[\frac{1-J_h}{J_h} q^{\eta-1}\left[\tilde{I}_h (1-\kappa_h)^{-\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and q, so ξ_h is obtained.

A3.2 Results for other variables

The performance of the model on other key variables is summarized in Table A3. The baseline model does a good job in predicting 96% of the rise in relative wage, and 86% of the

Table A3: Actual and Predicted Values for key variables

		q	l_h	v_h	β_l	β_h	β
	Data 1980	1.44	0.14	0.24	0.59	0.56	0.58
(1)	Data 2008	1.94	0.21	0.29	0.53	0.65	0.56
	Model 1980	matched	matched	matched	matched	matched	matched
(2)	Model 2008	1.92	0.20	0.28	0.52	0.65	0.56
	Counterfactual (keeping all else constant at 1980)						
(3)	$A_l/A_h \uparrow$	2.08	0.20	0.32	0.60	0.61	0.60
(4)	$\phi \downarrow$	2.11	0.16	0.26	0.59	0.62	0.60
(5)	$\xi_j \downarrow$	2.37	0.16	0.22	0.52	0.60	0.54
(6)	$\kappa_j \downarrow$	2.12	0.16	0.27	0.61	0.65	0.63
(7)	$\zeta \uparrow$	1.07	0.13	0.23	0.56	0.56	0.56

rise in the low-skill efficiency labour share and 80% of the rise in the value-added share of the high-skill services. Consistent with the data, it predicts a fall in labour income share in the goods sector and a rise in labour income share in the service sector. Finally, it generates a decline in aggregate labour income shares that matches the data.

A3.3 Alternative elasticity parameters

The elasticity parameters in the baseline are set to the values used in the related literature. This section considers alternative values for these elasticities. Given the calibration procedures, changing the elasticity parameters will change the values for other parameters. In the interest of space, we do not report those values. These parameter values are available upon request.

A3.3.1 The elasticity of substitution across high-skill and low-skill labour, ε

As discussed in the main text, there is no direct estimate for ε in our model but there are evidence suggest that it is likely to be small. We explain the logic behind using a value of 0.2 as baseline, here we examine the key results on low-skill wage stagnation and its divergence from aggregate productivity for a lower degree of consumption complementarity $\varepsilon = 0.5$. An increase in ε implies that the model requires a higher growth in A_{lh} to match the observed

Table A4: Data and Model Predictions, $\varepsilon = 0.5$, 1980-2008 Cumulative Growth, % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	33	61	21	23	m	45	-2.7
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_g/A_s \uparrow$	21	44	19	8.1	68	55	-13
(5)	$\xi_j \downarrow$	28	52	19	29	51	17	21

growth in relative prices, as a results other parameters are also affected.

As shown in both Table A4 the baseline results (2) are not affected given the calibration procedures. The more important question is whether it will affect the role played by our between-sector mechanism driven by faster productivity growth in the low-skill sector, i.e. a rise A_{lh}. As shown in (3), the basic mechanism remains important for the stagnation in low-skill real wage and it continues to account for a significant fraction of real divergence and wage inequality. Compared to the baseline results in Table 3 and Table 4, it predicts a slightly faster rise in the low-skill real wage and a slightly smaller fraction of the real divergence. Compared to the role played by the labour displacing technical change in (5), its advantage remains in predicting a rise in the relative prices of high-skill services, which can account for the differential trends in the marginal product of low-skill labour and a rise in the relative cost of living.

A3.3.2 The elasticity of substitution across capital and high-skill labour, ρ

The estimate of $\rho = 0.67$ in Krusell et al. (2000) is for the aggregate economy using data for 1963-1992. We can also infer the elasticity of substitution across capital and high-skill labour ρ using the equilibrium condition (35) and data on income shares and relative input prices, Using the equilibrium condition (35), the response in relative income shares to changes in relative prices of high-skill and capital input is

$$\ln\left(\frac{I_{jT}/\left(1-\beta_{jT}\right)}{I_{j0}/\left(1-\beta_{j0}\right)}\right) = (1-\rho)\ln\left(\frac{\chi_T}{\chi_0}\right), \tag{A12}$$

where by definition, $\chi = w_h/q_k = \phi(w_h/p_l)$, so its growth can be obtained from data on the relative price of capital and the high-skill wage deflated by price of low-skill sector. Given

Table A5: Data and Model Predictions, $\rho = 0.5$, 1980-2008 Cumulative Growth, % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	61	20	23	m	45	-3.0
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_g/A_s \uparrow$	21	38	14	8.5	63	50	-16
(5)	$\xi_j \downarrow$	27	47	15	28	46	14	16

the data in 1, equation (A12) implies ρ is 0.39 using income shares from the low-skill sector and 0.59 using income shares from the high-skill sector, which give an average of 0.49. If we were to use the aggregate income shares instead, equation (A12) implies $\rho = 0.48$. Thus we report the results for $\rho = 0.5$ in Table A5. It shows that the results for the full model (row 2) is almost identical to those in Table 3 and Table 4. The contribution of the uneven productivity growth to the real divergence and low-skill wage stagnation is also similar.

A3.3.3 The elasticity of substituion across low-skill and high-skill labour, η

The estimate of $\eta = 1.4$ in Katz and Murphy (1992) is for the aggregate economy using data for 1963-1987. For a similar period, 1963-1992, Krusell et al. (2000) finds $\eta = 1.67$ and $\rho = 0.67$ for the nested aggregate production function including capital. Using more recent data, abstracting from capital, Acemoglu and Autor (2012) find values within the range 1.6–1.8. Higher η implies a smaller exogenous decline in ξ_l is needed to account for the decline in labour income shares in the low-skill sector. Table A6 reports the results for $\eta = 2.0$. It shows the basic mechanism ($A_g/A_s \uparrow$) has a more important role in accounting for the divergence as the required decline in ξ_l reduced to -0.46% compared to -0.93% in the baseline. As in the baseline, the basic mechanism is important for generating the differential trends in the marginal product of low-skill while the labour displacing technical change ($\xi_j \downarrow$) is needed for the decline in the labour income share.

Table A6: Data and Model Predictions, $\eta = 2.0$, 1980-2008 Cumulative Growth, % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	60	20	24	m	44	-3.4
		Counterfactual (keeping all else constant at 1980)						
(3)	$A_g/A_s \uparrow$	23	31	6.7	11	52	36	-19
(5)	$\xi_j \downarrow$	19	45	22	19	45	22	22