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Adrian Buss and Savitar Sundaresan

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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JEL Classification: G11, G14, G23

Keywords: passive investing, Informational efficiency, Risk Taking, asset allocation, Asset Pricing

Adrian Buss - adrian.buss@insead.edu
INSEAD and CEPR

Savitar Sundaresan - s.sundaresan@imperial.ac.uk
Imperial College London and CEPR

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More Risk, More Information: How Passive Ownership Can Improve Informational Efficiency*

Adrian Buss Savitar Sundaresan

This version: May 31, 2020

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Passive investors, such as exchange-traded funds (ETFs) or index funds, are major shareholders of almost all publicly traded companies. For example, in the United States, they hold, on average, more than 17% of the shares outstanding; though with large cross-sectional variation across firms.[1] Not surprisingly, this development has spurred an intense debate among market participants about the impact of passive investing on financial markets; in particular, on the informativeness of stock prices and, hence, capital allocation efficiency.

A prominent view is that passive ownership negatively affects the informational content of stock prices because passive investors lack incentives to gather firm-specific information. Yet, empirically, price informativeness is often higher for firms that tend to have a higher share of passive owners. For example, Bai, Philippon, and Savov (2016) and Farboodi, Matray, Veldkamp, and Venkateswaran (2019), respectively, document that firms with high shares of institutional investors as well as large firms have more informative stock prices. In particular, changes in a firm's ownership structure will naturally also induce firms and active investors to revise their investment and information choices which, in turn, can promote informational efficiency. Consequently, the implications of the "secular rise" in passive investing observed in recent years are likely considerably more intricate than the "intuitive" argument above might suggest and could vary substantially in the cross-section.

This paper's objective is to provide new theoretical insights into the relationship between passive ownership and the informativeness of stock prices as well as firms' real investment decisions. Indeed, we identify a novel economic mechanism through which an increase in passive ownership, perhaps counter-intuitively, can lead to an improvement in price informativeness—not just in the cross-section of stocks but even in aggregate. In a nutshell, passive owners' inelastic demand lowers a firm's cost of capital, inducing it to take on more risk. This increase in riskiness, in turn, incentivizes active investors to acquire more private information and, hence, price informativeness goes up.

We develop a novel noisy rational expectations equilibrium (REE) model in which investors' optimal portfolio and information choices, stock prices, and firms' real investment policies are jointly determined in equilibrium. The model is shaped by three central fea-

[1] For instance, in 2017, the proportion of shares held by passive investors varied between 7.1% and 29.8% for firms in the S&P500 and between 0.01% and 36.6% for firms in the Russell 3000 (see, e.g., Adib 2020).

tures. First, the model explicitly accounts for passive ownership; allowing for variations in ownership in the cross-section and in aggregate. Second, firms take into account the ownership structure of their equity when deciding on their investments in growth opportunities. Third, active investors optimally choose the precision of their private information. Otherwise, the model is kept as simple and standard as possible to convey the economic mechanisms in the clearest possible way.

In the first step, we use our model to study the implications of *cross-sectional variations in passive ownership*.[2] We start by analyzing the impact of passive ownership on firms' real investment policies, that is, their investments in growth opportunities. Intuitively, a higher passive ownership reduces the number of shares that active investors have to hold in equilibrium and, hence, the risk each active investor has to bear. This lowers a firm's cost of capital or, equivalently, its marginal cost of investing in growth opportunities. As a result, firms with high passive ownership more aggressively invest in risky growth opportunities; that is, they accept more risk. This naturally drives up the mean and the variance of their cash flows (compared to identical firms with low levels of passive ownership).

We then turn to the implications of passive ownership for the informativeness of stock prices. We show that the resultant higher cash flow variance of firms with larger shares of passive owners induces active investors to acquire more precise private information. As a result, the average private signal precision and, in turn, price informativeness, increase with passive ownership—a result unique to our setting. Next, we characterize the impact of passive ownership on stock prices and returns. Higher passive ownership leads to higher stock prices through a the combination of three effects: (1) a higher aggregate demand, (2) a higher expected cash flow from increased firm-level investment, and (3) higher price informativeness (i.e., the lower price discount commanded by active investors). Stock return variances and excess returns are also positively related to passive ownership,[3] which is driven by the resultant higher cash flow variance (which, in the case of stock return variance, dominates the opposing effect of higher price informativeness). In contrast, because of the higher aggregate demand, a stock's Sharpe ratio usually declines in passive ownership.

[2]For this exercise, we keep the share of passive investors fixed. Hence, cross-sectional differences in passive investors' aggregate stock demand can only arise from differences in their average demand (i.e., the intensive margin of their trading).

[3]The exception is the (extreme) case in which a stock's net supply—after accounting for the demand of passive investors—becomes very small. In this case, a stock's expected excess return might decline because the risk each active investor has to bear diminishes.

Finally, we show that the difference in the trading profits of informed and uninformed investors are increasing in passive ownership (driven by the higher private signal precision).

In the second step, we use our economic framework to work out the *aggregate implications of a rise in passive ownership*. For that purpose, we vary the share of passive investors in the economy (or, equivalently, the aggregate assets under management of passive funds). In general, a rise in aggregate passive ownership leads to stronger dispersion in real investment policies, informational efficiency, and stock prices across firms. In particular, firms with a large average demand by passive investors (e.g., those that are part of broad stock market indices) are the main beneficiaries, whereas firms with a weak passive investor demand might even experience "liquidity crashes" as the share of passive investors rises.

The analysis also highlights that the aggregate impact of a rise in the share of passive investors critically depends on whether passive owners crowd out uninformed or informed investors. Indeed, if a rise in passive owners results in a decline in the share of uninformed investors, firms in aggregate invest more in growth opportunities. Consequently, informed investors acquire more precise information so that the informativeness of stock prices improves, both in aggregate and for a majority of stocks. This, in turn, also leads to an increase in aggregate stock prices.

In contrast, if passive owners crowd out informed investors, firms invest in aggregate less aggressively in growth opportunities because of a higher cost of capital (driven by a decline in aggregate precision). Moreover, for a large majority of firms, price informativeness deteriorates as aggregate passive ownership increases. This is the result of the deterioration in information aggregation, that is, the "intuitive" effect usually associated with passive investing. As a result, aggregate stock prices also decline.

Finally, we extend our analysis to the case of benchmarked investors. This extension serves two purposes. First and foremost, it allows us to expand our analysis to a wider set of institutional investors and, thus, broadens the scope of our analysis. Second, implicitly, it demonstrates that our findings are robust to allowing for an endogenous passive investor demand that is sensitive to cash flow variance and, hence, interacts with firms' real investment choices (in contrast to the exogenous demand assumed in our main model).

Overall, our model generates a rich set of novel and unique predictions regarding the impact of passive ownership on price informativeness, stock prices and returns, and firms' corporate decisions. A first, brief look at the existing empirical evidence lends support to many of our predictions. However, a broader empirical analysis of the key implications of passive investing would clearly be beneficial. We view our framework as a benchmark for such analyses.

Technically, our framework integrates a tractable corporate finance addition into an otherwise standard noisy REE framework. Doing so allows for "supply-side" adjustments by firms in response to variations in the ownership structure in financial markets. Thus, firm characteristics, namely, the mean and variance of firms' cash flows, are endogenous in our model, which, as demonstrated, can give rise to new and surprising economic forces. Yet the model remains tractable and yields closed-form solutions for all quantities of interest.

The paper contributes to three strands of the theoretical literature.[4] First, it relates to the literature studying the impact of institutional investors on financial markets with exogenous (symmetric) information. Cuoco and Kaniel (2011) and Basak and Pavlova (2013) explicitly model institutional investors' relative performance concerns and document that their portfolio choices push up the prices of stocks in a benchmark. Buffa, Vayanos, and Woolley (2020) study a setting with endogenous asset management contracts and document the impact of institutional investors on asset prices. Buffa and Hodor (2019) highlight how benchmarking can create spillovers across assets. While these papers focus on institutional investors in general, Chabakauri and Rytchkov (2019) and Baker, Chapman, and Galleyer (2020) explicitly analyze indexing. Chabakauri and Rytchkov (2019) carefully document two opposing forces of indexing—a "lockstep trading effect" and a risk-sharing effect—and study their impact on asset returns and welfare. Baker, Chapman, and Galleyer (2020) show how a decrease in the fee for passive investing can negatively impact the overall efficiency of the economy due a corresponding decline in the number of activists (who, in their model, play an essential monitoring role through the use of "voice"). In this paper, we explicitly model passive investors and, importantly, allow for endogenous information and real investment choices. Doing so delivers many new insights into the impact of institutional investors on firms' choices and financial markets. Our paper also borrows from Kashyap, Kovrijnykh,

[4]We relegate a discussion of the link to the empirical literature studying the impact of (passive) institutional investors to Section 6.

Li, and Pavlova (2020), who show that firms within an index accept riskier projects than do firms outside of an index. We deliver a similar result using a different framework and examine its implications for information collection and aggregation.

Second, the paper connects to the literature on institutional investors and their information choices. Closest to our work, Bond and García (2019) study the impact of "indexing" on welfare, price efficiency, and participation; documenting complementarities in indexing investors' participation decisions. While both their and our paper highlight positive implications of passive investing, the underlying economic mechanisms are markedly different in that their effects arise from better risk-sharing in the presence of passive investors (akin to a Hirshleifer (1971) effect) whereas our results arise from the fact that firms take into account the ownership structure in financial markets when choosing their real investment policies. Also, in their model, the key determinant of the informativeness of asset prices is the share of active investors; instead, we let active investors choose the precision of their information (taking their share as given).

Cong and Xu (2019) explicitly model the introduction of ETFs. They document higher price variability and co-movements, and lower asset-specific but higher factor information in prices following the introduction of an ETF. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) use the state of the business cycle to predict institutional investors' information choices and link these choices to observable patterns of mutual-fund managers' portfolio investments and returns. Breugem and Buss (2019), Kacperczyk, Nosal, and Sundaresan (2018), and Huang, Qiu, and Yang (2019) study the impact of institutional investors on price informativeness. Breugem and Buss (2019) show that benchmarking reduces the equity premium and, therefore, the incentives of active investors to collect information. In Kacperczyk, Nosal, and Sundaresan (2018), an increase in passive investing reduces the relative size of active investors and their market power such that active investors shift their attention toward fewer stocks, an action that lowers aggregate price informativeness. Finally, in Huang, Qiu, and Yang (2019), institutional investors (are endogenously) more risk averse and, hence, trade less aggressively on their information which harms information aggregation. A common theme of these papers is the focus on the negative consequences of institutionalization. In contrast, we highlight that passive ownership positively correlates with price informativeness in the cross-section if one allows firms to take into account the ownership structure in financial markets when choosing their real investment policies.

Third, the paper complements the literature on "feedback effects," which studies how financial markets affect firms' real investment decisions through firm managers *learning* from market prices.[5] For example, Goldstein and Yang (2017, 2019) discuss the impact of information disclosure on market efficiency. Chen, Goldstein, and Jiang (2007), Foucault and Frésard (2012, 2014), Edmans, Goldstein, and Jiang (2015), and Dessaint, Foucault, Frésard, and Matray (2018) discuss managers' learning in models with feedback effects and provide empirical support for this channel. Goldstein, Schneemeier, and Yang (2020) discuss a fundamental "mismatch" between managers' and traders' optimal choice of private information and highlight its impact on price and real efficiency. Importantly, while we are also interested in how financial markets affect firms' decision, our economic mechanism is markedly different in that it operates through the ownership structure, not the information environment. Indeed, to distinguish our mechanism from feedback effects, managers cannot learn from stock prices in our model.

The rest of the paper is organized as follows. Section 1 introduces our main economic framework. Section 2 characterizes the equilibrium in the economy. In Sections 3 and 4, we discuss the cross-sectional and aggregate implications of passive ownership, respectively. Section 5 extends our analysis to the case of benchmarked investors. Finally, Section 6 summarizes the empirical implications and relates them to existing and new empirical evidence. Section 7 concludes. All proofs have been relegated to the appendix.

1 Economic Framework

This section introduces our main economic framework, which incorporates a real investment problem into a competitive REE model of joint portfolio and information choice (as in Verrecchia 1982). The following key features knit together our framework: multiple stocks, endogenous real investment policies, and three types of investors (informed active, uninformed active, and passive).

[5] Confer with Bond, Edmans, and Goldstein (2012) for an excellent survey of the literature.

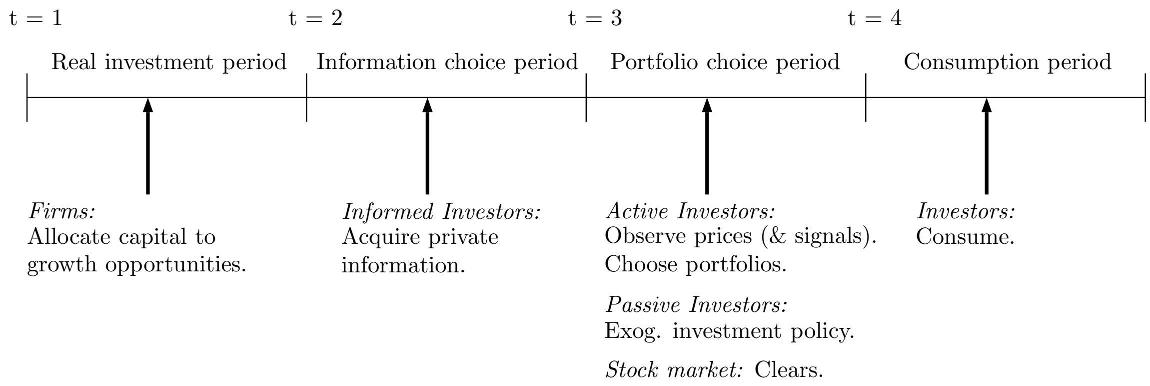

Figure 1: Timing. The figure illustrates the sequence of the events.

1.1 Model

Timing.

We consider a static model, which we break up into four subperiods, as illustrated in Figure 1. In period 1, the *real investment period*, firms decide how much capital to allocate to growth opportunities. In period 2, the *information choice period*, informed investors choose the precision of their private signals about firms' cash flows, taking into account their real investment decisions in period 1. In period 3, the *portfolio choice period*, investors set up their portfolios. While active investors select optimal portfolios (after observing public stock prices and/or their private signals), passive investors simply adhere to an (exogenous) investment policy. Prices are set such that markets clear. In period 4, the *consumption period*, payoffs are realized and investors consume the proceeds of their investments.

Investment opportunities.

There exist multiple financial securities that are traded competitively in financial markets: a riskless asset (the "bond") and N risky assets (the "stocks"). The bond has a gross payoff of R_f units of the consumption good in period 4 (normalized to 1 in the following) and is available in perfectly elastic supply. It also serves as the numéraire, with its price normalized to one. The stocks are modeled as claims to random payoffs $X_n, n \in \{1, \ldots, N\}$, which are only observable in period 4. The supply of each stock, θ_n, is finite and endogenous. Stock prices are denoted by P_n.

Investors.

There is a continuum of atomistic investors with mass one that we separate into three groups: (1) a fraction $\Gamma^{\mathcal{P}}$ of passive investors, $\mathcal{P}$; (2) a fraction $\Gamma^{\mathcal{I}}$ of informed active investors, $\mathcal{I}$; and (3) a fraction $\Gamma^{\mathcal{U}}$ of uninformed active investors, $\mathcal{U}$. Each investor, $i \in \{\mathcal{P}, \mathcal{I}, \mathcal{U}\}$, is endowed with initial wealth, $W_{0,i}$, which we normalize (without loss of generality) to 1.

Passive investors, $i \in \mathcal{P}$, adhere to an exogenous investment policy determined by, for example, the weight of a stock in an (sub)index. The fraction of total shares outstanding of stock n held by passive investors ("passive ownership") is denoted by $\theta_n^{\mathcal{P}}$. *Informed (active) investors*, $i \in \mathcal{I}$, can freely invest in all financial assets and can acquire private information about the stocks' payoffs. They have constant absolute risk aversion (CARA) preferences $u(W_i) = -(1/\rho)\exp(-\rho W_i)$ over terminal wealth W_i, with absolute risk aversion ρ. *Uninformed (active) investors*, $i \in \mathcal{U}$, can also freely invest in all financial assets but have no access to private information. They also have CARA preferences with absolute risk aversion ρ.

There also exist noise (liquidity) traders with random, not explicitly modeled, stock demand. This assumption, which is standard in the literature, prevents prices from fully revealing information acquired by informed investors and, thus, preserves the incentives of informed investors to acquire private information in the first place. Specifically, noise traders' demand—per unit of stock supply—$Z_n \sim \mathcal{N}(0, \sigma_Z^2)$, $n \in \{1, \ldots, N\}$ is normally and independently distributed across stocks.[6]

Firms.

There exist N firms, each of which is linked to one of the stocks traded in financial markets. Firms are endowed with assets in place (whose value we normalize to zero) and have access to growth opportunities. We follow Subrahmanyam and Titman (1999) and model the cash

[6] One might argue that an increase in passive ownership leads to either an increase in the intensity of noise trading (e.g., if passive investing "adds" noise, as in Chinco and Fos 2019) or a decrease in its intensity (e.g., if noise traders are risk-averse hedgers who reduce their intensity as cash flow variance goes up, as in Bhattacharya and Spiegel 1991). To differentiate our economic mechanisms from changes in noise trading (which are difficult to establish), we set the mean and volatility of noise traders' demand to be the same for all stocks.

flows of their growth opportunities, X_n, as

$$X_n = A_n\, I_n - \frac{c}{2}\, I_n^2, \qquad c > 0; \tag{1}$$

where I_n denotes firm n's investment in growth opportunities, and $A_n \sim \mathcal{N}(\mu_A, \sigma_A^2)$ denotes productivity which is assumed to be independently distributed across stocks and from noise traders' demands. In general, the mean, $\mu_n = \mu_A\, I_n - c\,(I_n^2/2)$, and the variance, $\sigma_n^2 = \sigma_A^2\, I_n^2$, of a firm's cash flow are increasing in the investment I_n, though marginal benefits decline and, at some point, turn negative because of decreasing returns to scale.

Firms finance their investments in growth opportunities by issuing equity. Specifically, we denote the number of shares issued by firm n by δ_n and the total number of shares outstanding by $\theta_n = 1 + \delta_n$. Each firm (manager) decides on its investment in growth opportunities, $I_n \geq 0$, in order to maximize its expected stock price, S_n.[7]

Information structure.

Informed investors and firms are endowed with unbiased priors. To clearly separate the impact of the ownership structure (our story) from managers' learning from stock prices (the "feedback" channel), we assume that firms do not learn from stock prices when making their real investment decisions. In period $t = 2$, informed investors can acquire private information about firms' cash flows. For example, they may study financial statements, gather information about consumers' taste, hire outside financial advisers, or subscribe to proprietary databases. Formally, each investor $i \in \mathcal{I}$ chooses the precision, $q_{i,n}$, of her private signal $Y_{i,n} = X_n + \varepsilon_{i,n}$, $\varepsilon_{i,n} \sim \mathcal{N}(0, 1/q_{i,n})$ to be received in period $t = 3$.[8] Higher precision reduces the posterior uncertainty regarding a stock's payoff but increases the information acquisition costs. We follow Verrecchia (1982) and assume that information costs are a function of investors' precision choice: $\kappa(q_{i,n})$; in particular, we assume that the

[7] In the appendix (specifically in the proof of Theorem 3), we show that our results are robust to the use of debt financing. In fact, if debt is riskless, the stock price, S_n, as well as a firm's optimal real investment policy, I_n, are independent of the financing choice; that is, the Modigliani-Miller theorems hold.

[8] Alternatively, one could assume that informed investors received a private signal about productivity A_n. In the appendix (specifically, the proof of Theorem 1), we discuss that this does not affect any of our results.

cost function is identical for all stocks and continuous, increasing, and strictly convex with $\kappa(0) = 0$.[9,10]

The expectation and variance conditional on prior beliefs are denoted by $\mathbb{E}[\,\cdot\,]$ and $\mathbb{V}\text{ar}(\,\cdot\,)$. Investors' expectations and variance conditional on their time 3 information set $\mathcal{F}_i$ are denoted by $\mathbb{E}[\,\cdot\,|\,\mathcal{F}_i]$ and $\mathbb{V}\text{ar}(\,\cdot\,|\,\mathcal{F}_i)$.

1.2 Investors' optimization problems and equilibrium

In the portfolio choice period ($t = 3$), informed and uninformed active investors, $i \in \{\mathcal{I}, \mathcal{U}\}$, respectively, choose the number of shares of stocks, $\{\theta^{\mathcal{I}}_{i,n}\}$ and $\{\theta^{\mathcal{U}}_{i,n}\}$, in order to maximize their expected utility, conditional on their private signal precision ($q^{\mathcal{U}}_{i,n} = 0$ for uniformed investors), their information set $\mathcal{F}_i$, and firms' real investment policies $\{I_n\}$:

$$U_{3,i}\big(\{I_n\}, \{q_{i,n}\}, \mathcal{F}_i\big) = \max_{\{\theta^{\mathcal{I}/\mathcal{U}}_{i,n}\}} \mathbb{E}\left[-\frac{1}{\rho}\exp\big(-\rho\, W_i\big)\,\middle|\, \mathcal{F}_i\right], \tag{2}$$

with terminal wealth, W_i, being given by[11]

$$W_i = W_{0,i}\, R_f + \sum_{n=1}^{N} \theta^{\mathcal{I}/\mathcal{U}}_{i,n} \left(\frac{X_n}{\theta\ \ _n} - PR_f\right) - \sum_{n=1}^{N} \kappa(q_{i,n}). \tag{3}$$

In the information choice period ($t = 2$), each informed investor, $i \in \mathcal{I}$, chooses the precision of her private signals, $q_{i,n}$, in order to maximize her expected utility, taking the

[9]This cost function seems most relevant given our focus on the (domestic) equity fund-management industry. In particular, it guarantees that informed investors acquire information about a large set of stocks in their investment mandate—as they do in practice. Benamar, Foucault, and Vega (2020) provide direct empirical support for such a cost function by showing that investors' demand for information about macroeconomic factors increases with uncertainty. Notably, our results remain qualitatively unchanged if, instead, we rely on an additive information capacity constraint as Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) use for their analysis of the mutual fund industry (see also Footnote 15). However, the model becomes considerably less tractable. In contrast, an entropy constraint usually favours specialization (or, with CARA preferences, leads to indeterminacy), that is, informed investors would only learn about a single stock; contrary to the empirical evidence on fund managers' information choices.

[10]While $\kappa(0) = 0$ guarantees the existence of an interior solution, the convexity assumption captures the idea that each new improvement in precision is more costly than the previous one.

[11]This follows from solving the budget equation of period 1: $W_{0,i} = \sum_{n=1}^{N} \theta^{\cdot}_{i,n}\, P_n + \theta^{\cdot}_{i,0}$ for $\theta^{\cdot}_{i,0}$ (number of shares of the bond) and plugging $\theta^{\cdot}_{i,0}$ into terminal wealth $W_i = \sum_{n=1}^{N} \theta^{\cdot}_{i,n}\, X_n + \theta^{\cdot}_{i,0}\, R_f - \sum_{n=1}^{N} \kappa(q_{i,n})$.

firms' real investment policies, I_n, as given:

$$U_{2,i}(\{I_n\}) = \max_{\{q_{i,n}\}\geq 0} \mathbb{E}\Big[U_{3,i}\big(\{I_n\}, \{q_{i,n}\}, \mathcal{F}_i\big)\Big], \tag{4}$$

where the expectation is taken over all possible realizations of her private signals $\{Y_{i,n}\}$ and public stock prices $\{P_n\}$.

Finally, in the real investment period ($t = 1$), each firm chooses the investment in growth opportunities, $I_n \geq 0$, in order to maximize the market value of the firm in period 1, while anticipating the investors' information and portfolio choices in subsequent periods:

$$S_n = \max_{\{I_n\}\geq 0} \mathbb{E}\big[P_n\big]. \tag{5}$$

Equilibrium definition.

We restrict our attention to equilibria that are symmetric in investors' portfolio and information choices. A rational expectations equilibrium is defined by real investment choices $\{I_n\}$, information choices $\{q_{i,n}\}, i \in \mathcal{I}$ as well as portfolio choices $\{\theta^{\mathcal{I}}_{i,n}\}$ and $\{\theta^{\mathcal{U}}_{i,n}\}$, $i \in \{\mathcal{I}, \mathcal{U}\}$, and prices $\{P_n\}$, $n \in \{1, \ldots, N\}$ such that

1. Taking prices as given, $\{\theta^{\mathcal{I}}_{i,n}\}$ and $\{q_{i,n}\}$ solve informed investor i's maximization problems (2) and (4), and $\{\theta^{\mathcal{U}}_{i,n}\}$ solves uninformed investor i's maximization problem (2).
2. I_n solves firm n's maximization problem (5).
3. Expectations are rational; that is, the average precision of private information implied by aggregating informed investors' precision choices equals the level assumed in the optimization problems (2), (4), and (5).
4. Aggregate demand equals aggregate supply.

Note that, in equilibrium, stock prices play a "triple" role: they clear the security markets, aggregate and disseminate informed investors' private information, and "feed back" to firms' real investment decisions.

2 Equilibrium Characterization

We now characterize the equilibrium in the economy; working backward from investors' portfolio and information choices to firms' real investment decisions.

2.1 Portfolio choice and equilibrium prices

Solving for investors' optimal asset demand, aggregating their demand, and imposing market clearing yields

Theorem 1. *There exists a unique linear rational expectations equilibrium. Specifically, conditional on firms' real investment policies, $\{I_n\}$, and informed investors' information choices, $\{q_{i,n}\}$, the equilibrium stock price is given by*

$$\theta_n \, P_n = \frac{1}{\bar{h}_n} \left(\frac{\mu_n}{\sigma_n^2} - \rho \, \bar{\theta}_n^{\mathcal{A}} \right) + \frac{1}{\bar{h}_n} \left(\bar{h}_n - \frac{1}{\sigma_n^2} \right) X_n + \frac{1}{\bar{h}_n} \left(\frac{\rho}{1 - \Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}} \, \bar{q}_n}{\rho \, \sigma_Z^2} \right) Z_n, \tag{6}$$

$$\text{where} \quad h_{0,n} \equiv \frac{1}{\sigma_n^2} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2 \, \sigma_Z^2}, \quad \bar{q}_n \equiv \frac{1}{\Gamma^{\mathcal{I}}} \int^{\mathcal{I}} q_{i,n} \, di, \quad \bar{h}_n \equiv h_{0,n} + \frac{\Gamma^{\mathcal{I}}}{1 - \Gamma^{\mathcal{P}}} \, \bar{q}_n, \tag{7}$$

$$\bar{\theta}_n^{\mathcal{P}} \equiv \frac{\theta_n^{\mathcal{P}}}{\Gamma^{\mathcal{P}}}, \quad \text{and} \quad \bar{\theta}_n^{\mathcal{A}} \equiv \frac{1 - \theta_n^{\mathcal{P}}}{1 - \Gamma^{\mathcal{P}}}. \tag{8}$$

Active investors' optimal stock holdings equal

$$\theta^{\cdot}_{i,n} = \theta_n \, \frac{\mathbb{E}\left[X_n \,|\, \mathcal{F}_i\right] - \theta_n \, P_n}{\rho \, \mathbb{V}ar\left(X_n \,|\, \mathcal{F}_i\right)}, \qquad i \in \{\mathcal{I}, \mathcal{U}\}. \tag{9}$$

Active investors' optimal demand for the stock, $\theta^{\cdot}_{i,n}$, in (9) follows the standard mean-variance portfolio rule. It is independent of an investor's initial wealth, $W_{i,0}$, and positively related to the mean and the precision of her posterior beliefs regarding payoff X_n.

Also, the equilibrium stock price, P_n, in (6), has the familiar structure of, for example, Hellwig (1980) and Verrecchia (1982) ("scaled" by the total supply of shares outstanding, θ_n). Moreover, the variables defined in (7) and (8) lend themselves to intuitive interpretations. $h_{0,n}$ characterizes the precision of public information or, equivalently, that of uninformed investors and is equal to the sum of the precision of prior beliefs, $1/\sigma_n^2$, and the precision of the public price signal, $\left(\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2\right) / \left(\rho^2 \, \sigma_Z^2\right)$. $\bar{q}_n$ measures the average precision of the private information of the informed investors. Consequently, $\bar{h}_n$ governs the average aggregate precision of informed and uninformed (active) investors. Finally, $\bar{\theta}_n^{\mathcal{P}}$ and $\bar{\theta}_n^{\mathcal{A}}$ in (8) govern the average fraction of shares of firm n held by passive and (informed and uniformed) active investors, respectively.

The equilibrium stock price (6) also allows us to immediately compute the precision of the stock price signals.

Lemma 1. *Conditional on informed investors' information choices, $\{q_{i,n}\}$, and firms' real investment policies, $\{I_n\}$, the precision of the public stock price of firm n is given by*

$$PI_n \equiv h_{0,n} - \frac{1}{\sigma_n^2} = \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\, \sigma_Z^2}.$$

Price informativeness, PI_n, is increasing in the average signal precision, $\bar{q}_n$.

2.2 Information choices

While Theorem 1 and Lemma 1 take the information environment as given, information choices are actually an endogenous outcome of the model. That is, in period $t = 2$, informed investors choose the precision of their private signals, $\{q_{i,n}\}$, while anticipating their optimal portfolio choice and the informativeness of stock prices in the trading round ($t = 3$). The average private signal precision, $\bar{q}_n$, is determined in the following theorem.

Theorem 2. *Conditional on firms' real investment policies, $\{I_n\}$, the average private signal precision, $\bar{q}_n$, is the unique solution to*

$$2\,\kappa'\left(\bar{q}_n\right) = \frac{1}{\rho}\left(\frac{1}{\sigma_n^2} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\, \sigma_Z^2} + \bar{q}_n\right)^{-1}. \tag{10}$$

Hence, the average private signal precision is increasing in the cash flow variance; formally, $d\bar{q}_n/d\sigma_n^2 > 0$.

At the optimum, the marginal cost of more precise private information, $2\kappa'\left(\bar{q}_n\right)$, equals the marginal benefit, which is governed by the inverse of informed investors' average posterior precision, $h_{0,n}+\bar{q}_n$, and risk tolerance, $1/\rho$. In particular, any decline in the precision of public information, $h_{0,n}$, increases an informed investor's incentives to acquire private information. Accordingly, an increase in cash flow variance, σ_n^2, shifts up the marginal benefits of private information, leading to a higher average private signal precision in equilibrium.

2.3 Real investment choices

The key new feature of our framework is that we allow for endogenous real investment decisions. Hence, the fundamental variance, σ_n^2, taken as given by Theorem 2, is, in fact, an endogenous outcome of the model. In particular, in period $t = 1$, each firm chooses the optimal investment in growth opportunities, $I_n \geq 0$, that will maximize its stock price (5) in period 1, while anticipating investors' optimal portfolio and information choices in periods 2 and 3. This results in the following real investment policies:

Theorem 3. *There exists an optimal investment in growth opportunities, $I_n > 0$, characterized by*

$$\mu_A - R_f - c\, I_n = \underbrace{\rho\, \bar{\theta}_n^{\mathcal{A}}}_{\equiv C_1} \times \underbrace{2\, I_n\, \sigma_A^2\, \frac{1}{\bar{h}_n^2} \left(-\frac{d\bar{h}_n}{d\sigma_n^2} \right)}_{\equiv C_2}, \qquad \textit{with} \; -\frac{d\bar{h}_n}{d\sigma_n^2} \geq 0. \tag{11}$$

At the optimum, the marginal benefit of investing in growth opportunities equals the marginal cost. Intuitively, the marginal benefit is given by the increase in the mean payoff, $\mu_A - R_f - c\, I_n$. The marginal cost derives from the higher price discount that risk-averse investors command in response to an increase in posterior variance $1/\bar{h}_n$ (resulting from the higher cash flow variance $\sigma_n^2 = I_n^2 \sigma_A^2$). Marginal costs can be decomposed into two components: first, C_1, the sensitivity of firm value to posterior variance, $dS_n/d(1/\bar{h}_n)$, and second, C_2, the sensitivity of posterior variance to investments in growth opportunities, $d(1/\bar{h}_n)/dI_n$. While the first component, C_1, does not vary with a firm's allocation of capital to growth opportunities, the second component, C_2, and, thus, marginal costs, are usually increasing in the investment, I_n.[12]

[12]In fact, depending on the convexity of the information-cost function κ, marginal costs might start to decline at some point (i.e., follows a hump-shaped pattern). We relegate the discussion of the implications of this phenomenon to Section 4.3.

3 The Cross-Sectional Implications of Passive Ownership

In this section, we now study the impact of *cross-sectional* variations in passive ownership on firms' real investment decisions, informational efficiency, and stock prices. To do so, we exploit the between-firm heterogeneity in passive ownership in our model. Specifically, for our main comparative statics analysis, we vary the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$; keeping the share of passive investors, $\Gamma^{\mathcal{P}}$, fixed.

3.1 Real investment

In the first step, we analyze how passive ownership affects firms' real investment policies. The following proposition summarizes our key findings.

Theorem 4. *As the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$, increases, the marginal costs of investing in growth opportunities shift down. Hence, firms with higher passive ownership invest more in growth opportunities; formally, $dI_n/d\bar{\theta}_n^{\mathcal{P}} > 0$.*

Intuitively, a higher average demand from passive investors lowers the average number of shares that active investors have to hold in equilibrium, $\bar{\theta}_n^{\mathcal{A}}$, and, accordingly, the amount of risk each active investor has to bear. Thus, active investors command a smaller price discount (per unit of risk); that is, the marginal cost of investing in growth opportunities or, equivalently, the cost of capital, declines.[13,14]

Panel A of Figure 2 illustrates the resultant decline in marginal costs. Naturally, the lower costs imply larger investments in growth opportunities for firms with more passive owners (as highlighted by the intersection with marginal benefits). Indeed, as discussed in Theorem 4 and shown in Panel B, the optimal investment, I_n, is monotonically increasing in passive investors' average demand, $\bar{\theta}_n^{\mathcal{P}}$. This mechanism aligns with the intuition behind the "benchmark inclusion subsidy" discussed in Kashyap, Kovrijnykh, Li, and Pavlova (2020).

[13]Alternatively, one can think of the decline in marginal costs as being driven by the passive investors' inelastic demand, which renders the firm value, S_n, less sensitive to posterior variance and, hence, drives down the first component of the marginal costs (C_1, in (11)). In contrast, the second component of the marginal costs, C_2, is unaffected by variations in the average demand of passive investors because their demand does not affect the equilibrium signal precision, $\bar{q}_n$.

[14]In our setting, idiosyncratic risk is priced and, hence, the decline in the cost of capital obtains through this channel. Note, however, that the same would apply in a setting with systematic risk. In particular, in that case, an increase in the average demand of passive investors would reduce the *sensitivity* of a firm's stock price to systematic risk and, thus, the cost of capital (even though passive investing does not change the aggregate price of risk).

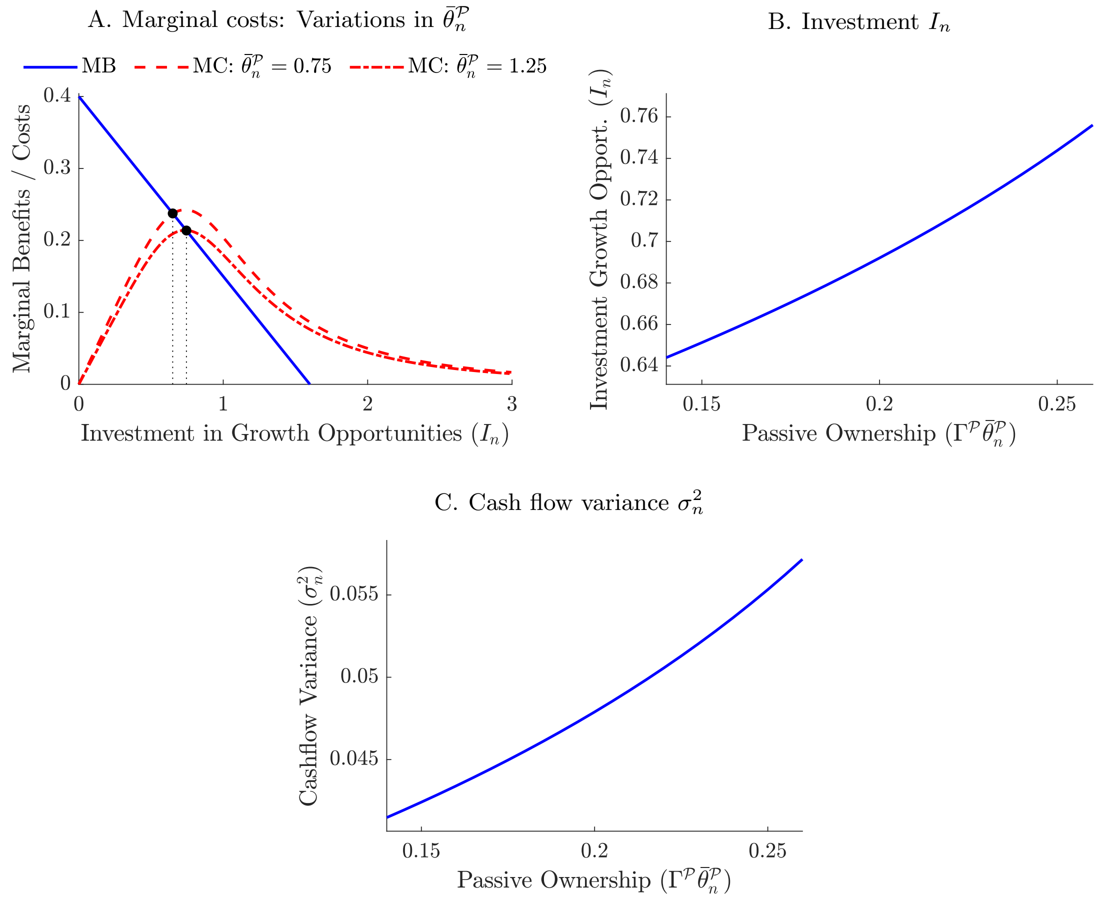

Figure 2: Real investment decisions. The figure depicts the impact of passive ownership on firms' real investment policies. Panel A illustrates how the marginal cost of investing in growth opportunities and, hence, real investment, I_n, vary with average passive investor demand $\bar{\theta}_n^{\mathcal{P}}$. Panels B and C depict a firm's investment in growth opportunities, I_n, and its cash flow variance, σ_n^2 as a function of the average demand of passive investors $\bar{\theta}_n^{\mathcal{P}}$, respectively. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/4$, and a quadratic information-cost function: $\kappa(q) = 0.004\,q^2$.

The larger capital allocation to growth opportunities by firms with higher passive ownership naturally translates into a higher expected cash flow and a higher cash flow variance. For the variance, σ_n^2, this is illustrated in Panel C of Figure 2.

Lemma 2. *As the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$, increases, the mean and the variance of the cash flow X_n increase; formally, $d\mu_n/d\bar{\theta}_n^{\mathcal{P}} > 0$ and $d\sigma_n^2/\bar{\theta}_n^{\mathcal{P}} > 0$.*

3.2 Informational efficiency

We now turn to the impact of passive ownership on the informativeness of stock prices.

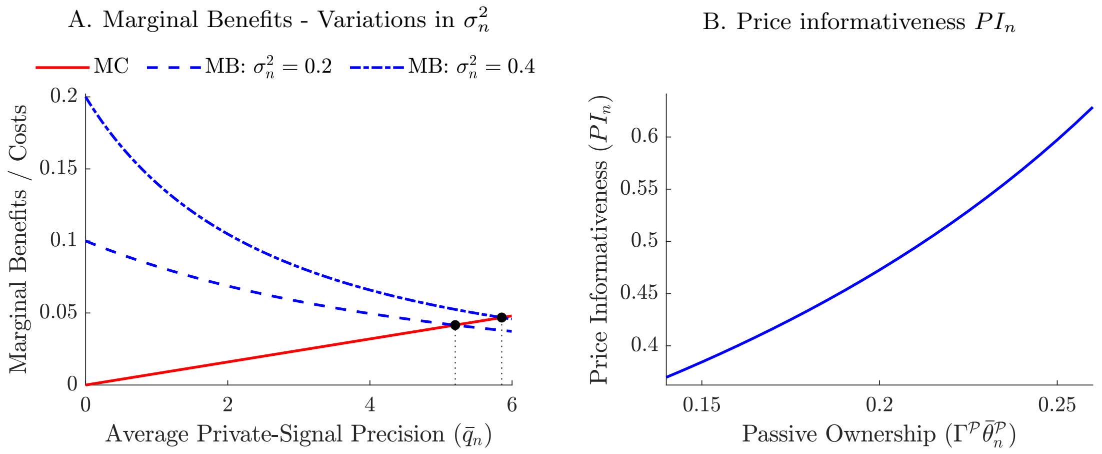

Figure 3: Informational efficiency. The figure depicts the impact of passive ownership on informational efficiency. Panel A depicts how the marginal benefits of private information and, hence, the equilibrium information choice, vary with cash flow variance, σ_n^2. Panel B plots price informativeness, PI_n, as a function of the average demand of passive investors $\bar{\theta}_n^{\mathcal{P}}$. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/4$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

Theorem 5. *As the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$, increases, the average precision of informed investors' private information and price informativeness increase; formally, $d\bar{q}_n/d\bar{\theta}_n^{\mathcal{P}} > 0$ and $dPI_n/d\bar{\theta}_n^{\mathcal{P}} > 0$.*

Notably, price informativeness *increases* in the proportion of shares held by passive investors. To understand the economic intuition behind this result, recall that firms with a high share of passive owners, $\bar{\theta}_n^{\mathcal{P}}$, invest more in growth opportunities and, hence, have a higher cash flow variance, σ_n^2 (see Panels B and C of Figure 2). The higher cash flow variance lowers the precision of public information, $h_{0,n}$, and, hence, increases the marginal benefit of private information.[15] Consequently, in equilibrium, informed investors acquire more precise private information; that is, the average private signal precision goes up, as illustrated in Panel A of Figure 3. This, in turn, pushes up price informativeness. Accordingly, the precision of public stock prices is monotonically increasing in the fraction of shares held by passive owners (Panel B of Figure 3).

3.3 Asset prices

Passive ownership also affects equilibrium stock prices and returns.

[15] A similar result would obtain with an additive information capacity constraint as employed by Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). In particular, Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) show that the marginal value of allocating an increment of capacity is increasing in the cashflow variance. Benamar, Foucault, and Vega (2020) provide direct empirical evidence for this effect.

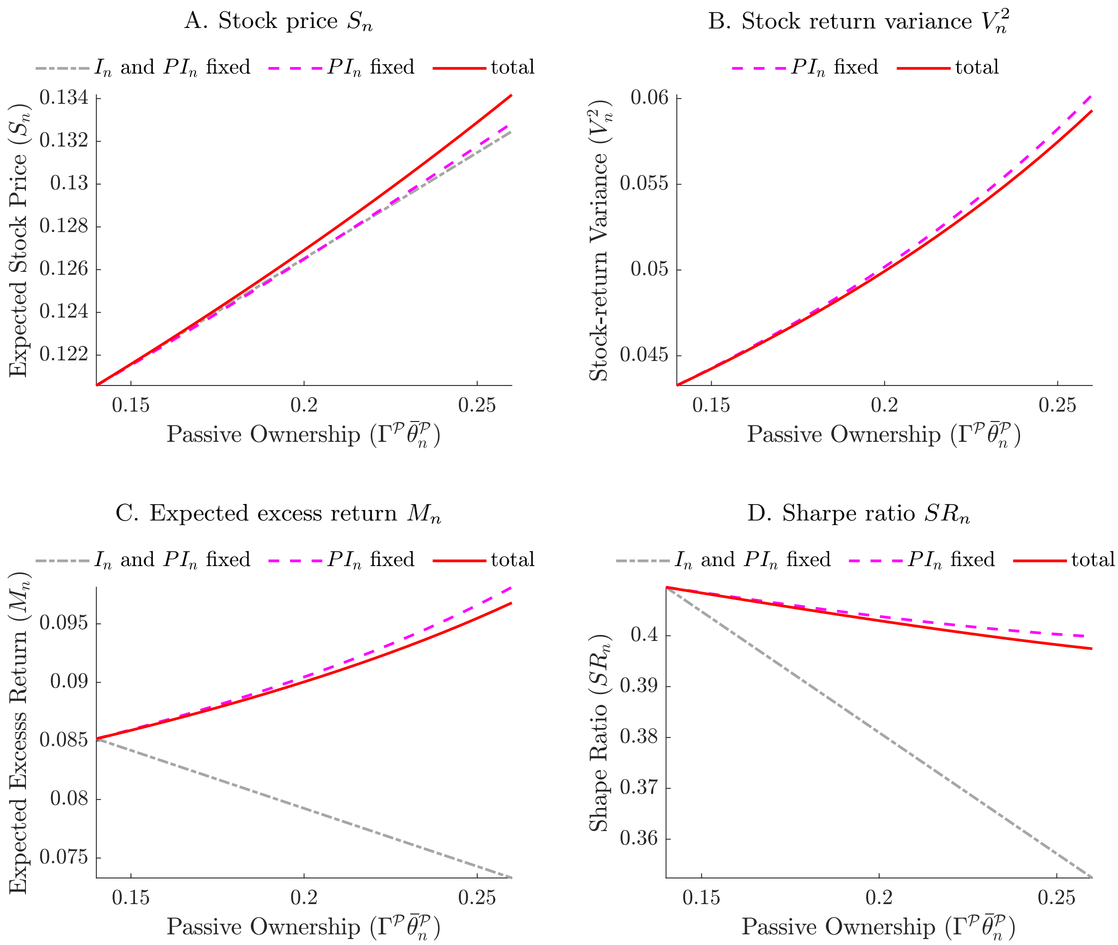

Figure 4: Stock prices and returns. The figure depicts the impact of passive ownership on stock prices and stock returns. It plots the (expected) stock price, S_n (Panel A), the stock return variance V_n^2 (Panel B), the expected excess return, M_n (Panel C), and the Sharpe ratio, SR_n (Panel D), as functions of the average demand of passive investors $\bar{\theta}_n^{\mathcal{P}}$. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/4$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

Theorem 6. *Firm n's (unconditional expected) stock price $S_n \equiv \mathbb{E}[P_n]$ is given by*

$$S_n = \mu_n - I_n - \rho\, \bar{\theta}_n^{\mathcal{A}} \frac{1}{\bar{h}_n}. \tag{12}$$

The stock price S_n is increasing in the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$; formally, $dS_n/d\bar{\theta}_n^{\mathcal{P}} > 0$.

Intuitively, the expected stock price, S_n, in (12), is given by a firm's expected cash flow (μ_n) minus the investment I_n and a risk discount $\left(-\rho\, \bar{\theta}_n^{\mathcal{A}} \frac{1}{\bar{h}_n}\right)$. A combination of three effects explains the increase in the stock price with passive ownership. First, an increase in passive investors' average demand lowers the average number of shares to be held by active investors, $\bar{\theta}_n^{\mathcal{A}}$, and, accordingly, the amount of risk each active investors has to bear. Thus, active investors command a lower price discount (i.e., the cost of capital declines), which pushes up the price (keeping investment, I_n, and price informativeness, PI_n, fixed). Second, as the average demand of passive investors goes up, so does the investment in growth opportunities, leading to a higher expected cash flow (μ_n) and, hence, a higher price (fixing price informativeness, PI_n). Third, the higher price informativeness for firms with higher shares of passive owners increases aggregate posterior precision, $\bar{h}_n$, and, hence, lowers the price discount active investors command. Panel A of Figure 4 illustrates the increase in the stock price with passive ownership and decomposes it into its three components.

Notably, the stock price reaction to changes in passive ownership is asymmetric; that is, the price increases considerably more for high passive-ownership firms than it declines for low passive-ownership firms. For example, in the illustration in Panel A, the increase is more than 6% for $\bar{\theta}_n^{\mathcal{P}} = 1.3$ whereas the decline is around -4% for $\bar{\theta}_n^{\mathcal{P}} = 0.7$ (in each case measured relative to the average firm with $\bar{\theta}_n^{\mathcal{P}} = 1.0$).

Not surprisingly, passive ownership also affects the stock return moments.

Theorem 7. *The (unconditional) expected excess return $M_n \equiv \mathbb{E}[X_n - P_n R_f]$ and the (unconditional) stock return variance $V_n^2 \equiv \mathbb{V}ar(X_n - P_n R_f)$ of stock n are given by*

$$M_n = \rho\, \bar{\theta}_n^{\mathcal{A}}\, \frac{1}{\bar{h}_n}, \qquad \textit{and} \qquad V_n^2 = \frac{1}{\bar{h}_n^2}\left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\,\bar{q}_n + \frac{\rho^2\,\sigma_Z^2}{(1-\Gamma^{\mathcal{P}})^2}\right).$$

As the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$, increases, the stock return variance, V_n^2 increases; formally, $dV_n^2/d\bar{\theta}_n^{\mathcal{P}} > 0$.

An increase in passive ownership unambiguously leads to an increase in the stock return variance. This is the result of two opposing forces, both of which stem from the larger investment in growth opportunities for firms with high passive ownership. Specifically, while the resultant increase in the cash flow variance pushes up the stock return variance (keeping price informativeness, PI_n, fixed), the corresponding increase in price informativeness lowers the return variance. In equilibrium, the first effect dominates, and, hence, the stock return variance is higher for firms with a large proportion of shares in the hands of passive investors, as illustrated in Panel B of Figure 4.

A stock's expected excess return, M_n, is affected by three economic forces. First, an increase in passive investors' average demand lowers the amount of risk active investors have to bear and, hence, the risk premium they command (fixing investment, I_n, and price informativeness, PI_n). Second, the higher investment in growth opportunities for firms with high passive ownership implies a higher posterior variance ($1/\bar{h}_k$), which, in turn, leads to a higher excess return (fixing price informativeness, PI_n). Third, the increase in price informativeness lowers the posterior variance and, hence, the expected excess return. For practically all relevant cases, the second (positive) effect dominates, implying that stocks with high passive ownership tend to have higher expected excess returns (relative to otherwise identical stocks with low passive ownership). Panel C of Figure 4 illustrates this. The exception is the case in which a stock's net supply—after accounting for the aggregate demand of passive investors—becomes small. In this case, the excess return starts to decline because the risk each active investor has to bear diminishes.[16]

The exact same forces also drive stock n's Sharpe ratio, $SR_n \equiv M_n/\sqrt{V_n^2}$, which is illustrated in Panel D. That is, while the lower amount of risk active investors have to bear (keeping investment, I_n, and price informativeness, PI_n, fixed) and the higher price informativeness imply a decline in the Sharpe ratio, the additional risk-taking of firms with high passive ownership pushes up the Sharpe ratio (fixing price informativeness, PI_n). Usually, the demand effect dominates, and, hence, the Sharpe ratio declines.

3.4 Trading profits

Active investors' trading profits also vary with the fraction of passive owners. In particular, as illustrated in Figure 5, informed and uninformed investors' gross profits from trading stock n, $TP_{i,n} \equiv \mathbb{E}\left[\theta_{i,n}\left(\frac{X_n}{\theta_n} - P_n\right)\right]$, are (usually) increasing in passive ownership. For uninformed investors, this is simply the result of the higher expected excess return of stocks with high passive ownership (which dominates the opposing effect of a decline in public precision $h_{0,n}$). For informed investors, an additional positive effect arises from the higher

[16] In fact, as passive investors' aggregate demand approaches the aggregate supply, the expected excess return converges to zero, because active investors have to bear no risk.

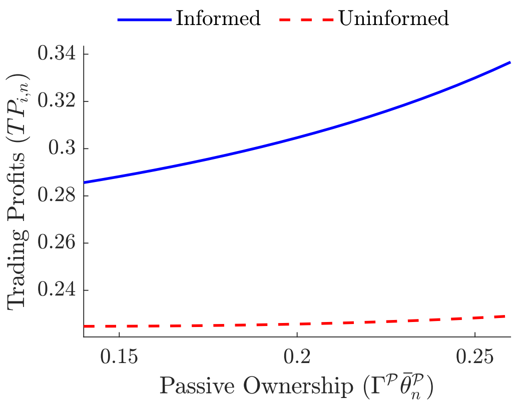

Figure 5: Trading profits. The figure depicts the impact of passive ownership on trading profits; specifically, it plots informed and uninformed investors' trading profits, $TP_{i,n}$ as functions of the average demand of passive investors $\bar{\theta}_n^{\mathcal{P}}$. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/4$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

private signal precision. Consequently, as passive ownership rises, the gross trading profits of informed investors increase relative to those of uninformed investors.[17]

Theorem 8. *As the average fraction of shares held by passive investors, $\bar{\theta}_n^{\mathcal{P}}$, increases, the difference in informed and uninformed investors' expected profits from trading stock n, $TP_{i,n}^{\mathcal{I}} - TP_{i,n}^{\mathcal{U}}$, increase; formally, $d(TP_{i,n}^{\mathcal{I}} - TP_{i,n}^{\mathcal{U}})/d\bar{\theta}_n^{\mathcal{P}} > 0$.*

4 The Implications of a Rise in Aggregate Passive Ownership

Next, we focus on the *aggregate* implications of passive ownership; specifically, we consider the impact of a rise in the assets under management of passive investment funds on firms' real investment policies, informational efficiency, and stock prices.

For our comparative statics analysis, we vary the share of passive investors in the economy, $\Gamma^{\mathcal{P}}$.[18] We proceed in two steps by separately discussing the cases in which passive owners "displace" uninformed and informed active investors.

[17] Cujean (2020) provides an important discussion on the (empirical) link between fund-manager performance and their informational advantages; showing that, because better informed managers take larger positions, their alpha is noisier and, hence, statistical significance and persistence of alpha concentrate in under-performing funds.

[18] Intuitively, one can think of the proportion of aggregate capital managed by passive investors as the extensive margin of their trading and of their average demand for a stock as the stock-specific intensive margin of their trading.

4.1 Passive owners displacing uninformed investors

We start our analysis with a discussion of the case in which passive owners crowd out *uninformed* investors; that is, a rise in the share of passive investors, $\Gamma^{\mathcal{P}}$, results in a decline in the share of uninformed investors, $\Gamma^{\mathcal{U}}$.

The following theorem summarizes the main implications for firms' real investment policies and cash flows.

Theorem 9. *Fixing the share of informed investors, $\Gamma^{\mathcal{I}}$, an increase in the share of passive investors, $\Gamma^{\mathcal{P}}$, leads to an increase (decline) in firms' investment in growth opportunities, I_n, and, hence, in cash flow variance, σ_n^2, for firms with an average passive investor demand of $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{U}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{U}}$), with $\xi^{\mathcal{U}} < 1$.*

For firms with strong average demand from passive investors ($\bar{\theta}_n^{\mathcal{P}} > 1.0$), an increase in the share of passive investors further reduces the average number of shares to be held by active investors in equilibrium and, hence, the risk each active investor has to bear. Thus, the cost of capital declines. For firms with weak demand ($\bar{\theta}_n^{\mathcal{P}} < 1.0$), the opposite applies. As a result, an increase in the share of passive investors leads to stronger dispersion in firms' real investment decisions, as illustrated in Panel A of Figure 6.

To understand the aggregate impact, note that the decline in the share of uninformed investors implies an increase in the average posterior precision among uninformed and informed investors, $(\Gamma^{\mathcal{I}}\bar{q}_n)/(\Gamma^{\mathcal{I}} + \Gamma^{\mathcal{U}})$ and, hence, in the aggregate posterior precision $\bar{h}_n$.[19] This pushes down the marginal costs for *all* firms, and, thus, in aggregate, firms invest more aggressively in growth opportunities (Panel A), and the average cash flow variance increases; formally $\xi^{\mathcal{U}}) < 1.0$.[20]

Corollary 1. *Fixing the share of informed investors, $\Gamma^{\mathcal{I}}$, an increase in the share of passive investors, $\Gamma^{\mathcal{P}}$, leads to an increase (decline) in price informativeness, PI_n, for firms with an average passive investor demand $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{U}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{U}}$).*

[19] Indeed, neither informed investors' optimal information choices $\bar{q}_n$ (characterized by (10)) nor price informativeness, PI_n, depend on the share of uninformed investors, $\Gamma^{\mathcal{U}}$. Hence, these two components of the aggregate posterior precision, $\bar{h}_n$, remain unchanged.

[20] To measure the aggregate impact, we simply rely on an (equal-weighted) average across firms (with $0.7 \leq \bar{\theta}_n^{\mathcal{P}} \leq 1.3$). The results are qualitatively unchanged if one focuses on an "average firm" with $\bar{\theta}_n^{\mathcal{P}} = 1.0$.

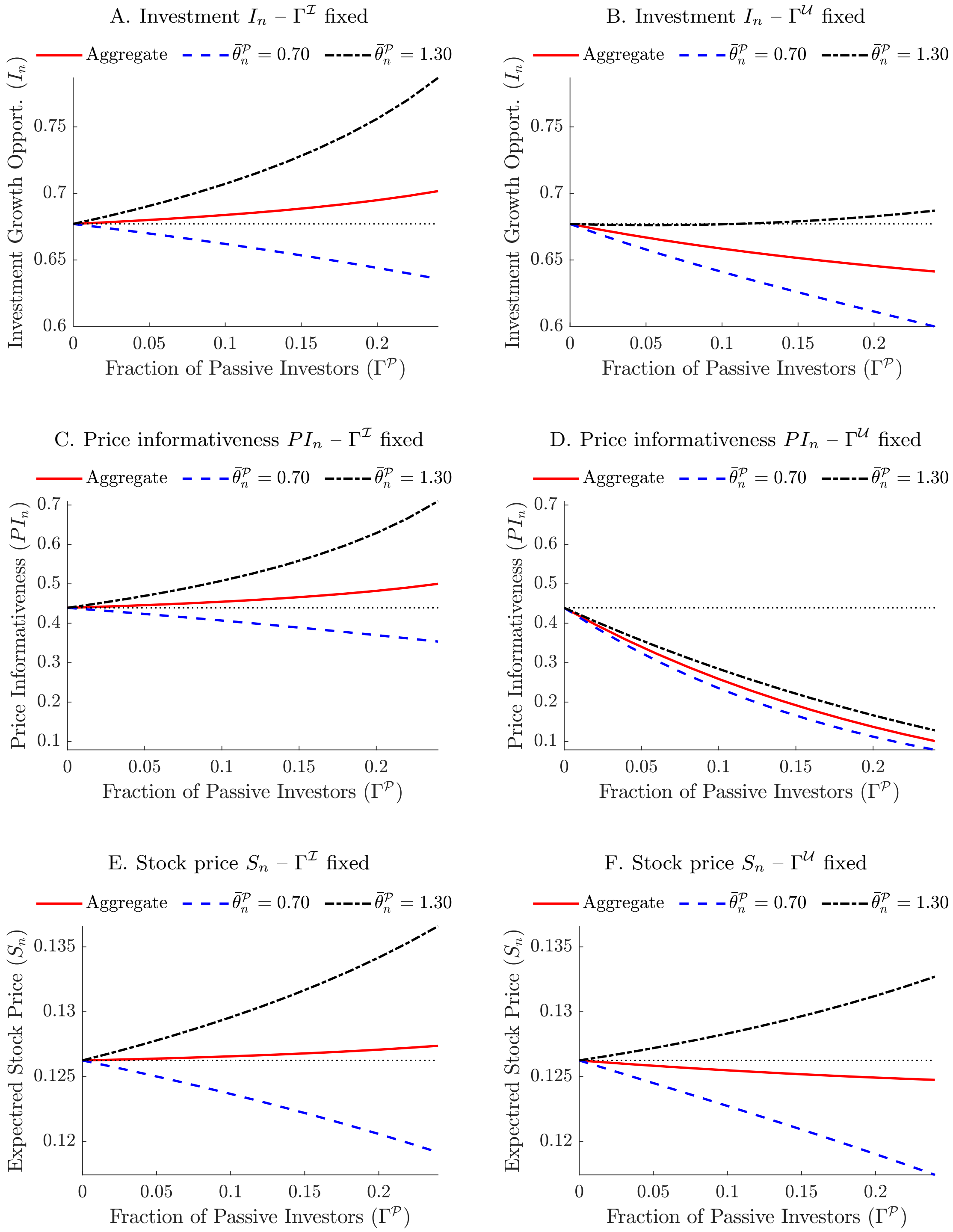

Figure 6: The impact of a rise in the share of passive investors. The figure illustrates the implications of variations in the share of passive investors, $\Gamma^{\mathcal{P}}$. Panels A, C, and E plot firms' investment in growth opportunities I_n, price informativeness PI_n, and the stock price S_n as functions of the share of passive investors $\Gamma^{\mathcal{P}}$ while keeping the share of informed investors, $\Gamma^{\mathcal{I}}$, fixed—both in aggregate and for firms with a low and a high passive investor demand $\bar{\theta}_n^{\mathcal{P}}$. Panels B, D, and F plot the same quantities while keeping the share of uninformed investors, $\Gamma^{\mathcal{U}}$, fixed. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{I}} = 0.5$, $c = 1/4$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

The higher average cash flow variance incentivizes informed investors to acquire more precise information, and, hence, in aggregate, price informativeness increases as the share of passive investors rises (Panel C). Moreover, the higher expected cash flows (together with higher informativeness) usually lead to a higher aggregate stock price (Panel E) and, because of the higher average cash flow variance, to a higher average stock return variance (not shown). Naturally, the more pronounced differences in firms' real investment policies translate into a higher dispersion in price informativeness (Panel C), stock prices (Panel E), and stock return moments (not shown) across firms.

In summary, if passive owners displace uninformed investors, real investment, price informativeness, and stock prices increase, both in aggregate and for a majority of firms. The largest benefits thereby accrue to firms with strong passive investor demand (e.g., firms that are part of broad stock market indices). Interestingly, an increase in the share of passive investors implies an increase in the expected utility of informed investors (relative to that of uninformed investors), due to the higher average private signal precision. This could reinforce a decline in the share of uninformed investors (and, potentially, further improve price informativeness).

4.2 Passive owners displacing informed investors

We now turn to the case in which passive owners crowd out *informed* investors; that is, a rise in the share of passive investors, $\Gamma^{\mathcal{P}}$, results in a decline in the share of informed investors, $\Gamma^{\mathcal{I}}$.

Intuitively, as the share of passive investors increases and, hence, the dispersion in passive ownership rises, the differences in firms' real investment policies again become more pronounced, as illustrated in Panel B of Figure 6. Accordingly, the dispersion in price informativeness, stock prices, and stock returns again increases in the share of passive owners (see, e.g., Panels D and F).

However, the aggregate implications are quite different.

Theorem 10. *Assume* $\left(-\frac{d^2\bar{h}_n}{d\sigma_n^2 d\Gamma^{\mathcal{P}}}\right) > 0$.[21] *Fixing the share of uninformed investors,* $\Gamma^{\mathcal{U}}$*, an increase in the share of passive investors,* $\Gamma^{\mathcal{P}}$*, leads to an increase (decline) in firms'*

[21] This is a sufficient condition for the theorem to hold. Because the aggregate posterior precision, $\bar{h}_n$, is declining in the share of passive investors, the theorem can hold even if the condition is violated. Indeed,

investment in growth opportunities, I_n, and, hence, in the cash flow variance, σ_n^2, for firms with average passive investor demand $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{I}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{I}}$), with $\xi^{\mathcal{I}} > 1$.

In particular, an increase in the share of passive investors now *increases* the marginal costs of investing in growth opportunities for all firms, and, hence, in aggregate, firms invest less (Panel B of Figure 6). This is the result of a deterioration in information aggregation (driven by the lower number of informed investors) which causes a decline in posterior precisions, $\bar{h}_n$. Accordingly, the average cash flow variance also drops.

Finally, the impact on price informativeness is as follows:

Corollary 2. *Assume $\left(-\frac{d^2\bar{h}_n}{d\sigma_n^2 d\Gamma^{\mathcal{P}}}\right) > 0$. Fixing the share of informed investors, $\Gamma^{\mathcal{I}}$, an increase in the share of passive investors, $\Gamma^{\mathcal{P}}$, leads to an increase (decline) in price informativeness, PI_n, for firms with an average passive investor demand $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{I}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{I}}$).*

Intuitively, the decline in cashflow variance weakens informed investors' incentives to acquire private information which, in turn, lowers the average private signal precision and price informativeness. Importantly, this effect is considerably amplified by the deterioration in information aggregation. Consequently, as the share of passive investors increases, there is a sharp decline in price informativeness, not only in aggregate but for a large majority of firms (Panel D). This decline in price informativeness, in turn, leads in aggregate to a decline in stock prices (Panel D) and an increase in stock return variances (not shown).

To summarize, if passive owners displace informed investors, the negative consequences of lower price informativeness and a lower stock price now apply to a much larger set of firms—a result of the deterioration in information aggregation. Moreover, in general, the dispersion across firms widens. In contrast to the former setting, an increase in the share of passive investors now implies a decline in the expected utility of informed investors (relative to that of uninformed investors) which could lead to a reinforcement of the initial decline in the share of informed investors (and, potentially, lead to a further deterioration in price informativeness).

in all our numerical simulations (over a wide range of parameter values), marginal costs increased with the share of passive investors.

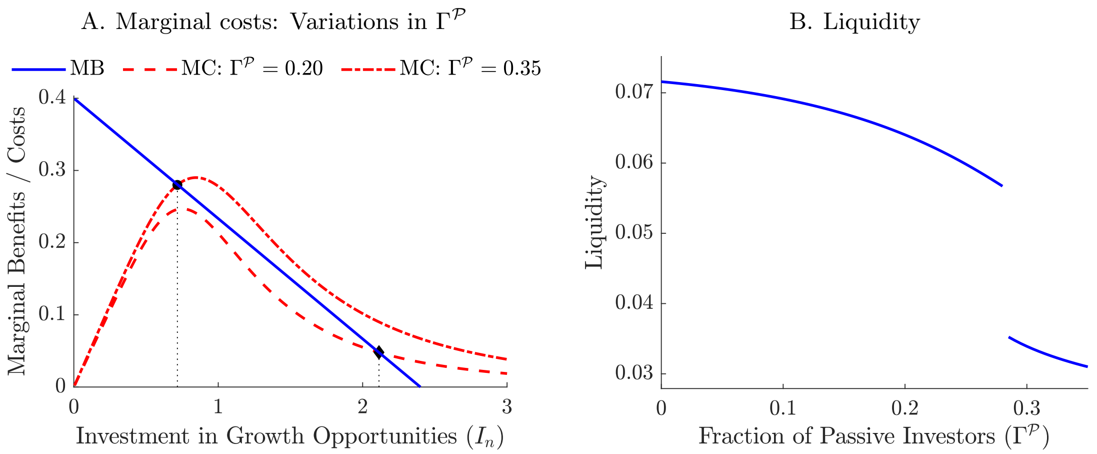

Figure 7: Liquidity crashes. The figure illustrates a liquidity crash for a firm with low average demand from passive investors ($\bar{\theta}_n^{\mathcal{P}} = 0.75$). Panel A illustrates how the marginal cost of investing in growth opportunities varies with the share of passive investors, $\Gamma^{\mathcal{P}}$. Panel B plots the stock's liquidity, which is measured as the sensitivity of the stock price, P_n, to noise traders' demand, Z_n, as a function of the share of passive investors, $\Gamma^{\mathcal{P}}$. The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/6$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

4.3 Liquidity crashes

For firms with low average demand from passive investors ($\bar{\theta}_n^{\mathcal{P}} \ll 1.0$), the marginal costs of investing in growth opportunities (or, equivalently, the cost of capital) unambiguously increase with the share of passive investors. This can lead to a "liquidity crash."

To understand the intuition behind this result, note that, depending on the convexity of the information-cost function κ, the marginal cost of investing in growth opportunities might start to decline and converge to zero as the investment, I_n, increases.[22] That is, in response to a firm's larger investment in growth opportunities and, hence, a larger cash flow variance, informed investors' incentives to acquire more precise private information strengthen. If the resultant increase in average posterior precision, $\bar{h}_n$, is strong enough, marginal costs start to decline. In some sense, firms "free ride" on the information choices of informed investors, who bear the (information) costs. As a result, marginal costs follow a hump-shaped pattern, as illustrated in Panel A.[23]

[22]This result is specific to the case with endogenous information choice. In the absence of information choice, marginal costs are monotonically increasing in the investment, I_n; confer also the discussion in Footnote 34.

[23]Note, there might even be multiple local maxima (i.e., points at which the marginal benefits "cross" the marginal costs from above); though the global maximum is usually unique. Specifically, the set of parameter values for which there are multiple global maxima has zero probability.

An increase in the share of passive investors, specifically, the accompanying upward shift in marginal costs, can then lead to an abrupt change in a firm's investment policy, I_n, and, hence, in a stock's liquidity (measured as the sensitivity of the stock price, P_n, with respect to the noise Z_n).[24] Specifically, as illustrated in Panel A of Figure 7, while a "high-investment, high-liquidity equilibrium" might be obtained for a small share of passive investors, a "low-investment, low-liquidity" equilibrium might be obtained for a large share of passive investors. Hence, as illustrated in Panel B, a liquidity crash occurs as the share of passive investors increases.

5 Extension: Benchmarking

In this section, we now extend our analysis to the case of benchmarked investors. This serves two purposes. First and foremost, it allows us to expand the implications of our work to a broader set of institutional investors. Second, it demonstrates that our findings are robust to an endogenous passive investor demand (that is sensitive to cash flow variance and, thus, firms' real investment policies).

5.1 Model and equilibrium

The model differs from our main economic framework along a single dimension: we explicitly model the portfolio choice of "passive" institutional investors. In particular, we follow Breugem and Buss (2019) and assume that benchmarked investors, $i \in \mathcal{B}$, have CARA preferences (with absolute risk aversion ρ) over "compensation" C_i:

$$C_i = W_i + \nu \sum_{n=1}^{N} \gamma_{i,n} \left(X_n - \theta_n \, P_n\right),$$

where W_i denotes terminal wealth (as defined in (3)), $\sum_{n=1}^{N} \gamma_{i,n} \left(X_n - \theta_n \, P_n\right)$ denotes the return of investor i's benchmark, and ν governs the strength of her benchmarking concerns.

[24] Similar liquidity crashes are discussed in Cespa and Foucault (2014) and Cespa and Vives (2015). In Cespa and Foucault (2014), liquidity dry-ups and large drops in price informativeness arise from spillover effects from one asset to another. Honkanen and Schmidt (2019) provide empirical support for such cross-asset learning. Cespa and Vives (2015) discuss the possibility of multiple equilibria (a high-information and a low-information equilibrium) and link their existence to the precision in public information. They also connect their theory to sudden liquidity dry-ups observed in the data (e.g., during the latest financial crisis).

Intuitively, benchmarking creates an incentive for institutional investors to attain a high return when their benchmark performs well, with the tracking error that they are willing to "tolerate" declining in the strength of their benchmarking concerns, ν.

Consequently, benchmarked (passive) investors are now fully optimizing, rational investors who select their portfolio in order to maximize expected utility, conditional on information from public stock prices (i.e., $\mathcal{F}_i = \{P_n\}, i \in \mathcal{B}$).[25] Specifically, in equilibrium, the optimal stock demand of the investor $i \in \mathcal{B}$ is given by

$$\theta^{\mathcal{B}}_{i,n} = \theta_n \frac{\mathbb{E}\left[X_n \,|\, \mathcal{F}_i\right] - \theta_n P_n}{\rho \, \mathbb{V}\mathrm{ar}\left(X_n \,|\, \mathcal{F}_i\right)} + \theta_n \, \nu \, \gamma_{i,n}. \tag{13}$$

The optimal demand comprises two components: first, the standard mean-variance demand, which is declining in the posterior variance (i.e., a firm's investment in growth opportunities), and, second, an inelastic hedging demand that arises from investors' desire to acquire stocks that positively covary with the benchmark.

The expected equilibrium stock price, $S_n = \mathbb{E}[P_n]$, is equal to

$$S_n = \mu_n - \frac{c}{2} I_n^2 - \rho \left(1 - \theta^{\mathcal{B}}_n\right) \frac{1}{\bar{h}_n}, \tag{14}$$

where $\bar{h}_n \equiv h_{0,n} + \Gamma^{\mathcal{I}} \, \bar{q}_n$ denotes the average precision among all investors (with $h_{0,n}$ and $\bar{q}_n$ defined in (7)) and $\theta^{\mathcal{B}}_n \equiv \Gamma^{\mathcal{B}} \, \nu \int^{\mathcal{B}} \gamma_{i,n} \, di$ denotes the aggregate hedging demand of benchmarked investors. It has the familiar form of (12), with the average "residual" demand of all investors $(1 - \theta^{\mathcal{B}}_n)$ replacing that of active investors $(\bar{\theta}^{\mathcal{A}}_n)$ in the price-discount component $(-\rho \left(1 - \theta^{\mathcal{B}}_n\right) \frac{1}{\bar{h}_n})$.

5.2 The impact of passive benchmarking

All our main results remain unchanged in the case of benchmarking.

Theorem 11. *An increase in the aggregate benchmarking demand of investors, $\theta^{\mathcal{B}}_n$, leads to an increase in firms' investment in growth opportunities, I_n, the cash flow variance, σ^2_n,*

[25] The assumption that benchmarked investors can only learn from stock prices is made for ease of exposition. The results are qualitatively unchanged if one were to allow passive investors to acquire private information.

the average private signal precision, $\bar{q}_n$, price informativeness, PI_n, the stock price, S_n, and the stock return variance, V_n^2; formally $dI_n/d\theta_n^{\mathcal{B}} > 0$, $d\sigma_n^2/d\theta_n^{\mathcal{B}} > 0$, $d\bar{q}_n/d\theta_n^{\mathcal{B}} > 0$, $dPI_n/d\theta_n^{\mathcal{B}} > 0$, $dS_n/d\theta_n^{\mathcal{B}} > 0$, and $dV_n^2/d\theta_n^{\mathcal{B}} > 0$.

The underlying economic mechanisms closely resemble those discussed in the preceding sections. That is, an increase in the aggregate benchmarking demand (or, equivalently, benchmarked investors' average demand) lowers the number of shares that investors have to hold in equilibrium for nonbenchmarking (speculative) reasons and, hence, the risk they have to bear. Accordingly, as the aggregate benchmarking demand rises, the marginal costs of investing in growth opportunities shift down, and, hence, firms invest more, as illustrated in Panel A of Figure 8.[26] The resultant higher cash flow variance, σ_n^2, in turn, incentivizes informed investors to acquire more precise private information (i.e., the average private signal precision, $\bar{q}_n$, goes up); leading to higher price informativeness, PI_n, for firms with a strong benchmarking demand (Panel B). As before, a combination of three effects produces the increase in the stock price S_n: (1) a stronger aggregate demand, (2) a higher expected cash flow, and (3) higher price informativeness. Finally, the stock return variance, V_n^2, goes up because of the increase in cash flow variance (which dominates the corresponding increase in price informativeness).

The implications of variations in the aggregate size of benchmarked investors are also very similar. That is, in general, an increase in the share of benchmarked investors, $\Gamma^{\mathcal{B}}$, leads to more pronounced differences in firms' real investment policies, price informativeness, and stock prices because of a widening gap in the aggregate benchmarking demand across firms. Moreover, an increase in the share of benchmarked investors leads to a increase (decline) in aggregate price informativeness if uninformed (informed) active investors are crowded out (Panel C). The implications for firms' corporate policies and stock prices (returns) follow accordingly.

[26] Intuitively, a strengthening of investors' benchmarking concerns, ν, leads to a further reduction in the risk that investors have to bear and, hence, amplifies the effect.

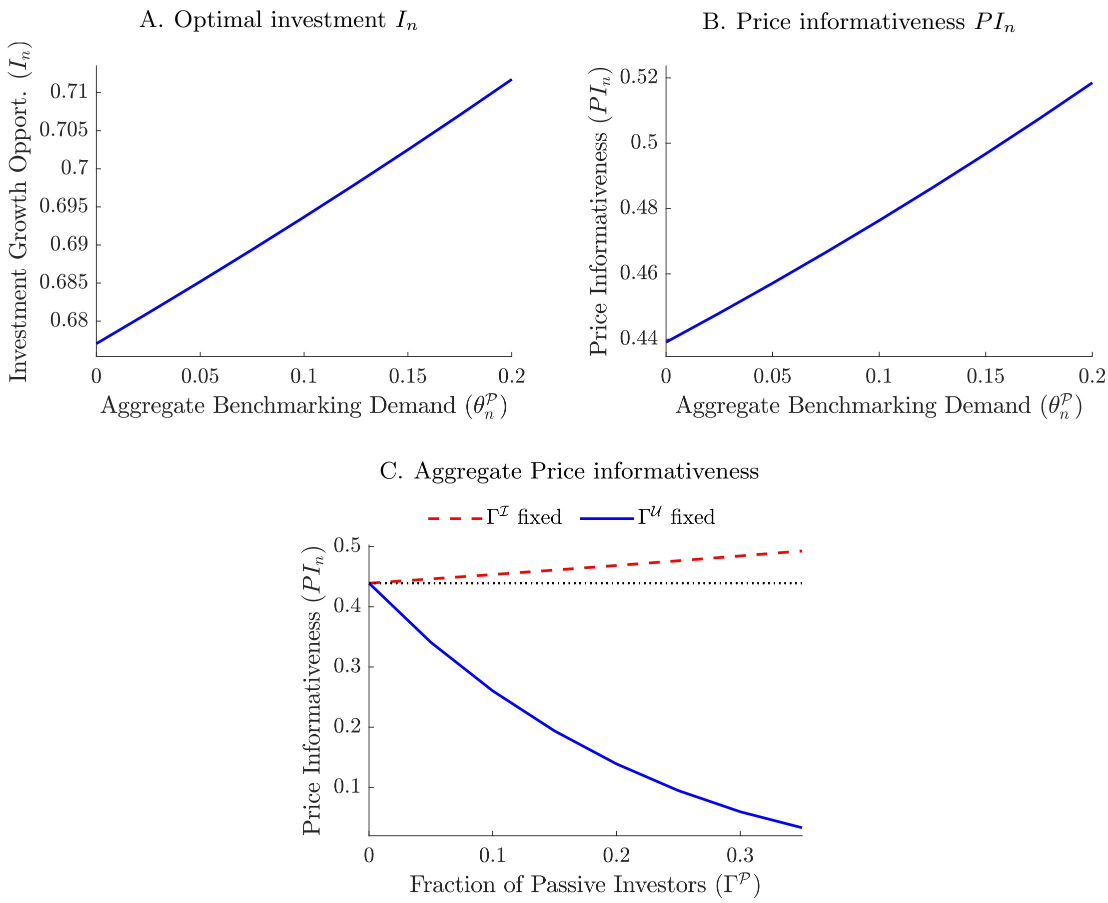

Figure 8: The impact of benchmarking. The figure illustrates the impact of variations in the aggregate benchmarking demand. It plots the investment in growth opportunities, I_n (Panel A) and price informativeness, PI_n (Panel B), as functions of the aggregate benchmarking demand, $\theta_n^{\mathcal{B}}$. Panel C depicts aggregate price informativeness as a function of the share of benchmarked investors $\Gamma^{\mathcal{B}}$ for the case in which benchmarked investors displace uninformed investors ($\Gamma^{\mathcal{I}}$ fixed) and the case in which benchmarked investors displace informed investors ($\Gamma^{\mathcal{U}}$ fixed). The graphs are based on the following parameter values: $R_f = 1$, $\rho = 2$, $\sigma_Z = 0.25$, $\mu_A = 1.4$, $\sigma_A = 0.1$, $\Gamma^{\mathcal{P}} = 0.25$, $\Gamma^{\mathcal{I}} = 0.5$, $\Gamma^{\mathcal{U}} = 0.25$, $c = 1/4$, $\mu = 1$, and a quadratic information-cost function: $\kappa(q) = 0.004\, q^2$.

6 Empirical Implications

Our model has implications for investors, financial markets, and firms. In particular, it delivers a variety of novel predictions regarding the informativeness of stock prices, asset prices and returns, as well as corporate decisions—in the cross-section but also in the time-series. In the following, we summarize the key predictions of our model, discuss existing and new empirical evidence that lends support to these predictions, and outline avenues of future research on the topic.

6.1 Price Informativeness

One of our central cross-sectional predictions is that, perhaps counter-intuitively, firms with high passive ownership should have more informative stock prices (see Theorem 5 and Panel B of Figure 3). This prediction is unique to our framework and would not obtain in traditional frameworks à la Verrecchia (1982); indeed, it is arising from firms' responses to changes in their ownership structure—the distinct feature of our framework.

While a comprehensive test of this prediction is outside the scope of this paper, a simple explanatory empirical analysis provides some initial support for this claim. Using the data set prepared by Kacperczyk, Sundaresan, and Wang (2020), we find that U.S. firms with high shares of passive owners tend to have more informative stock prices.[27] In particular, when sorting firm-year observations into quintiles by passive ownership and measuring price informativeness separately for each "bucket," a strong positive relation between passive ownership and the informativeness of stock prices obtains, as illustrated in Figure 9.

Similar results have been discussed in the literature. For example, Bai, Philippon, and Savov (2016) document that firms with high shares of institutional investors have more informative stock prices (compared to otherwise identical firms with low share of institutional owners). Note that while the authors do not distinguish between passive and active institutional investors, Theorem 11 highlights that our prediction should also apply for benchmarked institutional investors. Likewise, Farboodi, Matray, Veldkamp, and Venkateswaran (2019) document that the stock prices of large firms, which naturally also tend to have more passive owners, are more informative.[28] In fact, both their evidence and our prediction are more subtle. That is, Farboodi, Matray, Veldkamp, and Venkateswaran (2019) find that large growth firms have particularly informative stock prices—a result that endogenously arises in our model because firms with high passive ownership have more informative prices and invest more in growth opportunities. Dávila and Parlatore (2019) also provide empirical evidence that relative price informativeness increases with market

[27] While Kacperczyk, Sundaresan, and Wang (2020) study the impact of foreign institutional investors on price informativeness and, hence, rely on firms from 40 different countries, we focus on U.S. firms only; in order to eliminate the impact of cross-country differences. In total, our sample includes more than $5,000$ publicly traded firms and $42,701$ firm-year observations for a period from 2000 to 2016 (during which passive ownership became a prevalent feature of financial markets). See Kacperczyk, Sundaresan, and Wang (2020) for a detailed discussion of the data.

[28] Interestingly, our explanatory analysis suggests that—even after controlling for firm size and general institutional ownership—firms with higher shares of passive ownership tend to have more informative prices—for all levels of firm size (small/medium/big) and all levels of institutional holdings (low/medium/high).

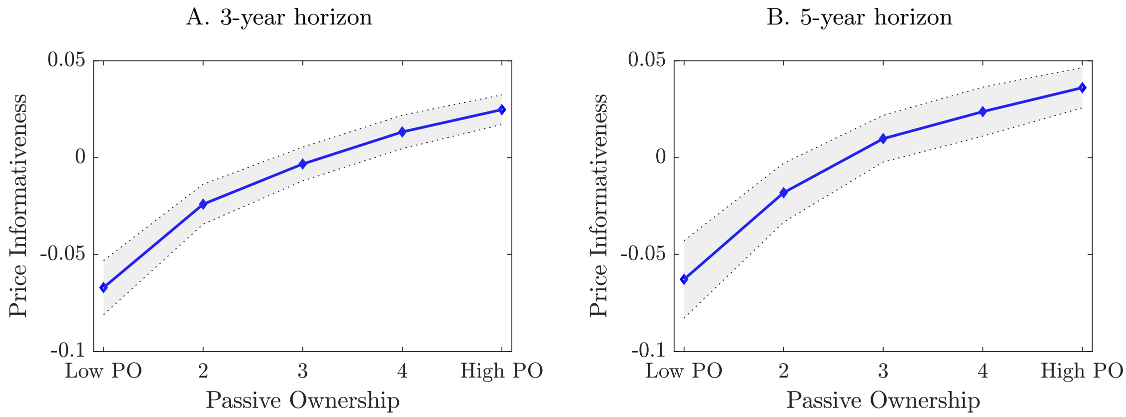

Figure 9: Price informativeness versus passive ownership. The figure depicts price informativeness for quintiles of passive ownership for U.S. publicly traded firms. Panels A and B report the results for 3- and 5-year forecasting horizons, respectively. The analysis is based on the data set prepared by Kacperczyk, Sundaresan, and Wang (2020). Passive ownership is measured, using data from FactSet, as the fraction of shares outstanding held by index funds, ETFs, and quasi-indexers (following the classifications of Bushee and Noe 2001). Passive ownership ranges from below 5% in the first quintile ("Low PO") to about 35% for the last quintile ("High PO"). Price informativeness is measured using the proxy developed by Bai, Philippon, and Savov (2016) that captures the extent to which firms' current stock prices reflect their future cash flows. Specifically, within each passive-ownership quintile, we estimate (pooled OLS) regressions of future earnings on today's market prices and a set of standard controls. Price informativeness is then calculated as the coefficient estimate on today's prices multiplied by the cross-sectional standard deviation of stock prices. Confidence intervals are based on the standard error on the coefficient; scaled by the cross-sectional standard deviation.

capitalization. Last, using index inclusion in MSCI indices, Kacperczyk, Sundaresan, and Wang (2020) show that passive investors increase price informativeness (though less than active investors).

We do not go beyond this brief examination of the data here and, clearly, do not attempt to establish causality. There might be other (hidden) variation that explains the higher price informativeness for firms with high shares of passive owners. To examine the prediction more formally, one could make use of the Russell 1000/2000 cut-off; exploiting that passive ownership is higher for large firms in the Russell 2000 than for small firms in the Russell 1000.[29] Initial supporting empirical evidence in this regard is provided by Glosten, Nallareddy, and Zou (2020). Employing a difference-in-difference approach to Russell 1000/2000 index reconstitutions, the authors find that the "change in short-run informational efficiency for firms moving from the Russell 1000 to the Russell 2000 is positive

[29] Using index additions and deletions as an exogenous shock is another possibility; however, this seems more challenging. First, measuring price informativeness on a stock level is complicated (though, recently, Dávila and Parlatore 2019 have developed a novel proxy). Second, index inclusion brings more analyst coverage and (media) attention; making it complicated to identify the impact of investors' information production.

and significantly greater than the change in short-run informational efficiency for firms moving from the Russell 2000 to the Russell 1000 index." Related, Huang, O'Hara, and Zhong (2018) argue that industry-ETFs reduce the post-earnings-announcement drift, suggesting an improvement in market efficiency.

Finally, by documenting a strong positive correlation between information demand and uncertainty, Benamar, Foucault, and Vega (2020) provide direct empirical support for the economic mechanism that generates higher price informativeness for stock with high passive ownership in our model. In particular, they study investors' demand for information about macroeconomic factors ahead of influential economic announcements and find a substantial increase in demand.

In the time-series, our model unambiguously predicts a widening gap in price informativeness across firms as the share of passive investors increases. This is consistent with the finding in Bai, Philippon, and Savov (2016) that "the gap in informativeness has expanded just as the gap in institutional share has grown." Related empirical evidence is report by Farboodi, Matray, Veldkamp, and Venkateswaran (2019) who argue that "the past few decades have been marked by diverging trends in informativeness," with price informativeness rising (falling) for large (small) firms and further diverging for large-growth firms (relative to large-value firms). Our predictions regarding the aggregate impact of passive ownership critically depend on the type of investors that is crowded out. If uninformed investors are displaced by passive owners, our theory predicts an improvement in aggregate price informativeness, consistent with the increase in the informativeness of the stock prices of firms in the S&P500, as documented in Bai, Philippon, and Savov (2016).[30]

6.2 Stock Prices and Returns

Our model also generates a rich set of cross-sectional asset pricing predictions. First, our model implies a strong index effect. That is, it predicts that firms that are added to an index and, hence, see their share of passive owners increase, should have persistently higher stock prices (see Theorem 6). While index effects can already arise in models without asymmetric

[30] Appel, Gormley, and Keim (2016) as well as Kacperczyk, Sundaresan, and Wang (2020) examine, among other things, how a firm's ownership structure changes following index additions. Interestingly, their results seem to indicate that passive investors crowd out uninformed retail investors. Specifically, they document a concomitant increase in the share of passive and active investors.

information (see, e.g., Cuoco and Kaniel 2011 and Basak and Pavlova 2013), our prediction is a bit more subtle in that the index effect is asymmetric (Panel A of Figure 4), with the price increase for index additions being stronger (than the decline for deletions). Second, our model predicts more volatile stock returns for firms with higher shares of passive owners (see Theorem 7). Third, we predict that firms with high passive ownership should usually have higher expected excess returns.

An extensive literature, starting with the work by Harris and Gurel (1986) and Shleifer (1986), lends support to the prediction of an index effect. Notably, consistent with our prediction, Chen, Noronha, and Singal (2004) document an asymmetric price response (though no permanent decline for deleted firms). Exploiting exogenous changes in index membership, Ben-David, Franzoni, and Moussawi (2018) find empirically that stocks with higher passive ownership display significantly higher return volatility and that "stocks with high ETF ownership earn a significant risk premium of up to 56 basis points monthly." There is also a large literature documenting that institutional ownership, in general, increases return volatility (see, e.g., Bushee and Noe 2000 and Sias 1996) which further supports our predictions.

Another asset-pricing prediction of the model is that the dispersion in firms' stock prices and returns should go up as the aggregate size of passive ownership increases—a testable implication that future research might want to explore.

Finally, the model also delivers predictions regarding the profits and decisions of informed, active investors (e.g., fund managers). For example, a testable prediction of the model is that, all else equal, fund managers that concentrate on trading in stocks with large passive ownership should be able to earn higher profits (Theorem 8). Moreover, an exogenous shock through which uninformed investors get displaced by passive owners should attract new active investors as their utility compared to that of uninformed investors increases.

6.3 Corporate Policies

Some of our most interesting and novel predictions relate to firms' financing costs, corporate decisions, and risk taking. In particular, according to the model, stocks with high shares of passive owners (or, equivalently, large shares of benchmarked institutional investors) should have a lower cost of capital, invest more in risky growth opportunities, and raise more (external) capital. While important in their own right, these predictions are also essential to our economic mechanism.

A key challenge for empirical work on these topics is that one has to control for many (hidden) factors that jointly affect the ownership structure and firms' investment and financing decisions; naturally giving rise to many endogeneity (selection) concerns. Moreover, firms' cost of capital are notoriously hard to estimate. Despite these difficulties, some paper have examined these links and, indeed, provide some evidence in favour of our predictions. For example, Massa, Peyer, and Tong (2005) use a firm's addition to the S&P500 as an instrument for (a decline in) its cost of capital. The authors find that, consistent with our predictions, index inclusion is associated with higher levels of corporate investment and equity issuance. Related, Bena, Ferreira, Matos, and Pires (2017) carefully analyze the impact of institutional ownership on corporate investment and employment for firms in 30 countries. They document a strong positive association between institutional ownership and firm-level capital expenditures and hiring (using index additions as an exogenous shock to institutional ownership).

Notably, Bena, Ferreira, Matos, and Pires (2017) also show that R&D development increases significantly after index inclusion. Corroborating empirical evidence is provided by Harford, Kecskés, and Mansi (2018) who report that a larger share of by passive ownership leads to more R&D investment. Thus, both papers provide initial suggestive evidence regarding enhanced risk taking of firms with large shares of (passive) institutional owners; as predicted by our framework.[31]

7 Conclusion

Passive investing is a cornerstone of financial markets today, with passive owners holding, on average, about 20% of U.S. publicly traded companies' shares outstanding. Recently, however, market participants and policy makers have raised concerns regarding the impact of passive ownership on financial markets and corporate decisions.

[31] A brief examination of the dataset prepared by Kacperczyk, Sundaresan, and Wang (2020) also suggests that firms with high passive ownership exhibit higher R&D expenditures in the future.

In this paper, we study the impact of passive ownership on real investment policies, price informativeness, and stock prices. For that purpose, we develop a novel economic framework in which firms' real investment decisions, investors' portfolios, and information choices as well as stock prices are jointly determined in equilibrium. Three key features shape the model: (1) cross-sectional and time-series variations in passive ownership, (2) firms taking into account the ownership structure in financial markets when making investment decisions, and (3) active investors optimally choosing the precision of their private information.

We demonstrate that price informativeness might correlate positively with passive ownership in the cross-section. In particular, passive ownership lowers the cost of capital and, hence, encourages firms to allocate more capital to risky growth opportunities. This effect, in turn, induces informed investors to devote more resources to information acquisition, thereby increasing the informational content of the stock price. Firms with high passive ownership also have higher stock prices, more volatile stock returns and usually higher expected excess returns.

An increase in aggregate passive ownership (i.e., the share of passive investors), in general, leads to stronger dispersion in real investment policies, stock prices and returns, and price informativeness across firms. Interestingly, if passive investors crowd out uninformed investors, price informativeness increases—in aggregate. Instead, if informed investors are displaced, information aggregation deteriorates and, hence, price informativeness declines, both in aggregate and for a large majority of firms.

Similar to how the feedback literature has highlighted the impact of learning from stock prices on corporate decisions, our work highlights the impact of the ownership structure in financial markets on these decisions. Indeed, a variety of natural extensions of our framework should deliver further insights. For example, one could explicitly model the underlying investment problem of retail investors; letting them choose between investing in passive and active investment funds (and, potentially, the respective contracts). Also, incorporating a feedback mechanism into our framework could lead to interesting interactions between managers' learning and the ownership structure. Moreover, while the existing literature provides (suggestive) empirical evidence for a variety of the testable predictions of our model, a broader empirical analysis of the impact of passive ownership is clearly warranted;

in particular, given the uninterrupted growth of passive investing. We view our framework as a benchmark to guide such empirical analyses.

Appendix

All derivations are provided for the general case of $R_f \neq 1$ and a non-zero-mean noise-trader demand, $Z_n \sim \mathcal{N}\left(\mu_Z, \sigma_Z^2\right)$.

Proofs for Section 2

Theorem 1

In the following, we take firms' real-investment choices, $\{I_n\}$ (or, equivalently, μ_n, σ_n^2, and θ_n) as well as informed investors' information choices, $\{q_{i,n}\}$, $i \in \mathcal{I}$, as given.

We conjecture (and later verify) that each stock's price is a linear function of its payoff, X_n, and noise traders' demand, Z_n:

$$\theta_n \, P_n R_f = a_n + b_n \, X_n + d_n \, Z_n. \tag{A1}$$

Each informed investor, $i \in \mathcal{I}$, receives two sets of (unbiased) signals. First, her private signals $Y_{i,n} = X_n + \varepsilon_{i,n}$; with precision $q_{i,n}$.[32] Second, the public price signals $\frac{\theta_n P_n R_f - a_n - d_n \mu_Z}{b_n} = X_n + \frac{d_n}{b_n}\left(Z_n - \mu_Z\right)$; with precision $\frac{b_n^2}{d_n^2 \, \sigma_Z^2}$. Hence, given her informed set $\mathcal{F}_i = \{\{Y_{i,n}\}, \{P_n\}\}$, her posterior beliefs are given by:

$$\hat{\mu}_{i,n}^{\mathcal{I}} \equiv \mathbb{E}\left[X_n \,|\, \mathcal{F}_i\right] = \frac{1}{h_{i,n}}\left(\frac{\mu_n}{\sigma_n^2} + q_{i,n} Y_{i,n} + \frac{b_n^2}{d_n^2 \, \sigma_Z^2} \frac{\theta_n P_n R_f - a_n - d_n \mu_Z}{b_n}\right); \tag{A2}$$

$$h_{i,n}^{\mathcal{I}} \equiv \mathbb{V}\text{ar}\left(X_n \,|\, \mathcal{F}_i\right) = \frac{1}{\sigma_n^2} + q_{i,n} + \frac{b_n^2}{d_n^2 \, \sigma_Z^2}. \tag{A3}$$

Accordingly, the posterior mean and precision of the beliefs of an uninformed investor, $\hat{\mu}_{i,n}^{\mathcal{U}}$ and $h_{i,n}^{\mathcal{U}}$, $i \in \mathcal{U}$, respectively, are given by (A2) and (A3) with $q_{i,n} = 0$.

With normally distributed prices and signals, active investors' beliefs ($i \in \{\mathcal{I}, \mathcal{U}\}$) are also normally distributed. Hence, informed investors' optimal stock demand, $\theta_{i,n}^{\mathcal{I}}$, is given by the standard mean-variance demand (adjusted for the total number of shares outstanding,

[32] Alternatively, one can assume that informed investors receive a signal $Y_{i,n}$ about A_n: $Y_{i,n} = A_n + \varepsilon_{i,n}$. Such a signal can be written as an unbiased signal about X_n as $I_n Y_{i,n} - \frac{c}{2} I_n^2 = X_n + I_n \varepsilon_{i,n}$, with precision $\hat{q}_{i,n} = q_{i,n}/I_n^2$. Our results remain qualitatively unchanged in this case, as an increase in I_n implies an increase in the marginal benefits of private information; in fact, all else equal, this specification strengthens the mechanism quantitatively.

θ_n) (9) which, using (A2) and (A3), can be written as:

$$\begin{aligned}\theta^{\mathcal{I}}_{i,n} &= \theta_n \frac{h^{\mathcal{I}}_{i,n}}{\rho} \frac{1}{h^{\mathcal{I}}_{i,n}} \left(\frac{\mu_n}{\sigma^2_n} + q_{i,n} Y_{i,n} + \frac{b^2_n}{d^2_n\, \sigma^2_Z} \frac{\theta_n P_n R_f - a_n - d_n \mu_Z}{b_n} \right) - \frac{h^{\mathcal{I}}_{i,n}}{\rho} \theta_n\, P_n R_f \\ &= \theta_n \frac{1}{\rho} \left(\frac{\mu_n}{\sigma^2_n} + q_{i,n} Y_{i,n} - \frac{b_n (a_n + d_n \mu_Z)}{d^2_n\, \sigma^2_Z} - \theta_n\, P_n R_f \underbrace{\left(h^{\mathcal{I}}_{i,n} - \frac{b_n}{d^2_n\, \sigma^2_Z} \right)}_{= \frac{1}{\sigma^2_n} + q_{i,n} + \frac{b_n (b_n - 1)}{d^2_n\, \sigma^2_Z}} \right).\end{aligned}$$

The optimal demand of uninformed investors ($i \in \mathcal{U}$), $\theta^{\mathcal{U}}_{i,n}$, follows accordingly by setting $q_{i,n} = 0$.

Market clearing requires that aggregate demand equals aggregate supply:

$$\theta_n\, \theta^{\mathcal{P}}_n + \int^{\mathcal{I}} \theta^{\mathcal{I}}_{i,n}\, di + \int^{\mathcal{U}} \theta^{\mathcal{U}}_{i,n}\, di + \theta_n\, Z_n = \theta_n.$$

Plugging in the informed and uninformed investors' demand $\theta^{\mathcal{I}}_{i,n}$ and $\theta^{\mathcal{U}}_{i,n}$, substituting the private signal $Y_i = X_n + \varepsilon_{i,n}$ (with $\int \varepsilon_{i,n}\, di = 0$), and using the definitions (7) as well as (8), yields:

$$\begin{aligned}\theta^{\mathcal{P}}_n &+ \frac{1 - \Gamma^{\mathcal{P}}}{\rho} \left(\frac{\mu_n}{\sigma^2_n} - \frac{b_n (a_n + d_n \mu_Z)}{d^2_n\, \sigma^2_Z} \right) + \frac{1}{\rho} \underbrace{\int^{\mathcal{I}} q_{i,n} \left(X_n + \varepsilon_{i,n} \right) di}_{\Gamma^{\mathcal{I}} X_n \bar{q}_n} \\ &- \theta_n\, P_n R_f \frac{1 - \Gamma^{\mathcal{P}}}{\rho} \left(\frac{1}{\sigma^2_n} + \frac{\Gamma^{\mathcal{I}}}{1 - \Gamma^{\mathcal{P}}} \bar{q}_n + \frac{b_n (b_n - 1)}{d^2_n\, \sigma^2_Z} \right) + Z_n = 1,\end{aligned}$$

which can be solved for $\theta_n\, P_n R_f$:

$$\begin{aligned}\theta_n\, P_n R_f = &\left(\frac{1}{\sigma^2_n} + \frac{\Gamma^{\mathcal{I}}}{1 - \Gamma^{\mathcal{P}}} \bar{q}_n + \frac{b_n (b_n - 1)}{d^2_n\, \sigma^2_Z} \right)^{-1} \times \\ &\left(\frac{\rho}{1 - \Gamma^{\mathcal{P}}} \left(Z_n - 1 + \theta^{\mathcal{P}}_n \right) + \frac{\Gamma^{\mathcal{I}}}{1 - \Gamma^{\mathcal{P}}} \bar{q}_n X_n + \frac{\mu_n}{\sigma^2_n} - \frac{b_n (a_n + d_n \mu_Z)}{d^2_n\, \sigma^2_Z} \right).\end{aligned} \tag{A4}$$

Consequently, the signal-to-noise ratio is given by $b_n/d_n = \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho}$ which can be used, together with definitions (7), to simplify the multiplicative factor in (A4):

$$\frac{1}{\sigma_n^2} + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\,\bar{q}_n + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}^2}{\rho^2\,\sigma_Z^2} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,d_n\,\sigma_Z^2} = \bar{h}_n - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,d_n\,\sigma_Z^2} = \frac{\bar{h}_n\,\rho\,d_n\,\sigma_Z^2 - \Gamma^{\mathcal{I}}\bar{q}_n}{\rho d_n\,\sigma_Z^2}.$$

Matching the coefficients of (A1) to those of (A4) and evoking definitions (7), yields:

$$d_n = \frac{\rho d_n\,\sigma_Z^2}{\bar{h}_n\,\rho\,d_n\,\sigma_Z^2 - \Gamma^{\mathcal{I}}\,\bar{q}_n}\,\frac{\rho}{1-\Gamma^{\mathcal{P}}} \iff \frac{\rho^2\,\sigma_Z^2}{1-\Gamma^{\mathcal{P}}} + \Gamma^{\mathcal{I}}\,\bar{q}_n = \bar{h}_n\,\rho\,\sigma_Z^2\,d_n$$

$$\iff d_n = \frac{1}{\bar{h}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\right)$$

and, hence,

$$\frac{b_n}{d_n} = \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho} \iff b_n = \frac{1}{\bar{h}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\right)\frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho}$$

$$\iff b_n = \frac{1}{\bar{h}_n}\left(\frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\,\bar{q}_n + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2}\right) = \frac{1}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right).$$

Hence, the constant, a_n, is given by:

$$a_n = \frac{\rho d_n\,\sigma_Z^2}{\bar{h}_n\,\rho\,d_n\,\sigma_Z^2 - \Gamma^{\mathcal{I}}\,\bar{q}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}}\left(\theta_n^{\mathcal{P}} - 1\right) + \frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho}\,\frac{a_n + d_n\,\mu_Z}{d_n\,\sigma_Z^2}\right)$$

$$\iff a_n \frac{\bar{h}_n\,\rho\,\sigma_Z^2\,d_n}{\bar{h}_n\,\rho\,d_n\,\sigma_Z^2 - \Gamma^{\mathcal{I}}\bar{q}_n} = \frac{\rho d_n \sigma_Z^2}{\bar{h}_n \rho d_n \sigma_Z^2 - \Gamma^{\mathcal{I}}\bar{q}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}}\left(\theta_n^{\mathcal{P}} - 1\right) + \frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho}\,\frac{\mu_Z}{\sigma_Z^2}\right)$$

$$\iff a_n = \frac{1}{\bar{h}_n}\left(\underbrace{\frac{\rho}{1-\Gamma^{\mathcal{P}}}\left(\theta_n^{\mathcal{P}} - 1\right)}_{=-\rho\,\bar{\theta}_n^{\mathcal{A}}} + \frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho}\,\frac{\mu_Z}{\sigma_Z^2}\right).$$

Plugging the coefficients a_n, b_n, and d_n into the conjectured price function (A1) recovers (6) (with $\mu_Z = 0$ and $R_f = 1$).

Lemma 1

The precision of the public price signal ("price informativeness") is given by:

$$PI_n \equiv \frac{b_n^2}{d_n^2\,\sigma_Z^2} = \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2} = h_{0,n} - \frac{1}{\sigma_n^2}.$$

It is unaffected by variations in $\bar{\theta}_n^{\mathcal{P}}$: $dPI_n/d\bar{\theta}_n^{\mathcal{P}} = 0$.

Theorem 2

In the following, we take firms' real-investment choices, $\{I_n\}$ (or, equivalently, μ_n, σ_n^2, and θ_n) as given.

In the information-choice period, the optimization problem of an informed investor is given by (4), or, equivalently:

$$\max_{\{q_{i,n}\}} \mathbb{E}\left[-\exp\left(-\rho\left(\mathbb{E}[W_i^* \,|\, \mathcal{F}_i] - \frac{\rho}{2}\mathbb{V}\mathrm{ar}\left(W_i^* \,|\, \mathcal{F}_i\right)\right)\right)\right], \tag{A5}$$

where W_i^* denotes terminal wealth (3), taking into account the *optimal* period-3 portfolio choice (9):

$$W_i^* = W_{0,i}\, R_f + \sum_{n=1}^{N} \theta_n\, h_{i,n}^{\mathcal{I}} \frac{\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n\, R_f}{\rho} \left(\frac{X_n}{\theta_n} - PR_f\right) - \sum_{n=1}^{N} \kappa(q_{i,n}).$$

In particular, the time-3 expectation and variance of W_i^* are given by:

$$\begin{aligned}
\mathbb{E}[W_i^* \,|\, \mathcal{F}_i] &= W_{0,i}\, R_f - \sum_{n=1}^{N}\kappa(q_{i,n}) + \sum_{n=1}^{N} h_{i,n}^{\mathcal{I}} \frac{\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n\, R_f}{\rho}\, \mathbb{E}\left[X_n - \theta_n\, PR_f \,|\, \mathcal{F}_i\right] \\
&= W_{0,i}\, R_f - \sum_{n=1}^{N}\kappa(q_{i,n}) + \sum_{n=1}^{N} h_{i,n}^{\mathcal{I}} \frac{(\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n\, R_f)^2}{\rho} = W_{0,i}\, R_f - \sum_{n=1}^{N}\kappa(q_{i,n}) + \sum_{n=1}^{N} \frac{z_{i,n}^2}{\rho}; \\
\mathbb{V}\mathrm{ar}(W_i^* \,|\, \mathcal{F}_i) &= \sum_{n=1}^{N} \left(h_{i,n}^{\mathcal{I}}\right)^2 \frac{(\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n\, R_f)^2}{\rho^2} \frac{1}{h_{i,n}} = \sum_{n=1}^{N} \frac{z_{i,n}^2}{\rho^2};
\end{aligned}$$

where $z_{i,n} \equiv \sqrt{h_{i,n}^{\mathcal{I}}}\,(\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n\, R_f)$ denotes an informed investor's time-3 expected Sharpe ratio of trading stock n.

Consequently, the optimization problem (A5) can be written as:

$$\max_{\{q_{i,n}\}} -\exp\left(-\rho\left(W_{0,i}\, R_f - \sum_{n=1}^{N}\kappa(q_{i,n})\right)\right)\mathbb{E}\left[\exp\left(-\frac{1}{2}\sum_{n=1}^{N} z_{i,n}^2\right)\right], \tag{A6}$$

where the last term governs the expectation of the exponential of a squared-normal variable. To compute this expectation, we can use Brunnermeier (2001, page 64):

$$\mathbb{E}[\exp(\omega^T A\,\omega + b^T\,\omega + d)] = |I_N - 2\,\Sigma\,A|^{-\frac{1}{2}} \exp\left(\frac{1}{2} b^T\ (I - 2\,\Sigma\,A)^{-1}\ \Sigma\,b + d\right)$$

for $\omega \sim \mathcal{N}(0,\Sigma)$. Note, however, that $\{z_{i,n}\}$ are not mean-zero random variables; only $\{z_{i,n} - \mathbb{E}[z_{i,n}]\}$ have a mean of zero. So, we first need to expand and rewrite:

$$\sum_{n=1}^{N} z_{i,n}^2 = -\frac{1}{2}\left((z_i - \mathbb{E}[z_i])^T\, I_N\, (z_i - \mathbb{E}[z_i]) + 2\mathbb{E}[z_i]^T\, (z_i - \mathbb{E}[z_i]) + \sum_{n=1}^{N} \mathbb{E}[z_{i,n}]^2\right),$$

where $z_i \equiv [z_{i,1} \ldots z_{i,N}]^T$ denotes an $N \times 1$ vector of the stocks' Sharpe ratios. In particular, in the notation of Brunnermeier, we have $\omega = z_i - \mathbb{E}[z_i] \sim \mathcal{N}(0,\Sigma)$ with Σ denoting a diagonal matrix of variances $\mathbb{V}\mathrm{ar}(z_{i,t})$ and the coefficients are given by $A = -\frac{1}{2} I_N$, $b = -\mathbb{E}[z_i]$ and $d = -\frac{1}{2}\sum_{n=1}^{N} \mathbb{E}[z_{i,n}]^2$.

Hence, the expectation of the exponential of the squared Sharpe ratio is given by:

$$\begin{aligned}
\mathbb{E}\left[\exp\left(-\frac{1}{2}\sum_{n=1}^{N} z_{i,n}^2\right)\right] &= \underbrace{|I_N + \Sigma|^{-\frac{1}{2}}}_{=\left(\prod_{n=1}^{N}(1+\mathbb{V}\mathrm{ar}(z_{i,n})\right)^{-\frac{1}{2}}} \\
&\quad \times \exp\left(\frac{1}{2}(-\mathbb{E}[z_i])^T \underbrace{(I+\Sigma)^{-1}\,\Sigma}_{\left[\frac{\mathbb{V}\mathrm{ar}(z_{i,n})}{1+\mathbb{V}\mathrm{ar}(z_{i,n})}\right]_{\mathrm{Diag}(N\times N)}} (-\mathbb{E}[z_i]) - \frac{1}{2}\sum_{n=1}^{N}\mathbb{E}[z_{i,n}]^2\right) \\
&= \left(\prod_{n=1}^{N}(1+\mathbb{V}\mathrm{ar}(z_{i,n})\right)^{-\frac{1}{2}} \exp\left(\frac{1}{2}\sum_{n=1}^{N}\frac{\mathbb{V}\mathrm{ar}(z_{i,n})\mathbb{E}[z_{i,n}]^2}{1+\mathbb{V}\mathrm{ar}(z_{i,n})} - \frac{1}{2}\sum_{n=1}^{N}\mathbb{E}[z_{i,n}]^2\right) \\
&= \prod_{n=1}^{N}\left\{(1+\mathbb{V}\mathrm{ar}(z_{i,n}))\exp\left(\frac{\mathbb{E}[z_{i,n}]^2}{1+\mathbb{V}\mathrm{ar}(z_{i,n})}\right)\right\}^{-\frac{1}{2}}, \qquad \text{(A7)}
\end{aligned}$$

which is based on the time-2 expectation and variance of $z_{i,n}$: $\mathbb{E}[z_{i,n}]$ and $\mathbb{V}\mathrm{ar}(z_{i,n})$. To compute this expectation and variance, define $u_{i,n} \equiv \sqrt{h_{i,n}^{\mathcal{I}}}\, z_{i,n} = h_{i,n}^{\mathcal{I}}\,(\hat{\mu}_{i,n}^{\mathcal{I}} - \theta_n\, P_n R_f)$.

The time-2 expectation of $u_{i,n}$ can easily be computed as:

$$\mathbb{E}[u_{i,n}] = h^{\mathcal{I}}_{i,n}\left(\mu_n - \mathbb{E}[\theta_n\, P_n R_f]\right) = h^{\mathcal{I}}_{i,n}\left(\mu_n - \mu_n - \frac{\rho}{\bar{h}_n}\left(\frac{\mu_Z}{1-\Gamma^{\mathcal{P}}} - \bar{\theta}^{\mathcal{A}}_n\right)\right)$$

$$= \frac{h^{\mathcal{I}}_{i,n}}{\bar{h}_n}\,\rho\left(\bar{\theta}^{\mathcal{A}}_n - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right). \tag{A8}$$

For the computation of its variance, we first have to derive $u_{i,n}$. After replacing the realized signals by their time-2 counterparts ($Y_{i,n} = X_n + \varepsilon_{i,n}$ and $\frac{\theta_n P_n R_f - a_n - d_n \mu_Z}{b_n} = X_n + \frac{d_n}{b_n}\left(Z_n - \mu_Z\right)$), one can write the posterior mean (A2) as:

$$\hat{\mu}^{\mathcal{I}}_{i,n} = \frac{1}{h^{\mathcal{I}}_{i,n}}\left(\frac{\mu_n}{\sigma_n^2} + q_{i,n}(X_n + \varepsilon_{i,n}) + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2}\left(X_n + \frac{\rho}{\Gamma^{\mathcal{I}}\,\bar{q}_n}\left(Z_n - \mu_Z\right)\right)\right)$$

$$= \frac{1}{h^{\mathcal{I}}_{i,n}}\left\{\left(\frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\mu_Z\right) + \underbrace{\left(q_{i,n} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2}\right)}_{=h^{\mathcal{I}}_{i,n} - \frac{1}{\sigma_n^2}} X_n + q_{i,n}\,\varepsilon_{i,n} + \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\, Z_n\right\}.$$

Substituting this expression and the equilibrium price (6) into $u_{i,n}$ yields:

$$u_{i,n} = \left\{\left(\frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\mu_Z\right) + \left(h^{\mathcal{I}}_{i,n} - \frac{1}{\sigma_n^2}\right) X_n + q_{i,n}\,\varepsilon_{i,n} + \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\, Z_n\right\}$$

$$- h^{\mathcal{I}}_{i,n}\left\{\frac{1}{\bar{h}_n}\left(\frac{\mu_n}{\sigma_n^2} - \rho\bar{\theta}^{\mathcal{A}}_n - \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho}\frac{\mu_Z}{\sigma_Z^2}\right) + \frac{1}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right) X_n + \frac{1}{\bar{h}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho\sigma_Z^2}\right) Z_n\right\}$$

$$= \left(1 - \frac{h^{\mathcal{I}}_{i,n}}{\bar{h}_n}\right)\left(\frac{\mu_n}{\sigma_n^2} - \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho}\frac{\mu_Z}{\sigma_Z^2}\right) + \frac{h^{\mathcal{I}}_{i,n}}{\bar{h}_n}\rho\,\bar{\theta}^{\mathcal{A}}_n + q_{i,n}\,\varepsilon_{i,n} +$$

$$X_n\frac{1}{\sigma_n^2}\left(-1 + h^{\mathcal{I}}_{i,n}\sigma_n^2 - h^{\mathcal{I}}_{i,n}\sigma_n^2 + \frac{h^{\mathcal{I}}_{i,n}}{\bar{h}_n}\right) + Z_n\frac{\rho}{\Gamma^{\mathcal{I}}\,\bar{q}_n}\left(PI_n - \frac{h^{\mathcal{I}}_{i,n}}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right)\right).$$

Hence, the time-2 conditional variance of $u_{i,n}$ is given by:

$$\mathbb{V}\mathrm{ar}(u_{i,n}) = q_{i,n}^2 \frac{1}{q_{i,n}} + \frac{1}{\sigma_n^4} \sigma_n^2 \left(\frac{h_{i,n}^{\mathcal{I}}}{\bar{h}_n} - 1 \right)^2 + \sigma_Z^2 \frac{\rho^2}{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2} \left(h_{i,n}^{\mathcal{I}} \left(\frac{1}{\bar{h}_n \sigma_n^2} - 1 \right) + PI_n \right)^2$$

$$= h_{i,n}^{\mathcal{I}} + \frac{1}{\sigma_n^2} \frac{\left(h_{i,n}^{\mathcal{I}}\right)^2}{\bar{h}_n^2} + \frac{1}{PI_n} \left(h_{i,n}^{\mathcal{I}}\right)^2 \left(\frac{1}{\bar{h}_n \sigma_n^2} - 1 \right)^2 - 2\, h_{i,n}^{\mathcal{I}}$$

$$= \frac{\left(h_{i,n}^{\mathcal{I}}\right)^2}{\bar{h}_n^2} \left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}\, \bar{q}_n}{1-\Gamma^{\mathcal{P}}} + \frac{\rho^2 \sigma_Z^2}{\left(1-\Gamma^{\mathcal{P}}\right)^2} \right) - h_{i,n}^{\mathcal{I}}, \tag{A9}$$

where we used in the last step:

$$\frac{1}{\sigma_n^2} + \frac{\bar{h}_n^2}{PI_n} \left(\frac{1}{\bar{h}_n \sigma_n^2} - 1 \right)^2 = \frac{1}{\sigma_n^2} + \frac{1}{PI_n} \left(\frac{1}{\sigma_n^2} - \bar{h}_n \right)^2 = \frac{1}{\sigma_n^2} + \frac{1}{PI_n} \left(- \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}} \bar{q}_n - PI_n \right)^2$$

$$= \frac{1}{\sigma_n^2} + \left(\frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\left(1-\Gamma^{\mathcal{P}}\right)^2 PI_n} + 2 \frac{\Gamma^{\mathcal{I}}\, \bar{q}_n}{1-\Gamma^{\mathcal{P}}} + PI_n \right) = \bar{h}_n + \frac{\Gamma^{\mathcal{I}}\, \bar{q}_n}{1-\Gamma^{\mathcal{P}}} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\left(1-\Gamma^{\mathcal{I}}\right)^2 PI_n}.$$

Consequently, the expectation, the variance, and the ratio of the squared expectation and the variance of $z_{i,n}$ are given by:

$$\mathbb{E}[z_{i,n}] = \frac{1}{\sqrt{h_{i,n}^{\mathcal{I}}}} \frac{h_{i,n}^{\mathcal{I}}}{\bar{h}_n} \rho \left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}} \right); \tag{A10}$$

$$\mathbb{V}\mathrm{ar}(z_{i,n}) = \frac{1}{\bar{h}_n^2} \left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}\, \bar{q}_n}{1-\Gamma^{\mathcal{P}}} + \frac{\rho^2 \sigma_Z^2}{\left(1-\Gamma^{\mathcal{P}}\right)^2} \right) h_{i,n}^{\mathcal{I}} - 1 \equiv A_{1,n}\, h_{i,n}^{\mathcal{I}} - 1; \tag{A11}$$

$$\frac{\mathbb{E}[z_{i,n}]^2}{1+\mathbb{V}\mathrm{ar}(z_{i,n})} = \rho^2 \left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}} \right)^2 \left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}\, \bar{q}_n}{1-\Gamma^{\mathcal{P}}} + \frac{\rho^2 \sigma_Z^2}{\left(1-\Gamma^{\mathcal{P}}\right)^2} \right)^{-1} \equiv A_{2,n}. \tag{A12}$$

Finally, plugging (A7) into the objective function (A6) and substituting (A11) as well as (A12), yields the following objective function:

$$\max_{\{q_{i,n}\}} -\exp\left(-\rho\left(W_{0,i}\,R_f - \sum_{n=1}^{N}\kappa(q_{i,n})\right)\right)\prod_{n=1}^{N}\left(A_{1,n}\,h^{\mathcal{I}}_{i,n}\,\exp\left(A_{2,n}\right)\right)^{-\frac{1}{2}}$$
$$\iff \max_{\{q_{i,n}\}} -\exp\left(\rho\sum_{n=1}^{N}\kappa(q_{i,n})\right)\underbrace{\prod_{n=1}^{N}\left(h_{0,n}+q_{i,n}\right)^{-\frac{1}{2}}}_{\equiv B\equiv B'_n\,h^{\mathcal{I}}_{i,n}}. \tag{A13}$$

Thus, the first-order condition with respect to $q_{i,n}$ is given by:

$$-\exp\left(\rho\sum_{n=1}^{N}\kappa(q_{i,n})\right)\rho\,\kappa'(q_{i,n})\,B+\left(-\exp\left(\rho\sum_{n=1}^{N}\kappa(q_{i,n})\right)\right)B'_n\left(-\frac{1}{2}\right)\left(h^{\mathcal{I}}_{i,n}\right)^{-\frac{3}{2}}=0,$$

such that an informed investor's optimal information choice, given arbitrary signal-precision choices by the other informed investors, is characterized by:

$$2\,\kappa'\left(q_{i,n}\right)=\frac{1}{\rho}\left(\frac{1}{\sigma_n^2}+\frac{\left(1-\Gamma\right)^2\,\bar{q}_n^2}{\rho^2\,\sigma_Z^2}+q_{i,n}\right)^{-1}=\frac{1}{\rho}\,\frac{1}{h_{0,n}+q_{i,n}}. \tag{A14}$$

Intuitively, the marginal benefits of acquiring information shift up as σ_n^2 increases:

$$2\,\kappa''(q_{i,n})\,\frac{dq_{i,n}}{d\sigma_n^2}=\frac{1}{\rho}\,(-1)\left(h_{0,n}+q_{i,n}\right)^{-2}\left(\frac{-1}{\sigma_n^4}+\frac{dq_{i,n}}{d\sigma_n^2}\right)$$
$$\iff \frac{dq_{i,n}}{d\sigma_n^2}\left(2\,\kappa''(q_{i,n})+\frac{1}{\rho}\left(h_{0,n}+q_{i,n}\right)^{-2}\right)=\frac{1}{\rho}\frac{1}{\sigma_n^4}\left(h_{0,n}+q_{i,n}\right)^{-2}\iff\frac{dq_{i,n}}{d\sigma_n^2}>0.$$

In equilibrium, precision choices are *mutual* best response functions (because each investor's information choice $q_{i,n}$ affects $\bar{q}_n$ and, at the same time, $\bar{q}_n$ affects $q_{i,n}$). Hence, in a symmetric equilibrium, average private-signal precision, $\bar{q}_n$, is determined by plugging $\bar{q}_n$ into an investor's best-information-response function (A14). This yields:

$$2\,\kappa'(\bar{q}_n)=\frac{1}{\rho}\frac{1}{h_{0,n}+\bar{q}_n}=\frac{1}{\rho}\frac{1}{\bar{h}^{\mathcal{I}}_n}, \tag{A15}$$

where $\bar{h}_n^{\mathcal{I}} \equiv h_{0,n} + \bar{q}_n$ denotes the average precision of informed investors. This recovers (10).

Note that the left-hand side of (A15) is increasing in $\bar{q}_n$ (because $\kappa''(\bar{q}_n) > 0$) whereas the right-hand side is decreasing in $\bar{q}_n$ $\left(\partial/\partial\bar{q}_n = \frac{-1}{\rho\left(\bar{h}_n^{\mathcal{I}}\right)^2}\left(1 + \frac{2\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n}{\rho^2\,\sigma_Z^2}\right) < 0\right)$. Hence, there exists a unique solution.

Taking the derivative of both sides of (A15) with respect to σ_n^2, yields:

$$2\,\kappa''(\bar{q}_n)\,\frac{d\bar{q}_n}{d\sigma_n^2} = \frac{1}{\rho}\,(-1)\frac{1}{\left(\bar{h}_n^{\mathcal{I}}\right)^2}\left(\frac{1}{\sigma_n^4}(-1) + \frac{d\bar{q}_n}{d\sigma_n^2} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2\,2\,\bar{q}_n}{\rho^2\sigma_Z^2}\frac{d\bar{q}_n}{d\sigma_n^2}\right)$$

$$\iff \frac{d\bar{q}_n}{d\sigma_n^2}\left(2\,\kappa''(\bar{q}_n) + \frac{1}{\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2}\left(1 + \underbrace{\frac{\left(\Gamma^{\mathcal{I}}\right)^2\,2\,\bar{q}_n}{\rho^2\sigma_Z^2}}_{=\frac{2\,P I_n}{\bar{q}_n}}\right)\right) = \frac{1}{\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2}\frac{1}{\sigma_n^4}$$

$$\iff \frac{d\bar{q}_n}{d\sigma_n^2} = \frac{\bar{q}_n}{\bar{q}_n + 2PI_n + 2\,\kappa''(\bar{q}_n)\,\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2\,\bar{q}_n}\,\frac{1}{\sigma_n^4} > 0; \qquad \left(< \frac{1}{\sigma_n^4}\right). \tag{A16}$$

Moreover, because the right-hand side of (10) does not depend on $\bar{\theta}_n^{\mathcal{P}}$, one gets $d\bar{q}_n/d\bar{\theta}_n^{\mathcal{P}} = 0$.

Theorem 3

For the computations of the expected stock price, S_n, note that it holds: $\theta_n\,S_n = \mathbb{E}[\theta_n P_n R_f]$, with $\theta_n = (1 + (I_n/\mathbb{E}[P_n]))$, and, hence, $S_n = \mathbb{E}[\theta_n P_n R_f] - I_n R_f$. Evoking the expectation of (6), we get:

$$S_n = \frac{1}{\bar{h}_n}\left(\frac{\mu_n}{\sigma_n^2} - \rho\bar{\theta}_n^{\mathcal{A}} - \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho}\frac{\mu_Z}{\sigma_Z^2}\right) + \frac{1}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right)\mu_n + \frac{1}{\bar{h}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho\sigma_Z^2}\right)\mu_Z - I_n R_f$$

$$= \mu_n - I_n R_f - \frac{\rho}{\bar{h}_n}\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right),$$

which, after plugging in $\mu_n = \mu_A\,I_n + \frac{c}{2}\,I_n^2$, $\mu_Z = 0$, and $R_f = 1$, yields (12).[33]

The first-order condition of S_n with respect to I_n is given by:

$$\frac{dS_n}{dI_n} = \mu_A - R_f - c\,I_n - \left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\frac{\rho}{\bar{h}_n^2}\,(-1)\,\frac{d\bar{h}_n}{dI_n} = 0, \tag{A17}$$

[33] Note, that the stock price value, S_n, is decreasing in posterior variance, $1/\bar{h}_n$, if and only if $\bar{\theta}_n^{\mathcal{A}} - \mu_Z/(1-\Gamma^{\mathcal{P}}) > 0$. In the following, we will concentrate on this case. This remark is reminiscent of Footnote 16.

with $\frac{d\bar{h}_n}{dI_n} = \frac{d\bar{h}_n}{d\sigma_n^2}\frac{d\sigma_n^2}{dI_n} = 2\sigma_A^2 I_n \frac{d\bar{h}_n}{d\sigma_n^2}$ and, using (A16):

$$\frac{d\bar{h}_n}{d\sigma_n^2} = -\frac{1}{\sigma_n^4}\frac{\bar{q}_n \frac{\Gamma^{\mathcal{U}}}{1-\Gamma^{\mathcal{P}}} + 2\,\kappa''(\bar{q}_n)\,\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2\bar{q}_n}{\bar{q}_n + 2PI_n + 2\,\kappa''(\bar{q}_n)\,\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2\bar{q}_n} < 0. \tag{A18}$$

Accordingly, define the marginal benefits of investing in growth opportunities, MB, and the respective marginal costs, MC, as:[34]

$$MB(I_n) = \mu_A - R_f - c\,I_n; \tag{A19}$$

$$MC\left(I_n, \bar{\theta}_n^{\mathcal{P}}\right) = \left(\frac{1-\theta_n^{\mathcal{P}}}{1-\Gamma^{\mathcal{P}}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\frac{\rho}{\bar{h}_n^2}\,2\sigma_A^2 I_n\left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right). \tag{A20}$$

While marginal benefits are non-negative if and only if $I_n \leq (\mu_A - R_f)/c$, marginal costs are positive for $I_n > 0$. Hence, the optimal allocation of capital to growth opportunities, $I_n \geq 0$, must lie in the closed interval $[0, (\mu_A - R_f)/c]$. Outside that interval marginal costs are positive whereas marginal benefits are negative; ruling out an optimum. Thus, the Bolzano-Weierstrass Extreme Value Theorem guarantees the existence of a global maximum. In particular, the global maximum is either a local maximum or situated at one of the edges of the closed interval.

Note, however, that, for $I_n = 0$, the marginal benefits are positive, whereas the marginal costs are zero. Hence, there exists an $\epsilon > 0$ such that

$$\int_0^{\epsilon}\left(MB(I_n) - MC\left(I_n, \bar{\theta}_n^{\mathcal{P}}\right)\right)\,dI_n > 0.$$

Therefore, $I_n = 0$ cannot be the global maximum. In contrast, for $I_n = (\mu_A - R_f)/c$, the marginal benefits are zero but the marginal costs are positive. Hence, there exists an $\epsilon > 0$

[34] With exogenous information (i.e., $\bar{q}_n = 0$), the marginal costs (A20) are given by:

$$MC\left(I_n, \bar{\theta}_n^{\mathcal{P}}\right)^{exog.} = \rho\left(\frac{1-\theta_n^{\mathcal{P}}}{1-\Gamma^{\mathcal{P}}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\sigma_n^4\left(2\,\sigma_A^2\,I_n\,\frac{-1}{\sigma_n^4}(-1)\right) = \underbrace{2\,\sigma_A^2\,\rho\left(\frac{1-\theta_n^{\mathcal{P}}}{1-\Gamma^{\mathcal{P}}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)}_{>0} I_n,$$

and, hence, monotonically increasing in I_n—driven by the resultant increase in cashflow variance, σ_n^2.

such that

$$\int_{\frac{\mu_A - R_f}{c} - \epsilon}^{\frac{\mu_A - R_f}{c}} \left(MB(I_n) - MC\left(I_n, \bar{\theta}_n^{\mathcal{P}}\right)\right) dI_n < 0.$$

Thus, $I_n = (\mu_A - R_f)/c$ also cannot be the global maximum. Consequently, the global maximum has to be at an interior point, $I_n \in (0, (\mu_A - R_f)/c)$ and, hence, is characterized by (11).

Debt Financing (Modigliani-Miller)

In case of riskfree debt financing, the total supply of the stock, θ_n, is equal to 1. Furthermore, in the final period, the firm has to repay the loan used to finance the investment I_n; hence, the expected cashflow, μ_n, is equal to $\mu_A\, I_n - (c/2)\, I_n^2 - I_n\, R_f$. Taking the expectation of (6)—under the assumption of $\theta_n = 1$—and setting $\mu_n = \mu_A\, I_n - (c/2)\, I_n^2 - I_n\, R_f$, yields:

$$\begin{aligned}
S_n &= \frac{1}{\bar{h}_n}\left(\frac{\mu_n}{\sigma_n^2} - \rho\,\bar{\theta}_n^{\mathcal{A}} - \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho}\frac{\mu_Z}{\sigma_Z^2}\right) + \frac{1}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right)\mu_n + \frac{1}{\bar{h}_n}\left(\frac{\rho}{1-\Gamma^{\mathcal{P}}} + \frac{\Gamma^{\mathcal{I}}\,\bar{q}_n}{\rho\,\sigma_Z^2}\right)\mu_Z \\
&= \mu_n - \frac{\rho}{\bar{h}_n}\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right) = \mu_A\, I_n - (c/2)\, I_n^2 - I_n\, R_f - \frac{\rho}{\bar{h}_n}\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right);
\end{aligned}$$

which recovers the stock price in the case of equity financing (12). Hence, the expected stock price, S_n, and, accordingly, firms' real-investment decisions (and all other equilibrium quantities) are independent of their financing choices; that is, the Modigliani-Miller theorems hold.

Proofs for Section 3

Theorem 4

The proof for Theorem 4 follows by contraction. In particular, without loss of generality, assume $\bar{\theta}_L^{\mathcal{P}} < \bar{\theta}_H^{\mathcal{P}}$. Moreover, assume $I_H^* < I_L^*$ for the two respective global maxima, $I_L^* \equiv I_n\left(\bar{\theta}_L^{\mathcal{P}}\right)$ and $I_H^* \equiv I_n\left(\bar{\theta}_L^{\mathcal{P}}\right)$.

For illustration purposes, first consider the case in which there exists a single local maximum, I_L^*. This case is illustrated in Panel A of Figure A1. Then, $\forall \hat{I} \leq I_L^*$, it holds

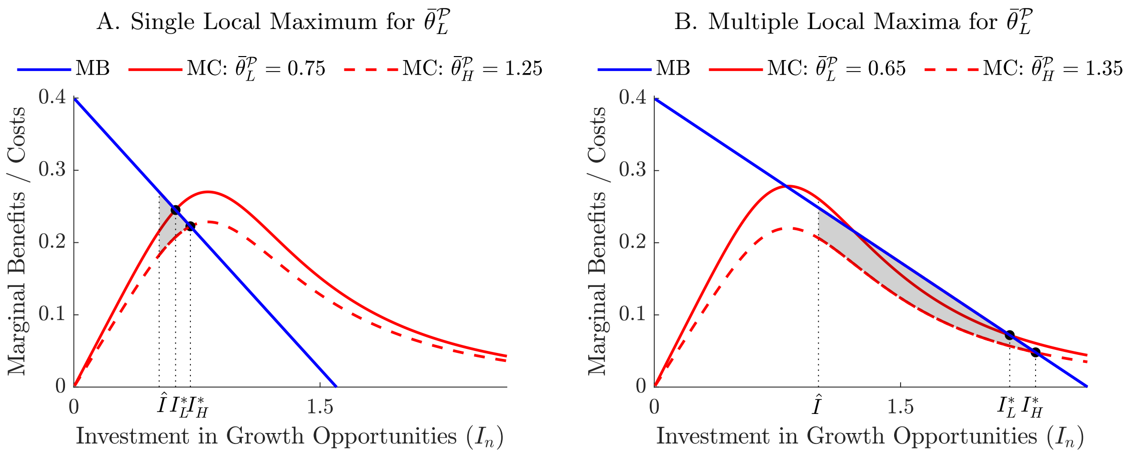

Figure A1: Real-investment choices and passive investors' average demand. The figure illustrates why the optimal investment in growth opportunities, I_n, is increasing in the average demand of passive investors, $\bar{\theta}_n^{\mathcal{P}}$. In particular, it depicts the marginal benefits of investing in growth opportunities, MB, as well as—for two levels of the average demand of passive investors, $\bar{\theta}_L^{\mathcal{P}}$ (low) and $\bar{\theta}_H^{\mathcal{P}}$ (high)—the marginal costs, MC, as functions of the capital allocation, I_n. Panel A illustrates the case of a single local maximum whereas Panel B illustrates the case of multiple local maxima. The integral of the difference between marginal benefits and marginal costs over $[\hat{I}, I_H^*]$ for the case of a high passive investor' demand is highlighted in grey.

that $MB(\hat{I}) > MC\left(\hat{I}, \bar{\theta}_L^{\mathcal{P}}\right) > MC\left(\hat{I}, \bar{\theta}_H^{\mathcal{P}}\right)$.[35] For the same reason, there exists an $\epsilon > 0$ such that $MB(I_L^* + \epsilon) > MC(I_L^* + \epsilon, \bar{\theta}_H^{\mathcal{P}})$. Consequently, it holds that:

$$\int_{\hat{I}}^{I_L^*+\epsilon} \left(MB(I_n) - MC\left(I_n, \bar{\theta}_H^{\mathcal{P}}\right)\right) dI_n > 0,$$

and hence, $S\left(I_L^* + \epsilon, \bar{\theta}_H^{\mathcal{P}}\right) > S\left(\hat{I}, \bar{\theta}_H^{\mathcal{P}}\right)$. Consequently, $\hat{I} \leq I_L^*$ cannot be the global maximum and, for the "true" optimum, I_H^*, it has to hold: $I_H^* > I_L^*$.

Now, consider the general case—with potentially multiple local maxima for $\bar{\theta}_L^{\mathcal{P}}$. This is illustrated for the case of two local maxima in Panel B of Figure A1. Because I_L^* is the global maximum, it holds that:

$$\int_{\hat{I}}^{I_L^*} \left(MB(I_n) - MC\left(I_n, \bar{\theta}_L^{\mathcal{P}}\right)\right) dI_n > 0, \qquad \forall \hat{I} < I_L^*.$$

Because marginal costs are, conditional on I_n, declining in $\bar{\theta}_n^{\mathcal{P}}$, it follows that:

$$\int_{\hat{I}}^{I_L^*} \left(MB(I_n) - MC\left(I_n, \bar{\theta}_H^{\mathcal{P}}\right)\right) dI_n > 0 \qquad \forall \hat{I} < I_L^*.$$

[35]It is easy to see, from (A20), that, conditional on I_n, marginal costs are declining in the average holdings of passive investors, $\bar{\theta}_n^{\mathcal{P}}$) and, hence, that is holds $\forall I_n$: $MC\left(I_n, \bar{\theta}_L^{\mathcal{P}}\right) > MC\left(I_n, \bar{\theta}_H^{\mathcal{P}}\right)$.

Moreover, there exists an $\epsilon > 0$ such that $MB(I_L^* + \epsilon) > MC(I_L^* + \epsilon, \bar{\theta}_H^{\mathcal{P}})$. Taken together, this implies that:

$$\int_{\hat{I}}^{I_L^*+\epsilon} \left(MB(I_n) - MC\left(I_n, \bar{\theta}_H^{\mathcal{P}}\right)\right) dI_n > 0.$$

and hence, $S\left(I_L^* + \epsilon, \bar{\theta}_H^{\mathcal{P}}\right) > S\left(\hat{I}, \bar{\theta}_H^{\mathcal{P}}\right)$. Consequently, $\hat{I} \leq I_L^*$ cannot be the global maximum and, for the "true" optimum, I_H^*, it has to hold: $I_H^* > I_L^*$.

Thus, in general, for $\bar{\theta}_L^{\mathcal{P}} < \bar{\theta}_H^{\mathcal{P}}$, it must hold $I_L^* < I_H^*$, or, equivalently, $\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}} > 0$.

Lemma 2

The total derivatives of μ_n and σ_n^2 with respect to $\bar{\theta}_n^{\mathcal{P}}$ can easily be computed as:

$$\frac{d\mu_n}{d\bar{\theta}_n^{\mathcal{P}}} = \frac{d\mu_n}{dI_n}\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}} = \underbrace{(\mu_A - cI_n)}_{>0}\underbrace{\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{>0} > 0;$$

$$\frac{d\sigma_n^2}{d\bar{\theta}_n^{\mathcal{P}}} = \frac{d\sigma_n^2}{dI_n}\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}} = \underbrace{(\sigma_A^2\, 2\, I_n)}_{>0}\underbrace{\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{>0} > 0;$$

where we used that investment, I_n, is increasing in $\bar{\theta}_n^{\mathcal{P}}$.

Theorem 5

Because investment, I_n, is increasing in $\bar{\theta}_n^{\mathcal{P}}$ and the average signal precision increases in prior variance (A16), we get:

$$\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}} = \underbrace{\frac{d\bar{q}_n}{d\sigma_n^2}}_{>0}\underbrace{\frac{d\sigma_n^2}{dI_n}}_{>0}\underbrace{\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{>0} > 0;$$

$$\frac{dPI_n}{d\bar{\theta}_n^{\mathcal{P}}} = \frac{dPI_n}{d\bar{q}_n}\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}} = \underbrace{\frac{\left(\Gamma^{\mathcal{I}}\right)^2\, 2\,\bar{q}_n}{\rho^2\,\sigma_Z^2}}_{>0}\underbrace{\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{>0} > 0.$$

Theorem 6

Taking the derivative of S_n in (12) yields:

$$\frac{dS_n}{d\bar{\theta}_n^{\mathcal{P}}} = (\mu_A - R_f - c\, I_n)\,\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}} - \rho\left(\frac{1}{\bar{h}_n}\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\bar{\theta}_n^{\mathcal{P}}} + \left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\frac{-1}{\bar{h}_n^2}\frac{d\bar{h}_n}{d\sigma_n^2}\frac{d\sigma_n^2}{dI_n}\frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}}\right)$$

$$= \frac{dI_n}{d\bar{\theta}_n^{\mathcal{P}}}\left((\mu_A - R_f - cI_n) - 2\,\sigma_A^2\, I_n\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\frac{\rho}{\bar{h}_n^2}\left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right)\right) - \frac{\rho}{\bar{h}_n}\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\bar{\theta}_n^{\mathcal{P}}}$$

$$= \underbrace{-\frac{\rho}{\bar{h}_n}}_{<0}\,\underbrace{\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\bar{\theta}_n^{\mathcal{P}}}}_{<0} > 0;$$

where we evoked the optimal real-investment condition (11) in the last step (equivalent to the Envelope theorem).

Theorem 7

The unconditional return variance, V_n^2, is given by:

$$V_n^2 \equiv \mathbb{V}\mathrm{ar}(X_n - \theta_n\, P_n R_f) = \mathbb{E}\left[(X_n - \theta_n\, P_n R_f)^2\right] - \mathbb{E}\left[X_n - \theta_n\, P_n R_f\right]^2$$

$$= \mathbb{E}\left[\mathbb{E}\left[(X_n - \theta_n\, P_n R_f)^2\,|\,\mathcal{F}_i\right]\right] - M_n^2$$

$$= \underbrace{\mathbb{E}\left[\mathbb{V}\mathrm{ar}\left(X_n\,|\,\mathcal{F}_i\right)\right]}_{=\frac{1}{h_{i,n}^{\mathcal{I}}}} + \mathbb{E}\left[\Big(\underbrace{\mathbb{E}[X_n\,|\,\mathcal{F}_i] - \theta_n\, P_n R_f}_{=\frac{u_{i,n}}{h_{i,n}^{\mathcal{I}}}}\Big)^2\right] - M_n^2$$

Using (A8) and (A9), we get:

$$\mathbb{E}\left[\frac{u_{i,n}^2}{\left(h_{i,n}^{\mathcal{I}}\right)^2}\right] = \frac{1}{\left(h_{i,n}^{\mathcal{I}}\right)^2}\left(\mathbb{V}\mathrm{ar}(u_{i,n}) + \mathbb{E}[u_{i,n}]^2\right)$$

$$= \frac{1}{\bar{h}_n^2}\left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\,\bar{q}_n + \rho^2\left(\frac{\sigma_Z^2}{(1-\Gamma^{\mathcal{P}})^2} + \left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)^2\right)\right) - \frac{1}{h_{i,n}^{\mathcal{I}}}.$$

Hence, the unconditional return variance, V_n^2, is given by:

$$V_n^2 = \frac{1}{\bar{h}_n^2}\left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\,\bar{q}_n + \frac{\rho^2\,\sigma_Z^2}{(1-\Gamma^{\mathcal{P}})^2}\right),$$

and its derivative with respect to $\bar{\theta}_n^{\mathcal{P}}$ is equal to:

$$\begin{aligned}\frac{dV_n^2}{d\bar{\theta}_n^{\mathcal{P}}} &= \frac{\partial V_n^2}{\partial \bar{h}_n}\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}} + \frac{\partial V_n^2}{\partial \bar{q}_n}\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}} \\ &= \underbrace{\left(\frac{-2}{\bar{h}^3}\left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\bar{q}_n + \frac{\rho^2\,\sigma_Z^2}{(1-\Gamma^{\mathcal{P}})^2}\right) + \frac{1}{\bar{h}_n^2}\right)}_{\frac{1}{\bar{h}_n^2}\left(1-\frac{2}{\bar{h}_n}\left(\bar{h}_n + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\bar{q}_n + \frac{\rho^2\,\sigma_Z^2}{(1-\Gamma^{\mathcal{P}})^2}\right)\right)<0}\underbrace{\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{<0} + \frac{\Gamma^{\mathcal{I}}}{1-\Gamma^{\mathcal{P}}}\frac{1}{\bar{h}_n^2}\underbrace{\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{>0} > 0.\end{aligned}$$

In particular, taking the derivative of the informed investors' optimal information choice with respect to $\bar{\theta}_n^{\mathcal{P}}$ yields:

$$\begin{aligned}&\frac{1}{\rho}\frac{-1}{\left(\bar{h}_n^{\mathcal{I}}\right)^2}\frac{d\bar{h}_n^{\mathcal{I}}}{d\bar{\theta}_n^{\mathcal{P}}} = 2\kappa''(\bar{q}_n)\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}} \\ \iff\ &\frac{d\bar{h}_n^{\mathcal{I}}}{d\bar{\theta}_n^{\mathcal{P}}} = -2\,\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2\,\kappa''(\bar{q}_n)\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}} < 0,\end{aligned}$$

which, together with $\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}} = \frac{d\bar{h}_n^{\mathcal{I}}}{d\bar{\theta}_n^{\mathcal{P}}} - \frac{\Gamma^{\mathcal{U}}}{1-\Gamma^{\mathcal{P}}}\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{P}}}$ implies $\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}} < 0$.

Also, the unconditional expected excess return, M_n, is given by:

$$M_n \equiv \mathbb{E}[X_n - \theta_n\,P_n R_f] = \mu_n - \left(\mu_n - \frac{\rho}{\bar{h}_n}\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right)\right) = \frac{\rho}{\bar{h}_n}\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}}\right).$$

Note, the excess return is positive if and only if $\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma^{\mathcal{P}}} > 0$. Its derivative with respect to $\bar{\theta}_n^{\mathcal{P}}$ is given by:

$$\frac{dM_n}{d\bar{\theta}_n^{\mathcal{P}}} = \frac{\partial M_n}{\partial\bar{\theta}_n^{\mathcal{P}}} + \frac{\partial M_n}{\partial \bar{h}_n}\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}} = \underbrace{\frac{\rho}{\bar{h}_n}\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\bar{\theta}_n^{\mathcal{P}}}}_{<0} + \underbrace{\rho\left(\bar{\theta}_n^{\mathcal{A}} - \frac{\mu_Z}{1-\Gamma}\right)}_{>0}\underbrace{\frac{-1}{\bar{h}_n^2}}_{<0}\underbrace{\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{P}}}}_{<0},$$

which can be greater or smaller than zero depending on the strength of the two opposing forces.

Theorem 8

Active investors' trading profits, in general, are given by:

$$TP_{i,n} \equiv \mathbb{E}\left[\theta_{i,n}\left(\frac{X_n}{\theta_n} - P_n\right)\right] = \frac{h_{i,n}}{\rho}\mathbb{E}\big[(\mathbb{E}\left[X_n \,|\, \mathcal{F}_i\right] - \theta_n P_n R_f)\big] = \frac{1}{\rho}\,\mathbb{E}\left[z_{i,n}^2\right],$$

with $z_{i,n}$ (defined as in the proof of Theorem 2) denoting an active investor's time-3 expected Sharpe ratio of trading in stock n. In particular, using (A10) and (A11), we get:

$$\mathbb{E}\left[z_{i,n}^2\right] = \mathbb{V}\mathrm{ar}(z_{i,n}) + \mathbb{E}\left[z_{i,n}\right]^2 = h_{i,n}\left(A_{1,n} - 1 + \frac{1}{\bar{h}_n^2}\,\rho^2\left(\left(\bar{\theta}_n^{\mathcal{A}}\right)^2 - \frac{\mu_Z}{1-\Gamma}\right)^2\right).$$

Consequently, the difference in the trading profits of informed investors and uninformed investors ($q_{i,n} = 0$) is equal to:

$$TP_{i,n}^{\mathcal{I}} - TP_{i,n}^{\mathcal{U}} = q_{i,n}\left(A_{1,n} - 1 + \frac{1}{\bar{h}_n^2}\,\rho^2\left(\left(\bar{\theta}_n^{\mathcal{A}}\right)^2 - \frac{\mu_Z}{1-\Gamma}\right)^2\right), \tag{A21}$$

which, given $dq_{i,n}/d\bar{\theta}_n^{\mathcal{P}} > 0$, is increasing in the average demand of passive investors.

Proofs for Section 4

Keeping I_n and $\bar{\theta}_n^{\mathcal{A}}$ fixed, the derivative of the marginal costs (A20) with respect to $\Gamma^{\mathcal{P}}$ is given by:

$$\frac{dMC}{d\Gamma} = \underbrace{\left(2\,\sigma_A^2\,I_n\,\rho\right)}_{\equiv\eta>0}\left\{\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\Gamma^{\mathcal{P}}}\underbrace{\frac{1}{\bar{h}_n^2}\left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right)}_{>0} + \underbrace{\bar{\theta}_n^{\mathcal{A}}\frac{-2}{\bar{h}_n^3}\left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right)}_{<0}\frac{d\bar{h}_n}{d\Gamma^{\mathcal{P}}} + \underbrace{\bar{\theta}_n^{\mathcal{A}}\frac{1}{\bar{h}_n^2}}_{>0}\left(-\frac{d^2\bar{h}_n}{d\sigma_n^2 d\Gamma^{\mathcal{P}}}\right)\right\}. \tag{A22}$$

The derivative of $\bar{\theta}_n^{\mathcal{A}}$ with respect to $\Gamma^{\mathcal{P}}$ is equal to:

$$\frac{d\bar{\theta}_n^{\mathcal{A}}}{d\Gamma^{\mathcal{P}}} = \frac{d\,\frac{1-\Gamma^{\mathcal{P}}\,\bar{\theta}_n^{\mathcal{P}}}{1-\Gamma^{\mathcal{P}}}}{d\Gamma^{\mathcal{P}}} = \frac{1}{1-\Gamma^{\mathcal{P}}}(-\bar{\theta}_n^{\mathcal{P}}) + \frac{(-1)(-1)}{(1-\Gamma^{\mathcal{P}})^2}\,(1-\Gamma^{\mathcal{P}}\,\bar{\theta}_n^{\mathcal{P}})$$

$$= \frac{1}{(1-\Gamma^{\mathcal{P}})^2}\left((1-\Gamma^{\mathcal{P}})(-\bar{\theta}_n^{\mathcal{P}}) + 1 - \Gamma^{\mathcal{P}}\bar{\theta}_n^{\mathcal{P}}\right) = \frac{1}{(1-\Gamma^{\mathcal{P}})^2}(1-\bar{\theta}_n^{\mathcal{P}}) = \begin{cases} > 0 & \text{if } \bar{\theta}_n^{\mathcal{P}} < 1; \\ < 0 & \text{if } \bar{\theta}_n^{\mathcal{P}} > 1; \\ = 0 & \text{if } \bar{\theta}_n^{\mathcal{P}} = 1. \end{cases}$$

That is, as one might expect, as the share of passive investors increases, the average number of shares active investors have to hold declines (increases) if the average demand of passive investors exceeds (is below) 1.0.

Theorem 9 and Corollary 1

If the share of uninformed investors remains fixed, informed investors' information choice is unaffected by variations in the share of passive investors. Hence, the average private signal precision, $\bar{q}_n$, as well as the precision of informed investors, $\bar{h}_n^{\mathcal{I}}$, is unchanged. As a result, we get:

$$\frac{d\bar{h}_n}{d\Gamma^{\mathcal{P}}} = \Gamma^{\mathcal{I}}\,\bar{q}_n \frac{1}{(1-\Gamma^{\mathcal{P}})^2} > 0.$$

and, using (A18):

$$\left(-\frac{d^2\bar{h}_n}{d\sigma_n^2 d\Gamma^{\mathcal{P}}}\right) = \underbrace{\frac{1}{\sigma_n^4}\,\frac{\bar{q}_n}{\bar{q}_n + 2PI_n + 2\,\kappa''(\bar{q}_n)\,\rho\,\left(\bar{h}_n^{\mathcal{I}}\right)^2\,\bar{q}_n}}_{>0}\,\underbrace{\left(\frac{-1}{1-\Gamma^{\mathcal{P}}} - \frac{\Gamma^{\mathcal{U}}}{(1-\Gamma^{\mathcal{P}})^2}\right)}_{<0} < 0.$$

Consequently, both the second and third term in (A22) are negative whereas the first term depends on the value of $\bar{\theta}_n^{\mathcal{P}}$. Hence, there exists a $\xi^{\mathcal{U}} < 1$ such that marginal costs decline (increase) for $\bar{\theta}_n^{\mathcal{P}} > \xi^{\mathcal{U}}$ ($\bar{\theta}_n^{\mathcal{P}} \leq \xi^{\mathcal{U}}$). Following the reasoning of the proof of Theorem 4, this implies an increase (decrease) in the investment in growth opportunities for $\bar{\theta}_n^{\mathcal{P}} > \xi^{\mathcal{U}}$ ($\bar{\theta}_n^{\mathcal{P}} \leq \xi^{\mathcal{U}}$) which, in turn, using Lemma 2, implies an increase (decrease) in cashflow variance. Derivations along the line of Theorem 5 then immediately imply that the average private signal precision, $\bar{q}_n$, and price informativeness, PI_n, are increasing in $\Gamma^{\mathcal{P}}$ for $\bar{\theta}_n^{\mathcal{P}} > \xi^{\mathcal{U}}$ ($\bar{\theta}_n^{\mathcal{P}} \leq \xi^{\mathcal{U}}$).

Theorem 10 and Corollary 2

Now, keep the share of uninformed investors fixed. Rewriting (10) as $\bar{h}_n^{\mathcal{I}} = \frac{1}{\rho} \frac{1}{2\,\kappa'(\bar{q}_n)}$ and evoking (A16), one can establish that, conditional on I_n, it holds:

$$\frac{d\bar{h}_n^{\mathcal{I}}}{d\Gamma^{\mathcal{P}}} = \frac{1}{2\rho} \frac{-1}{(\kappa'(\bar{q}_n))^2}\, \kappa''(\bar{q}_n) \frac{d\bar{q}_n}{d\Gamma} < 0;$$

where we employed that the derivative of (10) with respect to $\Gamma^{\mathcal{P}}$ implies:

$$\begin{aligned}
&2\,\kappa''(\bar{q}_n)\, \frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}} = \frac{1}{\rho}\,(-1) \frac{1}{\bar{h}_n^2} \left(\frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}} + \frac{(1-\Gamma^{\mathcal{P}})^2\, 2\,\bar{q}_n}{\rho^2\sigma_Z^2} \frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}} + \frac{2\,(1-\Gamma^{\mathcal{P}})\,(-1)\,\bar{q}_n^2}{\rho^2\sigma_Z^2} \right)\\
\iff\ & \frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}} \left(2\,\kappa''(\bar{q}_n) + \frac{1}{\rho\,\bar{h}_n^2} \left(1 + \frac{2\,PI_n}{\bar{q}_n} \right) \right) = \frac{1}{\rho\,\bar{h}_n^2} \frac{2PI_n}{1-\Gamma^{\mathcal{P}}}\\
\iff\ & \frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}} = \frac{\bar{q}_n}{\bar{q}_n + 2PI_n + 2\,\kappa''(\bar{q}_n)\,\rho\,\bar{h}_n^2\,\bar{q}_n} \frac{2PI_n}{1-\Gamma^{\mathcal{P}}} > 0; \qquad \left(< \frac{2PI_n}{1-\Gamma^{\mathcal{P}}} \right). \qquad \text{(A23)}
\end{aligned}$$

Moreover, note that $\bar{h}_n = \bar{h}_n^{\mathcal{I}} + \bar{q}_n \left(-\frac{\Gamma^{\mathcal{U}}}{1-\Gamma^{\mathcal{P}}} \right)$. This can be used to show that, conditional on I_n, it holds:

$$\frac{d\bar{h}_n}{d\Gamma^{\mathcal{P}}} = \underbrace{\frac{d\bar{h}_n^{\mathcal{I}}}{d\Gamma^{\mathcal{P}}}}_{<0} + \underbrace{\frac{d\bar{q}_n}{d\Gamma^{\mathcal{P}}}}_{>0} \underbrace{\left(-\frac{\Gamma^{\mathcal{U}}}{1-\Gamma^{\mathcal{P}}} \right)}_{<0} + \underbrace{\bar{q}_n\,\Gamma^{\mathcal{U}}}_{>0} \underbrace{\frac{-1}{(1-\Gamma^{\mathcal{P}})^2}}_{<0} < 0. \qquad \text{(A24)}$$

Accordingly, if $\left(-\frac{d^2\bar{h}_n}{d\sigma_n^2\, d\Gamma^{\mathcal{P}}} \right) > 0$, both the second and third term in (A22) are positive whereas the first term depends on the the value of $\bar{\theta}_n^{\mathcal{P}}$. Hence, there exists a $\xi^{\mathcal{I}} > 1$ such that marginal costs decline (increase) for $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{I}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{I}}$). Following the reasoning of the proof of Theorem 4, this implies an increase (decline) in the investment in growth opportunities for $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{I}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{I}}$) which, in turn, using Lemma 2, implies an increase (decrease) in cashflow variance. Derivations along the line of Theorem 5 then immediately imply that the average private signal precision, $\bar{q}_n$, and price informativeness, PI_n, are increasing (declining) in $\Gamma^{\mathcal{P}}$ for $\bar{\theta}_n^{\mathcal{P}} \geq \xi^{\mathcal{I}}$ ($\bar{\theta}_n^{\mathcal{P}} < \xi^{\mathcal{I}}$).

Proofs for Section 5

Theorem 11

We again start with conjecture (A1) for the equilibrium stock price, $\theta_n P_n R_f$. Consequently, active investors' posterior beliefs continue to be characterized by posterior mean and variance, (A2) and (A3), and their demand is given by standard mean-variance demand (9). Moreover, with normally-distributed prices and CARA-preferences, the demand of benchmarked investors is given by (13)—with beliefs characterized by (A2) and (A3) (with $q_{i,n} = 0$).[36] That is, passive investors "undo" the benchmarking by means of their hedging component.

Aggregating investors' demand, imposing market clearing, and matching the coefficients of the conjectured price function yields:

$$\theta_n P_n R_f = \frac{1}{\bar{h}_n}\left(\frac{\mu_n}{\sigma_n^2} + \rho\left(\theta_n^{\mathcal{P}} - 1\right) - \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho}\frac{\mu_Z}{\sigma_Z^2}\right) + \frac{1}{\bar{h}_n}\left(\bar{h}_n - \frac{1}{\sigma_n^2}\right)X_n + \frac{1}{\bar{h}_n}\left(\rho + \frac{\Gamma^{\mathcal{I}}\bar{q}_n}{\rho\sigma_Z^2}\right)Z_n,$$

$$\text{with} \quad h_{0,n} \equiv \frac{1}{\sigma_n^2} + \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2}, \quad \bar{q}_n \equiv \int^{\mathcal{I}} q_{i,n}\, di, \quad \bar{h}_n \equiv h_{0,n} + \Gamma^{\mathcal{I}}\,\bar{q}_n,$$

$$\text{and} \quad \theta_n^{\mathcal{B}} = \nu\,\Gamma^{\mathcal{B}} \int^{\mathcal{B}} \gamma_{i,n}\, di.$$

Accordingly, price informativeness is equal to $PI_n = \frac{\left(\Gamma^{\mathcal{I}}\right)^2 \bar{q}_n^2}{\rho^2\,\sigma_Z^2}$.

The information-choice problem of informed investors remains unchanged (though the variables $A_{1,n}$ and $A_{2,n}$ take on slightly different values) and, hence, the equilibrium private-signal precision is characterized by (10). Accordingly, it holds that $d\bar{q}_n/d\theta_n^{\mathcal{B}} > 0$ and, hence, $dPI_n/d\theta_n^{\mathcal{B}} > 0$.

[36] See, among others, Admati and Pfleiderer (1997) and Breugem and Buss (2019).

The expected stock price, S_n, can again be computed as $\mathbb{E}[P_n R_f] - I_n R_f$ which yields (14) (with $\mu_Z = 0, R_f = 1$). Taking the first-order condition with respect to I_n, implies:[37]

$$\mu_A - R_f - c\, I_n = \rho\, \left(1 - \theta_n^{\mathcal{B}}\right)\, 2\, I_n\, \sigma_A^2\, \frac{1}{\bar{h}_n^2} \left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right), \tag{A25}$$

$$\text{with} \quad -\frac{d\bar{h}_n}{d\sigma_n^2} = \frac{1}{\sigma_n^4}\, \frac{\bar{q}_n\, \left(1 - \Gamma^{\mathcal{I}}\right) + 2\, \kappa''\left(\bar{q}_n\right)\, \rho\, \bar{q}_n\, \left(h_{i,n}^{\mathcal{I}}\right)^2}{\bar{q}_n + 2\, P I_n + 2\, \kappa''\left(\bar{q}_n\right)\, \rho\, \bar{q}_n\, \left(h_{i,n}^{\mathcal{I}}\right)^2} \geq 0,$$

where we used expression (A16) to replace the derivative of $d\bar{q}_n/d\sigma_n^2$ (which remains unchanged).

Marginal benefits are non-negative if and only if $I_n \leq (\mu_A - R_f)/c$ and marginal costs (RHS of (A25)) are always non-negative. Hence, one can show (following the proof of Theorem 3) that there exists an optimal investment, $I_n \in (0, (\mu_A - R_f)/c)$, that fulfils (A25). Similarly, one can show that the Modigliani-Miller theorems continue to hold.

Moreover, keeping investment I_n fixed, marginal costs are declining in the aggregate benchmarking demand $\theta_n^{\mathcal{B}}$. This can be directly used to show that $dI_n/d\theta_n^{\mathcal{B}} > 0$ (following along the lines of the proof for Theorem 4) and, hence, $d\sigma_n^2/d\theta_n^{\mathcal{B}} > 0$.

The derivative of the stock price, S_n, in (14), with respect to the aggregate benchmarking demand is given by:

$$\begin{aligned}\frac{dS_n}{d\theta_n^{\mathcal{B}}} &= \left(\mu_A - R_f - cI_n\right) \frac{dI_n}{d\theta_n^{\mathcal{B}}} - \rho \left(\frac{1}{\bar{h}_n} + \left(1 - \theta_n^{\mathcal{B}} - \mu_Z\right) \frac{-1}{\bar{h}_n^2}\, 2\sigma_A^2\, I_n \left(-\frac{d\bar{h}_n}{d\sigma_n^2}\right) \frac{dI_n}{d\theta_n^{\mathcal{B}}}\right) \\ &= \frac{\rho}{\bar{h}_n}\, \frac{dI_n}{d\theta_n^{\mathcal{B}}} > 0.\end{aligned}$$

Finally, the expected excess return and the stock-return variance are given by:

$$M_n = \frac{\rho}{\bar{h}_n}\, \left(1 - \theta_n^{\mathcal{B}} - \mu_Z\right), \quad \text{and} \quad V_n^2 = \frac{1}{\bar{h}_n^2}\, \left(\bar{h}_n + \Gamma^{\mathcal{I}}\, \bar{q}_n + \rho^2\, \sigma_Z^2\right),$$

[37] $1 - \theta_n^{\mathcal{B}} - \mu_Z > 0$ is the relevant case here because this guarantees that the stock price is declining in $\bar{h}_n$ and that the expected excess return $M_n = (\rho/\bar{h}_n)\, \left(1 - \theta_n^{\mathcal{B}} - \mu_Z\right)$ is positive.

with the derivative of the stock-return variance with respect to the aggregate benchmarking demand being given by:

$$\frac{dV_n^2}{d\theta_n^{\mathcal{B}}} = \underbrace{\left(\frac{-2}{\bar{h}^3} \left(\bar{h}_n + \Gamma^{\mathcal{I}} \bar{q}_n + \rho^2 \, \sigma_Z^2 \right) + \frac{1}{\bar{h}_n^2} \right)}_{\frac{1}{\bar{h}_n^2} \left(-1 - \frac{2}{\bar{h}_n} \left(\Gamma^{\mathcal{I}} \bar{q}_n + \rho^2 \, \sigma_Z^2 \right) \right) < 0} \underbrace{\frac{d\bar{h}_n}{d\bar{\theta}_n^{\mathcal{B}}}}_{<0} + \Gamma^{\mathcal{I}} \frac{1}{\bar{h}_n^2} \underbrace{\frac{d\bar{q}_n}{d\bar{\theta}_n^{\mathcal{B}}}}_{>0} > 0.$$

References

Adib, F.-Z. F., 2020, "Passive Aggressive: How Index Funds Vote on Corporate Governance Proposals," working paper.

Admati, A., and P. Pfleiderer, 1997, "Does It All Add Up? Benchmarks and the Compensation of Active Portfolio Managers," *Journal of Business*, 70(3), 323–350.

Appel, I. R., T. A. Gormley, and D. B. Keim, 2016, "Passive investors, not passive owners," *Journal of Financial Economics*, 121(1), 111 – 141.

Bai, J., T. Philippon, and A. Savov, 2016, "Have financial markets become more informative?," *Journal of Financial Economics*, 122(3), 625–654.

Baker, S., D. Chapman, and M. Galleyer, 2020, "Activism and Indexing in Equilibrium," working paper.

Basak, S., and A. Pavlova, 2013, "Asset prices and institutional investors," *American Economic Review*, 103(5), 1728–58.

Ben-David, I., F. Franzoni, and R. Moussawi, 2018, "Do ETFs Increase Volatility?," *Journal of Finance*, 73(6), 2471–2535.

Bena, J., M. A. Ferreira, P. Matos, and P. Pires, 2017, "Are foreign investors locusts? The long-term eects of foreign institutional ownership," *Journal of Financial Economics*, 126(1), 122–146.

Benamar, H., T. Foucault, and C. Vega, 2020, "Demand for Information, Uncertainty, and the Response of U.S. Treasury Securities to News," *Review of Financial Studies*, forthcoming.

Bhattacharya, U., and M. Spiegel, 1991, "Insiders, Outsiders, and Market Breakdowns," *Review of Financial Studies*, 4(2), 255–282.

Bond, P., A. Edmans, and I. Goldstein, 2012, "The real effects of financial markets," *Annual Review of Financial Economics*, 4(1), 339–360.

Bond, P., and D. García, 2019, "The Equilibrium Consequences of Indexing," working paper.

Breugem, M., and A. Buss, 2019, "Institutional investors and information acquisition: Implications for asset prices and informational efficiency," *Review of Financial Studies*, 32(6), 2260–2301.

Brunnermeier, M., 2001, *Asset pricing under asymmetric information: Bubbles, crashes, technical analysis, and herding.* Oxford: Oxford University Press.

Buffa, A., and I. Hodor, 2019, "Institutional Investors, Heterogeneous Benchmarks and the Comovement of Asset Prices," working paper.

Buffa, A., D. Vayanos, and P. Woolley, 2020, "Asset Management Contracts and Equilibrium Prices," *CEPR Discussion Papers.*

Bushee, B., and C. Noe, 2000, "Corporate Disclosure Practices, Institutional Investors, and Stock Return Volatility," *Journal of Accounting Research*, 38, 171–202.

Bushee, B., and C. Noe, 2001, "Do institutional investors prefer near-term earnings over long-run value?," *Contemporary Accounting Research*, 18(2), 207–246.

Cespa, G., and T. Foucault, 2014, "Illiquidity Contagion and Liquidity Crashes," *Review of Financial Studies*, 27(6), 1615–1660.

Cespa, G., and X. Vives, 2015, "The Beauty Contest and Short-Term Trading," *Journal of Finance*, 70(5), 2099–2154.

Chabakauri, G., and O. Rytchkov, 2019, "Asset Pricing with Index Investing," working paper.

Chen, H., G. Noronha, and V. Singal, 2004, "The Price Response to S&P 500 Index Additions and Deletions: Evidence of Asymmetry and a New Explanation," *Journal of Finance*, 59(4), 1901–1930.

Chen, Q., I. Goldstein, and W. Jiang, 2007, "Price Informativeness and Investment Sensitivity to Stock Price," *Review of Financial Studies*, 20(3), 619–650.

Chinco, A., and V. Fos, 2019, "The sound of many funds rebalancing," *Working paper.*

Cong, W., and D. Xu, 2019, "Rise of Factor Investing: Asset Prices, Informational Efficiency, and Security Design," working paper.

Cujean, J., 2020, "Idea sharing and the performance of mutual funds," *Journal of Financial Economics*, 135(1), 88 – 119.

Cuoco, D., and R. Kaniel, 2011, "Equilibrium Prices in the Presence of Delegated Portfolio Management," *Journal of Financial Economics*, 101(2), 264–296.

Dávila, E., and C. Parlatore, 2019, "Identifying Price Informativeness," working paper.

Dessaint, O., T. Foucault, L. Frésard, and A. Matray, 2018, "Noisy Stock Prices and Corporate Investment," *Review of Financial Studies*, 32(7), 2625–2672.

Diamond, D. W., and R. E. Verrecchia, 1981, "Information aggregation in a noisy rational expectations economy," *Journal of Financial Economics*, 9(3), 221 – 235.

Edmans, A., I. Goldstein, and W. Jiang, 2015, "Feedback Effects, Asymmetric Trading, and the Limits to Arbitrage," *American Economic Review*, 105(12), 3766–3797.

Farboodi, M., A. Matray, L. Veldkamp, and V. Venkateswaran, 2019, "Where has all the big data gone?," *Working paper.*

Foucault, T., and L. Frésard, 2012, "Cross-Listing, Investment Sensitivity to Stock Price, and the Learning Hypothesis," *Review of Financial Studies*, 25(11), 3305–3350.

——— , 2014, "Learning from Peers' Stock Prices and Corporate Investment," *Journal of Financial Economics*, 111(3), 554–577.

Glosten, L., S. Nallareddy, and Y. Zou, 2020, "ETF Activity and Informational Efficiency of Underlying Securities," *Management Science*, forthcoming.

Goldstein, I., J. Schneemeier, and L. Yang, 2020, "Learning and Efficiency with Market Feedback," working paper.

Goldstein, I., and L. Yang, 2017, "Information disclosure in financial markets," *Annual Review of Financial Economics*, 9, 101–125.

——— , 2019, "Good disclosure, bad disclosure," *Journal of Financial Economics*, 131(1), 118–138.

Harford, J., A. Kecskés, and S. Mansi, 2018, "Do long-term investors improve corporate decision making?," *Journal of Corporate Finance*, 50, 424 – 452.

Harris, L., and E. Gurel, 1986, "Price and Volume Effects Associated with Changes in the S&P 500 List: New Evidence for the Existence of Price Pressures," *Journal of Finance*, 41(4), 815–829.

Hellwig, M. F., 1980, "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22(3), 477–498.

Hirshleifer, J., 1971, "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 61(4), 561–74.

Honkanen, P., and D. Schmidt, 2019, "Learning from Noise? Price and Liquidity Spillovers around Mutual Fund Fire Sales," working paper.

Huang, S., M. O'Hara, and Z. Zhong, 2018, "Innovation and informed trading: Evidence from industry ETFs," *Working paper.*

Huang, S., Z. Qiu, and L. Yang, 2019, "Institutionalization, Delegation, and Asset Prices," *Working paper.*

Kacperczyk, M., S. Sundaresan, and T. Wang, 2020, "Do Foreign Investors Improve Market Efficiency?," *Review of Financial Studies*, forthcoming.

Kacperczyk, M., S. Van Nieuwerburgh, and L. Veldkamp, 2016, "A rational theory of mutual funds' attention allocation," *Econometrica*, 84(2), 571–626.

Kacperczyk, M. T., J. B. Nosal, and S. Sundaresan, 2018, "Market power and price informativeness," .

Kashyap, A., N. Kovrijnykh, J. Li, and A. Pavlova, 2020, "The Benchmark Inclusion Subsidy," working paper.

Massa, M., U. Peyer, and Z. Tong, 2005, "Limits of Arbitrage and Corporate Financial Policy," working paper, CEPR Discussion Paper No. 4829.

Shleifer, A., 1986, "Do Demand Curves for Stocks Slope Down?," *Journal of Finance*, 41(3), 579–590.

Sias, R. W., 1996, "Volatility and the Institutional Investor," *Financial Analysts Journal*, 52(2), 13–20.

Subrahmanyam, A., and S. Titman, 1999, "The going-public decision and the development of financial markets," *Journal of Finance*, 54(3), 1045–1082.

van Nieuwerburgh, S., and L. Veldkamp, 2009, "Information Immobility and the Home Bias Puzzle," *Journal of Finance*, 64(3), 1187–1215.

——— , 2010, "Information Acquisition and Under-Diversification," *Review of Economic Studies*, 77(2), 779–805.

Verrecchia, R., 1982, "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, 50(6), 1415–1430.