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Abstract

We develop a novel framework that features loss amplification through firm-bank linkages. We use it to study optimal intervention in a lockdown that creates cash shortfalls to firms, which must borrow from banks to avoid liquidation. Firms’ increase in debt reduces firms’ output due to moral hazard. Banks need safe collateral to raise funds. Without intervention, aggregate risk constrains bank lending, increasing its cost and amplifying output losses. Optimal government support must provide sufficient aggregate risk insurance, and can be implemented with transfers to firms and fairly-priced guarantees on banks’ debt. Non-priced bank debt guarantees and loan guarantees are suboptimal.

JEL Classification: G01, G20, G28

Keywords: COVID-19, liquidity, firm's leverage, Financial Intermediation, Government interventions

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Firm-bank linkages and optimal policies in a lockdown*

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Abstract

We develop a novel framework that features loss amplification through firm-bank linkages. We use it to study optimal intervention during a lockdown. In the model, a lockdown creates cash shortfalls to firms, which need to borrow from banks to avoid liquidation. Firms' increase in debt reduces firms' output due to moral hazard. Banks need safe collateral to raise funds. Without intervention, aggregate risk constrains bank lending, increasing its cost and amplifying firm output losses. We find that optimal government support must provide sufficient aggregate risk insurance, and can be implemented with a combination of transfers to firms and fairly-priced guarantees on banks' debt. Instead, non-priced bank debt guarantees and bank loan guarantees are suboptimal policies.

JEL Classification: G01, G20, G28

Keywords: Covid-19, cash shortfall, firms' debt, moral hazard, bank equity, aggregate risk, government interventions

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1 Introduction

The lockdown measures implemented to contain the Covid-19 pandemic have led to cash-flow shortages for businesses of an unprecedented magnitude. To survive the crisis many firms need to raise large amounts of new funding. Should firms manage to obtain this funding, their post-lockdown leverage will significantly increase (Crouzet and Gourio [2020] and Carletti et al. [2020]),[1] which could drag economic recovery down due to debt overhang and other agency problems (Brunnermeier and Krishnamurthy [2020]). At the same time, Figure 1 shows that US banks have tightened their lending standards in the last months up to levels not observed since the 2008-09 crisis, which suggests that it could be difficult or very expensive for firms to obtain funding exactly when it is most necessary. The observed tightening in bank lending supply might result from the meaningful risk of large losses on bank assets due to the Covid-19 crisis, which could rapidly erode their current solid capitalization (Blank, Hanson, Stein, and Sunderam [2020]).

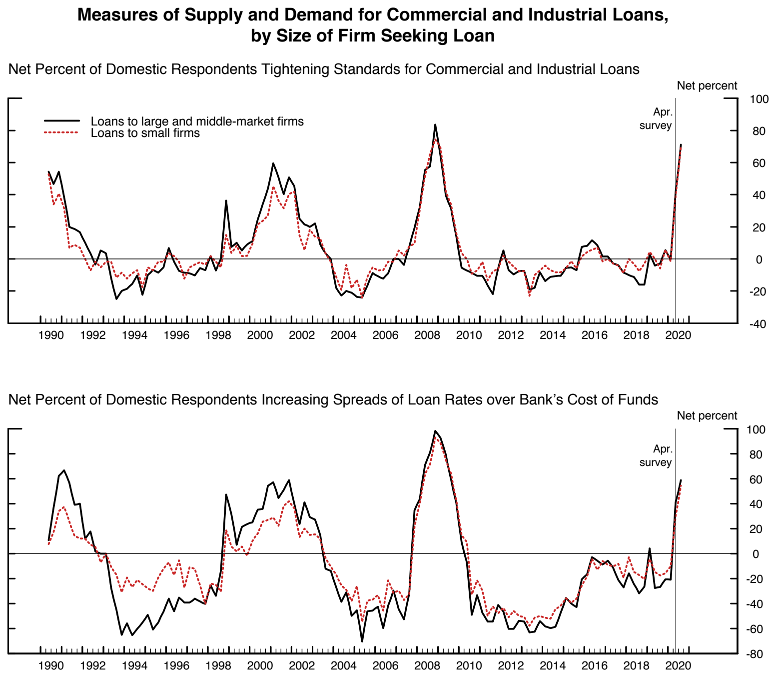
Governments have responded with an array of policies to avoid a wave of corporate defaults due to liquidity problems. Some of the policies introduced, like transfers to firms, directly cover firms' liquidity needs while others, like guarantees on bank loans or reductions in bank capital requirements, help indirectly by supporting bank lending. The type of policy affects whether the government faces immediate or contingent disbursements. It also has an impact on firms' leverage and bank risk exposures in the medium term, which could in turn affect output and the overall cost of government interventions in a non-trivial manner. We ask, have governments optimally used their available budget to support firms during lockdowns?

This paper develops a new theoretical framework of bank intermediation and real activity that considers financing frictions both at the firm and the bank level. The model is then used to study optimal policy design in a lockdown that creates cash-flow losses to firms which need to obtain new funding from banks in order to survive.[2] The framework features a novel firm-bank feedback mechanism that leads to amplification of the initial output losses. We emphasize the importance of the provision of sufficient aggregate risk insurance necessary

[1] For US public corporations, Crouzet and Gourio [2020] quantify that the additional debt needed to offset the decline in earnings could lead to a doubling of the share of highly levered firms by the middle of 2021. Using a representative sample of Italian firms, Carletti, Oliviero, Pagano, Pelizzon, and Subrahmanyam [2020] estimate that, under a three-month lockdown scenario, firms will face an aggregate annual profit drop amounting to 10% of the Italian GDP in 2018, and 17% of firms will have negative net worth.

[2] Our focus is on small and medium sized firms which rely on bank financing, and which are likely to be more impacted by lockdowns. For instance, the exercise in Carletti et al. [2020] shows that small and medium-sized enterprises are considerably more likely to enter in financial distress.

Figure 1: Bank lending standards



Source: The July 2020 Senior Loan Officer Opinion Survey on Bank Lending Practices.

to remove bank financing constraints. We find that some of the observed interventions, like loan guarantees or relaxations of capital requirements, could provide insufficient aggregate risk insurance when the budget of the government (and so the size of the intervention) is not large enough. We instead show that optimal interventions can be implemented with a combination of direct transfers to firms and fairly-priced guarantees on bank debt.

We show that the initial output losses created by a lockdown can be amplified through firm-bank linkages when two frictions are present in financial markets. First, an increase in firms' debt reduces firms' output and the value of their outstanding debt. This happens in our model because entrepreneurs are subject to moral hazard when they raise external funds.[3] Second, banks need safe collateral to raise funds. This happens in our model because end-investors with available funds have an absolute preference for safety. The diversification of firms' idiosyncratic risks allows banks to issue some safe debt, but aggregate risk limits bank lending supply.[4] Our novel amplification mechanism works as follows. Firms need to obtain new financing from banks in order to survive the lockdown. Since bank lending

[3]The reduction in firm value due to increases in leverage would also result from debt overhang problems à la Myers [1977].

[4]This bank funding friction, which in particular implies that banks cannot issue equity, aims at capturing that during episodes of high economic uncertainty investors tend to have a strong preference for high quality riskless assets. While we emphasize this market driven borrowing constraint, bank leverage and aggregate lending could also be limited by aggregate risk due to regulatory frictions.

is constrained, firms obtain funds at a high cost and end-up with high debt obligations. The implied rise in firms' leverage reduces their output and the value of their debt. Banks in turn suffer value losses on their outstanding loans, and find it more difficult to create the safe collateral necessary to raise funds. Thus, bank lending constraints get further tightened and the initial lockdown losses get further amplified through firm-bank balance sheets linkages.

We analyze in this context the design of aggregate welfare maximizing interventions by a government with some limited fiscal capacity. We find that policies that provide sufficient aggregate risk insurance are able to remove banks' funding constraints. Thus, optimal policies must sufficiently expose the government to tail risk. Optimal policies can be implemented through the combination of transfers to firms and guarantees on banks' debt that banks fairly reimburse upon good aggregate shocks (in which they make profits). Transfers reduce firms' new financing needs and fairly priced bank debt guarantees make their residual financing needs as cheap as possible at zero expected cost for the government. We are not aware of any government that has implemented our model implied optimal two-instrument intervention toolkit as a response to the Covid-19 crisis. In contrast, the response in most jurisdictions consists of the combination of transfers to firms, bank loan guarantees and non-priced bank debt guarantees.[5] Our final result is that this policy toolkit is generally suboptimal. This is because, in absence of any compensation from the private to the public sectors following good aggregate shocks, a government with a low (expected) budget would not be able to provide sufficient insurance against bad aggregate shocks.

We build a stylized competitive model of bank intermediation between savers, that only invest in safe debt, and firms with risky projects. Banks are able to issue safe debt by holding diversified portfolios of loans to firms. But, due to the presence of aggregate risk, banks' lending supply is constrained. We consider identical firms and banks with some existing balance sheets at the lockdown date. We model the lockdown as an exogenous liquidity need experienced by firms that has to be covered to avoid liquidation, which destroys part of firms' project value, as in Holmstrom and Tirole [1998]. We depart from that paper by assuming that firms do not have a pre-committed credit line and have to rely ex-post on funding from banks, which in turn need to raise new safe debt. Yet, an increase in firms' debt generates a moral hazard problem, as it reduces entrepreneurs' incentives to take non-

[5]Supervisory authorities have responded to the Covid-19 shock by releasing the counter-cyclical capital buffers regulated banks have built in the last years, and this decision has not been accompanied with an increase in deposit insurance fees. In the context of our model in which investors impose a maximum leverage constraint that ensures banks' debt is riskless, such capital buffer release could be interpreted as the introduction of non-priced bank debt guarantees.

observable effort. Thus, firm debt leads to higher project risk and lower expected output.[6] The increase in project risk also creates losses for the banks on their existing loan portfolios, reducing the safe collateral value of their legacy loans. This limits their capability to issue additional safe debt and constrains the lending supply, making new financing to firms more expensive. So, firms must end-up more indebted to obtain the needed funds, which in turn further aggravates entrepreneurs' moral hazard. Thus, the model exhibits an amplification channel of expected output losses that operates through the balance sheet linkages between firms and banks. When the liquidity shock is small, firms obtain financing but their expected output gets reduced. When the liquidity shock is larger, inefficiencies associated with moral hazard become so severe that new funding dries up and firms are liquidated.

The model gives rise to a tension between firms and banks' equity (continuation) values. In order to survive, firms need to promise some of their future payoffs to obtain enough bank funding. The bank in turn can only pledge a fraction of those payoffs as collateral to borrow from savers. In equilibrium, the bank must appropriate of enough firms' payoffs to be able to raise the needed funds by firms, which leads the bank to obtain lending rents on its scarce equity despite competition. This distribution of payoffs away from firms to the bank aggravates moral hazard and reduces the very same value of firm payoffs used as collateral by banks, requiring an even larger distribution of payoffs and amplifying the output losses.

We consider a government with an exogenously given maximum expected budget that intervenes in order to contain the amplification of the initial output losses generated by the lockdown. We first show that optimal intervention design must expose the government to sufficient tail risk. The reason is that banks need additional loss absorption capacity against aggregate shocks in order to intermediate new funds from investors, who demand safety, to firms. Yet, such increase in loss absorption capacity requires an increase in banks' equity value that must arise from a larger appropriation of firms' expected output, which in turn reduces entrepreneurs' skin-in-the-game and output. A government substitutes the need of banks' loss absorption capacity and limits the amplification of output losses through balance sheet linkages by providing sufficient aggregate risk insurance in the economy, that is, by injecting funds upon bad aggregate shocks.

Our second result is that optimal interventions can be implemented in a decentralized competitive environment through the combination of transfers to firms, that exhaust the government budget, and big enough government guarantees on banks' debt that banks fairly

[6] We assume a binomial pay-off structure for firms' projects so that the model does not distinguish between different forms of outside financing such as debt or outside equity. In this respect, the model set-up seems suitable for small firms for which there is no separation between ownership and control.

reimburse in the future (and thus have a zero expected cost). The guarantee is designed as an obligation for the government to meet any shortfall between the banks' asset returns and their safe debt promises upon a sufficiently bad aggregate shock. Fairly priced guarantees on banks' debt lead to transfers from the government to private agents upon bad aggregate shocks, in which the guarantee is executed, and from private agents to the government upon good aggregate shocks, in which the guarantee is reimbursed. They thus provide aggregate risk insurance in the economy. When the size of the guarantee is large enough, in equilibrium, the banks' funding constraint is not binding and bank lending is as cheap as possible. The government's provision of sufficient aggregate risk insurance thus eliminates scarcity rents associated with the aggregate loss absorption role of banks' equity, maximizing firms' skin-in-the-game and output.[7]

While transfers to firms are part of the financial policy response in support to firms during the Covid-19 crisis for many governments, the provision of fairly priced guarantees on banks' debt liabilities is not. Governments have instead relied on the introduction of guarantees in bank loans to firms. Most supervisory authorities in addition have released the capital buffers banks were required to build in the aftermath of the 2007-09 financial crisis. Capital buffer releases allow banks with access to deposit insurance to operate with higher leverage, and could be interpreted within our model as an extension of non-priced bank debt guarantees.[8] Both bank loan guarantees and non-priced bank debt guarantees provide aggregate risk insurance as they induce larger government disbursements under bad aggregate shocks. Our final result is that those alternative types of guarantee provide a suboptimal substitute for fairly priced bank debt guarantees. The reason is that, in absence of reimbursements from private agents to the government upon good shocks that provide some compensation for the disbursements upon bad shocks associated with guarantees, the government's capability to provide aggregate risk insurance would be limited by its intervention budget. Governments with a low (expected) budget would hence not be able to achieve optimal interventions.

In presence of political economy constraints that prevent governments to intervene through policies that include some form of reimbursement upon future good shocks, the use of non-priced bank debt guarantees (or a release of capital buffers for banks funded with insured deposits) would be nevertheless preferable to the use of bank loan guarantees. The reason

[7]It is possible to prove that an alternative optimal intervention toolkit consists of the combination of transfers to firms with the purchase by the government of fairly priced junior debt or equity issued by banks. The two later policies constitute in fact a substitute for fairly priced debt guarantees in the the provision of aggregate risk insurance in the economy.

[8]Regulated banks have, in practice, to pay fees for access to deposit insurance. Yet, these fees have not been increased with the release of capital buffers.

is that the former leads to government disbursements only upon bad shocks. While, due to idiosyncratic firm's risk, the latter "waste" some of the scarce government's budget in disbursements upon good shocks.

Related literature From a modeling perspective, our paper is mostly related to Holmstrom and Tirole [1998]. That paper focuses on the ex-ante design of contracts for liquidity provision when firms anticipate the possibility of liquidity shocks and, due to moral hazard problems, face constraints on their ex-post external financing capacity. We focus instead on an aggregate unexpected liquidity shock and the ex-post liquidity provision given existing firms and banks' legacy debts. We assume in addition that banks are funded with safe debt, which limits their supply of lending to firms, aggravating the firms' moral hazard problem. The interplay between these two frictions gives rise to amplification mechanisms affecting policy design that are absent in Holmstrom and Tirole [1998].[9]

This paper is also related to theoretical contributions in which frictions give rise to external financing constraints for both banks and firms. Holmstrom and Tirole [1997], Repullo and Suarez [2000], Rampini and Viswanathan [2019], highlight how shocks to the net worth of one of the set of agents gets amplified due to balance sheet linkages, but do not consider optimal intervention design, which is the focus of our paper.[10] The optimality of government transfers to firms or banks during crises is analyzed in Villacorta [2020], which shows in a dynamic macroeconomic model that the optimal transfer target depends on how negative shocks affect the distribution of net worth between banks and firms.[11]

Our paper belongs to the growing literature that analyzes optimal interventions by fiscal or monetary authorities during the Covid-19 crisis.[12] The initial contributions have focused on the role of fiscal and monetary policy interventions in macroeconomic models in which the lockdown gives rises to supply shocks that get amplified through demand factors (Guerrieri, Lorenzoni, Straub, and Werning [2020] and Caballero and Simsek [2020]), or creates falls in demand in some sectors which could potentially propagate to other sectors (Faria-e

[9] Another difference from the set-up in Holmstrom and Tirole [1998] is that we consider a continuous moral hazard problem, so that output not only depends on whether firms are able to continue but also on their overall external claims upon continuation.

[10] In this respect, our paper is also related to Arping, Lóránth, and Morrison [2010], which analyze the optimality of supporting firms' investment through co-funding or loan guarantees in a set-up in which banks' net worth is not relevant.

[11] It is possible to prove that in our model transfers to banks are always weakly dominated by transfers to firms.

[12] A strand of papers has focused instead on the optimal health policy response given the interaction between the evolution of the pandemic and the macroeconomy (for example, Eichenbaum, Rebelo, and Trabandt [2020], Alvarez, Argente, and Lippi [2020], Acemoglu, Chernozhukov, Werning, and Whinston [2020], Jones, Philippon, and Venkateswaran [2020], Correia, Luck, and Verner [2020]).

Castro [2020] and Bigio, Zhang, and Zilberman [2020]). Our paper abstracts from aggregate demand factors and instead focuses on shock amplifications stemming from balance sheet linkages between firms and banks. Regarding the focus on support policies to firms, the closest paper to ours is Elenev, Landvoigt, and Van Nieuwerburgh [2020b], which builds on the dynamic macroeconomic framework with constrained firms and banks developed in Elenev, Landvoigt, and Van Nieuwerburgh [2020a], and assesses quantitatively the effectiveness of the different corporate relief programs introduced in the US. The paper finds that forgivable bridge loans, which simulate the Paycheck Protection Program and could be interpreted as direct transfers in our model, are more effective than purchases of risky corporate debt, which simulate the Corporate Credit Facilities, and partial bank loan guarantees, which simulate the Main Street Lending Program. Our paper contributes to these findings by highlighting the optimality of introducing new policies that provide aggregate risk insurance and are fairly reimbursed in the future, such as the issuance of fairly priced bank debt guarantees.[13]

While in our model banks are unregulated and subject to a market imposed maximum leverage constraint, an alternative equivalent modeling approach would consist of regulated banks with access to fully insured deposits and subject to a regulatory maximum leverage constraint. From this perspective, our paper is also related to the literature that studies leverage regulatory requirements and the implications for banks' liquidity creation through deposits and risk-taking in lending. Begenau [2020] and Begenau and Landvoigt [2018] develop quantitative frameworks to assess the optimal capital requirements of regulated banks that directly manage production in the economy. We instead introduce moral hazard frictions between banks and firms and focus on optimal policy design in the presence of balance sheet linkages between these two sectors.

Finally, this paper is also related to the literature on optimal intervention design during financial or banking crises (see for example, Bruche and Llobet [2014], Philippon and Schnabl [2013], Segura and Suarez [2019]). In those papers, output losses result from asymmetric information or debt overhang problems that originate in the financial sector, and government interventions directly target this sector. In our paper instead output losses stem from firms' moral hazard problems, but we still find a role for the use of policies that target the financial sector, as they indirecly mitigate firms' moral hazard problems by reducing firms' funding cost.

[13] Elenev, Landvoigt, and Van Nieuwerburgh [2020b] also show that transfers would be more effective if they were contingent on firms liquidity needs. The mechanism that drives the optimality of our policy is orthogonal to theirs. In fact, in our model, firms have identical liquidity needs.

The rest of the paper is organized as follows. Section 2 describes the the model set-up. Section 3 characterizes the optimal Social Planner allocations given the government's budget. Section 4 describes how a government can implement optimal allocations in a decentralized manner with a mix of transfers to firms and fairly priced guarantees on banks' debt. Section 5 analyzes whether other types of guarantees introduced in many jurisdictions as a response to the Covid-19 crisis allow to achieve optimal allocations. Section 6 concludes. The proofs of the formal results in the paper are in the Appendix.

2 Model set-up

We first describe the set-up, sequence of events and pay-offs in an economy with no lockdown. We then describe the economy with a lockdown, which is the focus of the paper.

2.1 The benchmark model with no lockdown

Consider an economy with two dates, $t = 0, 1$, and four classes of agents with a zero discount rate: savers, a measure one of entrepreneurs that own firms, a banker that owns a competitive bank that intermediates funds from savers to firms, and a government. All agents except from savers are risk-neutral. Savers have deep pockets and are infinitely risk-averse agents who derive linear utility from consumption at their worst-case scenario (same preferences as in Gennaioli, Shleifer, and Vishny [2013]).[14] Since savers derive zero marginal utility from risky exposures, we can assume that they only invest in riskless assets, which as described below only the bank can issue. Both firms and the bank are run in the interest of their owners, and, at the initial date, have assets and liabilities in place resulting from some unmodeled prior decisions.

Firms At $t = 0$, each firm has a project in place and some debt liabilities. In order to continue the project, the firm has to incur an operating cost ρ at $t = 0$. In absence of a lockdown, such cost is paid by the firm out of the revenue $r_0 \geq \rho$ that the project generates at $t = 0$. To fix our ideas, we assume that $r_0 = \rho$.

Conditional on incurring the operating cost, the project has a pay-off at $t = 1$ of $A > 0$ in case of success, and of zero in case of failure. The success or failure of a project at $t = 1$

[14] For a given set Ω of states of nature at $t = 1$, Gennaioli et al. [2013] define the utility U derived by an infinitely risk-averse agent from a stochastic consumption distribution $(c_1(\omega))_{\omega\in\Omega}$ at $t = 1$ as $U \equiv \min_{\omega\in\Omega} c_0 + c_1(\omega)$.

depends on a firm-specific shock and an aggregate shock that are described below. The probability that the project succeeds is denoted with p and satisfies $p \in [0, p_{\max} < 1]$, where $p_{\max} < 1$. The success probability coincides with the unobservable effort intensity exerted by its entrepreneur between $t = 0$ and $t = 1$. We henceforth refer to the success probability p as the entrepreneur's effort, and also as the project's risk under the understanding that lower values of p correspond to riskier projects. An effort p entails the entrepreneur a disutility cost given by a function $c(p) \geq 0$ satisfying:

Assumption 1. *$c(0) = 0, c'(0) = 0, c'(p_{max}) \geq A, c''(p) > 0$, and $c'''(p) \geq 0$.*

The first-best entrepreneur's effort, denoted with $\overline{p}$, maximizes expected project pay-off net of effort cost, which we compactly refer to as expected output, and is given by:

$$\overline{p} = \arg\max_{p'} \left\{ p'A - c(p') \right\}. \tag{1}$$

Assumption 1 implies that $\overline{p} \in (0, p_{\max}]$ and is determined by the first order condition:

$$c'(\overline{p}) = A. \tag{2}$$

Each firm has at $t = 0$ debt with notional value denoted b_0 that has to be repaid at $t = 1$. This debt is held by the bank that is described next.

Firms' moral hazard The unobservability of the effort choice creates a moral hazard problem. Specifically, for a general debt promise $b \in [0, A]$ at $t = 1$, the entrepreneur's optimal risk choice, denoted $\hat{p}(b)$, maximizes its residual claim net of effort costs:

$$\begin{aligned} \hat{p}(b) &= \arg\max_{p'} \left\{ p'(A - b) - c(p') \right\} \iff & (3) \\ c'(\hat{p}(b)) &= A - b. & (4) \end{aligned}$$

Assumption 1 implies that:

Lemma 1. *For given debt promise $b \in [0, A]$, the effort p chosen by the firm is a function $\widehat{p}(b)$ satisfying*

$$\frac{d\widehat{p}(b)}{db} < 0, \frac{d\left[\widehat{p}(b)A - c(\widehat{p}(b))\right]}{db} < 0, \hat{p}(0) = \bar{p}, \hat{p}(A) = 0,$$

and there exists $b_{max} \in (0, A)$ such that

$$\frac{d\left[\widehat{p}(b)b\right]}{db} > 0 \text{ if and only if } b \in [0, b_{max}).$$

The lemma states that as the total debt promise increases, the projects become riskier (p decreases) and their net pay-off falls. The reason is that when b is larger, the entrepreneur has less incentives to undertake the costly effort because the value created by this action is to a larger extent appropriated by the bank. Moreover, the lemma states that the expected value $\widehat{p}(b)b$ of debt with promise b is increasing in this variable only below a threshold $b_{\max}$. Beyond it, the moral hazard is so severe that additional increases in b reduce the expected value of the debt.

We assume that:

Assumption 2. $b_0 < b_{max}$.

The assumption implies that firms' have some capability to increase the overall value of their debt. We denote $p_0 \equiv \hat{p}\left(b_0\right)$ the firm's risk choice under no lockdown.

Finally, a firm that does not incur the operating cost must liquidate its project and obtains a recovery value of R at $t = 0$. The firm then uses its available funds, amounting to $\rho + R$, to repay debt b_0 and the residual $\left(\rho + R - b_0\right)^+$ is distributed to the entrepreneur.

We make two assumptions:

Assumption 3. $R = p_0 b_0$.

This assumption implies that the expected value of the debt equals the liquidation value of the firm. This would result from the possibility of debt renegotiation in which the outside option of the creditor (the bank, described next) is to liquidate the project and seize R.

Assumption 4. $\rho < \overline{\rho} \equiv p_0 A - c(p_0) - R$.

This assumption implies that the operating cost of the project is low enough so that it is socially efficient to continue the project given the firm's debt and the risk choice it induces.[15] It also implies that entrepreneurs find optimal to continue their projects.

Summing up, in absence of a lockdown firms use their $t = 0$ revenues to pay their operating costs, and continue their projects with risk p_0.

Bank At $t = 0$, there is a representative bank that holds the portfolio of firms' debt with promise b_0 and risk p_0 described above. The bank is funded with deposits with a notional promise d_0 that are due at $t = 1$. Savers hold the bank deposits because they are riskless as the bank takes advantage of the diversification opportunities in the economy, which we describe next.

Specifically, at $t = 1$, an aggregate shock θ that affects the pay-off of all firms' projects is realized. Conditional on the realization of θ, the success probability of a project with

[15] Assumption 3 implies that $\overline{\rho} = p_0(A - b_0) - c(p_0)$, so $\overline{\rho} > 0$ from the optimality condition (3).

risk choice p is θp. Hence, when $\theta > 1$ ($\theta < 1$) the conditional probability of a success is larger (lower) than its unconditional value. In addition, conditional on θ, project pay-offs are independent across firms. The support of the aggregate shock is $[\underline{\theta}, 1/p_{\max}]$, with $\underline{\theta} \in (0,1)$, and its distribution $F(\theta)$ satisfies $E[\theta] = 1$.

We have thus that, for a given aggregate shock θ at $t = 1$, the pay-off of the banks' portfolio of debt is $\theta p_0 b_0$. The pay-off of the banks' assets is thus increasing in θ, with a minimum for $\theta = \underline{\theta}$. Crucially, while the lowest pay-off at $t = 1$ of each of the debt contracts issued by firms is zero, the diversification of their idiosyncratic risks renders the lowest pay-off of the bank's debt portfolio strictly positive.

We assume that:

Assumption 5. $d_0 = \underline{\theta} p_0 b_0$.

The assumption states that the bank's deposits d_0 equal the bank's debt portfolio return in the worst aggregate shock. Hence, the bank deposits are safe and their amount is maximum.

The bank plays no active role at $t = 0$ in the benchmark no lockdown economy.

2.2 Model set-up with a lockdown

We now describe the economy with a lockdown at $t = 0$. The only difference relative to the set-up described in the previous section is that the lockdown reduces firms' revenues at $t = 0$ to r_0= 0. The lockdown thus results in a liquidity shortfall of size ρ for each firm.[16]

Assumption 3 implies that $R < b_0$, so the entrepreneur obtains no value in case of project liquidation. Hence, entrepreneurs will attempt to borrow from the bank ρ units of funds to pay for their operating expenditures. But this would increase their overall debt and aggravate moral hazard problems in effort, reducing expected output in the economy (Lemma 1).

Government support policies and their objective We consider a government whose objective is to limit aggregate welfare losses. We focus on a government that can support firms both directly through transfers and indirectly through fairly priced guarantees on the bank's liabilities. The government can spend in expectation at most an exogenously given amount of funds $X \geq 0$, and anticipates how interventions affect the competitive financing provided by the bank to firms, the firms' risk choice and expected output.

[16]We assume for simplicity that the distribution of the aggregate shock θ does not change as a result of the lockdown, but our results are robust to allowing such distribution to deteriorate provided firms' continuation remains socially efficient.

We assume that:

Assumption 6. *$X \leq \rho$.*

The assumption ensures that the government intervention does not go beyond reducing firms' initial indebtedness.

Before presenting in detail the government's intervention tools, we devote next Section to discuss the problem of a Social Planner (SP) that can choose allocations in the economy to maximize aggregate welfare. We will later prove that the government is able to implement the SP optimal solution in a decentralized manner.

3 The Social Planner optimal allocations

In this Section, we consider the problem of an aggregate welfare maximizer SP that can decide whether firms continue or are liquidated, and, in case of continuation, chooses how to fund the firms' operating cost ρ at $t = 0$, fixes government transfers to and from private agents at $t = 1$, and allocates consumption across agents at $t = 1$. The SP is constrained insofar as: *i)* she cannot choose entrepreneurs' risk; *ii)* she must allocate riskless consumption to savers; *iii)* she must respect the participation constraints of savers, whose required net return is zero, the bank, that has the option to liquidate the firms, and the government, whose expected net transfers are upper bounded by X.

Formally, a SP allocation is described by a tuple $\Gamma = (d_L, \tau_L, c_E(z,\theta), c_B(\theta), c_D, \tau(\theta), p)$ consisting of: the funding mix for the firms' operating cost ρ at $t = 0$ described by new bank deposits $d_L \geq 0$ and a government transfer $\tau_L \geq 0$, consumptions at $t = 1$ by entrepreneurs, $c_E(z,\theta) \geq 0$, which are contingent on the realization $z = A, 0$ of their project and the aggregate shock θ, consumption of the bank, $c_B(\theta) \geq 0$, which is contingent on θ, consumption of depositors, $c_D \geq 0$, a (positive or negative) transfer at $t = 1$ from the government to private agents, $\tau(\theta)$,[17] and a risk choice by entrepreneurs, p.

A SP allocation Γ induces firms' continuation if it satisfies the following *continuation compatibility* constraints:

- Firms can finance their operating cost:

$$d_L + \tau_L = \rho. \tag{5}$$

[17]Since the SP allocation describes all private agents consumption, it is not necessary to be explicit about which private agents receive the transfer. The size of the transfer determines overall aggregate private consumption as described below (see (6)). Also, a negative $\tau(\theta)$ represents a transfer from private agents to the government. Again, it is not necessary to be explicit about which private agents make the transfer.

- Aggregate θ-contingent private consumption at $t = 1$ equals firms' payoff plus government transfers:

$$\theta p c_E(A, \theta) + (1 - \theta p) c_E(0, \theta) + c_B(\theta) + c_D = \theta p A + \tau(\theta), \tag{6}$$

  where the expression takes into account that firms' project risk is p, and conditional on θ a measure θp of the firms have successful projects.

- Old and new depositors receive safe consumption and at least a zero net return:

$$d_0 + d_L \leq c_D. \tag{7}$$

- The bank's expected consumption is not below that under firms' liquidation:

$$E[c_B(\theta)] \geq R - d_0. \tag{8}$$

- The government net expected cost does not exceed its budget:

$$\tau_L + E[\tau(\theta)] \leq X. \tag{9}$$

- Entrepreneurs' risk choice p is optimal given their consumption allocation:

$$p = \arg\max_{p'} \left\{ E\left[\theta p' c_E(A, \theta) + (1 - \theta p') c_E(0, \theta)\right] - c(p') \right\}. \tag{10}$$

For a continuation compatible allocation Γ, social welfare, denoted $Y(\Gamma)$, is given by

$$Y(\Gamma) = E\left[\theta p c_E(A, \theta) + (1 - \theta p) c_E(0, \theta) + c_B(\theta) + c_D - \tau(\theta)\right] - d_L - \tau_L - c(p). \tag{11}$$

The first term accounts for the expected aggregate consumption at $t = 1$ net of government transfers. The second and third terms capture the consumption at $t = 0$ that savers and the government forego to finance the firms' operating cost. The fourth term accounts for the disutility from entrepreneurs' effort.

Using the operating cost funding constraint (5) and the aggregate consumption constraint (6), and that $E[\theta] = 1$, we can express (11) as:

$$Y(\Gamma) = pA - c(p) - \rho, \tag{12}$$

which states that aggregate welfare equals expected firms' output minus effort and operating costs. Hence, social welfare in case of continuation only depends on the effort choice p of the entrepreneurs. From Assumption 1, we have that social welfare is strictly increasing in p for $p < \overline{p}$, where $\overline{p}$ is the first-best effort level described in (1).

The effort optimality condition in (10) implies that effort is given by:

$$c'(p) = E\left[\theta \left(c_E(A, \theta) - c_E(0, \theta)\right)\right], \tag{13}$$

The expression equalizes the marginal cost of effort to its marginal benefit, which amounts to the sensitivity of the entrepreneur's consumption to effort induced by the allocation Γ.

3.1 Optimal allocations inducing continuation

Suppose for the time being that there exist continuation compatible SP allocations. We informally characterize next the one that maximizes welfare.[18] From Assumption 1, the welfare expression (12) and the effort condition (13), we have that the SP will choose the continuation compatible allocation Γ that maximizes the entrepreneurs' sensitivity of consumption to effort, $E\left[\theta\left(c_E(A,\theta)-c_E(0,\theta)\right)\right]$.[19] Hence, she allocates entrepreneurs consumption only upon success of their projects, that is, $c_E(0,\theta)=0$ for all θ. In addition, in order to allocate entrepreneurs as much consumption as possible upon success, we have from the aggregate consumption condition (6), that the SP should minimize consumption of savers and the bank, and maximize the government transfers. This implies that the participation constraints of these agents given in (7) - (9) should be binding.

For convenience, we define the average project payoff of a successful entrepreneur that is allocated to outsiders as

$$b(\Gamma)=A-E[\theta c_E(A,\theta)]. \tag{14}$$

When $c_E(0,\theta)=0$, we have from (4) and (13) that entrepreneurs' effort is given by:

$$p=\hat{p}(b(\Gamma)), \tag{15}$$

where the properties of the function $\hat{p}(.)$ are exhibited in Lemma 1.

Using that in an optimal allocation $c_E(0,\theta)=0$ and (7) - (9) are binding, we have taking expectations in (6), that the optimal average project payoff allocated to outsiders, $b(\Gamma)$, must satisfy:

$$\underbrace{\underbrace{R}_{\text{Legacy investors}}+\underbrace{\rho}_{\text{Oper. cost}}}_{\text{Required value for continuation}}=\underbrace{\underbrace{\hat{p}(b(\Gamma))b(\Gamma)}_{\text{Projects' value to outsiders}}+\underbrace{X}_{\text{Gov.}}}_{\text{Actual value for continuation}}. \tag{16}$$

The expression states that the value that is allocated upon firm continuation to legacy investors ($R-d_0$ to the bank, and d_0 to old savers) and used to pay the operating cost (ρ), must equal the sum of the firms' value allocated to outsiders ($\hat{p}(b(\Gamma))b(\Gamma)$) and the government's contribution (X).

Equation (16) determines the optimal average payoff allocated to outsiders upon firms' success, $b(\Gamma)$, and the associated entrepreneurs' effort, $\hat{p}(b(\Gamma))$, as a function of the government's budget, X. It also shows how the initial cash-flow losses ρ implied by the lockdown

[18]For a formal proof of the arguments done in the next paragraphs before the statement of Proposition 1 see the proof of that proposition in the Appendix.

[19]The SP would nevertheless restrict to allocations such that $E\left[\theta\left(c_E(A,\theta)-c_E(0,\theta)\right)\right]\leq A$, since otherwise effort would be above its first-best level. It is easy to prove that (2) and Assumption 1 imply that such restriction is always satisfied for continuation compatible allocations.

get amplified. In absence of $t = 0$ revenues, the financing of the operating cost requires, at $t = 1$, a higher project payoff allocation to outsiders, $b(\Gamma)$. Yet, this induces a lower effort choice $p = \hat{p}(b(\Gamma))$, which reduces expected output and partially offsets the effect of the increase of $b(\Gamma)$ on the firms' value allocated to outsiders, $\hat{p}(b(\Gamma))b(\Gamma)$. Hence, the payoff $b(\Gamma)$ allocated to outsiders has to be further increased, which leads to an additional effort reduction. This feedback effect amplifies the initial loss. The amplification could be as strong as to render firms' continuation unfeasible (notice that, from Lemma 1, the projects' value allocated to outsiders has a maximum at $b(\Gamma) = b_{\max}$). The government transfers help in mitigating the amplification effects: as the government budget X increases, the average firms' payoff $b(\Gamma)$ allocated to outsiders gets reduced, which increases entrepreneurs' effort $\hat{p}(b(\Gamma))$ and social welfare.

The next result follows.

Proposition 1. *Let $\rho < \overline{\rho}$ be the firms' cash-flow problem and $X \leq \rho$ the government's budget. There exists a threshold $\underline{X}(\rho) \in [0, \rho)$ such that the set of continuation compatible allocations is non empty if and only if $X \in [\underline{X}(\rho), \rho]$. For $X \in [\underline{X}(\rho), \rho]$, a continuation compatible allocation Γ maximizes social welfare among the set of continuation compatible allocations if and only if:*

- *Entrepreneurs' consumption upon project failure is zero, $c_E(\theta, 0) = 0$.*
- *The participation constraints of depositors, the bank and the government in (7) - (9) are binding.*

In addition, the firms' risk choice $p^*(X)$ under any optimal continuation compatible SP allocation is strictly increasing, concave in X and $p^*(X = \rho) = p_0$. Finally, $\underline{X}(\rho)$ is (weakly) increasing in ρ and is strictly positive if $\rho > \underline{\rho}$, with $\underline{\rho} < \overline{\rho}$.

The proposition states that there exist SP allocations that allow the continuation of the firms provided the government has a sufficiently large budget, that is increasing in the size of the cash-flow shock. When continuation is feasible, the optimal intervention that induces continuation minimizes the welfare costs from the entrepreneurial moral hazard by granting the maximum expected consumption to the entrepreneurs compatible with the participation constraint of depositors, the bank and the government and allocates entirely such consumption to entrepreneurs upon the success of their projects. Finally, when the government has a larger budget, the SP is able to provide more consumption to entrepreneurs upon success, which increases their effort and welfare. The latter results are illustrated in Figure 2, which exhibits a numerical example in which the operating cost is not too high ($\rho < \underline{\rho}$), so that continuation is achievable even if $X = 0$.

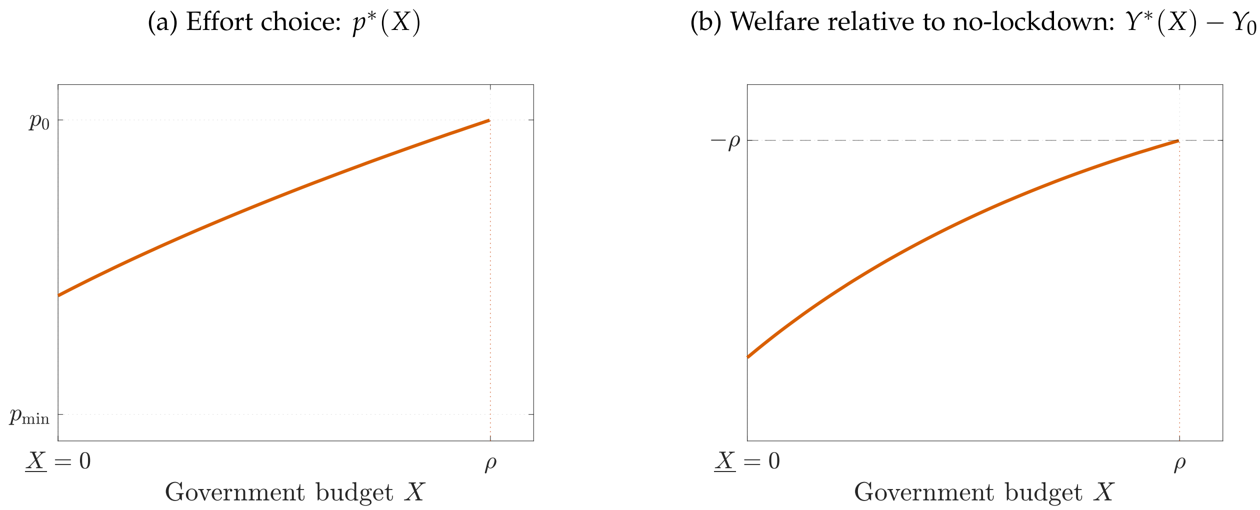


Figure 2: Effort choice and social welfare given government budget

Notes. Effort choice and social welfare difference relative to no-lockdown under any optimal SP allocation Γ for government budget X. Social welfare $Y^*(X)$ for the allocation Γ is defined in (12) and social welfare in the no-lockdown benchmark is $Y_0 = p_0 A - c(p_0)$. The exogenous parameter values used in the numerical illustration are: $A = 1.2$, $c(p) = 0.7p^2$, $\theta \sim U[0.4, 1.6]$, $b_0 = 0.17$, $\rho = 0.075$.

3.2 Optimality of continuation versus liquidation

We next analyze whether, for a government budget such that continuation is feasible ($X \geq \underline{X}(\rho)$, from Proposition 1), the SP finds indeed optimal to continue firms. If firms are liquidated, social welfare amounts to their recovery value, R, so that firms' continuation is optimal if and only if:

$$R + \rho \leq p^*(X)A - c\left(p^*(X)\right). \tag{17}$$

We can show that if for $X = 0$ project continuation is feasible, then continuation is also optimal.[20] This is because, in absence of net transfers from the government to private agents, the social and private value from continuation are the same. Yet, when $X > 0$, the value appropriated upon continuation by private parties exceeds in X the social value from continuation (which also includes the government's costs), so that in some situations continuation could be feasible but not optimal. In those cases, the suboptimal continuation of firms could be interpreted as a form of government induced zombie lending.

We have the next result.

Proposition 2. *Let $\rho < \overline{\rho}$ be the firms' cash-flow problem. There exists a threshold $\widetilde{X}(\rho) \in [\underline{X}(\rho), \rho)$ such that optimal SP allocations are continuation compatible if and only if the government budget*

[20] Suppose that $X = 0$ and there exists a feasible continuation allocation Γ. From (14) and (13), we can write $\hat{p}(b(\Gamma))b(\Gamma) = p(A - E[\theta c_E(A, \theta)]) = p(A - c'(p))$. Thus, (16) implies $R + \rho = p(A - c'(p)) \geq pA - c(p)$, where the last equality arises as $c(p) \leq pc'(p)$ from Assumption 1.

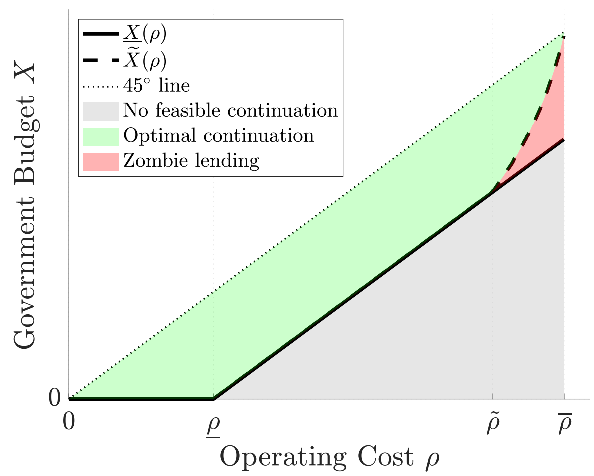


Figure 3: Optimality of firms' continuation given operating cost and government budget

Notes: The figure exhibits the region of optimality of firms' continuation for given ρ and X. In the region labeled "Zombie lending" firms' continuation is feasible but not optimal. The value of the fixed exogenous parameters coincides with that in Figure 2 (except for ρ which is a variable in this figure).

satisfies $X \in [\widetilde{X}(\rho), \rho]$. *Moreover,* $\underline{X}(\rho) < \widetilde{X}(\rho)$ *if and only if* $\rho < \tilde{\rho}$, *with* $\tilde{\rho} \in [\underline{\rho}, \overline{\rho})$ *so that in these cases for* $X \in (\underline{X}(\rho), \widetilde{X}(\rho))$ *suboptimal allocations may lead to the continuation of zombie firms.*

The proposition, whose results are illustrated in Figure 2, states that firms' continuation is optimal only when the government budget is sufficiently large relative to the operating cost (green region). When the government budget is relatively small, amplification of output losses cannot be contained and the firms' liquidation becomes optimal. This can happen either because continuation is not feasible (grey region) or because only the continuation of "zombie" firms whose entrepreneurs would have very little skin-in-the-game would be feasible (red region).

3.3 The government's aggregate risk insurance role

We have so far allowed entrepreneurs' consumption to depend both on idiosyncratic firm risk $z = A, 0$ and aggregate risk θ. Yet, due to the linear utility of the bank, government and entrepreneurs, and the exogeneity of aggregate shocks to entrepreneurs' actions, we can easily prove from (13) that there exist optimal allocations exhibiting aggregate risk independent individual entrepreneur consumption, that is: $c_E(A, \theta) = c_E(A)$, $c_E(0, \theta) = 0$ for all θ. We focus in the rest of the section on this restricted set of SP allocations and show the importance of the provision of aggregate risk insurance by the government. Typical contracts between banks and firms are only contingent on firms' idiosyncratic outcomes and not on aggregate variables. Therefore, these results motivate the government tools presented in Section 4 for the decentralized implementation of optimal allocations, and allow to understand why other government policies discussed in Section 5 are suboptimal.

Consider an optimal continuation compatible SP allocation with θ-independent entrepreneur consumption. Each entrepreneur's consumption can be described by the pay-off b allocated to outsiders upon his firm's success. Since $d_0 + d_L = c_D$, we can write from (6) the expected and worst aggregate shock contingent overall consumption by depositors and the bank as:

$$d_0 + d_L + E[c_B(\theta)] = pb + E[\tau(\theta)], \tag{18}$$

$$d_0 + d_L + c_B(\underline{\theta}) = \underline{\theta} pb + \tau(\underline{\theta}). \tag{19}$$

The LHS in these expressions highlights that the consumption allocated to depositors is constant, while the RHS shows that the firms' project value allocated to outsiders under the worst aggregate shock ($\underline{\theta} pb$) gets reduced relative to its expected value (pb). Such fall must be accommodated by the bank and/or the government. In fact, subtracting (19) from (18) we have the following aggregate risk insurance accounting identity:

$$\underbrace{(1-\underline{\theta})pb}_{\text{Required agg. risk insurance}} = \underbrace{E[c_B(\theta)] - c_B(\underline{\theta})}_{\text{Bank provision agg. risk ins.}} + \underbrace{\tau(\underline{\theta}) - E[\tau(\theta)]}_{\text{Gov. provision agg. risk ins.}}. \tag{20}$$

The LHS of this expression is the reduction of the firms' value allocated to outsiders under the worst shock relative to its expected value, which can be interpreted as the aggregate risk insurance necessary to make savers' consumption riskless. The RHS includes the sum of the reduction in the consumption of the bank under the worst shock relative to its expected consumption and the increase in the government transfers under the worst shock relative to its expected value. Those variations can be interpreted (if positive) as the aggregate risk insurance provided by the bank and the government, respectively.

Using that $d_L + \tau_L = \rho$ and that (8) - (9) are binding, we have from (18) that

$$\underbrace{(1-\underline{\theta})pb}_{\text{Required agg. risk insurance}} = (1-\underline{\theta})(R + \rho - X), \tag{21}$$

which expresses the required aggregate risk insurance as a fraction $1-\underline{\theta}$ of the overall value required for firms' continuation ($R+\rho$) minus the government expected transfer (X). Also, using $c_B(\underline{\theta}) \geq 0$, we have that:

$$\underbrace{E[c_B(\theta)] - c_B(\underline{\theta})}_{\text{Bank provision agg. risk ins.}} \leq R - d_0 = (1-\underline{\theta})R, \tag{22}$$

which states that an upper bound to the aggregate risk insurance provided by the bank is a fraction $1-\underline{\theta}$ of the value that legacy investors must obtain.

The next result follows.

Lemma 2. *Let* Γ *be an optimal SP allocations with aggregate risk independent individual entrepreneur consumption. The government's aggregate risk insurance satisfies:*

$$\tau(\underline{\theta}) - E[\tau(\theta)] \geq (1-\underline{\theta})(\rho - X). \tag{23}$$

The lemma provides a lower bound on the aggregate risk insurance provided by the government in optimal allocations with θ-independent entrepreneur consumption. The lower bound is strictly positive when the government's budget is not enough to cover entirely operating costs ($X < \rho$). The intuition is that the maximum aggregate risk insurance the bank can provide allows only to make legacy deposits riskless (which, recall, were riskless in the no lockdown economy), but additional aggregate risk insurance is needed in order to make the new deposits riskless. Such additional insurance must be provided by the government. Notice also from (23) that a government with a low budget must compensate its inability to directly finance the firms' operating cost with a higher provision of aggregate risk insurance in the economy.

Finally, what happens if the government does not provide sufficient aggregate risk insurance, that is, if $\tau(\underline{\theta}) - E[\tau(\theta)] < (1-\underline{\theta})(\rho - X)$? Notice that, conditional on firms' continuation, sufficient aggregate risk insurance must be provided to ensure the full repayment of deposits under bad aggregate shocks. If the government does not provide sufficient insurance, then, inverting the steps taken above, one can prove that the aggregate risk insurance provided by the bank should increase, that is: $E[c_B(\theta)] - c_B(\underline{\theta}) > R - d_0$. But then $E[c_B(\theta)] > R - d_0$, that is, the bank would appropriate more value. When the government is constrained in its capability to provide aggregate risk insurance, the SP finds increasing bank value as the only way to create the necessary aggregate risk insurance in the economy. From (18), this requires an increase of the firms' value allocated to outsiders, reducing entrepreneurs' effort and social welfare.

4 Implementation with decentralized government policies

In this Section, we focus on the decentralized implementation of SP optimal allocations. Recall from the model set-up that the bank provides standard loan financing to the firms whose repayment is contingent on firms' idiosyncratic risk. As a result, entrepreneurs' consumption is aggregate risk insensitive, and from Lemma 2 government policies must provide sufficient aggregate risk insurance. With this background motivation, we consider a government that makes transfers to firms and provides aggregate risk insurance in the economy by granting fairly priced guarantees on the bank's deposits.

4.1 Government support policies and competitive equilibrium

A government support policy is described by a pair (τ_L, κ) consisting of a government transfer to each firm of $\tau_L \leq X$ units of funds at $t = 0$, and a fairly priced government guarantee on bank deposits described by an aggregate shock parameter $\kappa \in [\underline{\theta}, 1]$ such that the government insures the repayment of deposits at $t = 1$ for aggregate shocks $\theta \in [\underline{\theta}, \kappa]$, and requires fair compensation to the bank for aggregate shocks $\theta > \kappa$.[21] We say that a higher κ corresponds to a larger deposit guarantee as it allows the bank to enjoy deposit insurance for a larger set of aggregate shocks. Notice that the expected cost for the government of the policy (τ_L, κ) is τ_L if there is firms' continuation, and zero otherwise.

For a support policy (τ_L, κ), firms need $\rho - \tau_L \geq 0$ units of bank funding at $t = 0$. If the bank provides such financing in exchange of a promised repayment $b_L \geq 0$ at $t = 1$, then firms continue and the new debt promise b_L adds to the existing one b_0, so that from (3) the firms' risk choice increases, $\hat{p}(b_0 + b_L) \leq \hat{p}(b_0) = p_0$. The bank must issue $\rho - \tau_L$ units of new deposits to finance lending to firms.

The promise b_L for the residual financing is *feasible* if:

- The bank deposits are safe given the guarantee:
$$d_0 + \rho - \tau_L \leq \kappa \hat{p}(b_0 + b_L)(b_0 + b_L). \tag{24}$$
The LHS of the inequality above captures the bank's promise to old and new depositors. The RHS captures that the government's deposit insurance makes deposits safe for aggregate shocks $\theta \leq \kappa$, allowing safe deposit issuance up to a fraction κ of the expected value of the bank's debt portfolio. The guarantee hence relaxes the maximum leverage constraint imposed by savers.

- The bank finds optimal to grant the new financing rather than liquidating the firms:
$$\Pi(b_L) \geq R - d_0, \tag{25}$$
where $\Pi(b_L)$ denotes the bank's expected profits when new lending is granted and the RHS is the bank's payoff in case of firms' liquidation. Since the government guarantee is fairly priced, we have that:
$$\Pi(b_L) = \hat{p}(b_0 + b_L)(b_0 + b_L) - d_0 - \rho + \tau_L. \tag{26}$$

Finally, whenever a feasible promise b_L for the residual financing exists given a policy (τ_L, κ), we define the *competitive promise* $b_L^*(\tau_L, \kappa)$ as the feasible promise with lowest b_L. Notice that

[21] Since $E[\theta] = 1$, for any guarantee $\kappa \leq E[\theta] = 1$ the bank has a sufficient residual claim contingent on shocks $\theta > \kappa$ to reimburse the government for the deposit insurance provided for shocks $\theta \leq \kappa$. See Appendix A for a formal description of the contingent transfers $\tau(\theta)$ associated with a fairly priced guarantee $\kappa \in [\underline{\theta}, 1]$.

such promise maximizes the firms' profits. It can be proved that the competitive promise arises as the outcome of Bertrand competition in the lending market between the bank and a potential new bank entrant that can also benefit from the government guarantee on deposits.

4.2 The competitive bank residual financing

Consider a government intervention (τ_L, κ) with $\tau_L \leq X$ and $\kappa \in [\underline{\theta}, 1]$. Notice $\tau_L = 0$ and $\kappa = \underline{\theta}$ corresponds to no intervention. We next analyze the outcome of the competitive financing of firms' residual funding needs. In order to gain more intuition on how the bank intermediates between savers and firms, we can use Assumption 5 to rewrite the bank's maximum leverage constraint in (24) as:

$$\rho - \tau_L \leq \underline{\theta} \left[\hat{p} \left(b_0 + b_L \right) \left(b_0 + b_L \right) - \hat{p} \left(b_0 \right) b_0 \right] + \left(\kappa - \underline{\theta} \right) \hat{p} \left(b_0 + b_L \right) \left(b_0 + b_L \right). \tag{27}$$

The inequality above can be interpreted as the bank's lending constraint. The LHS captures the new deposits the bank has to issue to finance firms. The two terms in RHS exhibit how the bank can fund those deposits. The first one captures the additional deposits the bank can issue from its new lending if its leverage remained fixed at $\underline{\theta}$. From Lemma 1, this term is increasing in b_L for $b_L \in [0, b_{\max} - b_0]$ so that the bank has some capability to issue new deposits even in absence of deposit guarantees. The second term, which is increasing in the size κ of the deposit guarantee, captures the additional deposit issuance capability due to the increase in leverage allowed by the guarantee. For given κ, the RHS has a maximum at $b_L = b_{\max} - b_0$, so the bank has a maximum capacity to provide deposits. Hence, when $\rho - \tau_L$ is high continuation might only be feasible for high guarantee size κ.

Suppose that there exists a feasible promise b_L. Since provided $b_L \in [0, b_{\max} - b_0]$ the constraints (24) and (25) get relaxed as b_L increases, it is easy to prove that the competitive promise b_L^* makes at least of one of the constraints binding.

Suppose for the time being that the bank's leverage constraint in (24) is binding. We have that the bank's expected profits in (26) can be rewritten as the following function of (τ_L, κ):

$$\Pi(\tau_L, \kappa) = \left(d_0 + \rho - \tau_L \right) \frac{1 - \kappa}{\kappa}. \tag{28}$$

The expression shows that the bank's profits amount to the product of its deposits ($d_0 + \rho - \tau_L$) and a term that captures the rents the bank obtains per unit of deposits ($(1 - \kappa)/\kappa$). For given guarantee κ, the bank's profits are increasing in the funding $\rho - \tau_L$ demanded by firms because the maximum leverage constraint faced by the bank prevents competition from reducing to zero the lending rents the bank obtains. In addition, the rents per unit of deposit obtained by the bank are decreasing in the deposit guarantee κ. The intuition is that an increase in the deposit guarantee, relaxes the bank's funding constraint allowing it to expand its supply of lending (equation (27)). As a result, the competitive bank reduces the promise b_L^*, which leads to a reduction in its profits. Hence, the government provision of aggregate risk insurance through fairly priced deposit guarantees κ allows to reduce the rents the bank obtains from providing new lending to firms.

Notice from (28) that for κ sufficiently close to one, the profits of a bank that were to maximize its leverage given the deposit guarantees would approach to zero, in which case its participation constraint (25) would not be satisfied. Hence, there is a maximum level of guarantees $\overline{\kappa}$ above which the competitive b_L^* makes the participation constraint (25) binding while the maximum leverage constraint (24) is slack. Further increases in the deposit guarantee do not lead neither to increases in bank leverage nor to reductions in b_L^*. Thus, there is a limit to the support that can be given to firms by granting deposit guarantees.

Building on these intuitions we obtain the next result.

Proposition 3. *Let $\underline{X}(\rho) < \rho$ be the continuation feasibility threshold defined in Proposition 1. There exist two increasing functions $\underline{\kappa}(\ell), \overline{\kappa}(\ell) \in [\underline{\theta}, 1)$ defined in the interval $\ell \in [0, \rho - \underline{X}(\rho)]$, with $\underline{\kappa}(\ell) < \overline{\kappa}(\ell)$ for $0 < \ell < \rho - \underline{X}(\rho)$ and $\underline{\kappa}(\ell) = \underline{\theta}$ for ℓ in a neighborhood of zero, such that interventions (τ_L, κ) lay in one of these regions:*

- *If $\tau_L < \underline{X}(\rho)$, or $\tau_L \geq \underline{X}(\rho)$ and $\kappa < \underline{\kappa}(\rho - \tau_L)$: Firms do not obtain bank lending and are liquidated.*
- *If $\tau_L \geq \underline{X}(\rho)$ and $\kappa \in [\underline{\kappa}(\rho - \tau_L), \overline{\kappa}(\rho - \tau_L))$: Firms obtain bank lending and the bank's leverage constraint is binding. The competitive promise $b_L^*(\tau_L, \kappa)$ and the bank's profits $\Pi(\tau_L, \kappa)$ are strictly decreasing in τ_L and κ.*
- *If $\tau_L \geq \underline{X}(\rho)$ and $\kappa > \overline{\kappa}(\rho - \tau_L)$: Firms obtain bank lending and the bank's leverage constraint is not binding. The competitive promise $b_L^*(\tau_L, \kappa)$ is strictly decreasing in τ_L and constant in κ, and $\Pi(\tau_L, \kappa) = R - d_0$.*

The proposition describes how government support policies affect firms' access to financing and the bank's profits. The results are illustrated in Figure 4, in which firms obtain financing only in the colored regions. For a given deposit guarantee, the financing that firms can get is limited by either the bank's capability to raise new deposits (leverage constraint (24), *LC*) or by its willingness to provide lending (participation constraint (25), *PC*). The function $\underline{\kappa}(\rho - \tau_L)$ (orange line) represents the minimum guarantee that allows the bank to obtain the residual financing needs given the limit imposed by the bank's *LC*. The function $\overline{\kappa}(\rho - \tau_L)$ (green dashed line) instead represents the maximum deposit guarantee for which

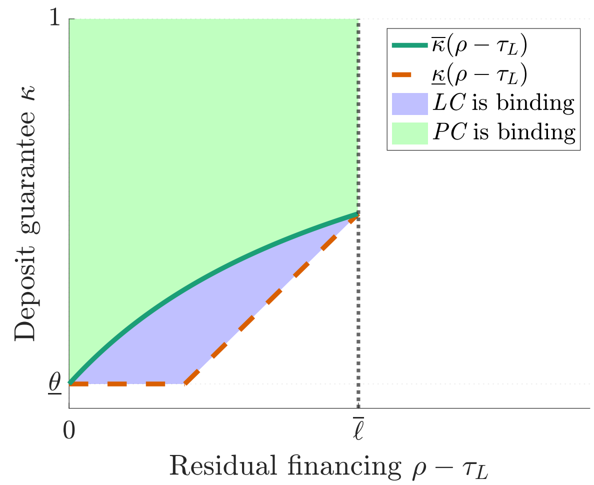


Figure 4: Bank's leverage and participation constraints for given intervention

Notes: The figure exhibits the three regions described in Proposition 3. *LC* refers to the leverage constraint (24) and *PC* to the participation constraint (25). Outside the colored areas continuation is not feasible. The level $\overline{\ell}$ defines the maximum new lending (new deposits) feasible, defined by: $\Pi(b_L = b_{\max} - b_0, \overline{\ell}) = \hat{p}\,(b_{\max})\, b_{\max} - d_0 - \overline{\ell} = R - d_0$. Notice that $\overline{\ell} = \rho - \underline{X}(\rho)$ when $\underline{X}(\rho) > 0$. Parameter values coincide with those in Figure 2 (except for ρ which is a variable in this figure).

a bank that chooses maximum leverage satisfies its *PC*. Hence, in the purple region, the deposit guarantee is large enough to afford banks' sufficient lending capability, but not too large so that the bank chooses maximum leverage and still obtains some profits relative to liquidation. As the guarantee κ increases in this purple region, the associated competitive promise b_L^* and bank profits go down. The government is in fact providing larger aggregate risk insurance, which reduces the equilibrium loss absorption capacity the bank must have, and hence the value of its equity. If the guarantee increases further, the economy enters into the green region, in which the bank's *LC* becomes slack. The reason is that if the bank were to choose maximum leverage, the financing to firms would be so cheap that the bank's *PC* would not be satisfied. An increase in the guarantee κ in this green region, has no effect on bank's leverage choice nor on b_L^*.

4.3 Optimality of the government toolkit and optimal policy mix

We now show that a government with support policies (τ_L, κ) with $\tau_L \leq X, \kappa \in [\underline{\theta}, 1]$ is able to achieve SP optimal allocations interventions and characterize optimal policies (τ_L, κ) as a function of the government's budget X.

Consider a government budget satisfying $X \geq \widetilde{X}(\rho)$, so that from Proposition 2 optimal SP allocations induce continuation, and define the support policy (τ_L, κ) with $\tau_L = X$ and

$\kappa \geq \overline{\kappa}(\rho - X)$, where $\overline{\kappa}(.)$ is defined in Proposition 3. The policy (τ_L, κ) is optimal as it satisfies the conditions in Proposition 1. In fact, taking into account that $\widetilde{X}(\rho) \geq \underline{X}(\rho)$, Proposition 3 implies that (τ_L, κ) induces continuation and makes the bank's participation constraint binding. The remaining three optimality conditions in Proposition 1 are also trivially met. First, in case of project failure, the entrepreneur defaults on the loan and its consumption equals zero, that is: $c_E(\theta, 0) = 0$. Second, since savers provide funding to banks in a competitive market, their participation constraint is binding. Third, the government's budget constraint is binding because $\tau_L = X$.

The next result follows.

Proposition 4. *Let $\widetilde{X}(\rho)$ and $\overline{\kappa}(l)$ the objects defined in Proposition 2 and 3, respectively. If $X \in [\widetilde{X}(\rho), \rho]$, a government support policy (τ_L, κ) induces an optimal SP allocation if and only if $\tau_L = X$ and $\kappa \geq \overline{\kappa}(\rho - X)$. Such support policies induce firms' continuation, bank's leverage equal to $\overline{\kappa}(\rho - X)$ and government provision of aggregate risk insurance equal to $(1 - \underline{\theta})(\rho - X)$. If $0 \leq X < \widetilde{X}(\rho)$ then no government support $(\tau_L = 0, \kappa = \underline{\theta})$ induces liquidation and optimal SP allocations.*

The proposition states that the policy toolkit (τ_L, κ) allows to implement optimal SP allocations through the competitive bank intermediation of funds from savers to firms. Indeed, the government can use its entire (expected) budget to grant transfers to firms and combine them with sufficiently large fairly priced guarantees to bank deposits. The guarantees provide aggregate risk insurance in the economy, allowing the competitive bank to increase its leverage to provide cheap financing to the firms. When the guarantee is large enough ($\kappa \geq \overline{\kappa}(\rho - X)$), the bank does not obtain any rents from new lending as the loss absorption capacity provided by its equity is not any more scarce, and entrepreneurs' effort and aggregate welfare are maximized.[22] The support policy (τ_L, κ) implements an optimal allocation. Notice that the proposition states that the government's provision of aggregate risk insurance, which amounts to the cost of deposit insurance under the worst aggregate shock, is equal to $(1 - \underline{\theta})(\rho - X)$. From Lemma 2, the government is providing the minimum aggregate risk insurance necessary for optimality.

Figure 5 further illustrates the importance of the provision of aggregate risk insurance by comparing the outcome under an optimal $(\tau_L = X, \kappa \geq \overline{\kappa}(\rho - X))$ policy with that under a policy that relies solely on transfers to firms $(\tau_L = X, \kappa = \underline{\theta})$ for different values of the government's budget X. The first difference in the outcome induced by the two policies is that for low X, the firms' continuation is only feasible when the policy includes deposit

[22]Notice instead that any policy in which $\kappa < \overline{\kappa}(\rho - X)$, represented by the purple region in Figure 3, generates rents to the bank, which makes the competitive b_L^* inefficiently high and induces a suboptimal effort choice and welfare.

guarantees.[23] For larger values of X, the two policy toolkits allow the firms' continuation but the competitive promise b_L^* for the residual financing $\rho - X$ demanded by firms is lower when deposit guarantees are used (Panel 5a). This is because deposit guarantees provide aggregate risk insurance that allows the bank to increase its leverage (Panel 5b). When only transfers are used, the bank must provide all the aggregate risk insurance to finance the new deposits, leading to higher bank profits relative to those in the no-lockdown economy, which coincide with those under optimal policies with deposit guarantees (Panel 5c). Finally, the bank rents, induced when only transfers are used, make lending expensive and increase the firms' value allocated to outsiders, aggravating moral hazard problems and amplifying initial output losses (Panel 5d). Notice that as the government budget increases further the differences between the outcomes under the two policy interventions get narrowed. The reason is that as the amount of new deposits raised by the bank is reduced, the need of the aggregate risk insurance provided by the government through deposit guarantees also diminishes.

5 Government policies with other guarantees

We have seen in the previous Section that transfers to firms and fairly priced guarantees on bank deposits constitute an optimal policy toolkit. Yet, while the former policy has been used in many jurisdictions in response to the Covid-19 crisis, the latter policy has not. Governments have instead relied on the introduction of guarantees in bank loans to firms. Also, most supervisory authorities have released capital buffers, whose effect in the context of our model would be equivalent to the introduction bank deposit guarantees that do not have to be reimbursed upon good shocks.

In this section, we analyze the capability to achieve optimal allocations under two alternative government intervention toolkits in which fairly priced bank deposit guarantees are substituted with non-priced bank deposit guarantees and bank loan guarantees, respectively. We find that, when the government budget is low, these policies provide a suboptimal substitute to fairly priced deposit guarantees because they do not provide sufficient aggregate risk insurance.

[23] This happens when $\rho \in (\underline{\theta}\underline{\rho}, \underline{\rho})$, so the operating cost is high enough such that $\underline{\kappa}(\rho) > \underline{\theta}$, but still $\underline{X}(\rho) = 0$.

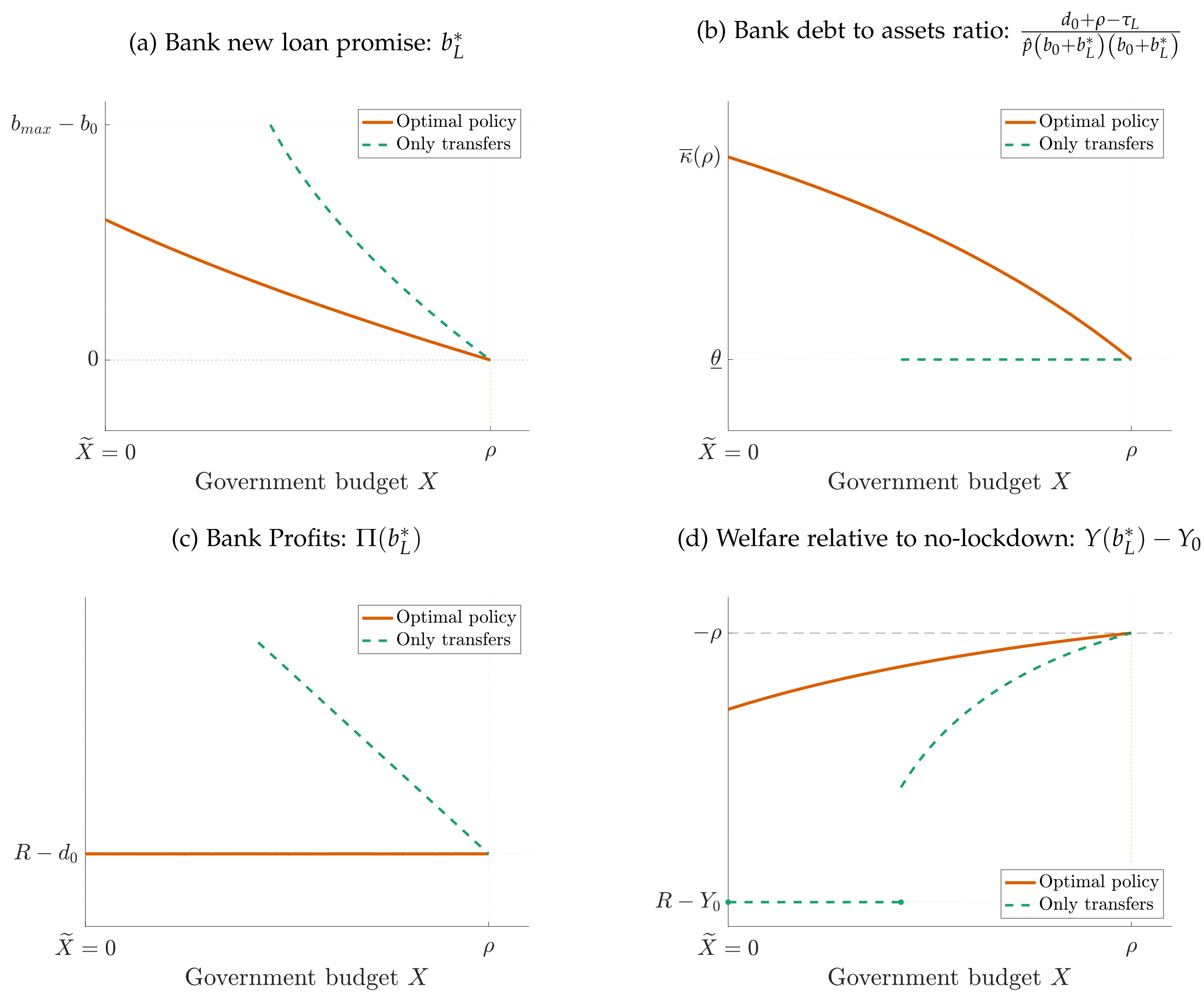


Figure 5: Equilibrium under optimal policies and only transfers for given government budget.

Notes: The figure exhibits the competitive new loan promise, b_L^* , bank debt to assets ratio, $\frac{d_0+\rho-\tau_L}{\hat{p}(b_0+b_L^*)(b_0+b_L^*)}$, bank profits, $\Pi(b_L^*)$, and welfare difference relative to the no-lockdown benchmark, $Y(b_L^*) - Y_0$, under an optimal policy ($\tau_L = X, \kappa \geq \overline{\kappa}(\rho - X)$) and an only-transfers policy ($\tau_L = X, \kappa = \underline{\theta}$) as a function of the government budget X. The value of the exogenous parameters coincides with that in Figure 2.

Alternative toolkit 1: Transfers and non-priced deposit guarantees

As first alternative intervention toolkit, we consider $(\tau_L, \kappa_{\text{free}})$ policies consisting of a transfer $\tau_L \geq 0$ to firms and a bank deposit guarantee described by the variable $\kappa_{Free} \geq \underline{\theta}$ with the only difference that the government grants it for free. Notice that the bank's maximum leverage constraint under this intervention remains as that in (24), while the fact that the deposit guarantee is not repaid by the bank would be captured in its expression for profits that would include an additional term to those in (26) capturing the value of the deposit insurance.[24]

For a policy (τ_L, κ_{Free}) with $\tau_L < \rho, \kappa_{Free} > \underline{\theta}$ that allows firms' continuation, we denote b_L^* the competitive promise in exchange for the bank's residual financing $\rho - \tau_L$. If the bank's maximum leverage constraint is binding for the promise b_L^*, the government's θ-contingent transfer to the bank at $t = 1$ to pay the deposit insurance is

$$\tau(\theta) = (d_0 + \rho - \tau_L - \theta\hat{p}(b_0 + b_L^*)(b_0 + b_L^*))^+ = (d_0 + \rho - \tau_L)\,\frac{(\kappa_{\text{free}} - \theta)^+}{\kappa_{\text{free}}}, \tag{29}$$

where the second equality uses that (24) is binding. The transfer $\tau(\theta)$ is decreasing in θ and equals zero for $\theta > \kappa_{\text{free}}$. Thus, the government provision of aggregate risk insurance is:

$$\tau(\underline{\theta}) - E[\tau(\theta)] = \left(\frac{\kappa_{\text{free}} - \underline{\theta}}{E[(\kappa_{\text{free}} - \theta)^+]} - 1\right) E[\tau(\theta)] > 0. \tag{30}$$

The expression shows that the aggregate risk insurance provided by the government is proportional to its expected expenditure, $E[\tau(\theta)]$, which in turn is limited by its budget, $E[\tau(\theta)] \leq X$. While non-priced deposit guarantees provide some insurance and will allow to improve welfare relative to the sole use of transfers, when the government budget X is sufficiently small relative to the operating cost ρ, the government will not be able to provide the minimum aggregate risk insurance given in (23) necessary to achieve optimality. The toolkit $(\tau_L, \kappa_{\text{free}})$ is hence, in general, suboptimal.[25]

Alternative toolkit 2: Transfers and bank loan guarantees

As second alternative intervention toolkit, we consider (τ_L, γ) policies consisting of a transfer $\tau_L \geq 0$ to firms and a guarantee on the new bank lending to firms described by the fraction $\gamma \in [0, 1]$ of the new lending promise b_L that the government pays to the bank in case of a firm's failure.

For a policy (τ_L, γ), the analogous constraint to (24) establishing the maximum bank

[24]Appendix A provides a more complete formal description of the two alternative policy toolkits discussed in this section.

[25]Yet, it is possible to prove that when X is close to ρ, the government can provide enough insurance and $(\tau_L, \kappa_{\text{free}})$ policies can achieve optimal allocations.

leverage constraint imposed by savers is

$$d_0 + \rho - \tau_L \leq \underline{\theta}\hat{p}\left(b_0 + b_L\right)\left(b_0 + b_L\right) + \left(1 - \underline{\theta}\hat{p}\left(b_0 + b_L\right)\right)\gamma b_L. \tag{31}$$

The second term in the RHS of this expression captures that under the shock $\underline{\theta}$ a fraction $1 - \underline{\theta}\hat{p}(b_0 + b_L)$ of the bank debt portfolio defaults and the bank obtains the government guaranteed amount γb_L. Loan guarantees thus relax the bank's leverage constraint, allowing the bank to provide cheaper financing to firms (lower b_L), which increases welfare.

For a policy (τ_L, γ) with $\tau_L < \rho, \gamma > 0$ that allows firms' continuation, we can denote b_L^* the new lending promise. The government's θ-contingent transfer to the bank at $t = 1$ to repay loan guarantees is

$$\tau(\theta) = \left(1 - \theta\hat{p}(b_0 + b_L^*)\right)\gamma b_L^*, \tag{32}$$

which is a decreasing function in θ and strictly positive for all θ. The government provision of aggregate risk insurance through loan guarantees is:

$$\tau(\underline{\theta}) - E[\tau(\theta)] = \frac{(1 - \underline{\theta})\hat{p}(b_0 + b_L^*)}{1 - \hat{p}(b_0 + b_L^*)} E[\tau(\theta)] > 0. \tag{33}$$

As in our discussion above, the government provision of aggregate risk insurance through loan guarantees is limited by its budget, so it will not provide enough insurance when it has a low budget. The toolkit (τ_L, γ) is hence, in general, also suboptimal.[26]

Comparison of the policy toolkits. Figure 6 compares, for different government budgets, the outcome under the optimal use of the three intervention toolkits: *i)* direct transfers and fairly priced deposit guarantees (τ_L, κ), in orange solid-line, *ii)* direct transfers and non-priced deposit guarantees $(\tau_L, \kappa_{\text{free}})$, in purple dashed-line, and *iii)* direct transfers and banks loan guarantees (τ_L, γ), in black dotted-line. Notice that the plot of the dash and dotted-lines does not start at a zero government budget because, for the parameters in the numerical illustration, firms' continuation is not optimal under the alternative intervention toolkits.

Panel 6a exhibits the share of the government budget that is used for direct transfersunder the optimal policy mix in each intervention toolkit. By construction, fairly priced deposit guarantees have a zero expected cost for the government, which can use its entire budget to grant transfers. In contrast, the government uses some of its budget for the provision of the two other types of guarantees. The figure shows in fact that when the government's budget is low it is optimal to allocate the budget entirely to the provision of guarantees, while transfers are also used when the budget is larger. The reason is that, from Lemma

[26] Again, it is possible to prove that when X is close to ρ, the government can provide enough aggregate risk insurance insurance and (τ_L, γ) policies can achieve optimal allocations.

2, when the government has a low budget the economy has large needs of aggregate risk insurance, which is provided by the guarantees.

Panel 6b shows the aggregate risk insurance provided under each intervention. Recall that the insurance provided under optimal (τ_L, κ) interventions equals, from Lemma 2 and Proposition 4, the minimum insurance provision required to achieve optimal allocations. In contrast, the insurance provided by optimal $(\tau_L, \kappa_{\text{free}})$ and (τ_L, γ) interventions is limited by the government's budget. So, when it is low, they provide less insurance than necessary to achieve optimal allocations. As the budget increases, the insurance provided by these policies increases, while the minimum insurance provision required for optimality decreases. For large enough government budget, the alternative toolkits provide sufficient insurance to achieve optimal allocations. Finally, notice that for low budget, the insurance provided under optimal $(\tau_L, \kappa_{\text{free}})$ interventions is larger than under optimal (τ_L, γ) interventions. The reason is that guarantees on bank deposits only give rise to disbursements to satisfy a shortfall between the payoff of the bank's assets and its overall amount of deposits, so that by definition they lead to the minimum disbursement that ensures the safety of the deposits. Loan guarantees are instead a less targeted way to provide aggregate risk insurance since, due to the presence of idiosyncratic firm risk, they imply a government disbursement even when aggregate shocks are positive and the bank makes profits at the final date. So, part of the budget is used to transfer resources to the bank upon good shocks, which "wastes" some of the scarce government budget at expense of transfers during bad shocks.

Panels 6c and 6d illustrate how aggregate risk insurance provision affects bank profits and social welfare, respectively. When the alternative intervention toolkits do not provide sufficient aggregate risk insurance by the government (that is, for low budget X), the bank must complement the shortfall through the creation of additional loss absorption capacity out of its portfolio of debt to firms. This requires an increase in the equilibrium promise of firms to the bank b_L^*, so that the the safe collateral value of the bank assets $\underline{\theta}\hat{p}(b_0 + b_L^*)(b_0 + b_L^*)$ increases enough to grant the bank additional capability to repay deposits under all contingencies. Thus, the scarcity of loss absorption in the economy to back intermediation from savers to firms, increases firms' borrowing cost, inducing some bank profits and amplifying welfare losses due to the aggravation of firms' moral hazard. Conversely, when the alternative intervention toolkits provide sufficient aggregate risk insurance (that is, for large budget), they minimize bank profits and maximize welfare.

Two final comments on the relationship between the optimal policy mix (Panel 6a), provision of aggregate risk insurance (Panel 6b), bank profits (Panel 6c), and welfare (Panel 6d) under the alternative toolkits are worth. First, it is optimal to make use of direct transfers to firms under the alternative toolkits only when sufficient aggregate risk insurance has already been granted through guarantees, so that bank profits are minimum and welfare is maximum. The reason is that, once the intervention provides sufficient aggregate risk insurance, the financial frictions imposed by savers demanding safety are not longer binding, and additional (bank deposit or loan) guarantees do not induce the bank to increase its leverage further and pass on cheaper financing to firms. As the reduction of the firms funding cost through guarantees loses traction, a government with a large budget must thus combine them with some direct transfers to firms. Second, for a low budget government, the larger capability to provide aggregate risk insurance through $(\tau_L, \kappa_{\text{free}})$ interventions relative to (τ_L, γ) interventions translates into lower bank profits and higher welfare. In presence of political economy constraints that prevent the government to intervene through policies that include some form of reimbursement upon future good shocks, it is thus preferable to rely on bank debt guarantees rather than loan guarantees.

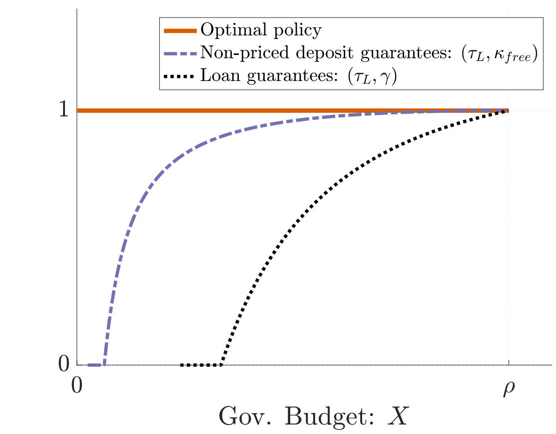
6 Conclusion

We analyze optimal design of interventions of a government with a limited fiscal budget to support firms facing liquidity needs during a lockdown. The analysis is conducted in a novel competitive model of financial intermediation with financial frictions at both the firm and bank level and highlights the amplification of losses through the balance sheets linkages between firms and banks.

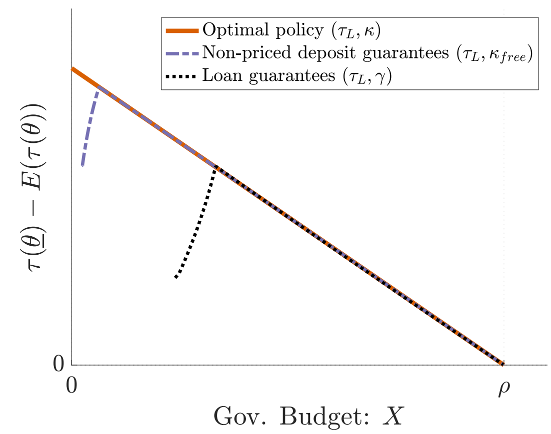
Two crucial assumptions drive the results in our model. First, the increase in debt required by firms to survive the lockdown reduces firms' equity, creating moral hazard frictions that affect both firms' output and the return of banks' loan portfolios. Second, banks must finance their lending to firms from investors which demand safety. The presence of both idiosyncratic and aggregate risk gives a role to banks' equity in addition to firms' equity. The need for aggregate risk absorption capacity to intermediate funds restricts lending supply and leads to equilibrium rents on the scarce banks' equity, making firm borrowing more expensive and further amplifying output losses.

Government interventions aimed at limiting output losses are optimal when their design provides sufficient government exposure to aggregate risk. This constitutes a substitute for the loss absorption role of bank equity, which eliminates bank intermediation rents and reduces borrowing costs and overall firms' indebtedness. Optimal interventions can be implemented with a mix of direct grants to firms and fairly priced guarantees on bank debt.

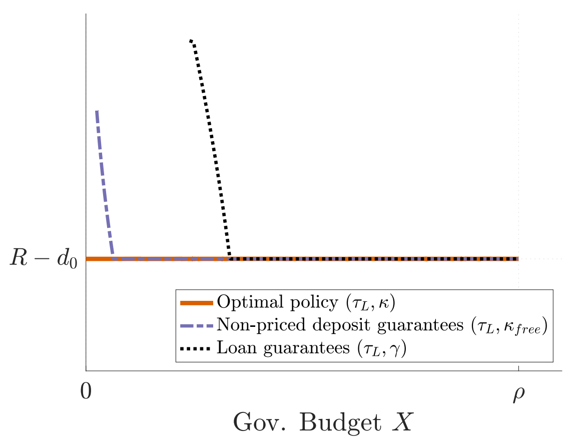
(a) Share of budget used for direct transfers: τ_L/X



(b) Aggregate risk insurance: $\tau(\underline{\theta}) - E[\tau(\theta)]$



(c) Bank profits: Π



(d) Welfare relative to no-lockdown: $Y - Y_0$

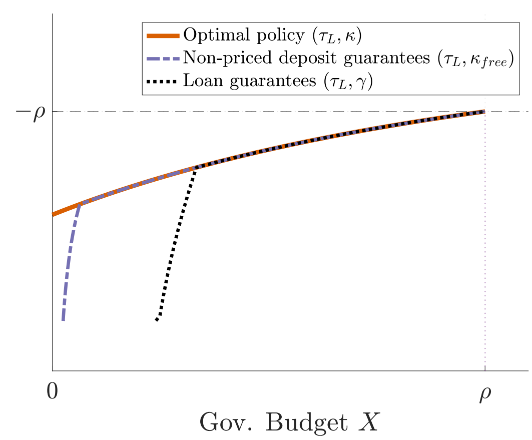


Figure 6: Comparison of intervention toolkits

Notes. Share of budget used for direct transfers τ_L/X, provision of aggregate risk insurance $\tau(\underline{\theta}) - E\tau(\theta)$, bank profits Π, and welfare difference relative to the no-lockdown benchmark $Y - Y_0$, under an optimal policy $(\tau_L = X, \kappa = \overline{\kappa}(\rho - X))$ (solid orange line), the optimal mix of transfers with non-priced deposit guarantees$(\tau_L, \kappa_{\text{free}})$ (dashed purple line), and the optimal mix of transfers with loan guarantees (τ_L, γ) (black dotted line) as a function of government budget X. Parameters' values coincide with that in Figure 2.

The latter provide aggregate risk insurance in the economy, and are repaid by banks upon good shocks, thereby reducing the overall cost of the public intervention.

Finally, some of the common intervention toolkits deployed during the current Covid-19 crisis are not optimal for governments with a low budget as they provide insufficient aggregate risk insurance. That is the case in particular when grants to firms are combined with either non-priced bank debt guarantees or bank loan guarantees. These guarantees induce larger government disbursements upon bad aggregate shocks, and thus constitute a form of aggregate risk insurance in the economy. Yet, since they are not reimbursed by the private sector, they consume part of the government budget that as a result cannot be used to grant transfers to firms.

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A Description of government interventions (Details)

In this Appendix, we provide additional details about the government intervention toolkits discussed in the paper.

Fairly priced deposit guarantees. Let (τ_L, κ) be an intervention belonging to the toolkit described in Section 4.2, where $\tau_L \in [0, X]$ is a transfer to each firm and $\kappa \in [\underline{\theta}, 1]$ represents a fairly priced government guarantee on bank deposits. We next describe the transfers at $t = 1$ from the government to the bank associated with the guarantee.

Given (τ_L, κ), firms need $\rho - \tau_L$ units of additional funds from the bank. Suppose the bank lends the required funds to the firms by issuing $\rho - \tau_L$ units of new deposits, and let b_L^* be the associated promised repayment at $t = 1$. The firm's risk choice is $\hat{p}\left(b_0 + b_L^*\right)$ and the safety of the deposits given the guarantee implies that $d_0 + \rho - \tau_L \leq \kappa \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right)$. The transfers at $t = 1$ from the government to the bank implied by the guarantee κ, denoted with $\tau(\theta|\tau_L, \kappa)$, are given by:

- For $\theta \in [\underline{\theta}, \kappa] : \tau(\theta|\tau_L, \kappa) = \min\left\{d_0 + \rho - \tau_L - \theta \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right), 0\right\}$.
- For $\theta > \kappa$: $\tau(\theta|\tau_L, \kappa) = -\min\left\{\pi, \theta \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right) - d_0 - (\rho - \tau_L)\right\}$ where π is the unique solution to

$$\int_{\kappa}^{\theta_{max}} \min\left\{\pi, \theta \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right) - d_0 - (\rho - \tau_L)\right\} = \int_{\underline{\theta}}^{\kappa} \min\left\{d_0 + \rho - \tau_L - \theta \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right), 0\right\}. \tag{34}$$

Notice that the existence of a solution to the equation results from the fact that $\kappa \leq 1$.

Alternative toolkit 1: Non-priced deposit guarantees. Let $(\tau_L, \kappa_{\text{free}})$ be an intervention belongind to the first toolkit described in Section 5, where $\tau_L \geq 0$ is a transfer to the firms and $\kappa_{\text{free}} \geq \underline{\theta}$ represents a non-priced government guarantee on bank deposits. Let again b_L^* denote the competitive debt promise for the new funding to firms. Analogously to the description above, we have that the transfers at $t = 1$ from the government to the bank implied by the guarantee κ_{free} given the transfer τ_L, denoted $\tau(\theta|\tau_L, \kappa_{\text{free}})$, are given by:

- For $\theta \in [\underline{\theta}, \kappa_{\text{free}}] : \tau(\theta|\tau_L, \kappa_{\text{free}}) = \min\left\{d_0 + \rho - \tau_L - \theta \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right), 0\right\}$.
- For $\theta > \kappa_{\text{free}}$: $\tau(\theta|\tau_L, \kappa_{\text{free}}) = 0$.

The government budget constraint is written as

$$\tau_L + E[\tau(\theta)] = \tau_L + E\left[\left(d_0 + \rho - \tau_L - \theta \hat{p}(b_0 + b_L^*)(b_0 + b_L^*)\right)^+\right] \leq X.$$

The safe deposit constraint for this intervention is the same as in (24). Since the deposit guarantee is not repaid by the bank in this case, the expression for the bank profits in (26) is replaced by

$$\Pi(b_L) = E\left[\left(\theta \hat{p}\left(b_0 + b_L\right)\left(b_0 + b_L\right) - \left(d_0 - \rho + \tau_L\right)\right)^+\right]. \tag{35}$$

If the leverage constraint (24) is binding, we can express the bank profits in (35) as

$$\Pi(b_L^*|\tau_L, \kappa_{\text{free}}) = E\left[\left(\theta\hat{p}\left(b_0 + b_L^*(\tau, \kappa_{\text{free}})\right)\left(b_0 + b_L^*(\tau, \kappa_{\text{free}})\right) - (d_0 + \rho - \tau)\right)^+\right] \tag{36}$$

$$= E\left[\left(\theta\frac{d_0 + \rho - \tau}{\kappa_{\text{free}}} - (d_0 + \rho - \tau)\right)^+\right], \tag{37}$$

so that we have the following compact expression for the bank profits as a function of the intervention $(\tau_L, \kappa_{\text{free}})$:

$$\Pi(\tau_L, \kappa_{\text{free}}) = (d_0 + \rho - \tau_L)\frac{E\left[(\theta - \kappa_{\text{free}})^+\right]}{\kappa_{\text{free}}}. \tag{38}$$

The expression is analogous to (28) and shows that, when the leverage constraint is binding, the bank profits is decreasing in τ_L and κ_{free}. An increase in κ_{free}, allows the bank to obtain more deposits for given unit of assets (collateral), expands the supply of bank lending, and allow the reduction of b_L^*, reducing bank profits. Similar arguments to the ones in Section 4.2 imply that there exists a maximum $\overline{\kappa}_{\text{free}}$ above which the bank participation constraint is binding and further increases in κ_{free} have no effect on b_L^*.

Alternative toolkit 2: Loan guarantees. Let (τ_L, γ) be an intervention belongin to the second toolkit described in Section 5, where $\tau_L \geq 0$ is a government transfer and $\gamma \in [0, 1]$ represents a guarantee on the new bank lending. Let again b_L^* denote the competitive debt promise for the new funding to firms. Analogously to the description above, we have that the transfers at $t = 1$ from the government to the bank implied by the guarantee γ given the transfer τ_L, denoted $\tau(\theta|\tau_L, \gamma)$, are given by:

- $\tau(\theta|\tau_L, \gamma) = (1 - \theta\hat{p}(b_0 + b_L^*))\,\gamma b_L^*$, which captures that the government pays to the bank γb_L^* for the fraction $(1 - \theta\hat{p}(b_0 + b_L^*))$ of firms that default for given θ.

The government budget constraint can be written as:

$$\tau_L + E[\tau(\theta)] = \tau_L + E\left[(1 - \theta p^*)\,\gamma(b_0 + b_L^*)\right] \leq X. \tag{39}$$

In this case, the maximum leverage constraint is described by (31), while the expression for the bank profits is replaced by

$$\Pi(b_L^*|\tau_L, \gamma) = E\left[\theta\hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right) + \left(1 - \theta\hat{p}\left(b_0 + b_L^*\right)\right)\gamma\left(b_0 + b_L^*\right) - d_0 - \rho + \tau_L\right]. \tag{40}$$

From (40), we have that bank profits are increasing in both the transfer τ_L and the guarantee γ for given b_L^*. So, when the bank participation constraint is binding, an increase in γ or in τ_L would both generate a reduction in the loan promise b_L^*. And, using (39), we have that both policies are substitutable in that case.

B Proofs of Lemmas and Propositions

Proof. **Lemma 1.**

We split the proof in a sequence of steps.

i) From Assumption 1 and equations (2) and (4) we have: $\widehat{p}'(b) = -1/c'(\hat{p}(b)) < 0$, $\widehat{p}'(0) = \overline{p}$, and $\widehat{p}'(A) = 0$.

ii) From *i)* and Assumption 1, we have that $\frac{\mathrm{d}[\widehat{p}(b)A - c(\widehat{p}(b))]}{\mathrm{d}b} < 0$.

iii) From *i)* and Assumption 1, we have that $\widehat{p}(b)b$ is concave in b and equals zero at $b = 0$ and at $b = A$. So, it has a maximum at $b_{\max} \in (0, A)$.

□

Proof. **Proposition 1.**

We split the proof in a sequence of steps.

i) Any continuation compatible allocation satisfies $E\left[\theta\left(c_E(\theta, A) - c_E(\theta, 0)\right)\right] \leq A$.

Let Γ' be an allocation for which $E\left[\theta\left(c'_E(\theta, A) - c'_E(\theta, 0)\right)\right] > A$. From (2), (13) and Assumption (1), we have that $p(\Gamma') > \overline{p}$. Also, from (1), we have that $pA - c(p)$ is decreasing in p for $p > \overline{p}$. It is easy to see then that the allocation Γ' is dominated by the planner reducing $c_E(\theta, A)$ by a small amount and instead increasing either the consumption of savers, the bank or the government, so all participation constraints would remain satisfied. The new allocation generates an small reduction of p and an increase of $Y(\Gamma)$ (see (12)), so that Γ' cannot be optimal.

ii) In any optimal continuation compatible allocation, $c_E(\theta, 0) = 0$ almost surely.

Denote Γ the allocation. Suppose on the contrary that there exists a subset Δ of aggregate shocks with positive measure such that $c_E(\theta, 0) > 0, \forall\theta \in \Delta$. Consider an allocation Γ' that only differs from Γ in: i) $c'_E(\theta, 0) = 0, \forall\theta \in \Delta$; ii) p' is given by the condition (10) ii) $\forall\theta \in \Delta$, $\tau'(\theta)$ is the solution to the aggregate consumption equation (6) given the new $c'_E(\theta, 0)$, p' and the other variables as in Γ. We have by construction that $p' > p$ and $\tau'(\theta) < \tau(\theta), \forall\theta \in \Delta$ so that Γ' is continuation compatible. Using *i)* we have that $p' \leq \overline{p}$ and we conclude that $Y(\Gamma') > Y(\Gamma)$.

iii) In any optimal continuation compatible allocation, (7) - (9) are binding.

Denote Γ the allocation. Suppose at least one of the constraints (7) - (9) is not binding. Suppose to fix our ideas that (7) is not binding. Let $\epsilon > 0$ small such that $d_0 + d_L < c_D - \epsilon$. Consider an allocation Γ' that only differs from Γ in: i) $c'_E(\theta, A) = c_E(\theta, A) + \epsilon, \forall\theta$; ii) $c'_D = c_D - \epsilon$; iii) p' is given by the condition (10); iv) $\forall\theta$, $\tau'(\theta)$ is the solution to the aggregate consumption equation (6) given the new $c'_E(\theta, A), c'_D, p'$ and the other variables as in Γ. Then, as in the item above Γ' is continuation compatible and $Y(\Gamma') > Y(\Gamma)$.

Finally when either (8) or (9) are not binding the arguments are analogous.

iv) The function $p\left[A - c'(p)\right]$ is concave with a unique maximum at $p_{min} = \hat{p}(b_{max}) < p_0$.

Immediate from Assumption 1, Lemma 1 and the definition in (4).

v) Let

$$\underline{X}(\rho) = \max\left\{R + \rho - \hat{p}(b_{max})b_{max}, 0\right\}. \tag{41}$$

Then $\underline{X}(\rho) \in [0, \rho)$ and is the threshold defined in the Proposition.

We have that

$$\hat{p}(b_{max})b_{max} > p_0 b_0 = R, \tag{42}$$

where the inequality uses that $b_0 < b_{\max}$ and the last equality uses Assumption 3. Hence we have that $\underline{X}(\rho) \in [0, \rho)$.

Suppose that $X < \underline{X}(\rho)$. Recall that if there exists a continuation compatible allocation for this value of X, then the value of $b(\Gamma)$ in the optimal allocation Γ must satisfy equation (16). But by construction there is no solution to that equation when $X < \underline{X}(\rho)$.

Suppose that $X \in [\underline{X}(\rho), \rho]$. We are going to construct a continuation compatible allocation Γ. In fact we are going to construct an optimal one. Define b as the unique solution to the equation (16) in the interval $b \in [b_0, b_{\max}]$, and let $p = \hat{p}(b)$ be the associated risk choice, we have that $p \in [p_{min}, p_0]$. Define $\forall\theta, c_E(\theta, A) = c'(p), c_E(\theta, 0) = 0, c_B(\theta) = R - d_0$,and set the constants $\tau_L = 0, d_L = \rho, c_D = d_0 + \rho$. Finally, define $\forall\theta$, $\tau(\theta)$ as the solution to the aggregate consumption equation (6) given the other private consumptions. We have by construction that Γ satisfies the condition for continuation compatibility (and also those for optimality).

vi) Let $\underline{\rho} = \hat{p}(b_{\max})b_{\max} - R$. Then $\underline{\rho} \in (0, \overline{\rho})$ and it is the threshold defined in the Proposition.

We have by construction that $\underline{X}(\rho) > 0$ if and only if $\rho > \underline{\rho}$. We have from (42) that $\underline{\rho} > 0$. We also have using Assumptions 1, 3, 4 and $p_{min} < p_0 < \overline{p}$ the inequalities:

$$\underline{\rho} + R = \hat{p}(b_{\max})b_{\max} < p_{min}A - c\left(p_{min}\right) < p_0 A - c\left(p_0\right) = \overline{\rho} + R, \tag{43}$$

and $\underline{\rho} < \overline{\rho}$.

vii) The function $p^*(X)$ defined as the risk choice $p^*(X) = \hat{p}(b^*(X))$ associated to the unique solution of $b^*(X)$ to (16) with $p^*(X) \geq p_{min}$ for $X \in [\underline{X}(\rho), \rho]$ is strictly increasing and concave.

Suppose $X \in [\underline{X}(\rho), \rho)$. Differentiating implicitly (16), we have:

$$0 = \frac{d\left(p(A - c'(p)\right)}{dp}\frac{dp^*(X)}{dX} + 1. \tag{44}$$

Taking into account that $p^*(X) \geq p_{min}$ and that from *iv)* the function $p(A - c'(p))$ is decreasing in p for $p \geq p_{min}$, we conclude that $\frac{dp^*(X)}{dX} > 0$. Differentiating again with respect to X, and using from *iv)* that $p(A - c'(p))$ is concave we conclude that $p^*(X)$ is concave too.

$$0 = \frac{d\left(p(A - c'(p)\right)}{dp}\frac{dp^*(X)}{dX}\left(A - c'\left(p^*(X)\right) - p^*(X)c''\left(p^*(X)\right)\right) + 1. \tag{45}$$

□

Proof. **Proposition 2**

The proof is split in a sequence of steps. We rely on results and notation from Proposition 2 and its proof.

For $\rho \leq \overline{\rho}$ and $X \in [\underline{X}(\rho), \rho]$ we denote with $p^*(X, \rho)$ the risk choice associated to the unique solution of $b^*(X, \rho)$ to (16) with $p^*(X, \rho) \geq p_{min}$. Notice that in this proof we allow for $\rho = \overline{\rho}$. We also denote $Y_C(X, \rho) = p^*(X, \rho)A - c\left(p^*(X, \rho)\right) - \rho$, the maximum welfare that can be achieved under firms' continuation.

i) The function $Y_C(X, \rho)$ is strictly increasing (decreasing) in X (ρ) for $\rho \leq \overline{\rho}, X \in [\underline{X}(\rho), \rho]$.

Taking into account that $p^*(X, \rho) \leq p_0 < \overline{p}$, it suffices to prove that $p^*(X, \rho)$ is strictly increasing (decreasing) in X (ρ). The first statement has already been proven and the second can be proved analogously.

ii) We have $Y_C(\rho, \rho) \geq R$ for all $\rho \leq \overline{\rho}$.

By construction $Y_C(\rho, \rho) = p_0 A - c(p_0) - \rho$, so the result follows from Assumption 4.

iii) Let $\widetilde{X}(\rho) = \min\left\{X \in [\underline{X}(\rho), \rho] : Y_C(X, \rho) \geq R\right\}$. Then, $\widetilde{X}(\rho)$ is the threshold defined in the Proposition.

Taking into account that $p^*(X, \rho)$ is strictly increasing in X we have that for all $X \geq \widetilde{X}(\rho)$ it is optimal to induce continuation.

iv) We have $\widetilde{X}(\rho) < \rho$ for $\rho < \overline{\rho}$.

From the definition of $\overline{\rho}$ we have for $\rho < \overline{\rho}$ that $Y_C(\rho, \rho) = p_0 A - c(p_0) - \rho > R$, and by continuity of $p^*(X, \rho)$ we get that $\widetilde{X}(\rho) < \rho$.

v) For ρ sufficiently close to $\overline{\rho}$ then $\widetilde{X}(\rho) > \underline{X}(\rho)$.

By continuity, it suffices to prove that $Y_C(\underline{X}(\overline{\rho}), \overline{\rho}) < R$. In fact, we have by construction that $p^*(\underline{X}(\overline{\rho}), \overline{\rho}) = p_{min} < p_0$. And using Assumption 4 we have that:

$$R = p_0 A - c(p_0) - \overline{\rho} > p_{min} A - c(p_{min}) - \overline{\rho} = Y_C(\underline{X}(\overline{\rho}), \overline{\rho}). \tag{46}$$

vi) If $\underline{X}(\rho) = 0$ then $\widetilde{X}(\rho) = \underline{X}(\rho)$.

If $\underline{X}(\rho) = 0$, we have that $R + \rho \leq \hat{p}(b_{max}) b_{max}$, and that there exists a solution for $p^*(0, \rho) \in [p_{\min}, p_0]$, then

$$R \leq \hat{p}(b_{max}) b_{max} - \rho < p_{min} A - c(p_{min}) - \rho < p^*(0, \rho) A - c(p^*(0, \rho)) - \rho = Y_C(0, \rho). \tag{47}$$

vii) If $\rho_1 < \rho_2 < \overline{\rho}$ and $\widetilde{X}(\rho_1) > \underline{X}(\rho_1)$, then $\widetilde{X}(\rho_2) > \underline{X}(\rho_2)$.

Suppose that $\rho_1 < \rho_2 < \overline{\rho}$ and $\widetilde{X}(\rho_1) > \underline{X}(\rho_1)$. Then, we have that *vi)* implies that $\underline{X}(\rho_1) > 0$ and thus $p^*(\underline{X}(\rho_1), \rho_1) = p_{min}$. From Proposition 1 we have that $\rho_1 < \rho_2$ implies that $\underline{X}(\rho_2) > \underline{X}(\rho_1) > 0$ and thus $p^*(\underline{X}(\rho_2), \rho_2) = p_{min}$. This implies that $Y_C(\underline{X}(\rho_2), \rho_2) = Y_C(\underline{X}(\rho_1), \rho_1) < R$ and hence $\widetilde{X}(\rho_2) > \underline{X}(\rho_2)$.

viii) There exists $\tilde{\rho} \in [\underline{\rho}, \overline{\rho})$ such that $\widetilde{X}(\rho) > \underline{X}(\rho)$ if and only if $\rho > \tilde{\rho}$.

This is an immediate consequence of *v), vi), vii)* and the fact that by continuity of the function $Y_C(X, \rho)$ the set $\rho < \overline{\rho}$ such that $\widetilde{X}(\rho) > \underline{X}(\rho)$ is an open set.

□

Proof. **Lemma 2.**

The lemma is a direct consequence of (20), (21) and (22).

□

Proof. **Proposition 3**

We refer to the leverage constraint (24) as *LC*, and to the participation constraint (25) as *PC*.

In the proposition, the function $\underline{\kappa}(\ell)$ represents the threshold for the deposit insurance for which the *LC* is feasible for given a residual financing ℓ, while the function $\overline{\kappa}(\ell)$ represent the threshold for which the *LC* is slack because otherwise the bank profits would be lower that $R - d_0$.

Lets first define over the interval $\ell \in [0, \rho - \underline{X}(\rho)]$ the function:

$$\underline{\kappa}(\ell) = \min \left\{ \underline{\theta}, \frac{d_0 + \ell}{\hat{p}\left(b_{\max}\right)\left(b_{\max}\right)} \right\}.$$

From Assumption 5 and Lemma 1, we have that $d_0 < \hat{p}(b_0) b_0 < \hat{p}(b_{\max}) b_{\max}$. So $\underline{\kappa}(\ell) = \underline{\theta}$ for ℓ close to zero, and it is increasing in ℓ.

Now, lets define over the interval $\ell \in [0, \rho - \underline{X}(\rho)]$ the function:

$$\overline{\kappa}(\ell) = \frac{d_0 + \ell}{R + \ell},$$

which corresponds to the value of κ that makes the profits defined in (28) equal to $R - d_0$ for given residual financing $\ell = \rho - \tau_L$. From Assumption 3 and 5, we have that $\overline{\kappa}(0) = \underline{\theta}$, and it is increasing in ℓ with $\overline{\kappa}(\ell) < 1$.

We can see that, for $\ell > 0$, $\underline{\kappa}(\ell) < \overline{\kappa}(\ell)$ if and only if $\ell < \hat{p}\left(b_{\max}\right)\left(b_{\max}\right) - R$, which from (41) corresponds to $\ell < \rho - \underline{X}(\rho)$.

The following steps show that $\underline{\kappa}(\ell)$ and $\overline{\kappa}(\ell)$ are the thresholds define in the Proposition, and that the properties in the proposition hold.

i.1) If $\tau_L < \underline{X}(\rho)$, then firms are liquidated.

Since $\tau_L > 0$, from (41), we must have that $\underline{X}(\rho) = R + \rho - \hat{p}(b_{max})b_{max} > 0$.

Suppose that there exists a feasible continuation promise $b_L^*(\tau_L, \kappa)$. From (26), we have that

$$\Pi(b_L^*) = \hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right) - d_0 - \rho - \tau_L \leq \hat{p}\left(b_0 + b_{\max}\right)\left(b_0 + b_{\max}\right) - d_0 - (\rho - \tau_L) < R - \underline{X}(\rho) - d_0 - \tau_L < R - d_0,$$

which contradicts the *PC*.

For the rest of the proof, assume that $\tau_L \geq \underline{X}(\rho)$:

i.2) If $\kappa < \underline{\kappa}(\rho - \tau_L)$: firms are liquidated.

Suppose that there exists a feasible continuation promise $b_L^*(\tau_L, \kappa)$. We have that:

$$\kappa\hat{p}\left(b_0 + b_L^*\right)\left(b_0 + b_L^*\right) \leq \kappa\hat{p}\left(b_0 + b_{\max}^*\right)\left(b_0 + b_{\max}^*\right) < d_0 + \rho - \tau_L,$$

where the last inequality follows from the definition of $\underline{\kappa}(\rho - \tau_L)$. The above inequality contradicts the *LC*.

ii) If $\kappa \in [\underline{\kappa}(\rho - \tau_L), \overline{\kappa}(\rho - \tau_L))$: Firms obtain bank lending and the bank's leverage constraint is binding. The competitive promise $b_L^*(\tau_L, \kappa)$ and the bank's profits $\Pi(\tau_L, \kappa)$ are strictly decreasing in τ_L and κ.

Since $\kappa \geq \underline{\kappa}(\rho - \tau_L)$, analogous steps to *i.2)* show that there exists values for the loan promise for which the *LC* is satisfied. Let $b_L^{LC} \in [0, b_{\max}]$ be the value for which the *LC* is binding, so

$$\kappa\hat{p}\left(b_0 + b_L^{LC}\right)\left(b_0 + b_L^{LC}\right) = d_0 + \rho - \tau_L. \tag{48}$$

From Lemma 1, we have that b_L^{LC} is unique, and that any other value $b_L^{'} \in [0, b_{\max}]$ that satisfies the constraint must be higher, that is: $\kappa\hat{p}\left(b_0 + b_L^{'}\right)\left(b_0 + b_L^{'}\right) > d_0 + \rho - \tau_L \Leftrightarrow b_L^{'} > b_L^{LC}$. Thus, b_L^{LC} is the minimum value for which the *LC* is satisfied. Moreover, from (28), we have that for such loan promise b_L^{LC}, the bank value equals $\Pi^{LC}(\tau_L, \kappa) = (d_0 + \rho - \tau_L)\left(\frac{1-\kappa}{\kappa}\right) > R - d_0$, where the inequality holds when $\kappa < \overline{\kappa}(\rho - \tau_L)$ by definition of $\overline{\kappa}(\ell)$. Therefore, the b_L^{LC} is the minimum loan promise for which the *LC* and *PC* are satisfied, then the competitive promise $b_L^*(\tau_L, \kappa) = b_L^{LC}$. Equations (48) and (28) imply that $b_L^*(\tau_L, \kappa)$ and $\Pi(\tau_L, \kappa)$ are strictly decreasing in τ_L and κ.

iii) If $\kappa > \overline{\kappa}(\rho - \tau_L)$: Firms obtain bank lending and the bank's leverage constraint is not binding. The competitive promise $b_L^*(\tau_L, \kappa)$ is strictly decreasing in τ_L and constant in κ, and $\Pi(\tau_L, \kappa) = R - d_0$.

Since $\kappa > \overline{\kappa}(\rho - \tau_L) \geq \underline{\kappa}(\rho - \tau_L)$, analogously to *ii)*, let $b_L^{LC} \in [0, b_{\max}]$ be the value that makes the *LC* binding. We have that $\Pi^{LC}(\tau_L, \kappa) = (d_0 + \rho - \tau_L)\left(\frac{1-\kappa}{\kappa}\right) < R - d_0$ because $\kappa > \overline{\kappa}(\rho - \tau_L)$, so in this case the *PC* is not satisfied.

From (26) and Lemma 1, we have that $\Pi(b_L)$ is strictly increasing in $b_L \in [0, b_{\max}]$. So, let b_L^{PC} be the unique value that makes the PC constraint binding:

$$\hat{p}\left(b_0 + b_L^{PC}\right)\left(b_0 + b_L^{PC}\right) = R + (\rho - \tau_L). \tag{49}$$

Thus, we have that b_L^{PC} is the minimum value that satisfies the PC, and that $b_L^{PC} > b_L^{LC}$, so the LC is also satisfied. Therefore, the competitive promise $b_L^*(\tau_L, \kappa) = b_L^{PC}$. From (49), we have that $b_L^*(\tau_L, \kappa)$ is strictly increasing in τ_L and independent of κ.

□

Proof. **Proposition 4.**

The proposition has two statements depending on the value of X, we prove each separately.

i) If $X \in [\widetilde{X}(\rho), \rho]$, a government support policy (τ_L, κ) induces an optimal SP allocation if and only if $\tau_L = X$ and $\kappa \geq \overline{\kappa}(\rho - X)$. Such support policies induce firms' continuation, bank's leverage equal to $\overline{\kappa}(\rho - X)$ and government provision of aggregate risk insurance equal to $(1 - \underline{\theta})(\rho - X)$.

The first sentence of the statement is a direct consequence of Proposition 1 and the arguments in section 4.3. From Proposition 3 we have that for $\kappa \geq \overline{\kappa}(\rho - X)$, the bank chooses leverage $\overline{\kappa}(\rho - X)$.

Finally, under the lowest aggregate shock the government injects: $\tau(\underline{\theta}) = d_0 + \rho - X - \underline{\theta}\hat{p}\,(b_0 + b_L)\,(b_0 + b_L)$, while since the guarantee is fiscally neutral: $E[\tau(\theta)] = 0$. Using that the PC constraint is binding, (49) implies that the government insurance provided equals $\tau(\underline{\theta}) - E[\tau(\theta)] = d_0 - \underline{\theta}R + (1 - \underline{\theta})(\rho - X) = (1 - \underline{\theta})(\rho - X)$, where the second equality follows from Assumption 5. Notice that the policy satisfies the minimum provision required by Lemma 2.

ii) If $0 \leq X < \widetilde{X}(\rho)$ then no government support $(\tau_L = 0, \kappa = \underline{\theta})$ induces liquidation and optimal SP allocations.

From Proposition 2, if $\widetilde{X}(\rho) > 0$, then the optimal SP allocation induces liquidation. Also, if $\widetilde{X}(\rho) > 0$, then $\underline{X}(\rho) > 0$. From Proposition 3, we have that under the no intervention policy $(\tau_L = 0, \kappa = \underline{\theta})$, firms are liquidated.

□