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FORWARD INTEREST RATES AS PREDICTORS OF FUTURE US AND UK SPOT RATES BEFORE AND AFTER THE 2008 FINANCIAL CRISIS

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A feature of the financial crisis rarely mentioned in the academic literature is that forward interest rates remained persistently higher than future spot rates. Yet according to the expectations hypothesis forward interest rates are unbiased predictors of future spot rates. More general theories attribute the forecast errors to term premia. This paper examines whether these theories can explain data for the US and UK that spans the financial crisis and whether alternative approaches provide better forecasts. The main findings are that these theories break down after the financial crisis and, not unexpectedly, that the forecast errors are due mainly to monetary policy.

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Forward interest rates as predictors of future US and UK spot rates before and after the 2008 financial crisis

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Abstract

A feature of the financial crisis rarely mentioned in the academic literature is that forward interest rates remained persistently higher than future spot rates. Yet according to the expectations hypothesis forward interest rates are unbiased predictors of future spot rates. More general theories attribute the forecast errors to term premia. This paper examines whether these theories can explain data for the US and UK that spans the financial crisis and whether alternative approaches provide better forecasts. The main findings are that these theories break down after the financial crisis and, not unexpectedly, that the forecast errors are due mainly to monetary policy.

1 Introduction

A feature of the financial crisis rarely mentioned in the academic literature is that forward interest rates remained persistently much higher than future spot rates. As forward rates are frequently used to forecast future spot rates - the Bank of England's macroeconomic models, for example, used forward rates as their expected future spot rates - this merits investigation. What was its cause? Can it be explained by forward interest rate theory? Was it the result of the financial crisis? Did it also happen prior to the crisis? Can forward rate forecasts be improved by using additional information? Would an entirely different approach to forecasting future spot rates based on time series analysis be better?

The issue of the usefulness of forward interest rates for predicting future spot rates has a long history. A common assumption based on the Expectations Hypothesis is that forward rates are predicted to give unbiased estimates of future spot rates. This was investigated by, for example, Macauley (1938), Hickman (1942) and Culbertson (1975) who rejected the hypothesis. Later studies by Mankiw and Miron (1986), Mishkin (1988), Hardouvelis (1988) and Buser, Karolyi and Sanders (1996) found for the US that the forecasting power varied with the maturity horizon and the time period of the data. A common finding is that forward rates rates have a better forecasting performance at short horizons. Dominguez and Novales (2002), using later data for eight countries, found that for horizons of up to twelve months forward rates gave better forecasts of future interest rates than current spot rates.

The Expectations Hypothesis is based on the assumption that bonds are riskless and so there are no term premia. As pointed out by Fama (1976, 1984), once term premia are allowed, forward rates are not predicted to give unbiased estimates of future spot rates. More generally, the deviation of future spot rates from forward rates is attributed to the presence of a term premium plus changes in the term premium over time until maturity. If these changes are small, or do not vary much over time, then the forecasting error is almost exactly equal to the term premium plus a constant. The implication is that to obtain unbiased forecasts of future spot rates we need to use risk-adjusted forward rates. As term premia tend to increase with time to maturity, the forecast error in unadjusted forward rates will be greater, the longer is the forecasting horizon. This seems to explain the finding of better forecasting performance at shorter horizons. The finding that forecasting performance varies at different time periods can be attributed to term premia changing due to underlying economic conditions.

It is of interest to determine whether these findings are replicated in recent data and whether this could explain the persistence of forward interest rates above future spot rates since the financial crisis. We also investigate a number of related issues. What does the deviation between forward interest rates and future spot rates imply for the theory of forward rates? Does the theory perform better before the financial crisis? What are the main causes of these deviations or forecast errors? Do past spot rates have better information about future spot rates than forward rates? Is there any other information that would improve forecasts, such as macroeconomic variables?

Ultimately, we want to know how best to forecast spot rates and hence to price financial contracts. If, for example, future spot rates exceed forward rates then a forward contract to borrow in the future would be profitable as it would cost less than borrowing at the future spot rate. But if, as seems to be the case after the financial crisis, future spot rates are less than forward rates then the contract would be loss-making. These losses might, of course, be the price of aversion to risk that theory suggests. The findings in this study might go some way to help understanding the strengths and weaknesses of forward rate contracts and how forecasts of future spot rates might be improved. We address all of these questions in this paper using monthly data for the United States from 1986.1-2019.12 and for the U.K. from 1998.1-2012.12.

The paper is set out as follows. In Section 2 we provide a brief theory of forward interest rates. The data are described and presented graphically in Section 3. The focus is on the differences between the data before and after the financial crisis. In Section 4 we provide new evidence on the Expectations Hypothesis and how this is affected by the financial crisis. In Section 5 we examine whether the more general theory of forward interest rates can explain the findings in the previous section. In Section 6 we ask whether better forecasts of future spot rates might be obtained by taking a times series approach that is akin to weak rationality. In Section 7 we assess whether using additional macroeconomic information improves forward rate forecasts of the future spot rate. In Section 8 we examine what might be the main macroeconomic factors that brought about the forecasting failures of forward interest rates. In Section 9 we summarise what our findings imply for the usefulness of forward interest rates in forecasting future spot rates and for the other questions posed in this paper.

2 Forward rates in the term structure of interest rates

A forward interest rate contract can be thought of as consisting of selling a zero-coupon bond with face value 1 that matures in $n+1$ periods and at the same time buying a zero-coupon bond that matures in n periods with the aim of investing this for one period at the spot interest rate prevailing at that time. This contract involves risk as the future spot rate is unknown. For a risk-neutral investor, the expected return from this portfolio is zero and implies that the expected future spot rate equals the certain return on buying and selling these bonds, i.e. the forward rate.

More formally, if $P_{t,n}$ is the price of an n-period zero-coupon bond at time t, with value at maturity of 1, $R_{t,n}$ is the corresponding yield to maturity and $f_{t,t+n}$ is the forward rate from $t+n$ to $t+n+1$ constructed at time t then $P_{t,n}$ can be expressed either as the discounted value

$$P_{t,n} = \frac{1}{[1+R_{t,n}]^n}$$

or, in terms of forward rates, as

$$P_{t,n} = \frac{1}{(1+f_{t,t})...(1+f_{t,t+n-1})} = \prod_{i=0}^{n-1} \frac{1}{1+f_{t,t+i}}$$

It then follows that

$$1+f_{t,t+n} = \frac{P_{t,n}}{P_{t,n+1}} = \frac{[1+R_{t,n+1}]^{n+1}}{[1+R_{t,n}]^n}$$

Or, taking logs, we obtain the approximation

$$\begin{aligned} f_{t,t+n} &= p_{t,n} - p_{t,n+1} \\ &= -nR_{t,n} + (n+1)R_{t,n+1} \end{aligned}$$

where $p_{t,n} = \ln P_{t,n}$. As $P_{t,0} = 1$ and $P_{t,1} = \frac{1}{1+s_t}$ the one period ahead forward rate satisfies $1 + f_{t,t} = \frac{P_{t,0}}{P_{t,1}} = 1 + s_t$ where s_t is the spot rate. With one period remaining on the bond, and its value at maturity known, the one-period ahead forward rate is equal to the spot rate, i.e. $f_{t,t} = s_t$.

The return from holding a bond for one period is given by $h_{t,n}$, where

$$\begin{aligned} 1+h_{t,n} &= \frac{P_{t+1,n-1}}{P_{t,n}} \\ &= \frac{(1+R_{t+1,n-1})^{-(n-1)}}{(1+R_{t,n})^{-n}} \end{aligned}$$

Hence, taking logs

$$h_{t,n} \simeq p_{t+1,n-1} - p_{t,n} = nR_{t,n} - (n-1)R_{t+1,n-1}$$

The no-arbitrage condition for bonds after adjusting for risk is

$$E_t[h_{t,n}] = s_t + \rho_{t,n}$$

where $\rho_{t,n}$ is the risk premium on an n-period bond at time t. Hence

$$E_t[h_{t,n}] = nR_{t,n} - (n-1)E_t[R_{t+1,n-1}] = s_t + \rho_{t,n}$$

This implies that

$$\begin{aligned} R_{t,n} &= \frac{n-1}{n} E_t[R_{t+1,n-1}] + \frac{1}{n}(s_t + \rho_{t,n}) \\ &= \frac{1}{n} \sum_{i=0}^{n-1} E_t[s_{t+i} + \rho_{t+i,n-i}] \end{aligned}$$

Noting that $\rho_{t+n,0} = 0$ for all $n \geq 0$, it follows that

$$\begin{aligned} f_{t,t+n} &= -nR_{t,n} + (n+1)R_{t,\,n+1} \\ &= -\sum\nolimits_{i=0}^{n-1} E_t[s_{t+i} + \rho_{t+i,n-i}] + \sum\nolimits_{i=0}^{n} E_t[s_{t+i} + \rho_{t+i,n+1-i}] \\ &= E_t s_{t+n} + \rho_{t+n,1} + \sum\nolimits_{i=0}^{n-1} E_t[\rho_{t+i,n+1-i} - \rho_{t+i,n-i}] \qquad (1) \end{aligned}$$

In the Expectations Hypothesis (EH) there are no risk premia, hence

$$f_{t,t+n} = E_t s_{t+n} \qquad (2)$$

More generally it is necessary to take the risk premia into account. Thus, in equation (1), the n-period ahead forward rate forecasts the future spot rate together with the current period's risk premium and the sum of one-period changes to all of the expected risk premia over the lifetime of the n-period bond as its time to maturity declines. An approximation to this would omit the last term, or would replace it with the average of the above changes in the risk premia $\overline{\Delta\rho}$ giving

$$f_{t,t+n} \simeq \alpha + E_t s_{t+n} + \rho_{t+n,1} \qquad (3)$$

where $\alpha = -n\overline{\Delta\rho}$. Thus, the risk-adjusted forward rate $f_{t,t+n} - \rho_{t+n,1}$ will give approximately an unbiased estimate of s_{t+n}. The deviation of the future spot rate from the forward rate (the "forecasting error") is approximately

$$\begin{aligned} s_{t+n} - f_{t,t+n} &\simeq \alpha - \rho_{t+n,1} + \varepsilon_{t+n} \qquad (4) \\ \varepsilon_{t+n} &= s_{t+n} - E_t s_{t+n} \end{aligned}$$

i.e. it is approximately minus the term premium on an n-period bond.

3 The data

In order to be able to relate forward rates, which are published on a daily basis by the U.S. Treasury and by the Bank of England, to macroeconomic data, which is available at best on a monthly basis, it is necessary to convert forward rates to a monthly basis. UK data are available for only a limited period of time. For the U.S. we use forward rates from 1986m1-2019m12 based on maturities up to 30 months. For the UK, a complete set of data for maturities up to 60 months is available but is only published for the period 1998m1-2012m12. Crucially, both data sets span the financial crisis of 2007-8 which has a big impact on both.

Figure 1

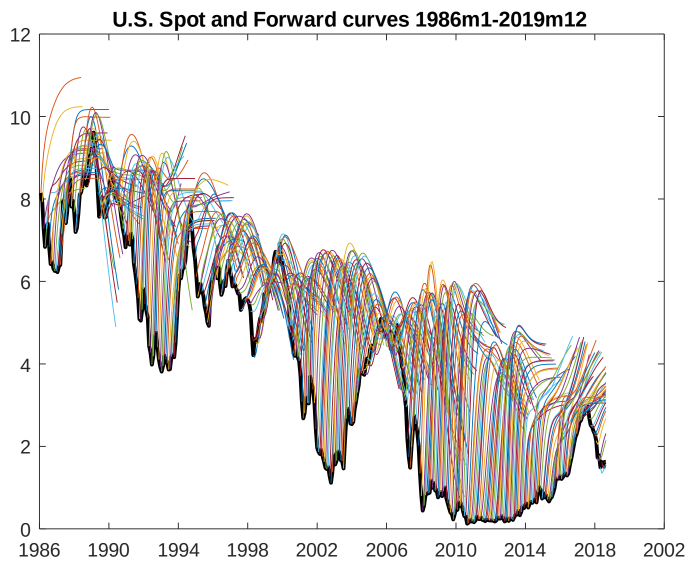
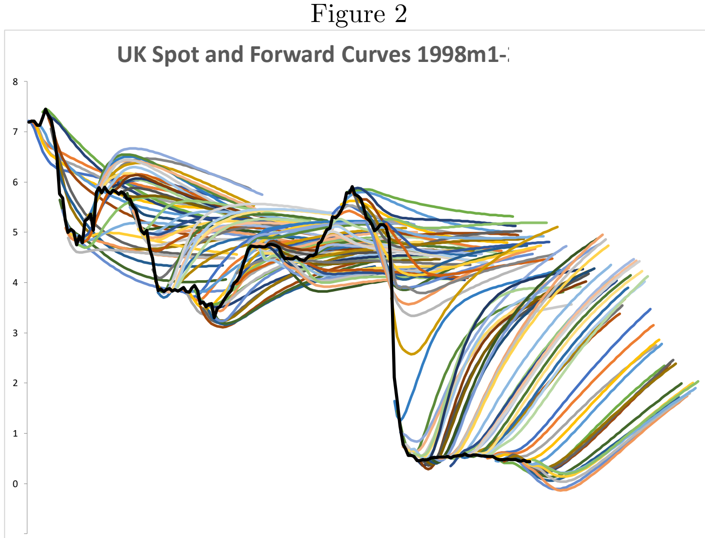


Figure 2



Figures 1 and 2 provide the key evidence that informs all of the more formal tests to be carried out later. They plot for each month the spot rate and the set of forward rates at that date for horizons of 1 to 30 months ahead (US) or 1 to 60 months ahead (UK). The spot rates are highlighted as the heavier line. If the forward rates were good forecasts of the future spot rate they should lie close to the spot rate. Instead, and significantly, they deviate increasingly with the forecast horizon. For the US, the forward rate curves tend to lie persistently above the spot rate and do so increasingly as the horizon increases. This is even more pronounced after the financial crisis in 2008m9. For the UK, before the financial crisis, the forward rate curves tend to mean revert and so miss the fluctuations in the spot rate. After the crisis, like the US forward rates, they bear little relation to the future spot rate as they persistently forecast that the spot rate will rise when it didn't. The possibility that forward rates provide unbiased forecasts of future spot rates already seems unlikely.

Figure 3. U.S. Spot and Forward rates 1986m1-2019m12

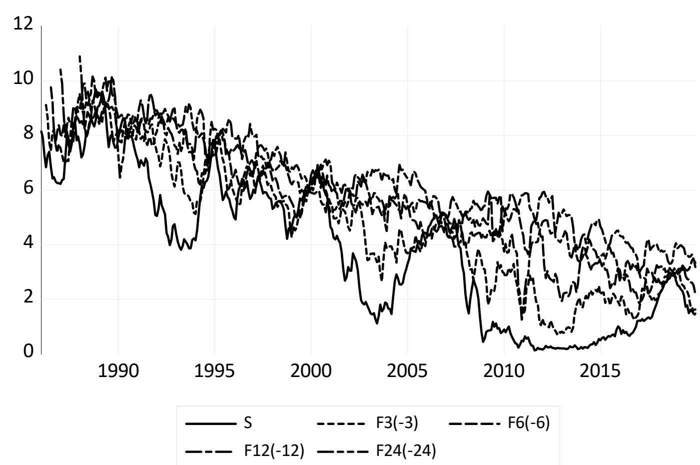
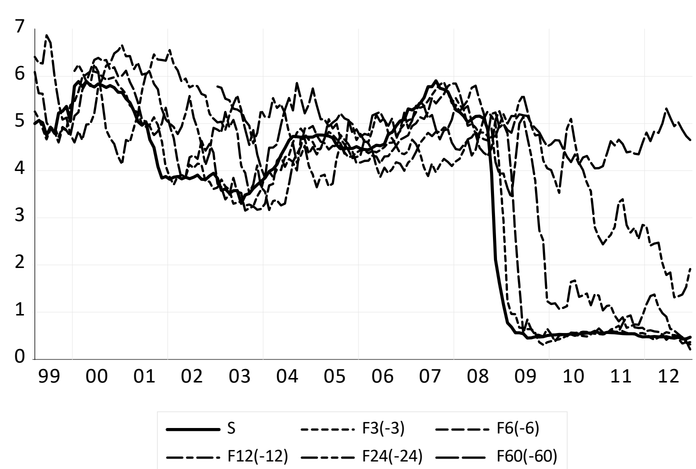


Figure 4. U.K. Spot and Forward rates 1989m1-2012m12



Our later empirical analysis focuses on horizons of 3, 6, 12 and 24 months. In Figures 3 and 4 we plot for each month the spot rate and these forward rates for the U.S. and U.K. i.e. s_{t+n} and $f_{t,t+n}$ for $n = 1, 3, 6, 12, 24$ and, in addition, for the UK, 60 months. The forward rates are aligned with the spot rates, i.e. they are lagged by the length of their forecast horizon so that at each point of time the forward rate is a forecast of the spot rate at that date. Perfect forecasts would result in a single line. For both countries the two graphs show a sharp fall in spot rates after the financial crisis, but not in the forward rates. For the US the forward rates are above the spot rate for most of the data period, and considerably above after 2008. For the UK forward rates are generally above the spot rate until 2003 and then far above from 2008. Just before the financial crisis, from 2005 to 2008, they are usually higher. According to the theory of forward interest rates the negative values of the deviations can be attributed, not to forecasting errors, but to risk premia. The greater the deviation of forward rates above the spot rate, the larger is the contribution of risk premia. This would imply that risk premia were larger after the financial crisis than before.

The deviations s_{t+n}-$f_{t,t+n}$ can be seen more easily from Figures 5 and 6. They are generally negative and even more so after the financial crisis. For the US they also show a degree of cyclicality before the crisis.

Figure 5. US Deviations of forward rates from future spot rates, 1986m1-2019m12

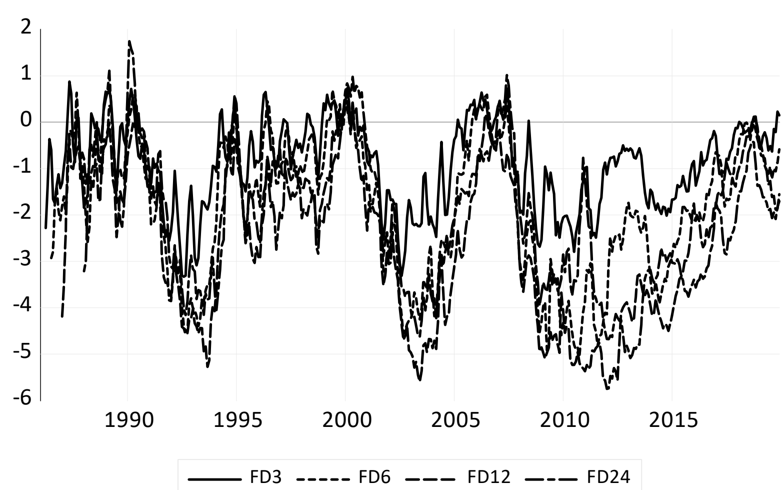


Figure 6. UK Deviations of forward rates from future spot rates, 1998m1-2012m12

Table 1. Forecast errors

	US				*UK*			
	$1986m6-2008m8$		$2008m9-2012m12$		$1997m1-2008m8$		$2008m9-2012m12$	
Horizon	*Mean*	*AR*(1)	*Mean*	*AR*(1)	*Mean*	*AR*(1)	*Mean*	*AR*(1)
3	−0.800 (0.978)	0.90	−1.160 (0.752)	0.92	−0.032 (0.382)	0.52	−0.108 (0.442)	0.58
6	−1.521 (1.418)	0.96	−2.488 (1.394)	0.96	−0.151 (0.797)	0.80	−0.197 (0.572)	0.55
12	−2.088 (1.548)	0.96	−3.313 (1.676)	0.98	−0.439 (1.213)	0.93	−0.653 (0.645)	0.63
24	−1.651 (1.408)	0.95	−3.012 (1.094)	0.96	−0.909 (1.810)	0.97	−2.019 (0.675)	0.80

Note: Standard errors are in parentheses

Table 1 reports $s_{t+n} - f_{t,t+n}$, i.e. the forecast error or deviation from the future spot rate - approximately minus the term premium for an n-period bond - together with its first-order partial autocorrelation coefficient (the first-order autocorrelation coefficient for an AR(1)). The mean errors are all negative indicating that forward rates are greater than the corresponding future spot rate - i.e. the term premium is positive - and they increase with the forecast horizon. Although large for the US, none are significantly different from zero for either the US or the UK as their standard deviations are large and increase considerably with the forecast horizon. The forecast errors are highly serially correlated. For the US a unit root cannot be rejected at the 5% level at all horizons. They are also very close to a random walk implying that changes in the term premium are independent; they also have means close to zero. Nor can a unit root be rejected for the UK at horizons above 6 months. Changes in the term premium are a first-order autogression with an autogressive coefficient that is always small but increasing with the horizon. Their means are close to zero.

The broad picture that emerges is that the forward deviations (forecast errors or term premia) are highly persistent but changes in the term premia are not persistent, especially for the US. The average levels of the term premia are not significantly different from zero due to their large standard deviations, while their changes are both small and insignificant. The lack of significance of the forecast errors indicates that forward rates might still be unbiased forecasts of future spot rates as the expectations hypothesis suggests. The persistence of the term premia is more consistent with the general theory of forward rates. The lack of persistence of changes in the term premia together with their small size suggests that the simpler form of the general theory - equation (3) - might be adequate for use in empirical analysis.

4 The Expectations Hypothesis for forward interest rates

According to the expectations hypothesis (EH), forward rates are predicted to be unbiased estimates of future spot rates. We have reported that for each forecast horizon the mean forecast errors are not significantly different from zero which supports the EH. However, Figure 1 shows that US forward rates have a systematic tendency to exceed spot rates; the longer the horizon, the greater is the gap. Figure 2 also shows that before the crisis, UK forward rates, while not being accurate forecasts of future spot rates, fluctuated between being above and below spot rates. After the crisis, UK forward rates also greatly exceed the spot rate.

Further evidence is provided by tests of $E_t s_{t+n} = f_{t,t+n}$ through the model

$$s_{t+n} - f_{t,t+n} = \alpha + (\beta - 1) f_{t,t+n} + e_{t+n} \tag{5}$$

Unbiasedness implies that $\alpha = 0$ and $\beta = 1$. OLS estimates of $\beta - 1$ together with two standard deviation bands are shown for the US and the UK in Figures 7 and 8.

Figure 7

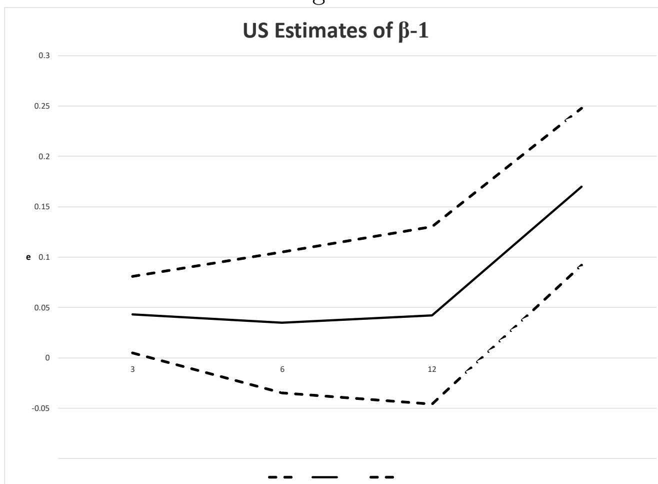
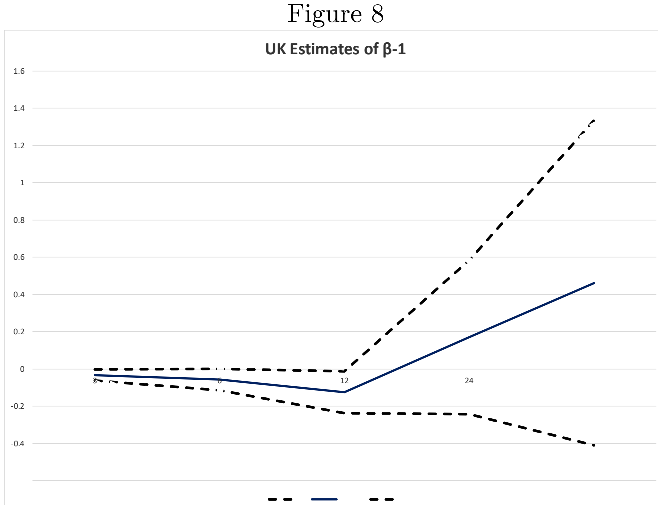


Figure 8



The estimates of β for the US and the UK are similar. They are just below 1 for forecast horizons up to 12 months, but not significantly different. The intercept α is always negative and significantly different from zero. For the 24-month horizon the estimate of β is greater than 1, but again not significantly different from 1. The estimates suggest that with a small mean correction forward rates would give unbiased forecasts of the future spot rate for horizons up to 12 months.

Based on the whole sample, therefore, the EH is not rejected for short horizons. But it is clear from Figures 1-6 that the forecasting performances of forward rates are very different before and after the financial crisis, i.e. from September 2008. These tests ignore this break. The estimates of β could be different over the two periods and could affect the very different forecasting performances before and after the crisis. We might, for example, expect forward rates to forecast better before the crisis when financial conditions were "normal", though not during, or possibly after, the crisis. To examine this we use dummy variables to split the data into before and after the financial crisis.This enables us to test for any significant break between the two periods. To obtain estimates for just the data after the crisis it is necessary to add the coefficients.

For the US we estimate the model

$$s_{t+n} - f_{t,t+n} = \alpha + Dum1_t + (\beta - 1) f_{t,t+n} + \gamma Dum1_t * f_{t,t+n} + e_{t+n} \qquad (6)$$

where $Dum = 0$ $(1986.1 - 2008.8)$ and 1 $(2008.9 - 2019.12)$. For the UK we estimate

$$\begin{aligned} s_{t+n} - f_{t,t+n} &= \alpha + Dum2_t + Dum3_t + (\beta - 1) f_{t,t+n} + \gamma Dum2_t * f_{t,t+n} \qquad (7) \\ &\quad + \gamma Dum3_t * f_{t,t+n} + e_{t+n} \end{aligned}$$

where $Dum1 = 2$ $(2008.9 - 2009.3)$ and 0 elsewhere, and $Dum3 = 1$ $(2009.4 - 2012m12)$ and 0 elsewhere. The reason for this different treatment of the financial crisis is that interest rates adjusted more slowly in the UK following the financial crisis than did those for the US, which fell rapidly. The US therefore appears to have two regimes whereas the UK seems to have three. The briefness of the UK's second regime will make the estimates over this period unreliable. We will not therefore comment on these estimates. The purpose on having this regime is to avoid it distorting the post-crisis estimates. The results are reported in Table 2.

Table 2: Tests of unbiasedness for US and UK forward rates

	US				*UK*			
n	3	6	12	24	3	6	12	24
c	−0.867 [4.33]	−0.870 [2.57]	−1.612 [3.61]	−1.168 [3.27]	0.473 [2.90]	0.743 [2.13]	2.185 [3.97]	1.827 [1.58]
$Dum1$	0.721 [2.28]	2.411 [4.77]	4.190 [6.68]	2.975 [4.09]				
$Dum2$					−0.016 [0.05]	−5.768 [2.00]	3.458 [1.11]	14.297 [0.92]
$Dum3$					−0.296 [0.96]	−0.793 [1.88]	−2.403 [3.84]	1.783 [0.68]
$f_{t,t+n}$	0.010 [0.302]	−0.098 [2.01]	−0.067 [1.11]	−0.073 [1.41]	−0.104 [3.18]	−0.186 [2.65]	−0.522 [4.76]	0.154 [0.67]
$Dum1 * f_{t,t+n}$	−0.496 [4.24]	−1.089 [9.33]	−1.329 [11.31]	−1.156 [6.90]				
$Dum2 * f_{t,t+n}$					−0.253 [3.04]	1.046 [1.71]	−1.200 [1.61]	2.611 [0.82]
$Dum3 * f_{t,t+n}$					−0.216 [0.45]	0.178 [0.56]	0.378 [2.21]	−0.562 [0.97]
$Mean\ s_{t+n} - f_{t,t+n}$	−0.928	−1.878	−2.544	−2.133	−0.057	−0.170	−0.501	−1.108
SE	0.892	1.231	1.311	1.217	0.345	0.709	1.023	1.722
DW	0.16	0.08	0.07	0.08	0.75	0.31	0.16	0.06

Notes: US: 1986.1-2019.12; UK: 1998.1-2012.12. Dum1=1 (2008.10-2019.12) Dum2=1 (2008.9-2009.3) Dum3=1 (2009.4-2012.12), t-statistics are in brackets

The results for the US and the UK are now very different. For the US, before the crisis, and once intercept-adjusted, the estimates of β are generally not significantly different from 1. Forward rates therefore give unbiased estimates of future spot rates. But after the crisis there is a significant break in the estimates of β at all horizons. The negative sign on the coefficient of $f_{t,t+n}$ after the crisis indicates that the implied estimates of β are significantly less than unity. After the crisis forward rates greatly exceeded the future spot rate. To compensate for this β needs to be less than unity. The Durbin-Watson statistics indicate high positive serial correlation in the disturbances at all horizons. The forecasting errors are therefore very persistent. Put another way, after the crisis, forward rates fail to adjust to the new spot rate regime. From the perspective of the EH, this is a surprising finding as it implies persistent and systematic forecast errors.

In contrast to the US, the results for the UK show that before the crisis, but except at the 24-month horizon, the estimates of β are significantly less than 1. Forward rates, therefore, give strongly biased estimates of future spot rates. There are no significant breaks in the estimates of β after the crisis, except at the 12-month horizon. Nonetheless, with the exception of the 3-month horizon, combining the estimates before and after the crisis, to give estimates just for the post-crisis data, indicates that values of β are closer to, and not significantly different from, 1.

To summarise, the results for the US and the UK are very different once allowance is made for the financial crisis. The US results before the crisis do not reject the EH but after the crisis they strongly reject the it. This applies at all horizons. The UK results imply that the EH is rejected before and after the crisis. The rejection is strongest at shorter horizons. Thus support for the EH is only found for the US data before the crisis but, with the exception of the 3-month horizon, not afterwards. Even then there is strong residual serial correlation suggesting persistent forecasting errors. As the EH is unable to provide an explanation of most of these findings we turn to the general theory of forward interest rates.

5 The general theory of forward interest rates

The most likely explanation of the above results is the general theory of forward rates which predicts that forward rates will not be unbiased forecasts of future spot rates. It says that the forecast errors at different horizons are a measure of the expected term premia for different maturities as in equation (3), possibly adjusted for changes in the term premia over the remaining lifetime of the bond, equation (1). The high persistence of the forecast errors reported earlier and the substantial residual serial correlation in the equations used to test the expectations hypothesis is consistent with the term premia for each maturity changing slowly over time.

If we re-write equation (1) as

$$s_{t+n} - f_{t,t+n} = \alpha + (\beta - 1) f_{t,t+n} - \rho_{t,n} - \sum_{i=0}^{n-1} E_t[\rho_{t+i+1,n-i} - \rho_{t+i,n-i}] + \varepsilon_{t+n} \tag{8}$$

then a negative estimate of $\beta - 1$ (even though in fact $\beta = 1$) would be produced by omitted-variable bias due to a positive correlation between $f_{t,t+n}$ and $\rho_{t,n} + \sum_{i=0}^{n-1} E_t[\rho_{t+i+1,n-i} - \rho_{t+i,n-i}]$. The large biases for the US after the financial crisis could then be attributed to high term premia at all maturities which raise forward rates and worsen the ability of forward rates to forecast future spot rates. A similar conclusion applies to the UK which has large biases prior to the financial crisis.

If we re-interpret equation (3) as approximately $s_{t+n} - f_{t,t+n} = -\rho_{t,n}$ then the problem of explaining the forecast errors would become one of modelling the term premia. This would take us into the vast literature on the term structure where there is no consensus on how best to model the term premia. The main findings are that the term structure can be almost fully explained by three factors: a shift factor (the short rate), a slope factor (the term spread) and a curvature factor (for example, the change in the term spread). These three factors explain about 70%, 20% and 5%, respectively, of the variations in yields, see for example Dai and Singleton (2000). In latent factor models of the term structure these factors also explain the term premia. These factor models are not, however, designed for forecasting. Their focus is on within-sample explanations of the term structure, and for this they use the whole sample of current and future yields.

General equilibrium models of the term structure explain the term premia predominantly by consumption growth, and inflation, for example, Balfoussia and Wickens (2007). But due to the low variance of consumption growth and inflation and the high variance of holding period returns, the fit is poor and the variance explained by the explanatory variables needs amplifying by having a high coefficient on consumption growth, usually interpreted as the coefficient of relative risk aversion. Adding output and money growth as factors improves fit a little - Ang and Piazzesi (2003) - but assuming non-additive utility does not, Smith, Sorensen and Wickens (2008). The problem for our purposes, as shown by Balfoussia and Wickens, is that the general equilibrium model gives very poor forecasts of future yields. Rather than employing these models of the term structure to model the term premia, therefore, we maintain our focus on forecasting future spot rates by using time series forecasting models that seek to exploit only current information in the term structure.

6 A time series approach to forecasting future spot rates

Fama (1976, 1984) found that for the two periods 1953-1974 and 1959-1982 including the current spot rate, which is available when forward rates are calculated, in addition to the forward rate, improved the forecasts of future spot rates and, moreover, predicted better that forward rates alone. Given that both the spot rate and the forward rates are non-stationary and close to random walks (or martingales), and hence highly persistent, it is not surprising that the current spot rate might have information about future spot rates. We therefore add the spot rate to equations (6) and (7) giving equations of the form

$$s_{t+n} - f_{t,t+n} = \alpha + (\beta - 1) f_{t,t+n} + \gamma s_t + e_{t+n} \tag{9}$$

This equation is based only on current information on forward and spot rates. If we allow, in addition, for past information then it can be interpreted as being based on weakly rational expectations. We also allow once more for possible breaks in the model before and after the financial crisis. The results for the US and UK are reported in Table 3.

Table 3: Forecasting deviations of US and UK spot from forward rates including current spot rates

	US				*UK*			
n	3	6	12	24	3	6	12	24
c	0.194 [1.47]	0.225 [1.00]	−0.749 [2.01]	−1.146 [3.20]	0.474 [2.95]	0.833 [2.47]	2.160 [3.86]	−1.377 [1.27]
$Dum1$	0.059 [0.29]	0.655 [1.94]	2.720 [5.06]	2.971 [4.08]				
$Dum2$					−1.423 [2.33]	46.380 [2.55]	10.624 [0.64]	−15.856 [0.93]
$Dum3$					−0.243 [0.76]	−0.866 [2.00]	−2.436 [3.52]	1.262 [0.51]
$f_{t,t+n}$	−1.007 [20.26]	−0.940 [19.00]	−0.633 [9.61]	−0.129 [1.88]	−0.092 [0.70]	0.164 [1.08]	−0.564 [3.24]	0.958 [3.51]
$Dum1 * f_{t,t+n}$	−0.104 [1.05]	−0.261 [3.09]	−0.734 [6.92]	−1.129 [6.41]				
$Dum2 * f_{t,t+n}$					−1.261 [3.22]	2.449 [2.86]	−0.406 [0.22]	0.591 [0.06]
$Dum3 * f_{t,t+n}$					0.171 [0.20]	−0.255 [0.33]	0.516 [1.00]	−1.134 [1.50]
s_t	0.960 [22.26]	0.860 [22.05]	0.604 [13.02]	0.065 [1.23]	−0.012 [0.09]	−0.364 [2.57]	0.047 [0.31]	−0.916 [4.75]
$Dum1 * s_t$	0.006 [0.07]	−0.103 [1.09]	−0.087 [0.69]	0.019 [0.18]				
$Dum2 * s_t$					1.240 [2.68]	−11.536 [2.81]	−1.918 [0.45]	2.289 [0.21]
$Dum3 * s_t$					−0.475 [0.57]	0.418 [0.87]	−0.123 [0.31]	0.722 [1.56]
$Mean\ s_{t+n} - f_{t,t+n}$	−0.928	−1.878	−2.544	−2.133	−0.057	−0.170	−0.501	−1.108
SE	0.550	0.793	1.078	1.216	0.340	0.684	1.032	1.601
DW	0.41	0.19	0.09	0.08	0.87	0.38	0.15	0.09

Notes: US: 1986.1-2019.12; UK: 1998.1-2012.12. Dum1=1 (2008.10-2019.12) Dum2=1 (2008.9-2009.3) Dum3=1 (2009.4-2012.12). t-statistics are in brackets.

Adding the current spot rate to equations (6) and (7) dramatically changes the results reported in Table 2 for both countries. For the US the spot rate is highly signficant except at the 24-month horizon, and the residual standard errors are reduced, implying improved forecasts of the future spot rate; but there is still strong residual correlation. The coefficients for the forward rate before the financial crisis are now negative and highly significant. The most pertinent finding, however, is that the coefficients on the forward rate and the spot rate are approximately equal but of opposite sign, and steadily approach zero as the horizon lengthens.

The explanation, for this is that the model is reflecting the time series properties of the variables. Both the forward rates and the spot rate are non-stationary I(1) processes and are cointegrated with cointegrating vector $(1, -1)$ so that the explanatory variables form a single I(0) variable $s_t - f_{t\,t+n}$. The equation has become, therefore, a pure time series model for integrated data and bears little relation to the EH, which it strongly rejects.

After the financial crisis, there is a significant break in the estimates of the coefficients of the forward rate - except at the 3-month horizon - but there is no break in the coefficients on the spot rate. The coefficients on the forward rates become even more negative after the crisis. Again there is a time series interpretation of this, as the coefficients on the forward rate now seem to be offsetting $f_{t,t+n}$ in the dependent variable thereby making the effective dependent variable just s_{t+n}. In the following section we explore this further.

No clear pattern emerges for the UK results except at the 24-month horizon. Before the financial crisis the estimates are similar to those for the US except that the sign of $s_t - f_{t,t+24}$ is different. After the crisis there is a significant break in both coefficients with the result that the variables still form the cointegrating vector $s_t - f_{t,t+24}$ but it has the opposite sign from that before the crisis. In other words, after the crisis $s_t - f_{t,t+24}$ is no longer significant and we are left with pure forecasting errors for $s_{t+24} - f_{t,t+24}$.

Including the current spot rate, as proposed by Fama, clearly improves forecasts of future spot rates but it leaves what is, in effect, a pure time series model with little relation to the EH. As found previously, this is particularly true after the financial crisis when the forecast errors are greatest.

Given these results, the notion that spot rates are best forecast with pure time series models and that the information set may be based on weakly rational expectations may be explored further. First, as previously noted, both spot and all forward rates are indistinguishable and very close to random walks (or martingales). Second, cointegration analysis does not reject that, for both countries and all n, $s_t - f_{t,t+n}$ is a cointegrating vector and therefore I(0). It follows that if $\alpha = 0$ and $\beta = 1$ then equation (5) can be rewritten as the vector error correction model

$$\begin{aligned} s_{t+n} - f_{t,t+n} &= s_t - f_{t,t+n} + u_{t+n} \\ u_{t+n} &= \Sigma_{i=0}^{m} \Delta s_{t-i} + \Sigma_{i=0}^{m} \Delta f_{t-i,t+n-i} + e_{t+n} \end{aligned} \tag{10}$$

where u_{t+n} is I(0). This would explain why for the US the coefficients on s_t and $f_{t,t+n}$ sum close to zero and why these coefficients are so large. It also implies that $f_{t,t+n}$ can be omitted from the equation leaving a pure time series model in the spot rate. These effects get stronger the greater is n. The same phenomenon occurs for the UK results with the difference that it gets weaker as n increases. This may have something to do with the stronger effect of the break in the data at longer horizons due to the financial crisis. These considerations reinforce the conclusion that such a time series model is not a suitable vehicle for testing unbiasedness. They also show that current spot rates might have useful information about future spot rates.

Nonetheless, the serially correlated residuals show that, as a forecasting equation for future spot rates, it can clearly be improved by taking this into account. To obtain an equation with best fit, adding the lagged dependent variable might appear to be best but, of course, such an equation could not serve as a forecasting equation as the lagged dependent variable is not known at time t. As only current information is allowable, under weak rationality this means current and lagged values of the spot rate and the forward rate. Given the non-stationarity of s_t and $f_{t,t+n}$ these variables should be included as changes. In effect, therefore, we are turning the previous question around as we are no longer asking whether the current spot rate has additional information to the forward rate, but vice-versa. As only one lag of the spot rate is found to be significant, and lags of the forward rate are not significant, the forecasting equation is formulated as

$$s_{t+n} - s_t = \alpha + \beta f_{t\,t+n} + \gamma s_t + \delta \Delta s_t + e_{t+n}. \tag{11}$$

The estimates are reported in Table 4.

Table 4. Time series forecasts of future US and UK spot rates

	US				*UK*			
n	3	6	12	24	3	6	12	24
c	0.138 [1.09]	0.033 [0.15]	−1.246 [3.35]	−2.276 [7.01]	0.418 [2.68]	1.129 [3.39]	2.086 [3.82]	4.196 [4.11]
$Dum1$	0.021 [0.11]	0.515 [1.48]	2.864 [5.15]	4.216 [5.51]				
$Dum2$					0.038 [0.14]	−0.582 [1.09]	−1.770 [1.72]	−4.489 [1.32]
$Dum3$					−0.013 [0.04]	−0.731 [1.19]	−1.444 [1.92]	−3.648 [2.20]
$f_{t,t+n}$	−0.005 [0.16]	0.070 [1.46]	0.402 [6.11]	1.026 [15.46]	0.865 [6.32]	0.658 [4.32]	0.830 [5.43]	0.202 [0.87]
$Dum1 * f_{t,t+n}$	−0.048 [0.65]	−0.191 [2.14]	−0.703 [5.98]	−1.293 [6.89]				
$Dum2 * f_{t,t+n}$					−0.619 [5.71]	−0.688 [3.10]	−0.343 [0.82]	0.109 [0.10]
$Dum3 * f_{t,t+n}$					−0.699 [1.27]	−0.524 [0.58]	−0.774 [1.48]	−0.171 [0.29]
s_t	−0.032 [0.89]	−0.120 [3.35]	−0.360 [8.20]	−0.939 [17.67]	−0.956 [7.21]	−0.922 [6.51]	−1.342 [10.00]	−1.243 [7.52]
Δs_t	0.467 [4.49]	0.367 [2.36]	0.724 [3.39]	0.604 [2.43]	0.175 [1.13]	0212 [0.71]	0.044 [0.10]	−0.659 [0.95]
R^2	0.064	0.059	0.191	0.500	0.619	0.439	0.468	0.362
SE	0.537	0.803	1.097	1.271	0.324	0.664	1.010	1.534
DW	0.62	0.27	0.16	0.13	1.00	0.38	0.15	0.08

Notes: US: 1986.1-2019.12; UK: 1998.1-2012.12. Dum1=1 (2008.10-2019.12) Dum2=1 (2008.9-2009.3) Dum3=1 (2009.4-2012.12). t-statistics are in brackets.

The results for the US indicate that for horizons of six months or more current changes in the spot rate are significant in forecasting the future spot rate. Although the current spot rate is also significant, prior to the financial crisis it tends to be offset by the forward rate which is also signficant. This is consistent with the spot rate being approximately a random walk and with the cointegration of spot and forward rates with cointegrating vectors $s_t - f_{t,t+n}$. However, after the financial crisis, the coefficient on the forward rate decreases and is close to zero for all horizons. The equation then becomes almost a pure time series model in the spot rate with no additional information from the forward rate. The proportion of s_{t+n} - as opposed to $s_{t+n} - s_t$ - explained by the model is between 0.85 and 0.96. Nonetheless, all of the equations still show considerable residual serial correlation.

The results for the UK have many similarities. The current spot rate is highly signficant at all horizons but the change in the spot rate is not significant. Apart from the 3-month horizon, the forward rate is also highly significant before the crisis. After the crisis, the coefficient on the forward rate is reduced. Like the US before the crisis, the forward rate tends to be substantially offset by the current spot rate, again reflecting the near cointegration of the two variables. After the crisis the forard rate plays little role and the model is close to being a pure time series model in the spot rate, but with unexplained residual serial correlation.

To summarise, these results suggest that, before the financial crisis, using the current spot rate as well as the forward rate improves the forecasts of the future spot rate for both the US and the UK. For the US, the change in the spot rate is also significant. But after the crisis, for both countries, the equation reduces to roughly a pure time series model in the spot rate. The presence of significant residual serial correlation for both countries at all horizons indicates that the within sample fit of the model could be improved by taking this into account, but it would not necessarily improve the forecasting performance of the model if based solely on current information. Common responses to serially correlated disturbances, such as adding the lagged dependent variable, or having autocorrelated disturbances, are ruled out as they involve information not available when the forecast is made in period t. The use of weakly rational expectations has improved the forecasts but, as the earlier discussion suggests, the serially correlated residuals leave open the possibility that the forecasts could be further improved by including additional information such as time-varying term premia.

7 Can macroeconomic information improve forward rate forecasts?

A possible source of additional information is the use current macroeconomic variables. As mentioned earlier, studies of the term structure have used macroeconomic variables as observable factors in modelling yields and term premia. We chose not to try to model term premia in this way and instead maintain a focus on forecasting future spot rates by using time series forecasting models that seek to exploit only current information. But this does not rule out the possibility that the macroeconomic variables used in term structure models might contain current information about future spot rates. We therefore consider whether the use of monthly cpi inflation and the rates of growth of the index of industrial production and M1 would improve the forward rate forecasts. We add these variables to the model used to test the EH, equations (6) and (7). We also take account of the possibility that their contributions may vary between before and after the financial crisis. The results are reported in

Tables 5 and 6; the earlier results in Table 2 that are without macroeconomic variables are included in the tables for ease of comparison.

Table 5: US forward rate forecasts with macroeconomic information

n	3		6		12		24	
c	-0.867 [4.33]	-0.868 [4.39]	-0.870 [2.57]	-1.111 [3.36]	-1.612 [3.61]	-1.963 [4.42]	-1.168 [3.27]	-1.242 [3.48]
$Dum1$	0.721 [2.28]	0.750 [2.26]	2.411 [4.77]	2.731 [5.44]	4.190 [6.68]	4.578 [7.34]	2.975 [4.09]	3.353 [4.26]
$f_{t,t+n}$	0.010 [0.302]	0.007 [0.24]	-0.098 [2.01]	-0.070 [1.44]	-0.067 [1.11]	-0.039 [0.64]	-0.073 [1.41]	-0.092 [1.75]
$Dum1 * f_{t,t+n}$	-0.496 [4.24]	-0.493 [4.24]	-1.089 [9.33]	-1.109 [9.73]	-1.329 [11.31]	-1.353 [11.57]	-1.156 [6.90]	-1.211 [-6.43]
π_t		0.402 [1.73]		0.599 [1.81]		0.782 [2.17]		0.310 [0.90]
$Dum1 * \pi_t$		0.010 [0.03]		-0.597 [1.24]		-0.745 [1.44]		-0.062 [0.13]
$\Delta \ln IP_t$		0.124 [1.23]		0.285 [2.04]		0.318 [2.07]		0.544 [3.71]
$Dum1 * \Delta \ln IP_t$		-0.141 [0.97]		-0.338 [1.70]		-0.384 [1.77]		-0.473 [2.19]
$\Delta \ln M1_t$		-0.373 [4.80]		-0.536 [4.90]		-0.388 [3.23]		0.002 [0.02]
$Dum1 * \Delta \ln M1_t$		0.267 [2.48]		0.400 [2.68]		0.313 [1.92]		-0.074 [0.48]
$Mean\ s_{t+n} - f_{t,t+n}$	-0.928	-0.928	-1.878	-1.878	-2.544	-2.544	-2.133	-2.133
SE	0.892	0.862	1.231	1.186	1.311	1.284	1.217	1.201
DW	0.16	0.32	0.08	0.24	0.07	0.16	0.08	0.16

Notes: 1986.1-2019.12. Dum1=1 (2008.10-2019.12). t-statistics are in brackets.

For the US including the macroeconomic variables makes little difference to the estimates of the coefficients of the forward rates reported in Table 2. Before the financial crisis the EH is generally not rejected, but after the crisis it is strongly rejected. Nonetheless, we find that all three macroeconomic variables are significant and hence would improve the forecasts. The strongest impact prior to the crisis is made by the rate of growth of M1. For horizons below 24 months it has a significant negative effect prior to the financial crisis. After the crisis there is a significant difference in its impact. The coefficients are now positive and almost completely offset those before the crisis, implying that after the crisis money growth has little impact. Inflation appears to have a significantly positive effect for the whole data period at horizons below 24 months, there being no significant break after the crisis. While money growth and inflation are most signficant at shorter horizons, output growth is more significant at longer horizons, but only prior to the crisis. Before the crisis it has a positive effect that increases in size with the horizon. After the crisis these effects are much reduced as the coefficients are significantly negative.

Table 6: UK forward rate forecasts with macroeconomic information

n	3		6		12		24	
c	0.473	0.501	0.743	1.089	2.185	3.889	1.827	−2.623
	[2.90]	[2.51]	[2.13]	[2.91]	[3.97]	[6.87]	[1.58]	[1.90]
$Dum2$	−0.016	−0.233	−5.768	−4.772	3.458	2.290	14.297	−12.523
	[0.05]	[0.62]	[2.00]	[1.81]	[1.11]	[0.77]	[0.92]	[0.75]
$Dum3$	−0.296	−0.323	−0.793	−1.149	−2.403	−4.091	1.783	2.814
	[0.96]	[0.96]	[1.88]	[2.51]	[3.84]	[6.32]	[0.68]	[0.88]
$f_{t,t+n}$	−0.104	−0.131	−0.186	−0.149	−0.522	−0.881	0.154	0.399
	[3.18]	[3.09]	[2.65]	[1.92]	[4.76]	[7.65]	[0.67]	[1.46]
$Dum2 * f_{t,t+n}$	−0.253	−0.160	1.046	0.681	−1.200	−0.921	2.611	2.191
	[3.04]	[1.73]	[1.71]	[1.22]	[1.61]	[1.31]	[0.82]	[0.65]
$Dum3 * f_{t,t+n}$	−0.216	−0.194	0.178	0.130	0.378	0.710	−0.562	−0.830
	[0.45]	[0.39]	[0.56]	[0.41]	[2.21]	[4.09]	[0.97]	[1.24]
π_t		0.133		−0.021		−0.070		−0.366
		[1.56]		[0.13]		[0.28]		[0.70]
$Dum3 * \pi_t$		0.142		−0.053		0.246		0.438
		[0.88]		[[0.15]		[0.43]		[0.38]
$\Delta \ln IP_t$		0.043		−0.039		−0.043		−0.111
		[1.23]		[0.56]		[0.42]		[0.52]
$Dum3 * \Delta \ln IP_t$		−0.046		0.024		0.037		0.042
		[0.68]		[0.19]		[0.20]		[0.09]
$\Delta \ln M1_t$		0.127		−0.578		−0.107		−0.259
		[2.72]		[6.34]		[0.74]		[0.57]
$Dum3 * \Delta \ln M1_t$		−0.126		0.605		0.017		0.072
		[1.54]		[3.84]		[0.07]		[0.13]
$Mean\ s_{t+n} - f_{t,t+n}$	−0.057	−0.057	−0.170	−0.170	−0.501	−0.501	−1.108	−1.108
SE	0.345	0.334	0.709	0.641	1.023	0.936	1.722	1.816
DW	0.75	0.86	0.31	0.56	0.16	0.22	0.06	0.09

Notes: 1998.1-2012.12. Dum2=1 (2008.9-2009.3) Dum3=1 (2009.4-2012.12)
t-statistics are in brackets.

The results for the UK are also similar to those in Table 2. Before the crisis the EH is strongly rejected and after the crisis a signficant break is found only at the 12 month horizon. Of the three macroeconomic variables, only money growth is significant, and this only for the 3 and 6 month horizons. After the crisis even money growth appears to have a negligible effect.

To summarise, including macroeconomic variables improves the forecasting performance but only prior to the financial crisis - which was a period of normal financial times. They have little or no effect after the crisis. Once again, therefore, we find that the results for the US are consistent with the EH before the crisis but not afterwards, and those for the UK are not consistent with the EH before the crisis. After the crisis neither set of results is consistent with the EH. Instead they point either to financial markets continuing, wrongly, to forecast that monetary easing would be brought to an end and spot rates would therefore rise or, applying the theory of forward rates, they sought a much higher term premium.

8 What causes the poor forecasting performance of forward rates?

If we take the forward rate as given and ignore any other current information that might help forecast future spot rates, such as macroeconomic variables, then the forecast errors are due to factors affecting the future spot rate that are not anticipated when forward rates are calculated. We consider whether these factors might be the future values of the three macroeconomic variables - inflation, output growth and money growth. In the previous section we included the current values of these variables in our information set. Another interpretation of those results is that they are a weakly rational forecast of the future values of these variables. By using the actual values of these variables we are combining these forecasts with the subsequent innovations.

We estimate the following equation

$$\begin{aligned} s_{t+n} - f_{t,t+n} &= \alpha + (\beta - 1) f_{t,t+n} + \Sigma_{i=0}^{n-1} \gamma_i \pi_{t+n-i} \\ &+ \Sigma_{i=0}^{n-1} \delta_i \Delta \ln IP_{t+n-i} + \Sigma_{i=0}^{n-1} \theta_i \Delta \ln M1_{t+n-i} + e_{t+n} \end{aligned} \quad (12)$$

For long horizons the equation would require too many lags to be estimated and, even if estimatable, too cumbersome to report in full. We therefore adopt a different approach. Instead we report in Table 7 the proportion of the variance of the forecast error attributable to each macroeconomic factor and the significance of each factors' contribution.

Table 7. Contributions of macroeconomic variables to deviations

	US				*UK*			
	3	6	12	24	3	6	12	24
$f_{t,t+n}$	-0.026 [1.41]	-0.083 [2.75]	-0.219 [5.72]	-0.287 [7.58]	-0.083 [5.02]	-0.190 [5.40]	-0.276 [1.95]	-1.670 [3.53]
π	13.5 (0.00)	7.3 (0.00)	17.4 (0.00)	23.4 (0.00)	4.2 (0.47)	28.0 (0.00)	45.5 (0.01)	79.0 (0.02)
$\Delta \ln IP$	1.6 (0.55)	0.7 (0.92)	2.1 (0.76)	3.0 (0.75)	19.4 (0.01)	9.3 (0.04)	6.5 (0.98)	19.6 (0.96)
$\Delta \ln M1$	58.7 (0.00)	66.9 (0.00)	54.0 (0.00)	45.8 (0.00)	57.8 (0.00)	31.8 (0.00)	3.6 (0.99)	13.3 (0.99)
R^2	0.252	0.426	0.524	0.674	0.289	0.515	0.387	0.577

Notes: US: 1986.1-2019.12; UK: 1998.1-2012.12. row 1 is the coefficient of $f_{t,t+n}$ and its t-statistic; rows 2-4 give the proportions of R^2 explained by the macro variables and their p-values; row 5 gives the R^2, the proportion of deviations explained by equation (12).

Table 7 is consistent with the results in Tables 5 and 6, namely, that US money growth is by far the main factor causing the deviations of the foward rate from the future spot rate. Inspection of when over the forecast horizon the macroeconomic variables have their greatest effect we find that the growth of money in every future period prior to the horizon is significant. Money growth is also the main factor for the UK for the 3 and 6 month horizons. Inflation is another significant factor for both countries at all horizons but explains a smaller proportion of the forecast errors. Unlike money growth, it is only inflation shortly before end of each horizon that is significant. Output growth is not significant for the US at any horizon but is significant for the UK at the 3 and 6 month horizons, again immediately prior the the end of the horizon. It may be noted that the R^2 statistics show that nearly half of the variation in the forecast errors are unexplained by macroeconomic variables in equation

(12). The residuals again have high serial correlation which we can attribute to unexplained components. The estimates of the forward rate strongly reject the EH at all horizons.

The main conclusions to draw from these results are that unanticipated macroeconomic factors, especially money growth, are a principal reason why forward rates give poor forecasts of future spot rates and that the impacts of inflation and output growth on spot rates occur shortly before the end of the horizon. The significance of the macroeconomic variables in Tables 5 and 6 seems likely to be due to them forecasting the later impact of that variable. After the crisis these forecasts deteriorated. Interpreting the results involving macroeconomic variables from the viewpoint of the theory, gives an alternative perspective on term premia from that obtained from estimates of the term structure based on yields. Here the future macroeconomic variables can be interpreted as capturing changes in the term premia and hence the deviations of forward rates from future spot rates. In order of their importance they are money growth, inflation and output growth.

9 Conclusions

We can now hazard some answers to the questions posed at the start of this investigation. It was noted early in the paper that the key piece of information, which the various later tests do little more than formalise, is provided by the plots of the forward curves, Figures 1 and 2. While US forward curves tended to exceed the spot rate throughout the sample, prior to the financial crisis UK forward curves tend to be quite flat and to mildy mean revert, thereby missing fluctuations in the spot rate, but not by a substantial margin. After the crisis the UK forward curves, like the US curves, strongly mean revert and completely fail to forecast future spot rates which stayed persistently low. We can therefore summarise our findings into before and after the financial crisis.

We found that before the financial crisis, when financial conditions were "normal", the EH was not usually rejected for the US but was rejected for the UK. After the crisis it was rejected for the US and the UK. The issue that then arises is whether these rejections were due to the deviations of forward rates from the future spot rate being due to risk premia, and hence consistent with the general theory of forward interest rates. Or were they due to pure and persistent forecasting errors?

Before the crisis the significance of macroeconomic variables, especially the rate of growth of the money supply and inflation, might be interpeted as acting as proxies for the term premium and hence offering some support for the general theory. This is consistent with the results for the US before the crisis, but not for the UK. There is also signficant residual serial correlation in both sets of results which the macroeconomic variables are not eliminating. After the crisis, the size of the deviations and the failure to find variables, including macroeconomic variables, that are signficant suggests that such theoretical considerations were dominated by a failure of expectations that caused large forecasting errors. The evidence in Table 7 indicates that the major cause of these forecasting errors was monetary policy: an unexpectedly high rate of money growth accompanied by an unexpectedly low rate of inflation.

Following Fama, we examined whether the current spot rate had useful additional information about future spot rates. We found that once the current spot rate was included the whole character of the model changed. It was no longer a representation of either the EH, or the general theory. Instead, it had the properties of a pure time series model for non-stationary variables. In particular, we found that the current spot rate and the forward rate formed a significant cointegrating residual with cointegrating vector $(1, -1)$ and that, as a result, the coefficient on the forward rate no longer retained its theoretical interpretation. We therefore reformulated the equation as a pure forecasting model for the change in the spot rate over the same horizons and adopted the approach of weakly rational expectations. We found for the US - but not the UK - that the change in the spot rate is also signficant. The resulting model reduced the residual standard error for the US by between 30-40%. That for the UK was reduced only a little. Despite this improvement, the equations still had highly serially correlated residuals implying persistent forecasting errors. The common response to this, such as adding the lagged dependent variable, or having autocorrelated disturbances, are ruled out as they are not observable when the forecast is made.

We have found, therefore, that forward interest rates are of limited use in forecasting future spot rates even in normal financial conditions. They tend to mean revert rather than anticipate the fluctuations in spot rates. To be consistent with the theory of forward rates it would be necessary for risk premia to offset these fluctuations. The time series evidence seems to support the deviations of forward rates from future spot rates being due to a lack of information when pricing bonds.

These findings have important implications for the profitability of forward contracts where the aim is to borrow at forward rates today and, on the maturity of the n-period bond, use the proceeds to invest in the future spot rate. This was especially true for the contracts that expired after the financial crisis. Such a contract has been shown to have been loss-making. However, if we could have reversed the contract it would have offered an arbitrage opportunity.

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