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DECENTRALIZED TRADE, ENTREPRENEURIAL INVESTMENT AND THE THEORY OF UNEMPLOYMENT

Melvyn G Coles

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Centre for Economic Policy Research
25–28 Old Burlington Street
London W1X 1LB
Tel: (44 171) 878 2900
cepr@cepr.org

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ABSTRACT

Decentralized Trade, Entrepreneurial Investment and the Theory of Unemployment*

This paper considers an equilibrium model of unemployment in a labour market where all vacancies are advertised in a newspaper. Unemployment occurs in occupations that are short on vacancies. New vacancies are created by entrepreneurial search and investment, so it may take some time before an unemployed worker finds a job. Wages are determined by bargaining. A unique rational expectations equilibrium is shown to exist. The unemployment-vacancy dynamics are consistent with so-called Beveridge curves. Individual unemployment spells can be long – especially in low turnover markets – while markets with high turnover experience large variations in unemployment and little wage variation. Although this latter case appears to exhibit 'sticky wages', this market outcome is (asymptotically) fully efficient. Although the appropriate government policy is to subsidize entrepreneurial investment (there is a wage bargaining distortion and a search externality), simulations show that the required subisidies are very small for appropriate parameter values. A *laissez-faire* policy is (almost) optimal even with unemployment spells as long as a year in some markets.

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Melvyn G Coles
Department of Economics
University of Essex
Colchester CO4 3SQ
UK
Tel: (44 1206) 873 333
Email: mcole@essex.ac.uk

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NON-TECHNICAL SUMMARY

This paper describes a new dynamic equilibrium model of unemployment. It not only explains unemployment persistence as an equilibrium phenomenon, it also obtains the appropriate data correlations shows that in booms unemployment is low, wages are high and vacancies are plentiful, while the converse is true in recessions. Furthermore, it does not use the so-called 'matching approach' to obtain these results. Being an equilibrium model, it can also calculate market efficiency. Most surprisingly, despite two economic distortions – wages are determined by bargaining and so are not necessarily competitive, while entrepreneurs do not take into account that by creating a new vacancy they make an unemployed worker better off – for sensible parameter values, the market outcome and the socially efficient outcome are virtually identical. This implies that from an efficiency perspective, the government should not intervene in such markets. This is particularly relevant as the model is consistent with average unemployment spells being longer than a year in low turnover markets during recessions. On equity grounds, the government might offer unemployment compensation to unemployed workers caught in this situation. Although not shown here, in the short run increasing unemployment compensation makes the currently unemployed better off, but this has adverse consequences in the long run. Making the unemployed better off raises their reservation wage which chokes off entrepreneurial investment. In the long run, total unemployment has to rise so that the expected value of being unemployed falls back to its original level. It is only there that entrepreneurial investment rates match the entry rate of unemployed workers. In the long run, an unemployment compensation policy is fundamentally flawed: it does not raise the expected value of being unemployed, while it does raise average unemployment and the cost of operating such a policy.

The model uses the following basic construction. Entrepreneurs search for profit-making opportunities. Given the current market wage, if they find an opportunity where the investment cost is low enough, they will take it, which then creates a new vacancy in the market. Otherwise there is no new vacancy. There is also a positive flow of unemployed workers into the market (such as school-leavers and workers who have just lost their jobs). In this framework, unemployment increases while the entry rate of unemployed workers exceeds the current investment rate of entrepreneurs. It is found that equilibrium wages fall as the number of unemployed increases. The economy is therefore stable – if there is high unemployment, competitive forces cause wages to fall, which, by stimulating greater investment rates, also causes unemployment to fall over time (on average). The equilibrium adjustment is slow, however, because new vacancies are only created as entrepreneurs find new profit-making opportunities. The model therefore explains unemployment persistence. It also obtains the appropriate data correlations – that during booms unemployment is low, wages are high and vacancies are plentiful, while the converse is true in recessions.

The efficiency result (described above) is particularly provocative in the light of the following three empirical predictions:

(a) In (locally-segmented) labour markets with high turnover, the wage level depends largely on the ratio of the number of entrepreneurs to the expected entry rate of unemployed workers. For arbitrarily high turnover rates, the market wage does not depend on the current unemployment level. Although this outcome looks like 'sticky wages', this is not the case. For reasons explained in the paper, such markets are efficient and market intervention is undesirable. The intuition for this wage inelasticity is that with high turnover, the rate at which new vacancies arrive is high. Although the number unemployed may also be high, the high entry rate of new vacancies implies that each unemployed worker need not wait long to obtain a job. As a result, their reservation wage is little affected by the current unemployment figures. Furthermore, the fact that wages do not adjust to the level of unemployment implies that the variance of unemployment is very high (it follows a random walk).

(b) In (locally-segmented) labour markets with low turnover, unemployment durations in recessions may average over a year. New vacancies arrive so slowly that with many other workers chasing those vacancies, the chances of finding work in the near future becomes very low. In such markets, wages fall significantly as unemployment increases and this wage response reduces the varation in unemployment. There is significant wage risk, however – unemployed workers in low turnover markets are much worse off in recessions than they are in booms.

(c) The model captures the despair of being unemployed when unemployment is high. In this model, when there are several people chasing the same job, the entrepreneur offers a low wage which leaves each applicant indifferent to accepting the job or continuing to look for work. In that case, each worker may question whether it is worth looking for a job in the first place.

1 Introduction

Markets where different agents enter the market at different points in time are necessarily incomplete markets. Prices cannot be predetermined by a Walrasian Auctioneer as not all agents are initially present in the market. The central issue is to understand how entry and matching takes place in such markets and to consider how prices are determined over time. The focus of the analysis is to ask whether such markets are efficient and to examine the potential role of government policy.

This paper considers an equilibrium model of decentralized trade in a labour market where all vacancies are advertised in a newspaper and all job seekers read these notices - there are no matching frictions. But even so, unemployment occurs as an equilibrium phenomenon in occupations which are short on vacancies. Such unemployed workers have to wait for an entrepreneur to discover a profit making opportunity, invest in it and thereby create a new vacancy. The unemployment-vacancy dynamics generated by this framework are non-trivial and are consistent with so-called Beveridge curves. Individual unemployment spells can be long - especially in low turnover markets. As detailed below, this approach offers new insights into the nature of decentralized trade which are quite different to those found in the standard matching approach.

The standard matching framework, as initially developed by Diamond (1982), Mortensen (1982) and Pissarides (1990), has become a useful tool for analyzing optimal government policy in labour markets. As there are real bargaining distortions and matching externalities in such markets, it suggests a positive role for government policy.[1] Diamond (1982) originally considered a model of decentralised trade which had two frictions : an entry friction (agents first had to search for goods) and a matching friction between traders. Subsequently, the matching approach has retained the matching frictions but has supposed production (the entry of new vacancies) is per-

[1]See Hosios (1990) and Layard, Nickell and Jackman (1992) for a full policy discussion of this literature. Also see Gale (1986),(1987) and Rubinstein and Wolinsky (1990) who analyse conditions when such markets replicate the competitive outcome. Related work includes Rubinstein and Wolinsky (1985), Wolinsky (1987) and Binmore and Herrero (1988).

fectly elastic. We do the opposite and focus on how entry frictions affect decentralised trade in the absence of matching frictions.

There are several empirical reasons why this view of decentralized trade is of interest. The first reason is anecdotal but illustrates the nature of the market considered. After moving to Essex, the author wished to furnish his new house with second-hand furniture. Naturally, search focussed on local second hand stores and newpaper ads. In particular, there was a local store which acted as a marketplace for several sellers. The first visit there was very successful and several purchases were made. The next visit, a week later was much less successful. The store had more or less the same stock of furniture from the previous week which had already been rejected as unsuitable. This continued for several months until one week, the store had the ideal table which was purchased immediately. The manager said the table had only come in that morning.

This example does not imply that matching frictions are not important. Indeed, it is because there are large search and transport costs that obtaining a (relatively cheap) table did not warrant a nationwide search. But the matching function approach does not describe the above trading sequence. When I first entered the store, it was long on dressers (and so I bought one) but was short on (desirable) tables, and so I had to wait for one to arrive later. Delays to trade in this (local) market were not due to matching frictions (I could observe their complete stock at all points in time), but due to entry frictions of new agents. Trade occured exactly when a new agent entered on the short side of a market.

The results of Coles and Smith (1995) also support the approach developed here. Using UK Job Center data, they find that the matching probabilities of those workers who have been unemployed more than one month are significantly and positively correlated with the flow of new vacancies into the market, but are uncorrelated with the current stock of vacancies. Furthermore, the matching probabilities of those who have been unemployed less than a week are significantly and positively correlated with the current stock of vacancies. This result is consistent with the above anecdote. Some labour market entrants are lucky, find they are on the short-side of their particular occupation and find immediate employment from the stock of current vacancies.

Others have to wait for a suitable vacancy to enter the market.

Thirdly, consider figure 1 which graphs the data used by Blanchard and Diamond (1989) to estimate the matching function. We can use this data to infer the flow of new vacancies by assuming

$$\text{flow of new vacancies} = \text{total hires} + \text{change in stock of vacancies.}$$

As the change in the stock of vacancies is tiny relative to total hires (over the month), this data implies that the flow of new vacancies over the month almost equals total hires. Futhermore the volatility of monthly hires (and the implied flow of new vacancies) against the smoothness of the stock of vacancies implies that the flow of vacancies is much more important than the stock variable for explaining matching rates. The following model is consistent with this interpretation of the data.

The model analysed here considers a representative occupation in which all vacancies and all workers are identical. Each unfilled vacancy requires exactly one worker. There is turnover where new vacancies are created by entrepreneurial search and investment, while there is an exogenous inflow of unemployed workers. If there is exactly one worker and one vacancy in the market, the terms of trade are negotiated by bilateral bargaining. If there are more agents on one side of the market than on the other, prices are determined by Bertrand competition. Given this price determination procedure, each entrepreneur's decision to invest takes into account the physical cost of the investment (which varies with each opportunity), the expected time taken to fill the vacancy and the wage that will be negotiated. The paper establishes a non-trivial existence and uniqueness proof of a Rational Expectations Equilibrium where the bilateral bargaining wage is determined by strategic bargaining.

The properties of the equilibrium are particularly surprising for such a simple framework. Because there are bargaining and search distortions in the market analyzed, the market equilibrium does not achieve the Social Planner's first best solution. The appropriate corrective taxation is to provide investment subsidies which are conditioned on the current level of unemployment. However, for natural parameter values it is found that :

(i) the choice of bargaining power in the bilateral bargaining game has little effect on equilibrium prices.[2] The surplus to be divided by the bilateral bargaining game is essentially the loss in expected discounted revenue by delaying trade until a third party enters the market. This surplus is small for sensible turnover numbers and discount rates.

(ii) Although there is a bargaining distortion and a search externality, the market is (almost) fully efficient, even for markets with relatively low turnover and high unemployment durations. The reason is that the bilateral bargaining outcome is dominated by the arrival rate of new agents rather than by bargaining power. The resulting price reflects the social value of the worker. Away from the bilateral bargaining case, Bertrand competition keeps the market quasi-competitive. As the negotiated wages reflect the social value of workers at all levels of unemployment, the privately optimal investment decisions of entrepreneurs reflect the socially optimal ones. Indeed, with arbitrarily high turnover rates the market outcome is fully efficient.

(iii) Markets with high turnover experience large fluctuations in the number unemployed but negotiated wages change little, i.e. wages are not sensitive to changes in the number unemployed. In fact, if α denotes the rate at which entrepreneurs find investment opportunities and g denotes the rate at which unemployed workers enter the market, the equilibrium wage level is largely determined by their ratio α/g. The greater the value of α/g, the higher the equilibrium wage level. This outcome is very different to that in the standard matching approach. There the unemployment stock to vacancy stock ratio plays the central role.

(iv) Markets with reasonably low turnover levels (say a new worker (or vacancy) enters on average every ten days) will have average unemployment durations which last months rather than weeks, and in periods of high unemployment may reach as long as a year.

One trivial criticism of this framework is that it predicts either zero unemployment or zero vacancies in the market. But aggregation over many such markets - where markets may be distinct by occupation (accountants do not want truck driving jobs) or

[2] In the strategic bargaining game with random alternating offers, the probability that the worker is chosen by nature to make the next offer can be interpreted as worker bargaining power (see Binmore et al (1986)).

location (accountants in city A do not want accountancy jobs in city B) will generate data which has a positive number of vacancies and a positive number of unemployed workers. The distinguishing feature of course is that no match exists between the current stock of unemployed and the current stock of vacancies - each is on the long side of their own particular market. The flow of new entrants drives the matching process in this case. Aggregating over such markets, total vacancies ($\sum_i V_{it}$) will covary negatively with total unemployment ($\sum_i U_{it}$), corresponding to the so-called Beveridge curve.[3] Of course, in this context, such data do not identify a matching function as there are no matching frictions by assumption.

There is a small directly related pricing literature. Taylor (1995) considers a formal game theoretic model of price formation along the lines described here but with fixed entry rates. There are no efficiency implications. Coles and Muthoo (1995) consider a more complicated model based on the same idea.

2 The Framework.

We consider equilibrium trade in a labour market within the context of an infinite horizon framework. It is assumed the market can be partitioned into distinct occupations and that workers cannot change occupation. In the representative occupation, there are entrepreneurs who hold unfilled vacancies and unemployed workers who are appropriately qualified to fill them. All such vacancies are identical, as are the unemployed workers. There are no matching frictions. If there is a positive number of vacancies and unemployed workers, it is assumed that in equilibrium, these vacancies and unemployed workers instantaneously match until one side of the market has zero agents. In that case, let $L \in \mathbb{N}$ index the state of the market at any point in time. $L \geq 0$ implies there are zero vacancies and L unemployed workers, while $L \leq 0$ signifies there are zero unemployed workers and -L vacancies. Hence if $L > 0$, the market is long in workers and short on vacancies for this particular occupation (and conversely for $L < 0$). Assume L is observed by all agents.

[3] This follows trivially from the fact $U_{it}V_{it} = 0$ in each market and that U_{it}, V_{it} are non-negative integers, so that $\text{cov}(U_{it}, V_{it}) = -(EU_{it})(EV_{it}) < 0$.

Assume all entrepreneurs (who hold vacancies) and workers are risk neutral and have the same discount rate $r > 0$. Unmatched agents obtain a zero flow payoff. The expected discounted revenue of a filled vacancy is normalized to unity.

Consider $L > 0$ and suppose a new vacancy enters the market. Via some wage bargaining process described below, the entrepreneur holding this vacancy negotiates a wage with one of the workers. If w_L denotes the equilibrium wage agreement in this state, the payoffs to the entrepreneur and the worker who gets the job are $1 - w_L$ and w_L respectively. It is assumed that the entrepreneur and worker write a binding contract which fixes the wage paid at this level in all future periods - there are no subsequent renegotiations (even though L will change value over time).[4] Once a vacancy is filled by a worker, the vacancy and the worker concerned leave the market for ever. The worker cannot subsequently be "poached" by another vacancy offering a higher wage. Similarly for $L < 0$ when a new unemployed worker enters the market.

Time is continuous. New unemployed workers enter the market at an exogenous Poisson rate with parameter $g > 0$ [i.e for small time period $\Delta > 0$, $g\Delta$ denotes the probability that a single worker enters this market]. The entry of new vacancies is described by entrepreneurial search, where there is a (large) fixed number of independent entrepreneurs who search for profit making opportunities. On aggregate, these entrepreneurs discover such opportunities at a constant Poisson rate $\alpha > g$. For each discovered opportunity, there is an associated investment cost $c \geq 0$ to exploit it. Furthermore, some opportunities are more costly to exploit than others. Given a discovery, its investment cost c is considered as an independent random draw with distribution F, which is assumed to be continuous and strictly increasing on the support [0,1]. By paying c, the entrepreneur holds a vacancy which he wishes to fill with

[4] Alternatively, the wage contract may describe a sequence of payments which rise with tenure. Wage bargaining determines where the worker starts on that ladder. By deferring payments to the future, the contract reduces the worker's incentive to quit should L fall. A different formulation could allow renegotiation. The fixed wage would then be replaced by a sequence of spot wages which depend on the current value of L. Although this would change the wage process, it is not clear that it would much change the equilibrium turnover dynamics. Examining the renegotiation case is an interesting variation which is left for future research.

one worker. If the entrepreneur declines to pay the investment cost, the opportunity is lost forever - there is no recall.

For tractability, we assume that given c and L, an entrepreneur invests if and only if the expected discounted profit by opening a vacancy exceeds its investment cost.[5] Let V_L denote the expected discounted payoff of an unemployed worker (for L > 0), and Π_L denote the expected discounted profit of an unfilled vacancy (for L < 0). The entrepreneur's investment rule is therefore assumed to be :

if $L > 0$, invest if and only if $c \leq 1 - w_L$;

if $L \leq 0$, invest if and only if $c \leq \Pi_{L-1}$.

Hence given L, entrepreneurs use a reservation investment strategy c_L where $c_L = 1 - w_L$ (if $L > 0$) and $c_L = \Pi_{L-1}$ (if $L \leq 0$). This implies that at any point in time, new vacancies enter the market according to a Poisson process with parameter $\alpha F(c_L)$.

The terms of trade between an unfilled vacancy and an unemployed worker are determined depending on the number of unfilled vacancies and unemployed workers which are currently in the market. First consider L ≥ 2 and suppose a new vacancy enters the market (if a new worker enters, the only effect is that L increases by one). Bertrand competition across workers implies $w_L = V_{L-1}$, the vacancy is immediately filled and L decreases by one. Conversely, suppose L ≤ -2 and a new unemployed worker enters the market. Bertrand competition across entrepreneurs implies $1 - w_L = \Pi_{L+1}$, a vacancy is immediately filled and L increases by one.

[5] This restriction implicitly assumes that once an entrepreneur has filled a vacancy he leaves the market for good and is replaced by a new entrepreneur. If we assume the entrepreneur remains in the market, he ignores that by reducing the stock of unemployed, he changes the value of search. If the number of entrepreneurs M is large, this is a small effect. The entrepreneur already does not take into account that he changes the value of search for the other M-1 entrepreneurs - a standard common resource externality. This assumption simply magnifies this externality effect by a factor of M/(M-1), which is small if M is large. The numerical simulations below suggest this is a very small effect for appropriate turnover numbers. Extending the model to analyse formally the search problem of entrepreneurs makes the model analytically intractable. Equilibrium is then characterised by a pair of non-linear second order difference equations (describing the search strategy c_L and the wage profile w_L) which creates a much more complicated fixed point problem.

The more complicated case arises when there is exactly one vacancy and one unemployed worker currently in the market - the bilateral bargaining problem. The next section simply assumes that w $\in [0,1]$ describes the wage outcome in the bilateral bargaining game. It characterises a Rational Expectations Equilibrium (REE) conditioned on that wage outcome. Section 4 closes the model by assuming w is determined by strategic bargaining and section 5 considers some numerical simualtions on the resulting equilibrium.

3 A Rational Expectations Equilibrium

Define $V_L(w)$ as the value of being an unemployed worker in a REE when there are $L > 0$ unemployed workers, and trade occurs at price $w \in [0,1]$ in the bilateral bargaining game. Similarly for $\Pi_L(w)$. A REE is formally defined as a sequence of functions $V_L(w)$ [for $L \geq 1$], $\Pi_L(w)$ [for $L \leq -1$] and wage agreements $w_L(w)$ which satisfy :

(A) the Bertrand bargaining equations : $w_L = V_{L-1}$ for $L \geq 2$, $w_L = 1 - \Pi_{L+1}$ for $L \leq -2$,

(B) V_L, Π_L are the agent's expected payoffs given trade occurs at prices $\{w_L\}$ defined by (A), w $\in [0,1]$ is the bilateral bargaining wage outcome and g, $\alpha F(c_L)$ define the entry rates of new workers and new vacancies respectively.

(C) $w_L \in [0,1]$ for all $L \neq 0$.

(C) must hold in a REE as the value of remaining unmatched must be positive (as the flow payoff to being unmatched is zero). All unemployed workers should therefore refuse a job offer if $w_L < 0$ while entrepreneurs should refuse to hire workers if $w_L > 1$.

Any solution to conditions (A)-(C) characterises a REE. Much of this section establishes the following Theorem.

Theorem 1 :

For any $r, \alpha, g > 0$ and w $\in [0,1]$, there exists a unique REE.

Lemmas 1a and 1b below obtain conditions which characterise $V_L(w)$ and $\Pi_L(w)$. Lemmas 2a and 2b will prove that there is a unique solution to those conditions. Proving the Theorem is then straightforward.

Lemma 1a (Characterization of $V_L(w)$)

Given $w \in [0,1]$, then in any REE, V_L must satisfy :

$$rV_L = g[V_{L+1} - V_L] + \alpha F(1 - V_{L-1})[V_{L-1} - V_L] \tag{1}$$

for $L = 1, 2, 3$.....subject to the boundary conditions

$$V_0 = w \tag{2}$$

$$V_L \in [0,1] \text{ for all } L > 0. \tag{3}$$

Proof in Appendix

(1) is a standard recursive equation describing V_L. Bertrand competition implies the worker obtains no surplus while L > 1. Indeed, if w = 0, so that the worker also gets no surplus in the bilateral bargaining game, then (1)-(3) imply $V_L = 0$ for all L > 0. However, if w $\in (0,1)$, so that the worker obtains positive surplus in the bilateral bargaining game, lemma 2a below shows that $V_L > 0$ for all L > 0. V_L describes the value of the worker's option to wait for unemployment to fall to zero, whereupon the worker obtains positive surplus in the bilateral bargaining game. Of course, it is the discount rate r and the entry rate of new vacancies and workers which determine the value of this option. (1)-(3) essentially computes this price for all L > 0.

The same argument implies the corresponding conditions for $\Pi_L(w)$.

Lemma 1b

Given $w \in [0,1]$, then in any REE, Π_L must satisfy :

$$r\Pi_L = g[\Pi_{L+1} - \Pi_L] + \alpha F(\Pi_{L-1})[\Pi_{L-1} - \Pi_L] \tag{4}$$

for $L = -1, -2, -3$.... subject to the boundary conditions

$$\Pi_0 = 1 - w \tag{5}$$

$$\Pi_L \in [0,1], \tag{6}$$

The next lemma shows that (1) has an unstable forward looking root and that a unique saddle path solution exists which satisfies (1)-(3). Given w, there is a unique REE for $L \geq 1$.

Lemma 2a (Existence and Uniqueness of $\mathbf{V}_L(w)$)

For any $w \in [0,1]$, there exists a unique solution $V_L(w)$ to (1)-(3). Furthermore :
(a) if $w \in (0,1)$;

(i) $V_L \in (0, w)$ for all $L \geq 1$,

(ii) V_L is strictly decreasing in L where $lim_{L\to\infty} V_L = 0$,

(b) if $w = 0$ or 1 then $V_L = 0$ for all $L \geq 1$.

Proof in Appendix.

Not surprisingly the more unemployed workers there are waiting for a vacancy, the worse off each unemployed worker is. Because these workers are worse off, the lower the negotiated wage in the Bertrand game. As the number unemployed grows arbitrarily large, the negotiated wage becomes arbitrarily close to zero - the workers obtain zero surplus.

The proof used to establish lemma 2a is easily modified to prove the analogous results for Π_L which we state without proof.

Lemma 2b (Existence and Uniqueness of $\Pi_L(w)$)

For any $w \in [0,1]$, there exists a unique solution $\Pi_L(w)$ to (4)-(6). Furthermore :
(a) if $w \in [0,1)$,

(i) $\Pi_L \in (0, 1-w)$ for all $L \leq -1$,

(ii) Π_L is strictly increasing in L where $lim_{L\to-\infty} \Pi_L = 0$,

(b) if $w = 1$, $\Pi_L = 0$ for all $L \leq -1$.

We now formally establish Theorem 1. By construction, lemmas 2a and 2b imply there is a unique candidate REE. Furthermore, the proof of lemmas 1a and 1b imply that $V_L(w)$ defined in lemma 1a (with $w_L(w) = V_{L-1}(w)$ for L > 0) and $\Pi_L(w)$ defined by lemma 1b (and $w_L(w) = 1 - \Pi_{L+1}(w)$ for L < 0) satisfy (A)-(C) and hence a REE exists.

The next section will determine w. However we first consider the dynamic properties of a REE which are independent of w. Given any w $\in (0,1)$, lemmas 2a and

2b imply c_L is strictly increasing in L where $c_L \rightarrow 0$ as $L \rightarrow -\infty$ and $c_L \rightarrow 1$ as L $\rightarrow \infty$. Thus the greater the value of L, the greater the entry rate of new vacancies. Define L* by $L^* = max\{L : \alpha F(c_L) \leq g\}$, which exists and is unique for $\alpha > g$. L evolves according to a stationary Markov process and hence varies stochastically over time. However, if $L > L^*$, the entry rate of vacancies exceeds the entry rate of new unemployed workers and so L is expected to decrease over time (and vice versa for $L < L^*$). Given L, the market dynamics are stable in that the expected dynamics always revert towards L^*. Although not necessarily equal to the mean of the ergodic distribution, L^* must lie within one of its mode.

Define w^e as the solution to $\alpha F(1-w^e) = g$. It follows that w^e is an increasing function of α/g, a term we shall refer to as entrepreneurial activity. Suppose given L that $w_L \geq w^e$. This implies $L \leq L^*$ and so L is expected to increase over time. Future wage agreements are expected to be closer in value to w^e. The converse is true if $w_L \leq w^e$. The equilibrium dynamics of L imply that negotiated wages center around w^e. This suggests that an increase in w will not result in higher wages in the long-run. Average unemployment adjusts so that negotiated wages hover around w^e. As we will find in the next two sections, entrepreneurial activity (α/g) plays a central role in wage determination.

4 Market Equilibrium

For simplicity, we use the results of Binmore (1987) and Binmore, Rubinstein and Wolinsky (1986) to model the bilateral bargaining wage outcome. In the market described above, if a worker and entrepreneur are bargaining bilaterally then over (small) time period $\Delta > 0$, another new agent enters the market with probability $(\alpha F(\Pi_{-1}) + g)\Delta$.[6] If a new worker enters the market, Bertrand competition implies the payoffs to the worker and entrepreneur become $V_1, 1 - V_1$ respectively, while if a new vacancy enters, their payoffs are $1-\Pi_{-1}, \Pi_{-1}$ respectively. Binmore et al (1986)

[6]If this bargaining pair reach immediate agreement, then $L = 0$. If they do not and an entrepreneur enters the market, the Bertrand game implies the entrepreneurs receive an expected payoff of Π_{-1}. Π_{-1} therefore defines the entrepreneur's reservation extraction cost when two agents are bargaining bilaterally, regardless of whether they reach immediate agreement or not.

show that in the limit as $\Delta \to 0$, the wage outcome of the strategic bargaining game limits to the solution given by the axiomatic Nash bargaining equation when each agent's Nash threatpoint is their expected payoff through perpetual disagreement during the bilateral bargaining game (also see Coles and Wright (1996) for further discussion). In the above context, this implies the worker's Nash threatpoint has to be $[\alpha F(\Pi_{-1})(1-\Pi_{-1})+gV_1]/(r+\alpha F(\Pi_{-1})+g)$. Similarly for the entrepreneur. The appropriate bilateral bargaining wage outcome, denoted w*, is therefore given by

$$w^* = argmax \left[w - \frac{\alpha F(\Pi_{-1})(1-\Pi_{-1})+gV_1}{r+\alpha F(\Pi_{-1})+g}\right]^{1-\theta}\left[1-w-\frac{\alpha F(\Pi_{-1})\Pi_{-1}+g(1-V_1)}{r+\alpha F(\Pi_{-1})+g}\right]^{\theta}$$

where $\theta \in [0,1]$ is typically referred to as the bargaining power of the entrepreneur. Solving this implies

$$w^* = \frac{r(1-\theta)+\alpha F(\Pi_{-1})[1-\Pi_{-1}]+gV_1}{r+\alpha F(\Pi_{-1})+g} \tag{7}$$

w* is a weighted average of (1-θ), $[1-\Pi_{-1}]$ and V_1. $(1-\theta)$ would be the negotiated wage if there were no breakdown of the bilateral bargaining game. [1-Π_{-1}] is the worker's payoff should a new vacancy enter, and V_1 is his payoff should a new worker enter. The relative weights depend on the rate at which each breakdown occurs.

The central reason for using (7) is that it restricts the set of REE to equilibria which are dynamically consistent. To see this, suppose we arbitrarily assumed that in the bilateral bargaining game, the players negotiate w = 0. But if w = 0, Bertrand competition implies $0 < \Pi_{-1} < 1$ (by lemma 2b). Rather than accept a wage w = 0, the worker should reject it and gamble that the next entrant is a new vacancy in which case the worker will obtain a strictly positive payoff. w = 0 is inconsistent with strategic behaviour. Similarly, if $\theta < 0$ in (7) a worker would be better off refusing to trade at w* in the bilateral bargaining game, gambling on the next entrant being a new vacancy. The same argument holds for the entrepreneur when $\theta > 1$. Only if $0 \leq \theta \leq 1$ do both the worker and the entrepreneur obtain positive surplus by trading at w*. Hence (7) with $\theta \in [0,1]$ identifies all market equilibria where both agents prefer to trade at that price than wait for the next entrant. Furthermore, Binmore et al (1986) show how this outcome can be sustained as the equilibrium of a dynamic, forward looking strategic bargaining game.

Conditional on $\theta \in [0,1]$, define a Market Equilibrium (ME) as a REE with w = w*; i.e. $\{w, w_L(w), V_L(w), \Pi_L(w)\}$ satisfy :

(D) $\{w_L(w), V_L(w), \Pi_L(w)\}$ define a REE given $w \in [0,1]$, and

(E) w satisfies the fixed point condition $w = w^*(w)$, where

$$w^*(w) = \frac{r(1-\theta) + \alpha F(\Pi_{-1}(w))[1 - \Pi_{-1}(w)] + gV_1(w)}{r + \alpha F(\Pi_{-1}(w)) + g}$$

By section 3, ME requires identifying the fixed point defined by (E) where $V_1(w)$ and $\Pi_{-1}(w)$ are given by lemmas 1a and 1b.

Theorem 2

For any r,g,$\alpha > 0$ and $\theta \in [0,1]$ a ME exists and is unique. Furthermore $w_L \in (0,1)$ for all L.

Proof in Appendix.

Having established existence and uniqueness of a ME, the next section considers its reduced form properties.

5 Numerical Simulations

There are some simple analytical comparative statics.[7] But to understand how the equilibrium distributions of L and of wages depends on the underlying market parameters $\theta, \alpha/r, g/r$, we consider some simple illustrative numerical simulations. Assume c is uniformly distributed and a discount rate of 5% per annum. We consider a representative occupation - say the market for accountants - and consider two cases ; a large city with high turnover (both g and α are high) and a small town with low turnover (low g and α). There is no job/worker migration. In the large city, on average one new unemployed accountant enters the market every day (which in a ME will also imply one new accountancy vacancy will arrive approximately each day). In the small town with low turnover, one new accountant enters the market every

[7]Firstly, an increase in θ causes w to decrease in a ME. Secondly as $\alpha, g \to 0, w \to 1\text{-}\theta$.

10 days.[8] The chosen parameter values are therefore $r = 0.05$ and $g = 365$ or 36.5, where one unit of time corresponds to a year. We shall also consider the effects of changes in entrepreneurial activity, where three values of α are considered, $\alpha = 1.5g$, $2g$ and $3g$ for each given value of g.[9] The corresponding values of w^e are 1/3, 1/2 and 2/3 respectively. Throughout we will set $\theta = 0.5$ but shall comment on how the simulations change if $\theta = 1$ (when the firm has all the bargaining power).

Notice that r/g is small in all cases, even for relatively low turnover rates.[10] Discounting over the expected time interval between consecutive entrants is small. The following result was suggested by the simulations reported below and describes two central properties of a ME.

Proposition 1 In a ME as $g, \alpha \to \infty$ but with g/α fixed :

(i)$w_L \to w^e$ for all L finite

(ii) the dynamics of L (conditional on a new entrant) tend to a random walk. Its ergodic variance becomes unboundedly large.

Proof in Appendix

Notice that in this limiting equilibrum, wages are not only independent of 1-θ (worker bargaining power), they also do not depend on L (the number unemployed). Wages are "sticky" in the sense that they are not sensitive to changes in the number unemployed. They are determined by expected flows rather than stocks and in particular depend only on the level of entrepreneurial activity.

As suggested by Proposition 1, the simulations below will show that as turnover rates increase wage variation decreases and the variance of L increases. The intuition is that if L (approximately) follows a random walk and entry rates are arbitrarily high, then the time cost to wait for market conditions to improve becomes arbitrarily

[8] An alternative interpretation is that these turnover numbers represent the thickness of markets for different occupations - for example g large may correspond to the market for unskilled labour in a local labour market, g small may correspond to the international market for brain surgeons.

[9] If $\alpha \leq g$, the entry rate of vacancies is always less than the entry rate of unemployed workers. Hence over time, the number unemployed must grow arbitrarily large and the value of being unemployed tends to zero.

[10] r/g =1 implies a new entrant arrives every 20 years.

small. Hence all wages limit to the same value. Proposition 1 proves that the limiting wage must be w^e in a ME, which implies the entry rate of vacancies equals the entry rate of unemployed workers. As wages do not adjust to variations in L, L follows a random walk and the limiting market outcome is unstable in the sense that L has unbounded variance.

Although Proposition 1 only proves that wages are independent of bargaining power when turnover rates are arbitrarily high, the numerical results below find that bargaining power has little effect on wages (in equilibrium) even for relatively low turnover rates. To understand why, notice by (7) that the surplus to be divided in the bilateral bargaining game is the difference between reaching immediate agreement and waiting for a third party to enter the market (whereupon trade occurs via Bertrand competition). For r/g small, this surplus is small which explains why changes in bargaining power will have very little effect on market prices and hence on the equilibrium unemployment dynamics.

The first table reports the equilibrium wage w for the chosen parameter values. As in all the tables reported below, comparing rows tells us the effect of increasing entrepreneurial activity, comparing columns tells us the effect of increasing scale (i.e. large v small cities).

Insert Table 1 here.

The striking result is that the bilateral bargaining wage is very close to w^e, even when there is relatively low turnover. Consistent with Proposition 1, the greater the rate of turnover, the closer w is to w^e. Giving the firm all the bargaining power ($\theta = 1$) only marginally changes these values. For example, with $\alpha = 1.5g$ and $\theta = 1$ the equilibrium wage falls by 0.2% to 0.3425 (with high turnover) and by 1.2% to 0.3483 (with low turnover). These changes are typical. The entry rates of competing workers and entrepreneurs dominates the determination of the bilateral bargaining wage - bargaining power plays little role in a ME for sensible parameter values.

Table 2 gives descriptive statistics of the ergodic distribution of L. The square brackets record [mode, mean, variance], while the round brackets record the symmetric 95% confidence interval (L_l, L_h) where $P(L < L_l) = P(L > L_h) = 0.025$.

Insert Table 2 here.

The expected value of L clearly falls as entrepreneurial activity increases. The mean of L is positive for $\alpha < 2g$ and is negative for $\alpha > 2g$. High entrepreneurial economies tend to have both higher wages and lower unemployment in equilibrium. Changing the bargaining power again has little effect on these numbers. For example with $\alpha = 1.5g$ and low turnover, raising θ from 0.5 to 1 implies the wage w falls by 1.2% as described above, but the mode only falls by one to 2, while the mean only falls by 0.6 to 5.0. The variance is unchanged at 144. A (large) change in bargaining power has a marginal effect on the distribution of L.

A second interesting result is that the distribution of L is skewed - towards positive unemployment if $\alpha < 2g$ and towards positive vacancies if $\alpha > 2g$. This occurs regardless of the chosen bargaining power. The reason for this is that the market has different "recovery rates" when $L \gg L^*$ and $L \ll L^*$. $L \gg L^*$ implies $c_L \simeq 1$ and so the expected rate at which L falls is $\alpha - g$. Conversely, if $L \ll L^*$, $c_L \simeq 0$ and the expected rate at which L increases is g. If $\alpha - g < g$, i.e $\alpha < 2g$, the market "recovers" more slowly when $L \gg L^*$ than when $L \ll L^*$, which explains the right skewness of the ergodic distribution of L. High unemployment periods tend to last longer when $\alpha/g < 2$. The converse holds when $\alpha/g > 2$. Notice that this argument is distribution independent.

Comparing high to low turnover cities, the variance of L is much greater in the high turnover market which again is consistent with Proposition 1. L is getting closer to a pure random walk. The next table describes the equilibrium variation in wages. Each bracket here records $[w_{L_u}, w_{L_l}]$, which are the equilibrium wages negotiated at the two ends of the confidence interval defined in the previous table. This approximately gives a 95% confidence interval for negotiated wages.

Insert Table 3 here

Not surprisingly, there is equilibrium wage variation. A worker who enters the market when there are many vacancies can negotiate a better wage contract than someone who enters the market when there are many unemployed. This risk is greatest in the low turnover markets. Wage (or utility) dispersion in the low turnover market is roughly double that in high turnover markets. This again reflects Proposition 1 where as turnover increases, all wages converge to the ergodic center wage. Of course, the small variation in w in high turnover markets is responsible for the large variation in L.

The final table computes the expected payoffs of the two types of agents (using the ergodic distribution probabilities) and the expected duration of unemployment. The square bracket records the expected value of being an unemployed worker, and the expected value of finding an investment opportunity (computed as $E_L \int_0^{c_L} [c_L - c] \; dF(c)$). The round brackets record the expected duration of unemployment on entering the market and the expected duration of unemployment when $L = 50$ (when there is unusually high unemployment).[11]

Insert Table 4 here.

Again, consistent with Proposition 1, the expected value of being unemployed converges to w^e as turnover increases. An increase in entrepreneurial activity has two main effects. The first is that the unemployed become better off while the value of an investment opportunity decreases. This occurs because of the wage adjustments described in Table 1. An increase in entrepreneurial activity also decreases the expected duration of unemployment, which is consistent with the distribution shifts described in Table 2.

Workers are slightly better off in high turnover cities while entrepreneurs are very slightly worse off. More interestingly, there is a dramatic fall in the expected duration of unemployment in higher turnover cities, even though L varies more widely in such markets. For example, if $\alpha = 2g$, the expected duration of unemployment is approximately 10 days in the high turnover city, while it is 6 weeks in the low turnover city. This unemployment duration statistic is also highly skewed to the right. The probability the market is long in vacancies when a worker first enters this market is approximately 0.5 (for these parameter values). Hence, unless the worker is lucky and a suitable vacancy already exists (in which case the worker experiences no

[11] These calculations assume that if there are $L > 0$ unemployed workers when a new vacancy arrives, the probability that a particular worker gets the job is $1/L$.

unemployment), the expected duration of unemployment is approximately 3 weeks in the high turnover city and 3 months in the low turnover city. Being unemployed is much riskier in low turnover markets.

If there are fifty unemployed workers, the expected duration of unemployment is six weeks in the high turnover city and is almost one year in the low turnover city. Although job destruction is not formally modelled here, this suggests that large cities with high turnover can absorb large unemployment shocks more quickly than small cities. Large and small cities have different risk characteristics.

6 The Social Planner's Problem

The previous section has pointed out that in equilibrium, workers can experience extended spells of unemployment. In particular, Table 3 shows the value of being unemployed can be quite low in low turnover cities and unemployment spells can be quite long. To determine the efficiency of this market outcome, we need to consider the Social Planner's problem.

Appendix C fully describes the SP's problem and obtains conditions which characterise his reservation investment cost, denoted c_L^p. It shows that in general the market outcome is not efficient, though it is possible to achieve the first best solution by a system of investment subsidies s_L. However, to understand the structure of these optimal investment subsidies, we continue to consider the above numerical simulations.

For the high turnover cases, it turns out that the market outcome is virtually fully efficient. This reflects the asymptotic result that as $r \to 0$ (or equivalently, as $\alpha, g \to \infty$) the market solution and the Social Planner's solution converge to the same outcome $F(c_L), F(c_L^p) \to g/\alpha$.[12] This is perhaps not surprising as the market becomes frictionless as $r \to 0$, while bargaining power cannot distort the bilateral bargaining wage.

[12] As $r \to 0$, the SP becomes arbitrarily patient. Given workers arrive at rate g, matching them with a job requires exploiting fraction g/α of all investment opportunities. Being arbitrarily patient, the least cost method of doing this is to use a reservation extraction cost c^p defined by $\alpha F(c^p) = g$. By Proposition 1, this is the same as the limiting market solution.

However, what is surprising is that the market is (almost) fully efficient in the low turnover case as well. The following graph depicts the optimal set of subsidies for the case $\alpha = 1.5g$ and low turnover. We highlight this case as it is the one where unemployment durations were the longest and the market was skewed towards positive unemployment. Intuition might have suggested that this market outcome was inefficient. The graph plots the optimal subsidies for $\theta = 0.5$ (which requires positive subsidies) and $\theta = 1$ (which requires negative subsidies (a tax)).

Insert Figure 1 here.

For $\theta = 0.5$, workers have too much bargaining power in the sense that 1-$\theta > w^e$. The bilateral bargaining wage outcome is too high. In the uncorrected ME, firms tend to underinvest and therefore the government's optimal policy is to subsidize investment. Conversely, if $\theta = 1$, the workers have too little bargaining power where $1 - \theta < w^e-$ the bilateral bargaining wage is too low. Firms therefore tend to overinvest in the uncorrected ME and so the government should respond by imposing an investment tax. However, the startling result is the size of the subsidies - they are very small. For $\theta = 0.5$, the largest subsidy occurs at $L = 1$, but this is still less than 1% of $c_0^p \simeq 2/3$. The loss in Social Welfare by not using corrective subsidies is small. For example, at L = 1, the value of the market solution without the subsidies is the same as that of the Social Planner's solution to 5 significant figures (after which computing rounding error and truncation error become problematic). This result is typical.

Even for reasonably low turnover rates the market solution is remarkably efficient. In particular, notice that the market outcome and the Social Planner's solution have not converged to the asymptotically efficient extraction rate of $g/\alpha = 0.667$. Table 3 shows that at the upper 95% confidence tail (where $L = 31$), the extraction probability is $1 - w_L = 0.762$, which is also (approximately) the Social Planner's extraction probability. This value is not close to $0.66\bar{7}$. This efficiency result is not due to asymptotic convergence.

The appropriate intuition is that even for relatively low turnover rates, bargaining power has little effect on the bilateral bargaining wage. For r/g small, this price is dominated by market forces (by the relative arrival rates of new entrants) which therefore reflects the shadow value of a worker when $L = 1$. Bertrand competition away from the bilateral bargaining case keeps the market quasi-competitive. As the negotiated wages reflect the worker's social value everywhere, there is no investment externality. The market (almost) replicates the SP solution, even for relatively low turnover numbers.

7 Conclusion

This paper has considered an equilibrium model of unemployment with entrepreneurial search and wage bargaining but with no matching frictions. In this framework, spells of unemployment are experienced by workers when there are zero vacancies for their particular occupation and they have to wait for an appropriate vacancy to enter the market. This is a different explanation of unemployment than the one provided by the standard matching approach - there appropriate vacancies always exist but workers face matching frictions to locate them.

The most striking result is that for reasonable numerical values, the value of the market outcome (almost) equals the value of the Social Planner's optimum. Even though unemployed workers in low turnover markets may experience extended unemployment spells which can be up to a year in length, the market is not acting inefficiently. Of course central to this result was the way the bilateral bargaining outcome was modelled. That choice ensured the wage was responsive to market conditions and hence potentially reflected the shadow value of the worker. If we had arbitrarily imposed a bilateral bargaining wage of 0.5, this efficiency result would disappear but this would be inconsistent with strategic behaviour. A second interesting result is that this paper explains why in equilibrium markets can experience wide variations in unemployment, and yet the equilibrium wage hardly changes. Furthermore, this is not an inefficient outcome.

Many extensions suggest themselves. Allowing wages to be renegotiated would change the equilibrium wage dynamics but it is not clear that it would change the turnover behaviour much. Indeed, there might be an interesting equivalence result. Perhaps the most important extension is to allow the inflow of unemployed workers to vary stochastically. It is well documented that over the business cycle, job destruction is relatively volatile (see for example Davis and Haltiwanger (1992).[13] One simple variation of the model above is to allow g to take two values (high and low) and suppose that g flips between these two values according to a Poisson process (say there are exogenous productivity shocks on filled vacancies). Initial work shows that if g drops to its low value, the wage level immediately jumps up which lowers the inflow of vacancies. But the ensuing flow of new vacancies exceeds g. The stock of vacancies tends to rise, the stock of unemployment tends to fall and job creation exceeds job destruction during this phase. Conversely, if g increases, the wage immediately drops and the flow of vacancies increases, but does not rise as high as g. The stock of vacancies tends to decrease while unemployment tends to rise. Job destruction exceeds job creation during this phase. This simple example obtains the correct negative time series covariance between unemployment and vacancies, and is consistent with job creation being smoother than job destruction. However, such work is still preliminary.

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[13] Indeed, a challenge for the standard matching approach has been to explain this data - see Mortensen and Pissarides (1994) for example).

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9 APPENDIX A

To clarify the arguments used, the proofs of technical lemmas are collected separately into Appendix B.

Proof of Lemma 1a : First consider $L \geq 2$. For $\Delta > 0$ but arbitrarily small, (A) and (B) are satisfied if and only if V_L satisfies the Bellman equation,

$$\begin{aligned}(1+r\Delta)V_L &= \alpha F(1-V_{L-1})\Delta V_{L-1} + g\Delta V_{L+1} \\ &\quad +[1-\alpha F(1-V_{L-1})\Delta - g\Delta]V_L + o(\Delta),\end{aligned}$$

where $w_L = V_{L-1}$.With probability $\alpha\Delta$, an entrepreneur finds a profit making opportunity and, given $w_L = V_{L-1}$, invests with probability $F(1-V_{L-1})$. Given that outcome, the Bertrand equilibrium implies each unemployed worker receives an expected payoff of $V_{L-1}(w)$. With probability $g\Delta$, another unemployed worker enters the market and L increases to $L+1$. Otherwise the state remains unchanged. The $o(\Delta)$ term reflects the Poisson approximation for $\Delta > 0$. Rearranging and letting $\Delta \to 0$ implies (1) for $L > 1$.

Now consider $L = 1$. This time if a new vacancy enters the market, the negotiated wage equals w, and so the corresponding Bellman equation is

$$(1+r\Delta)V_1 = \alpha F(1-w)\Delta w + g\Delta V_2 + [1-\alpha F(1-w)\Delta - g\Delta]V_1 + o(\Delta).$$

Rearranging and letting $\Delta \to 0$ implies (1) with (2) as the appropriate boundary condition. Given $V_L = w_{L+1}$, (3) is equivalent to (C). ■

Proof of Lemma 2a: Lemmas A1-A4 below establish the lemma for $w \in (0,1)$, while lemma A5 will consider $w = 0$ and $w = 1$.

Fix $w \in (0,1)$ and consider the class of functions $W_L(k)$ which satisfy (1), i.e.

$$W_{L+1} = [1+\frac{r}{g}]W_L + \frac{\alpha}{g}F(1-W_{L-1})[W_L - W_{L-1}] \tag{8}$$

but subject to the initial conditions $W_0 = w$ and $W_1 = k$, where $k \in [0,1]$. Lemma A1 establishes that for any k, $W_L(k)$ either asymptotes to $+\infty, -\infty$ or zero. Lemmas A2, A3 and A4 then establish there is a unique value of $k \in [0,1]$, denoted k*, where $W_L(k^*) \to 0$ as $L \to \infty$ and that $W_L(k^*) \in (0,1)$ for all L. Hence there exists a unique solution to (1)-(3) defined by $V_L = W_L(k^*)$.

Lemma A1 : Given $W_{L-1} > 0$,

(i) if $W_L \leq 0$ then $W_{L+S} < W_{L+S-1}$ for all $S \geq 1$ and $W_{L+S} \to -\infty$ as $S \to \infty$.

(ii) if $W_L \geq W_{L-1}$then $W_{L+S} > W_{L+S-1}$ for all $S \geq 1$ and $W_{L+S} \to \infty$ as $S \to \infty$.

Proof : Follows directly by using an induction argument on (8). ■

Given $w \in (0,1)$, lemma A1 implies that $W_L(k)$ either asymptotes to $+\infty$ or $-\infty$ as $L \to \infty$, or describes a strictly positive, strictly decreasing sequence $\{W_L\}$. In this latter case, monotonicity and boundedness imply this sequence must attain its limit point and (8) implies its limit point $\underline{W} = 0$.

Define $I^+ = \{k : W_L \to \infty \text{ as } L \to \infty\}$, $I^- = \{k : W_L \to -\infty \text{ as } L \to \infty\}$ and $I^0 = \{k : W_L \to 0 \text{ as } L \to \infty\}$. Lemma A1 implies $1 \in I^+$ and $0 \in I^-$, so that these two sets are non-empty, and that I^+, I^- and I^0 form a complete partition of the line [0,1]. Lemmas A2-A4 provide further information on the structure of these sets.

Lemma A2

(i) If $lim_{L\to\infty} W_L(k) \to \infty$, then $[k,1] \subseteq I^+$, i.e. I^+ is connected

(ii) If $lim_{L\to\infty} W_L(k) \to -\infty$, then $[0,k] \subseteq$ I$^-$, i.e. I$^-$ is connected.

Proof in Appendix B

Lemma A3: $\sup I^- = \inf I^+$.

Proof in Appendix B

Let k^*=sup $I^- = \inf I^+$. Lemma A4 completes the proof by showing that I^-, I^+ have appropriate closure properties.

Lemma A4: For some $k^* \in (0,1)$, $I^0 = \{k^*\}$.

Proof in Appendix B

Lemma A4 establishes that I^0 contains a single element. Further lemma A1 implies $W_L(k^*)$ describes a strictly positive, strictly decreasing sequence, where $k^* \in$

$(0, w) \subset [0, 1]$, and hence $W_L(k^*) \in (0, 1)$ for all L. This completes the proof of Lemma 2a for $w \in (0, 1)$.

Lemma A5

If $w = 0$ or 1 then $V_L = 0$ for all $L > 0$ is the unique solution to (1)-(3).

Proof in Appendix B

This completes the proof of lemma 2a.

Proof of Theorem 2

Notice that the fixed point condition (E) can rearranged as :

(E') w satisfies the equation $\phi(w) = 1 - \theta$ where

$$\phi(w) = w + \frac{g}{r}[w - V_1(w)] + \frac{\alpha}{r}F(\Pi_{-1}(w))[w + \Pi_{-1}(w) - 1] \tag{9}$$

A ME is equivalently defined as any solution to (D) and (E').

Existence :

Lemma A6 below implies ϕ is a continuous function of w. Lemmas 2a and 2b imply $\phi(0) < 0 \leq 1 - \theta$ and $\phi(1) > 1 \geq 1 - \theta$. Hence by continuity there must exist w $\in (0, 1)$ where $\phi(w) = 1 - \theta$ which implies existence. Furthermore, lemmas 2a, 2b and (A) now imply $w_L(w) \in (0, 1)$ for all L.

Lemma A6 (Continuity of $V_1(w)$ and $\Pi_{-1}(w)$)

$V_1(w), \Pi_{-1}(w)$ as defined in lemmas 1a and 1b are continuous functions for all $w \in [0, 1]$.

Proof in Appendix B.

Uniqueness :

Together lemmas A7-A9 below imply that ϕ is a strictly increasing function of w which by (E') implies uniqueness of a ME.

Lemma A7

For all $w \in [0, 1]$, $w - V_1(w)$ is strictly increasing in w.

Proof : Follows directly from Lemma B1 (in Appendix B) with $L = 1$ and using $V_1(0) = 0$. ■

Lemma A8

For all $w \in [0,1]$, $1 - w - \Pi_{-1}(w)$ is positive and strictly decreasing in w.

Proof in Appendix B

Lemma A9

For all $w \in [0,1]$, $\Pi_{-1}(w)$ is strictly decreasing in w.

Proof Consider $L = -1$ and $w, w' \in [0,1]$ where $\Pi_{-1}(w) = \Pi_{-1}(w')$. Uniqueness of the saddle path solution to (4)-(6) implies $\Pi_{-2}(w) = \Pi_{-2}(w')$. Rearranging (4) it can then be shown that this implies $\Pi_0(w) = \Pi_0(w')$.[14] But the definition of Π_0 implies $w = w'$. Hence Π_{-1} is strictly monotonic in w, and since it must be positive and $\Pi_{-1}(1) = 0$, it must be strictly decreasing.

This completes the proof of Theorem 2. ■

Proof of Proposition 1 The definition of a market equilibrium implies the market outcome is a function of g/r,α/r and θ [given F]. Letting $\alpha, g \to \infty$ while holding α/g fixed is mathematically equivalent to holding α, g fixed and letting r $\to$ 0. We shall prove that for any $\epsilon > 0$ (arbitrarily small), $|w^* - w^e| < \epsilon$ for r small enough. The following four lemmas set up the appropriate limiting argument.

Lemma A10(a)

If $w \in [w^e + \epsilon, 1)$, then $V_1(w) \leq w - d$ for all $r > 0$, where

$$d = 0.5\epsilon[1 - \frac{\alpha}{g}F(1 - w^e - 0.5\epsilon)] > 0.$$

i.e. $V_1(w)$ is bounded away from w in the limit as $r \to 0$.

Proof in Appendix B

Lemma A11a : If $w \in (0, w^e - \epsilon]$, then $V_1(w) \to w$ as $r \to 0$.

Proof in Appendix B

The same arguments apply for Π_{-1} and we simply state the corresponding results.

Lemma A10b : If $w \in (0, w^e - \epsilon]$, then $\Pi_{-1}(w)$ is bounded away from $1 - w$ for all $r > 0$.

[14] This proof does apply for $V_1(w)$ as (1) does not imply a unique solution for V_0, given V_1 and V_2.

Lemma A11b : If $w \in [w^e + \epsilon, 1)$, then $\Pi_{-1}(w) \to 1 - w$ as $r \to 0$.

These lemmas now allow us to prove Proposition 1. Theorem 2 implies a ME exists for all $r > 0$ where $w \in (0,1)$ must satisfy (E′), i.e.

$$r(1-\theta) = rw + g[w - V_1(w)] + \alpha F(\Pi_{-1}(w))[\Pi_{-1}(w) - (1-w)]$$

Given $\epsilon > 0$, lemmas A10b and A11a imply that $w \in (0, w^e - \epsilon]$ cannot satisfy this condition for r small enough as $\Pi_{-1}(w)$ is bounded away from $1-w$ (and from zero[15]) while all other terms converge to zero. Similarly, $w \in [w^e + \epsilon, 1)$ cannot satisfy this relationship for r small enough as $V_1(w)$ is bounded away from w. Hence w must lie within an ϵ—neighbourhood of w^e for r small enough.

The remaining limiting statements follow by inspection of (1), (4) and the fact that $w \to w^e$ as $r \to 0$. In particular, it is straightforward to show that this implies $V_L - V_{L+1}, \Pi_{L-1} - \Pi_L \to 0$ (otherwise (1) and (4) imply these differences grow exponentially with L and hence do not converge). Hence $V_L \to w^e, \Pi_L \to 1 - w^e$ as $r \to 0$ for all L finite. This solution implies that L tends to a random walk as $r \to 0$,which completes the proof.

[15] For w < 1, a simple contradiction argument on (4), using lemma 2b, implies $\Pi_{-1}(w)$ cannot tend to zero as r → 0.

10 Appendix B

Proof of Lemma A2

(i) Given k, define $M = min\{L : W_L(k) \leq W_{L-1}(k)\}$ which implies $W_L(k)$ is decreasing in L for $L \leq M$, that $W_M(k) > 0$ and $W_{M+1}(k) > W_M(k)$. We now prove (i) holds by contradiction, i.e. suppose $\exists\ k' \in (k, 1]$ where $W_L(k')$ does not asymptote to $+\infty$. Hence by Lemma A1, $W_L(k')$ is strictly decreasing for all L.

Notice that $k' > k$ implies $W_1(k') > W_1(k)$ and $W_1(k') - W_0(k') > W_1(k) - W_0(k)$. Using an induction argument on (8), with the fact that both sequences are decreasing for all $L \leq M$, it can be established that $W_L(k') > W_L(k)$ and $W_{L+1}(k') - W_L(k') > W_{L+1}(k) - W_L(k)$ for all L $\leq M$. But by definition of M, this implies $W_M(k') > W_M(k) > 0$ and $W_{M+1}(k') - W_M(k') > W_{M+1}(k) - W_M(k) > 0$. But lemma A1 then implies $\lim_{L\to\infty} W_L(k') = \infty$, which is the required contradiction.

A similar argument establishes (ii). ■

Proof of Lemma A3

Let $k_0 = \sup I^-$ and $k_1 = \inf I^+$. Lemma A2 immediately implies that $k_0 \leq k_1$. We now prove lemma A3 by contradiction. Suppose lemma A3 is false and hence by lemma A2, $k_0 < k_1$. Then $(k_0, k_1) \subseteq I^0$. Now consider $k, k' \in (k_0, k_1)$ where $k' > k$ and lemma A1 implies $W_L(k), W_L(k')$ are both strictly positive, strictly decreasing sequences which asymptote to zero. However, an induction argument on (8) implies $W_L(k') > W_L(k)$ and $W_{L+1}(k') - W_L(k') > W_{L+1}(k) - W_L(k)$ for all $L > 0$. Hence $lim_{L\to\infty} W_L(k') - lim_{L\to\infty} W_L(k) > k' - k > 0$, which is the required contradiction as both sequences must asymptote to zero. Hence $k_0 = k_1$. ■

Proof of Lemma A4

Given lemmas A2, A3 and that I^+, I^-, I^0 form a complete partition of the line [0,1], lemma A4 is established by showing that I^-, I^+ are open sets at $k = k^*$.

Notice that by induction on (8) given the initial conditions, $W_L(k)$ is a continuous function of k for all $L > 0$ (finite). Now consider $k \in I^+$. By lemma 1, there exists some $M > 0$ where $W_{M-1}(k) > 0$. $W_M(k) \geq W_{M-1}(k)$ and (8) implies $W_{M+1}(k) \geq [1 + \frac{r}{g}] W_M(k)$. Now consider $k' = k - \epsilon$, where $\epsilon > 0$. By continuity, it follows that for ϵ small enough, $W_{M+1}(k') > W_M(k') > 0$ and hence by lemma A1, $k' \in I^+$. Hence I^+

must be an open set at $k = k^*$. Similarly for I^-. ■

Proof of Lemma A5 : If $w = 1$, (8) implies $W_2 = (1 + r/g)W_1$. But if $W_1 > 0$, this implies $W_2 > W_1$ and by lemma A1, W_L asymptotes to $+\infty$. Hence $W_1 = 0$ is the only possible solution and induction on (8) implies $W_L = 0$ for all L. Hence there is a unique solution to (1)-(3) given by $V_L = 0$.

If $w = 0$, (8) implies $W_2 = (1 + (\alpha + r)/g)W_1$. The same argument implies $V_L = 0$ for all $L \geq 1$ is the unique solution to (1)-(3). ■

Proof of Lemma A6

To prove continuity of V_1, we need the following preliminary result

Lemma B1

If $V_{L-1}(w') > V_{L-1}(w)$ then $V_{L-1}(w') - V_L(w') > V_{L-1}(w) - V_L(w)$.

Proof. By contradiction. Suppose not for some L, w, w' and so $V_L(w') - V_L(w) \geq V_{L-1}(w') - V_{L-1}(w)$. Notice this implies $V_L(w') > V_L(w)$. By (1),

$$\begin{aligned} V_{L+1}(w') - V_{L+1}(w) \;=\; & [1 + \frac{r}{g}][V_L(w') - V_L(w)] \\ & -\frac{\alpha}{g}F(1 - V_{L-1}(w')[V_{L-1}(w') - V_L(w')] \\ & +\frac{\alpha}{g}F(1 - V_{L-1}(w)[V_{L-1}(w) - V_L(w)] \end{aligned} \tag{10}$$

But $V_L(.)$ is a decreasing sequence (by Lemma 2a), and so F strictly increasing, $V_{L-1}(w') > V_{L-1}(w)$, $V_{L-1}(w') - V_L(w') \leq V_{L-1}(w) - V_L(w)$ and (10) imply $V_{L+1}(w') - V_{L+1}(w) \geq [1 + \frac{r}{g}][V_L(w') - V_L(w)]$. It also implies $V_{L+1}(w') > V_{L+1}(w)$. An induction argument on (10) now implies $V_{L+S+1}(w') - V_{L+S+1}(w) \geq [1 + \frac{r}{g}][V_{L+S}(w') - V_{L+S}(w)]$ for all $S > 0$, which contradicts that $V_{L+S}(x) \to 0$ as S$\to \infty$ for all $x \in [0, 1]$. ■

Putting $L = 1$ in lemma B1 implies $w' - V_1(w') > w - V_1(w)$ if $w' > w$, i.e. $w - V_1(w)$ is strictly increasing with w. Hence, lemma 2a implies $w - V_1(w)$ is bounded between zero and one.

Putting $L = 1$ in (1) implies

$$[r + g + \alpha F(1 - w)]V_1(w) - gV_2(w) = \alpha w F(1 - w) \tag{11}$$

If $w, w' \in [0,1]$, (11) implies

$$[r + \alpha F(1-w')][V_1(w') - V_1(w)] + g\,[[V_1(w') - V_1(w)] - [V_2(w') - V_2(w)]$$
$$= \alpha[F(1-w') - F(1-w)][w - V_1(w)] + \alpha F(1-w')[w' - w] \qquad (12)$$

Now suppose $|w' - w|$ is arbitrarily small. $0 \leq w - V_1(w) \leq 1$ and F continuous and bounded implies the RHS of (12) is arbitrarily small. Lemma B1 implies that if $V_1(w') > V_1(w)$ then $[V_1(w') - V_1(w)]-$ $[V_2(w') - V_2(w)] > 0$. Hence $V_1(w') - V_1(w)$ must be arbitrarily small, otherwise the LHS of (12) is not arbitrarily small. Hence $V_1(w)$ must be a continuous function for all $w \in [0,1]$. ■

The argument is easily adapted to establish that $\Pi_{-1}(w)$ is also a continuous function.

Proof of Lemma A8 : Using the same arguments used to prove lemma B1, it is straightforward to show that if $\Pi_L(w') > \Pi_L(w)$ then $\Pi_L(w') - \Pi_{L-1}(w') > \Pi_L(w) - \Pi_{L-1}(w)$. Putting L = 0 implies the lemma.

Proof of Lemma A10(a): By contradiction. Suppose for some $r > 0$, $V_1(w) > w-d$. Using the fact that V_L must describe a positive, strictly decreasing sequence (by lemma 2a), a simple induction argument implies $V_L > w^e + 0.5\epsilon\ [1+[\frac{\alpha}{g}F(1-w^e-0.5\epsilon)]^L]$ for all $L > 0$, where by (1), $V_L - V_{L+1} < d[\frac{\alpha}{g}F(1-w^e-0.5\epsilon)]^L$. But this contradicts $V_L \to 0$ as $L \to \infty$. ■

Proof of Lemma A11a :Choose any $\eta > 0$ (arbitrarily small) and consider $r < \delta$ where $\delta = \eta[\alpha F(1-w)-g]/(2w) > 0$. If $w - V_1 \geq \eta$, then induction on (1), using lemma 2a which implies V_L describes a positive decreasing sequence, implies $V_L - V_{L+1} > \eta$ for all $L > 1$. But such a sequence cannot converge to zero as $L \to \infty$. Hence $w - V_1 < \eta$, which implies the lemma. ■

11 Appendix C

Let W_L denote the value of the Social Planner's programming problem with L unmatched agents currently in the market. First suppose $L > 0$. If an entrepreneur finds an investment opportunity with cost c and the SP decides to realize the opportunity, a worker is immediately employed from the current pool of unemployed and the value of this outcome is $1 - c + W_{L-1}$. Alternatively, if the opportunity is not realized, the value of the SP's problem remains at W_L. Hence for Δ arbitrarily small and $L > 0, W_L$ is defined recursively by

$$[1 + r\Delta]W_L = g\Delta W_{L+1} + \alpha\Delta E\left[\max[W_L, 1 - c + W_{L-1}]\right] + [1 - g\Delta - \alpha\Delta]W_L + o(\Delta)$$

The SP's optimal strategy has the reservation cost property, where the opportunity is realized if and only if $c \leq c_L^p$ where $c_L^p = 1 + W_{L-1} - W_L$. Rearranging and letting $\Delta \to 0$, the above implies

$$(r+g)W_L = gW_{L+1} + \alpha \int_0^{c_L^p} F(c)dc$$

Similar arguments hold for $L < 0$ and $L = 0$.

Proposition 2

W_L satisfies the following conditions.

(i) If $L \geq 1$;

$$(r+g)W_L = gW_{L+1} + \alpha \int_0^{c_L^p} F(c)dc$$

where $c_L^p = 1 + W_{L-1} - W_L$.

(ii) If $L \leq -1$;

$$(r+g)W_L = g[1 + W_{L+1}] + \alpha \int_0^{c_L^p} F(c)dc$$

where $c_L^p = W_{L-1} - W_L$.

(iii) If $L = 0$;

$$(r+g)W_0 = gW_1 + \alpha \int_0^{c_0^p} F(c)dc$$

where $c_0^p = W_{-1} - W_0$,

subject to the boundary condition that W_L is bounded for all L.

Proof The recursive equations are obtained using standard arguments. Boundedness follows because $0 \leq W_L \leq \alpha/r$ for all L [where the upper bound is obtained by putting $L = \infty$ and $c = 0$ so that each investment opportunity gives the maximal payoff of unity.] ■

The method for identifying a solution is the same as for identifying a ME. Given W_0, the saddle path properties of the two recursive equations imply a unique solution for W_L for $L \geq 1$ and $L \leq -1$. Given the implied values for W_{-1} and W_1, the problem is to find a fixed point, where W_0 must also satisfy (iii) in Proposition 2. In what follows, we assume a solution exists which implies optimal reservation extraction levels $c_L^p \in [0, 1]$.

The central question is whether $c_L = c_L^p$. This requires that

(i) $1 - V_{L-1} = 1 + W_{L-1} - W_L$ for $L \geq 2$,

(ii) $1 - w = 1 + W_0 - W_1$, for $L = 1$, and

(iii) $\Pi_{L-1} = W_{L-1} - W_L$ for $L \leq 0$.

Clearly the difference equations describing Π_L, V_L and W_L are quite different and so in general, these conditions will not be satisfied. Essentially there are two distortions. The first is the wage bargaining distortion. w is negotiated through strategic bargaining, and does not necessarily equal $W_1 - W_0$, the true shadow value of the worker when $L = 1$. This not only impacts at $L = 1$ but via the difference equations, has real distortionary effects for all L. Secondly there is a common resource externality problem. When $L > 0$, an entrepreneur on deciding whether to invest or not, only asks whether the negotiated wage will be less than $1 - c$. He does not take into account that by hiring a worker, there are fewer workers available for the other entrepreneurs (see footnote 5).

Given the solution for c_L^p, the appropriate policy response of the government is to introduce a system of investment subsidies s_L. With the optimal subsidies, firms enter at rate $\alpha F(c_L^p)$. Let V_L^p denote the value of being unemployed in a ME with the Planner's optimal subsidies in place. Then V_L^p is defined recursively by

$$rV_L^p = g[V_{L+1}^p - V_L^p] + \alpha F(c_L^p)[V_{L-1}^p - V_L^p]$$

which is analogous to (1), and is solved subject to the same boundary conditions. Similarly for Π_L^p :

$$r\Pi_L^p = g[\Pi_{L+1}^p - \Pi_L^p] + \alpha F(c_L^p)[\Pi_{L-1}^p - \Pi_L^p]$$

and the definition of w^{*p}. Assuming a M.E. exists, the optimal investment subsidy is $s_L = c_L^p - (1 - V_{L-1}^p)$ for $L \geq 1$. Similarly for $L < 1$.

12 Tables and Figures.

Table 1 : Equilibrium Bilateral Bargaining Wage (w*)

Entrepreneurial Activity	High Turnover (g=365)	Low Turnover (g=36.5)
$\alpha = 1.5g$	0.3434	0.3523
$\alpha = 2g$	0.4989	0.4957
$\alpha = 3g$	0.6586	0.6477

Table 2 : Summary Statistics of the Equilibrium Ergodic Distribution of L

Entrepreurial Activity	High Turnover	Low Turnover
$\alpha = 1.5g$	[7 , 11.1 , 623]	[3 , 5.6 , 144]
	(-35 , 64)	(-16 , 31)
$\alpha = 2g$	[0 , 0.2 , 467]	[0 , 0.3 , 106]
	(-42 , 44)	(-20 , 21)
$\alpha = 3g$	[-6 , -8.5 , 385]	[-3 , -3.9 , 87]
	(-49, 29)	(-23 , 14)

Table 3 Equilibrium Wage Variation.

Entrepreneurial Activity	High Turnover	Low Turnover
$\alpha = 1.5g$	[0.288 , 0.387]	[0.238 , 0.441]
$\alpha = 2g$	[0.456 , 0.541]	[0.407 , 0.584]
$\alpha = 3g$	[0.630 , 0.694]	[0.588 , 0.723]

Table 4 - Expected Equilibrium Payoffs and Unemployment Duration

Entrepreurial Activity	High Turnover	Low Turnover
$\alpha = 1.5g$	[0.331 , 0.222]	[0.324 , 0.224]
	(0.044 , 0.125)	(0.217 , 1.03)
$\alpha = 2g$	[0.498 , 0.125]	[0.490 , 0.126]
	(0.023 , 0.118)	(0.114 , 0.902)
$\alpha = 3g$	[0.665 , 0.056]	[0.659 , 0.056]
	(0.011 , 0.109)	(0.055 , 0.767)

UNEMPLOYMENT (U), NEW HIRES (H), (ADJUSTED) VACANCIES (V) : USA, 1968 - 1981 (monthly)

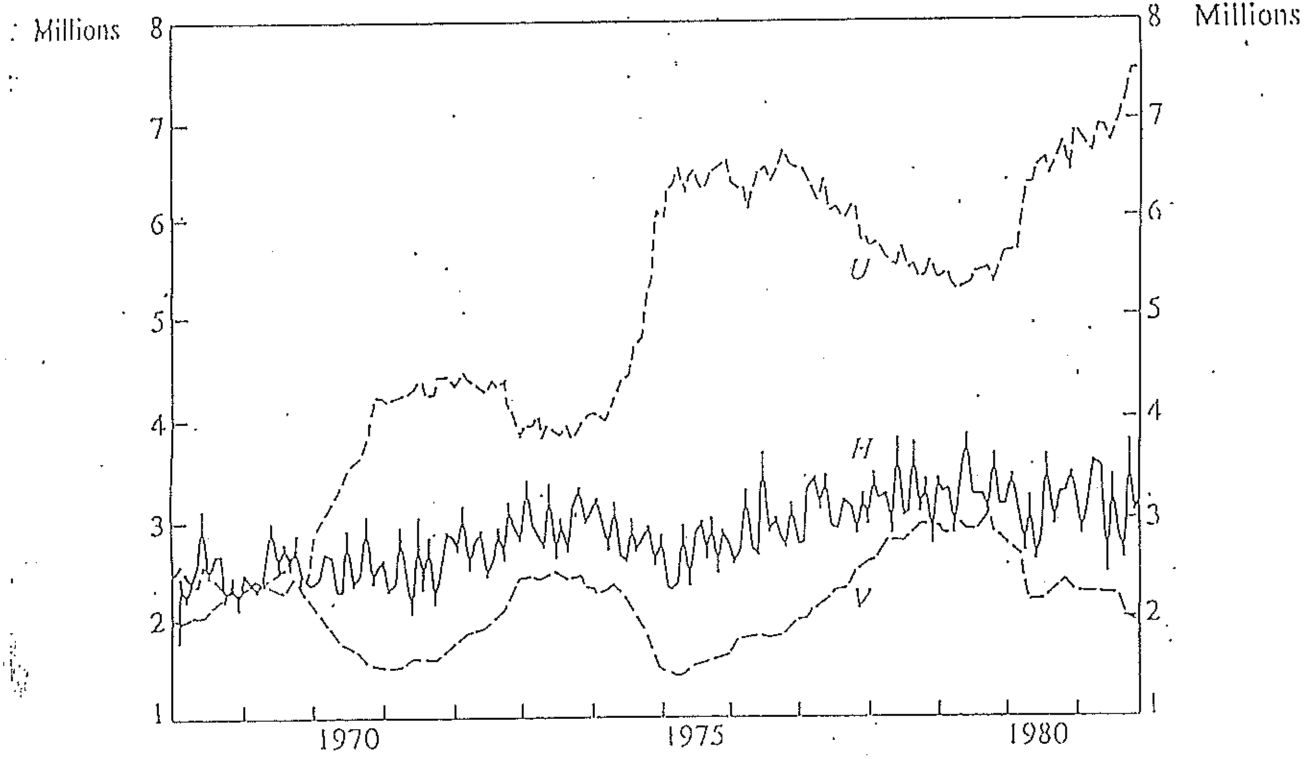

Figure 1 Optimal Investment Subsidies [r = 0.05, $\alpha = 1.5g$, Low Turnover]