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Nicolas Sahuguet

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Keywords: Comparative politics, Electoral rule, Contests, Candidate selection, Party Lists. proportional representation, party organization

Nicolas Sahuguet - nicolas.sahuguet@hec.ca
HEC Montreal and CEPR

Comparative Politics with Intraparty Candidate Selection*

Benoit S Y Crutzen[†]
Erasmus University Rotterdam

Nicolas Sahuguet[‡]
HEC Montréal

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Abstract

We develop a two-stage model in which parties select candidates before the election. Elections are under first-past-the-post (FPTP) or closed-list proportional representation (PR). Selection is competitive or non-competitive. With non-competitive selection, candidate effort is higher under FPTP. With competitive selection, effort is higher under PR. Under PR, competition motivates candidates to exert effort to be selected (as under FPTP) and to be ranked higher on the list. Empirical studies comparing electoral rules should consider how parties organize, to avoid omitted variable bias. The results also suggest that electoral rules influence how parties organize.

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[†]Department of Economics, Erasmus School of Economics, Erasmus Universiteit Rotterdam. crutzen@ese.eur.nl.

[‡]Department of Applied Economics, HEC Montréal. nicolas.sahuguet@hec.ca

1 Introduction

The theoretical literature on comparative politics predicts that electoral rules influence the behavior of politicians. When voters can compare candidates individually, as in the British first-past-the-post system, elections provide strong incentives for candidates to act in the interest of the electorate. To the contrary, when parties run as a team, voters can't select their favorite candidate. Closed list proportional representation, used for example in Israel or Spain, may lead candidates to free ride on their teammates and thus weaken their incentives to perform.

Candidate selection by parties provides another source of incentives to politicians. To stand for election, a candidate - be them an incumbent or not - must first win his party nomination. Indeed, in most democracies, political parties play the role of "gate keepers of the secret garden of politics," to borrow from Gallagher and Marsh (1988). More recently, Norris (2006) also points to the importance of parties in candidate selection, when she remarks that "the process of recruitment [...] is widely regarded as one of the most important residual functions of parties [...]."

Candidate selection criteria vary across democracies (Poguntke et al. 2016). Some countries, like Finland or the US, rely on decentralized processes and use primary elections to select candidates. Other countries, like Portugal and Spain, use a more centralized process, with the party elite or even just the party leader firmly in control of selection (see Montabes and Ortega, 1999, and Shomer, 2017). In some countries, like Norway, parties must respect legally established gender quotas. In India, the electoral law guarantees minority representation (see Kapoor and Magesan, 2018).

Somewhat surprisingly, this second source of incentives has not received much attention. The cross-country variation in candidate selection processes motivates our approach to include a selection stage in our comparative politics analysis. We wish to understand how electoral rules interact with candidate selection to influence the behavior of politicians.[1]

We develop a two-stage model in which parties select candidates, before the election

[1] There exists a growing literature on candidates' self-selection into the pool of candidates parties choose from. For a recent survey of this literature, see Dal Bo and Finan (2018). Our theory focuses on the stages of the game that follow such self-selection by eligible citizens.

takes place. Candidates invest time and resources into crafting policies that voters value (see Caillaud and Tirole 2002). This investment is modelled as a single costly effort decision.

In the legislative election stage, we compare British or US first-past-the-post (FPTP) to Israeli or Spanish closed-list proportional representation (PR). Under the first electoral rule, the number of constituencies is equal to the number of legislative seats. Each party lets one candidate compete for election in each constituency. Each district elects one candidate to the legislature, by plurality rule. Under PR, each party files a team of candidates. All teams compete to win as many seats as possible in a single, nationwide electoral district.

We model elections as imperfectly discriminating contests (see Tullock 1980). Under FPTP, a candidate wins his district with a probability proportional to the ratio of his effort over the sum of the efforts of all other candidates. Modelling elections under PR is more difficult as parties are teams, and not unitary actors *à la* Downs (1957). We build on the model of a team contest for multiple indivisible prizes of Crutzen, Flamand and Sahuguet (2020). A party wins a legislative seat according to the standard Tullock contest success function based on party aggregate efforts. A party's aggregate effort is the simple sum of the efforts of the candidates on its list. The probability that a party wins x seats then follows a simple binomial distribution with the number of seats in the legislature and the Tullock ratio as parameters. The model allows the mapping from votes to seats to match the empirical evidence (see Buisseret et al. 2019 for an analysis of elections in Sweden). As its seat share grows, a party finds it increasingly difficult to win an extra seat. The model is also tractable. This allows us to consider many real world features of elections.

Turning to the selection stage, for reasons of parsimony and tractability, we conflate all aspects of candidate selection into a single dimension: its degree of competition. As economists, we believe this dimension plays a central role. We thus consider two types of candidate selection processes: a competitive and a non-competitive one. Under the competitive process, selection depends only on the efforts of candidates. Candidates know that their effort influences not only their election chances, but also the probability that their party selects them to run in the election. Under FPTP, in every district, competition inside parties is a simple Tullock contest between candidates. Under PR, within each party, a pool of candidates compete for the highest possible ranks on the party list. Under non-competitive selection, the choice of candidates does not depend on effort. Under FPTP, in each district, a party candidate knows he is running in his district's election before exerting effort. Under PR, candidates know their position on the list before exerting effort.

We first compare candidates' efforts under the two electoral rules, when selection is not competitive. We show that PR provides weaker incentives than FPTP. This is in line with the previous literature, which points out some drawbacks of PR. To quote Persson et al. (2003, p. 961), under PR, "politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. Under proportional representation, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [as] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance". When parties select their candidates in a non-competitive fashion, our model replicates the above comparative politics result. Candidates at the top and bottom of the list exert little effort as effort has only a tiny impact on their probability of winning a seat. Candidates in the middle of the list have strong incentives. However, their high efforts do not compensate the low efforts of the other list members. Thus, a party's aggregate effort is higher under FPTP than PR.

We derive our second comparative politics result by comparing effort choices across the two electoral rules when selection is competitive. We solve for equilibrium efforts for any number of candidates at the selection stage. We show that, under FPTP, competition in each district between one or two candidates maximizes incentives. Under PR, we set this number equal to the number of seats available in the legislature. We find that a party's aggregate effort is higher under PR than under FPTP (and the same holds at the level of individual candidates, as under competitive selection all candidates exert the same effort, even under PR). Accounting for the incentives generated by selection at the party level thus overturns the result on the impact of electoral rules on average effort. The key reason for this reversal is that, under PR with competitive selection, candidates compete not just to be included on the party list, but to get the highest possible spot on the list.

These comparative politics results generate two implications. First, any empirical analysis that compares electoral rules should include variables about the way parties select their candidates. Otherwise, comparative politics results could suffer from an important omitted variable problem. This problem could explain why the empirical comparative politics literature does not reach any clear-cut conclusion about the impact of electoral rules.

Second, our results hint at a causal relationship between electoral rules and the way political parties organize. Under FPTP, the general election provides strong incentives to politicians, while these incentives are less powerful under PR. As a consequence, under FPTP, parties do not need to control the rules of the selection process to generate incentives. These findings thus predict that party elites want to retain more organizational control under PR than under FPTP, all else equal.

The rest of the paper is structured follows. The next section reviews the related literature. Section 3 presents the basic model and the next Section solves it. Section 5 derives the main comparative politics result. Section 6 presents several extensions and robustness checks and the last section concludes. An appendix contains all the proofs.

2 Related Literature

We contribute to the theoretical comparative politics literature that studies candidates' incentives under different electoral rules. The literature is vast but neglects the role of candidate selection. Important contributions on incentive aspects of elections include Bawn and Thies (2003), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Morelli (2004), Myersson (1993a, 1993b and 1999), Persson, Roland and Tabellini (2000) and Persson and Tabellini (1999, 2000 and 2003).[2] These theories do not all point in the same direction. Myerson (1993a) argues that incentives to offer targeted transfers to small subgroups of the electorate are similar across FPTP and PR if the number of parties is similar across electoral rules, but those incentives get worse under PR if the number of parties

[2]There is also a large theoretical and empirical literature that focuses on the selection of different types of candidates and on adverse selection. Recent contributions under FPTP include Snyder and Ting (2011) and Galasso and Nannicini (2011). Contributions that focus on selection issues under PR or multiple electoral rules include Besley et al. (2017), Buisseret et al. (2019), Buisseret and Prato (2019), Dal Bo et al. (2017), Gagliarducci et al. (2011), Galasso and Nannicini (2015), Hangartner et al. (2019), Mattozzi and Merlo (2015), Merilainen and Tukiainen (2018) and Myerson (1993b).

is larger under that rule. Lizzeri and Persico (2001) build on Myerson (1993a) to suggest that candidates' incentives are more aligned with voter preferences under PR in large electoral districts: politicians are less tempted to divert resources from the budget of a nationwide public good, to target inefficiently their electoral promises to subgroups of the population. In contrast, Persson and Tabellini (1999) hold the opposite view: incentives are stronger in elections under FPTP because voters can more effectively discipline politicians and make them accountable for their actions. Our paper develops a model in which politicians exert effort and is thus closer in spirit to Persson and Tabellini (1999). Our findings suggest that considering the way parties organize is important to understand the incentives of politicians under various electoral rules.

We model elections as contests (see Tullock (1980)), to obtain a tractable framework that replicates important aspects of the mapping from candidate choices to seats in both FPTP and PR systems. For elections under PR, we build on the team contest model of Crutzen, Flamand and Sahuguet (2020). Imperfectly discriminating contests are somewhat of a black box. Yet, the literature does not offer a model of elections under PR with strong microfoundations of voters' decisions. Standard alternatives such as the probabilistic model of Lindbeck and Weibull (1987) are subject to the same black box criticism.

As in Caillaud and Tirole (2002), the main function of a party is to select which eligible candidates to let run for election under its banner. Our focus on candidate selection is dictated by the fact that it is a key function of parties. For the US, Ranney (1975: 121, quoted by Ware 2002: 1, 95) also states that the way parties select their candidates is of first order importance for electoral outcomes: "[t]he adoption of the direct primary by the states from the early 1900s onward is [. . .] the most radical of all the party reforms adopted in the whole course of American history." We extend the existing literature by considering candidate selection under PR. The literature on candidates' incentives only focuses on the selection of a single candidate, either for a general election or for a district-specific one. Important contributions include Carillo and Mariotti (2001), Caillaud and Tirole (2002), Hirano, Snyder and Ting (2009), Castanheira, Crutzen and Sahuguet (2010) and Aragon (2014). Caillaud and Tirole (2002) is arguably the first model of party organization as a source of incentives for individual politicians.[3] Castanheira, Crutzen and Sahuguet (2010) extend the analysis of Caillaud and Tirole (2002) to allow for different political motives and general equilibrium effects. As in Castanheira et al. (2010), when selection is not competitive, candidates exert effort after learning that their party selected them for the election. To model competitive selection, we build on the seminal contribution in the theory of contests of Clark and Riis (1996).

A growing literature, in political science, analyses different characteristics of candidate selection processes, such as their degree of centralization or inclusiveness. Important contributions include, besides those cited above, Bille (2001), Hazan and Pennings (2001), Hazan and Rahat (2006, 2010), Katz and Mair (1994), Lundell (2004), Norris (1997, 2006), Rahat and Hazan (2001) and Shomer (2014, 2017). We contribute to this literature as we suggest that the degree of competition in the candidate selection process matters to understand political outcomes.

3 The Model

Consider a society with a mass K of voters, K odd and equal to 3 or more, and an election for K seats in parliament. We analyze two electoral rules: US- or British-style first past the post in K identical single member districts (FPTP hereafter) and Israeli-style proportional representation with closed party lists in a single nationwide district (PR hereafter).

Candidates belong to one of two parties L and R and choose effort e to maximize their expected utility. Candidates get a payoff $V \geq 0$ when they win a seat in the legislature. Candidates also get a payoff $M \geq 0$, when their party wins control of government. This second component captures the utility associated with the party of the candidate achieving its electoral goals. It is thus a proxy for the ideological color of candidates. The cost of effort is quadratic: $c(e) = \frac{1}{2}e^2$.

Under FPTP, an election takes place in every district between two party candidates. The result of the election depends on their efforts, e^L and e^R. Under PR, parties compete in the election via a list of K candidates. Voters can only cast their ballot for either list, they

[3] Alesina and Spear (1988) do not focus on how parties organize.

cannot vote for individual candidates. The aggregate effort of a party is defined as the sum of the efforts of all candidates on its list: $E^P = \sum_{m=1}^{K} e_m^P$. where e_m^P is the effort of the candidate of party P in mth position on the list. Parties' electoral success is a function of these aggregate efforts.

Within parties, we consider two candidate selection processes. When selection is non-competitive, the party chooses the candidates for the election based on candidate characteristics independent of effort. Under FPTP, the party selects one candidate for each district. Under PR, the party chooses K candidates and their order on the list. When selection is competitive, several candidates compete for selection on the basis of their effort.[4] Under FPTP, in each district, $n \geq 1$ candidates compete in a primary election for the right to run in the general election. Under PR, $N \geq K$ candidates compete nationwide for the right to be on the party list and for their position on the list. We now describe each scenario in detail.

3.1 First Past the Post

3.1.1 Non-competitive Selection

In each district, the two party candidates exert effort e^L and e^R. Party $L's$ candidate wins the election with probability:

$$p^L\left(e^L, e^R\right) = \lambda\left(\frac{e^L}{e^L + e^R}\right) + \frac{1-\lambda}{2}. \tag{1}$$

Parameter $\lambda \in [0,1]$ represents the importance of effort in the result of the election. When $\lambda = 1$, we have the lottery contest function. When $\lambda = 0$, the election is random: the result does not depend on candidate effort.

Party $L's$ candidate chooses effort e^L to maximize:

$$p^L\left[V + MP^L\left(\tfrac{K-1}{2}, K-1\right)\right] + M\left[\sum_{k=\frac{K+1}{2}}^{K-1} P^L\left(k, K-1\right)\right] - \tfrac{1}{2}\left(e^L\right)^2, \tag{2}$$

[4]Hirano and Snyder (2014) report that the issues candidates focus on and invest time and effort in during the two major US party primaries before legislative elections are typically also at the forefront of these legislative elections.

where $P^L(k, K-1) = C_k^{K-1}(p^L)^k(1-p^L)^{K-k-1}$ denotes the probability that party L wins k of the other $K-1$ seats

Candidates choose their effort considering the prospect of both getting elected and their party winning a majority of seats. The candidate's election is pivotal for the party winning a majority of seats when his party wins in exactly $\frac{K-1}{2}$ of the other $K-1$ districts. No matter the result of his election, the party of the candidate also wins a majority of seats when the party wins at least $\frac{K+1}{2}$ districts among the $K-1$ other districts.

As the maximization problem is symmetric both within and between parties, all candidates exert the same effort. We have:[5]

Proposition 1 *Under FPTP, when selection is non-competitive, in the symmetric equilibrium, candidates exert effort equal to* $\sqrt{\frac{\lambda(V+\bar{M})}{4}}$, *with* $\bar{M} = MC_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}$.

3.1.2 Competitive Selection

Before the election, parties select candidates on the basis of their efforts, for example in a primary election. We model the primary as a Tullock contest between $n > 1$ party candidates. Candidate i of party L wins the primary and represents his party in his district's general election with probability:

$$Q^{iL} = \frac{e^{iL}}{e^{iL} + \sum_{k \neq i} e^{kL}}, \tag{3}$$

where e^{kL} denotes the effort of candidate k of party L in the primary.

Candidate i chooses effort e^{iL} to maximize:

$$\begin{aligned}
&Q^{iL}p^L\left[V + MP^L\left(\tfrac{K-1}{2}, K-1\right)\right] + \left(1 - Q^{iL}\right)p_{-i}^L MP^L\left(\tfrac{K-1}{2}, K-1\right) \\
&+M\left[\sum_{j=\frac{K+1}{2}}^{K-1} P^L(j, K-1)\right] - \frac{\left(e^{iL}\right)^2}{2}.
\end{aligned} \tag{4}$$

The second term represents the payoff of winning a majority when candidate i loses the primary but the winner of that primary still ends up winning the district; we denote the probability of this event with p_{-i}^L.

In the symmetric equilibrium, all candidates exert the same effort. We have:

[5] All the proofs are in the Appendix.

Proposition 2 *Under plurality rule, when candidate selection is competitive among n candidates in each district, in the symmetric equilibrium, candidates exert effort equal to:*

$$e^* = \sqrt{V\left(\frac{n-1}{2n^2} + \frac{\lambda}{4n}\right) + \frac{\lambda}{4n}\bar{M}}.$$

If $\lambda < \frac{V}{\bar{M}+V}$ *, letting two candidates compete for selection* ($n = 2$) *maximizes effort.*
If $\lambda > \frac{V}{\bar{M}+V}$*, making selection non-competitive* ($n = 1$) *maximizes effort.*

The optimal number of candidates trades off two effects: a dilution and a competition effect. The dilution effect quickly counterbalances the competition effect as the number of candidates increases. Given this, parties severely restrict competition and do not let more than 2 candidates compete in the primary.

The effect of parameters λ, $\bar{M}$ and V on the optimal number of candidates deserves more comments. Parameter λ represents the relative elasticity of the general election result to candidates' effort with that of the primary election. When λ is small, the general election generates weak incentives and competitive selection becomes valuable. The ratio $\frac{V}{\bar{M}+V}$ represents the relative importance of individual and collective objectives for candidates. The more opportunistic are candidates – opportunistic behavior is stronger the larger is V compared to $\bar{M}$ – the more likely it is that λ is smaller than $\frac{V}{\bar{M}+V}$. If $\lambda < \frac{V}{\bar{M}+V}$, having a competitive selection stage provides better incentives to candidates. The more 'ideologically' motivated are candidates – the larger is $\bar{M}$ compared to V – the more likely that λ is larger than $\frac{V}{\bar{M}+V}$. If $\lambda \geq \frac{V}{\bar{M}+V}$ intraparty incentives are not useful; the dilution effect dominates as candidates exert effort to see their party win a majority rather than win a seat themselves.

3.2 Proportional Representation

Under PR, the result of the election depends on the two parties' aggregate efforts. The aggregate effort is the sum of efforts of all candidates on the list.[6] Specifically, the probability that party L wins a seat in the legislature is equal to $p^L = \lambda\frac{E^L}{E^L+E^R} + \frac{1-\lambda}{2}$. We assume that parties' seat shares in the legislature follow a binomial distribution. This Binomial-Tullock

[6]See Crutzen, Flamand and Sahuguet (2020) for a an aggregate production function with complementarities between candidates' individual efforts.

distribution is a natural extension of the Tullock contest function to the case of multiple prizes.[7] The probability that party L wins k seats is then $P^L(k) = C_k^K \left(p^L\right)^k \left(1-p^L\right)^{K-k}$.

3.2.1 Non-Competitive Selection

When selection is non-competitive, parties first select K candidates on the basis of some effort-independent characteristics and order them on a list, and then each selected candidate exerts effort. The candidate in mth position on the list of party L chooses effort e_m^L to maximize:

$$\sum_{s=m}^{K} P^L(s)\, V + \sum_{t=\frac{K+1}{2}}^{K} P^L(t)\, M - \frac{1}{2}\left(e_m^L\right)^2 \tag{5}$$

Incentives to get elected are mediated by the position on the party list. The prospect of helping the party reach a majority always matters for incentives. This is not the case under FPTP. Thus, partywide performance looms larger under PR. We have:

Proposition 3 *Under PR, when selection is non-competitive, in the symmetric equilibrium, candidates exert effort as a function of their position on the list:*

$$e_m^* = \frac{\sqrt{\lambda}}{2\sqrt{K\left(V + K\bar{M}\right)}} \left(mC_m^K \left(\frac{1}{2}\right)^{K-1} V + \bar{M} \right).$$

A party's aggregate effort is equal to $K\sqrt{\frac{\lambda\left(\frac{V}{K}+\bar{M}\right)}{4}}$.

Thus, under PR, when selection is not competitive, candidates at the top and bottom of a party list exert little effort, as the marginal effect of effort on their election probability is small. Candidates in the middle of the list exert highest effort. The distribution of efforts within parties is bell-shaped and symmetric about the median list position.

3.2.2 Competitive Selection

When selection is competitive, $N \geq K$ candidates compete for the K positions on the party list. As the allocation of seats follows the list, the first position on the list is more valuable

[7] Buisseret et al. (2019) analyze data from Swedish elections to estimate the probability that a candidate wins a seat as a function of his position on the list. The Binomial-Tullock distribution is consistent with their estimates.

than the second position and so on and so forth. We thus model the selection process as a contest between N candidates for K prizes of different values. We use the imperfectly discriminating contest model of Clark and Riis (1996).[8] Denote the effort of candidate i by e_i; the probability that i ends up in position k or higher on the party list is:

$$Q_i(k) = p_1 + \sum_{j=2}^{k} p_j \left[\prod_{s=1}^{j-1} (1 - p_s) \right], \tag{6}$$

where p_j is the probability that i ends up in position $j = 1, ..., k$ on the list and is given by the standard Tullock ratio contest success function among the candidates who have not yet been attributed a slot on the list. Thus, for the jth prize, candidate i competes with $N - j$ other party candidates and wins with probability:

$$p_j = \frac{e_i}{e_i + \sum_{k \neq i} e_k}, \#k = N - j. \tag{7}$$

We can interpret these probabilities as the result of a sequential process. Each candidate exerts effort at the beginning of the contest. Then, a Tullock contest with the contributions of the N contestants determines the winner of the first prize (the top spot the list). The winner and his contribution are then excluded. A Tullock contest with the contributions of the remaining $N - 1$ contestants determines the winner of the second prize. This process continues until all K prizes are awarded.

Candidates choose effort e_i^L to maximize:

$$\sum_{m=1}^{K} P^L(m)\, Q_i(m) V + \sum_{j=\frac{K+1}{2}}^{K} P^L(j)\, M - \frac{\left(e_i^L\right)^2}{2}. \tag{8}$$

Although K may not be the optimal number of candidates , we set $N = K$ for simplicity. We have:

[8] Clark and Riis (1996) is the standard model of contests with multiple prizes. The model is tractable and allows for closed-form solutions. The model also presents many desirable axiomatic properties (see Fu and Lu, 2012).

Proposition 4 *Under PR, when K candidates compete within each party to be ranked as high as possible on the list, in the symmetric equilibrium, candidates exert effort equal to:*

$$e^* = \sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K} \sum_{j=1}^{m} \frac{1}{K-j+1}\right]}. \tag{9}$$

4 Comparative Politics

Putting together Propositions 1-4 above, we derive our central comparative politics prediction:

Theorem 5 *The competitiveness of the selection process drives the ranking of electoral rules. When candidate selection is non-competitive, incentives are stronger under FPTP. When candidate selection is competitive, incentives are stronger under PR.*

The first part of the theorem is in line with previous results in the literature (see for example Persson, Tabellini and Trebbi 2003). Intuitively, under FPTP, the benefit of competitive selection is decreased by the dilution effect, while under PR, the party can reap the benefits of competitive selection without the need to increase the number of candidates and the associated dilution of incentives. Intraparty competition for the best spots on the party list turns out to provide strong incentives to all candidates. The literature seems to have overlooked this effect of party lists. Somewhat ironically, the supposedly unconditional perverse incentives of closed lists are typically at the center of critiques of PR. Our findings suggest that such perverse incentives may not materialize provided that parties allocate the positions on the list in a competitive fashion. Remark finally that, if candidates are purely ideological – if $M > V = 0$ – the two electoral rules generate the same incentives. Thus the above ranking of electoral rules when selection is non-competitive requires candidates to be opportunistic, at least to some extent.

Our findings also complement those of Myerson (1993b) and Buisseret and Prato (2019) on the effects of district magnitude. Both papers conclude that increasing the size of electoral districts leads to better outcomes for voters. Larger districts generate more *interparty* competition, which gives voters larger freedom of choice (Myerson), or allow for a better balancing of the objectives of voters and parties (Buisseret and Prato). Our model adds *intraparty* competition as a source of positive incentives associated with larger districts.

4.1 Implications

Our findings generate two fundamental implications. First, empirical comparative politics analysis should include data capturing candidates's selection, to avoid an important omitted variable problem. This data should focus not only on the formal and informal characteristics of the candidate selection process but also on the party leadership's goals and strengths. Indeed, it is rational for a farsighted party leadership to choose the selection procedure that maximizes the party's electoral success, as we assume in this paper. Yet, the leadership could use its power to select candidates for other reasons, such as favoring friends or factions or returning favors to other members of the party. This omitted variable problem could in itself be sufficient to explain why the existing empirical comparative politics evidence does not deliver clear-cut predictions. For example, Persson, Tabellini and Trebbi (2003, p.983) conclude their analysis by noting that "a comprehensive electoral reform, going from a Dutch-style electoral system [under proportional representation] with closed party lists in a single national constituency to a UK-style system with first past the post in one-member districts [would produce] a net result close to zero.". Yet, their empirical analysis does not consider how parties organize.

Existing data sets focus almost exclusively on the level of democratization, centralization and inclusiveness of candidate selection (see for example the Political Party Database introduced in Poguntke et al. 2016 and Scarrow et al. 2017, or the data sets in Kernell 2015, Lundel 2004 and Shomer 2014). Democratization refers to how democratic the selection system is. Centralization refers to the extent to which candidates selection is the sole remit of the party leader (fully centralized process) or, to the contrary, is left exclusively to the party's grass. roots and local branches (full decentralization). Inclusiveness refers to the size and reach of the selectorate choosing party candidates: a fully inclusive system allows any individual, be them registered party members or not, to vote; an exclusive system endows only a few party members "usually the party elite" with this task. Our theory suggests that the degree of competition in selection is also of primary importance. Whereas intuition suggests that competition and decentralization go hand in hand (see Bille 2001 and Lundel 2004), this need not be the case. For example, local party barons may end up controlling selection. Also, a democratic and inclusive system is no guarantee for selection to be competitive (see Hirano and Snyder 2014, Hassell 2016 and Crutzen and Sahuguet 2018 about US primaries).

Second and more generally, our results hint at a causal relationship between electoral rules and the way political parties organize. All else equal, under FPTP, the general election on its own incentivizes politicians substantially. Electoral incentives are less powerful in elections under PR. As a consequence, parties have less need to generate competition among candidates under FPTP. Under PR, parties have more to gain to become and remain well-organized machines. Political parties do take many forms. They go from the "American" model of the party as a decentralized and candidate-centered organization – Katz and Kolodny (1994) refers to the two main American parties as "empty vessels" given the dominant position individual incumbents seem to have in US politics, an aspect of US politics that has, if anything, grown in importance in the last twenty-five years – to European parties which are highly hierarchical organizations that regulate public affairs and in which, quite often, the party elite is in charge of selection. Of course, many forces to explain such differences. Our theory predicts that the electoral rule is one of them.

5 Discussion and Extensions

We first discuss a few key features of our model, then consider two important extensions.

We assume that voters care about politicians' effort because effort increases the quality of policy and decision making. By 'quality' we mean any characteristic of legislative work, policymaking and implemented policies that the electorate values (See Caillaud and Tirole 2002 or Castanheira, Crutzen and Sahuguet 2010). The assumption that candidates choose their effort only once – even though they participate in two contests – allows a direct comparison of effort under different systems. This assumption also has the benefit of simplifying the analysis and does find some empirical support: Hirano and Snyder (2014) offer evidence that candidates' choices in the primary also matter for the general election.

Under FPTP, voters only consider the efforts exerted by the two candidates in their district and neglect the efforts all other politicians in other districts. This assumption follows previous literature and allows us to replicate existing comparative politics results. Under the alternative assumption, that voters decide for which party to vote on the basis of the aggregate efforts of parties also under FPTP, then two electoral systems would lead to the same aggregate efforts when selection is non-competitive.[9] This is in line with the literature that argues that FPTP generates stronger incentives because of the closer link between politicians' decisions and voters' behavior. This advantage of FPTP disappears when voters base their decisions on the same inputs under different electoral systems.

Under FPTP, we assume that candidate selection takes place in each district independently and not at the national level. We do not observe such a grand contest in practice: district-specific party elites want and do influence selection in their district. Furthermore, in many democracies, like the US, candidates must be residents of their district, which hinders the cross-district mobility a national selection would require.

We assume that parameter λ, that drives the noisiness of the election is the same under PR and FPTP. We do not take a stand about which electoral system is more subject to noise. Of course, we can easily relax this assumption. Increasing the noise in the election leads to weaker incentives.

We now turn to two extensions of the model.

5.1 More Than Two Parties Under PR

PR typically displays a larger number of parties than FPTP. As candidates' incentives to exert effort are decreasing in the number of parties, FPTP can be associated to higher aggregate effort than PR irrespective of the degree of competition in the selection of candidates:

Proposition 6 *Suppose that 2 parties compete under FPTP and $Z > 2$ parties compete under PR. If Z is large enough, a party's aggregate effort is lower under PR than that under FPTP irrespective of the type of candidate selection process.*

[9] This result would obtain even though individual efforts would differ across the two systems, because of party lists under PR.

Proposition 6 suggests a novel trade-off in PR systems between the desire to represent citizen preferences and the need for incentives. Our theory also offers a rationale for the existence of broad, mainstream parties, as their coverage of many societal issues reduces the need for a large number of parties. Lizzeri and Persico (2005) also highlights the cost of an 'excessive' number of parties under PR.

5.2 Ideological Voters

In the model, parties do not take any ideological position. Candidates only differ in their choice of effort, and candidates' ideology appears only in the payoff parameter M. Allowing for more general voter ideological preferences would make the model intractable. Still, to understand the interaction between ideology and electoral incentives, we carry out the following thought experiment. We assume symmetry at the national level, so that electoral competition does not change under PR. Under FPTP, some districts are party strongholds: one party wins the election for sure. In other districts, competition is as in the basic model. When selection is not competitive, the candidate running in a stronghold district doesn't exert effort, as their election is certain. And the other party does not bother filing in a candidate. This implies that, when selection is not competitive, introducing ideology reduces incentives under FPTP but not under PR. Still, as long as there are not too many party strongholds, FPTP would still lead to more effort than PR.

When selection is competitive, candidates will exert effort even in party strongholds. Suppose there are s party strongholds, $s/2$ per party. Each candidate in a party stronghold knows that his party wins for sure in his party's $s/2$ strongholds (and loses for sure in the strongholds of the other party) and that his party probability of winning a majority of seats is $P^L\left(\frac{K-s+1}{2}, K-s\right)$ irrespective of whether or not the candidate is selected by the party. Thus, the problem each candidate faces in a party stronghold is to maximize:[10]

$$Q^{iL}V + MP^L\left(\tfrac{K-s+1}{2}, K-s\right) - \frac{\left(e^{iL}\right)^2}{2}. \tag{10}$$

[10]As long as the number of strongholds is small, we can consider that incentives in the other districts are unaffected by the presence of strongholds, that is, the effect of the presence of these strongholds on the value of $\bar{M}$ is negligible.

The FOC to a candidate's problem yields:

$$e^* = \sqrt{\frac{n-1}{n^2}V}, \tag{11}$$

which is maximized for $n = 2$, yielding $e^* = \sqrt{V/4}$.

Thus, in each party's stronghold, each candidate *of the advantaged party* exerts effort equal to e^* whereas no effort takes place in the other party. From above we know that, when ideology plays no role in the electorate, the optimal number of candidates in the intraparty selection process is either one or two. For both cases, as long as V is not too large compared to M, our comparative politics prediction still goes through.[11]

6 Conclusion

We develop a novel model of legislative elections preceded by an intraparty selection stage. Selection can be either competitive or not. When selection is non-competitive, FPTP generates stronger incentives than closed-list PR. To the contrary, when selection is competitive, PR generates stronger incentives than FPTP. Our results suggest that empirical analysis of the impact of electoral rules or any other institution affecting elections should include data about the organizational choices of parties, to avoid an omitted variable problem. Also, our results point to a causal relationship of electoral rules on the way parties organize.

One limitation of our approach is the large degree of symmetry in the model: all candidates are identical, both in terms of cost of effort and in terms of ability or quality. In Crutzen, Konishi and Sahuguet (2020), we extend the model to allow for heterogeneous candidates and derive a party's optimal list composition. We rationalize the recurrent observation that parties put their best candidates at the top of their list, and not around a party's marginal seat position, even though the marginal seat position generates the highest incentives to exert effort. A second important avenue for research is to go beyond our binomial-Tullock framework, to better microfound voters' decision under list PR and the as-

[11]The condition for our comparative politics result to go through is the same for both cases and is given by: $V < \frac{\lambda}{1-\lambda}\bar{M}$.

sociated distribution of seats. Such a task appears quite challenging, given the complexities of the game under list PR, but is also a very exciting topic for future research.

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Appendix : Proofs

Proof of Proposition 1

Parety L's Candidate chooses effort e^L to maximize:

$$\left(\tfrac{e^L}{e^L+e^R}+\tfrac{1-\lambda}{2}\right)\left[V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right]+M\left(\sum_{k=\frac{K+1}{2}}^{K-1}P^L\left(k,K-1\right)\right)-\tfrac{1}{2}\left(e^L\right)^2$$

The first-order condition yields:

$$\tfrac{e^R}{(e^L+e^R)^2}\lambda\left[V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right]-e^L=0.$$

Second order conditions are satisfied.

In the symmetric equilibrium, $P^L\left(\frac{K-1}{2},K-1\right)=C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$ and equilibrium effort is given by:

$$e^*=\sqrt{\frac{\lambda}{4}\left(V+MC^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}\right)}=\sqrt{\frac{\lambda}{4}\left(V+\bar{M}\right)}.$$

Each party's aggregate effort is then equal to:

$$E^*=Ke^*=K\sqrt{\frac{\lambda}{4}\left(V+\bar{M}\right)}.$$

Proof of Proposition 2

Candidate i chooses effort e^{iL} to maximize:

$$\begin{aligned}Q^{iL}p^L\left(V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right)+\left(1-Q^{iL}\right)\left(p^L_{-i}MP^L\left(\tfrac{K-1}{2},K-1\right)\right)\\+M\left(\textstyle\sum_{j=\frac{K+1}{2}}^{K-1}P^L\left(j,K-1\right)\right)-\tfrac{\left(e^{iL}\right)^2}{2}.\end{aligned}.$$

The first order condition yields:

$$\left[\frac{dQ^{iL}}{de^{iL*}}p_L+\frac{dp_L}{de^{iL*}}Q^{iL}\right]\left(V+MP^L\left(\tfrac{K-1}{2},K-1\right)\right)-\frac{dQ^{iL}}{de^{iL*}}\frac{e^{kL}}{e^{kL}+e^R}MP^L\left(\tfrac{K-1}{2},K-1\right)-e^{iL*}=0$$

We have:

$$\frac{dQ^{iL}}{de^{iL*}} = \frac{\sum_{j=1}^{n} e^{jL} - e^{iL}}{\left(\sum_{j=1}^{n} e^{jL}\right)^2}$$

$$\frac{dp_L}{de^{iL*}} = \frac{\lambda e^R}{\left(e^{iL} + e^R\right)^2}.$$

In the symmetric equilibrium, we get:

$$e^* = \sqrt{\left(\frac{n-1}{2n^2} + \frac{\lambda}{4n}\right) V + \frac{\lambda}{4n}\bar{M}}.$$

The second order condition is $\left(Q''p + 2Q'p' + Qp''\right)\left(V + \bar{M}\right) - Q''p^L_{-i} - 1$.

We also have that:

$$p' = \lambda \tfrac{p(1-p)}{e^L},\ Q' = \tfrac{Q(1-Q)}{e^L},\ p'' = \tfrac{\lambda p(1-p)}{\left(e^L\right)^2}\left(\lambda\left(1-2p\right) - 1\right) =,\ Q'' = \tfrac{-2}{\left(e^L\right)^2} Q^2\left(1-Q\right).$$

Both p'' and Q'' are negative but both Q' and p' are positive. Using our last derivations, the SOC can be rewritten as:

$$\left(V + \bar{M}\right)\left(\tfrac{-2}{\left(e^L\right)^2} Q^2\left(1-Q\right) p + 2\tfrac{\lambda(1-p)(1-Q)pQ}{\left(e^L\right)^2} + \tfrac{\lambda p(1-p)}{\left(e^L\right)^2}\left(\lambda\left(1-2p\right) - 1\right) Q\right) + \tfrac{2}{\left(e^L\right)^2} p^L_{-i} Q^2\left(1-Q\right)\bar{M} - 1$$
$$= \tfrac{\left(V+\bar{M}\right)Qp}{\left(e^L\right)^2}\left(-2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right) - 1\right)\right) + \tfrac{2}{\left(e^L\right)^2} p^L_{-i} Q^2\left(1-Q\right)\bar{M} - 1$$

The FOC implies that

$$\left(e^L\right)^2 = \left(Q(1-Q)p + Q\lambda p(1-p)\right)\left(V + \bar{M}\right)$$

Thus the SOC simplifies further to

$$\frac{-2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right) - 1\right)}{(1-Q) + \lambda(1-p)} + \frac{2p^L_{-i}/pQ\left(1-Q\right)}{\left((1-Q) + \lambda(1-p)\right)}\frac{\bar{M}}{\left(V + \bar{M}\right)} - 1$$

and to

$$-2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right) - 1\right) - \left(1-Q\right) - \lambda(1-p) + 2p^L_{-i}/pQ\left(1-Q\right)\frac{\bar{M}}{\left(V + \bar{M}\right)}$$
$$= -2Q\left(1-Q\right) + 2\lambda(1-p)(1-Q) + \lambda\left(1-p\right)\left(\lambda\left(1-2p\right)\right) - 2\lambda\left(1-p\right) - \left(1-Q\right) + 2p^L_{-i}/pQ\left(1-Q\right)\frac{\bar{M}}{\left(V + \bar{M}\right)},$$

which is negative.

Proof of Proposition 3

The candidate in mth position on the electoral list of party L chooses effort e_m^L to maximize:

$$\sum_{k=m}^{K} P^L(k)\,V + \sum_{j=\frac{K+1}{2}}^{K} P^L(j)\,M - \frac{1}{2}\left(e_m^L\right)^2.$$

The first order condition to the problem of that candidate is given by:

$$\begin{aligned}
e_m^L = {} & \lambda V \sum_{k=m}^{K} C_k^K P_L^{k-1}\left(1-P_L\right)^{K-k} k \tfrac{E^R}{\left(E^L+E^R\right)^2} \\
& - \lambda V \sum_{k=m}^{K} C_k^K \left(P_L\right)^k \left(1-P_L\right)^{K-k-1} \left(K-k\right) \tfrac{E^R}{\left(E^L+E^R\right)^2} \\
& + \lambda M \sum_{j=\frac{K+1}{2}}^{K} C_j^K P_L^{j-1}\left(1-P_L\right)^{K-j} j \tfrac{E^R}{\left(E^L+E^R\right)^2} \\
& - \lambda M \sum_{j=\frac{K+1}{2}}^{K} C_j^K \left(P_L\right)^j \left(1-P_L\right)^{K-j-1} \left(K-j\right) \tfrac{E^R}{\left(E^L+E^R\right)^2}
\end{aligned}$$

The first order condition evaluated in the symmetric equilibrium yields:

$$\begin{aligned}
e_m^* &= \frac{\lambda V}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{l=m}^{K}\left(2l-K\right)C_l^K + \frac{\lambda M}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{j=\frac{K+1}{2}}^{K}\left(2j-K\right)C_j^K \\
&= \frac{\lambda}{4E^*}\left(mC_m^K\left(\frac{1}{2}\right)^{K-1}V + \left(\frac{1}{2}\right)^{K-1}\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K M\right).
\end{aligned}$$

where the second line exploits the fact that $\sum_{l=j}^{K}\left(2l-K\right)C_l^K = jC_j^K$.

Then, sum the effort over all candidates on the list to derive $E^* = \sum_{m=1}^{K} e_m$:

$$\begin{aligned}
E^* &= \frac{\lambda V}{4E^*}\sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K-1} mC_m^K + \frac{\lambda M}{4E^*}\left(\left(\frac{K+1}{2}\right)\left(\frac{1}{2}\right)^{K-1} C_{\frac{K+1}{2}}^K M\right) \\
&= \frac{\lambda}{4E^*}\left(VK + MK\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K\left(\frac{1}{2}\right)^{K-1}\right)
\end{aligned}$$

Exploiting the fact that $C_k^n = \frac{n}{k}C_{k-1}^{n-1}$, we have that $\frac{K+1}{2}C^K_{\frac{K+1}{2}} = \frac{K+1}{2}\frac{2K}{K+1}C^{K-1}_{\frac{K-1}{2}} = KC^{K-1}_{\frac{K-1}{2}}$. We can then rewrite E^* as:

$$E^* = \frac{\sqrt{\lambda\left(VK + K^2\bar{M}\right)}}{2} = K\sqrt{\frac{\lambda}{4}\left(V/K + \bar{M}\right)}.$$

The second-order conditions are satisfied adapting the argument in Crutzen, Flamand and Sahuguet (2020) by adding the additional payoff $\bar{M}$.

Proof of Proposition 4

Candidate i in party L chooses effort e_i^L to maximize:

$$\sum_{m=1}^{K} Q_i(m)\left(P^L(m)\,V\right) + \sum_{j=\frac{K+1}{2}}^{K} P^L(j)\,M - \frac{\left(e_i^L\right)^2}{2}.$$

The first order condition is given by:

$$\sum_{m=1}^{K} Q_i(m)\frac{\partial P^L(m)}{\partial e_i^L}V + \sum_{j=\frac{K+1}{2}}^{K} \frac{\partial P^L(j)}{\partial e_i^L}M + \sum_{m=1}^{K}\frac{\partial Q_i(m)}{\partial e_i^L}P^L(m)V - e_i^L = 0.$$

In the symmetric equilibrium, we have:

$$Q_i^*(m) = \frac{m}{K};$$

$$\frac{\partial P^L(m)}{\partial e} = \lambda\frac{C_m^K}{4E^*}\left(2m-K\right)\left(\frac{1}{2}\right)^{K-1} = \lambda\frac{C_m^K}{Ke^*}\left(2m-K\right)\left(\frac{1}{2}\right)^{K+1};$$

Also:

$$p_j = \frac{1}{K-j+1}, 1-p_j = \frac{K-j}{K-j+1}, \frac{\partial p_j}{\partial e} = \frac{K-j}{\left(K-j+1\right)^2 e}$$

and thus we have:

$$\begin{aligned}
\frac{\partial Q_i(m)}{\partial e} &= \frac{\partial p_1}{\partial e} + \frac{\partial p_2}{\partial e}\left(1-p_1\right) + \frac{\partial p_3}{\partial e}\left(1-p_1\right)\left(1-p_2\right) + ... \\
&= \sum_{j=1}^{m} \frac{\partial p_j}{\partial e} \frac{1}{1-p_j} \left[\prod_{s=1}^{m} \left(1-p_s\right)\right] \\
&= \sum_{j=1}^{m} \frac{1}{e} \frac{K-j}{\left(K-j+1\right)^2} \frac{K-j+1}{K-j} \left[\prod_{s=1}^{m} \frac{K-s}{K-s+1}\right] \\
&= \frac{1}{e}\left[1-\frac{m}{K}\right] \sum_{j=1}^{m} \frac{1}{K-j+1}.
\end{aligned}$$

as

$$\prod_{s=1}^{m} \frac{K-s}{K-s+1} = 1 - \frac{m}{K}.$$

The FOC in the symmetric equilibrium can thus be rewritten as:.

$$\frac{\lambda V}{Ke^*} \sum_{j=1}^{K} \frac{j}{K} \left(C_j^K \left(2j-K\right) \left(\frac{1}{2}\right)^{K+1} \right) + \frac{\lambda M}{Ke^*} \sum_{j=\frac{K+1}{2}}^{K} \left(2j-K\right) C_j^K \left(\frac{1}{2}\right)^{K+1}$$
$$+ \frac{V}{Ke^*} \sum_{m=1}^{K} \left(C_m^K \left(\frac{1}{2}\right)^K \right) \left[\left(1-\frac{m}{K}\right) \sum_{j=1}^{m} \frac{1}{K-j+1} \right] - e^* = 0,$$

We know that $\sum_{j=x}^{K} \left(2j-K\right) C_l^K = x C_x^K$ and that $\frac{K+1}{2} C_{\frac{K+1}{2}}^K = \frac{K+1}{2} \frac{2K}{K+1} C_{\frac{K-1}{2}}^{K-1} = K C_{\frac{K-1}{2}}^{K-1}$. Thus $\frac{\lambda M}{Ke^*} \sum_{j=\frac{K+1}{2}}^{K} \left(2j-K\right) C_j^K \left(\frac{1}{2}\right)^{K+1} = \frac{\lambda M}{4Ke^*} \sum_{j=\frac{K+1}{2}}^{K} \left(2j-K\right) C_j^K \left(\frac{1}{2}\right)^{K-1} = \frac{\lambda \bar{M}}{4e^*}$.

To simplify $\sum_{m=1}^{K} \frac{m}{K} C_m^K \left(2m-K\right) \left(\frac{1}{2}\right)^{K+1}$, use the moment generating function for the binomial distribution to find:

$$\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K m = \frac{K}{2}, \text{ and } \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K m^2 = \left(K+K^2\right)/4.$$

Thus

$$\frac{\lambda V}{2K^2 e^*} \sum_{j=1}^{K} j C_j^K \left(2j-K\right) \left(\frac{1}{2}\right)^K = \frac{\lambda V}{4Ke^*}.$$

Finally:

$$\sum_{j=\frac{K+1}{2}}^{K} \left(2j-K\right) C_j^K = \frac{K+1}{2} C_{\frac{K+1}{2}}^K.$$

Therefore, the FOC simplifies to:

$$e^* = \sqrt{\frac{\lambda V}{4K} + \frac{\lambda \bar{M}}{4} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K} \sum_{j=1}^{m} \frac{1}{K-j+1}\right]}.$$

As in Fu and Lu (2009), for the above candidate equilibrium to be an equilibrium, we need to show that the most profitable deviation for a candidate, namely exerting zero effort, is not profitable. This is verified if M is not too large compared to V and λ is not too small.

The condition that needs to be satisfied for the deviation to zero effort to not be profitable is:

$$\begin{aligned} & V/4 + M/2 - (e^*)^2/2 \\ & \geq \\ & \left(\frac{\lambda(K-1)}{2K-1} + \frac{(1-\lambda)}{2}\right)^K V + \sum_{j=\frac{K+1}{2}}^{K} C_j^K \left(\frac{\lambda(K-1)}{2K-1} + \frac{(1-\lambda)}{2}\right)^j \left(1 - \frac{\lambda(K-1)}{2K-1} - \frac{(1-\lambda)}{2}\right)^{K-j} M, \end{aligned}$$

where the RHS of the inequality is a candidate's payoff when he exerts zero effort and thus ends up last on the party list.

Case 1: $M = 0$

When $M = 0$, the condition is satisfied if the term in V on the LHS is larger than the one on the RHS. To prove this, we proceed in three steps. First, remark that, for any value of λ, the RHS is never larger than $\left(\frac{1}{2}\right)^K V$ and is also strictly decreasing in K.

Second, For K equal to 3 and 5, it is easy to check by direct computation that the above condition is satisfied.

Last, e^* is increasing in K but it is never larger that $\sqrt{\frac{\ln 2}{2} V}$.[12] Thus, for any $K > 5$, the LHS is never smaller than $\left(\frac{1}{4} - \frac{\lambda}{8K} - \frac{\ln 2}{4}\right) V > 0.05V$ and is thus strictly larger than the RHS, which is strictly smaller than $\frac{V}{2^5} \simeq 0.03V$ for any $K > 5$. This concludes the proof that the deviation to zero effort is not profitable when $M = 0$.

[12] We provide the detailed derivation of these results in the proof of Theorem 5, where these characteristics of e^* are needed for our comparative politics result.

Case 2: $M > 0$

The term in M on the RHS may be large enough to violate the no deviation condition when λ is small. In that case, a sufficient condition for the no deviation condition to be satisfied provided M is not too large.

Proof of Theorem 5

We start by comparing efforts when selection is non-competitive. Comparing party aggregate efforts in propositions 1 and 3 yields the first part of the Theorem.

We are left with the comparison when selection is comeptitive. As all candidates exert the same effort within their partyu across the two electoral rules, we can compare individual efforts in this case. We thus need to compare effort under competitive FPTP,

$$e^* = \sqrt{V\left(\frac{n-1}{2n^2} + \frac{\lambda}{4n}\right) + \frac{\lambda}{4n}\tilde{M}},$$

and effort under competitive PR,

$$e^* = \sqrt{\tfrac{\lambda}{4}\left(\frac{V}{K} + \tilde{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\tfrac{K-m}{K}\sum_{j=1}^{m}\tfrac{1}{K-j+1}\right]}.$$

Remember from Proposition 2 that effort under competitive FPTP is maximized at $n = 1$ or $n = 2$ depending on whether $\lambda \gtrless \frac{V}{V+\bar{M}}$. We deal with each case in turn.

For the first case, for which $\lambda \geq \frac{V}{V+\bar{M}}$ $(n = 1)$, we need to compare $\sqrt{\frac{\lambda}{4}\left(V + \bar{M}\right)}$ to $\sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$. For the second case, for which $\lambda \leq \frac{V}{V+\bar{M}}$ $(n = 2)$, we need to compare $\sqrt{\left(\frac{1+\lambda}{8}\right)V + \frac{\lambda}{8}\bar{M}}$ to $\sqrt{\frac{\lambda}{4}\left(\frac{V}{K} + \bar{M}\right) + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$. In both cases, the term in $\bar{M}$ is weakly larger in the expression for effort under competitive PR. We can thus focus on the term in V in what follows.

Consider $\left(\frac{1}{2}\right)^K \sum_{m=1}^{K} C_m^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right] \equiv \Lambda(K)$ first. We use the following result about combinatorial sums of finite differences. Identity 14 in Spivey (2007) states that $\sum_{m=1}^{K} 2^{-K} C_m^K \left(H_K - H_{K-m}\right) = \sum_{m=1}^{K}\frac{1}{m2^m}$ with $H_K = \sum_{j=1}^{K}\frac{1}{j}$ being the K^{th} harmonic number. We then have:

$$\begin{aligned}
\Lambda(K) &= \sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K} C_m^K\,(1-m/K)\left(\sum_{j=1}^{m}\frac{1}{K+1-j}\right)\\
&= \sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K} C_m^{K-1}\,(H_K - H_{K-m})\\
&= \sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K} C_m^{K-1}\left(H_{K-1} - H_{K-1-m} + \frac{1}{K} - \frac{1}{K-m}\right)\\
&= \frac{1}{2}\sum_{m=1}^{K-1}\frac{1}{m2^m} + \frac{1}{2}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1} C_m^{K-1}\left(\frac{1}{K} - \frac{1}{K-m}\right)
\end{aligned}$$

Now:

$$\begin{aligned}
&\frac{1}{2}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1} C_m^{K-1}\left(\frac{1}{K} - \frac{1}{K-m}\right) = \frac{1}{K}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1} C_m^{K-1}\left(\frac{-m}{K-m}\right)\\
&= -\frac{1}{K^2}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1} C_m^{K} m = -\frac{1}{K}\left(1-\left(\frac{1}{2}\right)^{K-1}\right)
\end{aligned}$$

Thus

$$\left(\frac{1}{2}\right)^{K}\sum_{m=1}^{K} C_m^K\left[\tfrac{K-m}{K}\sum_{j=1}^{m}\tfrac{1}{K-j+1}\right] = \frac{1}{2}\left(\sum_{m=1}^{K-1}\left(\frac{1}{m2^m}\right) - \frac{1}{K} + \frac{1}{K}\left(\frac{1}{2}\right)^{K-1}\right).$$

This implies in turn that $\Lambda(K)$ is indeed increasing in K as we have:

$$\begin{aligned}
\Lambda(K+1) - \Lambda(K) &= \left(\frac{1}{(K)\,2^K} - \frac{1}{K+1} + \frac{1}{K+1}\left(\frac{1}{2}\right)^{K} + \frac{1}{K} - \frac{1}{K}\left(\frac{1}{2}\right)^{K-1}\right)\\
&= \frac{1}{2^K K}\frac{2^K-1}{K+1} > 0.
\end{aligned}$$

To see that $\lambda\frac{V}{4K} + \left(\frac{1}{2}\right)^K \sum_{m=1}^{K} V C_m^K\left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]$ is also increasing in K, simple algebra implies that:

$$\begin{aligned}
\frac{\lambda}{4(K+1)} + \Lambda(K+1) - \Big(\frac{\lambda}{4(K} + \Lambda(K)\Big) &= \lambda\left(\frac{1}{4(K+1)} - \frac{1}{4K}\right) + \frac{1}{2^K K}\frac{2^K-1}{K+1}\\
&= \frac{1}{K(K+1)}\left(\frac{2^K-1}{2^K} - \frac{\lambda}{4}\right);
\end{aligned}$$

For $K \geq 3$ and $\lambda \in [0,1]$, $\frac{2^K-1}{2^K} \geq 7/8 > \lambda$.

One can check by direct computation that, for $K = 3$, equilibrium effort under competitive PR is greater than that under competitive FPTP. The previous result shows that it is still the case for higher values of K.

Finally, for completeness and still ignoring $\bar{M}$, $\sqrt{\frac{\lambda}{4}\frac{V}{K} + V\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{K-m}{K}\sum_{j=1}^{m}\frac{1}{K-j+1}\right]}$ approaches $\sqrt{\frac{\ln 2}{2}V} \simeq 0.59\sqrt{V}$ as K goes to infinity, which is higher than maximal equilibrium effort under competitive FPTP, $\frac{\sqrt{V}}{2}$.

Proof of Proposition 6

For the sake of simplicity and wlog, we set M equal to 0. Equilibrium effort under FPTP when selection is non-competitive is $\frac{\sqrt{\lambda V}}{2}$. Let there be T identical parties under PR. The probability that party p wins l seats is given by:

$$P_p(l) = \frac{K!}{(K-l)!l!} \left(P_p\right)^l \left(1-P_p\right)^{K-l},$$

with $P_p = \frac{1-\lambda}{T} + \lambda \frac{E_p}{\sum_{j=1}^{T} E_j}$.

The problem for the candidate in position m on the list of party p is to maximise with respect to their own effort e_m^p:

$$\sum_{k=m}^{K} P_p(k)V - \frac{1}{2}\left(e_m^p\right)^2.$$

The first order condition to the problem of the candidate in position m on the list of party p is given by:

$$\begin{aligned} e_m^p = {} & \lambda V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k-1} \left(1-P_p\right)^{K-k} k \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2} \\ & - \lambda V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k} \left(1-P_p\right)^{K-k-1} (K-k) \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2} \end{aligned}$$

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal across parties and thus $E^{1*} = ... = E^{P*} = E^*$ and $P_p = 1/T$. We can simplify the above first order condition to find:

$$\begin{aligned}
e_m^L &= \frac{(T-1)\,\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[k \left(\frac{1}{T}\right)^{k-1} \left(\frac{T-1}{T}\right)^{K-k} - (K-k)\left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k-1} \right] \\
&= \frac{(T-1)\,\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\left(Tk - \frac{T\,(K-k)}{T-1}\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\
&= \frac{(T-1)\,\lambda V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\frac{T}{T-1}\,(Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\
&= \frac{\lambda V}{T E^*} \sum_{k=m}^{K} C_k^K \left[(Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right]
\end{aligned}$$

Exploiting the fact that $\sum_{k=m}^{K} C_k^K \left[(Tk-K)\left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{K-k}\right] = \left(\frac{1}{T}\right)^K (T-1)^{K-m+1}\, mC_m^K$, this simplifies further to:

$$e_m^L = \frac{\lambda V}{T E^*} \left(\frac{1}{T}\right)^{K} (T-1)^{K-m+1}\, mC_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K} \left(\frac{1}{T}\right)^K (T-1)^{K-m+1}\, mC_m^K = K\frac{T-1}{T}$, we get:

$$E^* = \frac{\lambda V}{T E^*} K \frac{T-1}{T} \iff E^* = \frac{\sqrt{K\,(T-1)\,\lambda V}}{T}$$

Comparing (21) to the party output under FPTP, $K\frac{\sqrt{\lambda V}}{2}$, we see that the result of the first part of Theorem 5 is reinforced.

We now turn to competitive selection. Effort under FPTP is equal to $\sqrt{\frac{(1+\lambda)V}{8}}$. Under PR, the first order condition to the problem faced by any politician i (in party p, say) is:

$$\left[\sum_{m=1}^{K} \frac{\partial P_p(l)}{\partial e_i} Q_i(m) + \sum_{m=1}^{K} P_p(m) \frac{\partial Q_i(m)}{\partial e_i}\right] V = e_i^*$$

As there are K candidates competing for one of the K list slots, the equilibrium probability of being offered slot m on the list is $Q_i^*(m) = \frac{m}{K}$;

Given that $P_p(m) = C_m^K \left(\frac{1-\lambda}{T} + \lambda \frac{E_i}{\sum_{j=1}^T E_j} \right)^m \left(1 - \frac{1-\lambda}{T} - \lambda \frac{E_i}{\sum_{j=1}^T E_j} \right)^{K-m}$, we have that, exploiting some of the algebra above:

$$\frac{\partial P_p(m)}{\partial e_i} = \frac{\lambda V}{TKe^*} C_m^K \left[(Tm - K) \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \right]$$

Also:

$$\frac{\partial Q_i(m)}{\partial e_{iL}} = \frac{1}{e_i^*} \left(1 - \frac{m}{K} \right) \sum_{j=1}^{m} \frac{1}{K-j+1}$$

Finally:

$$P_p^*(m) = C_m^K \left(\frac{E_i^*}{\sum_{j=1}^T E_j^*} \right)^m \left(1 - \frac{E_i^*}{\sum_{j=1}^T E_j^*} \right)^{K-m} = C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \tag{12}$$

Thus, in the symmetric equilibrium, effort is equal to:

$$\sqrt{\sum_{m=1}^{K} \frac{\lambda V}{TK} C_m^K \left[(Tm - K) \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \right] \frac{m}{K} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \left(1 - \frac{m}{K} \right) \sum_{j=1}^{m} \frac{1}{K-j+1}}$$

We know that $\sum_{m=1}^K C_m^K m^2 \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} = K(K-1) \left(\frac{1}{T} \right)^2 + \frac{K}{T}$. Thus the first term under the square root simplifies to $\frac{\lambda V (T-1)}{T^2 K}$ and equilibrium effort boils down to:

$$\sqrt{\frac{\lambda V \left(T-1 \right)}{T^2 K} + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \left(1 - \frac{m}{K} \right) \sum_{j=1}^{m} \frac{1}{K-j+1}}$$

We need to compare the above to $\sqrt{\frac{(1+\lambda)V}{8}}$. Given that both the first and second term in the square root making up equilibrium effort under PR are decreasing in T, there must be a value of T beyond which equilibrium effort under PR is smaller than that under FPTP.