ISSN 0265-8003

MARKET TRANSPARENCY, COMPETITIVE PRESSURE, AND PRICE VOLATILITY

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Discussion Paper No. 1472
September 1996

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ABSTRACT

Market Transparency, Competitive Pressure, and Price Volatility*

In this paper we analyse the role of asymmetric information between firms and consumers about market conditions. In standard models of oligopoly informational advantages of firms over customers do not play a role because all prices are observable. When customers are unable to observe all relevant prices in the market, however, they will attempt to infer the level of unobserved prices from those they can observe. This generates an incentive to use price policies to signal the price realizations of rivals. We show that even with arbitrarily low levels of uncertainty about marginal costs of production, equilibrium prices and price variability are strictly higher in a market with private information about costs. We show how firms can exploit this effect to relax competition through information exchange and analyse the role of advertising in such markets.

JEL Classification: D43, D82, D83, L1, L41
Keywords: market transparency, signalling, search, information exchange, advertising

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*This paper is produced as part of a CEPR research programme on *Market Structure and Competition Policy*, supported by a grant from the Commission of the European Communities under its Human Capital and Mobility Programme (no. ERBCHRXCT940653). We are grateful to Iñaki Aguirre, Ramón Caminal, Gabrielle Demange, Thomas Gehrig, Carmen Matutes, Regis Renault, Patrick Rey, Yossi Spiegel and Asher Wolinsky for very helpful discussions and suggestions. We have also benefited from the comments of seminar participants in Barcelona, Munich, Tel Aviv, the XI Jornadas de Economia Industrial in Madrid, the XX Simposio de Analisis Económico in Barcelona, the 1996 European Winter Meeting of the Econometric Society in Copenhagen, the 1996 ENTER Jamboree in Toulouse and the CEPR workshop on 'Applied Theory and Empirical Work in Industrial Organization', Champéry, 22/25 March 1996.

Submitted 5 August 1996

NON-TECHNICAL SUMMARY

The degree of market transparency in goods markets has been a central issue in some important competition policy cases. In particular, anti-trust authorities have often been concerned about information exchange between firms because such agreements create asymmetric information about market conditions between firms and their customers. At first sight such concerns appear to be hard to reconcile with economic theory. As long as consumers observe all prices it will not matter for market outcomes whether information about market conditions (like input prices and volume of demand) is private to the firms or not. This is the case because consumers make their choice only on the basis of prices. For given prices, information about market conditions does not change consumers' choices in a perfectly transparent market. We show in this paper, however, that concerns about asymmetric information are well founded once consumers cannot make complete price comparisons before their purchase decision. We show that with such imperfect market transparency on the side of consumers, asymmetric information between firms and consumers strictly reduces the competitiveness of markets.

Imperfect market transparency on the side of consumers may have two effects. First, it implies that not all consumers can observe all prices and therefore price comparisons are imperfect. As a result the demand functions faced by firms in the market are less elastic than under perfect market transparency and the expected price level is higher. This effect has been extensively studied in the literature on consumer search. The second effect of imperfect market transparency is that asymmetric information about market conditions may matter because consumers will try to infer this information from observed prices in order to estimate unobserved prices. This latter effect has received very little attention in the theoretical literature.

We show in this paper that the informational advantage of firms over customers can significantly reduce competitive pressure in the market beyond the demand elasticity effect. We use a simple duopoly model with differentiated products. Firms must post prices before consumers decide where to shop. Some consumers will know all the prices before making their shopping decision while others can only observe one price. Firms have correlated private information about a common shock to marginal costs at the time of setting prices, which should be interpreted as their knowledge about the situation in the input market. Correlation in the information implies that firms know more about the price decisions of competitors than those customers who cannot observe that price. The firm's information will be reflected in the price it sets, however, and consumers will try to infer the level of unobserved prices from the prices they can see. Consequently, the firm faces a signalling problem.

A firm which has information that the rival will set a high price must distort its own price upwards in order to credibly signal this fact to partially-informed consumers. The high price is credible because partially-informed consumers know that fully-informed consumers perfectly react to price differences between firms. As a result the expected price level will rise, helping duopolistic firms to push prices closer to the collusive outcome. This effect remains large even if the underlying cost uncertainty is small. The reason for this is that the variable about which there is signalling, i.e. the price of the rival, is endogenous to the problem. If one firm conditions on correlated private information this gives the other firm an incentive to signal the price set by its rival. Hence, conditioning on common information is a best response. In addition to lower competitive pressure this effect leads to larger price variability because the difference between high prices and low prices is greater than under symmetric information.

We extend our base model of imperfectly transparent markets with asymmetric information to analyse the impact of information exchange in this environment. We first show that the magnitude of the asymmetric-information effect becomes larger the higher the correlation in the private information of the firms. The reason for this is that more information about the price of a competitor is revealed through the price quoted by a firm if the underlying information on market conditions is more correlated. As a result a firm must increase the upward distortion in the price when correlation is higher in order to still convince customers that the general price level is high. Since the main effect of information exchange in our model is to create perfect correlation in the private information of firms, these results give some credence to the policy concern that information exchange may relax price competition by increasing the degree of asymmetric information between consumers and firms. This is in contrast to the large established literature on information exchange in oligopoly models (see Vives (1990), Raith (1993) and Kühn and Vives (1995) for an overview of this literature) in which information exchange does not affect the expected level of prices or output and the volatility of prices and output becomes small as the underlying cost uncertainty vanishes. The difference in results is entirely due to asymmetric information in imperfectly transparent markets since, in the absence of asymmetric information, our model falls into the class of models considered by this literature.

As we have discussed above the results of the model hold even if firms condition their price decisions on information that is irrelevant for payoffs, as long as this information is private to the firms. This fact allows a reinterpretation of information exchange in the model as pre-play communication about price setting in the market. For example, if consumers believe that with some probability firms meet privately and agree on setting a high price they will try to infer from the observed price whether such a meeting took place. The same equilibrium as in our model would exist because firms would have an incentive to signal through their price that there was a meeting and hence prices will be high. The mechanism of the paper can therefore be fairly broadly interpreted.

Lastly, we study the role of informative advertising in endogenizing the degree of market transparency using an advertising model similar to Bester and Petrakis (1995). While the basic intuition of our model persists, two new effects arise. First, there may be coordination problems in advertising. Second, for sufficiently low advertising costs perfectly separating equilibria fail to exist.

1 Introduction

The degree of market transparency in goods markets has been a central issue in some important competition policy cases.[1] In particular, anti-trust authorities have often been concerned about information exchange between firms because such agreements create asymmetric information about market conditions between firms and their customers. At first sight such concerns appear to be hard to reconcile with economic theory. As long as consumers observe all prices it will not matter for market outcomes whether information about market conditions (like input prices and volume of demand) is private to the firms or not. This is the case because consumers make their choice only on the basis of prices. For given prices, information about market conditions does not change the consumers' choice in a perfectly transparent market. However, we show in this paper that concerns about asymmetric information are well founded once consumers cannot make complete price comparisons before their purchase decision. We show that with such imperfect market transparency on the consumers side asymmetric information between firms and consumers strictly reduces the competitiveness of markets.

Imperfect market transparency on the side of consumers may have two effects. First, it implies that not all consumers can observe all prices and therefore price comparisons are imperfect. As a result the demand functions faced by firms in the market are less elastic than under perfect market transparency and the expected price level is higher. This effect has been extensively studied in the literature on consumer search. Stigler (1961) was probably the first to point out the importance of price comparisons for competition, while Diamond (1971) showed that the effect of search costs could lead to monopoly pricing and market breakdown. Later literature has explored this theme in a wide variety of models in an attempt to avoid the feature of market breakdown (see for example Salop and Stiglitz 1977, Reinganum 1979, Wilde and Schwartz 1979, Burdett and Judd 1983, Rob 1985 and Stahl 1989. See also Stiglitz 1989 for a survey). In all of these models the demand elasticity effect leads to price increases and in many cases price dispersion arises.

The second effect of imperfect market transparency is that asymmetric information about market conditions may matter because consumers will try to infer this information from observed prices in order to make estimates of unobserved prices. The literature on this issue is very small. Bénabou and Gertner (1993), Dana (1994) and Fishman (1996) have studied search models in which consumers have to solve a signal extraction problem when deciding whether to continue search. However, there is no strategic manipulation of information revelation in their models. The paper closest to ours is Daughety (1992) who explicitly consid-

[1] See the recent *Wood Pulp Case* (OJ L 85, 1985), in which the Commission of the European Community expressed concerns about market transparency.

ers the signaling problem of firms in a market with homogeneous consumers and individual bargaining about price. He obtains the result that signaling increases the search intensity and hence increases competitive pressure in the market. In our model we come to the opposite result because firms can convince customers not to search by setting sufficiently high prices.

We show in this paper that the informational advantage of firms over customers can significantly reduce competitive pressure in the market beyond the demand elasticity effect. We use a simple duopoly model with differentiated products. Firms have to post prices before consumers decide where to shop. Some consumers will know all the prices before making their shopping decision while others can only observe one price. Firms have correlated private information about a common shock to marginal costs at the time of setting prices, which should be interpreted as their knowledge about the situation in the input market. Correlation in the information implies that firms know more about the price decisions of competitors than those customers who cannot observe that price. However, the firm's information will be reflected in the price it sets and consumers will try to infer the level of unobserved prices from the prices they see. Consequently the firm faces a signaling problem.

A firm which has information that the rival will set a high price has to distort its own price upwards in order to credibly signal this fact to partially informed consumers. The high price is credible because partially informed consumers know that fully informed consumers perfectly react to price differences between the firms. As a result the expected price level will rise, helping duopolistic firms to push prices closer to the collusive outcome. This effect remains large even if the underlying cost uncertainty is small. The reason for this is that the variable about which there is signaling, i.e. the price of the rival, is endogenous to the problem. The mere fact of correlation allows firms to condition the price on correlated private information even if this information is not directly pay-off relevant. If one firm conditions on correlated private information this gives the other firm an incentive to signal the price set by its rival. Hence, conditioning on common information is a best response. In addition to lower competitive pressure this effect leads to larger price variability because the difference between high prices and low price is greater than under symmetric information.

The determination of an optimal strategy for a firm in our setting is very similar to models of quality signaling in monopoly (see Bagwell and Riordan (1991), Bagwell (1992)). Formally, the quality variable in demand in those models becomes the price realization of the rival firm in our model. What makes our context special is that the price realizations of the rival are themselves endogenously determined in equilibrium by the signaling incentives. This distinguishes our model from other work on signaling in duopoly (see Mailath (1989)). We show that the endogeneity of signaling incentives leads to a new type of multiplicity of equilibrium not encountered in other signaling models. The reason is that correlated private information is essentially used as a coordination device.

Firms may coordinate on setting a high price either when costs are low or when costs are high. Signaling gives an incentive to match a high price even when costs are low.

We extend our base model of imperfectly transparent markets with asymmetric information to analyze the impact of information exchange in this environment. We first show that the magnitude of the asymmetric information effect becomes larger the higher the correlation in the private information of the firms. The reason for this is that more information about the price of a competitor is revealed through the price a firm quotes if the underlying information on market conditions is more correlated. As a result a firm has to increase the upward distortion in the price when correlation is higher in order to still convince customers that the general price level is high. Since the main effect of information exchange in our model is to create perfect correlation in the private information of firms, these results give some credence to the policy concern that information exchange may relax price competition by increasing the degree of asymmetric information between consumers and the firms. This is in contrast to the large established literature on information exchange in oligopoly models (see Vives (1990), Raith (1993) and Kühn and Vives (1995) for an overview of this literature) in which information exchange does not affect the expected level of prices or output and the volatility of prices and output becomes small as the underlying cost uncertainty vanishes. The difference in results is entirely due to asymmetric information in imperfectly transparent markets since, in the absence of asymmetric information, our model falls into the class of models considered by this literature.

As we have discussed above the results of the model hold even if firms condition on payoff irrelevant information as long as this information is private to the firms. This fact allows a reinterpretation of information exchange in the model as pre-play communication about price setting in the market. For example, if consumers believe that with some probability firms meet privately and agree on setting a high price they will try to infer from the observed price whether such a meeting took place. The same equilibrium as in our model would exist because firms would have an incentive to signal through their price that there was a meeting and hence prices will be high. The mechanism of the paper can therefore be fairly broadly interpreted.

Finally we study the role of informative advertising in endogenizing the degree of market transparency using an advertising model similar to Bester and Petrakis (1995). While the basic intuition of our model persists, two new effects arise. First, there may be coordination problems in advertising. Second, for sufficiently low advertising costs perfectly separating equilibria fail to exist.

In section 2 we introduce the model and analyze the market in the absence of asymmetric information Section 3 explains the basic signaling problem and presents the main results of the paper. Section 4 applies our model to the issues of information exchange and pre-play communication. Section 5 discusses the role of advertising while section 6 concludes.

2 The Model

2.1 A Hotelling Model with Imperfectly Informed Consumers

There are two firms in the market. Both firms produces at the same random marginal cost c, which takes values $\bar{c}$ and $\underline{c}$, each with probability $\frac{1}{2}$, and has expectation c^e. For given expected marginal costs c^e we measure the degree of cost uncertainty by $\Delta \equiv \bar{c} - \underline{c} > 0$. Cost uncertainty should be thought of as representing volatility in the market price of an input common to both firms.

There is a continuum of consumers indexed by their type x, which is uniformly distributed on $[0, 1]$. A consumer of type x has reservation price $v - tx > 0$ for the good of firm 1 and reservation price $v - t(1 - x)$ for the good of firm 2. We assume imperfect market transparency about prices in the sense that some consumers cannot compare all prices before making a purchase decision. Consumers can sample randomly a fixed subset of prices and then have to make an unrevocable decision which store to visit. Once in the store they may buy or not buy, but they cannot switch to another store.[2]

A second important ingredient to our model is heterogeneity among consumers about how many prices of stores they can sample before making the shopping decision. There are some well informed and some badly informed buyers in the market. More formally, we assume that for each type of consumer x there is a proportion λ of consumers who can sample two prices before making a shopping decision. These consumers will be "fully informed" at the time of the purchase decision. The remaining fraction $(1-\lambda)$ of consumers of each type can only sample randomly one price of the two. Hence, a fraction $(1 - \lambda)/2$ of consumers becomes informed about price of firm i, p_i, but not about the price of firm j, p_j, before deciding which store to go to. We will call these consumers "partially informed" If consumers cannot observe the input prices of the firms, there is asymmetric information between firms and the consumers about market conditions.

Firms and consumers play a game in two stages. First firms observe the realization of costs and set prices. Then consumers obtain their information and decide on which store to go to and whether to buy or not. We assume that each firm i has to choose prices in the set $P = [0, v-t]$. This guarantees that consumers will always buy the product in the store that they go to. This assumption is without loss of generality as long as the proportion of fully informed consumers, λ, is sufficiently large. A strategy for firm i is a function $p_i : \{\underline{c}, \bar{c}\} \to P$, which maps realizations of the costs into price choices. Let $D_i(p_i, p_j)$ be the demand faced by firm i. Then firm i chooses $p_i(c)$ in order to maximize its expected

[2]This appears to be the simplest way of modelling imperfect market transparency. The basic mechanism at work should, however, carry over to a large class of search models.

profits given by

$$E\{(p_i - c)D(p_i, p_j) \mid c\}. \tag{1}$$

The actions of consumers in the second stage have a simple characterization. Denote the information partition of a consumer by I, which is the set of prices the consumer observes. The type of consumer within the group with information partition I who is indifferent between purchasing at shop 1 and at shop 2 is given by:

$$x(I) = \frac{1}{2} + \frac{E\{p_2 - p_1 \mid I\}}{2t} \tag{2}$$

For each consumer of type x and information partition I strategies are then fully characterized by the following: If $x > x(I)$ the consumer purchases from firm 2 at price $p_2(c)$. If $x < x(I)$ then the consumer purchases from firm 1 at price $p_1(c)$. Demand for the good of firm 1 from the segment of consumers with information partition I is given by $d(I) = \max\{0, \min\{x(I), 1\}\}$ and demand of firm 2 is given by $1 - d(I)$. Aggregating over the three types of consumers in the market, total demand for firm 1 is given by:

$$D_1(p_1, p_2) = \lambda d(\{p_1, p_2\}) + \frac{(1-\lambda)}{2}[d(\{p_1\}) + d(\{p_2\})] \tag{3}$$

which can, if $|E\{p_2 - p_1 \mid I\}| < t$ for all I, be rewritten as:

$$D_1(p_1, p_2) = \frac{1}{2} - (1+\lambda)\frac{p_1 - p_2}{4t} + (1-\lambda)\frac{E\{p_2 \mid p_1\} - E\{p_1 \mid p_2\}}{4t} \tag{4}$$

where $E\{p_i \mid p_j\}$ is the conditional expectation about price p_i of a consumer who has only observed price p_j. The second term in (4) refers to consumer reactions to observed prices, while the last term captures the effect from imperfect market transparency. Note that there are two effects arising from imperfect market transparency. First, increases in the price of good 1 will lead to decreases in demand proportional only to the number of consumers informed about the price of firm 1, $\frac{1+\lambda}{2}$. The $\frac{(1-\lambda)}{2}$ partially informed consumers do not react to price changes. We call this the "demand elasticity effect" and measure it below by the parameter $\tau \equiv \frac{1-\lambda}{1+\lambda}$. Second, changes in the price p_1 may also lead to changes in the expectations about the price of good 2 for those consumers who are only informed about the price of good 1, i.e. changes in $E\{p_2 \mid p_1\}$. We call this the "asymmetric information effect". This effect leads to a signalling problem in imperfectly transparent markets if there is asymmetric information between firms and consumers about costs.

The analysis below will be conducted using (4) as the demand function and not equation (3). This should be interpreted as an assumption that λ is "sufficiently close to 1". Indeed, it can be shown that there exists $\bar{\lambda} < 1$ such that for all $\lambda \in (\bar{\lambda}, 1)$ our propositions hold.[3]

[3]For expositional simplicity we have supressed an explicit calculation of such a bound $\bar{\lambda}$,

2.2 Symmetric Information: The Demand Elasticity Effect

Before we turn to the signaling problem that arises under asymmetric information between firms and partially informed consumers we will derive the equilibrium under symmetric information as a benchmark for our later results. Symmetric information means that consumers can, in addition to the prices they observe, see the realization of input prices, i.e. c. As a consequence the game between the firms and the consumers is a game of complete information and partially informed consumers do not need information contained in the price of firm i to infer the price of firm j in equilibrium. Hence, $E\{p_j(\cdot) \mid p_i, c\} = E\{p_j(\cdot) \mid c\} = p_j(c)$ in equilibrium. While there is no asymmetric information in this version of the model, imperfect market transparency does have an effect by reducing the price elasticity of demand. Partially informed consumers know the equilibrium price choices of all firms but can react to price deviations only in the price they observe.

For a given choice of strategy $p_j(\cdot)$ by firm j, firm i maximizes (1) which yields, imposing rational expectations, the necessary and sufficient condition for an optimal price:

$$\frac{1}{2} - \frac{p_i(c) - p_j(c)}{2t} - \frac{1+\lambda}{4t}[p_i(c) - c] = 0. \tag{5}$$

Using symmetry one derives directly the equilibrium price-cost margins:

$$p_i(c) - c = (1+\tau)t \tag{6}$$

The markup of price above marginal costs, $(1+\tau)t$, is decreasing in the degree of market transparency λ since τ decreases in λ. At full market transparency it is identical to the mark-up in Hotelling models with complete information, t. The demand elasticity effect of imperfect market transparency raises the price cost mark-up by τt. This increase in price cost margins is standard in search models. In the next section we will demonstrate that the asymmetric information effect is an equally important factor for raising prices above marginal costs.

2.3 The Equilibrium Concept with Asymmetric Information

With asymmetric information, the natural solution concept for our game is perfect Bayesian equilibrium. This means that we require that, given beliefs, strategies

which would add nothing to the content of the paper. The main problem we are excluding is that a firm may want to deviate to a high price to exploit customers that are not informed about its price. Such strategies are clearly suboptimal when the group of partially informed buyers is small. The restriction that λ is large enough avoids the problem of non-existence of pure strategy equilibria. The qualitative results of our paper should carry over to the mixed strategy equilibria that exist for smaller λ.

are optimal at every information set and that for given equilibrium strategies beliefs satisfy Bayes' rule wherever it applies. However, perfect Bayesian equilibrium puts no restrictions on off the equilibrium path beliefs which typically leads to a plethora of equilibria in signaling models. For this reason we will want to choose the appropriate refinement for our context.

When consumers in our model do not observe the cost realization c, firms are effectively signaling to consumers about the action of the rival. This creates a particular problem of refinement that is absent in more standard signaling models. To see this consider a market in which there is no asymmetric information between the consumers and the firms. Under these circumstances firms can reveal no new information to the consumers. Nevertheless, a continuum of perfect Bayesian equilibria may appear because, for any price p_i set by firm i, a different information set in the game is reached for consumers with information partition $I = \{p_i\}$. Hence, such consumers can attach different beliefs about p_j at every information set. Then it is possible to sustain equilibria with a high price for i simply because such consumers believe that $p_j = 0$ if the firm sets any other price but the equilibrium price p_i^*.

Intuitively such equilibria are not satisfactory because they allow consumers to draw inferences from the price p_i about the *strategy* of firm j. However, firms in our model move simultaneously and the strategy choice of firm j is not observed by firm i at the time of choosing its own strategy. We would think that a firm cannot convey information that it has not observed. Hence, it seems reasonable that the price of firm i can convey information about the cost c that firm i has observed before it chooses its price. However, it seems unreasonable that it can convey information about unobserved strategy choices. Therefore we assume[4]:

Assumption 1 *For every consumer with information partition $I = \{p_i\}$ beliefs about the strategy $p_j(\cdot)$ are constant across all information sets.*

Note that this assumption does not exclude the possibility of signaling information about the price of a rival. However, under this assumption a firm can only signal information related to the *realization* of the price p_j given the strategy $p_j(\cdot)$. Consumers are only allowed to draw inferences about the information on the state of the market that firm i has gathered and use this information to update their beliefs on the realization of c. Consumers take the strategy of firm j as given.

Under assumption 1 we can treat the determination of the best response of firm i to a strategy of firm j, $p_j(\cdot)$, as a standard signaling game between the firm and consumers, taking the equilibrium strategy of firm j as a parameter. As we

[4]The condition we impose is sometimes called "no signaling what you don't know" in the literature and Fudenberg and Tirole (1991) include it in their definition of perfect Bayesian equilibrium. We present this assumption separately here, because it may not be reasonable to impose when we study information exchange in section 4.

know from the signaling literature this game may still have a large multiplicity of equilibria. We will require that equilibria satisfy, in an appropriately modified fashion, the intuitive criterion of Cho and Kreps (1987). We will call an *Intuitive Equilibrium* a perfect Bayesian equilibrium of our game for which assumption 1 holds and in which strategies of the firms satisfy this intuitive criterion. Because we need some more notation to make this precise we introduce the formal definition further below.

3 Asymmetric Information, Competitive Pressure and Price Volatility

3.1 The Basic Signaling Problem

We show in this section that asymmetric information about market conditions between firms and partially informed consumers will significantly relax competitive pressure beyond the demand elasticity effect. Even in the limit as uncertainty becomes arbitrarily small, expected price cost margins strictly exceed $(1+\tau)t$. Furthermore, prices remain volatile even when the underlying uncertainty about production costs is arbitrarily small. This effect is generated by the price signaling incentives of firms operating in markets with uncertain costs and imperfect market transparency on the side of consumers.

To explain the signaling effect arising in the model, suppose that each firm i has a strategy that conditions on the cost realization c. Then the information about cost will be perfectly revealed to those partially informed consumers who only observe p_i through the price of firm i. For a given strategy $p_j(\cdot)$ these consumer can draw a perfect inference about the realization of the price of firm j. As a result, firm i has an incentive to convince partially informed customers that it is setting a high price because the price of the rival is expected to be high as well. This constitutes the signaling problem in the model.

Note that the consumer is not interested in learning about c itself. Such information is only relevant to the extent that it reveals the *price* charged by firm j. This makes the problem a non-standard signaling problem. From the point of view of the consumer, firm i is really signaling about the realization of the *price* of firm j (taking as given firm j's strategy). In this sense firm i is signaling about a parameter that is itself endogenous to the problem. One sees this most clearly supposing that firm j does not condition its strategy on the realization of costs. Then the information about c that is revealed in the price of firm i contains no information about the price of firm j and is therefore not payoff relevant for the customer. Hence, no signaling incentive exists for firm i in such a situation. More generally, the signaling incentives of firm i will depend endogenously on the strategy $p_j(\cdot)$ of firm j.

3.1.1 The Non-existence of Pooling Equilibria

From the above discussion it is easy to see that pooling by both firms across the states of the world is impossible for any $\Delta > 0$. Suppose there were an equilibrium in which firm j plays a pooling strategy, that is $p_j(c) = p_j^*$ for all c. Given that all customers know that this is the equilibrium strategy, they will not change their beliefs about price p_j, whatever price firm i charges. Hence, the best response of firm i to $p_j(c) = p_j^*$ is the optimal price given the cost level c and given p_j^*. However, it is straightforward to show, that for given p_j the best response price of firm i is strictly increasing in c, i.e. $p_i(\bar{c}, p_j^*) > p_i(\underline{c}, p_j^*)$. Hence, we have shown that the best response to a pooling strategy is always a strategy that fully reveals the cost level c:

Lemma 1 *Suppose assumption 1 holds and $\Delta > 0$. Then there exists no equilibrium in the game in which both firms play a pooling strategy.*

The reason for the non-existence of a symmetric pooling equilibrium is quite different from other models of signaling. In our model the parameter about which pricing policy is a signal for the consumer is the price realization of the rival. If there is pooling by firm j, the signaling problem for firm i disappears and the best response of firm i is undistorted given the pricing policy of the rival. Hence, the only symmetric equilibria that exist will have some degree of revelation of the information about market conditions.

While we could obtain the result of Lemma 1 imposing only assumption 1 on the solution, many pooling and separating equilibria will survive as in standard signaling games. To impose more structure on equilibrium outcomes we will adapt the intuitive criterion of Cho and Kreps (1987) to our game. Let $\beta_i \in B_i$ be the belief of a buyer who observes p_i about the realization of the cost level c, where B_i is the set of possible beliefs. It is convenient to define β_i^K as those beliefs in B_i that are concentrated on a particular realization $c = c(K)$, $c(K) \in \{\underline{c}, \bar{c}\}$. Let $\beta = (\beta_i, \beta_j)$ be the vector of beliefs of partially informed consumers. Since beliefs of consumers perfectly determine their purchase decision we will not make any distinction between consumer beliefs and actions. Let $\mathrm{p} \equiv (p_i(\cdot), p_j(\cdot))$ be the vector of strategies of firms i and j. We can then write the expected profits of firm i given c as $\pi_i(c, \mathrm{p}, \beta)$. Fix a perfect Bayesian equilibrium (p^*, β^*) and let $\pi_i^*(c) = \pi_i(c, \mathrm{p}^*, \beta^*)$ be the associated expected payoff of firm i when the cost level is c. We can now define our concept of equilibrium dominance.

Definition 1 *An action p_i of firm i is* equilibrium dominated *for c if*

$$\pi_i^*(c) > \max_{\beta_i \in B_i} \pi_i(c, p_i, p_j^*(\cdot), \beta_i, \beta_j^*)$$

Note that we are defining equilibrium domination for fixed strategies of firm j and the consumers who cannot observe price p_i. This seems to be the natural adaptation of the idea of equilibrium dominance of Cho and Kreps to our context. Now we can define the intuitive criterion in exactly the same way as Cho and Kreps, except that we base it on the above notion of equilibrium dominance.

Definition 2 *Fix a perfect Bayesian equilibrium* $(\mathbf{p}^*, \beta^*)$ *and let* π_i^* *be the associated vector of equilibrium payoffs for firm* i. *Let* $c(\bar{K})$ *be the complementary state to* $c(K)$ *(i.e.* $c(\bar{K}) = \bar{c}$ *if and only if* $c(K) = \underline{c}$*). Then the equilibrium fails the* intuitive criterion *if there is some* p_i *which is equilibrium dominated for* $c(K)$ *and*

$$\pi_i^*(c(\bar{K})) < \pi_i(c(\bar{K}), p_i, p_j^*(\cdot), \beta_i(c(\bar{K})), \beta_j^*)$$

Less formally the intuitive criterion says the following. Suppose firm i deviates to a price p_i that, in state $c(K)$, can never give it higher profits than in the candidate equilibrium but in state $c(\bar{K})$ does yield higher profits if buyers believe that it is state $c(\bar{K})$. Then buyers should attribute the observation of price p_i as evidence that the true state is $c(\bar{K})$.

With assumption 1 and our definition of the intuitive criterion we can solve the game by determining equilibria for the "best response signaling game" for each firm i given the strategy of its rival j. This is a fairly standard signaling game and it comes as no surprise that the intuitive criterion eliminates all remaining pooling equilibria. These are potential equilibria in which one firm uses a pooling strategy while the other firm conditions its price on the true cost level.

Lemma 2 *There exists no intuitive pooling equilibrium in the game.*

Proof. See Appendix ■

The idea of this result is that a firm facing a high price rival can always separate itself out. The reason is that it has lower costs of quoting a high price than a firm facing a low price rival. This comes about because fully informed consumers will react according to the price differences between the two firms which are greater in the latter case. Hence, to convince partially informed consumers that the firm faces a high price firm it suffices, by the intuitive criterion, to set a price high enough so that a firm facing a low price rival would never be willing to do so. In the proof we show that this argument is valid for any candidate pooling equilibrium.

3.1.2 Basic Properties of Separating Equilibria

While we have excluded the existence of pooling equilibria we will show below that a multiplicity of separating equilibria exists that is not due to beliefs off the equilibrium path. All intuitive separating equilibria have the property that, given the strategy of the rival firm, signaling costs are minimized. The multiplicity arises because of the endogeneity of signaling incentives as a consequence of oligopolistic interaction. We will show that the multiplicity is due to the fact that privately observed common information on costs is used by firms to correlate strategies.

To construct completely separating equilibria, consider first the problem of a best response of a firm i to the equilibrium strategy $p_j^*(\cdot)$ of its rival firm j. Since the equilibrium is perfectly separating, firm j sets a low price for one realization of cost and a high price for the other. Let $c(L)$ be the realization of the cost parameter for which firm j sets the low price and $c(H)$ the realization for which it sets a high price. Note that it is not possible to determine a priori whether $c(L) = \underline{c}$ or $c(L) = \bar{c}$. Firm j may set a high price in a state with low costs simply because it is signaling that the other firm is setting a high price. As long as cost differences are small signaling incentives will be determined by the way the rival conditions prices on his information. The state in which the firm will set a high price is therefore endogenously determined by the strategy of the rival firm. This means that typically multiple signaling equilibria exist.

It is useful for the arguments below to define the *full information price* p_i^K as the best response price for firm i to $(p_j^*(\cdot), \beta_j^*)$ given that $c = c(K)$ and beliefs are concentrated on state $c(K)$, i.e. $\beta_i = \beta_i^K$. Then the full information price is given by

$$p_i^K = \arg\max_{p_i} \pi_i(c(K), p_i, p_j^*(\cdot), \beta_i^K, \beta_j^*).$$

With this notation in place, we can obtain a property of the best response price function of firm i, which is familiar from many signaling models:

Lemma 3 *In any fully separating equilibrium of the game, firm i charges the "full information price" in the state $c = c(L)$, i.e. $p_i^*(c(L)) = p_i^L$*

Proof. Suppose to the contrary that $p_i^*(c(L)) \neq p_i^L$. Since the equilibrium is separating we have $\beta_i = \beta_i^L$ Now consider a deviation to p_i^L. If consumers do not change beliefs, profits are increased by definition of p_i^L Since profits given (c, p_i, p_j, β_j) are strictly greater for $\beta_i \neq \beta_i^L$, profits will also increase more from a move to p_i^L if beliefs change. This contradicts the assumption that $p_i^*(c(L)) \neq p_i^L$ is an equilibrium price. Hence $p_i^*(c(L)) = p_i^L$. ■

This is the typical result in signaling models. For the type that induces the most unfavorable beliefs in a separating equilibrium it can never be optimal to set any other price than the full information price. A separating equilibrium also has to satisfy the incentive compatibility condition that in the "low state" $c = c(L)$ a firm will not want to deviate to the price of the "high state", $p_i^*(c(H))$:

$$\pi_i(c(L), p_i^L, p_j^*(\cdot), \beta_i^L, \beta_j^*) \geq \pi_i(c(L), p_i^*(c(H)), p_j^*(\cdot), \beta_i^H, \beta_j^*) \tag{7}$$

Since π_i increases when more probability weight is put on higher prices of the rival, we have $\max_p \pi_i(c(L), p, p_j^*(\cdot), \beta_i^H, \beta_j^*) > \pi_i(c(L), p_i^L, p_j^*(\cdot), \beta_i^L, \beta_j^*)$. Together with the fact that $\pi_i(c(L), p, p_j^*(\cdot), \beta_i^H, \beta_j^*)$ is strictly concave in p this implies that there exist $\bar{p}$ and $\underline{p}$, such that $\pi_i(c(L), p_i^L, p_j^*(\cdot), \beta_i^L, \beta_j^*) < \pi_i(c(L), p, p_j^*(\cdot), \beta_i^H, \beta_j^*)$

if and only if $p \in (\underline{p}, \bar{p})$. As a consequence, in a separating equilibrium $p_i^*(c(H)) \notin (\underline{p}, \bar{p})$. Indeed, imposing the intuitive criterion yields a unique candidate for a symmetric separating equilibrium:

Lemma 4 *If the equilibrium is symmetric, the only separating equilibrium prices satisfying the intuitive criterion are* $p_i^*(c(L)) = p_i^L$ *and* $p_i^*(c(H)) = \max\{\bar{p}, p_i^H\}$.

Proof. See Appendix. ■

To understand Lemma 4 note that the intuitive criterion selects the separating equilibrium with the lowest signaling costs. When the incentive compatibility constraint (7) is binding in equilibrium the equilibrium price therefore must either be $\underline{p}$ or $\bar{p}$. Otherwise the firm sets the full information price p_i^H. In a symmetric equilibrium signaling a high price of a rival with a low price $\underline{p}$ is clearly not possible. Then the claim of Lemma 4 follows.

3.2 Equilibrium Behavior

With the preliminaries of the previous subsection we can now derive the behavior of the firms in symmetric separating equilibria using Lemmas 3 and 4. For any equilibrium we can write the optimal choice of price given the beliefs of partially informed buyers as a set of (slack) first order conditions for firm i's maximization problem. Imposing symmetry we can write this as:

$$[p^*(c) - c - Z(c)]\frac{1+\lambda}{4t} - D_i(p^*(c), p^*(c)) = 0 \tag{8}$$

for all c, since $p_i^*(c) = p_j^*(c) = p^*(c)$. The parameter $Z(c) \geq 0$ indicates the degree of slack in the first order condition of firm i at the equilibrium price $p^*(c)$. It can be usefully interpreted as the additional marginal cost of production induced by signaling. By Lemma 3 firm i chooses the full information price in the "low state" $c(L)$ and therefore $Z(c(L)) = 0$. The slack parameter is non-negative because, according to Lemma 4, the equilibrium price must (weakly) exceed the full information price in state $c(H)$. Expected "marginal cost due to signaling" for firm i is given by $E\{Z(c)\} = z$. From (8), noting that $D_i(p^*(c), p^*(c)) = \frac{1}{2}$, we obtain the equilibrium condition for the unconditional expected price p^e as

$$p^e = c^e + z + (1 + \tau)t \tag{9}$$

where z represents the additional mark-up on expected prices due to signaling. Writing, without loss of generality, $Z(c) = z[1 + (c - c^e)/(c^e - c(L))]$ and using (9) the first order conditions in (8) reduce to:

$$p^*(c) - p^e = \{1 + \frac{z}{c^e - c(L)}\}[c - c^e]. \tag{10}$$

Equation (10) gives the of reaction of prices to deviations of costs from the mean. If $z = 0$ this corresponds to the "symmetric information" response to cost realizations. This is the equilibrium price adjustment in the game with symmetric information of section 2.2. The second term in curly brackets depends on the equilibrium slack parameter z. It represents the asymmetric information effect on price adjustments to shocks.

The slack parameter z is determined by the incentive compatibility condition (7). If the incentive compatibility condition is binding, it is given by:

$$\begin{aligned} &[p^*(c(H)) - p^*(c(L))]D_i(p^*(c(L)), p^*(c(L))) \\ -&[p^*(c(H)) - c(L)][D_i(p^*(c(L)), p^*(c(L))) - D_i(p^*(c(H)), p^*(c(L))] = 0 \end{aligned} \tag{11}$$

We can now determine the value of the slack parameter in equilibrium by substituting from (9) and (10). This determines z as:

$$z = \frac{t}{2}\left[\frac{\tau(1+\tau)}{1-\tau}\right] + |c(L) - c^e| \tag{12}$$

Equations (9), (10) and (12) completely describe equilibrium prices in a symmetric separating equilibrium. Note that we obtain two potential equilibria when the incentive compatibility constraint is binding. There is one candidate equilibrium in which both firms set a low price when they observe a low cost realization, i.e. $c(L) = \underline{c}$, and one candidate equilibrium in which firms set a low price when they observe a high cost realization, $c(L) = \bar{c}$. Note that in the latter equilibrium z is higher which implies higher expected prices. The reaction to shocks, $p^*(c) - p^e$, will obviously be of opposite sign in the two equilibria but the absolute value is the same. To see this write (12) as $z = \bar{z} + |c(L) - c^e|$. Then, from (10), $p^*(c) - p^e$ is proportional to $\bar{z}/(c^e - c(L))$, which implies the claimed properties.

We have not yet shown that both equilibria will indeed exist. Nonexistence may come about because a firm under cost realization $c(H)$ can never be forced to signal if it is not worthwhile for it to do so. The distortion needed to achieve incentive compatibility for a firm with cost $c(L)$ may be so great that a firm with cost $c(H)$ prefers to induce false beliefs and set a lower price. Hence, to have existence we must have

$$\pi_i(c(H), p^*(c(H)), p^*(c(H)), \beta^H) \geq \max_p \pi_i(c(H), p, p^*(c(H)), \beta_i^L, \beta_i^H) \tag{13}$$

in equilibrium. Now we can prove

Proposition 1 *For every $\lambda < 1$ and each $\hat{c} \in \{\underline{c}, \bar{c}\}$ there exists $\tilde{\Delta} > 0$ such that, for all $\Delta \in (0, \tilde{\Delta})$, there exists a unique symmetric intuitive equilibrium with $\hat{c} = c(L)$ that perfectly separates between $c(L)$ and $c(H)$ for $i = 1, 2$. It is characterized by equations (9), (10) and (12).*

Proof. See Appendix.

Why can equilibria coexist in which firms condition differently on the information about costs that they receive? After all, at first sight it appears counterintuitive that prices should vary inversely with the level of costs. However, the level of cost realizations is unimportant for the firms when Δ is not too large. What matters is that uncertainty allows firms to set different prices for different realizations of costs. When some consumers cannot observe all prices, the fact that firm j conditions its strategy on costs creates private information about the *price* of firm j for firm i relative to its potential consumers. For small Δ firm i wants to set a high price whenever j sets a high price independently of the realization of costs.

In the absence of asymmetric information between firms and consumers competition would equalize the prices set in the two states in the limit as $\Delta \to 0$. However, the signaling distortion assures that this does not happen in the presence of asymmetric information. Indeed, firms could raise prices in our model by conditioning strategies on the realization of "sunspots" that are observable to the firms but not to the consumers. The only difference to proposition 1 would be that the "pooling equilibrium" in which firms do not condition on the sunspot would exist. Strictly positive cost differences Δ exclude the possibility of pooling but not the fact that privately observed costs are used as coordination devices for the firms. Since it is the correlation in the information of rival firms that matters for the result and not the size of the cost differences, we have:

Proposition 2 *Even arbitrarily small uncertainty about costs in the market strictly decreases the intensity of competition when there is asymmetric information between consumers and firms. More formally,*

$$\lim_{\Delta \to 0} |p^e - c^e| = (1 + \tau)t + \bar{z} > (1 + \tau)t. \tag{14}$$

Proposition 2 tells us that even in the limit as the difference between marginal costs tends to zero, the expected price stays strictly above the price that would be charged under symmetric information. But not only the price level is qualitatively different. The same is true for the volatility of prices. To see this, note that the prices for the two cost levels do not converge to the same level even in the limit as $\Delta \to 0$, since $\lim_{\Delta \to 0} |p^*(c) - p^e| = \bar{z} > 0$. Hence, even in the limit as underlying costs become almost certain, large price volatility remains in the market.

Proposition 3 *Even for arbitrarily small underlying cost uncertainty the variability of prices may be large. More formally*

$$\lim_{\Delta \to 0} E\{[p^*(c) - p^e]^2\} = |\bar{z}|^2 > 0 \tag{15}$$

Note that the effect on the expected price level and the effect of increased volatility of prices are decreasing in the degree of market transparency. If the market is more transparent it is more costly for a firm to deviate to a high price in the state $c(L)$ and so the distortion needed in the high state is lower. Hence, firms have less scope to credibly raise prices.

We have characterized up to now equilibria in which the size of the cost uncertainty is small relative to the degree of market transparency. However, imperfect market transparency does not always generate these types of results. When underlying cost differences are large relative to the degree of transparency of the market there are two effects. First, distortions in pricing in the state $c(H)$ may not be needed in equilibrium to separate between states when $c(H) = \bar{c}$. As market transparency becomes perfect the gains for the "low" type from mimicking the "high" type disappear. But at the same time the costs of mimicking a different type stay large because of the cost differences. Second, the equilibrium in which $c(H) = \underline{c}$ disappears. As market transparency becomes perfect the gains from convincing partially informed consumers that the rival's price is high converge to zero. However, to have a separating equilibrium the distortion in price must be at least as large as the cost difference. But then the firm has an incentive to deviate to a lower price and the downward incentive compatibility constraint will be violated. This argument is summarized in:

Proposition 4 *Fix $\Delta > 0$. Then there exists $\hat{\lambda} < 1$ such that for all $\lambda \in (\hat{\lambda}, 1)$, there exists a unique symmetric intuitive equilibrium. In this equilibrium the incentive compatibility constraint is not binding.*

Proof. See Appendix. ■

We have thus established that the signaling incentive in markets with asymmetric information between consumers and firms may lead to significant increases in the price level relative to markets with symmetric information.

4 Market Transparency and Information Exchange

In the previous section we have shown that asymmetric information about market conditions between firms and consumers matters in a market with imperfect market transparency. Our model, therefore, gives us a framework to evaluate the concern of anti-trust authorities that informational advantages of the firms may hurt consumers. Earlier literature on information exchange (see Kühn and Vives (1995), Raith (1993) for an overview) could not address this issue because it analyzed perfectly transparent markets. In all of these models there is no effect of information exchange on the *level* of expected prices. [5] We will show below

[5]The absence of an impact on the expected price level is an artefact of models with linear demand as analyzed in this literature. However, and in contrast to our results, the effect of information exchange on price levels is always of second order for small degrees of uncertainty when market transparency is perfect.

that with imperfect market transparency we will get substantial effects from information exchange on price levels despite maintaining all other assumptions in this literature.

4.1 Imperfectly Correlated Information

To analyze the effects of information exchange we first have to slightly generalize the model used in section 3. In that model the information of firms about market conditions is perfectly correlated because the cost level is known to the firms at the time of price setting. Therefore, there is no need for information exchange. In real markets, however, firms frequently have to set output prices before input prices are known. Hence, we assume that firms have to set prices before they have observed the realization of marginal costs. However, at the time of choosing prices, each firm i has received an independent, private signal s_i about the realization of marginal cost. This can be interpreted as the private information each firm has about the state of input markets. Given the realization of marginal costs c, the signal s_i takes the true value of the realization of c with probability $\frac{\rho+1}{2}$, and with probability $1-\frac{\rho+1}{2}$ the opposite value, $\rho \in [0,1]$. Hence, the expected value of costs given the signal is given by $E\{c \mid s_i\} = c^e + \rho[s_i - c^e]$. The parameter ρ measures the informativeness of the signal. If $\rho = 0$ the signal is completely uninformative while for $\rho = 1$ the signal perfectly reveals the true level of costs which yields the model of section 3 as a special case.

Instead of conditioning on the cost level firms now condition their strategies on the realization of their signal. This means that even in symmetric equilibria price choices will not be perfectly correlated among the firms. The signal of firm i, s_i, is an imperfect signal not only of its own cost but also of the signal received by firm j, s_j, and firm i's conditional expectation of firm j's signal is given by $E\{s_j \mid s_i\} = c^e + \rho^2[s_i - c^e]$. The parameter ρ^2 therefore measures the degree of correlation in the information of firms. In a separating equilibrium consumers who can only observe the price of firm i will be able to infer the realization of s_i and use this to update their beliefs about the realization of s_j and therefore the price $p_j(s_j)$ of firm j. Hence, the inference partially informed consumers make about the unobserved price is imperfect. In line with the notation in section 3 we will call $s(L)$ ($s(H)$) the realization of s_i for which the price of firm j is expected to be lower (higher). As is shown in the appendix all results of section 3 continue to hold for this modified model.

To obtain some more intuition about the changes induced by imperfect correlation of information consider the first order condition of the problem:

$$[p_i^*(s_i) - E\{c \mid s_i\} - Z(s_i)]\frac{1+\lambda}{4t} - E\{D_i(p_i^*(s_i), p_j^*(s_j)) \mid s_i\} = 0 \qquad (16)$$

This first order condition is exactly analogous to equation (8) in section 3 only that the marginal cost level is replaced by the expectation $E\{c \mid s_i\}$ and expected demand depends on the conditional expectation of $p_j(s_j)$ given s_i. The unconditional expected price in a symmetric equilibrium is again given by $p^c = c^c + z + (1+\tau)t$. Price adjustments to the information will now depend on the degree of correlation between the signal and the cost level, ρ, and the degree of correlation between the signals, ρ^2. In a symmetric equilibrium we have

$$p(s_i) - p^c = \frac{1}{1 + (1-\tau\rho^2)(1-\rho^2)}[\rho + \frac{z}{(c^c - s(L))}][s_i - c^c] \qquad (17)$$

and $z = \bar{z}(\rho^2) + \rho[s(L) - c^c]$, where

$$\bar{z}(\rho^2) = \left[1 + \frac{\rho^2}{1 + (1-\tau\rho^2)(1-\rho^2)}\right]^{-1} \left[\frac{\tau\rho^2}{1-\tau\rho^2}\right](1+\tau)t. \qquad (18)$$

Both the price level and the volatility of prices are positively affected by the degree of correlation in the information between the two firms, ρ^2. The more correlated the information of the firms is, the more information about the realization of the rival's signal is revealed in the price. Hence, the higher correlation between signals, the stronger the reaction of partially informed buyers to the information contained in the price and the greater the incentives for the firm to cheat. As a result, upward distortions in the high price become larger the stronger the correlation. To summarize:

Proposition 5 *The expected price in the market and the variance of equilibrium prices increases in the degree of correlation in the information of firms,* ρ^2

Proof. See Appendix. ■

This proposition already gives some intuition about the effects of information exchange on equilibrium prices. As far as information exchange leads to greater correlation in the information of firms we should expect it to increase the average price level. We confirm this intuition below in a formal model of information exchange.

4.2 Information Sharing

We model "information sharing" as a special trade association agreement in which firms commit to delegate information gathering about costs to the trade association. To simplify the discussion below we will make two restrictive assumptions. First, we assume that firms can only use information obtained from the trade association. Second, we assume that the signal s generated by the trade association contains no more information than the private signal s_i each firm received in the absence of the association. Hence, s takes the true value of c with probability $\frac{\rho+1}{2}$ and the opposite realization with probability $\frac{1-\rho}{2}$. We make this assumption simply to isolate the correlation effect of information exchange and discuss more realistic settings below.

As in the previous section equilibrium prices in a symmetric equilibrium satisfy the slack first order condition

$$[p^*(s) - E\{c \mid s\} - X(s)]\frac{1+\lambda}{4t} - E\{D(p^*(s), p^*(s)) \mid s\} = 0 \qquad (19)$$

where $X(s)$ refers to the "marginal cost due to slack" under information exchange. The effect of imperfect signals persists for the term $E\{c \mid s\}$. However, the signals firms receive are now perfectly correlated and $E\{D(p^*(s), p^*(s)) \mid s\} = \frac{1}{2}$ as in the case of observed costs of section 3. It should therefore be clear that the solution of the problem can be obtained from the solution in the previous subsection setting all terms in ρ^2 equal to 1, but leaving terms in ρ unchanged. Hence:

$$p^*(s) - E\{c \mid s\} = (1+\tau)t + X(s), \qquad (20)$$

where $X(s) = \{\bar{z}(1) + \rho[s(L) - c^e]\}[1 - \frac{s-c^e}{s(L)-c^e}]$. Terms in ρ refer to updating of beliefs about costs and are preserved. All terms that refer to beliefs about rivals are as in the model with perfect correlation. Information sharing on the basis of a trade association creates a common pool of information, i.e. perfect correlation in the information of the firms, which implies by proposition 5 an increase in expected prices and the variability of prices.

In the literature on information exchange trade associations are usually modeled in a different way. In particular, it is assumed that firms commit to exchange the private signals they observe (see Kirby 1988 or Vives 1990)[6] It should be clear that the effect of creating perfect correlation in the information of the firms will still persist. On top of this, two additional effects arise. First, information exchange improves the information of both firms about the cost realization. Second, firms are able to distinguish a greater number of states of the world. The first effect simply leads to a more precise estimate of c in equation (20) but does not qualitatively affect the result that information exchange increases the expected price level. The degree to which firms can raise price above the full information level only depends on the degree of correlation of their information. The second effect, however, leads to a larger multiplicity of equilibria and higher prices on average. To see this, suppose each firm i makes available its signal s_i to the competitor. Both firms can then condition their prices on the vector of realizations (s_i, s_j). This will have two consequences for equilibrium prices. First, the multiplicity of equilibria for small Δ will be enlarged because of the larger number of states that can be distinguished by the firms. Second, the full information price will only be charged in one of the four possible states while

[6]This kind of information exchange arrangement in the context of trade associations is quite common in practice (see Vives 1990). To credibly exchange cost information firms often submit their bills for inputs to the trade association which computes industry averages from the collected data. The aggregate data is then distributed to the firms (see Kühn and Vives 1995).

there is an upward deviation in all other states. This means that the distribution of prices will get more skewed towards higher prices the more states the firms can distinguish. Indeed, in the limit as $\Delta \to 0$ the distorted prices all converge to the same level in our model which strictly exceeds the full information price in each state.

To complete our discussion we will now argue that the incentives to exchange information may radically change relative to the standard literature if there is asymmetric information between firms and consumers. We will illustrate this for the information sharing technology we discussed at the beginning of the section although it should become clear that, based on the arguments in the previous paragraph, the argument carriers over to more standard models. As in the literature on information exchange we want to evaluate the incentives for information exchange by comparing the *ex-ante* profits before signals are known to the firms. Linearity in demand allows us to separate ex-ante profits into effects due to the level of expected prices and effects due to changes in the covariation of the deviations of price cost margins and demands from their expected levels:

$$\begin{aligned} & E\{(p_i - c)D_i(p_i, p_j)\} \qquad (21) \\ = \; & [p^e - c^e]D_i^e(p_i, p_j) \\ & +E\{[(p_i(s_i) - E\{c \mid s_i\}) - (p^e - c^e)][D_i(p_i, p_j) - D_i^e(p_i, p_j]\}. \end{aligned}$$

For the firm information exchange is only desirable, for given level of expected prices and quantities, if there is a larger positive correlation between price cost margins and demand after information exchange. In our special case of a Hotelling model, demand is always $\frac{1}{2}$ when firms have the same information and so there is no correlation between price cost margins and demand when there is information exchange. As a result we only have to analyze the sign of the last term in (21) for the case of no information exchange.

Consider first the situation without asymmetric information between consumers and the firm, i.e. consumers who observe p_i can also observe the signal s_i. Equilibrium prices are given by (9) and (10) setting $z = 0$. Hence, the expected value of prices is unaffected by information exchange as in the standard models. The sign of the variance term is positive. This is because for higher costs the price cost margins are lower and demand is lower. Consequently the expected profits of the firms are higher in the absence of information sharing when there is no asymmetric information.

When there is asymmetric information and Δ is not too large the situation is dramatically different. With small Δ the incentive compatibility constraint is binding, i.e. $z > 0$, and the expected price, p^e, is increased through information exchange. The variability effect of profits is strictly negative for Δ sufficiently small. Signaling tends to invert the usual correlation between price cost margins and demand. With signaling, price cost margins tend to be high when demand is low. The reason for this inverted correlation is that in the absence of information exchange sometimes a firm with high price faces a competitor with a low price. Hence, with asymmetric information both the price level effect and the effect due to correlations between price cost margins and changes in demand encourage information exchange. As a result we obtain:

Proposition 6 *In imperfectly transparent markets without asymmetric information as well as in perfectly transparent markets firms have no incentive to share information. In contrast, in imperfectly transparent markets with asymmetric information about market conditions firms strictly gain from exchanging information, at least for sufficiently small uncertainty* Δ.

Proof. See Appendix. ■

This proposition shows that in markets with asymmetric information between consumers and firms information sharing may have very different effects from those studied in the traditional information exchange literature. Firms have an incentive to create a common pool of information in order to increase the degree of asymmetric information. This increases signaling incentives and thus is a commitment device to raise prices in the market. We have thus demonstrated that information exchange may have the anticompetitive implications conjectured by antitrust practitioners. Furthermore, our analysis shows that the remedy of forcing firms to disclose their information about market conditions to consumers is an appropriate one.

It should be noted that our analysis suggests that the effect we have uncovered is of a larger order of magnitude than the effect in the traditional information exchange literature even when we consider models with non-linear demand. While in models with non-linear demand expected prices should be affected, the level effect on price is of second order relative to the size of the cost uncertainty Δ. In contrast, the effect we have uncovered is a first order effect that holds even for small degrees of uncertainty.

4.3 Pre-Play Communication

We have argued above that the important effect in our model is the possibility to use the signaling of private information to credibly coordinate on setting high prices in some states of the world. For this reason it is in the interest of the firms to create a common pool of information. Since the mechanism works even in the case of a sunspot which is privately observed by the firms, it is not even necessary that firms exchange real underlying cost data to relax competition. Indeed, we can reinterpret our model of information exchange as firms talking about which prices to set before making their pricing decisions.

To be slightly more formal about this consider a variation of our model in which there is some probability γ (in our base model $\gamma = \frac{1}{2}$) that firms have a chance to meet and decide on a common price. With probability $(1-\gamma)$ the meeting does not take place. Suppose that consumers cannot observe whether the firms meet or not. In this case partially informed consumers will try to infer from the price whether a meeting took place and whether as a result there is an agreement to quote high prices. There will clearly be an equilibrium in which firms quote high prices when they meet and low prices when they do not meet. Hence, there will be equilibria with the same properties as in our model because firms can condition their pricing policies on whether they have met or not. In this sense pre-play communication can matter in our model.

While the basic intuition of our model will carry over to pre-play communication it becomes problematic to reduce the equilibrium set using assumption 1. If there is communication between firms about the price to be set, there will be correlation in strategies and it is less convincing to assume that firms cannot signal the strategies of rivals to partially informed consumers. While this means that the equilibrium set to be considered may be significantly enlarged, the equilibria we derived will always survive and the basic insights are preserved.

5 Endogenous Market Transparency Through Advertising

In this section we endogenize the degree of market transparency by introducing the possibility of informative price advertising by firms. We will show that advertising has two additional effects in the presence of asymmetric information. First, it may create coordination problems between the firm. Second, the possibility of advertising may make fully separating equilibria impossible. To analyze these questions we will integrate advertising into the model of section 3 in the simplest possible way. A firm can inform either all consumers about its price, spending an amount a of advertising costs, or none. Advertising only matters for those consumers who can only sample one price and would be imperfectly informed in the absence of advertising. We will think of advertising as giving each consumer one sampling of a price for free in addition to their previous sampling possibilities. Hence, if at least one firm advertises in the market all consumers become perfectly informed about prices.

Before analyzing the effects of advertising with asymmetric information it is convenient to discuss, as a benchmark, the advertising model with symmetric information. This will also help to understand the workings of the model under asymmetric information. In addition, we replicate for our context the main results known from the literature on informative price advertising in duopoly, in particular those of Bester and Petrakis (1995).

5.1 Advertising with Symmetric Information

Consider our benchmark model of section 2.2 in which there is no asymmetric information between customers and firms. We will introduce the advertising technology described above and concentrate on symmetric equilibria in which both firms use the same price and advertising strategy. Let us suppose there is a symmetric equilibrium in which each firm advertises with probability q the price p^a and with probability $(1-q)$ it does not advertise and charges price p^n. If firm i decides to advertise, given that firm j uses the equilibrium strategy, firm i faces expected demand $E\{D_i(p_i, p)\} = \frac{1}{2} - \frac{1}{2t}[p_i - E\{p\}]$, where $E\{p\} = qp^a + (1-q)p^n$ is the expected price of firm j. The optimal advertising price for firm i is given by $p_i = \frac{1}{2}[t + c + E\{p\}]$ and, hence, the candidate equilibrium must satisfy

$$p^a = \frac{1}{2-q}[t + c + (1-q)p^n]. \tag{22}$$

Profits from advertising in the candidate equilibrium are given by

$$\Pi^a(q) = \frac{1}{2t}\left[\frac{t - p^n + c}{(2-q)} + [p^n - c]\right]^2 - a \tag{23}$$

Expected demand when firm i does not advertise is slightly more complicated. With probability q firm j advertises and demand is given by $\frac{1}{2} - \frac{1}{2t}[p_i - p^a]$. With probability $(1-q)$ there is no advertising and the demand structure is as in equation (4). It is then easy to derive profits conditional on not advertising for firm i as:

$$\Pi^n(q) = \frac{1}{2t}[p^n - c][\frac{2[t - (p^n - c)]}{2-q} + (p^n - c)], \tag{24}$$

where in (23) and (24) p^n is a decreasing function of q. For $q \in (0,1)$ to be an equilibrium advertising level, we must have:

$$\Pi^a(q) - \Pi^n(q) = \frac{1}{2t}\left[\frac{t - [p^n(q) - c]}{(2-q)}\right]^2 - a = 0 \tag{25}$$

It can be shown that $\Pi^a(q) - \Pi^n(q)$ is decreasing in q. Furthermore, $\Pi^a(1) - \Pi^n(1) = -a$, so that there can be no equilibrium in which both firms advertise with probability 1. This result should be clear since a firm's price is perfectly known if the other firm advertises and so there is a strict preference to save on advertising cost. Hence, we obtain:

Proposition 7 *There exists a unique symmetric equilibrium of the game with advertising under symmetric information. If $a \geq a^* = \frac{t\tau^2}{8}$ then $q = 0$ and there is no advertising in equilibrium. If $a < a^*$ then both firms advertise with positive probability $q \in (0,1)$ where the advertising probability solves*

$$\frac{t\tau^2}{8}\left[\frac{2(1-q)}{2 + q(3-q)\tau}\right]^2 = a \tag{26}$$

and the prices charged are given by (22) and

$$p^n - c = t(1+\tau)\frac{2}{2+q(3-q)\tau} \tag{27}$$

Proof. See Appendix ∎

We obtain from this the standard results of informative price advertising in duopoly. First it is immediate from (27) that the non-advertised price falls below the price in the absence of advertising, i.e. below $c + (1+\tau)t$. Second, the advertising incentives increase as market transparency goes down, i.e. as τ increases. Furthermore the advertising probability falls as advertising costs are increased. These properties replicate for our model the results of Bester and Petrakis (1995).

5.2 Advertising with Informational Asymmetries

We will now turn to the question under what circumstances symmetric intuitive equilibria exist that are fully separating. We will assume that costs are perfectly observable to the firms as in section 3. To simplify the arguments we will further assume that Δ is arbitrarily small. In a fully separating equilibrium much of the above analysis of advertising is still applicable. First note, that in a state $c(L)$ the full information actions are taken so that $p^n(c(L))$, $p^a(c(L))$, and $q(c(L))$ are determined as in the symmetric information case in the subsection above. This is simply the analog to Lemma 3 in our context which says that actions in the worst state will be the full information actions in a perfectly separating equilibrium. Since there is no asymmetric information when one firm advertises its price, the advertising price $p^a(c(H))$ in state $c(H)$ only depends on the non-advertising price $p^n(c(H))$ and on $q(c(H))$ but not on decisions in state $c(L)$. Hence, equation (22) continues to describe the optimal choice of an advertised price in state $c(H)$. As a consequence equation (25) continues to describe the advertising incentives at a candidate symmetric separating equilibrium in this state.

There is, however, one price that changes in the market with asymmetric information. The non-advertising price $p^n(c(H))$ is not determined by the optimal choice in state $c(H)$ but in order to satisfy the incentive compatibility condition for the firm in state $c(L)$. The price $p^n(c(H))$ in a separating equilibrium therefore does not depend on the probability of advertising $q(c(H))$ chosen by the firm in state $c(H)$. While in the symmetric information case p^n is decreasing in q and therefore the advertisement incentive in (25) is decreasing in q, $p^n(c(H))$ is constant in $q(c(H))$ and as a result the advertising incentive in the asymmetric information case is increasing in $q(c(H))$. This feature of the model drives the two effects that we analyze below.

Let us consider the range of advertising costs $a \in (a^*, \infty)$. For these advertising cost levels we know that the firms will not advertise in state L in a separating equilibrium by the above argument. Define a^{**} by:

$$a^{**} = \frac{1}{2t}\left[\frac{(p^{*}(c(H)) - c(H)) - t}{(2-q)}\right]^2 \tag{28}$$

where $p^{*}(c(H))$ is the price of section 3 where advertising was not possible. Note that $p^{*}(c(H)) - c(H) > p^{*}(c(L)) - c(L) > t$, so that $a^{**} > a^{*}$ For advertising costs a exceeding a^{**} there is clearly no incentive to advertise in state $c(H)$ even if the firm charges the non-advertising equilibrium price $p^{*}(c(H))$ and the rival does not advertise. Hence we obtain:

Proposition 8 *Suppose that $a > a^{**}$ Then there exists an equilibrium of the game with advertising such that no advertising occurs in equilibrium and equilibrium prices are equal to those in section 3.*

This proposition simply states that for sufficiently high advertising costs there is always an equilibrium in which advertising does not occur and the characterization of section 3 is valid. Note, however, that the signaling distortion in state $c(H)$ gives higher advertising incentives to the firm in state $c(H)$ than in state $c(L)$. Essentially, the profitability of undercutting another firm with an advertised price is higher in the high state because markups are higher due to the signaling incentive. This effect leads to $a^{**} > a^{*}$ and may cause both coordination problems in advertising and the impossibility of fully separating symmetric equilibria for low levels of advertising costs as we will now demonstrate.

5.2.1 Coordination Problems in Advertising

Despite the fact that for $a > a^{**}$ our analysis of section 3 still describes an equilibrium it is not always unique. We will now show that for some range $a \in (a^{**}, a^{+})$ there exist other equilibria with advertising. First, recall that for $a > a^{**}$ and Δ small the incentive compatibility constraint binds in a separating equilibrium. Therefore $p^{n}(c(H))$ is determined by the incentive compatibility constraint in state $c(L)$ whatever the advertising strategy in state H. The price $p^{n}(c(H))$ does not fall if there is advertising in state H.

Given that $p^{n}(c(H))$ is fixed, the relative profits of firm i from advertising and not advertising rise as the probability of firm j advertising, q, increases. Firm i looses more in profits from setting the incentive compatible non-advertising price $p^{n}(c(H))$ when q increases than it looses from setting $p^{a}(c(H))$. The reason is that firm i can adjust price $p^{a}(c(H))$ downwards, limiting the potential loss in market share from the other firm advertising with higher probability. This may lead to a coordination problem in advertising for the firms. Despite the fact that an equilibrium with no advertising exists, a firm may see itself forced to advertise because it expects the rival to advertise with positive probability. To see this consider an advertising cost $a^{**} + \epsilon$, with ϵ small. By construction of a^{**} we have

$\Pi^a(0) - \Pi^n(0) < 0$ and, since ϵ is small, there exists $\dot{q}$ such that $\Pi^a(q) - \Pi^n(q) > 0$ for all $q \geq \dot{q}$. By monotonicity of $\Pi^a(q) - \Pi^n(q)$ there will be a $q(a)$ close to 0, such that $\Pi^a(q(a)) - \Pi^n(q(a)) = 0$, so that firm i is indifferent between advertising at the price $p^a(c(H), q(a))$ and not advertising at price $p^n(c(H))$, given that firm j randomizes between these prices with probability $q(a)$ in state $c(H)$. Indeed, we get:

Proposition 9 *There exists an open interval of advertising costs (a^{**}, a^+) such that there exists an equilibrium which has fully separating non-advertising prices, in which firms do not advertise in state L and set the prices of section 3 and in which in state H firms advertise with probability $q(a)$ price $p^a(c(H), q(a))$ and with probability $1 - q(a)$ set the non-advertising price $p^n(c(H)) = p^*(c(H))$.*

Proof. Note, that by construction prices are optimal for the firm in state L and the price $p^a(c(H), q(a))$ is optimally chosen given the strategy of the rival firm. Also, if the firm chooses $p^*(c(H))$ when it does not advertise it is indifferent between advertising and not advertising in state H and randomizing with probability $q(a)$ is optimal. We have to check is whether firm i has an incentive to deviate from the non-advertising price $p^*(c(H))$. Note that for $q(a)$ small the optimal non-advertising price given any belief β is arbitrarily close to the optimal price at $q = 0$. Since we have shown in the proof of proposition 3 that $p^*(c(H)) > p^H$ and that the downward incentive compatibility is slack, this will also hold for $q(a)$ close to 0. Hence, there is no incentive for the firm in state $c(H)$ to deviate from its non-advertising price. Clearly, for a close to a^{**}, $q(a)$ is close to zero, so that this argument will be true for an open interval of advertising levels above a^{**}. ∎

We have shown here that there exists an open interval of advertising costs with $a > a^{**} > a^*$ such that there are multiple equilibria in the game with advertising. This multiplicity is due to a coordination problem that is *generated through signaling*. Because of signaling the non-advertising price does not respond to higher advertising by the rival. Since higher advertising levels by a rival therefore increase the advertising incentives for a firm it becomes possible that there is a no-advertising equilibrium in which firms strictly prefer not to advertise given that the other firm does not advertise which coexists with an advertising equilibrium sustained by the fact that the other firm advertises. Such coexistence is, however, limited because of the downward incentive compatibility constraint of the firm in state $c(H)$. If the advertising probability of a rival is very high, i.e. q close to 1, the loss from inducing unfavorable beliefs in the case of an intransparent market becomes very small and the firm in state $c(H)$ will set a non-advertising price that violates the incentive compatibility constraint for the firm in the $c(L)$ state.

5.2.2 The Impossibility of Fully Separating Equilibria for $a < a^{**}$

We have just argued that fully separating advertising equilibria may fail to exist alongside no-advertising equilibria because the downward incentive constraint for the firm in state $c(H)$ may be violated. This is true more generally for levels of advertising costs in the range (a^{*}, a^{**}). Note, that for advertising levels $a < a^{**}$, $\Pi^{a}(q) - \Pi^{n}(q) > 0$ for all $q \in [0, 1]$. However, there cannot be a symmetric equilibrium at which $q = 1$, since it is preferable for one firm to set $p^{n} = p^{a}$ and not advertise at all, since the observability of all prices is guaranteed. As a result, in equilibrium we must have $\Pi^{a}(q) - \Pi^{n}(q) \leq 0$. Hence, for $a < a^{**}$ we must have that $p^{n}(c(H)) < p^{*}(c(H))$. However, since $a > a^{*}$ the firms in state $c(L)$ will not advertise in equilibrium so that $p^{n}(c(H)) \geq p^{*}(c(H))$ to sustain incentive compatibility in a symmetric equilibrium. As a result there cannot exist a fully separating equilibrium for this range of advertising parameters.

Let us briefly discuss why full separation breaks down for lower advertising costs. As advertising incentives rise with lower advertising costs, the probability of a rival charging a low advertised price becomes larger. Hence, it becomes very costly to charge a high non-advertised price. At the same time the loss from inducing the wrong beliefs in uninformed consumers becomes low because the probability that consumers are informed is high at higher advertising levels. Hence, when the costs of advertising are low enough not advertised prices in the high state must fall, violating incentive compatibility in state L. The only intuitive equilibria that can exist for low advertising costs are therefore ones in which there is partial pooling by the firm in state $c(L)$. This implies that lower advertising costs bring down the non-advertised prices in state $c(H)$ but increase them in state $c(L)$, reducing thus the volatility of non-advertised prices. Advertising therefore does not eliminate the effects of asymmetric information but reduces price increases on average and stabilizes prices in the market.[7]

6 Conclusions

In this paper we have shown the importance of asymmetric information between the firms and consumers when there is imperfect market transparency on the side of consumers about prices. We have shown that asymmetric information may significantly relax competition beyond the demand elasticity effect of conventional search models. Even when the underlying uncertainty in input prices is small asymmetric information may create large increases in the price level and large price volatility. We have demonstrated that this effect can be exploited by firms through the creation of information exchange agreements or simple pre-play communication that is unobserved by consumers. Our model, therefore, gives some

[7] We have dropped a full analysis of equilibria in this range of parameters because the essential effects can already be understood from the discussion above.

support to competition policy concerns about the role of information exchange agreements in creating asymmetric information in markets to the detriment of consumers.

7 Appendix

In this appendix we will analyze a slightly generalized version of the model presented in section 2 that encompasses both the models used in section 3 and section 4. We assume here that firms have to set prices before they have observed the realization of marginal costs. However, at the time of choosing prices, each firm i has received an independent, private signal s_i about the realization of marginal cost. Given the realization of marginal costs c, the signal s_i takes the true value of the realization of c with probability $\frac{\rho+1}{2}$, and with probability $1-\frac{\rho+1}{2}$ the opposite value, $\rho \in [0,1[$. Hence, the expected value of costs given the signal is given by $E\{c \mid s_i\} = c^e + \rho[s_i - c^e]$. We will derive all the results for this general model. The results of section 3 (Lemma 1-Lemma 4 and Proposition 1-4) follow as special cases by setting $\rho = 1$.

The strategy $p_i(\cdot)$ maps from the realization of the signal $s_i \in \{\underline{c}, \bar{c}\}$ to the set P. We will denote by $s(H)$ the realization of the signal for which firm j sets a high price. The price $p_i(s(H))$ therefore denotes the price that firm i sets if it has received the signal $s_i = s(H)$ and knows that firm j would set a high price if he received signal $s_j = s(H)$. Otherwise we will adopt the same notation as in section 3 replacing c by s_i and $c(K)$ by $s(K)$.

Proof of Lemma 1:

Follows by the same argument as in the text.

Proof of Lemma 2:

By Lemma 1 there is no equilibrium in which both firms use a pooling strategy. Therefore we only have to consider a situation in which one firm uses a pooling strategy and the other firm conditions its strategy on its information. Without loss of generality suppose firm i sets the pooling price $p_i(s_i) = \dot{p}$ for all s_i in equilibrium. Then the equilibrium strategy for firm j satisfies:

$$p_j^*(s_j) = \frac{1}{1+\lambda}\left[t + \dot{p} - \frac{1-\lambda}{2} p_j^c\right] + \frac{E\{c \mid s_j\}}{2} \tag{29}$$

Hence, $p_j^*(\bar{c}) > p_j^*(\underline{c})$ for all $\Delta > 0$ and $p_j^*(s_j) = p_j^c + \frac{1}{2}E\{c - c^e \mid s_j\}$. Note that $s(H) = \bar{c}$ and $s(L) = \underline{c}$ as a consequence.

We now demonstrate that the pooling strategy $p_i(s_i) = \dot{p}$ violates the intuitive criterion for firm i. Let $\dot{\beta}_i$ be the belief of buyers in the candidate pooling equilibrium with $\dot{p}$ charged by firm i. Since partially informed buyers do not obtain any information from $\dot{p}$, beliefs $\dot{\beta}_i$ put the ex-ante probability $\frac{1}{2}$ on the high realization of s_i. Since demand is strictly increasing when buyers attach higher probability to a high realization of s_i, we have $\pi_i(s_i, \mathbf{p}, \beta_i^H, \beta_j) > \pi_i(s_i, \mathbf{p}, \hat{\beta}_i, \beta_j)$

By concavity of π_i in p_i there exists $\bar{p} > \hat{p}$ such that

$$\pi_i^*(\underline{c}) = \pi_i(\underline{c}, \dot{p}, p_j^*, \dot{\beta}_i, \beta_j^*) = \pi_i(\underline{c}, \bar{p}, p_j^*, \beta_i^H, \beta_j^*) \tag{30}$$

Hence all prices $p_i > \bar{p}$ are equilibrium dominated for $s_i = \underline{c}$. If in addition

$$\pi_i^*(\bar{c}) < \pi_i(\bar{c}, \bar{p}, p_j^*, \beta_i^H, \beta_j^*) \tag{31}$$

holds, there exist, by continuity of profits in p_i, prices slightly above $\bar{p}$ such that the intuitive criterion is violated. It remains to be shown that (31) is indeed satisfied.

First note that

$$\begin{aligned} \phi(s_i) &= \pi_i(s_i, \bar{p}, p_j^*, \beta_i^H, \beta_j^*) - \pi_i^*(s_i) \\ &= (\bar{p} - \hat{p})E\{D(\hat{p}, p_j^*(s_j)) \mid s_i\} \\ &+ (\bar{p} - E\{c \mid s_i\})[E\{D(\bar{p}, p_j^*(s_j)) \mid s_i\} - E\{D(\hat{p}, p_j^*(s_j)) \mid s_i\}] \end{aligned} \tag{32}$$

where the difference in conditional expected demands in the last line is independent of s_i. Using $p_j^*(s_j) = p_j^c + \frac{1}{2}E\{c - c^c \mid s_j\}$ we obtain:

$$\begin{aligned} &\phi(\bar{c}) - \phi(\underline{c}) \\ &= [E\{c \mid \bar{c}\} - E\{c \mid \underline{c}\}]\{\tfrac{1}{2}(\bar{p} - \hat{p})\tfrac{1+\lambda}{4t} \\ &-[E\{D(\bar{p}, p_j^*(s_j)) \mid s_i\} - E\{D(\hat{p}, p_j^*(s_j)) \mid s_i\}]\} \end{aligned} \tag{33}$$

Since $\phi(\underline{c}) = 0$ by definition of $\bar{p}$, the last line in (33) is strictly positive since the second term in (32) is strictly positive. Hence, $\phi(\bar{c}) - \phi(\underline{c}) = \phi(\bar{c}) > 0$, violating the intuitive criterion.□

Proof of Lemma 3:

Follows from the proof in the text by substituting $c(L)$ by $s(L)$.□

Proof of Lemma 4:

By Lemma 3 $p(s(L)) = p^L$. To prove Lemma 4 it remains to be shown that $p(s(H)) = \max\{\bar{p}, p^H\}$ in a symmetric intuitive equilibrium. Since in a separating equilibrium $\beta_i = \beta^H$ for $s_i = s(H)$ and since the incentive compatibility constraint must be satisfied in equilibrium we have $p(s(H)) \notin (\underline{p}, \bar{p})$. Furthermore, every price $p \notin [\underline{p}, \bar{p}]$ is equilibrium dominated for the firm that observes $s_i = s(L)$. Suppose $p^H \notin [\underline{p}, \bar{p}]$ and $p(s(H)) \neq p^H$. Then the intuitive criterion fails by setting $p_i = p^H$. Suppose $p^H \in [\underline{p}, \bar{p}]$. Then the intuitive criterion fails for $p_i \in (p(s(H)), \underline{p})$ if $p(s(H)) < \underline{p}$ and for $p_i \in (\bar{p}, p(s(H)))$ if $p(s(H)) > \bar{p}$. Hence, for $p^H \notin [\underline{p}, \bar{p}]$ we have only two possible equilibrium prices: $\bar{p}$ and $\underline{p}$. However, $\underline{p}$ can never be part of a symmetric equilibrium. To see this note that $\underline{p} < p^L$. This implies that firm i sets a low price for $s_i = s(H)$ violating symmetry. Thus, we have shown that, for each $s(L) \in \{\bar{c}, \underline{c}\}$, there is a unique candidate for a symmetric separating equilibrium with $p(s(H)) = \max\{p^H, \bar{p}\}$.□

Before proving the remaining propositions of sections 3 and 4 we briefly derive the candidate equilibrium strategies for a symmetric equilibrium. Differentiating and imposing symmetry the (slack) first order conditions can be written as:

$$[p^*(s_i) - E\{c \mid s_i\} - Z(s_i)]\frac{1+\lambda}{4t} - E\{D_i(p^*(s_i), p^*(s_j)) \mid s_i\} = 0 \tag{34}$$

for all s_i, where $Z(s_i)$ is the degree of slack in the first order condition in state s_i, which is 0 for $s_i = s(L)$. Let $E\{Z(s_i)\} = z(\rho)$. Since unconditional expected demand is $\frac{1}{2}$, expected price is given by:

$$p^e = c^e + z(\rho) + (1 + \tau)t \tag{35}$$

It can be shown that, given our assumptions, the optimal strategy for each firm i is a price function which is linear in $|s_i - c^e|$, the deviation of the signal realization s_i from its expected value. Hence, we can restrict ourselves to strategies $p(s_i) = p_i^e + \delta|s_i - c^e|$ where p^e and δ are constants. With this simplifications in place we can write expected demand of firm i when it observes the signal s_i and sets price $p_i^*(s(K))$ as

$$E\{D_i(p^*(s(K)), p^*) \mid s_i\} = \frac{1}{2} - (1+\lambda)\frac{1-\tau\rho^2}{4t}\delta^*[(s(K) - c^e) - \rho^2(s_i - c^e)], \tag{36}$$

The term in $s(K) - c^e$ comes from the price that the firm sets and term in $(s_i - c^e)$ refers to expectations about the price firm j sets in the market. In a separating equilibrium the firm has to set the price corresponding to its true signal s_i so that $s(K)$ in (36) is replaced by s_i. Then the term in brackets reduces to $(1 - \rho^2)(s_i - c^e)$. Note that the conditional expectation of demand will differ from $\frac{1}{2}$ whenever $\rho < 1$ since the firm is uncertain about the information received by its rival. Using (34) and (35) we can derive:

$$p^*(s_i) - p^e = \{\frac{1}{1 + (1 - \tau\rho^2)(1 - \rho^2)}[\rho + \frac{z(\rho)}{c^e - s(L)}]\}|s_i - c^e| \tag{37}$$

The slack parameter $z(\rho)$ is determined by the incentive compatibility condition which reduces to

$$\begin{aligned} &E\{[p^*(s(H)) - p^*(s(L))]D_i(p^*(s(L)), p^*(\cdot)) \\ &-[p^*(s(H)) - c][D_i(p^*(s(L)), p^*(\cdot)) - D_i(p^*(s(H)), p^*(\cdot))] \mid s_i = s(L)\} = 0 \end{aligned} \tag{38}$$

if it is binding. Substituting $p^*(K) = p^e + \delta^*|s(K) - c^e|$ and (35) in (38) one solves for $z(\rho)$ as:

$$z(\rho) = \left[1 + \frac{\rho^2}{1 + (1 - \tau\rho^2)(1 - \rho^2)}\right]^{-1} \left[\frac{\tau\rho^2}{1 - \tau\rho^2}\right](1 + \tau)t + \rho|s(L) - c^e| \tag{39}$$

Equations (35), (37) and (39) completely describe equilibrium prices in a symmetric separating equilibrium. Note that we obtain two potential equilibria when the incentive compatibility constraint is binding. It is convenient to define $\bar{z}(\rho^2)$ by $z(\rho) = \bar{z}(\rho^2) + \rho|s(L) - c^e|$.

Proof of Proposition 1:

For each configuration $s(L) = \underline{c}$ and $s(L) = \bar{c}$ there is only one candidate for an intuitive equilibrium given by equations (35), (37) and (39). There remain three facts to be shown to prove proposition 1. First, we have to show that for Δ small the incentive compatibility constraint is binding in both candidate equilibria. Second, we need to demonstrate that the downward incentive compatibility constraint is not binding. Third, we have to check that the intuitive criterion is not violated.

Step 1: Since $\lim_{\Delta\to 0}\rho|s(L)-c^e| = 0$ we have from (39) $\lim_{\Delta\to 0} z(\rho) = \bar{z}(\rho^2) > 0$ for all $\lambda < 1$. Hence, there exists $\bar{\Delta}_1$ such that for all candidate equilibria the incentive compatibility constraint is binding.

Step 2: The firm has no incentive to deviate from the equilibrium price for signal realizations is $s(H)$, i.e.

$$\pi_i(s(H), p(s(H)), p^*(\cdot), \beta_i^H \beta_j^*) \geq \max_p \pi_i(s(H), p, p^*(\cdot), \beta_i^L, \beta_j^*) \tag{40}$$

for all $\lambda < 1$. Let $p^+ = \arg\max_p \pi_i(s(H), p, p^*(\cdot), \beta_i^L, \beta_j^*) < p^H \leq p(s(H))$ be the price that a firm would optimally set if it would be faced with a realization of the signal $s(H)$ and would induce beliefs β^L Since we can always sustain the proposed equilibrium with off the equilibrium path beliefs β^L for all $p < p(H)$ in an intuitive equilibrium, we only have to ensure that the firm does not want to deviate to p^+ inducing beliefs β^L Rewriting (40) as

$$\begin{aligned} &[p(s(H)) - p^+]E\{D_i(p^+, p^*(\cdot)) \mid s(H)\} \\ &-E\{[p(s(H)) - c][D_i(p^+, p^*(\cdot)) - D_i(p(s(H)), p^*(\cdot))] \mid s(H)\} \geq 0, \end{aligned} \tag{41}$$

noting that by definition of p^+

$$E\{D_i(p^+, p^*(\cdot)) \mid s(H)\} = [p^+ - E\{c \mid s(H)\}]\frac{1+\lambda}{4t} \tag{42}$$

and using the fact that

$$D_i(p^+, p^*(\cdot)) - D_i(p(s(H)), p^*(\cdot)) = \frac{1+\lambda}{4t}\left[[p(s(H)) - p^+] - \tau\delta^*[s(H) - s(L)]\right], \tag{43}$$

(41) reduces to:

$$-[p(s(H)) - p^+]^2 + \tau\delta^*[s(H) - s(L)][p(s(H)) - E\{c \mid s(H)\}] \geq 0. \tag{44}$$

Now note that $\lim_{\Delta\to 0}[p(s(H)) - p^+] = \bar{z}(\rho^2)$, hence in the limit as $\Delta \to 0$, the left hand side of (44) can be written as

$$-\bar{z}^2 + [\frac{2\tau}{1+(1-\tau\rho^2)(1-\rho^2)}\bar{z}][(1+\tau)t + \bar{z} + \frac{1}{1+(1-\tau\rho^2)(1-\rho^2)}\bar{z}], \tag{45}$$

where we have suppressed the argument ρ^2 for convenience. Noting that $(1+\tau)t = \left[1 + \frac{\rho^2}{1+(1-\tau\rho^2)(1-\rho^2)}\right]\left[\frac{1-\tau\rho^2}{\tau\rho^2}\right]\bar{z}$ from (??), we obtain after some simple manipulations:

$$-\bar{z}^2 + \left[\frac{2}{\rho(1+(1-\tau\rho^2)(1-\rho^2))}\right]^2 \bar{z}^2 > \bar{z}^2\left[\frac{1}{\rho^2} - 1\right] \geq 0. \tag{46}$$

Since profits are continuous in Δ, the strict inequality continuous to hold for some strictly positive Δ, and there exists $\bar{\Delta}_2 > 0$ such that there are no incentives for downward deviations by the firm for $s_i = s(H)$.

Step 3: To complete the proof we have to show that $p(s(H))$ cannot be eliminated through the intuitive criterion through quoting some price $p \leq \underline{p}$. Note that $p(s(H))$ can only be eliminated for firm i if $\pi_i^*(s(H)) - \pi_i(s(H), p, p^*(\cdot), \beta_i^H, \beta_j^*) < 0$. However, we obtain that:

$$\begin{aligned} &\pi_i^*(s(H)) - \pi_i(s(H), p, p^*(\cdot), \beta_i^H, \beta_j^*) \\ &\quad \geq (1+\tau)t + E\{c \mid s(H)\} - p \\ &\quad\quad > p(s(L)) - p > 0 \end{aligned} \tag{47}$$

where the last inequality follows from the fact that $p \leq \underline{p} < p(s(L)) = (1+\tau)t + E\{c \mid s(L)\}$. This completes the proof of proposition 1.□

Proof of Proposition 2:

Follows directly from equations (35) and (39).

Proof of Proposition 3:

Follows directly from equation (37) and (39)

Proof of Proposition 4:

We will prove this proposition in two parts. First, we prove that for λ close enough to 1 there exists no equilibrium with $s(L) = \bar{c}$. Second, we prove that for an equilibrium with $s(L) = \underline{c}$ the incentive compatibility constraint is not binding.

Step 1: Note from equations (39) and (37) that

$$\lim_{\lambda \to 1} p(s_i) = \lim_{\lambda \to 1} p^c = E\{c \mid s(L)\} + t \tag{48}$$

for all s_i, which is the full information price for a firm that receives signal $s_i = s(L)$. Suppose $s(L) = \bar{c}$ and, given this, define p^+ as in the proof of proposition 1. Note that $\lim_{\lambda \to 1} p^+ = E\{c \mid \underline{c}\} + t < \lim_{\lambda \to 1} p^c$ Since for λ close to 1 a change in beliefs by partially informed consumers has virtually no effect on demand it must be the case that for such λ the price p^+ yields higher profits than $p(s(H))$.

Step 2: Suppose $s(L) = \underline{c}$. Then from (39) there exists $\hat{\lambda} < 1$ such that for all $\lambda > \hat{\lambda}$, $z(\rho) < 0$, a contradiction to the assumption that the incentive compatibility constraint is binding. In addition this implies that, for such λ, the incentive compatibility constraint is slack at the full information prices. Hence, the full information prices are equilibrium prices.□

Proof of Proposition 5:

Follows directly from differentiating equation (39) with respect to ρ^2

Proof of Proposition 6:

Using the fact that strategies are linear in the signals equilibrium payoffs in (21) can be rewritten as

$$[p^c - c^c]\frac{1}{2} - \frac{(1+\lambda)(1-\tau\rho^2)}{4t}[\delta^* - \rho|\delta^* E\{[s_i - c^c][s_i - s_j]\}, \tag{49}$$

where $E\{[s_i - c^e][s_i - s_j]\} = Var(s_i) - Cov(s_i, s_j) \geq 0$. When there is information exchange demand is always $\frac{1}{2}$ and there is no correlation between price cost margins and demand, i.e. $Var(s_i) - Cov(s_i, s_j) = 0$. When there is no information exchange $Var(s_i) - Cov(s_i, s_j) = [1 - \rho^2]\frac{\Delta^2}{4}$ and the sign of the variance term is determined by the sign of $-[\delta^* - \rho]\delta^*$ When there is no asymmetric information between consumers and the firm, i.e. consumers who observe p_i can also observe the signal s_i, equilibrium prices are given by (9) and (10) setting $z = 0$. Hence, the expected value of prices is unaffected by information sharing as in the standard models of information sharing. However, the sign of the variance term is determined from (10) by

$$-[\delta^* - \rho]\delta^* = \rho^2[1 - \frac{1}{\mu}]\frac{1}{\mu} > 0 \tag{50}$$

where $\mu = 1 + (1 - \tau\rho^2)(1 - \rho^2)$. Consequently the expected profits of the firms are higher in the absence of information sharing. When there is asymmetric information and when the incentive compatibility constraint is binding, i.e. $z > 0$, the expected price, p^e, is increased through information exchange. The variability effect of profits is given by

$$-[\delta^* - \rho]\delta^*(1 - \rho^2)\frac{\Delta^2}{4} = -\frac{1}{\mu}[z - \mu\rho[c^e - s(L)]]z(1 - \rho^2), \tag{51}$$

which is strictly negative for Δ sufficiently small. This proves the proposition.□

Proof of Proposition 7:

Given that firm j charges the equilibrium prices p^n and p^a conditional on advertising and non-advertising, the optimal non-advertising price for firm i maximizes:

$$(p_i - c)[q(\frac{1}{2} - \frac{1}{2t}[p_i - p^a]) + (1 - q)(\frac{1}{2} - \frac{1 + \lambda}{4t}[p_i - p^n])] \tag{52}$$

Maximizing, substituting for p^a and imposing symmetry yields (28) which defines $p^n(q)$. Substituting (28) in (26) we obtain that $\Pi^a(q) - \Pi^n(q)$ is decreasing in q. Since, for $a > a^*$, $\Pi^a(0) - \Pi^n(0) < 0$, there will not be any advertising in equilibrium and there is clearly no incentive to deviate from $p^n(0)$. For $a < a^*$, $\Pi^a(0) - \Pi^n(0) < 0$ and $\Pi^a(1) - \Pi^n(1) = -a < 0$. Hence there is a unique advertising level that makes the firms indifferent.□

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