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Abstract

Do firms seek to make the market transparent, or do they confuse consumers in their product perceptions? We show that the answer to this question depends decisively on preference heterogeneity. Contrary to the well-studied case of homogeneous goods, confusion is not necessarily an equilibrium in markets with differentiated goods. In particular, if the taste distribution is polarized, so that indifferent consumers are relatively rare, firms strive to fully educate consumers. By contrast, if the taste distribution features a concentration of indecisive consumers, confusion becomes part of the equilibrium strategies. The adverse welfare consequences of confusion can be more severe than with homogeneous goods, as consumers may not only pay higher prices, but also choose a dominated option, or inefficiently refrain from buying. Qualitatively similar insights obtain for political contests, in which candidates compete for voters with heterogeneous preferences.

JEL Classification: D43, L13, M30

Keywords: obfuscation, consumer confusion, differentiated products, price competition, polarized/indecisive preferences, Political Competition

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Preferences, Confusion and Competition

Andreas Hefti, Shuo Liu and Armin Schmutzler*

April 30, 2020

Abstract

Do firms seek to make the market transparent, or do they confuse the consumers in their product perceptions? We show that the answer to this question depends decisively on preference heterogeneity. Contrary to the well-studied case of homogeneous goods, confusion is not necessarily an equilibrium in markets with differentiated goods. In particular, if the taste distribution is *polarized*, so that indifferent consumers are relatively rare, firms strive to fully educate consumers. By contrast, if the taste distribution features a concentration of *indecisive* consumers, confusion becomes part of the equilibrium strategies. The adverse welfare consequences of confusion can be more severe than with homogeneous goods, as consumers may not only pay higher prices, but also choose a dominated option, or inefficiently refrain from buying. Qualitatively similar insights obtain for political contests, in which candidates compete for voters with heterogeneous preferences.

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1 Introduction

When purchasing goods such as smartphones, motor vehicles or insurance, consumers often make mistakes. To a certain extent, such mistakes reflect consumer confusion. This has been documented across various sectors, including retailing, financial services, utilities, telecommunication, the hospitality industry or health insurance (Eppler and Mengis, 2004; Walsh et al., 2007; Loewenstein et al., 2013; Kasabov, 2015).[1] Based on several decades of evidence, there is a broad consensus in marketing science that "consumers mix up, misidentify, or make wrong (i.e. illfounded) inferences about products and/or erroneous product selections" (Mitchell and Kearney, 2002, p. 357).

Firms can influence the degree of confusion through their own activities. On the one hand, firms can engage in measures to educate consumers: They can describe products transparently to facilitate comparison, or they can inform consumers about their true needs (e.g., by providing free trials). On the other hand, firms have means to confuse consumers. For instance, insurance companies may issue contracts with complicated premium-deductible schemes that impede comparisons with those of other firms. When advertising differentiated products, firms may emphasize irrelevant product details rather than those characteristics that really matter to consumers. Manufacturers of sophisticated products, such as smartphones, digital cameras, or laundry machines, may add attributes with unclear value to their products.

These issues are not specific to consumption choices. Like firms facing consumers, political candidates facing voters can reduce confusion by stating their policies clearly, or they can "becloud their policies in a fog of ambiguity" (Downs, 1957). The literature suggests that "complexity, obfuscation, vagueness, and uncertainty are permanent features of American electoral politics" (Gill, 2005, p. 372) and that candidates deliberately obscure their positions (Downs, 1957; Franklin, 1991; Tomz and Van Houweling, 2009; Rovny, 2012; Jacobson and Carson, 2015).

Do information providers, such as firms or political parties, seek to *educate* or *confuse* their audiences? The special case of homogeneous choice options may suggest that the answer is clear-cut. For example, oligopolistic producers of homogeneous goods suffer from the temptation to undercut each others' prices, resulting in a zero-profit equilibrium under well-known conditions.[2] The literature on behavioral industrial organization has shown that obfuscation techniques often allow firms to escape the "Bertrand trap":

[1] See "Why the confusion of the cell phone market has caused millions to switch," *Forbes*, May 2017, for a recent report about consumer confusion in the cell phone industry.

[2] Such an equilibrium arises, e.g., if the following conditions hold simultaneously: static interaction, identical and constant marginal costs, no capacity constraints, complete information (Tirole, 1988).

Suppliers of homogeneous goods can secure positive profits in environments where the same would be impossible if consumers were not confused.[3] In this paper, we show that firms need not benefit from, and may even be averse to consumer confusion in the case of *heterogeneous* choice options. This may seem surprising, as the scope for confusion with heterogeneous products is larger. For example, there can be many ways to present the differences between products, and the dimensions that firms emphasize are likely to influence the perceived valuations. Nevertheless, the incentives to confuse consumers are less clear-cut, because firms usually obtain positive profits in differentiated goods markets even without obfuscation. Accordingly, by blurring the perception of consumers, obfuscation may reduce rather than increase profits.

We introduce a general framework to uncover under which conditions strategic contestants (firms, political candidates, etc.), who compete for heterogeneous agents (consumers, voters, etc.), communicate their choice options clearly or ambiguously, respectively. The agents' true preferences are characterized by a distribution of match values for two contestants, who can influence their payoffs by choosing their *communication strategies* and *efforts*.

The chosen communication strategies jointly determine the agents' perception of product valuations, potentially introducing errors to their comparisons of choice options. *Agent confusion* arises if the perceived and true valuations disagree. We begin by assuming that communication strategies influence the comparison of alternatives by inducing stochastic perturbations, which do not affect the *average* valuation *differences*. Agent confusion then results in unsystematic decision mistakes, meaning that the agents cannot be systematically fooled.[4] These perception errors can have different origins, such as product complexity, attribute uncertainty or limited comparability because of framing effects; see Section 4.1 for an intuitive discussion and Appendix S.1 for a formalization.

Contrary to communication strategies, the chosen efforts have unambiguous effects on

[3]For instance, firms can benefit from hidden fees (Gabaix and Laibson, 2006; Ellison and Ellison, 2009; Heidhues et al., 2016), spurious differentiation resulting from the credulity of consumers (Spiegler, 2006), product complexity (Gabaix and Laibson, 2004) or combative marketing (Eliaz and Spiegler, 2011a,b), from coarse thinking (Mullainathan et al., 2008), incomparable price formats (Carlin, 2009; Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013; Spiegler, 2014), from increasing consumer search costs (Ellison and Wolitzky, 2012), or from consumers lacking (intertemporal) self-control (Heidhues and Kőszegi, 2017); see also Grubb (2015).

[4]The unbiasedness assumption can be seen to play a similar disciplining role in our analysis as the "conformity with the prior" assumption in Bayesian models of persuasion (Kamenica and Gentzkow, 2011), costly information acquisition (Caplin and Dean, 2015), or information design (Armstrong and Zhou, 2019).

how the agents evaluate the contestants. Examples of such efforts are price reductions (a lower price unambiguously increases the relative attractiveness for all consumers) or certain advertising measures (a more prominent option unambiguously captures relatively more attention). The decisive feature of our analysis is that the interaction between the communication strategies and the dispersion of true preferences determines the distribution of perceived valuations, and thereby the intensity of competition.

We formalize the above notions in a two-stage, non-cooperative, complete information game, where the contestants first simultaneously choose their communication strategies and then their efforts. Finally, each agent selects the contestant (buys a product, casts a vote) which she perceives as offering the higher value to her.

Main Result Our main finding is that agent preferences play a decisive role for whether confusion or education is supported as equilibrium outcome. While contestants always favor minimal competition, whether they can achieve this by confusing or educating agents depends critically on the true dispersion of opinions in the agent population. We identify intuitive properties of the preference distribution that determine whether contestants will engage in obfuscation or education. Specifically, we distinguish between *indecisive* preferences, for which, loosely speaking, indifferent agents are relatively common, and *polarized* preferences, for which strong opinions prevail. For instance, in a standard textbook Hotelling model with symmetric firms, consumer preferences are indecisive (polarized) if the density of the consumer distribution has a maximum (minimum) at the midpoint of the Hotelling interval.

Whether preferences are polarized or indecisive in a given setting is an empirical question; both cases seem to be relevant. To illustrate the properties, consider the hospitality industry. It is hard to imagine that most guests will be indifferent when faced with the choice between a "family" hotel and a "business" hotel – instead, most consumers will clearly prefer one alternative over the other, resulting in a polarized preference distribution. By contrast, there will be many more undecided consumers if the comparison is between two different business hotels, in line with indecisive preferences. In politics, polarized preferences prevail when partisan tendencies are pronounced – whether this is the case may vary across jurisdictions and time.

Education is the equilibrium outcome with polarized preferences if the range of decision mistakes is limited by the degree of true taste differentiation. This condition means that, even for the largest possible mistake, the agent with the strongest preferences for one contestant will not switch to the other contestant given identical efforts of the contestants. By contrast, there cannot be an equilibrium without agent confusion when preferences are indecisive. Even stronger results emerge if the communication profiles can be ordered in terms of the error dispersion they induce (e.g., by mean-preserving spreads). With polarized preferences, education then is the *only* equilibrium outcome. By contrast, with indecisive preferences the unique equilibrium features maximal confusion, and the equilibrium communication profile leads to a more extreme dispersion of perceived valuations compared to the true distribution. Finally, we consider the case where obfuscation possibilities are "massive" relative to true preferences, meaning that even the agents who are most loyal to a contestant could be confused enough to choose the dominated option. Then, confusion may arise in equilibrium even with polarized preferences.

Crucially, these results reflect the interplay between the shape of the true preference distribution and the effects of communication. Competition forces contestants to fight for the marginal agents, that is, for the agents who perceive the two options as equally valuable. The larger the mass of such perceptually indifferent agents, the fiercer is competition, and the less profitable the market becomes. When true preferences are indecisive, so that undecided "moderates" are common and "extremists" with strong opinions are rare, confusion reduces the mass of perceptually indifferent agents, as it converts more indecisive agents into extremists than vice versa. The opposite logic applies to the case of polarization, where undecided agents are rare and those with strong opinions are common. Then, *education* must decrease the mass of perceptually indifferent agents.

After presenting the general analysis, we derive some results that are more specific to our two applications. For price competition, the welfare analysis differs substantially from the homogeneous goods case studied by the literature. Absent a binding outside option, the main effect of consumer confusion in a homogeneous good setting is redistribution of rents from consumers to firms. By contrast, obfuscation implies that some consumers choose dominated options in a differentiated goods setting, resulting in an inefficient market outcome. We also show that policy measures directed at fostering competition, e.g., by means of product standardization, can backfire because they increase firms' incentives to confuse.

Further, we illustrate that the possibility of a binding outside option does not diminish the firms' incentives to obfuscate in the case of indecisive preferences: On the contrary, firms may deliberately choose to engage in obfuscation even if this means that some consumers (inefficiently) abstain from purchasing any product. This may help to understand why confusion remains prevalent in many cases, despite the development of a large body of "confusion reduction strategies" (Mitchell and Papavassiliou, 1999).[5] Likewise, our

[5]See Chernev et al. (2015) for a survey of related issues. The marketing literature has occasionally

main results continue to apply if consumers differ in their degree of sophistication, or in how prone to confusion they are (Walsh et al., 2007). All told, our analysis shows that with differentiated goods, consumer confusion is less likely to arise in equilibrium than with homogeneous goods, but if it occurs, its effects are more severe.[6]

When applied to political competition, our approach sheds new light on a long-standing question in the literature. Various authors have asked why political candidates or parties often choose ambiguous platforms rather than specifying their intended policies clearly (e.g., Shepsle, 1972; Aragones and Neeman, 2000; Callander and Wilson, 2008; Kartik et al., 2017). Our analysis highlights the role of voter heterogeneity for this decision. Candidates select ambiguous platforms if the share of indecisive voters is large. In such a case, obfuscation distorts the preference distribution in such a way that strong opinions become more common, and moderate views more rare. This is beneficial for candidates, as it reduces the subsequent intensity of the campaign. By contrast, with pre-existing polarization of the preference distribution, strategic candidates do not desire ambiguous platforms, because voter confusion backfires by increasing the share of indifferent voters.

The paper is organized as follows. Section 2 introduces the general framework. Section 3 presents the main results. Section 4 applies the general setting to product market competition. Section 5 contains the application to political economy. Section 6 concludes. All proofs and additional results are relegated to Appendices A and S.

2 The Model

A unit measure of agents needs to decide between the choice options offered by two contestants $i = 1, 2$. Agent preferences are characterized by a distribution of match values $(v_1^k, v_2^k) \in \mathbb{R}^2$ for the contestants, where the *match advantage* of contestant $i = 2$ for agent $k \in [0, 1]$ is given by $v_\Delta^k \equiv v_2^k - v_1^k$. The match advantages v_Δ^k are dispersed over the agent population according to an exogenously given distribution function G_0, which is commonly known by the contestants, and admits a zero-symmetric density function

conjectured that consumer confusion may serve to raise revenues (Mitchell and Papavassiliou, 1999; Mitchell and Kearney, 2002; Haan and Berkey, 2002), but this issue has not been further explored yet (Kasabov, 2015).

[6]Our results also contrasts with Spiegler (2019), who asks whether agents with mis-specified causal models can be systematically fooled as measured by biased expectations: We show that even if there is no average perception bias, contestants may exploit the agents due to a competition softening effect triggered by confusion.

g_0 (i.e., $g_0(x) = g_0(-x)\ \forall x \in \mathbb{R}$).[7] Each contestant can influence the agents' choices by means of two different instruments.

On the one hand, each contestant can choose a *communication strategy* $a_i \in A$ from some exogenously given set A. The respective communication profile $(a_1, a_2) = \mathbf{a} \in \mathcal{A} \equiv A^2$ influences the agents' perception of the match advantages. For example, the communication strategies could correspond to the ambiguity of the political platforms (see Section 5), or to the marketing campaigns that influence which associations consumers make when thinking about the product (Mullainathan et al., 2008). As we detail in Section 4.1, they could also amount to "presentation formats" that jointly influence how easy it is to compare the alternatives (Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013; Spiegler, 2014), or they could reflect "product complexity" as determined by the number of advertised product attributes (Mützel and Kilian, 2016). For any communication profile $\mathbf{a} \in \mathcal{A}$, the *perceived match advantage* $\tilde{v}^k_\Delta(\mathbf{a})$ of contestant $i = 2$ for agent k is determined according to

$$\tilde{v}^k_\Delta(\mathbf{a}) = v^k_\Delta + \varepsilon_{\mathbf{a}}. \tag{1}$$

where $\varepsilon_{\mathbf{a}}$ is a (possibly degenerate) random variable with distribution function $\Gamma_{\mathbf{a}}$, which is independent of v^k_Δ.[8] Expression (1) yields an analytically convenient reduced form to capture agent confusion, and one that is consistent with several key findings from marketing, consumer research and psychology, which we elaborate in Section 4.1 and Appendix S.1.[9]

On the other hand, each contestant can exert an "*effort*" $s_i \in \mathcal{S} \subset \mathbb{R}$ to persuade the agents to choose in his favor, given the dispersion of perceived match advantages $\tilde{v}^k_\Delta$. For example, such efforts could correspond to the advertising intensities of political candidates or firms, or they could represent (the negative of) product prices. Contrary to communication strategies, efforts always have homogeneous effects on how agents evaluate the contestants, where a higher s_i unambiguously increases every agent's evaluation of contestant i relative to his competitor. Specifically, agent k chooses contestant $i = 1$ if and only if $\tilde{v}^k_\Delta(\mathbf{a}) \leq s_1 - s_2$, that is, the perceived match advantage of contestant 2 is smaller than the effort advantage of contestant 1.

Let $G_{\mathbf{a}}$ denote the distribution function of the perceived match advantages $\tilde{v}^k_\Delta$ of contestant $i = 2$. For any given communication-effort profile $(\mathbf{a}, (s_1, s_2))$, the fraction of

[7] The formulation in terms of match advantages, rather than absolute values, highlights the comparative nature of the agents' thinking. Absent a binding outside option, this is without loss of generality (see Section 4.4).

[8] Our main insights do not hinge on the independence of $\varepsilon_{\mathbf{a}}$ and v^k_Δ; see Appendix S.5 for an example.

[9] Gabaix and Laibson (2004) and Kalaycı and Potters (2011) consider a similar additive structure of the decision utility in the case of homogeneous preferences.

agents who choose contestant $i = 1$, or the *market share* of this contestant, is equal to $G_{\mathbf{a}}(s_1 - s_2)$. Likewise, the market share of contestant $i = 2$ is $G_{\mathbf{a}}(s_2 - s_1) = 1 - G_{\mathbf{a}}(s_1 - s_2)$. We consider the following general form of the contestants' expected payoffs

$$\begin{aligned} \Pi_1^{\mathbf{a}}(s_1, s_2) &= R(s_1, s_2) G_{\mathbf{a}}(s_1 - s_2) - C(s_1), \\ \Pi_2^{\mathbf{a}}(s_1, s_2) &= R(s_2, s_1)\left(1 - G_{\mathbf{a}}(s_1 - s_2)\right) - C(s_2), \end{aligned} \tag{2}$$

where both $R : \mathbb{R}^2 \to \mathbb{R}_+$ and $C : \mathbb{R} \to \mathbb{R}_+$ are twice continuously differentiable.

The idea behind (2) is that, besides co-determining the market shares, the efforts could influence the revenue earned per unit of market share, and they could also be costly for the contestants. For example, if the contestants are political parties and s_i represents their campaigning intensity, then we can interpret the "market share" $G_{\mathbf{a}}$ as the share of favorable voters, $C(s_i)$ as the campaigning costs, and $R(s_i, s_j) = \bar{R} > 0 \ \forall s_i, s_j$, as the value of recruiting an additional voter. The contestants can also be firms, in which case we may use s_i to represent the advertising effort undertaken by i to persuade consumers to choose its product. Here, the formulation $R(s_i, s_j)$ allows that the efforts of both firms could jointly determine the willingness-to-pay of individual consumers, e.g., as in Von der Fehr and Stevik (1998).

In both examples, the effort expenditures are out-of-pocket costs and non-contingent, i.e., independent of success (market share). The above framework also includes the case where effort expenditures are of a purely implicit nature. As an example, consider two zero marginal-cost firms competing for consumers in prices $p_1, p_2 \geq 0$. In our framework, this can be accommodated by setting $\mathcal{S} = (-\infty, 0]$, $p_i = -s_i$ (hence $G_{\mathbf{a}}(s_1 - s_2) = G_{\mathbf{a}}(p_2 - p_1)$), $R(s_i, s_j) = -s_i$, and $C(s_i) = 0 \ \forall s_i, s_j$. In this example, a higher effort s_1 (i.e., a lower price p_1) increases the market share at the implicit cost of a lower revenue per consumer served. As a result, the (implicit) effort expenditures are directly related to the market share. Other applications with competitive firms are conceivable.[10]

We study the above setting as a two-stage complete information game played between the contestants, invoking the standard notion of Subgame Perfect Equilibrium (SPE) as the solution concept. In the *communication stage*, both contestants simultaneously choose their communication strategies $a_i \in A$. In the subsequent *effort stage*, they decide on how much effort s_i to exert.

Confusion or education? Our key question is under what conditions either confusion or agent education arises as a stable market outcome, intentionally induced by strategic

[10] For instance, the model of retail bank competition for customer deposits (see Freixas and Rochet, 2008) can be embedded in our framework by assuming that efforts correspond to the interest rates granted to depositors.

contestants. Specifically, can education be sustained as an equilibrium outcome if educating the agents is feasible for the contestants? If not, would the contestants want to induce as much agent confusion as possible? The following definition clarifies our notions of "confusion" and "education".

Definition 1 *We say that a communication profile* $\mathbf{a} \in \mathcal{A}$ *induces agent confusion (is obfuscating) if* $\tilde{v}_\Delta^k(\mathbf{a})$ *and* v_Δ^k *are not equal in distribution, i.e.,* $G_{\mathbf{a}} \neq G_0$*. A communication profile* $\mathbf{a} \in \mathcal{A}$ *induces agent education (is educating) if* $G_{\mathbf{a}} = G_0$*.*

Throughout the main analysis, we suppose that the effects of communication on the agents' perception of the match advantages are *unbiased*, meaning that $\varepsilon_{\mathbf{a}}$ in (1) is zero-symmetrically dispersed. In other words, we assume that for any given $\mathbf{a} \in \mathcal{A}$, either (i) $\varepsilon_{\mathbf{a}} = O$, where O is any random variable that is almost surely equal to zero, or (ii) the distribution function of $\varepsilon_{\mathbf{a}}$ satisfies $\Gamma_{\mathbf{a}}(x) = 1 - \Gamma_{\mathbf{a}}(-x)$ $\forall x \in \mathbb{R}$. We take $\Gamma_{\mathbf{a}}$ to have a density $\gamma_{\mathbf{a}}$ whenever $\varepsilon_{\mathbf{a}} \neq O$.

Intuitively, the zero-symmetry of $\varepsilon_{\mathbf{a}}$ implies that while the contestants may be able to increase or decrease the noise in the perception process, they cannot systematically bias the perceived match advantage distribution.[11] Such unbiasedness is consistent with how the notion of confusion is often used, for instance, in the marketing literature; see Section 4 and Appendix S.1. As an illustration, suppose that each contestant can choose product "features" (attributes, advertising slogans, labels,...), where each feature has an i.i.d. effect on the consumer evaluation of that product. Then, the number $a_i \in A \subset \mathbb{N}$ of features implemented corresponds to the communication strategy of contestant i, and quantifies i's contribution to agent confusion. The unbiasedness assumption means that some consumers value the addition of such additional features, whereas others are put off by the increase in complexity. Viewed through this lens, the core question of our paper is whether rational contestants would ever seek to implement features with such double-edged effects.

What matters for our analysis is that the contestants can possibly induce noise in the comparisons made by the agents. While in this paper we emphasize various behavioral explanations for confusion, we do not mean to exclude that similar perception errors could also arise from a highly sophisticated decision process.[12] As we show in Section

[11]This does not rule out that communication can bias the *levels* of the perceived match *values* $(\tilde{v}_1^k, \tilde{v}_2^k)$. We consider such a possibility in Section 5.2. Moreover, unbiasedness does not require that v_Δ and $\varepsilon_{\mathbf{a}}$ are independent; see Appendix S.5 for an illustrative example.

[12]As an example, it is a common result in models with noisy signals that Bayesian agents condition their actions on the signals they observe. That is, while the agents behave deterministically according to their posterior expectations, the expectations themselves are stochastic, as they depend on the particular

4.5, our results also apply with differentially sophisticated agents.

Together with the symmetry of G_0, the unbiasedness of $\varepsilon_{\mathbf{a}}$ implies that the perceived match advantage distribution, which is given by $G_{\mathbf{a}}(v) = \int G_0(v-e)d\Gamma_{\mathbf{a}}(e)$, with density $g_{\mathbf{a}}(v) = \int g_0(v-e)d\Gamma_{\mathbf{a}}(e)$, is itself zero-symmetric. These expressions reveal a simple interaction between true preferences, captured by G_0, and the perception errors induced by communication profile $\mathbf{a}$, which is decisive for whether confusion or education are equilibrium phenomena, as we show in the next section.

3 Equilibrium Analysis

We derive the SPE of the game by backward induction. Section 3.1 characterizes the symmetric second-stage effort equilibrium. Section 3.2 shows how preferences determine whether confusion can arise in equilibrium. Sections 3.3 and Section 3.4 sharpen the equilibrium predictions.

3.1 The Effort Stage

The payoff functions (2) together with the symmetry of G_0 and $\Gamma_{\mathbf{a}}$ imply that the contestants play a symmetric game in the competition stage for any given $\mathbf{a} \in \mathcal{A}$. The subsequent analysis concentrates on symmetric equilibria ($s_1 = s_2 = s$) in the effort stage.[13] For the equilibrium analysis, we require a technical assumption which, as we show in Sections 4 and 5, holds in our major applications to price and political competition.

Assumption 1 *The following conditions are satisfied:*

(A1.1) $\Pi_i^{\mathbf{a}}(s_i, s_j)$ *is strictly quasi-concave in* s_i, $\forall s_j \in \mathcal{S}$, $\mathbf{a} \in \mathcal{A}$ *and* $i, j = 1, 2$.

(A1.2) $\forall \mathbf{a} \in \mathcal{A}\ \exists\, s \in \mathcal{S}$ *such that* $z(s) = g_{\mathbf{a}}(0)$, *where*

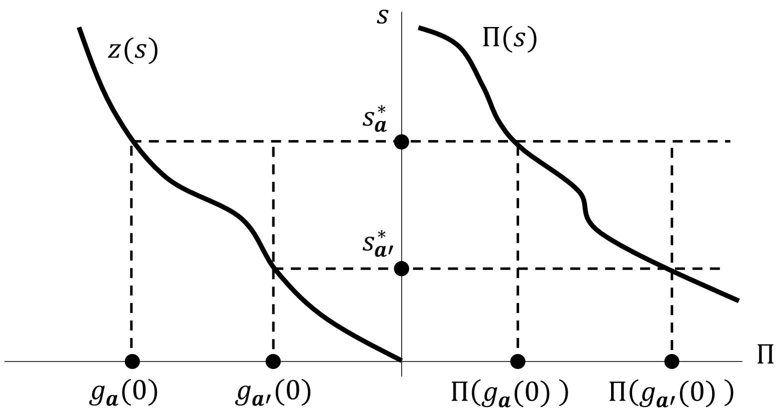
$$g_{\mathbf{a}}(0) \equiv \int g_0(-e)d\Gamma_{\mathbf{a}}(e) > 0, \quad z(s) \equiv \left(\frac{C'(s)}{R(s,s)} - \frac{R_1(s,s)}{2R(s,s)} \right),$$

and R_ℓ *is the partial derivative of* R *with respect to its* ℓ*-th argument.*

(A1.3) $z(s)$ *is strictly increasing, and* $R_1(s,s) + R_2(s,s) < 2C'(s)\ \forall s \in \mathcal{S}$.

signal that has realized. See Johnson and Myatt (2006) for an example, where noisy valuations arise as posterior expectations induced by more or less noisy advertising.

[13] A symmetric (stage) game with a differentiable structure, as the one studied here, always has a symmetric equilibrium, while asymmetric equilibria exist only under special circumstances (Hefti, 2017).



The figure shows the equilibria in the effort stage for two different values $\mathbf{a}, \mathbf{a}'$ (left), and the corresponding level of equilibrium payoffs (right).

Figure 1: Equilibrium in the effort stage

Assumption 1 guarantees the existence of a unique symmetric equilibrium in the effort stage. As the following result demonstrates, the measure $g_{\mathbf{a}}(0)$ of perceptually indifferent agents fully determines the effort equilibrium, reflecting the battle for the agents who *perceive* the two contestants as equally valuable.

Lemma 1 *Suppose that Assumption 1 holds. For any* $\mathbf{a} \in \mathcal{A}$ *a unique symmetric equilibrium exists, and both contestants choose* $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$*. The equilibrium payoff is strictly decreasing in* $g_{\mathbf{a}}(0)$ *for each contestant.*

Lemma 1 is illustrated in Figure 1. Assumption (A1.1) and (A1.2) jointly assure equilibrium existence, where the equilibrium effort $s^*_{\mathbf{a}}$ is characterized by the first-order condition $g_{\mathbf{a}}(0) = z(s^*_{\mathbf{a}})$. For any $\mathbf{a} \in \mathcal{A}$, there is a unique equilibrium effort level $s^*_{\mathbf{a}}$ because $z(s)$ is strictly increasing by (A1.3). This assumption amounts to the standard regularity condition that the equilibrium marginal benefits, as a function of s, must intersect with marginal costs from above. An exogenous increase in the measure of perceptually indifferent agents $g_{\mathbf{a}}(0)$ intensifies competition, resulting in a higher equilibrium effort level. Finally, the negative relation between the equilibrium payoffs and efforts (see the right panel of Figure 1) follows from the last part of (A1.3). This assumption captures that a possible increase in the contestants' payoffs due to higher equilibrium efforts must be dominated by higher effort expenditures.

3.2 Communication Behavior

We now analyze the equilibrium in the communication stage. The following properties of true preferences will be decisive for whether firms want to communicate or educate in this equilibrium.

Definition 2 *Let $\delta > 0$ be such that $[-\delta, \delta] \subset supp(g_0)$.*

(i) ***(Indecisiveness)*** *True match advantages are*

(a) weakly δ-indecisive if $g_0(0) > g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-indecisive if g_0 is strictly increasing (decreasing) on $[-\delta, 0]$ (on $[0, \delta]$), and

(c) strongly δ-indecisive if g_0 is strictly concave on $[-\delta, \delta]$.

(ii) ***(Polarization)*** *True match advantages are*

(a) weakly δ-polarized if $g_0(0) < g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-polarized if g_0 is strictly decreasing (increasing) on $[-\delta, 0]$ (on $[0, \delta]$), and

(c) strongly δ-polarized if g_0 is strictly convex on $[-\delta, \delta]$.

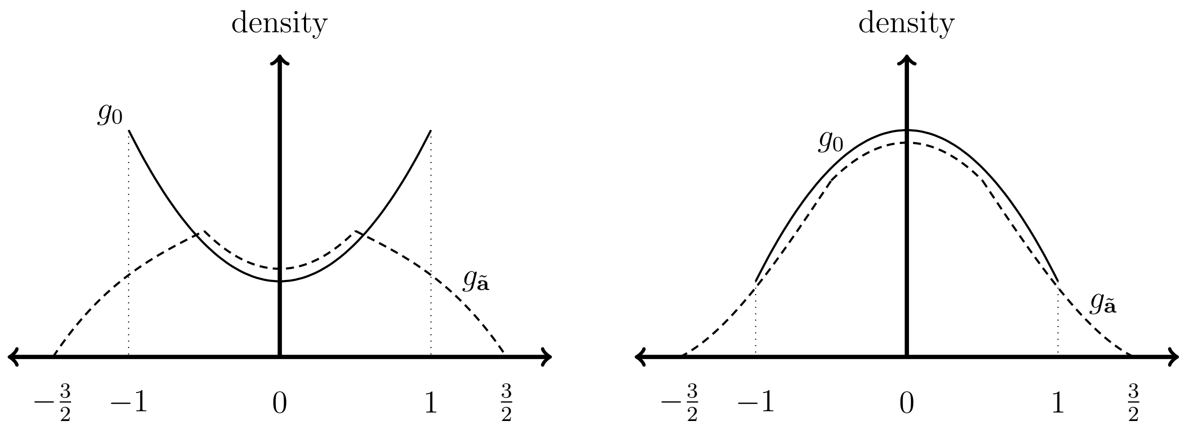
Strong δ-indecisiveness implies δ-indecisiveness and thus weak δ-indecisiveness. For δ-indecisive preferences, less pronounced valuation differences occur more frequently than more pronounced ones, while weakly δ-indecisive preferences only require that indifference ($v_\Delta = 0$) occurs more often than all other alternatives on $[-\delta, \delta]$. The relation between the different concepts of polarization is similar. Our most general result (Theorem 1) only requires the weakest notions of indecisiveness and polarization; the stronger concepts help to obtain equilibrium uniqueness and monotonicity (Theorem 2).

The analysis of the SPE requires additional structure on the communication technology. For definiteness, we assume that (a) agent education is among the feasible options for the contestants, and (b) each contestant can always force some confusion unilaterally, that is, choose an action such that confusion will emerge no matter what the opponent chooses. As we will sketch below, it is straightforward to adjust the analysis to the case that (a) or (b) is violated. To simplify the exposition, we adopt the convention that $A \subset \mathbb{R}_+$ and the communication strategy profile $\mathbf{0}$ is educating, i.e., $\varepsilon_{\mathbf{0}} = O$.

Assumption 2 *The set $\mathcal{A} \subset \mathbb{R}^2_+$ satisfies the following two conditions:*

(A2.1) $\mathbf{0} \in \mathcal{A}$.

(A2.2) $\forall i = 1, 2,\ j \neq i$ *and* $\forall a_j \in A$, $\exists\, a_i \in A$, *such that* $\varepsilon_{(a_i, a_j)} \neq O$.



(a) With polarization, $g_0(0) < g_{\tilde{\mathbf{a}}}(0)$.

(b) With indecisiveness, $g_0(0) > g_{\tilde{\mathbf{a}}}(0)$.

Figure 2: The heterogeneous effects of agent confusion, $\varepsilon_{\tilde{\mathbf{a}}} \sim U[-0.5, 0.5]$.

We are now ready to state our first main result.

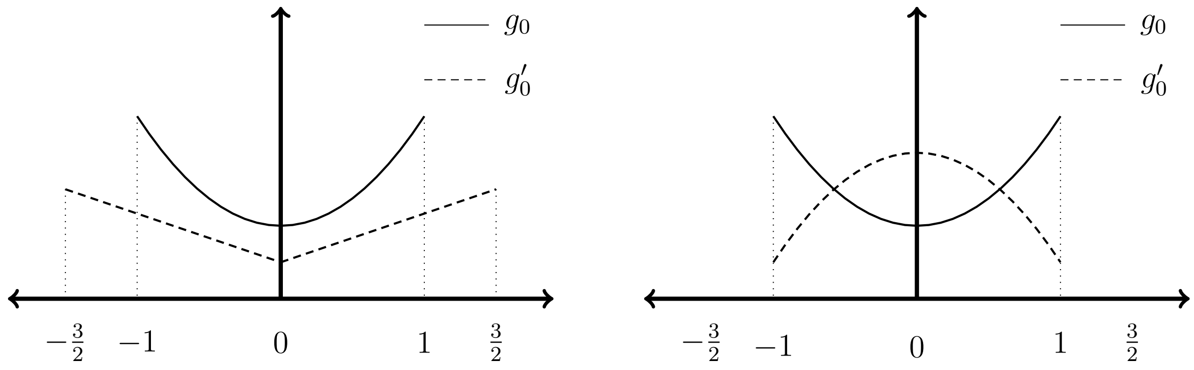
Theorem 1 *Suppose that Assumptions 1 and 2 hold.*

(i) If there exists $\delta > 0$ with supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true match values are weakly δ-polarized, then an SPE without consumer confusion exists.

(ii) If there exists $\delta > 0$ with supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true match values are δ-indecisive, then no SPE without consumer confusion exists.

Theorem 1 shows the decisive role of the shape of the true match value distribution for firms' communication strategies. The rationale is simple. Each obfuscating communication profile (i.e., $\varepsilon_{\mathbf{a}} \neq O$) distorts the perceived distribution of the match advantages over the agent population. Thereby, some truly indifferent consumers come to perceive one contestant as strictly superior, while some consumers who strictly favor one contestant over the other may become indifferent. By Lemma 1, the contestants benefit from confusion if and only if the former effect dominates the latter. With polarized match advantages, confusion pushes more agents towards indifference than vice versa, which intensifies the competition for market shares in the effort stage, as illustrated in Figure 2(a). In such a situation, therefore, both contestants have a strict incentive to avoid an obfuscated market. Because full agent education is feasible (condition (A2.1)), education must be part of an SPE.

By contrast, confusion successfully reduces the measure of perceptually indifferent agents if true match values are indecisive, as Figure 2(b) shows. Thus, even though the contestants are fully aware that some agents switch to the competitor in an obfuscated



(a) Unequal taste differentiation

(b) Equal taste differentiation

Figure 3: Taste differentiation and preference distribution

market, confusion is an effective means to soften competition. As any contestant can force some confusion on the market (condition (A2.2)), agent education cannot be supported as an equilibrium outcome.[14]

In the leading applications of our framework (product market competition and political candidates competing for votes), the (Lebesgue) measure $\lambda(supp(g_0))$ quantifies the agents' taste differentiation; the larger $\lambda(supp(g_0))$, the more differentiated the true tastes are. An interesting conclusion of Theorem 1 is that, under the conditions of the theorem, only the shape of g_0 matters for the type of equilibrium communication; taste differentiation *per se* plays no role. For example, if preferences are δ-indecisive, agent confusion is the only possible equilibrium, even with an arbitrarily large degree of taste differentiation. To illustrate, suppose that $supp(\gamma_{\mathbf{a}}) \subset [-1, 1]\ \forall \mathbf{a} \in \mathcal{A}$. The distributions g_0 and g_0' in Figure 3(a) differ in their degrees of taste differentiation, but in both cases an SPE with education exists. By contrast, g_0 and g_0' in Figure 3(b) have the same degree of taste differentiation, but an education equilibrium only exists in the latter case.

[14]If, contradicting condition (A2.2), agent education could be enforced unilaterally, then part (i) of Theorem 1 would be strengthened in that only SPE with agent education exist. More generally, without (A2.2) there could be SPE with agent education, regardless of the shape of the true match value distribution. Specifically, if, similar to Heidhues et al. (2016), each contestant could assure education by choosing some communication activity $a^e \in A$, i.e., $\varepsilon_{\mathbf{a}} = O$ if $a^e \in \{a_1, a_2\}$, then education (with both firms choosing a^e) would always be an equilibrium outcome. Note, however, that with polarized preferences any SPE featuring confusion will strictly dominate such an education equilibrium from the perspectives of the contestants.

3.3 How Much Confusion?

Up to this point we have put no order structure on the various distribution functions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$. Given that consumer confusion is of an unbiased nature, a natural starting point is to presume that the distribution functions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ can be ordered using the notion of a *mean-preserving spread* (MPS). Formally, for two random variables X and Y, Y is an MPS of X if Y has the same distribution as $X+\eta$, where $\eta \neq O$ and $E[\eta|X]=0$.[15] Intuitively, Y is a noisy version of X. The above assumptions trivially imply that any communication profile inducing confusion corresponds to an MPS of the distribution corresponding to an educating profile. We now require more generally that *all* noise distributions can be ordered via MPS.

Assumption 3 (MPS ordering) *$A \subset \mathbb{R}_+$ is compact, and $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = \mathbf{0}$. Moreover, $\forall \mathbf{a}, \mathbf{a}' \in \mathcal{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'}$ is an MPS of $\Gamma_{\mathbf{a}}$.*

Note that Assumption 3 implies Assumption 2 whenever $0 \in A$ and A contains more than one element. Moreover, agent confusion is maximal (minimal) according to the MPS order if both firms choose $\bar{a} \equiv \max A$ ($\underline{a} \equiv \min A$). In particular, the communication profile $(\bar{a}, \bar{a})$ induces maximal agent confusion as measured by the variance of the perceived match advantages.

Most examples we discuss in Appendix S.1 to support (1) feature the MPS ordering. For instance, consider the above-mentioned example where communication strategies correspond to the number of i.i.d features implemented by a contestant: The more such features are implemented, the more noisy the perceived valuations become. Another class of examples arises when the members of the family $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are *truncations* of each other. This order conveniently preserves essential features of the original distribution, such as log-concavity of the density or the shape of $\gamma_{\mathbf{a}}$. This essentially amounts to assuming that greater confusion increases the scope for the possible perception errors, while leaving the underlying stochastic principles behind the perception errors unaltered.[16]

Our second main result strengthens Theorem 1 by showing that under the MPS ordering a unique SPE exists, featuring either maximal or minimal confusion, depending on the distribution of true preferences.

[15] Rothschild and Stiglitz (1970) show that if the involved distribution functions have a uniformly bounded support, then the MPS ordering between distributions is equivalent to the order induced by *second-order stochastic dominance*. Müller (1998) shows how to extend this equivalence to the case of an unbounded support.

[16] For any given $\mathbf{a} \in \mathcal{A}$, $\Gamma_{\mathbf{a}}$ is the truncation $\varepsilon_{\mathbf{a}} \equiv \varepsilon|_{\varepsilon\in[-\omega_{\mathbf{a}},\omega_{\mathbf{a}}]}$ of a random variable ε with zero-symmetric density γ and $supp\,\gamma = (-\bar{\omega}, \bar{\omega})$, $0 \leq \omega_{\mathbf{a}} < \bar{\omega}$. Then, $\Gamma_{\mathbf{a}'}$ is an MPS of $\Gamma_{\mathbf{a}}$ iff $\omega_{\mathbf{a}'} > \omega_{\mathbf{a}}$.

Theorem 2 *Suppose that Assumptions 1 and 3 hold.*

(i) If there exists $\delta > 0$ such that supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are strongly δ-indecisive, then there exists a unique SPE, and confusion is maximal.

(ii) If there exists $\delta > 0$ such that supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are strongly δ-polarized, then there exists a unique SPE, and confusion is minimal.

By Assumption 3, a contestant i can always unilaterally enforce more (less) confusion in the MPS sense, provided that $a_i < \bar{a}$ ($a_i > \underline{a}$). Following the above logic, each firm will therefore choose its maximal (minimal) available a_i for indecisive (polarized) preferences. Therefore, both contestants coordinate on the communication profile that induces maximal confusion. The opposite reasoning applies with strongly polarized match values.

In Appendix S, we formulate a variant of Theorem 2 using an alternative order, which we call *two-sided single crossing*. This property is more restrictive than MPS, but it allows us to use indecisiveness (polarization) rather than strong indecisiveness (strong polarization). Finally, note that, appropriately modified, Theorem 2 applies even when $\mathbf{0} \notin \mathcal{A}$, so that it is not possible for the contestants to fully educate the agents. In such a case, both contestants would coordinate on the SPE featuring minimal agent confusion by playing $\underline{\mathbf{a}}$ in the communication stage under polarization.

3.4 Massive Confusion

Theorems 1 and 2 apply to situations where the scope for agent confusion is constrained by the degree of the existing taste differentiation, i.e., *supp*$(\varepsilon_{\mathbf{a}}) \subset$ *supp*(g_0) $\forall \mathbf{a} \in \mathcal{A}$. In other words, for equal second-stage efforts, obfuscation can never induce agents with the most extreme true valuations in favor of one contestant to switch to the other contestant. We next study an extension where such "massive" reversals in agent opinions are possible. Agent confusion is called *massive*, whenever *supp*$(g_0) \subsetneq$ *supp*$(\varepsilon_{\mathbf{a}})$ for some $\mathbf{a} \in \mathcal{A}$. Our next result uses the tractable case of uniformly distributed perception errors $\varepsilon_{\mathbf{a}}$ to show that, even with polarized preferences, contestants may choose to obfuscate the market if massive confusion is possible.

Theorem 3 *Suppose that Assumptions 1 and 3 hold, and $\varepsilon_{\mathbf{a}}$ is uniformly distributed on $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, where $\omega_{\mathbf{a}} \geq 0$, $\forall \mathbf{a} \in \mathcal{A}$. Let $\bar{\omega} \equiv \max_{\mathbf{a} \in \mathcal{A}} \omega_{\mathbf{a}}$. Then, the unique SPE features maximal confusion ($\omega_{\mathbf{a}^*} = \bar{\omega}$) if either (i) true match values are indecisive on supp(g_0), or (ii) true match values are polarized on supp(g_0) and $\bar{\omega}$ is sufficiently large.*

For indecisive agents, the prediction from Theorem 2 that contestants want to obfuscate as much as possible immediately generalizes to the case of massive confusion.[17] However, maximal confusion can now arise as the unique SPE, despite polarized preferences. The reason is that if contestants can induce sufficiently large potential differences in the perceived match advantages, then the measure of indifferent agents must eventually decrease, regardless of g_0.[18] Finally, the key insight from the literature on competition with boundedly rational consumers and *homogeneous* products, according to which obfuscation arises in equilibrium, can be seen as the limiting case of Theorem 3: In a situation where true match advantages are arbitrarily close to zero, any confusion is massive, so that contestants (i.e., firms) must benefit from introducing it.

Remark What drives the massive obfuscation result is the possibility to create extreme valuations that have no counterpart in the distribution of true valuations. This possibility appears particularly plausible when true preferences are close to homogeneous. However, there may be conceivable circumstances in which the agents' perceived valuations must remain contained in the support of the distribution of true valuations. For example, if individual consumers have a basic understanding of a market, but not of which option is best for them, the effects of confusion on perceived valuations may be bounded by the true valuation distribution. We discuss such a situation in Appendix S.5 by means of a Salop circle: Agents know the contestants' locations (e.g., the properties of the candidates' political platforms), but they do not know their own locations. We assume that, although individual agents may be confused about their true locations (and thus the true match advantages of the contestants), they understand the general situation that they must be located somewhere on the circle. In this model, there is a natural upper bound for how massive the agent confusion can be: At most, a contestant can make the agent located at the opposite end of the circle believe that they share the same location.

We show that for indecisive preferences the contestants' payoffs in the effort stage are inverse-U shaped in the extent of obfuscation. With polarization, we instead obtain an interior minimum. More generally, these findings suggest that in the Salop model, firms never desire to maximally confuse consumers in the sense that the perceived location of

[17] Note that the family of uniform distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ is ranked according to the MPS ordering. Hence Assumption 3 is only needed to assure a unique identification of the communication strategies in the SPE featuring maximal (minimal) confusion. Further, tedious algebra shows that, Theorem 3 extends to the case where $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ is a general family of truncation-ordered distributions, such that the different densities $\gamma_{\mathbf{a}}$ are again only distinguished by their supports.

[18] Note, however, that Theorem 3 relies on the assumption that agents do not choose an outside option. In Section 4.4, we allow for confusion that is so large that agents may opt for the outside option ex post.

a consumer bears no relation to his actual location.

4 Price Competition

In this section, we apply our general framework to study price competition between firms. In Section 4.1, we motivate our general confusion framework in the context of market competition. Section 4.2 applies the general framework to the case at hand. Section 4.3 considers welfare and regulation, dealing in particular with pitfalls of competition-enhancing regulation. In Section 4.4, we study the case of a binding outside option, and relate our model to the empirical literature on choice overload. Section 4.5 shows that our model applies to a situation where consumers differ in their degree of sophistication. Section 4.6 connects our approach to the advertising literature.

4.1 Consumer Perception Errors: Sources

Marketing, consumer research, perceptual psychology and neuroscience provide evidence of confusion that is consistent with the reduced-form (1) we use for our analysis. In the following, we provide an intuitive discussion of this evidence. In Appendix S.1 we show more rigorously how to formalize the respective notions in terms of our framework.

A large literature in marketing has documented consumer confusion resulting from information overload. Overload confusion occurs once consumers are "confronted with more product information and alternatives than they can process in order to get to know, to compare and to comprehend alternatives" (Walsh et al., 2007).[19] The general consequences of information overload, surveyed by Eppler and Mengis (2004) across fields as diverse as accounting, organizational science, marketing and consumer research, are unsystematic decision mistakes, a decreased decision accuracy, a lack of critical evaluation, a "failure to develop correct interpretations of various facets of a product or service" (Turnbull et al., 2000), and ambiguous perceptions by consumers (Solomon et al., 2014). More generally, the fact that confusion can have heterogeneous effects on the perceptions by different agents is consistent with well-established psychometric research on perceptual errors (Murray, 1993). Jacoby (1977) and Malhotra (1982) are among the first to point out that consumer confusion due to information overload emerges when choosing between complex products, where complexity is related to the number of attributes or

[19] The idea is rooted in psychology and neuroscience, where it is well understood that people make decision mistakes once the amount of the information they are exposed to surpasses a certain threshold (see, e.g., Miller, 1956).

surrounding marketing messages. Complexity confusion has been found in computers, mobile phones, automobiles, digital cameras, buildings or insurance policies (see Walsh et al., 2007; Kasabov, 2015; Mützel and Kilian, 2016), and has been associated with product packaging (Mitchell and Papavassiliou, 1999), or lengthy and complicated contracts involving "fine print" (see, e.g., Turnbull et al. (2000) for the case of mobile phones).

The emphasis of our model on relative valuations allows for the possibility that communication strategies not only affect the perceived valuation for a firm's own good, but also for the competitor's good. For instance, if one food brand uses the label "original" while another brand uses "authentic", consumers may be confused when comparing the brands (see e.g., Langer et al. (2007) for the confusing role of labels). In such cases, the valuations of the goods may be interdependent, rather than i.i.d.[20] In addition, the possibility that a communication strategy of a firm also affects the evaluation of the competitor's good is consistent with the approach of authors such as Carlin (2009), Piccione and Spiegler (2012), Chioveanu and Zhou (2013) or Spiegler (2014). These authors study homogeneous goods models where the mutual choice of a "frame", i.e., a way to present the price of a product, determines whether or not a consumer can compare products. The notion of framing can be incorporated into our setting by assuming that communication profiles induce a probability distribution over the different frames a consumer could adopt to compare the products (see Appendix S.1 for details).[21]

4.2 Main Results with Price Competition

The model of price competition and zero marginal-cost firms can be captured within our framework by specifying $\mathcal{S} = (-\infty, 0]$ and $R(s_i, s_j) = -s_i$, $C(s_i) = 0$, and let $p_i = -s_i$ be the price set by firm i. We conveniently refer to the effort stage as the *pricing* stage of the game. To apply the results from Section 3, we need to verify that the conditions in Assumption 1 are satisfied. (A1.3) always holds because

$$z(s) = -\frac{1}{2s}, \text{ and } R_1(s,s) + R_2(s,s) - 2C'(s) = -1 < 0, \forall s = -p \leq 0.$$

[20]This is consistent with the view that complexity is a synthetic phenomenon of all marketing messages interacting with each other, leading to a market level or "category complexity" (Mützel and Kilian, 2016).

[21]We could also have assumed that communication strategies amount to choosing the presentation of the final price ("price format") $\varepsilon_{\mathbf{a}}$, rather than (gross) match advantages as in (1). Then, the perceived price advantage of firm $i = 2$ by consumer k is $p^k_\Delta + \varepsilon_{\mathbf{a}}$, $p^k_\Delta \equiv p_1 - p_2$. That is, while consumers perceive the true possible price advantage of a firm with noise, each firm sets a deterministic price which a purchasing consumer in the end needs to pay. If price formats are chosen in the first stage and prices in the second, the model is formally equivalent to the one we studied. See Grubb (2015) for survey on models with noisy prices.

The following proposition provides a simple set of sufficient conditions on the distribution functions G_0 and $G_{\mathbf{a}}$ assuring that (A1.1) and (A1.2) hold, allowing us to identify the unique symmetric pure-strategy price equilibrium of the pricing stage.

Proposition 1 *Suppose that the following conditions are satisfied: (i) G_0 is log-concave on supp(g_0); (ii) g_0 is continuous at zero and $g_0(0) > 0$; (iii) If $\varepsilon_{\mathbf{a}} \neq O$, it has a density $\gamma_{\mathbf{a}}$ that is log-concave on supp($\gamma_{\mathbf{a}}$). Then Assumption 1 holds, and every subgame in the pricing stage has a unique symmetric pure-strategy equilibrium where both firms choose the price $p^*_{\mathbf{a}} = \frac{1}{2g_a(0)}$, $\forall \mathbf{a} \in \mathcal{A}$.*

The log-concavity conditions (i) and (iii) assure the strict quasiconcavity of the payoff (A1.1). Jointly with the technical condition (ii), the requirement that $\varepsilon_{\mathbf{a}}$ has a density function whenever it is not degenerate implies (A1.2), assuring that the equilibrium prices p^* are well-defined.

Proposition 1 directly shows that the equilibrium price $p^*_{\mathbf{a}}$ is determined by and decreasing in $g_{\mathbf{a}}(0)$, the measure of perceptually indifferent consumers. Since higher prices correspond to higher profits, firms prefer communication profiles that reduce the measure of perceptually indifferent consumers, consistent with Lemma 1. Based on this insight, it is straightforward to translate Theorems 1-3 to the price competition setting. In particular, SPE without consumer confusion exist for weak polarization, but not for weak indecisiveness. With stronger indecisiveness (polarization) conditions and suitable dispersion orders of the noise induced by the communication strategies, there is a unique SPE with maximal (minimal) confusion. Finally, if massive confusion is possible, confusion arises even with polarized preferences.[22]

Reflecting the logic of the general analysis, equilibrium forces push both firms to compete for the perceptually indifferent and therefore most price-sensitive consumers. If $g_{\mathbf{a}}(0)$ is low, there are only few such consumers. Thus, consistent with empirical observations (Ellison and Ellison, 2009), obfuscation is an effective means to lower price elasticities and increase markups if either tastes are indecisive or obfuscation can become massive (e.g., because products are near to homogeneous).

Competition on the line We now apply Proposition 1 to a well-known textbook example. Suppose that each consumer is characterized by a parameter $\theta \in \Theta = [-1, 1]$,

[22]Note that for any fixed support of preferences $[-v, v]$, firms always desire match advantages that are distributed in a polarized way, as these distributions soften competition most (see also Armstrong and Zhou, 2019). However, it is not possible to induce such distributions by means of confusion if true preferences are indecisive.

which is drawn from a commonly known distribution H with zero-symmetric density function $h(\theta) = \alpha\theta^2 + \frac{1}{2} - \frac{\alpha}{3}$ on Θ, where $\alpha \in \left[-\frac{3}{4}, \frac{6-3\sqrt{3}}{4}\right]$. The true match value of a type θ consumer for product $i \in \{1,2\}$ is $v_i^\theta = \mu - (x_i - \theta)^2$, where $\mu > 0$ is sufficiently large, and $x_1 = -1$, $x_2 = 1$ are the locations of the firms. Thus, θ can be interpreted as the location of the consumer on a (Hotelling) line, where the true match value is determined by a quadratic distance function. H translates into a distribution G_0 of true match advantages, where $G_0(x) = H(\frac{x}{4})$ $\forall x \in \mathbb{R}$. If $\alpha > 0$, the true match values are strongly polarized on the support of G_0, $[-4, 4]$. Conversely, if $\alpha < 0$, the true match values are strongly indecisive on $[-4, 4]$. More generally, $|\alpha|$ corresponds to the extent of polarization or indecisiveness, respectively. To capture obfuscation, suppose that the error distribution is uniformly distributed on an interval $[-\omega_\mathbf{a}, \omega_\mathbf{a}]$ where $\omega_\mathbf{a} < 4$ $\forall \mathbf{a} \in \mathcal{A}$, i.e., confusion cannot be massive.

Proposition 1 applies to this example.[23] Thus, a symmetric price equilibrium exists, given by $p_\mathbf{a}^* = \frac{1}{2g_\mathbf{a}(0)}$. This example allows to explicitly derive $g_\mathbf{a}(0)$, which yields

$$p_\mathbf{a}^* = \frac{288}{\alpha\omega_\mathbf{a}^2 + 72 - 48\alpha} > 0 \tag{3}$$

as equilibrium price. It is easily seen from (3) that the equilibrium price (and thus payoffs) increase in $\omega_\mathbf{a}$ if $\alpha < 0$ (indecisiveness), and decrease if $\alpha > 0$ (polarization), confirming the results from Theorems 1 and 2 under the respective assumptions on $\mathcal{A}$.[24]

The Logit model The Logit model is among the most frequently used models for the theoretical and empirical analysis of discrete choice. It is well known that whenever true tastes are described by a linear random utility model with an i.i.d. Gumbel distribution, a Logit demand system results (see Anderson et al., 1992). In such a case, the distribution of the true match advantages, v_Δ, follows a zero-mean logistic distribution. As this type of distribution is zero-symmetric, log-concave and features (global) indecisiveness on $\mathbb{R}$, our main analysis directly implies that only SPE featuring agent confusion can exist in a Logit context.

4.3 Welfare Implications

In a homogeneous goods setting, consumer confusion increases prices, and firms benefit at the expense of consumers. In our setting, goods are differentiated ($v_\Delta \neq O$), so that

[23] In particular, the parameter restriction on α assures the sufficiency of first-order conditions in the pricing stage; for details, see Proposition A1 in Appendix A.2.

[24] In the knife-edge case of uniformly distributed consumers ($\alpha = 0$), confusion has no price effect (unless it may become massive).

consumer confusion could, in principle, reduce prices. However, in such a case (when preferences are polarized), firms avoid obfuscating the market according to Theorems 1 and 2. More importantly, confusion can lead to inefficiency with differentiated goods, because some consumers might acquire a dominated product. For any equilibrium communication profile $\mathbf{a}^*$, the total expected welfare loss from mismatch is

$$L = 2\int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx \geq 0. \tag{4}$$

To understand (4), note that, if a consumer chooses the dominated option, the welfare loss is $|v_2 - v_1|$, i.e., the absolute difference of her true match values. Let $x = v_2 - v_1 > 0$ w.l.o.g. Then, $g_0(x)$ is the likelihood of type x and $\Gamma_{\mathbf{a}^*}(-x)$ is the probability that type x buys from the wrong firm. Expression (4) shows that equilibrium consumer confusion is necessary and sufficient for a positive welfare loss ($L > 0$) to occur. In view of the results in Sections 3.2 - 3.4, indecisive preferences imply an inefficient equilibrium outcome, while no such inefficiency arises with polarized preferences, as least as long confusion cannot become massive.

In Appendix S.4, we elaborate on the size of the welfare loss in the case of indecisive preferences. Specifically, we establish the intuitive result that the welfare loss (4) is monotonically increasing in the size of confusion if confusion follows a uniform distribution as in Proposition 3. In addition, we show in the example with competition on the line that an increase in the indecisiveness of preferences (captured by increasing $|\alpha|$) has an ambiguous effect on welfare.[25]

Regarding welfare, our analysis further informs the evaluation of regulations aimed at increasing competition between *incumbent* firms.[26] An example is a compulsory product standard or norm which increases the true similarity between products. In the example with competition on the line, such a regulation can be captured as a relocation of both firms towards the middle or, more precisely, as a truncation of the true match advantage distribution G_0. More generally, let $supp\, g_0 = [-\lambda, \lambda]$, where $\lambda > 0$ captures the true extent of product differentiation. Consider a policy with the effect of reducing this differentiation to $supp\, g_0^r = [-r, r]$, $0 < r < \lambda$, where g_0^r is a truncation of g_0.[27] Absent any

[25]The ambiguity follows from two competing effects. As $|\alpha|$ increases, more almost indifferent consumers buy the wrong product, but at the same time, these welfare losses are rather low. The former effect dominates (and the welfare loss increases in $|\alpha|$) only if obfuscation possibilities are large enough relative to true differentiation.

[26]In different settings, Spiegler (2006) and Hefti (2018) show that facilitating firm entry with boundedly rational consumers may lead to inefficiency.

[27]In the previous example with quadratic transportation costs, it is easy to see that if the new firm locations are given by $\{-r, r\}$, $r \in (0,1)$, then $supp\, g_0^r = [-4r, 4r] \subsetneq [-4, 4]$ and $g_0^r(x) = \frac{1}{4r}h(\frac{x}{4r})$

consumer confusion, it is easy to see that such a regulation successfully lowers equilibrium prices, as $g_0^r(0) > g_0(0)$, independent of the shape of g_0. The following result shows that, depending on consumer preferences, such regulations may have unintended, adverse effects on welfare due to consumer confusion.[28]

Proposition 2 *Suppose that Assumptions 1 and 3 are satisfied, and consider the regulation with* $0 < r < \lambda$ *outlined above. (i) If true preferences are strongly indecisive on* $\operatorname{supp} g_0$ *and* $\operatorname{supp} \gamma_{\mathbf{a}} \subset [-r, r]$, $\forall \mathbf{a} \in \mathcal{A}$, *the regulation strictly increases the welfare loss due to consumer confusion. (ii) If true preferences are polarized on* $\operatorname{supp} g_0$, *the regulation has no adverse welfare effects, unless possibly if confusion becomes massive.*

Intuitively, the qualitative shape of the true match advantage distribution is invariant to the above type of regulation. Therefore, the regulation does not affect whether or not confusion takes place by Theorem 2, at least as long as the scope of confusion is limited. However, when confusion does take place (i.e., with indecisive preferences), more consumers will be confused for any given valuation difference as the distribution of match advantages becomes more concentrated, while firms nevertheless obfuscate at maximal intensity. This implies that more consumers choose a dominated product following the regulation. By contrast, firms continue to educate consumers with polarized preferences, at least as long as massive confusion is infeasible.

Similar reasoning applies to changes in the environment that are not policy-induced. For instance, pundits believed that the Internet would lead to the "the death of distance" (Cairncross, 1997) in banking competition, as the possibility of online transactions was expected to dramatically reduce the importance of (geographical) distance between banks. By contrast, our model predicts that banks, competing in interest rates for depositors, are likely to counter such increasing competitive pressure by obfuscating the market.[29] This is consistent with the fact that, indeed, a pro-competitive effect of the Internet has not been observed in the data (Degryse and Ongena, 2005).

4.4 Abstaining from Purchases

The marketing literature has emphasized that confused consumers may inefficiently abstain from buying at all (Iyengar and Lepper, 2000; Iyengar et al., 2004; Eppler and

$\forall x \in [-4r, 4r]$.

[28] We focus entirely on consumer confusion as a new channel of inefficiency, leaving aside potential additional effects caused by the change of differentiation itself.

[29] As argued in Section 2 (see footnote 10), it is relatively straightforward to adopt our framework to capture such strategic competition in interest rates between banks.

Mengis, 2004; Bertrand et al., 2010; Chernev et al., 2015).[30] Consistent with this empirical observation, we next show that in the presence of binding outside options, firms may choose a confusing communication strategy even though this induces some consumers to exit the market. This provides an additional rationale for why the market does not eliminate confusion and its negative consequences.

In the spirit of the Hotelling model, suppose that the perceived match values are given by $\tilde{v}_1 = 1 + \frac{v}{2} + \frac{\varepsilon}{2}$ and $\tilde{v}_2 = 1 - \frac{v}{2} - \frac{\varepsilon}{2}$, where $v \in [-1, 1]$ is governed by a zero-symmetric distribution G_0 with density g_0. Further, all consumers have a reservation value normalized to zero. Thus, a consumer purchases the good with the higher perceived net value $\tilde{v}_j - p_j$ if this net value is non-negative, and does not purchase otherwise.[31] Compared to the previous analysis, each firm must take into account that a price increase may now come at the additional cost of losing some consumers to whom the firm actually is offering the better deal. The threat of exiting consumers becomes pertinent if prices are high. Therefore it is conceivable that the possibility of a binding outside option may discipline firms against obfuscating too much, e.g., in case of indecisive preferences.

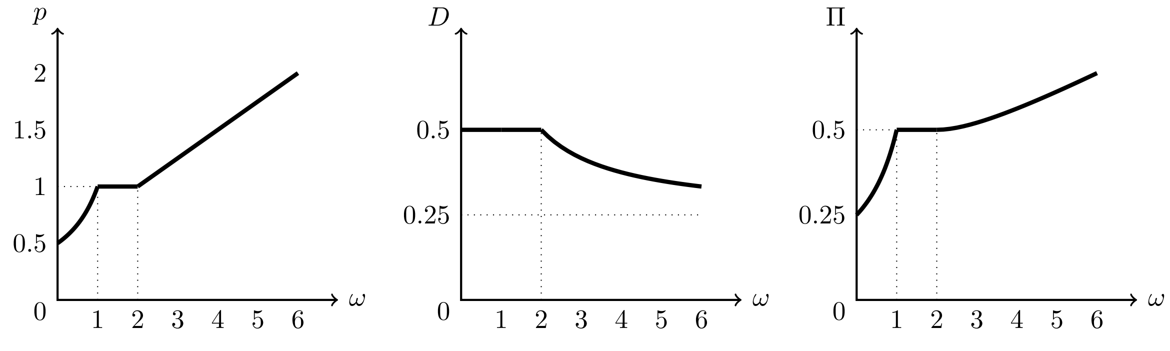
In the following, we explain the main equilibrium patterns predicted by the above model intuitively; see Appendix S.3 for a formal analysis. Figure 4 illustrates second-stage prices, firm demand and payoff if G_0 follows a simple "tent" distribution on $[-1, 1]$ (i.e., true preferences are indecisive),[32] and $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a} \in \mathcal{A}}$ is given by a family of uniform distributions that differ in their support $[-\omega, \omega]$, $\omega \geq 0$. As in Section 3.4 the parameter ω captures the intensity of confusion in a market. The figure shows that the equilibrium price and profit both are globally increasing functions of ω, and strictly so if ω is small ($\omega < 1$) or large ($\omega > 2$) enough. In particular, equilibrium profits remain increasing in ω despite an increasing fraction of exiting consumers.

The intuition behind the figure is as follows. As long as obfuscation cannot reduce the measure of perceptionally indifferent consumers enough ($\omega \leq 1$), prices remain so low that the outside option does not bind in equilibrium. In this case, the analysis is identical to the one of our main model: More confusion (a larger value of ω) increases prices and profits without reducing demand. For $\omega > 1$, the outside option may become binding for some

[30] In particular, the literature noted that product complexity, rather than the sheer number of products, is decisive for whether such "choice overload" emerges (Iyengar and Lepper, 2000; Lee and Lee, 2004; Scheibehenne et al., 2010; Chernev et al., 2015).

[31] In the above formulation, the true match values are (perfectly) negatively correlated, which implies that the distribution of the true match advantages, $v_\Delta = -v$, also is given by g_0. This property makes the formal analysis sufficiently tractable. Further, the normalization $supp(g_0) = [-1, 1]$ is not essential, but simplifies the presentation of results.

[32] The tent distribution has $g_0(x) = 1 + x$ for $x \in [-1, 0]$ and $g_0(x) = 1 - x$ for $x \in (0, 1]$.



(a) (b) (c)

Figure 4: (a) Price, (b) demand, and (c) payoff as a function of ω

consumers. Thus, the firms need to strategically balance the corresponding loss in demand against the benefits of higher prices. For intermediate confusion ($1 < \omega < 2$), this leads firms to choose their prices exactly such that no consumer decides to leave the market. By contrast, with sufficiently large confusion ($\omega > 2$) the competition for the perceptionally indifferent consumers softens so that the higher prices eventually compensate for the loss in demand.[33] In such a situation, firms increasingly exploit consumers with the most favorable views of their products at the cost of losing some less favorable consumers.

Figure 4 implies that, whenever obfuscation possibilities are sufficiently large ($\omega > 2$ is feasible), then a communication profile inducing maximal obfuscation must be part of an SPE.[34] Thus, our model rationalizes the possibility that consumers erroneously choose the outside option as an event that firms are willing to accept as part of their profit-maximizing strategies. Put differently, a binding outside option does not generally discipline firms against using confusing communication strategies; equilibrium payoffs overall are weakly increasing in ω, and become flat only for intermediate values of ω. Moreover, such firm behavior creates an additional source of market inefficiency, besides the inefficiency originating from the mismatch between some consumers and firms.

4.5 Consumer Sophistication

In research with behavioral agents, the population is frequently partitioned into two types: the "naive" and the "sophisticated", where the latter do not exhibit any behavioral bias. We now demonstrate that our main results also apply in the presence of agents with different degrees of sophistication. Let naiveté be an exogenous trait that is dispersed

[33] In this case, each firm prices as if it were a monopolist constrained only by the possibility that some consumers choose not to purchase the good.

[34] This result holds more generally, as we show in Appendix S.3.

over the consumer population according to the random variable ρ, such that $\tilde{v}_\Delta(\mathbf{a}) = v_\Delta + \rho\varepsilon_\mathbf{a}$, where $supp\,\rho \subset [0,1]$, and ρ, v_Δ and $\varepsilon_\mathbf{a}$ are independent for any given $\mathbf{a} \in \mathcal{A}$. A consumer with $\rho = 0$ is fully sophisticated, meaning that her perceptions are unaffected by the chosen communication strategies: $\tilde{v}_\Delta(\mathbf{a}) = v_\Delta$ $\forall \mathbf{a} \in \mathcal{A}$. By contrast, a larger value of ρ means less sophistication in that the possible distortions induced by $\varepsilon_\mathbf{a}$ are amplified.[35] Alternatively, ρ can be seen as a measure for the level of "confusion proneness" as introduced by Walsh et al. (2007), capturing that different consumers may be differ in how susceptible they are to obfuscation techniques. It is straightforward to use first-order conditions to verify that $p^*_\mathbf{a} = \frac{1}{2\hat{g}_\mathbf{a}(0)}$ results in the pricing stage, where $\hat{g}_\mathbf{a}(0) = \int\int g_0(\tilde{\rho}\tilde{\varepsilon})d\Gamma_\mathbf{a}(\tilde{\varepsilon})d\Gamma_\rho(\tilde{\rho})$ and Γ_ρ denotes the distribution function of ρ.[36] It is evident from this expression that the main firm-side incentives to obfuscate or educate still depend exclusively on the shape of g_0.[37]

4.6 Relation to the Advertising Literature

One interpretation of our results is that more precise information about the products increases prices and profits with indecisive preferences, but decreases them with polarized preferences. This contrasts with familiar results from the literature on informative advertising, surveyed in Bagwell (2007), where more information provided by firms typically reduces prices by intensifying competition. Our paper also relates to the literature on persuasive advertising, which emphasizes that persuasive advertising games have the structure of a prisoners' dilemma: Firms engage in costly advertising races, which, in equilibrium, do not affect prices and gross profits (see, e.g., Dixit and Norman, 1978; Von der Fehr and Stevik, 1998; Bagwell, 2007). In our setting, obfuscating communication strategies can be interpreted as activities that persuade some consumers at the cost of alienating others.[38] Our analysis shows that firms either refrain from such advertising measures (with polarized consumers) or use the measures to soften competition (with

[35]The information may be so complex that it cannot be fully assessed even by sophisticated agents (Eichenberger and Serna, 1996), in which case $\rho > 0$.

[36]The typical case of binary types occurs if all probability mass of Γ_ρ rests on $\rho = 0$ and $\rho = 1$, which yields $\hat{g}_\mathbf{a}(0) = \Gamma_\rho(0)g_\mathbf{a}(0) + (1 - \Gamma_\rho(0))g_0(0)$.

[37]In our framework, naive consumers impose a negative externality on sophisticated ones, independent of the shape of the preference distribution. This is because, in any case, the equilibrium measures taken by firms are such as to either confuse or educate the naive consumers, which increases the equilibrium price for sophisticated consumers as well. In fact, the desire for consumer education of firms and sophisticated consumers are exactly antipodal to each other.

[38]As an example, consider the cold-calls of tele-marketing agents (see Schumacher and Thysen, 2017).

indecisive consumers), which contrasts with the prisoners' dilemma situations.[39]

Finally, our results are related to Johnson and Myatt (2006), who show that a monopolist does not necessarily want to engage in measures related to advertising, marketing or product design that increase the heterogeneity of consumer valuations. In their monopoly analysis, the entire distribution of valuations matters for the optimal pricing strategy of a monopolist, while in our setting, in the absence of outside options the competitive forces imply that equilibrium prices depend only on the mass of perceptually indifferent consumers. Firms always desire more consumer heterogeneity in the sense of fewer indifferent and hence highly price-sensitive consumers. Our main result that the shape of the true match advantage distribution determines whether such heterogeneity is achievable by means of obfuscation or education has no counterpart in Johnson and Myatt (2006).

5 Competition for Voters

Communication strategies play a major role in political competition. A substantial literature has asked why political candidates often choose ambiguous platforms, rather than describe their policies exactly. In the standard Hotelling-Downs framework of political competition, ambiguous platforms indeed are always suboptimal when voters are risk-averse (Shepsle, 1972). By contrast, some authors have argued that ambiguity may be preferable, e.g., because it allows political candidates to maintain the flexibility to adapt to future circumstances (Aragones and Neeman, 2000; Kartik et al., 2017), while others have provided behavioral explanations, relying on context-dependent preferences (Callander and Wilson, 2008) or projection bias (Jensen, 2009).We contribute to the literature on strategic ambiguity in political competition by studying how the incentives of political candidates to confuse or educate potential voters about their platforms depend on the heterogeneity of true voter preferences. In Section 5.1, we directly apply our general analysis to the case of symmetric candidates. In Section 5.2, we relax the assumptions of perfectly symmetric contestants and unbiased perception errors.

5.1 Symmetric Political Parties

We consider the specification of the general model with $R(s_1, s_2) = 1$ and $C(s_i) = ks_i^\eta$, $\forall s_1, s_2 \in \mathcal{S} = \mathbb{R}_+$, where $k > 0$ and $\eta > 1$. Here, we interpret the market share as the share of votes, and we assume that the two political contestants, henceforth simply referred to as "parties", value the votes symmetrically. Parties are heterogeneous with

[39]It is easy to show that our results would qualitatively apply if we introduced obfuscation costs.

respect to their ideology, and voters have heterogeneous preferences over policies, captured by the distribution G_0 of true match advantages. Voters evaluate a party according to their perceived match advantages. The distribution of perceived match advantages, $G_{\mathbf{a}}$, is determined by the parties' communication strategies (a_1, a_2).[40] In particular, a party can avoid being precise, leading to a noisy perception, whereby some voters get a too positive impression of the party's value for them and others get a too negative impression. Parties not only influence election outcomes by their platforms. In addition, it is well known that the prominence in the media of a party has a strong persuasive effect on voting behavior. For instance, a candidate's comparative advantage in media exposure can lead undecided voters to favor him (Gerber et al., 2011; Gallego and Schofield, 2017). We capture this observation by interpreting s_i as advertising efforts, where the party with $s_i > s_j$ is more prominent and, consequently, wins more undecided voters.

Contrary to price reductions, efforts in this setting are unconditional, that is, they arise independent of the contestants' success in attracting market share. Nevertheless, our main insights about how the chosen communication strategies relate to the true dispersion of preferences carry over to political competition. Under suitable parameter restrictions (see Appendix A2), the equilibrium effort level $s^*_{\mathbf{a}}$ is described by the first-order equilibrium equation $g_{\mathbf{a}}(0) = C'(s^*_{\mathbf{a}})$. Hence, $s^*_{\mathbf{a}}$ is strictly increasing and equilibrium payoffs are strictly decreasing in the mass of perceptually indifferent voters $g_{\mathbf{a}}(0)$, reflecting increasing equilibrium effort levels and advertising expenditures .[41] Both parties therefore have a clear incentive to evade such intense competition. If Assumptions 2 and 3 hold, candidates will choose ambiguous platforms if preferences are indecisive (but not if they are polarized). In other words, a situation with indecisive preferences, i.e., with many truly undecided voters, provides a breeding ground for voter confusion.

5.2 Asymmetric Contestants

While our model allows for ex-post asymmetry of the contestants (after the choice of efforts), it assumes that they are symmetric ex ante (before the choice of communication strategies) and ex interim (between the choices of communication and efforts). We next elaborate on the robustness of the main insights of our paper with respect to these

[40] We can use higher values of a_i to capture greater ambiguity due to, for example, a larger set of policies (which leads to a larger support of the perceived match advantages), as is common in the literature.

[41] This is a special case of Lemma 1. We formally show in Appendix A.3 (see Proposition A2) that a symmetric equilibrium in the effort stage exists whenever the cost function is sufficiently convex (i.e., η is sufficiently large).

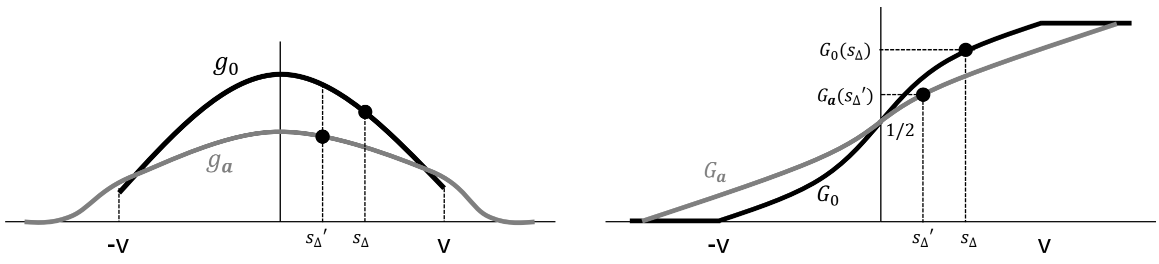


Figure 5: The effects of confusion with ex-ante asymmetric contestants indecisiveness

symmetry assumptions in the context of the voting model; similar reasoning applies to the price competition case.

Ex-ante asymmetry We first show that, with ex-ante asymmetric contestants, the shape of preferences has a similar effect on the nature of equilibrium as in the symmetric case. Suppose that $i = 1$ is a *strong* and $i = 2$ a *weak* contestant in the sense that $C_1'(s) < C_2'(s)$ for any $s > 0$. For a given distribution of perceived match advantages $G_{\mathbf{a}}$, the first-order conditions in the effort stage yield $g_{\mathbf{a}}(s_1 - s_2) = C_1'(s_1) = C_2'(s_2)$, showing that marginal costs are equated by equilibrium forces. It follows that we must have $s_1 - s_2 \equiv s_\Delta > 0$ in equilibrium, reflecting the advantage of the strong contestant. Because then also $G_{\mathbf{a}}(s_\Delta) > 1/2$, it follows that, despite unbiased perception errors, communication can lead to a redistribution of market shares and thereby possibly induce a conflict of interests between the contestants. Figure 5 illustrates the effects of agent confusion for indecisive preferences.[42]

As captured in the left part of Figure 5, agent confusion spreads out the density of the perceived match values $g_{\mathbf{a}}$ below g_0, and accordingly rotates the distribution $G_{\mathbf{a}}$ at 0 clockwise (displayed in the right part of Figure 5). On the one hand, both contestants benefit from confusion because the downward shift of $g_{\mathbf{a}}$ softens competition and allows them to choose lower equilibrium efforts. On the other hand, as we can see from the right part of Figure 5, the market share of the strong (weak) contestant would be larger (smaller) without agent confusion. Therefore, the strong contestant may have a mixed view about the benefits of agent confusion, while the weak contestant can only benefit.[43] The opposite logic applies with polarized preferences. Then, agent confusion can only harm the weak contestant because it intensifies competition, and the weak contestant loses some otherwise favorable agents to the strong competitor. In sum, the weak com-

[42]The suggestive insights portrayed by Figure 5 are supported by formal results (see Appendix S.6).

[43]Kalaycı and Potters (2011) observe a similar result in a Hotelling model with quality-differentiated products; also see Gabaix and Laibson (2004).

petitor unambiguously strives to educate (to confuse) in the case of polarized (indecisive) preferences, while the incentive is less clear-cut for the strong contestant.[44]

Ex-interim asymmetry Despite the many reasons suggesting an unbiased nature of the perception errors in the voters' comparison, the existence of communication profiles that lead to biased comparisons is conceivable. Are such biased communication profiles played in equilibrium provided that they are available? Do they overthrow the role of preferences that we identified in our previous results?[45] We study these questions in Appendix S.7 in a simple extension with binary communication strategies $A = \{0, 1\}$, where $a_j = 0$ corresponds to accurate communication, and $a_j = 1$ to communication that leads to noisy but biased comparisons in j's favor given that $a_{-j} = 0$. Preferences still play a key role for whether education or confusion results with possibly biased communication profiles: Mutual obfuscation remains the unique equilibrium with indecisive preferences, while education remains the only equilibrium outcome with polarized preferences if the contestants can unilaterally educate the agents, e.g., as in Heidhues et al. (2016).[46]

6 Conclusion

Our paper provides a theory explaining the strategic use of confusion by contestants, such as firms or political parties, who compete for agents with heterogeneous preferences. We find that the distribution of true agent preferences plays a decisive role for whether contestants choose an educating or a confusing communication strategy. Agent education emerges as an equilibrium outcome if true preferences are polarized, intuitively meaning that tastes are concentrated near the contestants' positions. By contrast, confusion arises for the case of indecisive preferences, characterized by a concentration of agents who are truly indifferent between the choice alternatives.

We also find that confusion can be an equilibrium outcome if it can be massive in the sense that the dispersion of true preferences is narrow relative to the perception errors that

[44]It is easy to see that a similar result emerges if $C_1(s) = C_2(s)$ but g_0 is shifted to the right, such that with equal effort, contestant $i = 1$ would obtain a strictly larger market share.

[45]Clearly, these issues are relevant for the price competition application as well: For instance, if Duracel's advertises its batteries as the "longest-living batteries", then all other batteries must be short-lived. If sufficiently many consumers are influenced by this logic, then this would be a clear case of biased communication.

[46]If education cannot be forced unilaterally, two possibilities arise. With very large biases, obfuscation becomes so attractive that contestants are trapped in a Prisoner's Dilemma, with mutual obfuscation as the sole equilibrium outcome. For biases of intermediate size, a coordination game results with two asymmetric equilibria, where exactly one firm obfuscates.

can be induced by obfuscation. In a product market setting with differentiated products, this generalizes the central insight from behavioral industrial organization that firms in homogeneous goods industries seek to obfuscate to escape the Bertrand trap: With homogeneous goods any confusion must be massive in our sense. Our analysis shows that, other than with homogeneous goods, firms do not always choose to confuse consumers, but if they do, its welfare effects are more detrimental. In particular, consumers can be harmed by choosing dominated options, paying higher prices or inefficiently forgoing purchases. The latter may help to understand why the consumer decision not to purchase any option due to confusion is a persistent empirical phenomenon.

Regulatory interventions may lead to undesired outcomes for reasons related to preference heterogeneity. For instance, regulations aiming at increasing the similarity of products to foster competition may be inefficient, because they can end up fostering confusion instead. Further, general restrictions of the acceptable communication strategies may backfire in situations with polarized preferences, where, in the absence of such restrictions, contestants would choose their communication to dissolve any pre-existing agent confusion.

Applied to political competition, our analysis shows that the distribution of voter preferences crucially affects whether parties choose ambiguous platforms: With many undecided voters such ambiguity may help to soften political competition as it moves the perceived valuations for the candidates towards polarization. At the same time, we find that, with pre-existing polarization, parties prefer to educate voters about their platforms. In sum, we find that the dispersion of preferences has similar implications for the incentives to confuse or educate in (political) contests or market competition with differentiated products.

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A Main Appendix

A.1 General Analysis: Proofs

Proof of Lemma 1 Let $\mathbf{a} \in \mathcal{A}$. By (A1.2) there exists $s^*_{\mathbf{a}} \in \mathcal{S}$ such that $z(s^*_{\mathbf{a}})$ is well-defined (i.e., $R(s^*_{\mathbf{a}}, s^*_{\mathbf{a}}) \neq 0$) and satisfies $z(s^*_{\mathbf{a}}) = g_{\mathbf{a}}(0)$, or, equivalently,

$$\frac{R_1(s^*_{\mathbf{a}}, s^*_{\mathbf{a}})}{2} + R(s^*_{\mathbf{a}}, s^*_{\mathbf{a}})g_{\mathbf{a}}(0) - C'(s^*_{\mathbf{a}}) = 0. \tag{A.1}$$

Moreover, $s^*_{\mathbf{a}}$ is uniquely determined because $z(s)$ is strictly increasing by (A1.3). Because $G_{\mathbf{a}}$ is zero-symmetric, (A.1) corresponds to the first-order condition for an interior symmetric equilibrium in the effort stage. By strict quasi-concavity (A1.1), we may conclude that the mutual choice of $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$ by both contestants constitutes the unique symmetric equilibrium in the effort stage. Because $z(s)$ is strictly increasing, the equilibrium effort $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$ is strictly increasing in $g_{\mathbf{a}}(0)$. Since by (A1.3) we have

$$\frac{d\Pi^{\mathbf{a}}_i(s,s)}{ds} = \frac{R_1(s,s) + R_2(s,s)}{2} - C'(s) < 0,$$

the equilibrium payoff must be strictly decreasing in $g_{\mathbf{a}}(0)$. ■

Proof of Theorem 1 Part (i): For any $\mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}} \neq O$, we have

$$g_{\mathbf{a}}(0) = \int_{supp(\varepsilon_{\mathbf{a}})} g_0(e)d\Gamma_{\mathbf{a}}(e) > \int_{supp(\gamma_{\mathbf{a}})} g_0(0)d\Gamma_{\mathbf{a}}(e) = g_0(0), \tag{A.2}$$

where the inequality follows from $supp(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$, the zero-symmetry of $\Gamma_{\mathbf{a}}$, and the fact that the true preferences are weakly polarized. Hence, by Lemma 1 we can conclude that $\Pi^{\mathbf{a}}_i(s^*_{\mathbf{a}}, s^*_{\mathbf{a}}) < \Pi^{\mathbf{0}}_i(s^*_{\mathbf{0}}, s^*_{\mathbf{0}})$ for all $\mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}} \neq O$. It then immediately follows that any choice of $\mathbf{a} \in \mathcal{A}$ for which $\varepsilon_{\mathbf{a}} = O$ must be an equilibrium of the communication stage, followed by $s^*_1 = s^*_2 = s^*_{\mathbf{0}}$ in the competition stage. In such SPE, communication strategies are thus chosen such that no agent confusion results, and by (A2.1) at least one such communication profile exists. For part (ii), the inequality in (A.2) is reversed by weak indecisiveness of the agents' preferences. Hence, by Lemma 1, any communication profile with $\varepsilon_{\mathbf{a}} \neq O$, followed by the symmetric equilibrium $s^*_{\mathbf{a}}$ would be strictly preferred by the contestants than the respective outcome under $\varepsilon_{\mathbf{a}} = O$. (A2.2) then assures that any possible SPE must involve agent confusion. ■

Proof of Theorem 2 Part (i): By Theorem 1, for this part of the proof it is without loss to assume that $\mathbf{0} \notin \mathcal{A}$, since even if it is available the communication profile $\mathbf{a} = \mathbf{0}$

will not be chosen in any SPE. Take any $\mathbf{a}, \mathbf{a}' \in \mathcal{A}$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. By Assumption 3, $\varepsilon_{\mathbf{a}'}$ has the same distribution as $\varepsilon_{\mathbf{a}} + \eta$, where $\eta \neq O$. Thus

$$\begin{aligned} g_{\mathbf{a}'}(0) &= \int_{-\delta}^{\delta} g_0(-e) d\Gamma_{\mathbf{a}'}(e) \\ &= E\left[g_0(\varepsilon_{\mathbf{a}'})\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}'}\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}} + \eta\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &< E\left[g_0\left(E[\varepsilon_{\mathbf{a}} + \eta|\varepsilon_{\mathbf{a}}]\right)\right] \\ &= E\left[g_0\left(\varepsilon_{\mathbf{a}}\right)\right] \\ &= \int_{-\delta}^{\delta} g_0(-e) d\Gamma_{\mathbf{a}}(e) \\ &= g_{\mathbf{a}}(0). \end{aligned} \tag{A.3}$$

The third line follows from the law of iterated expectations, the fourth because $\varepsilon_{\mathbf{a}'}$ and $\varepsilon_{\mathbf{a}} + \eta$ are equal in distribution, and the fifth line follows from Jensen's inequality because g_0 is strictly concave on $[-\delta, \delta] \supset supp(\varepsilon_{\mathbf{a}})$. Hence, $g_{\mathbf{a}}(0)$ achieves its minimum on $\mathcal{A}$ if and only if $\mathbf{a} = (\bar{a}, \bar{a})$. By Lemma 1, this also maximizes the payoffs of the contestants in the competition stage. Hence, $(\bar{a}, \bar{a})$ must be part of an SPE. Moreover, $(\bar{a}, \bar{a})$ is the only possible equilibrium outcome, because for any alternative $(a_1, a_2) \in \mathcal{A}$, a forward-looking contestant with $a_i < \bar{a}$ would always want to deviate to $\bar{a}$.

Part (ii): If g_0 is strictly convex, the inequality in (A.3) is reversed. By Lemma 1, any $\varepsilon_{\mathbf{a}'}$ which is MPS of $\varepsilon_{\mathbf{a}}$ therefore is payoff-dominated by $\varepsilon_{\mathbf{a}}$. Further, any $\varepsilon_{\mathbf{a}} \neq O$ trivially is an MPS of O. Thus, regardless of whether $\mathbf{0} \in \mathcal{A}$ or not, $\mathbf{a}^* = (\underline{a}, \underline{a})$ is the only possible equilibrium outcome. Indeed, setting $a_i = \underline{a}$ is a dominant action for each contestant i, because $\forall a_j \in A$, with any alternative $a_i > \underline{a}$ the resulting $\varepsilon_{(a_i, a_j)}$ is a MPS of $\varepsilon_{(\underline{a}, a_j)}$, which can only lead to a lower equilibrium payoffs in the competition stage. ■

Proof of Theorem 3 For every $\omega \geq 0$, let

$$g_\omega(0) \equiv \int_{-\omega}^{\omega} g_0(\varepsilon) d\Gamma_\omega,$$

where Γ_ω is the uniform distribution on $[-\omega, \omega]$:

$$\Gamma_\omega(x) = \begin{cases} 1 & \text{if } x > \omega, \\ \frac{x+\omega}{2\omega} & \text{if } x \in [-\omega, \omega], \\ 0 & \text{otherwise.} \end{cases}$$

By construction and the assumptions of the theorem, $\Gamma_{\mathbf{a}}(x) = \Gamma_{\omega_{\mathbf{a}}}(x)\ \forall x \in \mathbb{R}, \mathbf{a} \in \mathcal{A}$.

Consider first case (i). We start by showing that $g_\omega(0)$ is strictly decreasing in ω on $[0, +\infty)$. Since match values are indecisive on $supp(g_0)$ and $g_0(x) = 0\ \forall x \notin supp(g_0)$, we have $g_0(\varepsilon) < g_0(0)\ \forall \varepsilon \neq 0$. It then follows that, $\forall \omega > 0$,

$$g_\omega(0) = \int_{-\omega}^{\omega} g_0(\varepsilon)\Gamma_\omega < \int_{-\omega}^{\omega} g_0(0) d\Gamma_\omega = g_0(0).$$

Further, since $g_\omega(0)$ is differentiable with respect to ω for all $\omega > 0$, we have

$$\begin{aligned}
\frac{\partial g_\omega(0)}{\partial \omega} &= \frac{g_0(-\omega)}{2\omega} + \frac{g_0(\omega)}{2\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(-\varepsilon) d\varepsilon \\
&= \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\varepsilon) d\varepsilon \\
&< \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\omega) d\varepsilon \\
&= \frac{g_0(\omega)}{\omega} - \frac{g_0(\omega)}{\omega} \\
&= 0,
\end{aligned}$$

where the inequality follows that match values are indecisive on $supp(g_0)$, g_0 is zero-symmetric, and $g_0(x) = 0\ \forall x \notin supp(g_0)$. By Lemma 1, there exists a unique symmetric equilibrium in which both contestants choose the effort $s^*_{\mathbf{a}} = z^{-1}(g_{\omega_{\mathbf{a}}}(0))$ following every $\mathbf{a} \in \mathcal{A}$ in the effort stage . Since $g_\omega(0)$ is strictly decreasing in ω, $g_{\omega_{\mathbf{a}}}(0)$ is minimized at $\omega_{\mathbf{a}} = \bar{\omega}$. Therefore, the equilibrium payoff is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$ which, as a consequence of Assumption 3, implies that the unique SPE features maximal confusion.

Next, consider case (ii). We first prove the following two lemmas.

Lemma A1 *If $supp(g_0)$ is bounded, then* $\lim_{\omega \to +\infty} g_\omega(0) = 0$.

Proof. Since $supp(g_0)$ is bounded, we must have $supp(g_0) \subset [-\omega, \omega]$ for sufficiently large ω. As a result,

$$\lim_{\omega \to +\infty} g_\omega(0) = \lim_{\omega \to +\infty} \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = \lim_{\omega \to +\infty} \int_{supp(g_0)} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = 0. \qquad \square$$

Lemma A2 *If $supp(g_0)$ is bounded, then $g_\omega(0)$ is strictly decreasing in ω for all $\omega >$* $\sup supp(g_0)$

Proof. Since $supp(g_0)$ is bounded, we must have $\sup supp(g_0) < +\infty$, and $g_0(\omega) = 0$ for all $\omega > \sup supp(g_0)$. It then follows that, for every $\omega > \sup supp(g_0)$,

$$\frac{\partial g_\omega(0)}{\partial \omega} = \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon = -\int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon < 0. \qquad \square$$

Now consider any match value distribution that is polarized on its support $supp(g_0)$. By definition, g_0 is strictly decreasing on $(\inf supp(g_0), 0]$ and is strictly increasing on $[0, \sup supp(g_0))$. This implies that the support of g_0 must be bounded, as otherwise we would have

$$\int_{supp(g_0)} g_0(x)dx = \int_{-\infty}^{+\infty} g_0(x)dx \geq \int_{-\infty}^{+\infty} g_0(0)dx = +\infty,$$

contradicting the definition of g_0 as a density function. Applying Lemmas A1 and A2, we can conclude that $\lim_{\omega \to +\infty} g_\omega(0) = 0$ and that $g_\omega(0)$ is strictly decreasing in ω on $(supp(g_0), +\infty)$. Hence, given $g_0(0) > 0$ there must exist $\hat{\omega} > 0$, such that if $\omega \geq \hat{\omega}$, then $g_\omega(0) \leq g_0(0)$ and it is further decreasing as ω increases. Therefore, if $\bar{\omega}$ is sufficiently large, the subgame equilibrium payoff is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$ which, by Assumption 3, implies that the unique SPE then features maximal confusion. ■

A.2 Price Competition: Proofs and Additional Results

Proof of Proposition 1 First, we argue that the log-concavity assumptions in (i) and (iii) imply that each firm i's expected payoff $\Pi_i^{\mathbf{a}}(p_i, p_j) = p_i G_{\mathbf{a}}(p_j - p_i)$ is strictly quasi-concave in p_i, $\forall \mathbf{a} \in \mathcal{A}$ and $p_j \geq 0$. To see this, first note that $\forall \mathbf{a} \in \mathcal{A}$, the distribution function $G_{\mathbf{a}}$, which is defined by

$$G_{\mathbf{a}}(x) = \int_{supp(\varepsilon_{\mathbf{a}})} G_0(x - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon), \forall x \in \mathbb{R},$$

is log-concave on $supp(g_{\mathbf{a}})$. This is trivial if $\varepsilon_{\mathbf{a}} = O$, since in this case we have $G_{\mathbf{a}} = G_0$, and G_0 is log-concave on $supp(g_0)$ as condition (i) assumes. If $\varepsilon_{\mathbf{a}} \neq O$, then by (iii) it has a log-concave density $\gamma_{\mathbf{a}}$, and we thus have $G_{\mathbf{a}}(x) = \int_{supp(\varepsilon_{\mathbf{a}})} G_0(x - \varepsilon)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon, \forall x \in \mathbb{R}$. Then, we can again conclude that $G_{\mathbf{a}}$ is log-concave on $supp(g_{\mathbf{a}})$, because the convolution of log-concave functions is also log-concave.[47] Further, since the function $f(p) = p$ is strictly log-concave on $[0, +\infty)$, for all $p_j \geq 0$ the profit function $\Pi_i^{\mathbf{a}}(p_i, p_j)$ is strictly log-concave (and hence strictly quasi-concave) in p_i on $\left[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\right]$, where, for every $p_2 \geq 0$, we define

$$\underline{p}(p_2) \equiv \sup \{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\}, \text{ and } \bar{p}(p_2) \equiv \inf \{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\},$$

and similarly, for every $p_1 > 0$,

$$\underline{p}(p_1) \equiv \sup \{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\}, \text{ and } \bar{p}(p_1) \equiv \inf \{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\}.$$

[47] For an overview of the properties of log-concave distributions, see Bagnoli and Bergstrom (2005).

Since $\Pi_i^{\mathbf{a}}(p_i, p_j) = p_i$ is strictly increasing if $p_i \leq \max\{\underline{p}(p_j), 0\}$, and $\Pi_i^{\mathbf{a}}(p_i, p_j) = 0$ $\forall p_i \geq \bar{p}(p_j)$, we can conclude that $\forall p_j \geq 0$, $\Pi_i^{\mathbf{a}}(p_i, p_j)$ is strictly quasic-concave in p_i on the entire domain $[0, +\infty)$.

Next, we prove that (A1.2) is satisfied. Since in the current setting we have

$$z(s) = -\frac{1}{2s} \;\forall s = -p \leq 0,$$

it suffices to show that $g_{\mathbf{a}}(0) > 0$ $\forall \mathbf{a} \in \mathcal{A}$. If $\varepsilon_{\mathbf{a}} = O$, then $g_{\mathbf{a}}(0) = g_0(0) > 0$ as condition (ii) directly implies. If $\varepsilon_{\mathbf{a}} \neq O$, then again by (iii) it has a density $\gamma_{\mathbf{a}}$ which is log-concave on its support. By definition, it follows that $supp(\gamma_{\mathbf{a}})$ must be a convex set, i.e., an interval on $\mathbb{R}$. It then follows that $0 \in supp(\gamma_{\mathbf{a}})$, for if $0 \notin supp(\gamma_{\mathbf{a}})$ then $supp(\gamma_{\mathbf{a}})$ must reside entirely either in $(-\infty, 0)$ or in $(0, \infty)$, contradicting the symmetry of $\Gamma_{\mathbf{a}}$ at zero. The zero-symmetry of $\Gamma_{\mathbf{a}}$ further assures that $supp(\gamma_{\mathbf{a}})$ is an interval symmetric around zero. By (ii), g_0 is continuous and strictly positive at the point $x = 0$. Hence, there must exist $\delta > 0$ such that $g_0(x) > 0$ $\forall x \in [-\delta, \delta]$. In particular, we can choose this $\delta > 0$ so small to assure that $[-\delta, \delta] \subset supp(\gamma_{\mathbf{a}})$. Accordingly, we have

$$g_{\mathbf{a}}(0) = \int_{-\infty}^{+\infty} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon \geq \int_{-\delta}^{\delta} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon > 0.$$

The existence of a unique symmetric pure-strategy equilibrium in every subgame then immediately follows from Lemma 1. ■

Proof of Proposition 2 (i) Because $g_0(\cdot, r)$ is strongly indecisive on its support, the only SPE involves maximal confusion by Theorem 2. Let $L(\mathbf{a}, r)$ denote the welfare loss (4) for a given communication profile $\mathbf{a}$ and regulation r. Then

$$L(\mathbf{a}, r) = \frac{2}{2G_0(r) - 1} \int_0^{\varepsilon} x\Gamma_{\mathbf{a}}(-x)g_0(x)dx > 2\int_0^{\varepsilon} x\Gamma_{\mathbf{a}}(-x)g_0(x)dx = L(\mathbf{a}, \lambda),$$

showing that the welfare loss increases for $r < \lambda$. For (ii), note that the unique SPE involves full education also under the regulation as long as $supp\,\gamma_{\mathbf{a}} \subset supp\,g_0(\cdot, r)$, $\forall \mathbf{a} \in \mathcal{A}$, by Theorem 2, meaning that $L(\mathbf{0}, r) = L(\mathbf{0}, \lambda) = 0$. If the regulation leads to a sufficiently tight support of $g_0(\cdot, r)$, then Theorem 3 shows that maximal confusion becomes the unique SPE, at least if $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ is given by a family of uniform distributions. ■

Proposition A1 *Consider the model with competition on the line. Suppose that $\alpha \leq \hat{\alpha} \equiv (6 - 3\sqrt{3})/4\lambda^3$, (A1.3) holds, and $supp(\gamma_{\mathbf{a}}) \subset [-4\lambda^2, 4\lambda^2]$ $\forall \mathbf{a} \in \mathcal{A}$.*

(i) If Assumption 2 also holds, then there exists (does not exist) an SPE without consumer confusion if $\alpha > 0$ ($\alpha < 0$).

(ii) If Assumption 3 also holds and $\alpha \neq 0$, then there exists a unique SPE. This SPE features minimal (maximal) consumer confusion if $\alpha > 0$ ($\alpha < 0$).

Proof of Proposition A1 We start by arguing that if $\alpha \leq \hat{\alpha}$, then (A1.1) holds, that is, G_0 is log-concave on its support. To show this, we will make use of Lemma A3 below, which states that H is log-concave on its support if $\alpha \leq \hat{\alpha}$.

Lemma A3 *If $\alpha \leq \hat{\alpha}$, then H is log-concave on $[-\lambda, \lambda]$.*

PROOF. If $\alpha \leq 0$, the statement of the lemma immediately follows because in these cases the density function h is log-concave, which is sufficient (but not necessary) for the distribution function H to be log-concave on $[-\lambda, \lambda]$.

Suppose now that $\alpha \in (0, \hat{\alpha}]$. We will show that H remains to be log-concave despite that the density function h is actually log-convex. By continuity, it suffices to show that H is log-concave on the open interval $(-\lambda, \lambda)$. Since h is differentiable on $(-\lambda, \lambda)$, H is log-concave on this interval if and only if for all $\theta \in (-\lambda, \lambda)$,

$$\begin{aligned} & h'(\theta)H(\theta) - (h(\theta))^2 \leq 0 \\ \iff\ & 2\alpha\theta \cdot \left(\frac{1}{3}\alpha\theta^3 + \beta\theta + \frac{1}{2}\right) \leq \left(\alpha\theta^2 + \beta\right)^2 \\ \iff\ & \frac{2}{3}\alpha^2\theta^4 + 2\alpha\beta\theta^2 + \alpha\theta \leq \alpha^2\theta^4 + \beta^2 + 2\alpha\beta\theta^2 \\ \iff\ & -\frac{1}{3}\alpha^2\theta^4 + \alpha\theta \leq \left(\frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)^2. \end{aligned} \tag{B.1}$$

Given that $\alpha > 0$, the inequality obviously holds when $\theta \leq 0$. But given that $\theta > 0$, the LHS of (B.1) is increasing in θ on $[0, \lambda]$, since

$$\left(-\frac{1}{3}\alpha^2\theta^4 + \alpha\theta\right)' = -\frac{4}{3}\alpha^2\theta^3 + \alpha \geq -\frac{4}{3}\alpha^2\lambda^3 + \alpha > 0,$$

where the last inequality holds as $\hat{\alpha} < 3/(4\lambda^3)$. Hence, inequality (B.1) holds for all $\theta \in (-\lambda, \lambda)$ if and only if

$$\begin{aligned} & -\frac{1}{3}\alpha^2\lambda^4 + \alpha\lambda \leq \frac{1}{4\lambda^2} + \frac{\alpha^2\lambda^4}{9} - \frac{\alpha\lambda}{3} \\ \iff\ & -\frac{4}{9}\alpha^2\lambda^4 + \frac{4}{3}\alpha\lambda \leq \frac{1}{4\lambda^2} \\ \iff\ & -\alpha^2\lambda^4 + 3\alpha\lambda \leq \frac{9}{16\lambda^2} \\ \iff\ & \left(\alpha\lambda^2 - \frac{3}{2\lambda}\right)^2 \geq \frac{27}{16\lambda^2}. \end{aligned} \tag{B.2}$$

Since $\lambda > 0$ and $\hat{\alpha} \leq 3/(2\lambda^3)$, (B.2) is further equivalent to $\alpha \leq \frac{6-3\sqrt{3}}{4\lambda^3} = \hat{\alpha}$. □

Since $G_0(x) = \Pr(4\lambda\theta \leq x) = \Pr\left(\theta \leq \frac{x}{4\lambda}\right) = H\left(\frac{x}{4\lambda}\right)$ $\forall x \in \mathbb{R}$, and the function $t(x) = x/(4\lambda)$ is increasing and concave in x, G_0 is log-concave on $[4\lambda a, 4\lambda b]$ if H is log-concave on $[a, b] \subset \mathbb{R}$. Hence, by Lemma A3, G_0 is log-concave on $[-4\lambda^2, 4\lambda^2]$ provided that $\alpha \leq \hat{\alpha}$.

It is straightforward to check that all other conditions in Assumption 1 are satisfied. Hence, by Lemma 1, we can conclude that there exists a unique pure-strategy equilibrium in every pricing subgame, where each firm chooses the same price $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$ $\forall \mathbf{a} \in \mathcal{A}$. The statements of the proposition then immediately follow from Theorems 1 and 2. ■

A.3 Political Competition: Formal Analysis

The following proposition presents sufficient conditions for the existence of a symmetric second-stage equilibrium in efforts in the model of Section 5.

Proposition A2 *Consider the voting application and suppose that the following conditions are satisfied: (i) $\exists\, M \geq 0$, such that for almost all $x \in supp(g_0)$, $g_0(x) > 0$, $g_0'(x)$ exists, and $\left|\frac{g_0'(x)}{g_0(x)}\right| \leq M$; (ii) If $\varepsilon_{\mathbf{a}} \neq O$, it has a density $\gamma_{\mathbf{a}}$; (iii) $\forall \mathbf{a} \in \mathcal{A}$, $C'^{-1}(g_{\mathbf{a}}(0)) < \max\{k, 1/k\}$. Then there exists η^* such that if $\eta \geq \eta^*$, every subgame in the competition stage has a unique symmetric pure-strategy equilibrium, where both candidates choose the effort $s^*_{\mathbf{a}} = C'^{-1}(g_{\mathbf{a}}(0))$ $\forall \mathbf{a} \in \mathcal{A}$.*

Proof of Proposition A2 First, note that since

$$\Pi_i^{\mathbf{a}}\left(\max\{k, 1/k\}, s_j\right) \leq 1 - k(\max\{k, 1/k\})^\eta \leq 0,$$

no candidate will ever choose an effort level higher than $\max\{k, 1/k\}$. We now argue that if η is sufficiently large, then $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in $s_i \in [0, \max\{k, 1/k\}]$, $\forall s_j \in \mathbb{R}_+$, $\mathbf{a} \in \mathcal{A}$ and $i = 1, 2$. This will be proved by using the well-known fact that a twice differentiable real-valued function f, defined on some open interval $X \subset \mathbb{R}$, is strictly quasi-concave if $f'(x) = 0$ implies $f''(x) < 0$ for any $x \in X$.

Take any $\mathbf{a} \in \mathcal{A}$ and $s_2 \in \mathbb{R}_+$. Note that by conditions (i) and (ii),

$$g_{\mathbf{a}}(s_1 - s_2) = \int_{supp(\varepsilon_{\mathbf{a}})} g_0(s_1 - s_2 - \varepsilon)d\Gamma_{\mathbf{a}}(\varepsilon)$$

is differentiable. Now suppose that $\frac{\partial \Pi_1^{\mathbf{a}}(s_1, s_2)}{\partial s_1} = 0$, for some $s_1 \in (0, \max\{k, 1/k\})$, i.e.,

$$g_{\mathbf{a}}(s_1 - s_2) - k\eta s_1^{\eta - 1} = 0. \tag{C.1}$$

Then, we have

$$\frac{\partial^2 \Pi_1^{\mathbf{a}}(s_1, s_2)}{\partial s_1^2} = g'_{\mathbf{a}}(s_1 - s_2) - k\eta(\eta - 1)s_1^{\eta-2}$$
$$= g'_{\mathbf{a}}(s_1 - s_2) - (\eta - 1)g_{\mathbf{a}}(s_1 - s_2)s_1^{-1}.$$

Since (C.1) also implies that $g_{\mathbf{a}}(s_1 - s_2) > 0$, we further have $\frac{\partial^2 \Pi_1^{\mathbf{a}}(s_1,s_2)}{\partial s_1^2} < 0$ if and only if

$$s_1 \cdot \frac{g'_{\mathbf{a}}(s_1 - s_2)}{g_{\mathbf{a}}(s_1 - s_2)} < \eta - 1. \tag{C.2}$$

By condition (i) and $s < \max\{k, 1/k\}$, the LHS of (C.2) is bounded from above by $\max\{kM, M/k\}$. Thus the inequality (C.2) must hold whenever η is sufficiently large, implying that $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in s_i on the open interval $(0, \max\{k, 1/k\})$. By continuity, it is also strictly quasi-concave on $[0, \max\{k, 1/k\}]$.

Next, from the first-order conditions, we obtain a unique candidate for a symmetric pure-strategy equilibrium, where $s_1 = s_2 = s_{\mathbf{a}}^* = C'^{-1}(g_{\mathbf{a}}(0))$. By condition (iii), $s_{\mathbf{a}}^* < \max\{k, 1/k\}$. Hence, $(s_1, s_2) = (s_{\mathbf{a}}^*, s_{\mathbf{a}}^*)$ is indeed an equilibrium when $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in s_i on $[0, \max\{k, 1/k\}]$. In particular, this is the case whenever η is sufficiently large. ■

S Supplementary Material

This appendix contains a number of extensions of the main analysis. In Section S.1, we elaborate on the foundations for the perceived match advantages (1). In Section S.2, we present the two-sided single crossing ordering of the perception errors, and show that it implies similar results as the MPS ordering. Section S.3 formally analyzes the model with outside option from Section 4.4, and Section S.4 elaborates on the size of the welfare loss due to confusion in the context of price competition. In Section S.5, we study confusion about needs in case of a Salop model, as discussed informally at the end of Section 3.4. Finally, Sections S.6 and S.7 consider the cases of ex-ante and ex-interim asymmetric contestants, respectively.

S.1 Decision Noise: Examples and Foundations

In this section, we provide several different foundations for our framework based on insights from psychology, cognitive science and marketing. We will illustrate some leading examples in the following general random utility setting. Suppose that the perceived match values of the agents for a contestant $i \in \{1, 2\}$ are distributed according to $\tilde{v}_i = v_i + \varepsilon_i(\mathbf{a})$, where v_i is the distribution of true valuations for contestant i over the agent population, $\varepsilon_i(\mathbf{a})$ is a random variable, and $\mathbf{a} \in \mathcal{A}$ is a communication profile. Then, the distribution of the perceived *match advantages* of contestant $i = 2$ is given by (1) with $v_\Delta \equiv v_2 - v_1$ and $\varepsilon_\mathbf{a} \equiv \varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$.[48] For easy reference, we let $\hat{\mathcal{H}}$ denote the set of all random variables with a 0-symmetric and log-concave density function. As before, O denotes a constant random variable that yields $x = 0$ with probability one, and $\mathcal{H} \equiv \hat{\mathcal{H}} \cup O$.

Product Complexity and Information Overload The intuitive arguments from Section 4.1 about *overload confusion*, e.g., due to product complexity in terms of lengthy contracts, numerous marketing messages or product attributes can be easily expressed with a random utility model. Suppose that each contestant can choose a number of "features" to implement in the marketing process. Each feature involves imperfect mental information processing due to cognitive capacity limitations, which results in a noisy

[48] This decomposition of $\varepsilon_\mathbf{a}$ is suitable for some but not all examples we develop below. In this respect, it is helpful to note that expressing the effects of communication strategies directly in terms of $\varepsilon_\mathbf{a}$, rather than via $\varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$, is without loss of generality in the sense that any given zero-symmetric random variable can always be decomposed as a sum of two zero-symmetric random variables, and vice-versa (Rubin and Sellke, 1986).

attribution of the product's valuation, where some agents overestimate the value of a given feature to them, whereas others underestimate it.[49] Then, confusion due to product complexity can be captured in a simple way by assuming that each implemented feature has an i.i.d. effect on an agent's evaluation of the product, determined by a random variable $Z_s \in \hat{\mathcal{H}}$. If contestant j implements n_j such features, the perception noise is determined as

$$\varepsilon_j(n_j) = \sum\nolimits_{s=0}^{n_j} Z_s, \tag{S.1}$$

where $\varepsilon_j(n_j) \in \mathcal{H}$ if $n_j > 0$, and $\varepsilon_j(0)$ is degenerate.[50] Then, the number of features implemented ($n_j \geq 0$) corresponds to the communication strategy of contestant j. The broad meaning of (S.1) is that more features result in more unsystematic perception errors across agents.[51] If $\varepsilon_1, \varepsilon_2$ are independent and determined by (S.1), then $\varepsilon(n_1, n_2) \equiv \varepsilon_2(n_2) - \varepsilon_1(n_1) \in \mathcal{H}$, and $\varepsilon(n_1, n_2) \in \hat{\mathcal{H}}$ iff $n_1 + n_2 > 0$. Because $\varepsilon(n_1', n_2')$ is a mean-preserving spread of $\varepsilon(n_1, n_2)$ whenever $n_1' + n_2' > n_1 + n_2$, the type of obfuscation process captured by (S.1) has the additional feature that the resulting distributions ε can be ordered in the sense of MPS.[52]

A possible limitation of the above model is that in reality different features may affect consumer perception differentially, possibly with dependencies across features. We can adapt the model to encompass the case of possible dependencies among the implemented features. Formally, let $Z = \{Z_1, ...Z_K\}$ denote a set of random variables, where the random vector $(Z_1, ..., Z_K)$ has a joint density function $f(z_1, ..., z_K)$ that is coordinate-wise symmetric, i.e.

$$f(z_1, ..., z_k, ..., z_K) = f(z_1, ..., -z_k, ..., z_K), \quad \forall (z_1, ..., z_K) \in supp(f), \forall k = 1, ..., K.$$

A communication strategy corresponds to a selection $M_j \subset Z$ of features implemented by firm j, which affects the product valuation according to

$$\varepsilon_j(M_j) = \sum_{k \in M_j} Z_k. \tag{S.2}$$

[49]Unsystematic evaluation errors resulting from sufficiently rich information stimuli due to cognitive limitations are a well established fact for human behavior at least since Miller (1956).

[50]The former follows because an independent sum of random variables with log-concave and symmetric densities again produces a random variable with symmetric and log-concave density.

[51]Model (S.1) could be further modified to capture that confusion occurs only once a sufficient number of features have been implemented (i.e., information is sufficiently rich). This could be achieved by introducing a threshold value $\bar{n} \in \mathbb{N}$ such that $\varepsilon_j(n_j) = \sum_{s=0}^{n_j} Z_s$ if $n_j > \bar{n}$ and $\varepsilon_j(n_j) = O$ otherwise.

[52]Note that (S.1) also allows for a non-cognitive explanation, where the "features" are perceived without error but of heterogeneous valuations to the agents, but some agents like certain features that others dislike in a way that the effects cancel out across the agent population.

The set of marketing strategies A now corresponds to all possible selections, i.e., any marketing activity a_j belongs to the power set of Z, where $a_j = \emptyset$ means that no feature is chosen and ε_j is degenerate. If the density of Z is log-concave and coordinate-wise symmetric, so is the density of any non-empty selection M_j, meaning that $\varepsilon_j(M_j)$ is symmetric and log-concave as well. Assuming independence between $\varepsilon_1(M_1)$ and $\varepsilon_2(M_2)$ implies that $\varepsilon(M_1, M_2) = \varepsilon_2(M_2) - \varepsilon_1(M_1) \in \mathcal{H}$, and $\varepsilon(M_1, M_2) \in \hat{\mathcal{H}}$ if and only if $M_j \neq \emptyset$ for some $j \in \{1, 2\}$. Finally, the current set of marketing technology can be partially ordered according to MPS. In particular, $\varepsilon(M_1', M_2')$ is a mean-preserving spread of $\varepsilon(M_1, M_2)$ whenever $M_j \subsetneq M_j'$, $j = 1, 2$.

Limited comparability and framing Marketing sometimes emphasizes that product complexity should be viewed as a synthetic phenomenon of *all* marketing messages interacting with each other, leading to a market level or "category complexity" (Mützel and Kilian, 2016). Likewise, "ambiguity confusion" (Walsh et al., 2007) occurs if different information about various products leave consumers with many possible interpretations. In case of product labels, evidence indicates that not only the sheer number of labels can be a source of confusion, but also their contents *relative to each other and across brands*. For example, a two-year study by the British Food Advisory Committee concluded that labels like "fresh", "original" or "pure", which are frequently used to describe food products, result in consumer confusion because they seem similar but still can mean quite different things.[53] Likewise, the "Nutrition facts label" introduced in the 1980's in the US, originally intended to allow consumers to make more informed food choices, has apparently turned out to be a source of consumer confusion.[54] In sum, the chosen marketing activities (labels, ads, design aspects,...) jointly affect the individual evaluations of each product in the sense that $\varepsilon_j = \varepsilon_j(a_1, a_2)$, $j = 1, 2$. For instance, if one food brand uses the label "original" while another brand uses "authentic", the comparison of the two labels by consumers may cause confusion, which could have been avoided if both firms had coordinated on the same label.

A related point is made by Piccione and Spiegler (2012) and Chioveanu and Zhou

[53] To quote the principal policy advisor at the British Consumer Association: "Labels are all too often more of a marketing gimmick than a way of providing meaningful information to help consumers make." See the article "Report reveals food label confusion" published on DailyMail.

[54] In a recent online article ("The Nutrition Facts: Food Label Confusion", July 2016), GlobalVision, a company specializing in packaging and labeling, stressed that "unless you work in the industry, it is very difficult to decipher food labels accurately." Confusion in the comparison of products due to food labels has also been experimentally verified by Leek et al. (2015).

(2013) in a homogeneous goods model, where the mutual choices of "frames", specifically ways to present the price of a product, determines whether or not a consumer can *compare* the prices of both products. If a comparison can be made, the consumer chooses the cheaper product; otherwise the consumer picks at random. Our framework can accommodate a notion of limited comparability by supposing that the chosen communication profile determines the frame by which a consumer compares the products, and as such the extent to which an adequate comparison can be made.[55] To formalize this idea, suppose that each communication profile $(a_1, a_2) \in \mathcal{A}$ induces a frame, i.e., a random variable $\varepsilon(a_1, a_2) \in \mathcal{H}$. The frame captures the range and distribution of the possible errors an individual consumer can make in her product comparison. For example, a frame that facilitates a comparison is such that $\varepsilon(a_1, a_2)$ has much of its probability mass around zero. Accordingly, with such a frame the consumers are more likely to make only small comparative mistakes.

Spiegler (2014) also considers a homogeneous-goods model, where two firms simultaneously choose their marketing messages, which jointly determine the *distribution* of the frames a single consumer could adopt. The adopted frame, in turn, determines the choice probabilities of each firm. It is possible to implement this idea in our framework as well. Let $F \subset \mathbb{R}$ be the set of possible frames, where each $f \in F$ corresponds to a deterministic way how a consumer draws a product comparison. The actual frame adopted by the consumer after being exposed to a communication profile $\mathbf{a}$ is unknown to the firms, while they know the probability distribution $\varepsilon_{\mathbf{a}}$ over the possible frames induced by $\mathbf{a}$. It is easy to see that under the respective assumptions on F and $\varepsilon_{\mathbf{a}}$, $\mathbf{a} \in \mathcal{A}$, an error structure that is consistent with our framework results.[56]

Product and preference uncertainty Another source of confusion in the evaluation of products is the case of *inadequate* product information. A lack of communication by the firm may imply that consumers are forced to form conjectures about the value of a product and its attributes. Suppose that each firm can decide how much qualified information to display to consumer regarding their product. The less information is provided, the more consumers need to guess the relevant valuation of the product for

[55] Given the additive structure of (1) we can interpret the chosen frames as affecting net valuations or the price percepts.

[56] As a more technical remark, a property called Weighted Regularity (WR) plays a critical role for the equilibrium analysis in the homogeneous-good model of Piccione and Spiegler (2012) and Spiegler (2014). It is quite easy to see that WR may or may not be satisfied under the assumptions we imposed on our model, meaning that WR is not critical for our analysis.

them. To illustrate, let $A = [0, 1]$ and suppose that consumer guessing for the value of firm j's product is depicted by a random variable $\varepsilon_j(a_j)$ with a uniform distribution on $[-a_j, a_j]$. Then, $a_j = 0$ corresponds to the case that firm j provides all relevant information, and less information (higher a_j) leads to more noisy product perception.

This example highlights a connection to the literature on informative advertising (Bagwell, 2007). In that literature, information typically is of a binary nature, where a consumer can either be perfectly informed about a product with all its relevant attributes, or entirely uninformed about its existence. In our case, consumers always know that both products exist, but require information to judge the extent to which the product matches their needs. Our model then asks when firms may deliberately abstain from providing consumers with sufficient information to annihilate product uncertainty from a market.

Further, if consumers have a general understanding of the market structure, this could restrict the type of inference they draw during their product evaluations. As an illustration, suppose that consumers understand that they are located on a Hotelling line with firms sitting at the opposed edges, so they are aware of the negative correlation between true valuations. The communication profiles therefore can only affect perception in a way that preserves this correlation. Thus, the random variables $\varepsilon_1, \varepsilon_2$ are perfectly negatively correlated, such that $\varepsilon_1 = -\varepsilon_2$, where $\varepsilon_1 \in \mathcal{H}$. This is essentially a model where marketing has the effect of randomly moving each consumer around her true location on the Hotelling line. The interpretation is that marketing may either aid or obstruct consumers from properly orienting themselves in a market whose structure they principally are capable of understanding.

Spurious correlations In the last example, the negative correlation of valuation shocks originated from a basic understanding of consumers about the market structure. However, it is also conceivable that consumers form spurious correlations between their evaluations of the product, independent of the true market structure. As an illustration, suppose that a firm chooses to present its product in a simplistic way, while its competitor emphasizes, in detail, how many features its product has. Some consumers may come to believe that the second product is better as it seems to offer many functionalities, while other consumers value positively the apparent simplicity of the first product. However, in such a situation consumers with a better impression about the second product may also tend to have a worse view of the first product as being too simple (and vice-versa). For example, digital cameras for amateurs sometimes even feature more pixels, typically heavily advertised, than professional cameras, to give the impression that the camera is able to shoot particularly sharp photos. Likewise, if a firm advertises in superlatives, such as Duracel claiming to have the "longest-living batteries", this may lead some consumers to believe that other batteries must be worse. These examples suggest a spurious correlation of the following type: If product A is so good, then B must be really bad.

Any such spurious correlation can be captured by the joint distribution of the random vector $(\varepsilon_1, \varepsilon_2)$. An interesting question to ask is how much correlation the firms desire, if they can influence it through the communication profiles. For example, if for any $(a_1, a_2) \in A$, the random vector $(\varepsilon_1(a_1, a_2), \varepsilon_2(a_1, a_2))$ is jointly normal,

$$(\varepsilon_1, \varepsilon_2) \sim N\left(0, \begin{pmatrix} \sigma_{11} & \sigma_{12}(a_1, a_2) \\ \sigma_{12}(a_1, a_2) & \sigma_{22} \end{pmatrix}\right),$$

or a 0-symmetric truncation thereof, then $\varepsilon = \varepsilon_2 - \varepsilon_1$ also is normal with a variance that depends on the correlation between $\varepsilon_1, \varepsilon_2$. An interesting insight is that a more negative correlation leads to a greater dispersion of opinions as measured by the variance.[57] In this sense, the firms' desire for obfuscation leads to an increasingly polarized evaluation culture, where consumers judge the products in a way that a better impression of product A also implies a worse impression of product B.

S.2 Two-Sided Single Crossing Ordering

In this section, we discuss the two-sided single crossing (TSC) ordering of the distributions $\{\Gamma_{\mathbf{a}}\}$. Formally, let Γ, Γ' be two zero-symmetric distribution functions with supports $[-\omega, \omega]$ and $[-\omega', \omega']$, respectively. We say that Γ' is *more dispersed* than Γ in the sense of TSC, denoted by $\Gamma' \succ_{TSC} \Gamma$, if either (i) Γ' has a density function γ' while Γ is degenerate at zero, or (ii) Γ also has a density function γ, $\omega' \geq \omega$ and $\forall e, e' \in [0, \omega')$ with $e' > e$,

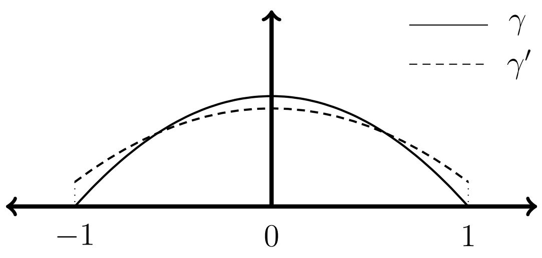
$$\gamma'(e) - \gamma(e) \geq 0 \Longrightarrow \gamma'(e') - \gamma(e') > 0. \tag{S.3}$$

In words, (S.3) requires that the two densities intersect at most once in $(-\omega', 0]$ and $[0, \omega')$, respectively; see Figure S.1.

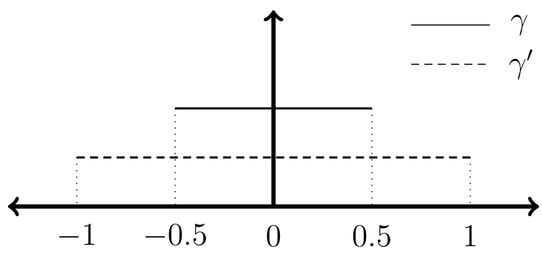
Assumption S1 (TSC ordering) *$A \subset \mathbb{R}_+$ is compact, and $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = \mathbf{0}$. Moreover, $\forall \mathbf{a}, \mathbf{a}' \in \mathcal{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'} \succ_{TSC} \Gamma_{\mathbf{a}}$.*

The following theorem shows that we obtain the same type of result as with the MPS ordering (Theorem 2).

[57] This type of reasoning generalizes beyond the normal case if the family of densities resulted by the marketing strategies in $\mathcal{A}$ can be ordered alone by their variance.



(a) $\Gamma' \succ_{TSC} \Gamma$, $supp(\gamma') = supp(\gamma)$.



(b) $\Gamma' \succ_{TSC} \Gamma$, $supp(\gamma') \supsetneq supp(\gamma)$.

Figure S.1: Examples of TSC orderings

Theorem S1 *Suppose that Assumptions 1 and S1 hold.*

(i) If there exists $\delta > 0$ such that $supp(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are δ-indecisive, then there exists a unique SPE, and confusion is maximal.

(ii) If there exists $\delta > 0$ such that $supp(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are δ-polarized, then there exists a unique SPE, and confusion is minimal.

Proof: Consider part (i). Similar to the proof of Theorem 2, for this part of the proof it is without loss to assume that $\mathbf{0} \notin \mathcal{A}$. Take any $\mathbf{a}, \mathbf{a}'$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. Let $supp(\varepsilon_{\mathbf{a}}) = [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $supp(\varepsilon_{\mathbf{a}'}) = [-\omega_{\mathbf{a}'}, \omega_{\mathbf{a}'}]$. Assumption S1 implies that there exists a unique $\hat{e} \in (0, \omega_{\mathbf{a}'})$, such that $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) < 0$ $\forall e \in [0, \hat{e})$, and $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) > 0$ $\forall e \in (\hat{e}, \omega_{\mathbf{a}'}]$. Since g_0 is strictly decreasing on $[0, \omega_{\mathbf{a}'}] \subset [0, \delta]$, we further have

$$\begin{aligned} \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e)\left(\gamma_{a'}(e) - \gamma_a(e)\right) de &< \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(\hat{e})\left(\gamma_{a'}(e) - \gamma_a(e)\right) de \\ &= \int_0^{\hat{e}} g_0(\hat{e})(\gamma_a(e) - \gamma_{a'}(e)) de \\ &< \int_0^{\hat{e}} g_0(e)(\gamma_a(e) - \gamma_{a'}(e)) de, \end{aligned} \tag{S.4}$$

where the equality makes use of the fact that, by symmetry and $\omega_{\mathbf{a}} \leq \omega_{\mathbf{a}'}$, we have

$$\frac{1}{2} = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}'}(e) de = \int_0^{\omega_{\mathbf{a}}} \gamma_{\mathbf{a}}(e) de = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}}(e) de.$$

Exploiting again the symmetry of g_0, $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{a}'}$, we further have

$$\begin{aligned}
&\int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\gamma_{\mathbf{a}'}(e)de - \int_{-\omega_{\mathbf{a}}}^{\omega_{\mathbf{a}}} g_0(-e)\gamma_{\mathbf{a}}(e)de \\
=& \int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\
=& \, 2\int_{0}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\
=& \, 2\left[\int_{0}^{\hat{e}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] \\
=& \, 2\left[\int_{0}^{\hat{e}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] < 0,
\end{aligned}$$

where the last inequality follows from (S.4). We have thus shown that $g_{\mathbf{a}'}(0) < g_{\mathbf{a}}(0)$ for any feasible $\mathbf{a} \neq \mathbf{a}'$ with $\mathbf{a} \leq \mathbf{a}'$. Hence, if preferences are δ-indecisive, $g_{\mathbf{a}}(0)$ must be uniquely minimized at $\mathbf{a}^* = (\bar{a}, \bar{a})$. By arguments analogous to the case with MPS ordering (Theorem 2), we can conclude that there exists a unique SPE, and $\mathbf{a}^* = (\bar{a}, \bar{a})$ is the unique equilibrium outcome in the first stage. The proof for part (ii) is analogous, and thus omitted. ■

The MPS Theorem 2 and Theorem S1 cannot be ranked according to their generality. First, if the distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are ordered by the TSC criterion, they are also ordered by the MPS criterion, while the converse generally is false. Second, we only need to impose indecisive or polarized match values with the TSC ordering in Theorem S1, while we need their strong counterparts with the MPS ordering in Theorem 2.

S.3 The Role of Outside Options

In this section we analyze the SPE in the model with outside options from Section 4.4, using the more general formulation where perceived match values of a consumer k are $\tilde{v}_1^k = \frac{m}{2} + \frac{v^k}{2} + \frac{\varepsilon^k}{2}$, $\tilde{v}_2 = \frac{m}{2} - \frac{v^k}{2} - \frac{\varepsilon^k}{2}$, where $m > 0$ is an exogenous constant, and v^k is symmetrically distributed over $[-1, 1]$ with distribution G_0 and density g_0. The parameter $m > 0$ is relevant only for the decision whether to buy any good, where a larger value of m means that, ceteris paribus, a consumer is more likely to purchase a good. The example in the main text corresponds to the special case $m = 2$.

Assumption S2 *G_0 is log-concave on supp $g_0 = [-1, 1]$, g_0 is continuous on $[-1, 1]$ with $g_0(0) > 0$, and $\forall \mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}} \neq O$, $\varepsilon_{\mathbf{a}}$ has a density $\gamma_{\mathbf{a}}$ that is log-concave on supp$(\gamma_{\mathbf{a}})$.*

In the current setting, the distribution of v^k coincides with the distribution of the true match advantages. As a result, Assumption S2 implies Conditions (i)-(iii) of Proposition 1. These conditions assured the existence of a unique symmetric equilibrium in the pricing stage of the game without outside option for any given communication profile $\mathbf{a} \in \mathcal{A}$, and play a similar role here. We begin our analysis with a characterization of the symmetric equilibria in the pricing stage.

Proposition S1 *Suppose that Assumption S2 holds. In the above game with outside option, there exists a unique symmetric pure-strategy equilibrium in the pricing stage, and* $\forall \mathbf{a} \in \mathcal{A}$ *both firms choose the price*

$$p^*_{\mathbf{a}} = \begin{cases} \frac{1}{2g_{\mathbf{a}}(0)} & \text{if } g_{\mathbf{a}}(0) > \frac{1}{m}, \\ \frac{m}{2} & \text{if } g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right], \\ p^M_{\mathbf{a}} > \frac{m}{2} & \text{otherwise}, \end{cases} \tag{S.5}$$

where $p^M_{\mathbf{a}}$ *solves the monopoly problem* $\max_{p \geq 0} \; \Pi^M_{\mathbf{a}}(p) \equiv p\left(1 - G_{\mathbf{a}}(2p - m)\right)$. *The equilibrium demand of each firm is strictly less than* $1/2$ *if and only if* $g_{\mathbf{a}}(0) < \frac{1}{2m}$.

All proofs are at the end of this section. The competition resulting from the presence of sufficiently many perceptually indifferent consumers ($g_{\mathbf{a}}(0) > 1/m$) disciplines both firms to choose an equilibrium price for which the outside option is non-binding for every consumer.[58] For lower values of $g_{\mathbf{a}}(0)$, competition is less intense, yielding a strong temptation to increase prices. As long as $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$, both firms settle exactly at the price $p = m/2$ that just keeps every consumer in the market.[59] By contrast, for $g_{\mathbf{a}}(0) < \frac{1}{2m}$ firms increase prices even though some consumers exit the market.

In essence, the pricing pattern identified by Proposition S1 determines what type of communication strategies the firms choose in the first stage. Specifically, if $g_{\mathbf{a}}(0) \geq \frac{1}{m}$ $\forall \mathbf{a} \in \mathcal{A}$, the SPE identified by Theorems 1 and 2 apply given the respective assumptions on g_0. Further, if true preferences are strongly indecisive on $supp\, g_0 = [-1, 1]$, $g_{\mathbf{a}}(0) \geq \frac{1}{2m}$ $\forall \mathbf{a} \in \mathcal{A}$ and $\{\Gamma_{\mathbf{a}}\}$ verify an MPS order, maximal obfuscation is always an SPE outcome.[60] Most importantly, Proposition S1 suggests that the firms may desire to confuse on a massive scale so that they can then exploit some local monopoly power, even though many consumers choose to exit.

[58] See Armstrong and Zhou (2019) for a similar result in a different setup.

[59] Absent a binding outside option, both firms would increase their price above $m/2$.

[60] The only difference to the case without outside option is that uniqueness of equilibrium may fail, despite an MPS ordering. In particular, any $\mathbf{a} \in \mathcal{A}$ inducing a value $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$ is an SPE with second-stage price $p^*_{\mathbf{a}} = m/2$.

To confirm this idea, we now analyze the tractable case where $\Gamma_{\mathbf{a}}$ follows a uniform distribution with support $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\forall \mathbf{a} \in \mathcal{A}$, and $g_0(0) > \frac{1}{m}$. The latter assumption implies that without confusion the equilibrium price $p_0^* = \frac{1}{2g_0(0)}$ is such that the outside option is non-binding for all consumers. In addition, we assume that $m \geq 2$, which simplifies the presentation of results. We first apply Proposition S1 to derive the equilibrium price, firm demand and payoff as a function of consumer confusion ω given that confusion is massive ($\omega \geq 1$).

Corollary S1 *Let $m \geq 2$ and suppose that Assumption S2 is satisfied. For any given $\omega > 0$, let Γ_ω follow a uniform distribution with support $[-\omega, \omega]$, and let p_ω denote the symmetric equilibrium price in the pricing game given consumer confusion Γ_ω. Likewise, let d_ω and Π_ω denote the corresponding demand and payoff of each firm. Then $p_\omega, d_\omega, \Pi_\omega : \mathbb{R}_{++} \to \mathbb{R}$ are continuous functions of ω, and if confusion is massive ($\omega \geq 1$), we have*

$$p_\omega = \begin{cases} \omega & \text{if } \omega \in \left[1, \frac{m}{2}\right) \\ \frac{m}{2} & \text{if } \omega \in \left[\frac{m}{2}, m\right] \\ \frac{\omega+m}{4} & \text{if } \omega > m \end{cases}, \quad d_\omega = \begin{cases} \frac{1}{2} & \text{if } \omega \in [1, m] \\ \frac{\omega+m}{4\omega} & \text{if } \omega > m \end{cases},$$

and $\Pi_\omega = p_\omega d_\omega$. If $1 \leq \omega \leq m$ all consumers buy a product; if $\omega > m$ the fraction of consumers leaving the market is $L(\omega) = \frac{\omega - m}{2\omega} > 0$, which is strictly increasing in ω with $\lim_{\omega \to \infty} L(\omega) = 1/2$.

Corollary S1 shows that equilibrium prices and payoffs are increasing and unbounded in the range of confusion ω, despite an increasing fraction of consumers who abstain from acquiring any product.

We now ask how the intensity of confusion $\omega = \omega_{\mathbf{a}}$ is determined by strategically behaving firms that fully anticipate the profit schedule Π_ω resulting from the various feasible confusion intensities. We impose a structure on the mapping $\mathbf{a} \mapsto \omega_{\mathbf{a}}$ that is consistent with the MPS order Assumption 3: $A \subset \mathbb{R}_+$ is compact, $0 \in A$, and $\omega : \mathcal{A} \to \mathbb{R}_+$ is such that $\omega_{\mathbf{a}'} > \omega_{\mathbf{a}}$ iff $\mathbf{a}' \geq \mathbf{a}$ and $\mathbf{a}' \neq \mathbf{a}$.[61] Further, define $\bar{a} \equiv \max A > 0$ and $\bar{\omega} \equiv \omega_{\bar{\mathbf{a}}}$ as the maximally feasible confusion. Given this structure, we now show that the SPE follow the same pattern as identified by Theorems 1-3 except that the chosen communication strategies may lead to consumer exit from the market.

Consider first the case where $\bar{\omega} \leq 1$, such that massive confusion is not feasible. If preferences are indecisive on $[-1, 1]$, then $g_{\mathbf{a}}(0) = \frac{1}{2\omega_{\mathbf{a}}} \int_{-\omega_{\mathbf{a}}}^{\omega_{\mathbf{a}}} g_0(e)de$ is strictly decreasing

[61] It is easily checked that the uniform distribution verifies the TSC order criterion (see Appendix S1). A simple example is given by $\omega(\mathbf{a}) = z(a_1 + a_2)$ where z is any strictly increasing function with $z(0) = 0$.

in the intensity of confusion $\omega_{\mathbf{a}}$, and $g_{\mathbf{a}}(0) \geq \frac{1}{2\bar{\omega}}$ $\forall \mathbf{a} \in \mathcal{A}$. Because $m \geq 2$ also assures $\frac{1}{2\bar{\omega}} > \frac{1}{m}$, it follows that $g_{\mathbf{a}}(0) > \frac{1}{m}$ $\forall \mathbf{a} \in \mathcal{A}$, and Proposition S1 implies that $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$, $\forall \mathbf{a} \in \mathcal{A}$. This shows that with indecisive preferences and $\bar{\omega} \leq 1$ (i) any SPE is such that no consumer leaves the market, (ii) there cannot be an SPE without confusion (as in Theorem 1), and (iii) maximal confusion is the unique SPE outcome in case of strongly indecisive preferences (as in Theorem 2). If preferences are polarized on $[-1,1]$, then $g_{\mathbf{a}}(0)$ must be strictly increasing in the intensity of confusion $\omega_{\mathbf{a}}$. Together with $g_0(0) > \frac{1}{m}$ it follows from Proposition S1 that $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$, $\forall \mathbf{a} \in \mathcal{A}$. Thus, (i) any SPE is such that no consumer leaves the market, (ii) education always is an SPE outcome (as in Theorem 1), and education is the unique SPE outcome with strongly polarized preferences (as in Theorem 2).

The following result allows for the possibility that massive confusion may arise. The main point is that maximal confusion becomes the unique SPE outcome if confusion can become massive enough, even though a substantial portion of consumers chooses not to buy at all.

Proposition S2 *Consider the above example where $\Gamma_{\mathbf{a}}$ follows a uniform distribution for each $\mathbf{a} \in \mathcal{A}$. If G_0 is indecisive on supp g_0, then a unique SPE with maximal confusion always exists, and a fraction* $\max\{\frac{\bar{\omega}-m}{2\bar{\omega}}, 0\}$ *of consumers leaves the market. If G_0 is polarized on supp g_0, then maximal confusion is an SPE outcome whenever $\bar{\omega} \geq \frac{m}{2}$, and the unique SPE whenever $\bar{\omega} > m$, in which case a fraction $\frac{\bar{\omega}-m}{2\bar{\omega}} \in (0, \frac{1}{2})$ of consumers leaves the market.*

S.3.1 Proofs

Proof of Proposition S1 For any given $\Gamma_{\mathbf{a}}$, the demand function of firm 1 is

$$\begin{aligned}
D_{\mathbf{a}}(p_1, p_2) &= \int \Pr\left(\tilde{v}_1^k - p_1 \geq \max\{\tilde{v}_2^k - p_2, 0\}\right) d\Gamma_{\mathbf{a}} \\
&= \int \Pr\left(v \geq p_1 - p_2 - e,\ v \geq 2p_1 - m - e\right) d\Gamma_{\mathbf{a}}(e) \\
&= \int \min\left\{\Pr\left(v \geq p_1 - p_2 - e\right), \Pr\left(v \geq 2p_1 - m - e\right)\right\} d\Gamma_{\mathbf{a}}(e) \\
&= 1 - \int \max\left\{G_0\left(p_1 - p_2 - e\right), G_0\left(2p_1 - m - e\right)\right\} d\Gamma_{\mathbf{a}}(e).
\end{aligned}$$

Recall that, for all $x \in \mathbb{R}$,

$$G_{\mathbf{a}}(x) = \int G_0(x-e) d\Gamma_{\mathbf{a}}(e), \text{ and } g_{\mathbf{a}}(x) = \int g_0(x-e) d\Gamma_{\mathbf{a}}(e).$$

Thus, for all $p \geq 0$, we have

$$D_{\mathbf{a}}(p,p) = \begin{cases} \frac{1}{2} & \text{if } p \leq \frac{m}{2}, \\ 1 - G_{\mathbf{a}}(2p-m) & \text{if } p > \frac{m}{2}. \end{cases} \tag{A.6}$$

Let $\Pi_1^{\mathbf{a}}(p_1,p_2) = p_1 D_{\mathbf{a}}(p_1,p_2)$. For every $p_2 > 0$, the function $\Pi_1^{\mathbf{a}}$ is differentiable in p_1 almost everywhere. In particular, if $p_1 < m - p_2$, such that $D_{\mathbf{a}}(p_1,p_2) = 1 - G_{\mathbf{a}}(p_1 - p_2)$, we have

$$\frac{\partial \Pi_1^{\mathbf{a}}(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(p_1-p_2) - p_1 g_{\mathbf{a}}(p_1-p_2),$$

which is also the left derivative of $\Pi_1^{\mathbf{a}}(p_1,p_2)$ at $p_1 = m - p_2$. Similarly, if $p_1 > m - p_2$, such that $D_{\mathbf{a}}(p_1,p_2) = 1 - G_{\mathbf{a}}(2p_1 - m)$, we have

$$\frac{\partial \Pi_1^{\mathbf{a}}(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(2p_1-m) - 2p_1 g_{\mathbf{a}}(2p_1-m),$$

which is also the right derivative of $\Pi_1^{\mathbf{a}}(p_1,p_2)$ at $p_1 = m - p_2$.

Since log-concavity is preserved under convolution, the function $G_{\mathbf{a}}$ is log-concave on its support $supp(g_{\mathbf{a}})$. In addition, since $G_{\mathbf{a}}$ is a distribution function, its log-concavity also holds on $[0,+\infty)$. Hence, for all $p_2 > 0$ and $\mathbf{a} \in \mathcal{A}$, the demand function $D_{\mathbf{a}}(p_1,p_2)$ must be log-concave in p_1 on both $[0, m-p_2]$ and $[m-p_2,+\infty)$. Note that we are not claiming that $D_{\mathbf{a}}(p_1,p_2)$ is log-concave in p_1 on the entire interval $[0,+\infty)$. In what follows, we will show that although the global log-concavity of the demand function is not assured, Assumption S2 is still sufficient to guarantee the existence of a unique symmetric equilibrium in every subgame of the pricing stage.

First, suppose that $g_{\mathbf{a}}(0) > \frac{1}{m}$. Suppose also that firm 2 is choosing $p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}$. Then, for any $p_1 \leq \frac{m}{2}$ the whole market is guaranteed to be covered (i.e., every consumer will buy from one of the firms). In addition, since

$$\left.\frac{\partial \Pi_1^{\mathbf{a}}(p_1,p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{1}{2g_{\mathbf{a}}(0)}<\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{1}{2g_{\mathbf{a}}(0)} \cdot g_{\mathbf{a}}(0) = 0,$$

and the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is strictly quasi-concave in p_1 on $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$, and $g_{\mathbf{a}}(0) > \frac{1}{m}$ implies that $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ maximizes the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ over the range $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$. We now argue that, in addition,

$$\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right) < \Pi_1^{\mathbf{a}}\left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right) \ \forall p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}.$$

To see this, note that if $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$, then some consumers choose their outside options even though they would prefer firm 1 over firm 2. Therefore, a deviation to

$p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be more profitable than it would have been in the case without outside option. But then, as we have shown in Lemma 1 and Proposition 1, in the absence of the outside option, choosing $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ actually uniquely maximizes firm 1's expected profits over $[0, +\infty)$ given that its competitor plays $p_2 = \frac{1}{2g_{\mathbf{a}}(0)}$. This implies that deviating to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be profitable either in the presence of the outside option. Therefore, $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ must be a global maximum of the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$, and $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ indeed constitutes an equilibrium in the pricing subagme. It is easy to see that this is the only symmetric equilibrium with a price strictly less than $\frac{m}{2}$. In addition, since

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) < \frac{1}{2} - 1 < 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave, even a monopoly firm would not choose a price $p \geq \frac{m}{2}$. Hence, when $g_{\mathbf{a}}(0) > \frac{1}{m}$, there cannot be any symmetric equilibrium in which both firms choose a price larger than $\frac{m}{2}$. As a result, $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) > \frac{1}{m}$.

Next, consider the case $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$. Taking $p_2 = \frac{m}{2}$ as given, we will show that $p_1 = \frac{m}{2}$ is a best response for firm 1. As mentioned, the profit function $\Pi_1(p_1, p_2)$ is differentiable in p_1 on $\mathbb{R}_{++} \backslash \{m - p_2\}$ and semi-differentiable at the point $p_1 = m - p_2$. In particular, we have

$$\left.\frac{\partial^- \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{m}{2} \cdot g_{\mathbf{a}}(0) = \frac{1}{2} - \frac{m}{2} \cdot g_{\mathbf{a}}(0) \geq 0,$$

and

$$\left.\frac{\partial^+ \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - m g_{\mathbf{a}}(0) = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq 0.$$

Since $\Pi_1^{\mathbf{a}}\left(p_1, \frac{m}{2}\right)$ is strictly quasi-concave on both $\left[0, \frac{m}{2}\right]$ and $\left[\frac{m}{2}, +\infty\right)$, the above inequalities imply that $p_1 = \frac{m}{2}$ is a maximum of $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2}\right)$ on each of these two intervals. This shows that $p_1 = \frac{m}{2}$ is a global maximum of $\Pi_1^{\mathbf{a}}\left(p_1, \frac{m}{2}\right)$ on $[0, +\infty)$. Hence, if $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$, the game in the pricing stage admits a symmetric equilibrium with $p_1 = p_2 = \frac{m}{2}$. Further, as $g_{\mathbf{a}}(0) \leq \frac{1}{m}$ a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ cannot exist. To see this, note that a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ involves full market coverage (as $p_1 < m - p_2$), and must be a solution to the first order condition

$$1 - G_{\mathbf{a}}(0) - p_1 g_{\mathbf{a}}(0) = 0.$$

Thus $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$, where the condition $p_1 < \frac{m}{2}$ therefore is equivalent to $g_{\mathbf{a}}(0) > \frac{1}{m}$, contradiction.

In addition, because

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq \frac{1}{2} - \frac{1}{2} = 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave, even a monopoly firm would not choose a price strictly higher than $\frac{m}{2}$. Hence, no symmetric equilibrium with $p_1 = p_2 > \frac{m}{2}$ can exist either. In sum, this shows that $(p_1, p_2) = \left(\frac{m}{2}, \frac{m}{2}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right]$.

Finally, suppose that $g_{\mathbf{a}}(0) < \frac{1}{2m}$. Observe that in this case, we have $p_{\mathbf{a}}^M > \frac{m}{2}$, because

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave on $[0, +\infty)$. Now suppose that firm 2 plays $p_2 = p_{\mathbf{a}}^M$, and consider firm 1's profit function $\Pi_1(p_1, p_{\mathbf{a}}^M)$. Given the formula of the demand function and $p_{\mathbf{a}}^M > m - p_{\mathbf{a}}^M$, we have

$$\Pi_1^{\mathbf{a}}(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M) = \Pi_{\mathbf{a}}^M(p_{\mathbf{a}}^M) > \Pi_{\mathbf{a}}^M(p_1) \geq \Pi_1^{\mathbf{a}}(p_1, p_{\mathbf{a}}^M) \quad \forall p_1 \in [0, +\infty) \backslash \{p_{\mathbf{a}}^M\},$$

which further implies that $p_1 = p_{\mathbf{a}}^M$ is the unique best response for firm 1. Hence, $(p_1, p_2) = \left(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M\right)$ indeed constitutes an equilibrium in the pricing subgame where $g_{\mathbf{a}}(0) < \frac{1}{2m}$. Moreover, given $\frac{1}{2g_{\mathbf{a}}(0)} > \frac{m}{2}$, there cannot exist a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$. Since

$$\left.\frac{\partial^- \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} > \left.\frac{\partial^+ \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0,$$

$p_1 = p_2 = \frac{m}{2}$ does not constitute an equilibrium either. In conclusion, $(p_1, p_2) = \left(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) < \frac{1}{2m}$.

Taken together, the above derivations prove that the equilibrium price $p_{\mathbf{a}}^*$ is determined by (S.5). Further, (A.6) implies that market shares are below 1/2 iff $g_{\mathbf{a}}(0) < \frac{1}{2m}$, completing the proof. ■

Proof of Corollary S1 Let g_ω denote the density of the perceived match advantages given that the perception errors ε follow the uniform distribution Γ_ω. It is easily checked that for any $\omega > 0$, the density γ_ω is log-concave on its support. Thus, Assumption 4.4 is satisfied, and Proposition S1 applies to the case of uniformly distributed perception errors. Specifically, the equilibrium price is determined by (S.5), where we replace $p^*_{\mathbf{a}}$ by p_ω and $g_{\mathbf{a}}(0)$ by $g_\omega(0)$. For any given $\omega > 0$, we have $g_\omega(0) = \frac{1}{2\omega}\int_{-\omega}^{\omega} g_0(e)de$. Next, note that $g_\omega(0) = \frac{1}{2\omega}\int_{-1}^{1} g_0(e)de = \frac{1}{2\omega}$ whenever $\omega \geq 1$, which we assume in the following. Given that $\omega \geq 1$, the condition $g_\omega(0) > \frac{1}{m}$ is equivalent to $\omega < \frac{m}{2}$. Thus, for $\omega \in [1, \frac{m}{2})$, we must have $g_\omega(0) = \frac{1}{2\omega} > \frac{1}{m}$, and hence $p_\omega = \frac{1}{2g_\omega(0)} = \omega$ by (S.5). Next, given that $\omega \geq 1$, the condition $g_\omega(0) \geq \frac{1}{2m}$ is equivalent to $\omega \leq m$. Thus, for $\omega \in [\frac{m}{2}, m]$ also $g_\omega(0) \in [\frac{1}{2m}, \frac{1}{m}]$, and thus $p_\omega = \frac{m}{2}$ for $\omega \in [\frac{m}{2}, m]$ by (S.5). Finally, $g_\omega(0) < \frac{1}{2m}$ iff $\omega > m$, in which case (S.5) implies that p_ω is given by the monopoly price p^M_ω. We now show that if $\omega > m$, then $p_\omega = p^M_\omega = \frac{\omega+m}{4}$, independent of the shape of g_0. If p_ω denotes the solution of this monopoly problem, then p_ω must be determined by the first order condition $\frac{\partial \Pi^M_\omega(p_\omega)}{\partial p} = 0$, which can be simplified to

$$\omega - \frac{1}{2}\int_{2p_\omega - m - \omega}^{2p_\omega - m + \omega} G_0(s)ds - p\int_{2p_\omega - m - \omega}^{2p_\omega - m + \omega} g_0(s)ds = 0 \tag{S.6}$$

To calculate the value of p_ω we need to evaluate the two integrals in the previous expressions. By Proposition S1, we know that $p_\omega > \frac{m}{2}$, which implies that $2p_\omega - m + \omega > 1$ for the upper bounds of the two integrals (recalling that $\omega \geq 1$). We now conjecture (and ex post verify) that $2p_\omega - m - \omega < -1$. Recalling that $supp\, g_0 = [-1, 1]$ and $\int_{-1}^{1} G_0(s)ds = 1$ as a consequence of symmetry, (S.6) evaluates to $p_\omega = \frac{\omega+m}{4}$ given the presumption that $2p_\omega - m - \omega < -1$. It now is easily verified that the last inequality indeed is satisfied for $p_\omega = \frac{\omega+m}{4}$, confirming that $p^M_\omega = \frac{\omega+m}{4}$ must be the monopoly price. In sum, these steps show that p_ω must be as stated by Corollary S1. Continuity of p_ω in ω then is obvious for $\omega \in [1, \infty)$. As the equilibrium price p_ω is determined by (S.5), it follows from (S.5) and the previous result that p_ω is continuous on the entire range $\omega \in (0, \infty)$ whenever $g_\omega(0)$ is continuous in ω on this range. As $g_\omega(0) = \frac{1}{2\omega}\int_{-\omega}^{\omega} g_0(e)de$ for any $\omega > 0$, the last property is obviously verified.

Turning to equilibrium demand, the proof of Proposition S1 shows that as long as $1 \leq \omega \leq m$ (i.e., $g_\omega(0) \geq \frac{1}{2m}$) the equilibrium price is such that the outside option is not binding for (almost) all consumers, meaning that $d_\omega = 1/2$ for $\omega \in [1, m]$. If $\omega > m$, the equilibrium price is given by $p^M_\omega = \frac{\omega+m}{4}$, and $d_\omega = 1 - G_\omega(2p^M_\omega - m)$, which evaluates to $d_\omega = \frac{\omega+m}{4\omega}$. The expression for equilibrium profits $\Pi_\omega = p_\omega d_\omega$ then follows immediately. Continuity of d_ω and of $\Pi_\omega = p_\omega d_\omega$ in ω follow from the continuity of p_ω. Finally, the claims about $L(\omega)$ follows from $L(\omega) = 1 - 2d_\omega$. ■

Proof of Proposition S2 Suppose that preferences are indecisive on $[-1, 1]$. If $\bar{\omega} \leq m$, the existence of a unique SPE with maximal confusion immediately follows from Theorem S1. If $\bar{\omega} > m$, the same result holds because it is shown in Corollary S1 that Π_ω is strictly increasing in ω for $\omega \geq m$. It is then clear that no consumer leaves the market in equilibrium when $\bar{\omega} \leq m$, while a fraction $L(\bar{\omega}) = \frac{\bar{\omega}-m}{2\bar{\omega}} \in (0, \frac{1}{2})$ of consumers leaves the market when $\bar{\omega} > m$.

Next, consider the case of polarized preferences. Corollary S1 assures that Π_ω is (weakly) increasing in ω whenever $\omega > \frac{m}{2}$ in this case. The firms benefit from confusion relative to an educated market if $\Pi_0 = \frac{1}{4g_0(0)} < \Pi_\omega$, $\omega > 0$. Because $g_0(0) > 1/m$ and thus $\Pi_0 < \frac{m}{4}$, Corollary S1 shows that a sufficient condition for $\Pi_0 < \Pi_{\bar{\omega}}$ is that $\bar{\omega} \geq \frac{m}{2}$. Thus, if $\bar{\omega} \geq \frac{m}{2}$, then maximal confusion is an SPE outcome (while education is not). As Π_ω is strictly increasing in ω for $\omega \geq m$, it follows that maximal confusion is the unique SPE outcome whenever $\bar{\omega} > m$. ■

S.4 Welfare Loss: Additional Results

The following two propositions formalize the claims made in Section 4.3 about the size of the welfare loss in the case of indecisive preferences.

Proposition S3 *Consider the price competition application, and suppose that for any* $\mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ *is uniformly distributed on* $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\omega_{\mathbf{a}} > 0$, *whenever* $\varepsilon_{\mathbf{a}}$ *is non-degenerate. Then, the expected welfare loss* (4) *is strictly increasing in* $\omega_{\mathbf{a}}$.

Proof: Let $\kappa \equiv \sup supp\,(g_0)$. We can write the expected welfare loss from mismatch as a function of the degree of confusion:

$$L(\omega) = 2\int_0^{\min\{\omega,\kappa\}} \left[x \cdot \frac{-x+\omega}{2\omega} \cdot g_0(x)\right] dx = \int_0^{\min\{\omega,\kappa\}} \left[x\left(1 - \frac{x}{\omega}\right) g_0(x)\right] dx.$$

Taking the first derivative, we obtain

$$L'(\omega) = \int_0^{\kappa} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega \geq \kappa$, and

$$L'(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x) + \omega\left(1 - \frac{\omega}{\omega}\right) g_0(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega < \kappa$. Since by assumption G_0 is a non-degenerate distribution, $L'(\omega) > 0$ $\forall \omega > 0$. Hence, the expected welfare loss is strictly increasing in ω. ■

Proposition S4 *Consider the model with competition on the line. Suppose that* $\varepsilon_{\mathbf{a}}$ *is as in Proposition S3. If* $\omega_{\mathbf{a}} < \hat{\omega} \equiv 64/15$, *then the expected welfare loss* (4) *is strictly decreasing in* α. *If* $\omega_{\mathbf{a}} > \hat{\omega}$, *then the expected welfare loss is strictly increasing in* α.

PROOF: Since $G_0(x) = H\left(\frac{x}{4\lambda}\right)$ $\forall x \in \mathbb{R}$, the density function of G_0, which we denote as g_0, is given by

$$g_0(x) = \frac{1}{4\lambda} h\left(\frac{x}{4\lambda}\right) = \begin{cases} \frac{1}{4\lambda} \cdot \left(\alpha \left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right) & \text{if } x \in [-4\lambda^2, 4\lambda^2], \\ 0 & \text{otherwise.} \end{cases}$$

The welfare loss can now be written as a function of α:

$$\begin{aligned} L(\alpha) &= \int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right) \cdot \frac{1}{4\lambda} \cdot h\left(\frac{x}{4\lambda}\right)\right] dx \\ &= \frac{1}{4\lambda} \int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right)\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)\right] dx. \end{aligned}$$

Taking derivative with respect to α, we have

$$L'(\alpha) = \frac{1}{4\lambda} \int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx.$$

First, suppose that $\omega \leq 4\lambda^2$. In this case, we obtain

$$\begin{aligned} L'(\alpha) &= \frac{1}{4\lambda} \int_0^{\omega} \left[x\left(1 - \frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx \\ &= \frac{1}{4\lambda}\left[\int_0^{\omega}\left(\frac{x^3}{16\lambda^2} - \frac{\lambda^2 x}{3}\right) dx - \int_0^{\omega}\left(\frac{x^4}{16\lambda^2\omega} - \frac{\lambda^2 x^2}{3\omega}\right) dx\right] \\ &= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2} - \frac{\lambda^2 x^2}{6}\right)\Bigg|_0^{\omega} - \left(\frac{x^5}{80\lambda^2\omega} - \frac{\lambda^2 x^3}{9\omega}\right)\Bigg|_0^{\omega}\right] \\ &= \frac{1}{4\lambda}\left[\frac{\omega^4}{64\lambda^2} - \frac{\omega^4}{80\lambda^2} - \frac{\lambda^2\omega^2}{6} + \frac{\lambda^2\omega^2}{9}\right]. \end{aligned}$$

Hence, provided that $\omega \in (0, 4\lambda^2]$, we further have

$$L'(\alpha) < 0 \iff \left(\frac{1}{64\lambda^2} - \frac{1}{80\lambda^2}\right)\omega^2 < \frac{\lambda^2}{6} - \frac{\lambda^2}{9} \iff \omega < \frac{4\sqrt{10}}{3}\lambda^2.$$

Since $4\sqrt{10}/3 \approx 4.22 > 4$, it follows that $L'(\alpha) < 0$ whenever $\omega \leq 4\lambda^2$.

Next, consider the case where $\omega > 4\lambda^2$. Expanding the equation $L'(\alpha)$ again, we have

$$\begin{aligned} L'(\alpha) &= \frac{1}{4\lambda} \int_0^{4\lambda^2} \left[x\left(1 - \frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx \\ &= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2} - \frac{\lambda^2 x^2}{6}\right)\Bigg|_0^{4\lambda^2} - \left(\frac{x^5}{80\lambda^2\omega} - \frac{\lambda^2 x^3}{9\omega}\right)\Bigg|_0^{4\lambda^2}\right] \\ &= \frac{1}{4\lambda}\left[\left(\frac{4^4\lambda^8}{64\lambda^2} - \frac{4^2\lambda^6}{6}\right) - \frac{1}{\omega}\left(\frac{4^5\lambda^{10}}{80\lambda^2} - \frac{4^3\lambda^8}{9}\right)\right] \\ &= 4\lambda^5\left[\left(\frac{16}{64} - \frac{1}{6}\right) - \frac{\lambda^2}{\omega}\left(\frac{64}{80} - \frac{4}{9}\right)\right]. \end{aligned}$$

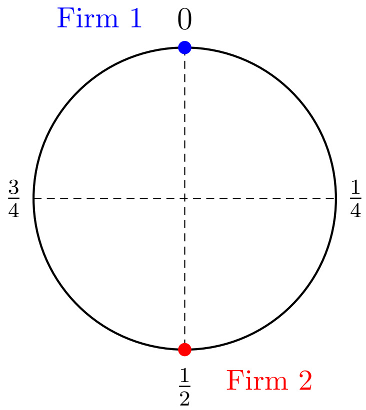


Figure S.2: The Salop circle

Hence, provided that $\omega > 4\lambda^2$, we further have

$$L'(\alpha) > 0 \iff \frac{\lambda^2}{\omega}\left(\frac{4}{5} - \frac{4}{9}\right) < \frac{1}{4} - \frac{1}{6} \iff \omega > \frac{64}{15}\lambda^2.$$

Note that $64/15 \approx 4.27 > 4$. We can now conclude that $L'(\alpha) < 0$ whenever $\omega < \hat{\omega} \equiv 64\lambda^2/15$, and $L'(\alpha) > 0$ whenever $\omega > \hat{\omega}$. ■

S.5 Confusion About Needs on a Salop Circle

In the following application, we use a Salop circle to further pursue the idea sketched at the end of Section 3.4 that the communication profiles influence how precisely the agents can learn their true needs. As the Salop model (Salop, 1979) has become a textbook-style workhorse model in IO and related fields, the subsequent analysis of strategic confusion or education upon a Salop circle is also interesting in itself.

Two firms are located at antipodal locations on a Salop circle, as illustrated in Figure S.2. Consumers are continuously and symmetrically distributed between the firms, where we indicate consumer locations in the clockwise direction with $\theta \in [0, 1)$. By symmetry, it suffices to specify the model only for the half-circle on the right-hand side. On this half-circle, consumers are dispersed over the $[0, 1/2]$-line according to the bounded function $h : \mathbb{R} \to \mathbb{R}_+$ with the following properties

$$\begin{aligned} &(1) \quad h(\theta) > 0 \Leftrightarrow \theta \in [0, 1/2]\,, \\ &(2) \quad h \text{ is symmetric at } \tfrac{1}{4}. \\ &(3) \quad \textstyle\int_0^{1/2} h(x)dx = 1/2, \\ &(4) \quad h \text{ is differentiable on } (0, 1/4) \cup (1/4, 1/2), \end{aligned} \tag{S.7}$$

The corresponding (half)-distribution function $H : \mathbb{R} \to [0, 1/2]$ is $H(\theta) = \int_0^\theta h(x)dx$,

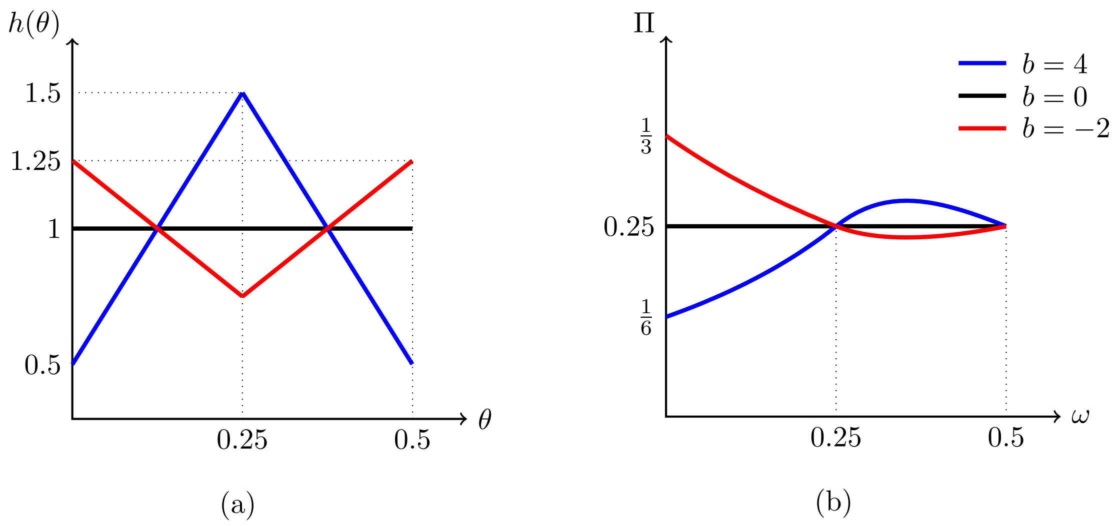


Figure S.3: (a) Preferences and (b) profits in the Salop model, $t = 1$.

$\forall \theta \in \mathbb{R}$. For a consumer located at $\theta \in [0, 1/2]$, the true match values are

$$v_1^\theta = \mu - t\theta \quad \text{and} \quad v_2^\theta = \mu - t(1/2 - \theta),$$

where $\mu, t > 0$ are parameters. The match advantage of firm 2 then is $v_\Delta^\theta = v_2^\theta - v_1^\theta = 2t(\theta - 1/4)$. The consumer would truly prefer firm 1 if and only if $p_2 - p_1 \geq v_\Delta^\theta$. We assume that μ is sufficiently large, such that every consumer will find it worthwhile to purchase one product in equilibrium.

In accordance with the logic behind Definition 2, we say that the consumer preference distribution features indecisiveness (polarization) on $[0, 1/2]$ if $h(\cdot)$ is strictly increasing (decreasing) on $(0, 1/4)$ and thus strictly decreasing (increasing) on $(1/4, 1/2)$. As a simple example, suppose that $h(\cdot)$ is piecewise linear,

$$h(\theta) = \begin{cases} 1 + b(\theta - 1/8) & \text{if } \theta \in [0, 1/4], \\ 1 - b(\theta - 3/8) & \text{if } \theta \in (1/4, 1/2], \\ 0 & \text{otherwise,} \end{cases} \tag{S.8}$$

where $|b| < 8$ to assure that $h(\theta) > 0$ on $[0, 1/2]$. Then, $b > 0$ corresponds to indecisive and $b < 0$ to polarized preferences, while $b = 0$ gives the standard textbook Salop model with uniformly distributed consumers. Figure S.3 (a) depicts the preference distribution for $b = 4, -2$ and 0.

Locational confusion As in Section 4, firms first choose their communication strategies $\mathbf{a} \in \mathcal{A}$, and then compete in prices. In the current setting, consumer confusion enters the model in form of i.i.d. shocks $\varepsilon_{\mathbf{a}}$ to true consumer locations θ. Specifically, confusion means that each consumer's perceived location is distorted around the true location θ by a 0-symmetric, uniformly distributed shock $e \in [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, where $\omega_{\mathbf{a}} \geq 0$ measures the size of confusion. The interpretation of this model is that communication strategies influence how well a consumer learns his true needs. If $\omega_{\mathbf{a}} = 0$ then $\hat{\theta} = \theta$ for each consumer $\theta \in [0, 1]$, meaning that communication allows each consumer to correctly learn her location. By contrast, if $\omega_{\mathbf{a}} = 1/2$, then $\hat{\theta} \in [0, 1)\ \forall\theta$, meaning that each consumer could find herself anywhere on the circle, independent of her true location. Note that obfuscation becomes *massive* in the sense of Section 3.4 whenever $\omega_{\mathbf{a}} > 1/4$, as then a consumer sitting exactly on a firm's location may, in principle, be so confused that she chooses the competitor's product. Nevertheless, $\omega_{\mathbf{a}} = 1/2$ corresponds to the natural upper bound of such massive obfuscation in the present model.

While the distribution of the perceived match advantages remains unbiased, here a notable difference to our main setting is that the true match advantages and the perception errors implied by locational confusion are not independent. Unbiasedness follows directly from the 0-symmetry of the locational shocks. The violation of independence occurs, in essence, because locational confusion confines the perceptions $\hat{\theta}$ to the circle, meaning that the range of perceived match advantages must always coincide with the range of true match advantages.[62]

Equilibrium analysis For a given $\omega_{\mathbf{a}} \in [0, 1/2]$ and given prices p_1, p_2, a firm's demand consists of those consumers who perceive the firm as offering the better deal. We now derive a formal expression for the expected demand of firm $j = 1$. As $\omega_{\mathbf{a}} \in [0, 1/2]$, $e \in [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $\theta \in [0, 1/2]$, it follows that $\theta + e \in [-1/2, 1]$. Thus, for each consumer $\theta \in [0, 1/2]$, the perceived distance to firm 1 is

$$\hat{d}_1 = \begin{cases} |\theta + e| & \text{if } \theta + e \leq 1/2, \\ 1 - (\theta + e) & \text{if } \theta + e > 1/2. \end{cases}$$

Fix prices $p_1, p_2 \geq 0$ and define $\Delta \equiv \frac{p_2 - p_1}{2t}$. If $|\Delta| \leq 1/4$, the market segment S_1 of firm 1 is $S_1 = \{\theta \in [0, 1] : \hat{d}_1 \leq \Delta + 1/4\}$. Further, $S_1 = \emptyset$ if $\Delta < -1/4$, and $S_1 = [0, 1]$ if $\Delta > 1/4$. A consumer transacts with firm 1 if her perceived location belongs to S_1. Hence, for $\Delta < 1/4$ the expected market demand of firm 1 from consumers on the right half-circle, $D_1 \in [0, 1/2]$, corresponds to the expected fraction of consumers for whom the

[62]To illustrate the violation of independence, let $t = 1$, $\omega_{\mathbf{a}} = 1/4$. Then, the perception errors, in terms of match advantages, implied by locational confusion for the consumer at $\theta = 0$ (hence $v^0_\Delta = 0$) have $supp\, \varepsilon^0_{\mathbf{a}} = [0, 1/2]$. By contrast, a consumer with location $\theta = 1/4$ (hence $v^{1/4}_\Delta = 0$) experiencing the same type of locational shock has $supp\, \varepsilon^{1/4}_{\mathbf{a}} = [-1/2, 1/2]$. Thus, $\varepsilon_{\mathbf{a}}$ and v_Δ cannot be independent.

true location is on $[0, 1/2]$ and the perceived locations is in S_1:

$$D_1(\Delta, \omega_{\mathbf{a}}) = \Pr\left(-1/4 - \Delta \leq \theta + \varepsilon_{\mathbf{a}} \leq 1/4 + \Delta\right) + \Pr\left(\theta + \varepsilon_{\mathbf{a}} \geq 3/4 - \Delta\right), \tag{S.9}$$

where the probabilities in (S.9) should be interpreted as conditional on $\theta \in [0, 1/2]$. The second term in (S.9) captures that some consumers with location in the segment $(1/4 + \Delta, 1/2]$, who actually would be better-off by choosing firm 2, may obtain perceived locations in the segment $(3/4, 1)$ for sufficiently large obfuscation $\omega_{\mathbf{a}}$, and then (erroneously) choose firm 1.

We now turn to the equilibrium analysis, relying on the standard first-order approach as is common in applications of the Salop model (see, e.g., Grossman and Shapiro (1984)).[63] For $|\Delta(p_1, p_2)| < 1/4$ and $\omega_{\mathbf{a}} \in [0, 1/2]$, the expected profit of firm $j = 1$ in the pricing stage is

$$\Pi_1(\Delta(p_1, p_2), \omega_{\mathbf{a}}) = 2p_1 D_1(\Delta(p_1, p_2), \omega_{\mathbf{a}}) = 2p_1 D_1\left(\frac{p_2 - p_1}{2t}, \omega_{\mathbf{a}}\right), \tag{S.10}$$

where $D_1(\Delta(p_1, p_2), \omega_{\mathbf{a}})$ is given by (S.9). A symmetric pricing equilibrium $p_{\mathbf{a}}$ in the pricing stage corresponds to a solution of $\frac{\partial}{\partial p_1}\Pi_1(p_{\mathbf{a}}, p_{\mathbf{a}}) = 0$. As shown in the proof of the next proposition, such a unique solution $p_{\mathbf{a}}$ exists for every $\omega_{\mathbf{a}}$, and we assume that $p_{\mathbf{a}}$ then also corresponds to the equilibrium price in the pricing stage. In the following, we show how firms' profit $\Pi^{\mathbf{a}} = \frac{p_{\mathbf{a}}}{2}$ in the symmetric pricing equilibrium of the pricing stage depends on the confusion parameter $\omega \in [0, 1/2]$ (we suppress the **a**-index for simplicity).

Proposition S5 *In the Salop model with locational confusion, the following cases can be distinguished:*

(i) (Indecisiveness) If the preference distribution is indecisive, there exists a unique $\omega_0 \in (1/4, 1/2)$ such that prices and profits increase strictly in ω up to ω_0, and decrease strictly thereafter. Moreover, prices and profits are minimized at $\omega = 0$.

(ii) (Polarization) If the preference distribution is polarized, there exists a unique $\omega_0 \in (1/4, 1/2)$ such that prices and profits decrease strictly in ω up to ω_0, and increase strictly thereafter. Moreover, prices and profits are maximized at $\omega = 0$.

(iii) (Uniform Dispersion) If the preference distribution is uniform, i.e., $h(\theta) = 1$ on $[0, 1/2]$ then the prices and profits are $p_\omega = t/2$ and $\Pi(\omega) = t/4$, $\forall \omega \in [0, 1/2]$.

[63]This approach essentially takes the existence of a symmetric price equilibrium as given, in that sufficiency of the first-order condition for profit maximization at a symmetric solution of $\frac{\partial \Pi_1}{\partial p_1} = 0$ in the pricing stage is presumed.

The proof is at the end of this section. Proposition S5 shows that as long as confusion cannot become massive ($\omega_{\mathbf{a}} \leq 1/4 \ \forall \mathbf{a} \in \mathcal{A}$), the dispersion of preferences on the Salop circle has the same implication for the effects of confusion as in the baseline model. In particular, firms are only harmed (can only benefit) from such confusion if preferences are polarized (indecisive). Intuitively, this is the case because competition on each half-circle of the Salop model is akin to competition on the line for small enough confusion.

If confusion becomes massive, however, the Salop model offers new insights. Specifically, firms cannot benefit from maximally confused consumers independent of whether preferences are indecisive or polarized: In the latter case they prefer education ($\omega = 0$) and in the former case the intermediate level of confusion given by ω_0. In the knife-edge case where consumers are uniformly distributed, confusion has no effects on prices and profits at all. These different cases are illustrated in Figure S.3 (b). The intuition is that massive confusion has two effects. First, some right-hand side consumers actually favoring firm $j = 1$ may become indifferent on the right-hand circle ($\hat{\theta} = 1/4$). Second, some consumers who are located on the left-hand circle and who favor firm $j = 1$ may be so confused as to become indifferent on the right-hand circle. The second effect is absent in a Hotelling model. With indecisive preferences, the share of consumers that become perceptually indifferent in the sense of the second effect increases in confusion, which explains why firms are eventually harmed by confusion once it becomes large enough. While this effects is partly reversed with polarized preferences, it is not possible to soften competition more with confusion as given by the case of educated consumers. Finally, confusion has no impact on competition with uniform preferences, because the average inflow and outflow of perceptually indifferent consumers exactly compensate each other in this case.

Proof of Proposition S5 Let $|\Delta| < 1/4$. For $\omega = 0$, (S.9) then yields

$$D_1(\Delta, 0) = \Pr(\theta \leq 1/4 + \Delta) + \Pr(\theta \geq 3/4 - \Delta) = H(1/4 + \Delta).$$

For $\omega \in (0, 1/2]$ we obtain

$$\begin{aligned} & D_1(\Delta, \omega) \\ = & \int_{-\omega}^{\omega} \tfrac{1}{2\omega} \left(H(1/4 + \Delta - e) - H(-1/4 - \Delta - e)\right) de + \int_{-\omega}^{\omega} \tfrac{1}{2\omega} \left(1/2 - H(3/4 - \Delta - e)\right) de \\ = & \tfrac{1}{2\omega} \left(\int_{1/4+\Delta-\omega}^{1/4+\Delta+\omega} H(\theta) d\theta - \int_{-1/4-\Delta-\omega}^{-1/4-\Delta+\omega} H(\theta) d\theta - \int_{3/4-\Delta-\omega}^{3/4-\Delta+\omega} H(\theta) d\theta \right) + 1/2. \end{aligned} \tag{S.11}$$

Then, (S.11) implies

$$\frac{\partial D_1(\Delta,\omega)}{\partial \Delta} = \frac{H(1/4+\Delta+\omega)-H(1/4+\Delta-\omega)}{2\omega} + \frac{H(-1/4-\Delta+\omega)-H(-1/4-\Delta-\omega)}{2\omega} + \frac{H(3/4-\Delta+\omega)-H(3/4-\Delta-\omega)}{2\omega}. \tag{S.12}$$

Using (S.12) and $p_2 = p_1 = p_\omega$ in the first-order condition then yields

$$p_\omega = \frac{t/2}{\frac{\partial}{\partial \Delta} D_1(0,\omega)} \tag{S.13}$$

as the unique solution. Then, the corresponding equilibrium profit $\Pi(\omega) \equiv p_\omega/2$ satisfies

$$sign\, \Pi'(\omega) = sign\, \frac{\partial p_\omega}{\partial \omega} = -sign\, Z(\omega), \qquad Z(\omega) \equiv \frac{\partial^2 D_1(0,\omega)}{\partial \Delta \partial \omega}. \tag{S.14}$$

Let $\omega \in (0,1/4)$. Noting that $h(\theta) = 0$ whenever $\theta \notin [0,1/2]$, we obtain from (S.12)

$$Z(\omega) = \frac{2\omega h(1/4+\omega) - (H(1/4+\omega) - H(1/4-\omega))}{2\omega^2}.$$

The symmetry of h at $\theta = 1/4$ implies

$$H\left(1/4+\omega\right) - H\left(1/4-\omega\right) = 2H\left(1/4+\omega\right) - 1/2. \tag{S.15}$$

Using this and $H(1/4) = 1/4$ in (S.15), we have

$$Z(\omega) \geq 0 \quad \Leftrightarrow \quad h\left(1/4+\omega\right)\omega \geq H\left(1/4+\omega\right) - 1/4 = \int_{1/4}^{1/4+\omega} h(\theta)d\theta. \tag{S.16}$$

Therefore we can establish the claims in (i), (ii) and (iii) for the case where $\omega \in (0,1/4)$.

(i) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta > \int_{1/4}^{1/4+\omega} h(1/4+\omega)d\theta = h(1/4+\omega)\omega$, thus $Z(\omega) < 0$ by (S.16). Hence prices and profits increase in obfuscation by (S.14) for $\omega \in (0,1/4)$.

(ii) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta < \int_{1/4}^{1/4+\omega} h(1/4+\omega)d\theta = h(1/4+\omega)\omega$, thus $Z(\omega) > 0$ by (S.16). Hence prices and profits decrease in obfuscation by (S.14) for $\omega \in (0,1/4)$.

(iii) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta = 1/4+\omega-1/4 = \omega$, and $h\left(\frac{1}{4}+\omega\right)\omega = \omega$, thus $Z(\omega) = 0$ by (S.16). Hence obfuscation has no effects on prices and profits by (S.14) for $\omega \in (0,1/4)$.

Next, suppose that $\omega \in (1/4,1/2]$.[64] Then, (S.12) gives

$$Z(\omega) = \frac{\omega\left(h(\omega-1/4) + h(3/4-\omega)\right) - \left(1 + H(\omega-1/4) - H(3/4-\omega)\right)}{2\omega^2}. \tag{S.17}$$

[64] The case $\omega = 1/4$ is not problematic, because $\partial D_1(0,\omega)/\partial\Delta$ is continuous at $\omega = 1/4$.

Using $h(\omega - 1/4) = h(3/4 - \omega)$ and $H(\omega - 1/4) = 1/2 - H(3/4 - \omega)$, the nominator of $Z(\omega)$ becomes

$$z(\omega) \equiv 2\omega h(3/4 - \omega) - 3/2 + 2H(3/4 - \omega). \tag{S.18}$$

Suppose now that h features indecisiveness (i). Then $h(1/2) < 1$, $h(1/4) > 1$, and

$$\lim_{\omega \downarrow 1/4} z(\omega) = \frac{h(1/2)}{2} - \frac{1}{2} < 0, \quad \text{and} \quad \lim_{\omega \uparrow 1/2} z(\omega) = h(1/4) - \frac{1}{2} > 0.$$

Further, for $\omega \in (1/4, 1/2)$ we have $z'(\omega) = -2\omega h'(3/4 - \omega) > 0$. These arguments, together with the continuity of $z(\omega)$, assure the existence of a unique $\omega_0 \in (1/4, 1/2)$, such that for $\omega \in [1/4, 1/2]$

$$z(\omega), Z(\omega) \begin{cases} < 0 & \text{if } \omega < \omega_0, \\ = 0 & \text{if } \omega = \omega_0, \\ > 0 & \text{if } \omega > \omega_0. \end{cases}$$

It then follows from (S.14) that $\Pi(\omega)$ and p_ω must have a global maximum at $\omega_0 \in [1/4, 1/2]$. Note from (S.12) that $\frac{\partial D_1(0,1/2)}{\partial \Delta} = 1$, which by (S.13) implies that $p_{1/2} = t/2$, and $\Pi(1/2) = t/4$. If $\omega = 0$, then $\frac{\partial D_1(0,0)}{\partial \Delta} = h(1/4)$, and thus $p_0 = t/(2h(1/4))$, and $\Pi(0) = t/(4h(1/4))$. Because $h(1/4) > 1$ with indecisive preferences, prices and profits must be minimal at $\omega = 0$, which completes the proof for (i). Case (ii) can be proved similarly. For (iii), note that if $h(\theta) = 1$ on $(1/4, 1/2]$, then $z(\omega) = 0$ on $(1/4, 1/2]$. ■

S.6 Ex-Ante Asymmetric Contestants

In this section, we consider the formal model on which the intuitive discussion about ex-ante asymmetric contestants in Section 5.2 is based on. We suppose that every choice of communication profile $\mathbf{a}$ determines a parameter $\omega = \omega(\mathbf{a}) \in [0, \bar{\omega}]$ of the density function $\gamma(\cdot, \omega)$ of the perception errors ε, where $\bar{\omega}$ is exogenously given.

The following technical assumptions are imposed on $\gamma(\cdot, \omega)$. First, $supp\, \gamma(\cdot, \omega) \subset supp\, \gamma(\cdot, \omega')$ whenever $\omega < \omega'$. Second, $\gamma(\cdot, \omega)$ is a zero-symmetric and log-concave C^1-density function on its support for any given $\omega \in (0, \bar{\omega})$. Third, $\gamma(\cdot, \omega')$ is an MPS of $\gamma(\cdot, \omega)$ whenever $\omega' > \omega$. We also take $\gamma(x, \omega)$ to be continuously differentiable in ω at any $x \in supp\, \gamma(\cdot, \omega)$. We denote the distribution and density of the perceived match advantages for any given as $G(\cdot, \omega)$ and $g(\cdot, \omega)$, respectively. Further, we let $G(x, 0) \equiv G_0(x)$ and $g(x, 0) \equiv g_0(x)$.

The payoff functions are given by (2) with $R(s_1, s_2) = 1$, where we replace $G_{\mathbf{a}}(s_1 - s_2)$ by $G(s_1 - s_2, \omega)$ given our notational convention, where $G(s_1 - s_2, \omega) = \int_{-\infty}^{s_1 - s_2} g(s, \omega) ds$ is the market share of contestant 1 for effort profile (s_1, s_2). Further, we assume that C_1, C_2 are C^2-functions with $C_j'(s), C_j''(s) > 0$ for any $s > 0$ and $C_j(0) = 0$, $j = 1, 2$. We assume that payoffs are strictly quasi-concave in own strategies for any given $\omega \in [0, \bar{\omega})$, meaning that $g'(s_1 - s_2, \omega) - C_j''(s_j) < 0$ whenever $g(s_1 - s_2, \omega) = C_j'(s_j)$. For a given $\omega \in [0, \bar{\omega})$, an interior effort equilibrium $s_1(\omega), s_2(\omega)$ is then determined by the first-order system

$$g(s_1 - s_2, \omega) = {C_1}'(s_1), \qquad g(s_1 - s_2, \omega) = {C_2}'(s_2). \tag{S.19}$$

In the following we consider ex-ante asymmetry of the contestants in terms of ranked cost functions, where $C_1'(s) < C_2'(s)$ for any $s > 0$. We refer to $j = 1$ as the *strong*, and to $j = 2$ as the *weak* contestant, respectively. Our main result in this section shows that the incentive to obfuscate or educate is quite unambiguous for the weak contestant.

Proposition S6 *If, for some sufficiently large $\delta > 0$, g_0 is strongly indecisive on $[-\delta, \delta]$, the weak contestant unambiguously desires maximal agent confusion ($\omega = \bar{\omega}$). If g_0 is strongly polarized on $[-\delta, \delta]$, the weak contestant unambiguously desires minimal agent education ($\omega = 0$).*

Proof of Proposition S6 We prove the proposition in a serious of lemmas. Note that in for follow, we take for granted the existence of a unique effort equilibrium $s_1(\omega), s_2(\omega) > 0$ for any given $\omega \in [0, \bar{\omega})$.[65]

Lemma S1 *For any given $\omega \in [0, \bar{\omega}]$, $s_1(\omega) > s_2(\omega)$ and $\Pi_1(\omega) > \Pi_2(\omega)$.*

Proof: By (S.19), $C_1'(s_1) = C_2'(s_2)$ in equilibrium, from which $s_1 > s_2$ follows. Equilibrium payoffs are $\Pi_1(\omega) = G(s_1 - s_2, \omega) - C_1(s_1)$ and $\Pi_2(\omega) = 1 - G(s_1 - s_2, \omega) - C_2(s_2)$. The fact that $s_1 > s_2$ implies $G(s_1 - s_2, \omega) > 1/2$. Then

$$\Pi_1(\omega) \geq \frac{1}{2} - C_1(s_2) \geq \frac{1}{2} - C_2(s_2) > \Pi_2(\omega),$$

where the second inequality follows that $C_1(0) = C_2(0)$ and $C_1'(s) < C_2'(s)$ for all $s > 0$. □

[65] Our formal analysis below can be extended to show that, actually, the strong quasi-concavity assumption already assures that at most one equilibrium can exist in the effort game.

Define $s_\Delta(\omega) \equiv s_1(\omega) - s_2(\omega)$. Note that for sufficiently large $\delta > 0$ (which we assumed), $s_\Delta(\omega) \in [0, \delta - \bar{\omega})\ \forall \omega \in [0, \bar{\omega}]$.

Lemma S2 *Suppose that g_0 is strongly indecisive (polarized) on $supp\, g_0$. For any given $x \in [0, \delta - \bar{\omega})$, $g(x, \omega)$ is strictly decreasing (increasing) in ω.*

Proof: The requirement $x \in [0, \delta - \bar{\omega})$ assures that $supp\, g_\omega(x) \subset (-\delta, \delta)$ for any $\omega \in (0, \bar{\omega}]$. The claims follow from the proof of Theorem 2 by replacing $g_{\mathbf{a}'}(0)$ with $g(x, \omega')$ and $g_{\mathbf{a}}(0)$ with $g(x, \omega)$ in (A.3). □

Lemma S3 *Suppose that g_0 is strongly indecisive (polarized) on $supp\, g_0$. For any given $x \in (0, \delta - \bar{\omega})$, $\omega, \omega' \in [0, \bar{\omega}]$ with $\omega < \omega'$, $G(x, \omega') < (>) G(x, \omega)$.*

Proof: The claim follows from Lemma S2 because

$$G(x, \omega) = \frac{1}{2} + \int_0^x g(s, \omega) ds > \frac{1}{2} + \int_0^{sx} g(s, \omega') ds = G(x, \omega'). \qquad \square$$

Lemma S4 *If g_0 is strongly indecisive on $supp\, g_0$, then $s_1'(\omega), s_2'(\omega) < 0$. If g_0 is strongly polarized on $supp\, g_0$, then $s_1'(\omega), s_2'(\omega) > 0$*

Proof: The assumptions imposed in the current model assure that (S.19) is a system of C^1 functions, so by the Implicit Function Theorem, we have for each $j = 1, 2$,

$$s_j{}'(\omega) = \frac{C_{-j}{}''}{A} \frac{\partial g(s_\Delta, \omega)}{\partial \omega}, \qquad A \equiv C_1{}'' C_2{}'' + (C_1{}'' - C_2{}'') \frac{\partial g(s_\Delta, \omega)}{\partial s_\Delta},$$

where $A > 0$ is implied by strong quasi-concavity. Hence $sign\, s_j'(\omega) = sign \left(\frac{\partial g(s_\Delta, \omega)}{\partial \omega} \right)$, and the claim for the indecisive case follows because $g(s_\Delta, \omega)$ is strictly decreasing in ω by Lemma S2.[66] The claim for the polarized case holds because then $g(s_\Delta, \omega)$ is strictly increasing in ω. □

[66]Strictly spoken, the strict monotonicity in Lemma S2 allows us only to conclude that $\frac{\partial g(s_\Delta, \omega)}{\partial \omega} \leq 0$. As $\frac{\partial g(s_\Delta, \omega)}{\partial \omega} = 0$ can never occur on any arbitrary small interval around ω, we ignore the knife-edge case where $\frac{\partial g(s_\Delta, \omega_0)}{\partial \omega_0} = 0$ for some ω_0.

Finally, we apply the Envelope Theorem to see how equilibrium payoffs respond to marginal changes of ω:

$$\Pi_1{}'(\omega) = -g(s_1 - s_2, \omega)s_1{}'(\omega) + \frac{\partial G(s_1 - s_2, \omega)}{\partial \omega},$$
$$\Pi_2{}'(\omega) = -g(s_1 - s_2, \omega)s_2{}'(\omega) - \frac{\partial G(s_1 - s_2, \omega)}{\partial \omega}. \quad (S.20)$$

Because $g(\cdot) > 0$, Lemmas S3 and S4 imply that $\Pi_2(\omega)$ is strictly increasing (decreasing) on $[0, \bar{\omega}]$ if g_0 is strongly indecisive (strongly polarized). ■

The comparative statics (S.20) shows that agent confusion increases equilibrium payoffs by a competition-softening effect. In the symmetric model, where $s_1 = s_2$ and, accordingly, $G(s_1 - s_2, \omega) = 1/2$, this is the only force, explaining why both firms unambiguously benefit from agent confusion. By contrast, the market share effect always works in opposite directions for the two contestants whenever $G(s_1 - s_2, \omega) \in (1/2, 1)$, showing a potential conflict of interest in the contestants' desire for confusion or education. Nevertheless, it turns out that the sensitivity and market share effects always work in the same direction for the weak, and always in opposite directions for the strong contestant. In particular, the weak (strong) contestant always gains (loses) market share if ω increases in case of indecisive preferences, and vice-versa in case of polarized preferences.

S.7 Ex-Interim Asymmetry

In this section, we consider a simple extension allowing for the possibility that biased perception errors $\varepsilon_{\mathbf{a}}$, leading to ex-interim asymmetries of the contestants, may result as a consequence of the chosen communication strategies. Suppose that $A = \{0, 1\}$, so that $\mathcal{A}$ consists of four ordered pairs, associated with four random variables $\varepsilon_{\mathbf{a}}$, each with *supp* $(g_{\mathbf{a}}) \subset$ *supp* (g_0). Then, $a_j = 0$ means that j communicates in a neutral way, while $a_j = 1$ means that j's communication is obfuscating with the possible effect of biasing perception towards the firm. We assume that $\varepsilon_{0,0} = O$, $\varepsilon_{1,1} \neq O$ is zero-symmetric, and $\varepsilon_{1,0} = -\varepsilon_{0,1}$. The symmetry of $\varepsilon_{1,1}$ captures that a communication profile where *both* contestants seek to bias valuations in their favor (e.g. both exaggerate the valuations of their offers) results in unbiased agent confusion.[67]

[67]For example, $\varepsilon_{1,1}$ always is zero-symmetric if $\varepsilon_{1,1} = \varepsilon^2 - \varepsilon^1$ and $\varepsilon^1, \varepsilon^2$ are iid. In particular, let $\varepsilon_{\mathbf{a}} \equiv \varepsilon^2_{a_2} - \varepsilon^1_{a_1}$, where $\varepsilon^1_{a_1}, \varepsilon^2_{a_2}$ are independent. Set $\varepsilon^j_0 = O$, and let ε^2_1 be uniform on $[0, \omega]$, and ε^1_1 be

		Contestant 2	
		$a = 0$	$a = 1$
Contestant 1	$a = 0$	Π_{00} , Π_{00}	Π_{01} , Π_{10}
	$a = 1$	Π_{10} , Π_{01}	Π_{11} , Π_{11}

Figure S.4: Equilibrium payoffs in effort stage game

Suppose that for any such given $\varepsilon_{\mathbf{a}}$ the effort subgame following $\varepsilon_{\mathbf{a}}$ has a unique effort equilibrium, with corresponding equilibrium payoffs indicated by $\Pi_{\mathbf{a}}$, as depicted in the game matrix of Figure S.4.

In any SPE, the choice of communication profile must induce a Nash equilibrium in that game. If $\varepsilon_{1,0} = \varepsilon_{0,1}$ is zero-symmetric and $\varepsilon_{1,0} \neq O$, this is just a special case of our main model and, by Theorem 1, preferences alone are decisive for the type of SPE that results. It is easily observed that the same equilibrium pattern holds if the bias induced by $a_j = 1$ is "weak" in the sense that $\varepsilon_{1,0}$ remains close to zero-symmetrically distributed. Then, the unilateral advantage of a biased communication profile is dominated by the perception noise it induces. As long as $\Pi_{00} < (>) \Pi_{10}$ continues to hold, education can never be (always is) an SPE in case of indecisive (polarized) preferences.

If perception errors are strongly biased in favor of $j = 1$ if $a = (1, 0)$ is chosen (and $j = 2$ is equally favored for $a' = (0, 1)$), it becomes conceivable that $\Pi_{10} > \Pi_{00} > \Pi_{01}$. This reflects a redistribution of the perceived match advantages in favor of $j = 1$. Consider first the case of indecisive preferences. Because a potential bias in favor of the competitor can be annihilated by choosing $a = 1$, and pure perception noise is beneficial with indecisive preferences, it follows that $\Pi_{11} > \Pi_{01}$. Thus, the only equilibrium involves mutual obfuscation ($a_1 = a_2 = 1$), similar to the case with unbiased perception errors.

Now consider the case of polarized preferences, meaning that $\Pi_{00} > \Pi_{11}$. The type of SPE now depends crucially on the obfuscation technology. In particular, full agent education remains the unique equilibrium prediction if consumer education can be unilaterally enforced. Then, $\varepsilon_{\mathbf{a}} = O$ whenever $\mathbf{a} \neq (1, 1)$, meaning that both contestants earn Π_{00}, whenever at least one contestant chooses $a = 0$. Then, any $\mathbf{a} \neq (1, 1)$ constitutes a Nash equilibrium in the first stage, and agents are fully educated. If however,

uniform on $[-\omega, 0]$, $\omega > 0$. Note that then $\varepsilon_{1,1}$ has a zero-symmetric density (a "tent" distribution) on $[-\omega, \omega]$.

education cannot be unilaterally enforced, $\Pi_{10} > \Pi_{00}$ implies that agent education cannot be sustained as an SPE outcome. The type of equilibrium then depends on whether $\Pi_{01} > \Pi_{11}$ or $\Pi_{01} < \Pi_{11}$. If $\Pi_{01} < \Pi_{11}$, correcting the bias is more valuable than the loss in payoffs incurred from mutual obfuscation due to polarized tastes. Then, the contestants are trapped in a Prisoner's Dilemma, with mutual obfuscation as the sole equilibrium outcome, which is both inefficient and harmful for the contestants. If instead $\Pi_{01} > \Pi_{11}$, then the contestants end up in a Coordination game with two (pure-strategy) equilibria, where one contestant earns the rents from a biased communication profile. In sum, agent preferences play a decisive role for the equilibrium outcome also with potentially biased communication strategies. In particular, neither a Prisoner's Dilemma nor a Coordination game can arise with indecisive tastes.