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**UNEXPECTED EFFECTS:
UNCERTAINTY, UNEMPLOYMENT, AND
INFLATION**

Pontus Rendahl and Lukas Freund

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Abstract

This paper studies the role of uncertainty in a search-and-matching framework with risk-averse households. A mean-preserving spread to future productivity contracts current economic activity even in the absence of nominal rigidities, although the effect is reinforced by the presence of the latter. That is, uncertainty shocks carry both contractionary demand- and supply effects. The reason is that a more uncertain future increases the precautionary behavior of households, which reduces interest rates and contracts demand. At the same time, as future asset prices becomes more volatile and positively covary with aggregate consumption, households demand a larger risk premium, which puts negative pressure on current asset values and thereby contracts supply. Thus, in comparison to a pure negative demand shock, an uncertainty shock puts less deflationary pressure on the economy and, as a result, renders a flatter Phillips curve.

JEL Classification: J64, E21, E32

Keywords: uncertainty, unemployment, inflation, search frictions

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Unexpected Effects: Uncertainty, Unemployment, and Inflation*

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April 22, 2020

Abstract

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1 Introduction

The effects of uncertainty on macroeconomic activity are evasive. While empirical studies generally conclude that elevated uncertainty leads to a decline in economic activity, the theoretical literature is more contentious.¹ The primary reason is that uncertainty triggers a rise in the precautionary motive to save, which, in equilibrium, can lead to an investment boom, and thereby *expand*, instead of *contract*, economic activity. Indeed, [Basu and Bundick \(2017\)](#) argue that an increase in uncertainty is expansionary unless nominal rigidities divert the increase in desired saving into a decrease in goods demand, which thereby causes a contraction in economic activity.²

This paper revisits these theoretical aspects using a search-and-matching (SaM) framework with risk-averse households, both in the presence and absence of nominal rigidities.³ In this setting, a rise in uncertainty lowers economic activity even when prices are flexible. The reason is that increasing uncertainty – modeled as a perceived rise in the volatility of future aggregate labor productivity – generates a strong negative comovement between the marginal utility of consumption and the firm-value (or equity price). This negative comovement causes a rise in the risk premium for equity and thereby lowers asset prices.⁴ With lower asset prices there is less entry of firms, higher unemployment, and a decline in output. The pronounced negative comovement between asset returns and the marginal utility of consumption arises because of the long-horizon valuation of a firm; as firms pay a fixed cost to enter the market, they obtain quasi-rents for an extended period of time, and their (equity) value therefore depends on the entire expected sequence of future productivity. This long-horizon valuation of firms contrasts markedly with a standard real business cycle (RBC) model in which returns, *ceteris paribus*, are unaffected by productivity beyond the immediate future (cf. [Barro and King \(1984\)](#)). Thus, the rise in the risk premium materializes because of a large increase in the expected volatility of asset prices, not because of a large increase in the volatility of the marginal utility of consumption.

However, the above mechanism does not operate in isolation. Indeed, under flexible prices

¹See, among many others, [Jurado et al. \(2015\)](#), [Baker et al. \(2016\)](#), [Leduc and Liu \(2016\)](#) and [Bloom et al. \(2018\)](#). [Bloom \(2014\)](#) and [Fernández-Villaverde and Guerrón-Quintana \(2020\)](#) provide excellent surveys. Although the effect of uncertainty on *real* economic activity is empirically well established, the impact on inflation remains contested. [Fernández-Villaverde et al. \(2015\)](#), [Leduc and Liu \(2016\)](#), [Basu and Bundick \(2017\)](#), and [Oh \(2019\)](#) find that inflation tends to fall after an uncertainty shock. [Meinen and Roehle \(2018\)](#) find the reaction of prices to be ambiguous. [Alessandri and Mumtaz \(2019\)](#) argue that uncertainty shocks are inflationary in normal times, although disinflationary during financial crises. In view of such mixed results, [Castelnuovo \(2019\)](#) concludes in a survey that more work is needed to understand the response of inflation to uncertainty shocks.

²[Basu and Bundick \(2017, p. 941\)](#) state “[...] higher uncertainty increases desired saving. Higher desired saving is expansionary if full employment of all factors is guaranteed, but may be contractionary if output is demand-determined,” which requires a slow adjustment of prices.

³See [Diamond \(1982\)](#), and [Mortensen and Pissarides \(1994\)](#).

⁴We refer to the *firm value*, *equity price*, and *asset price* interchangeably. Interest rates will be directly referred to as such.

there are two countervailing forces that dampen, but do not overturn, the main result. First, and as previously mentioned, a rise in uncertainty is associated with a precautionary motive to save. This motive operates in the opposite direction to the risk premium, as prudent agents value *all* assets – including equity – more highly when faced with a more uncertain future. The result is upward pressure on asset prices, as well as downward pressure on the risk-free real interest rate. Second, SaM models exhibit an asymmetry in the labor market such that larger fluctuations in economic activity lead to higher (average) unemployment (Hairault *et al.*, 2010).⁵ The primary reason is that the measure of *newly* employed workers depends on the product of the job finding rate and the amount of job seekers. As these components covary negatively over the cycle, more uncertainty leads to a higher expected future unemployment rate, which thereby lowers current consumption. Simply put, recessions are worse than booms are good. With preferences that exhibit a desire to smooth consumption over time, this latter feature reinforces the motive to save through an *asymmetry-discounting channel*, putting further upward pressure on asset prices and, again, reducing the risk-free rate.

Price rigidities add several twists to this story. First, the precautionary motive now operates in the opposite direction, *destabilizes* the equilibrium, and amplifies the decline in economic activity. The reason is familiar. With an increased desire to save, the willingness to purchase consumption goods falls, and firms operating under monopolistic competition reduce their prices in an attempt to restore demand. With nominal rigidities, these price adjustments are insufficient to maintain the original equilibrium, however, and there is a decline in output alongside a decline in inflation. Second, the asymmetry-discounting channel – which helps to stabilize the equilibrium under flexible prices – turns into a destabilizing *asymmetry-demand channel*, in which the motive to smooth consumption over time operates analogously, but not identically, to the precautionary channel, and hence contracts demand even further. Thus, with sticky prices, the three identified mechanisms work in the same direction, and the contraction in output correspondingly becomes markedly larger. The long-horizon valuation of firms is again a contributing factor, as persistent shocks lead to future contractions, which again feeds in to current asset prices, and magnifies the response in the present.

Our approach to disentangling the (simultaneously) operating transmission mechanisms reveals that uncertainty shocks are distinct from other disturbances to the economy, both from an observational point of view, as well for the conduct of macroeconomic policy. First, uncertainty shocks contain *both* contractionary demand and supply components. As a result, they render a flatter Phillips curve compared to analogous, regular, demand shocks.⁶ This result obtains because of the supply component – the risk premium channel. To understand why, envisage the consequences

⁵See also Jung and Kuester (2011), Petrosky-Nadeau *et al.* (2018), and Dupraz *et al.* (2019).

⁶Two notes are in place: First, with the Phillips curve we refer to the *observed* relationship between inflation and unemployment, not to the parameters of a structural equation. Second, we model a demand shock as a persistent rise in the nominal interest rate that gives rise to identical dynamics in terms of real economic activity as uncertainty shocks.

of a regular negative demand shock. In response to a contraction in demand, firms reduce their prices to restore the original equilibrium. With nominal rigidities, however, this adjustment is incomplete, and the result is a contraction in both demand and supply, with lower inflation. But if the risk premium simultaneously increases, asset prices fall, fewer firms enter, and supply contracts even in the absence of any changes in prices. Thus, with supply already partly contracted, the reduction in inflation required to restore the equilibrium is smaller. As a consequence, an increase in uncertainty leads to a reduction in output akin to a demand shock, but with a less pronounced fall in inflation.⁷ Second, from a policy perspective we show that a sufficiently aggressive response from the monetary authority can, under certain conditions, mimic the flexible price outcome following an uncertainty shock (cf. [Blanchard and Galí, 2007](#)). However, we also reveal the existence of uncomfortable tradeoffs between stabilizing output and inflation in the present and the expected future. These can have especially profound implications when price adjustments are costly in terms of economic resources.⁸

Our contribution is related to two distinct literatures, pertaining respectively to SaM models with risk-averse households and nominal rigidities, as well as the role of uncertainty in shaping the dynamics of economic aggregates. With respect to the former, our model closely follows that of [Leduc and Liu \(2016\)](#), with a few important differences.⁹ First, as emphasized in [Den Haan *et al.* \(2020\)](#), [Leduc and Liu's \(2016\)](#) analysis partly relies on an hitherto unexplored mechanism, through which uncertainty gives rise to a (small) contraction in economic activity even with flexible prices and risk neutrality. [Den Haan *et al.* \(2020\)](#) show that this mechanism operates via forward-looking wage-setting in conjunction with the nonlinearities of the matching function (but not through option-value considerations, as conjectured in [Leduc and Liu \(2016\)](#)). Thus, to confine attention to the key channels at work, wages are determined through alternating offers instead of Nash bargaining; an assumption that eliminates the forward-looking nature of wage-setting, and suppresses this channel entirely (cf. [Hall and Milgrom \(2008\)](#)). Second, given the results in [Den Haan *et al.* \(2020\)](#), we put a much stronger emphasis on uncovering the principles and mechanisms at play in this class of models, instead of a quantitative analysis that aligns the model with the data as closely as possible.

The literature on uncertainty shocks is too vast to comprehensively discuss here, but a few remarks are warranted to contextualize our analysis.¹⁰ [Basu and Bundick \(2017\)](#) analyze an

⁷Indeed, under plausible parameter configurations the risk premium channel is strong enough relative to demand forces to cause a *rise* in inflation.

⁸[Lepetit \(2020\)](#) explores the interaction of search frictions and nominal rigidities and highlights a significant trade-off between inflation and unemployment stabilization when the economy is exposed to shock to the level of productivity.

⁹Our analysis is, furthermore, consistent with [Hall's \(2017\)](#) argument – developed within the context of the SaM paradigm – that fluctuations in financial discounts represent an important driver of unemployment dynamics. We specifically highlight the role of time-varying uncertainty about the future as a factor behind such fluctuations.

¹⁰A by no means exhaustive list of references in addition to the ones mentioned below includes: [Bloom \(2009\)](#); [Andreasen \(2012\)](#); [Bachmann and Bayer \(2013\)](#); [Mongey and Williams \(2017\)](#); [Ghironi and Ozhan \(2019\)](#); [Bonciani and Oh \(2019\)](#); [Sedláček \(2019\)](#); [Berger *et al.* \(2019\)](#) and [Bachmann *et al.* \(2019\)](#).

otherwise standard RBC model with uncertainty shocks. They show that even with recursive preferences and a large coefficient of risk aversion, the model predicts an expansion in response to an increase in uncertainty. The reason is that the precautionary motive is sufficiently strong to generate an investment boom, and that nominal rigidities are necessary to divert the associated decline in consumption demand into a fall in output.^{11,12} In contrast, the framework explored in this paper relies on the idea that firms are long-lived, and that persistent shocks therefore significantly affect asset values. The result is a large positive correlation between consumption and asset returns that increases the risk premium, and contracts output even when prices are flexible. Moreover, in the presence of nominal rigidities, the asymmetric employment dynamics inherent in SaM models give rise to an additional channel that amplifies the decline in output further. As this latter aspect does not rely on a convex marginal utility it reflects a distinct mechanism from prudence. Relatedly, [Born and Pfeifer \(2020\)](#) study the conditionally countercyclical comovement between markups and uncertainty that is integral to the mechanism explored by [Basu and Bundick \(2017\)](#) and others in the data and find that the only margin along which (price) markups increase following an uncertainty shock is the extensive labor margin, indicating the potential relevance of SaM models with nominal rigidities for analyzing the effects of uncertainty.

Lastly, [Fernández-Villaverde *et al.* \(2011\)](#) analyze a flexible price, small open economy for emerging markets, and show that uncertainty with respect to the exogenous real interest rate exerts a contractionary effect on output. The reason is that greater volatility of the exogenous real interest rate makes foreign debt less attractive. And as foreign debt act as a hedge against idiosyncratic, domestic, productivity shocks, uncertainty makes investment in domestic capital less appealing, and output subsequently contracts.¹³ In similarity to the framework analyzed in this paper, [Fernández-Villaverde *et al.* \(2011\)](#) show that volatility in exogenous interest rates give rise to a risk premium on domestic investment. In contrast, however, they rely on sizeable fluctuations in exogenous interest rates, that characterize emerging markets to a larger extent than developed economies. This papers considers a closed economy in which interest rates are endogenously determined, and that exhibits volatility due to the uncertainty of the underlying fundamentals.

Focusing our analysis on a relatively simple theoretical model – rich enough to capture uncertainty effects on both unemployment and inflation but omitting a manifold of quantitatively relevant features – is integral to delivering the intended value-added; to provide a deeper understanding of the mechanisms operating in SaM models when exposed to uncertainty shocks, revealing their

¹¹When labor supply and demand clears on a spot market, the equilibrium hours worked increase, since greater uncertainty strengthens a general desire to self-insure, including through an expansion in “precautionary labor supply”.

¹²This mechanism rely strongly on the feature of “countercyclical markups”, which has been thoroughly explored in the literature; see, for instance, [Born and Pfeifer \(2014a\)](#), [Fernández-Villaverde *et al.* \(2015\)](#), [Castelnuovo and Pellegrino \(2018\)](#), [Cesa-Bianchi and Fernandez-Corugedo \(2018\)](#), and [Bianchi *et al.* \(2019\)](#).

¹³[Christiano *et al.* \(2014\)](#) and [Chugh \(2016\)](#) instead confine attention to the interaction between uncertainty and credit frictions, and show that this can have profound consequences.

empirical and potential policy implication.¹⁴ We show that uncertainty shocks are neither akin to aggregate demand shocks nor are best described by focusing on supply-side effects only; indeed, they carry direct implications for both, leading to a flatter Phillips curve. This insight may bear on the interpretation of phenomena in the data such as the “missing disinflation” puzzle highlighted by Hall (2011); and it may also affect the appropriate stance of monetary policy.

2 Theoretical Framework

The economy is populated by a unit measure of households; a competitive sector of intermediate goods firms producing a homogeneous input good; a monopolistically competitive sector of retail good producers, whose outputs are aggregated in a competitive final goods sector; and a monetary authority which sets the policy interest rate.¹⁵ Real quantities are defined in terms of the final good, and are – unless otherwise stated – denoted by lower case letters. Time is discrete and denoted $t = 0, 1, 2, \dots$

2.1 Households

In a given period t , a household can either be employed, n_t , or unemployed, u_t . The market for idiosyncratic employment risk is complete and the representative household – or simply *the household* – is comprised of a measure of n_t members that are working, and u_t members that are not. Non-employed members of the household may find a job even within the period they get displaced. Thus, the measure of the household’s members that are searching for a job in the beginning of a period is $u_t^s = u_{t-1} + \delta n_{t-1}$, where δ denotes an exogenous separation rate. The measure of employed individuals working in period t is therefore given by $n_t = f_t u_t^s + (1 - \delta)n_{t-1}$, where f_t denotes an endogenously determined job finding rate. The real wage is denoted w_t and, as there is no home production, total labor income is given by $w_t n_t$.

¹⁴There are many other potential channels through which uncertainty may impact economic activity that the model considered here does *not* allow for. To mention but a few, real options effects are absent in the setting considered here but present in Schaal’s (2017) model. By modeling nominal rigidities as arising from Rotemberg adjustment costs we largely abstract from precautionary pricing by product-price and wage setters that arise under Calvo prices (cf. Born and Pfeifer (2014a), Fernández-Villaverde *et al.* (2015), and for the important distinction between Rotemberg and Calvo foundations for the nonlinear new-Keynesian Phillips curve, Oh (2019)). And Cacciatore and Ravenna (2018) foreground the consequences of occasionally binding constraints on downward wage adjustment in generating state-dependent amplification of uncertainty shocks, a mechanism that would complement the effects we draw attention to in a setup with flexible wages.

¹⁵In the regular new-Keynesian (NK) literature *retail firms* are normally instead referred to as *intermediate goods firms* (see, for instance, Chapter 2 in Gali (2015)). However, as models with an underlying SaM structure uses an additional layer of firms/goods, we follow the terminology of Ravenna and Walsh (2008), Leduc and Liu (2016), and many others, and refer to these firms as retail firms.

In addition to labor income, the household enters the period with nominal bonds, B_{t-1} , and equity a_{t-1} . Equity is valued at the cum-dividend price J_t . However, as a fraction, δ , of firms goes out of business in each period, the total value of the household's equity position is $J_t a_{t-1} (1 - \delta)$. The household also receives profits from several other sources. Since none of these cannot affect, nor be affected by, the household's decisions, we summarize their total profits in the variable \tilde{d}_t , which is, for the moment, treated as given (see section 2.5 for a more detailed description).

The household may use the resources available in period t – i.e. labor income, bond and equity holdings, and the additional profits – to either consume the final good, c_t ; purchase new equity, a_t , at the ex-dividend price $J_t - d_t$; or purchase nominal bonds, B_t , at the price $1/(P_t R_t)$, where P_t denotes the aggregate price level, and R_t the gross nominal interest rate.

Thus, the budget constraint of the household is

$$c_t + a_t(J_t - d_t) + \frac{B_t}{P_t R_t} = w_t n_t + \tilde{d}_t + \frac{B_{t-1}}{P_t} + a_{t-1}(1 - \delta)J_t, \quad t = 0, 1, 2, \dots, \quad (1)$$

where a_{-1} and B_{-1} are given.

Subject to the above budget constraint, the household decides on a process, $\{c_t, a_t, B_t\}_{t=0}^{\infty}$, to maximize the expected present discounted value of lifetime household utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (2)$$

where E_0 denotes the mathematical expectation operator conditional on time $t = 0$ information; the parameter $\beta \in (0, 1)$ represents the subjective discount factor, and the period utility function, $u(\cdot)$, satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The first order conditions associated with the household's problem are given by a bond Euler equation

$$u'(c_t) = \beta E_t \left[\frac{R_t}{\Pi_t} u'(c_{t+1}) \right], \quad (3)$$

as well as an Euler equation for equity

$$u'(c_t) = \beta E_t \left[\frac{J_{t+1}(1 - \delta)}{J_t - d_t} u'(c_{t+1}) \right]. \quad (4)$$

Rearranging the latter and defining $\Lambda_{t,t+1} = \beta u'(c_{t+1})/u'(c_t)$ as the stochastic discount factor gives the asset pricing equation

$$J_t = d_t + E_t [\Lambda_{t,t+1} J_{t+1} (1 - \delta)]. \quad (5)$$

This asset pricing equation for equity will play an integral part of the equilibrium outcome, as intermediate goods producing firms generate dividends $d_t = x_t z_t - w_t$, where z_t denotes the marginal product of a worker, and x_t the relative price of intermediate goods. Thus, if intermediate goods producers generate a dividend process $\{d_t\}_{t=0}^{\infty}$, their asset price, or firm value, is determined by equation (5). This asset price will, in turn, determine firm entry, as discussed in the next section.

Lastly, we define the risk-free real interest rate as

$$R_t^{rf} = \frac{1}{E_t[\Lambda_{t,t+1}]}, \quad (6)$$

and the risk premium on equity as

$$RP_t = \frac{E_t[J_{t+1}](1 - \delta)}{J_t - d_t} - R_t^{rf}. \quad (7)$$

2.2 Firms

2.2.1 Intermediate goods producers

There is a large number of potential intermediate goods producing firms, but a finite measure of operating (or active) firms. The firms use labor as the only input to production in a constant returns to scale technology, producing a homogenous good. Thus, without any loss of generality we assume that each active firm employs precisely one worker. As a consequence, the measure of active intermediate firms equals the employment rate, n_t .

An active firm produces z_t units of intermediate goods, where z_t represents a workers marginal product. These goods are sold to final goods firms at price x_t , and the firms pay workers the wage w_t . Hence, each intermediate good firms generate (real) profits of $x_t z_t - w_t$. As a consequence, the value of an intermediate good producing firm is given by

$$J_t = x_t z_t - w_t + E_t[\Lambda_{t,t+1} J_{t+1} (1 - \delta)]. \quad (8)$$

Potential intermediate goods firms may enter the market by posting a vacancy. The (marginal) cost of posting a vacancy is denoted κ , which result in the firm meeting a searching household with probability h_t . Thus, the free-entry condition is given by

$$\kappa = h_t J_t. \quad (9)$$

We assume that the aggregate resources devoted to vacancy-posting – i.e. κv_t , where v_t denotes the aggregate amount of vacancies posted in period t – is rebated back to the households. That is, the households are assumed to own the “vacancy-posting agencies.”

Lastly, there are exogenous stochastic processes for both labor productivity, z_t , and the standard deviation of labor productivity shocks, $\sigma_{z,t}$. Both are modeled as AR(1) processes:

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \sigma_{z,t-1} \varepsilon_{z,t}, \quad (10)$$

$$\sigma_{z,t} = (1 - \rho_\sigma)\sigma_z + \rho_\sigma \sigma_{z,t-1} + \varepsilon_{\sigma_{z,t}}. \quad (11)$$

Importantly, the standard deviation of the innovation to productivity, $\varepsilon_{z,t}$, is time-varying. The parameters $\rho_z \in (-1, 1)$ and $\rho_\sigma \in (-1, 1)$ measure the persistence of the first- and second-moment shocks, respectively. Additionally, σ_z is the steady-state value of the standard deviation of the innovation to productivity. Both shocks $\varepsilon_{z,t}$ and $\varepsilon_{\sigma_{z,t}}$ are normally distributed with σ_{ε_z} set to unity; $\sigma_{\varepsilon_\sigma}$ will be calibrated.¹⁶

2.2.2 Final and retail goods producers

Final goods firms are perfectly competitive and use retail goods as the only input. However, as retailers operate under monopolistic competition, they take into account the demand schedule implied by the final goods firms' optimal production decisions. We therefore discuss both sectors under the same section, starting with the final goods producers.

Final goods producers. The final consumption good, y_t , is produced using a constant elasticity of substitution (CES) production function according to

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

where $y_t(i)$ denotes the retail good produced by firm i , with $i \in [0, 1]$. The parameter η denotes the elasticity of substitution between the differentiated retail goods.

Let $p_t(i)$ denote the relative price associated with retail good i . The optimization problem facing

¹⁶This specification of the stochastic processes is common in the literature but deviates from [Leduc and Liu \(2016\)](#) in two respects. First, the process for z_t is in levels rather than logs to prevent the expected value of productivity to be different from the deterministic steady-state value through a Jensen's inequality effect. Second, under the timing assumption in equation (10), which is common in the uncertainty shock literature (e.g., [Bloom \(2009\)](#) or [Schaal \(2017\)](#)), volatility shocks have a delayed impact on the distribution of labor productivity shocks. We observe that the level process (11) (which is common, see e.g., [Fernández-Villaverde et al. \(2011\)](#)) does not restrict $\sigma_{z,t}$ from taking on negative values; this is strictly speaking inconsistent with the definition of a standard deviation. In practical applications, this is not of material concern, however, because the policy functions are locally approximated around the positive deterministic steady-state value $\bar{\sigma}_z$. Accordingly, using a log process for $\sigma_{z,t}$ instead, turns out to produce no noticeably different results.

the final goods producers is then given by

$$\max_{y_t(i)_{i \in [0,1]}} \left\{ P_t y_t - \int_0^1 p_t(i) y_t(i) \right\},$$

where P_t denotes the aggregate price level/index.

The first order conditions to this optimization problem give rise to the demand schedule

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\eta} y_t, \quad (12)$$

with the associated price index

$$P_t = \left(\int_0^1 p_t(i)^{\frac{1}{1-\eta}} di \right)^{1-\eta}.$$

Retail goods producers. Differentiated retail goods are produced using the homogeneous intermediate good as the single input. The technology is such that one unit of the intermediate good produces one unit of the retail good. As the relative price of the intermediate good *in terms of the final good* is given by x_t , retailers make per-period profits¹⁷

$$\frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i). \quad (13)$$

Since we at times will consider a situation in which retailers cannot adjust prices frictionlessly, but only may do so by incurring a cost, a more general formulation for the retailers profits is given by

$$\hat{d}_t = \frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i) - \frac{\Omega_p}{2} \left(\frac{p_t(i)}{p_{t-1}(i)\Pi} - 1 \right)^2 y_t, \quad (14)$$

where $\Pi_t = P_t/P_{t-1}$ denotes the gross inflation rate. Thus, the period profits \hat{d}_t nest equation (13) in the special case of $\Omega_p = 0$.

Using a pricing relation analogous to equation (5), but denoting the asset price of retailers as $\hat{V}_t(p_t(i))$ yields

$$\hat{V}_t(p_t(i)) = \hat{d}_t + E_t [\Lambda_{t,t+1} \hat{V}_{t+1}(p_{t+1}(i))]. \quad (15)$$

Taking into account the demand schedule in equation (12), as well as the definition of the per-period profits in equation (14), the first order condition associated with optimizing the firm value above is

¹⁷We can equivalently think of x_t as the real marginal cost facing the retailer.

given by the new-Keynesian Phillips curve

$$x_t = \frac{\eta - 1}{\eta} + \frac{\Omega_p}{\eta} \left\{ \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right) - E_t \left[\Lambda_{t,t+1} \frac{y_{t+1}}{y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right] \right\}, \quad (16)$$

in which we have assumed symmetry, such that $p_t(i) = P_t$.

As previously mentioned, we assume that the aggregate resources devoted to price changes – the last term in equation (14) – are rebated back to the households. That is, the households are assumed to own the “price-adjusting agency.”

2.3 Labor markets

As already discussed in Section 2.1 the measure of unemployed workers searching for a job in period t is given by $u_t^s = u_{t-1} + \delta n_{t-1}$. And as discussed in Section 2.2.1 there is a measure v_t of aggregate vacancies posted by intermediate goods firms. Matches in the labor market, M_t , are then determined according to a standard Cobb-Douglas function,

$$M_t = \psi (u_t^s)^\alpha (v_t)^{1-\alpha}, \quad (17)$$

where $\alpha \in (0, 1)$ denotes the elasticity of matches with respect to job seekers u_t^s and ψ scales the matching efficiency. The implied hiring rate, h_t , is therefore

$$h_t = \frac{M_t}{v_t} = h(\theta_t) = \psi \theta_t^{-\alpha}, \quad (18)$$

where θ indicates labor market tightness which is given by

$$\theta_t = \frac{v_t}{u_t^s} = \frac{v_t}{1 - (1 - \delta)n_{t-1}}. \quad (19)$$

Analogously, the job finding probability for a searching worker is given by

$$f_t = \frac{M_t}{u_t^s} = f(\theta_t) = \psi \theta_t^{1-\alpha}. \quad (20)$$

Notice that $h(\theta)$ is strictly decreasing in θ while $f(\theta)$ is strictly increasing.

We can accordingly formulate the law of motion for employment as

$$n_t = f_t u_t^s + (1 - \delta)n_{t-1}, \quad (21)$$

$$= h_t v_t + (1 - \delta)n_{t-1}, \quad (22)$$

$$= M_t + (1 - \delta)n_{t-1}. \quad (23)$$

Together with the law of motion for employment, the equilibrium aggregate measure of vacancies posted in any given period, v_t , is endogenously determined as the solution to the free-entry condition in equation (9), which is here repeated to explicitly account for the relationship between the asset price, J_t , and labor market tightness, θ_t ,

$$\kappa = h(\theta_t)J_t. \quad (24)$$

2.3.1 Wage setting

Search frictions in the labor market imply that a matched firm and worker generate a joint surplus, giving rise to a situation of bilateral monopoly. This latter aspect leaves wages, without any further theory, undetermined. To this end, we consider a wage-setting protocol determined by *alternating offers*. This contrasts with the more common practice of wage-setting through *Nash bargaining*; we explain our choice below.¹⁸ Wage setting based on alternating offers stems from the observation that severing a match is not a credible threat; indeed the worker and the firm will always reach an agreement within the period the meeting occurs. Common knowledge of this feature implies that future variables bear no consequence on the currently agreed wage.

The alternating-offers game takes place in fictional time, in which each time-period is of length Δ . If the worker has the opportunity of proposing a wage, w_t , she will offer the highest possible value that the firm will accept. That is, the wage will yield the worker a maximum value of $\bar{v}_w = w_t$, and the firm a minimum value of $\underline{v}_f = x_t z_t - w_t$. However, as the firm can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the minimum value must also satisfy $\underline{v}_f = e^{-\Delta\omega} \times \bar{v}_f$, where \bar{v}_f denotes the firm's maximum value, and $e^{-\Delta\omega}$ the discount factor.

Conversely, if the firm has the opportunity of proposing a wage, w'_t , it will yield the firm a maximum value of $\bar{v}_f = x_t z_t - w'_t$, and the worker a minimum value of $\underline{v}_w = w'$. Again, as the worker can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the worker's minimum value must also satisfy $\underline{v}_w = \Delta\hat{\chi} + e^{-\Delta(1-\omega)} \times \bar{v}_w$, where $\Delta\hat{\chi}$ represents the flow utility the worker receives by not working. Notice that the worker and the firm discounts fictional time differently; a higher value of ω renders workers more patient which will play to the worker's advantage, and vice versa.

The above set-up provides six (linear) equations in six unknowns. Solving this system and letting Δ approach zero gives rise to a unique (subgame perfect) wage that is agreed upon immediately

$$w_t = \omega x_t z_t + (1 - \omega)\chi, \quad (25)$$

¹⁸See Binmore *et al.* (1986) for a discussion of the relationship between these alternative arrangements; Hall and Milgrom (2008) provide an in-depth exploration of the implications in a SaM model similar to the one considered here.

with $\chi = \hat{\chi}/(1 - \omega)$. That is, the wage agreed by alternating offers is a combination of the firm's revenues and the flow consumption-value the worker receives by delaying agreement.¹⁹

The primary reason for adopting alternating offers rather than the traditional Nash bargaining is that even when the mutually agreed-upon wages coincide in deterministic steady state, the two specifications can give rise to profoundly different dynamics in response to uncertainty shocks. In particular, Den Haan *et al.* (2020) show that the Nash-bargained wage carries a forward-looking component that has some peculiar implications when analyzing uncertainty shocks. While these issues are too intricate to be discussed here, it suffices to note that holding x_t constant (i.e. under flexible prices), the alternating offers formulation in equation (25) allows us to focus on those nonlinearities that are intrinsic to the matching process, without confounding the results from those arising from any other nonlinearities that are specific to the wage bargain, nor imposing that wages are completely rigid.

2.4 Monetary policy

The monetary authority sets the nominal interest rate, R_t , according to the Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \log\left(\frac{y_t}{y}\right). \quad (26)$$

In the presence of nominal rigidities, monetary policy can stimulate employment and production by cutting the interest rate, R_t . A lower interest rate increases demand for the final good through the bond Euler equation in (3). Increased demand for final goods leads retail firms to set higher prices and to increase demand for intermediate goods, putting upward pressure on the relative price of intermediate goods, x_t . To the extent that the increase in marginal revenues, $x_t z_t$, is not entirely offset by an increase in wages, w_t , the intermediate firms posts additional vacancies until the free-entry condition (24) is satisfied, that is, until the probability of filling a vacancy, $h(\theta_t)$, has decreased sufficiently to restore the free-entry condition.

In the case of flexible prices the above chain is broken. In particular, retail firms then adjust prices sufficiently to render the *real interest rate* unaffected (as inflation expectations change), which entirely offsets the initial increase in demand. Indeed, under flexible prices, i.e. when $\Omega_p = 0$, it is trivial to see from equation (16) that the relative intermediate goods price, x_t , is constant at $x = (\eta - 1)/\eta$, which implies that there is also no additional entry after a monetary policy intervention (that is, monetary policy is neutral). Nominal rigidities are necessary to prevent these

¹⁹This wage coincides exactly with the protocol proposed by Jung and Kuester (2011), which sets wages by maximizing the Nash product $(w_t - \chi)^\omega (x_t z_t - w_t)^{1-\omega}$. Hall and Milgrom (2008) proposes a bargaining specification that partially insulates wages from variations in labor market tightness. Equation (25) is a special case insofar as this isolation is complete. Additionally, it leaves the wage unresponsive to movements in the marginal utility of consumption.

price movements from operating fully.

2.5 Market clearing and equilibrium

Since all firms use a constant returns to scale technology – alongside with the fact that intermediate goods use only labor as an input, retail firms use only intermediate goods, and final goods firms use only retail goods – aggregate output is given by $y_t = z_t n_t$.

As mentioned in section 2.1, the household makes additional profits, \tilde{d}_t . These profits are in excess of the dividends arising from the ownership of intermediate firms, and instead include per-period profits from retailers, vacancy-posting agencies, and price adjusting firms. Aggregate profits arising from vacancy-posting agencies are equal to κv_t . Moreover, the aggregate profit arising from retailers net of price adjustment costs is

$$\frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i).$$

Using the fact that in a symmetric equilibrium $p_t(i) = p_t(j) = P_t$, alongside with the demand relation in equation (12) together with $y_t = z_t n_t$, reveals that these profits amount to $z_t n_t (1 - x_t)$. Thus,

$$\tilde{d}_t = \kappa v_t + z_t n_t (1 - x_t). \quad (27)$$

A summarizing definition of equilibrium follows.

Definition 1. A competitive equilibrium is a process of prices $\{J_t, R_t, \Pi_t, x_t, w_t\}_{t=0}^{\infty}$ and quantities $\{c_t, B_t, \theta_t, n_t, a_t\}_{t=0}^{\infty}$ such that,

- (i) $\{c_t, B_t, a_t\}_{t=0}^{\infty}$ solves the household's problem.
- (ii) Asset prices $\{J_t\}_{t=0}^{\infty}$ satisfy the asset pricing equation in (5).
- (iii) Labor market tightness, $\{\theta_t\}_{t=0}^{\infty}$, satisfies the free-entry condition $\kappa = h(\theta_t) J_t$.
- (iv) Employment, $\{n_t\}_{t=0}^{\infty}$, satisfies the law of motion
$$n_t = [(1 - n_{t-1}) + \delta n_{t-1}] f(\theta_t) + (1 - \delta) n_{t-1}.$$
- (v) Wages, $\{w_t\}_{t=0}^{\infty}$, satisfy equation (25).
- (vi) The gross nominal interest rate, $\{R_t\}_{t=0}^{\infty}$, satisfies the Taylor rule in equation (26).
- (vii) Relative prices for intermediate goods and inflation, $\{x_t, \Pi_t\}_{t=0}^{\infty}$, satisfy the Phillips curve in equation (16).
- (viii) Bond markets clear, $B_t = 0$.

(ix) *Equity market clear*, $a_t = n_t$.

(x) *Intermediate goods markets clear* $y_t(i) = z_t n_t$.

Using the equilibrium relations $B_t = 0$ and $n_t = a_t$, the household's budget constraint is

$$c_t + n_t(J_t - d_t) = w_t n_t + \tilde{d}_t + n_{t-1}(1 - \delta)J_t.$$

Rearranging and using that fact that $d_t = x_t z_t - w_t$ gives

$$c_t + J_t(n_t - n_{t-1}(1 - \delta)) = n_t x_t z_t + \tilde{d}_t.$$

Using the law of motion for employment in equation (22), and the definition of \tilde{d}_t above reveals that

$$c_t + \kappa v_t = n_t x_t z_t + \kappa v_t + z_t n_t (1 - x_t),$$

or simply $y_t = c_t = z_t n_t$.

Notice that aggregate consumptions is therefore not affected by the amount of vacancies created, nor the costs associated with price adjustments. This is indeed intentional; as we are exploring the role of uncertainty on behavior, any resource draining activity, such as price adjustments, may, somewhat mechanically, alter the marginal utility of consumption. We will explore the role of such activities in Section 4.2.2 and the appendix.

2.6 Numerical implementation

Below we outline the benchmark parameterization of the model, which largely follows that of [Leduc and Liu \(2016\)](#). We then briefly discuss the key elements of the solution method, and how the main results are illustrated.

2.6.1 Parameterization

One period in the model is equivalent to one quarter. The period utility function is given by

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma},$$

where $u(c) = \ln(c)$ if $\gamma = 1$. In the benchmark specification the coefficient of relative risk aversion γ , is set to one. This is a conservative choice as the low value of risk aversion tends to downplay the role of uncertainty, and we explore the effects of larger values for the purpose of completeness in the appendix. The discount factor β is set to 0.99 which implies an annual real interest rate of 4

percent in the steady state. We set the elasticity of substitution between differentiated retail product, η , to 10 which matches a steady state markup of 11 percent (Basu and Fernald, 1997).

Following Petrongolo and Pissarides (2001) the elasticity of the matching function, α , is set to 0.5. The matching efficiency parameter, ψ , is set to target an unemployment rate of 6.4%. According to the Job Openings and Labor Turnover Survey (JOLTS) the average monthly job separation rate is about 3.5 percent, which suggests a quarterly separation rate, δ , of about 0.1. To calibrate κ , we use the law of motion for employment in equation (22) and find that the measure of vacancies in the steady state is 0.134. Following Leduc and Liu (2016) we normalize the steady state value of labor productivity, z , to one, and then set κ such that the total cost of vacancy-posting is equal to 2 percent of steady state output.

We deviate from Leduc and Liu (2016) in some of the parameters governing wage setting. In particular, as discussed in section 2.3.1 we adopt an alternating offers framework as opposed to conventional Nash bargaining, necessitating an alternative calibration strategy. As in their work, given a steady state value of labor market tightness of $\theta = 0.848$, the free-entry condition in equation (24), alongside with the previously calibrated parameters, pins down the steady state asset value, J . Together with a steady state inverse markup equal to $x = (\eta - 1)/\eta = 0.9$, and a normalized steady state value of labor productivity $z = 1$, equation (8) implies a steady state wage, w , of 0.8782. Different from Leduc and Liu (2016), we choose the worker's bargaining weight ω and the "strike value" χ to target a steady-state elasticity of labor market tightness with respect to productivity, $\eta_{\theta,z}$, equal to 19.1 (cf. Shimer (2005, Table 1)). This elasticity is crucial in determining the magnitude of dynamic changes in the model and, for a given value of α , is directly related to the fundamental surplus fraction (see Ljungqvist and Sargent (2017)) through $\eta_{\theta,z} = xz/[\alpha[(xz - \chi)]]$. Using this relation we can back out a value of χ equal to 0.8058. The bargaining weight, ω , is then immediately pinned down by the steady state version of the wage relationship (25) as $\omega = (w - \chi)/(xz - \chi) = 0.7687$. In Section 4.3, we examine the sensitivity of our numerical results for combinations of ω and χ that generate different, and in particular, larger, values for $\eta_{\theta,z}$.

The parameter governing price stickiness, Ω_p , is set to 103.81 which gives rise to a slope of the Phillips curve that is equal to that of an implied model with Calvo pricing – solved using a first-order approximation – with a price resetting duration of four quarters. The parameters of the Taylor rule, ϕ_π and ϕ_y , are set to 1.5 and 0, respectively. Here we deviate from Leduc and Liu (2016) who set $\phi_y = 0.2$, as this choice tends to interact with assumptions about the nature of price adjustments in ways that we subject to a detailed in analysis in section 4.2.

Finally, the persistence and volatility of the productivity shock, ρ_z and σ_z , are set to the empirically relevant values of 0.95 and 0.01 respectively, which imply a standard deviation of productivity equal to 0.023. The persistence and volatility of the uncertainty shock, ρ_σ and σ_σ , are

Table 1: Calibrated parameters

Parameter	Interpretation	Value	Source/steady state target
γ	Coefficient of relative risk aversion	1	Convention
β	Discount factor	0.99	Annual real interest rate of 4%
ψ	Efficiency of matching	0.645	Unemployment rate of 6.4%
η	Elasticity of substitution	0.645	Markup of 11%
δ	Separation rate	0.1	JOLTS database
ω	Workers bargaining power	0.7687	Steady-state wage relation
α	Elasticity of $f(\theta)$	0.5	Petrongolo and Pissarides (2001)
κ	Vacancy-posting cost	0.14	2 percent of steady state output
Ω_p	Price adjustment cost	103.81	4 quarters price resetting duration
ϕ_π	Taylor rule parameter for inflation	1.5	Taylor principle/Convention
ϕ_y	Taylor rule parameter for output	0	Convention
χ	Income while delaying bargaining	0.8058	Elasticity of tightness of 19.1
ρ_z	Persistence of productivity	0.95	Leduc and Liu (2016)
ρ_σ	Persistence of uncertainty	0.76	Leduc and Liu (2016)
σ_z	St. dev. of productivity shock	0.01	Leduc and Liu (2016)
σ_σ	St. dev. of uncertainty shock	0.392	Leduc and Liu (2016)
Π	Steady state inflation rate	0.005	Annual inflation rate of 2 percent

Notes. This table lists the parameter values of the model. The calculations and targets are described in the main text. One period in the model corresponds to one quarter.

set equal to those estimated by [Leduc and Liu \(2016\)](#) using a structural vector autoregressive model, and are given by 0.76 and 0.392 respectively.

2.6.2 Solution method

We solve the model by third-order pruned perturbation. There are three reasons underlying this choice: First, a perturbation method of at least the third order is necessary to obtain policy functions that contain volatility shocks as independent arguments; that is, a third-order approximation allows the second moments of both exogenous and *endogenous* variables to affect expectations. Second, a third-order perturbation (or higher) allows us to consider the asymmetric effects that are intrinsic to SaM models such as the one considered here. Third, as a third-order perturbation method is also used in [Leduc and Liu \(2016\)](#), it is straightforward to compare results without concerns regarding computational discrepancies.

For most of our results we will follow [Fernández-Villaverde *et al.* \(2011\)](#) and [Born and Pfeifer \(2014b\)](#) and consider impulse response functions (IRFs) that isolate the *pure uncertainty* effect resulting from higher volatility. That is, we focus on the effect uncertainty has on expectations, and

how such changes in expectations trickle through to actual decisions, but ignore *materialized* shocks to the *level* of the exogenous processes. As such, we focus on the effect of uncertainty itself, and not on that of more extreme realizations of productivity shocks. To be more precise, let $f(\cdot)$ represent the policy function for, say, employment. That is, $n_t = f(n_{t-1}, z_t, \sigma_{z,t})$. The *pure uncertainty* IRF is then given by $n_{t+s} = f(n_{t+s-1}, z, \sigma_{z,t+s})$, for $s = 0, 1, \dots$

The pure uncertainty IRFs provide a transparent insight into how uncertainty *per se* affects the economy. But there are also non-negligible nuances to take into consideration, as in a non-linear model the pure uncertainty IRF may strongly diverge from the (rational) expectations path households possess of the same variable. Where appropriate we therefore complement this first type of IRF with the *total volatility* IRF, which is given by $n_{t+s} = E_t[f(n_{t+s-1}, z_{t+s}, \sigma_{z,t+s})]$, for $s = 0, 1, \dots$. That is, we integrate over possible future realizations of the model’s innovations.²⁰ Pure uncertainty effects emerge from agents’ responses in the present to changed expectations about the future. The total volatility IRF describes precisely the path of these expectations. All IRFs are computed around the ergodic mean in the absence of shocks (EMAS), which is also known as the risky (or stochastic) steady state (e.g., Coeurdacier *et al.*, 2011). Appendix C provides further details.

3 Transmission Mechanisms

To illustrate the channels through which uncertainty affects unemployment and inflation we proceed in three steps. First we analyze the model under the assumptions of risk neutrality and flexible prices. This allows us to illustrate some basic properties of volatility effects that will assist the subsequent analyses. In particular, we show that SaM models respond asymmetrically to productivity shocks over the business cycle, and that the pure uncertainty IRFs for some variables – which abstract from materialized outcomes of productivity – can differ substantially from their corresponding expectations. We proceed, secondly, by illustrating the transmission mechanism of the model with risk aversion and flexible prices. The primary cause underpinning a contractionary effect of increased uncertainty on unemployment is a rise in the risk premium on equity that lowers incentives for vacancy creation by firms. Third, we unveil the mechanisms at play when prices are sticky. Sticky prices give rise to a demand effect that relate both to prudence in preferences and to asymmetries employment dynamics. Together, these two forces exacerbate the effects observed under flexible prices, rendering the contractionary effect of uncertainty more pronounced.

²⁰These moments are calculated using the technique of Andreassen *et al.* (2018).

3.1 Some insights from a risk neutral model

While the baseline model considers risk-averse households, analyzing the model under risk neutrality affords some useful insights.²¹ The first of these is that under flexible prices, risk neutrality, and alternating offers bargaining, a mean-preserving spread to the distribution of productivity shocks affects neither the realized firm value, J_t , nor the expectation of future asset values. To focus intuition, and since we will consider IRFs for second-moment shocks at the steady state, suppose that materialized productivity is $z_t = z = 1, \forall t$. Then using the asset pricing relation in equation (8) with $\Lambda_{t,t+1} = 1, \forall t$, together with the wage-setting in equation (25), assuming flexible prices such that $x_t = x$, and given the mean-preserving spread assumption $E_t[z_{t+s}] = z_t = z$, we arrive at the asset value²²

$$J_t = (1 - \omega) \frac{xz - \chi}{1 - \beta(1 - \delta)}. \quad (28)$$

Thus, the asset value is linear in productivity, and it holds that $E_t[J_{t+s}] = J_t = J$.

For other endogenous variables, however, expectations may be altered even when materialized values are unaffected by a change in volatility. Thus, the free-entry condition in equation (24) can be rewritten using the specific functional forms and some simple algebraic manipulations as

$$\theta_t = \left(\frac{\Psi}{\kappa} J_t \right)^{\frac{1}{\alpha}}. \quad (29)$$

Thus, even when J_t is linear in productivity, Jensen's inequality implies that labor market tightness is convex. Thus,²³

$$E_t[\theta_{t+s}] > \left(\frac{\Psi}{\kappa} E_t[J_{t+s}] \right)^{\frac{1}{\alpha}} = \left(\frac{\Psi}{\kappa} J_{t+s} \right)^{\frac{1}{\alpha}} = \theta_{t+s}$$

That is, ceteris paribus, a mean-preserving spread to future productivity leads to an expected increase in future labor market tightness. However, if the shocks to productivity do not materialize, the resulting outcome for labor market tightness is nil. Hence, $E_t[\theta_{t+s}] > \theta_{t+s}$.

Finally, the law of motion for employment, n_t , is given by

$$n_t = \underbrace{(1 - n_{t-1} + \delta n_{t-1})}_{u_t^s} f_t + (1 - \delta) n_{t-1}, \quad (30)$$

with steady state value $n = f / (f(1 - \delta) + \delta)$. The first term, $u_t^s \times f_t$, represents newly formed

²¹For a more detailed examination of the results sketched here we refer to Den Haan *et al.* (2020).

²²We have ruled out exploding paths, such that $\lim_{s \rightarrow \infty} [\beta(1 - \delta)]^s E_t[J_{t+s}] = 0, \quad t = 0, 1, \dots$

²³See Hairault *et al.* (2010) for an early analysis of this result.

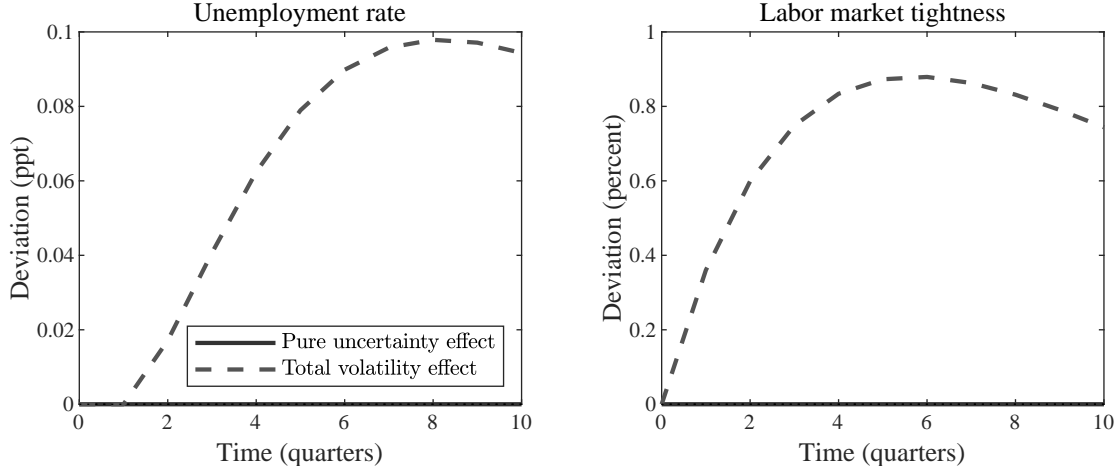


Figure 1: Realized vs perceived volatility: labor market tightness and employment

Notes: The figure illustrates the IRFs for a one standard-deviation shock to volatility under risk neutrality and flexible prices.

employment relationships and underpins an inherent asymmetry of employment dynamics in SaM models. For in expansions, the measure of job-seekers, u_t^s , is small and the job-finding rate, f_t , is large. The opposite is true in recessions. Thus, even if the job-finding rate varies symmetrically over the business cycle, its decline in recessions has a more pronounced effect on employment than its rise in expansions. Other things equal, this asymmetry leads to a lower expected employment rate in more volatile times. Put somewhat simplistically, good times are simply not as good as bad times are bad.

The following proposition concisely relates this intuitive description to the nonlinear dependence of employment on productivity in a special case.²⁴

Proposition 1. *Suppose that productivity is constant, $z_t = z_{t+1} = \dots = z$, and $\alpha \geq 1/2$. Then $n(z)$ is a strictly concave function.*

Proof. See Appendix A. □

As Proposition 1 relies on steady-state properties, Figure 1 numerically illustrates the associated dynamic response of the unemployment rate, $u_t = 1 - n_t$, and labor market tightness to an increase in volatility in the risk-neutral case. Even when the *pure uncertainty* effect on unemployment is nil (solid line), agents anticipating greater future volatility in productivity nonetheless *expect* a

²⁴While the proposition focuses on $\alpha \geq 1/2$, Den Haan *et al.* (2020) indicate that only for very low values of α is the job finding rate, f_t , sufficiently convex to outweigh the force of u_t^s and f_t being inversely related such that a rise in uncertainty lead to an initial brief dip in expected unemployment. See Jung and Kuester (2011, Proposition 1) for a useful complementary account of employment asymmetries in SaM models in terms of ergodic properties.

persistent rise in unemployment (dashed line). We will see next that in the presence of *risk aversion*, this expectation translates into non-zero pure uncertainty effects.

3.2 The transmission mechanism under flexible prices

While the preceding section highlighted that increases in anticipated, or perceived, volatility do not cause an increase in unemployment when households are risk neutral, this picture changes significantly once risk aversion is allowed for. Figure 2 shows the pure uncertainty effects of a one standard-deviation shock to volatility under log utility and flexible prices. The outcome is a decline in economic activity, a rise in the unemployment rate, and a reduction in both inflation and the risk-free real interest rate. Importantly, there is also a marked increase in the risk premium on equity. Thus, as the economy recedes, prices for safe assets increase, while those on risky assets decline.

3.2.1 Unemployment

To understand which forces drive this outcome, it is instructive to first focus on real economic activity, and subsequently turn to the interest rate and inflation dynamics. To this end, we decompose the equilibrium equity price as

$$\begin{aligned} J_t &= (1 - \omega)(xz - \chi) + (1 - \delta)E_t[\Lambda_{t,t+1}J_{t+1}] \\ &= (1 - \omega)(xz - \chi) + (1 - \delta)\{E_t[\Lambda_{t,t+1}]E_t[J_{t+1}] + Cov_t(\Lambda_{t,t+1}, J_{t+1})\}, \end{aligned} \quad (31)$$

where the absence of t -subscripts indicate that the relative price of intermediate goods, x_t , is constant in the absence of nominal rigidities in the retail sector, and there are no materialized shocks to productivity, z_t . Thus, the only remaining moving parts are those pertaining to expectations; both of the stochastic discount factor and future asset prices, as well as their covariance.

As shown in Figure 2, the risk-free real interest rate declines, which implies that the expected stochastic discount factor, $E_t[\Lambda_{t,t+1}]$ increases. This is due to two reasons. First, with preferences exhibiting prudence (that is, marginal utility being convex) an increase in uncertainty raises the expected marginal benefits of resources in the future, which sets off a precautionary motive to save. This precautionary motive puts upward pressure on the price of equity. Second, the employment asymmetries outlined in Proposition 1 imply that a decline in expected future employment is imminent, which further reinforces a perceived increase in the marginal benefits of savings through a desire to smooth consumption over time (as reflected in marginal utility being decreasing in consumption). This adds a further positive force pressing up the equity price.

Given that both the above-mentioned mechanisms put positive pressure on equity prices, the third and final force – captured by the covariance term between the stochastic discount factor and the

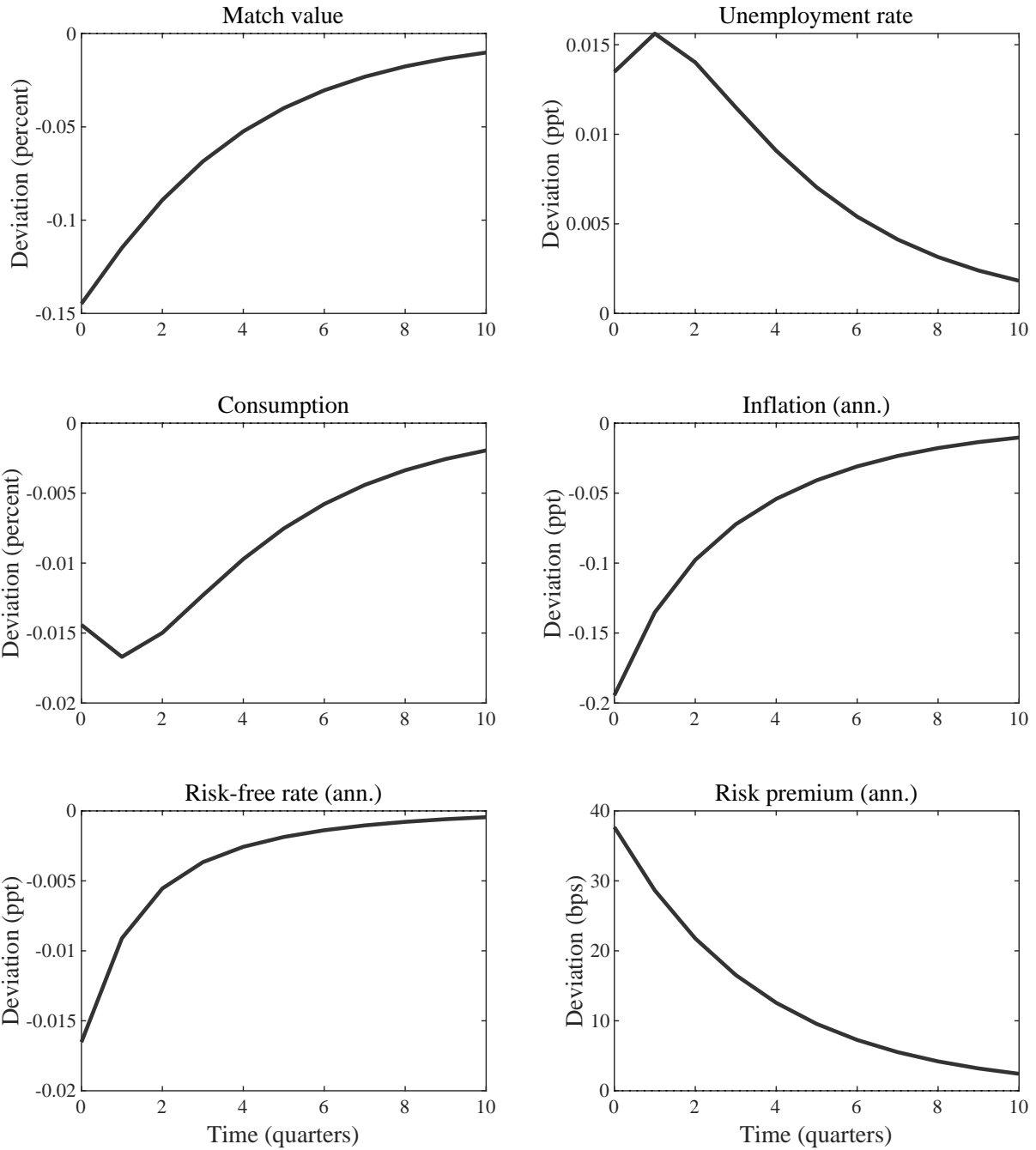


Figure 2: Uncertainty effects under flexible prices

Notes: The figure illustrates the pure uncertainty IRFs for a one standard-deviation shock to volatility under risk aversion and flexible prices.

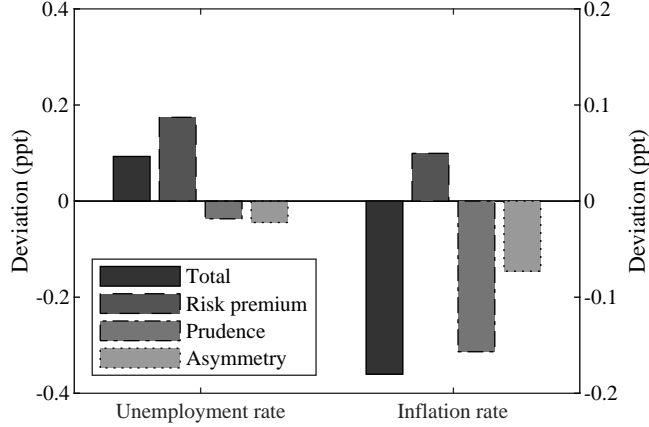


Figure 3: Decomposition of cumulative effects under flexible prices

Notes: The figure illustrates the cumulative effect of the various transmission mechanisms under flexible prices on two macroeconomic aggregates: unemployment (left axis) and inflation (right axis). The computations are described in the main text.

future price of equity – must react negatively. Using the definition of the risk premium in equation (7) together with the decomposed asset price above reveals a tight relationship between the risk premium and the covariance term

$$RP_t = -Cov_t(\Lambda_{t,t+1}, J_{t+1}) \frac{(1 - \delta)}{(J_t - d_t) E_t[\Lambda_{t,t+1}]}$$

Thus, as the covariance term declines, the risk premium increases; and it must move sufficiently to offset the rise in the expected stochastic discount factor. As a consequence, the main mechanism which brings the economy down is a fall in equity prices brought about by a rise in the risk premium that results from a decline in the covariance between the stochastic discount factor and future equity prices. Of course, as shocks are persistent, this mechanism is expected to repeat itself in the future, and there is a reinforcing effect arising from an additional anticipated decline in future equity prices, which puts additional downward pressure on current prices, and so on.

This story is not without economic appeal. A rise in uncertainty brings about a motive to save; both because of prudence, and the nonlinear dynamics of employment. This enhanced motive to save would, in isolation, put upward pressure on equity prices and result in an expansion. However, as consumption and asset prices are positively correlated, there is a negative covariance between future asset prices and the stochastic discount factor, indicating that equity indeed is a poor asset for hedging against this increase in risk. If this channel dominates the former – which it does under the baseline calibration – the result is an increase in unemployment alongside a rise in the risk premium.

To provide a quantitative account for these mechanisms, the left graph in Figure 3 shows a decomposition of the cumulative rise in unemployment along the IRF. To conduct this decomposition,

we first solve the model using equation (31) but *suppressing the covariance term to be zero*. The difference between the baseline result and the outcome of this exercise is due to the dynamics of the risk premium. As can be seen from the figure, the rise in risk premium puts significant upward pressure on the unemployment rate. Second, we repeat the above exercise, but additionally using a *linear approximation of the marginal utility* around the deterministic steady state value of consumption. As a linear marginal utility exhibits certainty equivalence, the difference between this exercise and the previous one accounts for the effect of prudence. Figure 3 reveals that prudence brings forth a negative, i.e., stabilizing, effect on the unemployment rate. Lastly, absent both prudence and the risk premium, the remaining dynamics are those pertaining to the nonlinearities in the law of motion of employment, and reflect the asymmetries inherent in SaM models. Again, these asymmetries gives rise to an additional negative effect on the unemployment rate.²⁵

3.2.2 Inflation and nominal interest rate

While movements in both inflation and the nominal interest rate are immaterial for real economic activity in a flexible price setting, understanding their dynamics may not only be of independent interest, but will also prove useful to unveil the mechanisms at play in the presence of nominal rigidities.

As outlined in the previous section, an increase in uncertainty renders a decline in the risk-free real interest rate, as both prudence and employment asymmetries push up the expected stochastic discount factor, $E_t[\Lambda_{t,t+1}]$. This movement in the risk-free rate stands in marked contrast to the returns on equity which increase due to the rise in the risk premium. While a reduction in the risk-free interest rate can materialize both due to a decline in the nominal rate, or because of a rise in *expected inflation*, the Taylor rule in equation (26) reveals that the nominal rate will only be lowered if there is a reduction in *current inflation*. Thus, the real interest rate falls as the nominal interest rate declines more than expected inflation, which leads to a reduction in the nominal rate that is sufficiently pronounced to outweigh the perceived decline in future inflation.

Movements in the risk premium are not unimportant to this story. In particular, as the risk premium rises, equity prices fall, the unemployment rate increases, and current private consumption declines. As a consequence, the rise in the expected discount factor is less pronounced than it would be in the absence of a variable risk premium, and the fall in both the nominal interest rate and inflation is therefore somewhat muted.²⁶ As will become apparent, this mechanism will give rise to a flatter Phillips curve than would be observed under regular demand shocks (see Section 4.1).

²⁵As the model is highly nonlinear, these mechanisms interact with each other, and the decomposition is not unique. We have computed different permutations of the ordering of closing down channels, as well as an alternative approach that proceeds by linearizing the law of motion for employment. The differences between the exercises are negligible.

²⁶Another, more heuristic, way of seeing this is that the movements in the covariance term in equation (31) is akin to a negative supply shock, which are commonly associated with inflationary pressure.

Lastly, following the same logic as in the previous section, the right graph in Figure 3 decomposes the cumulative response of inflation into its three driving forces. Both prudence and employment asymmetries contribute to a fall in inflation, while the risk premium is indeed inflationary. In the baseline setting the two former effects dominates the latter, and there is an overall decline in inflation.

3.3 Sticky prices and the role of demand

Figure 4 shows the results corresponding to Figure 2 but with sticky prices. Notice that the graph containing the risk premium has been replaced by the relative price of intermediate goods, x_t , which is now time-varying.²⁷ As can be seen from the figure the total effects qualitatively line up with those of Figure 2, but they are quantitatively more pronounced.²⁸ The reason is that two of the previously stabilizing forces – prudence and the employment asymmetries – are now destabilizing.

The reason nominal rigidities destabilize these forces follows a familiar new-Keynesian (NK) narrative. The rise in uncertainty puts upward pressure on the expected stochastic discount factor and thereby downward pressure on the risk-free real interest rate. However, as the monetary authority is constrained in its reaction by the Taylor rule, the nominal interest rate does not change unless there is visible disinflation. Thus, absent a decline in inflation the real interest rate would be left unchanged, and demand for final goods would fall short of supply. The reduction in demand, however, encourages retail firms to lower their prices. Because of price-adjustment costs their response is muted, which results in a decline in the demand for, and the price of, intermediate goods, x_t . As a consequence, the equity price falls, there is less entry and vacancy posting, less production, and supply approaches the reduced level of demand. At the same time, the reduction in the price level leads to disinflation and thereby a reduction in real and nominal interest rates, which serves to mute the initial fall in demand. This process ends when there is an equal decline in both the demand and supply for goods, and the equilibrium is restored. In contrast to the case with flexible prices, the equilibrium equity price is now lower partly as a result of a decline in the relative price for intermediate goods, which is driven by demand, and partly as a result of an increase in the risk premium. Thus, the same mechanisms that stabilized the economy under flexible prices – those that put upward pressure on the expected stochastic discount factor – are now, via the aggregate demand channel, destabilizing.

There are a few nuances to this story, however, that deserve to be highlighted. First, it may appear surprising that a seemingly short-lived decline in the relative price of intermediate goods

²⁷We will return to the dynamics of the risk premium in section 4.1.

²⁸In Sections 4.2 and 4.3 we highlight additional factors – relating to the conduct of monetary policy and the nature of price adjustment costs as well as perturbations of the wage bargaining parameters – that give rise to stronger amplification still.

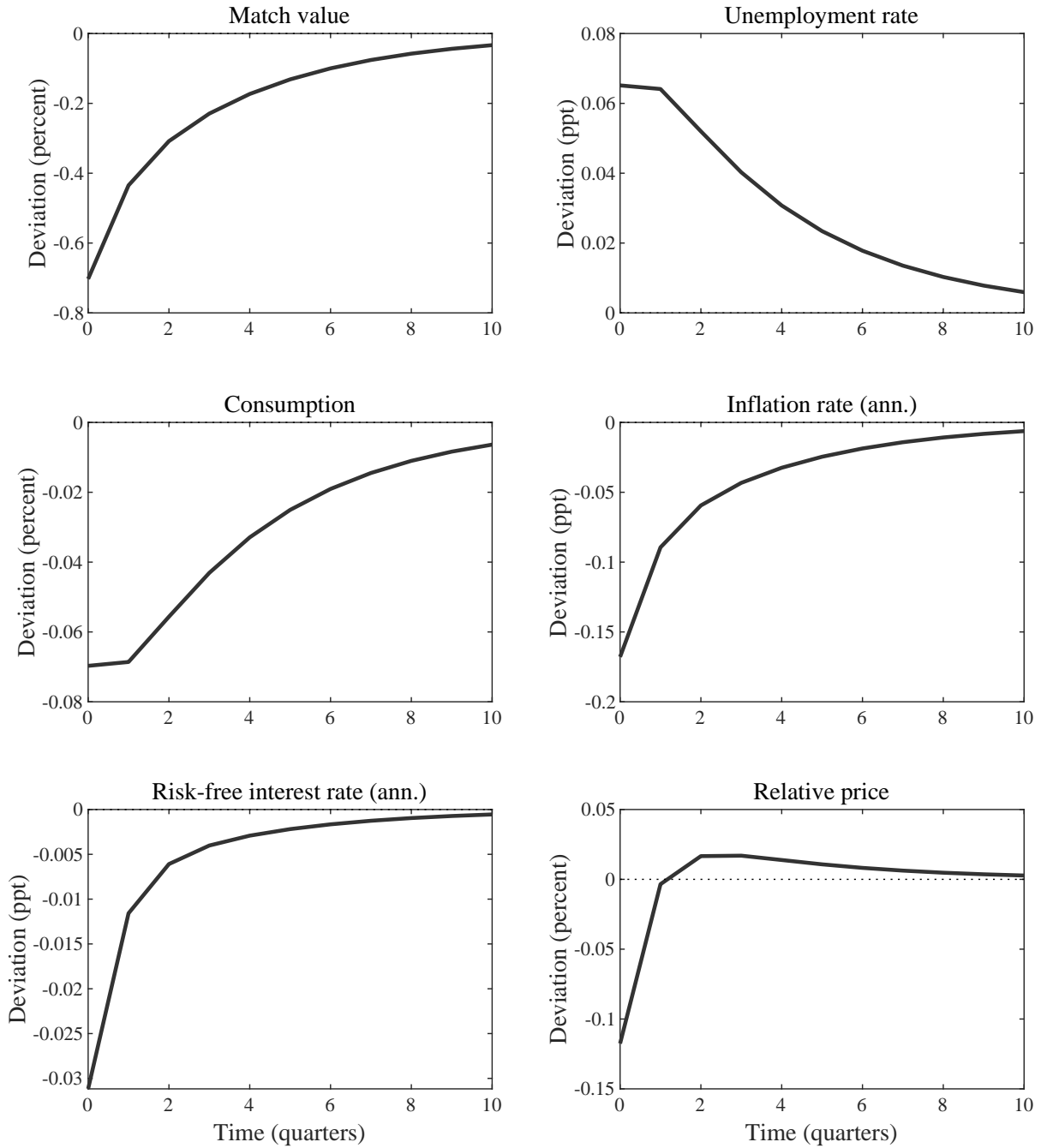


Figure 4: Uncertainty effects under sticky prices

Notes: The figure illustrates the IRFs for a one standard-deviation shock to volatility under risk aversion and sticky prices.

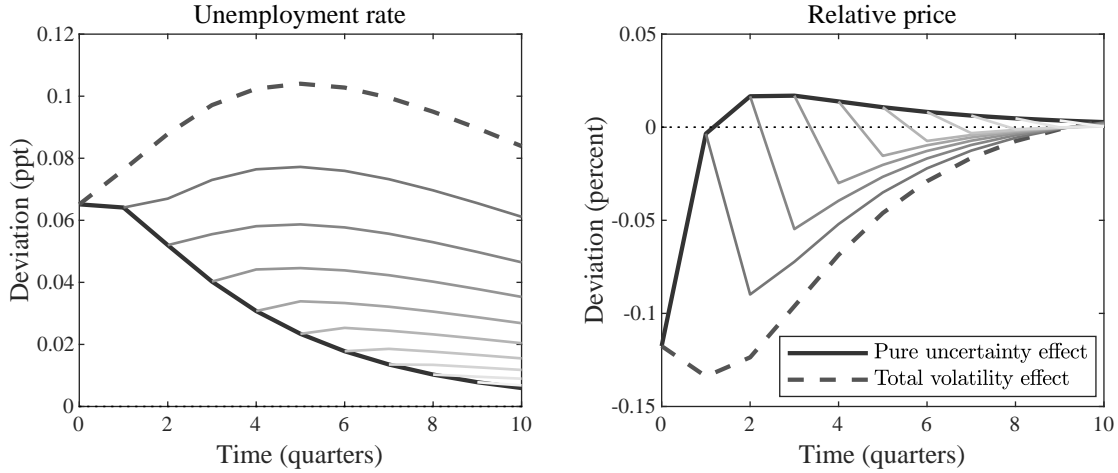


Figure 5: Beliefs about and effects on unemployment and relative price

Notes: The figure illustrates both the total volatility and the pure uncertainty effect of a one standard-deviation shock to volatility under sticky prices. The thin grey lines represent the total volatility effect starting from $t = 1, 2, \dots$ and, thus, capture the evolution of agents' expectations over the period of elevated uncertainty.

can lead to such a dramatic increase in the propagation of shocks. It does not. Indeed, Figure 5 reveals that agents *expect* relative prices to remain depressed for an extended period of time. The primary reason is that the employment asymmetries outlined in Proposition 1 lead agents to anticipate further adverse demand consequences in the future, which provides the foundation for an outlook of a persistently depressed relative price. And as the equity price is forward-looking, an expected, persistent decline in the relative price puts severe negative pressure on the equity price already in the present.²⁹

Second, the inflation rate and the risk-free interest rate display subtly different dynamics than under flexible prices. More precisely, inflation declines by less, while the risk-free rate declines by more. This is because flexible prices allows for larger adjustments in the inflation rate, which leads to a pronounced decline in *expected inflation*. When prices are flexible, the inflation rate declines markedly on impact, therefore, and is also expected to decline even further. As a consequence, while the nominal interest rate reacts according to the Taylor rule, the real interest rate falls by less under flexible prices as inflation is *expected* to remain low for a considerable amount of time.³⁰

Figure 6 illustrates the decomposition described in Section 3.2, Figure 3, but now under sticky

²⁹Thus, even though the logic outlined in the preceding paragraph qualitatively follows the NK *narrative*, there are reasons to believe that it is quantitatively quite different. To focus intuition, envisage a situation in which news about a negative demand shock in period $t + 1$ arrives in period t . In addition, assume that the monetary authority acts so aggressively in period t that prices – both inflation, Π_t and the relative price, x_t – remain constant (see Section 4.2.1); it is not expected to act that way in period $t + 1$, however. Then in a standard NK framework these news would bear no consequences on real economic activity in period t . With forward-looking asset prices, however, the corresponding decline in J_{t+1} transmits into the present, leading to a contraction in output already in period t .

³⁰Recall that the real interest rate is determined by the nominal rate and expected, rather than current, inflation.

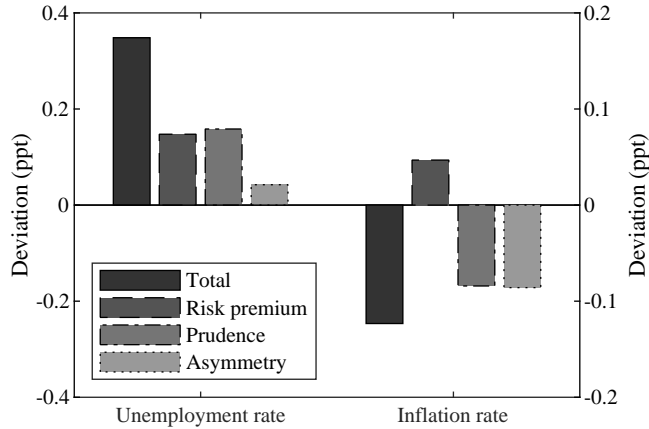


Figure 6: Decomposition of cumulative effects under sticky prices

Notes: The figure illustrates the cumulative effect of the various transmission mechanisms under sticky prices on two macroeconomic aggregates: unemployment (left axis) and inflation (right axis). The computations are described in the main text.

prices. While the decomposition is less transparent in the current setting, as each mechanism is itself interacted with the nominal rigidities, there are a few lessons to be learned from this exercise.³¹ First, and foremost, both the precautionary motive, due to the increase in uncertainty, and the motive to intertemporally smooth consumption (because of the asymmetric employment dynamics) are now operating in the *opposite* direction compared to the flexible prices benchmark. The reason is outlined at the beginning of this section, and hinges on the demand effects that arise due to nominal rigidities; what were previously stabilizing forces are now destabilizing. Second, the risk premium still contributes to the decline in economic activity, but its effect on unemployment is relatively smaller than the demand side effects.

Lastly, the inflationary pressure stemming from the increase in the risk premium remains positive. The reason is that the shortfall in demand must, in equilibrium, be met by an equal shortfall in supply. Under the standard NK logic the latter happens through reduction in retail prices that leads to a decline in the relative price of intermediate goods, x_t , which in turn contracts supply. This process continues until the equilibrium is restored. In the current setting, however, supply contracts even in the absence of any movements in the relative price. The reason is that as the risk premium rises, asset prices fall, entry of intermediate goods producing firms declines, and there is less supply of intermediate goods even in the absence of relative price adjustment. As a consequence, there is less need for the economy to operate through other price margins – including retail and intermediate goods prices – and the deflationary pressure is therefore suppressed. As we will see, this latter feature gives rise to some dynamics that distinguishes uncertainty shocks from more conventionally

³¹We deliberately abstract from any effects associated with “precautionary pricing” by linearizing the NK Phillips curve in equation (16). In Appendix B.1.3 we show such effects to be quantitatively small in the present setting.

modeled aggregate demand shocks.

4 Implications

A cursory reading of Figure 4 suggests that uncertainty shocks affect economic activity no differently from regular (aggregate) demand shocks, such as contractionary monetary policy. Indeed, both inflation and the risk-free real interest rate decline, output contracts, and the unemployment rate rises. This section evaluates to which extent uncertainty shocks differ from aggregate demand shocks – in terms of observational variables as well as implications for macroeconomic policy.

4.1 Are uncertainty shocks aggregate demand shocks?

To address this question, we modify the Taylor rule in equation (26) to include a shock to monetary policy, and reverse engineer a persistent rise in the nominal interest rate such that the impulse response function of unemployment *exactly* coincides with that of Figure 4.³² The effect on inflation and on the risk premium is documented in Figure 7, which makes clear that uncertainty shocks have a relatively muted effect on inflation, and a more pronounced effect on the risk premium.

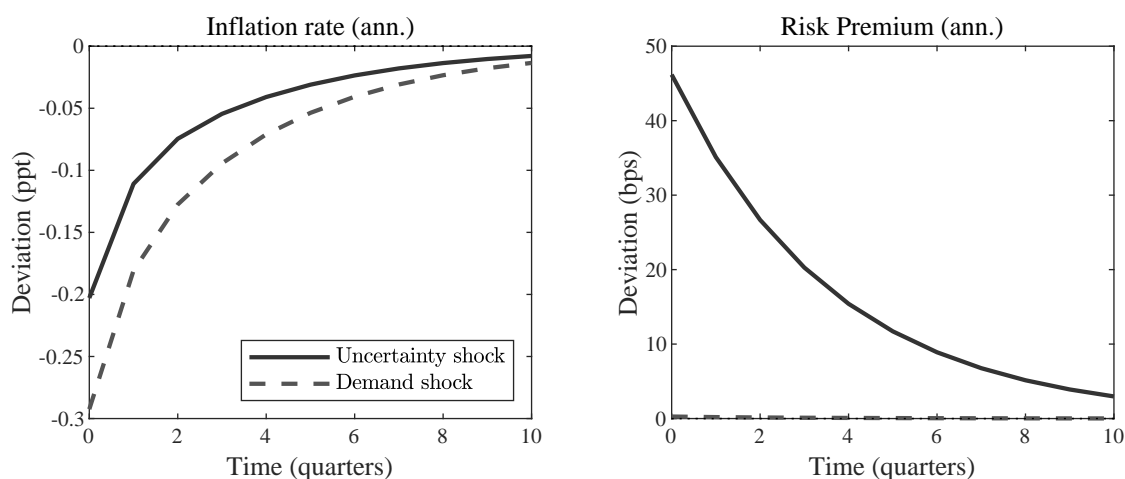


Figure 7: Demand vs. uncertainty shocks: impulse response functions

Notes: The figure illustrates the pure uncertainty effect of a one standard-deviation shock to the volatility of productivity shocks (solid line) and the impact of a shock to the nominal interest rate (dashed line). The persistence and magnitude of that interest rate shock are chosen so as to reproduce the response of unemployment to the pure uncertainty shock.

The reason is quite straightforward. An interest rate hike reduces demand for final goods through the bond Euler equation. Facing lower demand, retail firms reduce their prices, leading to an overall

³²That is, the sequence of interest rate shocks is such that the resulting effect on real economics activity is identical to that resulting from an uncertainty shock.

decline in the price level. As prices are sticky, however, the resulting price-adjustment is incomplete, and retailers demand fewer intermediate goods. As a consequence, the relative price of intermediate goods, x_t , falls, leading to lower asset values, which then contracts supply. This process continues until the (goods) market is in equilibrium, at a lower level of economic activity.

An uncertainty shock operates through comparable mechanisms, but with one pronounced difference: As the risk premium increases, asset values fall even without any adjustment to the relative price, x_t . This contracts entry, hiring, and reduces the supply of intermediate goods. Thus, in order to restore the equilibrium, less pronounced price adjustments are needed (and therefore less disinflation), which leads to a smaller decline in the relative price, x_t . Put simply, disinflation materializes to bring supply towards demand. But as an uncertainty shock contracts supply even in the absence of any price movements, less disinflation is needed to bring markets back to equilibrium.

This result is not a mere curiosity. For one thing, the results in figure 7 relies on logarithm utility and a Taylor rule that gives no weight to output. With a higher degree of risk aversion, the rise in the risk premium is even larger. And as discussed in the following section, allowing for $\phi_y > 0$ strengthens the force of supply relative to demand channels. In fact, for plausible regions in the parameter space, an uncertainty shock can even lead to inflation instead of disinflation.³³

Furthermore, Figure 8 illustrates the implied observable relationship between deviations of inflation and unemployment from their means – or Phillips curve slopes – resulting from simulations of the economy under *either* demand or pure uncertainty shocks.³⁴ As can be seen from that figure, uncertainty shocks lead to a flatter Phillips curve than demand shocks. This result obtains even under logarithmic utility. For higher values of the coefficient of risk aversion, the pattern is more pronounced.

The corollary emerging from this analysis is that to the extent that a decline in economic activity is primarily driven by a highly uncertain future, the model suggests that the “missing disinflation” should not come as a surprise (see, for instance, Hall (2011) and Lindé and Trabandt (2019)).³⁵ Similarly, the theoretical perspective offered here offers a rationale for the empirically ambiguous response of inflation to uncertainty shocks, even when that on real economic activity is well-established (cf. Footnote 1).

³³See Figure B.3 in Appendix B.1, for instance.

³⁴We run the simulations for 6,000 periods, of which we discard the first 1,000 to avoid initial values issues.

³⁵The “missing disinflation” puzzle is encapsulated by John C. Williams’ statement about the 2008-2009 experience in the United States (Williams (2010, p. 8)): “The surprise [about inflation] is that it’s fallen so little, given the depth and duration of the recent downturn. Based on the experience of past severe recessions, I would have expected inflation to fall by twice as much as it has.” At the same time, the sharp spike in various measures of uncertainty during the Financial Crisis has frequently been cited in explanations of the depths and persistence of the ensuing Great Recession (e.g., Basu and Bundick (2017) and Bloom *et al.* (2018)).

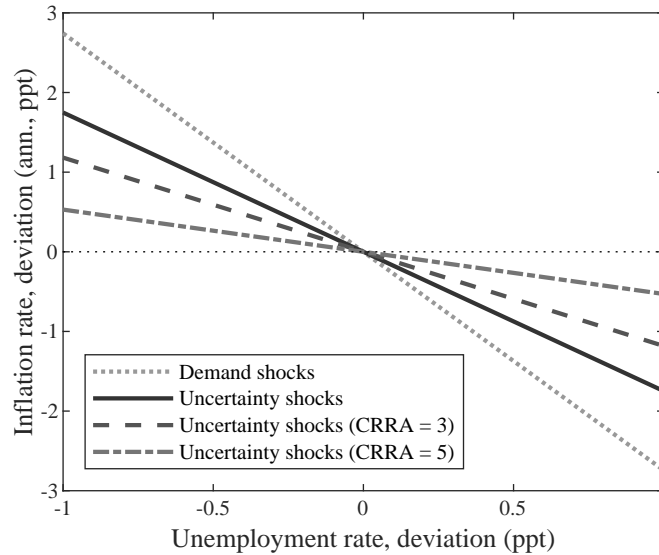


Figure 8: Demand vs. uncertainty shocks: Phillips curve slopes

Notes: The figure shows linear regression lines fitted to simulated demeaned data of unemployment and inflation, under the baseline calibration with sticky prices. The lines labeled “uncertainty shocks” relate to simulations with normally distributed innovations to the standard deviation of productivity shocks only; all other shocks are shut down. The “demand shocks” line refers to a simulation with only shocks to the level of the interest rate; the average size of shocks to the interest rate is such that the implied standard deviation of unemployment is equal to the baseline uncertainty-only simulation.

4.2 Implications for monetary policy

With flexible prices, the conduct of monetary policy is innocuous, and merely serves to separate nominal from real interest rate movements. When prices are sticky, however, monetary policy affects real economic activity and can have profound implications regarding the response of the economy to various shocks. This section makes two distinct points in this respect. First, we show that the divine coincidence holds. That is, by responding sufficiently aggressively to inflation the monetary authority can mimic the flexible price outcome (cf. [Blanchard and Galí, 2007](#)). Second, we reveal complex tradeoffs between stabilizing expectations for employment and inflation that have quantitatively significant implications especially when price adjustments are costly in terms of economic resources.

4.2.1 The divine coincidence

Figure 9 illustrates the effect of the uncertainty shock, varying the aggressiveness of the monetary authority’s response to inflation. The solid line shows the benchmark result under sticky prices from Figure 4, and the dashed lines shows the results as ϕ_π takes on higher values, eventually approaching infinity. The dotted line illustrates the flexible price benchmark result from Figure 2.

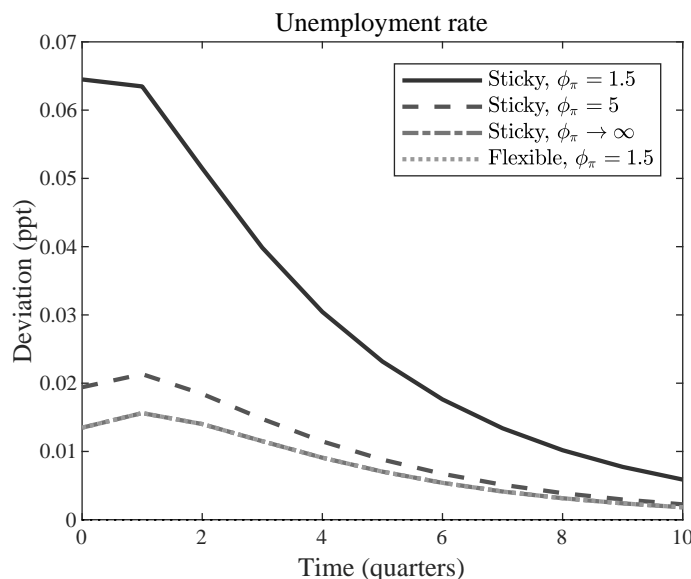


Figure 9: The divine coincidence

Notes: The figure illustrates the response of unemployment to an uncertainty shock under sticky prices, varying the aggressiveness of the monetary authority’s response to inflation.

As can be seen in the figure, the effect of an uncertainty shock on unemployment approaches that observed under flexible prices as the monetary authority response more aggressively to inflation. That is, by stabilizing inflation the monetary authority successfully stabilizes output as well; a result known as the divine coincidence (Blanchard and Galí, 2007).

The reason for this is twofold. First, sticky prices tend to dampen the effect of supply shocks. The reason is that a negative productivity shock leads to a decline in supply of intermediate goods that, absent any price movements, falls short of demand. As a consequence, the relative price of intermediate goods, x_t , increases, and retail firms raise their prices. The result is a rise in inflation and the real interest rate, and, as x_t increases, a decline in employment that is muted relative to the flexible price outcome. This feature is present when the monetary authority responds relatively passively to inflation, which limits the transmission of supply disturbances to changes in demand.³⁶ As a consequence, prices rigidities then link the (limited) movements in demand into to real economic activity, which subdues the response. A more aggressive monetary policy puts a lid on this mechanism, as the link between supply and demand becomes more pronounced. In the limit, i.e., as the coefficient ϕ_π approaches infinity, demand strictly follows supply (Say’s law), inflation is entirely stabilized, and the effect of a supply shock under sticky prices coincides with that under flexible prices.

Second, having established that a sufficiently aggressive monetary policy ensures that demand effects are immaterial to economic activity in response to supply disturbances, a similar argument

³⁶With “passively” we refer to the situation in which ϕ_π is finite; not when it is below one.

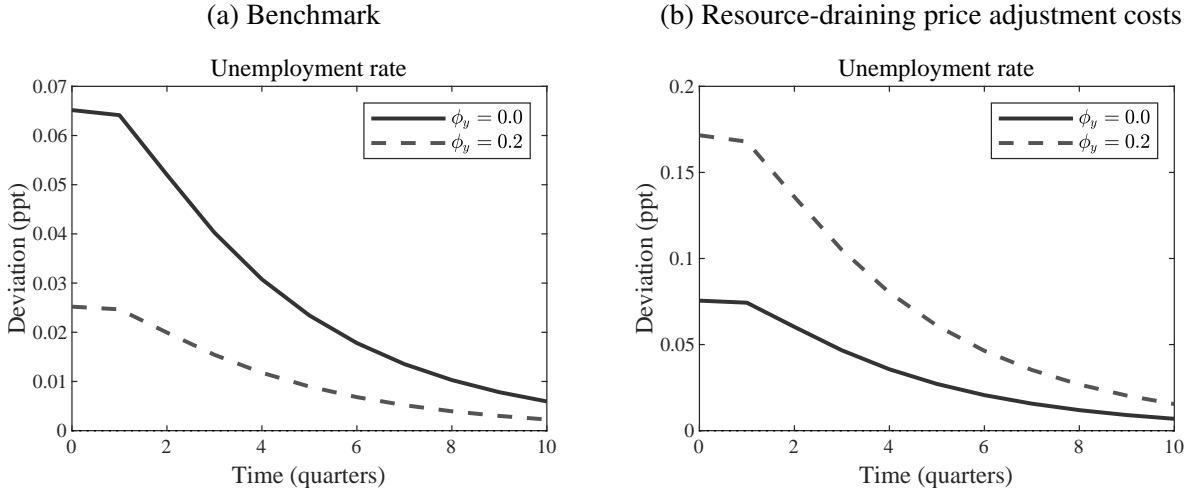


Figure 10: Taylor rule with active response to output

Notes: The figure illustrates the response of unemployment to an uncertainty shock under sticky prices and log utility, varying the aggressiveness of the monetary authority’s response to output. The left-hand panel assumes that all output is available for consumption, whereas the right-hand panel assumes that price-adjustment costs divert real resources.

applies to the effect of uncertainty regarding future shocks. In particular, a rise in uncertainty curbs demand. With a sufficiently passive monetary policy, the shortfall in demand leads to disinflation, a fall in the relative price for intermediate goods, x_t , and ultimately to a decline in supply and equilibrium economic activity. Once the monetary authority responds sufficiently aggressively, however, disinflation is not tolerated, and the nominal interest rate is immediately lowered such that demand coincides with supply without any movements in neither the price level nor the relative price, x_t . Yet, as the risk premium increases, supply does indeed contract, demand follows, and the equilibrium response to real economic activity coincides with that of flexible prices. Thus, a more aggressive response to inflation ensures that the equilibrium mimics the flexible price outcome when exposed to shocks to the *level* of productivity. Knowing this, the households’ expectations in period t also coincide with those of the flexible price outcome. By virtue of this effect on expectations, the same aggressiveness can curb the outcome already in the present.

4.2.2 Complex tradeoffs

The baseline analysis sets the Taylor rule coefficients, ϕ_π and ϕ_y , to 1.5 and zero, respectively. Yet it is not uncommon to allow for ϕ_y to be greater than zero. Indeed, both [Leduc and Liu \(2016\)](#) and [Basu and Bundick \(2017\)](#) consider $\phi_y = 0.2$ their baseline.³⁷

Figure 10a illustrates that the recessionary impact of elevated uncertainty on unemployment is much less severe if the monetary authority responds not only to inflation but also to output (with

³⁷Basu and Bundick (2017) assume that ϕ_y applies to output in deviation from its own lag, rather than its deviation from the deterministic steady state.

$\phi_y = 0.2$). This is for two reasons. First, $\phi_y > 0$ directly implies a more accommodative policy stance given the contraction in output in the present. Second, expectations for future production are stabilized, which mutes the force of the asymmetry-demand channel. To see the latter feature, notice first that the direct impact of, say, an increase in the productivity of the intermediate goods sector is to push down the intermediate price of inputs, x_t . However, the same shock also increases income and stimulates demand, which instead puts upward pressure on the relative price, x_t . The reverse holds true for negative productivity shocks. By responding to output, policymakers can limit the second effect, engineer a relatively more countercyclical relative price, and thus dampen business cycle volatility in hiring (see Lepetit (2020) for an detailed discussion). Given the asymmetric employment dynamics of the model, the expected *employment gap* – caused by the anticipation of greater future volatility – and resulting pessimism about the future, is then smaller in absolute magnitude.³⁸

The implications of setting $\phi_y > 0$ for *inflation* are more intricate in two respects. First, notice that as uncertainty shocks tend to push inflation and output in the same direction, responding more aggressively to either variable helps stabilizing the other one as well. Yet in response to a productivity shock, the inflation rate and output move in *opposite* directions. Accordingly, by stabilizing *future* output the monetary authority also leaves *future* inflation relatively more volatile.³⁹ Thus, the monetary authority faces a tradeoff between stabilizing either future real or nominal dynamics, even when this is not the case with respect to today’s economy in which, by construction, only uncertainty shocks materialize.

Second, when price adjustments are costly in resource terms, then stochastic volatility renders the tradeoffs facing the monetary authority more complex. To see this, suppose we relax the assumption, maintained thus far, that Rotemberg adjustment costs subtracted from retail firms’ profits are rebated to the household as a lump-sum. Then the GDP identity is instead given by $y_t = c_t + ac_t$, where ac_t represents the convex adjustment cost

$$ac_t = \frac{\Omega_p}{2} \left(\frac{p_t(i)}{p_{t-1}(i)\Pi} - 1 \right)^2 y_t.$$

Figure 10b illustrates that under this assumption the economy may even fall into a *deeper* recession following an uncertainty shock when the monetary authority actively responds to output. This result stands in marked contrast to Figure 10a. The key insight is that by setting $\phi_y > 0$ the monetary authority may stabilize expected future *production*, but now tolerating greater future inflation variability no longer guarantees stabilizing future *consumption*. The convex nature of

³⁸Notice that in a standard NK model without asymmetric employment dynamics, the expectation for more volatile employment would typically not affect the conditional mean.

³⁹This statement implicitly rests on the premise that overall dynamics are more strongly influenced by shocks to levels than by shocks to second moments. In practice, this is the case.

adjustment costs implies that greater expected future price adjustments translate into lower expected consumption for any given level of production. As before, expectations for lower future consumption then ripple through the economy, leading to a fall in current demand but also the expectation of lower future asset prices. To summarize, with resource draining price-adjustment costs, an increase in anticipated volatility leaves households more pessimistic about future consumption, either because employment is expected to be significantly more volatile – when the monetary authority ignores output fluctuations – or because inflation is expected to be more variable – when the Taylor rule does give weight to output. The monetary authority thus faces an uncomfortable tradeoff between expected future output volatility and inflation volatility; both of which contribute to exacerbating the effects of an uncertainty.

4.3 Sensitivity analysis

While the emphasis of our analysis lies on uncovering principles and mechanisms through which uncertainty shapes the dynamics of unemployment and inflation, we conclude by highlighting how quantitative results vary if we perturb the benchmark parameterization. In particular, the extent to which uncertainty shocks cause mild or severe recessions in the SaM model with risk aversion and price stickiness crucially hinges on the parameter χ , which in the alternating offer bargaining game over wages represents the worker’s outside option.⁴⁰

To illustrate this property, we solve the model repeatedly for a grid of values for χ and, each time, compute the cumulative response of unemployment to the uncertainty shock. Figure 11 shows the result: The cumulative impact of an uncertainty shock on the unemployment rate monotonically increases in χ . This holds true both in the absence of sticky prices (where the risk premium effect becomes stronger for larger values of χ) and with nominal rigidities added to the model (in which case demand effects become worse as well). Indeed, in either case the relationship depicted is distinctively convex. To give a sense of magnitudes, our benchmark value of $\chi \approx 0.81$ implies that the cumulative impact on the unemployment rate of a one standard-deviation uncertainty shock is 0.09 percentage points under flexible prices, and 0.35 percentage points in the presence of nominal rigidities. With $\chi = 0.85$, on the other hand, the latter value rises to 1.3 percentage points.⁴¹ The relatively modest magnitude of the effects shown in the analysis thus far is therefore not a necessary limitation of the model, but instead, reflects our conservative benchmark parameterization (cf.

⁴⁰Appendix B.1 provides further sensitivity checks.

⁴¹The value $\chi = 0.85$ is not picked randomly; it is the value implied by the steady-state version of the wage equation (25) when imposing a bargaining weight of $\omega = 0.5$ given $z = 1$ and $x = 0.9$. To offer some complementary perspective, the standard deviation of the quarterly real return on the S&P 500 over the past 40 years is slightly above 0.07. Even with a value of $\chi = 0.85$, the model-implied volatility of the return on equity is about four times lower. These data offer suggestive evidence that the uncertainty effects implied by our baseline parameterization represent a conservative lower bound.

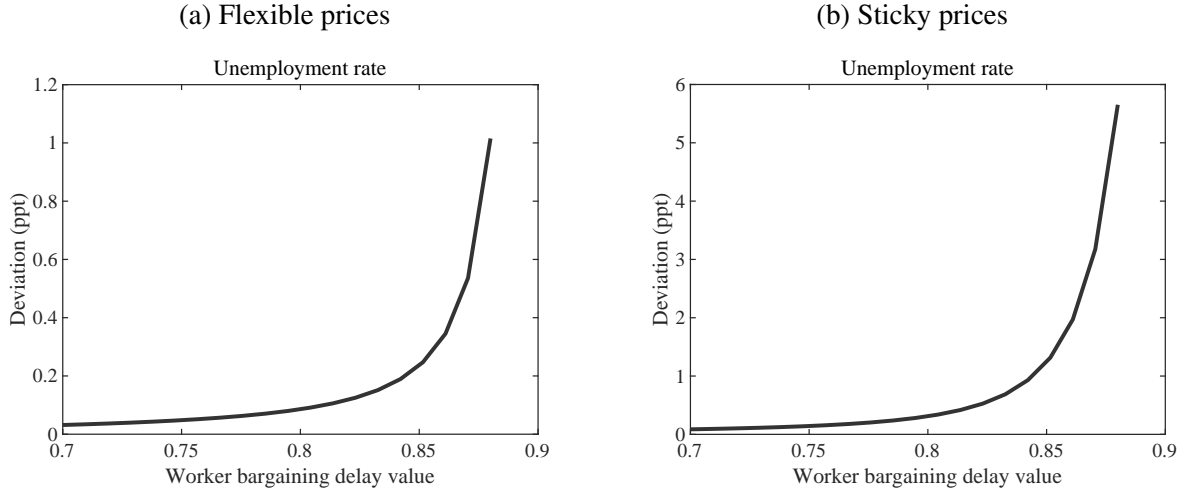


Figure 11: Sensitivity of results under flexible and sticky prices

Notes: The figure illustrates the cumulative response of unemployment to a one standard-deviation uncertainty shock, varying the flow consumption-value the worker receives by delaying agreement during the alternating offers bargaining process, χ . The left-hand panel assumes flexible prices, whereas the right-hand panel considers the sticky price setting.

Hagedorn and Manovskii (2008)).

The reasons for this result relate to the size of the fundamental surplus $xz - \chi$ (Ljungqvist and Sargent, 2017). Intuitively, if the fundamental surplus is small relative to output, a given percentage change in productivity induces a greater percentage change in resources devoted to vacancy creation (and, hence, a larger responses of employment).⁴² This same logic carries through to the analysis of uncertainty shocks, with a smaller fundamental surplus translating into more unemployment volatility. There is an additional nuance, though. The asymmetry-demand channel is only operative if employment is sufficiently volatile for agents to downgrade their expectations for future average employment by a significant amount. Higher volatility makes it more likely that the model gives rise to what Petrosky-Nadeau *et al.* (2018) describe as “endogenous disasters”. The anticipation of such an outcome leads agents to change their behavior – in terms of consumption, savings, and required risk premia – which gives rise to strong pure uncertainty effects in the present.

5 Concluding Remarks

This paper has studied the role of uncertainty a search-and-matching (SaM) model with risk-averse households, both in the presence and absence of nominal rigidities. In both cases, an increase in uncertainty contracts output, but the effect is more pronounced when prices are rigid. The results arise through three distinct channels: the risk premium; the precautionary motive to save; and the

⁴²Specifically, the fundamental surplus is the primary determinant of the steady-state elasticity of labor market tightness with respect to productivity, $\eta_{\theta,z} = \frac{1}{\alpha} \frac{xz}{xz - \chi}$.

asymmetries in the labor market (in conjunction with a desire for consumption smoothing). In the flexible price setting, both the precautionary motive and the asymmetries in the labor market provide put upward pressure on output, as the associated demand for saving trickles into an increase in the valuation of asset prices, expands entry, and reduces the unemployment rate. In sharp contrast, the risk premium channel instead puts negative pressure on asset prices. The reason is that the return on equity correlates negatively with the marginal utility of consumption, and thereby provides a poor hedge against uncertainty. Even with a moderate degree of risk aversion, the risk premium channel is strong enough to counter the two other countervailing channels, leading to an overall reduction in economic activity.

When prices are sticky, the decline in demand that follows an increased desire to save leads to countercyclical markups, and both the precautionary motive and the labor market asymmetries operate in the opposite direction compared to the flexible price scenario. The risk premium channel is largely unaffected by this alteration, and operates (observationally) as an additional negative supply disturbance. As in most models with nominal rigidities, supply disturbances move output and inflation in opposite directions. Thus, an uncertainty shock gives rise to a flatter Phillips curve compared to regular demand shocks, potentially providing a novel lens through which the “missing disinflation” can be viewed (see [Hall \(2011\)](#)).

While our analysis is deliberately centered around a stripped-down theoretical framework, it nevertheless points to several modeling features that may amplify uncertainty shocks to an empirically relevant extent.⁴³ For example, the asymmetry-demand channel rest on the interaction of risk aversion, on the one hand, and asymmetric employment dynamics in frictional labor markets, on the other hand.⁴⁴ [Dupraz et al. \(2019\)](#) propose a theory in which nominal downward rigidity in wage-setting means that economic fluctuations are drops below the economy’s full potential ceiling. Uncertainty shocks in such a “plucking model of business cycles” enriched with nominal rigidity would give rise to effects that parallel the asymmetry-demand channel and could be quantitatively relevant. Similarly, recursive preferences in conjunction with a high degree of risk aversion may also magnify the effects considerably ([Epstein and Zin, 1989](#)).

The primary feature underlying our results is the long-lasting nature of firms. In particular, in a SaM framework, firms pay a fixed cost to enter the market by posting a vacancy, and subsequently accumulate quasi-rents for a sustained period of time, such that the expected profits – net of the fixed cost – are zero. Consequently, a persistent shock – or, indeed, an *anticipated* shock – affects

⁴³Quantitative evaluations of uncertainty shocks have highlighted the difficulty estimated theoretical models have in generating large uncertainty effects that match empirical evidence. [Born and Pfeifer \(2014a\)](#) specifically highlight the need for mechanisms offering “asymmetric amplification” in the sense of strengthening the propagation of second-moment shocks more than that of level shocks.

⁴⁴Empirically, labor market asymmetries of this type are well-documented (e.g., [McKay and Reis \(2008\)](#), [Benigno et al. \(2015\)](#), and [Ferraro \(2018\)](#)). [Dupraz et al. \(2019\)](#) suggest, however, that search frictions may not by themselves be sufficient to quantitatively match the asymmetries observed in the data.

firms' quasi-rents for an extended period of time, which generates large fluctuations in the asset value already in the present, and thereby also returns and incentives for job creation. This contrasts markedly with a standard RBC model, in which productivity beyond the immediate future bear no direct consequences on current returns (Barro and King, 1984). We believe this distinct feature may have strong implications for a number of issues in macroeconomics beyond those analyzed in this paper, including policy (such as forward guidance), demand propagation, and more generally the management of agents' expectations.

Appendix A

A.1 Proofs

A.1.1 Proof of Proposition 1

In terms of the steady state job finding rate, f , employment is given by

$$n(f) = \frac{f}{f(1 - \delta) + \delta}.$$

As a consequence

$$\frac{\partial n}{\partial f} = \frac{\delta}{[f(1 - \delta) + \delta]^2},$$

which is positive and monotonically decreasing. Thus $n(f)$ is increasing and strictly concave in f .

The free-entry condition further implies that

$$f(z) = \psi^{\frac{1}{\alpha}} \kappa^{\frac{1-\alpha}{\alpha}} J(z)^{\frac{1-\alpha}{\alpha}}.$$

Thus, as $J(z)$ is a linear function (see equation (28) in the main text), $\alpha \geq 1/2$ is sufficient to guarantee that $f(z)$ is weakly concave, and $n(z)$ is therefore a strictly concave function of z . \square

Appendix B

B.1 Additional results

B.1.1 Shock persistence and the risk premium

The analysis in the main text stressed that the long-horizon valuation of firms in the search-and-matching (SaM) framework plays a crucial role in rationalizing the transmission channels through which uncertainty affects both unemployment and inflation. We drew particular attention of the presence of a risk premium, increases in which operate akin to a negative supply shock.

To further illustrate this idea, we solve the model repeatedly for a two-dimensional grid of different values for the persistence of productivity, ρ_z , and the persistence of the standard deviation of productivity shocks, ρ_{σ_z} . The benchmark values are 0.95 and 0.76, respectively. For each parameter combination we compute the response of the risk premium to an uncertainty shock both on impact and cumulatively along the IRF. Figure B.1 plots the result. It can be seen that while the magnitude of the rise in the risk premium on impact is a function of ρ_z only, a more persistent rise in uncertainty, as indicated by higher values of ρ_{σ_z} greatly magnifies the cumulative increase in the risk premium.

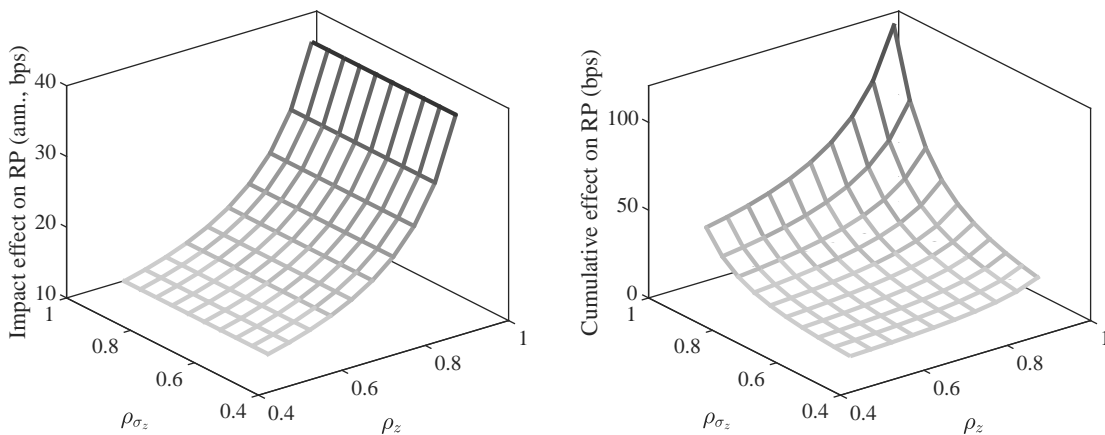


Figure B.1: The response of the risk premium as a function of shock persistence

Notes: The figure illustrates the effect of a one standard-deviation pure uncertainty shock on the risk premium, both on impact and cumulatively, as a function of the persistence of productivity, ρ_z , and the persistence of the standard deviation of productivity shocks, ρ_{σ_z} . The model is solved assuming flexible prices and log utility.

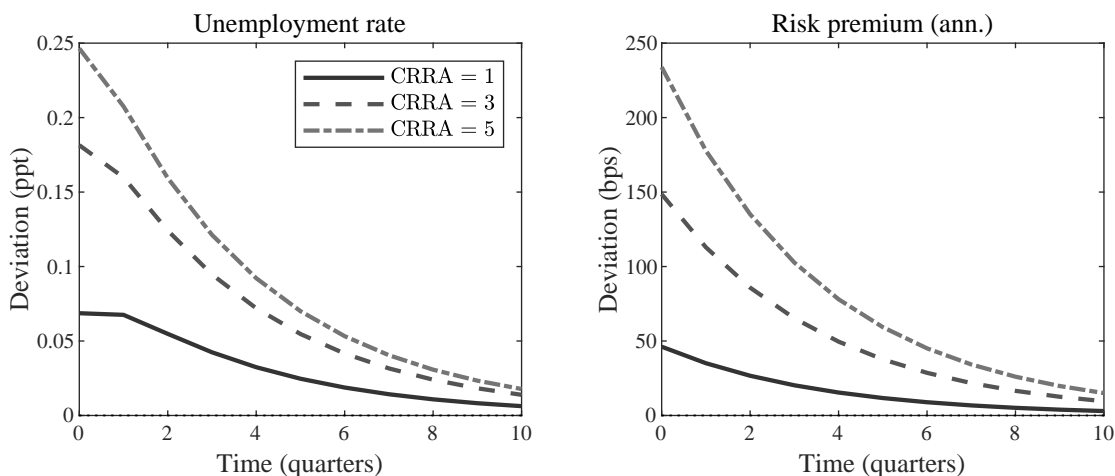


Figure B.2: Greater risk aversion worsens the recession under sticky prices

Notes: The figure illustrates the pure uncertainty IRFs for a one standard-deviation shock to volatility under sticky prices, varying the coefficient of relative risk aversion (CRRA). The baseline case with log utility corresponds to the line associated with “CRRA = 1.”

B.1.2 Greater risk aversion and the risk premium

Our benchmark parameterization assumes log utility and, thus, a fairly low value of risk aversion that tends to downplay the role of uncertainty in shaping economic activity. To complement the results offered in the main text, Figure B.2 shows the effects of a pure uncertainty shock under sticky prices and allowing for three different values of the coefficient of relative risk aversion (CRRA). The figure makes two simple points. Firstly, the extent to which households demand an (even) greater risk premium to compensate them for holding risky equity claims when they anticipate elevated levels of future volatility is greater for higher degrees of risk aversion. And, secondly, because of the negative impact this dynamic has on asset prices (the match value), an uncertainty shock then consequently also lowers entry by more, with adverse consequences for unemployment.

B.1.3 Precautionary pricing

When firms are subject to nominal rigidities, an increase in uncertainty about future demand conditions may give rise to precautionary pricing that renders markups more countercyclical.^{B.1} Intuitively, such behavior arises when firms’ marginal revenue product exhibits convexity; it is more costly for a given firm to set too low a price relative to its competitors (more units need to be sold at a sub-optimally low price) compared to setting it suboptimally high (the higher price per unit partially compensated for fewer units sold).

^{B.1}For details on precautionary pricing, we refer to Fernández-Villaverde *et al.* (2015), Oh (2019), and Born and Pfeifer (2020).

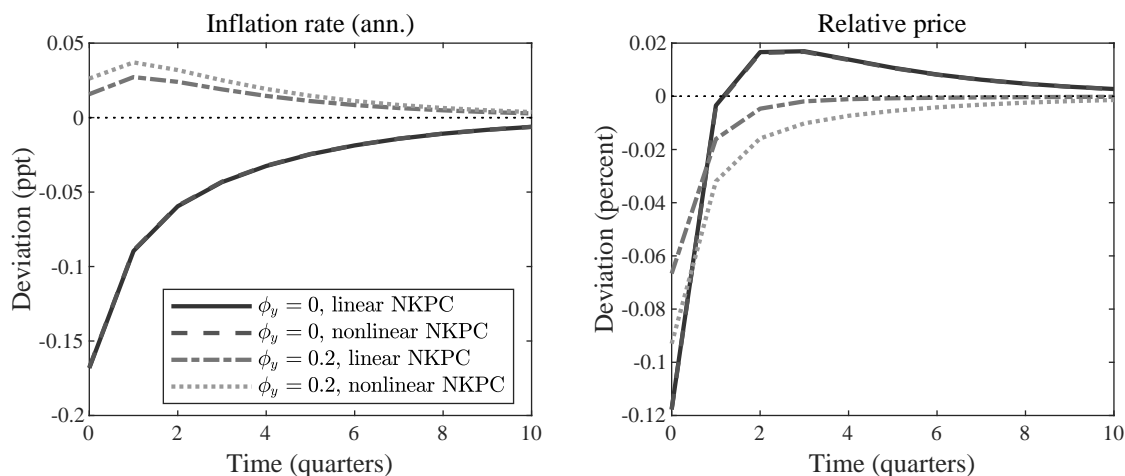


Figure B.3: The limited role of precautionary pricing

Notes: The figure illustrates the IRFs for a one standard-deviation shock to volatility under log utility and sticky prices. The different IRFs vary according to the coefficient on output in the Taylor rule, ϕ_y and differ in whether the structural new-Keynesian Phillips curve equation is linearized or not when solving the model.

To avoid confounding the key mechanisms of interest in our analysis with such effects, all results in the main text are derived by imposing a linearized version of the new-Keynesian Phillips curve (NKPC) in equation (16). This approach eliminates nonlinear terms that could potentially generate an upward pricing bias (cf. Fernández-Villaverde *et al.* (2015, Section VI)). To verify the robustness of our results, Figure B.3 plots the usual pure uncertainty IRFs under the benchmark parameterization, but illustrating also the outcome when the underlying model is solved such that the nonlinearity of the NKPC is preserved.

The primary point Figure B.3 serves to make is to illustrate why we consider our baseline assumption not only a useful simplification, in that it strengthens tractability, but also an innocuous one. As can be seen from the figure, nonlinearities in the NKPC play virtually no role when $\phi_\pi = 1.5$ and $\phi_y = 0.0$, and only a very minor role if that latter parameter is raised to 0.2 (and, consequently, future inflation is expected to be relatively more volatile). Consistent with the principles of precautionary pricing, to the extent that there do exist nonlinearities in the NKPC, they bias inflation upward and the relative price (i.e., the inverse relative markup) downward; the latter generally exerts a negative effect on job creation. To the extent that nonlinearities in the NKPC *do* exert upward pressure on inflation, this only goes to reinforce our finding that uncertainty shocks should give rise to a markedly ‘flatter’ relationship between realized unemployment and inflation compared to regular demand shocks.

A secondary point that the figure serves to re-emphasize is that because uncertainty shocks directly affect both demand and supply, it puts less disinflationary pressure on the economy than a pure negative demand shock. In particular, we see that with or without precautionary pricing, when

$\phi_y = 0.2$ then even for a modest coefficient of relative risk aversion, $\gamma = 1$, the uncertainty shock is, in fact, inflationary. The reasons for this result are discussed in Section 4.2 of the main text.

B.1.4 Resource-draining vacancy posting costs

In the benchmark model we assume that vacancy posting costs are rebated to the household. If one were to suppose, instead, that these expenditures subtract from consumption in the resource constraint – that is, $c_t + \kappa v_t = z_t n_t$ – then this gives rise to an additional transmission channel that operates under both flexible and sticky prices. As vacancies v_t are convex in productivity, a spike in expected volatility lowers the expected level of output available for consumption. Such an expectation then sets off a greater desire for savings in the present. Quantitatively, this channel turns out to be insignificant (figures are available upon request), so that we abstract from it in order to focus on the key transmission channels of interest.

B.1.5 Phillips curve plots

Because uncertainty shocks combine features of demand shocks with supply features, an economy perturbed by uncertainty shocks will display a Phillips curve relationship that appears flatter than the same economy subject to “traditional” demand shocks. In this appendix section we offer several results complementing the discussion in the main text.

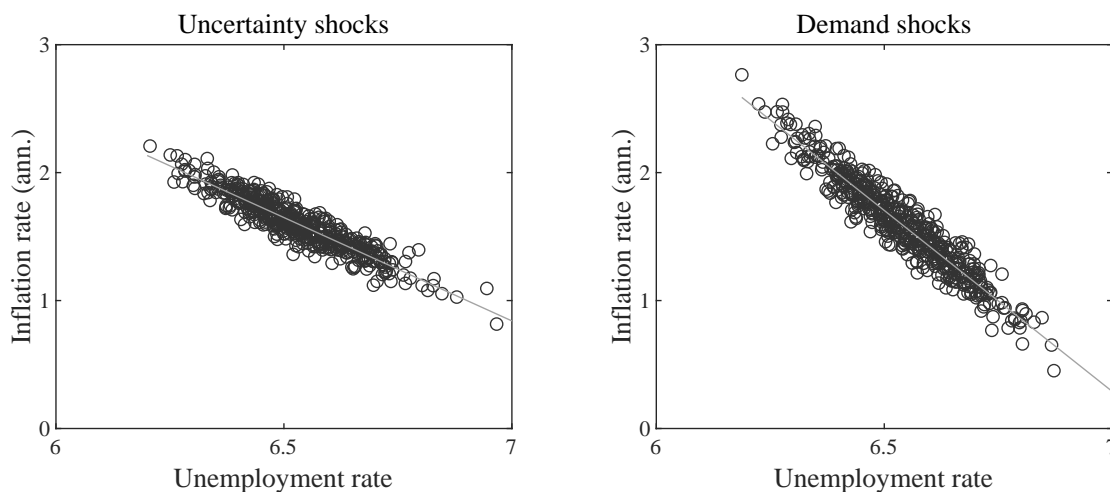


Figure B.4: Demand vs. uncertainty shocks: scatter plots

Notes: The figure shows 500 simulations of an economy hit by normally distributed shocks to either the perceived, but not materialized, volatility of shocks to productivity (left panel) or to the level of the interest rate (right panel). The underlying policy functions are approximated at third order under log utility and sticky prices.

First, while in the main text we focused on the fitted regression lines, Figure B.4 shows scatter plots of realizations in unemployment and (annualized) inflation for an economy simulated over

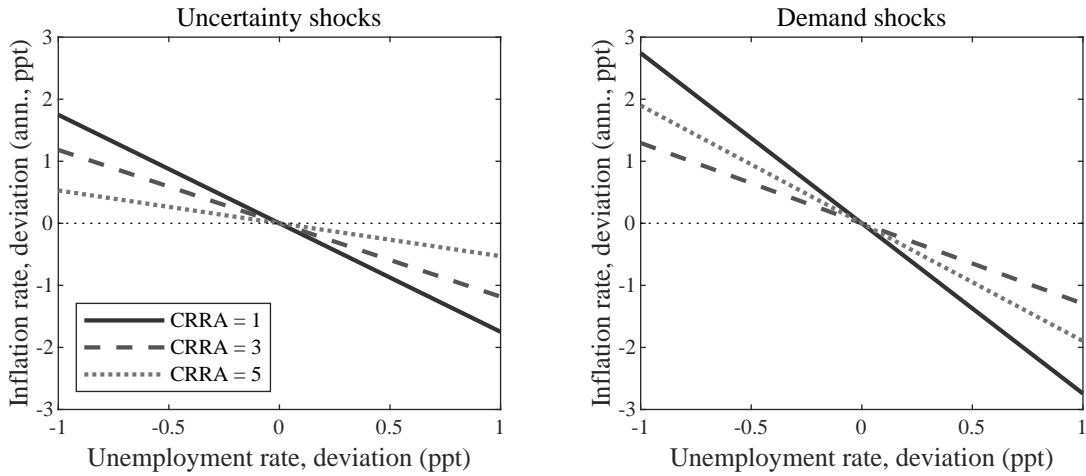


Figure B.5: Demand vs. uncertainty shocks: degrees of risk-aversion

Notes: The figure shows linear regression lines fitted to simulated demeaned data of unemployment and inflation, under the baseline calibration with sticky prices. The lines labeled *uncertainty shocks* relate to simulations with normally distributed innovations to the standard deviation of productivity shocks only; all other shocks are shut down. The “demand shocks” line refers to a simulation with only shocks to the level of the interest rate; the average size of shocks to the interest rate is such that the implied standard deviation of unemployment is equal to the uncertainty-only simulation in the log-utility case.

500 periods. Specifically, the left-hand panel describes an economy that is subject only to normally distributed innovations to the volatility of productivity shocks; all other shocks are shut down. Each dot then represents a pair (u_t, Π_t^{ann}) for one period t . The right-hand panel repeats the exercise, but allowing only for shocks to the level of the nominal interest rate. To facilitate direct comparison, and because interest rate shocks tend to have disproportionately stronger effects on the economy than uncertainty shocks in this model, we scaled the average size of shocks such that the implied standard deviation of the unemployment rate is equal across both cases. As anticipated, the simulations show that the Phillips Curve implied by uncertainty shocks implies changes in the unemployment rate to be associated with smaller variations in inflation than is the case following demand shocks.

Second, Figure B.5 demonstrates that the slope of the (observed) Phillips curve relationship in an economy subject to uncertainty shocks is flatter when the coefficient of relative risk aversion (CRRA), captured by parameter γ , is greater than the benchmark value of unity. For completeness, Figure 8 clarifies that such a change in parameterization may also flatten that relationship observed conditional on interest rate shocks. The reason is that under the assumed functional form for utility an increase in γ necessarily also decreases the elasticity of intertemporal substitution (EIS). As can be seen in the figure, however, the impact on the slope of the Phillips curve seems to be nonlinear in γ in the case of demand shocks, as it flattens for $\gamma = 3$ but steepens if γ increases further. The same is not observed for uncertainty shocks. Lastly, we note that the tight link between EIS and CRRA could be broken by adopting recursive preferences, as is done in many papers on uncertainty shocks

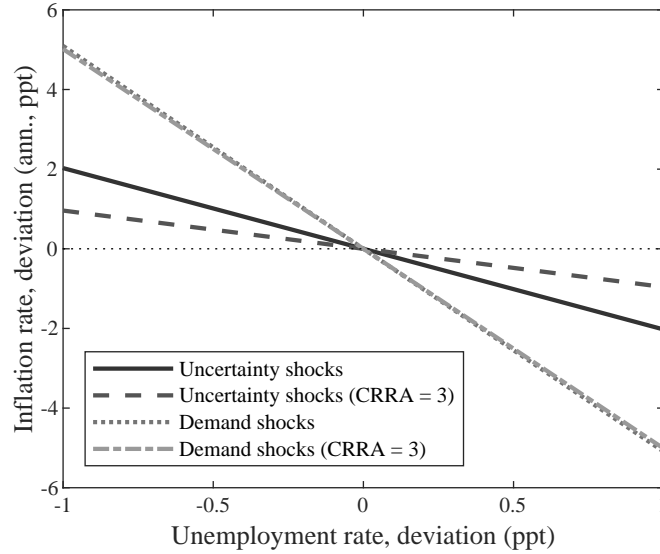


Figure B.6: Demand vs. uncertainty shocks: Interest rate shocks

Notes: The figure shows linear regression lines fitted to simulated demeaned data of unemployment and inflation, under the baseline calibration with sticky prices. The lines labeled “uncertainty shocks” relate to simulations of an economy that is subject only to normally distributed innovations to the standard deviation of interest rate shocks; all other shocks are shut down. The “demand shocks” economy only experiences shocks to the level of the interest rate, instead.

(e.g., [Basu and Bundick \(2017\)](#), but also see [de Groot et al. \(2018\)](#)). Such an analysis would be interesting but extends beyond the scope of this paper.

Finally, we analyze whether our results for the slope of the Phillips curve, which were based on uncertainty over future productivity shocks, generalize to uncertainty regarding future demand shocks. Specifically, we extend the Taylor rule to include a persistent interest rate shock that is determined by stochastic processes analogous to those we used for productivity (as used also in [Ghironi and Ozhan \(2019\)](#)). Figure B.6 shows the Phillips curve slopes resulting from simulations of an economy subject to shocks to either the level or the perceived volatility of the nominal interest rate, respectively. For illustrative purposes we used the same parameterization that [Basu and Bundick \(2017\)](#) use for their (time) preference shock process.^{B.2} The figure suggest that, analogous to the the case of uncertainty over future productivity shocks, classic demand shocks render a steeper Phillips curve than shocks to the anticipated volatility of future demand shocks do.

^{B.2}That is, the persistence parameters are $\rho_{e_R} = 0.93564$ and $\rho_{\sigma_R} = 0.74227$, respectively, with average volatility $\sigma_{e_R} = 0.0026251$ and a standard deviation of the volatility shock equal to $\sigma_{\sigma_R} = 0.0025022$. We could alternatively have used preference shocks. Notice, though, that the associated transmission channels would be somewhat more intricate (and not as generically associated with the demand side), because the household discount factor directly enters not only the Euler equation for bonds, which is where the nominal interest rate appears, but also the Euler equation for equity and the NKPC.

Appendix C

C.1 Methodological notes

This appendix summarizes a few concepts to clarify the numerical approach taken in the main text.

C.1.1 Definitions

Consider a dynamic and stochastic (discrete time) system made up of just one endogenous variable, y , that is subject to the exogenous shock ε ; the ideas extend to higher-dimensional systems. Write the policy function for y_t defining optimal decisions given state y_{t-1} and shock ε_t as $y_t = g(y_{t-1}, \varepsilon_t)$. To complete the notational setup, denote the past history of shocks by $\Omega_{\varepsilon,t} \equiv \{\dots, \varepsilon_{t-2}, \varepsilon_{t-1}\}$ and future realizations of shocks by $\Omega_{\varepsilon,t}^f \equiv \{\varepsilon_{t+1}, \varepsilon_{t+2}, \dots\}$.

The deterministic steady state (DSS) of a system refers to the fixed point of that system provided all stochastic elements are removed forever. In other words, it is the state reached in the absence of shocks and expecting no future risk. The, with some misuse of notation, the deterministic steady-state y^{DSS} satisfies $g(y_t, \varepsilon_t = 0 | \Omega_{\varepsilon,t}^f = \{0, \dots\}) - g(y_{t+1}, \varepsilon_{t+1} = 0 | \Omega_{\varepsilon,t+1}^f = \{0, \dots\}) = 0 \quad \forall t$, and we can write it as $y^{\text{DSS}} = g(y, 0 | \Omega_{\varepsilon}^f = \{0, \dots\})$.

The stochastic steady state (SSS) of a system, on the other hand, is that point in the state-space where agents would choose to remain if there are no shocks in that period but possibly in the future. That is, the stochastic steady-state satisfies $g(y_t, \varepsilon_t = 0) - g(y_{t+1}, \varepsilon_{t+1} = 0) = 0 \quad \forall t$, and, hence, $y^{\text{SSS}} = g(y, 0)$.

Finally, assuming the system satisfies stationarity and ergodicity, the ergodic mean with shocks (EMWS, also referred to as “ergodic mean” *simpliciter*) corresponds to the theoretical mean of the process when shocks evolve normally: $y^{\text{EMWS}} = E[y_t]$.

The SSS is sometimes also referred to as the “ergodic mean in the absence of shocks” (EMAS), because we can think of it also as average value in a long sample when shock realizations are zero yet agents take into account the possibility of shocks occurring. That is, $E[y_t | \varepsilon_t = 0]$. This way of thinking about the SSS is also informative about the method by which we can find the SSS. Unlike for the DSS, we cannot simply ignore randomness. Fortunately, though, we can compute the SSS using simulation-based methods – just as we would do for the EMWS. First, iterate on $y_{t+1} = g(y_t, \varepsilon_t = 0) \quad T$ times, where T is large, starting at y^{DSS} . Note that all shock realizations are zero, but each period, agents do not know that this will be the case going forward. Given the resulting sample $\{y_s\}_{s=1}^T$, we approximate $\hat{y}^{\text{SSS}} = y_{B+1}$, where B is the number of burn-in periods needed for the process to converge from the DSS to the SSS. By the definition of a steady state, $y_{B+1} = y_{B+2}$ and we can equivalently say that $\hat{y}^{\text{SSS}} = \frac{1}{T-(B+1)} \sum_{l=B+1}^T y_l$.

C.1.2 Computation of IRFs

In general, when nonlinear methods are used to solve a model, IRFs will depend on both the sequence of future shocks and the point in the state space at which the IRFs is started, i.e., the past history of shocks. Given the additional complexities, [Koop *et al.* \(1996\)](#) suggest the use of “Generalized Impulse Response Functions” (GIRFs). The GIRF of variable y at a time $t + l$ after a shock ε_t and conditional on the history of shocks $\Omega_{\varepsilon,t}$ is given by

$$GIRF_l(\varepsilon_t, \Omega_{\varepsilon,t}) = E_t[y_{t+l} | \varepsilon_t, \Omega_{\varepsilon,t}] - E_t[y_{t+l} | \varepsilon_t = 0, \Omega_{\varepsilon,t}].$$

This constitutes a “representative” IRFs at the ergodic mean in the sense that future shock realizations are averaged out.

The method of [Fernández-Villaverde *et al.* \(2011\)](#) we employ differs from this approach in two respects ([Born and Pfeifer, 2014b](#)). First, we condition on the future realizations of shocks being zero. Second, and consistent with this, we start the IRFs at the EMAS rather than the EMWs. The associated IRFs can be defined as follows

$$\begin{aligned} IRF_l(\varepsilon_t, \Omega_{\varepsilon,t}) &= [y_{t+l} | \varepsilon_t, \Omega_{\varepsilon,t} = \{\dots, 0\}, \Omega_{\varepsilon,t}^f = \{0, \dots\}] \dots \\ &\quad - [y_{t+l} | \varepsilon_t = 0, \Omega_{\varepsilon,t} = \{\dots, 0\}, \Omega_{\varepsilon,t}^f = \{0, \dots\}]. \end{aligned}$$

Note that we may drop the expectations operators, since everything is deterministic.

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