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FISCAL RULES AS BARGAINING CHIPS

Facundo Piguillem and Alessandro Riboni

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Abstract

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JEL Classification: H2, H6, D72

Keywords: fiscal rules, Government Debt, legislative bargaining, Political Polarization, Government shutdown=

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Fiscal Rules as Bargaining Chips *

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April 28, 2020

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Most fiscal rules can be overridden by consensus. We show that this does not make them ineffectual. Since fiscal rules determine the outside option in case of disagreement, the opposition uses them as “bargaining chips” to obtain spending concessions. We show that under some conditions this political bargain mitigates the debt accumulation problem. We analyze various rules and find that when political polarization is high, harsh fiscal rules (e.g., government shutdown) maximize the opposition’s bargaining power and leads to lower debt accumulation. When polarization is low, less strict fiscal limits (e.g, balanced-budget rule) are preferable. Moreover, we find that the optimal fiscal rules could arise in equilibrium by negotiation. Finally, by insuring against power fluctuations, negotiable rules yield higher welfare than strict ones.

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1 Introduction

Can fiscal rules be effective even when they are not respected? Various explanations have been put forth to explain the steady growth in public debt in the past decades. Frictions such as political polarization and turnover have been shown to generate incentives to accumulate debt in ways reminiscent of hyperbolic discounting, and therefore fiscal rules are considered optimal.¹ Recently, fiscal rules have spread worldwide, partly as a response to the fiscal legacy of the Great Recession.

Fiscal rules are usually embedded in statutory norms or constitutional laws. However, compliance is not guaranteed. In particular, it is often possible for politicians to override the rules if there is consensus. A fitting example is given by the harsh caps on discretionary spending introduced by the U.S. Budget Control Act (BCA) of 2011. These limits were not written in stone: the BCA also stipulated the procedure by which Congress could amend them. So far these caps have yet to be adhered to; in each year since 2011, Congress passed bipartisan legislation raising them.

It is tempting to assume that fiscal rules are ineffective when not respected. Contrary to this, we argue that fiscal rules may improve outcomes even when the possibility of overridden them exists. The fact that the rules are the default option when legislators disagree changes the incentive to override them in the first place. Indeed, we show that their effectiveness may even approach the optimal outcome.

When analyzing incentives to over-accumulate debt we need to take a stand on what is the underlying friction. There are many alternative theories, such as Persson and Svensson (1989), Alesina and Tabellini (1990) and Battaglini and Coate (2008), with distinctive and interesting predictions. However, all of them share an essential feature generating the friction: preference misalignment between current and future governments. Therefore, we consider the simplest strategic model of debt relying solely on this friction. As in Alesina and Tabellini (1990), two parties alternate in power and must decide how to allocate two public goods financed by taxes and debt. The parties differ in terms of the desired composition of spending: each party would like to allocate most (or all) of the budget to only one of the goods. As is well-known, political turnover and preference misalignment result in the overissuance of debt. If there is a positive probability of being voted out of office, the incumbent prefers spending according to her own preferences rather than transferring resources to the future.

¹For a survey of political frictions generating over-accumulation of debt see Alesina and Passalacqua (2015) and Yared (2019). For the optimality of rules see Amador et al. (2006) and Halac and Yared (2014).

We introduce two key features. First, we assume that policies are the result of negotiations among elected policymakers. In the U.S., for instance, both the executive and legislative powers must agree.² Second, we study the impact of a broad range of fiscal rules. We model fiscal rules as determining an upper bound to the total spending. We assume that *this limit is negotiable*: it can be temporarily raised with the opposition’s consent. If, however, the parties do not reach an agreement, the incumbent may allocate spending within the limit.

Before proceeding, it is important to review actual rules. In reality fiscal rules can take two forms. First, *procedural rules* regulate the process by which decisions are made; including the reversion policy when legislators cannot agree on a new budget. Second, there are *numerical restrictions* to fiscal outcomes, such as budget balanced and spending caps.³ Since compliance is problematic, without an enforcing mechanism numerical rules are only effective if policymakers do not agree to waive them. As a result, numerical constraints are similar to “budgetary reversion” rules: *they both set a threat point in budget negotiations*.⁴

Negotiable rules are widespread. Balanced budget laws can be suspended by a supermajority vote.⁵ In the U.S., caps on discretionary spending are also negotiable because Congress can raise them through the regular legislative process. The debt ceiling constrains government borrowing unless Congress agrees to raise it. A “government shutdown” is an example of procedural rule: if the legislature cannot agree on the budget, the default reversion policy for discretionary spending is zero. The key element of these rules is that they determine a threat point in budget negotiations and, thus, allow the opposition to use them as “*bargaining chips*.” American politics offers several examples of fiscal rules being used as leverage to push budgetary agendas. In 2011, for instance, Republicans used the debt ceiling threat to influence the Obama administration’s spending plans. Recently, the Democrats threatened a government shutdown to force the withdrawal of Trump’s immigration proposals.⁶

²Even when the executive and the legislative majority belong to the same party, institutional rules such as filibuster may endow the opposition with de-facto veto power.

³In the 90’s only a few countries (e.g. Germany, Japan and the U.S.) had numerical limits on budgetary aggregates. According to the IMF fiscal rule dataset, by 2012 the number of countries with numerical fiscal rules had risen to 76 (Budina et al. (2012)).

⁴Throughout this paper, we use the term fiscal (or budget) rules to denote both numerical rules and procedures. Notice that the “IMF definition” of fiscal rule includes only numerical rules. For a broader definition, which includes also procedural rules, see Drazen (2002) and Alesina and Perotti (1999).

⁵In several countries, budget balance laws have escape clauses (such as wars, recessions, and natural disasters) and can be overridden with a qualified majority. As a result, enforcement of the rules is problematic. Even in Germany, compliance with the golden rule, prohibiting borrowing in excess of investment, has been weak since its introduction. See Baumann and Kastrop (2007).

⁶For instance, Rep. Luis Guterrez (D-Ill.) stated: “*I’m not saying we should shut down the government, but if you want a budget with Democratic votes, then it’s got to have some Democratic priorities*” (Washington

Considering the previous discussion, we study how different fiscal rules, acting as “threat points,” affect bargaining outcomes. We address two main questions. First, we investigate whether negotiable rules limit large debt buildups. Second, we study which fiscal rule is most effective in favoring inter-party compromise and in reducing debt.

With respect to the first question, we show that fiscal rules have the following effects. When a rule is present, the party in power offers concessions to the opposition to avoid its application. As a result of this political bargain, budgets are less skewed towards the preferences of the incumbent government. The effect on debt accumulation is less straightforward. To garner the support of the opposition, the government must spend more on the good that the opposition prefers. This implies that fiscal rules could lead to more spending (and more debt) compared to a situation with no fiscal rules. There are, however, two other effects to consider. First, political compromise raises the cost of debt because the opposition agrees to increase debt only if she is sufficiently compensated. Second, the incumbent realizes that when she is out of power the other party will also reach for a compromise to override the rule. The expectation that in the future the other party will partly share the total resources increases the incumbent’s benefit of transferring resources to the future. We show that the first effect is outweighed by the two other effects. As a result, debt accumulation is reduced when parties negotiate in the shadow of fiscal rules.

The second key contribution of the paper is to study the impact of the off-equilibrium threat determined by the fiscal rule. In particular, we analyze how equilibrium debt changes when the fiscal limit tightens. One might worry that stringent fiscal rules (i.e., a government shutdown or harsh spending caps) would have such a punitive “threat point” that the incumbent would be able to override the fiscal limit without much compromise. We find instead that when political polarization is high (i.e., the two parties have opposite preferences), it is optimal to have stringent fiscal rules which drastically reduce public spending in case of disagreement. However, when polarization is low, the optimal budget rule calls for a less punitive threat point, (e.g., a balanced budget).

These different results are due to the manner in which budget rules affect the value of the outside option in different environments. When there is high polarization, the opposition knows that in case of disagreement the incumbent will primarily make cuts on the goods valued by the opposition. Thus, a harsh fiscal rule (e.g., government shutdown) is optimal because it postpones spending, making it possible for the opposition to consume in future

Post, 9 October 2017). In 2017 President Trump also threatened a government shutdown to force Congress to pass a bill to finance the border wall with Mexico.

periods should she become the incumbent. With her bargaining power strengthened, the opposition is able to appropriate more resources each period, decreasing the variability of the distribution of spending and, therefore, reducing the incentive to overspend.

Instead, when political polarization is low, the intuition that punitive rules can be “cheaply” waived by the incumbent can be applied. In this case, a stringent fiscal limit would make the acceptance constraint non-binding: the opposition prefers letting the incumbent act as a policy dictator to enforcing the fiscal limit. This occurs because a policy dictator in a low-polarization environment would choose a relatively even allocation of resources and no excessive indebtedness. Thus, a way to endow the opposition with more bargaining power is to increase the amount of resources available upon disagreement (i.e., budget balance). Because a breakdown in negotiation would result in an even split of resources, the off-equilibrium consumption path will be smooth and characterized by low debt accumulation, which is highly valued by the party currently out of power.

Since negotiable rules do not completely eliminate the debt problem, we study whether it would be desirable for both parties to “harden” the rules by making them non-negotiable. When the limits cannot be waived, fiscal rules cannot be used as a “bargaining chip.” We show that a commitment to a “hard” budget balance is not optimal. On the one hand, it would eliminate inefficient debt. But on the other, the incumbent would no longer have an incentive to compromise with the opposition. Hence, under non-negotiable rules the allocation of spending will be heavily skewed towards the executive’s preferences. This generates excessive intertemporal variation in public spending, which is *ex ante* inefficient. Taken together, the second effect dominates and hard rules yield lower welfare than negotiable ones.

We also examine the stability of fiscal rules by analyzing the incumbent’s temptation to “break” them: i.e., to override the fiscal limit without seeking an agreement. In the past decade, Democrats and Republicans alike have been weighing the possibility of sidestepping the Senate’s filibuster in order to enact their agendas. While this is tempting from the majority party’s perspective, it would likely compromise future bipartisanship. We find that rising political polarization does not necessarily imply that the rules sustaining compromise will be broken: higher polarization raises the short-run temptation to unilaterally “break” the rule, but it also increases the value of future compromise. This may explain why, in spite of increased political polarization, most U.S. senators are reluctant to get rid of the filibuster.

Finally, the interaction between fiscal rules and politicians temptation to over-spend delivers novel testable implications. When policies are negotiated, we obtain richer and more complex comparative statics compared to the canonical model with alternating policy dic-

tators. This is because we must also take into account how the parameters affect the opposition’s bargaining power. For instance, we show that the standard intuition that political persistence leads to lower debt accumulation does not necessarily extend to a model with bargaining. When power is more persistent, the opposition’s chances of taking power decrease, making it less valuable for her to transfer resources to the next period. Since it is cheaper to obtain the opposition’s consent, a larger share of spending goes to the incumbent, who may find it optimal to accumulate *more* debt as her reelection chances improve. Therefore, our findings suggest that when studying the empirical effect of political turnover on debt, it is important to condition on the number and type of constraints faced by the incumbent.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents the infinite-horizon model. In Subsection 3.2, we derive results in a tractable two-period model. Section 4 studies the optimal fiscal rules and Section 5 how they could arrived and be sustained in equilibrium. Section 6 provides some comparative statics and discusses the mapping of the model to the U.S. budget procedure. The conclusion follows. All proofs are in the Appendix.

2 Literature Review

This paper is closely related to the strategic-debt literature (e.g., Alesina and Tabellini (1990), Persson and Svensson (1989), Amador (2003), Debortoli and Nunes (2013) Aguiar and Amador (2011), and Chatterjee and Eyigungor (2016)). A well-known result in this literature is that political turnover and polarization generate a debt-bias. Politicians who face uncertain prospects for re-election overspend in the current period and raise debt to tie the hands of future policymakers. In a recent contribution Battaglini and Coate (2008) assume that policies are made through legislative bargaining. In this environment, legislators in the minimum winning coalition do not fully internalize the tax burden of spending decisions and thus approve targeted transfers to their districts. In addition, fearing that they might not be included in future coalitions, legislators have an incentive to transfer resources from the future to the present, leading to an over-accumulation of debt.⁷ Bouton et al. (2016) study the joint determination of debt and entitlement programs (such as pensions and health care). Through entitlements, governments pre-commit a fraction of future resources to a

⁷In Lizzeri (1999), voters favour candidates who propose a transfer of resources to the present because they fear that in the future, these resources will be offered to others. Bisin et al. (2015) study politicians’ incentives to accumulate debt when voters are time inconsistent.

particular use. Entitlements therefore provide an additional instrument to constrain future governments, which weakens the incentive to use debt.

The literature on fiscal rules has greatly expanded in recent years. Azzimonti et al. (2016) study the impact of a budget balance rule in the context of a calibrated version of Battaglini and Coate's model. When there are shocks, fiscal rules impose a trade-off between the cost resulting from a constrained response to the shocks, and the benefit in terms of debt discipline. In contrast to our analysis Azzimonti et al. (2016) study budget balance rules that are strict and cannot be overridden. We study instead how the possibility of override leads to political bargaining. Moreover, we focus on a wider set of rules. Our mechanism works mainly through off-equilibrium threats as in Taschereau-Dumouchel (2017) who analyzes wage bargaining between firms and workers. In his setup workers use the threat of unionization to obtain wage concessions. In Bouton et al. (2016) constraints on debt lead to increased entitlements since debt and entitlement are strategic substitutes. Dosis and Kirpalani (2017) study fiscal rules in federal governments.⁸ When there are fiscal rules at the local level, a lax central government ends up revealing its type earlier (by choosing to not enforce the rules) relative to an environment without rules. Interestingly, because of this effect they show that fiscal rules may exacerbate over-borrowing by local governments. Coate and Milton (2019) characterize the optimal fiscal limit when overrides are possible. They consider a principal-agent model in which the voter's preferred taxation changes stochastically. The politician can either abide by the rule or ask voters for permission to override it. They find conditions under which the optimal limit with override differs from that without. Unlike us, their economy is static and they do not study debt. In their model, fiscal limits take into account the need for flexibility, while in our model, preferences are constant over time.

Halac and Yared (2018) study fiscal rules in a world economy with integrated capital markets. They compare coordinated rules, chosen jointly by a group of countries, and uncoordinated rules, chosen independently by each country. In their model, rules affect countries not only by limiting their borrowing (and their flexibility to respond to shocks) but also by reducing interest rates. Halac and Yared (2017) drop the assumption that fiscal rules can be perfectly enforced and study fiscal rules which are self-enforcing, so that complying with the rule is preferable to the punishment that follows a breach.⁹ In Halac and Yared (2014, 2017,

⁸See Poterba (1996) and Alt and Lowry (1994) on fiscal rules in U.S. states. Grembi et al. (2016) study the impact of fiscal rules in Italian municipalities.

⁹On fiscal rules in a monetary union, see Chari and Kehoe (2007) and Beetsma and Uhlig (1999). Foarta (2018) analyzes fiscal rules in a banking union. Alfaro and Kanczuk (2017) and Hatchondo et al. (2015) study fiscal rules in models of debt and default. The political economy of default is studied in, among others,

2018) politicians have an exogenous present-bias for spending. In our model the present-bias arises endogenously as a consequence of political turnover and interacts with the fiscal rule. Thus, fiscal rules have a direct effect in reducing the temptation to accumulate debt that is not present in the aforementioned papers.

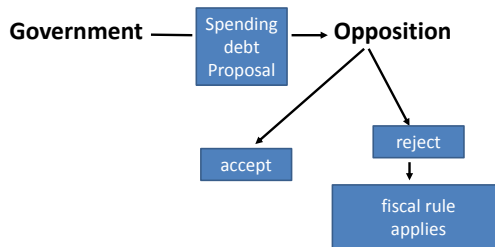
In this paper, we find that political bargaining in the shadow of fiscal rules leads to lower debt accumulation. In other contexts, however, the need to reach a political compromise may lead to higher debt. For instance, in the “war of attrition” model by Alesina and Drazen (1991), a strong government that can lead without compromise with the opposition finds it easier to reduce the deficit, by making the opposition suffer a larger stabilization cost. Existing empirical evidence (e.g., Milesi-Ferretti et al. (2002)) points out that coalition governments are associated with higher debt. This evidence does not necessarily contradict our results. First, this finding could be explained by other variables, such as proportional representation, which are correlated with the incidence of coalition governments. Second, this evidence often refers to coalitions that are weak (with a small majority) while we study coalitions between the two main parties.

3 The Model

Consider an infinite horizon economy. Public spending is financed by current taxes and debt. There are two parties (A and B) that stochastically alternate in power and negotiate policies in the shadow of a fiscal rule. In our model, the fiscal rule (discussed in further detail below) specifies the threat point in the negotiation between the parties. Bargaining in period t , with $t = 1, 2, \dots, \infty$, unfolds as follows (see Figure 1):

- (i) At the beginning of period t , one of the parties is elected and becomes the incumbent.
- (ii) The incumbent makes a take-it-or-leave-it offer to the opposition, specifying spending levels and debt.
- (iii) The opposition accepts the proposal if and only if doing so makes her at least as well-off as if she rejected.
- (iv) If the opposition rejects the proposal, the fiscal rule must apply to the current period. If there is more than one policy which satisfies the rule, the policy choice is up to the incumbent.

Figure 1: **Timing**



In period $t + 1$ the government at time t remains in power with probability $q \in [0, 1]$. With complementary probability $(1 - q)$ the opposition at t becomes the government at $t + 1$.

Tax revenue is exogenous and equal to τ in all periods. Although this assumption is not crucial for the main results, it greatly simplifies the exposition. If tax revenue were endogenous and both parties suffered equally from taxation, the following analysis would remain with minor modifications.¹⁰

Parties A and B are ideological, i.e., they represent the interests of different constituencies. There are two types of public goods and the two parties have different preferences over the desired composition of public spending: each party would like to allocate most (or all) of the budget to one of the two public goods. We let g^A and g^B denote, respectively, the good that is preferred by party A and that which is preferred by party B . For example, when the parties have different geographically based constituencies, g^A and g^B could represent district-specific public projects. The per-period utilities of parties A and B are given by:

$$\begin{aligned}
 u_A(g^A, g^B) &= u(g^A) + \theta u(g^B) \\
 u_B(g^A, g^B) &= u(g^B) + \theta u(g^A)
 \end{aligned}$$

where $u(g^j)$, with $j = A, B$, is a CRRA utility function:

$$u(g^j) = \frac{(g^j)^{1-\sigma}}{1-\sigma}$$

where $\sigma \in [0, 1]$ is the coefficient of relative risk aversion. The parameter $\theta \in [0, 1]$ captures the degree of political polarization. When $\theta = 0$, a party derives no utility from the

Dovis et al. (2016) and Azzimonti and Quadrini (2016).

¹⁰This holds under the assumption that the cost of taxation is given by $u(a - \tau)$, for some $a > 0$. See below for the definition of the function u . If we considered parties with heterogeneous preferences for taxation, the model would be less tractable, but this would not substantially alter the main results.

good favored by the other party, implying maximum disagreement about the composition of spending. As $\theta \rightarrow 1$, disagreement disappears.

We could also incorporate a minimum required, or previously committed, spending level $\bar{g} > 0$. By redefining spending as $\hat{g}^j = g^j - \bar{g}$ and revenues as $\hat{\tau} = \tau - 2\bar{g}$, we would obtain the same solution as long as $\tau > 2\bar{g}$. In this sense we can interpret g^j as “uncommitted” spending, so that an outcome with $g^j = 0$ should be interpreted as losing the possibility of freely allocating resources rather than an equilibrium without spending.

Throughout, we focus on stationary Markov-Perfect equilibria. Parties discount the future at rate β . We write the problem recursively, with current debt b as state variable. Since we assume that parties are symmetric, it is not necessary to specify the identity of the party in power and we thus distinguish parties only by whether they are in power or not. Let I (O) denote the party that is currently in power (out of power). Let $V_I(b)$ and $V_O(b)$ denote, respectively, the value functions of the incumbent and opposition. We define the continuation utility of party $j = I, O$ as:

$$W_j(b) \equiv qV_j(b) + (1 - q)V_{-j}(b) \quad (1)$$

We denote the good that is favored by the current incumbent g^I and the good that is favored by the current opposition g^O . The party in power solves:

$$V_I(b) = \max_{\{g^I, g^O, b'\}} \{u_I(g^I, g^O) + \beta W_I(b')\} \quad (\text{P1})$$

$$s.t. \quad \tau - (1 + r)b + b' - g^I - g^O \geq 0 \quad (\text{BC})$$

$$u_O(g^I, g^O) + \beta W_O(b') \geq m(b) \quad (\text{AC})$$

$$\underline{b} \leq b' \leq \bar{b}$$

$$V_O(b) = u_O(g_I^*(b), g_O^*(b)) + \beta W_O(\mathcal{B}^*(b))$$

The constraint (BC) is the government’s budget constraint, where b is current debt and b' are new debt issuances. The interest rate is exogenous and equal to r . We assume that b' must be smaller than the natural debt limit: $\bar{b} = \tau/r$. This implies that it is always feasible to pay the outstanding debt. For \underline{b} we can assume any negative number, including $-\infty$.

Inequality (AC) is the acceptance constraint: the opposition accepts the “take-it-or-leave-

it” proposal if and only if her utility is greater than or equal to $m(b)$.¹¹ The expression $m(b)$ is the opposition’s “threat point,” which depends on the fiscal rule. If the opposition rejects the incumbent’s proposal, total spending must satisfy the following fiscal limit:

$$g^I + g^O \leq \alpha(\tau - rb) \tag{2}$$

This rule prescribes that in case of disagreement total spending cannot exceed an exogenous proportion $\alpha \in [0, 1]$ of the net income flow, where a lower value of α yields a more stringent rule. If the opposition accepts a proposal that exceeds $\alpha(\tau - rb)$, the fiscal limit is waived only in the current period: any future override will require another agreement between parties. When $\alpha = 1$, expression (2) corresponds to a balanced budget rule that can be waived by consensus: the incumbent cannot spend more than $\tau - rb$ without the opposition’s approval. From (BC), this implies that debt does not grow. A rule $\alpha = 0$ is akin to a fiscal rule that prescribes a government-shutdown in case the parties do not agree. More generally, $\alpha = 0$ captures discretionary spending, that is, spending programs that must be approved each year and are ended if both parties cannot agree (Bowen et al. (2014)).

It is important to stress that the rule (2) does not specify the composition of public spending in case the fiscal rule applies. We assume that the way of meeting the fiscal rule’s requirements is at the discretion of the majority party. That is, in case of disagreement, the majority party acts as a dictator in choosing how to allocate spending, but with limited resources. With this assumption we capture the idea that when it is necessary to implement spending cuts, the executive can exercise her discretionary power to implement cuts on items that she does not particularly value.

Computing the policy in case of disagreement is immediate. Since the incumbent is free to choose the spending mix satisfying (2), it selects g^I and g^O to meet the static first-order condition: $g^O = \theta^{\frac{1}{\sigma}} g^I$. Therefore, we can write the opposition’s value of disagreement as:

$$m(b) = (\theta + \theta^{\frac{1-\sigma}{\sigma}})u(g^I) + \beta W_O(b^s) \tag{3}$$

where b^s is the debt level implied by the application of the rule. Because the rule (2) is

¹¹In Appendix C.5 we solve for an alternative bargaining protocol in which the proposal is made by the opposition. This may capture a situation in which the incumbent has a slim majority, which raises the bargaining power of the opposition. We show there that results are qualitatively unchanged.

satisfied with equality, debt is determined by:

$$b'^s = b + (\alpha - 1)(\tau - rb) \quad (4)$$

Thus, upon disagreement, debt stays constant, when $\alpha = 1$, or decreases, when $\alpha < 1$. The assumption $\alpha \leq 1$ is not essential, but it guarantees that the rule matters, so that both parties are willing to compromise in order to bypass it. If α were sufficiently above one, the incumbent would be unconstrained and act as a policy dictator. The outcome would then be equivalent to that of Alesina and Tabellini (1990). Moreover, as we show in Section 4, in our model a rule with $\alpha > 1$ would never be optimal.

The bargain between parties is non-trivial because the incumbent and the opposition have opposite incentives. Besides disagreeing on the composition of spending, they also disagree on the dynamic allocation of resources. Since the incumbent is not guaranteed to remain in power, she would like to transfer resources from tomorrow to today. Conversely, the opposition counts on the possibility of becoming the incumbent in the future. Consequently, she would like to transfer resources from today to tomorrow. Borrowing terminology from the hyperbolic-discounting literature, the party in power is present-biased, while the opposition is future-biased (this is formally shown in Appendix C.1).

3.1 Markov Perfect Equilibrium

Solving for the politico-economic equilibrium amounts to finding three functions: (i) $g_I^*(b)$, the spending level for the good preferred by the incumbent (ii) $g_O^*(b)$, the spending level for the good preferred by the opposition and (iii) debt dynamics: $b' = \mathcal{B}^*(b)$. Knowing the spending levels, we can compute the *consumption ratio* between the two goods as:

$$\gamma^*(b) = \frac{g_O^*(b)}{g_I^*(b)} \quad (5)$$

When the opposition's bargaining power is weak, the spending composition is more biased towards the preferences of the party in power and $\gamma^*(b)$ is small.

Throughout, we assume $\beta(1+r) = 1$, so that the incentives to run debt arise only from political considerations. In fact, it is immediate to show that the *social optimum* is to keep the debt level constant without issuing new debt, i.e., $b' = b$. A constant debt allows smooth spending over time. Notice that if initial debt is zero, the planner's solution implies that there is no incentive to accumulate debt. This result is obtained regardless of the planner's weights

on each party. If the social planner is utilitarian (equal weights), spending on both goods will be the same: $g^I = g^O = (\tau - rb)/2$ in all periods (See Appendix A.1). To summarize, the infinite horizon's analysis is carried out under the following assumptions.

Assumption 1. *Let $\theta \in [0, 1]$, $\beta \in [0, 1)$, $q \in [0, 1]$, and $\beta(1 + r) = 1$.*

Using a guess-and-verify approach, we compute the Markov equilibrium in which debt is the payoff-relevant state variable. We find that policy rules are linear in the net income flow.

Proposition 1 *Let a fiscal rule $\alpha \in [0, 1]$ be given. The Politico-Economic Equilibrium is characterized by two equilibrium factors of proportionality: $\nu \in [1, (1 + r)/r]$ and $p \in [1, 2]$. The proportion of resources allocated to the incumbent's preferred good is a linear function of tax revenue after interest payments:*

$$g^I = \frac{\nu}{p}(\tau - rb) \quad (6)$$

Total spending is given by:

$$g^I + g^O = \nu(\tau - rb) \quad (7)$$

The evolution of debt satisfies:

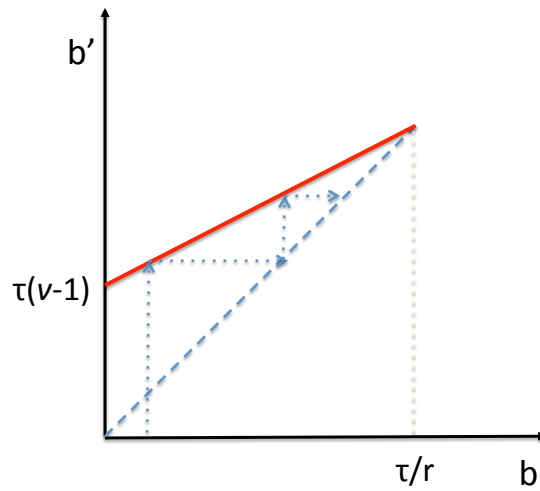
$$b' = b + (\nu - 1)(\tau - rb) \quad (8)$$

Proof: Appendix A.

From (8), the debt growth rate is governed by ν . The higher is ν , the more severe is the debt problem. Coefficient p determines how current spending is allocated between the incumbent and the opposition. From equations (5), (6) and (7) one can solve for the equilibrium consumption ratio: $\gamma = p - 1$. When $p \rightarrow 2$, the consumption ratio converges to one, implying that spending is shared equally among parties. In Section 4 we write down the conditions that implicitly define the equilibrium coefficients ν and p . Such conditions depend, non-trivially, on all of the model's parameters (e.g., polarization, curvature in the utility function, and the fiscal rule). We stress a key feature of our characterization: ν and p are constant and do not depend on current debt. Thus, $\gamma = g^O/g^I$ is also independent of debt and therefore remains constant over time. Intuitively, the equilibrium coefficients are constant because preferences are homothetic and the problem is symmetric.

Recall that under the utilitarian social optimum $\nu = 1$ and $p = 2$. The politico-economic equilibrium differs from it along two dimensions. First, the debt growth rate is strictly positive ($\nu > 1$). As a result, spending is inefficiently front-loaded. Second, the party in power appropriates a larger share of resources than the opposition ($p < 2$). Therefore, political turnover generates inefficient intertemporal risk-sharing. In Section 4 we analyze how, by varying α , the politico-economic equilibrium approaches the social optimum.

Figure 2: **Debt Dynamics**



In Figure 2 we illustrate the debt dynamics implied by (8). On the vertical axis, we plot future debt b' as a function of current debt. When $\nu > 1$, debt grows until it reaches the steady state, which is the natural debt limit. Since interest payments increase over time, from (7) the spending on both goods must progressively decrease. In the limit, the entire revenue will be used to pay interest and spending will be zero. Under the alternative interpretation of g^j as uncommitted spending, we can think about the limit outcome as governments losing the ability to freely allocate resources. After years of irresponsible behavior the government ends up with all resources committed to debt payments and fundamental functions and must either pay or default. In Section 4 we show that under all fiscal rules debt reaches the same steady state, τ/r . The rules will, however, affect transition dynamics by changing the intercept of the debt function. Note that a smaller intercept implies that the politico-economic equilibrium is closer to the planner's solution.

3.2 Two-period Model

To highlight the main mechanisms behind our results, we solve a model with two periods: $t = 1, 2$. To make the analysis in this section more transparent, we focus on extreme political polarization: each party cares only about one good. This case with $\theta = 0$ is the most interesting because the debt-accumulation problem is the most severe. To avoid cluttered notation, we also assume that $r = 0$, $\beta = 1$ and that there is no initial debt: $b_1 = 0$. Summarizing, the analysis for the two-period model is carried out under the following assumptions.

Assumption 2 (two-period model) *Let $\theta = 0$, $r = 0$, $\beta = 1$, and $b_1 = 0$.*

Under the above assumption, the government's period-1 budget constraint is:

$$g_1^I + g_1^O \leq b_2 + \tau \tag{9}$$

where b_2 denotes the quantity of debt at the end of period 1. In the second period all debt must be paid and new debt cannot be issued. The period-2 budget constraint is given by:

$$g_2^I + g_2^O + b_2 \leq \tau. \tag{10}$$

We assume that $b_2 \leq \tau$, so that it is always feasible to pay the outstanding debt at $t = 2$.

Before proceeding, it is instructive to compute the solution for when the party in power is a policy dictator unconstrained by fiscal rules, which is the standard approach in the strategic-debt literature. The problem of the (alternating) dictator can be easily solved backwards. At $t = 2$, the available resources are $(\tau - b_2)$. When there is full polarization, the dictator spends $(\tau - b_2)$ for her preferred public good and 0 for the other public good. Proceeding backwards, we use the final period solutions and the fact that the budget constraints hold with equality to obtain the following first-order condition for debt at $t = 1$:

$$u'(\tau + b_2) = qu'(\tau - b_2) \tag{11}$$

The left-hand side of (11) is the gain from issuing one more unit of debt while the right-hand side is the expected cost in terms of the need to cut future expenditure. Note that the planner's first-order condition is given by $u'(\tau + b_2) = u'(\tau - b_2)$, and therefore $b_2 = 0$. If the incumbent is always in power ($q = 1$) it is immediate that there is no over-accumulation of debt. Suppose instead that in the first period, the incumbent believes that there is some probability of being turned out of office. If in the second period she happens to be out

of power, the composition of public spending will be chosen according to the opposition's preferences. As a result, the incumbent does not fully internalize the value of future resources, which leads her to overspend in the first period.¹² The higher the probability of losing power, the larger the debt. Moreover, a higher value of the coefficient of relative risk aversion implies that the dictator wants to smooth consumption over both periods, which leads to lower debt. The next proposition re-states a well-known result.

Proposition 2 (alternating dictators) *When the incumbent is a policy dictator, debt is strictly positive. Debt is decreasing in q and σ .*

Proof: See Appendix B.1.

Next, we introduce fiscal rules into the two-period model and analyze how rules reduce the incentive to over-accumulate debt. The political bargain in the first period unfolds as shown in Figure 1. Unless there is a political consensus, public good spending in period 1 must satisfy the following condition:

$$g_1^I + g_1^O \leq \alpha\tau \quad (12)$$

Under the assumption that $b_1 = 0$, the above inequality coincides with the fiscal rule (2). Regarding bargaining at $t = 2$, we will assume that in the final period available resources are shared exogenously. More specifically, at $t = 2$ both parties negotiate and a proportion $\gamma'/(1 + \gamma')$ of total resources (with $\gamma' \in [0, 1]$) is spent on the public good favored by the opposition while the remaining is spent on the public good favored by the incumbent. That is, for any level of debt b , the spending composition in the final period is:

$$g_2^I = \frac{\tau - b_2}{1 + \gamma'} \quad \text{and} \quad g_2^O = \gamma' \frac{\tau - b_2}{1 + \gamma'} \quad (13)$$

The higher the value of γ' , the higher the opposition's *future* bargaining power, which is the outcome of the repeated bargaining in the infinite horizon economy, where γ' is endogenously determined. Taking γ' as exogenous at this stage allows us to disentangle the effects due to present bargaining and those due to future expected bargaining. We will later choose γ' to replicate the infinite horizon equilibrium.

¹²As shown by Tabellini and Alesina (1990), when $\theta > 0$ there is also an “insurance” effect which goes in the opposite direction. Because $u(\cdot)$ is concave, the incumbent has also the incentive to lower debt to smooth consumption. Following the strategic-debt literature, we assume parameters such that the “insurance” effect is dominated: $\sigma \in [0, 1]$. In the Appendix, we prove Proposition 2 for the general case $\theta \in [0, 1]$.

Now we turn to the incumbent's first-period problem, which can be written as:

$$\begin{aligned} \max_{\{b_2, g_1^O\}} & \{u(\tau + b_2 - g_1^O) + qu(g_2^I) + (1 - q)u(g_2^O)\} \\ \text{s.t.} & \quad u(g_1^O) + qu(g_2^O) + (1 - q)u(g_2^I) \geq m_1 \end{aligned} \quad (14)$$

Inequality (14) is the acceptance constraint, absent in the dictator's problem. The default utility of the opposition in case of disagreement is:

$$m_1 = u(\bar{g}_1^O) + qu\left(\gamma' \frac{\tau - \bar{b}_2}{1 + \gamma'}\right) + (1 - q)u\left(\frac{\tau - \bar{b}_2}{1 + \gamma'}\right) \quad (15)$$

where \bar{g}_1^O and \bar{b}_2 denote the spending and debt that the executive would set in case of disagreement. These values depend crucially on the fiscal rule in place. When $\theta = 0$, it is clear that upon disagreement the incumbent will choose $\bar{g}_1^O = 0$ and therefore the evolution of debt will be given by the budget rule, $\bar{b}_2 = -(1 - \alpha)\tau$. This implies that total available resources in the second period will be $\tau - \bar{b}_2 = (2 - \alpha)\tau$, a decreasing function of α .

Since (14) holds with equality, the acceptance constraint implicitly defines a function $g_1^O = G(b_2)$, which is increasing in debt:

$$G'(b_2) = \frac{1}{1 + \gamma'} \left[\frac{qu'(g_2^O)\gamma' + (1 - q)u'(g_2^I)}{u'(G(b_2))} \right] \quad (16)$$

When the agenda setter increases b_2 , she must also increase g_1^O to keep the opposition indifferent. This is intuitive: the opposition needs to be compensated for going into the next period with fewer resources.

By substituting the solution $G(b_2)$ into the above maximization problem we obtain the first order condition with respect to b_2 , which equalizes the marginal benefit of an extra unit of debt with its marginal cost:

$$(1 - \underbrace{G'(b_2)}_1)u'(\tau + b - \underbrace{G(b_2)}_2) = \underbrace{(1 - q)u'\left(\frac{(\tau - b_2)\gamma'}{1 + \gamma'}\right)}_3 \frac{\gamma'}{1 + \gamma'} + \underbrace{qu'\left(\frac{\tau - b_2}{1 + \gamma'}\right)}_4 \frac{1}{1 + \gamma'} \quad (17)$$

We now compare (17) with the policy dictator's first-order condition (11). The differences between the two first-order conditions are underbraced in equation (17) and described below.

1. One extra unit of debt does not translate into one extra unit of consumption because additional $G'(b_2)$ units must be given to the opposition as compensation. Compared to

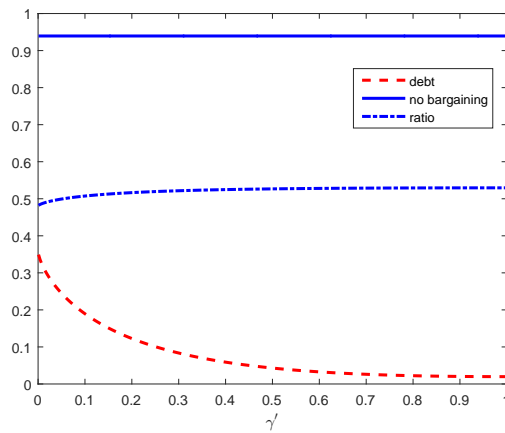
a model with alternating dictators, this channel reduces the incentive to raise debt.

2. Consumption at $t = 1$ is $\tau + b_2 - G(b_2)$ instead of $\tau + b_2$, which increases the marginal utility of consumption. If the party in power wants to maintain a given level of current consumption in the first period, she must increase debt.
3. As long as $\gamma' > 0$, the incumbent realizes that increasing debt not only reduces her consumption for when she stays in power, but also for when she is out of power. The extra value of future resources thus reduces the incentive to accumulate debt.
4. In case power is maintained, future resources will be partly appropriated by the opposition, which increases the incentive to accumulate debt.

Because the four channels do not all go in the same direction, the overall bargaining's effect is ambiguous. Note that channels (1) and (2) are driven by negotiations at time $t = 1$, while (3) and (4) are driven by the expectation of future bargaining.

Note the analogy between γ and γ' . Since the consumption ratio reflects relative bargaining power, one can interpret γ and γ' as the opposition's *current* and *future* bargaining power. Solving the two-period model amounts to determining the equilibrium values of γ and b_2 as a function of γ' . In Figure 3 we present an example illustrating how b_2 and γ vary with γ' . In drawing this figure, we assume $\sigma = 0.2$ and that the current fiscal rule is a government-shutdown (the results are qualitatively unchanged for other values of α). As a benchmark, we also plot the debt level in the alternating dictator model.

Figure 3: **Bargaining and Debt**



The first observation from Figure 3 is that debt is smaller than in the standard model with alternating dictators. Moreover, the budget is less skewed towards the preferences of the party in power. However, the composition of spending is not egalitarian. Because the incumbent has agenda-setting power, she obtains a larger share of consumption than the opposition. Note that debt is decreasing in γ' : the expectation of future bargaining increases the value of future resources and reduces the incentive to accumulate debt.

Let $\gamma = \Gamma(\gamma')$ be the equilibrium mapping from the future consumption ratio to the current one. In Proposition 1 we showed that in the infinite horizon economy $\gamma = \gamma'$. Hence, to obtain sharper and more meaningful predictions we focus on equilibrium outcomes satisfying $\gamma = \gamma^* = \Gamma(\gamma^*)$, the fixed point of $\Gamma(\cdot)$. In Figure 3 this would correspond to the intersection of γ' and the 45 degree line. Then we have:

Proposition 3 (*bargaining*) *Let a fiscal rule $\alpha \in [0, 1]$ be given. Suppose that $\gamma' = \gamma^* = \Gamma(\gamma^*)$. Then the fiscal rule is waived with the opposition's consent. Moreover, in equilibrium:*

- a) *Debt is lower than debt with alternating dictators.*
- b) *Debt is positive.*

Proof: See Appendix B.2.

Proposition 3 states that bargaining reduces the accumulation of debt but cannot eliminate it. Note that the constraint (12) does not hold on the equilibrium path. Nevertheless, fiscal rules influence debt accumulation. The existence of a fiscal rule induces the incumbent to reach a compromise with the opposition in order to bypass the rule. As stated in Proposition 3, political compromise leads to smaller debt. This result is not specific to the two-period model: under some additional mild assumptions, we state an equivalent result for the infinite-horizon model in Appendix A.6.

4 Infinite Horizon and the Optimal Rule

In this section we show that the intuition of the simple two-period model extends to the infinite-horizon and show how rules affect debt accumulation. We also characterize the fiscal rule α that maximizes welfare. To do so, we reduce the equilibrium characterization to a system of two non-linear equations with two unknowns: ν and $\phi_O \equiv [p - 1]^{1-\sigma} + \theta$. Recall from Proposition 1 that the coefficient ν governs the rate of debt growth, while p (hence ϕ_O)

determines how current spending is allocated between the two parties. We present here the two equations when $q = 1/2$, leaving for the appendix the general equations.

$$\phi_O = \max \left\{ \phi_O^s, \left(\frac{p\alpha}{p^s\nu} \right)^{1-\sigma} \phi_O^s + \frac{\beta(1+\theta)(1-\theta+\phi_O) [(1-r(\alpha-1))^{1-\sigma} - (1-r(\nu-1))^{1-\sigma}]}{2(1-\beta(1-r(\nu-1))^{1-\sigma})} \right\} \quad (18)$$

$$\frac{1}{\nu} = \frac{(1-\beta)(1+\phi_O-\theta)}{2[1-\beta(1-r(\nu-1))^{1-\sigma}]} \frac{[1+(\phi_O-\theta)^{\frac{\sigma}{1-\sigma}}]}{[1+(\phi_O-\theta)^{\frac{1}{1-\sigma}}]} (1-r(\nu-1))^{-\sigma} \quad (19)$$

Here the elements indexed by s correspond to the alternating dictator's solution: as shown in Appendix A.2, $p^s = 1 + \theta^{\frac{1}{\sigma}}$ and $\phi_O^s = \theta + \theta^{\frac{1-\sigma}{\sigma}}$. We show in Appendix A.5 that equation (18) represents the acceptance constraint in the incumbent's problem (P1), while equation (19) ensures that the Euler equation is satisfied. The expression ϕ_O is the endogenous weight in the opposition's utility function: using the equilibrium consumption ratio (5), the opposition's utility can be written as $u_O(g^I, g^O) = \phi_O u(g^I)$. A lower bound for the endogenous weight is given by ϕ_O^s , the opposition's weight in the alternating-dictator model. In the presence of inter-party compromise the weight ϕ_O raises above ϕ_O^s . For this reason, in what follows we will refer to ϕ_O as the opposition's equilibrium bargaining power. A key result, which we prove in Appendix A.6, is that ν and ϕ_O are negatively related.¹³ When the bargaining power of the opposition increases, political power is smoothed over time, which decreases ν and reduces the incentive to accumulate debt. Using this result we are able to prove the analogous to Proposition 3 for this more general model.

The max operator appears in (18) because, depending on the parameters, the acceptance constraint may not bind. When it does not $\phi_O = \phi_O^s$, as in (3), so the equilibrium coincides with alternating dictator's solution. Equations (18) and (19) are necessary, but not sufficient. Some solutions do not generate equilibrium allocations, but rather local minima. In the numerical results that we present below we make sure that the solutions are actual equilibria.

A natural question is whether there are multiple equilibria. Even if the restriction to Markov Perfect equilibria generally reduces the number of equilibria, uniqueness is difficult to achieve in dynamic problems. We address this issue in online Appendix C.6, where we show that for some combination of parameters there exist more than one equilibrium. We find numerically that in addition to the equilibrium with inter-party compromise, in which the acceptance constraint is always binding, the alternating-dictator's equilibrium might also exist. Intuitively, the expectation of no future compromise lowers the opposition's outside

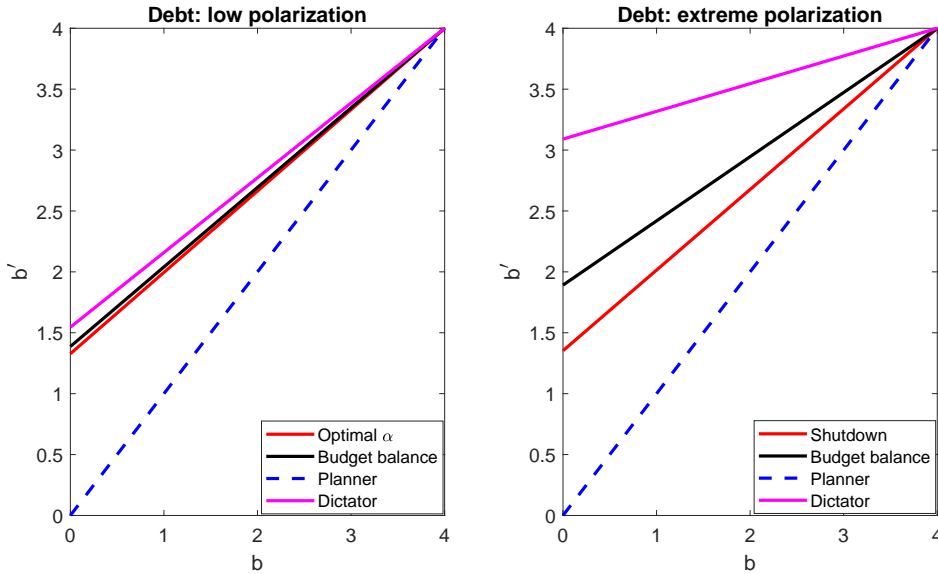
¹³This is proven under the following sufficient condition: $\sigma \geq \beta/2$.

option, rendering the current acceptance constraint non-binding. We also show that this multiplicity does not arise with either low θ or high β . Moreover, the set of parameters that generates multiplicity of equilibria is not convex, which complicates a formal proof. From now on, when we refer to an equilibrium solution, we refer to the solution in which the acceptance constraint binds: $\phi_O > \phi_O^s$.

The debt dynamics, determined by ν , is plotted in Figure 4. In this figure we depict both, the debt dynamics in the canonical model with alternating dictators and in the bargaining model under various fiscal rules (budget balance and government shutdown). We consider two cases, low polarization (left panel) and high polarization (right panel). Figure 4 shows that rules are not followed on the equilibrium path, but they nevertheless affect the dynamics. The largest deviations from the optimum arise when polarization is high. When polarization is low, the debt problem is not severe and fiscal rules offer only a slight improvement compared to the dictator's solution. When polarization is maximal, bargaining has is more important and rules drastically reduce the debt accumulation problem.

Figure 4 illustrates a key result: not all values of α are equally effective. The right panel of Figure 4 shows that when polarization is high (low θ), a government shutdown is more effective than a budget balance rule in reducing debt. When instead polarization is low, the opposite is true, left panel of Figure 4. We will discuss these results in more detail below.

Figure 4: Debt Dynamics



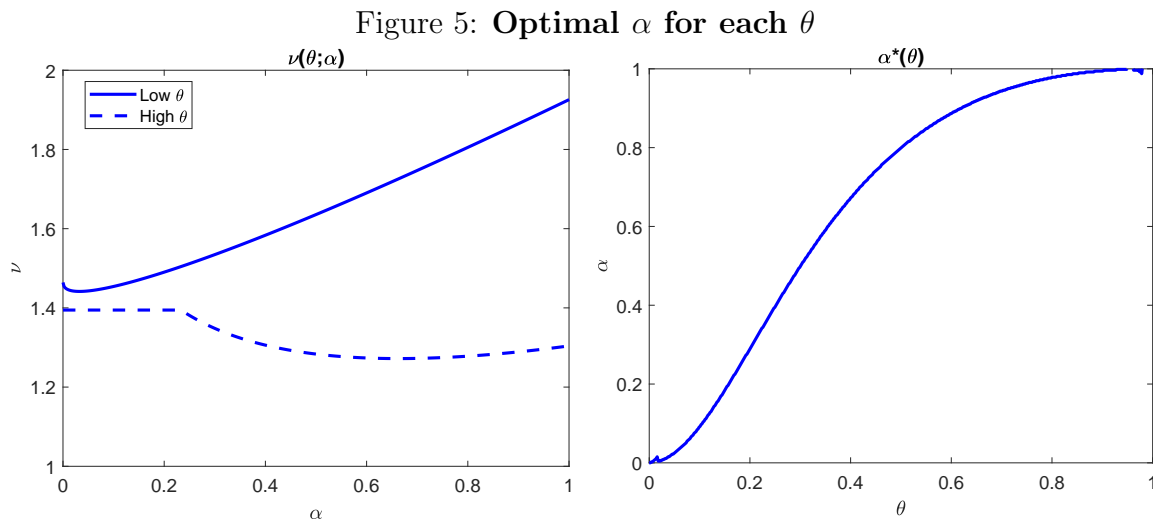
Next, we characterize the optimal fiscal rule, denoted by α^* . By optimal we mean the

fiscal rule that maximizes the opposition’s bargaining power ϕ_O . In Appendix A.7 we show that the optimal rule also minimizes the growth rate of debt and maximizes the expected social welfare for any arbitrary welfare weights. This result is obtained because, by a quick inspection of the system (18)-(19), α affects the equilibrium directly only through its impact on ϕ_O . The effect of the fiscal rule on the remaining equilibrium objects is indirectly channeled through ϕ_O .

Proposition 4 (optimal rule)

- a) The optimal fiscal rule ranges from shutdown to balanced budget: $0 \leq \alpha^*(\theta) \leq 1$.
- b) As $\theta \rightarrow 1$, $\alpha^*(\theta) \rightarrow 1$. As $\theta \rightarrow 0$, $\alpha^*(\theta) \rightarrow 0$.
- c) Suppose that the opposition’s continuation utility $W_O(b; \theta)$ is increasing in θ , then $\alpha^*(\theta)$ is increasing in θ .

Proof: Appendix A.7.



In the appendix we provide further details about the characterization. In the right panel of Figure 5, we plot the optimal fiscal rule α^* for all levels of polarization, fixing $\sigma = q = 1/2$. As stated in Proposition 4 the optimal rule is increasing in θ , starting at zero when $\theta = 0$ and reaching 1 when $\theta = 1$. *Harsh spending limits (i.e., government shutdown) are optimal when*

polarization is extreme, while less strict fiscal limits (i.e., budget balance) become optimal as polarization dissipates. Finally, $\alpha^* \in [0, 1]$, so it is never optimal to set $\alpha^* > 1$.¹⁴

In the left panel of Figure 5, we show how α affects ν for two values of the polarization parameter: high and low, both strictly positive and smaller than one. We see there that different α 's have substantially different effects on debt accumulation depending on θ . When polarization is high, the debt problem is more severe when α is close to one. Conversely, when polarization is low, debt growth is higher for lower levels of α .

To understand the intuition behind these results, notice that α affects the opposition's bargaining power differently for different polarization levels. When θ is close to zero (continuous blue line), upon disagreement the executive cuts the opposition's preferred goods as much as possible. The way to increase the opposition's bargaining power in this case is to reduce spending and move resources to the future. The opposition's utility then increases due to the fact that she can use these resources if her party becomes the new executive. This is why a very stringent threat point (e.g., government shutdown) is optimal when political preferences are highly polarized. More formally, this can be understood by analyzing how the opposition's bargaining power is determined. Looking at equation (18), notice that when the acceptance constraint is non-binding, ϕ_O is the sum of two terms: the first term reflects the opposition's current utility upon disagreement, while the second reflects her continuation utility. The parameter α increases the first term but lowers the second. When θ is zero, the first term vanishes because ϕ_O^s becomes zero. Only the dynamic term remains.

Suppose instead that θ is close to one (dashed blue line). In this case, upon disagreement the incumbent is willing to allocate resources to the goods that the opposition values. Thus, ϕ_O^s is relatively large, which makes the first term in (18) more important. Fiscal rules with low α increase the availability of future resources, but at the cost of present consumption. Because preferences are concave, this lowers the opposition's value of disagreement. Thus, for low values of α the acceptance constraint is non-binding. With low polarization, the opposition is better off by letting the incumbent act as a de-facto dictator and consume what she has to offer. That is, when θ is high, the policy outcome of an alternating dictator generates enough consumption smoothing for the opposition to make it preferable to the application of a rule with low α . This is what generates the flat part of the dashed blue curve: in that range, debt growth (ν) equals that of an alternating dictator's equilibrium. Fiscal rules with larger α , however, provide a better balance between current and future

¹⁴This result depends on the assumption that a constant level of debt is optimal. Under different assumptions, e.g., $\beta(1+r) < 1$, it might be optimal to have $\alpha^* > 1$

consumption, and thus make the acceptance constraint once again binding.

5 Choosing the Rules

So far, we have considered environments in which α is predetermined and taken as given by both parties. Given the results in Sections 4, several natural questions arise. Would a fiscal rule arise in equilibrium? Given that the fiscal rule is always waived, would it be desirable for both parties to eliminate the possibility of override? Finally, would the incumbent have the incentive to unilaterally “break” the rule? In what follows, we provide some answers to these questions. In order to keep the analysis tractable, in this section we assume that $q = 1/2$.

5.1 Bargaining over the Fiscal Rule

We make the fiscal rule endogenous by assuming that the incumbent can propose not only b' , but also α' , the fiscal rule for the *following* period. Taking debt and the current fiscal rule α as given, in each period the parties bargain over the new debt issuance and the rule that will apply in the next legislative session. If there is no agreement, total spending cannot exceed $\alpha(\tau - rb)$ and the current rule will apply to the next period: $\alpha' = \alpha$. The incumbent solves:

$$V_I(\alpha, b) = \max_{\{g^I, g^O, b', \alpha'\}} \{u_I(g^I, g^O) + \beta W_I(\alpha', b')\} \quad (\text{P2})$$

$$s.t. \quad \tau - (1+r)b + b' - g^I - g^O \geq 0 \quad (\text{BC})$$

$$u_O(g^I, g^O) + \beta W_O(\alpha', b') \geq m(\alpha, b) \quad (\text{AC})$$

$$\underline{b} \leq b' \leq \bar{b}$$

$$V_O(\alpha, b) = u_O(g_I^*(\alpha, b), g_O^*(\alpha, b)) + \beta W_O(\alpha'^*(\alpha, b), \mathcal{B}^*(\alpha, b))$$

The only difference between the above problem and (P1) from Section 3, is that in (P2) α is an additional endogenous state variable. We can state the following proposition:

Proposition 5 (*optimal endogenous rules*) *Suppose $q = 1/2$. Starting from any (α, b) , the two parties will adopt the optimal fiscal rule, α^* , characterized in Proposition 4.*

To understand the previous result, first notice that when $q = 1/2$, from (1), we have:

$$W_I(\alpha', b') = W_O(\alpha', b') = \frac{1}{2}V_I(\alpha', b') + \frac{1}{2}V_O(\alpha', b') \equiv W(\alpha', b'). \quad (20)$$

Letting μ be the multiplier of the (AC) constraint, the first-order condition with respect to α' is:

$$\beta(1 + \mu(\alpha, b)) \frac{\partial W(\alpha', b')}{\partial \alpha'} = 0; \quad \forall \alpha, b$$

The proposed rule is independent of the current α and maximizes the continuation value of both parties. As a result, the fiscal rule coincides with the optimal one: it maximizes the ex-ante social welfare function. Proposition 5 is both powerful and intuitive. By adopting the optimal rule from tomorrow onwards, the incumbent maximizes her continuation utility. At the same time, by improving the opposition's continuation utility, the incumbent relaxes the acceptance constraint, which reduces what the incumbent must concede to the opposition in order to waive the current rule.

To conclude, we have shown that when fiscal institutions are chosen before knowing the incumbent's identity, the optimal rule arises in equilibrium. In Section 5.3, we will examine the stability of rules by studying whether the incumbent, once in office, has the incentive to unilaterally “break” them.

5.2 Hard vs. Soft Rules

We examine whether it would be desirable for both parties to “harden” the fiscal rule by eliminating the possibility of override, focusing on balanced budget laws. We compare expected welfare under soft and hard budget balanced rules. More specifically, under both types of rule, we compute the (ex-ante) discounted sum of utility from time 1 to infinity: $W(b) = \frac{1}{2}V_I(b) + \frac{1}{2}V_O(b)$. Given that both parties have the same probabilities of becoming the next incumbent, expected welfare is the same. The optimality of a strict balanced budget law is not obvious. On the one hand, it would eliminate the debt accumulation problem. On the other, when rules are “hard”, the incumbent has no incentive to reach a compromise with the opposition. Thus, in every period the incumbent would choose her preferred spending mix, leading to excessive consumption volatility and no insurance against political risk.

Under our preference assumption, value functions are proportional to $u(\tau - rb)$. Thus, whether soft rules are preferable to hard rules is independent of the level of debt. Let μ denote the Lagrange multiplier of the acceptance constraint. In Appendix A.8, we show that the constant of proportionality for the soft rule is:

$$A^S = \frac{(1 + \theta)\nu^{1-\sigma}}{\underbrace{2[1 - \beta(1 - r(\nu - 1))^{1-\sigma}]}_A} \frac{1 + \left(\frac{\theta + \mu}{1 + \theta\mu}\right)^{\frac{1-\sigma}{\sigma}}}{\underbrace{\left[1 + \left(\frac{\theta + \mu}{1 + \theta\mu}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}}_B} \quad (21)$$

The constant of proportionality for the hard rule is:

$$A^H = \frac{(1 + \theta)}{\underbrace{2(1 - \beta)}_C} \frac{(1 + \theta^{(1-\sigma)/\sigma})}{\underbrace{(1 + \theta^{1/\sigma})^{1-\sigma}}_D} \quad (22)$$

The soft rule generates larger welfare if $A^S > A^H$. There are two effects counteracting each other. On the one hand, with soft rules there is better intra-temporal allocation of resources due to political bargaining. This is captured by the terms B and D in the above expressions. It is simple to verify that $B > D$ as long as $\mu > 0$. On the other hand, the soft rule generates faster debt growth compared to the hard rule, which is captured by the fact that $A \leq C$. Under the assumption that $(1 + r)\beta = 1$, we can write inequality $A \leq C$ as:

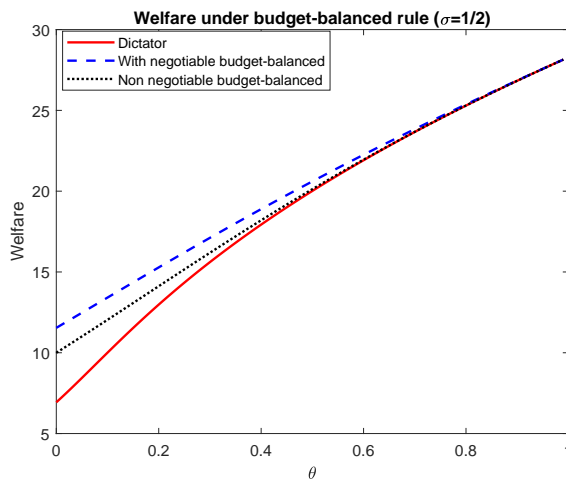
$$(1 - \beta)\nu^{1-\sigma} \leq 1 - \beta^\sigma(1 - \nu(1 - \beta))^{1-\sigma} \quad (23)$$

Notice that when $\nu = 1$ (i.e., debt does not grow) we obtain $A = C$ and thus the intra-temporal smoothing effect dominates, so that the soft rule is more efficient. However, ν is larger than 1 when $\beta \in (0, 1)$ and $\theta < 1$. Therefore, it is possible to show that $A^S > A^H$ when ν is sufficiently low. Since ν is endogenous, the comparison between (21) and (22) is highly involved. In Proposition 6 we are able to provide analytical results by making some parameter restrictions. We show that soft rules dominate hard rules when $\sigma = 1/2$ and β is close to either zero or one. However, the result is much more general and our numerical simulations show that the conditions of Proposition 6 can be significantly weakened.

Proposition 6 (*Inefficiency of hard rules*) *Assume $\sigma = 1/2$ and $q = 1/2$. When β is sufficiently close to either zero or one, soft budget rules generate higher welfare than “hard” rules.*

When β approaches one, there are two effects which tend to make soft rules preferable. First, because the opposition becomes more forward looking, the incumbent must grant

Figure 6: **Soft vs hard rules**



more generous spending concessions. This implies a better distribution of resources in every period, which has a direct impact on welfare. Second, the larger discount factor reduces the over-accumulation of debt because of more forward-looking behavior. On the other extreme, when β goes to zero, the assumption $(1+r)\beta = 1$ plays an important disciplinary role. Since the interest rate, r , approaches infinity as β goes to zero, the natural debt limit, τ/r also converges to zero when the discount factor is very small. In other words, even though the executive would like to accumulate debt, it becomes so expensive that in equilibrium the executive avoids it.

Figure 6, drawn assuming $q = 1/2$ and $\sigma = 1/2$, shows expected welfare under the dictator’s model, when rules are flexible and when they are hard. It shows that for all $\theta < 1$ a hard budget balance law is indeed suboptimal and allowing a supermajority override is ex-ante desirable for both parties. However, imposing rules, whether hard or flexible, is preferable to no rules at all.

5.3 Sustainable Rules

Throughout the analysis we have assumed that both parties must agree in order to override the fiscal limit. It is tempting, however, for the majority party to unilaterally disregard any rule that forces inter-party compromise. The Trump administration, for instance, has been openly discussing the possibility of using the “nuclear option” to sidestep the Senate’s

filibuster.¹⁵ This option basically reduces the majority requirement in the Senate from three-fifths to simple majority rule. Republican senators have been reluctant to use the “nuclear option” on legislative matters. A possible reason is that they anticipate that such a rule change will create a precedent that can be used by Democrats to bypass the filibuster in the future once they regain power. In light of this, it seems appropriate to appeal to an equilibrium concept other than Markovian Equilibria. Therefore, we study this issue by assuming that breaking the rule in the current period will trigger a reversion to a no-compromise equilibrium, in which whoever is in power is expected to break the rule and decide under full discretion. More specifically, taking as given the current rule α , the incumbent can decide to override the fiscal limit either unilaterally or by reaching an agreement with the opposition. In the former case, the rule is “broken”: the incumbent does not need to satisfy the acceptance constraint.¹⁶ If the majority party breaks the rule, the fiscal rule is henceforth disregarded. Thus, the incumbent trades off more discretion today for less compromise in the future.

In Appendix C.2 we provide some details about the possible outcomes. As one may expect, the opposition never wants rules to be broken: opposition parties are “rule lovers.” This is fairly intuitive because the opposition’s power stems from the possibility of using the fiscal rule as a “bargaining chip.” But what about the incumbent? It is important to bear in mind that from the perspective of the incumbent the decision to unilaterally waive the rule has short-run benefits and long-term costs. It is beneficial in the short run because today’s fiscal limit can be overridden without making concessions to the opposition. However, this benefit comes at the cost of ending future compromise by triggering a reversion to the alternating-dictator equilibrium. The incumbent does not break the rule when

$$\underbrace{\beta[W_I(b, \theta; \alpha) - W_I^s(b, \theta)]}_{\text{Long-run cost}} \geq \underbrace{u_I(g_s^I(\theta), g_s^O(\theta)) - u_I(g^I(\theta; \alpha), g^O(\theta; \alpha))}_{\text{Short-run benefit}} \quad (24)$$

where the index s refers to the alternating dictator’s solution.

Given the increasing polarization of American politics, a growing concern is that ideological conflict will eventually lead to the erosion of checks and balances, including the

¹⁵The most direct approach to eliminate the filibuster would be to formally change the text of Senate Rule 22, which would require the support of two-thirds of the senators. A less straightforward way is through a parliamentary ruling, informally known as the “nuclear option.” The advantage of this approach is that it can be triggered with support from only a simple majority of senators. In both 2013 and 2017, the Senate employed the “nuclear option” to end debate on judicial nominations.

¹⁶For tractability we assume that the incumbent does not have the option to choose a different rule.

elimination of the Senate’s filibuster. To investigate this issue, we examine how θ affects inequality (24). When $\theta = 1$, i.e., parties’ preferences are aligned, the social optimum is obtained both in the politico-economic equilibrium and in the alternating dictators’ solution. As a result, both sides of (24) are equal to zero. Lowering θ (increasing polarization) has non-trivial effects. On the one hand, it increases the short-run benefit of a unilateral waive because spending concessions to the opposition provide less utility to the incumbent. On the other hand, the long-term cost is also increasing in polarization: the lower the θ , the higher the benefit of smoothing consumption against political risk. Because θ affects both sides of (24) in a nonlinear way, the set of θ ’s in which (24) holds is not necessarily convex.

The left panel of Figure 7 illustrates how θ affects the incumbent’s incentive to break the rule for two alternative values of β and a shutdown-rule. The value of compromising with the opposition is in blue, while the value of breaking the rule and then switching to the alternating dictator’s equilibrium is in red. The dashed curves represent an environment with high β , while the solid curves correspond to an environment with low β . When polarization is high, we numerically find that a short-sighted incumbent prefers breaking the rule, while an incumbent with high β values compromise more. Since the benefits from compromise are reaped in the future, a low β intuitively raises the temptation to break the rule.¹⁷ As polarization decreases, Figure 7 illustrates that the difference between the two values becomes monotonically smaller. However, this “monotonicity” result does not carry over to all possible environments. In Appendix C.2 we show that for another combination of parameters the set of θ ’s for which inequality (24) holds is not convex.

Summing up, the general takeaway from this section is that there are incentives to break the rule, but the question of when, and under which conditions, is mostly quantitative.

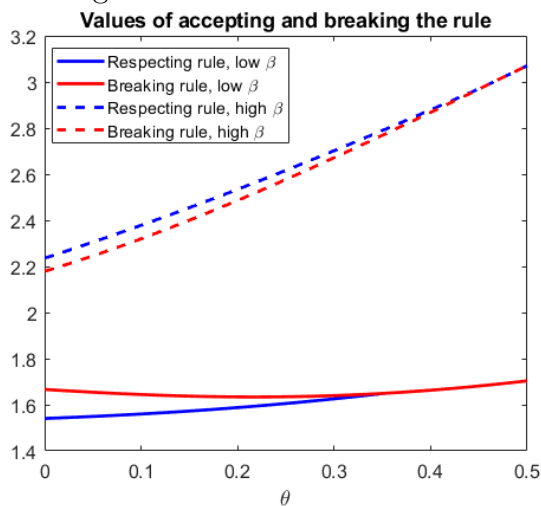
6 Discussion and Extensions

6.1 Comparative Statics

The validity of a theory depends on its empirically testable hypotheses. From this point of view, the “standard” model with alternating dictators has straightforward empirical predictions. We now show that the model we present is more ambiguous regarding its predictions.

¹⁷We have stressed the direct effect of β . There are, however, also indirect effects. For instance, when β is higher, the opposition becomes more reluctant to run debt, which increases the spending concession that the incumbent needs to offer. This leads to more inter-party compromise, which changes both sides of (24).

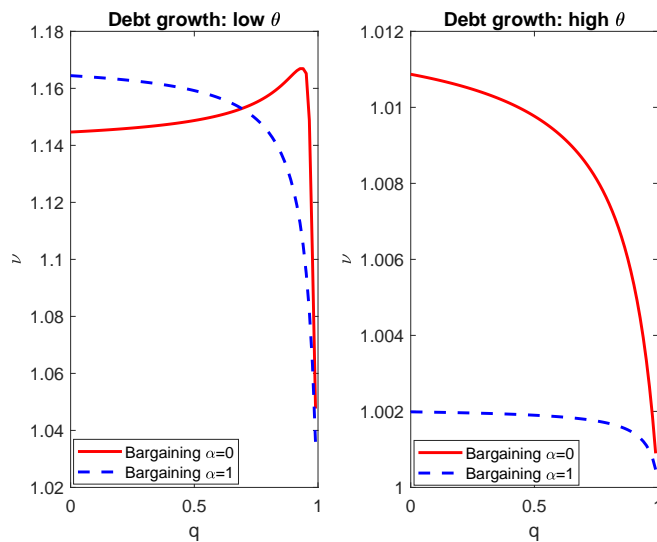
Figure 7: Sustainable Rules



For instance, if one regresses debt growth on turnover, our theory implies that anything is possible. The reason for this ambiguity is that the effects of the deep parameters of the model are channeled through the specific fiscal rule in place and its interactions with the rest of the parameters. Once we condition on the existing rule and the remaining parameters, the predictions are sharper. We examine how political turnover affects debt, which delivers richer comparative statics than the canonical model characterized in Proposition 2. In the standard model with alternating dictators, when the incumbent is more likely to stay in power (larger q), the cost of debt is more internalized, which lowers debt. Bargaining brings about new effects. In particular, when studying the comparative statics we also need to take into account how the parameters affect the acceptance constraint. When q increases, the opposition is less likely to be in power in the next period, weakening the opposition's incentives to transfer resources to the future. From equation (3), this decreases the opposition's bargaining power and makes debt cheaper for the incumbent. In our setting, these novel effects coexist with the standard ones, making the comparative statics ambiguous. In contrast to the standard intuition arising from the canonical model, we find that under some conditions increasing q may lead to *more* debt. Figure 8 shows there are cases ($\sigma = 0.5$, low θ and $\alpha = 0$) where debt is hump-shaped in political persistence.

This is just one example illustrating the complexity of the problem. It is possible to obtain similar ambiguous comparative statics for other parameters. For instance, in Appendix C.3, we show that the comparative statics with respect to σ (relative risk aversion) also differs

Figure 8: Turnover Probability



from Proposition 2: under certain parameters, σ affects debt in a non-monotonic way.

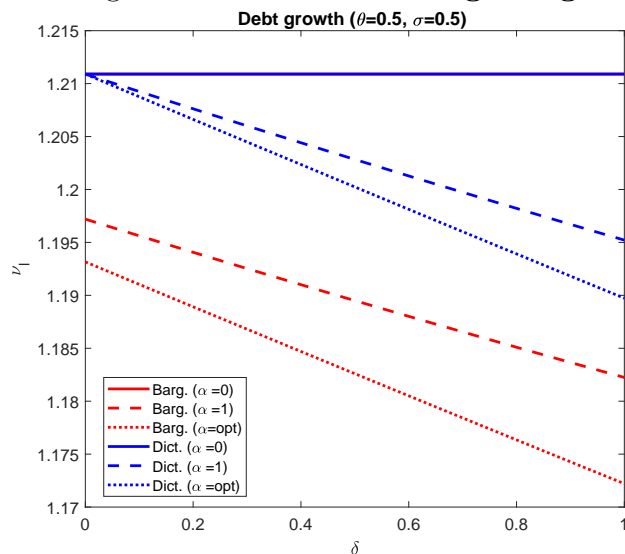
All in all, our findings suggest that when studying the empirical effect of parameters on debt, it is important to condition, among other things, on the type of fiscal rule in place and on the incumbent's margin of victory, which may affect the incumbent's need to compromise with the opposition. These theoretical results are ripe for future empirical investigation.

6.2 Occasional Bargaining

Finally, one may be concerned that checks and balances do not always work, so that when they are ineffective, the incumbent can reap all of the benefits of past discipline. In this section, we show that this is not the case when checks and balances are expected to be effective again in the future. We extend the model of Section 3 by fixing $q = 1/2$ and assuming that with probability $(1 - \delta)$ the government is not constrained by the fiscal rule: it can spend more than $\alpha(\tau - rb)$ without the opposition's consent. With complementary probability δ , the opposition must agree to override the fiscal limit. The probability $1 - \delta$ could capture the chances that an election results in a solid majority for the winning party. For example, in the U.S. this occurs when the president's party controls the executive branch and both houses of the legislative branch.¹⁸

¹⁸Unified party control occurred in 6 of the 24 congresses between 1969 and 2016. However, unified governments in which the president enjoys a filibuster-proof majority in the Senate are rare. In the post-war

Figure 9: Occasional Bargaining



In each period the economy can be in either one of two states: in the bargaining state (B) or in the “dictator” state (D). Whether or not the incumbent needs to compromise with the opposition is known at the beginning of the period. The model in Section 3 and the alternating-dictator model are special cases of the current setup with δ equal to one and zero, respectively. In Appendix C.4 we describe the formal derivations of the model, while in this section we provide an example that represents the general result. Figure 9 illustrates debt growth (ν) as a function of δ in both states. Notice that the allocations at $\delta = 0$ represent the case in which the incumbent is always a dictator. Thus, the solutions in state D at $\delta = 0$ are a natural benchmark. Furthermore, the allocations at $\delta = 1$ correspond to the case in which there is always bargaining. Then, the solutions in state B at $\delta = 1$ are the other natural benchmark. The figure makes clear that all of the solutions lie somewhere in between the alternating dictator’s solution and the bargaining solution. In state D, when the incumbent does not have to bargain, debt is lower than in the alternating dictator’s model. The intuition for this result is straightforward. For any $\delta > 0$, the incumbent knows that when she is out of power, with some positive probability she obtains a larger share of the future resources. This increases her valuation of future consumption, and therefore leads her to borrow less. Again, future bargaining plays a key role in making incumbents internalize future consequences. Moreover, for any δ , debt in the B state is lower than in the D state: overspending and debt are reduced when the incumbent needs the opposition’s approval to period, it occurred only in 6 years: in 1963-1966 (under Kennedy-Johnson) and in 1977-1979 (under Carter).

override the fiscal limit. Finally, in any given state, debt is decreasing in δ : when inter-party compromise is more likely to occur in the future, the incumbent issues a lower amount of debt. Summing up, the debt problem is less severe when inter-party compromise is either needed in the current period or is expected in the future.

6.3 U.S. Fiscal Rules

How does our setting relate to real-life politics? In this section, we discuss several budget rules and describe their connection to the model. We focus primarily on U.S. budget rules, as the bargaining framework that we have delineated fits well with American politics. There are two important reasons for which U.S. policymaking often requires an agreement between the two main parties. First, staggered elections imply that the executive and legislative branches do not necessarily coincide. Indeed, in recent years, divided party government has become the new normal. Second, U.S. Senate rules permit a senator to debate over a proposed piece of legislation so as to delay or entirely prevent a decision from being made. To bring the debate to a close, a three-fifths majority vote is needed. As a result, for many decisions the U.S. upper chamber operates under a supermajority rule.¹⁹

In what follows, we will discuss the following rules: government shutdown provision, debt ceiling, and discretionary spending caps. As will be discussed below, the three budget rules define a threat point in the negotiation between Congress and the president.

Government shutdown. Discretionary spending (e.g., national defense, foreign aid, education and transportation) is controlled through the appropriations process. As such, it requires that new funding legislation is passed and signed into law. When the government is divided, appropriations require an agreement between Congress and the president. A reversion point automatically sets the budget in the event that the legislature cannot agree on a new one by the beginning of the new fiscal year. Currently, the default reversion point for discretionary spending in the U.S. is zero. This implies a “shutdown” of federal government agencies and programs relying on annual funding appropriations. In a “shutdown,” federal agencies must discontinue all non-essential discretionary functions until new legislation is passed and signed into law.²⁰ Unlike the U.S., most countries have alternative reversion points to avoid a shut-

¹⁹Recently, the Senate has introduced special rules, “majoritarian exceptions,” to limit the debate (see Reynolds (2017)). The most common way to avoid a filibuster in the budget process is to use the so-called “budget reconciliation” procedure. In the past decade, the simple-majority vote has also been introduced to reduce debate time for nominees to executive and judicial branches.

²⁰Since the enactment of the U.S. government’s current appropriations process in 1976, there has been a total of 22 funding gaps in the federal budget, 10 of which have led to federal employees being furloughed.

down in the absence of a new budget. In a sample of 165 countries, Cox (2013) shows that in 28% of the countries the executive's proposal would automatically come into force, usually for a limited period of time (e.g. Finland, Germany, and Japan). Alternatively, in 46% of the sample the budget would revert to a modified version of last year's budget.²¹

Debt Ceiling and Balanced Budget Laws. The debt ceiling is a legal limit on the total nominal amount of federal debt that the U.S. government can carry at a given time. The ceiling constitutes a “soft” constraint: it restricts government spending unless Congress agrees to raise the limit. In periods of divided government (or when the majority party has a slim margin in the Senate), the opposition is able to use the debt ceiling to increase its bargaining leverage. The threat point in case of negotiation breakdown is that the limit is kept in place. To change the debt ceiling, specific legislation must be enacted, and the President must sign it into law. Since its establishment, the debt ceiling has increased roughly 100 times. By setting a nominal limit on debt, the debt ceiling is an uncommon form of fiscal rule across the world.²² Under the lens of Section 5.1's results, we can interpret this “tendency” to set the debt ceiling at a soon-to-be binding level as the optimal choice of the future threat point. Unlike the U.S., most countries connect their debt ceiling to their GDP and/or have balanced budget rules. Balanced budget rules are, however, also (either de facto or de iure) negotiable. In fact, escape clauses often allow the temporary suspension of these limits (usually these include natural disasters or acute economic slowdowns or recessions). These escape clauses are easier to activate when there is political consensus.²³ In the U.S., the balanced budget amendments that have been proposed (so far, unsuccessfully) in Congress include provisions requiring a supermajority vote to allow an excess of outlays over receipts.

Discretionary Spending Limits. Facing growing concerns over debt levels, the Budget Control Act (BCA) of 2011 introduced enforcement mechanisms mandating specific fiscal outcomes. In particular, it established harsh caps on discretionary spending from 2012 through 2021. There are currently separate annual limits for defense and non-defense discretionary spending. Congress may modify or repeal any aspect of BCA procedures at its discretion, but such changes require the enactment of legislation. On this matter, expedited procedures

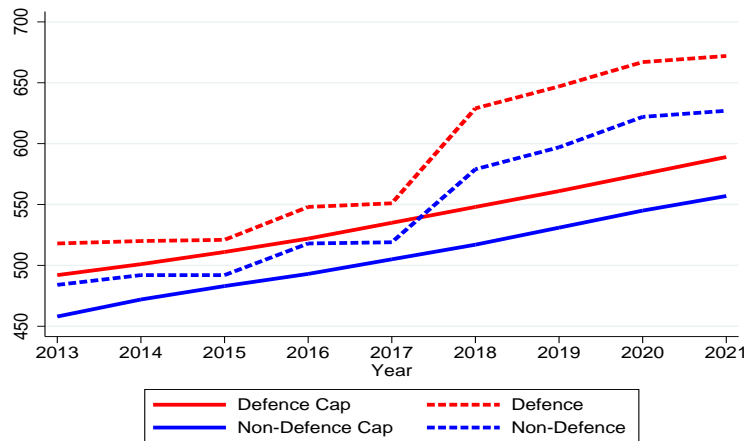
²¹Reversions to last year's budget also favor the executive in most of the world's constitutions. This is primarily because the executive can either impound funds or transfer them across budgetary categories under the reversion. In other words, the reversion is not simply last year's budget.

²²Denmark also has a statutory debt limit, while Australia abolished the debt ceiling in 2013.

²³For example, Switzerland and Italy have escape clauses which are activated with special majorities. In other cases, independent fiscal bodies assess the suitability and the timing of these escape clauses. Even in these cases, one would expect that if there is enough political consensus, it would be easier to trigger the escape clauses.

banning the filibuster are not allowed: in the Senate, a vote of three-fifths is needed to bring debate to a close. In Figure 10, the solid lines illustrate the spending caps (in billions) for defense and non-defense spending as established by the 2011 BCA; the dashed lines illustrate how these caps were amended as a result of several pieces of legislation, such as the American Taxpayer Relief Act of 2012, and the Bipartisan Budget Acts of 2013, 2015, 2018, and 2019.²⁴ When Congress passed the discretionary budget caps in 2011, many commentators viewed the decision to impose very low caps as pointless, at best, and possibly counter-productive. Our theory (Proposition 4) suggests that this may have favored inter-party compromise: if caps had been set at a higher level, Republicans and Democrats would still have reached an agreement to amend them, but this would have resulted in less compromise and more debt.

Figure 10: **Discretionary Spending**



7 Conclusion

In recent years, the number of countries adopting fiscal rules has continued to increase. Since most fiscal rules can be overridden by consensus, the effectiveness of rules is widely debated. We show that the possibility of override does not make fiscal rules irrelevant. Since fiscal rules determine the outside option in case of disagreement, the opposition uses fiscal rules

²⁴Members of Congress have insisted that the so-called “parity principle” should be applied for any legislation changing the limits: defense and non-defense spending caps should be changed by equal proportional amounts. In practice, spending increases have, to some degree, privileged non-defense spending under President Obama and defense spending under President Trump. Source for Figure 10 is Table 1 in <https://crsreports.congress.gov/product/pdf/R/R44874>.

as “bargaining chips.” In exchange for the opposition’s consent to raise debt and bypass the rule, the party in power offers spending concessions to the other party. This political bargain has two main implications. First, debt becomes more costly to accumulate, because the opposition will only agree to bypass the fiscal rule in exchange for more spending on her preferred public goods. Second, the expectation of future compromise increases the benefit of transferring resources to the future. All in all, we show that these two channels reduce the incentive for inefficient debt accumulation. Moreover, since budgets are less skewed towards the incumbent’s preferences, we find that the possibility of override improves welfare by insuring against power fluctuations.

In the wake of the recent US budget crisis, leading to a gap in budget funding and a near default, various commentators have questioned the usefulness of rules such as the government shutdown or the debt ceiling. Along the same line, many have criticized the unrealistically low spending limits imposed by the Budget Control Act of 2011. A widely held view is that these rules create unneeded uncertainty and can potentially lead to worse fiscal outcomes. In this paper, we have argued that there are also reasons to hold a more favorable view. In a highly polarized political system, these rules could push conflicting parties to compromise, leading to lower debt. These results are obtained in a model which abstracts from bargaining inefficiencies and delays. We leave these extensions to future research.

Before concluding, we stress that we do not explicitly consider mandatory spending (Social Security, Medicare, etc). Unlike discretionary spending, which is subject to annual appropriations, funding for mandatory programs is determined by the number of eligible recipients, which is specified by law. As such, mandatory programs continue year after year unless Congress agrees to change the law. In the model, this would imply that the allocation that was implemented in the previous period constitutes an additional state variable, greatly complicating the analysis. Various fiscal rules have been adopted to limit the growth of these programs. For example, under the pay-as-you-go rule, Congress must pay for new mandatory spending by reducing other entitlement spending or by increasing revenues. Punitive threats like the sequester (automatic across-the-board cuts) are meant to insure budget neutrality of new mandatory programs. Like the fiscal rules studied in the paper, these rules can be overridden by consensus. For instance, the Senate has waived the pay-as-you-go rule 14 times since 1993.²⁵ Studying how budget rules interact with mandatory spending would be an important extension for future research.

²⁵See the 2018 CRS Report, RL31943. Budget Enforcement Procedures: The Senate Pay-As-You-Go (PAYGO) Rule. by Bill Heniff Jr.

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APPENDIX

A Infinite Horizon: Proof of Propositions 1, 4 and 6

To solve the infinite-horizon model, we proceed by steps. In Section A.1, as a benchmark, we compute the planner's problem. In Section A.2, we return to the politico-economic problem and solve the incumbent's static problem to decide the spending allocation. In Section A.3, we determine the value of disagreement for the opposition. In Sections A.4, A.5 and A.6, we compute the solution of the dynamic problem using a guess-and-verify-method and we write down the conditions that define the equilibria. This concludes the proof of Proposition 1. In Section A.7, we study how rules affect debt accumulation. Finally, in Section A.8 we prove Proposition 6.

A.1 The planner's problem.

As a benchmark, consider the problem of a social planner who equally cares about all constituents. We denote by $V^P(b)$ the planner's value function. The optimal allocation solves:

$$V^P(b) = \max_{\{g^I, g^O, b'\}} \{u_I(g^I, g^O) + u_O(g^I, g^O) + \beta V^P(b')\}$$

$$\begin{aligned} \text{s.t. } \tau - (1+r)b + b' - g^I - g^O &\geq 0 \\ \underline{b} \leq b' &\leq \bar{b} \end{aligned}$$

Recalling that $\beta(1+r) = 1$, it is straightforward to show that the social planner keeps the debt level constant over time. The solution is characterized by

$$\begin{aligned} b'(b) &= b & (25) \\ g_I^P &= \frac{\tau - rb}{2} \end{aligned}$$

$$\begin{aligned} g_O^P &= \frac{\tau - rb}{2} \\ V^P(b) &= \frac{2(1+\theta)}{1-\beta} u\left(\frac{\tau - rb}{2}\right) & (26) \end{aligned}$$

A.2 Static Problem

The political problem can be split in two sub-problems: a static problem to decide the spending allocation and a dynamic one to decide b' . Let μ denote the multiplier of the acceptance constraint. From the first-order conditions with respect to g^O and g^I , it follows that

$$g^O = \left(\frac{\theta + \mu}{1 + \theta\mu} \right)^{\frac{1}{\sigma}} g^I; \quad \text{and} \quad g^O + g^I = \underbrace{\left[1 + \left(\frac{\theta + \mu}{1 + \theta\mu} \right)^{\frac{1}{\sigma}} \right]}_{\equiv p} g^I = pg^I \quad (27)$$

One can write the utilities of the incumbent and of the opposition, respectively, as:

$$u_I(g^I, g^O) = \underbrace{\left[1 + \theta \left(\frac{\theta + \mu}{1 + \theta\mu} \right)^{\frac{1-\sigma}{\sigma}} \right]}_{\equiv \phi_I} \frac{(g^I)^{1-\sigma}}{1-\sigma} = \phi_I u(g^I) \quad (28)$$

$$u_O(g^I, g^O) = \underbrace{\left[\theta + \left(\frac{\theta + \mu}{1 + \theta\mu} \right)^{\frac{1-\sigma}{\sigma}} \right]}_{\equiv \phi_O} \frac{(g^I)^{1-\sigma}}{1-\sigma} = \phi_O u(g^I) \quad (29)$$

From (27) and (29) we can write

$$p = 1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}} \quad (30)$$

Instead of expressing ϕ_I and ϕ_O as a function of μ , using the acceptance constraint, which is assumed binding, one can write:

$$\phi_O = \frac{m(b) - \beta W_O(b')}{u(g^I)} \quad \text{and} \quad \phi_I = 1 + \theta(\phi_O - \theta) \quad (31)$$

Recalling the definition of W_I from (1), and using (27), (28), (30) and (31), we rewrite the incumbent's problem as:

$$\begin{aligned} V_I(b) &= \max_{g^I, b'} \{ \phi_I u(g^I) + \beta W_I(b') \} \\ \text{s.t.} \quad &\tau - (1+r)b + b' - [1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}}] g^I \geq 0 \\ &\underline{b} \leq b' \leq \bar{b} \end{aligned}$$

where ϕ_O , which is given by (31), reflects the opposition's bargaining power.

A.3 Fiscal Rules

Before proceeding we need to specify how the budget rule affects the outside option of the opposition, $m(b)$. A budget rule is summarized by the parameter $\alpha \in [0, 1]$. Given $(\tau - rb)$, α determines total spending available for the two public goods. That is, the fiscal rule requires

$$g^O + g^I \leq \alpha(\tau - rb). \quad (32)$$

Inequality (32) must hold unless the two parties reach a consensus. If there is no consensus, the incumbent is free to choose the spending mix satisfying (32). Since upon disagreement the incumbent chooses the spending allocation, the optimal allocation g^I and g^O is the same as before with $\mu = 0$, i.e., $g^O = \theta^{\frac{1}{\sigma}} g^I$ and $g^O + g^I = \left(1 + \theta^{\frac{1}{\sigma}}\right) g^I = p^s g^I$ where

$$p^s \equiv 1 + \theta^{\frac{1}{\sigma}}. \quad (33)$$

Therefore, we can write

$$m(b) = \phi_s u(g^I) + \beta W_O(b'^s) \quad (34)$$

where

$$\phi_O^s \equiv \theta + \theta^{\frac{1-\sigma}{\sigma}} \quad (35)$$

and

$$b'^s \equiv b + (\alpha - 1)(\tau - rb) \quad (36)$$

Equation (3) is the value of the outside option for the opposition and plays a fundamental role in the following derivations.

A.4 First-Order Necessary Condition

Using (30) and (31), we write

$$\begin{aligned}
 V_I(b) &= \max_{g^I, b'} \left[1 + \theta \left(\frac{m(b) - \beta W_O(b')}{u(g^I)} - \theta \right) \right] u(g^I) + \beta W_I(b') \\
 \text{s.t. } \tau - (1+r)b + b' - \left[1 + \left(\frac{m(b) - \beta W_O(b')}{u(g^I)} - \theta \right)^{\frac{1}{1-\sigma}} \right] g^I &\geq 0 \\
 \underline{b} \leq b' &\leq \bar{b}
 \end{aligned}$$

or

$$\begin{aligned}
 V_I(b) &= \max_{g^I, b'} (1 - \theta^2)u(g^I) + m(b)\theta + \beta W_I(b') - \beta \theta W_O(b') \\
 \text{s.t. } \tau - (1+r)b + b' - g^I - (1 - \sigma)^{\frac{1}{1-\sigma}} [m(b) - \beta W_O(b') - \theta u(g^I)]^{\frac{1}{1-\sigma}} &\geq 0 \\
 \underline{b} \leq b' &\leq \bar{b}
 \end{aligned}$$

Let λ denote the Lagrange multiplier of the resource constraint. Taking the first-order conditions with respect to g^I and b' :

$$(1 - \theta^2)u'(g^I) - \lambda + \lambda(1 - \sigma)^{\frac{1}{1-\sigma}} \frac{1}{1 - \sigma} [m(b) - \beta W_O(b') - \theta u(g^I)]^{\frac{\sigma}{1-\sigma}} \theta (g^I)^{-\sigma} = 0$$

$$\beta W_I'(b') - \theta \beta W_O'(b') + \lambda + \lambda(1 - \sigma)^{\frac{1}{1-\sigma}} \frac{1}{1 - \sigma} [m(b) - \beta W_O(b') - \theta u(g^I)]^{\frac{\sigma}{1-\sigma}} \beta W_O'(b') = 0$$

or

$$\begin{aligned}
 (1 - \theta^2)u'(g^I) - \lambda + \lambda(\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \theta &= 0 \\
 \beta W_I'(b') - \theta \beta W_O'(b') + \lambda + \lambda(\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} g_I^\sigma \beta W_O'(b') &= 0
 \end{aligned}$$

Then,

$$\lambda = \frac{(1 - \theta^2)u'(g^I)}{1 - (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \theta}$$

Then,

$$\beta W'_I(b') - \theta \beta W'_O(b') + \frac{(1 - \theta^2) \beta W'_O(b') (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}}}{1 - (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \theta} = - \frac{(1 - \theta^2) u'(g^I)}{1 - (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \theta}$$

The first-order condition is therefore:

$$u'(g^I) = - \frac{\beta}{(1 - \theta^2)} \left\{ W'_I(b') (1 - \theta (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}}) - W'_O(b') \left(\theta (1 - \theta (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}}) - (1 - \theta^2) (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \right) \right\} \quad (37)$$

In the simplest case ($\theta = 0$), the above Euler equation becomes

$$u'(g^I) = -\beta W'_I(b') - \beta W'_O(b') \phi_O^{\frac{\sigma}{1-\sigma}}$$

This can be rewritten as:

$$u'(g^I) \left(1 + \frac{\beta W'_O(b') \phi_O^{\frac{\sigma}{1-\sigma}}}{u'(g^I)} \right) = -\beta W'_I(b')$$

When $\theta = 0$ the opposition's utility is $u(g_O) = \phi_O u(g_I)$. Then, $g_O = \phi_O^{\frac{1}{1-\sigma}} g_I$. We can therefore write $u'(g^I) \phi_O^{\frac{\sigma}{1-\sigma}} = u'(g_O)$

Then,

$$u'(g^I) \underbrace{\left(1 + \frac{\beta W'_O(b')}{u'(g^O)} \right)}_{<1} = -\beta W'_I(b')$$

Similarly to (16), $\beta W'_O(b')/u'(g^O)$ is what the incumbent must give to the opposition in exchange of additional unit debt. Using (1), the (BC) constraint and the definition of $W_I(b)$, one obtains

$$u'(\tau + b' - (1 + r)b - g_O) \left(1 + \frac{\beta W'_O(b')}{u'(g_O)} \right) = -\beta [qV'_I(b') + (1 - q)V'_O(b')]$$

The above equation is the equivalent of (17) for the infinite horizon model.

A.5 Equilibrium characterization

To find the Markov Perfect equilibrium we guess a solution and we verify that it satisfies all the optimality conditions. The key feature that we exploit is that with the proposed rule the government spending grows at a constant rate, rendering the value functions proportional to

the flow utility, $V_j(b) = a_j u(g^j)$ for some constants a_j , $j = I, O$. We guess and verify that the ratio g^O/g^I is constant for all t and for all debt levels. In an environment with homothetic preferences the value function is also homothetic and the opposition's bargaining power can be represented by a constant. We guess that spending is linear in $(\tau - rb)$:

$$g^I = \nu \frac{\tau - rb}{p} \quad g^O = (p - 1) \nu \frac{\tau - rb}{p}$$

$$V_I(b) = a_I u\left(\frac{\tau - rb}{p}\right); \quad V_O(b) = a_O u\left(\frac{\tau - rb}{p}\right)$$

where a , p and ν are constants to be determined.

Note that under this guess:

$$b' = b + (\nu - 1)(\tau - rb)$$

The lower ν , the smaller the incentive to accumulate debt. Using the guess:

$$a_I u\left(\frac{\tau - rb}{p}\right) = \phi_I u\left(\nu \frac{\tau - rb}{p}\right) + \beta [qa_I + (1 - q)a_O] u\left(\frac{\tau - rb}{p}\right) (1 - r(\nu - 1))^{1-\sigma}$$

$$a_O u\left(\frac{\tau - rb}{p}\right) = \phi_O u\left(\nu \frac{\tau - rb}{p}\right) + \beta [qa_O + (1 - q)a_I] u\left(\frac{\tau - rb}{p}\right) (1 - r(\nu - 1))^{1-\sigma}$$

Therefore,

$$a_I = \phi_I \nu^{1-\sigma} + \beta [qa_I + (1 - q)a_O] (1 - r(\nu - 1))^{1-\sigma}$$

$$a_O = \phi_O \nu^{1-\sigma} + \beta [(1 - q)a_I + qa_O] (1 - r(\nu - 1))^{1-\sigma}$$

Let $a = a_I + a_O$, adding up the previous equations:

$$a = \frac{(\phi_O + \phi_I) \nu^{1-\sigma}}{1 - \beta (1 - r(\nu - 1))^{1-\sigma}} \quad (38)$$

Define

$$W(b) \equiv V_I(b) + V_O(b).$$

Therefore the continuation values functions can be written as:

$$W_I(b) = \left[\frac{a_I}{a} q + \frac{a_O}{a} (1 - q) \right] W(b') = \zeta_I W(b')$$

$$W_O(b) = \left[\frac{a_I}{a} (1 - q) + \frac{a_O}{a} q \right] W(b') = \zeta_O W(b')$$

With this definitions the Euler equation (37) becomes:

$$u'(g^I) = -\frac{\beta}{(1 - \theta^2)} \left[\zeta + (\zeta_O - \theta \zeta_I) (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \right] W'(b')$$

where $\zeta \equiv \zeta_I - \theta \zeta_O$. Given a , the derivatives of the value function are:

$$W'(b) = \frac{-ra}{p} u' \left(\frac{\tau - rb}{p} \right)$$

$$W'(b') = \frac{-ra'}{p} u' \left(\frac{\tau - rb}{p} \right) (1 - r(\nu - 1))^{-\sigma}$$

Since $a' = a$, the Euler equation becomes:

$$u'(g^I) = \frac{\beta}{(1 - \theta^2)} \left[\zeta + (\zeta_O - \theta \zeta_I) (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \right] \frac{ra}{p} u' \left(\frac{\tau - rb}{p} \right) (1 - r(\nu - 1))^{-\sigma}$$

Under Assumption 1 we have $\beta r = 1 - \beta$, thus, using the guesses:

$$\nu^{-\sigma} = \frac{(1 - \beta) a}{(1 - \theta^2) p} \left[\zeta + (\zeta_O - \theta \zeta_I) (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \right] (1 - r(\nu - 1))^{-\sigma} \quad (39)$$

Define

$$\rho = \frac{(1 - \beta) a}{(1 - \theta^2) p} \left[(\zeta_I - \theta \zeta_O) + (\zeta_O - \theta \zeta_I) (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}} \right] \quad (40)$$

We obtain:

$$\nu = \frac{1 + r}{\rho^{\frac{1}{\sigma}} + r} \quad (41)$$

Thus, for a given ϕ_O , using (31), (38) and the definition of ν we have a fixed point in ρ . It is easy to show that there is a solution for all $\phi_O \in [\theta, 1 + \theta]$ and, using a , that in an equilibrium with $\phi_O < 1 + \theta$, $\nu > 1$ which implies $\rho < 1$.

Now, we solve for ϕ_O which depends on the budget rule. Recall that:

$$\phi_O = \frac{\phi_O^s u(g^{I,s}) + \beta [W_O(b'^s) - W_O(b')]}{u(g_I)}$$

where p^s , ϕ_O^s and b'^s are given by (33), (35) and (36). Thus,

$$\phi_O = \left(\frac{p}{p^s}\right)^{1-\sigma} \left(\frac{\alpha}{\nu}\right)^{1-\sigma} \phi_O^s + \zeta_O \frac{a\beta}{u(g^I)} [W((\tau - rb'^s)/p) - W((\tau - rb')/p)]$$

We rewrite as:

$$\phi_O \nu^{1-\sigma} = \left(\frac{p}{p^s}\right)^{1-\sigma} \alpha^{1-\sigma} \phi_O^s + \beta \zeta_O a [(1 - r(\alpha - 1))^{1-\sigma} - (1 - r(\nu - 1))^{1-\sigma}] \quad (42)$$

Knowing, ϕ_O , we can compute p as follows

$$p = 1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}} \quad (43)$$

Then, we can state our main result:

For any budget rule α , a Markov-perfect equilibrium is fully characterized by the factors ϕ_O , ν and p that simultaneously solve equations (41), (42) and (43).

A.6 Debt in the Infinite Horizon Model

Re-statement of Proposition 3 for the full model. *Suppose $q = 1/2$ and $\sigma \geq \beta/2$, Let a fiscal rule, $\alpha \in [0, 1]$, be given. In each period t the incumbent proposes a policy that does not satisfy the fiscal limit and the proposal is accepted by the opposition. In equilibrium:*

- a) *Debt is lower than debt with alternating dictators.*
- b) *Debt is positive.*

Proof: Notice that assuming $q = 1/2$ and replacing p from (43) and a from (38) in equation (39) we obtain:

$$\nu^{-\sigma} = \frac{(1 - \beta)}{2} \frac{(1 + \phi_O - \theta)\nu^{1-\sigma}}{1 - \beta(1 - r(\nu - 1))^{1-\sigma}} \frac{[1 + (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}}]}{[1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}}]} (1 - r(\nu - 1))^{-\sigma}$$

Define:

$$\begin{aligned} f(\phi_O) &\equiv \frac{(1 + \phi_O - \theta)[1 + (\phi_O - \theta)^{\frac{\sigma}{1-\sigma}}]}{2[1 + (\phi_O - \theta)^{\frac{1}{1-\sigma}}]} \\ X &\equiv 1 - r(\nu - 1) \end{aligned}$$

We need to show: (a) debt is lower under bargaining than with alternating dictators and (b) there is still over accumulation of debt. To prove (a) we show that ν is decreasing in ϕ_O . Since in any equilibrium in which the acceptance constraint is binding it must be true that $\phi_O > \phi_O^s$, this delivers the result. To prove (b) we show that $\nu = 1$ is not a solution to the above equation for any ϕ_O . To proceed first use the above definitions to write:

$$1 - \beta X^{1-\sigma} = f(\phi_O)(1 - \beta)\nu X^{-\sigma}$$

Since $r = (1 - \beta)/\beta$, $X = \frac{1-(1-\beta)\nu}{\beta}$, then we have:

$$1 - \beta X^{1-\sigma} = f(\phi_O)(1 - \beta X)X^{-\sigma}$$

$$X^\sigma = f(\phi_O) + \beta X (1 - f(\phi_O)) \quad (44)$$

Lemma A1. For all $\theta \in [0, 1]$, $f(\phi_O) \in [1/2, 1]$ and $f'(\phi_O) > 0$.

Proof: Define $\tilde{\phi} = \phi_O - \theta$. Note that $\tilde{\phi} \in [0, 1]$ since $\phi_O \in [\theta, 1 + \theta]$. Then we can rewrite f as:

$$f(\phi_O) = \frac{1}{2} \left[1 + \frac{\tilde{\phi} + \tilde{\phi}^{\frac{\sigma}{1-\sigma}}}{1 + \tilde{\phi}^{\frac{1}{1-\sigma}}} \right]$$

Since $\tilde{\phi} \geq 0$ it follows that $f \geq 1/2$ and $f(1 + \theta) = 1$. So, we are left to show $f'(\phi_O) > 0$ in the interval $[\theta, 1 + \theta]$. The derivative is given by:

$$f'(\phi_O) = \frac{1}{2} \frac{\left(1 + \frac{\sigma}{1-\sigma} \tilde{\phi}^{\frac{\sigma}{1-\sigma}-1}\right) \left(1 + \tilde{\phi}^{\frac{1}{1-\sigma}}\right) - \left(\tilde{\phi} + \tilde{\phi}^{\frac{\sigma}{1-\sigma}}\right) \frac{1}{1-\sigma} \tilde{\phi}^{\frac{1}{1-\sigma}-1}}{\left(1 + \tilde{\phi}^{\frac{1}{1-\sigma}}\right)^2}$$

The sign of the derivative is:

$$\text{sign}(f') = 1 + \frac{\sigma}{1-\sigma} \tilde{\phi}^{\frac{2\sigma-1}{1-\sigma}} + \tilde{\phi}^{\frac{1}{1-\sigma}} + \frac{\sigma}{1-\sigma} \tilde{\phi}^{\frac{2\sigma}{1-\sigma}} - \frac{1}{1-\sigma} \tilde{\phi}^{\frac{1}{1-\sigma}} - \frac{1}{1-\sigma} \tilde{\phi}^{\frac{2\sigma}{1-\sigma}}$$

$$\text{sign}(f') = 1 - \tilde{\phi}^{\frac{2\sigma}{1-\sigma}} + \frac{\sigma}{1-\sigma} \left[\tilde{\phi}^{\frac{2\sigma-1}{1-\sigma}} - \tilde{\phi}^{\frac{1}{1-\sigma}} \right]$$

As long as $\tilde{\phi} \leq 1$ we have that $1 - \tilde{\phi}^{\frac{2\sigma}{1-\sigma}} \geq 0$ and $\tilde{\phi}^{\frac{2\sigma-1}{1-\sigma}} - \tilde{\phi}^{\frac{1}{1-\sigma}} \geq 0$ because $\frac{2\sigma-1}{1-\sigma} \leq \frac{1}{1-\sigma}$. ■

Returning to equation (44), let $h(X)$:

$$h(X) = X^\sigma - f(\phi_O) - \beta X(1 - f(\phi_O))$$

Notice that $h(0) < 0$ and $h(1) \geq 0$ as long as $f \leq 1$. Because h is continuous it follows that there exists a solution $X \in (0, 1]$.

Suppose $\sigma \geq \beta/2$. Then there exists a unique solution $X \in [0, 1]$. This follows from the fact that h is monotone increasing in $[0, 1]$. Notice that

$$h'(X) = \sigma X^{\sigma-1} - \beta(1 - f(\phi_O))$$

By contradiction, suppose $h'(X) < 0$, then it must be true that:

$$\sigma X^{\sigma-1} < \beta(1 - f(\phi_O)) \Rightarrow X > \left(\frac{\sigma}{\beta(1 - f(\phi_O))} \right)^{\frac{1}{1-\sigma}}$$

But then, $X > 1$ because

$$\sigma \geq \beta(1 - f(\phi_O))$$

If $f(\phi_O) = 1$, the statement is true for all σ . Since the lower bound for f is $1/2$, we reach a contradiction for all $\sigma \geq \beta/2$, independently of the value for ϕ_O .

To show part (a) of Proposition 3, total differentiate equation (44), at the solution:

$$h'(X)dX = (1 - \beta X)f'(\phi_O)d\phi_O$$

Because both $h'(X) > 0$ and $(1 - \beta X) > 0$, and due to Lemma $f'(\phi_O) > 0$, it follows that $\frac{dX}{d\phi_O} > 0$. But since $X = 1 - r(\nu - 1)$, then $\frac{d\nu}{d\phi_O} < 0$. ■

A.7 Optimal Rule and Proof of Proposition 4

What is the optimal α ? Let α^* be the value of α that maximizes the equally weighted sum of the value functions of the incumbent and opposition:

$$\alpha^* = \operatorname{argmax}_{\{\alpha\}} \left\{ \frac{1}{2}V_I(b; \alpha) + \frac{1}{2}V_O(b; \alpha) \right\}$$

$$\alpha^* = \operatorname{argmax}_{\{\alpha\}} \left\{ \frac{1}{2}a_I u \left(\frac{\tau - rb}{p} \right) + \frac{1}{2}a_O u \left(\frac{\tau - rb}{p} \right) \right\}$$

Notice that α affects the equilibrium only through equation (42), while the effect of ν is indirect because ν depends on ϕ_O through equation (39). Also, because the value functions

are proportional to $u(\tau - rb)$, the maximizer is independent of τ and b . Thus, the first-order necessary condition is:

$$\left[\left(\frac{\partial a_I}{\partial \nu} \frac{\partial \nu}{\partial \phi_O} + \frac{\partial a_I}{\partial \phi_O} + \frac{\partial a_O}{\partial \nu} \frac{\partial \nu}{\partial \phi_O} + \frac{\partial a_O}{\partial \phi_O} \right) + \frac{(\sigma - 1)}{p^{\sigma-2}} [a_I + a_O] \frac{\partial p}{\partial \phi_O} \right] \frac{\partial \phi_O}{\partial \alpha} = 0$$

This implies that all the critical points of ϕ_O are also critical points of the welfare function. This results is independent of the assumed welfare weights. We choose equal weights because it generates simpler expressions. The fiscal rule that maximizes bargaining power also maximizes the joint welfare of the two parties. As long as the term in squared brackets is not zero the mapping is one to one. In what follows we proceed under this assumption and we look for the fiscal rule that maximizes ϕ_O .

Using equation (42), taking the first order condition, and keeping in mind that ζ_O , ν and p are functions of ϕ_O , we obtain:

$$\begin{aligned} \frac{\partial \phi_O}{\partial \alpha} \nu^{1-\sigma} &= -\phi_O (1-\sigma) \nu^{-\sigma} \frac{\partial \nu(\phi_O)}{\partial \phi_O} \frac{\partial \phi_O}{\partial \alpha} \\ &+ (1-\sigma) \alpha^{-\sigma} \phi_O^s \left(\frac{p}{p_s} \right)^{1-\sigma} + (1-\sigma) \alpha^{1-\sigma} \phi_O^s \left(\frac{p}{p_s} \right)^{-\sigma} \frac{1}{p_s} \frac{\partial p(\phi_O)}{\partial \phi_O} \frac{\partial \phi_O}{\partial \alpha} \\ &+ \beta \left[(1-r(\alpha-1))^{1-\sigma} - (1-r(\nu-1))^{1-\sigma} \right] \frac{\partial [a(\phi_O)\zeta_O(\phi_O)]}{\partial \phi_O} \frac{\partial \phi_O}{\partial \alpha} \\ &- r(1-\sigma)\beta a \zeta_O (1-r(\alpha-1))^{-\sigma} + r(1-\sigma)\beta a \zeta_O (1-r(\nu-1))^{-\sigma} \frac{\partial \nu(\phi_O)}{\partial \phi_O} \frac{\partial \phi_O}{\partial \alpha} \end{aligned}$$

Since at the optimum $\frac{\partial \phi_O}{\partial \alpha} = 0$, there is an analogous to the envelope theorem: only the direct effect matters; indirect effects vanish. Rearranging the equation we obtain:

$$\begin{aligned} (\alpha^*)^{-\sigma} \phi_O^s \left(\frac{p}{p_s} \right)^{1-\sigma} &= r\beta \zeta_O(\alpha^*) a(\alpha^*) (1-r(\alpha^*-1))^{-\sigma} \\ \alpha^* \left(\frac{p_s}{p} \right)^{\frac{1-\sigma}{\sigma}} &= \left(\frac{\phi_O^s}{(1-\beta)a(\alpha^*)\zeta_O(\alpha^*)} \right)^{\frac{1}{\sigma}} \left(\frac{1-(1-\beta)\alpha^*}{\beta} \right) \end{aligned} \quad (45)$$

As a result:

$$\alpha^* = \frac{\frac{1}{\beta} \left(\frac{\phi_O^s}{(1-\beta)a(\alpha^*)\zeta_O(\alpha^*)} \right)^{\frac{1}{\sigma}}}{\left(\frac{p_s}{p} \right)^{\frac{1-\sigma}{\sigma}} + \frac{(1-\beta)}{\beta} \left(\frac{\phi_O^s}{(1-\beta)a(\alpha^*)\zeta_O(\alpha^*)} \right)^{\frac{1}{\sigma}}}$$

Replacing a in the above

$$\alpha^* = \frac{\left(\frac{(1-\tilde{\beta})}{(1-\beta)} \frac{\phi_O^s \zeta_O^{-1}}{(\phi_O + \phi_I) \nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\beta \left(\frac{p_s}{p} \right)^{\frac{1-\sigma}{\sigma}} + (1-\beta) \left(\frac{(1-\tilde{\beta})}{(1-\beta)} \frac{\phi_O^s \zeta_O^{-1}}{(\phi_O + \phi_I) \nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}$$

where $\tilde{\beta} = \beta(1-r(\nu-1))^{1-\sigma}$. Using the equilibrium ϕ_O^s with full discretion:

$$\alpha^* = \frac{\left(\frac{(1-\tilde{\beta})}{(1-\beta)} \frac{(\theta + \theta^{\frac{1-\sigma}{\sigma}}) \zeta_O^{-1}}{(1+\theta)(1-\theta + \phi_O) \nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\beta \left(\frac{p_s}{p} \right)^{\frac{1-\sigma}{\sigma}} + (1-\beta) \left(\frac{(1-\tilde{\beta})}{(1-\beta)} \frac{(\theta + \theta^{\frac{1-\sigma}{\sigma}}) \zeta_O^{-1}}{(1+\theta)(1-\theta + \phi_O) \nu^{1-\sigma}} \right)^{\frac{1}{\sigma}}}$$

Equation (45) can be used to derive a shaper characterization. First, consider the case in which $\theta = 0$. We have shown that both a and ζ_O are always positive and that $p \geq p^s \geq 1$, for all θ , while $\phi_O^s = 0$. As a result, (45) immediately implies that when $\theta = 0$, $\alpha^* = 0$.

Second, consider the case with $\theta = 1$. Because this solution is equivalent to the planner's problem we have that $p^s = p$, $\nu = 1$, $\phi_I = \phi_O = \phi_O^s$ and $a_I = a_O = a/2$, this implies that

$$\frac{\phi_O^s}{(1-\beta)a(\alpha^*)\zeta_O(\alpha^*)} = 1$$

and therefore equation (45) generates:

$$\alpha^* = \left(\frac{1 - (1-\beta)\alpha^*}{\beta} \right)$$

Which is satisfied if $\alpha^*(\theta = 1) = 1$.

To show that $\alpha^*(\theta)$ is increasing, define:

$$F(\alpha^*, \theta) = \alpha^* \left(\frac{p_s^{1-\sigma}}{\phi_O^s} \right)^{\frac{1}{\sigma}} - \left(\frac{p^{1-\sigma}}{(1-\beta)a\zeta_O} \right)^{\frac{1}{\sigma}} \left(\frac{1 - (1-\beta)\alpha^*}{\beta} \right)$$

Since α^* solves $F(\alpha^*(\theta), \theta) = 0$, it follows that:

$$\frac{\partial \alpha^*}{\partial \theta} = -\frac{\frac{\partial F(\alpha^*, \theta)}{\partial \theta}}{\frac{\partial F(\alpha^*, \theta)}{\partial \alpha^*}}$$

We now show that $\frac{\partial F(\alpha^*, \theta)}{\partial \theta} < 0$ and $\frac{\partial F(\alpha^*, \theta)}{\partial \alpha^*} > 0$ which completes the proof of Proposition 4. Differentiating $F(\cdot)$ we obtain:

$$\begin{aligned} \frac{\partial F(\alpha^*, \theta)}{\partial \alpha^*} &= \left(\frac{p_s^{1-\sigma}}{\phi_O^s} \right)^{\frac{1}{\sigma}} + \frac{(1-\beta)}{\beta} \left(\frac{p^{1-\sigma}}{(1-\beta)a\zeta_O} \right)^{\frac{1}{\sigma}} > 0 \\ \frac{\partial F(\alpha^*, \theta)}{\partial \theta} &= -\frac{1}{\sigma} \left(\frac{\phi_O^s}{p_s^{1-\sigma}} \right)^{-\frac{1}{\sigma}-1} \frac{\partial \frac{\phi_O^s}{p_s^{1-\sigma}}}{\partial \theta} \\ &\quad - (1-\beta) \left(\frac{1-(1-\beta)\alpha^*}{\sigma\beta} \right) \left(\frac{(1-\beta)a\zeta_O}{p^{1-\sigma}} \right)^{-\frac{1}{\sigma}-1} \frac{\partial \frac{a\zeta_O}{p^{1-\sigma}}}{\partial \theta} < 0 \end{aligned}$$

The first inequality is straightforward because all the components are positive. The second inequality follows because $\frac{\partial \frac{\phi_O^s}{p_s^{1-\sigma}}}{\partial \theta} = \frac{\partial}{\partial \theta} \left(\frac{\theta + \theta^{\frac{1-\sigma}{\sigma}}}{(1+\theta^{1/\sigma})^{1-\sigma}} \right) > 0$ and $\frac{\partial \frac{a\zeta_O}{p^{1-\sigma}}}{\partial \theta} = \frac{\partial}{\partial \theta} \left(\frac{qa_O + (1-q)a_I}{p^{1-\sigma}} \right) = u((\tau - rb)^{-1}) \frac{\partial W_O(b; \theta)}{\partial \theta} > 0$, so that as long as the opposition's continuation value is increasing in θ , the optimal fiscal rule is also increasing in θ . ■

A.8 Proof of Proposition 6

Throughout, suppose $q = 1/2$. To compute A^H , note that when the fiscal rule is not negotiable, the incumbent's available resources are given by $(\tau - rb)$. The incumbent allocates a proportion $\frac{\theta^{1/\sigma}}{1+\theta^{1/\sigma}}$ of the total resources to the opposition. This generates an expected (total) flow utility of

$$\frac{[1 + \theta\theta^{(1-\sigma)/\sigma} + \theta + \theta^{(1-\sigma)/\sigma}]}{2(1 + \theta^{1/\sigma})^{1-\sigma}} u(\tau - rb)$$

Cancelling terms and abstracting from $u(\tau - rb)$ we obtain A^H .

To compute A^L , note that expected welfare under the soft rule is

$$\frac{1}{2} W(b) = \frac{1}{2} au \left(\frac{\tau - rb}{p} \right)$$

Using (38), (30) and (31) we obtain

$$\frac{1}{2}W(b) = u(\tau - rb) \frac{(1 + \theta)\nu^{1-\sigma}}{2[1 - \beta(1 - r(\nu - 1))^{1-\sigma}]} \frac{(1 + \phi^O - \theta)}{[1 + (\phi^O - \theta)^{\frac{1}{1-\sigma}}]^{1-\sigma}} \quad (46)$$

After using (28) and (29) we obtain the expression in the main text. As discussed in there, a sufficient condition for soft rules to yield higher welfare is that debt growth, determined by ν , is sufficiently low. Using equation (39), ν solves:

$$[1 - \beta(1 - r(\nu - 1))^{1-\sigma}](1 - r(\nu - 1))^\sigma = \frac{(1 - \beta)\nu [1 + (\phi^O - \theta)^{\frac{\sigma}{1-\sigma}}](1 + \phi^O - \theta)}{2 [1 + (\phi^O - \theta)^{\frac{1}{1-\sigma}}]} \quad (47)$$

Let

$$A(\beta, \nu) := \frac{[1 + (\phi^O - \theta)^{\frac{\sigma}{1-\sigma}}](1 + \phi^O - \theta)}{[1 + (\phi^O - \theta)^{\frac{1}{1-\sigma}}]}$$

Recall that $(1 + r)\beta = 1$, assume $\sigma = 1/2$ and define:

$$X := (1 - r(\nu - 1))^{1/2}$$

Notice that equation (47) can be written as:

$$\frac{2X}{(1 - \beta)}(1 - \beta X) = \nu \frac{(1 + \phi^O - \theta)^2}{[1 + (\phi^O - \theta)^2]} \quad (48)$$

Using the result in the expression for A^S when $\sigma = 1/2$, we obtain:

$$A^S = \frac{1}{2}(1 + \theta) \frac{(2X)^{1/2}}{[(1 - \beta)(1 - \beta X)]^{1/2}}$$

Using this rewriting we have that $A^S \geq A^H$ if and only if:

$$\frac{2X}{(1 - \beta X)} \geq \frac{1}{(1 - \beta)} \frac{(1 + \theta)^2}{(1 + \theta^2)}$$

or

$$X \geq \frac{1}{\frac{2(1-\beta)(1+\theta^2)}{(1+\theta)^2} + \beta}$$

We show that this condition is satisfied both, when $\beta \rightarrow 0$, in which case $X \rightarrow \frac{1}{2} \frac{(1 + (\frac{\theta + \mu}{1 + \theta \mu}))^2}{[1 + (\frac{\theta + \mu}{1 + \theta \mu})^2]} >$

$\frac{1}{2} \frac{(1+\theta)^2}{[1+\theta^2]}$, $\forall \theta \in (0, 1)$; and when $\beta \rightarrow 1$, in which case $X \rightarrow 1$ and $\frac{1}{\frac{2(1-\beta)(1+\theta^2)}{(1+\theta)^2} + \beta} \rightarrow 1$

To see this, using the definition of X and $A(\beta, \nu)$ we can write (48) as

$$X - \beta X^2 = (1 - \beta X^2) \frac{A(\beta, \nu)}{2}$$

Solving for this equation we obtain:

$$X = \frac{1}{\beta(A(\beta, \nu) - 2)} \left[-1 \mp \sqrt{1 + \beta A(\beta, \nu)(A(\beta, \nu) - 2)} \right]$$

First, notice that X must be positive, thus only the positive root can be a solution. Now, when $\beta \rightarrow 1$ the positive root in the last equation implies $X = 1$, while, since $A(\beta, \nu)$ is bounded below and above. As $\beta \rightarrow 0$, we have that $X \rightarrow \frac{A(\beta, \nu)}{2}$. More specifically, as β goes to zero, ν is forced to go to one. In fact, knowing that $(\nu - 1)\tau$ (the intercept of the debt function) must be smaller than the natural debt limit, and using the assumption that $(1 + r)\beta = 1$, we obtain that $\nu \leq 1/(1 - \beta)$. Then, when β goes to zero, r goes to infinity and ν goes to one. ■

B Two-period Model: Proofs

B.1 Proof of Proposition 2

We solve the alternating-dictator model by backward induction. Under Assumption 1, in the second period the available resources are $(\tau - b_1)$. The time-2 dictator spends $(1 - \tilde{\theta})(\tau - b_1)$ for her favorite public good and $\tilde{\theta}(\tau - b_1)$ for the other public good, where $\tilde{\theta}$ is given by

$$\tilde{\theta} = \frac{\theta^{\frac{1}{\sigma}}}{1 + \theta^{\frac{1}{\sigma}}} \quad (49)$$

The first period problem can be written as:

$$\max_{\{b_1, g_1^O\}} \left\{ u(\tau + b_1 - g_1^O) + \theta u(g_1^O) + [q + (1 - q)\theta]u((1 - \tilde{\theta})(\tau - b_1)) + [\theta q + (1 - q)]u(\tilde{\theta}(\tau - b_1)) \right\}$$

Notice that the choice between g_1^O and g_1^I is a static decision, so it can be solved independently from the dynamic decision. It is straightforward to show that the static problem in period 1 generates an analogous allocation to period 2: $g_1^O = \tilde{\theta}(\tau + b_1)$ and $g_1^I = (1 - \tilde{\theta})(\tau + b_1)$.

Replacing these expressions in the previous problem and taking derivatives, we obtain the first-order condition (11), where

$$\frac{\Omega(\theta)}{\Theta(\theta)} = q + \frac{(1-q)(\theta + \theta^{\frac{1-\sigma}{\sigma}})}{1 + \theta^{\frac{1}{\sigma}}}$$

Define

$$\Lambda := \frac{\Omega(\theta)}{\Theta(\theta)}$$

From (11), when Λ is larger, debt is lower. We therefore study how parameters affect Λ . First, we show that debt is decreasing in q :

$$\frac{\partial \Lambda}{\partial q} = 1 - \frac{\theta + \theta^{\frac{1-\sigma}{\sigma}}}{1 + \theta^{\frac{1}{\sigma}}} > 0$$

Since $\theta + \theta^{\frac{1-\sigma}{\sigma}} < 1 + \theta^{\frac{1}{\sigma}}$ can be written as $\theta > \theta^{\frac{1}{\sigma}}$. The inequality holds because $\theta < 1$ and $\sigma \leq 1$.

We now study the effect of polarization on debt. We compute

$$\frac{\partial \Lambda}{\partial \theta} = (1-q) \frac{\left(1 + \frac{1-\sigma}{\sigma} \theta^{\frac{1-2\sigma}{\sigma}}\right) \left(1 + \theta^{\frac{1}{\sigma}}\right) - \frac{1}{\sigma} \theta^{\frac{1}{\sigma}-1} (\theta + \theta^{\frac{1-\sigma}{\sigma}})}{\left(1 + \theta^{\frac{1}{\sigma}}\right)^2}$$

This derivative is positive when $\theta < 1$ and $\sigma \leq 1$. To see this we can rewrite it as:

$$\frac{\partial \Lambda}{\partial \theta} = (1-q) \left[1 - \theta^{\frac{2(1-\sigma)}{\sigma}} + \frac{1-\sigma}{\sigma} \left(\theta^{\frac{1-2\sigma}{\sigma}} - \theta^{\frac{1}{\sigma}} \right) \right]$$

Which is strictly positive since the first two terms in the square brackets add up to a positive value and the last term is also positive.

Finally, when $\theta = 0$ it is immediate from the dictator's first order condition that a higher σ leads to a smaller debt. ■

B.2 Proof of Proposition 3

Let γ be the equilibrium ratio of initial consumptions: $\gamma := \frac{g_1^O}{g_1^I}$. Reorganizing the first order condition (17):

$$u'(g_1^I) = \frac{1}{1 + \gamma'} [qu'(g_2^I) + (1 - q)u'(g_2^O)\gamma'] - G'(b_1)u'(g_1^I)$$

$$u'(g_1^I) = \frac{1}{1 + \gamma'} [qu'(g_2^I) + (1 - q)u'(g_2^O)\gamma'] + \frac{1}{1 + \gamma'} \left[\frac{qu'(g_2^O)\gamma' + (1 - q)u'(g_2^I)}{u'(g_1^O)} \right] u'(g_1^I)$$

Knowing that (9) and (10) are satisfied with equality, we write the first-order condition as

$$\frac{u'(\frac{\tau+b_1}{1+\gamma})}{u'(\frac{\tau-b_1}{1+\gamma})} = \frac{1}{1 + \gamma'} \left[q + (1 - q)\frac{u'(g_2^O)}{u'(g_2^I)}\gamma' + q\frac{u'(g_2^O)}{u'(g_2^I)}\frac{u'(g_1^I)}{u'(g_1^O)}\gamma' + (1 - q)\frac{u'(g_1^I)}{u'(g_1^O)} \right] \quad (50)$$

First, we show that debt is reduced. To do this, we impose the consistency requirement $\gamma = \gamma'$. With this restriction, it is easy to see that $\frac{u'(g_2^O)}{u'(g_2^I)} = u'(\gamma') = u'(\gamma) = \frac{u'(g_1^O)}{u'(g_1^I)}$. Then,

$$\begin{aligned} \frac{u'(\tau + b_1)}{u'(\tau - b_1)} &= \frac{1}{1 + \gamma} [q + (1 - q)u'(\gamma)\gamma + q\gamma + (1 - q)u'(\gamma^{-1})] \\ \frac{u'(\tau + b_1)}{u'(\tau - b_1)} &= q + (1 - q)\frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma} \end{aligned}$$

From the last equation it is straightforward that debt grows at a smaller rate than in the dictator's problem. The last term in the right-hand side of the above equation ensures it. Loosely speaking, it is analogous to increasing the discount factor from q in the dictator's problem to $q + (1 - q)\frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma}$ in the bargaining problem. Since $\gamma > 0$ simple algebra confirms point (a) of Proposition 3.

To prove that debt is still positive, we only need to show that $\frac{[u'(\gamma)\gamma + u'(\gamma^{-1})]}{1 + \gamma} \leq 1$, or:

$$\gamma^{1-\sigma} + \gamma^\sigma \leq 1 + \gamma$$

This is true for all $\gamma > 0$ and $\sigma \leq 1$. To see this, consider the function $f(\gamma) = \gamma^{1-\sigma} + \gamma^\sigma - 1 - \gamma$. We want to show that $f(\gamma) \leq 0$ for all γ . Rewriting the function as $f(\gamma) = (1 - \gamma^{1-\sigma})(\gamma^\sigma - 1)$ it is clear that $f(\gamma)$ is negative and it equals zero only when $\gamma = 1$. ■