DISCUSSION PAPER SERIES

DP14588

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FINANCIAL ECONOMICS

ISSN 0265-8003

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Discussion Paper DP14588
Published 09 April 2020
Submitted 08 April 2020

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Abstract

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JEL Classification: C15, D81, G12

Keywords: recursive utility, Asset Pricing, Equity premium puzzle, Risk-Free Rate Puzzle

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Valuation Risk Revalued*

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First Draft: July 20, 2018

This Draft: April 1, 2020

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1 INTRODUCTION

In standard asset pricing models, uncertainty enters through the supply side of the economy, either through endowment shocks in a Lucas (1978) tree model or productivity shocks in a production economy model. Recently, several papers introduced demand side uncertainty or "valuation risk" as a potential explanation of key asset pricing puzzles (Albuquerque et al., 2016, 2015; Creal and Wu, 2017; Maurer, 2012; Nakata and Tanaka, 2016; Schorfheide et al., 2018). In macroeconomic parlance, valuation risk is typically referred to as either a discount factor or time preference shock.[1]

The literature contends valuation risk is an important determinant of key asset pricing moments when it is embedded in Epstein and Zin (1989) recursive preferences. We show the success of valuation risk rests sensitively on the way it is introduced. In particular, we examine two specifications—Current (the specification used in the asset pricing literature) and Revised (our preferred alternative)—and show they come to very different conclusions. Moreover, we identify four desirable properties of Epstein-Zin recursive preferences that the current specification violates and the revised specification satisfies, which cautions against continuing to use the current preferences.

The first property of recursive preferences pertains to comparative risk aversion. It says that, holding all else equal, an increase in the coefficient of relative risk aversion (RA, γ) equates to an increase in a household's risk aversion. We show this property does not hold when the intertemporal elasticity of substitution (IES, ψ) is below unity under the current specification. An increase in γ equates to a decrease, rather than an increase, in risk aversion, flipping its standard interpretation.[2]

The second property is that preferences are well-defined with unitary IES. The IES measures the responsiveness of consumption growth to a change in the real interest rate. An IES of 1 is a focal point because this is when the substitution and wealth effects of an interest rate change exactly offset. We show this property does not hold under the current specification in the literature.

The third property is that recursive preferences nest time-separable log-preferences when $\gamma = \psi = 1$. We show the current specification does not always nest log preferences in this case because it can even generate extreme curvature and risk-aversion when γ and ψ are arbitrarily close to 1.

The final property is that equilibrium moments are continuous functions of the IES over its domain. We show there is a discontinuity under the current specification. When the IES is marginally above unity, households require an arbitrarily large equity premium and an arbitrarily small risk-free rate, while an IES marginally below unity predicts the opposite. This is because the utility function exhibits extreme concavity with respect to valuation risk when the IES is marginally above unity and extreme convexity on this dimension when the IES is marginally below unity.

[1] Time preference shocks have been widely used in the macro literature (e.g., Christiano et al. (2011); Eggertsson and Woodford (2003); Justiniano and Primiceri (2008); Rotemberg and Woodford (1997); Smets and Wouters (2003)).

[2] The distinction between Epstein and Zin (1989) recursive preferences and constant relative risk aversion (CRRA) utility is that in the former, ψ and γ are distinct structural parameters, whereas in the latter $\gamma = 1/\psi$.

The discontinuity is relevant because there is a tension between the finance and macroeconomics literatures as to whether the IES lies above or below unity. Setting the IES to 0.5, as is common in the macroeconomics literature, can inadvertently result in a sizable negative equity premium.[3] Imagine two researchers who want to estimate the IES set the domain to $[0, 1)$ and $(1, \infty)$, respectively. The estimates in the two settings would diverge due to the discontinuity. Therefore, awareness of these issues is important even if researchers continue to use the current preferences.

In a business cycle context, de Groot et al. (2018) propose a revised Epstein-Zin preference specification for valuation risk in which the time-varying weights in the CES time-aggregator sum to 1, a restriction the current specification does not impose. Under this revised specification there is a well-defined equilibrium when the IES is 1 and asset prices are robust to small variations in the IES. Continuity is preserved because the weights in the time-aggregator always sum to unity. Another interpretation is that the time-aggregator maintains the well-known property that a CES aggregator tends to a Cobb-Douglas aggregator as the elasticity approaches 1. The current specification violates the restriction on the weights so the limiting properties of the CES aggregator break down. In summary, the revised specification is consistent with the four desirable properties.

This paper makes two key contributions. First, it analytically shows the preference specification profoundly affects the equilibrium determination of asset prices. For example, the same RA and IES can lead to very different values for the equity premium and risk-free rate and comparative statics, such as the response of the equity premium to the IES, switch sign. Taken at face value, the current specification resolves the equity premium (Mehra and Prescott, 1985) and risk-free rate (Weil, 1989) puzzles in our baseline model with *i.i.d.* cash-flow risk. Under the revised specification, valuation risk has a smaller role, RA is implausibly high, and the puzzles resurface.

Second, using a simulated method of moments (SMM), this paper empirically re-evaluates the role of valuation risk in explaining asset pricing and cash-flow moments. We find after estimating a sequence of increasingly rich models under the revised specification, the role and contribution of valuation risk change dramatically relative to the literature. However, valuation risk under the revised specification consistently improves the ability of the models to match moments in the data.

We begin by estimating the Bansal and Yaron (2004) long-run risk model (without time-varying uncertainty) without valuation risk and find it significantly under-predicts the standard deviation of the risk-free rate, even when these moments are targeted. When we introduce valuation risk, it accounts for roughly 40% of the equity premium, but at the expense of over-predicting the standard deviation of the risk-free rate. After targeting the risk-free rate dynamics, valuation risk only accounts for about 5% of the equity premium. Therefore, we find it is crucial to target these dynamics

[3] Hall (1988) and Campbell (1999) provide empirical evidence for an IES close to zero. Basu and Kimball (2002) find an IES of 0.5 and Smets and Wouters (2007) estimate a value of roughly 0.7. In contrast, van Binsbergen et al. (2012) and Bansal et al. (2016) estimate models with Epstein-Zin preferences and report IES values of 1.73 and 2.18.

to accurately measure the contribution of valuation risk. Valuation risk is also able to generate the upward sloping term structure for real Treasury yields found in the data, whereas cash-flow risk alone predicts a counterfactually downward sloping term structure. While valuation risk (with or without the targeted risk-free rate moments) improves the fit of the long-run risk model, the model still fails a test of over-identifying restrictions. This is because the model fairs poorly in matching the low predictability of consumption growth from the price-dividend ratio, the high standard deviation of dividend growth, and the weak correlation between dividend growth and equity returns.

We consider two extensions that improve the model's fit: (1) an interaction term between valuation and cash-flow risk (a proxy for general equilibrium demand effects) following Albuquerque et al. (2016) (henceforth, "Demand" model) and (2) stochastic volatility on cash-flow risk as in Bansal and Yaron (2004) (henceforth, "SV" model). In a horse race between these extensions, we find the Demand model wins and passes the over-identifying restrictions test at the 5% level. However, the two extensions are complements and the combined model passes the test at the 10% level. This is because the demand extension lowers the correlation between dividend growth and equity returns, while the SV extension offsets the effect of higher valuation risk on risk-free rate dynamics. Targeting longer-term rates further increases the relative improvement of the combined model.

Our paper also makes an important technical contribution. It is common in the literature to estimate asset pricing models with a simulated method of moments (e.g., Adam et al., 2016; Albuquerque et al., 2016; Andreasen and Jørgensen, 2019). We build on this methodology in two ways. One, we run Monte Carlo estimations of the model and calculate standard errors using different sequences of shocks, whereas estimates in the literature are typically based on a particular sequence of shocks. This approach allows us to obtain more precise estimates and account for differences between the asymptotic and sampling distributions of the parameters. Two, we use a rigorous two-step procedure to find the global optimum that uses simulated annealing to obtain candidate draws and then recursively applies a nonlinear solver to each candidate. We find that without applying such rigor, the algorithm would settle on local optima and potentially lead to incorrect inferences.

Related Literature This paper builds on the growing literature that examines the role of valuation risk in asset pricing models. Maurer (2012) and Albuquerque et al. (2016) were the first. They adopt the current preference specification and find valuation risk accounts for key asset pricing moments, such as the equity premium. Albuquerque et al. (2016) also focus on resolving the correlation puzzle (Campbell and Cochrane, 1999). Schorfheide et al. (2018) use a Bayesian mixed-frequency approach that targets entire time series rather than specific moments, but they do not target the term structure. They focus on one model with three SV processes, but where valuation risk and cash-flow risk are always independent. We examine in-depth the role of valuation risk by estimating a sequence of increasingly rich models with long-run cash-flow risk, some of which include general equilibrium demand effects. We find the term structure moments are informative about the role of valuation risk and the data prefers models with demand effects. Creal and Wu (2017) focus on bond premia. They also use the current specification, but valuation risk is tied to consumption and inflation and does not have an independent stochastic element. They find the slope of the yield curve is largely explained by valuation risk, given an IES estimate equal to 1.02.

Nakata and Tanaka (2016) and Kliem and Meyer-Gohde (2018) study term premia in a New Keynesian model using the current specification. The former calibrate the IES to 0.11 and generate a negative term premium. The latter estimate the IES with a prior in the $[0, 1]$ range and obtain a value of 0.09. Both findings are a consequence of the asymptote, as we show analytically. In contrast with the literature, Rapach and Tan (2018) and Bianchi et al. (2018) use the revised specification and estimate a real business cycle model. They find valuation risk still explains a large portion of the term premium because demand shocks interact with the production side of the economy.[4]

The paper proceeds as follows. Section 2 lays out desirable properties of recusive preferences and the consequences of the valuation risk specification. Section 3 discusses asset pricing implications. Section 4 describes our estimation method. Section 5 quantifies the effects of valuation risk in our baseline model with *i.i.d.* cash-flow risk. Section 6 estimates the basic long-run risk model with and without valuation risk. Section 7 extends the long-run risk model to include valuation risk shocks to cash-flow growth and stochastic volatility on cash-flow risk. Section 8 concludes.

2 Epstein-Zin Preferences with Discount Factor Shocks

2.1 Background

Epstein and Zin (1989) preferences generalize standard expected utility time-separable preferences. Current-period utility is defined recursively over current-period consumption, c_t, and a certainty equivalent, $\mu_t(U_{t+1})$, of next period's random utility, U_{t+1}, as follows:

$$U_t = W(c_t, \mu_t(U_{t+1})), \tag{1}$$

where $\mu_t \equiv g^{-1}(E_t g(U_{t+1}))$, W is the *time-aggregator*, and g is the *risk-aggregator*. W and g are increasing and concave and W and μ_t are homogenous of degree 1. Note that $\mu_t(U_{t+1}) = U_{t+1}$ if there is no uncertainty, and $\mu_t(U_{t+1}) \leq E_t[U_{t+1}]$ if g is concave and future outcomes are uncertain. Most of the literature considers the following functional forms for W and g:

$$g(z) \equiv (z^{1-\gamma} - 1)/(1-\gamma), \qquad \text{for } 1 \neq \gamma > 0, \tag{2}$$

$$W(x, y) \equiv \left((1-\beta)x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \quad \text{for } 1 \neq \psi > 0. \tag{3}$$

When $\gamma = 1$, $g(z) = \log(z)$ and when $\psi = 1$, $W = x^{1-\beta}y^{\beta}$. Therefore, the time-aggregator is

[4] Two other strands of the literature have interesting connections to our work. One, disaster risk (see Barro, 2009 and Gourio, 2012) can generate variation in the stochastic discount factor analogous to valuation risk. Two, Bansal et al. (2014), identify "discount rate risk" as a component of risk premia distinct from cash-flow and volatility risks.

a CES function that converges to a Cobb-Douglas function as $\psi \to 1$.[5] It is also common in the literature to see the time-aggregator written without the $(1-\beta)$ coefficient on x as follows:

$$W(x,y) \equiv \left(x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}. \tag{3'}$$

In this case, (3') is undefined when $\psi = 1$. This is because the weights in the time-aggregator do not sum to 1. Nevertheless, the exact specification of W does not affect equilibrium behavior.[6]

Result 1. *Utility function* (1) *with time-aggregator* (3) *or* (3') *represents the same preferences.*

Result 1 holds because it is possible to switch between (3) and (3') with a positive monotonic transformation that multiplies the utility function by $(1-\beta)^{1/(1-1/\psi)}$.[7] To see this, note that the intertemporal marginal rate of substitution (equivalently, the stochastic discount factor) is given by

$$m_{t+1} \equiv \left(\frac{\partial U_t}{\partial c_{t+1}}\right) \bigg/ \left(\frac{\partial U_t}{\partial c_t}\right) = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{U_{t+1}}{\mu_t\left(U_{t+1}\right)}\right)^{1/\psi-\gamma}. \tag{4}$$

Since μ_t is homogenous of degree 1, applying the positive monotonic transformation to U_{t+1} in the both numerator and denominator leaves the intertemporal marginal rate of substitution unchanged.[8]

The results thus far are standard, but they lay the groundwork for the discussion that follows. Valuation risk involves introducing discount factor shocks—exogenous stochastic time-variation in β. Whether one works with (3) and replaces both instances of β with $a_t\beta$ (where a_t is a log-normal mean zero stationary AR(1) stochastic process) or one works with (3') and replaces the only instance of β with $a_t\beta$ is *not* innocuous, even though one might conclude it is from Result 1. The specification matters and in what follows we will describe the consequences of these choices.

To determine a preferred specification of valuation risk, we first establish four desirable properties of standard Epstein-Zin preferences *without* discount factor shocks, and then assess whether the two specifications of Epstein-Zin preferences with discount factor shocks satisfy each of them.

Property 1. *γ is a measure of comparative risk aversion.*

Suppose there are two households, A and B, with Epstein-Zin preferences as defined above. The two households are identical in every way except in preference parameter γ. If γ measures risk aversion, then household A is more risk averse than household B if and only if $\gamma^A > \gamma^B$.

Property 2. *ψ is a measure of the IES and preferences are well defined with unit IES.*

[5]The functional form for g implies $\mu_t = (E_t U_{t+1}^{1-\gamma})^{1/(1-\gamma)}$ when $\gamma \neq 1$ and $\mu_t = \exp(E_t \log(U_{t+1}))$ when $\gamma = 1$.

[6]Kraft and Seifried (2014) prove the continuous-time analog of recursive preferences (stochastic differential utility, Duffie and Epstein, 1992) is the continuous-time limit of recursive utility if the weights in the time-aggregator sum to 1.

[7]This is similar to the common practice of writing CRRA utility as $u(c) = c^\alpha/\alpha$ instead of $u(c) = (c^\alpha - 1)/\alpha$, even though the omitted constant term is necessary when proving the limit as $\alpha \to 0$ is given by $u(c) = \log(c)$.

[8]An equivalent observation is that time-preference is independent of the $(1-\beta)$ coefficient. In an environment without consumption growth and without risk, time-preference is captured by the discount factor (i.e., $m_{t+1} = \beta$).

The IES is defined as the responsiveness of consumption growth to a change in the real interest rate. A rise in the real interest rate induces both a substitution effect (consumption today becomes relatively more expensive, decreasing current consumption) and an income effect (a saver feels wealthier, increasing current consumption). The substitution and income effects exactly offset when $\psi = 1$. Therefore, a unitary IES is an important focal point for any model of preferences.[9]

Property 3. *When $\gamma = \psi = 1$, Epstein-Zin preferences are equivalent to time-separable log-preferences given by $U_t = (1-\beta)\log(c_t) + \beta E_t U_{t+1}$ or, alternatively, $U_t = \log(c_t) + \beta E_t U_{t+1}$.*

Property 3 is a special case of the more general property that when $\gamma = 1/\psi$, Epstein-Zin preferences simplify to standard expected utility time-separable preferences. However, time-separable log preferences are a staple of economics textbooks, so this provides another useful benchmark.

Property 4. *Equilibrium moments are continuous functions of the IES, ψ, over its domain $\mathbb{R}^+$.*

This final property relates to the discussion of time-aggregator (3) versus (3'). Adopt (3') and suppose $x = 1$ and $y > 0$. In this case, $\lim_{\psi\to 1^-} W = 0$ and $\lim_{\psi\to 1^+} W = +\infty$. Therefore, (3') exhibits a discontinuity. However, as discussed, this discontinuity does not affect the intertemporal marginal rate of substitution, (4), and, as a result, does not materialize in equilibrium moments.

2.2 Discount Factor Shocks

There are two ways to introduce discount factor shocks into the Epstein-Zin time-aggregator. The first is denoted the "[C]urrent specification" and given by

$$W^C(x, y, a_t) \equiv \left((1-\beta)x^{1-1/\psi} + a_t\beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}. \tag{3C}$$

The second is denoted the "[R]evised specification" and given by

$$W^R(x, y, a_t) \equiv \left((1-a_t\beta)x^{1-1/\psi} + a_t\beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}. \tag{3R}$$

The current specification is commonly adopted in the literature. Its use is not surprising since, at face value, it is the natural extension of discount factor shocks to expected utility time-separable preferences given by $U_t = u(c_t) + a_t\beta E_t U_{t+1}$. The specifications, however, are *not* equivalent.[10]

Result 2. *Utility function* (1) *given* (3C) *does not, in general, reflect the same preferences as* (3R).

To demonstrate this result, we show there is no positive monotonic transformation that maps the two specifications. Define $\tilde{U}_t^C = (\frac{1-a_t\beta}{1-\beta})^{1/(1-1/\psi)} U_t^C$, so the transformed preferences are given by

$$\tilde{U}_t^C = \left((1-a_t\beta)c_t^{1-1/\psi} + a_t\beta\mu_t\left(\tilde{a}_{t+1}^{1/(1-1/\psi)}\tilde{U}_{t+1}^C\right)^{1-1/\psi}\right)^{1/(1-1/\psi)}, \tag{5}$$

[9] A unitary IES is also the basis of the "risk-sensitive" preferences in Hansen and Sargent (2008, Section 14.3).

[10] The presence of the $(1-\beta)$ coefficient in (3C) is irrelevant but we include it for symmetry. The domain of a_t is constrained to ensure the time-aggregator weights are always positive. With (3C), $a_t > 0$. With (3R), $0 < a_t < 1/\beta$.

where $\tilde{a}_{t+1} \equiv (1 - a_t\beta)/(1 - a_{t+1}\beta)$. The revised preferences are given by

$$U_t^R = \left((1 - a_t\beta)c_t^{1-1/\psi} + a_t\beta\mu_t \left(U_{t+1}^R\right)^{1-1/\psi}\right)^{1/(1-1/\psi)}. \tag{6}$$

Therefore, the equivalence only exists if $a_{t+1} = a_t$ for all t. Comparing (5) and (6), there are two striking features of the current specification. One, it has more risk since $\tilde{a}_{t+1}$ introduces additional variance. Two, it has more curvature in the certainty equivalent since $\tilde{a}_{t+1}$ is raised to $1/(1-1/\psi)$.

To gain further insight, we make a few simplifying assumptions. First, suppose $c_{t+1} = 1$ and $\Delta_{t+j} \equiv c_{t+j}/c_{t+j-1} = \Delta > 1$ for all $j \geq 2$. Second, suppose $a_{t+j} = 1$ for $j = 0$ and $j \geq 2$, but a_{t+1} is a random draw. The terms inside the expectations operators contained in μ_t are given by

$$\bar{U}_C(a_{t+1}) \equiv g(U_{t+1}^C) = g\left((1 - \beta + a_{t+1}\beta\bar{x})^{1/(1-1/\psi)}\right), \tag{7}$$

$$\bar{U}_R(a_{t+1}) \equiv g(U_{t+1}^R) = g\left((1 - a_{t+1}\beta + a_{t+1}\beta\bar{x})^{1/(1-1/\psi)}\right), \tag{8}$$

where $\bar{x} = \Delta^{1-1/\psi}(1-\beta)/(1-\beta\Delta^{1-1/\psi})$. One source of intuition is to examine the curvature of (7) and (8) with respect to a_{t+1} by defining an Arrow-Pratt type measure of risk aversion given by

$$\mathcal{A}^j \equiv -(\bar{U}_j''(a_{t+1})/\bar{U}_j'(a_{t+1}))|_{a_{t+1}=1},$$

where $j \in \{C, R\}$. The curvatures of the current and revised specifications are given by

$$\mathcal{A}^C = \left(\frac{\gamma - 1/\psi}{1 - 1/\psi}\right)\beta\Delta^{1-1/\psi} \quad \text{and} \quad \mathcal{A}^R = \left(\frac{\gamma - 1/\psi}{1 - 1/\psi}\right)\frac{\beta}{1-\beta}\left(\Delta^{1-1/\psi} - 1\right). \tag{9}$$

To visualize these results, Figure 1 plots state-space indifference curves following Backus et al. (2005). Suppose there are two equally likely states for $a_{t+1} \in \{a_1, a_2\}$. The 45-degree line represents certainty. We plot (a_1, a_2) pairs, derived in Appendix A, that deliver the same utility as the certainty equivalent. A convex indifference curve implies aversion with respect to valuation risk.

Result 3. *The current specification violates Property 1 when $\psi < 1$ because increasing γ leads to a fall in risk aversion. In contrast, the property is never violated under the revised specification.*

Result 3 states that under the current specification, a higher RA can lead to a fall in risk aversion ($\partial\mathcal{A}^C/\partial\gamma < 0$) for $\psi < 1$. Visually, this is captured in the top-row of Figure 1. Under the current specification, with $\psi = 0.95$, an increase in γ from 0.1 to 3 causes the indifference curve to become less convex, indicating a decrease in risk aversion. When $\psi = 1.05$, the opposite occurs. In contrast, under the revised specification, $\partial\mathcal{A}^R/\partial\gamma > 0$ for all ψ, consistent with Property 1.

Result 4. *The current preferences become extremely concave with respect to valuation risk as $\psi \to 1^+$ and extremely convex as $\psi \to 1^-$ and are undefined when $\psi = 1$, violating Property 2. In contrast, the curvature of the revised preferences is continuous and increases only modestly in ψ.*

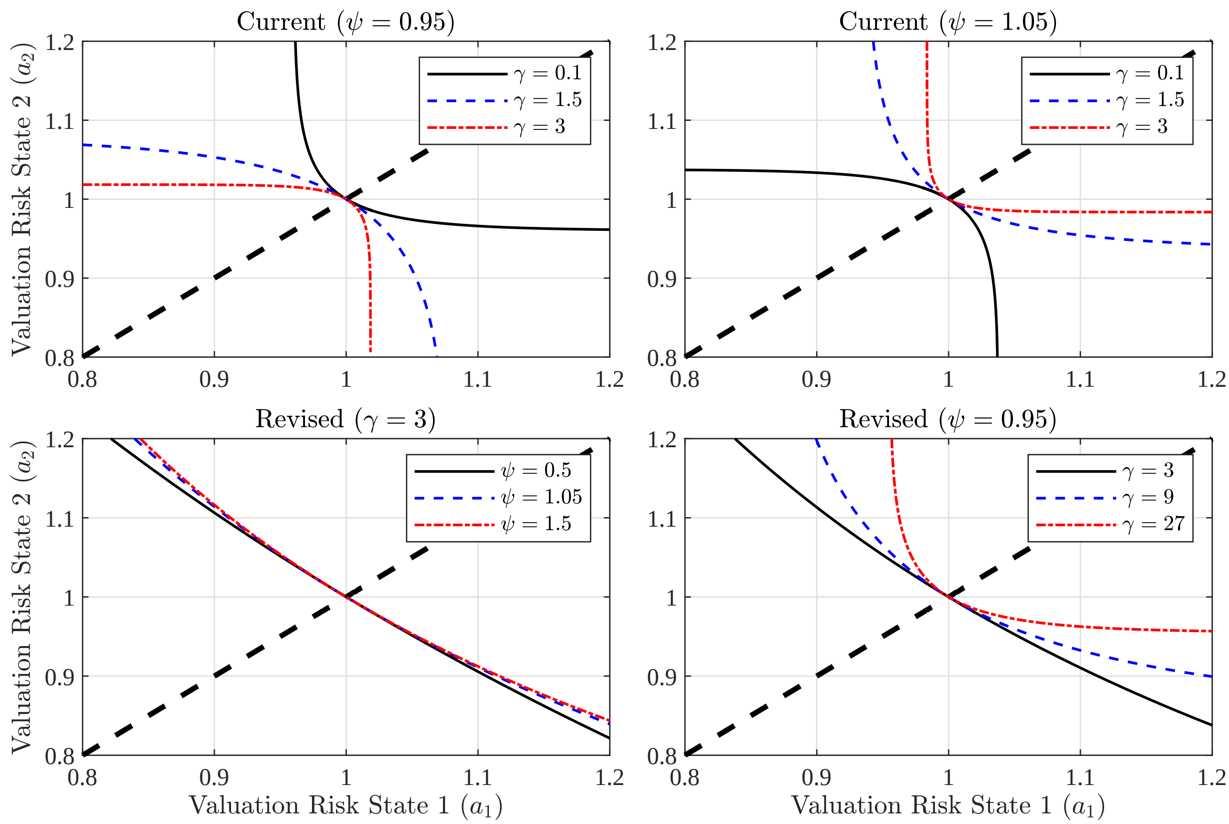

Figure 1: State-space indifference curves. We set $\beta = 0.9975$ and $\Delta = 1.0015$.

Result 4 states that under the current specification, risk aversion is very sensitive to the calibration of the IES. This is concerning since Epstein-Zin-type preferences are designed to separate risk attitudes from timing attitudes. Under the current specification, curvature and hence risk attitudes are primarily determined by the IES parameter. The revised specification resolves this problem.

One source of intuition is to examine an alternative version of the current specification given by

$$W^A(x, y, a_t) \equiv \left((1 - a_t\beta)x^{1-1/\psi} + \beta y^{1-1/\psi}\right)^{1/(1-1/\psi)}, \tag{3A}$$

where a_t only appears in the first position. A priori, if we accept the current specification, then (3A) should be an acceptable alternative. The curvature of the alternative specification is given by $\mathcal{A}^A = -\left(\frac{\gamma-1/\psi}{1-1/\psi}\right)\frac{\beta}{1-\beta}\left(1 - \beta\Delta^{1-1/\psi}\right)$, which has almost the exact opposite properties as $\mathcal{A}^C$ because the preferences become extremely convex with respect to valuation risk as $\psi \to 1^+$ and extremely concave as $\psi \to 1^-$. Since $\mathcal{A}^R = \mathcal{A}^C + \mathcal{A}^A$, the extreme curvature observed in both the current and alternative specifications broadly cancel out under the revised specification.[11]

Result 5. *Suppose $\gamma = 1 - \epsilon$ and $1 - 1/\psi = \epsilon^2$. As $\epsilon \to 0$, the current specification violates Property 3, whereas the revised specification converges to $U_t = (1 - a_t\beta)\log c_t + a_t\beta E_t U_{t+1}$.*

[11] Appendix B shows (3A) is isomorphic to (3C) with a small change in the timing of the discount factor shock.

Result 5 summarizes our investigation of Property 3 under valuation risk. If we begin with log-preferences and introduce discount factor shocks, then $U_t = (1-a_t\beta)\log(c_t)+a_t\beta E_t U_{t+1}$ or $U_t = \log(c_t) + a_t\beta E_t U_{t+1}$ and there is no curvature with respect to valuation risk ($\mathcal{A} = 0$). Therefore, when $\gamma = \psi = 1$, Epstein-Zin preferences under valuation risk should always reduce to one of these utility functions and the stochastic discount factor should reduce to $m_{t+1} \equiv a_t\beta(\frac{1-a_{t+1}\beta}{1-a_t\beta})\frac{c_t}{c_{t+1}}$ or $m_{t+1} \equiv a_t\beta\frac{c_t}{c_{t+1}}$. We show in Appendix C that this occurs under the revised specification, but *not* under the current specification when ψ approaches 1 at a faster rate than γ. Furthermore, suppose we calculate the limit as $\epsilon \to 0$, assuming $\gamma = 1-\epsilon$ and $1-1/\psi = \epsilon^2$ to ensure ψ converges to 1 at a faster rate than γ. The current specification still exhibits extreme curvature with respect to valuation risk even though both γ and ψ become arbitrarily close to 1 as in the log-preference case.

3 Consequences for Asset Pricing

Thus far, we have described the alternative valuation risk specifications in terms of properties related to the curvature of the utility function. This section applies these ideas to asset pricing moments using our baseline asset pricing model and analyzes their consequences for Property 4.

3.1 Baseline Asset-Pricing Model This section describes our baseline model with *i.i.d.* cash-flow risk. Later sections will introduce richer features into the model. Each period t denotes 1 month. There are two assets: an endowment share, $s_{1,t}$, that pays income, y_t, and is in fixed unit supply, and an equity share, $s_{2,t}$, that pays dividends, d_t, and is in zero net supply. A representative household chooses $\{c_t, s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$ to maximize utility (1) with time aggregator (3C) or (3R).[12]

The representative household's choices are constrained by the flow budget constraint given by

$$c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} = (p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}, \tag{10}$$

where $p_{y,t}$ and $p_{d,t}$ are the endowment and dividend claim prices. The optimality conditions imply

$$E_t[m^j_{t+1}r_{y,t+1}] = 1, \quad r_{y,t+1} \equiv (p_{y,t+1} + y_{t+1})/p_{y,t}, \tag{11}$$

$$E_t[m^j_{t+1}r_{d,t+1}] = 1, \quad r_{d,t+1} \equiv (p_{d,t+1} + d_{t+1})/p_{d,t}, \tag{12}$$

where $r_{y,t+1}$ and $r_{d,t+1}$ are the gross returns on the endowment and dividend claims, and

$$m^C_{t+1} \equiv a^C_t\beta\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\left(\frac{(U^C_{t+1})^{1-\gamma}}{\mu_t(U^C_{t+1})}\right)^{1/\psi-\gamma}, \tag{13}$$

$$m^R_{t+1} \equiv a^R_t\beta\left(\frac{1-a^R_{t+1}\beta}{1-a^R_t\beta}\right)\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\left(\frac{(U^R_{t+1})^{1-\gamma}}{\mu_t(U^R_{t+1})}\right)^{1/\psi-\gamma}. \tag{14}$$

[12] Kollmann (2016) introduces a time-varying discount factor into Epstein-Zin preferences in similar way as our revised specification. In that setup, however, the discount factor is a function of endogenously determined consumption.

To permit an approximate analytical solution, we rewrite the optimality conditions as follows

$$E_t[\exp(\hat{m}^j_{t+1} + \hat{r}_{y,t+1})] = 1, \tag{15}$$
$$E_t[\exp(\hat{m}^j_{t+1} + \hat{r}_{d,t+1})] = 1, \tag{16}$$

where a hat denotes a log variable. The log stochastic discount factor is given by

$$\hat{m}^j_{t+1} = \theta \log \beta + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{c}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}, \tag{17}$$

where $\theta \equiv (1-\gamma)/(1-1/\psi)$. The second term captures the direct effect of valuation risk on the stochastic discount factor, where $\omega^C = 0$, $\omega^R = \beta$, and $\hat{a}_t \equiv \hat{a}^C_t \approx \hat{a}^R_t/(1-\beta)$. Valuation risk also has an indirect effect through the return on the endowment. The log preference shock, $\hat{a}_{t+1}$, follows

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1},\ \varepsilon_{a,t+1} \sim \mathbb{N}(0,1), \tag{18}$$

where $0 \leq \rho_a < 1$ is the persistence of the process and $\sigma_a \geq 0$ is the shock standard deviation. We apply a linear approximation to the asset returns following Campbell and Shiller (1988) to obtain

$$\hat{r}_{y,t+1} = \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}, \tag{19}$$
$$\hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}, \tag{20}$$

where $\hat{z}_{y,t+1}$ is the log price-endowment ratio, $\hat{z}_{d,t+1}$ is the log price-dividend ratio, and

$$\kappa_{y0} \equiv \log(1+\exp(\hat{z}_y)) - \kappa_{y1}\hat{z}_y, \quad \kappa_{y1} \equiv \exp(\hat{z}_y)/(1+\exp(\hat{z}_y)), \tag{21}$$
$$\kappa_{d0} \equiv \log(1+\exp(\hat{z}_d)) - \kappa_{d1}\hat{z}_d, \quad \kappa_{d1} \equiv \exp(\hat{z}_d)/(1+\exp(\hat{z}_d)), \tag{22}$$

are constants that are functions of the steady-state price-endowment and price-dividend ratios.

To close the model, the processes for log-endowment and log-dividend growth are given by

$$\Delta\hat{y}_{t+1} = \mu_y + \sigma_y \varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{23}$$
$$\Delta\hat{d}_{t+1} = \mu_d + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{24}$$

where μ_y and μ_d are the steady-state growth rates, $\sigma_y \geq 0$ and $\psi_d\sigma_y \geq 0$ are the shock standard deviations, and π_{dy} determines the covariance between consumption and dividend growth. At this point, cash-flow growth is *i.i.d.* Later sections will introduce other empirically relevant features.

The asset market clearing conditions imply $s_{1,t} = 1$ and $s_{2,t} = 0$, so the resource constraint is $\hat{c}_t = \hat{y}_t$. Equilibrium includes sequences of prices $\{\hat{m}_{t+1}, \hat{z}_{y,t}, \hat{z}_{d,t}, \hat{r}_{y,t+1}, \hat{r}_{d,t+1}\}_{t=0}^{\infty}$, quantities $\{\hat{c}_t\}_{t=0}^{\infty}$, and exogenous variables $\{\Delta\hat{y}_{t+1}, \Delta\hat{d}_{t+1}, \hat{a}_{t+1}\}_{t=0}^{\infty}$ that satisfy (15)-(20), (23), (24), and the resource constraint, given the state of the economy, $\{\hat{a}_0\}$, and shock sequences, $\{\varepsilon_{y,t}, \varepsilon_{d,t}, \varepsilon_{a,t}\}_{t=1}^{\infty}$.

We posit the following solutions for the price-endowment and price-dividend ratios:

$$\hat{z}_{y,t} = \eta_{y0} + \eta_{y1}\hat{a}_t, \quad \hat{z}_{d,t} = \eta_{d0} + \eta_{d1}\hat{a}_t, \tag{25}$$

where $\hat{z}_y = \eta_{y0}$ and $\hat{z}_d = \eta_{d0}$. We solve the model with the method of undetermined coefficients. Appendix D derives the SDF, a Campbell-Shiller approximation, the solution, and key asset prices.

3.2 Asset Pricing Moments

We begin with a brief discussion of the asset pricing implications of the model without valuation risk. In particular, we review how Epstein-Zin preferences, by separating risk attitudes from timing attitudes, aid in matching the risk-free rate and equity premium. We then compare these moments under the current and revised valuation risk preferences.

3.2.1 Conventional Model

In the original Epstein-Zin preferences, there is no valuation risk ($\sigma_a = 0$). If, for simplicity, we further assume endowment and dividend risks are perfectly correlated ($\psi_d = 0$; $\pi_{dy} = 1$), then the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((1/\psi - \gamma)(1-\gamma) - \gamma^2)\sigma_y^2/2, \tag{26}$$

$$E[ep] = \gamma\sigma_y^2, \tag{27}$$

where the first term in (26) is the subjective discount factor, the second term accounts for endowment growth, and the third term accounts for precautionary savings. Endowment growth creates an incentive for households to borrow in order to smooth consumption. Since both assets are in fixed supply, the risk-free rate must be elevated to deter borrowing. When the IES, ψ, is high, households are willing to accept higher consumption growth so the interest rate required to dissuade borrowing is lower. Therefore, the model requires a fairly high IES to match the low risk-free rate in the data.

With CRRA preferences, higher RA lowers the IES and pushes up the risk-free rate. With Epstein-Zin preferences, these parameters are independent, so a high IES can lower the risk-free rate without lowering RA. The equity premium only depends on RA. Therefore, the model generates a low risk-free rate and modest equity premium with sufficiently high RA and IES parameter values. Of course, there is an upper bound on what constitute reasonable RA and IES values, which is the source of the risk-free rate and equity premium puzzles. Other prominent model features such as long-run risk and stochastic volatility à la Bansal and Yaron (2004) help resolve these puzzles.

3.2.2 Valuation Risk Model Comparison

We now turn to the model with valuation risk. Figure 2 plots the average risk-free rate, the average equity premium, and κ_1 (i.e., the marginal response of the price-dividend ratio on the equity return) under both preference specifications. For simplicity, we remove cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and assume the time preference shocks are *i.i.d.* ($\rho_a = 0$). Under these assumptions, the assets are identical so $(\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}) =$

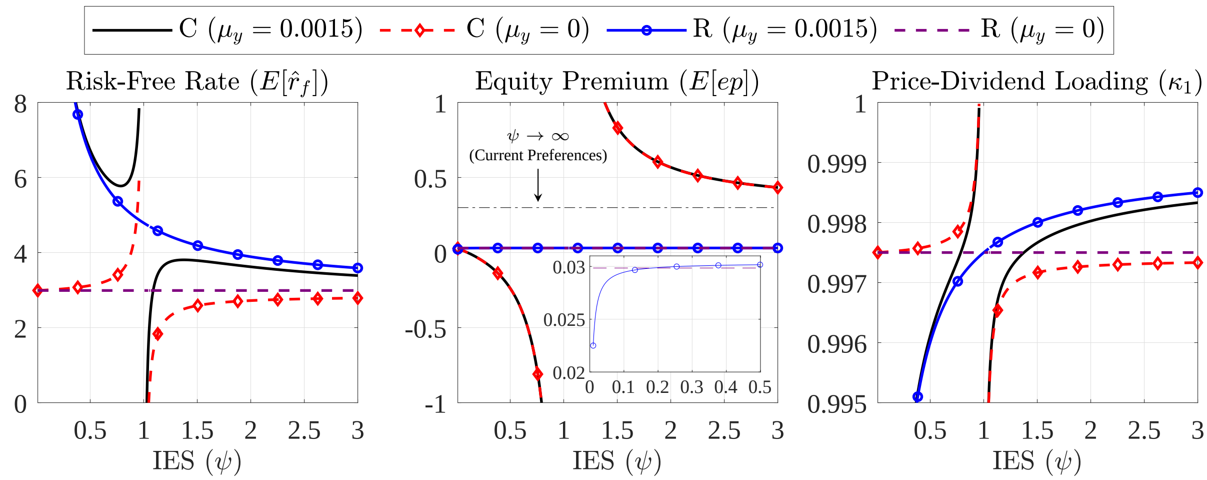

Figure 2: Equilibrium outcomes in the model without cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and *i.i.d.* preference shocks ($\rho_a = 0$) under the current (C) and revised (R) preference specifications. We set $\beta = 0.9975$, $\gamma = 10$, and $\sigma_a = 0.005$.

$(\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}) \equiv (\kappa_0, \kappa_1, \eta_0, \eta_1)$. We plot the results with and without cash-flow growth (μ_y).

In Figure 2, the current preferences are given by the solid-black (positive endowment growth) and red-diamond (no endowment growth) lines. In both cases, the average risk-free rate and average equity premium exhibit a vertical asymptote when the IES is 1. The risk-free rate approaches positive infinity as the IES approaches 1 from below and negative infinity as the IES approaches 1 from above. The equity premium has the same comparative statics with the opposite sign, except there is a horizontal asymptote as the IES approaches infinity. These results occur because of the extreme curvature of the utility function when ψ is close to 1 as described in the previous section.[13]

Analytics provide similar insights. The average risk-free rate and equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + (\theta - 1)\kappa_1^2\eta_1^2\sigma_a^2/2, \tag{28}$$

$$E[ep] = (1 - \theta)\kappa_1^2\eta_1^2\sigma_a^2, \tag{29}$$

and the log-price-dividend ratio is given by $\hat{z}_t = \eta_0 + \hat{a}_t$ (i.e., the loading on the preference shock, η_1, is 1). Therefore, when the household becomes more patient and $\hat{a}_t$ rises, the price-dividend ratio rises one-for-one on impact and returns to the stationary equilibrium in the next period. Since η_1 is independent of the IES, there is no endogenous mechanism that prevents the asymptote in θ from influencing the risk-free rate or equity premium. Since $0 < \kappa_1 < 1$, θ dominates both of these moments when the IES is near 1. The following result describes the comparative statics with the IES.

Result 6. *Suppose $\gamma > 1$. The current preferences violate Property 4. As $\psi \to 1^+$, $\theta \to -\infty$, so $E[\hat{r}_f] \to -\infty$ while $E[ep] \to +\infty$. As $\psi \to 1^-$, $\theta \to +\infty$, so $E[\hat{r}_f] \to +\infty$ while $E[ep] \to -\infty$.*

[13]Pohl et al. (2018) find the errors from a Campbell-Shiller approximation of the nonlinear model can significantly affect equilibrium outcomes. Appendix E proves that the vertical asymptote also occurs in the fully nonlinear model.

Therefore, small and reasonable changes in the value of the IES (e.g., from 0.99 to 1.01) can result in dramatic changes in the predicted values of the average risk free rate and average equity premium. It also illustrates why valuation risk seems like such an attractive feature for resolving the risk-free rate and equity premium puzzles. As the IES tends to 1 from above, θ becomes increasingly negative, which dominates other determinants of the risk-free rate and equity premium. In particular, with an IES slightly above 1, the asymptote in θ causes the average risk-free rate to become arbitrarily small, while making the average equity premium arbitrarily large. Bizarrely, an IES marginally below 1 (a popular value in the macro literature), generates the opposite predictions. As the IES approaches infinity, $1-\theta$ tends to γ. Therefore, even when the IES is far above 1, the last term in (28) and (29) is scaled by γ and can still have a meaningful effect on asset prices.

In Figure 2, the revised preferences are given by circle-blue (positive endowment growth) and dashed-black (no endowment growth) lines. In both cases, the average risk-free rate and average equity premium are continuous in the IES, regardless of μ_y. When $\mu_y = 0$, the endowment stream is constant. This means the household is indifferent about the timing of when the preference uncertainty is resolved, so both κ_1 and the average equity premium are independent of the IES. When $\mu_y > 0$, the household's incentive to smooth consumption interacts with uncertainty about how it will value the higher future endowment stream.[14] When the IES is large, the household has a stronger preference for an early resolution of uncertainty, so the equity premium rises as a result of the valuation risk (see the Figure 2 inset). Therefore, the qualitative relationship between the IES and the equity premium has different signs under the current and revised specifications. Moreover, the increase in the equity premium is quantitatively small and converges to a level well below the value with the current preferences. It is this difference in the sign and magnitude of the relationship between the IES and the average equity premium that will explain many of our empirical results.

In this case, the expressions for the average risk-free rate and equity risk premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((\theta-1)\kappa_1^2\eta_1^2 - \theta\beta^2)\sigma_a^2/2, \tag{30}$$

$$E[ep] = ((1-\theta)\kappa_1\eta_1 + \theta\beta)\kappa_1\eta_1\sigma_a^2. \tag{31}$$

Relative to the current specification, η_1, is unchanged.[15] However, both asset prices include a new term that captures the effect of valuation risk on current utility, so a rise in a_t that makes the household more patient raises the value of future certainty equivalent consumption and lowers the value of present consumption. The asymptote occurs under the current specification because it does not account for the effect of valuation risk on current-period consumption. With the revised

[14] Andreasen and Jørgensen (2019) show how to decouple the household's timing attitude from the RA and IES.

[15] Notice κ_1 is a function of the steady-state price-dividend ratio, z_d. When the IES is 1, $z_d = \beta/(1-\beta)$, which is equivalent to its value absent any risk. Therefore, when the IES is 1, valuation risk has no effect on the price-dividend ratio. This result points to a connection with income and substitution effects, which usually cancel when the IES is 1.

preferences, $\kappa_1 = \beta$ when $\psi = 1$, so the terms involving θ cancel out and the asymptote disappears.

Result 7. *The revised preferences satisfy Property 4, as $E[\hat{r}_f]$ and $E[ep]$ are continuous in ψ.*

When $\psi = 1$, valuation risk lowers the average risk-free rate by $\beta^2\sigma_a^2/2$ and raises the average equity return by the same amount. Therefore, the average equity premium equals $\beta^2\sigma_a^2$, which is invariant to the RA parameter. When $\psi > 1$, $\kappa_1 > \beta$, so an increase in RA lowers the risk-free rate and raises the equity return. As $\psi \to \infty$, the equity premium with the revised specification relative to the current specification equals $1 + \beta(1-\gamma)/(\gamma\kappa_1)$. This means the disparity between the predictions of the two models grows as RA increases. As a consequence, the revised preferences would require much larger RA to generate the same equity premium as the current preferences.

3.3 FURTHER DISCUSSION

The previous section shows the current and revised preferences generate different predictions. This section covers two miscellaneous questions readers may have.

Question 1: Is the valuation risk specification under CRRA preferences important?

Since we have demonstrated that the valuation risk specification is important under Epstein-Zin preferences, it is worth addressing whether the same is true under CRRA preferences. In particular, is the choice between $U_t = u(c_t) + a_t\beta E_t U_{t+1}$ and $U_t = (1 - a_t\beta)u(c_t) + a_t\beta E_t U_{t+1}$ important? In terms of first-order dynamics, both specifications generate the same impulse response functions with an appropriate rescaling of σ. The rescaling is by the factor $1 - \rho_a\beta$, where ρ_a is unchanged across the specifications. There is a numerically small difference in $E[\hat{r}_f]$ and $E[ep]$, which is easy to see by setting $\theta = 1$ in equations (28)-(31). This stems from the conditional expectation of a_{t+1}.

Question 2: Are the revised preferences the only viable alternative?

A potential alternative to the revised specification is the following:

$$V_t = W(c_t, a_t\mu_t) = [c_t^{1-1/\psi} + \beta(a_t\mu_t)^{1-1/\psi}]^{1/(1-1/\psi)}. \tag{32}$$

We refer to this specification as "disaster risk" preferences following Gourio (2012). That paper shows how a term like a_t can arise endogenously in a production economy asset pricing model.

Technically, since the disaster risk shock affects the certainty equivalent of future utility and does not alter the time-aggregator, these preferences are consistent with the four desirable properties described in Section 2. However, they do not represent a household's intrinsic time preference uncertainty. To appreciate why, once again set $\gamma = 1/\psi = 1$, giving $V_t = \log c_t + \log(a_t) + E_t V_{t+1}$. The model reduces to time-separable log-preferences with an additive shock term. As a result a_t disappears from any equilibrium condition, so the disaster risk preferences are not able to capture an exogenous change in the household's impatience, even though there is no plausible reason why a household with time-separable log-preferences cannot become more or less patient over time. This means valuation risk must be linked to time-variation in the discount factor, as in (5) and (6).

4 Data and Estimation Methods

We construct our data using the procedure in Bansal and Yaron (2004), Beeler and Campbell (2012), Bansal et al. (2016), and Schorfheide et al. (2018). The moments are based on seven time series from 1929 to 2017: real per capita consumption expenditures on nondurables and services, the real equity return, real dividends, the real risk-free rate, the price-dividend ratio, and the real 5- and 20-year U.S. Treasury yields. Nominal equity returns are calculated with the CRSP value-weighted return on stocks. We obtain data with and without dividends to back out a time series for nominal dividends. Both series are converted to real series using the consumer price index (CPI).

The nominal risk-free rate is based on the CRSP yield-to-maturity on 90-day Treasury bills, and the intermediate and long-term nominal Treasury yields are available on Morningstar Direct (formerly Ibbotson Associates). We first convert the nominal time series to a real series using the CPI. Then we construct an *ex-ante* real rate by regressing the *ex-post* real rate on the nominal rate and inflation over the last year. The consumption data is annual. To match this frequency, the monthly asset pricing data are converted to annual time series using the last month of each year.

Using the annual time series, our target moments, $\hat{\Psi}_T^D$, are estimated with a two-step Generalized Method of Moments (GMM) estimator, where $T = 87$ is the sample size.[16] Given the GMM estimates, the model is estimated with Simulated Method of Moments (SMM). For parameterization θ and shocks $\mathcal{E}$, we solve the model and simulate it $R = 1{,}000$ times for T periods. The model-implied analogues of the target moments are the median moments across the R simulations, $\bar{\Psi}_{R,T}^M(\theta, \mathcal{E})$. The parameter estimates, $\hat{\theta}$, are obtained by minimizing the following loss function:

$$J(\theta, \mathcal{E}) = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\theta, \mathcal{E})]'[\hat{\Sigma}_T^D(1 + 1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\theta, \mathcal{E})],$$

where $\hat{\Sigma}_T^D$ is the diagonal of the GMM estimate of the variance-covariance matrix.[17] We use Monte Carlo methods to calculate the standard errors on the parameter estimates. For different sequences of shocks, we re-estimate the structural model $N_s = 500$ times and report the mean and $(5, 95)$ percentiles. Appendix F and Appendix G provide more details about our data and estimation method.

The baseline model targets 15 moments: the means and standard deviations of consumption growth, dividend growth, equity returns, the risk-free rate, and the price-dividend ratio, the correlation between dividend growth and consumption growth, the autocorrelations of the price-dividend ratio and risk-free rate, and the cross-correlations of consumption growth, dividend growth, and equity returns. These targets are common in the literature and the same as Albuquerque et al. (2016), except we exclude 5- and 10-year correlations between equity returns and cash-flow growth. We omit the long-run correlations to allow a longer sample that includes the Great Depression period.

[16] In total, there are 89 periods in our sample, but we lose one period for growth rates and one for serial correlations.

[17] For the revised preferences, we impose the restriction $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$ when estimating the model parameters. This ensures the time-aggregator weights are positive in 99.997% of the simulated observations.

Many elements of our estimation procedure are common in the literature. We use a limited information approach to match empirical targets and SMM to account for short-sample bias that occurs because asset pricing models often have very persistent processes. To improve on the current methodology, we repeat the estimation procedure for different shock sequences. The estimations are run in parallel on a supercomputer. The literature typically estimates models once based on a particular seed and uses the Delta method to compute standard errors. While our approach has a higher computational burden, our estimates are independent of the seed and have more precise standard errors. The estimates allow us to numerically approximate the sampling distribution of the parameters and test whether they are significantly different across models. We also obtain a distribution of J values, which determine whether a model provides a significant improvement in fit over another model, and the corresponding p-values from a test of over-identifying restrictions.

5 ESTIMATED BASELINE MODEL

This section takes the baseline model from Section 3.1 and compares the estimates from the current and revised preference specifications. We fix the IES to 2.5, which is near the upper end of the plausible range of values in the literature.[18] This restriction helps us compare the estimates from the two preference specifications because the model fit, as measured by the J value, is insensitive to the value of the IES in the revised specification, but the unconstrained global minimum prefers an implausibly high IES. For example, the J value is only one decimal point lower with an IES equal to 10. Therefore, we are left with estimating nine parameters to match 17 empirical targets.

Table 1 shows the parameter estimates and Table 2 reports the data and model-implied moments for six variants of our baseline model: with and without targeting the yield curve (5- and 20-year average risk-free bond yields); with the current preferences; and with the revised preferences, with and without an upper bound on RA. For each parameter, we report the average and $(5, 95)$ percentiles across 500 estimations of the model. For each moment, we provide the mean and t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart.

We begin with the model that excludes the yield curve moments. In both preference specifications, the data prefers a very persistent valuation risk process with $\rho_a > 0.98$. In the current specification, the risk aversion parameter, γ, is 1.55. In the revised specification $\gamma = 74.23$, which is well outside what is considered acceptable in the asset pricing literature.[19] Both specifications generate a sizable equity premium (the estimates are about 1% lower than the empirical equity

[18] Estimation results with $\psi = 1.5$ and $\psi = 2.0$ for each specification considered below are included in Appendix H. In total, we estimate 54 variants of our model. Since each variant is estimated 500 times, there are $27{,}000$ estimations. The estimations are run in Fortran and the time per estimation ranges from 1-24 hours depending on model complexity.

[19] Mehra and Prescott (1985, p. 154) say "Any of the above cited studies... constitute an *a priori* justification for restricting the value of [RA] to be a maximum of ten, as we do in this study." Weil (1989, p. 411) describes $\gamma = 40$ as "implausibly" high. Swanson (2012) shows γ does not equate to risk aversion when households have a labor margin. Therefore, only in production economies can γ be reasonably above 10, where it is common to see values around 100.

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.55 (1.52, 1.58)	74.23 (70.95, 77.47)	10.00 (10.00, 10.00)	1.38 (1.35, 1.41)	98.17 (93.29, 103.03)	10.00 (10.00, 10.00)
β	0.9977 (0.9976, 0.9978)	0.9957 (0.9956, 0.9957)	0.9973 (0.9972, 0.9973)	0.9979 (0.9977, 0.9980)	0.9964 (0.9963, 0.9964)	0.9978 (0.9977, 0.9978)
ρ_a	0.9968 (0.9965, 0.9971)	0.9899 (0.9896, 0.9902)	0.9879 (0.9876, 0.9882)	0.9973 (0.9970, 0.9976)	0.9893 (0.9890, 0.9896)	0.9878 (0.9875, 0.9881)
σ_a	0.00031 (0.00030, 0.00033)	0.03547 (0.03491, 0.03596)	0.03880 (0.03832, 0.03927)	0.00028 (0.00027, 0.00030)	0.03653 (0.03597, 0.03703)	0.03891 (0.03845, 0.03939)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0016)	0.0021 (0.0020, 0.0021)	0.0010 (0.0009, 0.0010)	0.0010 (0.0010, 0.0011)	0.0016 (0.0016, 0.0017)	0.0005 (0.0004, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0058 (0.0057, 0.0059)	0.0058 (0.0057, 0.0060)	0.0058 (0.0058, 0.0058)	0.0056 (0.0054, 0.0057)	0.0060 (0.0059, 0.0061)
ψ_d	1.54 (1.43, 1.64)	0.97 (0.87, 1.07)	1.09 (0.97, 1.19)	1.52 (1.42, 1.61)	1.13 (1.04, 1.22)	1.02 (0.93, 1.13)
π_{dy}	0.815 (0.764, 0.872)	0.436 (0.400, 0.472)	0.617 (0.562, 0.674)	0.816 (0.759, 0.873)	0.613 (0.581, 0.639)	0.601 (0.546, 0.662)
J	29.27 (28.62, 29.98)	47.98 (47.62, 48.35)	55.55 (54.93, 56.11)	31.73 (31.05, 32.43)	49.99 (49.60, 50.41)	59.36 (58.85, 59.86)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table 1: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

premium) and a near zero risk-free rate. However, they significantly under-predict the standard deviation of dividend growth and over-predict the autocorrelation of the risk-free rate in the data.[20]

Using the analytical expressions for the average risk-free rate and equity premium (see (D.15) and (D.16) in Appendix D), it is possible to break down the fraction of each moment explained by cash-flow and valuation risk.[21] With the current specification valuation risk explains 98.9% and 99.2% of the risk-free rate and the equity premium, whereas with the revised preferences it explains only 63.1% and 79.0%. Since the estimate of the cash-flow shock standard deviation is unchanged, cash-flow risk has a bigger role in explaining the equity premium due to higher RA.

The revised specification has a significantly poorer fit than the current specification ($J = 48.0$ vs. $J = 29.3$), although both specifications fail the over-identifying restrictions test.[22] The poorer fit is mostly due to the model significantly over-predicting the volatility of the risk-free rate and

[20]The estimate of the valuation risk shock standard deviation, σ_a, is two orders of magnitude larger in the revised specification than the current specification. Recall that the valuation risk term in the SDF is given by $\hat{a}_t - \omega\hat{a}_{t+1}$. When the valuation risk shock is *i.i.d.*, the estimates of the shock standard deviation are very similar. However, as the persistence increases with the revised preferences, $SD_t[\hat{a}_t - \omega\hat{a}_{t+1}]$ shrinks, so σ_a rises to compensate for the extra term.

[21]The mean risk-free rate is given by $E[\hat{r}_{f,t}] = \alpha_1 + \alpha_2\sigma_a^2 + \alpha_3\sigma_y^2$ and the mean equity premium is given by $E[ep_t] = \alpha_4\sigma_a^2 + \alpha_5\sigma_y^2$ for some function of model parameters α_i, $i \in \{1, \dots, 5\}$. Therefore, the contribution of valuation risk to the risk-free rate and equity premium is given by $\alpha_2\sigma_a^2/(\alpha_2\sigma_a^2 + \alpha_3\sigma_y^2)$ and $\alpha_4\sigma_a^2/(\alpha_4\sigma_a^2 + \alpha_5\sigma_y^2)$.

[22]The test statistic is given by $\hat{J}^s = J(\hat{\theta}, \mathcal{E}^s)$, where $\mathcal{E}^s$ is a matrix of shocks given seed s. $J(\hat{\theta}, \mathcal{E})$ converges to a χ^2 distribution with $N_m - N_p$ degrees of freedom, where N_m is the number of empirical targets and N_p is the number of estimated parameters. The $(5, 95)$ percentiles of the p-values determine whether a model reliably passes the test.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.18)	2.01 (0.49)	1.89 (0.00)	1.98 (0.36)	1.95 (0.25)
$E[\Delta d]$	1.47	1.84 (0.38)	2.47 (1.04)	1.17 (−0.32)	1.25 (−0.23)	1.98 (0.52)	0.58 (−0.93)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.47)	3.56 (1.02)	3.49 (0.48)	3.52 (0.74)	3.60 (1.27)
$E[r_d]$	6.51	5.46 (−0.66)	5.59 (−0.58)	4.06 (−1.53)	4.78 (−1.08)	4.98 (−0.96)	3.37 (−1.96)
$E[r_f]$	0.25	0.25 (0.00)	0.36 (0.18)	1.06 (1.32)	0.09 (−0.26)	0.26 (0.00)	0.41 (0.26)
$E[r_{f,5}]$	1.19	1.25 (0.09)	1.76 (0.83)	2.18 (1.46)	0.92 (−0.40)	1.23 (0.05)	1.50 (0.46)
$E[r_{f,20}]$	1.88	3.19 (2.18)	3.49 (2.69)	3.33 (2.42)	2.57 (1.16)	2.29 (0.68)	2.63 (1.25)
$SD[\Delta c]$	1.99	1.99 (0.00)	2.00 (0.01)	2.00 (0.02)	2.00 (0.01)	1.92 (−0.16)	2.07 (0.16)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.13 (−3.28)	2.49 (−3.14)	3.44 (−2.80)	2.46 (−3.15)	2.45 (−3.16)
$SD[r_d]$	19.15	18.41 (−0.39)	13.65 (−2.90)	13.44 (−3.01)	18.47 (−0.36)	13.47 (−3.00)	13.11 (−3.18)
$SD[r_f]$	2.72	3.21 (0.96)	3.69 (1.92)	3.86 (2.25)	2.99 (0.53)	3.70 (1.92)	3.76 (2.04)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.16)	0.23 (−3.49)	0.48 (0.47)	0.24 (−3.32)	0.22 (−3.59)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.36)	0.88 (3.14)	0.95 (4.17)	0.89 (3.29)	0.88 (3.13)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.85)	0.83 (−1.30)	0.93 (0.75)	0.84 (−1.00)	0.83 (−1.32)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.32)	0.41 (−0.59)	0.50 (−0.19)	0.48 (−0.29)	0.48 (−0.27)	0.51 (−0.13)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)	0.09 (0.57)	0.09 (0.56)	0.09 (0.66)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.38)	0.18 (1.38)	0.18 (1.35)	0.19 (1.41)

Table 2: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

under-predicting the volatilities of the price-dividend ratio and equity return. The intuition is as follows. In the revised specification, risk-free rate volatility is relatively more sensitive to valuation risk than equity return volatility. Since the volatility of equity returns is higher than the volatility of the risk-free rate in the data, valuation risk alone does not allow the model to match these moments. Dividend growth volatility, however, cannot rise to compensate for the lack of the equity return volatility because the target correlation between equity returns and dividend growth is near zero.

The revised preferences not only have a worse fit, but the risk aversion parameter is implausibly large. When we restrict γ to a maximum of 10—the upper end of the values used in the asset pricing literature—the fit deteriorates further ($J = 55.6$ vs. 48.0). The primary source of the poorer fit is the larger estimate of the risk-free rate (1.1% vs. 0.4%) and lower equity return (4.1% vs. 5.6%).

Intuition suggests that valuation risk should also be informative about the long-term risk-free interest rates, not just the short-term rate. When longer-term moments are omitted from the estimation routine, both preferences over-predict the slope of the yield curve ($E[r_{f,20}] - E[r_f]$ is

2.9% and 3.1% for the current and revised preferences, relative to the 1.6% in the data). Once the yield curve moments are included, however, the slopes fall to 2.4% and 2.0%, respectively. For the revised preferences, this flattening of the yield curve is generated by a rise in RA. Overall, the inclusion of these moments worsens the fit of the model but does not materially change the results.

To summarize, our results demonstrate that introducing valuation risk to the baseline model in its revised form does not resolve the equity premium and risk-free rate puzzles. The rest of the paper examines whether revised valuation risk has a significant role in richer asset pricing models.

6 ESTIMATED LONG-RUN RISK MODEL

Long-run risk provides a well-known resolution to many asset pricing puzzles. This section introduces this feature into our baseline model and re-examines the marginal contribution of valuation risk with the revised preferences. To introduce long-run risk, we modify (23) and (24) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon_{y,t+1},\ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{33}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_{dy}\sigma_y \varepsilon_{y,t+1} + \psi_d \sigma_y \varepsilon_{d,t+1},\ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{34}$$

$$\hat{x}_{t+1} = \rho_x \hat{x}_t + \psi_x \sigma_y \varepsilon_{x,t+1},\ \varepsilon_{x,t+1} \sim \mathbb{N}(0,1), \tag{35}$$

where the specification of the persistent component, $\hat{x}_t$, follows Bansal and Yaron (2004). We apply the same estimation procedure as the baseline model, except there are three additional parameters, ϕ_d, ρ_x, and ψ_x. We also match up to five additional moments: the autocorrelations of consumption growth, dividend growth, and the equity return and two predictability moments—the correlations of consumption growth and the equity premium with the lagged price-dividend ratio.

The long-run risk model also prefers a high IES even though it does not significantly lower the J value. As a result, we continue to set the IES to 2.5 and estimate the remaining parameters. The parameter estimates are shown in Table 3 and the data and model-implied moments are reported in Table 4. The tables show the results for six variants of the model: with and without targeting both the yield curve and higher-order risk-free rate moments; with and without targeting the yield curve but always including higher-order risk-free rate moments; and with and without valuation risk.

We begin with the model without valuation risk and without the yield curve and risk-free rate moments (column 1). This is a typical model estimated in the literature. The model fails to pass the over-identifying restrictions test at the 5% level, signalling that the standard long-run risk model is insufficient to adequately describe the behavior of asset prices and cash flows. The parameter estimates are similar to the estimates in the literature. In particular, the data requires a small but very persistent shock that generates risk in long-run cash-flow growth ($\rho_x = 0.9988$; $\psi_x = 0.0260$).

The literature typically excludes the standard deviation and autocorrelation of the risk-free rate when estimating the long-run risk model because the model does not generate sufficient volatility

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.58 $(2.31, 2.84)$	2.63 $(2.36, 2.93)$	2.70 $(2.41, 2.96)$	2.54 $(2.25, 2.83)$	2.51 $(2.14, 2.84)$	2.33 $(1.99, 2.70)$
β	0.9990 $(0.9989, 0.9991)$	0.9980 $(0.9979, 0.9982)$	0.9990 $(0.9988, 0.9991)$	0.9989 $(0.9987, 0.9990)$	0.9985 $(0.9983, 0.9986)$	0.9984 $(0.9983, 0.9986)$
ρ_a	–	0.9817 $(0.9800, 0.9835)$	–	0.9548 $(0.9531, 0.9565)$	–	0.9569 $(0.9546, 0.9592)$
σ_a	–	0.0475 $(0.0452, 0.0498)$	–	0.0167 $(0.0161, 0.0173)$	–	0.0175 $(0.0167, 0.0184)$
μ_y	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0014, 0.0017)$
μ_d	0.0013 $(0.0009, 0.0016)$	0.0013 $(0.0009, 0.0016)$	0.0014 $(0.0012, 0.0017)$	0.0013 $(0.0009, 0.0016)$	0.0013 $(0.0010, 0.0015)$	0.0011 $(0.0008, 0.0014)$
σ_y	0.0041 $(0.0040, 0.0042)$	0.0041 $(0.0039, 0.0043)$	0.0049 $(0.0048, 0.0050)$	0.0041 $(0.0040, 0.0042)$	0.0046 $(0.0045, 0.0047)$	0.0038 $(0.0036, 0.0041)$
ψ_d	3.25 $(3.02, 3.47)$	2.78 $(2.53, 3.02)$	3.05 $(2.83, 3.25)$	3.17 $(2.92, 3.41)$	3.22 $(2.97, 3.45)$	3.36 $(3.01, 3.67)$
π_{dy}	0.588 $(0.322, 0.868)$	0.812 $(0.547, 1.120)$	0.122 $(-0.200, 0.418)$	0.666 $(0.416, 0.916)$	0.206 $(-0.148, 0.515)$	0.777 $(0.500, 1.059)$
ϕ_d	2.30 $(2.07, 2.51)$	1.55 $(1.44, 1.68)$	2.15 $(1.94, 2.34)$	2.19 $(1.97, 2.43)$	2.35 $(2.06, 2.62)$	2.34 $(2.05, 2.64)$
ρ_x	0.9988 $(0.9983, 0.9992)$	0.9994 $(0.9992, 0.9995)$	0.9977 $(0.9969, 0.9985)$	0.9990 $(0.9985, 0.9994)$	0.9976 $(0.9968, 0.9985)$	0.9990 $(0.9985, 0.9994)$
ψ_x	0.0260 $(0.0247, 0.0274)$	0.0261 $(0.0248, 0.0274)$	0.0314 $(0.0292, 0.0335)$	0.0255 $(0.0242, 0.0269)$	0.0303 $(0.0281, 0.0327)$	0.0249 $(0.0234, 0.0264)$
J	20.55 $(19.80, 21.30)$	14.29 $(13.86, 14.72)$	56.48 $(55.64, 57.39)$	19.59 $(18.96, 20.27)$	63.32 $(62.50, 64.15)$	24.50 $(23.78, 25.20)$
pval	0.009 $(0.006, 0.011)$	0.027 $(0.023, 0.031)$	0.000 $(0.000, 0.000)$	0.012 $(0.009, 0.015)$	0.000 $(0.000, 0.000)$	0.006 $(0.005, 0.008)$
df	8	6	10	8	12	10

Table 3: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

(a standard deviation of 0.51 vs. 2.72 in the data) and over-predicts the autocorrelation (0.96 vs. 0.68 in the data). Even when these two moments are targeted, as shown in column 3, long-run cash-flow risk is unable to significantly improve on these moments (the standard deviation rises to 0.68 and the autocorrelation falls to 0.95). The standard long-run risk model also fairs poorly on three additional moments: (1) the standard deviation of dividend growth (too low), (2) the correlation between dividend growth and the return on equity (too high), and (3) the predictability of consumption growth (too high). All of them are significantly different from their empirical targets.

Adding valuation risk (columns 2 and 4) significantly improves the fit of the model. With the restricted set of moments, the J value declines from 20.6 to 14.3. More importantly, the p-value from the over-identifying restrictions test rises from 0.01 to 0.03, even though the valuation risk model contains two more parameters than the standard model (6 degrees of freedom instead of 8).

Unlike cash-flow risk, valuation risk directly affects the time-series properties of the risk-free rate, which makes it important to target these moments in the estimation. In column 2, the model includes valuation risk but targets neither the standard deviation nor the autocorrelation of the risk-free rate. As a result, the estimated model significantly over-predicts both moments (the standard

		Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.89 (0.00)	1.89 (0.03)	1.89 (−0.01)	1.89 (0.00)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.53 (0.06)	1.54 (0.07)	1.71 (0.25)	1.50 (0.03)	1.52 (0.05)	1.35 (−0.13)
$E[z_d]$	3.42	3.42 (0.00)	3.40 (−0.18)	3.41 (−0.07)	3.42 (0.00)	3.42 (0.00)	3.43 (0.05)
$E[r_d]$	6.51	6.33 (−0.11)	6.44 (−0.05)	5.82 (−0.43)	6.43 (−0.05)	5.58 (−0.58)	6.31 (−0.13)
$E[r_f]$	0.25	0.26 (0.01)	0.26 (0.01)	0.26 (0.01)	0.25 (0.00)	1.40 (1.88)	1.19 (1.55)
$E[r_{f,5}]$	1.19	0.11 (−1.60)	0.99 (−0.30)	0.05 (−1.69)	0.25 (−1.40)	1.24 (0.07)	1.26 (0.09)
$E[r_{f,20}]$	1.88	−0.32 (−3.65)	0.94 (−1.56)	−0.53 (−4.00)	−0.15 (−3.37)	0.83 (−1.74)	0.98 (−1.49)
$SD[\Delta c]$	1.99	1.92 (−0.14)	1.96 (−0.07)	2.40 (0.84)	1.91 (−0.16)	2.22 (0.47)	1.76 (−0.48)
$SD[\Delta d]$	11.09	5.59 (−2.01)	4.64 (−2.36)	6.38 (−1.72)	5.42 (−2.07)	6.34 (−1.74)	5.32 (−2.11)
$SD[r_d]$	19.15	18.15 (−0.53)	19.75 (0.32)	18.92 (−0.12)	18.21 (−0.50)	19.02 (−0.07)	18.25 (−0.47)
$SD[r_f]$	2.72	0.51 (−4.36)	5.44 (5.36)	0.68 (−4.03)	2.82 (0.19)	0.61 (−4.16)	2.91 (0.38)
$SD[z_d]$	0.45	0.53 (1.29)	0.46 (0.10)	0.51 (0.98)	0.52 (1.14)	0.51 (1.00)	0.52 (1.16)
$AC[\Delta c]$	0.53	0.43 (−1.07)	0.46 (−0.74)	0.48 (−0.59)	0.43 (−1.07)	0.46 (−0.79)	0.42 (−1.18)
$AC[\Delta d]$	0.19	0.27 (0.76)	0.20 (0.12)	0.31 (1.16)	0.26 (0.65)	0.31 (1.13)	0.25 (0.59)
$AC[r_d]$	−0.01	0.00 (0.17)	−0.05 (−0.44)	0.00 (0.08)	−0.01 (0.02)	0.00 (0.07)	−0.01 (0.01)
$AC[r_f]$	0.68	0.96 (4.33)	0.84 (2.49)	0.95 (4.21)	0.69 (0.14)	0.95 (4.20)	0.70 (0.27)
$AC[z_d]$	0.89	0.94 (1.05)	0.90 (0.29)	0.93 (0.83)	0.94 (1.00)	0.93 (0.82)	0.94 (1.01)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.28)	0.51 (−0.14)	0.44 (−0.46)	0.49 (−0.23)	0.45 (−0.44)	0.50 (−0.20)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.32)	0.06 (0.18)	0.08 (0.50)	0.07 (0.29)	0.08 (0.53)	0.07 (0.28)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.07)	0.19 (1.44)	0.28 (2.53)	0.23 (1.96)	0.28 (2.49)	0.23 (1.89)
$Corr[ep, z_{d,-1}]$	−0.16	−0.17 (−0.04)	−0.13 (0.38)	−0.14 (0.25)	−0.17 (−0.01)	−0.14 (0.26)	−0.17 (−0.02)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.67)	0.59 (2.30)	0.69 (2.85)	0.65 (2.64)	0.68 (2.77)	0.65 (2.60)

Table 4: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

deviation is 5.44 vs. 2.72 in the data and the autocorrelation is 0.84 vs. 0.68 in the data). However, once these moments are targeted in the estimation (column 4), the standard deviation of the risk-free rate is 2.82 and the autocorrelation of the risk-free rate is 0.69, consistent with the data.

In both columns 2 and 4, the model closely matches the mean risk-free rate and equity return. However, the contribution of valuation risk is quite different across the various sets of moments. Recall that in the baseline model, valuation risk explains a sizable majority of the risk-free rate and equity premium. In column 2, valuation risk has a smaller but still meaningful contribution (48.2%

of the risk-free rate and 38.9% of the equity premium). In column 4, however, it explains very little of these moments (8.8% and 5.1%) because the model requires smaller and less persistent valuation risk shocks ($\rho_a = 0.9548$ and $\sigma_a = 0.0167$) to match the dynamics of the risk-free rate.[23]

Finally, we turn to the yield curve. In columns 1 and 3, which exclude valuation risk and do not target longer-term risk-free rates, the presence of cash-flow risk generates a (counterfactual) downward sloping yield curve. This is because households in the model dislike long-run risks to cash-flow growth and longer-term risk-free bonds provide additional insurance against these risks. Valuation risk, however, generates a positive term premium for longer-term risk-free bonds because it creates the possibility that households will revalue future cash flows. A longer-term asset increases exposure to this risk. This results in a lower price and higher return for risk-free assets with a longer maturity, leading to an upward sloping yield curve. In columns 2 and 4, which add valuation risk, the yield curve is humped shaped due to the competing effects of the two risks.

The failure of the long-run risk model to predict an upward sloping yield curve is not resolved by targeting the yield curve moments. In column 5, which excludes valuation risk but targets the yield curve moments, the yield curve remains downward sloping. However, the entire curve is raised, resulting in a short-term risk free rate of 1.4%. The addition of valuation risk (column 6) improves the slope of the yield curve, lowering $E[r_f]$ by 21 basis points and raising $E[r_{f,20}]$ by 15 basis points. However, the constraints imposed by also targeting the standard deviation and autocorrelation of the risk-free rate limit the role of valuation risk in fully matching the yield curve.

These results show that valuation risk does not unilaterally resolve the risk-free rate and equity premium puzzles, but the improvements in fit show that it helps match the data. Despite these improvements, the long-run risk model with valuation risk still performs poorly on the three moments listed above as well as the yield curve. Furthermore, all six specifications fail to pass the over-identifying restrictions test at the 5% level. The next section addresses these shortcomings.

7 ESTIMATED EXTENDED LONG-RUN RISK MODEL

We consider two extensions to the long-run risk model. First, we allow valuation risk shocks to directly affect cash-flow growth, in addition to their effect on asset prices through the SDF (henceforth, the "Demand" shock model). This feature is similar to a discount factor shock in a production economy model. For example, in the workhorse New Keynesian model, an increase in the discount factor looks like a negative demand shock that lowers interest rates, inflation, and consumption. Therefore, it provides another mechanism for valuation risk to help fit the data, especially the

[23] The contribution of valuation risk under the current preferences is larger than under the revised preferences. In the model without the higher-order risk-free rate or term structure moments, valuation risk under the current preferences explains 95.3% of the risk-free rate and 94.2% of the equity premium. If only the term structure moments are excluded, valuation risk explains a smaller percentage but it is still bigger than with the revised preferences (28.6% and 17.1%).

correlation moments.[24] Following Albuquerque et al. (2016), we modify (33) and (34) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon_{y,t+1} + \pi_{ya}\sigma_a\varepsilon_{a,t+1}, \tag{36}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{da}\sigma_a\varepsilon_{a,t+1}, \tag{37}$$

where π_{ya} and π_{da} control the covariances between valuation risk shocks and cash-flow growth.[25]

Second, we add stochastic volatility to cash-flow risk following Bansal and Yaron (2004) (henceforth, the "SV" model). SV introduces time-varying uncertainty. Bansal et al. (2016) show SV leads to a significant improvement in fit. An important question is therefore whether the presence of SV will affect the role of valuation risk. To introduce SV, we modify (33)-(35) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_{y,t}\varepsilon_{y,t+1}, \tag{38}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_{y,t}\varepsilon_{y,t+1} + \psi_d\sigma_{y,t}\varepsilon_{d,t+1}, \tag{39}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_{y,t}\varepsilon_{x,t+1}, \tag{40}$$

$$\sigma^2_{y,t+1} = \sigma^2_y + \rho_{\sigma_y}(\sigma^2_{y,t} - \sigma^2_y) + \nu_y\varepsilon_{\sigma_y,t+1}, \tag{41}$$

where ρ_{σ_y} is the persistence of the SV process and ν_y is the standard deviation of the SV shock.

Table 5 and Table 6 present estimates from three versions of the extended long-run risk model: (1) the SV model without valuation risk (columns 1 and 4), (2) the demand shock model (columns 2 and 5), and (3) the combination of the demand shock and SV models (columns 3 and 6). In each case, we report the results from including and excluding longer-term rates as targeted moments.

We begin with the models that exclude longer-term returns as targeted moments.[26] A key finding is that all three extensions improve on the p-values from the simpler long-run risk models in the previous section. Adding SV to the model without valuation risk increases the p-value from near zero (Table 3, column 3) to 0.02 (Table 5, column 1). The estimated SV process is very persistent ($\rho_{\sigma_y} = 0.9630$) and the shock is statistically significant, consistent with the literature. The improved fit largely occurs because SV helps match the higher-order risk-free rate moments (the standard deviation is 2.54 vs. 2.72 in the data and the autocorrelation is 0.69 vs. 0.68 in the data).

The Demand model increases the p-value from 0.012 (Table 3, column 4) to 0.096 (Table 5, column 2). Thus, the Demand model easily passes the over-identifying restrictions test at the 5% level. Consistent with the predictions of a production economy model, π_{ya} and π_{da} are negative in

[24] See, for example, Smets and Wouters (2003). However, without a carefully microfounded model, it is not clear whether $\varepsilon_{a,t+1}$ should be correlated with $\Delta\hat{y}_{t+1}$ or $\hat{x}_t$ (or both) and what restrictions should be placed on the shock coefficients. While there are limitations to using this reduced-form specification, it is very useful for informing what description of the shock processes best explain the data and for developing models with deeper microfoundations.

[25] With the inclusion of π_{ya} and π_{da}, π_{dy} and ψ_d are redundant so we exclude them from the Demand specifications.

[26] The No VR+SV model is the same model BKY estimate. In that paper, the model passes the over-identifying restrictions test at the 5% level, while in our case it does not. The key difference is that BKY do not target the correlations between cash-flows and the equity return. When we exclude these moments, our p-value jumps to 0.15.

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.58 (2.41, 2.74)	3.22 (2.99, 3.42)	6.51 (5.14, 8.05)	1.48 (1.25, 1.72)	3.42 (3.10, 3.78)	8.01 (7.09, 8.98)
β	0.9982 (0.9981, 0.9983)	0.9991 (0.9990, 0.9991)	0.9980 (0.9977, 0.9983)	0.9980 (0.9979, 0.9982)	0.9987 (0.9986, 0.9988)	0.9976 (0.9975, 0.9977)
ρ_a	–	0.9594 (0.9576, 0.9614)	0.9930 (0.9921, 0.9936)	–	0.9616 (0.9591, 0.9639)	0.9933 (0.9927, 0.9937)
σ_a	–	0.0185 (0.0179, 0.0193)	0.0288 (0.0275, 0.0296)	–	0.0194 (0.0186, 0.0203)	0.0286 (0.0278, 0.0291)
μ_y	0.0016 (0.0014, 0.0017)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0018)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0017)
μ_d	0.0013 (0.0010, 0.0016)	0.0015 (0.0012, 0.0016)	0.0015 (0.0014, 0.0017)	0.0002 (0.0000, 0.0006)	0.0013 (0.0011, 0.0015)	0.0015 (0.0013, 0.0017)
σ_y	0.0008 (0.0004, 0.0014)	0.0041 (0.0039, 0.0042)	0.0006 (0.0001, 0.0013)	0.0010 (0.0003, 0.0019)	0.0037 (0.0036, 0.0039)	0.0004 (0.0000, 0.0008)
ψ_d	2.99 (2.79, 3.19)	–	–	2.82 (2.58, 3.02)	–	–
π_{dy}	0.771 (0.503, 1.049)	–	–	0.773 (0.436, 1.099)	–	–
ϕ_d	1.90 (1.81, 2.00)	2.69 (2.54, 2.85)	2.84 (2.65, 2.99)	1.77 (1.67, 1.87)	3.21 (2.96, 3.46)	2.87 (2.76, 2.98)
ρ_x	0.9992 (0.9989, 0.9994)	0.9975 (0.9971, 0.9980)	0.9958 (0.9952, 0.9965)	0.9995 (0.9994, 0.9995)	0.9970 (0.9965, 0.9976)	0.9957 (0.9951, 0.9963)
ψ_x	0.0255 (0.0241, 0.0269)	0.0306 (0.0285, 0.0313)	0.0358 (0.0334, 0.0385)	0.0253 (0.0241, 0.0267)	0.0303 (0.0285, 0.0312)	0.0357 (0.0337, 0.0378)
π_{ya}	–	-0.055 $(-0.074, -0.038)$	-0.049 $(-0.064, -0.033)$	–	-0.037 $(-0.053, -0.022)$	-0.044 $(-0.059, -0.029)$
π_{da}	–	-1.036 $(-1.068, -1.003)$	-0.877 $(-0.905, -0.852)$	–	-1.011 $(-1.047, -0.975)$	-0.896 $(-0.920, -0.872)$
ρ_{σ_y}	0.9630 (0.9589, 0.9668)	–	0.7708 (0.5997, 0.8794)	0.9562 (0.9502, 0.9624)	–	0.5741 (0.4708, 0.6777)
ν_y	1.2e−5 (1.1e−5, 1.4e−5)	–	2.7e−5 (2.0e−5, 3.5e−5)	1.5e−5 (1.3e−5, 1.7e−5)	–	3.6e−5 (3.1e−5, 4.1e−5)
J	18.09 (17.38, 18.81)	13.52 (12.98, 14.04)	9.25 (8.85, 9.66)	25.02 (23.97, 26.09)	18.51 (17.97, 19.02)	10.08 (9.64, 10.54)
pval	0.021 (0.016, 0.026)	0.096 (0.081, 0.113)	0.161 (0.140, 0.182)	0.005 (0.004, 0.008)	0.047 (0.040, 0.055)	0.260 (0.229, 0.292)
df	8	8	6	10	10	8

Table 5: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates. The IES is 2.5.

the estimation. More specifically, a positive valuation risk shock, which makes households more patient, reduces consumption and dividend growth. In a direct horse race between the SV model and the Demand model, which have the same number of parameters, the Demand model wins. The superior fit of the Demand model comes from the fact that it better matches the high volatility of dividend growth and the low correlation between dividend growth and equity returns. The model is better able to match these moments because the volatility of dividend growth increases with π_{da} while partially offsetting the positive relationship between valuation risk and the return on equity.

The Demand+SV model (column 3) raises the p-value to 0.161, passing the over-identifying restrictions test at the 10% level. This result reveals that the two extensions to the long-run risk model are complements, rather than substitutes, which is not obvious *a priori* because both features

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.90 (0.05)	1.87 (−0.08)	1.89 (0.02)	1.96 (0.27)	1.89 (0.02)	1.91 (0.10)
$E[\Delta d]$	1.47	1.58 (0.11)	1.74 (0.28)	1.83 (0.38)	0.36 (−1.16)	1.61 (0.14)	1.78 (0.32)
$E[z_d]$	3.42	3.41 (−0.07)	3.40 (−0.16)	3.39 (−0.21)	3.48 (0.44)	3.41 (−0.09)	3.40 (−0.19)
$E[r_d]$	6.51	6.68 (0.10)	5.81 (−0.44)	5.78 (−0.46)	5.76 (−0.47)	5.52 (−0.62)	5.73 (−0.49)
$E[r_f]$	0.25	0.13 (−0.21)	0.36 (0.17)	0.19 (−0.11)	0.98 (1.20)	1.22 (1.58)	0.27 (0.03)
$E[r_{f,5}]$	1.19	−0.75 (−2.87)	0.31 (−1.30)	0.75 (−0.65)	1.43 (0.34)	1.25 (0.09)	1.45 (0.38)
$E[r_{f,20}]$	1.88	−2.13 (−6.66)	−0.12 (−3.31)	0.63 (−2.08)	1.21 (−1.10)	0.96 (−1.52)	1.57 (−0.51)
$SD[\Delta c]$	1.99	2.01 (0.03)	1.98 (−0.04)	2.09 (0.21)	2.12 (0.26)	1.75 (−0.49)	2.11 (0.25)
$SD[\Delta d]$	11.09	5.28 (−2.12)	7.60 (−1.28)	9.68 (−0.51)	5.24 (−2.14)	7.79 (−1.20)	9.84 (−0.46)
$SD[r_d]$	19.15	18.71 (−0.23)	18.31 (−0.44)	18.69 (−0.24)	18.22 (−0.49)	18.63 (−0.27)	18.58 (−0.30)
$SD[r_f]$	2.72	2.54 (−0.36)	2.97 (0.49)	2.69 (−0.07)	2.59 (−0.27)	3.05 (0.64)	2.61 (−0.22)
$SD[z_d]$	0.45	0.51 (0.91)	0.50 (0.81)	0.48 (0.46)	0.54 (1.44)	0.49 (0.69)	0.49 (0.58)
$AC[\Delta c]$	0.53	0.44 (−0.97)	0.43 (−1.07)	0.45 (−0.92)	0.45 (−0.91)	0.42 (−1.21)	0.45 (−0.90)
$AC[\Delta d]$	0.19	0.24 (0.45)	0.21 (0.20)	0.17 (−0.24)	0.24 (0.43)	0.22 (0.25)	0.17 (−0.21)
$AC[r_d]$	−0.01	−0.03 (−0.26)	0.02 (0.32)	−0.03 (−0.20)	0.01 (0.28)	0.01 (0.30)	−0.01 (0.03)
$AC[r_f]$	0.68	0.69 (0.08)	0.71 (0.49)	0.70 (0.25)	0.65 (−0.45)	0.72 (0.64)	0.71 (0.39)
$AC[z_d]$	0.89	0.93 (0.87)	0.93 (0.81)	0.91 (0.41)	0.94 (1.13)	0.92 (0.70)	0.91 (0.47)
$Corr[\Delta c, \Delta d]$	0.54	0.51 (−0.13)	0.49 (−0.24)	0.51 (−0.11)	0.52 (−0.08)	0.45 (−0.39)	0.49 (−0.21)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.18)	0.09 (0.59)	0.10 (0.79)	0.06 (0.12)	0.09 (0.63)	0.11 (0.83)
$Corr[\Delta d, r_d]$	0.07	0.21 (1.72)	0.13 (0.79)	0.06 (−0.06)	0.21 (1.71)	0.13 (0.75)	0.06 (−0.12)
$Corr[ep, z_{d,-1}]$	−0.16	−0.23 (−0.65)	−0.14 (0.26)	−0.12 (0.42)	−0.20 (−0.41)	−0.13 (0.37)	−0.11 (0.53)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.63)	0.66 (2.66)	0.62 (2.47)	0.67 (2.71)	0.65 (2.59)	0.62 (2.48)

Table 6: Extended long-run risk models. Data and average model-implied moments. t-statistics are in parentheses.

help match risk-free rate dynamics. It also occurs even though the two additional parameters in the model reduce the degrees of freedom and the critical value for the over-identifying restrictions test.

The model continues to fail on one key moment: the predictability of consumption growth given the price dividend ratio (i.e., $Corr[\Delta c, z_{d,-1}]$) remains too high (0.62 vs. 0.19 in the data). The overall improvement in fit occurs because the Demand+SV model does a much better job matching dividend growth dynamics. Specifically, it better matches the standard deviation of dividend growth (9.68 vs. 11.09 in the data) and the weak correlation between dividend growth and equity returns (0.06 vs. 0.07 in the data). In this model, valuation risk has a bigger role than in the Demand model ($\rho_a = 0.993$ vs. $\rho_a = 0.959$; $\sigma_a = 0.0288$ vs. $\sigma_a = 0.0185$), while the SV process is not as persistent ($\rho_{\sigma_y} = 0.771$ vs. $\rho_{\sigma_y} = 0.963$) as in the No VR+SV model. Also, σ_y is significantly smaller, so the contribution of consumption growth volatility from pure endowment risk is smaller when compared to the Demand model. The Demand model has trouble matching dividend growth dynamics while simultaneously matching risk-free rate dynamics. An expanded role of valuation risk is crucial for matching dividend growth dynamics. Without SV, this is not possible because it would cause the model to miss on the risk-free rate dynamics. Introducing SV, however, permits a lower σ_y, which helps offset the effect of valuation risk on the risk-free rate dynamics.

In terms of the yield curve, the No VR+SV and Demand models are both able to improve along this dimension. Once the long-term rates are targeted, the yield curve slope (i.e., $E[r_{f,20}] - E[r_f]$) rises from -2.0% to 0.2% with the No VR+SV model (column 4) and from -0.5% to -0.3% with the Demand model (column 5). However, in both cases, the yield curve is hump-shaped and the addition of the yield curve moments decreases the fit of the models as measured by the p-value. In the case of the Demand model, it no longer passes the test of over-identifying restrictions at the 5% level. In sharp contrast, the Demand+SV model improves in terms of the p-value when the yield curve moments are targeted from 0.16% to 0.26% and the yield curve is no longer hump-shaped (column 6). All three yield curve moments are insignificantly different from their data counterparts.

8 CONCLUSION

Although valuation risk has become the subject of a substantial body of research to address asset pricing puzzles, the literature has ignored the full implications of the current preference specification. This paper first documents four desirable properties of Epstein-Zin recursive preferences without valuation risk. It then shows the current valuation risk specification violates these properties because the distributional weights in the time-aggregator of the utility function do not sum to 1. In contrast, our revised preferences, which restrict the distributional weights, satisfy all four properties. These results caution against continuing to use the current specification in future work.

Under our revised preferences, valuation risk has a much smaller role in resolving the equity premium and risk-free rate puzzles. However, we find valuation risk still plays an important role in matching the standard deviation and autocorrelation of the risk-free rate as well as the yield curve. Furthermore, allowing valuation risk to directly affect cash-flow growth, similar to a production economy model, adds a source of volatility that significantly improves the empirical fit of the model and helps match the standard deviation of dividend growth and its correlation with equity returns.

Despite the importance of valuation risk, our paper and the literature is silent on its structural foundations. As a consequence, there are several open research questions. For example, what does it mean for a representative household to have a time-varying time-preference? Is there an economy with multiple (heterogenous) households that supports these preferences? Is there a decision-theoretic explanation and is it possible to back out the dynamics of a time-varying time-preference from experiments or data? We believe these questions are important avenues for future research.

REFERENCES

ADAM, K., A. MARCET, AND J. P. NICOLINI (2016): "Stock market volatility and learning," *The Journal of Finance*, 71, 33–82, https://doi.org/10.1111/jofi.12364.

ALBUQUERQUE, R., M. EICHENBAUM, V. X. LUO, AND S. REBELO (2016): "Valuation risk and asset pricing," *The Journal of Finance*, 71, 2861–2904, https://doi.org/10.1111/jofi.12437.

ALBUQUERQUE, R., M. EICHENBAUM, D. PAPANIKOLAOU, AND S. REBELO (2015): "Long-run bulls and bears," *Journal of Monetary Economics*, 76, S21–S36, https://doi.org/10.1016/j.jmoneco.2015.09.010.

ANDREASEN, M. AND K. JØRGENSEN (2019): "The importance of timing attitudes in consumption-based asset pricing models," *Journal of Monetary Economics*, forthcomong, https://doi.org/10.1016/j.jmoneco.2019.01.008.

BACKUS, D. K., B. R. ROUTLEDGE, AND S. E. ZIN (2005): "Exotic Preferences for Macroeconomists," in *NBER Macroeconomics Annual 2004, Vol. 19*, ed. by M. Gertler and K. Rogoff, MIT Press, NBER Chapters, 319–414.

BANSAL, R., D. KIKU, I. SHALIASTOVICH, AND A. YARON (2014): "Volatility, the macroeconomy and asset prices," *The Journal of Finance*, 69, 2471–2511, https://doi.org/10.1111/jofi.12110.

BANSAL, R., D. KIKU, AND A. YARON (2016): "Risks for the long run: Estimation with time aggregation," *Journal of Monetary Economics*, 82, 52–69, https://doi.org/10.1016/j.jmoneco.2016.07.003.

BANSAL, R. AND A. YARON (2004): "Risks for the long run: A potential resolution of asset pricing puzzles," *The Journal of Finance*, 59, 1481–1509, https://doi.org/10.1111/j.1540-6261.2004.00670.x.

BARRO, R. J. (2009): "Rare disasters, asset prices, and welfare costs," *American Economic Review*, 99, 243–64, https://doi.org/10.1257/aer.99.1.243.

BASU, S. AND M. KIMBALL (2002): "Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption," Manuscript, University of Michigan.

BEELER, J. AND J. Y. CAMPBELL (2012): "The long-run risks model and aggregate asset prices: An empirical assessment," *Critical Finance Review*, 1, 141–182, https://doi.org/10.1561/104.00000004.

BIANCHI, F., H. KUNG, AND M. TIRSKIKH (2018): "The origins and effects of macroeconomic uncertainty," NBER Working Paper 25386.

CAMPBELL, J. Y. (1999): "Asset prices, consumption, and the business cycle," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1C, chap. 19, 1231–1303.

CAMPBELL, J. Y. AND J. COCHRANE (1999): "Force of habit: A consumption-based explanation of aggregate stock market behavior," *Journal of Political Economy*, 107, 205–251, https://doi.org/10.1086/250059.

CAMPBELL, J. Y. AND R. J. SHILLER (1988): "The dividend-price ratio and expectations of future dividends and discount factors," *Review of Financial Studies*, 1, 195–228, https://doi.org/10.1093/rfs/1.3.195.

CHRISTIANO, L. J., M. EICHENBAUM, AND S. REBELO (2011): "When is the government spending multiplier large?" *Journal of Political Economy*, 119, 78–121, https://doi.org/10.1086/659312.

CREAL, D. AND J. C. WU (2017): "Bond risk premia in consumption-based models," Manuscript, University of Chicago Booth.

DE GROOT, O., A. RICHTER, AND N. THROCKMORTON (2018): "Uncertainty shocks in a model of effective demand: Comment," *Econometrica*, 86, 1513–1526, https://doi.org/10.3982/ECTA15405.

DUFFIE, D. AND L. G. EPSTEIN (1992): "Stochastic differential utility," *Econometrica*, 60, 353–394, https://doi.org/10.2307/2951600.

EGGERTSSON, G. B. AND M. WOODFORD (2003): "The zero bound on interest rates and optimal monetary policy," *Brookings Papers on Economic Activity*, 34(1), 139–235, https://doi.org/10.7916/D8S46PV9.

EPSTEIN, L. G. AND S. E. ZIN (1989): "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework," *Econometrica*, 57, 937–69, https://www.doi.org/10.2307/1913778.

——— (1991): "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis," *Journal of Political Economy*, 99, 263–86, https://doi.org/10.1086/261750.

GOURIO, F. (2012): "Disaster risk and business cycles," *American Economic Review*, 102, 2734–2766, https://doi.org/10.1257/aer.102.6.2734.

HALL, R. E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96, 339–357, https://doi.org/10.1086/261539.

HANSEN, L. P. AND T. J. SARGENT (2008): *Robustness*, Princeton, NJ: Princeton University Press.

JUSTINIANO, A. AND G. E. PRIMICERI (2008): "The time-varying volatility of macroeconomic fluctuations," *American Economic Review*, 98, 604–41, https://www.doi.org/10.1257/aer.98.3.604.

KLIEM, M. AND A. MEYER-GOHDE (2018): "(Un)expected monetary policy shocks and term premia," Deutsche Bundesbank Discussion Paper.

KOLLMANN, R. (2016): "International business cycles and risk sharing with uncertainty shocks and recursive preferences," *Journal of Economic Dynamics and Control*, 72, 115–124, https://www.doi.org/10.1016/j.jedc.2016.03.005.

KRAFT, H. AND F. T. SEIFRIED (2014): "Stochastic differential utility as the continuous-time limit of recursive utility," *Journal of Economic Theory*, 151, 528–550, https://doi.org/10.1016/j.jet.2013.12.007.

LUCAS, JR, R. E. (1978): "Asset prices in an exchange economy," *Econometrica*, 46, 1429–45, https://www.doi.org/10.2307/1913837.

MAURER, T. A. (2012): "Is consumption growth merely a sideshow in asset pricing?" Manuscript, Washington University in St. Louis.

MEHRA, R. AND E. C. PRESCOTT (1985): "The equity premium: A puzzle," *Journal of Monetary Economics*, 15, 145–161, https://doi.org/10.1016/0304-3932(85)90061-3.

NAKATA, T. AND H. TANAKA (2016): "Equilibrium yield curves and the interest rate lower bound," Finance and Economics Discussion Series, 2016-085.

NEWEY, W. K. AND K. D. WEST (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica*, 55, 703–708, https://www.doi.org/10.2307/1913610.

POHL, W., K. SCHMEDDERS, AND O. WILMS (2018): "Higher order effects in asset pricing models with long-run risks," *The Journal of Finance*, 73, 1061–1111, https://doi.org/10.1111/jofi.12615.

RAPACH, D. E. AND F. TAN (2018): "Asset pricing with recursive preferences and stochastic volatility: A bayesian DSGE analysis," Manuscript, Saint Louis University.

ROTEMBERG, J. J. AND M. WOODFORD (1997): "An optimization-based econometric framework for the evaluation of monetary policy," in *NBER Macroeconomics Annual 1997, Volume 12*, ed. by B. S. Bernanke and J. J. Rotemberg, MIT Press, 297–361.

SCHORFHEIDE, F., D. SONG, AND A. YARON (2018): "Identifying long-run risks: A bayesian mixed frequency approach," *Econometrica*, 86, 617–654, https://doi.org/10.3982/ECTA14308.

SMETS, F. AND R. WOUTERS (2003): "An estimated dynamic stochastic general equilibrium model of the euro area," *Journal of the European Economic Association*, 1, 1123–1175, https://doi.org/10.1162/154247603770383415.

——— (2007): “Shocks and frictions in us business cycles: a bayesian dsge approach,” *American Economic Review*, 97, 586–606, https://doi.org/10.1257/aer.97.3.586.

SWANSON, E. T. (2012): “Risk aversion and the labor margin in dynamic equilibrium models,” *American Economic Review*, 102, 1663–91, http://dx.doi.org/10.1257/aer.102.4.1663.

VAN BINSBERGEN, J. H., J. FERNÁNDEZ-VILLAVERDE, R. S. KOIJEN, AND J. RUBIO-RAMÍREZ (2012): “The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences,” *Journal of Monetary Economics*, 59, 634–648, https://doi.org/10.1016/j.jmoneco.2012.09.002.

WEIL, P. (1989): “The equity premium puzzle and the risk-free rate puzzle,” *Journal of Monetary Economics*, 24, 401–421, https://www.doi.org/10.3386/w2829.

A INDIFFERENCE CURVE DERIVATION

For compactness, define $\rho = 1 - 1/\psi$ and $\alpha = 1 - \gamma$. Then from (7) and (8) in the main paper

$$\bar{U}_C(a_{t+1}) \equiv g(U^C_{t+1}) = g\left((1 - \beta + a_{t+1}\beta\bar{x})^{1/\rho}\right),$$
$$\bar{U}_R(a_{t+1}) \equiv g(U^R_{t+1}) = g\left((1 - a_{t+1}\beta + a_{t+1}\beta\bar{x})^{1/\rho}\right),$$

where $g(U_{t+1}) = (E_t[U^\alpha_{t+1}])^{1/\alpha}$. The certainty equivalent is given by

$$\bar{U} = (1 - \beta + \beta\bar{x})^{1/\rho}.$$

Suppose there are two possible outcomes for a_{t+1}, denoted a_1 and a_2. Then

$$\bar{U}_C = \left(\frac{(1 - \beta + a_1\beta\bar{x})^{\alpha/\rho} + (1 - \beta + a_2\beta\bar{x})^{\alpha/\rho}}{2}\right)^{1/\alpha},$$
$$\bar{U}_R = \left(\frac{(1 - a_1\beta + a_1\beta\bar{x})^{\alpha/\rho} + (1 - a_2\beta + a_2\beta\bar{x})^{\alpha/\rho}}{2}\right)^{1/\alpha}.$$

Set $\bar{U}_C$ and $\bar{U}_R$ equal to the certainty equivalent, fix a_1, and solve for a_2 to obtain:

$$a^C_2 = \frac{\left(2\bar{U}^\alpha - (1 - \beta + a_1\beta\bar{x})^{\alpha/\rho}\right)^{\rho/\alpha} - (1 - \beta)}{\beta\bar{x}},$$
$$a^R_2 = \frac{\left(2\bar{U}^\alpha - (1 - a_1\beta + a_1\beta\bar{x})^{\alpha/\rho}\right)^{\rho/\alpha} - 1}{\beta(\bar{x} - 1)}.$$

We plot combinations of (a_1, a_2) under the current and revised preferences.

B Isomorphic Representations of the Current Specification

In the current literature, the preference shock typically hits current utility. If, for simplicity, we abstract from Epstein-Zin preferences, then the utility function and Euler equation are given by

$$U_t = \alpha_t u(c_t) + \beta E_t[U_{t+1}], \tag{B.1}$$

$$\beta E_t[(\alpha_{t+1}/\alpha_t)u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{B.2}$$

The shock follows $\Delta\hat{\alpha}_{t+1} = \rho\Delta\hat{\alpha}_t + \sigma_\alpha\varepsilon_t$, so the change in α_t is known at time t. Alternatively, if the preference shock hits future consumption, the utility function and Euler equation are given by

$$U_t = u(c_t) + a_t\beta E_t[U_{t+1}], \tag{B.3}$$

$$a_t\beta E_t[u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{B.4}$$

If the shock follows $\hat{a}_t = \rho\hat{a}_{t-1} + \sigma_a\varepsilon_t$, the two specifications are isomorphic because setting $a_t \equiv \alpha_{t+1}/\alpha_t$ in (B.4) yields (B.2). We use the second specification because it is easier to compare the current and revised preferences when the shock always shows up in the Euler equation in levels.

C Result 5 Proof

The results in this section apply a variant of the following four limits:

1. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon^2}\left(\left(E\left[(x\epsilon^2 c+1)^{1/\epsilon}\right]\right)^\epsilon - 1\right)\right] = E[cx]$
2. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon^2}\left(\left(E\left[(x\epsilon^2 c+x)^{1/\epsilon}\right]\right)^\epsilon - E[x]\right)\right]$ is undefined
3. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon}\left(\left(E\left[(x\epsilon c+1)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} - 1\right)\right] = E[cx]$
4. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon}\left(\left(E\left[(x\epsilon c+x)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} - E[x]\right)\right] = E[cx] + \mathcal{O}$

where x is an exogenous stochastic variable, c is a stochastic policy relevant variable, and $\mathcal{O} = E[x\log x] - E[x]\log(E[x])$ is an additive term that is independent of the policy relevant variable.

Case 1 Define $\gamma = 1-\epsilon$ and $1-1/\psi = \epsilon^2$. Then preferences are given by

$$U_t^j = \left(w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j\left(E_t\left[\left(U_{t+1}^j\right)^\epsilon\right]\right)^\epsilon\right)^{1/\epsilon^2}.$$

For simplicity, assume $a_{t+j} = 1$ and c_{t+j} is nonstochastic for $j \geq 2$. Defining $V_t^j = (U_t^j)^{\epsilon^2}$ implies

$$V_t^j = w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j\left(E_t\left[\left(V_{t+1}^j\right)^{1/\epsilon}\right]\right)^\epsilon,$$

$$V_{t+1}^j = \sum_{k=1}^{\infty} \left(\prod_{i=1}^{k-1} w_{2,t+i}^j\right) w_{1,t+k}^j c_{t+k}^{\epsilon^2}.$$

Combining these results then implies

$$V_t^j = w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j \left(E_t\left[\left(\sum_{k=1}^{\infty} \tilde{w}_{2,t+k}^j w_{1,t+k}^j c_{t+k}^{\epsilon^2}\right)^{1/\epsilon}\right]\right)^{\epsilon},$$

where $\tilde{w}_{2,t+k}^j \equiv \prod_{i=1}^{k-1} w_{2,t+i}^j$. Now define $W_t^j = (V_t^j - 1)/\epsilon^2$, so the utility function is given by

$$W_t = w_{1,t}^j u_t + w_{2,t}^j \left(E_t\left[\left(\tfrac{1}{\epsilon^2}\sum_{k=1}^{\infty} \tilde{w}_{2,t+k}^j w_{1,t+k}^j(\epsilon^2 u_{t+k}+1)\right)^{1/\epsilon}\right]\right)^{\epsilon} + \tfrac{w_{1,t}^j - 1}{\epsilon^2},$$

where $u_t = (c_t^{\epsilon^2} - 1)/\epsilon^2$ is a CRRA utility function that converges to $\log c_t$ as $\epsilon \to 0$.

Under the revised specification, $w_{1,t}^R = 1 - a_t\beta$ and $w_{2,t}^R = a_t\beta$. Therefore,

$$W_t = (1-a_t\beta)u_t + \tfrac{a_t\beta}{\epsilon^2}\left(\left(E_t\left[\left(\epsilon^2\sum_{k=1}^{\infty}\tilde{a}_{t+k}(1-a_{t+k}\beta)u_{t+k}+1\right)^{1/\epsilon}\right]\right)^{\epsilon} - 1\right), \tag{C.1}$$

where $\tilde{a}_{t+k} \equiv \prod_{i=1}^{k-1} a_{t+i}\beta$. Applying Limit 1, then implies

$$\lim_{\epsilon\to 0} W_t = (1-a_t\beta)\log c_t + a_t\beta E_t[\sum_{k=1}^{\infty}\tilde{a}_{t+k}(1-a_{t+k}\beta)\log c_{t+k}].$$

Under the current preferences, $w_{1,t}^C = 1-\beta$ and $w_{2,t}^C = a_t\beta$. Therefore,

$$W_t = (1-\beta)u_t + \tfrac{a_t\beta}{\epsilon^2}\left(\left(E_t\left[\left(\epsilon^2(1-\beta)\sum_{k=1}^{\infty}\tilde{a}_{t+k}u_{t+k} + 1 - \beta + a_{t+1}\beta\right)^{1/\epsilon}\right]\right)^{\epsilon} - \tfrac{1}{a_t}\right),$$

which does not converge to a log utility function as $\epsilon \to 0$ according to Limit 2.

Case 2 The assumption that $\gamma = 1-\epsilon$ and $1-1/\psi = \epsilon^2$ may appear contrived. What is important is that both γ and ψ tend to 1, but ψ approaches 1 at a faster rate. When they approach 1 at the same rate, then time-separable log utility results regardless of whether the preference specification.

To see this result, suppose $\gamma = 1-\epsilon$ and $\psi = 1+\epsilon$. Then utility is given by

$$U_t^j = \left(w_{1,t}^j c_t^{\frac{\epsilon}{1+\epsilon}} + w_{2,t}^j\left(E_t\left[(U_{t+1}^j)^{\epsilon}\right]\right)^{\frac{1}{1+\epsilon}}\right)^{\frac{1+\epsilon}{\epsilon}}.$$

Once again, assume $a_{t+j} = 1$ and c_{t+j} is nonstochastic for $j \geq 2$. Defining $V_t^j = (U_t^j)^{\frac{\epsilon}{1+\epsilon}}$ implies

$$V_t^j = w_{1,t}^j c_t^{\frac{\epsilon}{1+\epsilon}} + w_{2,t}^j\left(E_t\left[\left(\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j c_{t+k}^{\frac{\epsilon}{1+\epsilon}}\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}},$$

where $\tilde{w}_{2,t+k}$ is the same as Case 1. Define $W_t^j = (1+\epsilon)(V_t^j - 1)/\epsilon$. The utility function is given by

$$W_t = w_{1,t}^j u_t + w_{2,t}^j\left(E_t\left[\left(\tfrac{1+\epsilon}{\epsilon}\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j(\tfrac{\epsilon}{1+\epsilon}u_{t+k}+1)\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} + \left(\tfrac{1+\epsilon}{\epsilon}\right)(w_{1,t}^j - 1),$$

where $u_t = (c_t^{\epsilon/(1+\epsilon)} - 1)/(\epsilon/(1+\epsilon))$ is a CRRA utility function that converges to $\log c_t$ as $\epsilon \to 0$.

Under the revised specification, $w^R_{1,t} = 1 - a_t\beta$ and $w^R_{2,t} = a_t\beta$. Therefore,

$$W_t = (1 - a_t\beta)u_t + a_t\beta \left(\tfrac{1+\epsilon}{\epsilon}\right) \left(\left(E_t \left[\left(\tfrac{\epsilon}{1+\epsilon} \textstyle\sum_{k=1}^{\infty} \tilde{a}_{t+k}(1 - a_{t+k}\beta)u_{t+k} + 1 \right)^{1+\epsilon} \right] \right)^{\frac{1}{1+\epsilon}} - 1 \right),$$

where $\tilde{a}_{t+k}$ is defined above. Applying Limit 3, then implies (C.1).

Under the current preferences, $w^C_{1,t} = 1 - \beta$ and $w^C_{2,t} = a_t\beta$. Therefore,

$$W_t = (1 - \beta)u_t + a_t\beta \left(\tfrac{1+\epsilon}{\epsilon}\right) \left(\left(E_t \left[\left(\tfrac{\epsilon}{1+\epsilon}(1-\beta) \textstyle\sum_{k=1}^{\infty} \tilde{a}_{t+k}u_{t+k} + a^s_{t+1} \right)^{1+\epsilon} \right] \right)^{\frac{1}{1+\epsilon}} - E_t[a^s_{t+1}] \right)$$
$$+ a_t\beta \left(\tfrac{1+\epsilon}{\epsilon}\right) (E_t[a^s_{t+1}] - 1/a_t),$$

where $a^s_{t+1} \equiv 1 - \beta + a_{t+1}\beta$. Applying Limit 4, then implies

$$\lim_{\epsilon \to 0} W_t = (1 - \beta)\log c_t + a_t\beta(1 - \beta)E_t[\textstyle\sum_{k=1}^{\infty} \tilde{a}_{t+k} \log c_{t+k}] + \mathcal{O}_t.$$

where $\mathcal{O}_t = E_t[a^s_{t+1} \log a^s_{t+1}] - E_t[a^s_{t+1}] \log(E_t[a^s_{t+1}]) + a_t\beta(E_t[a^s_{t+1}] - 1/a_t) \lim_{\epsilon\to 0} \left(\frac{1+\epsilon}{\epsilon}\right)$ is an exogenous additive term that does not affect the household's optimality conditions.

D ANALYTICAL DERIVATIONS

Stochastic Discount Factor The Lagrangian for specification $j \in \{C, R\}$ is given by

$$U^j_t = \max \left[w^j_{1,t} c_t^{1-1/\psi} + w^j_{2,t} \left(E_t \left[\left(U^j_{t+1}\right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$
$$- \lambda_t(c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} - (p_{y,t} + y_t)s_{1,t-1} - (p_{d,t} + d_t)s_{2,t-1}),$$

where $w^C_{1,t} = 1-\beta$, $w^R_{1,t} = 1-a^R_t\beta$, $w^C_{2,t} = a^C_t\beta$, and $w^R_{2,t} = a^R_t\beta$. The optimality conditions imply

$$w^j_{1,t} \left(U^j_t\right)^{1/\psi} c_t^{-1/\psi} = \lambda_t, \tag{D.1}$$

$$w^j_{2,t} \left(U^j_t\right)^{1/\psi} \left(E_t \left[\left(U^j_{t+1}\right)^{1-\gamma} \right] \right)^{\frac{1/\psi-\gamma}{1-\gamma}} E_t \left[\left(U^j_{t+1}\right)^{-\gamma} \left(\partial U^j_{t+1}/\partial s_{1,t} \right) \right] = \lambda_t p_{y,t}, \tag{D.2}$$

$$w^j_{2,t} \left(U^j_t\right)^{1/\psi} \left(E_t \left[\left(U^j_{t+1}\right)^{1-\gamma} \right] \right)^{\frac{1/\psi-\gamma}{1-\gamma}} E_t \left[\left(U^j_{t+1}\right)^{-\gamma} \left(\partial U^j_{t+1}/\partial s_{2,t} \right) \right] = \lambda_t p_{d,t}, \tag{D.3}$$

where $\partial U^j_t/\partial s_{1,t-1} = \lambda_t(p_{y,t} + y_t)$ and $\partial U^j_t/\partial s_{2,t-1} = \lambda_t(p_{d,t} + d_t)$ by the envelope theorem. Updating the envelope conditions and combining (D.1)-(D.3) generates (11) and (12) in the main text.

Following Epstein and Zin (1991), we posit the following minimum state variable solution:

$$U^j_t = \xi_{1,t}s_{1,t-1} + \xi_{2,t}s_{2,t-1} \quad \text{and} \quad c_t = \xi_{3,t}s_{1,t-1} + \xi_{4,t}s_{2,t-1}. \tag{D.4}$$

where ξ is a vector of unknown coefficients. The envelope conditions combined with (D.1) imply

$$\xi_{1,t} = w^j_{1,t}(U^j_t)^{1/\psi} c_t^{-1/\psi}(p_{y,t} + y_t), \tag{D.5}$$

$$\xi_{2,t} = w^j_{1,t}(U^j_t)^{1/\psi} c_t^{-1/\psi}(p_{d,t} + d_t). \tag{D.6}$$

Multiplying (D.5) by $s_{1,t-1}$ and (D.6) by $s_{2,t-1}$ and then adding yields

$$U^j_t = w^j_{1,t}(U^j_t)^{1/\psi} c_t^{-1/\psi}((p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}), \tag{D.7}$$

which, after plugging in the budget constraint, (10), and imposing equilibrium, can be written as

$$(U^j_t)^{1-1/\psi} = w^j_{1,t} c_t^{-1/\psi}(c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t}) = w^j_{1,t} c_t^{-1/\psi}(c_t + p_{y,t}). \tag{D.8}$$

Imposing (D.8) on the utility function implies

$$w^j_{1,t} c_t^{-1/\psi} p_{y,t} = w^j_{2,t}\left(E_t\left[\left(U^j_{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}. \tag{D.9}$$

Solving (D.8) for U^j_t and (D.9) for $E_t[(U^j_{t+1})^{1-\gamma}]$ and then plugging into (13) and (14) implies

$$m^j_{t+1} = (x^j_t)^\theta (c_{t+1}/c_t)^{-\theta/\psi} r^{\theta-1}_{y,t+1}, \tag{D.10}$$

where $x^j_t \equiv w^j_{2t} w^j_{1t+1}/w^j_{1t}$. Taking logs of (D.10) yields (17), given the following definitions:

$$\hat{x}^C_t = \hat{\beta} + \hat{a}^C_t,$$

$$\hat{x}^R_t = \hat{\beta} + \hat{a}^R_t + \log(1 - \beta\exp(\hat{a}^R_{t+1})) - \log(1 - \beta\exp(\hat{a}^R_t)) \approx \hat{\beta} + (\hat{a}^R_t - \beta\hat{a}^R_{t+1})/(1-\beta),$$

and $\hat{a}_t \equiv \hat{a}^C_t = \hat{a}^R_t/(1-\beta)$ so the preference shocks with the current and revised specifications are directly comparable. It follows that $\hat{x}^j_t = \hat{\beta} + \hat{a}_t - \omega^j \hat{a}_{t+1}$ as in (17), where $\omega^C = 0$ and $\omega^R = \beta$.

Campbell-Shiller Approximation The return on the endowment is approximated by

$$\begin{aligned}
\hat{r}_{y,t+1} &= \log(p_{y,t+1} + y_{t+1}) - \log(p_{y,t}) \\
&= \log(y_{t+1}(p_{y,t+1}/y_{t+1}) + y_{t+1}) - \log(y_t(p_{y,t}/y_t)) \\
&= \log(y_{t+1}(\exp(\hat{z}_{y,t+1}) + 1)) - \hat{z}_{y,t} - \log(y_t) \\
&= \log(\exp(\hat{z}_{y,t+1}) + 1) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&\approx \log(\exp(\hat{z}_y) + 1) + \exp(\hat{z}_y)(\hat{z}_{y,t+1} - \hat{z}_y)/(1 + \exp(\hat{z}_y)) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&= \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}.
\end{aligned}$$

The derivation for the equity return, $\hat{r}_{d,t+1}$, is analogous to the return on the endowment.

Model Solution We use a guess and verify method. For the endowment claim, we obtain

$$
\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{y,t+1})]) \\
&= \log(E_t[\exp(\theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1-1/\psi)\Delta\hat{y}_{t+1} + \theta(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}))]) \\
&= \log\left(E_t\left[\exp\begin{pmatrix} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1-1/\psi)(\mu_y + \sigma_y \varepsilon_{y,t+1}) \\ +\theta\kappa_{y0} + \theta\kappa_{y1}(\eta_{y0} + \eta_{y1}\hat{a}_{t+1}) - \theta(\eta_{y0} + \eta_{y1}\hat{a}_t) \end{pmatrix}\right]\right) \\
&= \log\left(E_t\left[\exp\begin{pmatrix} \theta\hat{\beta} + \theta(1-1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) \\ +\theta(1-\omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\ +\theta(1-1/\psi)\sigma_y\varepsilon_{y,t+1} + \theta(\kappa_{y1}\eta_{y1} - \omega^j)\sigma_a\varepsilon_{a,t+1} \end{pmatrix}\right]\right) \\
&= \theta\hat{\beta} + \theta(1-1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + \tfrac{\theta^2}{2}(1-1/\psi)^2\sigma_y^2 \\
&\quad + \tfrac{\theta^2}{2}(\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2 + \theta(1-\omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t,
\end{aligned}
$$

where the last equality follows from the log-normality of $\exp(\varepsilon_{y,t+1})$ and $\exp(\varepsilon_{a,t+1})$.

After equating coefficients, we obtain the following exclusion restrictions:

$$
\hat{\beta} + (1-1/\psi)\mu_y + (\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + \tfrac{\theta}{2}((1-1/\psi)^2\sigma_y^2 + (\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2) = 0, \quad \text{(D.11)}
$$

$$
1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1) = 0. \quad \text{(D.12)}
$$

For the dividend claim, we obtain

$$
\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{d,t+1})]) \\
&= \log\left(E_t\left[\exp\begin{pmatrix} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + (\theta(1-1/\psi)-1)\Delta\hat{y}_{t+1} + \Delta\hat{d}_{t+1} \\ +(\theta-1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) + (\kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t}) \end{pmatrix}\right]\right) \\
&= \log\left(E_t\left[\exp\begin{pmatrix} \theta\hat{\beta} + (\theta(1-1/\psi)-1)\mu_y + \mu_d \\ +(\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1}-1)) \\ +(\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\ (\pi_{dy}-\gamma)\sigma_y\varepsilon_{y,t+1} + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1} \end{pmatrix}\right]\right) \\
&= \theta\hat{\beta} + (\theta(1-1/\psi)-1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1}-1)) \\
&\quad + (\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\
&\quad + \tfrac{1}{2}((\pi_{dy}-\gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2).
\end{aligned}
$$

Once again, equating coefficients implies the following exclusion restrictions:

$$
\begin{aligned}
&\theta\hat{\beta} + (\theta(1-1/\psi)-1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1}-1)) \\
&\quad +\tfrac{1}{2}((\pi_{dy}-\gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2) = 0, \quad \text{(D.13)}
\end{aligned}
$$

$$
\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1) = 0. \quad \text{(D.14)}
$$

Equations (D.11)-(D.14), along with (21) and (22), form a system of 8 equations and 8 unknowns.

Asset Prices Given the coefficients, we can solve for the risk free rate. The Euler equation implies

$$\hat{r}_{f,t} = -\log(E_t[\exp(\hat{m}_{t+1})]) = -E_t[\hat{m}_{t+1}] - \tfrac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}],$$

since the risk-free rate is known at time-t. The pricing kernel is given by

$$\begin{aligned}
\hat{m}_{t+1} &= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{y}_{t+1} + (\theta-1)\hat{r}_{y,t+1} \\
&= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) - \gamma\Delta\hat{y}_{t+1} + (\theta-1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (\theta(1-\omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\
&\quad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} - \gamma\sigma_y\varepsilon_{y,t+1} \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) + (1-\omega^j\rho_a)\hat{a}_t \\
&\quad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1}) - \gamma\sigma_y\varepsilon_{y,t+1},
\end{aligned}$$

where the last line follows from imposing (D.12). Therefore, the risk-free rate is given by

$$\begin{aligned}
\hat{r}_{f,t} = \gamma\mu_y - \theta\hat{\beta} - (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) - (1-\omega^j\rho_a)\hat{a}_t \\
- \tfrac{1}{2}\gamma^2\sigma_y^2 - \tfrac{1}{2}((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)^2\sigma_a^2.
\end{aligned}$$

Note that $\hat{r}_{f,t} = \log(E_t[\exp(\hat{r}_{f,t})])$. After plugging in (D.11), we obtain

$$\hat{r}_{f,t} = \mu_y/\psi - \hat{\beta} - (1-\omega^j\rho_a)\hat{a}_t + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma) - \gamma^2)\sigma_y^2.$$

Therefore, the unconditional expected risk-free rate is given by

$$E[\hat{r}_f] = -\hat{\beta} + \mu_y/\psi + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma) - \gamma^2)\sigma_y^2. \quad \text{(D.15)}$$

We can also derive an expression for the equity premium, $E_t[ep_{t+1}]$, which given by

$$\log(E_t[\exp(\hat{r}_{d,t+1} - \hat{r}_{f,t})]) = E_t[\hat{r}_{d,t+1}] - \hat{r}_{f,t} + \tfrac{1}{2}\operatorname{Var}_t[\hat{r}_{d,t+1}] = -\operatorname{Cov}_t[\hat{m}_{t+1}, \hat{r}_{d,t+1}],$$

where the last equality stems from the Euler equation, $E_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}]+\tfrac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}] = 0$. We already solved for the SDF, so the last step is to solve for the equity return, which given by

$$\begin{aligned}
\hat{r}_{d,t+1} &= \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1} \\
&= \kappa_{d0} + \kappa_{d1}(\eta_{d0} + \eta_{d1}\hat{a}_{t+1}) - (\eta_{d0} + \eta_{d1}\hat{a}_t) + \Delta\hat{d}_{t+1} \\
&= \mu_d + \kappa_{d0} + \eta_{d0}(\kappa_{d1}-1) + \eta_{d1}(\kappa_{d1}\rho_a - 1)\hat{a}_t + \kappa_{d1}\eta_{d1}\sigma_a\varepsilon_{a,t+1} + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}.
\end{aligned}$$

Therefore, the unconditional equity premium can be written as

$$E[ep] = \gamma\pi_{dy}\sigma_y^2 + (\theta\omega^j + (1-\theta)\kappa_{y1}\eta_{y1})\kappa_{d1}\eta_{d1}\sigma_a^2. \quad \text{(D.16)}$$

Long-term Bond Prices The pricing kernel can be written as

$$\hat{m}_{t+1} = m_0 + m_1 \hat{a}_t + m_2 \sigma_a \varepsilon_{a,t+1} + m_3 \sigma_y \varepsilon_{y,t+1},$$

where

$$m_0 \equiv \theta \hat{\beta} - \gamma \mu_y + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)), \qquad m_2 \equiv (\theta - 1)\kappa_{y1}\eta_{y1} - \theta \omega^j,$$
$$m_1 \equiv 1 - \omega^j \rho_a, \qquad m_3 \equiv -\gamma.$$

The 1-period bond price is given by

$$\hat{p}_t^{(1)} = -\hat{r}_{f,t} = \log(E_t[\exp(\hat{m}_{t+1})]) = m_0 + m_1 \hat{a}_t + m_2^2 \sigma_a^2/2 + m_3^2 \sigma_y^2/2.$$

The 2-period bond price is given by

$$\begin{aligned}
\hat{p}_t^{(2)} &= \log E_t[\exp(\hat{m}_{t+1} + \hat{p}_{t+1}^{(1)})] \\
&= \log E_t[\exp(m_0 + m_1 \hat{a}_t + m_2 \sigma_a \varepsilon_{a,t+1} + m_3 \sigma_y \varepsilon_{y,t+1} + \\
&\qquad m_0 + m_1(\rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1}) + m_2^2 \sigma_a^2/2 + m_3^2 \sigma_y^2/2)] \\
&= 2m_0 + m_1(1 + \rho_a)\hat{a}_t + (m_2 + m_1)^2 \sigma_a^2/2 + m_2^2 \sigma_a^2/2 + m_3^2 \sigma_y^2.
\end{aligned}$$

More generally, the price of any n-period bond for $n > 1$ is given by

$$\hat{p}_t^{(n)} = n m_0 + m_1 \textstyle\sum_{j=0}^{n-1} \rho_a^j \hat{a}_t + \frac{1}{2} \sum_{k=2}^{n} (m_2 + m_1 \sum_{j=0}^{n-k} \rho_a^j)^2 \sigma_a^2 + \frac{1}{2} m_2^2 \sigma_a^2 + \frac{n}{2} m_3^2 \sigma_y^2$$

and the risk-free return is given by $r_{f,t}^{(n)} = -\hat{p}_t^{(n)}/n$.

D.1 SPECIAL CASE 1 ($\sigma_a = \psi_d = 0$ & $\pi_{dy} = 1$) In this case, there is no valuation risk ($\hat{a}_t = 0$) and cash flow risk is perfectly correlated ($\Delta\hat{y}_{t+1} = \mu_y + \sigma_y \varepsilon_{y,t+1}$; $\Delta\hat{d}_{t+1} = \mu_d + \sigma_y \varepsilon_{y,t+1}$). Under these assumptions, it is easy to see that (D.15) and (D.16) reduce to (26) and (27) in the main text.

D.2 SPECIAL CASE 2 ($\sigma_y = 0$, $\rho_a = 0$, & $\mu_y = \mu_d$) In this case, there is no cash flow risk ($\Delta\hat{y}_{t+1} = \Delta\hat{d}_{t+1} = \mu_y$) and the time preference shocks are *i.i.d.* ($\hat{a}_{t+1} = \sigma_a \varepsilon_{a,t+1}$). Under these two assumptions, the return on the endowment and dividend claims are identical, so $\{\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}\} = \{\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}\} \equiv \{\kappa_0, \kappa_1, \eta_0, \eta_1\}$. Therefore, (D.15) and (D.16) reduce to (28) and (29) for the current specification and (30) and (31) for the revised specification.

The exclusion restriction, (D.12), implies $\eta_1 = 1$ so (D.11) simplifies to

$$0 = \hat{\beta} + (1 - 1/\psi)\mu_y + \kappa_0 + \eta_0(\kappa_1 - 1) + \tfrac{\theta}{2}(\kappa_1 - \omega^j)^2 \sigma_a^2. \tag{D.17}$$

First, recall that $0 < \kappa_1 < 1$. Therefore, the asymptote in θ will permeate the solution with the current preferences ($\omega^C = 0$). However, with the revised preferences ($\omega^R = \beta$), we guess and verify that $\kappa_1 = \beta$ when $\psi = 1$. In this case, (D.17) reduces to $\hat{\beta} + \kappa_0 + \eta_0(\beta - 1) = 0$. Combining with (21), this restriction implies that $\eta_0 = \log\beta - \log(1-\beta)$ and $\kappa_0 = -(1-\beta)\log(1-\beta) - \beta\log\beta$. Plugging the expressions for η_0, κ_0, and κ_1 into (21) and (D.17) verifies our initial guess for κ_1.

Alternatively, if utility is Epstein-Zin with a stationary preference shock on *current* utility, then

$$U_t = \left[a_t(1-\beta)c_t^{1-1/\psi} + \beta\left(E_t\left[(U_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \quad 1 \neq \psi > 0. \tag{D.18}$$

Since $w_{1,t} = a_t(1-\beta)$ and $w_{2,t} = \beta$, $x_t = \beta a_{t+1}/a_t$ and the pricing kernel is given by

$$\hat{m}_{t+1} = \theta\log\beta + \theta(\hat{a}_{t+1} - \hat{a}_t) - (\theta/\psi)\Delta\hat{c}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}. \tag{D.19}$$

Given this slight modification, the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((\theta-1)\kappa_1^2\eta_1^2 + \theta)\sigma_a^2/2, \tag{D.20}$$

$$E[ep] = ((1-\theta)\kappa_1\eta_1 - \theta)\kappa_1\eta_1\sigma_a^2. \tag{D.21}$$

Since $\eta_1 = -1$, there is once again no endogenous mechanism that prevents the asymptote in θ from influencing asset pricing moments, just like in (28) and (29) under the current specification.

E NONLINEAR MODEL ASYMPTOTE

Assuming $\mu_{t+1} \equiv y_{t+1}/y_t = d_{t+1}/d_t$, the (nonlinear) Euler equation is given by

$$z_t = \frac{a_t\beta}{1-\chi^j a_t\beta}\left(E_t\Big[\underbrace{\left(\left(1-\chi^j a_{t+1}\beta\right)\mu_{t+1}^{1-1/\psi}(1+z_{t+1})\right)^{\theta}}_{x_{t+1}}\Big]\right)^{1/\theta}, \tag{E.1}$$

where $\chi^C = 0$ and $\chi^R = 1$. Notice the asymptote disappears if $\mathrm{SD}(x_{t+1}) \to 0$ as $\psi \to 1$. The main text focuses on results from a Campbell and Shiller (1988) approximation of the model. In this appendix, we demonstrate three noteworthy results using the model's exact, nonlinear, form.

One, consider the case without valuation risk, so $a_t = 1$ for all t. The Euler equation reduces to

$$z_t = \beta(E_t[(\mu_{t+1}^{1-1/\psi}(1+z_{t+1}))^{\theta}])^{1/\theta}. \tag{E.2}$$

When $\psi = 1$, we guess and verify that $z_t = \beta/(1-\beta)$, so the price-dividend ratio is constant. This is the well know result that when the IES is 1, the income and substitution effects of a change in endowment growth offset. Therefore, the price-dividend ratio does not respond to cash flow risk.

Two, consider the case when a_t is stochastic under the revised preferences ($\chi^R = 1$) and either $\psi = 1$ (CRRA preferences) or $\mu_t = 1$ for all t (no cash-flow growth). In both cases, we guess and verify that $z_t = a_t\beta/(1-a_t\beta)$. The price dividend ratio is time-varying but independent of θ, so an asymptote does not affect equilibrium outcomes. Thus, the household is certainty-equivalent.

Three, consider what happens under the current preferences ($\chi^C = 0$), which do not account for the offsetting movements in $1-a_t\beta$. To obtain a closed-form solution for any IES, we assume $\mu_t = \mu$ and the preference shock evolves according to $\log(1 + a_{t+1}\eta) = \sigma\varepsilon_{t+1}$, where ε_{t+1} is standard normal. Under these assumptions, we guess and verify that the price-dividend ratio is given by

$$z_t = a_t\eta = a_t\beta\mu^{1-1/\psi}\exp(\theta\sigma^2/2). \tag{E.3}$$

In this case, θ appears in the price-dividend ratio, so the asymptote affects equilibrium outcomes. These results prove that the asymptote is not due to a Campbell-Shiller approximation of the model.

F Data Sources

We drew from the following data sources to estimate our models:

1. **[$RCONS$] Per Capita Real PCE (excluding durables)**: Annual, chained 2012 dollars. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 7.1.

2. **[$RETD$] Value-Weighted Return (including dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETD).

3. **[$RETX$] Value-Weighted Return (excluding dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETX).

4. **[CPI] Consumer Price Index for All Urban Consumers**: Monthly, not seasonally adjusted, index 1982-1984=100. Source: Bureau of Labor Statistics (FRED ID: CPIAUCNS).

5. **[RFR] Risk-free Rate**: Monthly, annualized yield calculated from nominal price. Source: Wharton Research Data Services, CRSP Treasuries, Risk-free Series (CRSP ID: TMYTM).

6. **[$RFR5$] 5-year U.S. Treasury Yield**: Monthly, intermediate-term, annualized. Source: Ibbotson Associates via Morningstar Direct, IA SBBI US IT (ID: FOUSA05XQC).

7. **[$RFR20$] 20-year U.S. Treasury Yield**: Monthly, long-term, annualized. Source: Ibbotson Associates via Morningstar Direct, IA SBBI US LT (ID: FOUSA05XQ8).

We applied the following transformations to the above data sources:

1. **Annual Per Capita Real Consumption Growth (annual frequency)**:

$$\Delta\hat{c}_t = 100\log(RCONS_t/RCONS_{t-1})$$

2. **Annual Real Dividend Growth (monthly frequency)**:

$$P_{1928M1} = 100, \quad P_t = P_{t-1}(1 + RETX_t), \quad D_t = (RETD_t - RETX_t)P_{t-1},$$
$$d_t = \textstyle\sum_{i=t-11}^{t} D_i/CPI_t, \quad \Delta\hat{d}_t = 100\log(d_t/d_{t-12})$$

3. **Annual Real Equity Return (monthly frequency)**:

$$\pi_t^m = \log(CPI_t/CPI_{t-1}), \quad \hat{r}_{d,t} = 100\textstyle\sum_{i=t-11}^{t}(\log(1 + RETD_i) - \pi_i^m)$$

4. **Annual Real Risk-free Rate (monthly frequency)**:

$$rfr_t = RFR_t - \log(CPI_{t+3}/CPI_t), \quad \pi_t^q = \log(CPI_t/CPI_{t-12})/4,$$
$$\hat{r}_{f,t} = 400(\hat{\beta}_0 + \hat{\beta}_1 RFR_t + \hat{\beta}_2\pi_t^q),$$

where $\hat{\beta}_j$ are OLS estimates from regressing the quarterly *ex-post* real rate, rfr, on the quarterly nominal rate, RFR, and inflation, π^q. The fitted values estimate the *ex-ante* real rate.

5. **5- and 20-year Real Risk-free Rate (monthly frequency)**:

$$rfrX_t = RFRX_t - \log(CPI_{t+12}/CPI_t), \quad \pi_t^a = \log(CPI_t/CPI_{t-12}),$$
$$\hat{r}_{f,X,t} = 100(\hat{\beta}_0 + \hat{\beta}_1 RFRX_t + \hat{\beta}_2\pi_t^a),$$

where $\hat{\beta}_j$ are the OLS estimates from regressing the annual *ex-post* real long-term rate, $rfr5$ or $rfr20$, on the annual nominal rate, $RFR5$ or $RFR20$, and inflation, π^a. The fitted values estimate the *ex-ante* real long-term rate.

6. **Price-Dividend Ratio (monthly frequency)**:

$$\hat{z}_{d,t} = \log(P_t/\textstyle\sum_{i=t-11}^{t} D_i)$$

We use December of each year to convert each of the monthly time series to an annual frequency.

G ESTIMATION METHOD

The estimation procedure has two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 10 lags. The second stage is a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The second stage is repeated for many different draws of shocks to obtain a sampling distribution for each parameter. The following steps outline the algorithm:

1. Use GMM to estimate the moments, $\hat{\Psi}_T^D$, and the diagonal of the covariance matrix, $\hat{\Sigma}_T^D$.
2. Use SMM to estimate the structural asset pricing model. Given a random seed, s, draw a T-period sequence of shocks for each shock in the model. Denote the shock matrix $\mathcal{E}_T^s$ (e.g., in the baseline model $\mathcal{E}_T^s = [\varepsilon_{y,t}^s, \varepsilon_{d,t}^s, \varepsilon_{a,t}^s]_{t=1}^T$). For $s \in \{1, \ldots, N_s\}$, run the following steps:
   (a) Specify a guess, $\hat{\theta}_0$, for the N_p estimated parameters and the parameter variance-covariance matrix, Σ_P, which is initialized as a diagonal matrix.
   (b) Use simulated annealing to minimize the loss function.
      i. For $i \in \{0, \ldots, N_d\}$, repeat the following steps:
         A. Draw a candidate vector of parameters, $\hat{\theta}_i^{cand}$, where

$$\hat{\theta}_i^{cand} \sim \begin{cases} \hat{\theta}_0 & \text{for } i = 0, \\ \mathbb{N}(\hat{\theta}_{i-1}, c_0 \Sigma_P) & \text{for } i > 0. \end{cases}$$

         We set c to target an acceptance rate of 30%. For the revised preferences, we restrict $\hat{\theta}_i^{cand}$ so that $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$. This ensures the utility function weights are positive in 99.997% of the simulated observations.
         B. Solve the Campbell-Shiller approximation of the model given $\hat{\theta}_i^{cand}$.
         C. Given $\mathcal{E}_T^s(r)$, simulate the monthly model R times for T periods. We draw initial states, $\hat{a}_0$, from $\mathbb{N}(0, \sigma_a^2/(1-\rho_a^2))$. For each repetition r, calculate the moments, $\Psi_T^M(\hat{\theta}_i^{cand}, \mathcal{E}_T^s(r))$, the same way they are calculated in the data.
         D. Calculate the median moments across the R simulations, $\bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand}, \mathcal{E}_T^s) = \text{median}\{\Psi_T^M(\hat{\theta}_i^{cand}, \mathcal{E}_T^s(r))\}_{r=1}^R$, and evaluate the loss function:

$$J_i^{s,cand} = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand}, \mathcal{E}_T^s)]'[\hat{\Sigma}_T^D(1+1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i^{cand}, \mathcal{E}_T^s)].$$

         E. Accept or reject the candidate draw according to

$$(\hat{\theta}_i^s, J_i^s) = \begin{cases} (\hat{\theta}_i^{cand}, J_i^{s,cand}) & \text{if } i = 0, \\ (\hat{\theta}_i^{cand}, J_i^{s,cand}) & \text{if } \min(1, \exp(J_{i-1}^s - J_i^{s,cand})/c_1) > \hat{u}, \\ (\hat{\theta}_{i-1}, J_{i-1}^s) & \text{otherwise,} \end{cases}$$

         where c_1 is the temperature and $\hat{u}$ is a draw from a uniform distribution. The lower the temperature, the more likely it is that the candidate draw is rejected.
      ii. Find the parameter draw $\hat{\theta}_{\min}^s$ that corresponds to $\min\{J_i^s\}_{i=1}^{N_d}$, and update Σ_P^s.

A. Discard the first $N_d/2$ draws. Stack the remaining draws in a $N_d/2 \times N_p$ matrix, $\hat{\Theta}^s$, and define $\tilde{\Theta}^s = \hat{\Theta}^s - \mathbf{1}_{N_d/2\times 1} \sum_{i=N_d/2}^{N_d} \hat{\theta}_i^s/(N_d/2)$.

B. Calculate $\Sigma_P^{s,up} = (\tilde{\Theta}^s)'\tilde{\Theta}^s/(N_d/2)$.

(c) Repeat the previous step N_{SMM} times, initializing at draw $\hat{\theta}_0 = \hat{\theta}^s_{\min}$ and covariance matrix $\Sigma_P = \Sigma_P^{s,up}$. Gradually decrease the temperature. Of all the draws, find the lowest N_J J values, denoted $\{J^{s,j}_{guess}\}_{j=1}^{N_J}$, and the corresponding draws, $\{\theta^{s,j}_{guess}\}_{j=1}^{N_J}$.

(d) For $j \in \{1, \ldots, N_J\}$, minimize the same loss function with MATLAB's `fminsearch` starting at $\theta^{s,j}_{guess}$. The resulting minimum is $\hat{\theta}^{s,j}_{\min}$ with a loss function value of $J^{s,j}_{\min}$. Repeat, each time updating the guess, until $J^{s,j}_{guess} - J^{s,j}_{\min} < 0.001$. The parameter estimates reported in the tables in the main paper, denoted $\hat{\theta}^s$, correspond to $\min\{J^{s,j}_{\min}\}_{j=1}^{N_J}$.

3. The set of SMM parameter estimates $\{\hat{\theta}^s\}_{s=1}^{N_s}$ approximate the joint sampling distribution of the parameters. We report its mean, $\bar{\theta} = \sum_{s=1}^{N_s} \hat{\theta}^s/N_s$, and $(5, 95)$ percentiles.

For all model specifications, the results in the main paper are based on $N_s = 500$, $R = 1{,}000$, $N_d = 20{,}000$, $N_{SMM} = 5$, and $N_J = 50$. N_p, c_0, and the temperatures, c_1, are all model-specific.

H Estimation Robustness

Baseline Model: $\psi = 2.0$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.46 (1.44, 1.48)	75.79 (72.61, 79.16)	10.00 (10.00, 10.00)	1.31 (1.29, 1.34)	98.91 (94.21, 103.85)	10.00 (10.00, 10.00)
β	0.9978 (0.9977, 0.9980)	0.9957 (0.9956, 0.9958)	0.9974 (0.9974, 0.9975)	0.9980 (0.9979, 0.9982)	0.9964 (0.9963, 0.9964)	0.9979 (0.9979, 0.9980)
ρ_a	0.9968 (0.9965, 0.9971)	0.9899 (0.9896, 0.9902)	0.9877 (0.9874, 0.9880)	0.9973 (0.9970, 0.9976)	0.9893 (0.9890, 0.9896)	0.9877 (0.9874, 0.9880)
σ_a	0.00031 (0.00030, 0.00033)	0.03554 (0.03504, 0.03605)	0.03907 (0.03864, 0.03955)	0.00028 (0.00027, 0.00030)	0.03657 (0.03604, 0.03709)	0.03918 (0.03869, 0.03963)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0016)	0.0020 (0.0020, 0.0021)	0.0010 (0.0009, 0.0010)	0.0010 (0.0010, 0.0011)	0.0016 (0.0016, 0.0017)	0.0005 (0.0004, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0058 (0.0057, 0.0059)	0.0058 (0.0057, 0.0060)	0.0058 (0.0058, 0.0058)	0.0055 (0.0054, 0.0057)	0.0060 (0.0059, 0.0062)
ψ_d	1.54 (1.43, 1.63)	0.97 (0.88, 1.07)	1.07 (0.96, 1.18)	1.52 (1.42, 1.61)	1.13 (1.04, 1.23)	1.01 (0.92, 1.12)
π_{dy}	0.816 (0.765, 0.870)	0.438 (0.405, 0.475)	0.606 (0.550, 0.668)	0.816 (0.760, 0.873)	0.614 (0.584, 0.645)	0.598 (0.545, 0.657)
J	29.27 (28.62, 29.98)	48.09 (47.73, 48.47)	56.08 (55.47, 56.67)	31.73 (31.05, 32.43)	50.04 (49.64, 50.46)	59.90 (59.38, 60.40)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table H.1: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates.

Moment	Data	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
		Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.19)	2.00 (0.45)	1.89 (0.00)	1.98 (0.36)	1.95 (0.22)
$E[\Delta d]$	1.47	1.84 (0.38)	2.45 (1.02)	1.15 (−0.34)	1.25 (−0.24)	1.97 (0.52)	0.56 (−0.95)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.48)	3.56 (1.02)	3.49 (0.48)	3.52 (0.74)	3.60 (1.27)
$E[r_d]$	6.51	5.46 (−0.66)	5.57 (−0.59)	4.03 (−1.55)	4.78 (−1.08)	4.98 (−0.96)	3.35 (−1.98)
$E[r_f]$	0.25	0.25 (0.00)	0.37 (0.19)	1.07 (1.34)	0.09 (−0.26)	0.26 (0.01)	0.42 (0.28)
$E[r_{f,5}]$	1.19	1.25 (0.09)	1.75 (0.83)	2.19 (1.46)	0.92 (−0.40)	1.23 (0.06)	1.51 (0.47)
$E[r_{f,20}]$	1.88	3.19 (2.19)	3.47 (2.65)	3.32 (2.40)	2.57 (1.16)	2.28 (0.68)	2.62 (1.24)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.99 (−0.01)	2.01 (0.04)	2.00 (0.01)	1.91 (−0.17)	2.08 (0.18)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.12 (−3.28)	2.47 (−3.15)	3.44 (−2.80)	2.46 (−3.15)	2.44 (−3.16)
$SD[r_d]$	19.15	18.41 (−0.39)	13.64 (−2.91)	13.39 (−3.04)	18.47 (−0.36)	13.46 (−3.00)	13.06 (−3.21)
$SD[r_f]$	2.72	3.21 (0.96)	3.70 (1.92)	3.87 (2.27)	2.99 (0.53)	3.70 (1.93)	3.77 (2.06)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.17)	0.23 (−3.52)	0.48 (0.47)	0.24 (−3.32)	0.22 (−3.62)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.35)	0.88 (3.12)	0.95 (4.17)	0.89 (3.28)	0.88 (3.11)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.86)	0.83 (−1.33)	0.93 (0.75)	0.84 (−1.00)	0.83 (−1.35)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.31)	0.41 (−0.59)	0.50 (−0.19)	0.48 (−0.29)	0.48 (−0.28)	0.51 (−0.13)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)	0.09 (0.57)	0.09 (0.55)	0.09 (0.67)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.37)	0.18 (1.38)	0.18 (1.35)	0.19 (1.41)

Table H.2: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

Baseline Model: $\psi = 1.5$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.31 (1.29, 1.32)	78.83 (75.37, 82.75)	10.00 (10.00, 10.00)	1.21 (1.19, 1.23)	100.12 (95.79, 104.96)	10.00 (10.00, 10.00)
β	0.9981 (0.9980, 0.9982)	0.9958 (0.9957, 0.9958)	0.9977 (0.9976, 0.9977)	0.9983 (0.9982, 0.9984)	0.9964 (0.9964, 0.9965)	0.9982 (0.9981, 0.9982)
ρ_a	0.9968 (0.9965, 0.9971)	0.9898 (0.9895, 0.9901)	0.9875 (0.9871, 0.9878)	0.9973 (0.9970, 0.9976)	0.9892 (0.9889, 0.9896)	0.9874 (0.9871, 0.9877)
σ_a	0.00031 (0.00030, 0.00033)	0.03566 (0.03515, 0.03618)	0.03946 (0.03898, 0.04000)	0.00028 (0.00027, 0.00030)	0.03665 (0.03608, 0.03722)	0.03959 (0.03915, 0.04004)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0016)	0.0020 (0.0020, 0.0021)	0.0009 (0.0009, 0.0010)	0.0010 (0.0010, 0.0011)	0.0016 (0.0016, 0.0017)	0.0004 (0.0004, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0057 (0.0056, 0.0058)	0.0059 (0.0057, 0.0060)	0.0058 (0.0058, 0.0058)	0.0055 (0.0054, 0.0056)	0.0061 (0.0059, 0.0062)
ψ_d	1.54 (1.44, 1.63)	0.98 (0.88, 1.09)	1.05 (0.95, 1.16)	1.52 (1.42, 1.61)	1.14 (1.05, 1.24)	0.99 (0.90, 1.09)
π_{dy}	0.816 (0.763, 0.873)	0.443 (0.409, 0.477)	0.600 (0.548, 0.662)	0.816 (0.759, 0.875)	0.617 (0.589, 0.647)	0.590 (0.535, 0.645)
J	29.27 (28.62, 29.98)	48.26 (47.90, 48.64)	57.00 (56.39, 57.59)	31.74 (31.06, 32.44)	50.11 (49.71, 50.54)	60.78 (60.28, 61.26)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table H.3: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.21)	1.99 (0.40)	1.89 (0.00)	1.98 (0.36)	1.93 (0.18)
$E[\Delta d]$	1.47	1.84 (0.38)	2.44 (1.01)	1.12 (−0.37)	1.25 (−0.24)	1.97 (0.52)	0.54 (−0.98)
$E[z_d]$	3.42	3.45 (0.18)	3.49 (0.49)	3.56 (1.03)	3.49 (0.48)	3.52 (0.74)	3.60 (1.27)
$E[r_d]$	6.51	5.46 (−0.66)	5.55 (−0.60)	4.00 (−1.57)	4.78 (−1.08)	4.98 (−0.96)	3.32 (−1.99)
$E[r_f]$	0.25	0.25 (0.00)	0.38 (0.20)	1.09 (1.38)	0.09 (−0.26)	0.27 (0.02)	0.44 (0.31)
$E[r_{f,5}]$	1.19	1.25 (0.09)	1.75 (0.81)	2.20 (1.49)	0.92 (−0.40)	1.23 (0.06)	1.53 (0.49)
$E[r_{f,20}]$	1.88	3.19 (2.19)	3.43 (2.58)	3.31 (2.38)	2.57 (1.16)	2.27 (0.65)	2.61 (1.22)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.97 (−0.05)	2.02 (0.06)	2.00 (0.01)	1.90 (−0.18)	2.09 (0.20)
$SD[\Delta d]$	11.09	3.47 (−2.79)	2.12 (−3.28)	2.44 (−3.16)	3.44 (−2.79)	2.47 (−3.15)	2.42 (−3.17)
$SD[r_d]$	19.15	18.41 (−0.39)	13.61 (−2.92)	13.28 (−3.09)	18.47 (−0.36)	13.46 (−3.00)	12.97 (−3.26)
$SD[r_f]$	2.72	3.21 (0.96)	3.70 (1.93)	3.88 (2.29)	2.99 (0.53)	3.70 (1.94)	3.78 (2.08)
$SD[z_d]$	0.45	0.46 (0.22)	0.25 (−3.19)	0.23 (−3.58)	0.48 (0.47)	0.24 (−3.33)	0.22 (−3.67)
$AC[r_f]$	0.68	0.95 (4.12)	0.90 (3.34)	0.88 (3.09)	0.95 (4.17)	0.89 (3.28)	0.88 (3.08)
$AC[z_d]$	0.89	0.92 (0.64)	0.85 (−0.87)	0.82 (−1.39)	0.93 (0.75)	0.84 (−1.02)	0.82 (−1.41)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.31)	0.41 (−0.59)	0.50 (−0.18)	0.48 (−0.29)	0.48 (−0.28)	0.51 (−0.12)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.57)	0.06 (0.23)	0.09 (0.61)	0.09 (0.57)	0.09 (0.56)	0.09 (0.67)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.41)	0.15 (1.03)	0.18 (1.37)	0.18 (1.38)	0.18 (1.36)	0.18 (1.40)

Table H.4: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

Long-Run Risk Model: $\psi = 2.0$

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.40 $(2.18, 2.62)$	2.58 $(2.33, 2.87)$	2.49 $(2.20, 2.75)$	2.43 $(2.19, 2.68)$	2.35 $(1.90, 2.75)$	2.29 $(1.96, 2.65)$
β	0.9992 $(0.9991, 0.9993)$	0.9983 $(0.9982, 0.9984)$	0.9992 $(0.9990, 0.9993)$	0.9991 $(0.9990, 0.9992)$	0.9987 $(0.9985, 0.9988)$	0.9987 $(0.9986, 0.9988)$
ρ_a	–	0.9811 $(0.9793, 0.9829)$	–	0.9537 $(0.9519, 0.9555)$	–	0.9562 $(0.9537, 0.9587)$
σ_a	–	0.0483 $(0.0460, 0.0507)$	–	0.0165 $(0.0159, 0.0171)$	–	0.0174 $(0.0164, 0.0184)$
μ_y	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0014, 0.0017)$
μ_d	0.0012 $(0.0009, 0.0015)$	0.0013 $(0.0009, 0.0016)$	0.0014 $(0.0011, 0.0017)$	0.0012 $(0.0009, 0.0015)$	0.0012 $(0.0009, 0.0015)$	0.0011 $(0.0007, 0.0014)$
σ_y	0.0041 $(0.0040, 0.0043)$	0.0040 $(0.0038, 0.0043)$	0.0050 $(0.0049, 0.0051)$	0.0041 $(0.0039, 0.0042)$	0.0046 $(0.0045, 0.0047)$	0.0037 $(0.0034, 0.0039)$
ψ_d	3.26 $(3.05, 3.47)$	2.89 $(2.66, 3.13)$	3.01 $(2.81, 3.18)$	3.25 $(3.01, 3.49)$	3.22 $(2.92, 3.50)$	3.53 $(3.18, 3.90)$
π_{dy}	0.593 $(0.354, 0.834)$	0.782 $(0.487, 1.114)$	0.132 $(-0.184, 0.419)$	0.640 $(0.392, 0.885)$	0.208 $(-0.147, 0.546)$	0.791 $(0.476, 1.110)$
ϕ_d	2.31 $(2.13, 2.51)$	1.65 $(1.53, 1.78)$	2.11 $(1.88, 2.30)$	2.27 $(2.06, 2.50)$	2.36 $(2.00, 2.69)$	2.50 $(2.19, 2.86)$
ρ_x	0.9990 $(0.9986, 0.9993)$	0.9994 $(0.9993, 0.9995)$	0.9981 $(0.9974, 0.9988)$	0.9990 $(0.9986, 0.9994)$	0.9979 $(0.9969, 0.9989)$	0.9991 $(0.9986, 0.9995)$
ψ_x	0.0255 $(0.0242, 0.0269)$	0.0260 $(0.0247, 0.0273)$	0.0306 $(0.0287, 0.0328)$	0.0252 $(0.0240, 0.0266)$	0.0296 $(0.0273, 0.0321)$	0.0246 $(0.0232, 0.0261)$
J	20.91 $(20.16, 21.71)$	14.36 $(13.93, 14.78)$	54.54 $(53.67, 55.47)$	19.91 $(19.25, 20.63)$	62.31 $(61.48, 63.17)$	25.31 $(24.57, 26.07)$
pval	0.007 $(0.005, 0.010)$	0.026 $(0.022, 0.030)$	0.000 $(0.000, 0.000)$	0.011 $(0.008, 0.014)$	0.000 $(0.000, 0.000)$	0.005 $(0.004, 0.006)$
df	8	6	10	8	12	10

Table H.5: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.88 (−0.03)	1.89 (0.02)	1.88 (−0.02)	1.89 (0.00)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.48 (0.01)	1.55 (0.08)	1.68 (0.21)	1.48 (0.01)	1.43 (−0.05)	1.30 (−0.18)
$E[z_d]$	3.42	3.43 (0.03)	3.40 (−0.17)	3.42 (−0.05)	3.43 (0.02)	3.43 (0.05)	3.43 (0.08)
$E[r_d]$	6.51	6.47 (−0.02)	6.46 (−0.04)	5.93 (−0.37)	6.49 (−0.01)	5.59 (−0.57)	6.32 (−0.12)
$E[r_f]$	0.25	0.30 (0.07)	0.26 (0.01)	0.28 (0.05)	0.26 (0.01)	1.46 (1.99)	1.23 (1.61)
$E[r_{f,5}]$	1.19	0.11 (−1.60)	0.96 (−0.34)	0.01 (−1.74)	0.21 (−1.45)	1.28 (0.12)	1.28 (0.12)
$E[r_{f,20}]$	1.88	−0.42 (−3.82)	0.79 (−1.80)	−0.74 (−4.34)	−0.29 (−3.60)	0.77 (−1.84)	0.94 (−1.55)
$SD[\Delta c]$	1.99	1.92 (−0.14)	1.94 (−0.10)	2.45 (0.95)	1.89 (−0.21)	2.22 (0.47)	1.69 (−0.62)
$SD[\Delta d]$	11.09	5.62 (−2.00)	4.79 (−2.30)	6.40 (−1.71)	5.50 (−2.04)	6.35 (−1.73)	5.40 (−2.08)
$SD[r_d]$	19.15	18.03 (−0.59)	19.79 (0.34)	18.74 (−0.21)	18.16 (−0.52)	18.90 (−0.13)	18.25 (−0.47)
$SD[r_f]$	2.72	0.64 (−4.11)	5.56 (5.60)	0.87 (−3.66)	2.83 (0.21)	0.77 (−3.86)	2.92 (0.39)
$SD[z_d]$	0.45	0.53 (1.34)	0.46 (0.08)	0.52 (1.12)	0.52 (1.17)	0.52 (1.11)	0.53 (1.19)
$AC[\Delta c]$	0.53	0.43 (−1.07)	0.46 (−0.75)	0.48 (−0.55)	0.43 (−1.09)	0.46 (−0.78)	0.42 (−1.21)
$AC[\Delta d]$	0.19	0.27 (0.77)	0.21 (0.20)	0.31 (1.16)	0.26 (0.69)	0.31 (1.14)	0.26 (0.63)
$AC[r_d]$	−0.01	0.01 (0.21)	−0.05 (−0.45)	0.00 (0.12)	−0.01 (0.04)	0.00 (0.11)	−0.01 (0.03)
$AC[r_f]$	0.68	0.96 (4.34)	0.84 (2.44)	0.96 (4.25)	0.69 (0.14)	0.95 (4.24)	0.70 (0.27)
$AC[z_d]$	0.89	0.94 (1.09)	0.90 (0.27)	0.93 (0.91)	0.94 (1.02)	0.93 (0.88)	0.94 (1.02)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.27)	0.50 (−0.18)	0.44 (−0.45)	0.48 (−0.26)	0.45 (−0.43)	0.49 (−0.21)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.30)	0.06 (0.21)	0.08 (0.47)	0.07 (0.29)	0.08 (0.50)	0.07 (0.29)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.09)	0.19 (1.51)	0.28 (2.54)	0.24 (2.01)	0.28 (2.49)	0.23 (1.93)
$Corr[ep, z_{d,-1}]$	−0.16	−0.17 (−0.10)	−0.13 (0.37)	−0.15 (0.16)	−0.17 (−0.04)	−0.15 (0.19)	−0.17 (−0.04)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.67)	0.60 (2.31)	0.69 (2.87)	0.65 (2.64)	0.68 (2.78)	0.64 (2.59)

Table H.6: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

Long-Run Risk Model: $\psi = 1.5$

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.05 $(1.92, 2.18)$	2.44 $(2.22, 2.68)$	2.08 $(1.86, 2.31)$	2.13 $(1.97, 2.34)$	2.10 $(1.67, 2.58)$	2.17 $(1.88, 2.56)$
β	0.9995 $(0.9994, 0.9995)$	0.9988 $(0.9987, 0.9990)$	0.9995 $(0.9994, 0.9995)$	0.9995 $(0.9994, 0.9995)$	0.9991 $(0.9989, 0.9992)$	0.9992 $(0.9991, 0.9993)$
ρ_a	–	0.9801 $(0.9781, 0.9820)$	–	0.9514 $(0.9490, 0.9538)$	–	0.9550 $(0.9521, 0.9581)$
σ_a	–	0.0497 $(0.0472, 0.0521)$	–	0.0160 $(0.0153, 0.0168)$	–	0.0172 $(0.0162, 0.0184)$
μ_y	0.0015 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0018)$	0.0015 $(0.0014, 0.0017)$	0.0015 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0014, 0.0017)$
μ_d	0.0011 $(0.0008, 0.0015)$	0.0013 $(0.0009, 0.0017)$	0.0013 $(0.0010, 0.0017)$	0.0012 $(0.0008, 0.0015)$	0.0010 $(0.0007, 0.0014)$	0.0010 $(0.0006, 0.0013)$
σ_y	0.0042 $(0.0040, 0.0044)$	0.0040 $(0.0037, 0.0043)$	0.0051 $(0.0050, 0.0052)$	0.0041 $(0.0039, 0.0043)$	0.0046 $(0.0044, 0.0047)$	0.0035 $(0.0032, 0.0037)$
ψ_d	3.22 $(3.01, 3.44)$	3.10 $(2.83, 3.38)$	2.94 $(2.78, 3.11)$	3.29 $(3.06, 3.53)$	3.26 $(2.94, 3.58)$	3.80 $(3.42, 4.23)$
π_{dy}	0.552 $(0.311, 0.798)$	0.740 $(0.414, 1.066)$	0.191 $(-0.091, 0.456)$	0.611 $(0.372, 0.895)$	0.223 $(-0.167, 0.591)$	0.808 $(0.464, 1.168)$
ϕ_d	2.29 $(2.12, 2.44)$	1.84 $(1.71, 1.98)$	2.02 $(1.85, 2.21)$	2.33 $(2.15, 2.51)$	2.41 $(2.04, 2.81)$	2.76 $(2.44, 3.15)$
ρ_x	0.9993 $(0.9991, 0.9995)$	0.9995 $(0.9993, 0.9995)$	0.9988 $(0.9983, 0.9993)$	0.9993 $(0.9990, 0.9994)$	0.9984 $(0.9974, 0.9993)$	0.9992 $(0.9987, 0.9995)$
ψ_x	0.0250 $(0.0238, 0.0263)$	0.0258 $(0.0246, 0.0270)$	0.0290 $(0.0275, 0.0307)$	0.0248 $(0.0236, 0.0259)$	0.0283 $(0.0263, 0.0307)$	0.0240 $(0.0227, 0.0255)$
J	21.89 $(21.04, 22.74)$	14.61 $(14.15, 15.08)$	52.00 $(51.02, 53.11)$	20.68 $(19.95, 21.47)$	61.09 $(60.20, 61.98)$	26.65 $(25.83, 27.48)$
pval	0.005 $(0.004, 0.007)$	0.024 $(0.020, 0.028)$	0.000 $(0.000, 0.000)$	0.008 $(0.006, 0.011)$	0.000 $(0.000, 0.000)$	0.003 $(0.002, 0.004)$
df	8	6	10	8	12	10

Table H.7: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.85 (−0.16)	1.89 (0.03)	1.86 (−0.09)	1.86 (−0.11)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.38 (−0.10)	1.55 (0.07)	1.58 (0.11)	1.42 (−0.05)	1.25 (−0.24)	1.20 (−0.28)
$E[z_d]$	3.42	3.44 (0.09)	3.40 (−0.16)	3.43 (0.02)	3.43 (0.08)	3.44 (0.13)	3.44 (0.13)
$E[r_d]$	6.51	6.85 (0.21)	6.47 (−0.03)	6.34 (−0.11)	6.77 (0.16)	5.66 (−0.53)	6.35 (−0.10)
$E[r_f]$	0.25	0.53 (0.45)	0.27 (0.02)	0.40 (0.24)	0.43 (0.29)	1.56 (2.14)	1.29 (1.70)
$E[r_{f,5}]$	1.19	0.29 (−1.33)	0.92 (−0.40)	0.05 (−1.68)	0.33 (−1.27)	1.33 (0.21)	1.31 (0.17)
$E[r_{f,20}]$	1.88	−0.42 (−3.82)	0.54 (−2.21)	−0.97 (−4.74)	−0.34 (−3.68)	0.69 (−1.98)	0.90 (−1.63)
$SD[\Delta c]$	1.99	1.98 (−0.03)	1.90 (−0.18)	2.51 (1.06)	1.90 (−0.19)	2.19 (0.40)	1.59 (−0.84)
$SD[\Delta d]$	11.09	5.70 (−1.97)	5.04 (−2.21)	6.36 (−1.73)	5.60 (−2.01)	6.36 (−1.73)	5.49 (−2.05)
$SD[r_d]$	19.15	17.81 (−0.71)	19.88 (0.39)	18.32 (−0.44)	17.98 (−0.61)	18.71 (−0.23)	18.20 (−0.50)
$SD[r_f]$	2.72	0.88 (−3.63)	5.75 (5.97)	1.18 (−3.03)	2.86 (0.27)	1.00 (−3.39)	2.93 (0.40)
$SD[z_d]$	0.45	0.54 (1.41)	0.46 (0.08)	0.54 (1.36)	0.53 (1.23)	0.53 (1.29)	0.53 (1.24)
$AC[\Delta c]$	0.53	0.44 (−1.03)	0.46 (−0.78)	0.49 (−0.49)	0.43 (−1.08)	0.46 (−0.81)	0.41 (−1.29)
$AC[\Delta d]$	0.19	0.28 (0.81)	0.23 (0.33)	0.31 (1.14)	0.27 (0.75)	0.31 (1.15)	0.26 (0.69)
$AC[r_d]$	−0.01	0.02 (0.32)	−0.05 (−0.47)	0.01 (0.23)	0.00 (0.12)	0.00 (0.17)	0.00 (0.07)
$AC[r_f]$	0.68	0.96 (4.37)	0.83 (2.36)	0.96 (4.33)	0.69 (0.17)	0.96 (4.28)	0.70 (0.28)
$AC[z_d]$	0.89	0.94 (1.15)	0.90 (0.26)	0.94 (1.05)	0.94 (1.07)	0.94 (0.97)	0.94 (1.04)
$Corr[\Delta c, \Delta d]$	0.54	0.48 (−0.29)	0.49 (−0.22)	0.45 (−0.40)	0.48 (−0.27)	0.45 (−0.43)	0.49 (−0.24)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.26)	0.06 (0.26)	0.08 (0.40)	0.07 (0.27)	0.08 (0.45)	0.07 (0.29)
$Corr[\Delta d, r_d]$	0.07	0.25 (2.14)	0.20 (1.64)	0.28 (2.51)	0.24 (2.07)	0.28 (2.49)	0.23 (1.98)
$Corr[ep, z_{d,-1}]$	−0.16	−0.18 (−0.21)	−0.13 (0.37)	−0.17 (−0.05)	−0.18 (−0.12)	−0.16 (0.06)	−0.17 (−0.08)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.68)	0.60 (2.32)	0.70 (2.89)	0.66 (2.65)	0.68 (2.77)	0.64 (2.56)

Table H.8: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

Extended Long-Run Risk Model: $\psi = 2.0$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.43 (2.29, 2.56)	3.02 (2.79, 3.27)	5.85 (4.67, 6.93)	1.47 (1.19, 1.73)	3.43 (3.00, 3.85)	7.46 (6.49, 8.50)
β	0.9985 (0.9984, 0.9986)	0.9992 (0.9992, 0.9993)	0.9984 (0.9982, 0.9986)	0.9983 (0.9982, 0.9985)	0.9989 (0.9989, 0.9990)	0.9979 (0.9978, 0.9980)
ρ_a	–	0.9586 (0.9565, 0.9606)	0.9925 (0.9908, 0.9934)	–	0.9614 (0.9589, 0.9639)	0.9931 (0.9916, 0.9938)
σ_a	–	0.0184 (0.0176, 0.0191)	0.0285 (0.0268, 0.0297)	–	0.0195 (0.0185, 0.0205)	0.0280 (0.0270, 0.0289)
μ_y	0.0016 (0.0014, 0.0018)	0.0015 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0017 (0.0015, 0.0019)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0017)
μ_d	0.0013 (0.0009, 0.0016)	0.0014 (0.0012, 0.0016)	0.0015 (0.0013, 0.0017)	0.0002 (0.0000, 0.0005)	0.0013 (0.0011, 0.0015)	0.0014 (0.0012, 0.0016)
σ_y	0.0007 (0.0003, 0.0014)	0.0041 (0.0040, 0.0043)	0.0006 (0.0001, 0.0014)	0.0010 (0.0003, 0.0018)	0.0036 (0.0034, 0.0038)	0.0003 (0.0000, 0.0007)
ψ_d	3.00 (2.81, 3.21)	–	–	2.87 (2.63, 3.09)	–	–
π_{dy}	0.754 (0.482, 1.038)	–	–	0.744 (0.417, 1.082)	–	–
ϕ_d	1.93 (1.84, 2.03)	2.68 (2.51, 2.87)	2.76 (2.60, 2.89)	1.82 (1.72, 1.93)	3.40 (3.06, 3.72)	2.73 (2.62, 2.85)
ρ_x	0.9993 (0.9991, 0.9995)	0.9978 (0.9973, 0.9983)	0.9965 (0.9958, 0.9971)	0.9995 (0.9994, 0.9995)	0.9971 (0.9965, 0.9977)	0.9966 (0.9960, 0.9972)
ψ_x	0.0253 (0.0241, 0.0266)	0.0290 (0.0276, 0.0305)	0.0342 (0.0319, 0.0367)	0.0256 (0.0244, 0.0268)	0.0293 (0.0277, 0.0308)	0.0338 (0.0317, 0.0361)
π_{ya}	–	−0.053 (−0.072, −0.035)	−0.051 (−0.067, −0.034)	–	−0.035 (−0.051, −0.020)	−0.046 (−0.062, −0.028)
π_{da}	–	−1.044 (−1.078, −1.008)	−0.866 (−0.897, −0.836)	–	−1.007 (−1.044, −0.971)	−0.894 (−0.921, −0.869)
ρ_{σ_y}	0.9608 (0.9559, 0.9651)	–	0.7758 (0.6417, 0.8724)	0.9524 (0.9453, 0.9596)	–	0.5004 (0.3552, 0.6236)
ν_y	1.3e−5 (1.2e−5, 1.5e−5)	–	2.7e−5 (2.1e−5, 3.5e−5)	1.5e−5 (1.3e−5, 1.8e−5)	–	4.0e−5 (3.4e−5, 4.5e−5)
J	18.44 (17.74, 19.18)	13.99 (13.40, 14.54)	9.77 (9.32, 10.22)	26.27 (25.13, 27.51)	19.32 (18.80, 19.86)	11.28 (10.78, 11.76)
pval	0.018 (0.014, 0.023)	0.082 (0.069, 0.099)	0.135 (0.116, 0.157)	0.004 (0.002, 0.005)	0.037 (0.031, 0.043)	0.187 (0.162, 0.214)
df	8	8	6	10	10	8

Table H.9: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.91 (0.08)	1.85 (−0.15)	1.89 (0.03)	1.98 (0.37)	1.89 (0.02)	1.92 (0.13)
$E[\Delta d]$	1.47	1.53 (0.06)	1.73 (0.27)	1.84 (0.38)	0.30 (−1.23)	1.58 (0.11)	1.72 (0.26)
$E[z_d]$	3.42	3.42 (−0.03)	3.40 (−0.16)	3.39 (−0.20)	3.49 (0.51)	3.41 (−0.07)	3.40 (−0.17)
$E[r_d]$	6.51	6.77 (0.16)	5.90 (−0.38)	5.83 (−0.43)	5.69 (−0.51)	5.49 (−0.64)	5.75 (−0.48)
$E[r_f]$	0.25	0.09 (−0.27)	0.45 (0.33)	0.15 (−0.18)	0.99 (1.22)	1.25 (1.64)	0.30 (0.08)
$E[r_{f,5}]$	1.19	−0.79 (−2.93)	0.37 (−1.21)	0.57 (−0.93)	1.43 (0.34)	1.27 (0.12)	1.51 (0.46)
$E[r_{f,20}]$	1.88	−2.35 (−7.03)	−0.17 (−3.40)	0.18 (−2.82)	1.14 (−1.22)	0.93 (−1.58)	1.45 (−0.70)
$SD[\Delta c]$	1.99	2.03 (0.09)	1.98 (−0.03)	2.12 (0.26)	2.15 (0.32)	1.69 (−0.64)	2.16 (0.34)
$SD[\Delta d]$	11.09	5.38 (−2.09)	7.58 (−1.28)	9.47 (−0.59)	5.40 (−2.08)	7.81 (−1.20)	9.61 (−0.54)
$SD[r_d]$	19.15	18.67 (−0.25)	18.14 (−0.53)	18.53 (−0.33)	18.27 (−0.46)	18.59 (−0.29)	18.33 (−0.43)
$SD[r_f]$	2.72	2.47 (−0.50)	2.99 (0.53)	2.66 (−0.12)	2.45 (−0.53)	3.06 (0.68)	2.55 (−0.34)
$SD[z_d]$	0.45	0.51 (0.93)	0.51 (0.91)	0.49 (0.56)	0.54 (1.47)	0.50 (0.72)	0.50 (0.78)
$AC[\Delta c]$	0.53	0.44 (−0.95)	0.43 (−1.06)	0.45 (−0.92)	0.45 (−0.85)	0.41 (−1.26)	0.45 (−0.88)
$AC[\Delta d]$	0.19	0.24 (0.49)	0.21 (0.20)	0.17 (−0.21)	0.25 (0.52)	0.22 (0.29)	0.17 (−0.22)
$AC[r_d]$	−0.01	−0.03 (−0.25)	0.02 (0.36)	−0.03 (−0.20)	0.01 (0.27)	0.02 (0.32)	0.00 (0.08)
$AC[r_f]$	0.68	0.69 (0.07)	0.71 (0.50)	0.70 (0.35)	0.65 (−0.48)	0.72 (0.66)	0.72 (0.57)
$AC[z_d]$	0.89	0.93 (0.89)	0.93 (0.88)	0.91 (0.51)	0.94 (1.13)	0.92 (0.72)	0.92 (0.62)
$Corr[\Delta c, \Delta d]$	0.54	0.51 (−0.13)	0.48 (−0.27)	0.52 (−0.10)	0.52 (−0.09)	0.45 (−0.41)	0.49 (−0.23)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.20)	0.09 (0.55)	0.10 (0.76)	0.06 (0.18)	0.09 (0.64)	0.10 (0.75)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.77)	0.14 (0.80)	0.07 (0.04)	0.22 (1.80)	0.13 (0.78)	0.06 (−0.05)
$Corr[ep, z_{d,-1}]$	−0.16	−0.23 (−0.67)	−0.15 (0.18)	−0.13 (0.33)	−0.20 (−0.41)	−0.13 (0.35)	−0.12 (0.42)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.64)	0.66 (2.66)	0.63 (2.52)	0.67 (2.73)	0.64 (2.57)	0.63 (2.53)

Table H.10: Extended long-run risk models. Data and average model-implied moments. t-statistics are in parentheses.

Extended Long-Run Risk Model: $\psi = 1.5$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.17 (2.06, 2.28)	2.61 (2.32, 2.96)	4.57 (3.67, 5.54)	1.69 (1.37, 2.28)	3.42 (2.90, 3.97)	5.99 (4.65, 8.05)
β	0.9990 (0.9990, 0.9991)	0.9995 (0.9995, 0.9995)	0.9991 (0.9989, 0.9992)	0.9988 (0.9987, 0.9989)	0.9994 (0.9993, 0.9994)	0.9986 (0.9984, 0.9988)
ρ_a	–	0.9564 (0.9536, 0.9592)	0.9898 (0.9860, 0.9929)	–	0.9611 (0.9582, 0.9641)	0.9848 (0.9728, 0.9926)
σ_a	–	0.0178 (0.0168, 0.0188)	0.0260 (0.0231, 0.0287)	–	0.0196 (0.0184, 0.0207)	0.0229 (0.0197, 0.0265)
μ_y	0.0016 (0.0014, 0.0018)	0.0015 (0.0014, 0.0016)	0.0016 (0.0015, 0.0017)	0.0017 (0.0015, 0.0019)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0017)
μ_d	0.0012 (0.0009, 0.0016)	0.0014 (0.0011, 0.0016)	0.0015 (0.0013, 0.0017)	0.0003 (0.0000, 0.0008)	0.0013 (0.0010, 0.0015)	0.0012 (0.0009, 0.0015)
σ_y	0.0007 (0.0003, 0.0013)	0.0041 (0.0039, 0.0043)	0.0006 (0.0000, 0.0013)	0.0013 (0.0004, 0.0027)	0.0033 (0.0031, 0.0035)	0.0003 (0.0000, 0.0008)
ψ_d	3.07 (2.88, 3.27)	–	–	2.98 (2.74, 3.19)	–	–
π_{dy}	0.692 (0.400, 0.970)	–	–	0.619 (0.152, 1.018)	–	–
ϕ_d	2.02 (1.91, 2.13)	2.65 (2.42, 2.92)	2.62 (2.44, 2.79)	1.95 (1.83, 2.08)	3.73 (3.33, 4.17)	2.57 (2.32, 2.99)
ρ_x	0.9994 (0.9993, 0.9995)	0.9984 (0.9978, 0.9989)	0.9978 (0.9972, 0.9983)	0.9995 (0.9992, 0.9995)	0.9973 (0.9966, 0.9980)	0.9983 (0.9976, 0.9989)
ψ_x	0.0253 (0.0242, 0.0265)	0.0276 (0.0262, 0.0291)	0.0305 (0.0283, 0.0326)	0.0259 (0.0246, 0.0279)	0.0285 (0.0266, 0.0301)	0.0287 (0.0269, 0.0308)
π_{ya}	–	−0.055 (−0.078, −0.035)	−0.053 (−0.071, −0.033)	–	−0.033 (−0.049, −0.018)	−0.055 (−0.076, −0.035)
π_{da}	–	−1.065 (−1.111, −1.024)	−0.891 (−0.954, −0.833)	–	−1.004 (−1.049, −0.964)	−0.975 (−1.040, −0.905)
ρ_{σ_y}	0.9545 (0.9489, 0.9601)	–	0.7629 (0.6252, 0.8652)	0.8894 (0.2121, 0.9462)	–	0.2819 (0.0259, 0.5148)
ν_y	1.4e−5 (1.3e−5, 1.6e−5)	–	2.8e−5 (2.2e−5, 3.6e−5)	2.0e−5 (1.5e−5, 4.8e−5)	–	4.1e−5 (3.5e−5, 4.7e−5)
J	19.38 (18.62, 20.24)	15.22 (14.53, 15.84)	11.18 (10.65, 11.77)	29.58 (28.18, 31.87)	20.70 (20.16, 21.28)	14.39 (13.82, 14.94)
pval	0.013 (0.009, 0.017)	0.055 (0.045, 0.069)	0.084 (0.067, 0.100)	0.001 (0.000, 0.002)	0.023 (0.019, 0.028)	0.073 (0.060, 0.087)
df	8	8	6	10	10	8

Table H.11: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.93 (0.15)	1.83 (−0.24)	1.90 (0.06)	1.98 (0.36)	1.89 (0.02)	1.93 (0.16)
$E[\Delta d]$	1.47	1.46 (−0.02)	1.68 (0.22)	1.78 (0.32)	0.43 (−1.09)	1.53 (0.05)	1.50 (0.03)
$E[z_d]$	3.42	3.43 (0.03)	3.40 (−0.13)	3.40 (−0.19)	3.49 (0.49)	3.42 (−0.04)	3.41 (−0.07)
$E[r_d]$	6.51	6.83 (0.20)	6.15 (−0.23)	6.00 (−0.32)	5.83 (−0.42)	5.45 (−0.66)	5.91 (−0.38)
$E[r_f]$	0.25	0.01 (−0.40)	0.66 (0.67)	0.09 (−0.27)	1.10 (1.39)	1.31 (1.73)	0.30 (0.08)
$E[r_{f,5}]$	1.19	−0.85 (−3.01)	0.53 (−0.98)	0.35 (−1.25)	1.45 (0.37)	1.30 (0.16)	1.69 (0.72)
$E[r_{f,20}]$	1.88	−2.69 (−7.58)	−0.18 (−3.42)	−0.48 (−3.92)	0.96 (−1.53)	0.87 (−1.66)	1.30 (−0.96)
$SD[\Delta c]$	1.99	2.07 (0.16)	1.97 (−0.04)	2.11 (0.24)	2.21 (0.45)	1.56 (−0.89)	2.05 (0.11)
$SD[\Delta d]$	11.09	5.60 (−2.01)	7.49 (−1.32)	8.89 (−0.80)	5.75 (−1.95)	7.84 (−1.19)	8.53 (−0.94)
$SD[r_d]$	19.15	18.70 (−0.24)	17.81 (−0.71)	18.13 (−0.54)	18.38 (−0.40)	18.51 (−0.33)	17.83 (−0.69)
$SD[r_f]$	2.72	2.31 (−0.81)	3.01 (0.57)	2.66 (−0.12)	2.03 (−1.36)	3.08 (0.72)	2.67 (−0.11)
$SD[z_d]$	0.45	0.51 (0.97)	0.52 (1.08)	0.50 (0.84)	0.54 (1.48)	0.50 (0.79)	0.52 (1.13)
$AC[\Delta c]$	0.53	0.45 (−0.91)	0.43 (−1.06)	0.44 (−0.95)	0.46 (−0.79)	0.41 (−1.34)	0.44 (−1.01)
$AC[\Delta d]$	0.19	0.26 (0.61)	0.21 (0.20)	0.17 (−0.19)	0.26 (0.69)	0.23 (0.32)	0.17 (−0.21)
$AC[r_d]$	−0.01	−0.03 (−0.27)	0.03 (0.43)	−0.02 (−0.15)	0.01 (0.19)	0.02 (0.34)	0.01 (0.24)
$AC[r_f]$	0.68	0.69 (0.07)	0.71 (0.51)	0.71 (0.48)	0.64 (−0.59)	0.73 (0.70)	0.73 (0.68)
$AC[z_d]$	0.89	0.93 (0.90)	0.94 (1.00)	0.93 (0.75)	0.94 (1.11)	0.93 (0.76)	0.93 (0.93)
$Corr[\Delta c, \Delta d]$	0.54	0.50 (−0.16)	0.48 (−0.28)	0.50 (−0.16)	0.50 (−0.17)	0.45 (−0.39)	0.49 (−0.23)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.23)	0.08 (0.47)	0.09 (0.59)	0.06 (0.24)	0.09 (0.65)	0.08 (0.43)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.89)	0.14 (0.83)	0.09 (0.24)	0.23 (2.00)	0.14 (0.81)	0.09 (0.29)
$Corr[ep, z_{d,-1}]$	−0.16	−0.23 (−0.67)	−0.16 (0.02)	−0.16 (0.09)	−0.20 (−0.40)	−0.14 (0.31)	−0.16 (0.05)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.66 (2.66)	0.66 (2.66)	0.65 (2.59)	0.67 (2.76)	0.64 (2.55)	0.65 (2.62)

Table H.12: Extended long-run risk models. Data and average model-implied moments. t-statistic are in parentheses.