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Abstract

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Learning over the Business Cycle: Policy Implications*

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April 2, 2020

Abstract

This paper studies the policy implications of the endogeneity of information about the state of the economy. The business cycle can be made less noisy, and more efficient, by incentivizing firms to vary their pricing and production decisions more with their beliefs about the state of the economy. This calls for countercyclical taxes complemented by a monetary policy that “leans against the wind.” The optimal policies trade-off allocative efficiency for informational efficiency.

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1 Introduction

Economic agents monitor macroeconomic statistics and market signals such as prices for clues about the state of the economy. But the informativeness of such signals is a function of other agents' behavior. How does this endogeneity of information affect the efficiency of the business cycle and the design of optimal policy?

The contribution of this paper is to address this question within a micro-founded, general-equilibrium, macroeconomic model. Our main lesson is that the optimal policy combines countercyclical taxes and a monetary policy that “leans against the wind.” A complementary, methodological contribution is to adapt the primal approach of the Ramsey literature to setting with both incomplete and endogenous information.

Preview. Our model shares the same core micro-foundations as the textbook New Keynesian model, except for three, main modifications. First, we let the nominal rigidity originate in an information friction: firms set their prices on the basis of dispersed, noisy information about the state of the economy. Second, we let the informational friction be also the source of a real rigidity: firms make some real production decisions on the basis of the same noisy information. And third, we let each firm observe various market signals or macroeconomic statistics whose informational content is endogenous to the choices of other firms.

Although subsets of these three features have appeared in previous works, their *combination* is novel to the literature—and it is essential for our results. We next describe the role played by each of them and explain how they shape the optimal policy.

The first feature, the information-driven nominal rigidity, needs no motivation: it is familiar from [Woodford \(2003a\)](#), [Mankiw and Reis \(2002\)](#), [Mackowiak and Wiederholt \(2009\)](#) and a large follow-up literature. But if it were not for the other two features, the policy problem would be trivial: the full-information first best would be implementable with a subsidy that offsets the monopoly distortion and a monetary policy that stabilizes the price level.

The logic is the same as in the textbook New Keynesian model: the optimal subsidy corrects the monopoly distortion and the optimal monetary policy neutralizes the nominal rigidity. That the nominal rigidity originates from an information friction rather than Calvo-like sticky prices makes no difference for this logic. Hence, if it were not for the two other features of our model, which are absent from the aforementioned works, the informational friction would have been inconsequential for welfare and there would be no interesting, new lesson for policy.¹

Thus consider the second feature, the information-driven real rigidity. This feature, which is borrowed from [Angeletos and La'O \(2010, 2020\)](#), guarantees that the full-information first

¹Like the textbook New Keynesian model, our model allows for lump-sum taxation. Without it, the requisite subsidy cannot be financed in a non-distortionary way and the first best is not attainable. But as shown in [Correia, Nicolini, and Teles \(2008\)](#), this does not upset the essence of the argument: it only replaces the first best with the kind of flexible-price second best characterized in [Lucas and Stokey \(1983\)](#).

best is unattainable regardless of the tax and monetary policies: relative to the first best, some misallocation in resources and some dispersion in relative prices is inevitable, and indeed desirable, given that firms must base at least some of their production decisions on heterogeneous information about the state of the economy.

This is where the third, and most novel, feature comes into play. The welfare bite of the aforementioned real rigidity naturally depends on how precise the available information is. And because this information is endogenous to the choices of others, a new role for policy emerges.

By incentivizing firms to respond more aggressively to variation in their beliefs about the state of the economy, a countercyclical tax on firm revenue or production improves the aggregation of information, thus also reducing the welfare losses due to incomplete information. But because such a tax instrument is too blunt, in a sense we explain below, it has to be complemented with a monetary policy that raises the interest rate above the natural rate during booms (and lowers it during recessions).

Mechanisms at work. To understand the precise role played by monetary policy, it is useful to compare our setting to that of [Angeletos and La'O \(2020\)](#). That paper has shown that the kind of information-driven real rigidity accommodated here redefines the concepts of the “divine coincidence” and the “output gap” that underlie the modern theory of optimal monetary policy. The optimal policy still aims at neutralizing the nominal rigidity, but this no longer coincides with minimizing the gap between equilibrium and first-best output, simply because the latter is no longer the right benchmark. Instead, the appropriate target level of output is one that displays both less sensitivity, or more inertia, with respect to innovations in underlying fundamentals (TFP) and a positive level of noise- or sentiment-driven fluctuations.

Whereas this previous work treats the information structure as exogenous, here we let it be endogenous—and this drives our novel policy conclusions. In [Angeletos and La'O \(2020\)](#), the appropriate gauge for aggregate output is modified for the reason already explained, but the basic policy guidelines remain unchanged: the optimal tax policy serves only the role of correcting the monopoly distortion and the optimal monetary policy serves only the role of neutralizing the nominal rigidity, or replicating the relevant flexible-price outcomes. In our setting, instead, the optimal policies strike a balance between these familiar goals and the novel goal of inducing a better aggregation of information through prices and quantity signals. In other words, the optimal policies trade-off allocative efficiency for informational efficiency.

Our main result is a characterization of this trade-off and its optimal resolution. In particular, we identify two “informational wedges” that serve as sufficient statistics of how the endogeneity of information affects the optimal policy mix. One of them relates to the learning through quantities, the other to the learning through prices. We then show that both wedges enter the determination of optimal taxes, whereas only the second enters the determination of monetary policy. We conclude that, in the realistic case in which both kinds of learning are

present, the optimal policy mix combines countercyclical taxes with a monetary policy that leans against the wind.

Countercyclical taxes serve the goal of improving the informational content of *both* quantity and price signals. By contrast, monetary policy is exclusively connected to the informational content of prices: if all learning were to take place through quantity signals, then optimal monetary policy would only serve the goal of neutralizing the nominal rigidity. Learning from prices is therefore essential for breaking the “divine coincidence” in our setting.

Let us explain the logic. When firms vary their production decisions more aggressively with their private information about the state of the economy, they must also vary their prices more aggressively (and in the opposite direction than their quantities), simply because they face downward-sloping demand curves. This means that when firms respond more aggressively to their own information, *both* the price and the quantity signals become more informative. To induce firms to internalize this information externality, the optimal policy must make firms’ expected net returns more sensitive to the state of the economy. Countercyclical taxes serve this goal, whether learning takes place through quantity signals, price signals, or both.

When price signals are absent and the information externality operates entirely through signals of production, a countercyclical tax on production alone does the job of incentivizing firms to use their information in the socially optimal way. In this case, monetary policy is left with the sole job of neutralizing the nominal rigidity—as desired when information is exogenous. The particular monetary policy that neutralizes nominal rigidities and implements flexible-price allocations is one that targets a negative correlation between the price level and aggregate output; that is, it “leans against the wind” ([Angeletos and La’O, 2020](#)).

When instead learning takes place also through prices, such a tax is not sufficient because it is too blunt. The ideal tax system subsidizes less the production choices that are free to adjust after prices have been set, relative to production choices determined at a similar time as prices; doing so increases the informational content of both quantity and price signals. But note that this would require not only an unrealistic knowledge of which firm choices are set under what information, but also an extremely fine-tuned, differential tax system.

Thus, a monetary policy that leans *even more* against the wind relative to the one that implements flexible prices mimics such a differential subsidy. And in contrast to a differential subsidy, this monetary policy implements the socially optimal allocation without requiring the policymaker to know the “IO details” of which production choices and prices are set when and on the basis of what information.

We conclude that a countercyclical tax on production alone improves learning through both quantities and prices, but it does not allow the planner to regulate separately the two kinds of learning. By contrast, combining such a tax with a state-contingent monetary policy facilitates a finer regulation of the two forms of learning.

Related Literature. Methodologically, our paper is at the crossroads of two literatures: the Ramsey literature on optimal taxation and optimal monetary policy (Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1994; Correia, Nicolini, and Teles, 2008); and the literature that studies the efficient decentralized use and aggregation of information (Vives, 1988; Angeletos and Pavan, 2007, 2009).

The latter literature develops and characterizes a notion of constrained efficiency for a certain class of incomplete-information, linear-quadratic games. Relative to those works, our analysis considers different and richer micro-foundations. We are primarily concerned with the characterization of a particular mix of fiscal and monetary policies, as opposed to an abstract notion of constrained efficiency. Nevertheless, the optimal policy problem in our paper can be understood by mapping it to a more abstract constrained-efficiency problem—one that is conceptually the same as that developed in Vives (1988) and Angeletos and Pavan (2007, 2009). We view this translation as an integral part of our contribution.

In so doing, we also offer a concrete example of how the primal approach from the Ramsey literature (Lucas and Stokey, 1983; Chari and Kehoe, 1999) can be adapted to the kind of endogenous-information settings that we are interested in. As in that literature, the analysis becomes much more transparent and straightforward once the policy problem is posed in terms of implementable allocations as opposed to policy instruments. Unlike that literature, however, we must also take into account how different implementable allocations are associated with different information structures, due to the endogeneity of the signals that agents observe about one another's choices. As a result, the implementability results that we develop in this paper entail a fixed-point relation between allocations and information structures. Since such a fixed-point relation is endemic to noisy rational expectations equilibrium (REE) settings, the primal approach we take in this paper could be of broader methodological value.²

The monetary aspect of our analysis is closely related to the following set of papers that study optimal monetary policy in the presence of informational frictions: Ball, Mankiw, and Reis (2005); Adam (2007); Lorenzoni (2010); Paciello and Wiederholt (2014); Angeletos and La'O (2020). All of these papers abstract from endogenous aggregation of information. With the exception of Angeletos and La'O (2020), they also equate the informational friction with a particular form of nominal friction. By contrast, the lessons we deliver in this paper hinge on the joint property that the informational friction interferes with real allocations even when there is no nominal friction (meaning either that prices are flexible or that monetary policy replicates flexible-price allocations) and that information is endogenously aggregated.

Finally, our focus on learning through prices brings to mind Amador and Weill (2010) and Vives (2017). In a micro-founded macro context, Amador and Weill (2010) show how such

²Complementary in this respect are also Laffont (1985) and Messner and Vives (2001). These papers do not take the Ramsey-like primal approach, but share the spirit of studying optimality directly over a particular set of permissible strategies.

learning can crowd out valuable private information in the absence of policy. We complement this work by characterizing the optimal mix of policies that help correct the problem. [Vives \(2017\)](#) studies how the informational externality through prices matters in a microeconomic context. Our context is obviously very different, as are our lessons for macroeconomic policy.

Layout. The rest of the paper is organized as follows. Section 2 introduces the baseline model, which abstracts from nominal rigidity and monetary policy. Section 3 studies constrained efficiency, implementability, and optimal policy in the baseline model. These results serve as a stepping stone for the analysis in Section 4. There, we extend the model so that the informational friction becomes the source of both real and nominal rigidity, and we proceed to study the optimal mix of fiscal and monetary policies in Section 5. Section 6 concludes.

2 The Baseline, Non-Monetary Model

We start with a stripped-down version of our framework, which abstracts from nominal rigidity and monetary policy. This serves three related goals. First, it permits a cleaner exposition of how the primal approach from the Ramsey literature can be adapted to both dispersed and endogenous information and, by the same token, how the policy problem of interest can be mapped to the more abstract problems studied in [Vives \(1988\)](#) and [Angeletos and Pavan \(2007, 2009\)](#). Second, it sheds light on the role of taxes. And third, it paves the way for the more complicated but also more interesting analysis that allows for nominal rigidity and non-neutral monetary policy.

Time and geography. Time is discrete and periods are indexed by $t \in \{0, 1, 2, \dots\}$. There is a representative household consisting of a consumer and a continuum of workers. There is a continuum of “islands”, indexed by $i \in I = [0, 1]$, which define the boundaries of local labor markets as well as the “geography” of information: information is symmetric within an island, but asymmetric across islands. Each island is inhabited by a representative firm, which specializes in the production of differentiated commodities.

Each period has two stages. In stage 1, the representative household sends a worker to each of the islands. Local labor markets then open, workers decide how much labor to supply, firms decide how much labor to demand, and local wages adjust so as to clear the local labor market. At this point, workers and firms in each island have perfect information regarding local productivity, but imperfect information regarding the productivity in other islands. After employment and production choices are sunk, workers return home and the economy transitions to stage 2. In stage 2, all information that was previously dispersed is aggregated and becomes publicly known, and commodity markets open. Quantities are now pre-determined by the exogenous productivities and the endogenous employment choices made during stage 1, but prices adjust so as to clear product markets.

The representative household. The utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \bar{\beta}^t \left[U(C_t) - \int_I V(n_{i,t}) di \right]$$

with $U(C) = \frac{1}{1-\gamma} C^{1-\gamma}$ and $V(n) = \frac{1}{\epsilon} n^\epsilon$, where $\gamma \geq 0$ parameterizes the income elasticity of labor supply (also, the reciprocal of the elasticity of intertemporal substitution), $\epsilon \geq 1$ parameterizes the Frisch elasticity of labor supply, $n_{i,t}$ is the labor of the worker who gets located on island i during stage 1 of period t , and C_t is aggregate consumption. The latter is the CES aggregator of all of the commodities that the household purchases and consumes during stage 2:

$$C_t = \left[\int_I c_{i,t}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

where $c_{i,t}$ is the quantity the household consumes in period t of the commodity produced by the representative firm on island i , and $\rho > 1$ is the elasticity of substitution across commodities of different islands.

The representative household receives labor income and profits from all the islands in the economy. Its budget constraint is thus given by the following:

$$\int_I p_{i,t} c_{i,t} di + B_{t+1} \leq \int_I \pi_{i,t} di + \int_I w_{i,t} n_{i,t} di + R_t B_t,$$

where $p_{i,t}$ is the nominal price of the commodity produced by the representative firm on island i , $\pi_{i,t}$ is the profit of that firm, $w_{i,t}$ is the nominal wage on island i , and R_t is the nominal gross rate of return on the riskless bond, and B_t is the amount of bonds held in period t .

The objective of each household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the worker-members of the household as solving a team problem: they share the same objective (household utility) but have different information sets when making their labor-supply choices. Formally, the household sends off during stage 1 its workers to different islands with instructions on how to supply labor as a function of (i) the information that will be available to them at that stage and (ii) the wage that will prevail in their local labor market. In stage 2, the consumer-member collects all of the income that the worker-members have collected and decides how much to consume in each of the commodities and how much to save (or borrow) in the riskless bond.

Firms. The output of the representative firm on island i during period t is given by

$$q_{i,t} = A_{i,t} (n_{i,t})^\theta$$

where $A_{i,t}$ is the productivity in island i , $n_{i,t}$ is the firm's employment, and $\theta \in (0, 1]$ is the degree of diminishing returns in production. The firm's realized profit is given by

$$\pi_{i,t} = p_{i,t} q_{i,t} - w_{i,t} n_{i,t}$$

Finally, the objective of the firm is to maximize its expectation of the representative consumer's valuation of its profit, namely, its expectation of $U'(C_t) \pi_{i,t}$.

Aggregates and market clearing. Labor markets operate in stage 1, while product markets operate in stage 2. The wage clears the labor market within each island so that labor supply equals labor demand. For the commodities, market clearing in each product market implies that consumption is equal to output: $c_{i,t} = q_{i,t} \quad \forall i$. Nominal prices are normalized so that the ideal price index, $P_t \equiv \left[\int p_{it}^{1-\rho} di \right]^{\frac{1}{1-\rho}}$, is fixed at 1. Aggregate output and employment are defined by, respectively,

$$Q_t \equiv \left[\int_I q_{i,t}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad N_t \equiv \int_I n_{i,t} di.$$

Aggregate and idiosyncratic productivity shocks. We assume that the island-specific productivities $A_{i,t}$ are log-normally distributed in the cross-section of islands:

$$a_{i,t} \equiv \log A_{i,t} = \bar{a}_t + \xi_{k,t}$$

where \bar{a}_t is the aggregate productivity shock and $\xi_{i,t}$ is an idiosyncratic, island-specific, shock. The aggregate shock is drawn from a Normal distribution with mean $\mu_{A,t}$ and variance $\sigma_{A,t}^2$, while the idiosyncratic shock is drawn from a Normal distribution with mean 0 and variance $\sigma_{\xi,t}^2$. The variables $\mu_{A,t}$, $\sigma_{A,t}$ and $\sigma_{\xi,t}$ are common knowledge in period t but need not be deterministic: they could be arbitrary functions of the (public) history of past productivity shocks.³ For future reference, we let $\kappa_{A,t} \equiv \sigma_{A,t}^{-2}$ and $\kappa_{\xi,t} \equiv \sigma_{\xi,t}^{-2}$.

Information. In stage 1, when key employment and production choices are made, the firms and workers that are located in any given island face uncertainty about what's going on in other islands. More specifically, these agents get to see the productivity of their own island but not the productivities of other islands. Because local productivities are correlated (through the aggregate productivity shocks), local productivity serves also as a noisy private signal of the distribution of productivities and information in other islands.⁴

In addition to this information, all firms and workers observe exogenous private and public signals about the underlying aggregate productivity. The private signal is given by

$$x_{i,t}^a = \bar{a}_t + \varepsilon_{i,t}^{xa},$$

and the public one by

$$z_t^a = \bar{a}_t + \varepsilon_t^{za},$$

where $\varepsilon_{i,t}^{xa} \sim \mathcal{N}(0, \sigma_{xa}^2)$ and $\varepsilon_t^{za} \sim \mathcal{N}(0, \sigma_{za}^2)$ are noise (the former one idiosyncratic, the latter one common). For future reference, we let $\kappa_{xa} \equiv \sigma_{xa}^{-2}$, $\kappa_{za} \equiv \sigma_{za}^{-2}$.

³For example, the special case where aggregate productivity follows a random walk could be nested by letting $\mu_t = \bar{a}_{t-1}$ and letting σ_t be a constant.

⁴The assumption that firms and workers know their own productivities perfectly is inessential; all of our results go through if we allow for uncertainty about local as well as aggregate productivity.

Finally, firms and workers also observe two *endogenous* signals about the production activity that is taking place in other islands, one public and one private. In particular, letting

$$Q_t \equiv \left[\int_I q_{i,t}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

measure aggregate output, the endogenous public and private signals are given by, respectively,

$$z_t^q = \log Q_t + \varepsilon_t^{zq} \quad \text{and} \quad x_{i,t}^q = \log Q_t + \varepsilon_{i,t}^{xq},$$

where $\varepsilon_t^{zq} \sim \mathcal{N}(0, \sigma_{zq}^2)$ and $\varepsilon_{i,t}^{xq} \sim \mathcal{N}(0, \sigma_{xq}^2)$ are noises, the first one common and the second one idiosyncratic across islands.

The signal z_t^q is meant to capture macroeconomic data released by various government agencies. For now, this corresponds to a statistic of aggregate GDP. In the extended monetary model (Section 4), we add a signal of the aggregate price level. And although we do not explicitly consider them, signals of aggregate employment or the interest rate introduce the same kind of public learning as that captured by z_t^q here.

The signal x_t^q , on the other hand, is meant to be a proxy for all kinds of private learning about the state of the economy. For instance, one can think of firms collecting private data about product conditions in particular markets, as in [Townsend \(1983\)](#) and [Amador and Weill \(2010\)](#), or of them extracting information from idiosyncratic market transactions, as in [Lucas \(1973\)](#). The exact modeling of such private sources of information is left outside the analysis, but the essential feature we do capture is their dependance on the behavior of others.⁵

3 Efficiency, Implementation, and Optimal Policy

In this section we define and characterize the relevant efficiency benchmark, study the informational externality that underlies it, and show how it can be implemented with taxes.

3.1 Some preliminaries, notation, and the efficiency definition

In addition to the usual resource constraint, the fictitious planner of our economy faces an informational constraint: the employment and production choices he can dictate (or incentivize) to any given island i must be measurable in the vector $(a_{i,t}, x_{i,t}^a, z_t^a, x_{i,t}^q, z_t^q)$. This measurability constraint epitomizes the informational friction faced by the market mechanism and the planner alike.

⁵The assumed signals can be thought as special cases of a more general class of signals of the form $\omega_t^q = \log Q_t + \varepsilon_t^{agg} + \varepsilon_t^{idio}$, where ε_t^{agg} is aggregate noise and ε_t^{idio} is idiosyncratic noise. Such signals could be the product of rational inattention over macroeconomic statistics: the measurement error is the source of aggregate noise and rational inattention is the source of idiosyncratic noise. Our results readily extend to such signals, modulo of course that we do not explicitly model the attention choice.

Next, note that because there is no capital, each period is completely separate from all other periods. For this reason we may drop the time t subscript and analyze the problem as if it were static. The time dimension will only matter when we study equilibrium implementation and, in particular, the determination of interest rates.

In principle, we could still consider any allocation in which the employment and output of an island are arbitrary functions of the aforementioned signals. For the analysis to remain tractable, however, we must restrict attention to allocations that preserve the Gaussian structure of the information structure. This is true as long as the employment and production choices of an island are log-linear in the private (island-specific) signals.

Indeed, as long as this is the case, we can guess and verify that the information that is available to any island i in stage 1 can be summarized by the triplet (a_i, x_i, z) , where a_i is the current local productivity; x_i is a Gaussian sufficient statistic for all the private (local) information about the aggregate state of the economy; and z is a Gaussian sufficient statistic for all the public information.

The precise definitions of these statistics, which are exogenous to the behavior of any single individual or island but are ultimately endogenous to the entire economy, will be provided shortly. For now, it suffices to note that the aforementioned tractability requirement boils down to restricting attention to the space of allocations in which the output of an island is a log-linear function of the triplet $\omega_i \equiv (a_i, x_i, z)$, or

$$q_i = q(\omega_i) = \exp\{\varphi_0 + \varphi_a a_i + \varphi_x x_i + \varphi_z z\}, \quad (1)$$

for arbitrary scalar coefficients $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z) \in \mathbb{R}^4$. Let Ω denote the set of all possible ω_i ; thus $\omega_i \in \Omega$.

For any such allocation, aggregate output is itself a log-linear function of the aggregate fundamental and the public statistic:

$$Q = Q(\bar{a}, z) = \exp\{\varphi'_0 + (\varphi_a + \varphi_x)\bar{a} + \varphi_z z\} \quad (2)$$

for some constant φ'_0 that differs from φ_0 due to the aggregate effects of heterogeneity.⁶ It follows that the endogenous signals x_i^q and z^q can be transformed into Gaussian signals about the underlying aggregate productivity, thus preserving the Gaussian structure of the information. The aforementioned sufficient statistics can then be constructed by taking, in effect, the projection of the aggregate productivity to the relevant signals.

We thus have that $x_i - \theta$ is idiosyncratic Gaussian noise with variance κ_x^{-1} , and similarly $z - \theta$ is aggregate Gaussian noise with variance κ_z^{-1} , where κ_x and κ_z denote the precisions of these two statistics. For familiar reasons, the precisions of these statistics can be expressed as

⁶See the Appendix for the exact characterization of the gap $\varphi'_0 - \varphi_0$ as a function of the CES parameter ρ and the variances of the idiosyncratic productivity and the idiosyncratic noises. For all essential purposes, however, one can safely ignore the “detail” that $\varphi'_0 \neq \varphi_0$.

the sum of the precisions of all the underlying, component signals. And because the precision of the information contained in the signals x_i^q and z^q about the underlying fundamental hinges on how strongly economic activity responds to it, κ_x and κ_z are endogenous to any allocation chosen by the planner, in the manner described below.

Lemma 1. *Take any log-linear allocation as in (1), with arbitrary $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z) \in \mathbb{R}^4$. The precisions of the sufficient statistics x and z generated by this strategy are given by*

$$\kappa_x = \sigma_\xi^{-2} + \sigma_{x_a}^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{x_q}^{-2} > 0 \quad \text{and} \quad \kappa_z = \sigma_{z_a}^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{z_q}^{-2} > 0. \quad (3)$$

To understand this result, note that the endogenous signals x_i^q and z^q about aggregate output can be transformed to simple Gaussian signals about the underlying aggregate productivity because aggregate output is itself a log-linear function of \bar{a} , as in (2). From this equation (2), note that the sum $\varphi_a + \varphi_x$ determines the sensitivity of aggregate output to the underlying aggregate productivity. But this implies that for any given exogenous noises, it is precisely this sensitivity $\varphi_a + \varphi_x$ that determines how much information the endogenous signals x_i^q and z^q contain about aggregate productivity: the more sensitive is aggregate output to aggregate productivity, the more informative these signals. It follows that the sum $\varphi_a + \varphi_x$ thus determines the precisions κ_x and κ_z of the overall private and public information.

Next, it is straightforward to show that, once we restrict attention to the class of allocations that satisfy (1), welfare can be expressed as a function of the strategy coefficients $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and the information precisions $\kappa = (\kappa_x, \kappa_z) \in \mathbb{R}_+^2$.

Lemma 2. *There exists a function $\mathcal{W} : \mathbb{R}^4 \times \mathbb{R}_+^2 \rightarrow \mathbb{R}$ such that, for any allocation that satisfies (1), welfare (ex ante utility) is given by $W = \mathcal{W}(\varphi; \kappa)$.*

With these observations in mind, we define constrained efficiency (within the log-linear class of strategies) as follows.

Definition 1. *A constrained efficient allocation is a pair (φ, κ) , consisting of a log-linear production strategy φ and precisions κ , that maximizes welfare subject to condition (3).*

Thus, the notion of constrained efficiency we adopt is similar to that of [Angeletos and Pavan \(2007, 2009\)](#), appropriately adapted to our micro-founded, business-cycle economy. Condition (1) serves only the need for tractability. Condition (3), on the other hand, is central, as it ensures that the planner must take into account how different allocations sustain different information structures.⁷

⁷At the same time, it is important to note that our notion of efficiency does not endow the planner with any communication channels in addition to those already available to the market: the planner is still prohibited from transferring information from one island to another in any way other than through the particular endogenous signals (x_i^q, z^q) .

3.2 Optimal Allocations

We now proceed to characterize the constrained efficient and the best implementable allocations. In general, these allocations could differ. We will show that this is not the case in our baseline model (although it is the case in our monetary extension).

Let us start with the constrained-efficient allocation. Recall that we can express welfare as $W(\varphi; \kappa)$ where $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and $\kappa = (\kappa_x, \kappa_z)$; a closed-form expression of this function is provided in the Appendix. Next, recall that the precisions induced by any given strategy are characterized by condition (3) in Lemma 1; let $K : \mathbb{R} \rightarrow \mathbb{R}_+^2$ denote the function that maps the sum $\bar{\varphi} \equiv \varphi_a + \varphi_x$ to the values of κ_x and κ_z seen in (3). We can then express the planner's problem as follows.

Planner's problem. *Choose a strategy $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and precisions $\kappa = (\kappa_x, \kappa_z)$ so as to maximize $W(\varphi; \kappa)$ subject to the constraint that $\kappa = K(\varphi_a + \varphi_x)$.*

The solution to this problem is complicated by the fact that this problem is non-concave and that a closed-form solution for the efficient strategy does not exist. Nevertheless, because the precisions depend on the strategy only through the sum $\varphi_a + \varphi_x$, we can bypass these complications by splitting the planner's problem in two steps. We summarize these steps as follows, and leave the detailed derivations to the Appendix.

Auxiliary Problem 1. *Choose $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ so as to maximize $W(\varphi; \kappa)$ subject to $\varphi_a + \varphi_x = \bar{\varphi}$ and let Δ be the Lagrange multiplier on this constraint.*

Auxiliary Problem 2. *Choose $\bar{\varphi}$ so as to maximize $W(\bar{\varphi}; K(\bar{\varphi}))$.*

The first step lets the planner optimize over the set of strategies subject to an additional constraint, namely that the sum $\varphi_a + \varphi_x$ equals $\bar{\varphi}$ for some arbitrary $\bar{\varphi}$. The second step then lets the planner optimize over $\bar{\varphi}$ and over the precisions that are induced by it. The first-step problem is strictly concave and, in fact, its first-order conditions can be reduced to a simple linear system. The solution to this problem leads to the conditions (5) and (6) below, which express the efficient strategy as a function of Δ , the Lagrange multiplier on the aforementioned auxiliary constraint. The second step then permits us to prove the existence of an efficient allocation, to interpret the wedge Δ as the shadow value of the informational externality, and to complete the full characterization of the efficient allocation.

Let us define

$$\beta \equiv \frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} > 1 \quad \text{and} \quad \alpha \equiv \frac{\frac{1}{\rho} - \gamma}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} < 1.$$

As it will be evident below, the coefficient β determines the elasticity of local output to variations in local productivity, while the coefficient α determines the elasticity of local output to variations in (expected) aggregate output.

Proposition 1. (i) A constrained efficient strategy always exists and is given by

$$\log q(\omega) = \varphi_0^* + \varphi_a^* a + \varphi_x^* x + \varphi_z^* z,$$

where the coefficients $(\varphi_a^*, \varphi_x^*, \varphi_z^*)$ and the associated precisions (κ_x^*, κ_z^*) are the fixed point to the following system:

$$\varphi_a^* = \beta \tag{4}$$

$$\varphi_x^* = \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \Delta \tag{5}$$

$$\varphi_z^* = \left\{ \frac{\kappa_z^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \Delta \tag{6}$$

$$\kappa_x^* = \sigma_\xi^{-2} + (\varphi_a^* + \varphi_x^*)^2 \sigma_{xq}^{-2}$$

$$\kappa_z^* = \sigma_{za}^{-2} + (\varphi_a^* + \varphi_x^*)^2 \sigma_{zq}^{-2}$$

for some Δ , which itself is proportional to the sum $\frac{\partial W}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}}$ and is strictly positive whenever $\alpha \neq 0$.

(ii) An equilibrium in the absence of policy (zero taxes) also exists and satisfies the same conditions as the efficient strategy above, replacing Δ with 0.

Part (i) of Proposition 1 characterizes the efficient strategy. Part (ii) contrasts it to the equilibrium in the absence of policy intervention (“laissez faire”). Together, these properties establish that policy intervention is warranted whenever $\Delta > 0$, which in turn is true whenever $\alpha \neq 0$.

Before elaborating on the economic meaning of this result, let us note that the result is intentionally silent about the constant φ_0 . This differs between the laissez-faire equilibrium and the constrained efficient allocation of a familiar reason that is of no interest here: the monopoly distortion. This distortion is orthogonal to the informational friction and can be corrected with an *acyclical* subsidy on production. In what follows, we disregard the monopoly distortion and concentrate on the informational friction and more specifically on the scalar Δ .

This scalar is a wedge that summarizes the impact of the informational externality on the efficient production strategy relative to the laissez-faire equilibrium—or, equivalently, relative to the allocation that would have maximized welfare had information been exogenous. Let us elaborate on this.

In the absence of taxes, the equilibrium allocation is described in Proposition 1 but without the wedge Δ . Note that the precisions κ_x^* and κ_z^* obtained at the efficient allocation above are higher than those obtained at the equilibrium, precisely because the planner induces a higher $\bar{\varphi}$. Had information been exogenous, Δ would have been zero and the planner would have chosen the same allocation as the equilibrium in the absence of taxes. This is because, in the economy under consideration, there is no misalignment between the privately and the socially optimal use of information, barring any informational externalities.

Any deviation from the equilibrium allocation thus involves a loss in terms of *allocative* efficiency: it reduces welfare for given information. But it is only this sacrifice that permits the planner to engineer an increase in the precision of the available information. Furthermore, such an increase is welfare improving precisely because the equilibrium use of information is efficient to start with. If the latter were not true, as for example in the case of [Morris and Shin \(2002\)](#), an improvement in the precision of the available information could map to a deterioration of welfare.

What then justifies the aforementioned sacrifice is precisely that these higher precisions contribute to higher welfare. In short, the planner trades off less *allocative* efficiency (i.e., less welfare for given information) for more *informational* efficiency (i.e., higher welfare via better information).

To be precise, the above argument establishes the direction of a *local* welfare improvement starting from the equilibrium without policy intervention. But such local arguments are not necessarily informative about the position of the *global* maximum when the planner's problem fails to be concave, as it is the case here.

A concrete example of how the local argument could fail is that the planner can induce a high precision for the endogenous signals by picking, not only a high enough positive value for the sum $\varphi_a + \varphi_x$, but also a sufficiently *negative* value for it. This is simply because the informativeness of the endogenous signals depends only on the absolute value of the sensitivity of aggregate output to aggregate productivity, not on the sign of this sensitivity. We can nevertheless rule out this possibility with a different, non-local argument: the planner can always achieve the same precision along with higher allocative efficiency by choosing the symmetrically positive value for $\varphi_a + \varphi_x$. This is because the value of $\varphi_a + \varphi_x$ that maximizes allocative efficiency—i.e., the equilibrium one—is positive to start with and the welfare function is symmetric around this point. Along with the fact that any positive value for $\varphi_a + \varphi_x$ that is lower than the equilibrium one is clearly suboptimal—for locally raising this value would have improved both allocative and informational efficiency.

Finally, the discussion above presumes that $\Delta > 0$, which is the case as long as $\alpha \neq 0$. When instead α is zero, Δ is also zero, and the need for policy intervention vanishes, for the simple reason that there is no value for knowing the aggregate state of the economy in the knife-edge case in which $\alpha = 0$. We expand on this point shortly.

3.3 The Informational Wedge

In Proposition 1, the informational externality, as measured by the sum $\frac{\partial W}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \varphi} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \varphi}$, is itself evaluated at the constrained efficient strategy. By the same token, the wedge Δ is jointly determined with the coefficients φ^* and the precisions κ^* . The details can be found in the Appendix. This complication prevents a closed-form characterization of Δ .

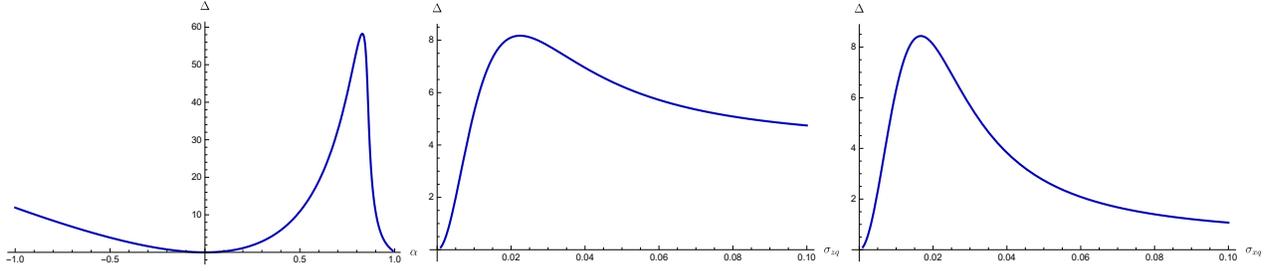


Figure 1. The wedge Δ for different values of α (Panel 1) and for different levels of noise in the endogenous public signal (Panel 2) and the endogenous private signal (Panel 3).

This complication does not matter for the *qualitative* properties of the efficient strategy discussed above, nor for the qualitative cyclical properties of the optimal policy that will be discussed shortly. For these it suffices to know that Δ is strictly positive as long as $\alpha \neq 0$.

However, in order to study the comparative statics of Δ with respect to the economy’s deep parameters, we must resort to numerical simulations. The model is sufficiently parsimonious that we may conduct a rather extensive exploration of its parameter space. In Figure 1, we use a particular parameterization to illustrate a few key findings which are qualitatively robust across a wide range of parameter values. For each of these simulations, we compute and plot the Δ that maximizes welfare. Although the model is too stylized to permit a serious quantitative evaluation, the particular parameterization used in this figure is reasonably realistic: we use conventional values for the underlying preference and technology parameters and empirically plausible values for the measurement errors in the endogenous signals.⁸

Consider first Panel A of Figure 1, which plots Δ as a function of the degree of strategic complementarity, or the GE feedback, as measured by α . When $\alpha = 0$, the GE feedback is muted and there is no informational externality: $\Delta = 0$. This coincides with our results stated in Proposition 1. In this case, firms care only about their own productivity, which they already know; they do not care about the state of the economy, because the positive demand effect of others’ production on their revenue is exactly offset by the negative effect on labor costs. As a result, the social value of learning about the state of economy is zero, and there is no informational externality. There is therefore no reason for policy to intervene when $\alpha = 0$.

Away from $\alpha = 0$, the value of learning about the state of the economy is instead strictly positive, and so is Δ . This is true whether firms’ actions are strategic complements ($\alpha > 0$) or strategic substitutes ($\alpha < 0$). In the former case, the positive effect of others’ production on a firm’s demand and revenue outweighs the negative effect on labor costs; firms thereby have an incentive to produce more when other firms produce more. In the latter case, the opposite properties are true. In both cases, however, firms care to know what others do and either do the same or the opposite. This explains why Δ is strictly positive on both sides of $\alpha = 0$.

⁸The parameterization used here is the same as that used in the monetary extension, modulo the exclusion of nominal rigidity and price signals. See Section 5.4 for a detailed description.

Interestingly, though, Panel A of Figure 1 shows that even if we restrict attention to the positive domain for α , i.e. the region of strategic complementarity, the wedge Δ is non-monotonic in α . As α increases from 0 to 1, the wedge Δ initially increases but eventually starts falling, converging to zero as α approaches 1.

To understand the downward-sloping region, note that when strategic complementarities are sufficiently strong, firms care a lot about coordinating their behavior. They thereby find it optimal to largely disregard any private information about the state of the economy and instead condition their behavior heavily on the public signal, even if the latter is a rather poor signal about underlying aggregate fundamentals. In the limit as $\alpha \rightarrow 1$, the public signal serves in equilibrium as a nearly perfect coordinating device, or signal of the behavior of others. This removes the planner's desire to distort equilibrium allocations in favor of improving learning. As for the endogenous private signals, these are optimally disregarded in favor of public signals in this limit, hence the value of improving their quality vanishes, too. As a result, Δ approaches zero as α approaches 1. This limit thereby has the same policy implication as when $\alpha = 0$, but the rationale is quite different.

Panels B and C of Figure 1 plot Δ for different levels of noise in the public and private endogenous signals, respectively. The relationship is again non-monotonic. When noise in either of these signals is zero, all firms observe the economy's fundamentals perfectly; as a result, there is no need to distort allocations to improve learning and $\Delta = 0$. As the noise in either of these signals moves away from zero, learning becomes imperfect and Δ increases.

To understand the downward-sloping region of Panel B, note that when the noise in the public signal becomes sufficiently high, holding the private noise fixed, firms endogenously disregard the public signal in favor of the private signal. As a result, Δ decreases. However, in the limit as the noise of the public signal tends to infinity, it is *as if* there were no public signal and only a private signal. In this case, firms learn purely from the private signal, but noise in the private signal implies a strictly positive informational externality. Therefore, as public signal noise approaches infinity, Δ falls and converges from above to a strictly positive constant. The same, but reverse, intuition holds true in Panel C as the noise in the private signal tends to infinity, holding the public noise fixed.

To sum up, this example highlights why the informational wedge is likely to be highest when both the degree of strategic complementarity and the measurement errors are "moderate." However, a quantitative translation of this statement is beyond the scope of this paper. But we will offer a back-of-the-envelope calculation of the wedge and of its footprint on both taxes and monetary policy within a calibrated version of the monetary extension in Section 4.

3.4 Implementation and Optimal Policy

Our notion of constrained efficiency allows the planner to choose allocations and associated information structures without any consideration of whether and how these outcomes can be implemented in a market-based equilibrium. We now show how such an allocation can in fact be achieved with countercyclical taxes.

Consider any combination of the following tax instruments: a linear tax $\tau^R(\bar{a}, z)$ on firm revenue or sales, a linear tax $\tau^L(\bar{a}, z)$ on household labor income, and a linear tax $\tau^C(\bar{a}, z)$ on household consumption (a sales tax that is uniform across commodities). These taxes are collected in stage 2 and can be contingent on the information that is publicly available at that time. To maintain tractability and guarantee that equilibrium allocations are log-Normal, these taxes are assumed to be log-linear functions of (\bar{a}, z) . Finally, we assume that in order for the government to balance its budget, the government has access to additional lump-sum taxes or transfers to the household. We define an equilibrium as follows.

Definition 2. *A (log-linear) equilibrium is the combination of signal precisions $(\kappa_x, \kappa_z) \in \mathbb{R}_+^2$ and a production strategy $q : \Omega \rightarrow \mathbb{R}_+$ as in (1), along with an employment strategy $n : \Omega \rightarrow \mathbb{R}_+$, tax rate functions $\tau^R, \tau^L, \tau^C : \mathbb{R}^2 \rightarrow \mathbb{R}$, a wage function $w : \Omega \rightarrow \mathbb{R}_+$, a price function $p : \Omega \times \mathbb{R}^2 \rightarrow \mathbb{R}$, and household consumption demand and labor supply functions, and such that:*

(i) *Given the signal precisions, the remaining elements constitute a competitive equilibrium in the sense that the production and employment choices are optimal for the firms and the households, the wages and the prices clear the labor and goods markets, and the government's budget constraint is satisfied in all states.*

(ii) *Given the production strategy, the signal precisions are generated according to (3).*

Next, we show that any such tax combination reduces to a single tax wedge between the marginal return and the marginal cost of labor. The key implementability constraint is then identified in the following lemma.

Lemma 3. *A production strategy is implementable with the aforementioned tax instruments if and only if it solves the following fixed-point problem:*

$$q(\omega)^{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \left(\frac{\rho-1}{\rho}\right) \theta A(\omega)^{\frac{\epsilon}{\theta}} \mathbb{E} \left[(1 - \tau(\bar{a}, z)) Q(\bar{a}, z)^{\frac{1}{\rho} - \gamma} | \omega \right] \forall \omega, \quad (7)$$

where $q(\omega)$ is the production of an island of type ω , $A(\omega)$ is its productivity, $Q(\bar{a}, z)$ is aggregate output, and $1 - \tau(\bar{a}, z)$ is the combined tax wedge induced by the aforementioned tax instruments.

The proof of this result is similar to the characterization of equilibrium in [Angeletos and La'O \(2010\)](#) but with the inclusion of a tax wedge. Basically, condition (7) means that the marginal cost of production in each island is equated to the local expectation of the marginal revenue product, net of taxes.

Let us express the tax wedge as follows

$$-\log(1 - \tau(\bar{a}, z)) = \tau_0 + \tau_A \bar{a} + \tau_z z, \quad (8)$$

for some known coefficients $(\tau_0, \tau_A, \tau_z) \in \mathbb{R}^3$. The coefficient τ_0 parameterizes the mean value of the tax and helps correct the monopoly distortion—a familiar function that, as already mentioned, is of no interest to us. Central for our purposes are instead the coefficients τ_A and τ_z . These coefficients determine the elasticities of the tax with respect to, respectively, the underlying fundamental and the public statistic; as we explain next, their ultimate function is to regulate how information is used and aggregated in equilibrium.⁹

To see how such contingent taxes can impact the decentralized use and aggregation of information, suppose the tax is negatively correlated with innovations in aggregate productivity and positively correlated with the common noise. Anticipating these correlations, firms choosing output according to equation (7) will have an incentive during stage 1 to react more strongly to any information they may have about aggregate productivity and less strongly to any information they may have about the common noise. It follows that firms will unambiguously increase their response to their private sources of information; whether they will at the same time reduce their response to common information then depends simply on whether the positive correlation of the tax with the underlying common noise is sufficiently strong relative to its negative correlation with respect to aggregate productivity. This explains why state-contingent taxes regulate both φ_x and φ_z , the sensitivities to private and public information.¹⁰

We can then deduce the relevant policy implications by characterizing the tax wedge that implements the efficient allocation. Let ε denote the noise in the public signal z .

Proposition 2. *There exists a state-contingent tax policy as in (8) that implements the efficient allocation. The optimal tax is countercyclical in either of the following senses: $\text{Corr}(\tau, \bar{a}) < 0$, $\text{Corr}(\tau, \bar{a}|z) < 0$, and $\text{Corr}(\tau, Q) < 0$. Moreover, the tax is positively correlated with the noise: $\text{Corr}(\tau, \varepsilon) > 0$.*

The countercyclicity of optimal taxes follows directly from comparing equilibrium and efficient allocations. Recall that, when information is endogenous, efficiency dictates the government to raise φ_x , the sensitivity of production to private information, so as to boost social learning. At the same time, it also dictates to lower φ_z , the sensitivity to public information, so as to preserve allocative efficiency. How can the tax system provide the agents with the right

⁹Here we have chosen to express the taxes as a function of \bar{a} and z . But since $\log Q$ and $\log N$ are both linear (and indeed non-colinear) combinations of \bar{a} and z , we could equivalently condition taxes on any pair among the set $\{\log Q(\bar{a}, z), \log N(\bar{a}, z), \bar{a}, z\}$.

¹⁰To be precise, the argument made above only explains how taxes impact individual firm incentives holding given the behavior of other firms; that is, they explain the impact of the policy on best responses, not on equilibrium. However, the equilibrium could fail to inherit the comparative-static properties of best responses only when the degree of complementarity is too strong (namely $\alpha > 1$), which is never the case here because the assumed micro-foundations guarantee that the economy is “sufficiently convex.”

incentives for these goals to materialize in equilibrium? For the agents to find it optimal to raise their response to their private information about aggregate productivity, it better be that they expect the tax to fall—and hence their net-of-tax returns to increase—with any positive innovations in aggregate productivity. And for them to find it optimal to decrease their response to public information, it better be that they expect the tax to increase with the public signal or, equivalently, with the noise in it. This explains why the optimal tax must be negatively correlated with \bar{a} and positively correlated with ε along the equilibrium.

Note that it is the combination of the two cyclical properties of the tax—its negative correlation with \bar{a} and its positive correlation with ε —that achieves full efficiency. However, it is only the negative correlation with \bar{a} that is the key instrument for increasing φ_x and thereby for boosting the aggregation of information over the business cycle. The positive correlation with the noise is instrumental only for reducing φ_z , which is necessary for counterbalancing the allocative inefficiency caused by the higher φ_x , but is irrelevant for the efficiency of learning.

4 The Monetary Model

In this section we consider the full version of our model which allows the informational friction to be the source of both real and nominal rigidity. This model sheds light on the joint determination of optimal fiscal and monetary policies.

Set up. We modify the baseline model in three dimensions. First, we introduce price rigidities. In particular, we assume that firms set nominal prices at the end of stage 1, while information is still dispersed, and cannot adjust them in stage 2, in response to the new information that becomes available at that stage. We refer to this scenario as “sticky prices” and to the alternative in which prices are free to adjust in stage 2 as “flexible prices.”¹¹

Second, we let firms make an additional labor-demand choice in stage 2 and, accordingly, we let households make a second labor-supply choice in that stage. This permits a firm’s output to respond to any surprise in its demand that obtains after the firm has set its price and, relatedly, allows monetary policy to have real effects. In particular, we let the production of the typical commodity be given by

$$y(\omega, \bar{a}, z) = A(\omega)n(\omega)^\theta l(\omega, \bar{a}, z)^{1-\theta},$$

¹¹This terminology is borrowed from [Angeletos and La’O \(2020\)](#) but is non-standard. What that work and our paper alike call “sticky prices” is often referred to in the related literature as “flexible prices with information constraints” in order to emphasize that there is no ad hoc nominal rigidity of the Calvo type. Nevertheless, the terminology proposed in [Angeletos and La’O \(2020\)](#) and adopted here is most appropriate for understanding the mapping between models in which the nominal rigidity originates from an informational friction, as the model used here and in related works (e.g., [Woodford, 2003b](#)), and the textbook New Keynesian framework, in which the nominal rigidity takes the Calvo-like form. The adopted terminology is also consistent with that in [Correia, Nicolini, and Teles \(2008\)](#) and [Correia et al. \(2013\)](#); in those works, “sticky firms” refer to a group of firms whose prices cannot be measurable in the concurrent aggregate productivity. Clearly, the essence is the same whether this restriction is interpreted as “pre-determined prices” or as “informationally-constrained prices.”

where $y(\omega, \bar{a}, z)$ denotes output, $n(\omega)$ denotes the labor input in stage 1, $l(\omega, \bar{a}, z)$ denotes the labor input in stage 2, and $\theta \in (0, 1)$; and, accordingly, we let the per-period utility of the representative household be given by

$$U(C(\bar{a}, z)) - \int \frac{1}{\epsilon} n(\omega)^\epsilon dF(\omega|\bar{a}, z) - \int \frac{1}{\epsilon} l(\omega, \bar{a}, z)^\epsilon dF(\omega|\bar{a}, z),$$

where $C(\bar{a}, z)$ is the same CES aggregator as that in the baseline model, $F(\cdot|\bar{a}, z)$ is the cdf of ω conditional on the aggregate state (\bar{a}, z) , and $\epsilon > 1$.

Note that, relative to the baseline model, the only new terms in both technology and preferences are those pertaining to $l(\omega, \bar{a}, z)$. Also note that second-stage labor can depend on the information that becomes available at that stage, which explains why l is a function of (\bar{a}, z) in addition to ω (unlike n which is only a function of ω).

Third, we let firms and workers in each island observe signals of the (nominal) prices set by firms in other islands in addition to signals of the (real) quantities. In particular, we denote by $Y(\bar{a}, z)$ and $P(\bar{a}, z)$ the real aggregate output and the nominal price level¹² and let each firm observe a total of four endogenous signals about these objects. Two of these signals are public and are given by

$$z^y = \log Y(\bar{a}, z) + \varepsilon^{zy} \quad \text{and} \quad z^p = \log P(\bar{a}, z) + \varepsilon^{zp},$$

where $\varepsilon^{zy} \sim \mathcal{N}(0, \sigma_{zy}^2)$ and $\varepsilon^{zp} \sim \mathcal{N}(0, \sigma_{zp}^2)$ are the respective noises. The remaining two are private and are given by

$$x_i^y = \log Y(\bar{a}, z) + \varepsilon_i^{xy} \quad \text{and} \quad x_i^p = \log P(\bar{a}, z) + \varepsilon_i^{xp},$$

where $\varepsilon_i^{xy} \sim \mathcal{N}(0, \sigma_{xy}^2)$ and $\varepsilon_i^{xp} \sim \mathcal{N}(0, \sigma_{xp}^2)$ are the respective noises. We interpret the former two signals as readily accessible and commonly known macroeconomic statistics, and the latter two as proxies for private learning about economic activity.

Substance and solution strategy. As in the baseline model, the essential issue here is the endogeneity of the information contained in these signals. If, instead, information had been exogenous, our framework would have been nested in that of [Angeletos and La'O \(2020\)](#) and their results would have guaranteed that the following key lessons from the New Keynesian framework ([Correia, Nicolini, and Teles, 2008](#)) would extend to the presence of informational frictions. That is, first, that optimal taxes are acyclical; and, second, that optimal monetary policy only neutralizes the nominal rigidity, or replicates flexible prices. From this perspective, our key contribution will be to show how the endogeneity of information upends these lessons.

But there is also a subtle technical difference from our baseline model due to the introduction of price signals. In the baseline model, *all* endogenous signals are well defined

¹² $Y(\bar{a}, z)$ is defined by the same CES aggregator as $C(\bar{a}, z)$, replacing $c(\omega, \bar{a}, z)$ with $y(\omega, \bar{a}, z)$, and $P(\bar{a}, z)$ is defined as the corresponding ideal price index. Clearly, $Y(\bar{a}, z) = C(\bar{a}, z)$ by market clearing in the goods market (or, equivalently, by resource feasibility).

for arbitrary allocations and regardless of whether such allocations were chosen directly by the planner or implemented via decentralized markets. The same is true here for the quantity signals, but not for the price signals: the latter are well defined *only* in the context of market-based implementations.

This precludes the solution strategy taken in the baseline model. There, we could define and characterize a natural efficiency benchmark without explicit consideration of implementability. Here, we must first characterize the set of the combinations of quantities, prices, and information structures that can be implemented as a market outcome given the available policy instruments; and only then can we proceed to identify the best allocation in this set.

Taxes and monetary policy. The above discussion brings to the forefront the question of what the available policy instruments are. We maintain the taxes from the baseline model but add monetary policy as a new instrument.

The following property extends from our baseline model to the present setting: a tax on firm revenue is equivalent to a tax on firm output, total employment, or payroll; a tax on household labor income or consumption; or any other tax that has a uniform impact across the two labor inputs. With this in mind, we think of the tax on firm revenue as a proxy for all of the above and specify it as follows:

$$-\log(1 - \tau(\bar{a}, z)) = \tau_0 + \tau_A \bar{a} + \tau_z z, \quad (9)$$

for some scalars (τ_0, τ_A, τ_z) and for some random variable z that is a sufficient statistic for all the available public information (similarly defined as in the baseline model). We finally describe monetary policy with the following policy rule for the nominal interest rate:

$$\log(1 + R(\bar{a}, z)) = \rho_0 + \rho_A \bar{a} + \rho_z z, \quad (10)$$

for some scalars (ρ_0, ρ_A, ρ_z) . Condition (9) is the same as (8) in the baseline model. Condition (10) is the analogue for monetary policy. In both of these conditions, a log-linear specification is employed in order to preserve the Gaussian property of the information structure.

In the following section, we first characterize the combinations of allocations, prices, and information structures that can be implemented with arbitrary policies of the aforementioned kind. We then identify the best allocation within this set. Finally, we characterize the policies that support it as an equilibrium. Before jumping into the formal arguments, it is useful to anticipate the following two properties, which help clarify the role played by monetary policy.

First, note that the coefficients ρ_z and ρ_A parameterize the response of monetary policy to, respectively, the public signal and the innovations in aggregate output. Since firms set prices in stage 1, when they know z but not \bar{a} , monetary policy can have real effects insofar as it responds to the variation in \bar{a} that is not spanned by z . By the same token, ρ_z has no impact on the real allocations and the information structure; only ρ_A matters. This conclusion contrasts with taxation, in which case both τ_z and τ_A have real effects.

Second, if prices had been flexible (i.e., free to adjust in stage 2), taxes would still have real effects, but monetary policy would not. Therefore, by letting prices be sticky (i.e., chosen in stage 1), we effectively *expand* the set of allocations and information structures that can be attained by the planner. In particular, an important intermediate step of the subsequent analysis will be to show that, for any given tax policy, there always exist a monetary policy that replicates the corresponding flexible-price outcomes, as well as monetary policies that induce different allocations and different information structures. A key final result will then be that the former kind of monetary policy is optimal when there are only quantity signals, whereas the latter kind becomes optimal once there are price signals.

5 Implementability and Optimality

This section contains our main results. We first characterize the allocations and prices that can be supported by the available policy instruments. This gives us the “primal” representation of the policy problem. We use this to identify the optimal allocation and the optimal information structure. We proceed to recover the policies that support this allocation and information structure in an equilibrium.

5.1 Implementability in the Monetary Model

As in the baseline model, we guess (and subsequently verify) that the relevant information available to any island in stage 1 can be summarized by the triplet $\omega = (a, x, z)$ with $\omega \in \Omega$, where a is local productivity; x is a Gaussian sufficient statistic for all private information regarding the aggregate state; and z is a Gaussian sufficient statistic summarizing all public information regarding the aggregate state. We similarly guess (and subsequently verify) that the employment and production levels of any island ω can be expressed as follows:

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_z z \quad \text{and} \quad \log l(\omega, \bar{a}, z) = l_0 + l_A \bar{a} + l_a a + l_x x + l_z z, \quad (11)$$

where $q(\omega)$ is now defined as $q(\omega) \equiv A(\omega)n(\omega)^\theta$, a composite of productivity and the first-stage labor choice, and where $(\varphi_0, \varphi_a, \varphi_x, \varphi_z) \in \mathbb{R}^4$ and $(l_0, l_A, l_a, l_x, l_z) \in \mathbb{R}^5$ are coefficients that are indirectly under the control of the planner.

Three clarifications are needed here. First, although $q(\omega)$ plays a similar technical role as in the baseline model, it now has a more subtle interpretation: instead of being the entire output of a firm or island, it is the component of it that is determined in stage 1, that is, it excludes the input that is free to adjust to monetary policy. Second, although the planner has control over the nine coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z; l_0, l_A, l_a, l_x, l_z) \in \mathbb{R}^9$ that parameterize the state dependence of the real allocations, this control is limited by certain implementability restrictions. These restrictions will be derived shortly.

The third clarification is the following. Since firms set prices in stage 1 along with first-stage labor (equivalently q), it would seem most natural to specify firms' behavior in terms of a pair of strategies for q and p . However, one can always recast a firm's pricing-setting choice as a choice of a state-contingent plan for how its flexible stage-2 input, l , will adjust to realized demand. For this reason we may specify firms' behavior as a pair of strategies for q and l . This recasting, which shifts the focus from price-setting choices to implementable allocations, works best for our purposes and is a defining feature of the primal approach to optimal policy.¹³

In fact, this equivalence can be inferred from our following equilibrium definition.

Definition 3. *A (log-linear) equilibrium is a combination of signal precisions $(\kappa_x, \kappa_z) \in \mathbb{R}^2$, a "sticky" price function $p : \Omega \rightarrow \mathbb{R}$, production/employment strategies $q : \Omega \rightarrow \mathbb{R}_+$, $n : \Omega \rightarrow \mathbb{R}_+$ and $l : \Omega \times \mathbb{R}^2 \rightarrow \mathbb{R}$ as in (11), policies $\tau : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $R : \mathbb{R}^2 \rightarrow \mathbb{R}$ as in (9) and (10), a wage function $w : \Omega \rightarrow \mathbb{R}_+$, and household consumption demand and labor supply functions, such that:*

(i) *Given the signal precisions and the tax and monetary policies, the quantities and the prices are optimal form the firms and the households, the labor and goods markets clear, the government's budget constraint is satisfied, and the nominal interest rate satisfies*

$$U'(C(\bar{a}, z)) = \bar{\beta} \mathbb{E} \left[(1 + R(\bar{a}, z)) \frac{P(\bar{a}, z)}{P(\bar{a}_{+1}, z_{+1})} U'(C(\bar{a}_{+1}, z_{+1})) \middle| \bar{a}, z \right] \quad (12)$$

where (\bar{a}_{+1}, z_{+1}) corresponds to the aggregate state in the following period.

(ii) *Given the production and pricing strategies, the signal precisions are generated accordingly.*

Our equilibrium definition is analogous to our earlier equilibrium definition for the baseline flexible-price model (Definition 2); the only modifications here are the following. First, because prices are now sticky, i.e. measurable in ω , second-stage labor must adjust in order for goods markets to clear. Second, the nominal interest rate is such that it satisfies the intertemporal Euler equation for the household (2), written here in recursive form. Third, we again impose that the precisions of the private and public signal are generated endogenously by the allocation; however, we have yet to show how they are generated. We turn to this consideration next.

Implementability results. The rest of this subsection is organized in three results. The first result (Lemma 4) characterizes the information structures that are induced by the aforementioned strategies under arbitrary coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z; l_0, l_A, l_a, l_x, l_z) \in \mathbb{R}^9$. The second result (Proposition 3) works out the implementability restrictions on these coefficients; together with the first result, this amounts to finding the set of the implementable combinations of allocations and information structures. The last result (Lemma 5) characterizes the associated prices.

We start with the first result, which is the analogue of Lemma 1 in the baseline model.

¹³For examples of the primal approach applied to models with nominal rigidity, see Correia, Nicolini, and Teles (2008) and Correia et al. (2013), as well as the related work of Angeletos and La'O (2020).

Lemma 4. *Take any pair of strategies as in (11) and let κ_x and κ_z denote the precisions of the sufficient statistics of, respectively, the private and the public information that obtain in equilibrium when all firms follow the aforementioned strategies. Then,*

$$\kappa_x = \sigma_{xa}^{-2} + \Phi^2 \sigma_{xy}^{-2} + \Psi^2 \sigma_{xp}^{-2} \quad \text{and} \quad \kappa_z = \sigma_{za}^{-2} + \Phi^2 \sigma_{zy}^{-2} + \Psi^2 \sigma_{zp}^{-2} \quad (13)$$

where

$$\Phi = \varphi_a + \varphi_x + (1 - \theta)(l_a + l_x + l_A) \quad \text{and} \quad \Psi = \frac{1}{\rho} (\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x)) \quad (14)$$

measure the elasticities of, respectively, the aggregate level of output and the aggregate price level with respect to the underlying fundamental, conditional on the public information.

The interpretation of (13) is straightforward: the terms $\kappa_{xy} \equiv \Phi^2 \sigma_{xy}^{-2}$ and $\kappa_{xp} \equiv \Psi^2 \sigma_{xp}^{-2}$ that show up in κ_x capture the precision of the *private* learning that obtains through the observation of, respectively, quantity and price signals; the corresponding terms in κ_z capture the corresponding *public* learning. Only (14) deserves some explanation. To this goal, note that (11) implies that aggregate output can be expressed, up to a constant, as

$$\log Y(\bar{a}, z) = (\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x + l_A)) \bar{a} + (\varphi_z + (1 - \theta)l_z) z.$$

Because z and, hence, also the second term above is known, observing the available quantity signal is akin to observing the first term above plus measurement error. This explains the interpretation of and the formula for Φ . The logic for Ψ is similar, except that it requires the characterization of the equilibrium price level. This missing piece follows from the last result of this subsection (Lemma 5) and from the more detailed analysis in the Appendix.

Let us move on to the second result, which identifies the relevant implementability restrictions. The exact definitions of α and β for the monetary model are in the proof of Proposition 3 in the Appendix. We also assume that the parameters are such that $\alpha > 0$. Although this assumption is required only for the characterization of optimal fiscal and monetary policy in Section 5.3, this is the most realistic case; we thus assume it from the outset.

Proposition 3. *(i) A pair of strategies as in (11) can be implemented as an equilibrium with an appropriate combination of a contingent linear tax and a monetary policy if and only if the following conditions are satisfied:¹⁴*

$$\varphi_a = \beta \quad (15)$$

$$l_a = \frac{1}{\theta} (\varphi_a - 1) \quad (16)$$

$$l_x + l_A \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_z} = \frac{1}{\theta} \varphi_x \quad (17)$$

$$l_z + l_A \frac{\kappa_z}{\kappa_A + \kappa_x + \kappa_z} = \frac{1}{\theta} \varphi_z \quad (18)$$

¹⁴There is also a restriction between φ_0 and l_0 , which we omit because it is of no interest: φ_0 and l_0 are irrelevant for both the endogenous precisions and business-cycle properties of the real allocations.

(ii) Had prices been flexible (i.e., free to adjust to realized demand), a pair of strategies as in (11) would have been implemented if and only if the following condition was satisfied in addition to conditions (15)-(18):

$$l_A = \frac{1}{\theta} \frac{\varphi_x \kappa_A + \kappa_x + \kappa_z}{\beta \kappa_x}. \quad (19)$$

Proposition 3 plays a similar role to the familiar “implementability” results in the Ramsey literature: it represents the optimal policy problem in terms of the allocations that are induced by the policy rather than the policy instruments themselves. In our context, it sheds light on how the planner can regulate the decentralized use of information and thereby also its aggregation through the available price and quantity signals.

As in the baseline model, the planner can induce any φ_x and φ_z she may desire by appropriately choosing the contingencies of the taxes. However, conditional on picking these coefficients, her control over the remaining coefficients is limited either entirely (with flexible prices) or partly (with sticky prices). In particular, if prices were flexible, the planner would have no further control: all remaining coefficients would be fixed functions of the chosen pair (φ_x, φ_z) and deep parameters. Instead, since prices are sticky, the planner has an extra degree of freedom: by appropriately choosing monetary policy, it can induce any l_A it wishes and thereby also influence the pair (l_x, l_z) , over and above its control on the pair (φ_x, φ_z) .

More specifically, whether prices are flexible or sticky, the absence of a differential tax on the two types of labor implies that the equilibrium necessarily equates the (expected) marginal rate of transformation between these two types of labor with the corresponding marginal rate of substitution in preferences. This explains why l_a, l_x and l_z are related to φ_a, φ_x and φ_z as in (15)-(18).¹⁵ Furthermore, if prices were flexible, once taxes had been set to achieve the desired φ_x and φ_z , the sensitivity of second-stage employment and production to the realized aggregate productivity would be pinned down by equating the realized marginal returns and costs of stage-2 labor. It is this restriction that gives (19). But since prices are sticky, this restriction is no more present: by designing the extent to which monetary policy accommodates the realized productivity shock, the planner can effectively choose any l_A she wishes, in addition to the free choice of the pair (φ_x, φ_z) . In other words, sticky prices amount to an extra degree of freedom.

Finally, note that, as the planner chooses a higher l_A , which amounts to a more accommodative monetary policy, firms respond optimally by raising the sensitivity of their own prices to the information they have about aggregate productivity when they set their prices, thus, partly offsetting the monetary policy. This explains why l_x and l_z are negatively related to l_A in the way defined by conditions (17) and (18), and why the planner has no control over l_x and l_z other than that afforded through the free choice of l_A . In short, when prices are sticky, monetary policy affords exactly one extra degree of freedom over choosing allocations.

¹⁵In particular, the equality of expected marginal rates of transformation and substitution gives $\mathbb{E}[\log l(\omega, \bar{a}, z)] = \text{const} + \log n(\omega)$, where *const* includes second-order terms; using then $\log q(\omega) = \log A(\omega) + \theta \log n(\omega)$ and noting that this condition must be satisfied for every $\omega = (a, x, z)$, gives the constraints in part (i) of Proposition 3.

So far we have shown that the set of implementable allocations and information structures is characterized by the combination of condition (11), Lemma 4 and Proposition 3. We conclude this subsection with the characterization of the prices that are associated with any element of this set.

By consumer optimality and market clearing in the goods markets, we have

$$\frac{p(\omega)}{P(\bar{a}, z)} = \left(\frac{c(\omega, \bar{a}, z)}{C(\bar{a}, z)} \right)^{-\frac{1}{\rho}} \quad \text{and} \quad c(\omega, \bar{a}, z) = y(\omega, \bar{a}, z) = q(\omega)l(\omega, \bar{a}, z)^{1-\theta}.$$

Combining these facts with (11), we reach the following result.

Lemma 5. *Pick any allocation and information structure that satisfy the combination of condition (11), Lemma 4 and Proposition 3. The associated equilibrium prices satisfy*

$$\log p(\omega) = \psi_0 + \psi_a a + \psi_x x + \psi_z z,$$

where ψ_a and ψ_x are pinned down by

$$\psi_a = -\frac{1}{\rho}(\varphi_a + (1 - \theta)l_a) \quad \text{and} \quad \psi_x = -\frac{1}{\rho}(\varphi_x + (1 - \theta)l_x),$$

while ψ_z is indeterminate.

Because any component of monetary policy that is public information at the moment prices are set cannot have any real effect, the dependence of monetary policy and prices to z is indeterminate. By contrast, the dependence of a firm's price on its own productivity and on its private signal are uniquely determined. To understand why, note first that, once the planner has picked a real allocation, there is a unique collection of relative prices that support it in equilibrium. Note next that the relative price of two firms, A and B , can move in a particular manner with the private information of firm A only if the nominal price of firm A moves in the exact same manner, simply because the nominal price of firm B can not possibly be contingent on the private information of another firm. Furthermore, this is true whether the relevant private information is a private signal about the aggregate state of the economy or merely the firm's own productivity. This provides the intuition for why ψ_a and ψ_x are uniquely pinned down once the corresponding production coefficients are fixed. The specific formulas given in the lemma above follow from the optimality conditions of the household, or equivalently the inverse demand functions faced by the firms.

A direct corollary of Proposition 3 and Lemma 5 is that, by controlling l_A and thereby l_x , monetary policy also influences ψ_x , the sensitivity of prices to local information. In particular, by (17), ψ_x is decreasing in l_A : the more firms expect monetary policy to accommodate the realized productivity shock, the higher their incentive to raise their prices in response to any private information they may have about aggregate productivity. This observation anticipates the role of monetary policy in regulating the aggregation of information through prices.

5.2 Optimal Allocations

We now proceed to identify the best implementable allocation and information structure. First, we express welfare as a function of the coefficients seen in condition 11 and the precisions of the endogenous sufficient statistics.

Lemma 6. *There exists a function $\mathcal{W} : \mathbb{R}^{11} \rightarrow R$ such that the expected utility of the representative household can be expressed as $\mathcal{W}(\varphi, l; \kappa_x, \kappa_z)$.*

Next, we use Lemma 4 and Proposition 3 to express the planner's problem as follows.

Planner's Problem. *Choose strategy coefficients $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and $l = (l_0, l_A, l_a, l_x, l_z)$ and precisions κ_x and κ_z so as to maximize $\mathcal{W}(\varphi, l; \kappa_x, \kappa_z)$ subject to (13) and (15)-(18).*

Like standard Ramsey policy problems, this problem imposes certain implementability constraints on the set of allocations that the planner can choose; as explained before, these constraints are summarized in conditions (15)-(18). Importantly, the planner takes into account that different allocations induce different information structures; this explains why the planner controls κ_x and κ_z , subject to condition (13).

The solution to this problem is characterized in the following result.

Proposition 4. *There exist scalars $\Delta_Y > 0$ and $\Delta_p > 0$, which depend on the information parameters, such that the optimal allocation satisfies*

$$\log q(\omega) = \varphi_0^* + \varphi_a^* a + \varphi_x^* x + \varphi_z^* z$$

$$\log l(\omega, \bar{a}, z) = l_0^* + l_A^* \bar{a} + l_a^* a + l_x^* x + l_z^* z$$

with following coefficients:

$$\varphi_a^* = \beta \tag{20}$$

$$\varphi_x^* = \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + (\Delta_y + \Delta_p) \tag{21}$$

$$\varphi_z^* = \left\{ \frac{\kappa_z^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} (\Delta_y + \Delta_p) \tag{22}$$

$$l_a^* = \hat{l}_a \tag{23}$$

$$l_x^* = \hat{l}_x + \left(\frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \right) \lambda \Delta_p \tag{24}$$

$$l_z^* = \hat{l}_z + \left(\frac{\kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \right) \lambda \Delta_p \tag{25}$$

$$l_A^* = \hat{l}_A - \lambda \Delta_p \tag{26}$$

where $(\hat{l}_a, \hat{l}_x, \hat{l}_z, \hat{l}_A)$ are the coefficients that would have obtained if prices were flexible (but taxes were fixed at their sticky-price optimal level) and λ is a positive scalar that is invariant to information parameters (it only depends only on preferences and technology parameters).

This result establishes that the impact of learning on the optimal implementable allocations resembles qualitatively the one in the baseline model. In particular, the scalars Δ_y and Δ_p are the Lagrange multipliers that measure the social value of increasing the precisions of, respectively, the quantity signals (z^y, x_i^y) and the price signals (z^p, x_i^p) .

Consider first the sensitivity of first-stage decisions to information. Modulo the new definitions of α and β , the coefficients given by the conditions (21) and (22) coincide with their counterparts in Proposition 1 if we let the total information wedge be $\Delta \equiv \Delta_y + \Delta_p$. The intuition is of course the same. The planner corrects the information externality and boosts social learning by raising the sensitivity of first-stage decisions to private information. At the same time, it preserves allocative efficiency by lowering the sensitivity to public information.

Let's now turn to second-stage decisions. Suppose for a moment that agents did learn from price signals (formally, let $\sigma_{z^p}^2 = \sigma_{x^p}^2 = \infty$). As a result, $\Delta_p = 0$ and by conditions (24)-(26) second-stage coefficients would coincide with their counterparts under flexible prices. In particular, condition (26) requires monetary policy to be set so as to replicate flexible prices.

To gain intuition for this result, remember that sticky prices provide the planner with an extra degree of freedom. Whether the planner uses this extra lever and, thus, moves away from the allocations that would obtain under flexible prices depends on whether the distorted allocations come with a further boost to social learning. This is the case when, and only when, prices serve as a signal of the state of the economy.

The argument above suggests that in the general case in which agents learn also from prices, optimal implementable allocations differ from those that would be consistent with flexible prices. More specifically, note that, as the planner increases φ_x to render social learning more effective, if monetary policy were replicating the flexible-price outcomes, by Proposition 3 this increase in φ_x would have been associated with an increase in l_A . Condition (26) establishes that it is actually optimal to reduce l_A relative to this benchmark. That is, the optimal level of output moves less with productivity than its flexible-price counterpart (with taxes).

There are two complementary ways to understand this result. One is in terms of monetary policy mimicking a missing tax instrument. Another is in terms of the optimal covariation between nominal prices and real economic activity that monetary policy should aim to implement. We discuss the first perspective here and the second in the next subsection.

To understand the first perspective, let us momentarily abstract from sticky prices and monetary policy. Let us also put aside how Proposition 4 was obtained in the first place, where the wedges Δ_y and Δ_p originate from, and what they capture. Instead, let us treat those wedges as exogenous and take as given that the planner has to implement the real allocations characterized in Proposition 4. What is the most direct way to achieve this goal if the planner has access to a completely unrestricted set of tax instruments?

The answer is that the planner should apply a differential subsidy on the early and the late production choices. In particular, the subsidy on early production choices should be higher

than the subsidy on later production choices. To read this property from Proposition 4, note first this: what we have called “flexible-price allocations” corresponds under the present perspective to the allocations implemented with a uniform subsidy to all production choices, i.e. to both n and l (equivalently q and l). It follows from condition (26) that relative to the case where all production choices are equally subsidized, the late production choices ought to be subsidized *less* by an amount proportional to Δ_p . Equivalently, n has to be subsidized in proportion to the sum $\Delta_y + \Delta_p$, but l has to be subsidized only in proportion to Δ_y .

From this perspective, a monetary policy that leans further against the wind relative to the one that implements flexible-price outcomes assumes the role of a differential subsidy. Such a monetary policy, by contracting during a boom and expanding during a recession, mimics the effect of introducing a procyclical subsidy that has a bigger footprint on the late production choices than on the earlier ones. This as-if differential tax partly offsets the uniform subsidy, so that in the end the effective subsidy on l is proportional only to Δ_y , whereas that on n is proportional to the sum $\Delta_y + \Delta_p$, as desired.

This echoes how monetary policy works in the textbook New Keynesian model: in that model, too, whenever monetary policy departs from flexible-price allocations, it does so only to mimic a missing tax instrument (Correia, Nicolini, and Teles, 2008). In particular, in the oft-considered case of a monetary policy that “leans against cost-push shocks,” the missing tax instrument is the uniform (across inputs) state-contingent subsidy needed to offset a time-varying monopoly. Here, this kind of tax instrument is allowed, so the rationale for this policy is different. Monetary policy still mimics *some* type of missing tax instrument, but in this case it is a subsidy that can differentiate between early and late choices.

Clearly, such a differential subsidy is hard to envision in the real world, especially given the difficulty of figuring out the exact timing of the different production choices and the information upon which they are based. This explains why we view a “direct” implementation implausible in practice. An “indirect” implementation via monetary policy helps bypass this problem for the following basic reason: insofar as certain production decisions take place at a similar time, or on the basis of similar information, as price-setting decisions, monetary policy will naturally have less control over these real decisions relative to those that *must* adjust after prices are fixed. In other words, monetary policy naturally has a differential real effect, akin to that of a differential tax. Importantly, monetary policy allows the planner to mimic the needed differential tax without explicit knowledge of which real decisions are made first or second, or which ones are most directly regulated by monetary policy.

This discussion explains the *mechanics* of the optimal monetary policy, but not the *function* served by it. To understand this, and to pave the way for the second perspective on what monetary policy accomplishes (which we turn to in the next subsection), we must ask why Proposition 4 holds in the first place, and in particular why optimality calls for a larger subsidy on n than on l . The abstract math is that n contributes to learning through both quantities and

prices, whereas l contributes only through quantities. The logic is that, because firms choose n at the same time and on the basis of the same information as p , a more information-sensitive n translates to more information revelation through both quantities and prices. By contrast, because l adjusts after prices have been fixed, a more information-sensitive l contributes to more learning only through quantities. It follows that internalizing the social learning through quantities calls for a procyclical subsidy on both n and l , whereas internalizing the social learning through prices call for a subsidy only on l , or equivalently for a monetary policy that mimics the aforementioned differential tax.

We expand on this logic and also formally characterize the policy mix that supports the optimal allocation in the next subsection.

5.3 Optimal Policy Mix

To characterize the optimal combination of fiscal and monetary policy, we simply combine the results in Propositions 3 and 4 to obtain the following result.

Proposition 5. *(i) The optimal tax is countercyclical, as in the baseline model. In particular,*

$$\tau_A^* + \tau_z^* = -\chi_1 \Delta_y - \chi_2 \Delta_p, \quad (27)$$

for some positive scalars χ_1, χ_2 .

(ii) The optimal monetary policy is less accommodative of the productivity shock than the policy that would have replicated the flexible-price allocations. In particular,

$$\rho_A^* = \hat{\rho}_A + \chi_3 \Delta_p, \quad (28)$$

where χ_3 is a positive scalar and $\hat{\rho}_A$ is the coefficient in the interest-rate rule that replicates flexible-price outcomes.

This result translates the informational wedges from Proposition 4 into policy prescriptions. Part (i) says that optimal taxes are countercyclical, part (ii) says that monetary policy deviates from the benchmark of replicating flexible prices towards further “leaning against the wind,” or raising interest rates more aggressively during booms.

The intuition behind part (i) is similar to that in our baseline model. The novelty is that there are now two informational wedges driving optimal taxes, one related to the learning through quantities and the other related to the learning through prices. This is because countercyclical taxes incentivize firms to react more aggressively to their information when making both their production and their pricing choices, thereby improving the learning through both channels.

The logic for taxes holds true even if monetary policy were restricted to replicate flexible-price outcomes. For any given information structure, replication of flexible-price outcomes maximizes allocative efficiency by minimizing relative-price distortions. But part (ii) of

Proposition 4 establishes that when and only when there is learning through prices, optimal monetary policy deviates from this familiar benchmark: it trades-off less allocative efficiency, or more relative-price distortions, for the sake of inducing more learning through prices.

In the previous subsection, we explained how the counter-cyclical of the optimal monetary policy can be understood in terms of mimicking a missing tax instrument. We now expand on a second, complementary perspective, which relates to why the optimal monetary policy “leans against the wind” in the sense of inducing a negative relation between prices and real economic activity—and in particular why this helps improve social learning.

To this goal, it is important to understand first the reference point from which the optimal monetary policy departs. For any given information structure, had monetary policy replicated flexible-price outcomes, it would have maximized allocative efficiency and minimized relative-price distortions. But it would *not* have stabilized the nominal price level. Instead, it would have allowed the nominal price level to move in the direction opposite that of real output, for the reason first explained in [Angeletos and La’O \(2020\)](#).

Let us first review this reason. Due to the real rigidity, it is efficient for the relative production of any two firms to vary with their relative information, or beliefs, about the state of the economy. Next, because the demand curve faced by any given firm is downward-sloping, in order to preserve efficient movements in relative production, a firm’s nominal price must move in the direction opposite its real quantity in response to any variation in its private information. But aggregate booms tend to correlate with a wave of optimism across a majority of firms, while efficiency dictates that the prices of these firms move in the direction opposite their quantities. As a result, the aggregate price level must be negatively correlated with aggregate output in order to preserve allocative efficiency.

Let us now relate this benchmark to our own result. If taxes were set to zero and monetary policy were to replicate flexible-price outcomes, nominal prices would have moved in the direction opposite aggregate output and aggregate productivity. In our context, this would have *already* allowed prices to aggregate and reveal information about the state of economy. Starting from this benchmark, the cost of a slightly different monetary policy in terms of allocative efficiency is only second order. But the benefit of inducing more learning is of first order—and this is precisely what the Δ_y and Δ_p wedges capture. Furthermore, because nominal prices are negatively related to real economic activity in the benchmark, the direction of the optimal deviation thereby is clear: it is optimal to raise *both* the positive correlation of the real quantities with the underlying fundamental and the corresponding negative correlation with prices.

For the reasons already explained, both goals can be accomplished in large part by a pro-cyclical subsidy, or counter-cyclical tax, on production. But insofar as there is learning through prices, monetary policy should lean even more against the wind relative to the flexible-price benchmark. If instead monetary policy had moved in the other direction, that of stabilizing the price level, it would have only impeded learning and reduce welfare.

Finally, had we considered a version of this model with fully flexible prices, i.e. zero nominal rigidity, but with learning from both quantities and prices, then the optimal allocation described above would be unattainable. Under flexible prices we would no more have the extra lever afforded by monetary policy, and consequently we would no more be able to use that lever to mitigate the distortion between first and second-stage input choices: equilibrium would necessarily equate their marginal rate of transformation with their corresponding marginal rate of substitution (as in condition 19). In this restricted case, there would still be value for countercyclical taxes, but the optimal tax would be less countercyclical than the one characterized under sticky prices. And by the same token, there would be less learning.

5.4 A Numerical Illustration

Our model is too stylized to permit a quantitative evaluation of the theoretical results obtained above. In our model all the learning takes place “within a period,” whereas in a more realistic model it would take place across periods. Notwithstanding this limitation, it is instructive to illustrate our results within a “calibrated” version of our model.

We focus on a case where learning is all private. This could be motivated in at least one of two ways. First, in the tradition of [Lucas \(1972\)](#), one may have a prior that most learning happens in a decentralized fashion. Second, in the tradition of [Sims \(2003\)](#), [Woodford \(2003b\)](#) and [Mackowiak and Wiederholt \(2009\)](#), one may argue that even if aggregate statistics are readily available, firms may pay little attention to them, or may effectively observe them with idiosyncratic noise. Formally, we set $\sigma_{zy} = \sigma_{zp} = 0$ and let all information be private.¹⁶

Consider first ϵ , the inverse of the Frisch elasticity of labor supply, and γ , which in models without capital captures mainly the income elasticity of labor supply. We follow [Woodford \(2003b\)](#) and set $\epsilon = 1.3$ and $\gamma = 0.2$. These values imply that the complete-information version is consistent with the empirical regularity that output and employment have the same cyclical behavior over the business cycle and real wages are relatively flat. We next set $\theta = 0.5$, which means roughly that half of production is fixed on the basis of incomplete information and half adjusts freely to realized aggregate demand. To calibrate the idiosyncratic risk in TFP, we refer to the NBER-CES Manufacturing Database, which computes a measure of TFP for all 6-digit NAICS manufacturing industries. This suggests a value for σ_a equal to 0.08, or an 8% standard deviation in firm-level TFP from year to year. For the aggregate shock, on the other hand, we set σ_A equal to 0.02, or a 2% standard deviation in aggregate TFP.

We do not have a strong prior on what the right parameterization of the degree of strategic complementarity is. This naturally depends on whether one interprets α narrowly, as a measure of the aggregate demand externalities allowed in our simple model, or more broadly as a

¹⁶The assumption $\sigma_{zp} = 0$ can also be motivated on purely empirical grounds, that the signal contained in inflation about real economic activity is almost negligible (e.g., [Angeletos, Collard, and Dellas, 2018](#)).

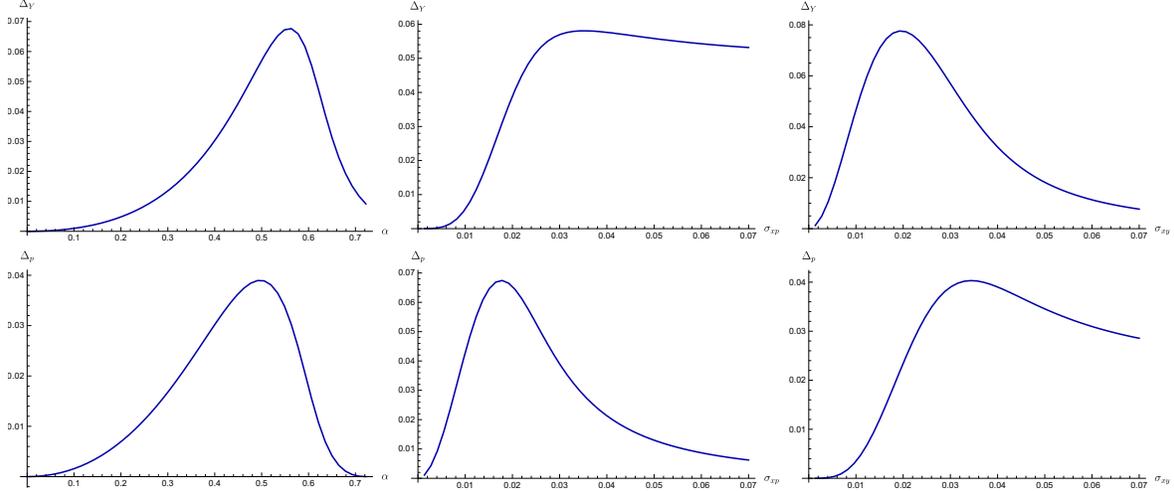


Figure 2. The two wedges (Δ_Y in the first row, Δ_p in the second) for different values of the strategic complementarity (first column), the noise in the price signal (second column) and the noise in the quantity signal (third column).

proxy for all other additional sources of complementarity left outside our model, such as those originating from financial frictions. Under the narrow interpretation, [Angeletos and La'O \(2010\)](#) argue that a value of α around or above 0.5 can be justified for an “elastic” neoclassical economy of the kind argued for in the RBC literature ([King and Rebelo, 1999](#)). We adopt this as our baseline value but explore how the informational wedges and the optimal policies vary as we vary α in the entire $[0, 1]$ range.

What about the noise in the available private signals, or the agents’ cognitive capacity in paying attention to such signals and extracting information from them? For our baseline, we set $\sigma_{xy} = \sigma_{xp} = 0.03$ on the basis of the following rationale. These values, along with our value for idiosyncratic TFP, imply that the overall signal-to-noise ratio in a firm’s overall information about aggregate output is equal to 1.¹⁷ This means that whenever aggregate output goes up by one unit, the average forecast of it goes up by half a unit. In other words, the size of the forecast error is commensurate to the size of the innovation in the forecast. Such a pattern is broadly consistent with the evidence from surveys of macroeconomic forecasts documented in [Coibion and Gorodnichenko \(2012, 2015\)](#). We thus use these values as our baseline, but also explore how the results vary as we vary the noise in either signal.

Figure 2 computes the wedges Δ_Y and Δ_p as a function of α and the levels of noise in the two endogenous signals. Both wedges in the monetary model behave qualitatively the same as the wedge in the baseline model.¹⁸

¹⁷The variance of a firm’s posterior about aggregate TFP is $(\sigma_a^{-2} + \sigma_{xy}^{-2} + \sigma_{xy}^{-2})^{-2}$, which under our parameterization is approximately $(0.02)^2$, or the same as σ_λ^2 . And since aggregate output is proportional to aggregate TFP, this verifies the claim made above.

¹⁸Also note that, when $\theta < 1$, there is an upper bound on α , which is obtained by taking the highest admissible value of ρ and is given by 0.72 under our parameterization. This explains the domain of α in the figure.

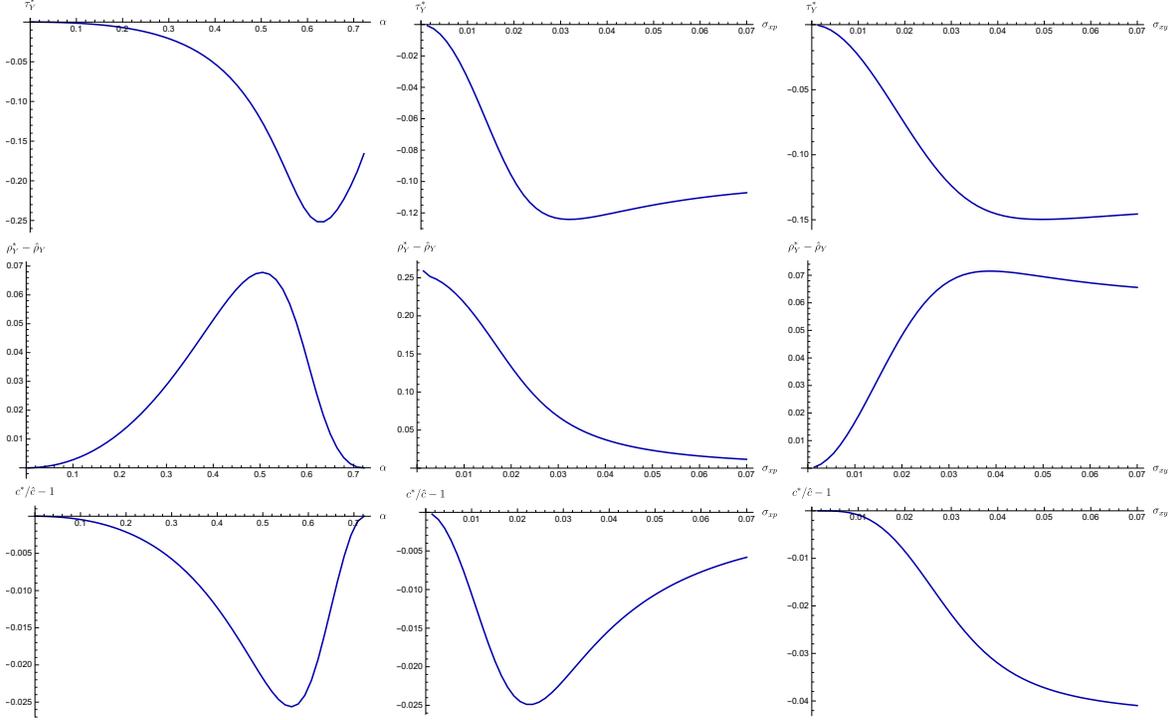


Figure 3. The countercyclicity of the tax (first row) and the optimal countercyclicity of monetary policy as manifested in interest rates (second row) and the output gap (third row), for different values of the strategic complementarity (first column), the level of noise in the price signals (second column), and the level of noise in the quantity signals (last column).

We now turn to optimal policy. Because aggregate output is a log-linear function of aggregate TFP, we can readily re-express the tax and the nominal interest rate that support the optimal allocation as follows:

$$-\log(1 - \tau) = \tau_Y^* \log Y \quad \text{and} \quad \log(1 + R) = \rho_Y^* \log Y.$$

Under this representation, τ_Y^* measures the optimal cyclical elasticity of the tax and ρ_Y^* measures the optimal cyclical elasticity of the nominal interest rate.¹⁹ Similarly, let $\hat{\rho}_Y$ denote the cyclical elasticity of the nominal interest rate required to replicate the corresponding flexible-price allocation. The difference $\rho_Y^* - \hat{\rho}_Y$ then provides a simple measure of the countercyclicity of monetary policy: the more positive this quantity is, the less accommodative the optimal monetary policy is over the business cycle. A second, complementary measure is the quantity $y_A^*/\hat{y}_A - 1$, where y_A^* and \hat{y}_A are the TFP elasticities of, respectively, the optimal and the flexible-price levels of aggregate output: the more negative this quantity is, the more negative the “output gap” is during a boom.

¹⁹These elasticities are given simply by $\tau_Y^* = \tau_A^*/y_A^*$ and $\rho_Y^* = \rho_A^*/y_A^*$, where τ_A^* and ρ_A^* are the corresponding elasticities in terms of productivity, as characterized in Proposition 5, and y_A^* is the elasticity of aggregate output to aggregate productivity along the optimal allocation.

Figure 3 illustrates how these two measures of the optimal countercyclicality of monetary policy, as well as the optimal countercyclicality of taxes, depend on the degree of strategic complementarity and the level of noise in the available quantity and price signals. Let us first comment on the signs of the measures seen in this figure. The negative value for τ_Y^* means that the optimal taxes are countercyclical. Similarly, the positive value for $\rho_Y^* - \hat{\rho}_Y$ and the negative value for $y_A^*/\hat{y}_A - 1$ both mean that optimal monetary policy “leans against the wind:” interest rates are higher than their flexible-price counterparts and the output gap is negative.

Let us next turn to the effect of α , the degree of strategic complementarity. Here, we see the policy translation of the non-monotonic pattern we documented earlier for the wedges: the value of both countercyclical taxes and countercyclical monetary policy is highest when α is neither too low nor too high. For extreme values of α , there is little such value either because firm decisions are nearly independent (for low α) or because the complementarity is so strong that firms optimally disregard any private information and hence there is little scope for social learning to start with (for high α).

The effect of the two levels of noises on the optimal policies also mirror their effects on the wedges. The only subtlety here is the following. As explained earlier on, the optimal monetary policy deviates from the benchmark of replicating flexible prices only insofar as there is learning through prices. It follows that when the noise in the price signals is sufficient large, such a deviation is not worthwhile, which in turn explains why both $\rho_Y^* - \hat{\rho}_Y$ and $y_A^*/\hat{y}_A - 1$ converge to zero as σ_{xp} alone goes to infinity. By contrast, because countercyclical taxes serve the dual role of improving the aggregation of information through both quantities and price signals, τ_Y^* stays bounded away from zero as either σ_{xp} or σ_{xy} becomes larger and larger. For τ_Y^* to vanish, both sources of learning have to be muted.

6 Conclusion

In this paper we have shown that the endogeneity of information contained in macroeconomic statistics and market outcomes about the state of the economy calls for both counter-cyclical taxes and a monetary policy that leans against the wind. We have explained how such policies incentivize firms to act more resolutely on their private information about the economy, thus improving the information revealed to other firms. We have distinguished two main channels of such learning, one through real quantities and another through nominal prices, and explained how each of them contributes to shaping optimal policy. Finally, we have tied the nature of the optimal monetary policy to the second channel: learning through prices.

Our model contained three key features: (i) a real rigidity due to informational friction; (ii) a nominal rigidity due to informational friction; and (iii) the endogenous aggregation of information. Although subsets of these features can be found in previous works, their *combination* is both novel to the literature and the key to our policy lessons.

The real rigidity, which was isolated in our baseline analysis, was essential for understanding why there is social value in improving the aggregation of information in the first place: were it not for the real aspect of the informational friction, a monetary policy that replicates flexible prices would have implemented the *complete-information* first-best outcomes, negating the value of any intervention. The nominal rigidity, on the other hand, was key to letting monetary policy be non-neutral and, hence, be able to assist in the aforementioned goal. And finally, were it not for the endogeneity of information aggregation, the state-contingency of taxes would be unnecessary and there would be no reason for monetary policy to depart from replicating the flexible-price benchmark.

In deriving these lessons, we have allowed the planner to vary taxes with the business cycle, thus providing economic agents with an incentive to respond more strongly to their private information about the state of the economy. In the New Keynesian literature, such state-contingent taxes are typically assumed away so as to open the door for monetary policy to stabilize the economy against inefficient fluctuations, such as movements in monopoly distortions. Such cost-push shocks are absent here, and so, too, is the standard rationale for either state-contingent taxes or monetary policy. Instead, state-contingent taxes and monetary policy are useful because they serve a novel function: they help internalize informational externalities and boost social learning.

Finally, our analysis has focused on the imperfection of information within the private sector, but has assumed that monetary policy itself can be contingent on the true state. Let us first clarify that this assumption does not require an informational asymmetry between the policymaker and the private sector: by the time the policymaker observes the state and adjust its policy, certain production and price-setting decisions have already been made.²⁰ What this assumption, however, does abstract from is the possible endogeneity of the information upon which the policymaker must act.

Suppose, in particular, that both taxes and monetary policy have to be contingent on noisy indicators of the macroeconomic outcomes, namely real output and the price level. The measurement error in these indicators will of course impede the policymaker's ability to achieve the kind of second best outcomes we have characterized in this paper: a third best would prevail. But unless the measurement error is really severe, we expect our policy lessons to be *reinforced* for the following reason: the kind of policies we characterized in this paper will facilitate not only more learning within the private sector but also more information transmission from the private sector to the policymaker. As a result, it should lead to better, more informed, policies. We leave the exploration of this idea for future work.

²⁰In our model, indeed, the state of nature is commonly observed by all firms and the policy maker at the moment taxes and interest rates adjust.

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Appendix

6.1 Proofs for Section 3

Proof of Lemma 1. Take any log-linear strategy of the form $q(a, x, z) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_z z$, for arbitrary coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z)$. The endogenous public signal is then given by

$$z^q = \log Q(\bar{a}, z) + \varepsilon^{zq}$$

where

$$\log Q(\bar{a}, z) = \varphi'_0 + \varphi_a a + \varphi_x x + \varphi_z z$$

is the log of aggregate output. It follows that the public signal z^q can be transformed into an unbiased Gaussian signal \tilde{z}^q about aggregate productivity, defined as follows:

$$\tilde{z}^q \equiv \frac{z^q - \varphi'_0 - \varphi_z z}{\varphi_a + \varphi_x} = \bar{a} + \tilde{\varepsilon}^{zq}$$

where $\tilde{\varepsilon}^{zq} \equiv \varepsilon^{zq}/(\varphi_a + \varphi_x)$. The precision of this signal is

$$\kappa_{zq} \equiv \frac{1}{\text{Var}(\tilde{\varepsilon}^{zq})} = (\varphi_a + \varphi_x)^2 \sigma_{zq}^{-2}.$$

Standard Bayesian updating then implies that the sufficient statistic z of available public information is given by a weighted average of the exogenous productivity signal z^a and the (normalized) endogenous output signal \tilde{z}^q :

$$z = \frac{\kappa_{za}}{\kappa_z} z^a + \frac{\kappa_{zq}}{\kappa_z} \tilde{z}^q,$$

where κ_{za} and κ_{zq} are the precisions of these two signals, while $\kappa_z = \kappa_{za} + \kappa_{zq}$ is the overall precision of the sufficient statistic z .

The analysis of the private signal x_i^q is similar: it can be transformed into an unbiased signal with precision $\kappa_{xq} = (\varphi_a + \varphi_x)^2 \sigma_{xq}^{-2}$.

Proof of Lemma 2. Take an arbitrary log-linear strategy of the form $q(a, x, z) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_z z$. For any coefficients $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and any precisions $\kappa = (\kappa_x, \kappa_z)$, the implied level of welfare (ex-ante utility) can be expressed as follows:

$$\mathbb{E}u = \mathcal{W}(\varphi; \kappa) \equiv \frac{1}{1 - \gamma} \exp V_c(\varphi; \kappa) - \frac{1}{\epsilon} \exp V_n(\varphi; \kappa), \quad (29)$$

where

$$\begin{aligned}
V_c(\varphi; \kappa) &\equiv (1 - \gamma) (\varphi_0 + (\varphi_a + \varphi_x + \varphi_z)\mu) \\
&\quad + \frac{1}{2} (1 - \gamma) \left(\frac{\rho - 1}{\rho} \right) \left[\frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{\varphi_a \varphi_x}{\kappa_x} \right] \\
&\quad + \frac{1}{2} (1 - \gamma)^2 \left[\frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z)^2}{\kappa_A} \right],
\end{aligned} \tag{30}$$

$$\begin{aligned}
V_n(\varphi; \kappa) &\equiv \frac{\epsilon}{\theta} (\varphi_0 + (\varphi_a + \varphi_x + \varphi_z - 1)\mu) \\
&\quad + \frac{1}{2} \frac{\epsilon^2}{\theta^2} \left[\frac{(\varphi_a - 1)^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{(\varphi_a - 1)\varphi_x}{\kappa_x} + \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z - 1)^2}{\kappa_A} \right].
\end{aligned} \tag{31}$$

Proof of Proposition 1. We prove the two parts of the proposition together. Recall from Lemma 1 that any given strategy induces a κ_x and a κ_z as functions of $\varphi_a + \varphi_x$; let $K_1(\varphi_a + \varphi_x)$ and $K_2(\varphi_a + \varphi_x)$ denote the first and second element of the vector $K(\varphi_a + \varphi_x)$. We can then express the planner's problem as follows:

Planner's problem. Choose $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ and $\kappa = (\kappa_x, \kappa_z)$ so as to maximize $W(\varphi; \kappa)$ subject to $\kappa = K(\varphi_a + \varphi_x)$.

To solve this problem, we proceed in two steps. The first step is to characterize the strategy that is optimal subject to the constraint that the sum $\varphi_a + \varphi_x$ is kept constant at some $\bar{\varphi} \in \mathbb{R}$ and accordingly the precisions κ_x and κ_z are kept constant at $\kappa_x = K_1(\bar{\varphi})$ and $\kappa_z = K_2(\bar{\varphi})$. The second step is to optimize over the sum $\bar{\varphi}$ and the precision κ_x and κ_z subject to the constraint that $\kappa_x = K_1(\bar{\varphi})$ and $\kappa_z = K_2(\bar{\varphi})$. The first step permits us to characterize the efficient allocation as a function of the Lagrange multiplier associated with the constraint $\varphi_a + \varphi_x = \bar{\varphi}$. The second step permits us to interpret this Lagrange multiplier as the shadow value of the informational externality, as well as to prove the existence of an efficient allocation and to complete its characterization by showing that this multiplier is strictly positive.

Thus consider the first step. Fix some $\bar{\varphi} \in \mathbb{R}$, let $\kappa = K(\bar{\varphi})$, and consider the following constrained problem:

Auxiliary problem 1. Choose φ so as to maximize $\mathcal{W}(\varphi; \kappa)$ subject to $\varphi_a + \varphi_x = \bar{\varphi}$.

Note that \mathcal{W} is differentiable in φ for fixed κ . Let $\tilde{\eta}$ denote the Lagrange multiplier for the constraint $\varphi_a + \varphi_x = \bar{\varphi}$. The first-order conditions for this problem are then the following:

$$\begin{aligned}
\varphi_0 &: 0 = \frac{\partial \mathcal{W}}{\partial \varphi_0} \\
\varphi_a &: 0 = \frac{\partial \mathcal{W}}{\partial \varphi_a} + \tilde{\eta} \\
\varphi_x &: 0 = \frac{\partial \mathcal{W}}{\partial \varphi_x} + \tilde{\eta} \\
\varphi_z &: 0 = \frac{\partial \mathcal{W}}{\partial \varphi_z}
\end{aligned}$$

Using the characterization of \mathcal{W} , the first of these conditions reduces to the following:

$$\varphi_0 : 0 = \exp V_c(\varphi; \kappa) - \frac{1}{\theta} \exp V_n(\varphi; \kappa). \quad (32)$$

This guarantees that $V_c = V_n - \log \theta$ at the efficient allocation and gives φ_0 as a function of $\varphi_a, \varphi_x, \varphi_z, \kappa_x, \kappa_z$ and exogenous parameters. Let $V \equiv V_c = V_n - \log \theta$ and let $\eta \equiv e^{-V} \tilde{\eta}$. The rest of the first-order conditions reduce to the following:

$$\begin{aligned} \varphi_a : 0 &= \left(\frac{\rho-1}{\rho} \right) \frac{\varphi_a}{\kappa_\xi} + \left(\frac{\rho-1}{\rho} \right) \frac{\varphi_x}{\kappa_x} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_z)}{\kappa_A} \\ &\quad - \frac{\epsilon(\varphi_a - 1)}{\theta \kappa_\xi} - \frac{\epsilon \varphi_x}{\theta \kappa_x} - \frac{\epsilon(\varphi_a + \varphi_x + \varphi_z - 1)}{\theta \kappa_A} + \eta \\ \varphi_x : 0 &= \left(\frac{\rho-1}{\rho} \right) \frac{\varphi_x}{\kappa_x} + \left(\frac{\rho-1}{\rho} \right) \frac{\varphi_a}{\kappa_x} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_z)}{\kappa_A} \\ &\quad - \frac{\epsilon \varphi_x}{\theta \kappa_x} - \frac{\epsilon(\varphi_a - 1)}{\theta \kappa_x} - \frac{\epsilon(\varphi_a + \varphi_x + \varphi_z - 1)}{\theta \kappa_A} + \eta \\ \varphi_z : 0 &= (1-\gamma) \frac{\varphi_z}{\kappa_z} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_z)}{\kappa_A} - \frac{\epsilon \varphi_z}{\theta \kappa_z} - \frac{\epsilon(\varphi_a + \varphi_x + \varphi_z - 1)}{\theta \kappa_A} \end{aligned}$$

For fixed η , this is a linear system of three equations in the three coefficients φ_a, φ_x and φ_z . Subtracting the first equation from the second, we obtain

$$\varphi_a^{**} = \frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \equiv \beta.$$

We can then solve the remaining two equations for φ_x and φ_z as follows:

$$\begin{aligned} \varphi_x^{**} &= \left\{ \frac{(1-\alpha)\kappa_x}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left\{ \frac{\kappa_x(\kappa_z + \kappa_A)}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right\} \eta \\ \varphi_z^{**} &= \left\{ \frac{\kappa_z}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left\{ \frac{\kappa_x \kappa_z}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right\} \eta \end{aligned}$$

Letting

$$\Delta \equiv \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left(\frac{\kappa_x(\kappa_z + \kappa_A)}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right) \eta,$$

gives conditions (5) and (6). Finally, note that Δ is just a rescaling of the Lagrange multiplier η , so we can think of Δ itself as the relevant Lagrange multiplier. Using then the above results along with the constraint $\varphi_a + \varphi_x = \bar{\varphi}$, we can express Δ (or equivalently η) as follows:

$$\Delta = \bar{\varphi} - \left\{ \frac{\kappa_x + \kappa_z + \kappa_A}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \right\} \beta. \quad (33)$$

Using this into conditions (5) and (6), we can obtain the optimal coefficients as functions of the sum $\bar{\varphi}$ and the precisions κ_x and κ_z . Let $\varphi(\bar{\varphi}; \kappa)$ denote this solution; for the rest of this proof, whenever we write φ , we mean $\varphi = \varphi(\bar{\varphi}; \kappa)$.

We can then express the level of welfare obtained at this solution also as function of the sum $\bar{\varphi}$ and the precisions κ_x and κ_z . In particular, using the FOC with respect to φ_0 , we get that

$$\mathcal{W}(\varphi; \kappa) = \left(\frac{\frac{\epsilon}{\theta} - 1 + \gamma \theta}{1 - \gamma} \frac{1}{\epsilon} \right) \exp V_c(\varphi; \kappa). \quad (34)$$

Since $\frac{\epsilon}{\theta} - 1 + \gamma > 0$ and $\frac{\epsilon}{\theta} > 0$, we can consider the following monotone transformation of welfare:

$$\mathcal{TW}(\varphi; \kappa) \equiv \frac{1}{1 - \gamma} V_c(\varphi; \kappa).$$

Using then the characterization of the efficient coefficients, we conclude that

$$\mathcal{TW}(\varphi(\bar{\varphi}; \kappa), \kappa) = W(\bar{\varphi}; \kappa) \equiv A(\kappa) - B(\kappa) (\bar{\varphi} - f(\kappa))^2, \quad (35)$$

where

$$B(\kappa) \equiv \frac{\epsilon}{2\theta(1 - \alpha)} \frac{\kappa_A + (1 - \alpha)\kappa_x + \kappa_z}{\kappa_x(\kappa_A + \kappa_z)} > 0$$

and

$$f(\kappa) \equiv \frac{\kappa_A + \kappa_x + \kappa_z}{(1 - \alpha)\kappa_x + \kappa_z + \kappa_A} \beta = \arg \max_{\bar{\varphi}} W(\bar{\varphi}; \kappa) = \arg \max_{\bar{\varphi}} \mathcal{W}(\varphi(\bar{\varphi}; \kappa), \kappa).$$

(The precise value of $A(\kappa)$ has no particular interest, so it is omitted.) This result has a simple interpretation. Note that $f(\kappa)$ identifies the sum $\bar{\varphi} = \varphi_a + \varphi_x$ that would have been efficient had information been exogenous (equivalently, $\varphi(f(\kappa); \kappa)$ are simply the coefficients of the efficient allocation when $\Delta = 0$). Hence, (35) expresses welfare as a monotone transformation of the quadratic distance between any value $\bar{\varphi}$ that the planner may choose and the one that would have been optimal from a purely allocative perspective. Clearly, the only reason that the efficient $\bar{\varphi}$ may differ from $f(\kappa)$ is the informational externality.

We now proceed to the second step, namely that of optimizing over the sum $\bar{\varphi} = \varphi_a + \varphi_x$ and the induced precisions $\kappa_x = K_1(\bar{\varphi})$ and $\kappa_z = K_2(\bar{\varphi})$. Letting

$$\bar{W}(\bar{\varphi}) \equiv W(\bar{\varphi}; \kappa(\bar{\varphi})),$$

the planner's problem reduces to the following unidimensional problem:

Auxiliary problem 2. Choose $\bar{\varphi} \in \mathbb{R}$ so as to maximize $\bar{W}(\bar{\varphi})$.

First, note that, because $f(\kappa) > 0$, it is necessarily the case that, for any given κ , $W(\bar{\varphi}; \kappa) > W(-\bar{\varphi}; \kappa)$ whenever $\bar{\varphi} > 0$. And because $\kappa(\bar{\varphi}) = \kappa(-\bar{\varphi})$, it is immediate that $\bar{W}(\bar{\varphi}) > \bar{W}(-\bar{\varphi})$ whenever $\bar{\varphi} > 0$, which means that it is never optimal to choose $\bar{\varphi} < 0$.

Next, we can show that

$$\frac{\partial W}{\partial \kappa_z} = \frac{\epsilon}{2\theta\kappa_z^2} \varphi_2(\bar{\varphi}; \kappa)^2 = \frac{\epsilon(\beta - (1 - \alpha)\bar{\varphi})^2}{2\theta(1 - \alpha)^2(\kappa_A + \kappa_z)^2}.$$

Along with the fact that κ_z is a quadratic function of $\bar{\varphi}$, this guarantees that

$$\frac{\partial W}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \bar{\varphi}} \rightarrow 0 \quad \text{as} \quad \bar{\varphi} \rightarrow \infty.$$

In words, the social value of a marginal increase in the precision κ_z of public information vanishes as this precision goes to infinity. A similar result holds for private information:

$$\frac{\partial W}{\partial \kappa_x} = \frac{\epsilon}{2\theta(1-\alpha)\kappa_x^2} \varphi_1(\bar{\varphi}; \kappa)^2 = \frac{\epsilon(\beta - \bar{\varphi})^2}{2\theta(1-\alpha)\kappa_x^2}$$

and hence

$$\frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}} \rightarrow 0 \quad \text{as } \bar{\varphi} \rightarrow \infty.$$

At the same time, because

$$\frac{\partial W}{\partial \bar{\varphi}} = -2B(\kappa) (\bar{\varphi} - f(\kappa))$$

and because $B(\kappa) \rightarrow \frac{\epsilon}{2\theta(1-\alpha)\kappa_x} > 0$ and $f(\kappa) \rightarrow \beta$ as $\kappa_z \rightarrow \infty$, we have that

$$\frac{\partial W}{\partial \bar{\varphi}} \rightarrow -\infty \quad \text{as } \bar{\varphi} \rightarrow \infty.$$

Combining, we conclude that

$$\frac{\partial \bar{W}(\bar{\varphi})}{\partial \bar{\varphi}} \rightarrow -\infty \quad \text{as } \bar{\varphi} \rightarrow \infty.$$

Along with the facts that $\bar{W}(\bar{\varphi})$ is continuous in $\bar{\varphi}$ and that it is without loss of optimality to restrict $\bar{\varphi} \in [0, \infty)$, this guarantees the existence of a solution to auxiliary problem 2 (and hence the existence of an efficient allocation).

Let $\bar{\varphi}^* \geq 0$ denote any such a solution. Since \bar{W} is differentiable, this solution must satisfy $\frac{\partial \bar{W}}{\partial \bar{\varphi}} = 0$. Using the definition of \bar{W} , this is equivalent to

$$\frac{\partial W}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}} = 0. \quad (36)$$

Note that the second and the third term are always non-negative. Whenever $0 \leq \bar{\varphi} < f(\kappa)$, the first term is strictly positive, so that the sum is also strictly positive; this rules out $\bar{\varphi}^* \in [0, f(\kappa))$. Moreover, when $\bar{\varphi} = f(\kappa)$, the first term is zero, but now the other two terms are strictly positive, so that the sum is also strictly positive; this rules out $\bar{\varphi}^* = f(\kappa)$. It follows that $\bar{\varphi}^* > f(\kappa)$ necessarily. From (33) and the definition of $f(\kappa)$, we have that, at the efficient allocation, $\Delta = \bar{\varphi}^* - f(\kappa)$. It follows that $\Delta > 0$, as claimed in the proposition.

Finally, that Δ (or equivalently η) represents the shadow value of the informational externality follows directly from the envelope condition of auxiliary problem 1, namely $\frac{\partial W}{\partial \bar{\varphi}} = -\eta$, along with the first-order condition of auxiliary problem 2, namely condition (36). Indeed, combining these two conditions gives

$$\eta = \frac{\partial W}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}},$$

which means that the Lagrange multiplier measures the social value of increasing the precision of available public information by increasing the sensitivity of allocations to local information.

Proof of Lemma 3. We consider a combination of the following tax instruments: a linear tax $\tau^R(\bar{a}, z)$ on firm revenue, a linear tax $\tau^L(\bar{a}, z)$ on household labor income, and a linear tax $\tau^C(\bar{a}, z)$ on household consumption (a sales tax that is uniform across commodities). To guarantee the existence of an equilibrium where the allocations are log-normal, these taxes are assumed to be log-linear functions of the (\bar{a}, z) :

$$\begin{aligned} -\log(1 - \tau^R(\bar{a}, z)) &= \tau_0^R + \tau_A^R \bar{a} + \tau_z^R z, \\ -\log(1 - \tau^L(\bar{a}, z)) &= \tau_0^L + \tau_A^L \bar{a} + \tau_z^L z, \\ \log(1 + \tau^C(\bar{a}, z)) &= \tau_0^C + \tau_A^C \bar{a} + \tau_z^C z. \end{aligned}$$

Given these taxes, the firm's realized net-of-tax profits are given by

$$\pi(\omega, \bar{a}, z) = (1 - \tau^R(\bar{a}, z)) p(\omega, \bar{a}, z) q(\omega) - w(\omega) n(\omega),$$

while the budget constraint of the household is given by

$$\begin{aligned} (1 + \tau^C(\bar{a}, z)) \int p(\omega, \bar{a}, z) c(\omega, \bar{a}, z) dF(\omega | \bar{a}, z) \\ = \int \pi(\omega, \bar{a}, z) dF(\omega | \bar{a}, z) + (1 - \tau^L(\bar{a}, z)) \int w(\omega) n(\omega) dF(\omega | \bar{a}, z) + T(\bar{a}, z) \end{aligned}$$

where $T(\bar{a}, z)$ is a lump-sum transfer or tax. (By the government budget, the latter is equal to the revenue from all the taxes.) It follows that the optimal labor supply of the typical worker on island ω is given by

$$n(\omega)^{\epsilon-1} = w(\omega) \mathbb{E} \left[(1 - \tau^L(\bar{a}, z)) \frac{U'(C(\bar{a}, z))}{(1 + \tau^C(\bar{a}, z)) P(\bar{a}, z)} \middle| \omega \right],$$

while the consumer's stochastic discount factor is given by $\frac{U'(Q(\bar{a}, z))}{(1 + \tau^C(\bar{a}, z)) P(\bar{a}, z)}$. The firm's objective is thus given by

$$\mathbb{E} \left[\frac{U'(Q(\bar{a}, z))}{(1 + \tau^C(\bar{a}, z)) P(\bar{a}, z)} \left((1 - \tau^R(\bar{a}, z)) P(\bar{a}, z) Q(\bar{a}, z)^{1/\rho} q(\omega)^{1-1/\rho} - w(\omega) n(\omega) \right) \middle| \omega \right].$$

Taking the FOC for the firm's problem, substituting the equilibrium wage, and guessing that the taxes and the allocations are jointly log-normal (which they are in the equilibrium we construct in the main text), we conclude that the equilibrium level of employment is pinned down by the following condition:

$$n(\omega)^{\epsilon-1} = \left(\frac{\rho-1}{\rho} \right) \mathbb{E} \left[\frac{\chi (1 - \tau^R(\bar{a}, z)) (1 - \tau^L(\bar{a}, z))}{1 + \tau^C(\bar{a}, z)} U'(Q(\bar{a}, z)) \left(\frac{q(\omega)}{Q(\bar{a}, z)} \right)^{-\frac{1}{\rho}} \left(\theta A(\omega) n(\omega)^{\theta-1} \right) \middle| \omega \right].$$

where χ is a constant that depends on second-order terms. The result then follows by defining the tax wedge as

$$1 - \tau(\bar{a}, z) \equiv \frac{\chi (1 - \tau^R(\bar{a}, z)) (1 - \tau^L(\bar{a}, z))}{1 + \tau^C(\bar{a}, z)}.$$

and substituting in for $n(\omega)$,

$$n(\omega) = \left(\frac{q(\omega)}{A(\omega)} \right)^{\frac{1}{\theta}}$$

Equivalently, the tax wedge is given by (8) with $\tau_0 \equiv -\log \chi + \tau_0^R + \tau_0^C + \tau_0^L$, $\tau_A \equiv \tau_A^R + \tau_A^C + \tau_A^L$, and $\tau_z \equiv \tau_z^R + \tau_z^C + \tau_z^L$.

The following lemma describes the set of strategies as in (1) that can be implemented by a tax policy as in (8). This is a key step to prove that the optimal allocations defined by (4)-(6) are implementable.

Lemma 7. *Consider any strategy as in (1). There exists a state-contingent tax policy as in (8) that implements this strategy as an equilibrium strategy, i.e., that satisfies condition (7), if and only if $\varphi_a = \beta$.*

Proof of Lemma 7. By Lemma 3, the equilibrium strategy must solve the following fixed point:

$$q(\omega)^{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \left(\frac{\rho - 1}{\rho} \right) \theta A(\omega)^{\frac{\epsilon}{\theta}} \mathbb{E} \left[\exp(-\tau_0 - \tau_A \bar{a} - \tau_z z) Q(\bar{a}, z)^{\frac{1}{\rho} - \gamma} | \omega \right] \quad (37)$$

with $Q(\bar{a}, z)$ given by (2). It follows that the equilibrium strategy is given by

$$\log q(\omega) = \hat{\varphi}_0(\tau) + \hat{\varphi}_a(\tau) a + \hat{\varphi}_x(\tau) x + \hat{\varphi}_z(\tau) z,$$

where

$$\hat{\varphi}_a(\tau) = \beta$$

$$\hat{\varphi}_x(\tau) = \left(1 - \frac{\theta}{\epsilon \alpha} \tau_A \right) \left(\frac{(1 - \alpha) \kappa_x}{(1 - \alpha) \kappa_x + \kappa_z + \kappa_A} \right) \frac{\alpha}{1 - \alpha} \beta \quad (38)$$

$$\hat{\varphi}_z(\tau) = \frac{1}{1 - \alpha} \left(\frac{\kappa_z}{\kappa_x} \hat{\varphi}_x(\tau) - \frac{\beta \theta}{\epsilon} \tau_z \right) \quad (39)$$

$$\hat{\varphi}_0(\tau) = \frac{1}{\frac{\epsilon}{\theta} + \gamma - 1} \left[-\tau_0 + \left(-\tau_A + \left(\frac{1}{\rho} - \gamma \right) (\hat{\varphi}_a(\tau) + \hat{\varphi}_x(\tau)) \right) \frac{\kappa_A}{\kappa_A + \kappa_x + \kappa_z} \mu \right. \\ \left. + \left(\frac{1}{\rho} - \gamma \right) \left(\frac{\rho - 1}{\rho} \right) \frac{(\hat{\varphi}_a(\tau) + \hat{\varphi}_x(\tau))^2}{2} \sigma_x^2 + \frac{1}{2} \left(\frac{1}{\rho} - \gamma \right)^2 (\hat{\varphi}_a(\tau) + \hat{\varphi}_x(\tau))^2 \sigma_0^2 \right. \\ \left. + \frac{1}{2} \tau_A^2 \sigma_0^2 - \tau_A \left(\frac{1}{\rho} - \gamma \right) (\hat{\varphi}_a(\tau) + \hat{\varphi}_x(\tau)) \sigma_0^2 + \log \left(\theta \frac{\rho - 1}{\rho} \right) \right]. \quad (40)$$

We now prove the claim in the lemma. Pick an arbitrary strategy for which $\varphi_a = \beta$ and let $(\varphi_0^\#, \varphi_x^\#, \varphi_z^\#)$ denote the remaining coefficients. From condition (38), there is a unique value for τ_A that induces $\hat{\varphi}_x(\tau) = \varphi_x^\#$; this is given by

$$\tau_A = \frac{\epsilon}{\theta} \left\{ \alpha - \varphi_x^\# \frac{(1 - \alpha) \kappa_x + \kappa_z + \kappa_A}{\beta \kappa_x} \right\}. \quad (41)$$

From (39), there is then a unique value for τ_z that induces $\hat{\varphi}_z(\tau) = \varphi_z^\#$; this is given by

$$\tau_z = \frac{\epsilon}{\beta\theta} \left\{ \frac{\kappa_z}{\kappa_x} \varphi_x^\# - (1 - \alpha) \varphi_z^\# \right\}. \quad (42)$$

Combining these two results, we have a unique pair (τ_A, τ_z) that induces the desired $(\varphi_x^\#, \varphi_z^\#)$. But then from (40) there is also a unique τ_0 that induces $\hat{\varphi}_0(\tau) = \varphi_0^\#$.

Proof of Proposition 2. From conditions (41) and (42) in the proof of Lemma 7, the optimal tax satisfies

$$\begin{aligned} \tau_A^* &= \frac{\epsilon}{\theta} \left(\alpha - \varphi_x^* \frac{(1 - \alpha)\kappa_x^* + \kappa_z^* + \kappa_A}{\beta\kappa_x^*} \right), \\ \tau_z^* &= \frac{\epsilon}{\beta\theta} \left(\frac{\kappa_z^*}{\kappa_x^*} \varphi_x^* - (1 - \alpha) \varphi_z^* \right). \end{aligned}$$

Using the characterization of φ_x^* and φ_z^* from Proposition 1, we get

$$\tau_A^* = -\lambda\Delta \quad \text{and} \quad \tau_z^* = \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \lambda\Delta, \quad (43)$$

where

$$\lambda \equiv \frac{\epsilon}{\beta\theta} \frac{(1 - \alpha)\kappa_x^* + \kappa_z^* + \kappa_A}{\kappa_x^*} > 0.$$

It follows that $\Delta > 0$ is both necessary and sufficient for every one of the following properties: $\tau_A^* < 0$, $\tau_A^* + \tau_z^* < 0$ and $\tau_z^* > 0$.

To interpret this result, note first that $z = \bar{a} + \varepsilon$, where ε is noise. The property that $\tau_A < 0$ means the that tax is negatively correlated with aggregate productivity for given common belief z ; that is, it is negatively correlated with the surprise component in realized aggregate productivity. At the same time, the property that $\tau_a + \tau_z < 0$ means that the tax is negatively correlated with aggregate productivity for given noise ε ; that is, the overall effect of the productivity shock is also negative. Next, the property that $\tau_z > 0$ means that the tax is positively correlated with the noise. Finally, to understand the overall cyclical behavior of the optimal tax, consider the covariance between the (log) tax and (log) output. Since

$$-\log(1 - \tau(\bar{a}, z)) = \tau_0^* + (\tau_A^* + \tau_z^*)\bar{a} + \tau_z^*\varepsilon \quad \text{and} \quad \log Q(\bar{a}, z) = \varphi_0^* + (\varphi_a^* + \varphi_x^* + \varphi_z^*)\bar{a} + \varphi_z^*\varepsilon,$$

their covariance is given by

$$Cov(-\log(1 - \tau), \log Q) = (\tau_A^* + \tau_z^*)(\varphi_a^* + \varphi_x^* + \varphi_z^*)Var(\bar{a}) + \tau_z^*\varphi_z^*Var(\varepsilon)$$

Using the fact that $Var(\bar{a}) = 1/\kappa_A$ and $Var(\varepsilon) = 1/\kappa_z^*$ and rearranging, we get

$$Cov(-\log(1 - \tau), \log Q) = (\tau_A^* + \tau_z^*)(\varphi_a^* + \varphi_x^*)\frac{1}{\kappa_A} + \left\{ \tau_A^*\frac{1}{\kappa_A} + \tau_z^*\frac{\kappa_A + \kappa_z^*}{\kappa_A\kappa_z^*} \right\} \varphi_z^*.$$

By (43), the last term is necessarily zero. Next, note that $\varphi_a^* + \varphi_x^*$ is necessarily positive, while $\tau_A^* + \tau_z^*$ is necessarily negative. We conclude that the tax is negatively correlated with aggregate output.

6.2 Proofs for Section 4

Proof of Lemma 4. From the consumer's optimal demand, we have that the (shadow) prices must satisfy

$$-\rho(\log p(\omega) - \log P(\bar{a}, z)) = (\log c(\omega, \bar{a}, z) - \log C(\bar{a}, z))$$

where

$$\begin{aligned}\log p(\omega) &= \text{const} + \psi_a a + \psi_x x + \psi_z z \\ \log P(\bar{a}, z) &= \text{const} + \psi_a \bar{a} + \psi_x \bar{a} + \psi_z z \\ \log c(\omega, \bar{a}, z) &= \text{const} + (\varphi_a + (1 - \theta)l_a)a + (\varphi_x + (1 - \theta)l_x)x + (\varphi_z + (1 - \theta)l_z)z + (1 - \theta)l_A \bar{a} \\ \log C(\bar{a}, z) &= \text{const} + (\varphi_a + (1 - \theta)l_a)\bar{a} + (\varphi_x + (1 - \theta)l_x)\bar{a} + (\varphi_z + (1 - \theta)l_z)z + (1 - \theta)l_A \bar{a}\end{aligned}$$

It follows that the following must hold for all (a, x, z, \bar{a}) :

$$-\rho(\psi_a a + \psi_x x - (\psi_a + \psi_x)\bar{a}) = (\varphi_a + (1 - \theta)l_a)a + (\varphi_x + (1 - \theta)l_x)x - (\varphi_a + (1 - \theta)l_a + \varphi_x + (1 - \theta)l_x)\bar{a},$$

which is true if and only if

$$\psi_a = -\frac{1}{\rho}(\varphi_a + (1 - \theta)l_a) \quad (44)$$

$$\psi_x = -\frac{1}{\rho}(\varphi_x + (1 - \theta)l_x) \quad (45)$$

Finally, note that the observation of $z^p = \log P(\bar{a}, z) + \varepsilon^p$ is equivalent to the observation of the unbiased Gaussian signal

$$\tilde{z}^p \equiv \frac{z^p - \text{const} - \psi_z z}{\psi_a + \psi_x} = \bar{a} + \tilde{\varepsilon}_p,$$

where $\tilde{\varepsilon}_p = \varepsilon_p / (\psi_a + \psi_x)$. We conclude that the precision of the public price signal is given by

$$\kappa_{zp} \equiv (\psi_a + \psi_x)^2 \sigma_{zp}^{-2} = \frac{1}{\rho^2} (\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x))^2 \sigma_{zp}^{-2}.$$

The proof for the public signal on $\log Y(\bar{a}, z)$ is given in the main text. We thus have

$$\kappa_{zy} \equiv (\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x + l_A))^2 \sigma_{zy}^{-2},$$

which, together with $\kappa_z = \sigma_{za}^{-2} + \kappa_{zy} + \kappa_{zp}$, gives the expression for κ_z in (13). Analogous arguments give the expression for κ_x .

Proof of Proposition 3. In equilibrium, $l(\omega, \bar{a}, z)$ adjusts in stage 2 so as to satisfy the the consumer's demand:

$$\frac{p(\omega)}{P(\bar{a}, z)} = \left(\frac{q(\omega) l(\omega, \bar{a}, z)^{1-\theta}}{C(\bar{a}, z)} \right)^{-\frac{1}{\rho}} \quad (46)$$

Solving for $l(\omega, \bar{a}, z)$ and substituting into the firm's objective, the latter reduces to the following:

$$\mathbb{E} \left[\frac{U'(C(\bar{a}, z))}{P(\bar{a}, z)} \left((1 - \tau(\bar{a}, z))C(\bar{a}, z)p^{1-\rho}P(\bar{a}, z)^\rho - w_2(\omega) \left(\frac{p}{P(\bar{a}, z)} \right)^{-\frac{\rho}{1-\theta}} \left(\frac{C(\bar{a}, z)}{q} \right)^{\frac{1}{1-\theta}} - w_1(\omega) \left(\frac{q}{e^a} \right)^{\frac{1}{\theta}} \right) \middle| \omega \right].$$

Note that this objective is strictly concave in $p^{1-\rho}$ and $q^{1/\theta}$, which guarantees that the FOCs are both necessary and sufficient and that they uniquely pin down the solution to the firm's problem for given wages. Next, note that the equilibrium wages satisfy

$$n(\omega)^{\epsilon-1} = w_1(\omega) \mathbb{E} \left[\frac{U'(C(\bar{a}, z))}{P(\bar{a}, z)} \middle| \omega \right] \quad \text{and} \quad l(\omega)^{\epsilon-1} = w_2(\omega) \mathbb{E} \left[\frac{U'(C(\bar{a}, z))}{P(\bar{a}, z)} \middle| \omega \right]$$

Solving these conditions for $w_1(\omega)$ and $w_2(\omega)$ and substituting the solutions into the first-order conditions for the firm's problem gives us the following two conditions for the equilibrium price and production choices taken in stage 1:

$$p(\omega)^{1-\rho+\frac{\rho\epsilon}{1-\theta}} = \frac{\mathbb{E} \left[P(\bar{a}, z)^{\frac{\rho\epsilon}{1-\theta}} C(\bar{a}, z)^{\frac{\epsilon}{1-\theta}} q(\omega)^{-\frac{\epsilon}{1-\theta}} \middle| \omega \right]}{(1-\theta) \mathbb{E} \left[\left(\frac{\rho-1}{\rho} \right) (1-\tau(\bar{a}, z))C(\bar{a}, z)^{1-\gamma}P(\bar{a}, z)^{\rho-1} \middle| \omega \right]} \quad (47)$$

$$q(\omega)^{\frac{\epsilon}{\theta}} = p(\omega)^{1-\rho} e^{\frac{\epsilon}{\theta}a} \theta \mathbb{E} \left[\left(\frac{\rho-1}{\rho} \right) (1-\tau(\bar{a}, z))C(\bar{a}, z)^{1-\gamma}P(\bar{a}, z)^{\rho-1} \middle| \omega \right] \quad (48)$$

Using (46), we can restate these conditions in terms of allocations alone as follows:

$$0 = n(\omega)^{\epsilon-1} - \mathbb{E} \left[\left(\frac{\rho-1}{\rho} \right) (1-\tau(\bar{a}, z))U'(C(\bar{a}, z)) \left(\frac{c(\omega, \bar{a}, z)}{C(\bar{a}, z)} \right)^{-\frac{1}{\rho}} \left(\theta \frac{c(\omega, \bar{a}, z)}{n(\omega)} \right) \middle| \omega \right]$$

$$0 = \mathbb{E} \left[l(\omega, \bar{a}, z) \left\{ l(\omega, \bar{a}, z)^{\epsilon-1} - \left(\frac{\rho-1}{\rho} \right) (1-\tau(\bar{a}, z))U'(C(\bar{a}, z)) \left(\frac{c(\omega, \bar{a}, z)}{C(\bar{a}, z)} \right)^{-\frac{1}{\rho}} \left((1-\theta) \frac{c(\omega, \bar{a}, z)}{l(\omega, \bar{a}, z)} \right) \right\} \middle| \omega \right]$$

Rearranging these conditions, we get

$$\mathbb{E} [l(\omega, \bar{a}, z)^\epsilon | \omega] = \frac{1-\theta}{\theta} e^{-\frac{\epsilon}{\theta}a} q(\omega)^{\frac{\epsilon}{\theta}} \quad (49)$$

$$q(\omega)^{\frac{\epsilon}{\theta}+\frac{1}{\rho}-1} = e^{\frac{\epsilon}{\theta}a} \theta \left(\frac{\rho-1}{\rho} \right) \mathbb{E} \left[(1-\tau(\bar{a}, z))C(\bar{a}, z)^{\frac{1}{\rho}-\gamma} l(\omega, \bar{a}, z)^{(1-\theta)\left(\frac{\rho-1}{\rho}\right)} \middle| \omega \right] \quad (50)$$

The first condition equates the (expected) marginal rates of transformation and substitution between l and n . We conclude that a set of allocations, prices and policies constitute an equilibrium if and only if the following hold: (i) the allocations and the tax policy satisfy conditions (49) and (50) along with the resource constraint

$$C(\bar{a}, z) = \left[\int \left(q(\omega)l(\omega, \bar{a}, z)^{1-\theta} \right)^{\frac{\rho-1}{\rho}} dF(\omega|\bar{a}, z) \right]^{\frac{\rho}{\rho-1}}; \quad (51)$$

(ii) the nominal prices satisfy condition (46); and (iii) the interest-rate rule satisfies the Euler condition

$$C(\bar{a}, z)^{-\gamma} = \bar{\beta} (1 + R(\bar{a}, z)) P(\bar{a}, z) \mathbb{E} \left[\frac{C(\bar{a}_{+1}, z_{+1})^{-\gamma}}{P(\bar{a}_{+1}, z_{+1})} \middle| \bar{a}, z \right]. \quad (52)$$

We now seek to translate conditions (46)-(52) in terms of the relevant coefficients that parameterize the allocations, prices and policy under a log-normal specification. Thus let

$$\begin{aligned}
\log q(\omega) &= \text{const} + \varphi_a a + \varphi_x x + \varphi_z z \\
\log l(\omega, \bar{a}, z) &= \text{const} + l_A \bar{a} + l_a a + l_x x + l_z z \\
\log C(\bar{a}, z) = \log Y(\bar{a}, z) &= \text{const} + c_A \bar{a} + c_z z \\
\log p(\omega) &= \text{const} + \psi_a a + \psi_x x + \psi_z z \\
\log(1 - \tau(\bar{a}, z)) &= \text{const} - \tau_A \bar{a} - \tau_z z \\
\log(1 + R(\bar{a}, z)) &= \text{const} + \rho_A \bar{a} + \rho_z z
\end{aligned}$$

for some coefficients $(\varphi_a, \varphi_x, \dots, \rho_A, \rho_z)$.

The resource constraint (51) is satisfied if and only if

$$c_A = (\varphi_a + \varphi_x) + (1 - \theta)(l_a + l_x + l_A) \quad (53)$$

$$c_z = \varphi_z + (1 - \theta)l_z. \quad (54)$$

Also, the interest rate is pinned down by the Euler equation (52). Taking the logs of both sides and using the fact that expectations about future variables are constant by the assumption of i.i.d. shocks, we can restate condition (52) as

$$\log(1 + R(\bar{a}, z)) = \text{const} - \gamma \log Y(\bar{a}, z) - \log P(\bar{a}, z).$$

Using the log-normal specification above, the latter condition is satisfied if and only if

$$\rho_A = -\gamma c_A - (\psi_a + \psi_x) \quad (55)$$

$$\rho_z = -\gamma c_z - \psi_z. \quad (56)$$

Next, we can rewrite the consumer's demand function as

$$-\rho(\log p(\omega) - \log P(\bar{a}, z)) = (\log c(\omega, \bar{a}, z) - \log C(\bar{a}, z))$$

where

$$\begin{aligned}
\log c(\omega, \bar{a}, z) &= \log q(\omega) + (1 - \theta) \log l(\omega, \bar{a}, z) \\
&= \text{const} + (\varphi_a + (1 - \theta)l_a)a + (\varphi_x + (1 - \theta)l_x)x + (\varphi_z + (1 - \theta)l_z)z + (1 - \theta)l_A \bar{a}
\end{aligned}$$

It follows that the following must hold for all (a, x, z, \bar{a}) :

$$-\rho(\psi_a a + \psi_x x - (\psi_a + \psi_x)\bar{a}) = (\varphi_a + (1 - \theta)l_a)a + (\varphi_x + (1 - \theta)l_x)x - (\varphi_a + (1 - \theta)l_a + \varphi_x + (1 - \theta)l_x)\bar{a}.$$

This is true if and only if

$$\psi_a = -\frac{1}{\rho}(\varphi_a + (1 - \theta)l_a) \quad \text{and} \quad \psi_x = -\frac{1}{\rho}(\varphi_x + (1 - \theta)l_x)$$

Finally, note that conditions (49) and (50) may be rewritten as follows:

$$\mathbb{E}[\log l(\omega, \bar{a}, z) | \omega] = \text{const} + \frac{1}{\theta} (\log q(\omega) - a) \quad (57)$$

$$\log q(\omega) = \text{const} + \beta a - k(\tau_A \mathbb{E}[\bar{a} | \omega] + \tau_z z) + \frac{\alpha}{\chi} \mathbb{E}[\log C(\bar{a}, z) | \omega] \quad (58)$$

where

$$\beta \equiv \frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta} - (\rho - 1)\nu} > 1, \quad \alpha \equiv \left(\frac{1}{\rho} - \gamma\right) \frac{\rho\nu\chi}{\frac{\epsilon}{\theta} - (\rho - 1)\nu},$$

$$\nu \equiv \frac{\epsilon}{\rho(\epsilon - 1 + \theta) + 1 - \theta} > \frac{1}{\rho}, \quad \chi \equiv \frac{\epsilon}{(\epsilon - 1 + \theta) + \gamma(1 - \theta)} > 0, \quad k \equiv \nu\rho\frac{\theta}{\epsilon}\beta > 0.$$

Clearly, condition (57) holds for all ω if and only if

$$l_a = \frac{1}{\theta} (\varphi_a - 1) \quad (59)$$

$$l_x = \frac{1}{\theta} \varphi_x - l_A \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_z} \quad (60)$$

$$l_z = \frac{1}{\theta} \varphi_z - l_A \frac{\kappa_z}{\kappa_A + \kappa_x + \kappa_z} \quad (61)$$

while condition (58) holds for all ω if and only if

$$\varphi_a = \beta \quad (62)$$

$$\varphi_x = -k\tau_A \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_z} + \frac{\alpha}{\chi} c_A \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_z} \quad (63)$$

$$\varphi_z = -k \left(\tau_A \frac{\kappa_z}{\kappa_A + \kappa_x + \kappa_z} + \tau_z \right) + \frac{\alpha}{\chi} \left(c_A \frac{\kappa_z}{\kappa_A + \kappa_x + \kappa_z} + c_z \right) \quad (64)$$

where c_A and c_z are given by (53)-(54).

Note that conditions (59) through (62) give the implementability constraints stated in the proposition, completing the proof of the necessity of these conditions for an allocation to be part of an equilibrium. We next prove sufficiency.

Pick arbitrary $(\varphi_x, \varphi_z, l_A)$ and let $(\varphi_a, l_a, l_x, l_z)$ satisfy conditions (59) through (62). Note that there is a unique $(\varphi_a, l_a, l_x, l_z)$ that has this property for any given $(\varphi_x, \varphi_z, l_A)$. Next, pick an arbitrary ψ_z and let $(c_A, c_z, \psi_a, \psi_x)$ be determined as in (53)-(45). Next, let (τ_A, τ_z) be the unique solution to (63)-(64); for future reference, this solution is given by

$$\tau_A = \frac{1}{\chi k} \left\{ \alpha c_A - \chi \frac{\kappa}{\kappa_x} \varphi_x \right\} \quad (65)$$

$$\tau_z = \frac{1}{\chi k} \left\{ \alpha c_z - \chi \left(\varphi_z - \varphi_x \frac{\kappa_z}{\kappa_x} \right) \right\} \quad (66)$$

where $\chi k > 0$. Finally, set (ρ_A, ρ_z) as in (55)-(56). By construction, the allocations, prices and policies defined in this way constitute an equilibrium, which completes the sufficiency argument.

Part (ii). The proof of this part is similar to that of part (i), except for one key difference: now the marginal costs and returns of stage-2 employment must be equated state-by-state, not just in expectation. In particular, we would have

$$n(\omega)^{\epsilon-1} = \left(\frac{\rho-1}{\rho}\right) \mathbb{E} \left[(1-\tau(\bar{a}, z)) U' (C(\bar{a}, z)) \left(\frac{c(\omega, \bar{a}, z)}{C(\bar{a}, z)}\right)^{-\frac{1}{\rho}} \left(\theta \frac{c(\omega, \bar{a}, z)}{n(\omega)}\right) \middle| \omega \right] \quad (67)$$

$$l(\omega, \bar{a}, z)^{\epsilon-1} = \left(\frac{\rho-1}{\rho}\right) (1-\tau(\bar{a}, z)) U' (C(\bar{a}, z)) \left(\frac{c(\omega, \bar{a}, z)}{C(\bar{a}, z)}\right)^{-\frac{1}{\rho}} \left((1-\theta) \frac{c(\omega, \bar{a}, z)}{l(\omega, \bar{a}, z)}\right) \quad (68)$$

It is this additional restriction that pins down l_A . (A detailed derivation is available upon request.)

Proof of Lemma 5. The proof is contained in the proof of Proposition 3. In particular, note that the arguments in the proof are silent about the value for ψ_z , which is thus indetermined.

Proof of Lemma 6. Take any allocation in which $\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_z z$ and $\log l(\omega, \bar{a}, z) = l_0 + l_A \bar{a} + l_a a + l_x x + l_z z$. Simple steps prove that welfare (ex-ante utility) is given by

$$\mathcal{W}(\varphi, l; \kappa_x, \kappa_z) = \frac{1}{1-\gamma} \exp V_c(\varphi, l, \kappa) - \frac{1}{\epsilon} \exp V_l(\varphi, l, \kappa) - \frac{1}{\epsilon} \exp V_n(\varphi, l, \kappa)$$

where $\varphi = (\varphi_a, \varphi_x, \varphi_z)$, $l = (l_a, l_x, l_z, l_A)$, and $\kappa = (\kappa_x, \kappa_z)$, and where

$$\begin{aligned} V_c(\varphi, l, \kappa) &\equiv (1-\gamma) (\varphi_0 + (1-\theta)l_0 + [(1-\theta)l_A + (\varphi_a + (1-\theta)l_a) + (\varphi_x + (1-\theta)l_x) + (\varphi_z + (1-\theta)l_z)] \mu) \\ &\quad + \frac{1}{2} (1-\gamma) \left(\frac{\rho-1}{\rho}\right) \left[\frac{(\varphi_a + (1-\theta)l_a)^2}{\kappa_\xi} + \frac{(\varphi_x + (1-\theta)l_x)^2}{\kappa_x} + 2 \frac{(\varphi_a + (1-\theta)l_a)(\varphi_x + (1-\theta)l_x)}{\kappa_x} \right] \\ &\quad + \frac{1}{2} (1-\gamma)^2 \left[\frac{(\varphi_z + (1-\theta)l_z)^2}{\kappa_z} + \frac{((1-\theta)l_A + (\varphi_a + (1-\theta)l_a) + (\varphi_x + (1-\theta)l_x) + (\varphi_z + (1-\theta)l_z))^2}{\kappa_A} \right] \\ V_l(\varphi, l, \kappa) &\equiv \epsilon (l_0 + (l_A + l_a + l_x + l_z) \mu) + \frac{1}{2} \epsilon^2 \left[\left(\frac{l_a^2}{\kappa_\xi} + \frac{l_x^2}{\kappa_x} + 2 \frac{l_a l_x}{\kappa_x} \right) + \frac{l_z^2}{\kappa_z} + \frac{(l_A + l_a + l_x + l_z)^2}{\kappa_A} \right] \\ V_n(\varphi, l, \kappa) &\equiv \frac{\epsilon}{\theta} (\varphi_0 + (\varphi_a + \varphi_x + \varphi_z - 1) \mu) \\ &\quad + \frac{1}{2} \frac{\epsilon^2}{\theta^2} \left[\frac{(\varphi_a - 1)^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{(\varphi_a - 1) \varphi_x}{\kappa_x} + \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z - 1)^2}{\kappa_A} \right]. \end{aligned}$$

Proof of Proposition 4. We henceforth consider a relaxed problem, where we ignore the constraint on φ_a imposed by (15); it will turn out that the solution to this relaxed problem satisfies this constraint, which means that the solution to the relaxed problem is also the solution to our initial problem.

The first-order conditions of the (relaxed) problem with respect to φ_0 and l_0 give

$$\begin{aligned} \varphi_0 &: 0 = \exp V_c - \frac{1}{\theta} \exp V_n \\ l_0 &: 0 = (1-\theta) \exp V_c - \exp V_l. \end{aligned}$$

Hence, at the optimal allocation, $\exp V \equiv \exp V_c = \frac{1}{\theta} \exp V_n = \frac{1}{1-\theta} \exp V_l > 0$. Let the Lagrange multipliers on the implementability constraints (16)-(18) be, respectively, $e^V \mu_a$, $e^V \mu_x$, and $e^V \mu_z$.

Next, as in the proof of Proposition 1, we can represent the informational externalities by two Lagrange multipliers, one for the sum $\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x + l_A)$, which determines κ_{zy} and κ_{xy} and, hence, the precision of the output signals; and another for the sum $\varphi_a + \varphi_x + (1 - \theta)(l_a + l_x)$, which determines κ_{zp} and κ_{xp} and, hence, the precision of the price signals. Let these multipliers be, respectively, $e^V \eta_Y$ and $e^V \eta_p$. We can then state the rest of the first-order conditions of the optimal policy problem as follows.

First, the conditions for the stage-1 strategy are the following:

$$\begin{aligned}
\varphi_a : 0 &= \left(\frac{\rho - 1}{\rho} \right) \left(\frac{(\varphi_a + (1 - \theta)l_a)}{\kappa_\xi} + \frac{(\varphi_x + (1 - \theta)l_x)}{\kappa_x} \right) \\
&\quad + (1 - \gamma) \frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \\
&\quad - \frac{\epsilon}{\theta} \left[\frac{(\varphi_a - 1)}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{(\varphi_a + \varphi_x + \varphi_z - 1)}{\kappa_A} \right] + \eta_Y + \eta_p - \frac{1}{\theta} \mu_a \\
\varphi_x : 0 &= \left(\frac{\rho - 1}{\rho} \right) \left(\frac{(\varphi_x + (1 - \theta)l_x)}{\kappa_x} + \frac{(\varphi_a + (1 - \theta)l_a)}{\kappa_x} \right) \\
&\quad + (1 - \gamma) \frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \\
&\quad - \frac{\epsilon}{\theta} \left[\frac{\varphi_x}{\kappa_x} + \frac{(\varphi_a - 1)}{\kappa_x} + \frac{(\varphi_a + \varphi_x + \varphi_z - 1)}{\kappa_A} \right] + \eta_Y + \eta_p - \frac{1}{\theta} \mu_x \\
\varphi_z : 0 &= (1 - \gamma) \left[\frac{(\varphi_z + (1 - \theta)l_z)}{\kappa_z} + \frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \right] \\
&\quad - \frac{\epsilon}{\theta} \left[\frac{\varphi_z}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z - 1)}{\kappa_A} \right] - \frac{1}{\theta} \mu_z
\end{aligned}$$

And second, the conditions for the stage-2 strategy are the following:

$$\begin{aligned}
l_a : 0 &= \left(\frac{\rho - 1}{\rho} \right) \left(\frac{(\varphi_a + (1 - \theta)l_a)}{\kappa_\xi} + \frac{(\varphi_x + (1 - \theta)l_x)}{\kappa_x} \right) \\
&\quad + (1 - \gamma) \left[\frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \right] \\
&\quad - \epsilon \left[\frac{l_a}{\kappa_\xi} + \frac{l_x}{\kappa_x} + \frac{(l_A + l_a + l_x + l_z)}{\kappa_A} \right] + \eta_Y + \eta_p + \frac{\mu_a}{1 - \theta} \\
l_x : 0 &= \left(\frac{\rho - 1}{\rho} \right) \left(\frac{(\varphi_x + (1 - \theta)l_x)}{\kappa_x} + \frac{(\varphi_a + (1 - \theta)l_a)}{\kappa_x} \right) \\
&\quad + (1 - \gamma) \left[\frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \right] \\
&\quad - \epsilon \left[\frac{l_x}{\kappa_x} + \frac{l_a}{\kappa_x} + \frac{(l_A + l_a + l_x + l_z)}{\kappa_A} \right] + \eta_Y + \eta_p + \frac{\mu_x}{1 - \theta} \\
l_z : 0 &= (1 - \gamma) \left[\frac{(\varphi_z + (1 - \theta)l_z)}{\kappa_z} + \frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \right] \\
&\quad - \epsilon \left[\frac{l_z}{\kappa_z} + \frac{(l_A + l_a + l_x + l_z)}{\kappa_A} \right] + \frac{\mu_z}{1 - \theta} \\
l_A : 0 &= (1 - \gamma) \left[\frac{((1 - \theta)l_A + (\varphi_a + (1 - \theta)l_a) + (\varphi_x + (1 - \theta)l_x) + (\varphi_z + (1 - \theta)l_z))}{\kappa_A} \right] - \epsilon \left[\frac{(l_A + l_a + l_x + l_z)}{\kappa_A} \right] \\
&\quad + \eta_Y + \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_z} \frac{\mu_x}{1 - \theta} + \frac{\kappa_z}{\kappa_A + \kappa_x + \kappa_z} \frac{\mu_z}{1 - \theta}
\end{aligned}$$

For any given η_Y and η_p , the combination of these seven FOCs with the three implementability constraints (16)-(18) defines a linear system of 10 equations in 10 unknowns, the allocation coefficients $(\varphi_a, \varphi_x, \varphi_z)$ and (l_A, l_a, l_x, l_z) and the implementability multipliers

(μ_a, μ_x, μ_z) . The solution to this system gives the following results: For the stage-1 allocation, we get

$$\varphi_a^* = \beta \quad (69)$$

$$\varphi_x^* = \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \delta_Y \eta_Y + \delta_p \eta_p \quad (70)$$

$$\varphi_z^* = \left\{ \frac{\kappa_z^*}{(1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} (\delta_Y \eta_Y + \delta_p \eta_p) \quad (71)$$

where

$$\begin{aligned} \delta_Y &\equiv \frac{(1-\alpha)\theta\kappa_x^*(\kappa_A + \kappa_z^*)}{(\gamma + \epsilon - 1)((1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A)} > 0 \\ \delta_p &\equiv \frac{(\epsilon\theta - \alpha(\gamma + \epsilon - 1 + \theta(1-\gamma)))\kappa_x^*(\kappa_A + \kappa_z^*)}{\epsilon(\gamma + \epsilon - 1)((1-\alpha)\kappa_x^* + \kappa_z^* + \kappa_A)} > 0. \end{aligned}$$

For the stage-2 allocation, we get

$$\begin{aligned} l_A^* &= \hat{l}_A - \lambda \delta_p \eta_p \\ l_a^* &= \hat{l}_a \\ l_x^* &= \hat{l}_x + \left(\frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \right) \lambda \delta_p \eta_p \\ l_z^* &= \hat{l}_z + \left(\frac{\kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \right) \lambda \delta_p \eta_p \end{aligned}$$

where

$$\begin{aligned} \hat{l}_A &\equiv \frac{(\kappa_A + \kappa_x^* + \kappa_z^*)\varphi_x^*}{\beta\kappa_x^*\theta} \\ \hat{l}_a &\equiv \frac{1}{\theta} (\varphi_a^* - 1) \\ \hat{l}_x &\equiv \frac{1}{\theta} \varphi_x^* - \hat{l}_A \frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \\ \hat{l}_z &\equiv \frac{1}{\theta} \varphi_z^* - \hat{l}_A \frac{\kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \end{aligned}$$

and where

$$\lambda \equiv \frac{(\gamma + \epsilon - 1)(\kappa_A + \kappa_x^* + \kappa_z^*)(\kappa_A + (1-\alpha)\kappa_x^* + \kappa_z^*)}{(1-\alpha)\theta(\gamma + \epsilon - 1 + \theta(1-\gamma))\kappa_x^*(\kappa_A + \kappa_z^*)} > 0.$$

Note that, by Proposition 3, $(\hat{l}_A, \hat{l}_a, \hat{l}_x, \hat{l}_z)$ identifies the stage-2 allocation that would obtain in the (unique) flexible-price equilibrium in which the stage-1 allocation is given by (69)-(71). Finally, for the implementability multipliers, we get

$$\mu_a = \mu_x = \mu_z = 0.$$

Letting

$$\Delta_Y \equiv \eta_Y \delta_Y \quad \text{and} \quad \Delta_p \equiv \eta_p \delta_p$$

completes the proof of all the conditions in the proposition.

What remains is to show that Δ_Y and Δ_p (or, equivalently, η_Y and η_p) are positive. In the remaining of the proof, to simplify expressions, we focus on the case in which endogenous information is only public, that is, $\kappa_{xy} = \kappa_{xp} = 0$. Note that

$$\begin{aligned}\eta_Y &= e^{-V} \frac{\partial \mathcal{W}}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \kappa_{zy}} \frac{\partial \kappa_{zy}}{\partial \varphi_x}, \\ \eta_p &= e^{-V} \frac{\partial \mathcal{W}}{\partial \kappa_z} \frac{\partial \kappa_z}{\partial \kappa_{zp}} \frac{\partial \kappa_{zp}}{\partial \varphi_x}.\end{aligned}$$

Next, note that $\frac{\partial \kappa_z}{\partial \kappa_{zy}} = \frac{\partial \kappa_z}{\partial \kappa_{zp}} = 1$. Also,

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = -(1-\gamma) \exp V_c \frac{(\varphi_z + (1-\theta)l_z)^2}{\kappa_z^2} + \frac{\epsilon}{(1-\theta)^2} \exp V_l \frac{((1-\theta)l_z)^2}{\kappa_z^2} + \frac{\epsilon}{\theta^2} \exp V_n \frac{\varphi_z^2}{\kappa_z^2}. \quad (72)$$

At the optimal allocation, we know that $\exp V \equiv \exp V_c = \frac{1}{\theta} \exp V_n = \frac{1}{1-\theta} \exp V_l$, as well as that $\mu_a = \mu_x = \mu_z = 0$. Using the first fact, we get

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \frac{e^V}{\kappa_z^2} \left\{ (\gamma-1) (\varphi_z + (1-\theta)l_z)^2 + \frac{\epsilon}{1-\theta} ((1-\theta)l_z)^2 + \frac{\epsilon}{\theta} \varphi_z^2 \right\}.$$

Using the second fact along with the FOCs with respect to (l_a, l_x, l_z, l_A) , we can express l_z as a function of φ_z and η_Y :

$$l_z = \frac{(1-\gamma)\varphi_z - \kappa_z \eta_Y}{(\epsilon-1+\theta) + \gamma(1-\theta)}.$$

It follows that

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = e^V \frac{1}{\beta} (1-\alpha) \left\{ \frac{\epsilon}{\theta} \frac{\varphi_z^2}{\kappa_z^2} + \frac{1-\theta}{\epsilon-1+\gamma} \eta_Y^2 \right\} > 0.$$

Finally, recall that $\frac{\partial \kappa_{zy}}{\partial \varphi_x} > 0$ if and only if $\varphi_a + \varphi_x + (1-\theta)(l_a + l_x + l_A) > 0$, while $\frac{\partial \kappa_{zp}}{\partial \varphi_x} > 0$ if and only if $\varphi_a + \varphi_x + (1-\theta)(l_a + l_x) > 0$. Combining these result, we conclude that $\Delta_Y > 0$ and $\Delta_p > 0$ if and only if the optimal allocation satisfies $\varphi_a + \varphi_x + (1-\theta)(l_a + l_x + l_A) > 0$ and $\varphi_a + \varphi_x + (1-\theta)(l_a + l_x) > 0$.

To prove this, we proceed in a similar fashion as in Proposition 1. Let

$$\bar{v} \equiv \left[\begin{array}{c} \frac{1}{\rho} [\varphi_a + \varphi_x + (1-\theta)(l_a + l_x)] \\ \varphi_a + \varphi_x + (1-\theta)(l_a + l_x + l_A) \end{array} \right] \quad \text{and} \quad v(\kappa) \equiv \left[\begin{array}{c} \frac{1}{\theta \rho} \frac{(\beta-1+\theta)(\kappa_A + \kappa_x + \kappa_z)}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \\ \frac{1}{\theta} \frac{(\beta-(1-\theta)(1-\alpha))(\kappa_A + \kappa_x + \kappa_z)}{(1-\alpha)\kappa_x + \kappa_z + \kappa_A} \end{array} \right].$$

It is immediate to show that $v_1(\kappa), v_2(\kappa) > 0$. As in Proposition 1, $\bar{v} - v(\kappa)$ is the distance between any value \bar{v} that the planner may choose and the one that would have been optimal from a purely allocative perspective. Moreover, welfare can be expressed as (a monotone transformation of) a quadratic form of this distance. In particular, using the FOCs with respect to φ_0 and l_0 , we get that welfare is given by

$$\mathcal{W}(\varphi, l, \kappa) = \frac{\epsilon-1+\gamma}{1-\gamma} \frac{1}{\epsilon} \exp V_c(\varphi, l, \kappa).$$

Since $\epsilon - 1 + \gamma > 0$ and $\frac{1}{\epsilon} > 0$, we can once again consider the following monotone transformation of welfare:

$$\mathcal{TW}(\varphi, l, \kappa) \equiv \frac{1}{1 - \gamma} V_c(\varphi, l, \kappa).$$

Using then the characterization of the optimal coefficients (φ, l) as functions of \bar{v} , we conclude that

$$\mathcal{TW}(\varphi, l, \kappa) = W(\bar{v}, \kappa) \equiv A(\kappa) + (\bar{v} - v(\kappa))' B(\kappa) (\bar{v} - v(\kappa)), \quad (73)$$

where $A(\kappa)$ is scalar that identifies the level welfare attained when $\bar{v} = v(\kappa)$, while $B(\kappa)$ is a 2-by-2 matrix that identifies the Hessian of the (transformed) welfare function W . The latter is given by the following:

$$\begin{aligned} B(\kappa) &\equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ b_{11} &\equiv -\frac{\epsilon\rho(1 - \theta + \rho(-1 + \epsilon + \theta))\kappa_x + \epsilon\rho(1 - \theta)(1 - \rho + \rho\epsilon)(\kappa_A + \kappa_z)}{2(-1 + \gamma + \epsilon)(1 - \theta)\kappa_x(\kappa_A + \kappa_x + \kappa_z)} \\ b_{22} &\equiv -\frac{\epsilon}{2(\kappa_A + \kappa_z)} - \frac{\epsilon^2\theta}{2(-1 + \gamma + \epsilon)(1 - \theta)(\kappa_A + \kappa_x + \kappa_z)} \\ b_{12} \equiv b_{21} &\equiv \frac{\epsilon^2\theta\rho}{2(-1 + \gamma + \epsilon)(1 - \theta)(\kappa_A + \kappa_x + \kappa_z)}. \end{aligned}$$

Note that $b_{11} < 0$ and that the determinant of $B(\kappa)$ is positive:

$$\begin{aligned} \det(B) &= b_{11}b_{22} - b_{12}b_{21} \\ &= \frac{\epsilon^3\theta\rho(1 - \rho + \rho\epsilon)}{4(-1 + \gamma + \epsilon)^2(1 - \theta)\kappa_x(\kappa_A + \kappa_x + \kappa_z)} \\ &\quad + \frac{\epsilon^2\rho(1 - \theta + \rho(-1 + \epsilon + \theta))\kappa_x + \epsilon^2\rho(1 - \theta)(1 - \rho + \rho\epsilon)(\kappa_A + \kappa_z)}{4(-1 + \gamma + \epsilon)(1 - \theta)\kappa_x(\kappa_A + \kappa_z)(\kappa_A + \kappa_x + \kappa_z)} \\ &> 0. \end{aligned}$$

It follows that the matrix $B(\kappa)$ is negative definite and, hence, the aforementioned quadratic form for welfare in (73) is also negative definite. The same type of arguments as in Proposition 1 then imply that the optimal \bar{v} is positive, and indeed higher than $v(\kappa)$, which in turn guarantees that $\Delta_Y > 0$ and $\Delta_p > 0$.

Finally, note that none of the derivations of this proposition required the assumption that $\alpha > 0$. That restriction will be used only in the proof of Proposition 5.

Proof of Proposition 5. We prove the proposition in reverse order.

Part (ii). At the optimal allocation, aggregate output is given by $\log Y(\bar{a}, z) = \text{const} + c_A^* \bar{a} + c_z^* z$, where $c_A^* = (\varphi_a^* + \varphi_x^*) + (1 - \theta)(l_a^* + l_x^* + l_A^*)$ and $c_z^* = \varphi_z^* + (1 - \theta)l_z^*$. If monetary policy were replicating the flexible-price allocations, then aggregate output would be given by $\log Y(\bar{a}, z) = \text{const} + \hat{c}_A \bar{a} + \hat{c}_z z$, where $\hat{c}_A = (\varphi_a^* + \varphi_x^*) + (1 - \theta)(\hat{l}_a + \hat{l}_x + \hat{l}_A)$ and $\hat{c}_z = \varphi_z^* + (1 - \theta)\hat{l}_z$. Analogous expressions hold for the price level.

Using condition (55) we then have

$$\rho_A^* - \hat{\rho}_A = \left(\frac{1}{\rho} - \gamma \right) (1 - \theta)(l_x^* - \hat{l}_x) - \gamma(1 - \theta)(l_A^* - \hat{l}_A).$$

From Proposition 4, $l_x^* - \hat{l}_x = \frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*}(\hat{l}_A - l_A^*)$, thus,

$$\begin{aligned} \rho_A^* - \hat{\rho}_A &= \left[\left(\frac{1}{\rho} - \gamma \right) \frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*} + \gamma \right] (1 - \theta) (\hat{l}_A - l_A^*) \\ &= \left[\frac{1}{\rho} \frac{\kappa_x^*}{\kappa_A + \kappa_x^* + \kappa_z^*} + \gamma \frac{\kappa_A + \kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \right] (1 - \theta) \lambda \Delta_p \\ &\equiv \chi_3 \Delta_p. \end{aligned}$$

Part (i). Consider now the optimal tax. From conditions (65) and (66), we know that the optimal tax satisfies

$$\begin{aligned} \tau_A^* &= \frac{1}{\chi k} \left\{ \alpha c_A^* - \chi \frac{\kappa_A + \kappa_x^* + \kappa_z^*}{\kappa_x^*} \varphi_x^* \right\} \\ \tau_z^* &= \frac{1}{\chi k} \left\{ \alpha c_z^* - \chi \left(\varphi_z^* - \varphi_x^* \frac{\kappa_z^*}{\kappa_x^*} \right) \right\}. \end{aligned}$$

Let

$$\begin{aligned} \hat{\tau}_A &\equiv \frac{1}{\chi k} \left\{ \alpha \hat{c}_A - \chi \frac{\kappa_A + \kappa_x^* + \kappa_z^*}{\kappa_x^*} \varphi_x^* \right\} \\ \hat{\tau}_z &\equiv \frac{1}{\chi k} \left\{ \alpha \hat{c}_z - \chi \left(\varphi_z^* - \varphi_x^* \frac{\kappa_z^*}{\kappa_x^*} \right) \right\}; \end{aligned}$$

these coefficients identify the tax policy that would be required in order to implemented the optimal stage-1 sensitivities, φ_x^* and φ_z^* , if monetary policy were replicating the flexible-price allocations associated with these stage-1 sensitivities. It is easy to verify that

$$\hat{\tau}_A = -\bar{\lambda} \Delta \quad \text{and} \quad \hat{\tau}_z = -\bar{\lambda} \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \Delta,$$

where $\Delta \equiv \Delta_Y + \Delta_p$ and $\bar{\lambda} \equiv \frac{(\epsilon\theta - \alpha(-1 + \gamma + \epsilon + \theta - \gamma\theta))^2(\kappa_A + (1 - \alpha)\kappa_x^* + \kappa_z^*)}{(1 - \alpha)\epsilon^2(-1 + \gamma + \epsilon)\theta\kappa_x^*} > 0$. Note that the optimal tax satisfies

$$\begin{aligned} \tau_A^* &= \hat{\tau}_A + \frac{1}{\chi k} \alpha (c_A^* - \hat{c}_A) \\ \tau_z^* &= \hat{\tau}_z + \frac{1}{\chi k} \alpha (c_z^* - \hat{c}_z). \end{aligned}$$

By Proposition 4,

$$\begin{aligned} c_A^* - \hat{c}_A &= (1 - \theta)(l_x^* - \hat{l}_x + l_A^* - \hat{l}_A) \\ &= -(1 - \theta) \frac{\kappa_A + \kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \lambda \Delta_p \end{aligned}$$

and

$$\begin{aligned} c_z^* - \hat{c}_z &= (1 - \theta)(l_z^* - \hat{l}_z) \\ &= (1 - \theta) \frac{\kappa_z^*}{\kappa_A + \kappa_x^* + \kappa_z^*} \lambda \Delta_p. \end{aligned}$$

Therefore,

$$\begin{aligned} \tau_A^* + \tau_z^* &= \hat{\tau}_A + \hat{\tau}_z - \frac{1}{\chi k} \alpha \frac{\kappa_A}{\kappa_A + \kappa_x^* + \kappa_z^*} \lambda \Delta_p \\ &= -\bar{\lambda} \left(1 + \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \right) (\Delta_Y + \Delta_p) - \frac{1}{\chi k} \alpha \frac{\kappa_A}{\kappa_A + \kappa_x^* + \kappa_z^*} \lambda \Delta_p \end{aligned}$$

and the statement of the proposition follows from letting $\chi_1 \equiv \bar{\lambda} \left(1 + \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \right) > 0$ and $\chi_2 \equiv \bar{\lambda} \left(1 + \frac{\kappa_z^*}{\kappa_A + \kappa_z^*} \right) + \frac{1}{\chi k} \alpha \frac{\kappa_A}{\kappa_A + \kappa_x^* + \kappa_z^*} \lambda > 0$ since $\alpha > 0$.