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Innovate to Lead or Innovate to Prevail: When do Monopolistic Rents Induce Growth?

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This paper extends the standard Schumpeterian model of creative destruction by allowing the cost of innovation for followers to increase in their technological distance from the leader. This assumption is motivated by the observation that the more technologically advanced the leader is, the harder it is for a follower to leapfrog without incurring extra cost for using leader's patented knowledge. Under this R&D cost structure, leaders have an incentive to play an "endpoint strategy": they increase their technological advantage, counting on the fact that followers will eventually stop innovating – allowing leadership to prevail. We find that several results in the standard model now fail to hold. In addition to the High Growth steady state in which only followers innovate, there now exist two other steady states: a Medium Growth (a source) and a Low Growth (a saddle) steady state, that feature both leaders and followers innovating. An increase in monopolistic rents or an extension of patent duration increases the likelihood that over time the economy converges to a low growth steady state.

JEL Classification: O31, O34, O41, L16

Keywords: Innovation, Persistent monopoly, Endogenous growth theory

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Innovate to Lead or Innovate to Prevail: When do Monopolistic Rents Induce Growth?*

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October 31, 2021

Abstract

This paper extends the standard Schumpeterian model of creative destruction by allowing the cost of innovation for followers to increase in their technological distance from the leader. This assumption is motivated by the observation that the more technologically advanced the leader is, the harder it is for a follower to leapfrog without incurring extra cost for using leader's patented knowledge. Under this R&D cost structure, leaders have an incentive to play an "endpoint strategy": they increase their technological advantage, counting on the fact that followers will eventually stop innovating – allowing leadership to prevail. We find that several results in the standard model now fail to hold. In addition to the High Growth steady state in which only followers innovate, there now exist two other steady states: a Medium Growth (a source) and a Low Growth (a saddle) steady state, that feature both leaders and followers innovating. An increase in monopolistic rents or an extension of patent duration increases the likelihood that over time the economy converges to a low growth steady state.

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1 Introduction

The Schumpeterian idea of creative destruction lies at the heart of the innovation-based growth theory (Grossman and Helpman, 1991; Aghion and Howitt, 1992). Under this view, growth is achieved by entrepreneurs or firms successively undertaking costly R&D to improve and replace each other's exiting product. There are two key ingredients to Schumpeter's theory of creative destruction. First, a successful innovator (leader), by launching a new product in the market, reveals the frontier knowledge embodied in the product to potential competitors (followers). The full knowledge spillover ensures a level playing field in the next round of innovation race for the next product. Second, to prevent this very knowledge spillover from enabling imitators to drive the market price of the new product down to the the marginal cost, monopoly rights must be granted to the leader whereby he can recoup his initial investment in R&D. The rich dynamics of competition, firm exit and turnover inherent in the Schumpeterian model makes it the bedrock on which a large and growing literature is based, where the outcomes from creative destruction among heterogeneous firms can be mapped to micro data (starting from Klette and Kortum (2004)).

In this paper, we call into question a key assumption of Schumpeterian model and show how some of its main implications get turned on their heads. The assumption is the assertion that full knowledge spillover enables all, leaders and followers alike, to compete equally in the innovation race for the next product. We believe that this assumption is too drastic and unrealistic. We claim that, even when a leader's knowledge is made public, for example through the patent's registration process, it is far from immediate that followers can effectively make a productive use of it. Because of the very nature of the patent system, any innovation that builds upon previously patented knowledge would face costly legal challenges by the industry leader and, in any case, the amount of "costless" knowledge that is revealed through a patent's application is always limited - as any student knows, knowledge is never really for free and learning is a costly, time consuming process.

Examples of the disadvantage that a follower faces trying to improve on a leader's product abound in history and at present. In 1769, the great inventor James Watt obtained a patent on his idea of a separate condenser in a steam engine, an improvement upon the Newcomen steam engine. Over the following thirty years while the patent lasted, steam engines were modified and improved by many of his peers: William Bull, Richard

Trevithick, Arthur Woolf, and Jonathan Hornblower. Yet none of these models made it to the market until 1804 after the patent expired, because, no matter how much better the newer models were, they had to use the idea of the separate condenser (Boldrin and Levine (2008) contains many more examples). Fast forward 200 years, in 2007, Apple and Samsung began their decade-long multi-million dollar patent war that spread across courts in ten countries around the globe. This is yet another example of the unequal footing from which a leader (then Apple) and a follower (then Samsung) were competing to bring about new products.

The discussion above suggests that innovation costs for a follower are at least as large as, if not larger than, those for a leader. Moreover, the technologically more advanced a leader is relative to the follower, the more costly it is for the follower to leapfrog him. At the extreme, when the technological distance is large enough, then the leader has achieved the "endpoint strategy" (Hörner (2004)) of pushing the innovation race to a state where any attempt by the follower to leapfrog the incumbent has become prohibitively expensive, and innovation efforts by both the leader and the follower cease. The asymmetry between leader's and follower's innovation capability has long been addressed by the theoretical microeconomic literature on races (tracing back to Harris and Vickers (1987); Budd et al. (1993), and Hörner (2004)). It is exactly the intuition developed in this literature that we add to an endogenous growth model. To put it simply, we embed the assumption of state-dependent asymmetric R&D costs into an otherwise standard endogenous growth model à la Grossman and Helpman (1991) (henceforth, GH), where followers' R&D efforts are aimed at leapfrogging the leaders, thus contributing to aggregate growth.

More formally, we study the effects of state-dependent innovation costs in a general equilibrium model with a continuum of industries, where in each industry a leader and a follower play a game of innovation. The state of the industry is the technological distance between the leader and a competitive fringe of followers, such that when followers fall behind in the innovation race they see their innovation costs rise. In this model, the balance growth path where only followers innovate (as in GH) is no longer the only one that can emerge in equilibrium. For certain range of parameters, the high-growth equilibrium of the GH type can coexist with "growth traps," which are low-growth equilibria where also leaders innovate provided that their technological gap with the followers is small. These equilibria are characterized by low growth, because the R&D effort by leaders has two opposite effects on the aggregate innovation rate. On one hand, it contributes to raising the innovation rate of an industry (the intensive margin of innovation) when the leader and its followers are close to each other. On the other, it increases the share of industries (the extensive margin) where leaders and followers are sufficiently far from each other that innovation drops to zero. In our model the positive effect on the intensive margin is always dominated by the negative effect on the extensive margin, so that the success of leaders' end-point strategies of kicking the followers out of the innovation race is both the reason for large innovation being done at times by leaders, and the cause of low aggregate growth.

Starting from an equilibrium path that converges to the high-growth steady state, we show that an increase in market power beyond a certain threshold will prompt the economy to change its course and converge towards a low-growth steady state. Along this path, aggregate innovation, the only form of investment in this model, drops over time. The drop occurs because the higher market power increases leaders' incentives to gain a technological advantage over their followers and secure a lasting dominance in the industry that discourages innovation by followers. As more industries over time become dominated by a persistent leader, aggregate innovation by both leaders and followers decline. The mechanism highlighted in this paper speaks in spirit to recent empirical evidence of increasing market power, declining competition, investment and business dynamism in the US (Haltiwanger, 2015; Gutiérrez and Philippon, 2017; Loecker et al., 2020).

There have been many papers in the literature that feature leaders innovating and persistent monopoly. Some of the papers emphasize the asymmetry of the technology of innovation between leaders and followers but without the technology being state-dependent (Barro and Sala-i-Martin, 1995; Segerstrom and Zolnierek, 1999; Segerstrom, 2007). Some focus on the asymmetry between leaders and followers in aspects of production other than innovation: customer's base (Stein, 1997) and overhead or expansion cost (Klette and Kortum, 2004; Aghion et al., 2019). Others examine asymmetry stemming from the particular market or game structure that leaders and followers are in, for example a leader's first mover advantage or initial market power (Gilbert and Newbery, 1982; Denicolò, 2001; Etro, 2004; Aghion et al., 2005). Among the aforementioned papers, several share our message that persistent monopolies can be detrimental to long-run growth, though it's clear that we reach the conclusion from very different mechanisms (Gilbert and Newbery, 1982; Aghion et al., 2005, 2019).[1]

[1] More recent quantitative models of innovation that allow leaders to have different innovation rates than followers both vertically and horizontally do it without fully endogenizing firm's innovation rates (Garcia-Macia et al., 2019; Atkeson and Burstein, 2019).

More closely related to our work is Acemoglu and Akcigit (2012) and Liu et al. (2019). Both papers have studied innovation efforts of leaders and followers that are dependent on the technological distance in a duopoly setting. Following the patent race literature, both focus on the step-by-step catchup whereby followers' cost disadvantage lies in having to "reinvent" every step the leader has taken. In contrast, our model features a competitive fringe of followers, whose cost of leapfrogging increases in the distance to the leader. In our view, the step-by-step catch up process seems less realistic than the costly leapfrogging as a description of the innovation behavior of followers. The presence of patent infringement threats forces followers to find new ways to produce better goods, albeit at a much higher cost, rather than to retrace the leader's footsteps at a lower speed. The main departure of our work from Acemoglu and Akcigit (2012) and Liu et al. (2019) is thus to zoom in on the costly leapfrogging with the additional benefit of providing a full analytical characterization of the possible equilibria.[2]

The rest of the paper proceeds as follow. Section 2 introduces the baseline model, Section 3 characterizes the steady states of the economy while Section 4 discusses its global dynamics properties. Section 5 provides simulations where we vary the policy parameters of the model and we discuss welfare implications. Section 6 presents two extensions of the baseline model. Conclusion follows.

2 The Model

The model is based on GH's seminal model of quality ladders. It is a continuous time infinite horizon model. There is a continuum of goods, indexed real numbers in a unit interval. There are two types of agents in the model, households and firms.

2.1 Households

There is a representative household who decides what to consume at each point in time, given its income. It is endowed with one unit of labor and supplies it inelastically. It owns the firms in the economy and hence receives a stream of profits from the firms. Its

[2]The richest version of Acemoglu and Akcigit (2012) (i.e. the "leapfrogging and infringement" extension) allows for both step-by-step slow catch-up and "frontier" R&D by followers, the latter of which resembles our cost assumption. In this version however the authors' results are mainly quantitative, where simulation results are influenced by multiple innovation processes of the followers.

wealth at time 0, W_0, is then the present value of the stream of profits and labor income it receives ad infinitum. At each instant, the household chooses the quantity, d_{it}, of each of the $i \in [0,1]$ goods to consume, taking as given the quality of each good, q_{it}, the price of each good, p_{it}, and the instantaneous interest rate r_t.

The household consumes C_t at time t, which is an aggregate of all varieties of goods:

$$\log C_t = \int_{[0,1]} \log\left(q_{it} d_{it}\right) di. \tag{1}$$

The functions $q_{it} > 0$ define the highest quality developed up to time t for good i. The household's lifetime utility is characterized by a time-additive log period utility function with a rate of time preference of ρ. It solves the following problem:

$$\max_{\{d_{it}, \forall i\}_{t=0}^{\infty}} \quad \int_0^{\infty} e^{-\rho t} \log C_t dt \tag{2}$$

$$\text{s.t.} \quad \int_0^{\infty} e^{-R_t} E_t dt \leq W_0, \tag{3}$$

where R_t is the compounded interest rate and E_t represents total spending at time t:

$$R_t = \int_0^t r(t')dt',$$
$$E_t = \int_{[0,1]} p_{it} d_{it} di.$$

The Cobb-Douglas form of the the consumption aggregate implies that the amount spent by the household on good i is the same across all products, giving

$$d_{it} = \frac{E_t}{p_{it}}.$$

The intertemporal Euler equation gives

$$\frac{\dot{E}_t}{E_t} + \rho = r_t. \tag{4}$$

Household's wealth $W(0)$ is given by

$$W_0 = \int_0^{\infty} e^{-R_t} \left[\Pi_t + w_t L_t + w_t \int_{i\in[0,1]} \omega_{it} \Lambda_{it}]di\right] dt,$$

where Π_t are aggregate profits received from firms, w_t is the wage paid to labor employed in the production sector, L_t, and ω_{it} is the wage premium paid to (skilled) labor employed in the R&D sector in industry i, Λ_{it}. We refer to $\Lambda(i)$ as the *intensive margin* of innovation in industry i. The role of these variables is explained in detail in Section 2.2, which lays out firms' problem. Here we simply specify that total labor is in fixed supply, normalized to unity

$$L_t + \int_{i \in [0,1]} \Lambda_{it} di = 1,$$

where L_t and Λ_{it} are all non-negative. We also assume that the intensive margin of innovation must be bounded above by some constant $\bar{\Lambda}$. The interpretation is simply that there is at most an amount $\bar{\Lambda}$ of workers in the economy with the necessary skill to perform R&D activities in any given industry. For example, there is a fixed supply of labor skilled in biomedical sciences available to the pharmaceutical industry, a fixed supply of labor skilled in computer science available to the information technology industry, and so on and so forth. Clearly, in this situation the household's optimal supply of skilled labor to R&D in an industry, $\Lambda^*(i)$, is the correspondence $\Lambda^*(i) = [0, \bar{\Lambda}]$ if the wage premium is equal to one, while $\Lambda^*(i) = \bar{\Lambda}$ whenever $\omega(i) > 1$. Modeling the supply of skilled labor as perfectly elastic up to $\overline{\Lambda}$ and perfectly inelastic afterwards has two advantages. First, when $\overline{\Lambda}$ does not bind, our model is equivalent to GH's model, so that our model nests GH as a special case. Second, this is a simple and intuitive way to introduce decreasing returns to innovation at the industry level (the cost of skilled labor becoming increasingly more expensive when an industry 's R&D reaches the $\bar{\Lambda}$ threshold). We take $\overline{\Lambda}$ as an arbitrarily large constant.[3]

2.2 Firms

Each product i corresponds to an industry. In each industry, there is a leader and a competitive fringe of followers. The leader in industry i has the technology to produce the state-of-the-art version q_{it} of product i. Such technology is protected by a patent, so that only the leader can produce the quality q_{it}. Leaders and followers also carry out R&D activities. A successful innovation by either a leader or a follower raises the state-of-the-art quality from q_{it} to γq_{it}, where $\gamma > 0$ is the distance between two consecutive rungs on

[3] On a technical note, to obtain an equilibrium under a discountinous labor supply correspondence, we proceed in two steps. First, we propose a continuously differentiable labor supply function with a parameter that governs the speed at which the supply increases as λ exceeds $\overline{\Lambda}$. Second, we let the parameter go to infinity to obtain the formulation of the $\bar{\Lambda}$ described above. Details can be found in Appendix A.

product i's quality ladder. The quality of a good can then be written as $q_{it} = \gamma^{s_{it}}$, with $s_{it} \in \mathbb{N}$ the number of rungs along the quality ladder that have been climbed up to time t in industry i. Detailed description of firms' production and R&D activities follow.

2.2.1 Production of Goods

The output y_{it} in industry i is produced using labor l_{it} according to a linear technology

$$y_{it} = l_{it}.$$

Since, for a given industry, products of different qualities are perfect substitutes, a leader who charges a markup over marginal cost of production labor anywhere between 1 and γ can put his followers out of business. Let the markup charged by leaders be $m \in [1, \gamma]$, which we interpret as a policy variable exogenously determined, as when, for instance, an antitrust authority limits the monopoly pricing power of the leaders. We will later investigate how the growth rate of the economy varies with the markup level m. The price at which leaders sell their products is therefore given by

$$p_{it} = m w_t, \text{for } m \in [1, \gamma], \tag{5}$$

where w_t is the wage rate of production labor. Without loss of generality, we normalize w_t to 1 so that, from now on, we express variables in terms of the period wage. The goods prices are then $p_{it} = m$ and profits of leaders can then be simply expressed as

$$\pi_{it} = (m-1) y_{it}.$$

At equilibrium prices, the household's demand for good i is given by

$$d_{it}^* = \frac{E_t^*}{m}.$$

Using the market clearing condition, $d_{it}^* = y_t^*(i)$, we conclude that all industries produce the same amount of output, Y_t, using the same amount of labor, L_t, given by

$$Y_t^* = L_t^* = \frac{E_t^*}{m}. \tag{6}$$

It follows then that, for all leaders, profits are given by

$$\Pi_t^* = (m-1)L_t^*. \tag{7}$$

2.2.2 Game of Innovation

Within each industry, leaders and followers play a game of innovation, and expectations about each other's future strategies determine current innovation efforts. We start by describing the innovation technologies available to the leaders and followers, which depend on the existing technological distance between the two parties, measured in number of steps on the quality ladder.

When the distance between a leader and a follower is one step, we maintain the GH assumption that both the leader and follower have the same R&D technology. That is, if a firm hires an amount λ of skilled workers to perform R&D, the firm experiences an arrival of a successful innovation at a Poisson rate $\Gamma(\lambda)$ given by

$$\Gamma(\lambda) = \chi\lambda, \text{ for } \chi > 0,$$

where χ is a parameter that governs the productivity in the R&D sector. The innovation technology displays constant returns at the firm's level.

When the technological distance between a leader and a follower is instead two or more steps, the follower can no longer innovate with the same technology as that used by the leader. Specifically, we assume that the cost of innovation for the follower who is two or more steps behind the leader is high enough that the follower stops innovating completely. In Appendix B, we show that this assumption is without loss of generality, because, under the assumption of linear innovation technologies, either a step-by-step catch-up or a fast catch-up process as in Acemoglu and Akcigit (2012) will give us the result that followers who are two or more steps behind the leader optimally choose not to innovate.

This structure of the innovation technology is meant to capture the idea that leapfrogging becomes increasingly difficult for followers when their technological distance from the industry leader increases. There are two complementary interpretations for this assumption.

The first is that every state-of-the-art version of a product incorporates elements from the previous versions, which are patented. If the follower's technology is not far from that of the leader (i.e. when the follower is only one step behind), then the follower is able to invent the a new product quality without having to incorporate in this new quality any technological element over which only the leader owns a patent. Indeed, patents can impose substantial legal and uncertainty costs for challengers. Therefore, when the followers own patents on quite obsolete technologies (i.e. when the follower is two steps behind the leader), then it is not possible for the follower to invent the state-of-the-art quality without having to incorporate elements that have already been patented by the leader. However, as in GH, leaders do not have any incentive to grant a license to a follower, whose innovation costs therefore become prohibitively large.

The second is that some free-of-charge knowledge spillover to followers does take place, but it takes time. If a leader is a lot more advanced in his stock of knowledge, then it takes a longer time for the knowledge spillover to complete. In this case, followers fall for some time behind the leader in the amount of R&D knowledge they can muster when the distance to the leader is larger.

When leaders and followers are one step apart, there are potential incentives for both to innovate. Followers innovate to replace the leader, as in GH. Leaders may also want to innovate for the pure goal of distancing themselves further from the followers. As the distance grows, innovation costs for followers rise and the followers stop threatening. The incumbent's leadership will then be secured for a long period of time through innovation in the current period. We refer to this strategy of the leader as an *endpoint* strategy.

We assume that when a leader is two steps ahead of a follower, the distance is reduced to one step at an exogenous (small) rate $\tau > 0$. When rising R&D costs are interpreted as driven by legal constraints imposed by patents, then τ can be thought as a policy variable that controls the legal term of patents. When rising R&D costs are tied to lack of full knowledge spillover, then τ indicates the frequency at which the spillover occurs. It is worth-noting that when we let τ go to infinity, we are back to the GH world where there is instant spillover of the innovation technology and leaders and followers can be at most one step apart.

We say that an industry is in the *contestable* state if the distance between a leader and his followers is equal to one step, and in the *non-contestable* state if the distance is two steps. We indicate with $\alpha_t \in [0,1]$ the share of industries that at a given time t are in the contestable state. Since innovation only takes place in a contestable state, we call α_t the *extensive margin* of innovation in the economy.

Mathematically, the combination of two types of firms (leader l or follower f) and of two possible distances (1 or 2) between firms, gives rise to four value function $V_\Delta^j(t)$, for $j \in \{l, f\}$ and $\Delta \in \{1,2\}$. When it does not create confusion, we omit the indication of the dependence of variables on time. Our four value functions satisfy, at any point of differentiability, the following differential equations

$$rV_2^l = \Pi + \tau(V_1^l - V_2^l) + \dot{V}_2^l \tag{8}$$

$$rV_2^f = \tau(V_1^f - V_2^f) + \dot{V}_2^f \tag{9}$$

$$rV_1^l = \max_{\lambda^l \geq 0} \Pi - \omega\lambda^l + \chi\lambda^l(V_2^l - V_1^l) + \chi\lambda^f(V_1^f - V_1^l) + \dot{V}_1^l \tag{10}$$

$$rV_1^f = \max_{\lambda^f \geq 0} -\omega\lambda^f + \chi\lambda^f(V_1^l - V_1^f) + \chi\lambda^l(V_2^f - V_1^f) + \dot{V}_1^f. \tag{11}$$

Equation (8) (Equation (9)) describes the value function of a leader (follower) who is two steps ahead (behind). This corresponds to a non-contestable state, where the endpoint part of the innovation game is reached due to the assumed high innovation cost to the follower and where thus both the leader and the follower optimally decide not to innovate. Note that the leader is the only one who makes profits and that the distance between the two is subject to the exogenous rate τ of reduction back to one step. Equations (10) and (11) describe the value functions of the two in a contestable state, where strictly positive innovation rates may still be chosen by both the leader and follower. The leader pays the cost of innovation, $\omega\lambda^l$, to increase the probability of enlarging the techonological gap and obtaining V_2^l, whereas the follower pays the cost $\omega\lambda^f$ to increase the probability of leapfrogging and obtaining V_1^l. The free entry condition for the competitive fringe of followers implies that

$$V_1^f(t) = V_2^f(t) = 0, \quad \forall t.$$

2.3 Equilibrium

In equilibrium, R&D strategies are symmetric across all industries. We focus on Markov equilibria. Therefore, at any given point in time, efforts λ_t^l and λ_t^f by leaders and followers, and the corresponding intensive margin Λ_t, are the same across all industries in the contestable state, so we can drop the index i form our notation. The evolution of the extensive margin is

$$\dot{\alpha}_t = (1 - \alpha_t)\tau - \alpha_t \chi \lambda_t^l. \tag{12}$$

The aggregate number of rungs on the quality ladder achieved at time t, $S_t = \int_{[0,1]} s_{it} di$, evolves according to

$$\dot{S}_t = \chi H_t \equiv \chi \Lambda_t \alpha_t, \tag{13}$$

where H_t is defined to be the total amount of skilled R&D labor employed at time t.

The definition of equilibrium in this model is standard.

Definition 1. *An equilibrium is given by prices* $\{r_t, w_t, p_{it}, \omega_t\}_{t=0}^{\infty}$, *innovation rates by leaders and followers* $\left\{\lambda_t^{l*}, \lambda_t^{f*}\right\}_{t=0}^{\infty}$, *functions* $\{E_t^*, L_t^*, \Lambda_t^*, S_t^*, Y_t^*, \Pi_t^*, \alpha_t^*\}_{t=0}^{\infty}$ *for aggregate expenditure, supply of production labor, supply of R&D labor, aggregate quality, output, profits and the extensive margin of innovation, such that*

i) Given prices and the evolution of Π_t^*, *the innovation rates* λ_t^{l*} *and* λ_t^{f*} *solve firms' innovation game.*

ii) Given aggregate expenditure E_t^* *and normalized wages* $w_t = 1$, Y_t^* *and* $p_t^* = m$ *are the optimal output and price level chosen by leaders in any industry. Correspondingly* $\Pi_t^* = (m-1)Y_t^*$ *are the profits of leaders.*

iii) Given prices and the evolution of aggregate profits, E_t^*, L_t^* *and* Λ_t^* *are, respectively, the optimal expenditure, and the optimal production and R&D labor supplies of households.*

iv) Given Λ_t^*, *the wage premium* ω_t *of firms in the contestable state is compatible with fixed supply of skilled labor and satisfies the complementary slackness condition:* $(\omega_t - w_t)(\bar{\Lambda} - \Lambda_t^*) = 0$. *Given* λ_t^{l*} *and an initial condition* $\alpha(0)$, *the extensive margin* α_t^* *satisfies (12). Given an initial condition* $S(0)$ *and the evolution of* $H_t^* = \alpha_t^* \Lambda_t^*$, *the aggregate quality* S_t^* *satisfies (13).*

v) Markets clear, i.e. $L_t^* = Y_t^*$, $E_t^*/m = Y_t^*$, $\Lambda_t^* = \lambda_t^{f*} + \lambda_t^{l*}$, $H_t^* = 1 - L_t^*$.

Note that, since in equilibrium the quantities of all goods are the same and equal to the production labor input, $d_{it} = L_t$, the log aggregate consumption can be written as

$$\log C_t = \int_{[0,1]} \log(q_{it} d_{it}) di = \int_{[0,1]} \log \gamma^{s_{it}} di + \log L_t = \log(\gamma) S_t + \log L_t.$$

The growth rate of consumption is therefore

$$\frac{\dot{C}_t}{C_t} = \log(\gamma)\dot{S}_t + \frac{\dot{L}_t}{L_t} = \log(\gamma)\chi H_t + \frac{\dot{L}_t}{L_t} = \log(\gamma)\chi \alpha_t \Lambda_t + \frac{\dot{L}_t}{L_t}.$$

The growth rate of aggregate consumption is then given, in equilibrium, by the sum of the growth rate of the production labor input and the growth of the aggregate output quality. We refer to the quantity $g_t = \log(\gamma)\dot{S}$ as the *rate of technological growth*.

The balanced growth path of this model is an equilibrium where aggregate consumption and quality, C_t and S_t, grow at the same rate g.

3 Steady States

Depending on the parameter values, the equilibrium economy can display up to three steady states, which we label using subscripts H, M or L to indicate whether a steady state is characterized by a high, medium or low value for the extensive margin of innovation, α.

3.1 The H Steady State

The highest possible steady state value for α^* is 1. In this steady state only followers innovate, and thus $\lambda^{l*} = 0$ and $\lambda^{f*} > 0$. As already discussed, by taking $\bar{\Lambda}$ large enough we can make sure that in a neighborhood of the steady state $\Lambda_t^* = \lambda_t^{f*} < \bar{\Lambda}$, giving the skill premium ω_t equal to 1. Hence, the first order condition for λ^f in a neighborhood of a H steady state imply that

$$V_1^l(t) = \frac{1}{\chi}.$$

The condition above naturally implies that $\dot{V}_1^l = 0$. Since $\lambda_t^{l*} = 0$, a straightforward substitution in the definition of V_1^l gives

$$\frac{r_t}{\chi} + \lambda_t^f = \Pi_t. \tag{14}$$

Combining the above equation with the facts that $\Pi = (m-1)L$, $\lambda^f = (1-L)/\alpha$ and $r = \rho + \dot{L}/L$, we obtain

$$\frac{\dot{L}}{L} = \chi \left[\left(m - 1 + \frac{1}{\alpha} \right) L - \frac{1}{\alpha} \right] - \rho. \tag{15}$$

Equation (15) defines the evolution of the economy around the H steady state, together with the condition

$$\dot{\alpha} = \tau(1-\alpha). \tag{16}$$

The H steady state is then characterized by

$$\alpha_H^* = 1; \tag{17}$$

$$L_H^* = \frac{\rho + \chi}{\chi m}; \tag{18}$$

$$\lambda_H^{f*} = \frac{(m-1)\chi - \rho}{\chi m}. \tag{19}$$

Linearizing the system, (15) and (16), we can show that the H steady state is a saddle. The non-negativity of λ_H^{f*} requires that $m > 1 + \frac{\rho}{\chi}$.[4]

The value to a leader who is hypothetically two steps ahead in the H steady state can be computed as

$$V_2^{l*} = \frac{(m-1)\frac{\rho+\chi}{m} + \tau}{\chi(\rho+\tau)}.$$

We also note that, to guarantee that indeed leaders do not want to innovate, so that they optimally choose $\lambda^{*l} = 0$, we must have $m < \overline{M}$, where $\overline{M}$ is defined so that $V_{2,H}^{l*} = 2/\chi$.

[4]For $m < 1 + \frac{\rho}{\chi}$, the steady state will be characterized by $\alpha_H^* = L_H^* = 1$ and $\lambda^{f*} = 0$, a case that we rule out for its lack of relevance.

One can show that

$$\overline{M} = \frac{\rho + \chi}{\chi - \rho - \tau}. \tag{20}$$

For the above to be a meaningful condition, we assume $\chi > \rho + \tau$.

3.1.1 The M and L Steady States

In the M and L steady states, both leaders and followers innovate in the contestable state. The first order conditions for λ^l and λ^f give $V_2^l(t) = 2\omega_t/\chi = 2V_1^l(t)$. Substituting these conditions into the value functions and after appropriate calculations we obtain the two equations:[5]

$$\begin{cases} \Pi = (2\lambda^f - \frac{\tau}{\chi})\omega \\ \dot{\omega} = (r + \tau - \chi\lambda^f)\omega. \end{cases} \tag{21}$$

The M and L steady states differ in whether the supply of R&D labor is exhausted or not. In the M steady state the industry-level R&D labor supply constraint is not binding. Hence, $\lambda_M^{l*} + \lambda_M^{f*} = \Lambda_M^* < \overline{\Lambda}$ and $\omega_M^* = 1$. In contrast, in the L steady state, the industry-level skilled labor supply binds at $\overline{\Lambda}$ and $\omega_L^* > 1$. In either steady state, the interest rate is ρ and the second equation of (21) implies followers innovate at the same intensity,

$$\lambda_M^{f*} = \lambda_L^{f*} = \frac{\rho + \tau}{\chi}. \tag{22}$$

In the M steady state, we can solve out the production labor from the first equation in (21), together with $\Pi_M^* = (m-1)L_M^*$ and $\omega_M^* = 1$:

$$L_M^* = \frac{2\lambda_M^{f*} - \frac{\tau}{\chi}}{m-1} = \frac{2\rho + \tau}{\chi(m-1)}. \tag{23}$$

In the L steady state, since the skilled labor supply binds, we have $\lambda_L^{l*} = \overline{\Lambda} - \frac{\rho+\tau}{\chi}$. The

[5]For detailed derivations, see Appendix A.2.

evolution of the extensive margin (equation (12)) then implies that in the steady state,

$$\alpha_L^* = \frac{\tau}{\tau + \chi\lambda_L^{l*}} = \frac{\tau}{\chi\overline{\Lambda} - \rho}.$$

This, in turn, pins down the production labor and profit in the L steady state,

$$L_L^* = 1 - \alpha_L^*\overline{\Lambda} = \frac{(\chi - \tau)\overline{\Lambda} - \rho}{\chi\overline{\Lambda} - \rho};$$
$$\Pi_L^* = (m-1)\frac{(\chi - \tau)\overline{\Lambda} - \rho}{\chi\overline{\Lambda} - \rho}.$$

With these inputs, we can solve out the value to a leader who is one step ahead from (10),

$$V_{1,L}^{l*} = \frac{\Pi_L^*}{\rho + \chi\lambda_L^{f*}} = (m-1)\frac{(\chi - \tau)\overline{\Lambda} - \rho}{(2\rho + \tau)(\chi\overline{\Lambda} - \rho)}.$$

For the L steady state to exist, $V_{1,L}^{l*} = \frac{\omega_L^*}{\chi} > \frac{1}{\chi}$. Define $\underline{\mathrm{M}}$ so that $V_{1,L}^{l*} = \frac{1}{\chi}$ for $m = \underline{\mathrm{M}}$. Therefore, the existence of the L steady state requires $m > \underline{\mathrm{M}}$, which is given by

$$\underline{\mathrm{M}} = 1 + \frac{(2\rho + \tau)(\chi\overline{\Lambda} - \rho)}{\chi\left((\chi - \tau)\overline{\Lambda} - \rho\right)}. \tag{24}$$

Comparing $\underline{\mathrm{M}}$ and $\overline{M}$, we have $\underline{\mathrm{M}} < \overline{M}$ if and only if

$$\overline{\Lambda} > \frac{\rho + \tau}{\chi},$$

a condition that, as usual, holds as long as $\overline{\Lambda}$ is sufficiently large.

3.2 Discussion

The results derived in the previous section can be collected as follows:

Proposition 1. *There are two constants $\underline{M} < \overline{M}$, defined by (20) and (24), such that*

i) For $m < \underline{M}$, only the H steady state exists. For $m > \overline{M}$ only the L steady state exists. For $m \in [\underline{M}, \overline{M}]$ the steady states H, M, L all exist. Over the interval the M steady state exists, α_M^* is increasing in m.

ii) The H and the L steady states have the saddle-path property, while the M steady state is a source. In particular, if $m > \overline{M}$, then, for any initial condition α_0, the economy always converges to the L steady state.

Proof. See Appendix A.2. □

It is worth pointing out that we have $\alpha_H^* > \alpha_M^* > \alpha_L^*$ when all three steady states exist, and hence our naming of these steady states. Moreover, the three steady states are not only ranked by their extensive margin of innovation, but are also ranked by their equilibrium growth rate, as indicated by the following corollary to Proposition 1.

Corollary 1. *For $m \in (\underline{M}, \overline{M})$, the steady state growth rates satisfy $g_H^* > g_M^* > g_L^*$. Moreover, the steady state growth g_L^* associated with $m > \overline{M}$ is smaller than the growth rate in any of the steady states for $m \in (\underline{M}, \overline{M})$.*

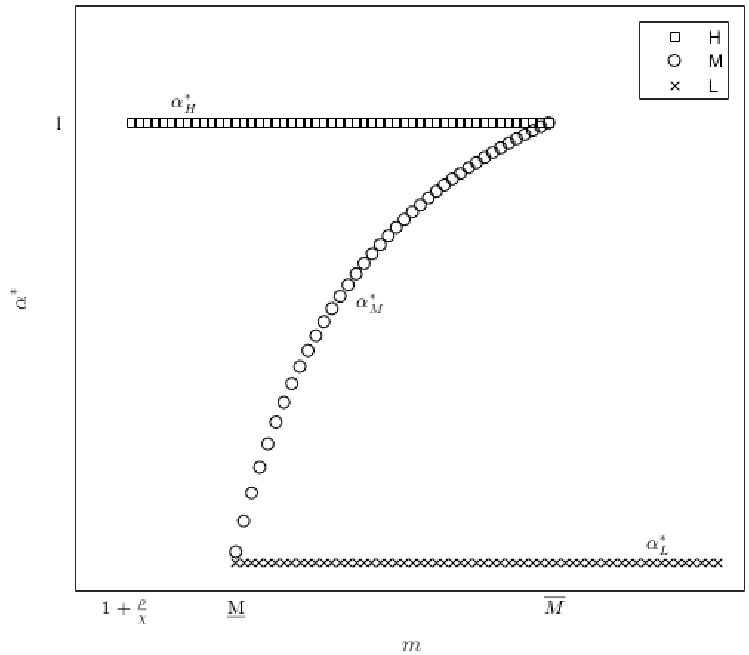
Proof. See Appendix A.3. □

The mechanism behind the structure of the steady states is based on the joint effect of two simple properties of the model: the effect of leaders' innovation on aggregate growth, and the effect of the net present value of monopoly on leaders' incentives to innovate.

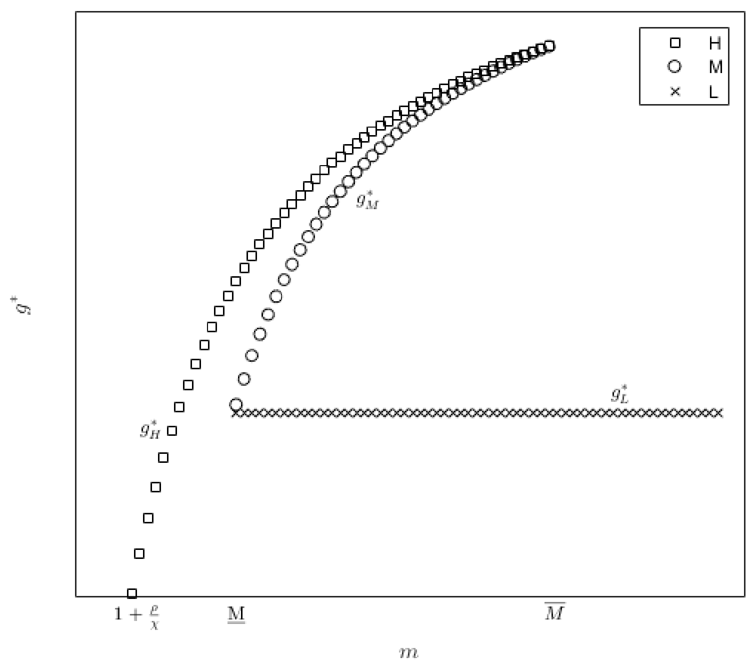
First, greater innovation by leaders is associated with lower long-run technological growth – the result outlined in Corollary 1. This is not surprising, since leaders' innovation is motivated by endpoint strategies, whose sole goal is to discourage innovation. The positive effect on growth from a higher intensive margin of innovation carried out by leaders in contestable states is more than offset by the greater fraction of industries that, in the long run, end up in non-contestable states with zero innovation. To see this, recall that the rate of technological growth is proportional to the product of the extensive and intensive margins. Using (12) to calculate the steady state value of the extensive margin, we obtain

$$g^* = \log(\gamma)\chi\alpha^*\Lambda^* = \log(\gamma)\chi\frac{\tau}{\tau + \chi\lambda^{l*}}\Lambda^* = \log(\gamma)\chi\frac{\tau}{\tau + \chi\lambda^{l*}}(\lambda^{f*} + \lambda^{l*}).$$

A larger innovation rate by leader is then associated with lower aggregate growth, provided that $\chi\lambda^{f*} > \tau$. This latter condition, which in our case always holds in equilibrium, simply requires that the probability of a successful innovation by followers in the contestable state is greater than the exogenous probability of a spillover (or of a patent expiration) in the non-contestable state.



(a) Steady State Extensive Margin of Innovation, α_i^* for $i = H, M, L$



(b) Steady State Growth Rate, g_i^* for $i = H, M, L$

Figure 3.1: Extensive Margin of Innovation and Growth Rate in the Steady States

Note: This figure illustrates the structure of the steady states of the model. In particular, it shows how the steady state extensive margin of innovation α^* and the steady state growth rate g^* vary as we vary the markup parameter m. For a discussion, see Section 3.2.

Second, a higher net present value of monopoly power increases leaders' incentives to innovate. This is intuitive, since a larger leadership value triggers more effort to secure it by means of endpoint strategies. Indeed, it is straightforward to show that the incremental value $V_2^l - V_1^l$ to a leader who successfully innovates is given by[6]

$$V_2^l - V_1^l = \frac{\chi\lambda^f}{(r+\chi\lambda^f)(r+\tau)}\Pi.$$

Leaders' endpoint strategies are incentivized when monopolist's profits Π are higher, when followers' innovation rates λ^f are higher, when interest rates r are lower, and when the externality intensity τ is lower. Profits are higher, for instance, when markups m are larger, which explains the result of Proposition 1, depicted in Figure 3.1. Panel (a) of the figure plots the steady state extensive margin of innovation α^* against m. As the figure shows, if markups m are too large, i.e. $m > \bar{M}$, then incentives for leaders to innovate are so strong that for any initial condition the economy converges to L, which is the steady state with lowest long-run growth (Panel (b)). A similar reasoning explains why when knowledge spillover are infrequent, then only the L steady state exists (notice that $\bar{M}$ is increasing in τ). Instead, a higher discounting ρ reduces the present value of profits, and thus discourages R&D by leaders, while higher innovation by followers, by increasing the threat to the incumbents, strengthens their incentives to play end point strategies.

For values of $m \in [\underline{M}, \bar{M}]$, the model features multiple steady states. The key to understanding this result is to link, in general equilibrium, the two properties discussed above. Fix a given $m \in [\underline{M}, \bar{M}]$ and begin by assuming that the economy is in a high growth steady state. Since a large fraction H of the labor input is devoted to R&D, production labor and period profits are low. With low profits, incentives for leaders to innovate fall. For similar reasons, innovation intensity for followers λ^{f*} is also reduced (in an H steady state the extensive margin of innovation is large, but the intensive margin is small).[7] This further depresses leaders' innovation incentives. Finally, under the log-utility assumption considered so far the steady state interest rate $r = \rho$ is independent of the steady state growth rate g. However, Section 6.1 shows that, when the elasticity of intertemporal substitution is greater than 1, then a larger growth rate raises the equilibrium interest rate, further dampening innovation incentives for leaders. In conclusion, if the economy is at a H steady state, then general equilibrium effects discourage innovation by leaders, and

[6] At the steady state, (10) implies $V_1^l = \frac{\Pi}{r+\chi\lambda^f}$ and (8)-(10) implies $V_2^l - V_1^l = \frac{\chi\lambda^f V_1^l}{r+\tau}$. Combining the two, we obtain the expression for $V_2^l - V_1^l$.

[7] If we compare (22) with (19) we note in fact that $\lambda_H^{f*} < \lambda_M^{f*} = \lambda_L^{f*}$ for $m < \bar{M}$.

since R&D by leaders is negatively associated with long run growth, then the high growth steady state is self-confirmed. A similar line or reasoning can be followed, for instance, to self-confirm an initial position at the L steady state. In particular, in the region with multiple steady states, the L equilibrium represents a "monopolistic growth trap."

4 Global Dynamics

We have established that the model contains three steady states, two of which are saddle-path stable while the other is unstable. One may now wonder whether there exist also other types of equilibria, such as cycles where leaders oscillate between innovating and not innovating. The proposition below establishes that no such equilibrium exists.

Proposition 2. *Consider any equilibrium where variables are continuous at all t, with the possible exception of times where λ_t^{*l} drops to zero. Then for $t \to \infty$ the economy must converge to one of the three steady states L, M, or H.*

Proof. See Appendix A.4. □

Note that Proposition 2 holds also for equilibria where the variables are allowed to change discontinuously at specific points. In this way, Proposition 2 encompasses the case of equilibrium paths that converge to the H steady state but that start at time 0 with strictly positive innovation rates $\lambda_0^{*l} > 0$ by leaders. These equilibria can in fact feature a point of discontinuity caused by the linearity of the R&D technology, which induces a "bang-bang" structure for the leaders' optimal innovation strategy with λ_t^{*l} jumping to zero at the time t when leaders' innovation ceases (incidentally, this causes the value function V_2^l to display a kink at such points).

4.1 Saddle Path Dynamics: Simulations

In this section, we provide an empirically plausible parametrization of the model and numerically simulate the saddle path convergence of the economy to either the H or the L steady state depending on the initial condition α_0.

We begin by considering an initial condition α in the interval (α_L^*, α_M^*) but close to α_M^*, and we construct the saddle path equilibrium that leads the economy to converge over time to the L steady state. This is obtained as follows. First, as the economy is initially close to the M steady state, both leaders and follows innovate and the intensive margin constraint $\bar{\Lambda}$ is not binding. Therefore the system (α_t, L_t) evolves as follows

$$\begin{cases} \dot{\alpha}_t = \rho\alpha_t + \frac{2\rho+\tau}{m-1} + \tau - \chi \\ L_t = L_M^* = \frac{2\rho+\tau}{\chi(m-1)} \end{cases} . \tag{25}$$

As soon as $\overline{\Lambda}$ binds, the system switches to

$$\begin{cases} \dot{\alpha}_t = \tau - \alpha_t(\chi\overline{\Lambda} - \rho) \\ L_t = 1 - \alpha_t\overline{\Lambda} \end{cases}$$

and the economy converges to the L steady state.

Alternatively, we can construct a converge path for the economy starting from an initial $\alpha_0 \in (\alpha_M^*, \alpha_H^*)$ that is sufficiently close to α_M^*. The system evolves according to (25) until leaders are indifferent between innovating and not innovating, i.e. until $V_2^l(t) = 2/\chi$. After that, leaders stop innovation and the equilibrium jumps to the saddle path that converges to the H steady state where only followers innovate. The system evolves according to:

$$\begin{cases} \dot{\alpha}_t = (1 - \alpha_t)\tau \\ \dot{L}_t / L_t = \chi\left((m-1)L_t - \frac{1-L_t}{\alpha_t}\right) - \rho \end{cases} .$$

We simulate the model under the parameters given in Table 4.1. A period in the model is one year. We set the subjective discount rate to 0.02. The skilled labor supply cap, $\overline{\Lambda}$, is calibrated to the percentage of college graduates among adult population in the US in 2017, not all of whom need work in the R&D sector. The rate of patent expiration, τ, is taken to be 0.05, which is the inverse of the term of patents (20 years). We set m to be in the interval $[\underline{M}, \overline{M}]$, so all three steady states exist. The step size γ of a successful innovation is chosen to ensure reasonable growth rates of the economy.

The saddle equilibrium path that converges to the L steady state is illustrated in Panel (a) of Figure 4.1. The initial condition of the extensive margin of innovation, α_0, is chosen to be just below α_M^*. Initially, around the M steady state, both leaders and followers innovate. While the extensive margin of innovation decreases over time, the intensive margin of innovation increases due to increasing innovation rates by leaders. As long as the skilled labor supply is not binding, the evolution of the two counteract each other perfectly, so that the rate of technological growth pinned down by the aggregate level of innovation, g_t, is constant and so is the size of the production sector, L_t. As soon as the skilled labor supply binds in contestable industries, the decline in the extensive margin takes over, the aggregate level of innovation declines and the production sector expands. Admittedly, the kink in g_t is driven by the fixed supply of skilled labor together with the linearity of the R&D technology at the firm level. In Section 6.2 we show that if we relax the assumption of the linear R&D cost structure, then g_t evolves smoothly as the economy converges to the L steady state.

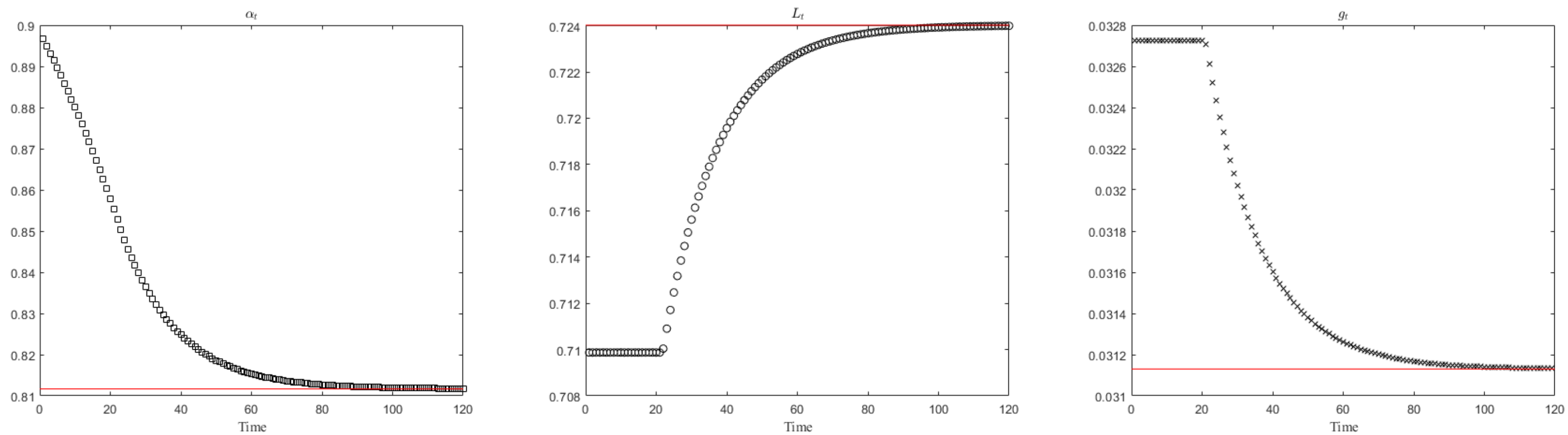
Panel (b) of the same figure shows the saddle equilibrium path that converges to the H steady state. Starting from an initial extensive margin of innovation just above the M steady state, the extensive margin of innovation increases, the intensive margin innovation decreases due to leaders innovating less, while the aggregate innovation stays constant, until the moment when leaders no longer find it profitable to innovate. At that point, the equilibrium jumps to the saddle path that converges to the H steady state. The extensive margin α_t moves continuously, though its rate of change has a kink at that point, while the production labor L_t has a discontinuous jump. Once this point of discontinuity is crossed, the extensive margin keeps increasing until all industries become contestable and the economy approaches the H steady state.

Parameter	Value	Justification
ρ	0.02	Convention
$\overline{\Lambda}$	0.34	Pct. of college graduates among adult population
τ	0.05	Term of patents, 20 years
χ	0.24	Ensure the existence of path to the H
m	1.53	The average of $\overline{M}$ and $\underline{M}$
γ	1.60	Ensure $\gamma \geq m$ and reasonable consumption growth rate

Table 4.1: Parameter Values

Note: This table reports the parameter values we use in the simulation of the baseline model and their justifications. For a discussion, see Section 4.1.

(a) Saddle Path to the L Steady State



(b) Saddle Path to the H Steady State

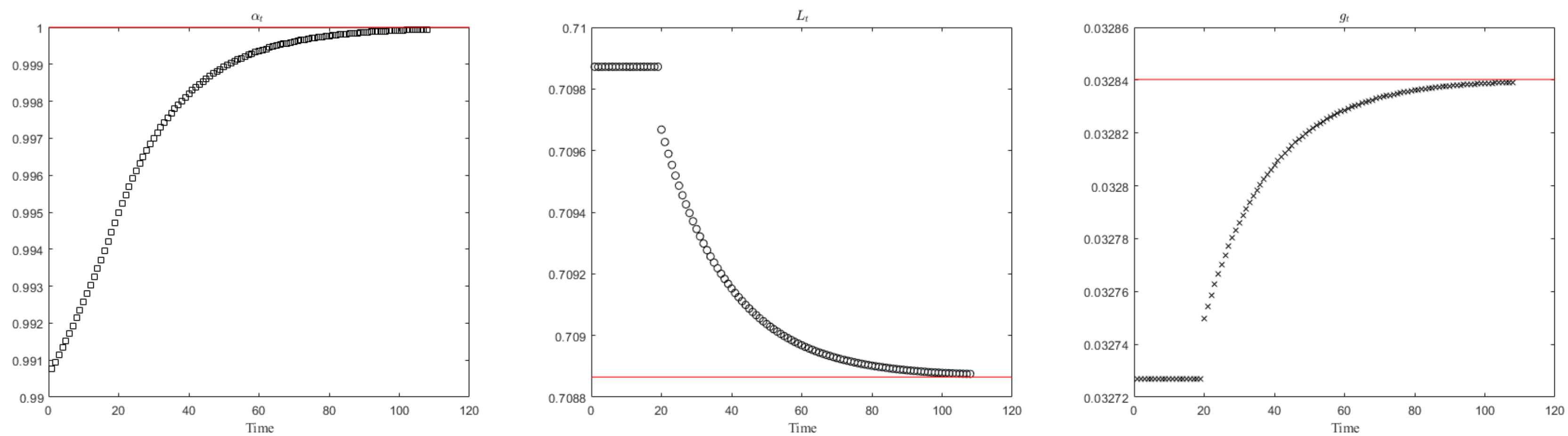


Figure 4.1: Equilibrium Saddle Paths

Note: This figure shows the simulated equilibrium paths to the L and the H steady state. In particular, we plot the evolution of the extensive margin of innovation α_t, the amount of production labor L_t, and the steady state growth rate g_t on their saddle paths to the steady state. The model is simulated under the parametrization given by Table 4.1. For a discussion, see Section 4.1.

5 Policy and Welfare Implications

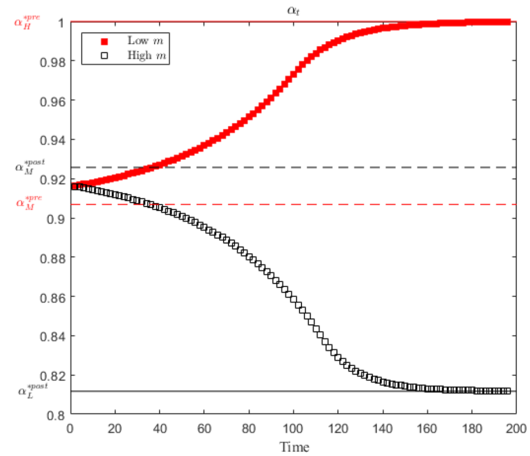
Should Medicare be allowed to bargain for better deals with drugs providers, effectively reducing the markup for pharmaceutical companies? What is the effect of longer patents' duration? These are all common policy questions that, in our model, involve setting the parameters m and τ. The results in the previous section show that, in general, the long-run growth rate of the economy responds in a non-linear way to changes in these parameters such as the mark-up m of the patent's expiration rate τ. These are both characteristics of a country's legal framework, which therefore provides a powerful set of constraints to the ability of a country to innovate and grow (Parente and Prescott, 2002).

To further shed light on this issue, this section explores the dynamic evolution of the economy under two policy experiments, one involving a change in m and one a change in τ. As before, to make our experiments more striking we look at knife-edge cases where the economy's initial condition α_0 is around the M steady state.

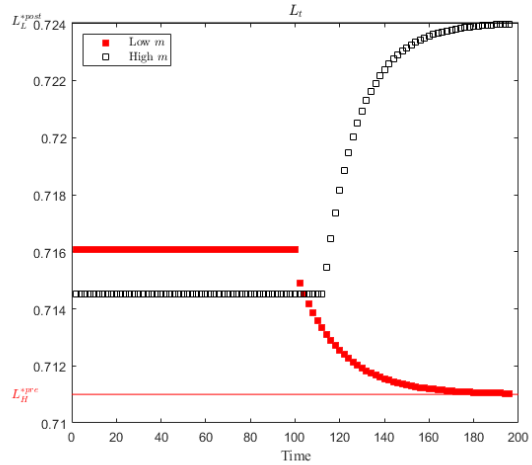
5.1 Raising the Markup Ceiling

In the first policy experiment, we raise the markup m slightly from 1.5237 to 1.5248 and simulate the equilibrium path from the same initial condition $\alpha_0 = 0.9162$ under the two different policy environments. The parametrization is otherwise identical to that in Table 4.1. The simulated equilibrium paths are found in Figure 5.1. Panels (a) to (c) illustrate the equilibrium behavior of the extensive margin of innovation α_t, the aggregate production labor L_t, the rate of technological growth g_t. Panel (d) in the same figure shows the ratio of the aggregate consumption in the high-m environment to the aggregate consumption in the low-m environment. To ease the reading of the figure, we use solid red to describe the low-m economy and hollow black to describe the high-m economy.

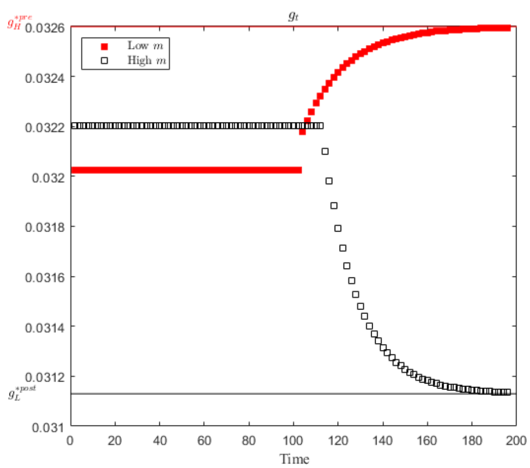
In the low-m economy, the initial condition α_0 is above the M steady state level of extensive margin, α_M^{*pre}, setting the economy on the saddle path to the H steady state. The H steady features innovation only by followers and in all industries which add up to a high level of aggregate innovation and growth. However, for the same initial condition, a slight increase in the mark-up m increases the M steady state level of the extensive margin, α_M^{*post}, which completely changes the equilibrium path of the economy. Now in fact the economy is set to converge to the L steady state, featuring a much lower level of aggregate innovation and growth.



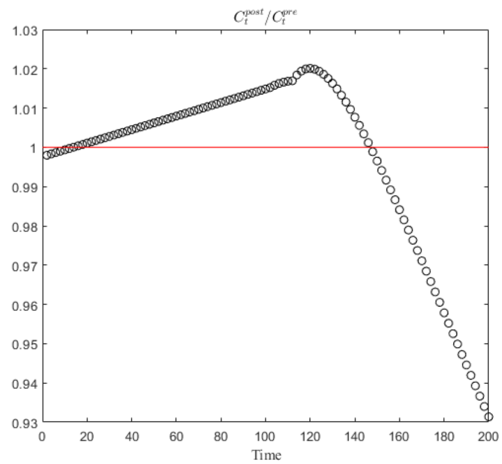
(a) Extensive margin of innovation



(b) Aggregate production labor



(c) Rate of technological growth



(d) Ratio of aggregate consumption

Figure 5.1: Policy Experiment: Raising m

Note: This figure illustrates how the saddle path of the economy can change upon a change in the policy variable m. The red lines in Panel (a)-(c) depict the evolution of the extensive margin of innovation α_t, the production labor L_t and the growth rate g_t on the saddle path to a H steady state. Upon an increase in the markup m, the economy however lands on a saddle path converging to the L steady state, as shown by the black lines in those panels. Panel (d) shows the ratio of consumption, period by period, before to after the increase in m. For a discussion, see Section 5.1.

To evaluate the aggregate consequence of such a policy change, in Panel (d) we track the aggregate consumption in the high-m environment relative to that in the low-m. It is noteworthy that raising the mark-up produces higher aggregate consumption growth in the short run, before the negative long-run effects kick in. The reason for the diverging short-run and long-run welfare implications of raising the markup ceiling is as follows. As monopoly profits are increased under a larger m, leaders respond by raising their effort in innovation while followers' optimal intensive margin of innovation remain unchanged.[8] On the other hand, higher intensive margin of innovation by the leaders increases the speed at which an industry escapes the contestable state and reduces the extensive margin of innovation (Panel (a)). Therefore, the intensive and extensive margin of innovation move in opposite direction in the short run after the policy change. Panels (b) and (c) tell us that the first of the two effects dominates in the short run so that aggregate innovation increases and aggregate labor employed in production decreases. Over time, the skilled labor employed in the R&D sector in those fewer and fewer contestable industries is exhausted, so the intensive margin of innovation at the industry level cannot increase while the extensive margin keeps decreasing. In the long run, the second effect clearly dominates, canceling out any short-run gain and leading to a permanently lower consumption growth.

5.2 Lengthening Patents' Duration

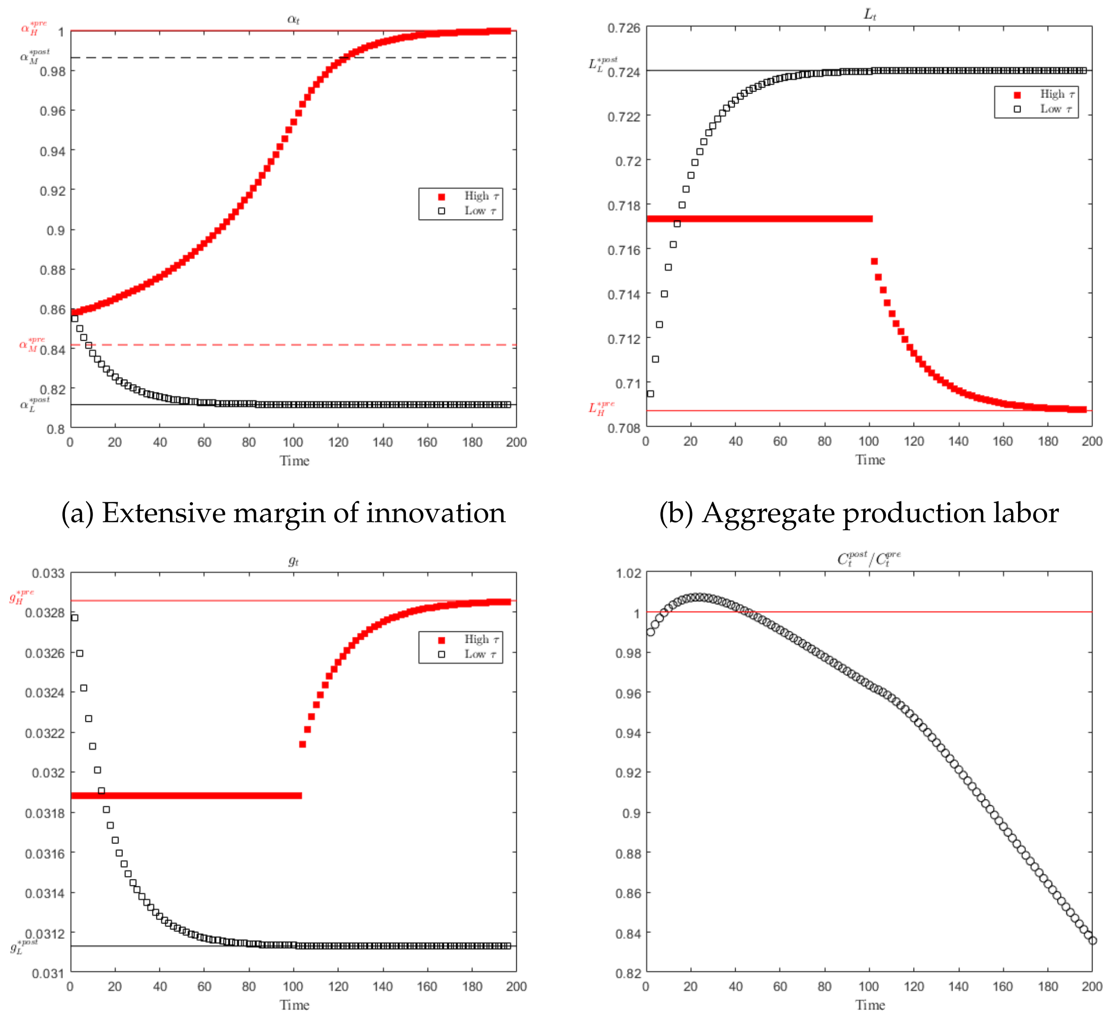
We also consider a policy experiment where τ is reduced marginally from 0.051 to 0.050. Consider an economy with an initial condition $\alpha_0 = 0.8576$. We plot the simulated saddle path to their respective steady state under the low-τ and the high-τ environment in Figure 5.3. The rest of the model parameters remain unchanged.

Before the policy that reduces τ, the initial extensive margin is above its M steady state level, α_M^{*pre}, which means that the economy is on the path to the H steady state. A reduction in τ increases the M steady state level of the extensive margin of innovation to a level above α_0. As a consequence, the economy is now set on the saddle path to the L steady state, with the supply of skilled labor becoming immediately binding. As a result, the extensive margin of innovation declines and so do the aggregate innovation and growth rate (Panel (a)-(c)).

Panel (d) of Figure 5.3 compares the aggregate consumption in the two equilibrium paths. The initial dip below 1 of the ratio is caused, as is in the previous experiment, by the initial

[8] Recall that around the M steady state, the fixed skill supply is not binding and hence $\omega_t = 1$. From the second equation of (21), it is implied that followers around the M steady state innovate at a constant intensity, $\lambda_t^f = \frac{\rho+\tau}{\chi}$.

contraction in L_0 corresponding to the increase in R&D activity. However, as before, after a temporary increase in consumption growth relative to the high-τ economy, the force of declining extensive margin dominates and consequently the consumption trajectory declines permanently relative to the high-τ economy.



(a) Extensive margin of innovation

(b) Aggregate production labor

(c) Rate of technological growth

(d) Ratio of aggregate consumption

Figure 5.3: Policy Experiment: Decreasing τ

Note: This figure illustrates how the saddle path of the economy can change upon a change in the policy variable τ. The red lines in Panel (a)-(c) depict the evolution of the extensive margin of innovation α_t, the production labor L_t and the growth rate g_t on the saddle path to a H steady state. Upon a reduction in the destruction rate τ, the economy however lands on a saddle path converging to the L steady state, as shown by the black lines in those panels. Panel (d) shows the ratio of consumption, period by period, before to after the increase in m. For a discussion, see Section 5.2.

5.3 Pareto Optimality

Equilibria featuring higher innovation may or may not be desirable from a welfare perspective. The optimal innovation rate depends in fact on the relation between the social benefit of a successful innovation, represented by γ, the cost of achieving a successful innovation, proxied by $1/\chi$, and the rate of time preferences, ρ.

The optimal innovation rate can be calculated from a planner' problem. The social planner splits the fixed supply of labor between production and R&D activities across industries to maximize the lifetime utility of the representative consumer. The problem is

$$\begin{aligned}
&\max_{l_{it},\Lambda_{it}} \int_0^\infty e^{-\rho t} \log C_t dt \\
\text{s.t.} \quad &\log C_t = \int_0^1 \log\left(q_{it} d_{it}\right) di \\
&d_{it} = l_{it} \\
&q_{it} = \gamma^{s_{it}} \\
&\dot{s}_{it} = \chi \Lambda_{it} \\
&\int_0^1 \left(l_{it} + \Lambda_{it}\right) di \leq 1 \\
&\Lambda_{it} \leq \overline{\Lambda}.
\end{aligned}$$

where as usual Λ_{it} represents the overall innovation effort in industry i and the last two constraints are the labor resource constraint and the fixed skill constraint.

Proposition 3. *The solution to the planner's problem is a unique steady state with consumption growth rate $g^{SP} = \chi \log(\gamma) - \rho$. Furthermore:*
(1) The decentralised H steady state features a suboptimally low growth rate $g_H^* < g^{SP}$ if and only if

$$m < \log(\gamma)\left(\frac{\chi}{\rho} + 1\right)$$

(2) The decentralised L steady state features a suboptimally high growth rate $g_L^* > g^{SP}$ if and only if

$$\tau > 1 - \frac{\rho}{\chi\overline{\Lambda}}\left(\frac{\chi\overline{\Lambda} - \rho}{\chi\log(\gamma)} + 1\right)$$

Proof. See Appendix A.5. □

In the H steady state of the decentralised economy only followers innovate and the steady state behaves similarly to the decentralised steady state in GH, where in addition $m = \gamma$. Under this condition, Proposition 3 implies that if $\gamma < \log(\gamma)\left(\frac{\chi}{\rho}+1\right)$ then the GH version of the H steady state features underinvestment in innovation. On the contrary, the when $\gamma > \log(\gamma)\left(\frac{\chi}{\rho}+1\right)$ the equilibrium features overinvesment. By tracing out the shape of these inequalities as a function of γ, one can see that, depending on the other model parameters, there can exists a range of values $[\gamma_1, \gamma_2]$ where the GH equilibrium features underinvestment for γ in that range, and overinvestment for γ outside of that range. Intuitively, if the positive knowledge externality from innovation (which increases with the logarithm of γ) outweighs the negative externality from business stealing effects (which increases linearly with the profit mark-up $m = \gamma$), then there is too little growth in the decentralised economy.

Turning again to the more general formulation of our model, the leader's mark-up can be taken to be a regulated value m strictly lower than γ. Based on the discussion above, this means that for a given $m < \gamma$ the negative welfare consequence of the business stealing effects are smaller than in the GH case with $m = \gamma$. Therefore the interval of values for γ where the H equilibrium features underinvestment shrinks relative to the GH case.[9]

In the event that the H equilibrium features overinvestment, then Proposition 3 states that the L steady state is characterized by overinvestment if and only if τ is sufficiently large. This is a straightforward consequence of the fact that the growth rate g_L^* in the the L steady state is strictly increasing in τ (Appendix 1), i.e. g_L^* is higher when the patent duration is shorter. On the other hand, when τ approaches zero then g_L^* also approaches zero, which is surely lower than the social optimum.

6 Extensions and Robustness

In this section, we consider two extensions of our stylized baseline model. In Section 6.1, we consider more general utility functions that belong to the constant intertemporal elasticity of substitution class. We discuss how the intertemporal elasticity of substitution affects our results. In Section 6.2, we relax the assumption of the linear cost of innovation and replace it with a quadratic cost of innovation. Since innovation costs are now strictly

[9] Yet, in all the numerical example and policy experiments in the previous sections the H steady state features underinvestment in innovation relative to the social optimum.

convex, we no longer need to impose a maximum supply of skilled labor at the industry level. We show that the structure of the steady states in the model with the quadratic cost resembles that in the baseline model, therefore making sure the linearity of the baseline model does not drive our key results in any way.

6.1 Relaxing Log Utility

In the baseline model, we assumed that households have a log period utility function, which amounts to assuming unit intertemporal elasticity of substitution. In this section, we relax this assumption by adopting a more general class of utility functions for households:

$$\int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt,$$

where $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution. All other elements of the model remain the same as in the baseline model. The consumption Euler equation becomes:

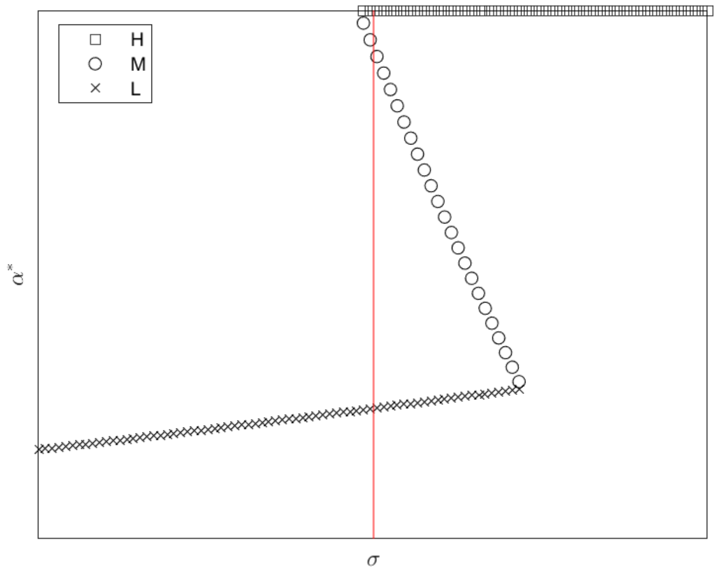
$$r_t = \rho + \sigma \frac{\dot{E}_t}{E_t} + (\sigma - 1) \log(\gamma) \dot{S}_t. \tag{26}$$

In a steady state, the relationship between the interest rate and the rate of technological growth is now

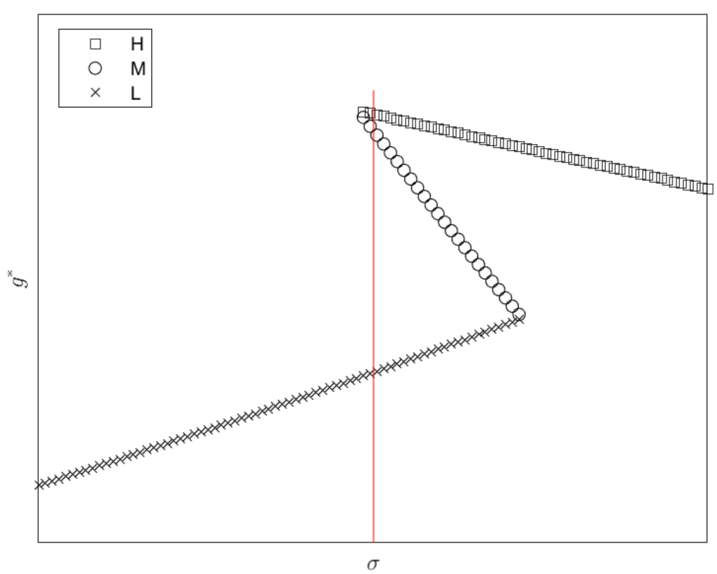
$$r^* = \rho + (\sigma - 1) g^*.$$

With a unit elasticity $\sigma = 1$ the steady state interest rate is equal to the rate of time preference ρ, as in the baseline model.

Under the more general utility function, the steady state interest rate depends positively (negatively) on growth when σ is larger (smaller) than unity. This implies that when $\sigma > 1$, in a steady state with high technological growth and innovation, the interest rate will also be high, which would tend to self-confirm the high-growth situation by discouraging innovation by leaders. As discussed in Section 3.2, this is one of the sources of the multiplicity of steady states in our model. On the other hand, when $\sigma < 1$, high aggregate growth and innovation would lead to low interest rates, which would encourage leaders to innovate and would thus decrease the share industries in the contestable state.



(a) Steady State Extensive Margin of Innovation



(b) Steady State Growth Rate

Figure 6.1: The Model with the Constant Intertemporal Elasticity of Substitution Preference

Note: This figure illustrates the structure of the steady states of the model extended to have constant intertemporal elasticity of substitution preference. In particular, it shows how the steady state extensive margin of innovation α^* and the steady state growth rate g^* vary as we vary the elasticity of substitution parameter σ. For a discussion, see Section 6.1.

This by itself is a force that, in the long-run, would tend to push the economy toward a low-growth situation, playing against the self-confirmation of a high-growth steady state. This reasoning show that if $\sigma < 1$ is small enough, then the economy may not display in fact a multiplicity of steady states.

We can characterize analytically the structure of the steady states for a range of σ.

Proposition 4. *There exist two constants $\underline{\sigma}_H < 1 < \overline{\sigma}_L$ such that for $\sigma \in (\underline{\sigma}_H, \overline{\sigma}_L)$ the economy has three steady states, H, M, and L. The H and L steady states are saddle path stable, while the M steady state is unstable. Moreover, for $1 \leq \sigma < \overline{\sigma}_L$, the three steady states are also ranked by their aggregate growth rates, $g_H^* > g_M^* > g_L^*$.*

Proof. See Appendix C.

Fix an $m \in \left(\underline{M}, \overline{M}\right)$ and all other parameters as in baseline model. There exists a range of σ, $(\underline{\sigma}_H, \overline{\sigma}_L)$ and $\underline{\sigma}_H < 1 < \overline{\sigma}_L$, in which there are three steady states. At $\underline{\sigma}_H$, the H and M steady states coincide where leaders become indifferent between innovating and not innovating. At $\overline{\sigma}_L$, the M and L steady states coincide where the constraints on $\bar{\Lambda}$ becomes just binding. Figure 6.1 illustrates the three steady states, their extensive margins of innovation and growth rates, as we vary σ. This figure is based on simulations of the model, keeping all parameters as in Table 4.1 and varying σ around unity. The red line denotes the steady states corresponding to a model with $\sigma = 1$ (i.e. the baseline model).

The figure shows that for low values of σ the only steady state that exists is the L steady state. For moderate values of σ around one, multiple steady states arise. However, when σ becomes too big, only H steady state survives. Beyond that point in fact the steady state interest rate is too high to warrant innovation by leaders.

6.2 Relaxing Linear Cost of R&D

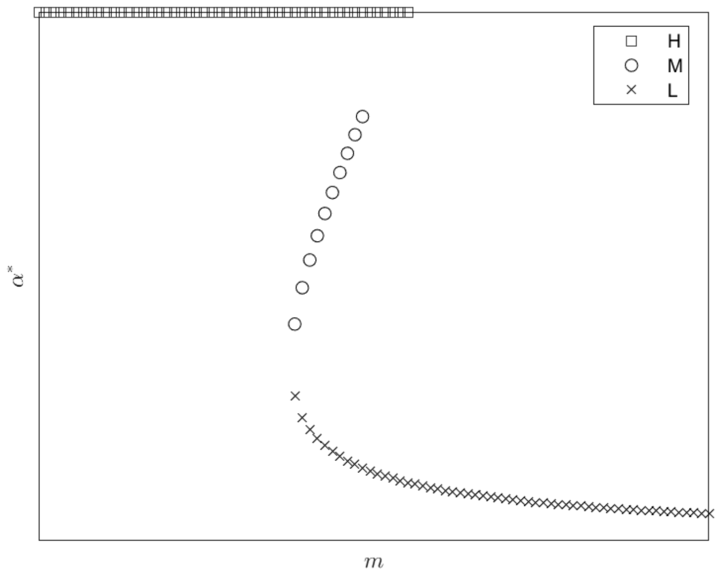
Another potential concern is whether the multiplicity of steady states, from which we derive subtle policy implications, could be driven by the linearity of the model. To address this issue, we modify the model to introduce a quadratic cost of innovation to both leaders and followers. Suppose the cost of innovation is the following:

$$\phi_j \lambda + \frac{1}{2} \theta_j \lambda^2, \quad j = 1, 2,$$

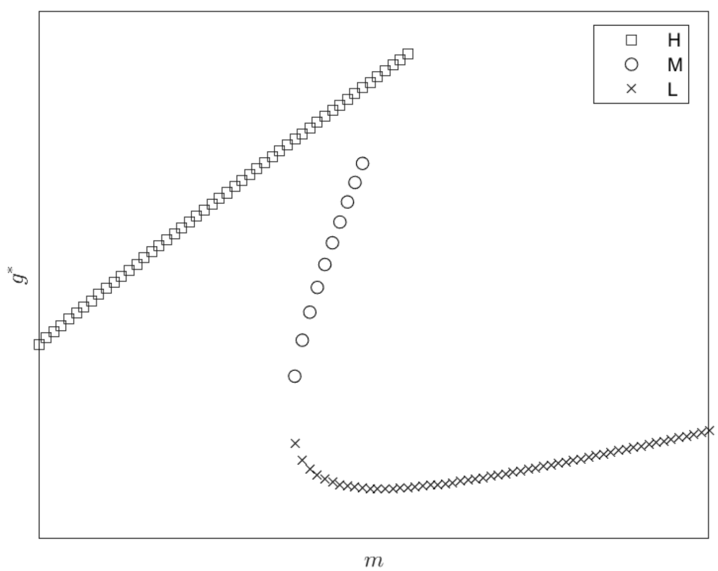
where $j = 1$ is for leaders and $j = 2$ is for followers and $\phi_j, \xi_j > 0$ are parameters of the model. Since this effectively imposes decreasing return on innovation at the firm level, we then abandon the assumption of maximum supply $\bar{\Lambda}$ of skilled labor. We solve and simulate this modified model and examine if the baseline key properties of the steady states survive these modifications.[10]

In Figure 6.3, we plot the steady state values of the extensive margin of innovation, α^*, and the growth rate, g^*, against different values of the markup m from the modified model. Comparing this figure to Figure 3.1 from the baseline model, we confirm that the structure of the steady states under the quadratic cost of innovation remains similar to that in the linear model.

[10]The mathematical derivations of the steady states in the model with quadratic costs are found in Appendix D.



(a) Steady State Extensive Margin of Innovation



(b) Steady State Growth Rate

Figure 6.3: The Model with Quadratic Costs of Innovation

Note: This figure illustrates the structure of the steady states of the model extended to have quadratic cost of innovation. In particular, it shows how the steady state extensive margin of innovation α^* and the steady state growth rate g^* vary as we vary the markup parameter m. For a discussion, see Section 6.2.

7 Conclusion

Traditional "new growth theory" models of endogenous growth deliver the result that higher monopoly power – higher markup or longer patent protection – leads to higher aggregate growth. In this paper, we show that this conclusion rests crucially on the assumption of complete and instantaneous knowledge spillover to followers. We believe this assumption to be dubious, and thus we study the case where the cost for followers to leap-frog the industry's leader increases in the leader's technological advantage.

We find that under our more general setting, the equilibrium properties of the economy change dramatically. First of all, instead of being characterized by just one steady state where only followers innovate (this is the High growth steady state of the traditional Schumpeterian models), the economy may now features two additional steady states where also leaders innovate, one with Medium and one with Low growth. The High and the Low growth steady states are both saddle path stable. The Low growth steady state is characterized by high but infrequent innovation effort by industry leaders, whose "endpoint strategy" is to acquire new patents in order to distance themselves from the followers, thus increasing the followers' innovation costs and pushing them out of the innovation race. Second, we find that when leaders are granted large monopolistic rents or long-lasting patent protection, then the economy features once again a unique steady state, but it's the Low growth steady state instead of the traditional High growth one. Allowing leaders to take advantage of excessively high markups and long patent protection is harmful to growth, as these conditions provide leaders with incentives to enact strategies aimed at stifling firms entry into their industry.

Our theoretical findings indicate that standard results of the "new growth theory" literature are not robust to the relaxation of the unrealistic assumption of complete and instantaneous knowledge spillover. Our results also provide a potential framework to interpret the recent empirical trends of increasing markups, reduced investment, and lower business dynamism.

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Appendix

A Technical Details of the Baseline Model

A.1 The Supply Function of Skilled Labor

To obtain an equilibrium when the supply function for specialized labor is perfectly elastic up to $\bar{\Lambda}$, and perfectly inelastic afterwards, we proceed in two steps. First, we postulate the existence of an exogenous supply function for the specialized labor given by

$$\omega = 1 + \theta\psi(\Lambda) \tag{A-1}$$

where $\theta > 0$ and $\psi(\cdot)$ is a C^1 function such that

$$\begin{array}{ll} \psi(\Lambda) = 0, & \text{for } \Lambda \leq \bar{\Lambda} \\ \psi', \psi'' > 0, & \text{for } \Lambda > \bar{\Lambda} \\ \psi(\Lambda) \to +\infty, & \text{as } \Lambda \to +\infty. \end{array} \tag{A-2}$$

Second, we take the limit of the resulting equilibrium as $\theta \to +\infty$. For the purpose of this paper, we take $\bar{\Lambda}$ to be an arbitrarily large constant.

A.2 The Proof of Proposition 1

The H steady state and its existence are established in the main text of the paper. Here we focus on the M and L steady states.

From the first-order conditions of the innovating leaders and followers, we have $2\omega_t/\chi = 2V_1^l(t) = V_2^l(t)$. Therefore, $2\dot{V}_1^l = 2\frac{\dot{\omega}_t}{\chi} = \dot{V}_2^l$. Substituting these conditions for the value functions and their derivatives into the equations defining V_1^l and V_2^l, we obtain two equations. The first is

$$\Pi_t = (2\lambda_t^f - \tau/\chi)\omega_t, \tag{A-3}$$

and the second equation is

$$\dot{\omega}_t = (r_t + \tau - \chi\lambda_t^f)\omega_t.$$

They comprise the system of equations of (21) in the paper.

From the first equation of (21), we have

$$\frac{\dot{\Pi}}{\Pi} = \frac{\dot{L}}{L} = \frac{\dot{Y}}{Y} = \frac{\dot{E}}{E} = r - \rho = \frac{2\chi\dot{\lambda}^f}{2\chi\lambda^f - \tau} + \frac{\dot{\omega}}{\omega} = \frac{2\chi\dot{\lambda}^f}{2\chi\lambda^f - \tau} + r + \tau - \chi\lambda^f$$
$$\Rightarrow \quad 2\dot{\lambda}^f = (2\lambda^f - \tau/\chi)\left(\lambda^f - \frac{\rho+\tau}{\chi}\right)\chi. \tag{A-4}$$

From the expression for Π, $2\lambda^f - \tau/\chi > 0$. Then in the steady state, $\lambda^{f*} = \frac{\rho+\tau}{\chi}$. Since $\Pi^* = (m-1)(1-\alpha^*\Lambda^*)$,

$$(m-1)(1-\alpha^*\Lambda^*) = \frac{2\rho+\tau}{\chi}\omega(\Lambda^*)$$
$$\Rightarrow \quad \alpha^* = \frac{1}{\Lambda^*}\left(1 - \frac{2\rho+\tau}{\chi(m-1)}\omega(\Lambda^*)\right) \equiv \nu_1(\Lambda^*).$$

From $\dot{\alpha} = (1-\alpha)\tau - \alpha\chi\lambda^l$, we have

$$0 = (1-\alpha^*)\tau - \alpha^*\chi\lambda^{l*} = (1-\alpha^*)\tau - \alpha^*\chi(\Lambda^* - \lambda^{f*}) = (1-\alpha^*)\tau - \alpha^*\chi(\Lambda^* - \frac{\rho+\tau}{\chi})$$
$$\Rightarrow \quad \alpha^* = \frac{\tau}{\Lambda^*\chi - \rho} \equiv \nu_2(\Lambda^*).$$

The system of equations

$$\begin{cases} \alpha^* = \frac{1}{\Lambda^*}\left(1 - \frac{2\rho+\tau}{\chi(m-1)}\omega(\Lambda^*)\right) \equiv \nu_1(\Lambda^*) \\ \alpha^* = \frac{\tau}{\Lambda^*\chi-\rho} \equiv \nu_2(\Lambda^*) \end{cases}, \tag{A-5}$$

when having two meaningful solutions, define the M and L steady states.

In the limit economy, let $\theta \to +\infty$. Then,

$$\nu_1(\Lambda^*) = \begin{cases} \frac{1}{\Lambda^*}\left(1 - \frac{2\rho+\tau}{\chi(m-1)}\right) & \textit{if } \Lambda^* < \overline{\Lambda} \\ -\infty & \textit{if otherwise} \end{cases}.$$

In Figure A-1, we plot ν_1 (for both a finite θ and for the limit when $\theta \to \infty$) and ν_2. One can show that for finite θ, ν_1 and ν_2 have at most two crossings, because $\frac{\nu_1'}{\nu_2'}$ increases in Λ.[11]. As $\theta \to \infty$, the lower crossing occurring at the binding skilled labor constraint, defining the L steady state, $\Lambda_L^* = \overline{\Lambda}$ and the higher crossing defines the M steady state, where $\omega_M^* = 1$.

Let θ go to infinity. Varying m shifts $\nu_1(\cdot)$ up and down. Let $\underline{M}$ be the m such that $\nu_1(\cdot)$ and $\nu_2(\cdot)$ have only one intersection at $\overline{\Lambda}$. This implies that if m is lower than $\underline{M}$, then the M and L steady states disappear. Let $\overline{M}$ be the m such that there are two intersections of $\nu_1(\cdot)$ and $\nu_2(\cdot)$, with the higher one corresponding to $\alpha^* = 1$ and the lower one corresponding to $\overline{\Lambda}$. This implies that if $m = \overline{M}$, then $\alpha_H^* = \alpha_M^* = 1$. If $m > \overline{M}$, then the H and M steady states disappear. We can show that

$$\underline{M} \to 1 + \frac{(\chi\overline{\Lambda} - \rho)(2\rho + \tau)}{\chi\left(\overline{\Lambda}(\chi - \tau) - \rho\right)}.$$

$$\overline{M} = \begin{cases} \frac{\chi+\rho}{\chi-\tau-\rho} & \text{if } \chi - \tau - \rho > 0 \\ +\infty & \text{if otherwise.} \end{cases}$$

In sum, when $m < \underline{M}$, there is only one H steady state.

[11] We have

$$\frac{\nu_1'}{\nu_2'} = \frac{\frac{2\rho+\tau}{\chi(m-1)}\theta\psi'\Lambda + \nu_1\Lambda}{\tau}\left(\chi - \frac{\rho}{\Lambda}\right)^2.$$

It can be shown that the first term's derivative with respect to Λ is $\frac{2\rho+\tau}{\chi(m-1)}\Lambda\psi'' > 0$. The second term is clearly increasing in Λ. Therefore, overall the ratio ν_1'/ν_2' is increasing in Λ.

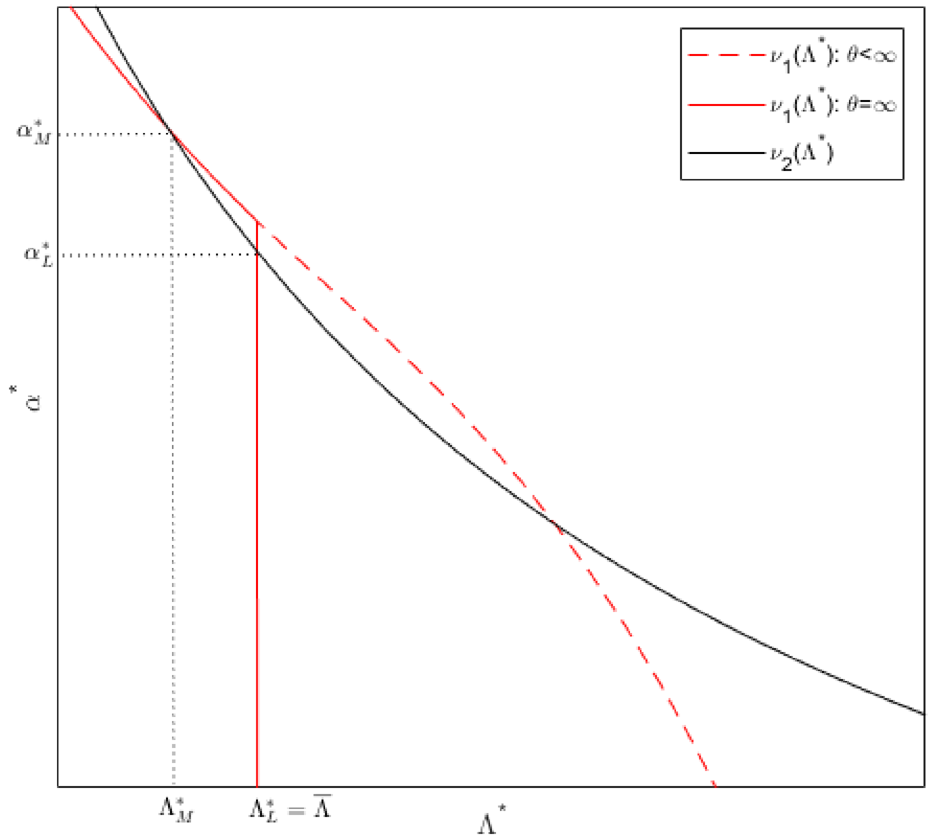

Figure A-1: The M and L Steady States

Note: This figure shows how the M and L steady states are determined. For details of sample selection, see Appendix A.2.

When $\underline{M} \leq m \leq \overline{M}$, there are three steady states, H, M, and L.

$$\begin{aligned} \alpha^*_H &= 1; \\ \alpha^*_M &\rightarrow \frac{\chi - \tau - \frac{2\rho+\tau}{m-1}}{\rho}; \\ \alpha^*_L &\rightarrow \frac{\tau}{\chi\overline{\Lambda} - \rho}. \end{aligned} \tag{A-6}$$

In the M steady state, $\Lambda^*_M < \overline{\Lambda}$ and $\omega^*_M = 1$. In the L steady state, $\Lambda^*_L = \overline{\Lambda}$ and $\omega^*_L > 1$.

When $m > \overline{M}$, there is only one L steady state.

The stability properties of the M and the L steady state are easily established. In the jargon of economy theory, a steady state is locally stable if, given an initial condition (in our case, an initial value for α_0) in the neighborhood of the steady state, there exists an equilibrium path converging to the steady state as $t \to \infty$. We know that in a neighborhood of either the M or the L steady state, the differential equation (A-4) must hold. In order to have λ_t^f converge to its steady state value $\lambda^{f*} = \frac{\rho+\tau}{\chi}$, we must necessarily have $\lambda_t^f = \frac{\rho+\tau}{\chi}$ for all t.[12] Substituting $\lambda_t^f = \frac{\rho+\tau}{\chi}$ into (A-3) and combine with $\Pi_t = (m-1)(1-\alpha_t\Lambda_t$, we have

$$\begin{aligned}\alpha_t &= \frac{1}{\Lambda_t}\left(1 - \frac{1\rho+\tau}{\chi(m-1)}\omega(\Lambda)\right) \\ &= \nu_1(\Lambda_t).\end{aligned}$$

In other words, starting from some α_0, a hypothetical convergence path coincides with the curve ν_1 in Figure A-1.

Recall the curve ν_2 in Figure A-1 describes the combination of α and Λ such that $\dot{\alpha} = 0$. This implies that, starting from any α_t below the ν_2 curve, we have $\dot{\alpha}_t < 0$ while starting from an α_t above the curve we have $\dot{\alpha}_t > 0$.

For any $\alpha \in (\alpha_L^*, \alpha_M^*)$, we have $\nu_1 > \nu_2$. It follows that, on the hypothetical converging trajectory, we must have $\dot{\alpha}_t < 0$. Hence, starting from any $\alpha_0 \in (\alpha_L^*, \alpha_M^*)$, there exists a unique initial value Λ_0 on ν_1 such that the equilibrium pair (α_t, Λ_t) travels southeast along the curve ν_1 and converges to the L steady state as $t \to \infty$. Similarly, pick any $\alpha_0 < \alpha_L^*$, we have $\dot{\alpha}_t > 0$ along the trajectory, which implies that there exists a unique equilibrium pair (α_t, Λ_t) which travels northwest along ν_1 and converges to the L steady state. Finally, for any $\alpha_0 > \alpha_M^*$ we have $\nu_1 < \nu_2$. Therefore, any path starting and lying on ν_1 is characterized by $\dot{\alpha}_t > 0$ for all t, which shows that there is no initial condition α_0 in the neighborhood of α_M^* for which we can find an equilibrium path converging to the M steady state. We then say that M is a source and L is a saddle.

Clearly, if $m > \bar{M}$ then the only steady state is L, and for any initial condition $\alpha_0 \in (0, 1]$ the only equilibrium is the one associated with the unique path converging to the L steady

[12]Recall that necessarily $2\lambda^f - \tau/\chi > 0$. If λ_t^f increases to the steady state value, then $\lambda_t^f > \frac{\rho+\tau}{\chi}$ and λ_t^f will increase without bound. If λ_t^f decreases to the steady state value, then $\lambda_t^f < \frac{\rho+\tau}{\chi}$ and λ_t^f will decrease to zero. Either is a contradiction.

state.

A.3 The Proof of Corollary 1

Firstly, note that aggregate R&D labor in the M and L can be expressed by

$$\alpha_i^* \Lambda_i^* = \frac{\tau \Lambda_i^*}{\tau + \chi(\Lambda_i^* - \lambda_i^{f*})}, \text{ for } i = M, L.$$

Since $\lambda_M^{f*} = \lambda_L^{f*}$ and $\Lambda_M^* < \Lambda_L^* = \overline{\Lambda}$, we conclude that $\alpha_M^* \Lambda_M^* < \alpha_L^* \Lambda_L^*$ under the assumption that $\tau < \chi - \rho < \chi$. Since $g_i^* = \log(\gamma)\alpha_i^* \Lambda_i^*$, we have $g_M^* > g_L^*$.

Secondly, comparing the aggregate production labor in the H and M steady states, (18) and (23), we find $L_H^* < L_M^*$ if and only if $m < \overline{M}$. Since $g_i^* = \log(\gamma)\chi(1 - L_i^*)$, we have $g_H^* > g_M^*$ when both exist.

Finally, we can easily solve out the growth rate in the L steady state: $g_L^* = \log(\gamma)\chi\frac{\tau\chi\overline{\Lambda}}{\chi\overline{\Lambda}-\rho}$, which is independent of m. Therefore, g_L^* is smaller than any growth rates in the M and H steady states for any $m \in \left(\underline{\mathrm{M}}, \overline{M}\right)$.

A.4 The Proof of Proposition 2

Define "Region I" the system of differential equations (15)-(16) where only followers innovate and "Region II" the system (A-3)-(A-4) where both leaders and followers innovate.

Assume that the initial conditions of the system do not coincide with either the steady states or the saddle paths of the two Regions. Then, starting from such initial conditions, a candidate equilibrium must necessary switch Region at least once, possibly featuring equilibrium cycles where the economy switches indefinitely between Regions (it is straightforward to show that within-Region cycles do not exist).

Indicate with T a time when the economy switches Region and consider a candidate equilibrium characterized by an infinite sequence $T_1, T_2, \ldots$ of switches between Regions. For brevity, in the remainder of the proof we assume that $\bar{\Lambda}$ is arbitrarily large, so that $\alpha(0) > \alpha_L^*$ and $\lambda^f(T) < \bar{\Lambda}$, implying $\omega(T) = 1$ at any T. Also, with a slight abuse of notation, we will indicate with $x(T)$ or $\dot{x}(T)$, respectively, the limit from the right of the function $x(T)$ or of its time derivative.

Without loss of generality we assume that at $T = T_1$ the economy switches from Region I to Region II. Because of the continuity assumption, we must have $\lambda^l(T) = 0$. There are now two possibilities that need to be considered separately.

The first is that $\lambda^f(T_1) \geq \lambda_M^{f*} = \frac{\rho+\tau}{\chi}$. In this case (A-4) implies that $\dot{\lambda}^f(T_1) \geq 0$. Also, since $\Pi = (m-1)L$, equation (A-3) implies $\dot{L}(T_1) \geq 0$. Moreover, since $\lambda^l(T_1) = 0$, then $\dot{\alpha}(T_1) > 0$. Given that $\lambda_t^l = \frac{1-L_t}{\alpha_t} - \lambda_t^f$ for any t, it follows that $\dot{\lambda}^l(T_1) < 0$. But this is not possible, since it would imply that $\lambda_t^l < 0$ at some time $t > T_1$.

For the rest of the proof we will then focus on the second possibility, that is $\lambda^f(T_1) < \lambda_M^{f*}$. Since $\Pi_t = (m-1)L_t$, then using equation (A-3) we have

$$\frac{\dot{L}}{L} = \frac{\dot{\Pi}}{\Pi} = \frac{2\dot{\lambda}^f}{2\lambda^f - \frac{\tau}{\chi}}$$

Substituting (A-4) gives

$$\frac{\dot{L}}{L} = \chi\lambda^f - \rho - \tau$$

Recalling that $\lambda^f(T_1) < \lambda_M^{f*}$, we conclude that $\dot{L} < 0$ and $\dot{\lambda}^f < 0$ while the system is in Region II after the switch at T_1.

Then, at the time $T_2 > T_1$ when the economy switches back to Region I, we have $\lambda^f(T_2) < \lambda^f(T_1)$, $L(T_2) < L(T_1)$, $\lambda^l(T_2) = \lambda^l(T_1) = 0$ and thus $\alpha(T_2) = \frac{1-L(T_2)}{\lambda^f(T_2)} > \alpha(T_1)$. Since α strictly increases while in Region I, we conclude that as the economy switches back and forth between the two regions it generates a strictly increasing sequence $\{\alpha(T_n)\}$. This sequence can only converge to α_H^* given that $\alpha(0) > \alpha_L^*$ and that in Region I $\dot{\alpha} > 0$ as long as $\alpha < \alpha_H^*$.

We therefore conclude that there is no equilibrium that features a cycle where the economy switches between Region I and Region II. Any equilibrium, if it exists, that at some time T_1 switches from Region I to Region II at some T_1 must converge in the limit to the H steady state.

A.5 The Solution to Planner's Problem

Using symmetry properties, we can drop the index i and write the social planner's problem as

$$\begin{aligned}
&\max_{\Lambda_t, L_t} \int_0^\infty e^{-\rho t} \left(S_t \log \gamma + \log L_t\right) \\
\text{s.t.} \quad &\dot{S}_t = \chi \Lambda_t \\
&\Lambda_t \leq \overline{\Lambda} \\
&\Lambda_t + L_t \leq 1.
\end{aligned}$$

Assume that the constraint $\bar{\Lambda}$ does not bind in equilibrium. Then the current value Hamiltonian is $H(S_t, \Lambda_t, \mu_t) \equiv S_t \log \gamma + \log(1 - \Lambda_t) + \mu_t \chi \Lambda_t$, where μ_t is the co-state variable. The optimality conditions are

$$\begin{aligned}
\frac{\partial H(S_t, \Lambda_t, \mu_t)}{\partial \Lambda} &= \frac{-1}{1 - \Lambda_t} + \mu_t \chi = 0 \\
\frac{\partial H(S_t, \Lambda_t, \mu_t)}{\partial S} &= \log \gamma = -\dot{\mu}_t + \rho \mu_t.
\end{aligned}$$

Differentiating the first equality with respect to time and combining it with the second equality yields

$$\frac{\dot{L}}{L} + \rho = \chi \log(\gamma) L.$$

In the steady state, $L^* = \frac{\rho}{\chi \log(\gamma)}$ and $\Lambda^* = 1 - \frac{\rho}{\chi \log(\gamma)}$. The consumption growth rate is $g^{SP} = \log(\gamma)\chi\Lambda^* = \chi \log(\gamma) - \rho$. We focus on the steady state comparisons, because in the social planner's problem, the steady state is a source. The equilibrium path of L_t in fact indicates that if $L_t > L^*$, then L_t will increase without bound and if $L_t < L^*$, then L_t will decrease without bound. In either case, L_t will eventually violate the boundary conditions that $0 \leq L_t \leq 1$.

The consumption growth rate in the H steady state in the decentralized economy is $g_H^* =$

$\log(\gamma)\frac{(m-1)\chi-\rho}{m}$. This means that $g^{SP} > g_H^*$ if and only if

$$m < \log(\gamma)\left(\frac{\chi}{\rho}+1\right).$$

The consumption growth rate in the L steady state in the decentralized economy is $g_L^* = \log(\gamma)\chi\frac{\tau\chi\overline{\Lambda}}{\chi\overline{\Lambda}-\rho}$. This means that $g^{SP} < g_L^*$ if and only if

$$\chi\log(\gamma)-\rho < \log(\gamma)\chi\frac{\tau\chi\overline{\Lambda}}{\chi\overline{\Lambda}-\rho} \Leftrightarrow \chi\log(\gamma)\left(\frac{(1-\tau)\chi\overline{\Lambda}-\rho}{\chi\overline{\Lambda}-\rho}\right) < \rho.$$

When $0 < \tau < 1$, which is the relevant case, the condition becomes:

$$\tau > 1-\frac{\rho}{\chi\overline{\Lambda}}\left(\frac{\chi\overline{\Lambda}-\rho}{\chi\log(\gamma)}+1\right).$$

These results are summarised in Proposition 3.

B Dynamic Race with Endogenous Steps

Assume that the innovation technologies used by followers are linear and that innovation costs for followers are increasing in the follower's lag from the leader.[13] We will show that, under these assumptions, the model with an exogenous maximum distance of two steps is in fact the equilibrium result of a model where the maximum distance is endogenous. This conclusion holds for both the "step-by-step catch-up" and the "fast catch-up" versions of the model.

To prove the claim, we need to consider only the problem of the follower, taking as given the value functions V_s^l of a leader s steps ahead, for $s = 1, 2, \ldots$. Also, to simplify the exposition, we can focus only on steady states. In the "step-by-step catch-up" we assume that a follower with lag $s > 1$ must first spend resources to close the gap to $s = 1$, and only then can try to leap-frog the leader. Instead, in the "fast catch-up case," followers can jump immediately from any state $s > 1$ to state $s = 1$, but they face innovation costs that are larger the greater the number of steps $s - 1$ that need to be filled.

[13] Note that we make no assumption about the form of the innovation costs of the leader.

Consider first the "fast catch-up" case. The value V_s^f of a follower $s \geq 2$ steps behind is given by the solution to

$$rV_s^f = \max_{\lambda_s^f \geq 0} -\lambda_s^f + (\chi_s \lambda_s^f + \tau_s)(V_1^f - V_s^f) + \lambda_s^l (V_{s+1}^f - V_s^f). \tag{B-1}$$

The value function of a follower $s = 1$ step behind solves the usual problem,

$$rV_1^f = \max_{\lambda_1^f \geq 0} -\lambda_1^f + \lambda_1^f (V_1^l - V_1^f) + \lambda_1^l (V_2^f - V_1^f). \tag{B-2}$$

Innovation costs $1/\chi_s$ are assumed to be increasing in the lag s, while the spillover intensity τ_s is assumed to be a decreasing sequence. We employ the normalizations $\chi_1 = 1$ and $\tau_1 = 0$.

For brevity we can appeal to an intuitive argument that, since innovation costs are increasing in the follower's lag, and spillover's intensities are decreasing, then $V_1^f \geq V_2^f \geq 0$, i.e. the follower is at least as well-off when he is one step behind the leader compared to when he is two steps behind.[14] Now, regardless of whether the condition $\lambda_1^f \geq 0$ is binding in the maximization of (B-2), and assuming that the optimal value of λ_1^f is finite, we have

$$V_1^f = \frac{\lambda_1^l}{r + \lambda_1^l} V_2^f.$$

Since at a steady state $r > 0$ and $\lambda_1^l \geq 0$, the equation above and the inequalities $V_1^f \geq V_2^f \geq 0$ are satisfied if and only if

$$V_1^f = V_2^f = 0.$$

Substituting $V_1^f = V_2^f = 0$ in (B-1) for $s = 2$, the solution to the maximization gives optimal values $\lambda_2^f = 0$ and $V_3^f = 0$. Iterating the procedure for $s = 3, 4, \ldots$ yields

$$V_s^f = \lambda_s^f = 0, \forall s > 1.$$

[14] The result that the value to the follower decreases with the lag is standard in models of races (see for instance Hörner (2004)). For the sake of our demonstration, we can make the (incorrect) assumption that $V_2^f > V_1^f \geq 0$. Then, optimality of (B-1) for $s = 2$ requires that $\lambda_2^f = 0$. Moreover, since $V_2^f \geq 0$ and $\tau_2 > 0$, then $V_3^f - V_2^f > 0$ and thus $V_3^f - V_1^f > 0$. Iterating the argument for $s = 3, 4, \ldots$, we would conclude that $V_{s+1}^f - V_s^f > 0$ and λ_s^f for any $s > 1$. Hence, followers never innovate at stages $s > 1$.

This concludes the proof that, provided that the optimal λ_s^f is finite, followers never innovate when they are more than one step behind the leader.

The proof for the "step-by-step catch-up" case is straightforward and follows a similar logic as the one outline above for the "fast catch-up" case. We know from the baseline model that $V_1^f = 0$. This means that no follower that is more that 1 step behind the leader would be willing to spend R&D resources to retrace, one by one, all the technological steps needed to close the gap to $s = 1$.

C The Model with Constant Intertemporal Elasticity of Substitution Utility

The representative household solves the following problem:

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt \tag{C-1}$$

$$\text{s.t.} \quad \int_0^\infty e^{-R_t} E_t dt \leq W(0), \tag{C-2}$$

where R_t is the compounded interest rate and E_t represents total spending at time t:

$$R_t = \int_0^t r(\tau) d\tau,$$
$$E_t = \int_{[0,1]} p_{it} d_{it} di.$$

The Cobb-Douglas form of the the consumption aggregate implies that the amount spent by the household on good i is the same across all products, giving

$$d_{it} = \frac{E_t}{p_{it}}.$$

Therefore, we can write the consumption aggregate as

$$\log C_t = \int_{[0,1]} \log\left(\frac{q_{it}}{p_{it}} E_t\right) di = \int_{[0,1]} \log\left(\frac{q_{it}}{p_{it}}\right) di + \log E_t \equiv \log Q_t + \log E_t,$$

where Q_t is proportional to the aggregate quality index and $C_t = Q_t E_t$.

We can rewrite the consumer's problem equivalently with a flow budget constraint, $\dot{a}_t = r_t a_t + I_t - E_t$, where a_t is the stock of savings (wealth) at time t and I_t is the total income (labor income and profit from firms) at time t. We set up the current value Hamiltonian, $\mathcal{H}(E_t, a_t, \mu_t) = \frac{(Q_t E_t)^{1-\sigma}}{1-\sigma} + \mu_t(r_t a_t + I_t - E_t)$. The first order conditions, $\frac{\partial \mathcal{H}}{\partial E_t} = 0$ and $\frac{\partial \mathcal{H}}{\partial a_t} = \rho\mu_t - \dot{\mu}_t$, imply

$$
\begin{aligned}
r_t &= \rho + \sigma \frac{\dot{E}_t}{E_t} + (\sigma - 1)\frac{\dot{Q}_t}{Q_t} \\
&= \rho + \sigma \frac{\dot{E}_t}{E_t} + (\sigma - 1)\log(\gamma)\dot{S}_t,
\end{aligned}
$$

which is (26) in the paper. Also note that in the special case of log period utility ($\sigma = 1$), we obtain the familiar $\frac{\dot{E}_t}{E_t} = r_t - \rho$.

Let's maintain all the parametric assumptions made in the baseline model and suppose $m \in \left(\underline{M}, \overline{M}\right)$ so three steady states exist under the baseline assumption $\sigma = 1$. We characterize the structure of the steady states in this environment when σ deviates from 1.

The H steady state. The highest feasible steady state value for α^* is one, since in this case $\lambda^{l*} = 0$. Let's first assume that around the H steady state we have $\lambda^{f*} > 0$. As usual, a steady state with high extensive margin will be associated with a low intensive margin Λ^*. By taking $\bar{\Lambda}$ large enough, we can make sure that in a neighborhood of the steady state $\Lambda_t^* = \lambda_t^{f*} < \bar{\Lambda}$, giving $\omega_t = 1$. Hence, the first order condition for λ^f in a neighborhood of a H steady state implies that

$$
V_1^l(t) = \frac{1}{\chi}.
$$

The condition above implies that, in a neighborhood of the H steady state, $\dot{V}_1^l = 0$. Since $\lambda_t^{l*} = 0$, a straightforward substitution in the definition of V_1^l gives

$$
\frac{r_t}{\chi} + \lambda_t^f = \Pi_t. \tag{C-3}
$$

Combining the above equation with the facts that $\Pi_t = (m-1)L_t$, $\lambda_t^f = (1-L_t)/\alpha_t$ and (26), we obtain

$$\frac{\rho}{\chi} + \frac{\sigma}{\chi}\frac{\dot{L}}{L} + (\sigma - 1)\log(\gamma)(1-L) + \frac{1-L}{\alpha} = (m-1)L. \tag{C-4}$$

Equation (C-4) defines the evolution of the economy around the H steady state, together with the condition

$$\dot{\alpha} = \tau(1-\alpha). \tag{C-5}$$

The H steady state is then characterized by

$$\begin{aligned}
\alpha_H^* &= 1; \\
L_H^* &= \frac{1+\rho/\chi+(\sigma-1)\log(\gamma)}{m+(\sigma-1)\log(\gamma)}; \\
\lambda_H^{f*} &= \frac{m-1-\rho/\chi}{m+(\sigma-1)\log(\gamma)}; \\
g_H^* &= \log(\gamma)\frac{\chi(m-1)-\rho}{m+(\sigma-1)\log(\gamma)}.
\end{aligned}$$

Linearizing the system (15), (16) we can show that the H steady state is a saddle. We maintain the assumption from the baseline model that $m > 1 + \rho/\chi$ to have a non-degenerate H steady state. Moreover, $\lambda_H^{f*} > 0$ and $L_H^* > 0$ jointly requires

$$\sigma > 1 - \frac{1+\frac{\rho}{\chi}}{\log(\gamma)} \equiv \underline{\sigma}_{H1}.$$

For the H steady state to exist, we in addition require $V_2^{l*} < \frac{2}{\chi}$ so that leaders indeed do not have incentive to innovate. This gives us

$$
\begin{aligned}
V_2^{l*} &= \frac{(m-1)L_H^* + \tau V_1^{l*}}{\rho + (\sigma-1)g_H^* + \tau} \\
&= \frac{(m-1)\chi\left[1+\rho/\chi + (\sigma-1)\log(\gamma)\right] + \tau(m + (\sigma-1)\log(\gamma))}{(\rho+\tau)\chi(m+(\sigma-1)\log(\gamma)) + (\sigma-1)\log(\gamma)(\chi(m-1)-\rho)\chi} < \frac{2}{\chi} \\
\Rightarrow \quad \sigma &> 1 - \frac{\left(\frac{\rho+\chi}{\chi-\rho-\tau} - m\right)(\chi-\rho-\tau)}{\log(\gamma)(\chi(m-1)+\tau)} \\
&= 1 - \frac{\left(\overline{M}-m\right)(\chi-\rho-\tau)}{\log(\gamma)(\chi(m-1)+\tau)} \equiv \underline{\sigma}_{H2}.
\end{aligned}
$$

We maintain the assumption from the baseline that $\chi > \rho + \tau$. For the set of parameters under which the H steady state exists in the baseline model (i.e. $m < \overline{M}$), the H steady state exists in this extended model as long as σ is greater than $\underline{\sigma}_{H2}$, which is a number less than 1.

To compare $\overline{\sigma}_{H1}$ and $\overline{\sigma}_{H2}$, we first note that

$$
\begin{aligned}
&\frac{\rho+\chi}{\chi} - \frac{\left(\overline{M}-m\right)(\chi-\rho-\tau)}{\chi(m-1)+\tau} \\
=&\frac{(2\chi-\tau)\chi\left(m-1-\frac{\rho}{\chi}\right)}{\chi\left(\chi(m-1)+\tau\right)},
\end{aligned}
$$

which has the same sign as $m-1-\frac{\rho}{\chi}$. Recall that $m > \underline{M} = 1 + \frac{(2\rho+\tau)(\chi\overline{\Lambda}-\rho)}{\chi((\chi-\tau)\overline{\Lambda}-\rho)}$ and $\underline{M}$ is decreasing in $\overline{\Lambda}$. This implies

$$
m > \underline{M} > \lim_{\overline{\Lambda}\to+\infty} \underline{M} = 1 + \frac{2\rho+\tau}{\chi-\tau} > 1 + \frac{\rho}{\chi}.
$$

Therefore, $m - 1 - \frac{\rho}{\chi} > 0$, $\frac{\rho+\chi}{\chi} > \frac{\left(\overline{M}-m\right)(\chi-\rho-\tau)}{\chi(m-1)+\tau}$, and in turn

$$
\underline{\sigma}_{H1} < \underline{\sigma}_{H2}.
$$

Define $\underline{\sigma}_H = \underline{\sigma}_{H2}$. The H steady state exists as long as $\sigma \geq \underline{\sigma}_H$. At $\underline{\sigma}_H$, leaders are indifferent between innovating or not. For a σ that is infinitesimally smaller than $\underline{\sigma}_H$, leaders will have strictly prefer to innovate.

The M and L steady states In the M and L steady states, both leaders and followers innovate at the contestable state. The first order conditions for λ^l and λ^f give $V_2^l(t) = 2\omega_t/\chi = 2V_1^l(t)$. Substituting these conditions into the value functions and after appropriate calculations we obtain the two equations:

$$\begin{cases} \Pi = (2\lambda^f - \frac{\tau}{\chi})\omega \\ \dot{\omega} = (r + \tau - \chi\lambda^f)\omega. \end{cases} \tag{C-6}$$

From the first equation in (C-6) and $\Pi = (m-1)L$ we can solve out L:

$$L = \frac{2\lambda^f - \tau/\chi}{m-1}\omega,$$

which, together with the labor market clearing condition, implies

$$1 - \frac{2\lambda^f - \tau/\chi}{m-1}\omega = \alpha(\lambda^f + \lambda^l). \tag{C-7}$$

From the second equation in (C-6), in the steady state $r + \tau - \chi\lambda^f = 0$. Combined with the Euler equation derived at the beginning of this section, we have

$$\rho + (\sigma - 1)g + \tau - \chi\lambda^f = 0. \tag{C-8}$$

The difference between an M and a L steady state is that in an M steady state, the skilled labor supply does not bind ($\lambda^l + \lambda^f < \overline{\Lambda}$) and the skill premium is one ($\omega = 1$), whereas in a L steady state, the opposite is true: $\lambda^l + \lambda^f = \overline{\Lambda}$ and $\omega > 1$.

Then we can use four equations to characterize an M steady state

$$\begin{cases} \alpha = \frac{\tau}{\tau+\chi\lambda^l} \\ g = \log(\gamma)\chi\alpha(\lambda^f+\lambda^l) \\ \rho+(\sigma-1)g+\tau-\chi\lambda^f = 0 \\ 1-\frac{2\lambda^f-\tau/\chi}{m-1} = \alpha(\lambda^f+\lambda^l) \end{cases} . \tag{C-9}$$

The first equation becomes from the evolution of the extensive margin α_t. The second equation is the definition of the growth rate. The third equation is (C-8). The last equation is (C-7), where $\omega = 1$ in an M steady state. From these four equations, we can solve for the M steady state endogenous variables: α^*_M, g^*_M, λ^{f*}_M and λ^{l*}_M.

We can use another set of four equations to characterize a L steady state

$$\begin{cases} \alpha = \frac{\tau}{\tau+\chi(\overline{\Lambda}-\lambda^f)} \\ g = \log(\gamma)\chi\alpha\overline{\Lambda} \\ \rho+(\sigma-1)g+\tau-\chi\lambda^f = 0 \\ 1-\frac{2\lambda^f-\tau/\chi}{m-1}\omega = \alpha\overline{\Lambda} \end{cases} . \tag{C-10}$$

From these four equations, we can solve for the L steady state endogenous variables: α^*_L, g^*_L, λ^{f*}_L and ω^*_L.

Let's focus on the M steady state first. Combining the second and fourth equation in (C-9), we have one equation that links g to λ^f:

$$g = \log(\gamma)\chi\frac{m-1-2\lambda^f+\tau/\chi}{m-1}. \tag{C-11}$$

Together with the third equation in (C-9), we can solve out the M steady state explicitly

$$\begin{aligned} \alpha^*_M &= \frac{(\chi-\tau)(m-1)-(2\rho+\tau)-2\tau(\sigma-1)\log(\gamma)}{\rho(m-1)-[\tau-\chi(m-1)](\sigma-1)\log(\gamma)}; \\ L^*_M &= \frac{2\rho+\tau+2\chi(\sigma-1)\log(\gamma)}{\chi(m-1)+2\chi(\sigma-1)\log(\gamma)}; \\ \lambda^{f*}_M &= \frac{(\sigma-1)\log(\gamma)\chi[m-1+\tau/\chi]+(m-1)(\rho+\tau)}{[m-1+2(\sigma-1)\log(\gamma)]\chi}; \\ g^*_M &= \log(\gamma)\frac{\chi(m-1)-2\rho-\tau}{m-1+2(\sigma-1)\log(\gamma)}. \end{aligned}$$

The λ_M^{l*} is implied in the last equation of (C-9). Rearranging terms,

$$\lambda_M^{l*} = \frac{\tau\left((m+1)\lambda_M^{f*} - m + 1 - \tau/\chi\right)}{(m-1)(\chi-\tau) + \tau - 2\chi\lambda_M^{f*}}, \tag{C-12}$$

which is increasing in λ_M^{f*}. All these endogenous variables are well-defined when $\sigma = 1$. Let's differentiate λ_M^{f*} with respect to $\sigma - 1$.

$$\frac{d\lambda_M^{f*}}{d(\sigma-1)} = \frac{\log(\gamma)(m-1)\chi\left[(m-1)\chi - 2\rho - \tau\right]}{[m-1+2(\sigma-1)\log(\gamma)]^2\chi^2}.$$

Recall that $m > \underline{M} = 1 + \frac{(2\rho+\tau)(\chi\overline{\Lambda}-\rho)}{\chi((\chi-\tau)\overline{\Lambda}-\rho)} > 1 + \frac{2\rho+\tau}{\chi}$. Hence,

$$\frac{d\lambda_M^{f*}}{d(\sigma-1)} > 0.$$

This means, as σ decreases below 1, both λ_M^{f*} and λ_M^{l*} will decrease until λ_M^{l*} becomes zero, at which point the M steady state coincides with the H steady state when leaders are indifferent between innovating and not innovating. To see this point, when $\lambda_M^{l*} = 0$, from (C-12), λ_M^{f*} becomes

$$\lambda_M^{f*} = \frac{m-1+\tau/\chi}{m+1}.$$

Evaluate the λ_H^{f*} at $\sigma = \underline{\sigma}_H$:

$$\lambda_H^{f*} = \frac{m-1-\rho/\chi}{m - \frac{(\overline{M}-m)(\chi-\rho-\tau)}{\chi(m-1)+\tau}} = \frac{m-1+\tau/\chi}{m+1} = \lambda_M^{f*}.$$

This also means, As σ rises above 1, both λ_M^{f*} and λ_M^{l*} will increase until the sum hits the fixed supply: $\lambda_M^{f*} + \lambda_M^{l*} = \overline{\Lambda}$. At this point, as we will show below, the M steady state coincides with the L steady state where $\omega = 1$.

Let's focus on the L steady state now. Combining the first two equations in (C-10) and cancelling out α, we have

$$\frac{g}{\log(\gamma)\chi\overline{\Lambda}} = \frac{\tau}{\tau + \chi(\overline{\Lambda} - \lambda^f)}.$$

Combining the above with the third equation in (C-10), we can infer the L steady state λ_L^{f*} from

$$\chi^2 \lambda_L^{f*2} - (\rho + 2\tau + \chi\overline{\Lambda})\chi\lambda_L^{f*} + (\rho + \tau)(\tau + \chi\overline{\Lambda}) + (\sigma - 1)\log(\gamma)\tau\chi\overline{\Lambda} = 0. \qquad \text{(C-13)}$$

Under our assumption of $m \in (\underline{M}, \overline{M})$, we know when $\sigma = 1$ there exists a well-defined L steady state. When $\sigma = 1$, the above quadratic has two roots: $\lambda^f = \frac{\rho+\tau}{\chi}$ and $\lambda^f = \overline{\Lambda} + \frac{\tau}{\chi}$ (omitted because it is greater than $\overline{\Lambda}$). The smaller root is the R&D intensity of the followers in the L steady state in the baseline model, λ_L^{f*}, and we also know in that steady state $\omega_L^* > 1$. As σ increases above 1, the quadratic function shifts up and the smaller root, λ_L^{f*}, increases, which in turn implies that α_L^* increases (see the first equation of (C-10)). Now from the fourth equation in (C-10), we deduce that the steady state ω_L^* must decrease. Therefore, as σ increases, the smaller root to (C-13) defines the L steady state level of λ^f until the implied ω_L^* decreases to 1, at which point the L steady state coincides with the M steady state where the constraint on skilled labor supply becomes just binding.

To see this point, note how the solution to (C-9) when $\lambda^f + \lambda^l = \overline{\Lambda}$ must also solve (C-10) when $\omega = 1$ and vice versa.

Finally, we show that the larger root of this quadratic equation (C-13) can never be a L steady state. Since σ only shifts the quadratic function up and down, the larger root will always be strictly larger than the λ_L^{f*} when L and M steady states coincide as we discuss above. Suppose the larger root, λ_2^f occurs in a L steady state. Then in that steady state, the extensive margin α must be larger than the extensive margin when L and M steady states coincide. This also means, ω in that steady state much be strictly smaller than the skill premium when L and M steady states coincide, which we know is 1. This contradicts the definition of a L steady state.

Let $\overline{\sigma}_L$ be the σ at which the L and M steady states coincide and let $\underline{\sigma}_L$ be the σ when the smaller root of the quadratic equation is $\frac{\tau}{2\chi}$ and $\underline{\sigma}_L < 1$. We have shown that for

$\sigma \in (\underline{\sigma}_L, \overline{\sigma}_L)$, the L steady state exists. λ_L^{f*} is given by the smaller root of equation (C-13) and the other steady state variables can be derived by

$$\begin{aligned} \alpha_L^* &= \frac{\tau}{\tau + \chi(\overline{\Lambda} - \lambda_L^{f*})}; \\ g_L^* &= \alpha_L^* \log(\gamma) \chi \overline{\Lambda}; \\ L_L^* &= 1 - \alpha_L^* \overline{\Lambda}. \end{aligned}$$

We next show that $\underline{\sigma}_L < \underline{\sigma}_H$, such that the L steady state is defined whenever the H steady state is defined and $\sigma < 1$. Substituting λ^f with $\frac{\tau}{2\chi}$ in (C-13), we can rearrange to obtain

$$(1 - \underline{\sigma}_L) \log(\gamma) = \frac{(2\rho + \tau)(2\chi\overline{\Lambda} + \tau)}{4\tau\chi\overline{\Lambda}}.$$

Recall $\underline{\sigma}_H = \underline{\sigma}_{H2}$ and

$$(1 - \underline{\sigma}_H) \log(\gamma) = \frac{\chi + \rho - m(\chi - \rho - \tau)}{\chi(m-1) + \tau}.$$

We can derive the following inequalities

$$\begin{aligned} (1 - \underline{\sigma}_H) \log(\gamma) &< \frac{(2\rho + \tau)\rho(\chi\overline{\Lambda} - \tau - \rho)}{2\chi(\rho + \tau)(\chi\overline{\Lambda} - \rho) - \tau^2\overline{\Lambda}\chi} \\ &= \frac{2\rho + \tau}{\chi\overline{\Lambda}} \frac{\rho(\chi\overline{\Lambda} - \tau - \rho)}{2(\rho + \tau)\left(\chi - \frac{\rho}{\overline{\Lambda}}\right) - \tau^2} \\ &< \frac{2\rho + \tau}{\chi\overline{\Lambda}} \frac{\rho(\chi\overline{\Lambda} - \tau - \rho)}{\tau(2\chi - \tau)}. \end{aligned}$$

The first inequality is obtained by replacing m by $\underline{M}$ since $(1 - \underline{\sigma}_H) \log(\gamma)$ decreases in m and $m > \underline{M}$. The second inequality is obtained by replacing $\left(\chi - \frac{\rho}{\overline{\Lambda}}\right)$ on the denominator by $\frac{\chi\tau}{\rho+\tau}$ since $\overline{\Lambda} > \frac{\rho+\tau}{\chi}$. Now, we have

$$\begin{aligned}
& (1-\underline{\sigma}_H)\log(\gamma) < (1-\underline{\sigma}_L)\log(\gamma) \\
\Leftarrow \quad & \frac{\rho(\chi\overline{\Lambda}-\tau-\rho)}{2\chi-\tau} < \frac{2\chi\overline{\Lambda}+\tau}{4} \\
\Leftrightarrow \quad & 4\rho(\chi\overline{\Lambda}-\tau-\rho) < (2\chi-\tau)(2\chi\overline{\Lambda}+\tau) \\
\Leftrightarrow \quad & 2\chi(2\rho-2\chi+\tau)\overline{\Lambda} < (2\chi-\tau)\tau + 4\rho(\rho+\tau),
\end{aligned}$$

which is always true. Because we maintain the assumption that $\chi > \rho + \tau$, the left hand side of the above inequality is negative where as the right hand side is positive. Hence, we conclude

$$\underline{\sigma}_L < \underline{\sigma}_H < 1.$$

This means, the L steady state is always defined for any $\sigma < 1$ under which the H steady state is also defined.

We summarize the discussions above into Proposition 4 in the paper, which is reproduced here. Maintain the assumption that $m \in (\underline{M}, \overline{M})$.

Proposition 4. There exist $\underline{\sigma}_H$ and $\overline{\sigma}_L$ such that $\underline{\sigma}_H < 1 < \overline{\sigma}_L$. For $\sigma \in (\underline{\sigma}_H, \overline{\sigma}_L)$, the economy has three steady states, H, M, and L. The H and L steady states are saddle path stable, while the M steady state is unstable. For $1 \leq \sigma < \overline{\sigma}_L$, the three steady states can be ranked by the aggregate growth rates, $g_H^* > g_M^* > g_L^*$.

We show how aggregate growth is ordered in the three steady states as described in the proposition. First, we show that the growth rate in the H steady state is always higher than that in the M steady state. Since $g_i^* = \log(\gamma)\chi(1-L_i^*)$ for $i = M, H$, it suffices to show that $L_H^* < L_M^*$.

$$\begin{aligned}
& L_H^* < L_M^* \\
\Leftrightarrow \quad & \frac{1+\rho/\chi+(\sigma-1)\log(\gamma)}{m+(\sigma-1)\log(\gamma)} < \frac{2\rho+\tau+2\chi(\sigma-1)\log(\gamma)}{\chi(m-1)+2\chi(\sigma-1)\log(\gamma)} \\
\Leftrightarrow \quad & (\sigma-1)\log(\gamma) > -\frac{\chi+\rho-m(\chi-\rho-\tau)}{\chi(m-1)+\tau} \\
\Leftrightarrow \quad & \sigma > \underline{\sigma}_H,
\end{aligned}$$

an assumption made in Proposition 4. Hence, we have $g_H^* > g_M^*$.

Next we order g_M^* and g_L^*. We first introduce the following Lemma which can be proved by contradictions.

Lemma. In the M and L steady states, we have $\lambda_M^{f*} \geq \lambda_L^{f*}$, if $\sigma \geq 1$.

Proof. The case of $\sigma = 1$ is discussed in the baseline model, in which case $\lambda_M^{f*} = \lambda_L^{f*}$. Suppose $\sigma > 1$ and we prove by contradiction. Suppose $\lambda_M^{f*} \leq \lambda_L^{f*}$. Since $\lambda_i^{f*} = \frac{r_i^* + \tau}{\chi}$, for $i = M, L$, it implies that $r_M^* \leq r_L^*$. Since $r_i^* = \rho + (\sigma - 1)g_i^*$, it implies that $g_M^* \leq g_L^*$. Since $g_i^* = \log(\gamma)\chi(1 - L_i^*)$, we have $L_M^* \geq L_L^*$. On the other hand, from the first equation of (C-6), it must be true that

$$L_M^* = \frac{2\lambda_M^{f*} - \frac{\tau}{\chi}}{m-1} < \frac{2\lambda_L^{f*} - \frac{\tau}{\chi}}{m-1} < \frac{2\lambda_L^{f*} - \frac{\tau}{\chi}}{m-1}\omega_L = L_L^*,$$

since $\omega_L^* > 1$. We reach a contradiction. □

This, together with the equation $\rho + (\sigma - 1)g_i^* + \tau - \chi\lambda_i^{f*} = 0$ for $i = M, L$, implies that $g_M^* > g_L^*$ as long as $\sigma > 1$. This concludes the proof for $g_H^* > g_M^* > g_L^*$ for $1 \leq \sigma < \overline{\sigma}_L$. The proof of the local stability properties of the steady states is available upon request.

D The Model with A Quadratic Cost of Innovation

Replace the linear cost of innovation in the baseline model with the following quadratic cost. In order to achieve an arrival rate of innovation of λ (to reduce notation we normalize $\xi = 1$), the firm needs to employ the following amount of skilled labor:

$$\phi_j\lambda + \frac{1}{2}\xi_j\lambda^2,$$

where ϕ_j and ξ_j are parameters of the cost function for leaders ($j = 1$) and followers ($j = 2$). The value functions of leaders and followers are given as follows.

$$rV_2^l = \Pi + \tau(V_1^l - V_2^l) + \dot{V}_2^l \tag{D-1}$$
$$rV_2^f = \tau(V_1^f - V_2^f) + \dot{V}_2^f \tag{D-2}$$
$$rV_1^l = \max_{\lambda^l \geq 0} \Pi - \phi_1\lambda^l - \frac{1}{2}\xi_1\lambda^{l2} + \lambda^l(V_2^l - V_1^l) + \lambda^f(V_1^f - V_1^l) + \dot{V}_1^l \tag{D-3}$$
$$rV_1^f = \max_{\lambda^f \geq 0} -\phi_2\lambda^f - \frac{1}{2}\xi_2\lambda^{f2} + \lambda^f(V_1^l - V_1^f) + \lambda^l(V_2^f - V_1^f) + \dot{V}_1^f \tag{D-4}$$

The FOCs imply

$$V_2^l - V_1^l = \phi_1 + \xi_1\lambda^l \tag{D-5}$$
$$V_1^l - V_1^f = \phi_2 + \xi_2\lambda^f. \tag{D-6}$$

D.1 Both Leaders and Followers Innovating

Focus on the steady states where both leaders and followers innovate. Subtracting (D-3) from (D-1) and rearranging,

$$(r + \tau + \lambda^l)(V_2^l - V_1^l) = \phi_1\lambda^l + \frac{1}{2}\xi_1\lambda^{l2} + \lambda^f(V_1^l - V_1^f),$$

where $V_2^l - V_1^l$ is given by (D-5) and $V_1^l - V_1^f$ is given by (D-6), and $r = \rho$ in a steady state. This implies the first equation that involves λ^f and λ^l:

$$(\rho + \tau)(\phi_1 + \xi_1\lambda^l) + \frac{1}{2}\xi_1\lambda^{l2} = \lambda^f(\phi_2 + \xi_2\lambda^f). \tag{D-7}$$

Subtracting (D-4) from (D-2) and rearranging,

$$V_2^f - V_1^f = \frac{\phi_2 \lambda^f + \frac{1}{2}\xi_2 \lambda^{f2} - \lambda^f (V_1^l - V_1^f)}{r + \tau + \lambda^l} = \frac{-\frac{1}{2}\xi_2 \lambda^{f2}}{r + \tau + \lambda^l}, \tag{D-8}$$

where the last equality follows from substituting $V_1^l - V_1^f$ by (D-6).

Subtracting (D-4) from (D-3) and rearranging,

$$(r + 2\lambda^f)(V_1^l - V_1^f) = \Pi^* - \phi_1 \lambda^l - \frac{1}{2}\xi_1 \lambda^{l2} + \lambda^l (V_2^l - V_1^l) + \phi_2 \lambda^f + \frac{1}{2}\xi_2 \lambda^{f2} - \lambda^l (V_2^f - V_1^f),$$

where $V_1^l - V_1^f$ is given by (D-6), $V_2^l - V_1^l$ is given by (D-5), $V_2^f - V_1^f$ is given by (D-8). Substituting these terms in the above equation, we have

$$(r + 2\lambda^f)(\phi_2 + \xi_2 \lambda^f) = \Pi^* + \frac{1}{2}\xi_1 \lambda^{l2} + \lambda^f \frac{(r+\tau)(\phi_2 + \frac{1}{2}\xi_2 \lambda^f) + \lambda^l (\phi_2 + \xi_2 \lambda^f)}{r + \tau + \lambda^l}. \tag{D-9}$$

Note that in a steady state where both leaders and followers innovate, the extensive margin is given by

$$\alpha^* = \frac{\tau}{\tau + \lambda^l}.$$

Then, profit in the steady state becomes

$$\begin{aligned}
\Pi^* &= (m-1)\left[1 - \alpha^* \left(\phi_1 \lambda^l + \frac{1}{2}\xi_1 \lambda^{l2} + \phi_2 \lambda^f + \frac{1}{2}\xi_2 \lambda^{f2}\right)\right] \\
&= (m-1)\left[1 - \frac{\tau}{\tau + \lambda^l} \left(\phi_1 \lambda^l + \frac{1}{2}\xi_1 \lambda^{l2} + \phi_2 \lambda^f + \frac{1}{2}\xi_2 \lambda^{f2}\right)\right],
\end{aligned}$$

which we can plug in (D-9) together with $r = \rho$ to obtain

$$\begin{aligned}
&\xi_2 \left[\frac{1}{2}\left(\frac{\rho + \tau + 2\lambda^l}{\rho + \tau + \lambda^l} - \frac{(m-1)\tau}{\tau + \lambda^l}\right) - 2\right] \lambda^{f2} - \left(\frac{(m-1)\phi_2 \tau}{\tau + \lambda^l} + \phi_2 \rho \xi_2\right) \lambda^f \\
&\qquad + (m-1)\left[1 - \frac{\tau}{\tau + \lambda^l}\left(\phi_1 \lambda^l + \frac{1}{2}\xi_1 \lambda^{l2}\right)\right] + \frac{1}{2}\xi_1 \lambda^{l2} - \rho \phi_2 = 0.
\end{aligned} \tag{D-10}$$

Equations (D-7) and (D-10) form a system of equations, from which we can solve for λ^l and λ^f, which give us the steady state λ^{l*} and λ^{f*}.

D.2 Only Followers Innovating

Now consider the steady state, where only followers innovate. In this steady state, $\alpha^* = 1$ and $\lambda^{l*} = 0$.

The value functions, (D-3) and (D-4), at the steady state become

$$\begin{aligned} rV_1^l &= \Pi + \lambda^f(V_1^f - V_1^l) \\ rV_1^f &= -\phi_2\lambda^f - \frac{1}{2}\xi_2\lambda^{f2} + \lambda^f(V_1^l - V_1^f). \end{aligned}$$

Taking the difference of the above two equations, we have

$$(r + 2\lambda^f)(V_1^l - V_1^f) = \Pi + \phi_2\lambda^f + \frac{1}{2}\xi_2\lambda^{f2}. \tag{D-11}$$

Note that the profit is given by

$$\Pi = (m-1)L = (m-1)\left(1 - \phi_2\lambda^f - \frac{1}{2}\xi_2\lambda^{f2}\right). \tag{D-12}$$

Plugging (D-6) and (D-12) in (D-11) and replace r by the steady state value ρ, we have

$$\begin{aligned} &(\rho + 2\lambda^f)(\phi_2 + \xi_2\lambda^f) = (m-1)\left(1 - \phi_2\lambda^f - \frac{1}{2}\xi_2\lambda^{f2}\right) + \phi_2\lambda^f + \frac{1}{2}\xi_2\lambda^{f2} \\ \Rightarrow &\xi_2\left(\frac{1}{2}m + 1\right)\lambda^{f2} + (\rho\xi_2 + m\phi_2)\lambda^f + \rho\phi_2 - m + 1 = 0. \end{aligned}$$

from which we can solve for the steady state value for λ^f, λ^{f*}.

From the corner solution for λ^f, we can infer that

$$V_2^l - V_1^l < \phi_1.$$

Taking the difference of V_2^l and V_1^l, we have

$$V_2^l - V_1^l = \frac{\lambda^{f*}}{\rho + \tau}(V_1^l - V_1^f).$$

From (D-11), we derive

$$\begin{aligned} V_2^l - V_1^l &= \frac{\lambda^{f*}}{\rho + \tau} \frac{\Pi + \phi_2 \lambda^{f*} + \frac{1}{2}\xi_2 \lambda^{f*2}}{\rho + 2\lambda^{f*}} \\ &= \frac{\lambda^{f*}}{\rho + \tau} \frac{(m-1) - (m-2)\left(\phi_2 \lambda^{f*} + \frac{1}{2}\xi_2 \lambda^{f*2}\right)}{\rho + 2\lambda^{f*}} < \phi_1. \end{aligned}$$

This is the condition for the existence of the steady state where leaders indeed do not innovate.