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An economy plagued by a slump and in a liquidity trap has some options to exit the crisis. We discuss helicopter money and other equivalent policies that can reflate the economy and boost consumption. Traditional helicopter money, via the joint cooperation between the treasury and the central bank, depends critically on the central bank fully guaranteeing treasury's debt. We show that the central bank can do helicopter money on its own, without any treasury's involvement.

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1 Introduction

"Let us suppose now that one day a helicopter flies over this community and drops an additional $1,000 in bills from the sky, which is, of course, hastily collected by members of the community. Let us suppose further that everyone is convinced that this is a unique event which will never be repeated." (Friedman, 1969)

Helicopters have been recently flying over many countries. Following the COVID-19 pandemic, the US government has approved a two trillion dollars support to the economy and the Federal Reserve has committed to unlimited quantitative easing among which purchases of the treasury's debt. The UK government has announced it would extend the size of the government's bank account at the central bank, known historically as the "Ways and Means Facility". The European Central Bank has also extended to unprecedented levels its asset purchase program. A possible implementation of Friedman's proposal is indeed to have the government doing a transfer to the citizens financed by issuing debt, which is in turn purchased by the central bank through more supply of money or reserves. Time will tell us whether this was true monetisation.

In his writing, Friedman's hypothetical experiment was meant to show the effectiveness of monetary policy on inflation. It is, indeed, odd to think that the central bank cannot control the price level. At the end of the day, the Fed's liabilities define exactly what a dollar is. By virtue of this definition, the Fed has the power to print dollars at will without facing any constraint. Since the value of a dollar in terms of goods is the inverse of the price level, the Fed can really throw from the sky as many dollar bills as needed to lower the value of money and reflate the price level. Helicopter money should work!

This suggestive idea has recently received considerable attention in academia and policy circles given that central banks across the globe have lost their conventional ammunitions, having slashed the nominal interest rate down to zero. Helicopter money has been discussed as a viable option to reflate the economy (see among others Bernanke, 2002 and 2003, Galì, 2020a and 2020b, Tuner 2013, 2016).

This paper describes an economy plagued by a slump due to an adverse demand shock in which even cutting the nominal interest rate down to zero does not bring the economy to full capacity, as in the framework of Krugman (1998). Fiscal policy has only access to lump-sum transfers as effective policy tools, like at the inception of the pandemic crisis, where health-policy measures induced a contraction in labor supply that could not be offset using other tools like spending or changes in tax rates.

We study helicopter money and other alternative, and equivalent, policies that can reflate the economy, boost aggregate demand and bring the economy out of the slump.

To analyse the spectrum of available policies, it is key to understand that the central bank's liabilities (money or reserves) are special since they are free of any nominal risk, by definition. These liabilities indeed define what a dollar actually is. Therefore, the central bank can create dollars and reserves at will to pay for its liabilities, without being subject to any solvency requirement. The treasury's liabilities, on the other hand, are in principle like the liabilities of any other agent in the economy. They are a promise to pay a given amount of dollars at maturity. As such, since the treasury cannot create dollars, the treasury's liabilities need to satisfy a solvency condition in order to be repaid and be nominally risk free.

The set of tools available to reflate the economy, indeed, changes depending on whether or not the treasury's liabilities are fully backed by the central bank, i.e. whether or not the special properties of the central bank's liabilities extend to the treasury's as well.

In the first case, when the treasury is backed by the central bank, helicopter money can be implemented in the traditional way. The treasury can make transfers to the private sector, or cut taxes, and finance these policies by issuing more debt. In this case, it does not really matter whether this debt is purchased by the central bank. The reason is that the treasury's debt has the same risk-free properties of the central bank's liabilities.[1] Key for the success of this combination of policies is that the treasury commit not to withdraw the short-run tax relief with higher taxes in the future. The increase in government's liabilities is therefore inflationary, lowers the real rate and stimulates aggregate demand.

The second case, in which the central bank does not back the treasury's liabilities, is quite relevant, because it describes well the current situation of the European Monetary Union where the treasuries of the several countries have to satisfy a solvency condition for the debt they issue, and where indeed such debt is assigned differentiated degrees of credit worthiness by rating agencies. A tax relief today should necessarily be offset by future taxes or by default on treasury's debt. With the treasury out of the picture, however, the central bank can still rely on some policy options to reflate the economy, and all those options are equivalent to the "traditional" account of helicopter money. We discuss three alternatives.

First, the central bank can reduce its net worth. This can be done in two ways. The central bank can write a check to the treasury to be fully rebated to the private sector. Alternatively, without involving the treasury, the central bank could just write off its credits, if any, to the private sector, therefore making a direct wealth transfer. In both cases, the private sector experiences an increase in its wealth, which pushes up consumption, aggregate demand and reflates the economy.

Second, the central bank could commit to systematically transfer a larger fraction of

[1]This is the case considered by Sims (2016), who rules out default on nominal public debt, based on the argument that "the government can print the money the debt promises".

seigniorage revenues to the treasury, and then to the private sector. Since seigniorage is a real resource – mirroring the private sector's costs of holding money – the central bank reduces the real backing of money by rebating it to private sector in a larger fraction, therefore reducing the value of money and increasing the price level.

Third, if it holds some real assets like gold, the central bank retains the ability to control the price level even in the polar case where seigniorage revenues are zero or they are entirely rebated to the private sector, as long as it commits to actively use its gold holdings to provide some real backing. In this scenario the central bank could reduce the amount of gold it commits to mobilize, which signals its willingness to redeem a smaller amount of its liabilities for gold, thereby depleting their exchange value and increasing the price level.

This paper is related to a recent literature that has studied liquidity trap and policy options. Krugman (1998) is our main inspiration for describing a simple model of a slump at the zero lower bound. With respect to his work, we characterize the long-run equilibrium and therefore the policies that can reflate the economy including helicopter money. Woodford (2000, 2001) is the reference for understanding the special role of the liabilities of the central bank as discussed also in recent work by Buiter (2014) and Benigno (2020). Benigno and Nisticò (2020), among others, analyse the implications of separating the treasury and the central bank for the control of inflation through central-bank balance-sheet policies.

Auerbach and Obstfeld (2005) and Buiter (2014) study experiments of helicopter drops in various models with different frictions. Along those lines, Galì (2020b) compares debt-financed versus money-financed fiscal cuts as well as the role of government purchases, and Di Giorgio and Traficante (2018) study the open-economy dimension of this comparison. Eggertsson and Woodford (2003) and Woodford (2012) stress the importance of forward guidance as an alternative way to reflate the economy out of a liquidity trap which can be equivalent in its outcome to the proposal of this work.

2 The Model

We consider a simple perfect-foresight, infinite-horizon, endowment monetary model.

2.1 Households

Households have inter-temporal preferences defined over consumption C and real money balances $m \equiv M/P$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \Big[U(C_t) + V\left(m_t\right) \Big] \tag{1}$$

in which β is the rate of time preference with $0 < \beta < 1$ and ξ is a preference shock. Utility from consumption, $U(\cdot)$, and real money balances, $V(\cdot)$ have standard concave properties with $V(\cdot)$ having a satiation level at $\bar{m}$, such that $V_m(m_t) = 0$ for $m_t \geq \bar{m} > 0$.

Households are subject to a flow budget constraint of the form

$$P_t C_t + M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq P_t Y + M_{t-1} + B_{t-1} + X_{t-1} + (1 + \delta Q_t) D_{t-1} - T_t, \qquad (2)$$

where Y is a constant endowment, P is the price level and T are lump-sum taxes levied by the Treasury. Four securities are available. Households can save or borrow in default-free bonds, B, and hold central bank's reserves, X; both securities pay the nominal default-free interest rate i. They can also save or borrow using long-term bonds, D, which pay a decaying coupon δ and sell at price Q. Finally, they can hold physical money M, which does not pay any interest rate; $M_t \geq 0$ and $X_t \geq 0$ whereas B and D can take positive values, in which case they are assets, or negative, in which case they are debt.

Borrowing possibilities are subject to a limit that prevents Ponzi schemes, which would otherwise allow for an infinite level of consumption. At each point in time t, a natural borrowing limit applies

$$-\Big(B_{t-1} + (1 + \delta Q_t) D_{t-1} + X_{t-1} + M_{t-1}\Big) \leq \sum_{T=t}^{\infty} R_{t,T}^{n} \left(P_T Y - T_T\right) < \infty, \qquad (3)$$

which can be equivalently written in real terms as

$$-\frac{B_{t-1} + (1 + \delta Q_t) D_{t-1} + X_{t-1} + M_{t-1}}{P_t} \leq \sum_{T=t}^{\infty} R_{t,T} \left(Y - \frac{T_T}{P_T}\right) < \infty, \qquad (4)$$

where $R_{t,T}$ is the real discount factor between period t and a generic period T with $T > t$ while $R_{t,t} \equiv 1$. The real discount factor is related to real interest rates according to

$$R_{t,T} \equiv \prod_{j=t}^{T-1} \frac{P_{j+1}}{P_j(1 + i_j)}$$

for $T > t$ whereas the nominal discount factor is given by

$$R_{t,T}^{n} \equiv \prod_{j=t}^{T-1} \frac{1}{(1 + i_j)}$$

with $R_{t,t}^{n} = 1$.

The borrowing limit (4) states that the real net debt position of households at time t, which is the term on the left hand side of the inequality, should not be larger than the present discounted value of their real net income. The latter should be finite, otherwise an infinite level of consumption would still be feasible. The constraint (4), or equivalently (3) in nominal terms, makes also sure that at any point in time the household can pledge enough assets, together with current and future net income to pay back the debt. If commitment to obligations is not questionable, therefore, such debt is paid with certainty. This requirement is also coherent with attributing the default-free nominal rate to the debt of households in writing the budget constraint (2). In what follows, any default-free nominal debt will have the same characteristics, i.e. to be repaid with certainty.

Households choose sequences for consumption and portfolio holdings $\{C_t,\, M_t,\, B_t,\, X_t,\, D_t\}_{t=t_0}^{\infty}$, with $C_t,\, M_t,\, X_t \geq 0$, to maximize utility (1) under the sequence of flow budget constraints (2) and borrowing limits (4), taking as given the sequence of prices, endowment and taxes $\{P_t,\, Q_t,\, R_{t_0,t},\, Y,\, T_t\}_{t=t_0}^{\infty}$ and initial conditions $M_{t_0-1},\, B_{t_0-1},\, X_{t_0-1},\, D_{t_0-1}$.

Solution of the above optimization problem implies, for any period t, a standard Euler equation restricting the intertemporal path of consumption

$$\xi_t U_c(C_t) = \beta(1+i_t)\frac{P_t}{P_{t+1}}\xi_{t+1}U_c(C_{t+1}), \tag{5}$$

the asset-pricing condition with respect to long-term bonds

$$Q_t = \beta\frac{P_t}{P_{t+1}}\frac{\xi_{t+1}U_c(C_{t+1})}{\xi_t U_c(C_t)}(1+\delta Q_{t+1}), \tag{6}$$

and the first-order condition with respect to money holdings

$$\frac{\xi_t U_c(C_t)}{P_t} = \frac{\xi_t}{P_t}V_m\left(\frac{M_t}{P_t}\right) + \beta\frac{\xi_{t+1}U_c(C_{t+1})}{P_{t+1}}, \tag{7}$$

which, by using (5), implies the following demand for real money balances

$$\frac{M_t}{P_t} \geq L(C_t, i_t) \tag{8}$$

where L is the liquidity-preference function $L(C_t, i_t) \equiv V_m^{-1}\left(U_c(C_t)\frac{i_t}{1+i_t}\right)$. Comparing (5) and (7), it follows that the nominal interest rate cannot be negative, $i_t \geq 0$. Equation (8) holds with equality whenever $i_t > 0$.

Finally, optimization also requires households to exhaust all their resources, thus implying the following intertemporal budget constraint to hold with equality

$$\sum_{t=t_0}^{\infty} R_{t_0,t}\left(C_t + \frac{i_t}{1+i_t}\frac{M_t}{P_t}\right) = \frac{W_{t_0}}{P_{t_0}} + \sum_{t=t_0}^{\infty} R_{t_0,t}\left(Y_t - \frac{T_t}{P_t}\right), \tag{9}$$

where W_t denotes the household's nominal financial wealth at the beginning of period t:

$$W_t \equiv B_{t-1} + (1+\delta Q_t)D_{t-1} + X_{t-1} + M_{t-1}.$$

2.2 The government

The government includes a central bank and a treasury. The central bank chooses a sequence for the default-free nominal interest rate i, nominal remittances to transfer to the treasury T^C, its short-term liabilities M^C and X^C in terms of cash and reserves, respectively, and the short-term and long-term assets to hold in its portfolio, respectively B^C and D^C, so as to satisfy the following flow budget constraint

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C. \tag{10}$$

For future reference, let us also define the central bank's nominal net worth

$$N_t \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} \tag{11}$$

and financial income

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) + r_t Q_{t-1} D_{t-1}^C = i_{t-1}(N_{t-1} + M_{t-1}^C), \tag{12}$$

where $1+r_t \equiv (1+\delta Q_t)/Q_{t-1}$ is the nominal return on long-term bonds between periods $t-1$ and t, and where in the second equality of equation (12) we have used (11) and $r_t = i_{t-1}$. The latter condition is implied by (5) and (6) in the perfect foresight equilibrium.

From an accounting perspective, M_t^C and X_t^C are the central bank's liabilities, but of a special type. As discussed by Woodford (2000, 2001), these liabilities are the unit of account of the monetary system, meaning that they define what a currency is. Because of this property, they are nominal default-free securities by definition, regardless of the central bank's policy. Indeed, while private debt is a claim on some dollar bills that the borrower needs to raise to be solvent, dollar bills issued by the central bank are only a claim on themselves. The central bank, and only the central bank, does not need to raise resources to repay its liabilities, also because it has the power to "print" them at will. Differently from household's debt, therefore, we do not need to require any solvency condition to be sure that they are paid with certainty.

This special property does not necessarily apply to the treasury, which chooses a sequence for taxes T, short-term nominal liabilities B^F and long-term ones D^F so as to satisfy its flow budget constraint

$$Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1+\delta Q_t) D_{t-1}^F + B_{t-1}^F - T_t - T_t^C, \tag{13}$$

taking as given asset prices and the nominal remittances T^C received by the central bank.

Like private debt, treasury bonds are also a claim on a given amount of dollar bills, which the treasury needs to raise to be solvent. It follows that the use of default-free interest rate in (13) is only accurate if treasury's debt is repaid with certainty, which requires a borrowing limit analogous to (3)

$$B_{t-1}^F + (1+\delta Q_t) D_{t-1}^F \leq \sum_{T=t}^{\infty} R_{t,T}^n (T_T + T_T^C) \tag{14}$$

or, equivalently, in real terms

$$\frac{B_{t-1}^F + (1+\delta Q_t) D_{t-1}^F}{P_t} \leq \sum_{(T=t}^{\infty} R_{t,T} \left(\frac{T_T}{P_T} + \frac{T_T^C}{P_T} \right). \tag{15}$$

The nominal (real) value of treasury's debt at a certain point in time should not be greater than the nominal (real) present discounted value of taxes and remittances received from the central bank. The borrowing limit (14), or (15), prevents the government from running a Ponzi scheme and at the same time is coherent with the default-free properties of treasury's debt, as specified in budget constraint (13). Violation of the borrowing limit would allow for infinite spending possibilities for the treasury, and in the case of this model for infinite transfers to households implying infinite consumption. From this perspective, the treasury is not different from private borrowers in the economy: either it is solvent and repays its debt, or eventually it has to default on it.

The borrowing limits above imply a restriction on the kind of tax policy that the treasury can run. An exogenous real tax policy, as often assumed in the fiscal theory of the price level, would lead to violation of (15) for a certain range of prices. To see this argument, consider a real tax policy of the type $T_t/P_t = \tau$, for all t. For a given path of remittances, low values of the price level would make the left-hand side of (15) higher than the right-hand side. At those price levels, the treasury would be unable to pay back its obligations. In this scenario, it has either to adjust its tax policy to ensure solvency – thus deviating from $T_t/P_t = \tau$ for some t – or to default on some of its debt, in which case the left-hand side would be adjusted for the seized portion and the relevant interest rate would include a premium compensating for default.

Constraints (14) or (15) are therefore appropriate requirements to add to treasury's borrowing possibilities to support the assumption that its debt is default-free, in line with the requirements that any other debtor should satisfy. They are realistic requirements, as well, since in practice financial markets – as well as rating agencies – evaluate the extent to which the treasury is solvent, just as they do for private borrowers. Central banks, instead, are not subject to the same scrutiny.

To further clarify the implications of our framework, and for the sake of comparison with the literature, we also consider an alternative possible institutional arrangement, which is the one implicitly assumed by the literature. This arrangement provides for the central bank to explicitly extend the special properties of its liabilities to the treasury. This is the only case in which treasury's debt can be properly regarded as nominally risk free regardless of its tax policy, because the central bank "backs" it with its own liabilities. This is possible if the central bank commits to either transfer enough resources to make treasury's debt always repaid or to purchase treasury's debt in any amount and even indefinitely by issuing its own liabilities. In this second case, the treasury can even in principle run Ponzi schemes without undermining the default-free properties of its liabilities. Either ways, this commitment makes (15) no longer a constraint on the path of taxes given prices.

In what follows, whenever constraint (15) applies, we assume that the treasury raises just enough resources to pay its obligations, and therefore (15) holds with equality.

2.3 Equilibrium

Equilibrium in the goods market implies that consumption is equal to output

$$C_t = Y.$$

Equilibrium in the asset markets, instead, implies that

$$B_t^F = B_t + B_t^C,$$

$$D_t^F = D_t + D_t^C,$$

$$M_t^C = M_t,$$

$$X_t^C = X_t,$$

for the four securities traded.

Using goods market equilibrium we can write the Euler equations as

$$(1 + i_t) = \frac{1}{\beta}\frac{\xi_t}{\xi_{t+1}}\frac{P_{t+1}}{P_t}, \tag{16}$$

$$Q_t = \frac{(1 + \delta Q_{t+1})}{(1 + i_t)}. \tag{17}$$

Using goods and asset market equilibria, equation (8) becomes

$$\frac{M_t}{P_t} \geq L(Y, i_t), \tag{18}$$

with equality whenever $i_t > 0$. The intertemporal budget constraint of the household, equation (9), can be written as

$$\frac{B_{t_0-1} + (1 + \delta Q_{t_0})D_{t_0-1} + X_{t_0-1} + M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0}\frac{\xi_t}{\xi_{t_0}}\left(\frac{T_t}{P_t} + \frac{i_t}{1+i_t}\frac{M_t}{P_t}\right). \tag{19}$$

When the central bank does not "back" the treasury's liabilities, we can combine equation (19) with (15) and get

$$\frac{B_{t_0-1}^C + (1 + \delta Q_{t_0})D_{t_0-1}^C - X_{t_0-1} - M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0}\frac{\xi_t}{\xi_{t_0}}\left(\frac{T_t^C}{P_t} - \frac{i_t}{1+i_t}\frac{M_t}{P_t}\right). \tag{20}$$

In this case, to complete the relevant equilibrium conditions, we add the flow budget constraint of the central bank

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1 + \delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C. \tag{21}$$

An equilibrium in this case is a set of non-negative sequences $\{P_t, i_t, Q_t, M_t, X_t, B_t^C, D_t^C, T_t^C\}_{t=t_0}^{\infty}$ satisfying (16), (17), (18), (20), (21), in which the central bank can specify four out of the following six sequences $\{i_t, M_t, X_t, B_t^C, D_t^C, T_t^C\}_{t=t_0}^{\infty}$, given the sequence of exogenous shocks $\{\xi_t\}_{t=t_0}^{\infty}$ and initial conditions $D_{t_0-1}^C$,$B_{t_0-1}^C$, X_{t_0-1}, M_{t_0-1}.[2] The remark to make at this point is that the treasury's policies are irrelevant in this scenario for the determination of prices: only the central bank can have a role. The requirement that remittances are non-negative

[2]Note that the equilibrium condition (20) does not restrict the equilibrium variables at each point in time but only in the long run, which explains why there are four degrees of freedom to choose policy.

further excludes any support from the treasury to the central bank.

When instead the central bank "backs" the treasury, the equilibrium condition (20) is replaced by (19), and (21) is replaced by the consolidation of the flow budget constraints (10) and (13):

$$Q_t D_t + \frac{B_t}{1+i_t} + M_t + \frac{X_t}{1+i_t} = (1+\delta Q_t)D_{t-1} + B_{t-1} + M_{t-1} + X_{t-1} - T_t. \tag{22}$$

In this case, an equilibrium is a set of sequences $\{P_t, i_t, Q_t, M_t, X_t, B_t, D_t, T_t\}_{t=t_0}^{\infty}$ with $\{P_t, i_t, Q_t, M_t, X_t, T_t\}_{t=t_0}^{\infty}$ non-negative satisfying (16), (17), (18), (19), (22), given the sequence of shocks $\{\xi_t\}_{t=t_0}^{\infty}$ and initial conditions D_{t_0-1},B_{t_0-1}, M_{t_0-1}, X_{t_0-1}. The central bank and the treasury can specify four out of the following six sequences $\{i_t, M_t, X_t, B_t, D_t, T_t\}_{t=t_0}^{\infty}$. The tax policy is now relevant for price determination, and the treasury plays then a role.

3 The liquidity trap

We use the model presented in the previous Section to characterize a liquidity trap, a condition under which at zero nominal interest rate there is an excess supply of goods, in the same spirit as Krugman (1998). Time t_0 has the interpretation of the short run. The economy will be stationary after, and including, period t_0+1, which is going to be labelled the long run. There are two important features that distinguish the short from the long run: 1) prices are rigid in the short run and flexible in the long run, 2) a preference shock is low in the short run and high in the long run.[3]

3.1 Short run

Prices are fully rigid and such that $P_{t_0} = P$, for a positive P. The preference shock at time t_0, ξ_{t_0}, is equal to ξ, whereas $\xi_t = \bar{\xi}$ with $\bar{\xi} > \xi$ for $t \geq t_0+1$. Given the assumption of rigid prices, goods market does not necessarily clear at time t_0 and consumption is determined by the Euler Equation

$$\begin{aligned} U_c(C_{t_0}) &= \beta(1+i_{t_0})\frac{P_{t_0}}{P_{t_0+1}}\frac{\xi_{t_0+1}}{\xi_{l_0}}U_c(C_{t_0+1}) \\ &= \beta(1+i_{t_0})\frac{P}{P_{t_0+1}}\frac{\bar{\xi}}{\xi}U_c(Y), \end{aligned} \tag{23}$$

[3]For illustrative purposes, the short run lasts only one period, though we can make it longer by extending the duration of price rigidity and/or of the shock. In the Appendix, we show some robustness along this dimension. See Section A.3.

where in the second line we have used the assumptions on the preference shock, $P_{t_0} = P$ and goods market equilibrium in the long run, $C_{t_0+1} = Y$.

Let us leave aside for now the determination of the long-run price level P_{t_0+1} and assume that its equilibrium value is $\bar{P}$, for a positive $\bar{P}$. Further assume that the distance between $\bar{\xi}$ and ξ is large enough so that, given P and $\bar{P}$, the following inequality holds

$$\beta \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi} > 1. \tag{24}$$

If this inequality holds, equation (23) implies that short-run consumption falls below output at any non-negative interest rate: the economy is in a slump.

Figure 1 shows this situation. In the space $(C_{t_0}, 1 + i_{t_0})$, the Euler equation (23) and the zero-lower bound (ZLB) on the nominal interest rate imply a downward-sloping aggregate demand curve (AD) that dies out at $i_t = 0$. The vertical red line displays the aggregate supply curve (AS), located at the level of the constant endowment Y. Starting from a stationary equilibrium where $C = Y$ and $1 + i = \Pi/\beta$ (point A in the figure), a negative demand shock, which creates a gap such that $\xi < \bar{\xi}$ and (24) holds, shifts the AD curve to the left into AD', inducing a downward pressure on current consumption. The central bank can exploit the downward slope of aggregate demand and cut the nominal interest rate to stimulate consumption as much as possible. To restore the equilibrium in the goods market, $C_{t_0} = Y$, the central bank would need to cut the nominal rate down to $1 + i_{t_0} = (\xi/\bar{\xi})(\bar{P}/(P\beta))$. However, if the size of the shock satisfies (24), the required cut in the nominal rate would violate the ZLB. As a consequence, the central bank cannot descend the AD' schedule beyond point B, where the economy is in a slump and experiences a shortage of demand:

$$\underline{C} = Y U_c^{-1} \left(\beta \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi} \right) < Y.$$

Equation (23) clarifies that the other possibility to restore the equilibrium in the goods market is to act on the future price level, reflating the economy, lowering the real rate and boosting consumption: in Figure 1, indeed, raising $\bar{P}$ shifts the aggregate demand schedule to the right into AD'' and the economy can reach the full-employment equilibrium E.[4]

[4]We should clarify that, although we analyse the policies to restart the economy once in a liquidity trap, the mechanisms we discuss are at work also for positive values of the nominal interest rates. In that scenario, however, the equivalence results we discuss are in general weaker.

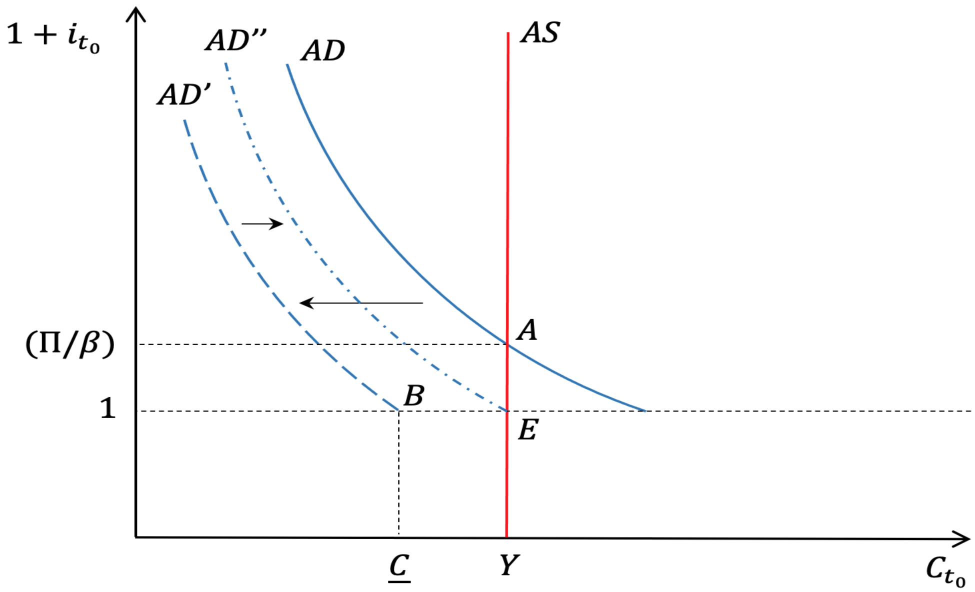

Figure 1: The effects of a negative preference shock $\xi < \bar{\xi}$: AD shifts to the left into AD' and the economy is in a slump ($\underline{C} < Y$) due to the ZLB, unless the economy is reflated by shifting AD' to the right into AD''.

3.2 Long run

In the long run, i.e. for $t \geq t_0 + 1$, prices are flexible and the preference shock is at the high level $\xi_t = \bar{\xi}$. Goods market clears and consumption is equal to output. We assume a simple interest-rate policy targeting a constant rate of inflation $\Pi > \beta$, through

$$1 + i_t = \frac{\Pi}{\beta}$$

for each $t \geq t_0 + 1$. Therefore $i_t > 0$. Substituting it into (5) and using $C_t = Y$ and $\xi_t = \bar{\xi}$ we obtain that

$$\frac{P_{t+1}}{P_t} = \Pi$$

for each $t \geq t_0 + 1$: inflation is constant after $t_0 + 1$ at the level Π targeted by the central bank.

Using the interest-rate policy into the equilibrium in the money market (8), we obtain that

$$\frac{M_t}{P_t} = L\left(Y, \frac{\Pi}{\beta} - 1\right)$$

for any $t \geq t_0 + 1$.

What is left to determine is the price at time P_{t_0+1} which is key to reflate the economy from the liquidity trap. The way P_{t_0+1} is determined depends on whether or not the central bank is backing the treasury. In case it is not, the remaining equilibrium conditions are (20) and (21). Equation (20), at time t_0+1, can be written as

$$\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}\left(\frac{T_t^C}{P_t}\right)=\mathcal{S}\left(Y,\Pi\right)+\frac{Q_{t_0}D_{t_0}^C+B_{t_0}^C-X_{t_0}-M_{t_0}}{P_{t_0+1}} \tag{25}$$

in which we have defined the present discounted value of real seigniorage as

$$\mathcal{S}(\Pi,Y)\equiv\frac{\Pi-\beta}{\Pi(1-\beta)}L\left(Y,\frac{\Pi}{\beta}-1\right).$$

To derive (25), we have used $i_{t_0}=0$ – that implies $Q_{t_0}=1+\delta Q_{t_0+1}$ – and noting that in equilibrium $R_{t_0+1,t}=\beta^{t-t_0-1}$. Under the same assumptions, equation (21) at time t_0 can be written as

$$Q_{t_0}D_{t_0}^C+B_{t_0}^C-M_{t_0}-X_{t_0}=(1+\delta Q_{t_0})D_{t_0-1}^C+B_{t_0-1}^C-X_{t_0-1}-M_{t_0-1}-T_{t_0}^C. \tag{26}$$

In case instead the central bank is backing the treasury, the relevant equilibrium conditions are (19) and (22), which imply

$$\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}\left(\frac{T_t}{P_t}\right)+\mathcal{S}(\Pi,Y)=\frac{B_{t_0}+X_{t_0}+M_{t_0}+Q_{t_0}D_{t_0}}{P_{t_0+1}}, \tag{27}$$

and

$$Q_{t_0}D_{t_0}+B_{t_0}+M_{t_0}+X_{t_0}=(1+\delta Q_{t_0})D_{t_0-1}+B_{t_0-1}+M_{t_0-1}+X_{t_0-1}-T_{t_0}, \tag{28}$$

respectively.

4 Central bank and treasury acting together

We consider first the case in which the central bank backs the treasury's debt, thus extending the default-free property of its own liabilities to the treasury's as well. Since treasury's debt is fully guaranteed by the central bank's "printing press", the treasury can run whatever fiscal policy it pleases in terms of the path of real or nominal taxes, since it is not restricted by any solvency constraint. The equilibrium price at time t_0+1 can then be determined by using equations (27) and (28). Note that since the central bank has set the interest rate to a target, we are left with three degrees of freedom to further specify policy. We assume that the treasury sets the path of taxes, $\{T_t\}_{t=t_0}^{+\infty}$, and how much to issue of short term debt net of central bank's holdings, $\{B_t\}_{t=t_0}^{+\infty}$; the central bank further sets the path of reserves, $\{X_t\}_{t=t_0}^{+\infty}$.

Given the interest-rate policy the path of money is endogenously determined by equilibrium in the money market (8).

What is critical for the determination of the price level in this case is the specification of the tax policy. As in the Fiscal Theory of the Price Level, the treasury can set a non-Ricardian (or active) tax policy $\{T_t/P_t = \tau_t\}_{t=t_0+1}^{+\infty}$ irrespective of the real value of its obligations. Equation (27), which is an equilibrium condition but not a solvency constraint, can then determine the price level P_{t_0+1} at, let's say, $\bar{P}$. It should be read in the following way. It is not the left-hand side of (27) that necessarily adjusts to match the right-hand side – i.e. the real value of the outstanding government's nominal liabilities at any equilibrium P_{t_0+1}. The other way round, indeed. The government can rely on two sources of real assets to back the exchange value of its nominal liabilities: the stream of real taxes and that of real seigniorage. Long-run prices therefore adjust to satisfy the equilibrium condition that the real value of assets match that of liabilities, given monetary and fiscal policies that determine the left-hand side of (27) and given the outstanding government's nominal liabilities at time $t_0 + 1$:

$$\bar{P} = \frac{B_{t_0} + X_{t_0} + M_{t_0} + Q_{t_0} D_{t_0}}{\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau_t + \mathcal{S}(\Pi, Y)}. \tag{29}$$

Equation (29) shows the alternative policy options to reflate the economy. Before discussing them, note that a non-Ricardian tax policy is essential for what follows. A Ricardian fiscal policy would lead to indeterminacy of the price level and, therefore, may not allow the policymaker to control long-run prices and boost the economy in the short run.

The first option to reflate the economy is to raise the numerator of (29), *ceteris paribus*. In the traditional narrative of the so called "helicopter money" the government (treasury or central bank) increases permanently the long-run nominal liabilities – namely B_{t_0}, X_{t_0}, or D_{t_0} – in order to finance a tax cut in the short run. Since the short-run nominal interest rate is zero, all these possibilities are equivalent, as implied by (28).[5] Indeed, given that all the government's liabilities have the special properties of the central bank's, B_{t_0}, X_{t_0} or D_{t_0} are always paid in full since they are guaranteed by the "printing press" of the central bank without any need to raise taxes or seigniorage revenues. And, indeed, taxes and seigniorage should not move (at least not proportionally) for an increase in government debt to produce an effect on long-run prices, as equation (29) clearly shows.

Moreover, equation (28) clarifies that the increase in government liabilities outstanding at t_0+1 can be generated by a tax cut at t_0 and therefore a larger current primary deficit. This larger deficit can equivalently be financed issuing either short-term or long-term treasury's debt, which can be held by either the private sector or the central bank. In the former case

[5] Some equivalence results are going to break at positive short-run interest rate, but not the overall argument on the general effectiveness of the policies proposed.

B_{t_0} or D_{t_0} increase for given X_{t_0}, while in the latter case the opposite occurs, as the central bank raises its liabilities to absorb the new issuance of treasury's debt, leaving unchanged the stock of debt held by the private sector (B_{t_0} and D_{t_0}). In the latter case, it does not really matter whether the central bank holds permanently the treasury's debt or writes it off, as discussed in Buiter (2014) and Galí (2020a, 2020b).

For all these policy options to succeed, equation (29) clarifies that it is important that the denominator does not change (at least not proportionally): the treasury should therefore commit to never undo the short-run tax relief.[6]

It is useful to visualize results using a simple AD–AS logic. To this end, we can use the equilibrium condition (9) and exploit some simplifications on preferences as outlined in the Appendix (namely log and separable preferences in consumption and real money balances), to write long-run consumption as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{B_{t_0}+X_{t_0}+M_{t_0}+Q_{t_0}D_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}\left(Y-\tau_t\right)\right\}, \tag{30}$$

for some positive preference parameter θ. This equation can be interpreted as a consumption demand equation relating long-run consumption to long-run prices. However, the channel through which prices affect consumption is not a conventional one since it acts through the nominal financial wealth held by the agent – the one that has the same default-free characteristics of central bank's liabilities. Assuming that the private sector is a net creditor with respect to the government (i.e. $B_{t_0}+X_{t_0}+M_{t_0}+Q_{t_0}D_{t_0} > 0$), then equation (30) implies a negative relationship between long-run prices and consumption. For a creditor, indeed, an increase in the price level reduces the real value of his/her assets pushing consumption down. This relationship is plotted in Figure 2 as an AD equation together with the AS equation of constant long-run output.

Consider now an increase in the government's nominal liabilities at time t_0. Since the agent is a net creditor, this raises the nominal financial wealth that agents carry into period t_0+1 and creates an excess demand of goods at the initial price level: the demand curve shifts to the right into AD'. In order for consumption to fall back to the level of the constant endowment, such excess demand stimulates an increase in the price level that reduce the real value of the financial assets held by the consumer and restore equilibrium in E'.[7]

[6] This is therefore an example of "unbacked fiscal expansion", in the words of Jacobson, Leeper and Preston (2019).

[7] Equation (29) suggests two alternative policy options to reflate the economy, which work through a reduction in the denominator. The first alternative is a treasury's commitment to lower real taxes in the long run, given an unchanged path of liabilities carried from t_0. The second alternative is a central bank's commitment to lower the present-discounted value of seigniorage revenues by changing the inflation target

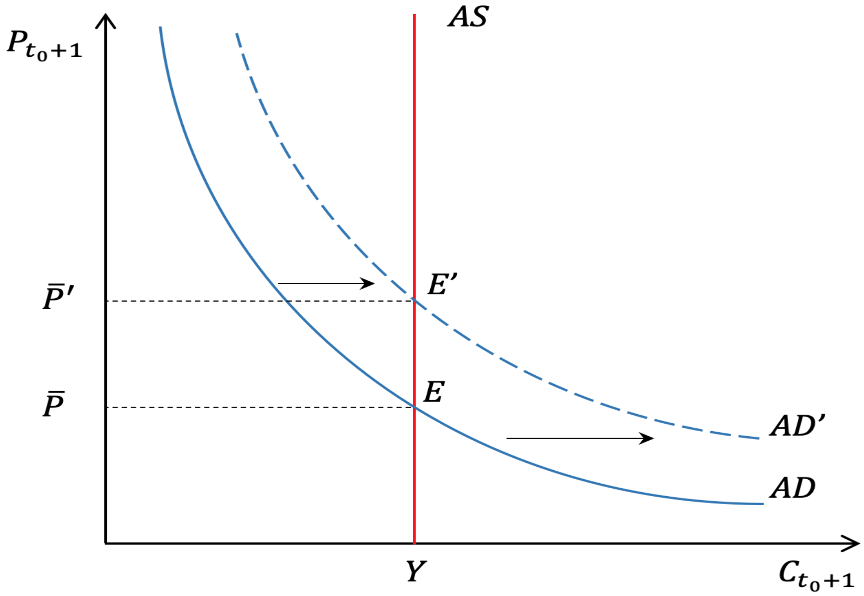

Figure 2: Reflating the economy when the government faces a consolidate budget constraint.

5 Central bank acting alone

Consider now the case in which the central bank does not back treasury's liabilities. The relevant equilibrium conditions to determine the price level are now (25) and (26), while the central bank has to specify three out of the four sequences $\left\{X_t, B_t^C, D_t^C, T_t^C\right\}_{t=t_0}^{\infty}$.

Equation (25) emphasizes two implications compared to the previous case that are key to understand the results that will follow. First, the relevant definition of nominal private wealth is now mirrored by the net financial position of the central bank only ($Q_{t_0} D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}$) instead of the whole government's, since the private sector no longer values treasury's liabilities as "relevant wealth" for its spending decisions. Second, what matters for price determination is now the path of central bank's remittances, instead of taxes. Analogously to the fiscal theory of the price level, the specification of the remittance policy is critical for determining the price level in the long-run and reflating the economy.

Consider first the case in which the central bank sets an exogenous and constant path for *nominal* remittances such that $\{T_t^C = T^C\}_{t=t_0+1}^{+\infty}$ with $T^C \geq 0$.[8] Equation (25) can then be

Π. The sign of the required change in Π is ambiguous, and depends on whether seigniorage evaluated at the target rate of inflation, Π, is increasing or decreasing in Π. Note also that this last channel is switched off in the simple AD–AS logic we employ. Indeed, the simplification used to derive the equation (30) – namely that preferences are logarithmic and separable with respect to consumption and real money balances – implies perfectly offsetting income and substitution effects that make seigniorage a function of income only, with no role for the inflation rate.

[8]The Appendix discusses the case where the central bank is able to control a stream of real remittances. See Section A.2. In that case, the central bank is committing to make a (fully indexed) monetary transfer such that it corresponds to a certain purchasing power in terms of consumption goods.

written as

$$\frac{\mathcal{T}^C}{P_{t_0+1}} = \mathcal{S}(\Pi, Y) + \frac{Q_{t_0}D^C_{t_0} + B^C_{t_0} - X_{t_0} - M_{t_0}}{P_{t_0+1}}. \tag{31}$$

where we defined

$$\mathcal{T}^C \equiv \frac{\Pi}{\Pi - \beta}T^C. \tag{32}$$

Equation (31) is again an equilibrium condition – not a solvency constraint, since X_{t_0} and M_{t_0} are default free by definition – which then determines the long-run price level $\bar{P}$ at

$$\bar{P} = \frac{\mathcal{T}^C - N_{t_0}}{\mathcal{S}(\Pi, Y)}, \tag{33}$$

where central bank's net worth (N) is defined by (11), and we consider the fact that at time t_0 the nominal interest rate is at the ZLB. The intuition behind equation (33) is simple and analogous to what discussed in the case of the FTPL, with one key difference: what is crucial is no longer the real valuation of the nominal liabilities of the whole government, but those of the central bank alone. Indeed, while in nominal terms the central bank's liabilities are default free by definition, as discussed, their *real* value depends instead on the real resources that the central bank can rely on to back them.

Although the central bank does not have taxation power, it extracts real resources from the households by issuing securities – namely money – that households hold also for their non-pecuniary services. The cost of holding real money balances is a real cost, as shown in the intertemporal budget constraint (9), which corresponds to the seigniorage revenues accruing to the central bank, $\mathcal{S}(\Pi, Y)$. By setting a constant interest-targeting policy, the central bank can fix the value of seigniorage independently of the current price level. Accordingly, long-run prices in equation (31) adjust to satisfy the equilibrium condition that the real value of central bank's assets $\mathcal{S}(\Pi, Y)$ match that of its net liabilities $(\mathcal{T}^C - N_{t_0})/P_{t_0+1}$. This happens exactly at the price level $\bar{P}$.

To complete the analysis, we now study how the central bank can steer the price level at $t_0 + 1$ to reflate the economy from the liquidity trap. Note that the law of motion of net worth is given by

$$N_t = N_{t-1} + \Psi_t - T^C_t, \tag{34}$$

where central bank's profits are defined in (12). Note also that, given $\mathcal{S}(\Pi, Y) > 0$, the numerator in equation (33) should be positive, to support a positive price level $\bar{P}$ at equilibrium. The remittance policy, therefore, should be set so as to ensure $\mathcal{T}^C > N_{t_0}$ regardless of the relative net financial position of the central bank versus the private sector.[9]

[9] If net worth N_{t_0} is negative, $\mathcal{T}^C$ can be also equal to zero.

The options to reflate the economy that are available to the central bank – and only to the central bank – are several. First, it could act on the numerator of (33), by cutting its net worth, *ceteribus paribus*. This can be accomplished by increasing short-run transfers to the treasury, as shown by (34), which implies on the one hand an increase in current base money and, on the other hand, lower current taxes for the private sector, and is therefore equivalent to the traditional narrative of helicopter money. An alternative, but equivalent, way of cutting nominal net worth would be for the central bank to write-off some of the assets in its portfolio. In particular, by writing off private securities from its balance sheet, the central bank can trigger a positive and reflationary wealth effect directly on the private sector, without any involvement of the treasury.

We can visualize the intuition behind the above discussion by using again an AD–AS logic. Note first that treasury's debt in this case is not at all considered relevant wealth by the private sector, since it is paid by future taxes levied on the private sector itself. Consumption demand, therefore, can be written by combining (9) and the solvency constraint (15) holding with equality, under the simplifying preference specification used in the Appendix, as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{\mathcal{T}^C - N_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}Y\right\}. \tag{35}$$

This demand function emphasizes two implications. The first is that the net nominal asset position of the consumer, which can be considered as wealth, mirrors that of the central bank only, i.e. the term $-N_{t_0}$ in equation (35). The second is that what at the end matters for the nominal wealth position of the household also includes the component of human wealth that is indirectly affected by the remittance policy of the central bank, through $\mathcal{T}^C$. If the latter satisfies $\mathcal{T}^C > N_{t_0}$ then the private sector is a net creditor with respect to the overall nominal wealth position that matters for its consumption. Accordingly, an increase in the price level at $t_0 + 1$ reduces the demand for consumption through a negative wealth effect, as before. The aggregate demand schedule is therefore again a downward-sloping function, as in Figure 2, and the same analysis applies. Cutting the central bank's net worth at time t_0 reflates the economy by expanding the demand for long-run consumption, and taking the economy into equilibrium E'.

Note that this proposal is different from those of Buiter (2014) and Galì (2020a, 2020b). In their case, indeed, the mechanism runs as follows. First, the treasury lowers taxes financing the cut with newly issued debt purchased by the central bank through an increase in its liabilities (reserves or money). Then, the central bank writes off the treasury's debt or, equivalently, it rolls it over permanently. What is important in their proposal is the lowering of taxes financed at the end by the increase in central bank's liabilities. Our analysis, instead, clarifies that there is no need to increase central bank's liabilities (money or reserves) and the treasury can be completely uninvolved as long as the central bank writes off private securities from its balance sheet.

Moreover, we point to additional and alternative options. The central bank indeed can reflate the economy also by committing to an increase in the present-discounted value of future remittances in the numerator of (33). The effect of this policy action would be equivalent to a cut in nominal net worth, as the nominal wealth that is relevant for private sector's spending decisions expands – as shown by the first term in (35) – thus pushing up prices along the same lines as before. A third option available to the central bank is to lower the present-discounted value of seigniorage by changing its inflation target Π, thus acting on the denominator of (33).[10] In this latter case, indeed, the central bank is reducing the amount of real assets that are backing its nominal liabilities, thus depleting their exchange value.

We now discuss the robustness of our results by considering other types of remittance policies. As it often happens, central banks transfer part of their financial income to the treasury. We consider then a more general nominal remittance policy of the form

$$T_t^C = T^C + \alpha \left(i_{t-1} M_{t-1} \right) \tag{36}$$

with a parameter α such that $0 \leq \alpha \leq 1$ and where $i_{t-1}M_{t-1}$ are seigniorage revenues at time t implied by the use of cash. Using it into (25), and considering $i_{t_0} = 0$ yields

$$\frac{\mathcal{T}^C}{P_{t_0+1}} = (1-\alpha)\mathcal{S}(\Pi, Y) + \frac{Q_{t_0} D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}}{P_{t_0+1}}, \tag{37}$$

where $\mathcal{T}^C$ is defined in (32). The above equation allows to uniquely determine the long-run price level $\bar{P}$ at:

$$\bar{P} = \frac{\mathcal{T}^C - N_{t_0}}{(1-\alpha)\mathcal{S}(\Pi, Y)}. \tag{38}$$

We can draw several implications. The first, general, implication is that the specification of the remittance policy is key for the determination of the price level. The second is that, when the central bank specifies its remittance policy as in (36), it can rely on an additional policy margin to affect the long-run price level and reflate the economy: the share of seigniorage revenues α that it systematically rebates to the treasury. The third, particularly important, implication is that the remittance rule (36), although it is specified in *nominal* terms, includes a term whose effects are equivalent to those of a *real* transfer. Indeed, rebating a share of seigniorage revenues linked to outstanding money liabilities, M, allows the

[10] If the economy is on the left side of the Laffer curve, a lower inflation target is able to trigger both effects – on the seigniorage and the discounted path of future nominal remittances – consistently with each other.

central bank to transfer – through the term $\alpha\left(i_{t-1}M_{t-1}\right)$ in (36) – a certain purchasing power in terms of consumption goods for each $t > t_0+1$. Indeed, note that $\alpha\left(i_{t-1}M_{t-1}\right)/P_t$ is equal to zero for $t = t_0 + 1$, since $i_{t_0} = 0$, and equal to $\alpha(\Pi - \beta)L(Y, \Pi/\beta - 1)/(\beta\Pi)$ for each $t > t_0 + 1$, and therefore independent of the price level.

We can intuit this using again an *AD–AS* logic. Combining (9), the solvency constraint (15) and the remittance policy (36) delivers the following *AD* schedule:

$$C_{t_0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{\mathcal{T}^C - N_{t_0}}{P_{t_0+1}} + \alpha\mathcal{S}(Y) + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}Y\right\} \tag{39}$$

where we also take into account that under the logarithmic specification of preferences, seigniorage is a function of income only. As the middle term in the curly brackets clarifies, committing to a permanently higher share α signals a permanent cut in the amount of *real* resources that are extracted from the private sector through seigniorage, which induces a positive wealth effect that shifts the aggregate demand to the right, as in Figure 2, and again pushes the price level up to equilibrium E'.

To better understand the role of α for price-level determination, recall that the present-discounted value of the seigniorage revenues are the only real asset that the central bank can rely on to back the exchange value of its currency when setting a remittance policy in nominal terms. As the central bank rebates parts of these seigniorage revenues to the private sector (via the treasury), it reduces the size of such real backing in its balance sheet, thus depleting the exchange value of its currency. Indeed, in the limiting case in which all seigniorage revenues are remitted to the treasury (i.e. $\alpha = 1$), the central bank has no real asset left to back the exchange value of its currency, and the price level becomes infinite, as clearly implied by equation (37).

There is one caveat to consider in the above analysis: the existence also of a non-monetary equilibrium in which the value of money is zero and prices are infinite. Although this equilibrium is dominated in welfare by the one with a finite price level, it is important to discuss its occurrence because it further emphasizes the relevance of seigniorage revenues for price determination, and for helicopter money. Recall that seigniorage depends on the demand of real money balances and therefore on the first-order condition (7). Inspection of (7) shows that a non-monetary equilibrium is possible if preferences are such that

$$\lim_{P_t\to\infty}\frac{1}{P_t}V_m\left(\frac{M_t}{P_t}\right) = 0. \tag{40}$$

Accordingly, one way to rule out the non-monetary equilibrium is by imposing an assumption on preferences that makes the above limit positive, as suggested by Obstfeld and Rogoff

(1983). Note, however, that our analysis of price determination is completely different from theirs since they posit a money-supply rule as opposed to the interest-rate policy we consider instead. Moreover, differently from all the literature on price determination, one of the key element in our context is the specification of central bank's remittances policy.

As we show in the next Section, the non-monetary equilibrium can also be ruled out without requiring any special assumption on the money-demand function, in a model in which the central bank holds also gold in its portfolio. In that environment, our proposal of central bank's helicopter money will also generalize to cases in which the central bank rebates all seigniorage revenues, or even in a cashless economy, when the seigniorage revenues are zero altogether.

6 An economy with gold

Consider an economy where there is a real asset (G) available for trade, like gold, in a constant supply $\bar{G}$, which provides utility benefits to households. Households have thus preferences defined over consumption C, real money balances m, and gold holdings g

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \Big[U(C_t) + V(m_t) + Z(g_t) \Big] \tag{41}$$

where the utility from the stock of gold in the hands of the household, $Z(\cdot)$, is increasing and strictly concave. Households maximize (41) subject to a sequence of flow budget constraints of the form

$$P_t C_t + M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t + P_t q_t (g_t - g_{t-1}) \\ \leq P_t Y + M_{t-1} + B_{t-1} + X_{t-1} + (1 + \delta Q_t) D_{t-1} - T_t, \tag{42}$$

where q_t is the real price of gold at time t. Solution of the optimization problem above requires, for any period t, the first-order conditions (5)–(7) plus the one for gold holdings, which implies the equilibrium real price of gold:

$$q_t = \frac{Z_g(g_t)}{U_c(C_t)} + R_{t,t+1} q_{t+1}. \tag{43}$$

The real price of gold depends on the utility benefits of gold in units of consumption goods and on the present-discounted value of its next-period level.

The other first-order condition that is affected compared to the benchmark model of the previous section is the intertemporal budget constraint of the household, which now reads as

$$\sum_{t=t_0}^{\infty} R_{t_0,t}\left(C_t + \frac{i_t}{1+i_t}\frac{M_t}{P_t} + (g_t - \bar{G})\frac{Z_g(g_t)}{U_c(C_t)}\right) = \frac{W_{t_0}}{P_{t_0}} + \sum_{t=t_0}^{\infty} R_{t_0,t}\left(Y_t - \frac{T_t}{P_t}\right) \tag{44}$$

in which nominal financial wealth at the beginning of period t is defined by:

$$W_t \equiv B_{t-1} + (1+\delta Q_t)D_{t-1} + X_{t-1} + M_{t-1} + P_t q_t (g_{t-1} - \bar{G}).$$

We allow gold to be held as an asset by the central bank in its balance sheet. Let g_t^C denote the central bank's gold portfolio, with

$$\bar{G} = g_t + g_t^C.$$

Under the assumption that the central bank does not back the treasury's liabilities, and therefore the solvency condition (15) holds, using goods and asset market equilibrium, we now obtain the key equation for price level determination at time t_0+1 – i.e. the equivalent of equation (25) – as

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\left(\frac{T_t^C}{P_t} + \frac{Z_g(g_t)}{U_c(Y)}g_t^C\right) = \mathcal{S}\left(Y,\Pi\right) + q_{t_0+1}g_{t_0}^C + \frac{Q_{t_0}D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}}{P_{t_0+1}}. \tag{45}$$

Using the equilibrium real price of gold (43), we can further write it as

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\left(\frac{T_t^C}{P_t}\right) = \mathcal{G}_{t_0} + \mathcal{S}\left(Y,\Pi\right) + \frac{Q_{t_0}D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}}{P_{t_0+1}} \tag{46}$$

where we defined

$$\mathcal{G}_{t_0} \equiv \sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\left[\frac{Z_g(\bar{G}-g_t^C)}{U_c(Y)}\left(g_{t_0}^C - g_t^C\right)\right]. \tag{47}$$

An important implication of equation (46) is that non-monetary equilibria are now ruled out without the need to make any special assumption on the preference toward liquidity, and thus even in the case in which equation (40) holds. Indeed, seigniorage revenues are no longer essential for the equilibrium price level to be determinate and finite. To see this, consider again the nominal remittance policy (36) and use it in equation (46), to obtain

$$\frac{\mathcal{T}^C}{P_{t_0+1}} = \mathcal{G}_{t_0} + (1-\alpha)\mathcal{S}(\Pi,Y) + \frac{Q_{t_0}D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}}{P_{t_0+1}}. \tag{48}$$

with $\mathcal{T}^C$ defined in (32). Note the difference with respect to (37) for the additional term $\mathcal{G}_{t_0}$. The equilibrium condition (48) is therefore no longer compatible with an infinite price level insofar as $\mathcal{G}_{t_0} \neq 0$. This condition just requires some trading in gold by the central bank, as (47) shows. Instead, a constant central bank's holding of golds implies $\mathcal{G}_{t_0} = 0$ and does not exclude the non-monetary equilibrium.

We can use (48) to determine the long-run price level $\bar{P}$ at

$$\bar{P} = \frac{\mathcal{T}^C - \widetilde{N}_{t_0}}{(1-\alpha)\mathcal{S}(\Pi, Y) + \mathcal{G}_{t_0}}, \tag{49}$$

where $\widetilde{N}_{t_0} \equiv Q_{t_0} D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}$ denotes the central bank's net worth in nominal securities, and we have also used $i_{t_0} = 0$. Even if seigniorage is zero, as in a cashless economy, or it is entirely rebated to the treasury ($\alpha = 1$), it is possible to determine the price level at a finite value, provided $\mathcal{G}_{t_0} \neq 0$. As we are going to show shortly, the slope of the household's aggregate demand is again downward sloping in the case $\mathcal{T}^C > \widetilde{N}_{t_0}$, therefore it should be required that $\mathcal{G}_{t_0} > 0$. In this case, indeed, the real backing would be provided by the possibility for the central bank to mobilize its gold portfolio to extract some real revenue from the private sector and therefore redeem some of its nominal liabilities for gold.

To clarify, consider the following example: the central bank starts with positive gold holdings, $g_{t_0-1}^C = \bar{g}^C > 0$, it keeps them constant until a generic period T, i.e. $g_t^C = \bar{g}^C$ for $t_0 \leq t < T$, and then sells a fraction $1-\lambda$ of them at time T, i.e. $g_t^C = \lambda \bar{g}^C$ for $t \geq T$, with with $\lambda \in [0, 1]$. In this scenario, using (43) and (47) we can show that the commitment to mobilize a fraction $1-\lambda$ of the gold portfolio at time T implies

$$\mathcal{G}_{t_0}(\lambda, T) = \beta^{T-t_0-1} \frac{Z_g(\bar{G} - \lambda \bar{g}^C)}{Z_g(\bar{G} - \bar{g}^C)} (1-\lambda)\bar{q}\bar{g}^C, \tag{50}$$

which is decreasing in T and (under certain conditions) λ,[11] and where $\bar{q} \equiv \frac{1}{1-\beta} \frac{Z_g(\bar{G}-\bar{g}^C)}{U_c(Y)}$ denotes the equilibrium real price of gold if the central bank held its initial portfolio indefinitely, i.e. $g_t^C = \bar{g}^C$ for all $t \geq t_0$. Equation (50) then shows that $\mathcal{G}_{t_0} > 0$ as long as the central bank commits to redeem its liabilities for gold in a finite time (i.e. $T < \infty$) and even in a relatively small amount, i.e. λ close to one.

To understand the policy options available to the central bank, we can build the intuition using again an *AD–AS* logic. Under the simplifying preference specification used in the Appendix, which we complement with $Z(g_t) = \vartheta \ln(g_t)$, we can write consumption demand,

[11] In particular, $\mathcal{G}_{t_0}$ is always decreasing in λ if preferences are logarithmic in gold holdings, while it is also in general as long as $\gamma(g) < \frac{\bar{G}-\lambda\bar{g}^C}{(1-\lambda)\bar{g}^C}$, i.e. if the household's coefficient of relative risk aversion in gold holdings $\gamma(g)$ is not too large compared to the amount of gold the central bank commits to sell, $(1-\lambda)\bar{g}^C/\bar{G}$.

by combining (44), the solvency constraint (15) holding with equality, and the remittance policy (36) as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta-\vartheta}\left\{\frac{\mathcal{T}^C - \widetilde{N}_{t_0}}{P_{t_0+1}} + q_{t_0+1}(\lambda,T)\bar{g} + \alpha\mathcal{S}(Y) + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}Y\right\}, \tag{51}$$

where $\bar{g} = \bar{G} - \bar{g}^C$ and $q_{t_0+1}(\lambda, T)$ denotes the equilibrium real price of gold at time t_0+1, as a function of the share of gold holdings $1-\lambda$ that the central bank commits to mobilize and the future date T at which it commits to do it, with:

$$\frac{q_{t_0+1}(\lambda,T)}{\bar{q}} \equiv \beta^{T-t_0-1}\left(\frac{Z_g(\bar{G}-\lambda\bar{g}^C)}{Z_g(\bar{G}-\bar{g}^C)} - 1\right) + 1. \tag{52}$$

The equilibrium real price of gold q_{t_0+1} is therefore increasing in both λ and T, given the concavity of $Z(\cdot)$, and is equal to $\bar{q}$ if the central bank does not commit to ever redeem its liabilities for gold (i.e. if either $\lambda = 1$ or $T \to \infty$). Moreover, it is also independent of the price level at t_0+1 and only depends on real factors, as also in general shown by equation (43).

Equation (51) then clearly shows that also in this environment, the same analysis of the previous section applies, from a qualitative perspective. Again, the relevant financial position of the household takes into account the component of human wealth influenced by the remittance policy of the central bank, through $\mathcal{T}^C$. If the latter satisfies $\mathcal{T}^C > \widetilde{N}_{t_0}$, the private sector is a net creditor with respect to the nominal wealth position that matters for its spending decisions. Accordingly, an increase in the price level at t_0+1 reduces the demand for consumption through a negative wealth effect, and the aggregate demand schedule is again a downward-sloping function, as in Figure 2. Cutting the central bank's net worth using nominal securities at time t_0, or committing to rebate a larger share of seigniorage revenues α, reflates the economy by expanding the demand for long-run consumption, and taking the economy into equilibrium E', along the lines discussed in the previous section.

Moreover, in this environment the central bank has access to yet another policy option, which acts on the denominator of (49). Indeed, a reduction in $\mathcal{G}_{t_0}$ signals a weaker willingness of the central bank to actively use its gold portfolio to back its nominal liabilities. The example introduced earlier clarifies that the central bank can do this in two ways. Either by committing to mobilize a smaller share of its gold portfolio at a given point in the future (i.e. a higher λ) or by delaying the future date at which it will use a given share of its gold holdings to redeem its nominal liabilities (i.e. a higher T). In either case the effects are analogous to those of an increase in α: the amount of real assets that the central bank is willing to use to back the central bank's nominal liabilities falls, thus depleting their exchange value and reflating the economy. In the AD–AS logic of Figure 2, lowering $\mathcal{G}_{t_0}$ by increasing either λ or T drives up the equilibrium real price of gold q_{t_0+1} – as implied by equation (52) – and therefore yields a positive wealth effect that shifts the aggregate demand outward and pushes up the equilibrium price, as implied by equation (51).

7 Conclusion

This paper studies the economics behind policies available to reflate an economy out of a slump. We discuss a set of policy actions that are all equivalent to the standard specification of "helicopter money", and characterize the alternative mechanisms at work depending in particular upon specific institutional arrangements between the central bank and the treasury.

We have kept our model as simple and tractable as possible. Several extensions can address the limitations of our analysis. First, a more elaborate dynamic extension could be helpful to understand the effectiveness of policies even in the medium run, and would allow to capture the endogenous duration of the ZLB policy depending on the policies used to reflate the economy, along the lines of Eggertsson and Woodford (2003). This extension could interestingly break the equivalence between some of the policies we discuss. A second important assumption of our framework is the lump-sum nature of transfers or taxes: this is motivated by the observation that fiscal policy can also be in a trap under certain shocks that bound the availability of effective tools to just lump-sum transfers.[12] This assumption diminishes the effectiveness of fiscal policy, when the central bank does not fully back its liabilities, because Ricardian equivalence holds. Assuming distortionary taxes or productive public spending can, in general, give more role to fiscal policy to boost the economy out of the slump, as discussed by Eggertsson (2011). It would be interesting to compare the effectiveness of alternative fiscal tools with those explored in this work.

[12] This was the case during the Great Lockdown.

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A Appendix

A.1 Derivation of equation (30)

We use the following preference specification:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\ln C_t + \theta \ln \frac{M_t}{P_t} \right].$$

Consider equation (9),

$$\sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(C_t + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right) = \frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(Y_t - \frac{T_t}{P_t} \right).$$

and recall that in the long run $\xi_t = \bar{\xi}$ for each $t \geq t_0 + 1$. Note that, under this preference specification,

$$\frac{M_t}{P_t} = \theta C_t \frac{1+i_t}{i_t}$$

and moreover that $R_{t_0+1,t} C_t = \beta^{t-t_0-1} C_{t_0+1}$. We can then write (9) as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(Y_t - \frac{T_t}{P_t} \right) \right\}.$$

Use the assumption of constant endowment and real tax policy $T_t/P_t = \tau_t$, we can write it as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{W_{t_0+1}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t} \left(Y - \tau_t \right) \right\}. \tag{A.1}$$

The above is the consumption demand, given income and policy, and for a given sequence of the real interest rate, captured by the discount factor $R_{t_0+1,t}$.

A.2 The case of real remittances

Consider the case in which the central bank can set a path for real remittances according to the rule

$$\frac{T_t^C}{P_t} = \tau_t^C + \alpha \frac{i_{t-1} M_{t-1}}{P_t} \tag{A.2}$$

Then we can write the equilibrium condition (25) as

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau_t^C = (1-\alpha)\mathcal{S}(Y,\Pi) + \frac{Q_{t_0}D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0}}{P_{t_0+1}},$$

which then determines the long-run price level $\bar{P}$ at

$$\bar{P} = \frac{N_{t_0}}{\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau_t^C - (1-\alpha)\mathcal{S}(Y,\Pi)}, \tag{A.3}$$

where $N_{t_0} \equiv Q_{t_0}D_{t_0}^C + B_{t_0}^C - X_{t_0} - M_{t_0} > 0$. In this scenario, the control of a real stream of resources through the remittance rule (A.2) is able to uniquely pin down the price level at finite values even in the limiting case $\alpha = 1$, or when the economy is cashless (i.e. $\mathcal{S} = 0$).

Notice however that the sign of the nominal net worth of the central bank (which mirrors the relevant net financial position of the private sector) becomes now key. Assume first that nominal net worth is positive. For the price level in (A.3) to be positive, the denominator of (A.3) should be also positive: the remittance policy should be set in this case so as to ensure that the stream of real remittances is smaller than the stream of real seigniorage.

The policy options available to the central bank to reflate the economy are again several. First, it could act on the numerator of (A.3), by raising its net worth, *ceteribus paribus*. This can be accomplished by reducing short-run transfers to the treasury, as shown by (34), which implies higher current taxes for the private sector.

Combining (9) and (15), under the real remittance policy and the simplifying preference specification used in the Appendix, allows to write the aggregate demand function as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta}\left\{-\frac{N_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} R_{t_0+1,t}(Y+\tau_t^C)\right\}. \tag{A.4}$$

This demand function clarifies that the net asset position of the consumer, which can be considered as wealth, mirrors that of the central bank only: if the central bank's net worth N_{t_0} is positive then the private sector is a net debtor with respect to the wealth position that matters for its consumption. The demand function has now a different shape simply because the consumer has a negative position with respect to financial wealth. For a net debtor, indeed, an increase in the price level reduces the real value of his/her obligations, thereby pushing up consumption: equation (A.4) now implies a positive relationship between long-run consumption and the price level (*AD* schedule in Figure 3).

Consider now an increase in the net asset position of the central bank, produced by a cut in remittances at t_0 (and therefore a short-run monetary contraction). In a specular way, this

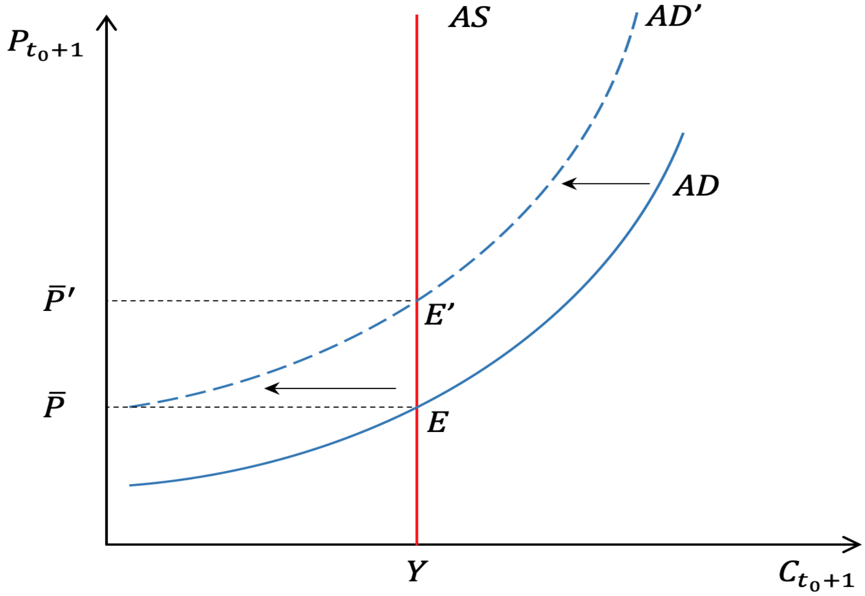

Figure 3: Reflating the economy when the central bank acts alone: the case of positive net worth.

implies a deterioration of the net debt position for the private sector and a negative wealth effect which induces a fall in demand, at the initial price level, and an excess supply of goods: the AD schedule shifts to the left into AD'. Since the agent is a net debtor, therefore, in order for the constant endowment to be entirely absorbed by consumption, such excess supply now stimulates a rise in the price level that can ease the real debt burden on consumers and stimulate their demand up to the point where it is equal to supply (i.e. $\bar{C} = Y$) in the new equilibrium E'.[13] This result does not depend on some of the simplifying assumptions of this Section, namely a two-period model with exogenous output, fully rigid prices in the short run and flexible in the long run. Indeed, this finding will extend unchanged to the benchmark New-Keynesian model.[14]

To get an intuition of this apparently counterintuitive result, we notice that it echoes a popular proposition in monetary economics, the "unpleasant monetary arithmetic" of Sargent and Wallace (1984). There, too, a monetary contraction in the short run ends up producing more inflation eventually. The parallel is interesting because the mechanism is technically similar, while its economic significance is very different. In Sargent and Wallace (1984), the

[13] The alternative policy options to achieve the same allocation work through changing the denominator of (A.3). The central bank could commit to reduce the present-discounted value of real remittances transferred in the long run, which at the end means higher taxes for the households. But the mechanism is similar as above, since the reduction in the present-discounted value of net income for the households deteriorates their overall wealth position at the initial price level. Therefore an increase in the price level is required in the new equilibrium to reduce the real value of the financial liabilities of the household and compensate the fall in human wealth. By the same logic, committing to an increase in future seigniorage revenues can now reflate the economy.

[14] Results are available upon request.

underlying key condition for the "unpleasant" result is fiscal dominance and an active fiscal policy: the monetary tightening in the short run sets the public debt on a diverging path; then, fiscal dominance and the need to restore solvency of the government in the long run imply that, eventually, the growth rate of money needs to increase in order to finance the fiscal deficit and offset the net financial position of the government. Importantly, in Sargent and Wallace (1984) the budget constraint of the public sector is consolidated, which rationalizes the need for money to adjust eventually: the central bank is backing the treasury's liabilities.

Here, instead, fiscal policy is passive, so we are in a monetary dominance world. And the central bank faces an independent and separate budget constraint. Where is the similarity then? Although fiscal policy is passive, the remittance policy is instead active. Indeed, the monetary dominance in this setting includes two sub-regimes: one in which remittances are passive to ensure stationary real net worth for any price level (which leaves the price level indeterminate), and one in which they are active, meaning they are unrelated to the path of net worth (and provide the anchor that determines the price level). In the latter case, the real net worth will follow an analogously diverging path, requiring the price level to adjust to restore equilibrium. In this case the short-run monetary tightening sets the real net worth "at the initial price level" on a diverging path, implying that the net financial position of the private sector keeps deteriorating. Long-run solvency of the private sector then requires, eventually – i.e. at $t_0 + 1$ – that the central bank reverts the tight money through higher nominal remittances that support a higher price level (consistently with its exogenous real remittances policy) and restore solvency of the private sector in real terms.

Anyhow, equations (A.3) and (A.4) suggest that there are other tools available to the central bank, which would also work and relate more directly to policy options discussed in the literature, such as "helicopter money". The central bank makes in this case a sufficiently large transfer to the private sector financing it through higher seigniorage in the future. The transfer should be large enough to turn its net worth negative, the numerator on the right-hand side of (A.3). As implied by equations (11) and (34), there are two ways to turn N_{t_0} negative. The first is to make a direct transfer by writing off some of the assets held from time $t_0 - 1$, the ones issued by the private sector. This has a direct positive wealth effect on the private sector without any involvement by the treasury. The second is to make an indirect transfer by increasing the remittances to the treasury or by writing off part of treasury's debt held in its portfolio. The larger resources obtained by the treasury can be rebated to the private sector through a matching tax cut, to satisfy equation (15) with equality.[15]

The private sector, therefore, experiences a positive wealth effect in both cases. It is

[15] See Benigno and Nisticò (2020) for proof that a tax rule satisfying restriction (15) with equality requires the treasury to rebate to the private sector any remittances received by the central bank, period by period.

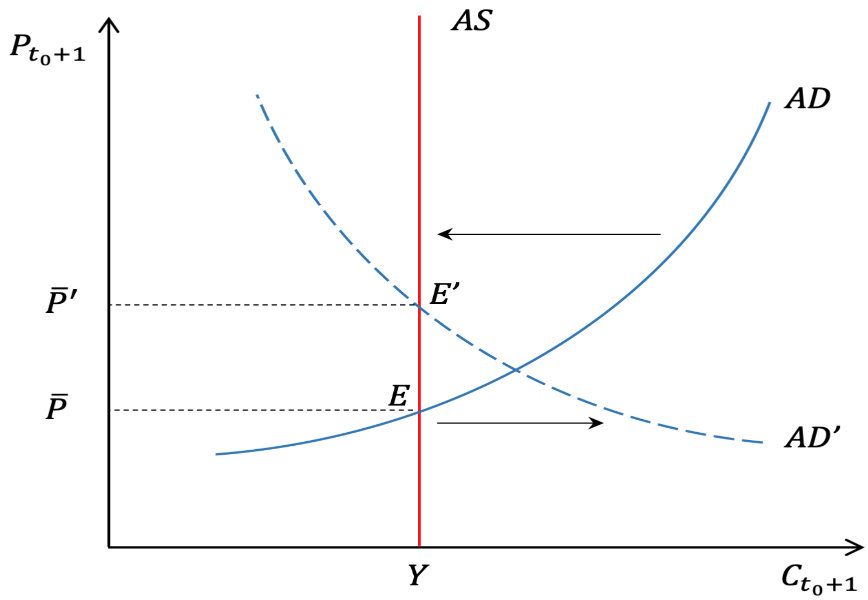

Figure 4: Reflating the economy when the central bank acts alone: "helicopter money" through negative net worth.

important to emphasize, however, that the key mechanism behind this version of "helicopter money" relies on turning the private sector into a net creditor with respect to the financial securities that can be considered as wealth. Under this condition, indeed, the excess demand of goods induced by the positive wealth effect is able to stimulate an increase in the price level that reduce the real value of the private net asset position and allow demand to meet supply. On the contrary, a positive wealth effect on the private sector that is not so large to make it a net creditor would not work in reflating this economy if the central bank controls a stream of *real* remittances. An increase in the price level, indeed, would improve the financial position of the private sector and exacerbate the excess demand even further. In this case, instead, a fall in the price level is required in order to worsen the net debt position and absorb the excess demand.

Equation (A.3), moreover, shows that the proposed policies should be complemented with further actions in order for the price level to be positive and consistent with an equilibrium. Indeed, if the numerator in (A.3) turns negative, so should the denominator. Therefore, it should be that

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \tau_t^C < (1-\alpha)\mathcal{S}\left(Y,\Pi\right),$$

which can be obtained by either lowering the present-discounted value of the remittances in the long run or by raising retained seigniorage revenues through a reduction in the share α, or an increase in the inflation target, if seigniorage is on the left side of the Laffer curve.

Figure 4 shows that these policies work in a similar way as the "helicopter money" experiment. Like the latter, indeed, the central bank's transfer at t_0 is reflected into an improvement in the long-run net financial position of the private sector and implies a positive wealth effect that boosts aggregate demand in $t_0 + 1$ and shifts the AD schedule to the right. However, in this case, key is that the improvement in the net financial position of the private sector is large enough to turn it into a net creditor. Indeed, the reason why upward pressures on aggregate demand turn out to be inflationary now (as opposed to before) is that turning the private sector into a net creditor not only shifts the AD schedule to the right, but it also flips it into a downward-sloping curve. It is precisely this switch in the slope of the AD schedule that allows the central bank to reflate the economy through an upward pressure on long-run aggregate demand: since the economy is already at full capacity, indeed, the surge in demand stimulates an increase in the price level that reduces consumer's real wealth and brings consumption back the output level.

A.3 Robustness

In this Section we discuss the robustness of our analysis along a longer duration of fixed prices. Let us consider a short run lasting two periods, t_0 and $t_0 + 1$, instead of one as in the benchmark case. The long run is therefore shifted forward in period $t_0 + 2$. The analysis can be easily generalized to a longer short run. As before, in the short run, prices are sticky, therefore $P_{t_0} = P_{t_0+1} = P$ and the preference shock is at the low level, $\xi_{t_0} = \xi_{t_0+1} = \xi$; in the long run, the preference shock is at the high level and therefore $\xi_t = \bar{\xi}$ for each $t \geq t_0 + 2$. Inflation is on target after $t_0 + 2$ and the price level at time $t_0 + 2$ is $\bar{P}$. By writing the Euler equation at time t_0 and using the simplifying assumption of log consumption utility we get

$$C_{t_0} = \frac{1}{\beta(1 + i_{t_0})} C_{t_0+1}$$

in which we have used the two assumptions that $P_{t_0} = P_{t_0+1} = P$ and $\xi_{t_0} = \xi_{t_0+1} = \xi$. At $t_0 + 1$ the Euler equation instead reads as

$$C_{t_0+1} = \frac{1}{\beta(1 + i_{t_0+1})} \frac{\xi}{\bar{\xi}} \frac{\bar{P}}{P} Y$$

where we used the appropriate specifications of prices and preference shocks between short and long run and we set $C_{t_0+2} = Y$. Combining the above two equations we get:

$$C_{t_0} = \frac{1}{\beta^2(1 + i_{t_0})(1 + i_{t_0+1})} \frac{\xi}{\bar{\xi}} \frac{\bar{P}}{P} Y.$$

Under the assumption that

$$\beta^2 \frac{P}{\bar{P}} \frac{\bar{\xi}}{\xi} > 1,$$

we can replicate the analysis of the previous sections and note that, even in case interest rates are zero in both periods t_0 and $t_0 + 1$, consumption remains below output in the short tun. Having set these short-term rates to zero, the only way policymakers can raise C_{t_0} is by lifting off the long-run price level $\bar{P}$. Therefore the analysis will follow similar lines of previous sections where what matters for the determination of the long-run price level is the government's asset/debt position that will be carried in period $t_0 + 2$.[16]

[16] The only, obvious, difference with the previous analysis is that the negative preference shock needs now to be stronger than in Section 2 in order for the ZLB to bind.