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EDUCATIONAL INEQUALITY, ASSORTATIVE MATING AND WOMEN EMPOWEREMENT

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Abstract

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JEL Classification: E0, E5, G01

Keywords: educational inequality, women empowerment, collective bargaining

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Abstract

Using PISA data for all waves and countries, it is shown that family cultural and economic background has bigger influence than school characteristics and quality on adolescents' math, reading and science scores. Women education, a proxy for women empowerment, has an added and increasing effect, when controlling for assortative mating. Their added value peaks at intermediate levels of education, but declines afterwards, when controlling for educational homogamy. A model with households' collective bargaining, warm glow preferences and human capital accumulation can rationalize the evidence. Through the lens of the model, mothers' higher impact is due either to higher devotion to child-rearing, which increases in presence of a gender wage gap, or to a within-household bargaining that raises in education, or else the empowerment externality.

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1 Introduction

Understanding the determinants of educational inequality is valuable, at the least so since this is a predominant part of income inequality, one of the most worrisome recent trends¹. Family background can influence educational attainments on top and above school systems, making it hard for a society to foster inter-generational mobility and revert inequality trends. Additionally, there is evidence of a correlation between mothers' and children's educational attainments, though causality hasn't been proven yet². Appraising the role of family, relatively to schools, and of women empowerment, in affecting children's educational attainment is the aim of this paper.

An accurate answer to such a question requires data with enough cross-sectional and time-series variation. A cross-country perspective is essential to avoid conditionality onto institutional and cultural backgrounds. And a comprehensive account requires data that allows the researcher to control for other likely determinants. Such requirements are comfortably satisfied by the PISA survey data set. This includes the coverage of up to 73 countries, several waves and numerous variables on family, school and other backgrounds. Not less important, PISA provides comparable, across countries and regions, scores in math, reading and science for adolescents. At last, the break up, for some key variables such as education, between mothers and fathers allows me to quantify the added contribution of the first, while controlling for assortative mating.

Results show that family cultural and economic background influences children's attainments significantly more than school characteristics³. An exception is teachers' quality. Family cultural background counts more than the economic one. Results are robust to the inclusion of either family income or wealth. Riveting is the fact that books and art imprints are more important than ICT devices. Furthermore, I find robust evidence that mothers' education, a widely agreed proxy for their empowerment, has an additional impact relatively

¹See OECD[45], Becker and Chiswick[8]. Educational inequality raises income inequality when its returns exponentially raise with it, see Goldin and Katz[34]

²See OECD[46]. Experimental evidence on this link is in Decker et. al. [23]

³Given the extensive availability of variables in the surveys, regressors are selected through principal component analysis to jointly minimize omitted variable bias and multi-collinearity.

to the fathers' one⁴. Unsurprisingly, mothers' impact raises with their education. In the baseline OLS specification it is possible that mothers' additional effect is confounded by the sorting of high income and high education couples. However, most compelling is the fact that mothers' impact is still significant even when controlling for assortative mating through the use of propensity score matching⁵. In a further specification I control specifically for educational homogamy, by including interaction dummies constructed by combining the six levels of parents' education. In this case mothers' additional effect peaks at intermediate levels and declines afterwards. This suggests that, when controlling for educational sorting, traditional channels such as higher mothers' devotion to child rearing might be predominant.

The results above can be rationalized through the lens of an OLG model where households' decisions are collectively bargained, in which parents hold warm glow preferences and invest in their children's education, and in which future generations' human capital depends upon their parents' one⁶. The last assumption provides the link between children's attainments and family background. The collective bargaining allows me to disentangle the separate role of women empowerment onto the educational investment decision for their children, particularly so in a specification where I allow for women's bargaining to depend upon their education ⁷. The model also includes the traditional trade-off between work-time, with market wages increasing with education, and child-rearing time, whose returns manifest in future generations' utility. This is important as this trade-off changes at different levels of mothers' education and implies that child-rearing devotion peaks at intermediate levels of education.

Analytically and numerically, it is shown that parents' education is a major determinant of their children's one. Higher labour earnings translate into higher investment in education, hence into higher human capital for future generations. In the model, mothers' human capital counts relatively more due to either higher devotion to child-rearing, an effect which is even

⁴Possible non linear effects are accounted for by refining parents educational background in six different levels.

⁵Abadie and Imbens[1]

⁶The model follows the tradition of Becker and Barro [7] and Barro and Becker [6].

⁷The endogenous bargaining follows the specification of de la Croix and Donckt [27]

higher in presence of gender wage gap, or when women's bargaining power increases with their education. In the devotion channel mothers' impact raises with education because of improved quality-time, but declines beyond a certain threshold when wages include a market premium for education⁸. The wage premium steepens the trade-off between working and child-rearing time. When women's bargaining depends on their education, the empowerment channel, mothers' added value comes from their ability to influence households' decisions, which includes investments in children's education. As the value functions of different genders in future generations are weighted by the genders' bargaining weights, mothers internalize the effects of the educational choices on female off-springs, who are themselves in charge of the child rearing. This gives raise to an empowerment externality. This second effect is however quantitatively small ⁹, to the extent that no additional preference weight is given to different genders in future generations and in presence of a wage gender gap ¹⁰

The rest of the paper proceeds as follows. Section 2 reviews the literature. Section 3 presents the data, empirical specifications, and their results. Section 4 presents the model and its results. Section 5 concludes.

2 Literature Review

Empirically the paper is linked to a group of papers addressing the determinants of educational attainments. Several papers study the link between educational attainments and parental income or wealth. Many focus on individual countries, by exploiting exogenous changes in regulations or other quasi-natural experiments¹¹. Interestingly, many find little role for economic background and conjecture that other inherited¹² or cultural characteristics might play a role. While the use of exogenous changes in regulations is compelling, the limit of

⁸The importance of parental time is uncovered in time-use surveys, see Aguiar and Hurst[3], and especially for highly educated mothers, see Bianchi et. al.[11] and McLanahan[41]

⁹Mothers' added value might likely come from additional channels, such as role models or other cultural transmissions. Those are not considered in my model since they are not quantifiable through the variables available in the PISA survey

¹⁰As standard in two sexes OLG models, parents invest less in their daughters' education since women devote less time to the labour market and this reduces the returns to girls' education

¹¹See Black, Devereux, and Salvanes[13], Bleakley and Ferrie[14], Chevalier, Denny, and McMahon[17], Dahl and Lochner[22], Hertz et al[37] or Rothstein and Wozny[52].

 $^{^{12}}$ Krapohl et. al. [44].

the single country perspective prevents to reach conclusions which are independent from the institutional or societal background. Separately other authors have examined the role of schools ¹³ or of local social networks ¹⁴. Contrary to all of those, this paper takes a cross-country perspective, confronts parental cultural¹⁵ and economic background and compares the roles of family and school. Moreover, it interacts and compares gender roles in family achievements. All of the aforementioned analyses are possible by using the PISA data.

Past empirical evidence exists on the positive correlation between mothers' and children's education and cultural values ¹⁶. This includes mostly field experiments ¹⁷. The cases are usually again country-specific. The break down, available in the PISA data, between mothers' and fathers' education allows me to identify their separate roles for the largest set of countries.

Theoretically, the paper is related to the literature that builds on the altruistic parents models from Becker and Barro[7] and Barro and Becker[6] and studies the consequences of intra-households' decisions for future generations. The model is extended also to account for women's bargaining power to depend upon their education level, an empowerment channel, that helps to account for mothers' added value to children's education (The specification with endogenous bargaining follows de la Croix and Donckt[27].). The collective bargaining set-up makes the model immune to the criticism surrounding the unitary decisions set-up¹⁸ and it allows me to separately identify the power of mothers and fathers in households' decisions¹⁹.

3 Empirical Analysis

The first goal of the empirical analysis is to assess the role of the family cultural and economic background against the role of school characteristics. The answer has important policy

¹³See Ellison and Swanson[31] among others

¹⁴Currie and Moretti^[21]

¹⁵See Giuliano et al.[33] for the importance of cultural attitudes.

¹⁶See OECD[46]

¹⁷Decker et. al.[23] or Dohmen et. al.[26]

¹⁸see Chiappori [18], [19] and Knowles [40] and Kalai [39]

 ¹⁹The importance of women's empowerment for intra-household allocations is examined in Edlund and Lagerlöf[30]and Miller[42] and in Doepke and Tertilt[24]for the choice of empowered political regimes.
 ²⁰See Pollack[49].

implications as it addresses the ability of societies in reverting inherited inequality trends. The second goal is to assess the additional contribution of mothers to children's education, after controlling for assortative mating. This control is important to rule out the possibility that the higher correlation between mothers' and off-springs' education is driven by parents' assortative mating or more specifically by their educational homogamy.

3.1 Data and Econometric Specification

The data for the analysis is collected from PISA - the Program for International Student Assessment²¹, which has been conducting tests on students aged between 15.3 and 16.2 and surveys of students, families and schools since 2003. Students are assessed in science, mathematics, reading, collaborative problem solving and financial literacy. I focus on the first three scores. The data set has several advantages. It is the only comparable cross-country data set on educational attainments, hence its use makes results unconditional on institutional or societal background. It provides temporal variation through many waves. And the three joint questionnaires on students, families, and schools provide a very broad range of possible scores' determinants, hence minimizing the risk of unobserved heterogeneity. For instance, variables on family characteristics include economic background, as well as cultural and other households' resources, parents' education, occupation and time devoted to children. School variables include information on infrastructure, teachers' quality, etc. Since several variables might provide overlapping information and to avoid multi-collinearity, regressors are selected based on a principal component analysis²². The extracted factors explain more than half of the variation in the group. A final advantage, particularly of the PISA score, is their comparability across countries and school systems. The scores are based on a two-hour computer- or paper-based test. Different groups of students answer different, yet overlapping, sets of items. This serves the purpose of adapting questions to different countries and institutional contexts. Furthermore, to obtain maximum comparability, the data is adjusted

²¹See https://www.oecd.org/pisa/pisaproducts/.

²²Combinations of related characteristics are subject to factor analysis with varimax rotation.

by computing multiple imputations, using so called plausible values ²³. Details on this are in Appendix A.

The baseline econometric specification regresses math, reading and science scores at a time on family and school factors, controlling for country fixed effects²⁴ and for the children's gender and reads as follows²⁵:

$$y_i = \mu_i + \beta_0 + \beta_1 M E D_i + \beta_2 F E D_i + \Pi_1 X_i + \Pi_2 Home_i + \Pi_3 School_i + \varepsilon_i \tag{1}$$

where the index *i* indicates the household, y_i is the PISA score of student *i*, and can alternatively be the math, the science or the reading score. Regressions are run either by pooling observations for all surveys from 2003 to 2013, which contain the same set of variables, or for the 2015 survey alone, which contains an expanded set of variables. The advantage of the pooling is the extensive temporal and cross-sectional variation. The advantage of using the 2015 survey is its expanded set of controls. The regressors include the variables MED_i and FED_i , which are dummies for mother's education and father's education, respectively. To allow for non-linear effects, six different education levels for each parent are considered. The group of regressors labelled as $Home_i$ contains variables related to domestic resources and family educational choices. To avoid multi-collinearity the set of regressors is selected through a principal component analysis (see Appendix A)²⁶. In the benchmark econometric specification family financial background is measured through wealth, as it is available for all countries. For robustness, regressions with income, instead of wealth, are also estimated. In this case the sample is limited to the set of reporting countries. (See Table 1 in Appendix A). The regressors labelled *School*_i represent factors for school characteristics and quality ²⁷

²³For robustness the regressions below are estimated on both imputed and non-imputed values. No significant difference emerges. See also Dustmann et. al.[29].

²⁴Since PISA scores have been made comparable across systems, country fixed effects capture the residual variation, such as local networks (see Currie and Moretti[21]).

²⁵The data are re-scaled and adjusted using REPEST STATA. See Appendix A.

²⁶Factors for the surveys pooled from 2003 to 2013 include cultural possessions at home, home educational resources, ICT resources, home possessions. For the 2015 survey additional factors are parents' communication, support in science, choice of school based on performance, on costs, on religion or teaching/pedagogical approaches.

²⁷Those include teacher/student ratio, percent of certified teachers, and with graduate degree, inadequacy of administrative stuff or of teaching stuff. The 2015 survey also includes school infrastructure, percentage of teachers with master or bachelor, total number of all teachers at school, class size, whether the school is public or not and the percentage of school funds coming from government.

Finally, the set of controls, X_i , includes student gender, student age, and whether he/she is native or not. In this and the following specifications robust standard errors are clustered at the school level.

Beyond the baseline specification two others are considered. Both allow me to separately identify the added value of mothers and to control for assortative mating or educational homogamy. The first, which controls for assortative mating in general, is two-stage procedure based on propensity score matching analysis. The second, which is more targeted to control for educational homogamy, includes in the baseline specifications interaction dummies obtained from combining the the parents' six education levels. Results are presented next and in sequence.

3.1.1 Results Benchmark Specification

The results for the benchmark specification and for the pooled waves 2003-2013 are reported in Table 3. Results are reported separately for math, science and reading. Each sub-column presents results for alternative measures of family economic background, namely wealth, income and home resources. Results for the 2015 wave are in Table 6. Therein financial background is measured by income. Results are as follows.

First, family characteristics, especially income, wealth, cultural possessions and educational resources have a bigger impact on children's educational attainments than school characteristics. Parents' education is generally significant and positive. The coefficients on school characteristics are very small and in many cases insignificant. This is even more so if the regressions include income or resources. Only exceptions are for the 2003-2013 sample, whereby the teacher/student ratio seems to be occasionally significant, and for the 2015 sample, whereby the total number of teachers, their graduate degree and the class size seem to play a role. Interestingly, for the 2015 sample, public schools have a negative and significant coefficient.

As for family variables, in the 2015 sample, cultural possessions are significant, while home possessions are not, an indication that cultural transmission is stronger. In both data samples, ICT resources are either insignificant or have a negative impact, possibly another indication of the importance of cultural in-heritage. Finally, coefficients on wealth and income are positive and significant for the 2003-2013 sample. Wealth becomes insignificant for the 2015 wave, hence Table 6 reports results only for income²⁸.

Next, I examine the separate roles of mothers' and fathers' education. For the pooled regression, the mothers' effect is larger in most of the cases. The fathers' effect falls mildly when income or resources are included in the regression²⁹. This is likely due to a larger correlation between fathers' education and job-status. In the 2015 wave regression the impact of fathers' education becomes insignificant. In this case it seems that the "time spent in child rearing" (communication, parents' support in sciences, etc.) is absorbing the fathers' contribution. Unfortunately, the survey does not specify the exact share of child-rearing per parent. To fully assess the added value of mothers, however, regressions should control for assortative mating or for educational homogamy, something that I will describe in the next section.

Furthermore, it is of interest to highlight additional results. Native and older students do better, female students tend to do better in reading, while males do better in math ³⁰ and the time that parents spend in general communication with their children produces better results than helping with science, the latter being a sign that independent learning has a value. Finally, the choice of school based on academic performance is more beneficial than the one based on economic reasons or religious beliefs.

3.2 Propensity Score Matching Estimator

Mothers' education might matter more since highly educated mothers tend to assort with highly educated and highly earning fathers. This might be due either to assortative mating along several dimensions, like, e.g., education, income, wealth or to just educational homogamy. Next, I therefore quantify mothers' added value controlling for both. I start with assortative mating in general through propensity score matching techniques. The latter allows researchers

 $^{^{28}}$ When more variables are included in the specifications, these seem to capture most of the effects channelled through wealth

 $^{^{29}\}mathrm{Results}$ are less significant for income due to more limited sample size.

 $^{^{30}\}mathrm{See}$ Guiso, Monte, Sapienza and Zingales [35] among others for similar results on this.

to control for possible confounding factors, including assortative mating. For the propensity score matching the control and the treatment group are divided into bins which share certain characteristics. The treatment group in our case consists of mothers with high education, the control group is identified by fathers' education levels. Treatment and control groups are linked based on shared characteristics. The estimator of the treatment effect for every bin is then simply the difference between the outcome variable for that bin and the outcome variable for its matched counter-part. The global estimator for the average treatment effect (ATE) is the sample average of the treatment effect, hence in this case all couples in which the mother has a high education ³¹.

With potentially many confounding factors, the curse of dimensionality typically complicates the construction of bins, which is based on the choice of covariates. I follow the method in Rosenbaum and Rubin [53], which uses a unified distance function, namely the propensity score. The latter is defined as the conditional probability of receiving a treatment, in this case mothers having a better education, given a set of explanatory variables. Implementation takes place through a two-stage procedure (see Cameron and Triverdi[15]). In the first step, a logit model for the treatment, mother's education, as the outcome variable is estimated using fathers' education, wealth and country fixed effects as counfounders. This parsimonious specification controls for assortative mating and possibly other major confounding factors, while limiting the bias due to arbitrary choices of the covariates. The fitted values from the first stage characterize the propensity score, which summarizes the pre-treatment controls. In the second stage, the following econometric specification is estimated with OLS³²:

$$y_i = \mu_i + \beta_0 + \beta_1 MED_i + \beta_2 PSCORE_i + \Pi_1 X_i + \Pi_2 Home_i + \Pi_3 School_i + \varepsilon_i$$
(2)

This second stage regression includes mothers' education, school variables and the previous controls, while the impact of other family variables is captured by the propensity score. Table 5 and 8 show results for the pooled regression and for the 2015 one, respectively. Both Tables

³¹The estimator exists and is consistent under certain conditions, which are the ignorability assumption, the overlap assumption and random sampling. See Abadie and Imbens[1].

³²Estimators are carried on a random choice of the imputed values.

unequivocally exhibit a higher coefficient on mothers' education for the math and science scores, the only exception being the coefficient on the reading score for the pooled regression.

3.3 Interaction Dummies for Assortative Mating

To make the above results independent from specific techniques, this section adopts an alternative specification, which consists in the inclusion of interaction dummies in the baseline regression, given by the combinations of the six education levels for mothers and fathers³³. This specification is also more directly tailored to control for educational homogamy. Table 4 and 7 show results for the usual samples. As before, mothers' contribution seems to prevail and increase with education levels. This time however, the pooled regression also shows that the effect of mothers' education peaks at intermediate levels. This puts forward two explanations. The first is a compositional effect, as females' presence in the highest levels of education is thin. The second is that mothers contribute more to child-rearing time, relatively to fathers. If so, this effect is more likely to emerge at intermediate levels. Females at the highest level of education face a steeper trade-off between the wage premium and the benefits of child rearing and they might marginally reduce the latter.

4 An OLG Model with Children's Education Investment and Empowered Women

Two main results emerge from the evidence above. Family variables have bigger influence than school ones; mothers' education adds value on the margin. In this section, I show that an OLG model with warm glow preferences, human capital accumulation and collective bargaining for households' decisions can rationalize and replicate both patterns³⁴. Warm glow preferences provide an incentive for parents' investment in education, as the latter affects future generations' utility. Human capital accumulation provides the link between parents' and children's education levels. Children's human capital depends on parents' education and on a production function that depends upon child-rearing time. It is assumed that

³³The omitted category in this setting is both parents having low education.

³⁴The model follows the tradition of Becker and Barro [6] and Barro and Becker [7]

mothers spend relatively more time than fathers in children's education³⁵. This is the first channel capturing mothers' added value. The collective bargaining³⁶ allows me to separate the role of mothers' bargaining power into children's education investment decisions. In an extension of the model, women's bargaining power is endogenized with respect to their human capital³⁷, i.e. more educated women are more empowered. As mothers' bargaining power raises, relatively to fathers', so does the weight that they put on female off-springs, who are themselves in charge of child-rearing. Internalizing this intra-generational externality, mothers bargain more strongly for investment in children's education, which in turn results into higher future generations' human capital. In the model, fertility and education decisions are endogenous in a way that embeds the traditional quantity-quality trade-off and a market wage, increasing with education, captures the classical work-time versus child-rearing time trade-off. The last two elements capture the non-linear nature through which mothers, at different levels of education, differently substitute child-rearing with work time, an effect uncovered in the regressions presented in section 3.3.

4.1 Value Functions and Resource Allocation

Each individual lives for two periods. In the first period they only accumulate human capital, while in the second men and women are randomly matched and form married couples that jointly choose the family consumption, the number of children and the off-springs' educational investment subject to the household's resource constraint. Due to collective bargaining, household members pool resources, but have separate value functions. Hence during adulthood, individuals allocate their time endowment between child-rearing and the labour market. Wages are set in a competitive labour market and increase with human

³⁵See Doepke and Tertilt[24] and others for a similar assumption. The realm of this assumption is confirmed even in recent studies, see Bianchi [10] and Bianchi, Robinson, and Milkie[11]

³⁶It has been argued that the non-unitary bargaining model is better suited to capture intra-households decisions, see Chiappori [18], [19], Knowles[40]. The collective-bargaining is the most reasonable for common households' decisions, such as children's education

³⁷I follow the formulation indicated in De la Croix and Donckt[27].

capital. Individual value functions read as follows:

$$V_t^i = u(c_t^i) + \beta n_t \mathbb{E}_t \left\{ \frac{(1 - \eta_{t+1}^m) V_{t+1}^f + \eta_{t+1}^m V_{t+1}^m}{2} \right\}$$
(3)

where i = f, m is the index for the female or male individual, β captures the degree of altruism towards future generations, c_t^i is individual consumption, n_t is the family's number of children. Parents derive utility also from their off-springs' value function. The weights on the future value functions derive from population size, which is assumed to be half for each gender, and by the respective gender's bargaining coefficients³⁸. No exogenous disparity is introduced among genders, meaning that male and female off-springs are equally weighted. This assumption allows me to isolate the impact of mothers' bargaining power on future generations welfare. Men inelastically supply all their time in the labour market, namely $t^m = 1$, while women bear the child-rearing³⁹. Given a fixed time cost per child, ϕ , women's time allocation at any time t is as follows: $t_t^f + \phi n_t + \phi(e_t^f + e_t^m) \leq 1$, where $\phi(e_t^f + e_t^m)$ is the time cost for educating daughters and sons and where e_t^f and e_t^m is the choice of educational investment⁴⁰. The remaining time is devoted to labour.

Labour earnings are: $w_t^m h_t^m$ and $w_t^f t_t^f h_t^f$, where $t^m = 1$. Wages are set in a competitive market and are a function of work-time and human capital. Firms hire both male and female workers in a competitive labour market and merge their labour supply into a Cobb-Douglas production function, $Y_t = A(h_t^m)^{1-\alpha} (t_t^f h_t^f)^{\alpha}$, implying that:

$$w_t^f = \alpha A_t (h_t^m)^{1-\alpha} (t_t^f h_t^f)^{\alpha-1}; w_t^m = (1-\alpha) A_t (h_t^m)^{-\alpha} (t_t^f h_t^f)^{\alpha}$$
(4)

The wage's dependence on human capital provides incentives to invest in it, but also creates a trade-off between child-rearing and working time.

³⁸These assumptions also make the value function recursive.

³⁹This follows various studies in the literature cited above and has supportive evidence in Bianchi [10]. The extension to the case in which both parents bear part of child-rearing would not change the main channels of the model.

⁴⁰An equivalent formulation would have $\phi(e_t^f + e_t^m)$ as a financial cost entering households' budget constraint. 13

Households choose $\left\{c_t^m, c_t^f, n_t, e_t^m, e_t^f\right\}$ to maximize the aggregate household's value function at every period t:

$$V^{h}(h_{t}^{f}, h_{t}^{m}) = \eta_{t}^{m} V^{m}(h_{t}^{f}, h_{t}^{m}) + (1 - \eta_{t}^{m}) V^{f}(h_{t}^{f}, h_{t}^{m})$$
(5)

which recursively reads as:

$$\begin{aligned} V^{h}(h_{t}^{f},h_{t}^{m}) &= \max_{\left\{c_{t}^{m},c_{t}^{f},n_{t},e_{t}^{m},e_{t}^{f}\right\}} \left\{\eta_{t}^{m}u(c_{t}^{m}) + (1-\eta_{t}^{m})u(c_{t}^{f}) + \\ & \frac{1}{2}\beta n_{t}\mathbb{E}_{t}\left[\eta_{t+1}^{m}V^{m}(h_{t+1}^{m},h_{t+1}^{\bar{f}}) + (1-\eta_{t+1}^{m})V^{f}(h_{t+1}^{f},h_{t+1}^{\bar{m}})\right]\right\} \end{aligned}$$

and where $h_t^{\bar{f}}$ and $h_t^{\bar{m}}$ denote the average human capital in the population⁴¹, subject to the the following budget constraint:

$$c_t^m + c_t^f \le w_t^m t_t^m h_t^m + w_t^f h_t^f (1 - \phi n_t - \phi(e_t^f + e_t^m))$$
(6)

and male and female human capital accumulation, which depends upon parents' human capital and upon educational investment through a Cobb-Douglas specification:⁴²:

$$h_{t+1}^{f} = (Be_{t}^{f})^{\delta} (h_{t}^{f})^{\gamma} (h_{t}^{m})^{1-\gamma}; h_{t+1}^{m} = (Be_{t}^{m})^{\delta} (h_{t}^{f})^{\gamma} (h_{t}^{m})^{1-\gamma}$$
(7)

where $0 < \gamma < 1$, B and δ affect the returns to education, namely $\frac{\partial h_{t+1}^f}{\partial e_t^f} = \delta B(Be_t^f)^{\delta-1}(h_t^f)^{\gamma}(h_t^m)^{1-\gamma}$ and $\frac{\partial h_{t+1}^f}{\partial e_t^f} = \delta B(Be_t^f)^{\delta-1}(h_t^f)^{\gamma}(h_t^m)^{1-\gamma}$. Note that, in parallel with the empirical specification, the technology for human capital accumulation is positively assortative ⁴³.

⁴¹Sons and daughters randomly marry children from other families, hence the average level of the human capital for spouses enters the future value function. See also Doepke and Tertilt[24] for a characterization of parents' optimization as a Nash game in which other parents' choices are taken as given.
⁴²See Ben-Porath [9], Heckman [36], Rosen [51]or Garcia and Heckman [32].

 $^{{}^{43}\}frac{\partial^2 h_{t+1}^f}{\partial h_t^f \partial h_t^m} = (Be_t^f)^{\delta} \gamma (1-\gamma) (h_t^f)^{\gamma-1} (h_t^m)^{-\gamma} > 0,$, which implies that he technology is super-modular with respect to education levels

4.2 Empowering Channel

In the benchmark model, bargaining weights are set exogenously. In an extended version of the model bargaining power is set as a function of human capital. I adopt the functional form proposed by de la Croix and Donckt[27], which reads as follows:

$$\eta_t^m = (1 - \zeta) \bar{\eta_t^m} + \zeta \frac{(h_t^m)^\mu}{(h_t^m)^\mu + (h_t^f)^\mu}$$
(8)

The term $(1 - \zeta)\eta_t^{\overline{m}}$ captures the component assigned by the society to men, while the second term captures the dependence of women's bargaining power upon their education. Women with higher education, by possessing higher critical understanding and psychological self esteem, also possess higher ability to negotiate and to affect households' decisions (see Kabeer[38] and more recently Murphy-Graham[43]).

4.1 Optimality and Equilibrium Conditions

The set of equilibrium conditions is derived for a general specification of the bargaining weight, which will be later specified as either exogenous or endogenous. Under a CES utility with an elasticity σ , the optimality condition for consumption sharing reads as follows:

$$\eta_t^m (c_t^f)^\sigma = (1 - \eta_t^m) (c_t^m)^\sigma.$$
(9)

Consumption functions are obtained by merging 9 with the budget constraint, 6^{44} :

$$c_t^f = \left[1 - \frac{(\eta_t^m)^{\frac{1}{\sigma}}}{(\eta_t^m)^{\frac{1}{\sigma}} + (1 - \eta_t^m)^{\frac{1}{\sigma}}}\right] \left[A_t(h_t^m)^{1 - \alpha}(h_t^f)^{\alpha}(1 - \phi n_t - n_t(e_t^f + e_t^m))^{\alpha}\right]$$
(10)

$$c_t^m = \left[\frac{(\eta_t^m)^{\frac{1}{\sigma}}}{(\eta_t^m)^{\frac{1}{\sigma}} + (1 - \eta_t^m)^{\frac{1}{\sigma}}}\right] \left[A_t(h_t^m)^{1 - \alpha}(h_t^f)^{\alpha}(1 - \phi n_t - n_t(e_t^f + e_t^m))^{\alpha}\right]$$
(11)

 $\overline{ {}^{44}\text{Note that}: w_t^m t_t^m h_t^m + w_t^f t_t^f h_t^f = } \\
= w_t^m h_t^m + w_t^f h_t^f (1 - \phi n_t - \phi(e_t^f + e_t^m)) = \\
= A_t (h_t^m)^{-\alpha} (t_t^f h_t^f)^{\alpha} = A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - \phi(e_t^f + e_t^m))^{\alpha}. \\
15$

Upon defining $\lambda_t = \frac{\eta_t^m}{(c_t^m)^{\sigma}}$ as the Lagrange multiplier on the budget constraint, 6, the first order conditions with respect to the two education levels read as follows:

$$\frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[\frac{Y_t}{t_t^f} \alpha n_t \right] + \frac{\beta}{2} n_t \eta_t^m \mathbb{E}_t \left[\frac{\partial V^m(h_{t+1}^m, h_{t+1}^{\bar{f}})}{\partial e_t^m} \right] = 0$$
(12)

$$\frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[\frac{Y_t}{t_t^f} \alpha n_t \right] + \frac{\beta}{2} n_t (1 - \eta_t^m) \mathbb{E}_t \left[\frac{\partial V^f(h_{t+1}^f, h_{t+1}^{\overline{m}})}{\partial e_t^f} \right] = 0$$
(13)

Using the envelope theorem, by which $\left[\frac{\partial V^i(h_{t+1}^i, h_{t+1}^j)}{\partial e_t^i}\right] = \lambda_t w_{t+1}^i \frac{\partial h_{t+1}^i}{\partial e_{t+1}^i}$, substituting for w_{t+1}^m from 4 and noting that $\frac{\partial h_{t+1}^i}{\partial e_{t+1}^i} = \delta(Be_{t+1}^i)^{\delta-1}(h_t^j)^{\gamma}(h_t^i)^{1-\gamma}$, the above conditions can be expressed as a function of the states. At last, the fertility choice is given by:

$$\frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[A_t(h_t^m)^{1-\alpha} (h_t^f)^{\alpha} \alpha (1 - \phi n_t - n_t(e_t^f + e_t^m))^{\alpha - 1} (\phi + e_t^f + e_t^m) \right]$$

$$= \frac{\beta}{2} \mathbb{E}_t \left\{ V^h(h_{t+1}^f, h_{t+1}^m) \right\}$$
(14)

where $\mathbb{E}_{t}\left\{V^{h}(h_{t+1}^{f}, h_{t+1}^{m})\right\} = \mathbb{E}_{t}\left\{\eta_{t+1}^{m}V^{m}(h_{t+1}^{m}, h_{t+1}^{\bar{f}}) + (1 - \eta_{t+1}^{m})V^{f}(h_{t+1}^{f}, h_{t+1}^{\bar{m}})\right\}$. The full list of first order conditions is in appendix B.

4.2 An Analytically Tractable Version of the Model

The model above can be solved analytically under logarithmic utility and by assuming a separable specification for the number of children⁴⁵. All together this results in the following recursive utility specification:

$$V^{i}(h_{t}^{i}, \bar{h}_{t}^{j}) = \ln(c_{t}^{m}) + \kappa \ln(n_{t}) + \frac{1}{2}\beta \mathbb{E}_{t} \left[\eta^{m} V^{m}(h_{t}^{m}, \bar{h}_{t}^{f}) + (1 - \eta^{m}) V^{f}(h_{t}^{f}, \bar{h}_{t}^{m}) \right]$$
(15)

One implication of the model solution is the following.

Proposition 1. Investment in children's education raises with the fraction of time that mothers devote to child-rearing, captured by the cost ϕ . Higher education levels and higher human capital of parents, both increase human capital for future generations through the accumulation equation.

⁴⁵Specifically, it is assumed that the utility reads as follows: $ln(c^i) + \kappa \ln(n_t)$.

Proof. See Appendix C for the model solution in terms of the policy functions. The latter, 39, and 40, imply that the cost ϕ increases education levels for both daughters and sons for any level of the other parameters.

The model implications outlined in proposition 1 are well in line with the evidence in section 3, which links children's educational attainments to parents' education and the family's cultural background. In the model, this is so for two reasons. First, parents' education enters the human capital accumulation, which also embeds an assortative mating component. Second, parents with higher human capital earn higher wage premia, hence also invest more in education. The magnitude of this effect also depends upon the size of δ , which, by affecting the returns to education, determines the incentives to invest in it. This can be clearly seen from equations 39, and 40 showing that off-springs' education raises with δ . This second channel is consistent with the link uncovered in the data between family wealth and/or income and children's educational attainments.

In the model with exogenous bargaining, the added value of mothers is channelled solely through the extra time spent in child-rearing. In the next section I relax some of the assumptions and solve the model numerically. This also allows me to give further insights in the mothers' added value.

4.3 Quantitative Results

Simulations of the extended model are used here to plot the policy function of children's human capital with respect to both parents' human capital separately. The calibration is chosen partly from past literature⁴⁶ and partly to match facts on education ratios. One period is taken to be 30 years. The parameters, B, and α , are set to 23.38 and 0.345, respectively. Jointly these values determine, for given values of δ and γ , the gender wage gap, which from Bureau of Labor Statistics data, is $w_t^f = 0.8w_t^m$, and the ratio of education expenditure to GDP, which in most countries is around 0.06. The parameter γ in the human capital accumulation is set equally for both spouses to make sure that their innate abilities to transmit human capital are the same. The parameter δ and the time cost parameter ϕ are set

 $^{^{46}\}mathrm{Mostly}$ de La Croix and Doepke
[28] or de La Croix and Donckt
[27]



Figure 1

Policy function of future generation human capital against parents' human capital separately. Case with exogenous bargaining power and baseline calibration

following de la Croix and Doepke [28]. The consumption elasticity of substitution, σ , is set to $\frac{1}{4}$. Following Becker and Barro[7] and Barro and Becker[6], a parameter on consumption, $1 - \sigma$, larger than the parameter on the number of children, guarantees a positive number of children in equilibrium. In the benchmark simulation I set $\eta_t^m = 0.5$. The case with symmetric bargaining power allows me at first to isolate the mothers' added value through child-rearing time. In the case with an endogenous bargaining weight, the parameter μ is set equal to 2 as in de La Croix and Donckt[27]. At last, aggregate productivity, A_t , follows an AR(1) process with persistence of 0.95. This is well in line with the RBC and growth literature.

Figure 1 shows the policy function of future generations' human capital plotted against the two endogenous state variables, namely the human capital of both parents separately. The figure clearly shows that future generations' human capital raises with both parents human capital. Furthermore, the contribution of the mother's human capital to children's education is higher than that of the father. Given the symmetry in preferences and bargaining power, the sole channel responsible for this is the relatively higher mothers' child-rearing time. Next, I examine the effects of the endogenous bargaining specifications. Figure 2 shows the results. As before, human capital of future generations depends positively upon both parents' levels



Figure 2 Policy function of future generation human capital against parents' human capital separately. Case with endogenous bargaining power and baseline calibration

and mothers' human capital counts more. In this case, mothers' human capital induces a slightly steeper gradient. The latter is induced by the increasing weight on female future generations. As the value functions of different genders in future generations are weighted by the genders' bargaining weights, mothers internalize the effects of the educational choices on female off-springs, who are themselves in charge of the child rearing. This gives raise to an empowerment externality. This second effect is however quantitatively small in the model, at least to the extent that no additional preference weight is given to different genders in future generations and in presence of a gender wage gap. Indeed, incentives to invest in daughters education are dampened by the lower labour market returns of women education. It is possible that women empowerement transmits into children education through additional channels, such as role models or other cultural transmissions. Those are not considered in my model since they are not quantifiable through the variables available in the PISA survey.

At last, I examine the impact of the wage gap, by changing the α at all education levels. An increase in wages steepens the trade-off between the cost of child-rearing and the wage premium. Figure 3 shows the results. The substitution between child-rearing and working time results in a reduction of children's education attainments.



Figure 3 Policy function of future generation human capital against parents' human capital separately. Case with exogenous bargaining power and wage gap

4.4 Non-Linear Effects of Human Capital

The regression presented in Table 4, shows that mothers' added value peaks at intermediate levels. Through the lens of the model this result can be rationalized as follows. Mothers are aware that their human capital improves the quality of time devoted to children. At the same time highly educated women forego a much larger wage premium. The two effects combined might explain the decline in the amount of time spent in child-rearing. To appreciate the non-linear effects of the wage premium, Figure 4 plots the usual policy function by assuming a jump in mothers' wage premium from 0.345 to 0.45 from a certain education threshold onward. Mothers' added value is still positive, however, it declines above the threshold, the reason being that mothers in this quantile reduce their child-rearing time. If the latter is not substituted by fathers, as assumed here, children's educational attainments decline.

5 Conclusions

Using the PISA data for all 72 countries and all past waves, this paper finds that family cultural and financial background counts more than school characteristics for children's educational attainments. A result that highlights the obstacles to reverse inter-generational



Figure 4 Policy function of future generation human capital against parents' human capital separately. Case with exogenous bargaining power and wage gap rising at specific education level

inequality. Mothers seem to have a separate added value, even when controlling for assortative mating or educational homogamy. Empirical results can be rationalized through an OLG model with warm glow preferences, collective bargaining in households' decisions and human capital accumulation. The model captures well the qualitative channels behind the empirical facts.

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6 Appendix A. PISA Data, Sampling and Scaling

The PISA study has a 2-step sampling procedure to guarantee a random sample. First, schools are chosen from a comprehensive national list with probabilities which are proportional to an estimated number of 15-year-old students enrolled in the school. In the second stage, the students within each schools are sampled, typically 42 students for computer-based tests or 35 students for paper-based tests. Survey weights, available at the PISA website, ensure that each sampled student represents the appropriate number of students in the full PISA population.

To guarantee cross-country comparability, the selection of questions in the PISA tests is based on a variant of matrix sampling (using different sets of items and different assessment modes), in which each student receives a sub-set of questions from a pool of items. Comparability across tests is guaranteed by using item response theory (IRT) scaling. Specifically, it is noted that some items require similar skills to be addressed. Results are then described in terms of distributions of skill performances in the population.

Comparability can be improved further by using the multiple imputation procedure or plausible values⁴⁷. Plausible values are drawn from an a posteriori distribution by combining the scaled test items (IRT) with a latent regression model using information from the student context questionnaire⁴⁸. The average estimator across plausible values is reported and the imputation error is added to the variance estimator, which allows to retrieve the unbiased estimations. Each of this paper's regression is using score values with and without imputations.

Family economic background is captured either by wealth or by income, the latter used for robustness reasons. Income is available for a sub-set of countries listed in Table 1. The data set for this case contains 42,691 students.

⁴⁷See PISA Technical Report[48].

⁴⁸Stata procedure REPEST is specifically designed to be used with the PISA data set.

Table 1 List of countries included in the estimations when family economic background is proxied by income.

3-character	Freq.	Percent	Cum.
CHL	7,053	8.18	8.18
DEU	6,504	7.54	15.72
DOM	4,740	5.50	21.22
ESP	6,736	7.81	29.03
FRA	$6,\!108$	7.08	36.12
GBR	$14,\!157$	16.42	52.54
GEO	$5,\!316$	6.17	58.70
HKG	$5,\!359$	6.22	64.92
KOR	$5,\!581$	6.47	71.39
LUX	$5,\!299$	6.15	77.54
MAC	$4,\!476$	5.19	82.73
MEX	7,568	8.78	91.50
PRT	$7,\!325$	8.50	100.00

This table displays the summary statistics for the demographic variables and outcomes for the waves 2003-2013. Observations are pooled across waves.

	Wealth	Possessions
father's education - group 1	0.00439^{*}	-0.0248^{***}
	(0.00235)	(0.00880)
father's education - group 2	0.0256***	-0.00359
	(0.00239)	(0.00987)
father's education - group 3	0.0344^{***}	-0.0172^{*}
	(0.00277)	(0.00990)
father's education - group 4	0.0831^{***}	0.0392^{***}
	(0.00235)	(0.0106)
father's education - group 5	0.421^{***}	0.298^{***}
	(0.00338)	(0.0146)
father's education - group 6	0.524^{***}	0.431^{***}
	(0.00298)	(0.0121)
wealth, std	0.0536^{***}	
	(0.000811)	
cultural possessions, std		0.0283^{***}
		(0.00298)
home educ.resources, std		-0.00947^{***}
		(0.00349)
ict resources, std		0.00802^{**}
		(0.00359)
home possessions, std.		0.0364^{***}
		(0.00535)
Country Fixed Effects	Yes	Yes
Year Fixed Effects	Yes	No
No. of Obs.	433715	27793
\mathbb{R}^2	0.311	0.344

Table 2Mother's Education correlated with Father's Educations, Waves 2003-2013

The outcome variable is equal to 1 if mother has university education, and zero otherwise. Controls include wealth indicator, standardized as z-score (column 1) or cultural possessions indicators, standardized as a z-score (column 2), time effects, country effects, gender, age and origin of the student. Estimations are performed using OLS, standard errors are clustered at the school level. Significance levels are * $p \le 0.10$, ** $p \le 0.05$, *** $p \le 0.01$.

Academic Performance of Children given Parental Education, Home and School Resources Table 3

			Math			Science			Read	
		wealth	income	resources	wealth	income	resources	wealth	income	resources
	mother's education - group 1	12.84^{***}	8.485^{**}	8.858	8.899^{**}	11.46^{***}	9.143	10.70^{***}	7.309	6.055
		(3.372)	(3.592)	(6.761)	(3.536)	(3.979)	(6.685)	(3.875)	(5.009)	(6.093)
	mother's education - group 2	11.97^{***}	2.164	4.323	8.780^{**}	1.999	0.0403	9.561^{**}	5.349	1.435
		(3.206)	(4.134)	(6.157)	(3.598)	(4.145)	(6.380)	(3.891)	(4.989)	(5.982)
	mother's education - group 3	21.08^{***}	16.25^{***}	15.64^{***}	19.19^{***}	17.10^{***}	14.64^{**}	18.62^{***}	16.95^{***}	12.25^{**}
		(3.080)	(4.109)	(6.066)	(3.510)	(4.217)	(6.189)	(3.878)	(5.066)	(5.846)
	mother's education - group 4	27.12^{***}	25.00^{***}	20.88^{***}	25.46^{***}	26.39^{***}	22.67^{***}	24.81^{***}	25.32^{***}	18.05^{***}
		(2.855)	(3.873)	(6.117)	(3.150)	(3.973)	(6.185)	(3.887)	(4.887)	(5.950)
	mother's education - group 5	26.53^{***}	24.28^{***}	23.16^{***}	26.11^{***}	26.44^{***}	24.47^{***}	25.86^{***}	30.71^{***}	23.19^{***}
		(3.227)	(4.122)	(6.564)	(3.732)	(4.052)	(6.475)	(3.711)	(5.298)	(6.452)
	mother's education - group 6	36.48^{***}	37.68^{***}	28.00^{***}	36.07^{***}	36.57^{***}	26.06^{***}	32.49^{***}	37.60^{***}	25.67^{***}
		(3.221)	(4.066)	(6.299)	(3.657)	(3.945)	(6.398)	(3.944)	(4.899)	(6.055)
	father's education - group 1	12.41^{***}	9.517^{**}	5.381	13.41^{***}	8.309^{**}	12.26^{*}	16.27^{***}	18.69^{***}	17.45^{***}
		(3.018)	(4.226)	(8.027)	(3.416)	(3.630)	(7.220)	(3.479)	(4.847)	(6.707)
	father's education - group 2	9.158^{***}	6.295	-2.851	8.221^{**}	2.012	-1.455	10.35^{***}	6.181	5.137
		(2.961)	(4.019)	(7.913)	(3.489)	(3.874)	(7.399)	(3.586)	(4.519)	(6.892)
	father's education - group 3	18.74^{***}	17.19^{***}	8.192	19.50^{***}	16.87^{***}	13.94^{*}	22.69^{***}	24.58^{***}	19.14^{***}
		(2.967)	(4.183)	(7.484)	(3.829)	(3.501)	(7.356)	(3.490)	(4.134)	(7.006)
3	father's education - group 4	23.63^{***}	23.84^{***}	13.26^{*}	22.54^{***}	20.75^{***}	14.02^{**}	25.68^{***}	29.87^{***}	22.60^{***}
0		(2.882)	(4.213)	(7.320)	(3.455)	(3.448)	(7.107)	(3.598)	(4.144)	(6.623)
	father's education - group 5	19.64^{***}	22.80^{***}	9.651	19.41^{***}	21.20^{***}	12.57^{*}	22.30^{***}	26.16^{***}	17.24^{**}
		(3.498)	(4.628)	(8.229)	(4.242)	(3.703)	(7.396)	(3.945)	(4.333)	(7.105)
	father's education - group 6	43.19^{***}	39.53^{***}	30.16^{***}	42.41^{***}	38.73^{***}	35.04^{***}	42.55^{***}	42.28^{***}	35.73^{***}
		(3.047)	(4.679)	(7.877)	(3.461)	(3.763)	(7.511)	(3.861)	(4.436)	(6.937)
	wealth, std	22.21^{***}			21.75^{***}			22.26^{***}		
		(0.732)			(0.702)			(0.718)		
	student's gender: 1 - female, 0 - male	-10.35^{***}	-11.00^{***}	-10.15^{***}	-3.828^{***}	-2.188	-3.127	34.20^{***}	36.01^{***}	35.49^{***}
		(1.005)	(1.666)	(2.404)	(0.969)	(1.548)	(2.082)	(0.944)	(1.656)	(1.948)
	age of student	16.17^{***}	22.30^{***}	19.91^{***}	15.89^{***}	21.81^{***}	18.40^{***}	16.49^{***}	19.87^{***}	15.90^{***}
		(1.464)	(2.027)	(2.872)	(1.536)	(2.144)	(2.876)	(1.596)	(2.000)	(2.600)
	native	25.17^{***}	17.25^{***}	11.13^{***}	32.53^{***}	26.12^{***}	21.80^{***}	28.05^{***}	16.49^{***}	9.459^{**}
		(1.974)	(2.959)	(4.044)	(2.083)	(3.313)	(4.036)	(2.234)	(3.497)	(4.100)
	teacher/student ratio	-0.247	1.107^{***}	1.906^{***}	-0.0998	0.935***	1.895^{***}	-0.0538	0.950***	2.000***
		(0.228)	(0.318)	(0.584)	(0.220)	(0.255)	(0.515)	(0.242)	(0.305)	(0.429)
	proportion of certified teachers	13.95^{**}	7.729	-0.122	13.10^{**}	8.966	13.15	14.18^{**}	3.186	1.251
		(6.755)	(6.691)	(11.84)	(6.048)	(7.251)	(10.77)	(6.169)	(7.468)	(11.14)
	proportion of teachers with grad degree	8.366^{**}	8.394^{*}	4.035	8.017^{**}	6.441^{*}	-5.930	7.418^{**}	8.546^{*}	-0.989
		(3.585)	(4.605)	(10.94)	(3.580)	(3.656)	(9.487)	(3.342)	(4.594)	(8.842)
	inadequate admin staff, std	-2.768^{**}	-4.919^{***}	-2.455	-2.645^{**}	-4.417^{***}	-2.932	-2.775^{**}	-5.880^{***}	-3.316^{*}
		(1.327)	(1.828)	(2.450)	(1.249)	(1.614)	(2.121)	(1.208)	(1.931)	(1.835)
	inadequate teaching staff, std	5.913***	1.702	-0.234	5.533***	1.822	0.295	4.753***	1.223	-0.575
		(1.019)	(1.824)	(2.559)	(1.029)	(1.608)	(2.255)	(1.065)	(1.668)	(1.931)
	household income - group 2		12.39^{***}	10.45^{***}		12.07^{***}	6.009 * * *		11.51^{***}	7.741^{***}

	wealth	income	resources	wealth	income	resources	wealth	income	resources
		(1.851)	(2.905)		(1.694)	(2.781)		(2.096)	(2.637)
household income - group 3		26.20^{***}	21.06^{***}		24.09^{***}	18.82^{***}		23.11^{***}	15.93^{***}
		(1.859)	(2.782)		(1.646)	(2.665)		(1.841)	(2.410)
household income - group 4		33.97^{***}	24.51^{***}		30.74^{***}	22.13^{***}		30.43^{***}	20.39^{***}
		(1.871)	(3.227)		(1.664)	(3.158)		(1.920)	(3.050)
household income - group 5		42.92^{***}	32.50^{***}		38.01^{***}	29.07^{***}		37.38^{***}	26.37^{***}
		(2.190)	(3.710)		(2.028)	(3.388)		(2.199)	(3.239)
household income - group 6		54.72^{***}	42.29^{***}		47.77^{***}	36.55^{***}		46.22^{***}	32.14^{***}
		(2.620)	(3.812)		(1.992)	(3.199)		(2.318)	(3.012)
cultural possessions, std			8.959^{***}			12.60^{***}			11.49^{***}
			(1.286)			(1.206)			(1.196)
home educational resources, std			11.42^{***}			12.44^{***}			10.68^{***}
			(1.441)			(1.335)			(1.239)
ict resourses, std			6.318^{***}			6.033^{***}			6.944^{***}
			(1.214)			(1.232)			(1.238)
home possessions, std.			-2.147			-8.831^{***}			-5.898^{***}
			(1.955)			(2.070)			(1.859)
Country Fixed Effects	Yes	Yes	$\mathbf{Y}_{\mathbf{es}}$	Yes	$\mathbf{Y}_{\mathbf{es}}$	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	No	Yes	$\mathbf{Y}_{\mathbf{es}}$	No	Yes	Yes	No
No. of Obs.	265929	81736	27793	265929	81736	27793	265929	81736	27793
$ m R^2$	0.370	0.372	0.284	0.292	0.311	0.245	0.305	0.336	0.276

31

(columns 2, 5 and 8) and income and household possessions indicator standardized as a z-score (columns 3, 6 and 9), time effects, country effects, gender, age and origin of the student. Each column represents OLS regression coefficients for the pooled dataset PISA waves 2003-2013. Estimations are performed using multiple imputation and weighting according to PISA data structure. Standard errors are clustered at the school level. Significance levels are * $p \le 0.05$, *** $p \le 0.01$. The outcome variables are PISA scores in math, science or reading. Controls include wealth indicator standardized as z-score (column 1, 4 and 7), income

	Math	Science	Read
mother's education - group 1	13.53***	14.41***	16.24***
	(4.407)	(4.212)	(5.119)
mother's education - group 2	17.70^{***}	18.28^{***}	19.83***
	(4.033)	(3.626)	(4.433)
mother's education - group 3	31.39^{***}	33.09***	31.16***
	(3.738)	(3.733)	(4.402)
mother's education - group 4	33.38^{***}	35.69***	34.88***
	(3.660)	(3.600)	(4.280)
mother's education - group 5	32.56^{***}	37.39***	34.69***
	(3.916)	(3.939)	(4.384)
mother's education - group 6	42.32***	46.80***	41.85***
	(3.920)	(3.891)	(4.336)
father's education - group 1	13.66***	16.68***	17.54***
	(3.889)	(3.877)	(4.698)
father's education - group 2	12.42^{***}	12.90^{***}	13.96***
	(3.636)	(3.805)	(3.883)
father's education - group 3	25.98^{***}	28.32***	29.96***
	(3.602)	(3.563)	(3.701)
father's education - group 4	28.84^{***}	29.34***	30.49^{***}
	(3.387)	(3.448)	(3.604)
father's education - group 5	26.86***	28.80***	30.54***
	(3.783)	(3.658)	(3.972)
father's education - group 6	45.84^{***}	47.45***	45.79***
~ •	(3.802)	(3.577)	(3.680)
No. of Obs.	265929	265929	265929

 Table 4

 Marginal Effects of Mother's and Father's Education. Data from pooled surveys

 2003-2013

The outcome variables are PISA scores in math (column 1), science (column 2) or reading (column 3). The table presents the marginal effects of mother's education (upper panel) and father's education levels after estimating OLS regression specification with interactions. Estimations are performed using multiple imputation and weighting according to PISA data structure, on the pooled dataset PISA waves 2003-2013. Controls include wealth indicator standardized as z-score, school characteristics such as teacher/student ration, proportion of certified teachers and teachers with graduate degree, standardized indicators of inadequate teaching and administrative staff, time effects, country effects, gender, age and origin of the student. Standard errors are clustered at the school level. standard errors are clustered at the school level. Significance levels are * $p \le 0.10$, ** $p \le 0.05$, *** $p \le 0.01$.

	Math	Science	Read
mother' educ: university	19.53***	19.90***	16.30***
	(4.841)	(5.043)	(4.869)
father' educ: university	14.78***	16.47^{***}	19.64^{***}
	(4.914)	(5.337)	(4.530)
wealth, std	20.17^{***}	18.00***	16.35^{***}
	(2.430)	(2.951)	(2.567)
student's gender: 1 - female, 0 - male	-12.39^{**}	-1.670	34.85^{***}
	(4.953)	(5.307)	(4.836)
age of student	11.60	21.41^{**}	8.812
	(7.833)	(8.580)	(7.821)
native	10.40^{*}	14.66^{**}	10.84^{*}
	(6.309)	(6.419)	(6.054)
teacher/student ratio	1.247^{**}	1.765^{***}	1.778^{***}
	(0.518)	(0.477)	(0.470)
proportion of certified teachers	13.47	10.45	7.686
	(10.39)	(9.653)	(12.25)
proportion of teachers with grad degree	13.00	13.99	14.33
	(9.250)	(11.27)	(8.947)
inadequate admin staff, std	-10.42^{***}	-12.10^{***}	-8.774^{***}
	(3.195)	(2.934)	(2.956)
inadequate teaching staff, std	0.618	0.315	3.216
	(2.969)	(2.925)	(3.031)
Country Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes
No. of Obs.	155061	155061	155061
\mathbb{R}^2	0.299	0.264	0.302

 Table 5

 Results when controlling for Propensity Score Matching Technique. Data from the pooled surveys 2003-2013

The outcome variables are PISA scores in math (column 1), science (column 2) or reading (column 3). The table presents the OLS regression coefficients weighted by propensity scores (2-step procedure). Estimations are performed on the pooled dataset PISA waves 2003-2013. Controls include wealth indicator standardized as z-score, school characteristics such as teacher/student ration, proportion of certified teachers and teachers with graduate degree, standardized indicators of inadequate teaching and administrative staff, time effects, country effects, gender, age and origin of the student. Standard errors are clustered at the school level. standard errors are clustered at the school level. Significance levels are * $p \le 0.10$, ** $p \le 0.05$, *** $p \le 0.01$.

Table 6Academic Performance of Children given Education, Home and School Resources.Data from the 2015 Survey

	Math	Science	Read
	5.440	2.338	5.235
mother's education - group 1	(5.260)	(4 227)	(4.465)
	(0.000)	(4.327)	(4.405)
mother's education - group 2	4.038	3.803	(1.011
	(4.351)	(4.283)	(4.436)
mother's education - group 3	15.75***	19.00***	23.53***
	(5.592)	(5.271)	(5.185)
mother's education - group 4	13.65^{***}	15.18^{***}	17.18^{***}
	(4.952)	(4.387)	(4.499)
mother's education - group 5	15.88^{***}	15.20***	17.40^{***}
	(4.994)	(4.431)	(5.016)
mother's education - group 6	13.63**	13.92***	16.03^{***}
	(5.588)	(5.164)	(5.290)
father's education - group 1	-6.033	-4.657	-7.069
	(5 796)	(4 749)	(5.532)
father's advantion group 2	0.210	2 522	2 200
lather's education - group 2	-0.310	2.525	(4.657)
	(5.414)	(4.096)	(4.657)
father's education - group 3	-3.731	4.630	2.924
	(5.973)	(4.938)	(5.153)
father's education - group 4	0.0593	6.325	5.787
	(5.393)	(4.682)	(4.599)
father's education - group 5	4.737	8.236^{*}	4.985
Ŭ.	(5.443)	(4.816)	(4.880)
father's education group 6	7 478	11 98**	8 521*
lather's education - group o	(5 422)	(4,624)	(4.012)
	(5.432)	(4.624)	(4.913)
student's gender: 1 - female, 0 - male	-8.337^{***}	-7.368^{***}	20.73^{***}
	(2.176)	(2.018)	(1.884)
Age	12.01^{***}	13.30^{***}	13.71^{***}
	(3.387)	(2.929)	(3.748)
native	24 63***	34 03***	25.61***
	(5 587)	(6.220)	(6.968)
	0.501)	10 46***	0.500)
cultural posessions, std	8.562	10.46	8.740
	(1.156)	(1.069)	(1.130)
home educ.resources, std	5.385^{***}	6.286^{***}	7.281^{***}
	(1.157)	(0.939)	(1.137)
ict resourses, std	4.917^{**}	4.354^{**}	3.928
	(2.148)	(2.199)	(2.460)
Standardized values of (homepos)	-3.348	-5.763^{*}	-2.550
	(2.652)	(2.959)	(3.191)
Standardized values of (parent communic con)	1.046**	1 602**	2 221***
Standardized values of (parent_communic_gen)	(0.800)	(0.765)	(0.820)
	(0.899)	(0.765)	(0.839)
Standardized values of (parent_support_scie)	-3.399^{***}	-3.028^{***}	-5.065^{***}
	(0.910)	(0.841)	(1.048)
parent_choose_sch_educperf_z	6.022^{***}	6.200^{***}	6.981***
	(0.997)	(0.818)	(0.928)
Standardized values of (parent_choose_sch_econom)	-5.884***	-5.475^{***}	-5.876^{***}
	(0.994)	(0.946)	(0.921)
Standardized values of (parent shoose ash approach)	4 740***	5 674***	4.560***
Standardized values of (parent_choose_sch_approach)	-4.749	-3.074	-4.500
	(1.081)	(1.008)	(1.120)
hh income, group 2	8.056***	9.278***	9.393***
	(2.639)	(2.827)	(2.837)
hh income, group 3	16.74^{***}	15.53^{***}	19.02^{***}
	(3.122)	(3.068)	(3.288)
hh income, group 4	16.28^{***}	15.29***	17.16^{***}
	(3.958)	(3.465)	(3.337)
hh income group 5	28 39***	25 41***	22 75***
ini nicolne, group 5	(4.280)	(2 212)	(4.680)
LL in any of the second s	(4.00U)	(0.010)	(4.009)
nn income, group b	45.35"""	40.05 ***	38.51***
	(3.572)	(3.486)	(3.726)
Standardized values of (school_infrustr)	3.756	5.876	6.382
	(5.449)	(4.548)	(5.289)
Student-Teacher ratio	0.401^{*}	0.263	0.340
	(0.242)	(0.202)	(0.207)
Total number of all teachers at school	0.205***	0.200***	0.995***
Total number of an reachers at school	(0.0400)	(0.0267)	(0.0276)
	(0.0400)	(0.0307)	(0.0376)
proportion of certified teachers	1.281	-1.501	-1.892
	(4.683)	(4.232)	(4.848)
proportion of teachers with grad degree	102.9^{***}	93.89***	94.14^{***}
	(28.03)	(27.18)	(30.03)
Index proportion of all teachers ISCED LEVEL 54 Master	7.941	10.057	12.64*
Index biobortion of an teachers for this this of a stasse		(6 542)	(6 749)
index proportion of an teachers 150ED DEVEL 5A Master	(6 594)		10.7491
	(6.524)	(0.343)	1 400
Index proportion of all teachers ISCED LEVEL 5A Bachelor	(6.524) -1.317	(6.543) -2.439	-1.433
Index proportion of all teachers ISCED LEVEL 5A Bachelor	(6.524) -1.317 (5.469)	(6.543) -2.439 (4.641)	(5.234)
Index proportion of all teachers ISCED LEVEL 5A Bachelor inadequate admin staff, std	(6.524) -1.317 (5.469) -0.893	(6.543) -2.439 (4.641) -1.487	(5.123) -1.433 (5.234) -0.456
Index proportion of all teachers ISCED LEVEL 5A Bachelor inadequate admin staff, std	(6.524) -1.317 (5.469) -0.893 (2.021)	$\begin{array}{c} (0.543) \\ -2.439 \\ (4.641) \\ -1.487 \\ (1.919) \end{array}$	$ \begin{array}{c} -1.433 \\ (5.234) \\ -0.456 \\ (2.159) \end{array} $

	(5.566)	(4.475)	(5.472)
Class Size	0.587^{***}	0.650^{***}	0.766^{***}
	(0.218)	(0.201)	(0.218)
Percent. total funding for school year comes from? Government	-0.0896*	-0.0977^{**}	-0.0522^{***}
	(0.0476)	(0.0413)	(0.0435)
public school $= 1$, private school $= 0$	-7.952^{*}	-11.43^{***}	-15.23^{***}
	(4.466)	(4.414)	(4.464)
Constant	158.9^{***}	152.7^{***}	150.3^{**}
	(53.01)	(46.02)	(59.33)
Country Fixed Effects	Yes	Yes	Yes
No. of Obs.	21710	21710	21710
\mathbb{R}^2	0.400	0.381	0.367

The outcome variables are PISA scores in math (column 1), science (column 2) or reading (column 3). The table presents the OLS regression coefficients. Estimations are performed on the PISA dataset wave 2015, using multiple imputation and weighting according to PISA data structure. Controls include wealth indicator standardized as z-score, measures of parental support and dimensions of school choice, home cultural possessions, ICT and educational resources at home, school characteristics such as teacher/student ration, proportion of certified teachers and teachers with graduate degree, standardized indicators of inadequate teaching and administrative staff, class size and information about school funding, time effects, country effects, gender, age and origin of the student. Standard errors are clustered at the school level. standard errors are clustered at the school level. Significance levels are * p≤0.10, ** p≤0.05, *** p≤0.01..

	Math	Science	Read
mother's education - group 1	-2.751	-4.997	-4.236
	(6.403)	(7.496)	(7.246)
mother's education - group 2	4.197	-1.426	1.941
	(5.474)	(6.490)	(6.273)
mother's education - group 3	12.86^{**}	13.60^{*}	18.51^{***}
	(6.532)	(7.231)	(7.040)
mother's education - group 4	12.56^{**}	12.69^{*}	14.46^{**}
	(5.533)	(6.680)	(6.440)
mother's education - group 5	13.22^{**}	11.10^{*}	10.77^{*}
	(5.723)	(6.684)	(6.491)
mother's education - group 6	12.50^{**}	9.314	12.13^{*}
	(5.544)	(6.883)	(6.671)
father's education - group 1	2.154	-2.253	-4.680
	(6.972)	(6.649)	(7.950)
father's education - group 2	9.204^{*}	8.684	10.54
	(4.902)	(5.439)	(6.518)
father's education - group 3	2.816	8.468	8.793
	(5.693)	(6.466)	(7.383)
father's education - group 4	13.55^{***}	14.94^{***}	15.68^{**}
	(5.134)	(5.685)	(6.595)
father's education - group 5	13.16^{**}	15.42***	14.05^{**}
	(5.229)	(5.889)	(6.872)
father's education - group 6	18.32***	14.95^{**}	13.12^{*}
	(5.269)	(5.929)	(7.009)
No. of Obs.	21710	21710	21710

 Table 7

 Marginal Effects of Mother's and Father's Education. Data from the 2015 survey

The outcome variables are PISA scores in math (column 1), science (column 2) or reading (column 3). The table presents the marginal effects of mother's education (upper panel) and father's education levels after estimating OLS regression specification with interactions. Estimations are performed using multiple imputation and weighting according to PISA data structure, PISA dataset wave 2015. Controls include wealth indicator standardized as z-score, measures of parental support and dimensions of school choice, home cultural possessions, ICT and educational resources at home, school characteristics such as teacher/student ration, proportion of certified teachers and teachers with graduate degree, standardized indicators of inadequate teaching and administrative staff, class size and information about school funding, time effects, country effects, gender, age and origin of the student. Standard errors are clustered at the school level. Significance levels are * $p \le 0.10$, ** $p \le 0.05$, *** $p \le 0.01$.

	Math	Science	Read
mother' educ: university	10.16***	8.854***	9.721**
	(1.255)	(1.248)	(1.253)
ather' educ: university	5.666***	4.729***	0.845
·	(1.426)	(1.430)	(1.476)
cultural posessions, std.	5.996***	8.362***	10.13***
and an possible, but	(0.848)	(0.834)	(0.839)
nome educ resources std	-1 556*	0.0271	1 870**
ionie eductiesources, std.	(0.880)	(0.880)	(0.800)
	(0.869)	(0.889)	(0.899)
ct resourses, std.	-5.879	-7.280	-7.300
	(1.529)	(1.515)	(1.522)
nome possessions, std.	19.30***	13.65***	10.19***
	(2.165)	(2.121)	(2.111)
parent communication, std.	4.581***	4.286^{***}	2.988**
	(0.745)	(0.722)	(0.725)
parent support science, std.	-5.178^{***}	-2.363^{***}	-3.769*
	(0.704)	(0.692)	(0.696)
arent_choose_sch_educperform, std.	4.401***	4.394^{***}	5.247^{*3}
	(0.666)	(0.660)	(0.680)
arent_choose_sch_econom, std.	-8.844^{***}	-8.529^{***}	-8.011**
,	(0.683)	(0.684)	(0.672)
arent_choose_sch_approach, std	-4 844***	-6.175***	-5.339*
arenteeneobelleapproach, btd.	(0.635)	(0.631)	(0.642)
ousshold income group 2	6 167***	7 570***	4.670*
ousenoid income, group 2	(0.207)	(0.200)	4.070
	(2.307)	(2.308)	(2.387)
ousehold income, group 3	22.33	19.06	26.45
	(2.460)	(2.416)	(2.536)
ousehold income, group 4	21.93***	19.45^{***}	22.70***
	(2.629)	(2.604)	(2.676)
ousehold income, group 5	34.51***	30.98^{***}	31.07^{**}
	(2.739)	(2.720)	(2.755)
ousehold income, group 6	46.37***	43.04^{***}	45.91^{**}
	(2.464)	(2.461)	(2.506)
tudent's gender: 1 - female, 0 - male	-13.74^{***}	-11.85^{***}	17.88***
	(1.188)	(1.183)	(1.194)
Age .	8.396***	9.827***	11.39***
0	(2.058)	(2.052)	(2.061)
ative	14 26***	17 91***	13.26**
	(3.116)	(3.028)	(3.221)
abool infractivity at the	5 505**	0.020)	6.620*
chool mirastructure, std.	5.595	0.104	(2.176)
	(2.241)	(2.143)	(2.170)
tudent-Teacher ratio	0.470***	0.283***	0.519*
	(0.0928)	(0.0909)	(0.0989)
otal number of all teachers at school	0.121***	0.118^{***}	0.131^{*}
	(0.0147)	(0.0151)	(0.0155)
roportion of certified teachers	2.777	1.719	-4.970^{*}
	(1.959)	(1.973)	(2.121)
roportion of teachers with grad degree	57.13***	41.73^{***}	20.61
	(12.32)	(13.14)	(13.35)
roportion of all teachers Master	-1.102	-1.536	4.250
-	(2.869)	(2.934)	(2.904)
roportion of all teachers Bachelor	-10 78***	-7 496***	-2.468
	(2.087)	(2.083)	(2 200)
adequate admin staff std	1 006**	-1.616*	_0.203)
laucquate aunin stail, Stu.	-1.000	(0.876)	-0.238
	(0.804)	(0.0/0)	(0.850)
hadequate teaching staff, std.	-8.966***	-10.41	-8.940*
	(2.411)	(2.275)	(2.317)
lass size	0.389***	0.468***	0.532^{*}
	(0.0885)	(0.0887)	(0.0937)
Percent government funding	-0.150^{***}	-0.150^{***}	-0.151^{*}
	(0.0218)	(0.0217)	(0.0216)
public school $= 1$, private school $= 0$	-3.206^{*}	-7.281^{***}	-8.092^{*}
	(1.767)	(1.766)	(1.787)
Country Fixed Effects	Yes	Yes	Yes
Je of Obe	10044	19944	10044
NO. 01 ODS.	18244	18244	18244
n	0.321	0.281	0.267

Table 8Matching Technique, Wave 2015

The outcome variables are PISA scores in math (column 1), science (column 2) or reading (column 3). The table presents the OLS regression coefficients weighted by propensity scores (2-step procedure), PISA dataset wave 2015. Controls include wealth indicator standardized as z-score, measures of parental support and dimensions of school choice, home cultural possessions, ICT and educational resources at home, school characteristics such as teacher/student ration, proportion of certified teachers and teachers with graduate degree, standardized indicators of inadequate teaching and administrative staff, class size and information about school funding, time effects, country effects, gender, age and origin of the student. Standard errors are clustered at the school level. Significance levels are * $p \le 0.10$, ** $p \le 0.05$, *** $p \le 0.01$.

7 Appendix B. Full List of Model Equations

The models features one exogenous state, namely the productivity shock A_t , and two predetermined states, namely human capital of both parents, h_t^m and h_t^f . Let us assume that t_t^m and $t_t^f = (1 - \phi n_t - \phi (e_t^f + e_t^m)).$

7.1 Model with Exogenous Bargaining

Budget and technology constraints:

$$c_t^m + c_t^f \le w_t^m t_t^m h_t^m + w_t^f h_t^f (1 - \phi n_t - \phi (e_t^f + e_t^m))$$
(16)

$$h_{t+1}^{f} = (Be_{t}^{f})^{\delta} (h_{t}^{f})^{\gamma} (h_{t}^{m})^{1-\gamma}$$
(17)

$$h_{t+1}^{m} = (Be_{t}^{m})^{\delta} (h_{t}^{f})^{\gamma} (h_{t}^{m})^{1-\gamma}$$
(18)

Competitive prices:

$$w_t^f = \alpha A_t (t_t^m h_t^m)^{1-\alpha} (t_t^f h_t^f)^{\alpha-1}; w_t^m = (1-\alpha) A_t (t_t^m h_t^m)^{-\alpha} (t_t^f h_t^f)^{\alpha}$$
(19)

Value functions:

$$V^{h}(h_{t}^{f}, h_{t}^{m}) = \eta_{t}^{m} V^{m}(h_{t}^{f}, h_{t}^{m}) + (1 - \eta_{t}^{m}) V^{m}(h_{t}^{f}, h_{t}^{m})$$
(20)

$$V_t^m = \frac{(c_t^m)^{1-\sigma}}{1-\sigma} + \beta b(n_t) n_t \frac{(1-\eta_{t+1}^m) V_{t+1}^f + \eta_{t+1}^m V_{t+1}^m}{2}$$
(21)

$$V_t^f = \frac{(c_t^f)^{1-\sigma}}{1-\sigma} + \beta b(n_t) n_t \frac{(1-\eta_{t+1}^m) V_{t+1}^f + \eta_{t+1}^m V_{t+1}^m}{2}$$
(22)

Above one can assume $b(n_t) = 1$ or $b(n_t) = n_t^{1-\varepsilon}$. Consumption functions:

$$c_t^f = \left[1 - \frac{(\eta_t^m)^{\frac{1}{\sigma}}}{(\eta_t^m)^{\frac{1}{\sigma}} + (1 - \eta_t^m)^{\frac{1}{\sigma}}}\right] \left[A_t (h_t^m)^{1 - \alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha}\right]$$
(23)

$$c_t^m = \left[\frac{(\eta_t^m)^{\frac{1}{\sigma}}}{(\eta_t^m)^{\frac{1}{\sigma}} + (1 - \eta_t^m)^{\frac{1}{\sigma}}}\right] \left[A_t(h_t^m)^{1 - \alpha}(h_t^f)^{\alpha}(1 - \phi n_t - n_t(e_t^f + e_t^m))^{\alpha}\right]$$
(24)

Fertility choice:

$$\frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[A_t (h_t^m)^{1-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} \alpha (\phi + e_t^f + e_t^m) \right]$$

$$= \frac{\beta}{2} \mathbb{E}_t \left\{ V^h (h_{t+1}^f, h_{t+1}^m) \right\}$$
(25)

Education choices:

$$0 = \frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[-A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} \alpha n_t \right] +$$
(26)
+ $\frac{1}{2} \beta \mathbb{E}_t \left\{ \left[(1 - \alpha) A_{t+1} (h_{t+1}^m)^{-\alpha} (t_{t+1}^f h_{t+1}^f)^{\alpha} \right] \left[\delta B (B e_{t+1}^m)^{\delta - 1} (h_t^f)^{\gamma} (h_t^m)^{1 - \gamma} \right] \right\}$ (27)
$$0 = \frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[-A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} \alpha n_t \right] +$$
(27)
+ $\frac{1}{2} \beta \mathbb{E}_t \left\{ \left[\alpha A_t (h_{t+1}^m)^{1 - \alpha} (t_{t+1}^f h_{t+1}^f)^{\alpha - 1} t_{t+1}^f \right] \left[\delta B (B e_{t+1}^f)^{\delta - 1} (h_t^f)^{\gamma} (h_t^m)^{1 - \gamma} \right] \right\}$

7.2 Model with Endogenous Bargaining

The functional form for endogenous bargaining is in equation 8 in the main text. The set of equations is the same as above, except for the first order conditions on the education levels, which now read as follows:

$$0 = \frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[-A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} \alpha n_t \right] +$$

$$+ \frac{1}{\beta} \left\{ \frac{\partial \eta_{t+1}^m (h_{t+1}^m, h_{t+1}^f)}{\partial \eta_{t+1}^m (h_{t+1}^m, h_{t+1}^f)} + \left[(1 - \alpha) A_{t+1} (h_t^m)^{-\alpha} (t_t^f + h_{t-1}^f)^{\alpha} \right] \right\} \left[\delta B (Be^m_{+1})^{\delta - 1} (h_t^f)^{\gamma} (h_{t-1}^m)^{1 - \gamma} \right]$$
(28)

$$+\frac{1}{2}\beta\left\{\frac{\partial\eta_{t+1}^{m}(h_{t+1}^{m},h_{t+1}^{m})}{\partial h_{t+1}^{m}}+\left[(1-\alpha)A_{t+1}(h_{t+1}^{m})^{-\alpha}(t_{t+1}^{f}h_{t+1}^{f})^{\alpha}\right]\right\}\left[\delta B(Be_{t+1}^{m})^{\delta-1}(h_{t}^{f})^{\gamma}(h_{t}^{m})^{1-\gamma}\right]$$

$$0 = \frac{\eta_t^m}{(c_t^m)^{\sigma}} \left[-A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} \alpha n_t \right] +$$

$$+ \frac{1}{2} \beta \left[-\frac{\partial \eta_{t+1}^m (h_{t+1}^m, h_{t+1}^f)}{\partial h_{t+1}^f} + \left[\alpha A_t (h_{t+1}^m)^{1 - \alpha} (t_{t+1}^f h_{t+1}^f)^{\alpha - 1} t_{t+1}^f \right] \right] \left[\delta B (B e_{t+1}^f)^{\delta - 1} (h_t^f)^{\gamma} (h_t^m)^{1 - \gamma} \right]$$
(29)

8 Appendix C. Proof Proposition 1

We shall solve the model under the utility specification in 15. We can re-write the budget constraint by substituting wages with their marginal productivity. This delivers:

$$c_t^m + c_t^f \le (1 - \alpha) A_t (h_t^m)^{-\alpha} (t_t^f h_t^f)^{\alpha} + \alpha A_t (h_t^m)^{1 - \alpha} (t_t^f h_t^f)^{\alpha - 1} h_t^f t_t^f$$
(30)

Summing up and substituting $t_t^f = (1 - \phi n_t - n_t(e_t^f + e_t^m))$, the above delivers:

$$c_t^m + c_t^f \le A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha}$$
(31)

Next, we shall write down the Lagrangian problem, which reads as follows:

$$\mathcal{L} = \eta \ln(c_t^m) + (1 - \eta) \ln(c_t^f) + \kappa \ln((n_t) + \frac{1}{2}\beta \mathbb{E}_t \left[\eta^m V^m(h_t^m, \bar{h_t^f}) + (1 - \eta^m) V^f(h_t^f, \bar{h_t^m}) \right] + \lambda_t \left[c_t^m + c_t^f - A_t(h_t^m)^{-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t(e_t^f + e_t^m))^{\alpha - 1} \right]$$
(32)

The first order conditions on male and female consumption deliver the following marginal condition:

$$\frac{\eta^m}{1-\eta^m} = \frac{c_t^m}{c_t^f} \tag{33}$$

The latter merged with the budget constraint, 31, delivers the following consumption functions for males and females:

$$c_t^f = \eta^m \left[A_t (h_t^m)^{1-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha} \right]$$
(34)

$$c_t^m = (1 - \eta^m) \left[A_t (h_t^m)^{1-\alpha} (h_t^f)^{\alpha} (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha} \right]$$
(35)

When males and females hold the same bargaining power consumption would be equalized. Since education and fertility choices affect the value function of future generations, their closed form solution can be obtained by a guess and verify procedure. Following Doepke and Tertilt[24], we can assume the following functional forms for the value functions:

$$V^{m}(h_{t}^{m}, h_{t}^{f}, \bar{h_{t}^{m}}, h_{t}^{f}) = a_{1} + a_{2}\ln(h_{t}^{m}) + a_{3}\ln(h_{t}^{f}) + a_{4}\ln(\bar{h_{t}^{m}}) + a_{5}\ln(h_{t}^{f})$$
(36)

$$V^{f}(h_{t}^{m}, h_{t}^{f}, \bar{h_{t}^{m}}, \bar{h_{t}^{f}}) = b_{1} + b_{2}\ln(h_{t}^{m}) + b_{3}\ln(h_{t}^{f}) + b_{4}\ln(\bar{h_{t}^{m}}) + b_{5}\ln(\bar{h_{t}^{f}})$$
(37)

Taking first order conditions of the Lagrangean with respect to education levels and the functional form for the value function we obtain the following margin condition:

$$e_t^m = e_t^f \frac{a_2}{b_3} \frac{\eta^m}{1 - \eta^m}$$
(38)

Taking the first order conditions of the Lagrangean with respect to fertility and education and using again the functional forms for the value function delivers the following education levels:

$$e_t^f = \frac{\frac{\beta}{2}\delta(1-\eta^m)b_3\phi}{\kappa - \frac{\beta}{2}\delta(a_2\eta^m + b_3(1-\eta^m))}$$
(39)

$$e_t^m = \frac{\frac{\beta}{2}\delta\eta^m a_2\phi}{\kappa - \frac{\beta}{2}\delta(a_2\eta^m + b_3(1 - \eta^m))}$$
(40)

Merging the first order condition with respect to fertility with the equilibrium education levels delivers:

$$\frac{\delta}{n_t} = \frac{\eta^m}{c_t^m} \left[A_t (h_t^m)^{1-\alpha} (h_t^f)^{\alpha} \alpha (1 - \phi n_t - n_t (e_t^f + e_t^m))^{\alpha - 1} (\phi + e_t^f + e_t^m) \right]$$
(41)

Rearranging the equation above delivers the following level of fertility:

$$n_t = \frac{\kappa - \frac{\beta}{2}\delta(a_2\eta^m + b_3(1 - \eta^m))}{\phi(\kappa + \alpha)}$$
(42)

Using the optimal education levels and fertility rates, one obtains the optimal time allocation for women:

$$t_t^f = \frac{\alpha}{\kappa + \alpha} \tag{43}$$

Plugging this into consumption delivers the following expressions:

$$c_t^m = \eta^m \left[A_t (h_t^m)^{-\alpha} (h_t^f)^{\alpha} (\frac{\alpha}{\kappa + \alpha})^{\alpha} \right]$$
(44)

$$c_t^f = (1 - \eta^m) \left[A_t (h_t^m)^{-\alpha} (h_t^f)^\alpha (\frac{\alpha}{\kappa + \alpha})^\alpha \right]$$
(45)

We shall now solve for the explicit expressions by deriving the closed form solution in the parameter of the value functions. This can be done by the method of undetermined coefficients. Specifically, I substitute in the value functions for men and women the solutions for male and female consumption and education levels derived above and the fertility level from equation 42. Next, I isolate the coefficients which relate to the variables h_t^m , h_t^f , $\bar{h_t^m}$ and $\bar{h_t^f}$ in the value function for male and I set them equal to a_2, a_3, a_4 and a_5 , respectively. I do the same for the female value functions. Furthermore, in each optimal value function I derive an expression for the constant coefficients and I set them equal to a_1 and b_1 for male and female, respectively. Next, solving the system of non-linear equations that emerges from the equalization, I get the following closed form solution for the coefficients (note that for reasons of generality here I am using different weights for male and female assigned to future generations, $beta^f$ and $beta^f$):

$$a_{2} = \frac{2(1-\alpha) + (1-\alpha)\gamma\beta^{f}(1-\eta^{m}) + \alpha(1-\gamma)\beta^{m}(1-\eta^{m})}{(2-(1-\gamma)\beta^{m}\eta^{m} - \gamma\beta^{f}\eta^{m})}$$
(46)

$$b_{2} = \left[(1-\alpha) + \frac{(1-\gamma)\beta^{f}(1-\eta^{m})}{(2-(1-\gamma)\beta^{m}\eta^{m} - \gamma\beta^{f}\eta^{m})} \right]$$
(47)

$$a_3 = \left[\alpha + \frac{\gamma \beta^m (1 - \eta^m)}{(2 - (1 - \gamma)\beta^m \eta^m - \gamma \beta^f \eta^m)}\right]$$
(48)

$$b_{3} = \frac{2\alpha + (1 - \alpha)\gamma\beta^{f}(1 - \eta^{m}) - \alpha(1 - \gamma)\beta^{m}\eta^{m}}{(2 - (1 - \gamma)\beta^{m}\eta^{m} - \gamma\beta^{f}\eta^{m})}$$
(49)

$$a_4 = \frac{1-\gamma}{\gamma} a_5 \tag{50}$$

$$b_4 = \frac{\beta^f}{\beta^m} a_4 \tag{51}$$

$$a_{5} = \left[\frac{\gamma \frac{\beta^{m}}{2}}{1 - \eta^{m} \frac{\beta^{m}}{2} - (1 - \eta^{m}) \frac{\beta^{m}}{2}}\right] \left[(\eta^{m} \alpha + (1 - \eta^{m})(1 - \alpha) + \frac{\eta^{m} \gamma \beta^{m} (1 - \eta^{m}) + (1 - \gamma) \beta^{f} (1 - \eta^{m})^{2}}{(2 - (1 - \gamma))\beta^{m} \eta^{m} - \gamma \beta^{f} (1 - \eta^{m}))}\right]$$
(52)

$$b_5 = \frac{\gamma}{1 - \gamma} b_4 \tag{53}$$

$$a_{1} = \left[\frac{2 - \beta^{f} \eta^{m}}{2 - (\eta^{m} \beta^{f} + (1 - \eta^{m}) \beta^{f})}\right] (M_{1} + M_{2}) + \left[\frac{\beta^{m} (1 - \eta^{m})}{2 - (\eta^{m} \beta^{f} + (1 - \eta^{m}) \beta^{f})}\right] (F_{1} + F_{2})$$
(54)

$$b_1 = \left[\frac{2 - \beta^m (1 - \eta^m)}{2 - (\eta^m \beta^f + (1 - \eta^m) \beta^f)}\right] (F_1 + F_2) + \left[\frac{\beta^f \eta^m}{2 - (\eta^m \beta^f + (1 - \eta^m) \beta^f)}\right] (M_1 + M_2)$$
(55)

where:

$$M_1 = F_1 = \ln(\eta^m A(\frac{\alpha}{\kappa + \alpha})^{\alpha}) + \kappa \ln\left[\frac{\kappa - \frac{\beta}{2}(a_2\eta^m + b_3(1 - \eta^m))}{\phi(\kappa + \alpha)}\right]$$
(56)

$$M_{2} = \frac{\beta^{m}}{2} \delta \ln(a_{2}\eta^{m}) \left[\eta^{m}(a_{2} + a_{4}) + (1 - \eta^{m})(b_{2} + b_{4})\right] +$$

$$+ \frac{\beta^{m}}{2} \delta \ln(b_{3}(1 - \eta^{m})) \left[\eta^{m}(a_{3} + a_{5}) + (1 - \eta^{m})(b_{3} + b_{5})\right] +$$

$$+ \frac{\beta^{m}}{2} \delta \ln \left[\frac{B\phi_{2}^{\beta}\delta}{\kappa - \frac{\beta}{2}(a_{2}\eta^{m} + b_{3}(1 - \eta^{m}))}\right]$$

$$\left[\eta^{m}(a_{2} + a_{3} + a_{4} + a_{5}) + (1 - \eta^{m})(b_{2} + b_{3} + b_{4} + b_{5})\right]$$
(57)

$$F_{2} = \frac{\beta^{f}}{2} \delta \ln(a_{2}\eta^{m}) \left[\eta^{m}(a_{2} + a_{4}) + (1 - \eta^{m})(b_{2} + b_{4}) \right] +$$

$$+ \frac{\beta^{f}}{2} \delta \ln(b_{3}(1 - \eta^{m})) \left[\eta^{m}(a_{3} + a_{5}) + (1 - \eta^{m})(b_{3} + b_{5}) \right] +$$

$$+ \frac{\beta^{f}}{2} \delta \ln \left[\frac{B\phi_{2}^{\beta}\delta}{\kappa - \frac{\beta}{2}(a_{2}\eta^{m} + b_{3}(1 - \eta^{m}))} \right]$$

$$\left[\eta^{m}(a_{2} + a_{3} + a_{4} + a_{5}) + (1 - \eta^{m})(b_{2} + b_{3} + b_{4} + b_{5}) \right]$$
(58)

Given the model solution, we can examine the behavior of the education levels, which are given by equations 39 and 40, once we substitute for the coefficients' solution. Given that the coefficients a_2 and b_3 are positive, the derivative of the optimal education levels, 39 and 40, with respect to ϕ , which proxies the extent of mothers' child-rearing time, are positive. This proves the first part of proposition 1. As for the second, substituting the equilibrium education levels, 39 and 40, into the human capital accumulation equation, 7, immediately shows the dependence of the future generations human capital on the parents' one.