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Abstract

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JEL Classification: D43, C72

Keywords: non-sequential search, simultaneous search, oligopoly, Search cost heterogeneity, differentiated products, non-uniform sampling, Prominence

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Simultaneous Search for Differentiated Products: The Impact of Search Costs and Firm Prominence*

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Abstract

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1 Introduction

The early consumer search literature, which dates back at least to the 1960s, was dominated by homogeneous product models and focused on how search costs limited consumer price discovery, which often resulted in price dispersion (Stigler, 1961; Burdett and Judd, 1983; Stahl, 1989). With the rise of the Internet, it has become evident that search frictions, by constraining not only price but also product choice sets, distort consumer choice further. To properly capture this important feature, the more recent consumer search literature has focused on modelling markets for differentiated products. Moreover, following Weitzman (1979) and Wolinsky (1983, 1986), the accent has been put on models of sequential consumer search (Anderson and Renault, 1999; Armstrong, Vickers and Zhou, 2009; Moraga-González and Petrikaitė, 2013).

This emphasis on sequential consumer search is not always justified because, depending on the context, *simultaneous* search, also referred to as non-sequential or fixed-sample-size search, may be superior to sequential search (Morgan and Manning, 1985). Further, empirically it seems that in some industries simultaneous search is more prevalent than sequential search. For example, recent work by De Los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2017) has shown that for books and car insurance sold online, observed consumer search patterns are consistent with simultaneous search.[1] Furthermore, because search decisions do not depend on search outcomes when searching simultaneously, obtaining closed-form expressions for purchase probabilities and market shares is relatively easy and this has made models of simultaneous search for differentiated products popular in recent empirical work (see, e.g., De Los Santos, Hortaçsu, and Wildenbeest, 2012; Honka, 2014; Moraga-González, Sándor and Wildenbeest, 2015; Pires, 2016, 2018; Ershov, 2018; Murry and Zhou, 2019; Lin and Wildenbeest, 2019; and Donna, Pereira, Pires, and Trindade, 2019).

Despite this, market models of simultaneous search for differentiated products remain understudied in the theoretical literature. The purpose of this paper is narrowing this gap by extending the literature on consumer search for differentiated products to allow for *simultaneous* consumer search and consumer search cost heterogeneity. Within this framework, we derive novel results concerning the impact of search costs on competition and the effect of firm prominence on prices.

To the best of our knowledge, Anderson, De Palma, and Thisse (1992, p. 246) is the only theoret-

[1] To be able to empirically distinguish between sequential and simultaneous search, De Los Santos, Hortaçsu, and Wildenbeest (2012) exploit data on the sequence of searches and focus on a crucial difference between the two search methods in terms how search outcomes affect search behavior: when consumers search sequentially, the decision to continue searching depends on the outcome of the search, while with simultaneous search consumers commit to a certain number of searches before seeing any search outcomes. Honka and Chintagunta (2017) propose a test that only requires data on consideration sets.

ical study of equilibrium pricing with simultaneous consumer search for differentiated products.[2] In their model, N firms offer differentiated products to consumers who initially do not know how much the products are worth to them. The value of the match between a consumer and a product is a random draw from the double exponential distribution. Only after paying a search cost, a consumer can learn the value she places on a given product. Firms are symmetric and all consumers have the same utility and search cost. The problem of a consumer is thus to choose how many products to inspect; after having learned the match values of the inspected products, the consumer picks the product that yields the highest utility.

In contrast to markets for homogeneous products in which consumers optimally choose to check the prices of at most two firms (Burdett and Judd, 1983; Janssen and Moraga-González, 2004), Anderson, De Palma, and Thisse show that with differentiated products, depending on the magnitude of the search cost, consumers may check the products of any number of firms (including all of them if the search cost is sufficiently low). Specifically, they show that the equilibrium price in the search model is equal to the Perloff and Salop's (1985) (full information) price that would prevail in a market where the number of competitors is equal to the sample size selected by consumers. In equilibrium the price is therefore insensitive to the number of sellers. Further, small increases in the search cost do not affect the equilibrium price; it is only when the search cost increases by a sufficiently large amount that consumers choose to inspect fewer products, which results in a higher equilibrium price. Furthermore, market settings in which some firms are more salient than others (cf. the prominence model of Armstrong, Vickers, and Zhou, 2009) are no different from symmetric market environments.

The three rigidities just mentioned, namely that prices respond neither to small changes in search costs, nor to variations in the number of competitors, nor to differences in the market saliency of the firms, are somewhat unsatisfactory model features and we believe they are responsible for the fact that models of simultaneous search for differentiated products have not been used much in the theoretical consumer search literature. To deal with these limitations, we propose to introduce search cost heterogeneity. When consumers differ in their costs of search, they optimally choose to inspect different numbers of products. Consumers with sufficiently low search costs, then, choose to check all products; consumers with higher search costs choose to inspect a subset of the products, the higher their search costs the smaller the subset of products they inspect; consumers with prohibitively high search costs do not search at all and drop out of the market altogether. A consumer search equilibrium is then a partition of the consumer population into subsets of consumers inspecting

[2]For an authoritative and up-to-date survey of firm pricing with consumer search, see Anderson and Renault (2018).

different numbers of products. From the point of view of an individual firm, consumers who check many products are more price sensitive than consumers who inspect just a few. Optimal pricing trades-off the incentives to extract profits from the less price sensitive consumers and the incentives to compete for the more price sensitive ones. As we vary the number of firms, or change the search cost distribution, the partition resulting from consumer equilibrium behavior changes smoothly, which also smoothly changes the equilibrium price. Further, as sampling becomes less uniform due to a firm's enhanced market prominence, the allocation of consumers to firms changes continuously and this is also reflected in the price equilibrium.

Using this new model, we derive the following results. We first study the characterization and existence of a symmetric pure-strategy price equilibrium. For duopoly, we establish the existence of equilibrium for the case in which the density of match values is uniform and the search cost density is arbitrary. More general results are hard to obtain because the payoff of an individual firm consists of the sum of the profit originating from the consumers who check only its product and the profit stemming from the consumers who check the two products. Even if each of these profit functions is well behaved, as it is known, the sum might not be well behaved. Intuitively, a symmetric pure-strategy equilibrium may fail to exist because an individual firm may find it profitable to deviate from a putative equilibrium price by significantly jumping up its price, thereby sacrificing profit from the consumers who check the products of the two firms in exchange for profit from the consumers who only check the deviants product. Such a deviation may be unprofitable under more general conditions. In particular, we show that a symmetric pure-strategy price equilibrium also exists when the distribution of match values is quadratic and convex and the distribution of search costs is quadratic and concave.

We then proceed to an examination of how the equilibrium price responds to increases in search costs in the duopoly model. When the search costs of all consumers are relatively low, that is, when the upper bound of the search cost distribution is not too high, all consumers inspect the products of the two firms and the price equilibrium is identical to that corresponding to a perfect information model *à la* Perloff and Salop (1985).[3] In such a case, a small increase in the search costs distribution (in the sense of first-order stochastic dominance) does not affect the equilibrium price.

For intermediate values of the upper bound of the search cost distribution, some consumers inspect one product and the rest of the consumers check both products. The proportions of consumers testing

[3]To be sure, in Perloff and Salop (1985) the market is fully covered in the sense that all consumers buy one of the existing products.

one or two products are endogenous and when search costs become higher, fewer consumers check the two products. This leads to a higher equilibrium price.

For large values of the upper bound of the search cost distribution, some consumers drop out of the market altogether, while the rest inspect either one product or both products. In such situations, we derive a necessary and sufficient condition under which higher search costs for all consumers result in a lower equilibrium price. This result, which extends insights from Moraga-González, Sándor, and Wildenbeest (2017a) to the present case of simultaneous search for differentiated products, arises because search costs affect both the intensive search margin (or search intensity) and the extensive search margin (or the decision to search at all). Regarding the extensive search margin, an increase in search costs tends to increase the elasticity of demand because high-search-cost consumers drop out of the market altogether whereas regarding the intensive search margin, an increase in search costs tends to decrease the elasticity of demand because consumers search less. Which of these two effects dominates depends on the properties of the search cost distribution. The necessary and sufficient condition we find in the simultaneous search setting is quite distinct from the condition that would arise under sequential search (as in Moraga-González, Sándor, and Wildenbeest, 2017a). In fact, it may occur that a change in search costs will have the opposite effect on equilibrium prices in the sequential search model than in the simultaneous search model. This observation has a major implication for the empirical researcher interested in the understanding of the impact of a reduction in search costs. Even if a mis-specification of the search protocol does not bias the estimation of the search cost distribution, counterfactual analysis of lower search costs may lead to wrong conclusions.

We identify a stochastic ordering of distributions, called the *reversed hazard rate* ordering, such that higher search costs result in lower (higher) prices when the search cost distribution exhibits the decreasing (increasing) reversed hazard rate property. The decreasing (increasing) reversed hazard rate property is equivalent to the notion of log-submodularity (log-supermodularity) of the cumulative distribution function. Intuitively, when the search cost distribution is log-submodular (log-supermodular) an increase in search costs is more (less) noticeable at lower than at higher quantiles, which implies that the share of consumers inspecting the two products relative to the share of consumers inspecting just one increases (decreases) and the equilibrium price correspondingly goes down (up). The log-supermodularity or log-submodularity of distributions is empirically testable using estimates of search cost distributions. Based on these tests, the effects of policies that improve search technologies and/or increase market transparency can then be predicted.

In Sections 5 and 6 we explore the robustness of these results. In Section 5 we consider the case of

N firms and provide a characterization of the price equilibrium. Again, the nature of the equilibrium depends on the magnitude of the upper bound of the search cost distribution. At one extreme, we have an equilibrium in which all consumers inspect all products; hence, the price is equal to Perloff-Salop's price and insensitive to small increases in search costs. At the other extreme, we have a situation in which not all consumers search. For intermediate levels of search costs, all consumers inspect at least one product in equilibrium. Drawing from a recent contribution by Choi and Smith (2017) about preservation of quasi-concavity under aggregation, we give conditions for the existence of equilibrium assuming search costs are sufficienctly spread. We show that, for any arbitrary search cost distribution, an equilibrium exists in markets with fewer firms than nine. With a larger number of firms, the existence of equilibrium is guaranteed provided that the marginal cost of production is sufficiently high. We also present numerical results based on a family of distributions that, depending on parameters, can be ranked according to the increasing or decreasing reversed hazard rate ordering and confirm that the equilibrium price may both increase or decrease in search costs when not all consumers choose to search. Otherwise, if all consumers inspect at least one product, higher search costs unambiguously lead to a higher equilibrium price.

In Section 6 we return to the duopoly model and examine the case in which the firms differ in the likelihood with which they are sampled by consumers (Hortaçsu and Syverson, 2004; De los Santos, 2018). Intuitively, non-uniform sampling creates a market asymmetry in favour of the salient firm because the consumers who visit it have higher search costs on average than the consumers who visit the non-salient firm. As a result, the salient firm charges a higher price and obtains higher profits than the non-salient one. Our result is consistent with McDevitt (2014), who finds that plumbing firms in Chicago with a name that begins with an A or a number, and are therefore more likely to be searched first when using the Yellow Pages, command a price premium that is 8.4 percent above the average.

In contrast to the study of prominence of Armstrong, Vickers, and Zhou (2009), in our model with simultaneous search market saliency does not hurt consumers. In fact, when one firm is prominent and is therefore visited by all the consumers who choose to inspect only one product, in the unique equilibrium both the prominent and the non-prominent firms charge lower prices and consumer surplus is thus higher than when the firms are equally likely to be visited by consumers. We present numerical results confirming this result for less extreme situations of saliency. Finally, our results regarding the effects of search costs on the equilibrium prices remain with non-uniform sampling, so both firms prices increase in search costs when all consumers search, while, depending on the search cost distribution, they can increase or decrease when the market is not covered.

The remainder of this paper is structured as follows. In the next paragraphs, we discuss the related literature. In Section 2, we set up the model; the main insights of the paper are developed for the case of duopoly so this section focuses on that case. Section 3 characterizes the pricing equilibrium and studies its existence and uniqueness. Section 4 analyses the effects of higher search costs. Section 5 presents the N-firm model and Section 6 shows how to model non-uniform sampling within the simultaneous search framework. Section 7 concludes. Finally, an Appendix contains the proofs not provided in the main text.

Related literature

The literature on consumer search can be classified in terms of the search protocol and whether or not products are horizontally differentiated. Most of the early papers are about homogeneous product markets. A key contribution is Diamond (1971), who demonstrated that when consumers search sequentially to discover lower prices for a homogenous product, the unique pricing equilibrium is the monopoly price. Stahl (1989) introduced a simple form of search cost heterogeneity into Diamond's framework (the well-known and much-used 'shoppers and non-shoppers' formulation) and derived an equilibrium with price dispersion. Dealing with more general forms of consumer search cost heterogeneity in models of sequential consumer search with homogeneous product sellers has proven to be quite difficult (Stahl, 1996).

Burdett and Judd (1983) used a model of simultaneous search to show that an equilibrium with price dispersion also exists in the absence of search cost heterogeneity. Janssen and Moraga-González (2004) extended the setting of Burdett and Judd to oligopoly and allowed for an atom of shoppers. Their main results are on the effects of entry. Hong and Shum (2006) were the first to introduce general forms of consumer search cost heterogeneity in Burdett and Judd's framework.[4] However, they did this for the purpose of estimation and did not provide existence of equilibrium or comparative statics results. Moraga-González, Sándor, and Wildenbeest (2017b) prove the existence of a mixed strategy equilibrium in such a model and present new results on the relationship between prices and the number of firms. In the online appendix of Moraga-González, Sándor, and Wildenbeest (2017a), an analysis of how search costs affect prices in such a model is provided.

Weitzman (1979) is the first paper that studies optimal consumer search for differentiated products. Wolinsky (1983, 1986) are early papers embedding sequential consumer search for differentiated

[4]See also Moraga-González and Wildenbeest (2008), Wildenbeest (2011), Moraga-González, Sándor, and Wildenbeest (2013), and Sanches, Silva Junior and Srisuma (2018).

products into market settings. These papers show that, because of product differentiation, monopoly pricing is not an equilibrium. Hence, product differentiation invalidates the Diamond paradox. Moreover, with infinitely many firms, because consumers have positive search costs, prices remain above the marginal cost (Wolinsky, 1986). Anderson and Renault (1999) developed the model further and proved that prices increase when search costs rise, the number of firms decreases or products become less differentiated, and that entry is excessive from a welfare perspective.

Wolinsky's model is nowadays regarded as the workhorse model of sequential search for differentiated products in the consumer search literature. As such, it has seen numerous extensions in recent years. One such extension is the study of prominence of Armstrong, Vickers, and Zhou (2009) (see also Wilson, 2010; Rhodes, 2011; Zhou, 2011 and Fishman and Lubensky, 2018). Their paper studies the effects of non-random search in a sequential search framework in which consumers first visit a firm—the so-called *prominent* firm—and, if unsatisfied with the offering of that firm, they proceed by searching randomly among the non-prominent firms. In contrast to our results on non-uniform sampling, Armstrong, Vickers, and Zhou show that the prominent product is offered at a lower price than the non-prominent ones, and that making a product prominent increases industry profit but lowers consumer surplus and welfare. Relatedly, Armstrong and Zhou (2011) and Haan and Moraga-González (2011) present models in which a seller's market prominence depends on its choice of strategic variables such as price or advertising intensity.

Also closely related to our model, Moraga-González, Sándor, and Wildenbeest (2017a) extend Wolinsky's model by allowing for arbitrary search cost densities. They provide conditions for existence and uniqueness of equilibrium and derive the comparative statics effects of higher search costs. Like in this paper, they find that prices can increase or decrease when search costs go up provided that some consumers choose to not search in equilibrium. Because they deal with sequential search, their sufficient conditions for prices to increase or decrease in search costs are based on properties of search cost densities (specifically, the likelihood ratio ordering), rather than of search cost distributions (reversed hazard rate ordering), which are weaker.

2 Model

In this section we present a *duopoly* model of firms selling horizontally differentiated products to consumers who search the market for satisfactory goods using a simultaneous search strategy.[5] The two firms produce the horizontally differentiated products at a marginal cost equal to r and choose

[5]The N-firm model is examined in Section 5.

their prices simultaneously to maximize profits. We focus on pure-strategy symmetric Nash equilibria (SNE); let p^* denote a SNE price.

There is a unit mass of consumers. A consumer m has tastes for a product i described by the following indirect utility function:

$$u_{im} = \begin{cases} \varepsilon_{im} - p_i & \text{if she buys product } i \text{ at price } p_i; \\ 0 & \text{otherwise.} \end{cases}$$

The parameter ε_{im} is a match value between consumer m and product i. The match value ε_{im} is assumed to be i.i.d. across consumers and products. Let F be the cumulative distribution function of ε_{im}, defined over the support $[0, \overline{\varepsilon}]$. We assume that the density function of match values, denoted f, is differentiable and log-concave.

Consumers search for a satisfactory product non-sequentially. This means that they first choose the number of firms to visit, including possibly none, in order to maximize expected utility. Once they have visited the desired number of firms, they buy from the store offering them the best deal, or else they do not buy anything. While deciding on the intensity of search, they hold correct conjectures about the equilibrium price. The total cost of search of a consumer with search cost c_m who searches $n = 0, 1, 2$ times is nc_m. Consumers have heterogeneous search costs. The distribution of search costs is denoted G and the density g; we assume that g is positive on the support $(\underline{c}, \overline{c})$. The lower bound $\underline{c}$ does not play much of a role so we will set it equal to 0 in much of what follows. The upper bound $\overline{c}$, by contrast, plays a very important role in the analysis that follows because it drives consumer search participation. We allow it to be sufficiently large.

To put our model in perspective, it is a duopoly version of the workhorse search model of Wolinsky (1986), but with search cost heterogeneity and simultaneous search instead of sequential search. The critical distinction between sequential and simultaneous search is that with simultaneous search consumers commit ex-ante to a number of searches. As mentioned in the introduction, only Anderson, De Palma, and Thisse (1992) have theoretically analyzed simultaneous search for differentiated products. In their model, all consumers have the same search cost and this results in an equilibrium where all of them inspect the same number of products. With arbitrary search cost heterogeneity, different consumers pursue distinct search strategies including the possibility of not searching at all. As a matter of fact, when the upper bound of the search cost distribution is sufficiently large, some consumers choose to directly consume the outside option (of zero) without inspecting any of the products on offer.

3 Equilibrium

In this section we characterize a symmetric pure-strategy Nash equilibrium. Let us start examining the problem of the consumers. Assume both firms charge a price $p^* \in [r, p^m]$, where p^m denotes the standard monopoly price. Because consumers have correct expectations about the equilibrium price, a consumer with search cost c that chooses to only inspect the product of one firm expects to obtain a utility equal to

$$U(1,c) = \Pr[\varepsilon \geq p^*][E[\varepsilon|\varepsilon \geq p^*] - p^*] - c = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c. \tag{1}$$

For a consumer to conduct at least one search, such an expected utility has to be positive. If an individual with search cost $c \in [0, \overline{c}]$ exists such that equation (1) is equal to zero, then this means that for some consumers it is not worthwhile to conduct a first search. Correspondingly, we define the critical search cost value:

$$c_0(p^*) \equiv \min \left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \right\}.$$

If $c_0(p^*)$ is strictly lower than the upper bound of the search cost distribution $\overline{c}$, a fraction of the consumer population will abstain from searching.

Consider now a consumer with search cost c for whom it is worth to conduct at least one search. This consumer has to choose between inspecting the product of one firm only or inspecting the products of the two firms. Let $z_2 \equiv \max\{\varepsilon_1, \varepsilon_2\}$ and note that the distribution of z_2 is $F(\varepsilon)^2$. Then, the utility a consumer expects to get when checking the two products is equal to

$$U(2,c) = \Pr[z_2 \geq p^*][E[z_2|z_2 \geq p^*] - p^*] - 2c = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) 2F(\varepsilon) f(\varepsilon) d\varepsilon - 2c. \tag{2}$$

Comparing this utility with that derived from inspecting only one product, she will prefer to visit the two firms provided that $U(2,c) > U(1,c)$, or

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) 2F(\varepsilon) f(\varepsilon) d\varepsilon - c \geq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon.$$

Correspondingly, we define the critical search cost value $c_1(p^*)$ above which and below $c_0(p^*)$ consumers prefer to inspect one product only:

$$c_1(p^*) \equiv \min \left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1] f(\varepsilon) d\varepsilon \right\}.$$

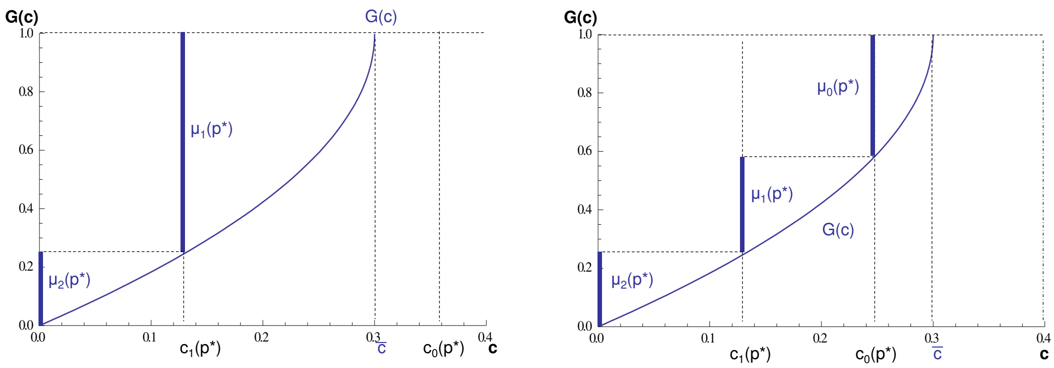
It is straightforward to check that $c_1(p^*) \leq c_0(p^*)$. Individuals with search cost below $c_1(p^*)$ prefer to search twice. Hence, the population of consumers can be split into three groups of consumers.

These three groups comprise consumers not searching at all, searching one time, and searching two times. Denoting the group of consumers searching k times by $\mu_k(p^*)$, we have:

$$\mu_0(p^*) = 1 - G(c_0(p^*)); \ \mu_1(p^*) = G(c_0(p^*)) - G(c_1(p^*)); \text{ and } \mu_2(p^*) = G(c_1(p^*)) \tag{3}$$

Notice that our assumptions on the search cost distribution imply that $\mu_2(p^*) \leq 1$ (with equality when $\bar{c}$ is low enough), while $0 \leq \mu_i(p^*) < 1$, $i = 0, 1$ (with equality when $\bar{c}$ is sufficiently low).

Figure 1 illustrates two of these cases (the third, less interesting, case has $\mu_2(p^*) = 1$). In Figure 1(a) we represent a case where all consumers search; in particular the vertical (blue) line denoted $\mu_1(p^*)$ depicts the share of consumers who search once, while the vertical (blue) line denoted $\mu_2(p^*)$ shows the fraction of consumers who search twice. Note that when $\bar{c}$ is very low, all consumers will search twice. In contrast, Figure 1(b) shows that for c_0 sufficiently small, a fraction of consumers $\mu_0(p^*)$ finds it optimal to refrain from searching.



(a) c_0 sufficiently large

(b) c_0 sufficiently small

Figure 1: Equilibrium search intensities and search costs

We now move to the problem of the firms. To characterize the symmetric pure-strategy equilibrium we start by deriving the payoff of a firm i that deviates from equilibrium pricing by charging a price $p_i \neq p^*$, given that the rival firm charges p^* and given consumer search behavior. The expected payoff to the deviant firm i is:

$$\pi_i(p_i; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2} \Pr[\varepsilon_i \geq p_i] + \mu_2(p^*) \Pr\left[\varepsilon_i - p_i \geq \max\{\varepsilon_j - p^*, 0\}\right]\right), \tag{4}$$

where Pr stands for probability. This payoff formula is easily understood. The per-consumer profit is $p_i - r$. Consumers who search only once happen to visit firm i with probability 1/2; these consumers buy firm i's product when the match values they obtain there are higher than the price p_i. Consumers who search twice only buy from firm i when firm i's deal is better than the rival's and the outside option of 0. Thus, the payoff can be seen as a weighted average of the perfect information monopoly payoff and duopoly payoff, though the weights do not sum up to 1.

When firm i deviates by charging a higher price than the rival, $p_i > p^*$, the payoff in equation (4) can be written as follows:[6]

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right). \tag{6}$$

The first order condition (FOC) in this case is:

$$\begin{aligned}\frac{d\pi_i(p_i)}{dp_i} &= \frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - p_i + p^*) f(\varepsilon)\, d\varepsilon \\ &\quad - (p_i - r)\left\{\frac{\mu_1(p^*)}{2}f(p_i) + \mu_2(p^*)\left[\int_{p_i}^{\overline{\varepsilon}} f(\varepsilon - p_i + p^*) f(\varepsilon)\, d\varepsilon + F(p^*)f(p_i)\right]\right\} = 0 \end{aligned} \tag{7}$$

Setting $p_i = p^*$ in equation (7), replacing $\mu_1(p^*)$ and $\mu_2(p^*)$ by their corresponding values in terms of the search cost distribution and rearranging, we obtain the necessary condition for a symmetric equilibrium price p^*. Let us define the function

$$H(p) \equiv N(p)G(c_1(p)) - D(p)G(c_0(p)), \tag{8}$$

where the functions $D(p)$ and $N(p)$ are given by

$$\begin{aligned} D(p) &\equiv -[1 - F(p) - (p - r) f(p)]; \\ N(p) &\equiv F(p)(1 - F(p)) - 2(p - r)\left(\int_p^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon + F(p)f(p) - \frac{1}{2}f(p)\right). \end{aligned}$$

The necessary condition for a symmetric equilibrium price p^* is

$$H(p^*) = 0. \tag{9}$$

Equation (9) cannot be solved for an explicit solution in p^*. However, we now note that a candidate equilibrium price $p^* \in [r, p^m]$ exists. We observe first that when we set $p = r$ we obtain

$$H(r) = (1 - F(r))[F(r)G(c_1(r)) + G(c_0(r))] > 0.$$

Second, if we set $p = p^m$ then we get that

$$H(p^m) = N(p^m)G(c_1(p^m)), \tag{10}$$

[6] When firm i deviates by charging a lower price, the payoff formula is different:

$$\pi_i(p_i < p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\left[1 - F(\overline{\varepsilon} + p_i - p^*) + \int_{p_i}^{\overline{\varepsilon} + p_i - p^*} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right]\right). \tag{5}$$

However, the condition that a symmetric price equilibrium must satisfy is the same as the one we derive below in equation (9).

just because the price p^m satisfies the first order condition for the monopoly problem: $1 - F(p^m) - (p^m - r) f(p^m) = 0$. The sign of $H(p^m)$ depends on the sign of $N(p^m)$, for which we can write:

$$\begin{aligned} N(p^m) &= F(p^m)(1-F(p^m)) - 2(p^m - r)\left(\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon + F(p^m)f(p^m) - \frac{1}{2}f(p^m)\right); \\ &= [1+F(p^m)][1-F(p^m)] - 2(p^m - r)\left(\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon + F(p^m)f(p^m)\right); \\ &= (p^m - r)\left[f(p^m)[1-F(p^m)] - 2\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon\right], \end{aligned} \tag{11}$$

where we have used the relation $1 - F(p^m) - (p^m - r) f(p^m) = 0$ once more. Upon observing equation (11) it follows that the sign of $H(p^m)$ depends on the sign of the expression inside the squared brackets. Let us define

$$M(p) \equiv f(p)[1-F(p)] - 2\int_{p}^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon.$$

Taking the derivative of M with respect to p gives $f'(p)(1-F(p)) + f(p)^2$, which is greater than zero by log-concavity of f (see Corollary 2 in Bagnoli and Bergstrom, 2005). Since M is increasing in p and is equal to zero when we set $p = \overline{\varepsilon}$, we conclude that $M(p^m) < 0$. Hence $H(p^m) < 0$.

Since H is a continuous function with $H(r) > 0$ and $H(p^m) < 0$, we conclude that for any log-concave density f, there exists a candidate price equilibrium $p^* \in [r, p^m]$. Note also that at the candidate equilibrium price p^* we must have $dH(p^*)/dp < 0$. Further, we can prove that:

Proposition 1 *Depending on the magnitude of the upper bound of the search cost distribution $\overline{c}$, in the simultaneous search duopoly model there may exist three types of SNE.*

(A) A SNE where all consumers search twice and firms charge a price given by the solution to

$$\frac{1}{2}(1 - F^2(p^*)) - (p^* - r)\left[\int_{p^*}^{\overline{\varepsilon}} f(\varepsilon)^2 d\varepsilon + F(p^*)f(p^*)\right] = 0. \tag{12}$$

This equilibrium is unique and exists provided that f is log-concave and

$$\overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1]f(\varepsilon)d\varepsilon. \tag{13}$$

(B) A SNE where a fraction $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon - p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ of consumers searches the two firms and the rest just one, in which case the equilibrium price p^* is given by the solution to equation (9). For this equilibrium to exist $\overline{c}$ must satisfy the inequality

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)f(\varepsilon)d\varepsilon \geq \overline{c} > \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1]f(\varepsilon)d\varepsilon, \tag{14}$$

and when F is the uniform distribution, an equilibrium surely exists.

(C) Finally, a SNE where a fraction $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ *of consumers searches the two firms, a fraction* $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)f(\varepsilon)d\varepsilon\right)-G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ *of consumers searches one firm only, and the rest do not search at all, in which case the equilibrium price* p^* *is given by the solution to equation (9). For this equilibrium to exist* $\overline{c}$ *must satisfy the inequality*

$$\overline{c} > \int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)f(\varepsilon)d\varepsilon, \tag{15}$$

and when F *is the uniform distribution, an equilibrium surely exists.*

Proof. See the Appendix.

Proving the existence of equilibrium when $\overline{c}$ is relatively large is challenging because the payoff of a firm consists of the sum of the payoff originating from the consumers who check only its product and the payoff stemming from the consumers who check the two products. Under the log-concavity of f, each of these payoffs is quasi-concave (which follows from an application of the Prékopa (1973) aggregation result in our setting). Despite this, unfortunately the sum of these payoffs may fail to be quasi-concave, which implies that we need to impose additional restrictions on the primitives of the model in order to guarantee the existence of a pure-strategy equilibrium. In the Appendix we show that, for any arbitrary search cost distribution G, the payoff of a firm is strictly concave in a firm's own price when match values are uniformly distributed, which ensures the existence of equilibrium.

For arbitrary distributions of match values F, the equilibrium may fail to exist. The problem is that an individual firm may find it profitable to deviate from a putative equilibrium price p^* by significantly jumping up its price, thereby sacrificing profit from the consumers who check the products of the two firms in exchange for profit from the consumers who only check the deviant's product. It is nevertheless possible to provide more general existence results. Intuitively, to rule out such a deviation, it is necessary that the share of consumers who only check one product is not very large. In the Appendix we show that when F is quadratic and convex and G is quadratic and concave, a pure-strategy symmetric equilibrium exists.

4 Higher search costs

We now study how the equilibrium price derived in Proposition 1 depends on the magnitude of search costs. To do this, we parametrize the search cost density by a scalar β and assume that an increase in β shifts the search cost distribution downwards, that is, an increase in β signifies an increase in search costs in the sense of first order stochastic dominance. Let $G(c;\beta)$ be a parametrized search cost CDF with $\partial G(c;\beta)/\partial\beta < 0$ and denote the price equilibrium corresponding to a given β by $p^*(\beta)$. We next study how the equilibrium price $p^*(\beta)$ responds to a change in β.

The first observation we make is that the price in Proposition 1(A), given by the solution to the FOC in equation (12), is completely independent of a small change in the search cost distribution. As mentioned above, this is because search costs are so low in this case that they do not restrict consumers' search behavior at all and, as a result, all consumers inspect the products of the two firms in equilibrium.

Cases (B) and (C) of Proposition 1 are more interesting. In both cases, if an equilibrium exists, the price is given by the solution to the FOC given by equation (9). Because we have parametrized G by β, let us denote by $H(p^*;\beta)$ the corresponding parametrized function defined by the FOC in equation (9). By the implicit function theorem, the comparative statics effect of an increase in search costs is then given by

$$\frac{dp^*(\beta)}{d\beta} = -\left.\frac{\partial H}{\partial\beta}\right/\frac{\partial H}{\partial p^*}\ . \tag{16}$$

We have already noted above that the denominator of equation (16), $\partial H/\partial p^*$, is negative. We now study the sign of the numerator of equation (16). For this we now distinguish between cases (B) and (C) in Proposition 1.

Consider first the situation in Proposition 1(B). In this case, the upper bound of the search cost distribution is neither too high nor too low, which implies that all consumers inspect at least one product (i.e., $G\left(c_0(p^*),\beta\right) = 1$) and some consumers inspect the two products (i.e. $\mu_2(p^*) = 1-\mu_1(p^*) = G(c_1(p^*),\beta) > 0$). In such a case, the numerator of equation (16) is

$$\frac{\partial H}{\partial\beta} = N(p^*)\frac{\partial G\left(c_1(p^*),\beta\right)}{\partial\beta} > 0,$$

where the sign follows from the facts that $D(p^*) < 0$ and existence of a candidate equilibrium implies that $N(p^*) < 0$. As a result, since $\partial H/\partial p^* < 0$ and $\partial H/\partial\beta > 0$, we have demonstrated that $dp^*(\beta)/d\beta > 0$. That is, an increase in search costs results in higher prices, which is the standard result in the consumer search literature. In the present case all consumers search, which implies that an increase in search costs only affects consumers' search intensity (the intensive search margin), and does not affect consumers' participation (the extensive search margin). When search costs increase, consumers search less, and prices go up. That consumers search less is reflected here by $G(c_1(p^*),\beta)$ falling in β, which, by definition, means that the fraction of consumers checking the two products decreases and, by implication, the fraction of consumers inspecting only one product increases. Facing fewer consumers who compare the products of the two firms after search costs increase and more consumers who only check one of the products, the sellers safely increase their prices.

Consider now the situation in case (C). In this situation $G(c_0(p^*), \beta) < 1$ and therefore for the numerator of equation (16) we have

$$\frac{\partial H}{\partial \beta} = N(p^*)\frac{\partial G(c_1(p^*), \beta)}{\partial \beta} - D(p^*)\frac{\partial G(c_0(p^*), \beta)}{\partial \beta}.$$

Using the equilibrium condition in equation (9), we can rewrite this as follows:

$$\begin{aligned}
\frac{\partial H}{\partial \beta} &= D(p^*)\frac{G(c_0(p^*), \beta)}{G(c_1(p^*), \beta)}\frac{\partial G(c_1(p^*), \beta)}{\partial \beta} - D(p^*)\frac{\partial G(c_0(p^*), \beta)}{\partial \beta} \\
&= D(p^*)\left[\frac{G(c_0(p^*), \beta)}{G(c_1(p^*), \beta)}\frac{\partial G(c_1(p^*), \beta)}{\partial \beta} - \frac{\partial G(c_0(p^*), \beta)}{\partial \beta}\right] \\
&= D(p^*)G(c_0(p^*), \beta)\left[\frac{1}{G(c_1(p^*), \beta)}\frac{\partial G(c_1(p^*), \beta)}{\partial \beta} - \frac{1}{G(c_0(p^*), \beta)}\frac{\partial G(c_0(p^*), \beta)}{\partial \beta}\right]. \qquad (17)
\end{aligned}$$

The sign of $\partial H/\partial\beta$ is therefore ambiguous; it depends on the values that the hazard rate G'_β/G takes at the cutoff points $c_0(p^*)$ and $c_1(p^*)$, where G'_β is short-hand notation for $\partial G/\partial\beta$. The interesting issue is that this derivative can be negative, in which case the equilibrium price will *decrease* when search costs increase. The next proposition summarizes our findings and provides a sufficient condition for the equilibrium price to decrease in search costs. We explain the intuition behind this result after stating it precisely.

Proposition 2 *Let $G(c;\beta)$ be a search cost CDF with positive density on $[0, \overline{c}]$ and with derivative $\partial G(\cdot)/\partial\beta < 0$. Then the comparative statics with respect to β of the SNE price of the non-sequential search duopoly model described in Proposition 1 is as follows:*

(A) The equilibrium price given by Proposition 1(A) is independent of β. Therefore, higher search costs do not have a bearing on the equilibrium price.

(B) The equilibrium price given by Proposition 1(B) unambiguously increases in β. Therefore, higher search costs always result in higher prices.

(C) The equilibrium price given by Proposition 1(C) decreases in β if and only if

$$\frac{1}{G(c_1(p^*), \beta)}\frac{\partial G(c_1(p^*), \beta)}{\partial \beta} - \frac{1}{G(c_0(p^*), \beta)}\frac{\partial G(c_0(p^*), \beta)}{\partial \beta} > 0. \qquad (18)$$

Moreover, if G'_β/G increases (decreases) in c, then the equilibrium price increases (decreases) in β. The price is independent of β if G'_β/G is constant in c.

The contrast between the results in parts (B) and (C) of the proposition is important in that it demonstrates once more that the standard (positive) association between search costs and prices is based on a restriction on the magnitude of search costs.[7] When search costs are initially low, an increase in search costs only affects the intensive search margin. Consumers face more difficulties when comparing products, which means they engage in less product comparison. Buyers who stop comparing products enlarge the group of buyers who do not, and this means an individual firm faces less elastic demand. Correspondingly, firms adjust their prices upwards.

However, when search costs are not restricted to be initially low, increases in search costs affect both the intensive and the extensive search margins. At the intensive search margin, the same effect as before occurs. The share of consumers who used to inspect the two products goes down and this tends to decrease the elasticity of demand an individual firm faces. However, at the extensive search margin, more consumers drop out of the market altogether when search costs go up, which changes the demand composition and the identity of the average consumer. Condition (18) is necessary and sufficient for an increase in search costs to raise rather than reduce the elasticity of demand; under this condition, higher search costs result in lower prices. A sufficient condition is that the hazard rate G'_β/G (or the elasticity of G with respect to β) be increasing in search costs. Under those conditions, an increase in search costs is more noticeable at higher percentiles of the search cost distribution than at lower, which implies that the effect on the extensive search margin is stronger than the effect on the intensive search margin.

In order to illustrate these arguments, we refer to Figure 2. In this figure we represent the effect of an increase in search costs on the intensive and extensive search margins. Initially consumer search costs are given by the blue search cost distribution. This search cost distribution has the property that G'_β/G is increasing in c.[8] The increase in search costs is represented by the shift from the blue distribution to the red one. As the graph shows, the increase in search costs is much more felt at the higher percentiles of the search cost distribution.

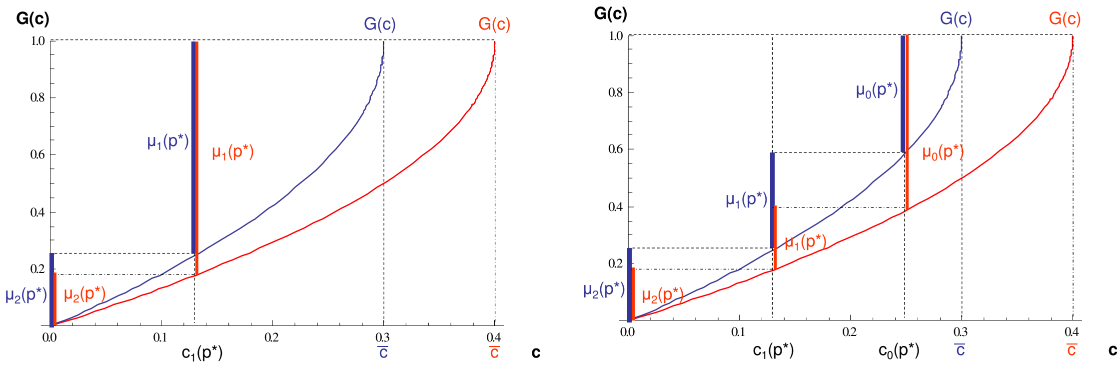
In Figure 2(a) we represent the case discussed in Proposition 2(B). Before the increase in search costs, the blue fractions of consumers $\mu_1(p^*)$ and $\mu_2(p^*)$ represent the equilibrium fractions of con-

[7]Moraga-González, Sándor, and Wildenbeest (2017a) present a similar finding for the standard model of sequential search for differentiated products (cf. Wolinsky, 1986). See also the online Appendix of that paper for an study of non-sequential search for homogeneous products (cf. Burdett and Just, 1983), as well as Fabra and Reguant (2018), who study price discrimination in such a setting. It is reassuring to learn that, when the market is not covered, the insight that prices can increase or decrease in search costs is robust to the search protocol (sequential vs. non-sequential), the type of product (differentiated vs. homogeneous) and the nature of the market equilibrium (pure vs. mixed strategies).

[8]We use the Kumaraswamy (1980) distribution in Figure 7 with parameters $a = 1$, $b = 1/2$, and upper bound $\beta = 0.3$. In Section 4.2 we formally introduce this distribution and demonstrate that G'_β/G is increasing in c for those parameter values.

sumers checking one and two products, respectively. Because here search costs are small for all consumers ($c_0(p^*) = \bar{c}$), they all check at least one product. Keeping prices constant, an increase in search costs results in a fall in the number of consumers who inspect the two products and, correspondingly, in an increase in the number of consumers who just check one. This lowers the demand elasticity so firms find it optimal to raise their prices.

Figure 2(b) shows the case discussed in Proposition 2(C). In this case search costs are sufficiently large ($c_0(p^*) < \bar{c}$) and a fraction of consumers $\mu_0(p^*)$ therefore opts out of the market altogether. When search costs increase, keeping prices fixed, the share of consumers who do not even start searching increases substantially. This causes the share of inelastic consumers to fall more than the share of elastic consumers; this demand composition effect increases overall elasticity of demand, and firms lower their prices as a result.



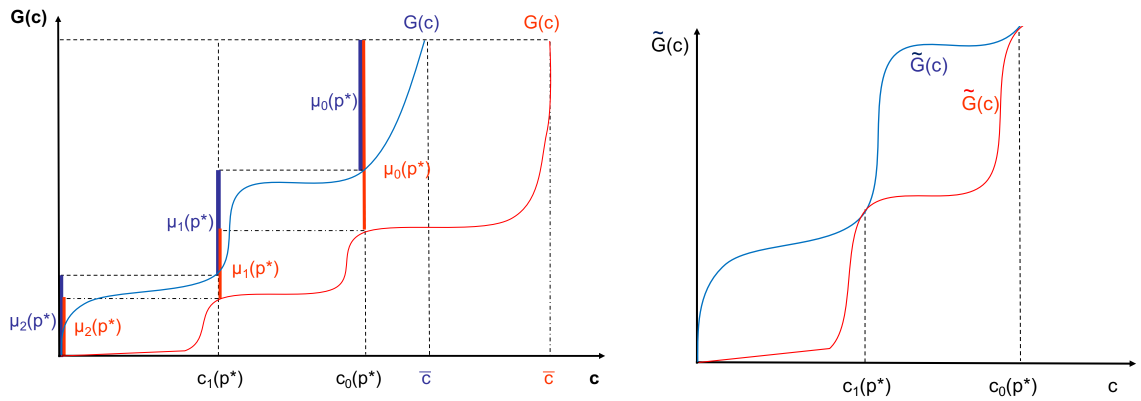
(a) c_0 sufficiently large

(b) c_0 sufficiently small

Figure 2: The effect of an increase in search costs

Moraga-González, Sándor, and Wildenbeest (2017a) present a related finding for the standard model of sequential search for differentiated products (cf. Wolinsky, 1986). They also show that, when search costs are not restricted to be low, higher search costs may result in a higher or lower equilibrium price and provide necessary and sufficient conditions for these effects to occur. Mathematically, the conditions they give is however quite different from that in condition (18). The main distinction relates to the nature of search in the two different models. While with simultaneous search what matters for pricing are the relative masses of consumers checking one or two products, with sequential search the entire density of the various consumer types who search in the market affects price. In fact, it may happen that a change in search costs will have the opposite effect on equilibrium prices in the sequential search model than in the simultaneous search model. We now provide two examples.

Let us start with a situation in which an increase in search costs results in a decrease in the equilibrium price under simultaneous search and in an increase in the equilibrium price under sequential search. We depict this situation in Figure 3. Note that consumer search behavior with the two search cost distributions depicted in Figure 3(a) is exactly the same as that with the search cost distributions depicted in Figure 2(b). That is, the corresponding shares of consumers checking one and two products are exactly the same in the two situations. As a result, the equilibrium prices are also identical and we know from the discussion above that an increase in search costs leads to a lower equilibrium price. Hence, although the search cost distributions are quite different in Figures 2(b) and 3(a), these differences make no impact on pricing under simultaneous search because all firms care about are the shares of consumers searching one or two times (and do not care about whether consumers within those shares have higher or lower search costs).



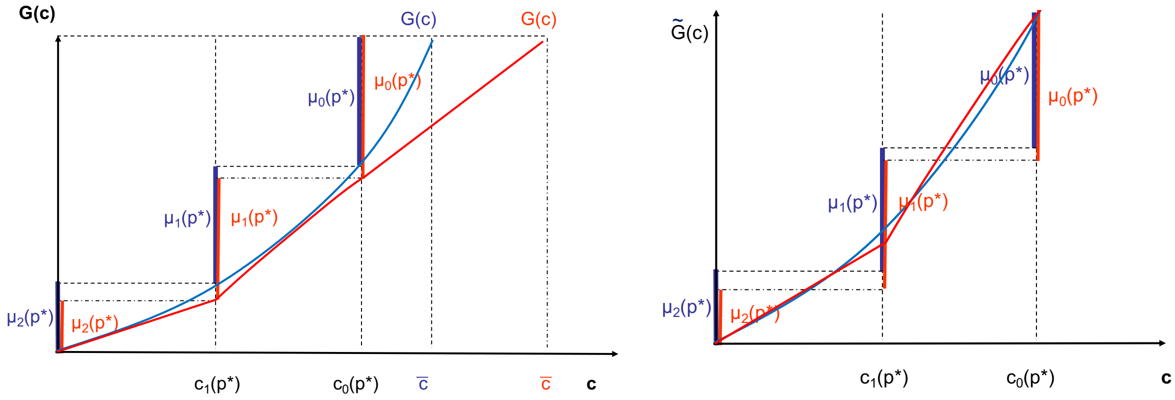
(a) Search cost distributions

(b) Search cost truncated distributions

Figure 3: The effect of an increase in search costs (b)

With sequential search things are quite different. While for the distributions in Figure 2(b) the equilibrium price would also decrease as search costs increase (since G'_{β}/G is increasing in c), the opposite holds for the distributions in Figure 3(a). This is because the truncated distributions corresponding to the two original search cost distributions, which are depicted in Figure 3(b), are also ranked according to the FOSD order (cf. Proposition 2 in Moraga-González, Sándor and Wildenbeest (2017)).

The opposite happens for the example in Figure 4. An increase in search costs would lead to a higher equilibrium price under simultaneous search and to a lower equilibrium price under sequential search. The first fact follows from the observation that an increase in search costs reduces the share of consumers comparing the two products and does not affect much the share of consumers checking one product only. The second fact from the behaviour of the truncated search cost distributions.



(a) Search cost distributions (b) Search cost truncated distributions

Figure 4: The effect of an increase in search costs (c)

The observation that an increase in search costs may lead the opposite effect on prices depending on the search protocol has a major implication for the empirical researcher interested in the understanding of the impact of a changes in search costs. Even if a mis-specification of the search protocol does not bias the estimation of the search cost distribution, counterfactual analysis of lower search costs may lead to wrong conclusions.

4.1 The reversed hazard rate stochastic ordering

In this section we relate the sufficient condition in Proposition 2(C) to the *reversed hazard rate ordering* of distributions (see Shaked and Shanthikumar, 2007), which is a well-known stochastic ordering.

Definition 1. *The distribution $G(c;\beta)$ has the increasing reversed hazard rate (IRHR) property if and only if for any $\beta' < \beta$,*

$$G(c,\beta)G(d,\beta') \leq G(c,\beta')G(d;\beta)$$

for any $c \leq d$ in the union of the supports of $G(c,\beta')$ and $G(c;\beta)$.

We now define distributions for which the reverse property holds, namely, that they have *decreasing reversed hazard rates*:

Definition 2. *The distribution $G(c;\beta)$ has the decreasing reversed hazard rate (DRHR) property if and only if for any $\beta' < \beta$,*

$$G(c,\beta)G(d,\beta') \geq G(c,\beta')G(d;\beta)$$

for any $c \leq d$ *in* $[0, \min\{\bar{c}(\beta), \bar{c}(\beta')\}]$.[9]

A few simple calculations reveal that for distributions with IRHR, the ratio G'_β/G increases in c, which is equivalent to the notion of *log-supermodularity* of the distribution function. On the contrary, for distributions with DRHR, the ratio G'_β/G decreases in c; this is then equivalent to *log-submodularity* of the distribution function.

Corollary 1 to Proposition 2C. *For log-supermodular (log-submodular) search cost distributions distributions, a FOSD increase in search costs results in an increase (decrease) in the equilibrium price given in Proposition 1(C).*

We note that the notions of IRHR (log-supermodularity) and DRHR (log-submodularity) take very simple forms in the common cases of additive and multiplicative shocks to search costs. In the case of multiplicative shocks, the search cost distribution is $G(c/(1+\beta))$, with $\beta \geq 0$. In this case, IRHR (DRHR) is identical to cg/G being decreasing (increasing), which is the same as decreasing (increasing) search cost elasticity of the cumulative distribution function G. In the case of additive shocks, the search cost distribution is $G(c-\beta)$, with $\beta \geq 0$. Re-defining the notion of DRHR on the set $[\max\{\underline{c}(\beta), \underline{c}(\beta')\}, \min\{\bar{c}(\beta), \bar{c}(\beta')\}]$ we note then that IRHR (DRHR) is equivalent to g/G being decreasing (increasing), which is the same as log-concavity (log-convexity) of the distribution function G.[10]

4.2 An illustrative example: The Kumaraswamy's distribution

The Kumaraswamy's (1980) distribution has a cumulative distribution function G and a probability distribution function g given by:

$$\begin{aligned} G(c) &= 1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b, \; c \in [0, \beta], \quad a, b > 0; \\ g(c) &= \frac{ab}{\beta}\left(\frac{c}{\beta}\right)^{a-1}\left[1 - \left(\frac{c}{\beta}\right)^a\right]^{b-1}. \end{aligned}$$

The Kumaraswamy distribution is often used as a substitute for the beta distribution (see Ding and Wolfstetter, 2011). An increase in β signifies an increase in search costs for all consumers. Depending

[9] Note that we define DRHR up to the minimum of the upper bounds of the supports of the search cost distributions $G(c, \beta)$ and $G(c, \beta')$. This is needed for compatibility of the DRHR ranking with the FOSD ranking of distributions.

[10] In Moraga-González, Sándor, and Wildenbeest's (2017a) model of sequential search for differentiated products, conditions based on the likelihood ratio ranking of densities are provided for a similar implication. Note that the likelihood ratio ordering, which relates to the density functions of random variables, is stronger than the reversed hazard rate ordering, which has to do with the cumulative distribution functions. A consequence of this distinction is that the implication that for search cost densities satisfying the decreasing likelihood property an increase in search costs leads to a lower equilibrium price cannot be applied to the case of additive shocks to search costs.

on the parameter b, this distribution can be log-supermodular ($b > 1$) or log-submodular ($0 < b < 1$). Then, we have the following:

Corollary to Proposition 2 *Assume that search costs are distributed on the interval* $[0, \beta]$ *according to the Kumaraswamy distribution. Then:*

(A) The equilibrium price in Proposition 1(A) is independent of β*.*

(B) The equilibrium price in Proposition 1(B) unambiguously increases in β*.*

(C) For all a*, the equilibrium price in Proposition 1(C) decreases in* β *if* $0 < b < 1$*, is constant in* β *if* $b = 1$*, and increases in* β *if* $b > 1$*.*

5 The N-firm model

The previous simultaneous search model with differentiated products can be generalized to the case of $N > 2$ firms. The problem of a consumer with search cost c is to choose a number k of firms to be sampled in order to maximize her expected utility:

$$\max_k \left\{ \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) k F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon - kc \right\}.$$

It can easily be checked that this problem is well-behaved and that a unique solution exists. Such a solution defines a partition of the consumer population into groups of buyers $\mu_k(p^*)$ that search $k = 0, 1, 2, \ldots, N$ firms, with $\sum_{k=0}^{N} \mu_k(p^*) = 1$; as above, some of these groups may have zero mass as the upper bound of the search cost distribution decreases.

In order to determine the size of these groups, let us define the critical search cost parameters

$$\begin{aligned} c_0(p^*) &= \min\left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \right\} \\ c_k(p^*) &= \min\left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(k+1) F(\varepsilon) - k \right] F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon \right\}, \ k = 1, 2, \ldots, N-1. \end{aligned}$$

The fractions of consumers searching k times are then given by the expressions:

$$\begin{aligned} \mu_0 &= 1 - G(c_0(p^*)) \\ \mu_k &= G(c_{k-1}(p^*)) - G(c_k(p^*)), \ k = 1, 2, \ldots, N-1 \\ \mu_N &= G(c_{N-1}(p^*)) - G(c_N(p^*)) = G(c_{N-1}(p^*)) \text{ since } c_N = 0. \end{aligned} \tag{19}$$

If $c_{N-1}(p^*) = \overline{c}$, then all consumers will inspect the N products in equilibrium and the situation will again resemble the perfect information model of Perloff and Salop (1985). When $c_{N-1}(p^*) <$

$\overline{c} = c_{N-2}(p^*)$, a fraction $\mu_N = G(c_{N-1}(p^*))$ of consumers will inspect the N products and the remaining consumers will each check $N-1$ randomly selected products. When $c_{N-2}(p^*) < \overline{c} = c_{N-3}(p^*)$, a fraction $\mu_N = G(c_{N-1}(p^*))$ of consumers will inspect the N products, a fraction $\mu_{N-1} = G(c_{N-2}(p^*)) - G(c_{N-1}(p^*))$ will check $N-1$ products, and the remaining consumers will each inspect $N-2$ randomly selected products. And so on and so forth.

Let $z_k \equiv \max\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k\}$. In general, the expected payoff of a firm i that deviates from the symmetric equilibrium price by charging a price $p_i \neq p^*$ is

$$\pi_i(p_i; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}\Pr[\varepsilon_i \geq p_i] + \sum_{k=2}^{N}\frac{k\mu_k(p^*)}{N}\Pr\left[\varepsilon_i - p_i \geq \max\{z_{k-1} - p^*, 0\}\right]\right) \tag{20}$$

As before, the demand of the deviant firm i stems from the various consumer groups, and a consumer who searches k times compares the offer of firm i with the offers of $k-1$ other firms.

For the case where the deviant firm charges a higher price than the rest of the firms, the expression in equation (20) becomes:[11]

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left[\frac{\mu_1(p^*)}{N}(1 - F(p_i)) + \sum_{k=2}^{N}\frac{k\mu_k(p^*)}{N}\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))^{k-1} f(\varepsilon)d\varepsilon\right]. \tag{21}$$

Taking the FOC gives:

$$\begin{aligned}
&\mu_1(p^*)(1 - F(p_i)) + \sum_{k=2}^{N} k\mu_k(p^*)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - p_i + p^*))^{k-1} f(\varepsilon)d\varepsilon - (p_i - r)\mu_1(p^*)f(p_i) \\
&- (p_i - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} (k-1)F(\varepsilon - p_i + p^*)^{k-2} f(\varepsilon - p_i + p^*)f(\varepsilon)d\varepsilon + F(p^*)^{k-1}f(p_i)\right) = 0.
\end{aligned} \tag{22}$$

After imposing symmetry, simplifying and rearranging we obtain:

$$\begin{aligned}
&\mu_1(p^*)\left[1 - F(p^*) - (p^* - r)f(p^*)\right] + \sum_{k=2}^{N} k\mu_k(p^*)\int_{p^*}^{\overline{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon)d\varepsilon \\
&- (p^* - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p^*}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^*)^{k-1}f(p^*)\right) = 0.
\end{aligned} \tag{23}$$

In the Appendix we show that a candidate equilibrium $p^* \in [r, p^m]$ exists.

Depending on the magnitude of the upper bound of the search cost distribution $\overline{c}$, there may exist $N+1$ types of equilibria:

1. When

$$\overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)\left[(N+1)F(\varepsilon) - N\right] F(\varepsilon)^{N-1} f(\varepsilon)d\varepsilon$$

[11] Downward price deviations lead to an expression similar to equation (5).

then all consumers search the N-firms in the market and the equilibrium price is given by the solution to the FOC in equation (23) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, \ldots, N-1$ and $\mu_N(p^*) = 1$. This equilibrium exists an is unique, as it is the same as that in Perloff and Salop (1985).

2. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(N+1)F(\varepsilon) - N\right] F(\varepsilon)^{N-1} f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[NF(\varepsilon) - (N-1)\right] F(\varepsilon)^{N-2} f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms and the rest of the consumers search $N-1$ firms, and the equilibrium price is given by the solution to the FOC in equation (23) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, \ldots, N-2$ and replacing $\mu_{N-1}(p^*)$ and $\mu_N(p^*)$ by their corresponding values in equation (19).

3. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[NF(\varepsilon) - (N-1)\right] F(\varepsilon)^{N-2} f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(N-1)F(\varepsilon) - (N-2)\right] F(\varepsilon)^{N-3} f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_{N-1}(p^*) = G(c_{N-2}(p^*)) - G(c_{N-1}(p^*))$ searches $N-1$ firms and the rest of the consumers search $N-2$ firms, and the equilibrium price is given by the solution to the FOC in equation (23) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, \ldots, N-3$ and replacing $\mu_{N-2}(p^*), \mu_{N-1}(p^*)$ and $\mu_N(p^*)$ by their corresponding values in equation (19).

4,5,...,N-1. So on and so forth.

N. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[2F(\varepsilon) - 1\right] F(\varepsilon) f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_k(p^*) = G(c_{k-1}(p^*)) - G(c_k(p^*))$ searches $k = 2, 3, \ldots, N-1$ firms and the rest of the consumers search just one firm. In this case the equilibrium price is given by the solution to the FOC (23) after replacing $\mu_1(p^*), \mu_2(p^*), \ldots, \mu_N(p^*)$ by their corresponding values in (19).

N+1. Finally, when

$$\overline{c} > \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_k(p^*) = G(c_{k-1}(p^*)) - G(c_k(p^*))$ searches $k = 1, 2, 3, \ldots, N-1$ firms and the rest of the consumers do not search at all. In this case the equilibrium price is given by the solution to the FOC (23) after replacing $\mu_1(p^*), \mu_2(p^*), \ldots, \mu_N(p^*)$ by their corresponding values in (19).

For the candidate price p^* to be a SNE, the payoff function in (20) must be quasi-concave in p_i. Except in case 1 above, the payoff of a firm consists of the profits derived from the various consumer groups. Using the well-known aggregation result of Prékopa (1973), it can be shown that each of the summands of the payoff function in (20) is quasi-concave. However, even if each element of the sum of payoffs is quasi-concave, the payoff function need not be quasi-concave. Drawing from a recent contribution by Choi and Smith (2017), we can provide conditions for existence. We do it for the most general case in which the upper bound of the search cost distribution $\overline{c}$ is sufficiently large. The result can easily adapted to prove the existence of equilibrium in alternative situations.

Proposition 3 *Suppose there are N firms in the market and that $\overline{c}$ is sufficiencly large. Then a candidate market equilibrium exists in which firms charge p^* and a fraction $\mu_k(p^*)$, $k = 0, 1, 2, ..., N$ of consumers checks the products of k firms, where the fractions $\mu_k(p^*)$ are given by (19). Suppose that the search cost distribution G is arbitrary and the distribution of match values F is uniform. Then if $N \leq 8$ an equilibrium surely exists, while if N is arbitrary an equilibrium surely exists whenever $r > \frac{N-3}{N+1}\overline{\varepsilon}$.*

Proof. See the Appendix.

Our proof builds on the novel insight by Choi and Smith (2017) that the weighted sum of quasi-concave functions is also quasi-concave if the increasing part of each is more risk averse than any decreasing part. To apply this result in our setting, we first verify that each of the summands in the payoff (20) is quasi-concave. After this, for two arbitrary summands, we identify the set of prices for which one summand is increasing and the other is decreasing. Finally, we show that the Choi and Smith's condition holds when either the number of firms is sufficiently low or the marginal cost is sufficiently large.[12]

We now turn to the question of how the equilibrium price is affected by an increase in search costs. To study how an increase in search costs affects the equilibrium price we proceed by solving the model numerically. We again focus on the most novel case, i.e. where search costs are sufficiently large so that not all consumers search in equilibrium. Assuming that search costs follow the Kumaraswamy distribution with upper bound β, we set $a = 1$, pick β sufficiently high so that all fractions of consumers defined above in equation (19) are strictly positive, and compute the price equilibrium and search intensities for various levels of the parameter b. For the case $N = 2$, our Proposition 2

[12] In our working paper, we provide evidence based on numerical solutions of the N-firm model using the Kumaraswamy distribution that higher search costs result in a lower, equal or higher equilibrium price exactly as in the duopoly case of Section 4.2.

shows mathematically that, after an increase in search costs, prices go down when the parameter b of the Kumaraswamy search cost distribution is less than 1; for $b = 1$, prices do not change; while for $b > 1$, prices increase. Table 1 shows that the same results are obtained in a market where $N = 5$, $r = 0$, and match values are uniformly distributed on the set $[0, 1]$.

	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
μ_0	0.7008	0.8467	0.8970	0.7910	0.8955	0.9303	0.8905	0.9465	0.9646
μ_1	0.1251	0.0651	0.0439	0.0900	0.0450	0.0300	0.0484	0.0233	0.0153
μ_2	0.0668	0.0341	0.0228	0.0463	0.0231	0.0154	0.0241	0.0118	0.0078
μ_3	0.0370	0.0187	0.0125	0.0253	0.0126	0.0084	0.0129	0.0064	0.0042
μ_4	0.0216	0.0108	0.0072	0.0146	0.0073	0.0048	0.0074	0.0036	0.0024
μ_5	0.0484	0.0243	0.0162	0.0325	0.0162	0.0108	0.0163	0.0081	0.0054
p^*	0.3504	0.3521	0.3526	0.3536	0.3536	0.3536	0.3567	0.3551	0.3545
π	0.0171	0.0087	0.0059	0.0120	0.0060	0.0040	0.0062	0.0030	0.0020
CS	0.0792	0.0402	0.0269	0.0544	0.0272	0.0181	0.0280	0.0138	0.0091
$CS/(1-\mu_0)$	0.2647	0.2626	0.2619	0.2606	0.2606	0.2606	0.2566	0.2587	0.2593
$Welfare$	0.1650	0.0842	0.0565	0.1144	0.0572	0.0381	0.0595	0.0291	0.0193

Table 1: Simultaneous search for differentiated products: price equilibrium and search intensities (Kumaraswamy distribution, $a = 1$)

The table also illustrates the impact of higher search costs on profits, consumer surplus, and welfare. What we see is that, even if higher search costs result in lower prices, consumer surplus goes down in search costs. This is clearly due to the impact higher search costs have on the extensive search margin, which is of first order. In fact, notice that conditional on searching, consumers benefit from higher search costs only because prices fall.

Another interesting result is that firm profits always decrease when search costs increase, even if prices go up. Once again, this is due to the impact of higher search costs on the extensive search margin.

6 Non-uniform sampling

Hortaçsu and Syverson (2004), De los Santos, Hortaçsu, and Wildenbeest (2012), and De los Santos (2018) have observed empirically that some firms are more salient than others and because of this consumers are more likely to encounter them when they search. In this section, we explore the implications of non-uniform sampling for pricing, firm profits and consumer surplus. In addition, we show that our results about the effect of higher search costs on the equilibrium price do not qualitatively depend on the assumption that firms are equally likely to be sampled.

Assume now that one of the firms, say firm 1, is more likely to be sampled than the other firm.

Let α be the probability with which a consumer who searches once comes across the offer of firm 1, with $\alpha \geq 1/2$. Correspondingly, $1-\alpha$ is the probability with which a consumer who searches once finds the offer of firm 2. Naturally, the case of $\alpha = 1/2$ corresponds to the symmetric model we have analyzed so far.

Let p_1^* and p_2^* be the equilibrium prices of the firms. For α different from 1/2, the firms will have asymmetric demands so we expect these prices to be different from one another.

6.1 Consumer search

We next characterize optimal consumer search behavior. As in the previous section, consider a consumer with search cost c that chooses to inspect only one product. The expected utility the consumer derives from searching once is:

$$\begin{aligned} U(1,c) &= \alpha Pr[\varepsilon_1 \geq p_1^*]E[\varepsilon_1 - p_1^* | \varepsilon_1 \geq p_1^*] + (1-\alpha)Pr[\varepsilon_2 \geq p_2^*]E[\varepsilon_2 - p_2^* | \varepsilon_2 \geq p_2^*] - c \\ &= \alpha \int_{p_1^*}^{\bar{\varepsilon}} (z-p_1^*)f(z)dz + (1-\alpha)\int_{p_2^*}^{\bar{\varepsilon}} (z-p_2^*)f(z)dz - c, \end{aligned} \tag{24}$$

where we have taken into account that with probability α the consumer will end up inspecting the product of firm 1, and with probability $1-\alpha$ the product of firm 2.

To compute the cutoff c_0 above which consumers will not search, we equalize the utility from searching once to the utility from not searching at all, which is zero, and solve for the corresponding critical search cost. Because such a cutoff may be higher than the upper bound of the search cost distribution, we define c_0 as:

$$c_0(p_1^*, p_2^*) \equiv \min\left\{\bar{c}, \alpha \int_{p_1^*}^{\bar{\varepsilon}} (z-p_1^*)f(z)dz + (1-\alpha)\int_{p_2^*}^{\bar{\varepsilon}} (z-p_2^*)f(z)dz\right\}.$$

Notice that $c_0(\cdot)$ is (potentially) a function of the prices of the two firms. The share of consumers not searching in the market is then:

$$\mu_0(p_1^*, p_2^*) = 1 - G(c_0(p_1^*, p_2^*)), \tag{25}$$

which can of course be equal to zero when $\bar{c}$ is low enough.

We now compute the share of consumers searching once. To do this, we look for the search cost of the consumer indifferent between searching once and searching twice. The expected utility from searching twice, denoted $U(2)$, is given by:

$$U(2,c) = Pr[\max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} \geq 0]E[\max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} | \max\{\varepsilon_1 - p_1^*, \varepsilon_2 - p_2^*\} \geq 0] - 2c. \tag{26}$$

Equating equation (24) to equation (26) and solving for the search cost gives the critical search cost value above which it is worth to search once and not twice. Because this critical search cost may be greater than the upper bound of the search cost distribution, we define c_1 as:

$$c_1(p_1^*, p_2^*) \equiv \min \left\{ \bar{c} \int_0^{\max\{\bar{\varepsilon}-p_1^*, \bar{\varepsilon}-p_2^*\}} z[f(z+p_1^*)F(z+p_2^*) + F(z+p_1^*)f(z+p_2^*)]dz - \alpha \int_0^{\bar{\varepsilon}-p_1^*} zf(z+p_1^*)dz \right.$$
$$\left. -(1-\alpha) \int_0^{\bar{\varepsilon}-p_2^*} zf(z+p_2^*)dz \right\}.$$

Assuming that $p_1^* > p_2^*$, something that we later check it holds in equilibrium, we have:

$$c_1(p_1^*, p_2^*) \equiv \min \left\{ \bar{c}, \int_0^{\bar{\varepsilon}-p_2^*} z[f(z+p_1^*)F(z+p_2^*) + F(z+p_1^*)f(z+p_2^*)]dz - \alpha \int_0^{\bar{\varepsilon}-p_1^*} zf(z+p_1^*)dz \right.$$
$$\left. -(1-\alpha) \int_0^{\bar{\varepsilon}-p_2^*} zf(z+p_2^*)dz \right\}.$$

The share of consumers searching once is then

$$\mu_1(p_1^*, p_2^*) = G(c_0(p_1^*)) - G(c_1(p_1^*, p_2^*)), \tag{27}$$

which again can be equal to zero when $\bar{c}$ is sufficiently small, and the share of consumers searching twice is

$$\mu_2(p_1^*, p_2^*) = G(c_1(p_1^*, p_2^*)). \tag{28}$$

6.2 Payoffs of the firms

We now move to the problem of the firms. We start by deriving the payoff of the firms when they deviate from equilibrium pricing. Consider first firm 1, the most salient firm. The expected payoff to firm 1 when deviating by charging a price $p_1 \neq p_1^*$ is:

$$\pi_1(p_1; p_1^*, p_2^*) = (p_1 - r)\left(\alpha \frac{\mu_1(p_1^*, p_2^*)}{2} \Pr[\varepsilon_1 \geq p_1] + \mu_2(p_1^*, p_2^*) \Pr\left[\varepsilon_1 - p_1 \geq \max\{\varepsilon_2 - p_2^*, 0\}\right]\right).$$

When firm 1 deviates but still charges a higher price than the rival's price so $p_1 > p_2^*$, this payoff can be written as follows:

$$\pi_1(p_1; p_1^*, p_2^*) = (p_1 - r)\left(\alpha \frac{\mu_1(p_1^*, p_2^*)}{2}(1 - F(p_1)) + \mu_2(p_1^*, p_2^*) \int_{p_1}^{\bar{\varepsilon}} F(\varepsilon - (p_1 - p_2^*))f(\varepsilon)d\varepsilon\right). \tag{29}$$

Notice that this payoff is similar to that in equation (5) although there are differences: one difference is that firm 1 now obtains a share α of the consumers who search once; the other difference is that now the shares of consumers searching once and twice depend on both equilibrium prices.

The FOC, after applying symmetry between the deviation price and consumer expectations, is:

$$\alpha\frac{\mu_1(p_1^*,p_2^*)}{2}(1-F(p_1^*))+\mu_2(p_1^*,p_2^*))\int_{p_1^*}^{\overline{\varepsilon}}F\left(\varepsilon-p_1^*+p_2^*\right)f\left(\varepsilon\right)d\varepsilon$$
$$-\left(p_1^*-r\right)\left[\alpha\frac{\mu_1(p_1^*,p_2^*)}{2}f(p_1^*)+\mu_2(p_1^*,p_2^*)\left(\int_{p_1^*}^{\overline{\varepsilon}}f\left(\varepsilon-p_1^*+p_2^*\right)f\left(\varepsilon\right)d\varepsilon+F(p_2^*)f(p_1^*)\right)\right]=0 \quad (30)$$

Consider now firm 2, the least salient firm. The expected payoff to the least salient firm when deviating by charging a price $p_2 \neq p_2^*$ is:

$$\pi_2(p_2;p_1^*,p_2^*)=(p_2-r)\left((1-\alpha)\frac{\mu_1(p_1^*,p_2^*)}{2}\Pr[\varepsilon_2\geq p_2]+\mu_2(p_1^*,p_2^*)\Pr\left[\varepsilon_2-p_2\geq\max\{\varepsilon_1-p_1^*,0\}\right]\right).$$

When $p_1^* > p_2 > p_2^*$, this payoff can be written more compactly as

$$\pi_2(p_2;p_1^*,p_2^*)=(p_2-r)\left((1-\alpha)\frac{\mu_1(p_1^*,p_2^*)}{2}(1-F(p_2))+\mu_2(p_1^*,p_2^*)\int_{p_2}^{\overline{\varepsilon}}F(\varepsilon-(p_2-p_1^*))f(\varepsilon)d\varepsilon\right).$$

The FOC, after applying symmetry between the deviation price and consumer expectations is:

$$(1-\alpha)\frac{\mu_1(p_1^*,p_2^*)}{2}(1-F(p_2^*))+\mu_2(p_1^*,p_2^*)\int_{p_2}^{\overline{\varepsilon}}F(\varepsilon-(p_2^*-p_1^*))f(\varepsilon)d\varepsilon$$
$$-\left(p_2^*-r\right)\left[(1-\alpha)\frac{\mu_1(p_1^*,p_2^*)}{2}f(p_2)+\mu_2(p_1^*,p_2^*)\left(\int_{p_2^*}^{\overline{\varepsilon}}f(\varepsilon-p_2^*+p_1^*)f(\varepsilon)d\varepsilon+F(p_1^*)f(p_2^*)\right)\right]=0 \quad (31)$$

6.3 Price equilibrium with non-uniform sampling

Computing a price equilibrium requires solving the system of FOCs given by equations (30)-(31) for p_1^* and p_2^*, after factoring the expressions for $\mu_1(p_1^*,p_2^*)$ and $\mu_2(p_1^*,p_2^*)$ given in equations (27)-(28). Unfortunately, the resulting system of equations is extremely complicated to deal with. To make further progress, let us assume that match values and search costs are uniformly distributed on $[0,1]$ and $[0,\overline{c}]$, respectively.

We now note than when the upper bound of the search cost distribution is small enough so that $c_1(p_1^*,p_2^*) = c_0(p_1^*,p_2^*) = \overline{c}$, then non-uniform sampling does not affect the price equilibrium. The reason is that all consumers find it worthwhile to sample the two products and the price equilibrium is again the same as in Perloff and Salop (1985). In what follows, therefore, we focus on situations where not all consumers search twice.

Consider first the case in which $\overline{c}$ is neither too low nor too high so that all consumers participate in the market. We will later characterize the region of search costs upper bounds for which this is true. In such a case,

$$c_1(p_1^*,p_2^*)=\frac{1}{6}\left((1-p_2^*)^2(1+3p_1^*-p_2^*)+3\alpha(p_1^*-p_2^*)(2-p_1^*-p_2^*)\right),$$

and it holds that $c_1(p_1^*, p_2^*) < c_0(p_1^*, p_2^*) = \bar{c}$. The corresponding share of consumers inspecting one of the products is

$$\mu_1(p_1^*, p_2^*) = 1 - \frac{3\alpha(p_1^* - p_2^*)(2 - p_1^* - p_2^*) + (1 - p_2^*)^2(1 + 3p_1^* - p_2^*)}{6\bar{c}},$$

and, obviously, $\mu_2(p_1^*, p_2^*) = 1 - \mu_1(p_1^*, p_2^*)$ is the share of consumers inspecting the two products.

The profits of the firms can then be written as:

$$\pi_1(p_1; \cdot) = \frac{(1 - p_1)p_1 \left[2\alpha + (1 - 2\alpha - p_1^* + 2p_2^*)(1 - \mu_1(p_1^*, p_2^*))\right]}{2};$$

$$\pi_2(p_2; \cdot) = \frac{(1 - p_2)p_2 \left[2(1 - \alpha) - (1 - 2\alpha - 2p_1^* + p_2)(1 - \mu_1(p_1^*, p_2^*))\right]}{2}.$$

Armstrong, Vickers, and Zhou (2009) have studied the special case in which one of the firms is prominent, which is obtained when setting $\alpha = 1$. For such a case, the corresponding FOCs for profits maximization are given by:

$$1 - 2p_1^* - \frac{\left[3{p_1^*}^2 - 4p_1^*p_2^* + 2p_2^* - 1\right]\left[3p_1^{*2} + 3p_1^*((1 - p_2^*)p_2^* - 3) + (3 - p_2^*)^2 p_2^* - 1\right]}{12\bar{c}} = 0; \qquad (32)$$

$$[p_1^*(4p_2^* - 2) + p_2^*(4 - 3p_2^*) - 1]\left[3p_1^{*2} + 3p_1^*((1 - p_2^*)p_2^* - 3) + (3 - p_2^*)^2 p_2^* - 1\right] = 0. \qquad (33)$$

We can demonstrate the following result:

Proposition 4 *Assume that one of the firms, say firm 1, is prominent, that is $\alpha = 1$. Also assume that match values and search costs are uniformly distributed on $[0, 1]$ and $[0, \bar{c}]$, respectively. Then, for any*

$$\bar{c} \in \left(\frac{8\sqrt{2} - 11}{3}, \frac{1}{8} + \frac{1}{18\sqrt{3}}\right],$$

there exists a unique price equilibrium (p_1^*, p_2^*) in pure-strategies given by the solution to the system of equations (32)-(33). In equilibrium, all consumers search; in particular, a share $\mu_1(p_1^*, p_2^*)$ inspect the product of the prominent firm only, and the rest check the two products. The equilibrium prices satisfy the inequality

$$p_2^* < p_1^* < p^*,$$

where p^* is the symmetric equilibrium price. As a result, market prominence increases consumer surplus.

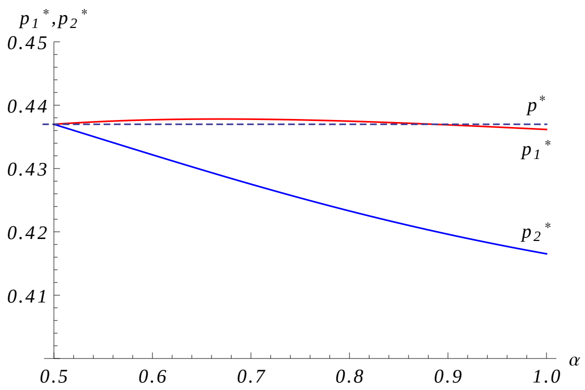
Proof. See the Appendix.

Proposition 4 shows that prominence, that is, an extreme case of non-uniform sampling, increases consumer surplus. The intuition behind this result is as follows. When one firm is more likely to be visited by the consumers who inspect just one product, this firm's pool of consumers becomes less elastic compared to the symmetric equilibrium situation. As a result, a firm that becomes more salient in the market tends to increase its price. By contrast, the rival firm's pool of consumers becomes more elastic compared to the symmetric equilibrium because there are disproportionately more consumers willing to inspect the two products of the firms. Thus, for this firm the situation is the opposite and hence it tends to decrease its price. In the limit case of Proposition 4 in which one firm is prominent, the reduction in the price of the non-prominent firm is so strong that the prominent firm also decreases its price. As a result, consumer surplus increases.

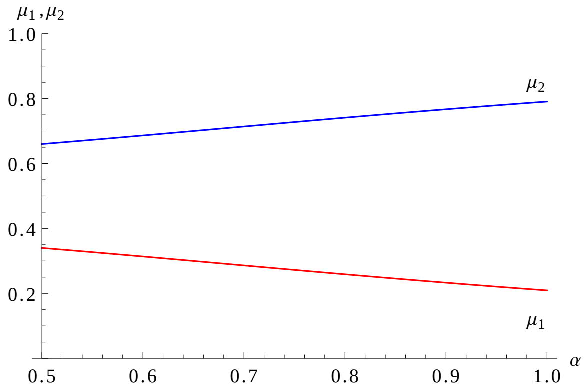
The insight in Proposition 4 that consumers gain from non-uniform sampling situations is not unique to the case $\alpha = 1$. However, for arbitrary α, it is very hard to prove this finding because the corresponding FOCs given by equations (30)-(31) are very difficult to analyze. We therefore proceed by solving the model numerically.

In Figure 5(a) we plot the equilibrium prices against the non-uniform sampling probability α. For $\alpha = 1/2$ we get the symmetric equilibrium p^*, which is depicted by the dashed line. The graph reveals that, as α increases, the salient firm first raises its price above the symmetric price but eventually decreases it, while the non-salient firm decreases it for any value of $\alpha > 0.5$. Around the $\alpha = 1/2$ case, the decrease of the non-salient firm's price is however quite sharp in comparison to the increase in the price of the salient firm. Despite the fact that consumers visit the salient firm with higher probability, on balance, this is good for consumers. In equilibrium, thus, they tend to search more, which can be seen in Figure 5(b). We plot the equilibrium profits in Figure 5(c). The profits of the salient firm increase as α goes up, while the non-salient firm's profits decrease. However, the increase in the profits of the salient firm is stronger than the decrease in the profits of the non-salient firm so the joint profits of the two firms rise. We plot consumer surplus in Figure 5(d) together with industry profits, and social welfare. We conclude that industry profits, consumer surplus, and welfare with non-uniform sampling are higher than with random search.

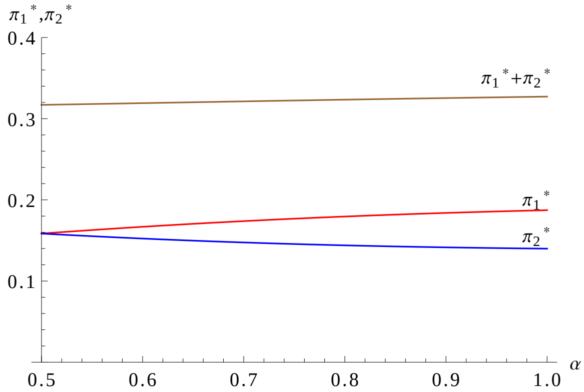
Consider now the case in which $\overline{c}$ is sufficiently high so that not all consumers participate in the market. In this case, we can repeat the numerical exercise above to show similar insights. In Figure 6(a) we plot the equilibrium prices against the non-uniform sampling probability α. The graph



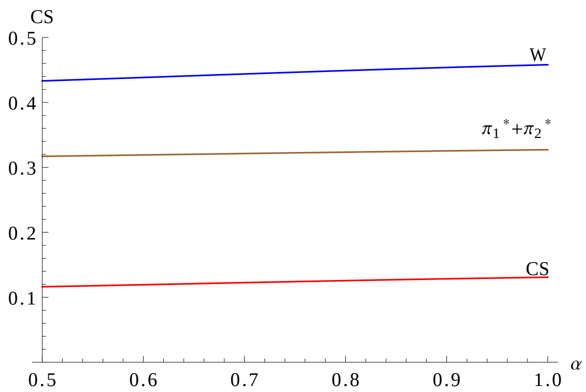
(a) Equilibrium prices



(b) Search



(c) Firms' profits

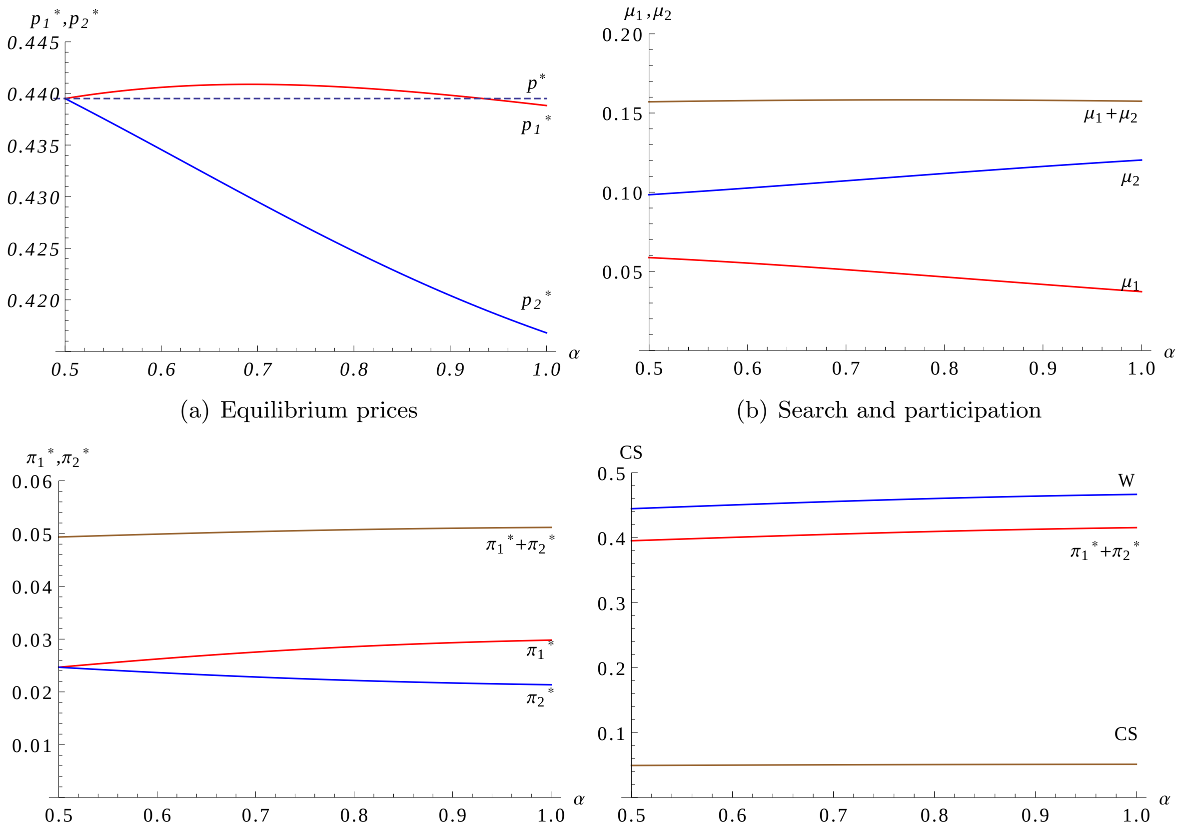


(d) Consumer surplus, industry profits, and social welfare

Figure 5: The effect of non-uniform sampling

reveals the same pattern as before: as α increases, the salient firm first raises its price above the symmetric price and then lowers it, while the non-salient firm decreases it. As before, the decrease of the non-salient firm's price is however quite sharp in comparison to the increase in the price of the salient firm. Despite the fact that consumers visit the salient firm with higher probability, on balance, this is good for consumers. This is seen in Figure 6(b), where we plot the shares of consumers searching once and twice. As shown in this figure, consumer participation goes up as α increases. Figure 6(c) shows the profits of the firms. The salient firm increases is profits as it becomes more salient, while the non-salient firm decreases its profits. As before, joint industry profits increase as sampling probabilities become more unequal. Figure 6(d) plots consumer surplus, joint profits, and social welfare. Again, with non-uniform sampling welfare is higher than with random search.

We finally show that our results regarding the effects of higher search costs also hold with non-uniform sampling. For this, let us assume that match values are uniformly distributed on $[0, 1]$ and that search costs are distributed on the interval $[0, \beta]$ according to the Kumaraswamy distribution.



(a) Equilibrium prices

(b) Search and participation

(c) Firms' profits

(d) Consumer surplus, industry profits, and social welfare

Figure 6: The effect of non-uniform sampling (market not fully covered)

Let us assume that β is sufficiently high. Table 2 shows the market equilibrium for three values of the (unequal) sampling probability α as well as different values of the parameter b of the search cost distribution. When $b = 1$, the equilibrium price does not depend on β; when $b = 1.5$, the the equilibrium price increases in β; finally, when $b = 0.5$, the equilibrium price decreases in β.

7 Conclusions

This paper has extended the literature on simultaneous search by allowing for differentiated products and consumer search cost heterogeneity. While such a framework has been the basis for a number of empirical applications in recent years, with the exception of Anderson, De Palma, and Thisse (1992), models of simultaneous search for differentiated products have not received much attention in the theoretical literature to date.

In contrast to Anderson, De Palma, and Thisse (1992) where all consumers have the same search cost, in our paper consumers choose to inspect different numbers of products before buying. The

	$\alpha = 1/2$								
	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
μ_0	0.7733	0.8843	0.9224	0.8429	0.9214	0.9476	0.9183	0.9599	0.9734
μ_1	0.0825	0.0426	0.0287	0.0586	0.0293	0.0195	0.0312	0.0151	0.0099
μ_2	0.1441	0.0729	0.0488	0.0983	0.0491	0.0327	0.0503	0.0248	0.0165
p_1^*, p_2^*	0.4387	0.4391	0.4392	0.4395	0.4395	0.4395	0.4402	0.4398	0.4397
π_1, π_2	0.0356	0.0181	0.0121	0.0246	0.0123	0.0082	0.0127	0.0062	0.0041
	$\alpha = 2/3$								
	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
μ_0	0.7719	0.8836	0.9219	0.8418	0.9209	0.9472	0.9177	0.9596	0.9733
μ_1	0.0738	0.0381	0.0257	0.0525	0.0262	0.0175	0.0279	0.0135	0.0089
μ_2	0.1542	0.0781	0.0523	0.1056	0.0528	0.0352	0.0542	0.0267	0.0177
p_1^*	0.4401	0.4405	0.4406	0.4408	0.4408	0.4408	0.4415	0.4411	0.4410
p_2^*	0.4306	0.4309	0.4310	0.4311	0.4311	0.4311	0.4316	0.4314	0.4313
π_1	0.0391	0.0199	0.0133	0.0271	0.0135	0.0090	0.0141	0.0069	0.0045
π_2	0.0334	0.0170	0.0114	0.0230	0.0115	0.0076	0.0119	0.0058	0.0038
	$\alpha = 1$								
	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
μ_0	0.7730	0.8841	0.9222	0.8425	0.9212	0.9475	0.9180	0.9598	0.9734
μ_1	0.0525	0.0270	0.01823	0.0371	0.0185	0.0123	0.0197	0.0095	0.0063
μ_2	0.1744	0.0887	0.0594	0.1202	0.0601	0.040	0.0622	0.0305	0.0202
p_1^*	0.4383	0.4386	0.4386	0.4388	0.4388	0.4388	0.4392	0.4390	0.4389
p_2^*	0.4167	0.4168	0.4168	0.4168	0.4168	0.4168	0.4168	0.4168	0.4168
π_1	0.0428	0.0219	0.01470	0.0298	0.0149	0.0099	0.0155	0.0076	0.0050
π_2	0.0309	0.0157	0.0105	0.0213	0.0106	0.0071	0.0110	0.0054	0.0035

Table 2: non-uniform sampling: search intensities, prices, and profits (Kumaraswamy distribution, $a = 1$)

consumer equilibrium is thus a partition of the set of consumers into subsets of buyers inspecting k products, $k = 0, 1, 2, \ldots, N$. Consequently, the aggregate demand of a typical firm stems from the demands of these distinct consumer groups. The more products inspected, the more price sensitive the consumer is. Absent the possibility of price discrimination, this poses a complicated pricing problem. For duopoly and triopoly, we have shown that when the search cost distribution is arbitrary and distribution of match values is uniform, the pricing problem is well behaved and a pure strategy Nash equilibrium always exists in this model. For the duopoly case, we have provided more general conditions under which a pure strategy equilibrium exists.

We have also studied the effects of increasing search costs on the equilibrium price. The typical assumption in the existing literature is that all consumers search or, equivalently, that all consumers have sufficiently low search costs. With arbitrary search cost heterogeneity, this assumption is, at the very least, questionable. We have shown that, depending on the nature of the search cost distribution, an increase in the search costs of all consumers may result in a lower or in a higher equilibrium price. The key to understanding this result is to recognize that an increase in search costs changes the composition of demand. When search costs increase, some consumers cease to buy. If there are relatively many of these consumers, the remaining consumers may on average be more elastic than before. We have first derived a necessary and sufficient condition for the equilibrium price to decrease (increase) in search costs. We have then shown that for distributions with the decreasing (increasing) reversed hazard rate property, which is equivalent to log-submodularity (log-supermodularity) of the cumulative distribution function, the equilibrium price will decrease (increase) as the costs of search of all consumers rise.

We have examined two extensions of the model. In the first extension, we have considered the case of N firms. We have characterized the symmetric pure-strategy equilibrium price and provided an existence result. Moreover, we have shown, using numerical methods, that the insights from the duopoly model regarding the effects of higher search costs generalize to the case of oligopoly. In the second extension, we have introduced non-uniform sampling, that is, the idea that some firms are more salient than others in the marketplace and therefore their products are more likely to be inspected by consumers than those of the rest of the firms. We have focused on the special case in which one of the firm products is prominent, that is, it is always inspected by the consumers who choose to inspect only one item. We have shown that the prominent firm charges a higher price than the rival firm. Moreover, the equilibrium prices of both firms are below the symmetric equilibrium price. Thus, market prominence works in favor of consumers. These results generalize to weaker forms of market saliency. Finally, as in the case with symmetric firms, we have verified numerically that when not all consumers choose to search in equilibrium, both prices can increase or decrease depending on the nature of the search cost density.

Appendix

Proof of Proposition 1. (A) If all consumers search both firms, the payoff function in equation (6) coincides with that in Perloff and Salop (1985):

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon.$$

From Caplin and Nalebuff (1991) we know that under log-concavity of f, this payoff function is quasi-concave in p_i and therefore p^* is the unique symmetric equilibrium price. In order for all consumers to search twice, we need that $c_1(p^*) = \overline{c}$, which is guaranteed under condition (13).

(B and C) When $\overline{c}$ is relatively large some consumers search once and some search twice. In such a case, the candidate equilibrium price is given by the solution to equation (9). We now note that the payoff in equation (6) involves the sum of two log-concave functions. Unfortunately, such a sum need not be quasi-concave, which implies that we need to impose additional restrictions on the primitives of the model in order to guarantee the existence of a pure-strategy equilibrium.[13] We now show that when the match values follow a uniform distribution, the payoff function in equation (6) is strictly concave. The second order derivative of equation (6) is:

$$\begin{aligned}
\frac{d^2\pi_i\left(p_i > p^*\right)}{dp_i^2} \quad = \quad & -2\frac{\mu_1(p^*)}{2} f(p_i) - 2\mu_2(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} f\left(\varepsilon - p_i + p^*\right) f\left(\varepsilon\right) d\varepsilon + F(p^*) f(p_i)\right) \\
& - \left(p_i - r\right)\left[\frac{\mu_1(p^*)}{2} f'(p_i) - \mu_2(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} f'\left(\varepsilon - p_i + p^*\right) f\left(\varepsilon\right) d\varepsilon \right.\right. \\
& \left.\left. + f(p^*) f(p_i) - F(p^*) f'(p_i)\right)\right] \qquad (34)
\end{aligned}$$

where f' denotes the derivative of f.

For the uniform distribution, we have $F(\varepsilon) = \varepsilon/\overline{\varepsilon}$, $f(\varepsilon) = 1/\overline{\varepsilon}$, and $f'(\varepsilon) = 0$. Plugging these values in equation (34) and simplifying gives:

$$\begin{aligned}
\frac{d^2\pi_i\left(p_i > p^*\right)}{dp_i^2} &= -\frac{\mu_1(p^*)}{\overline{\varepsilon}} - 2\mu_2(p^*)\left(\frac{\overline{\varepsilon} - p_i}{\overline{\varepsilon}^2} + \frac{p^*}{\overline{\varepsilon}^2}\right) + \left(p_i - r\right)\mu_2(p^*)\frac{1}{\overline{\varepsilon}^2}; \\
&= -\frac{\overline{\varepsilon}\mu_1(p^*) + \mu_2(p^*)(2\overline{\varepsilon} - 3p_i + 2p^* + r)}{\overline{\varepsilon}^2},
\end{aligned}$$

[13] This problem is quite common in search models where demand stems from various consumer types. For example, in the sequential search model of Anderson and Renault (1999) demand stems from consumers who happen to visit a firm for the first time, and from consumers who happen to walk away from a firm and later return to it to conduct a purchase. In their model, assuming that the density of match values f is increasing ensures existence and uniqueness of equilibrium.

which is clearly negative because p_i cannot be greater than the monopoly price, which in this case of the uniform distribution is given by $p^m = (\overline{\varepsilon} + r)/2$. In a similar way, we can compute the second order condition for prices $p_i < p^*$, which gives

$$\frac{d^2\pi_i\,(p_i < p^*)}{dp_i^2} = -\frac{1}{\overline{\varepsilon}}\,(\mu_1(p^*) + 2\mu_2(p^*)) < 0.$$

Because of strict concavity of the payoff function, we conclude that the equilibrium exists and is unique.

In order for consumers to search as prescribed in Proposition 1(B), we need that $c_1(p^*) < \overline{c} < c_0(p^*)$, which is guaranteed under condition (14). Finally, for consumers to search as prescribed in Proposition 1(C), we need that $c_0(p^*) < \overline{c}$, which gives condition (15). ■

Existence of equilibrium when F and G are quadratic.

The symmetric equilibrium of Proposition 1,B,C exists when F is quadratic and convex and G is quadratic and concave. To see this, we first note that $f' > 0$ ensures that the payoff (5) is strictly concave for $p_i < p^*$. This is because the second derivative of demand when $p_i < p^*$ is:

$$-f'(p_i)\left[\frac{\mu_1(p^*)}{2} + \mu_2(p^*)F(p^*)\right] - \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}+p_i-p^*} f(\varepsilon - p_i + p^*)f'(\varepsilon)d\varepsilon,$$

which is clearly negative for $p_i > p^*$. We now show that the payoff (6) is quasi-concave for $p_i > p^*$. To demonstrate this, we first write the demand corresponding to the payoff (6) as an integral:

$$d_i(p_i > p^*; p^*) = \int_{p_i}^{\varepsilon}\left[\frac{\mu_1(p^*)}{2} + \mu_2(p^*)F(\varepsilon - p_i + p^*)\right] f(\varepsilon)d\varepsilon.$$

From Prékopa (1973), this demand is log-concave in p_i if the term in square brackets is log-concave both in p_i and ε because the product of log-concave functions is log-concave and integration with respect to ε preserves log-concavity. Taking logarithms of the term in square brackets and writing out the Hessian matrix, it is readily seen that

$$\left(\frac{\mu_1\,(p^*)}{2\mu_2\,(p^*)} + F\,(\varepsilon - p_i + p^*)\right) f'\,(\varepsilon - p_i + p^*) - f^2\,(\varepsilon - p_i + p^*) < 0 \tag{35}$$

suffices for log-concavity in p_i and ε. Consider that match values are distributed on $[0, \overline{\varepsilon}]$ according to the quadratic and convex distribution function:

$$F\,(\varepsilon) = \frac{2 - a\overline{\varepsilon}^2}{2\overline{\varepsilon}}\varepsilon + \frac{a}{2}\varepsilon^2, \text{ with } a < \frac{6 - 4\sqrt{2}}{\overline{\varepsilon}^2}. \tag{36}$$

Assume for the moment that $\mu_1(p^*) < 2\mu_2(p^*)$. Later we will provide conditions under which this is true. If this is so, the expression on the LHS of (35) is less than:

$$2f'(\varepsilon - p_i + p^*) - f^2(\varepsilon - p_i + p^*) = 2a - \left(\frac{2 - a\overline{\varepsilon}^2}{2\overline{\varepsilon}} + a\varepsilon\right)^2 < 2a - \left(\frac{2 - a\overline{\varepsilon}^2}{2\overline{\varepsilon}}\right)^2.$$

The last expression is increasing in a so if it is negative for the highest admissible a then it is always negative. Substituting $a = \frac{6-4\sqrt{2}}{\overline{\varepsilon}^2}$ gives:

$$2a - \left(\frac{2 - a\overline{\varepsilon}^2}{2\overline{\varepsilon}}\right)^2\Bigg|_{a=\frac{6-4\sqrt{2}}{\overline{\varepsilon}^2}} = 0$$

so we conclude that (35) is negative.

It remains to find conditions under which $\mu_1(p^*) < 2\mu_2(p^*)$ which, using equation (3), is equivalent to:

$$G(c_0(p^*)) - 3G(c_1(p^*)) < 0, \tag{37}$$

where, using the distribution of match values in (36), the expressions for $c_0(p^*)$ and $c_1(p^*)$ are:

$$\begin{aligned} c_0(p^*) &= \frac{(\overline{\varepsilon} - p^*)^2(a\overline{\varepsilon}(\overline{\varepsilon} + 2p^*) + 6)}{12\overline{\varepsilon}} \\ c_1(p^*) &= -\frac{(\overline{\varepsilon} - p^*)^2\left[40p^* - \overline{\varepsilon}\left(a^2\overline{\varepsilon}(\overline{\varepsilon} - p^*)\left(\overline{\varepsilon}^2 + 3\overline{\varepsilon}p + 6p^{*2}\right) - 30ap^{*2} - 20\right)\right]}{120\overline{\varepsilon}^2} \end{aligned}$$

Consider the quadratic and concave search cost distribution:

$$G(c) = \frac{1 + b\overline{c}^2}{\overline{c}}c - bc^2, \text{ with } 0 < b < 1/\overline{c}^2.$$

Using this distribution to find parameters under which the inequality (37) holds is unfortunately intractable. Intuitively, the inequality tends to be satisfied when the curvature of the search cost distribution is higher. Therefore, let us take the case in which $b = 1/\overline{c}^2$. Normalizing $\overline{\varepsilon} = \overline{c} = 1$, we can write

$$\begin{aligned} &\frac{120}{(1-p^*)^2}\left[G(c_0(p^*)) - 3G(c_1(p^*))\right] = \\ &\frac{1}{40}\left(a^2(1-p^*)\left(6p^{*2} + 3p^* + 1\right) + 30ap^{*2} + 40p^* + 20\right)\cdot \\ &\left(a^2\left(6p^{*2} + 3p^* + 1\right)(1-p^*)^3 + 30ap^{*2}(1-p^*)^2 - 20\left(p^{*2}(3-2p^*) + 11\right)\right) \\ &+ 10(2ap^* + a + 6)\left(2 - \frac{1}{12}(1-p^*)^2(2ap^* + a + 6)\right) \end{aligned}$$

We can check the sign of this expression by plotting it in the admissible space $(a, p^*) \in \left(0, \frac{6-4\sqrt{2}}{\overline{\varepsilon}^2}\right) \times (0, 1/2)$:

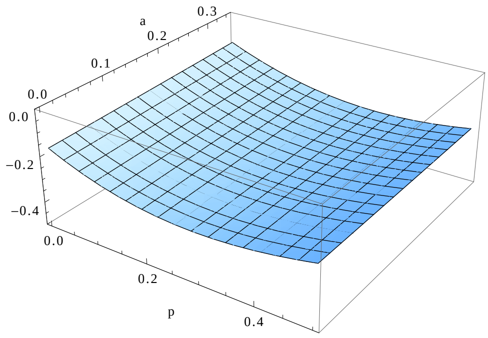


Figure 7: Plot of inequality (37)

The plot reveals that inequality (37) holds, which completes the proof of the claim. ■

Proof of Corollary of Proposition 2.

Parts (A) and (B) do not need any further clarification. Regarding part (C), we now show that, for any a and β, the Kumaraswamy distribution is log-supermodular for $b > 1$ and log-submodular for $0 < b < 1$. The case of $b = 1$ is special in that the distribution is both log-supermodular and log-submodular; in that case the equilibrium price remains constant as search costs increase.

Note that for the Kumaraswamy distribution it holds that

$$\frac{\partial G(c;\beta)}{\partial \beta} = -\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a\left(1-\left(\frac{c}{\beta}\right)^a\right)^{b-1} < 0;$$

correspondingly, the hazard ratio G'_β/G is

$$\frac{G'_\beta(c;\beta)}{G(c;\beta)} = -\frac{\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a\left(1-\left(\frac{c}{\beta}\right)^a\right)^{b-1}}{1-\left[1-\left(\frac{c}{\beta}\right)^a\right]^b}. \tag{38}$$

We now let

$$t \equiv 1-\left(\frac{c}{\beta}\right)^a.$$

Note that $t \in (0,1)$ and that t is monotonically decreasing in c. We can rewrite equation (38) as

$$\frac{G'_\beta}{G} = -\frac{ab(1-t)t^{b-1}}{\beta(1-t^b)},$$

and then take the derivative of G'_β/G with respect to t. This gives

$$\frac{d[G'_\beta/G]}{dt} = -\frac{abt^{b-2}(b-1-bt+t^b)}{\beta(1-t^b)^2}.$$

We now argue that this derivative is negative for all $b > 1$ and positive for all $0 < b < 1$. Consider first the $b > 1$ case. Let $h(t) \equiv b - 1 - bt + t^b$. Then $h(0) = b - 1 > 0$, $h(1) = 0$, and

$h'(t) = -b\left(1-t^{b-1}\right) < 0$. So h is monotonically decreasing and hence $h(t) > 0$ for any $t \in (0,1)$. As a result, G'_β/G decreases in t (and thus increases in c). By Proposition 2, this implies that when condition (15) holds, for the Kumaraswamy family of search cost distributions with parameter $b > 1$, the equilibrium price increases as search costs rise.

Second, assume $0 < b < 1$. In this case we have $h(0) = b - 1 < 0$, $h(1) = 0$ and $h'(t) = -b\left(1-t^{b-1}\right) > 0$. Hence $h(t) < 0$ for any $t \in (0,1)$. As a result, G'_β/G increases in t (and therefore decreases in c). By Proposition 2, this implies that when condition (15) holds, for the Kumaraswamy family of search cost distributions with parameter $0 < b < 1$, the equilibrium price decreases as search costs go up.

For completeness, let $b = 1$. Plugging $b = 1$ in (38) gives $G'_\beta/G = -a/\beta$, which is constant in c and therefore the equilibrium price when condition (15) holds does not vary with β. ■

Proof of Proposition 3.

We first prove that a candidate equilibrium price $p^* \in [r, p^m]$ exists in the N-firm simultaneous search model with differentiated products. For the case where the deviant firm charges a higher price than the rest of the firms, the expression in equation (20) becomes:[14]

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left[\frac{\mu_1(p^*)}{N}(1 - F(p_i)) + \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))^{k-1} f(\varepsilon) d\varepsilon\right]. \quad (39)$$

Taking the FOC gives:

$$\begin{aligned}&\mu_1(p^*)(1 - F(p_i)) + \sum_{k=2}^{N} k\mu_k(p^*) \int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - p_i + p^*))^{k-1} f(\varepsilon) d\varepsilon - (p_i - r)\mu_1(p^*) f(p_i) \\ &- (p_i - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} (k-1) F(\varepsilon - p_i + p^*)^{k-2} f(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon + F(p^*)^{k-1} f(p_i)\right) = 0.\end{aligned} \quad (40)$$

After imposing symmetry, simplifying and rearranging we obtain:

$$\begin{aligned}&\mu_1(p^*)\left[1 - F(p^*) - (p^* - r) f(p^*)\right] + \sum_{k=2}^{N} k\mu_k(p^*) \int_{p^*}^{\overline{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon \\ &- (p^* - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p^*}^{\overline{\varepsilon}} (k-1) F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^*)^{k-1} f(p^*)\right) = 0.\end{aligned} \quad (41)$$

[14] Downward price deviations lead to an expression similar to equation (5). The FOCs corresponding to upward and downward deviations are identical after imposing symmetry.

Note that when we set $p^* = r$, the LHS of this equation is strictly positive. We now show that when we set $p^* = p^m$, then it is strictly negative, which implies that there exists a candidate equilibrium price $p^* \in [r, p^m]$.

Since the monopoly price p^m satisfies $1 - F(p^m) - (p^m - r)f(p^m) = 0$, when we evaluate the LHS of the FOC at p^m we obtain:

$$\sum_{k=2}^{N} \mu_k(p^*) \left[1 - F(p^m)^k\right] - (p^m - r) \sum_{k=2}^{N} k\mu_k(p^*) \left(\int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^m)^{k-1} f(p^m) \right), \tag{42}$$

where we have used the fact that $\int_{p^m}^{\overline{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon = 1 - F(p^m)^k$.

We now claim equation (42) is negative. To show it, we first observe that

$$1 - F(p^m)^k = (1 - F(p^m)) \sum_{j=0}^{k-1} F(p^m)^j = (p^m - r) f(p^m) \sum_{j=0}^{k-1} F(p^m)^j,$$

where we have used again the monopoly pricing rule, and write equation (42) as follows:

$$(p^m - r) \left\{ f(p^m) \sum_{k=2}^{N} \mu_k(p^*) \frac{1 - F(p^m)^k}{1 - F(p^m)} - \sum_{k=2}^{N} k\mu_k(p^*) \left(\int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^m)^{k-1} f(p^m) \right) \right\}.$$

Putting terms together, this simplifies to

$$(p^m - r) \sum_{k=2}^{N} \mu_k(p^*) \left\{ f(p^m) \left[\frac{1 - F(p^m)^k}{1 - F(p^m)} - kF(p^m)^{k-1} \right] - k \int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon \right\}.$$

We now note that the expression within curly brackets is increasing in p^m. In fact, its derivative is equal to

$$\begin{aligned} &f'(p^m) \left[\frac{1 - F(p^m)^k}{1 - F(p^m)} - kF(p^m)^{k-1} \right] + f^2(p^m) \left[\frac{1 - F(p^m)^k - kF(p^m)^{k-1}}{(1 - F(p^m))^2} \right] \\ &= \frac{1 - F(p^m)^k - kF(p^m)^{k-1}(1 - F(p^m))}{(1 - F(p^m))^2} \left[f'(p^m)(1 - F(p^m) + f^2(p^m) \right] > 0 \end{aligned}$$

where the sign follows by log-concavity of f. Since it is increasing in p^m and it is equal to zero when we set $p^m = \overline{\varepsilon}$, we conclude it is always negative. This shows that a candidate equilibrium price $p^* \in [r, p^m]$ exists in the non-sequential search N-firm model with differentiated products.

We now prove that the candidate equilibrium is indeed an equilibrium under the conditions provided in the Proposition. In proving this part, we make use of the following result by Choi and Smith (2017).

Lemma 1 *(Choi and Smith 2017, Corollary 1). Suppose that $\gamma(x,y)$ is differentiable in x, it has no flat regions with respect to x and let $\gamma(x,y) = \gamma_I(x,y) + \gamma_D(x,y)$ be any decomposition such that $\gamma_I(x,y)$ is increasing and $\gamma_D(x,y)$ is decreasing in x for all y. If $\gamma_I(\cdot,y')$ is more concave than $-\gamma_D(\cdot,y'')$ for all y', y'', then $\int \gamma(x,y)\,dH(y)$ is quasi-concave, where H is a probability measure.*

To use this Lemma in our setting, let us rewrite the expected payoff of a firm i that deviates from the symmetric equilibrium price by charging a price $p_i \neq p^*$ as follows:

$$\pi_i(p_i;p^*) = \int_0^{\overline{c}} \gamma(p_i,c)\,dG(c)\,, \tag{43}$$

where

$$\gamma(p_i,c) = \begin{cases} (p_i - r)\Pr\left[\varepsilon_i - p_i \geq \max\{\max\left\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{k(c)-1}\right\} - p^*, 0\}\right] & \text{if } \; k(c) \geq 2, \\ (p_i - r)\Pr[\varepsilon_i \geq p_i] & \text{if } \; k(c) = 1. \end{cases} \tag{44}$$

where $k(c)$ denotes the optimal number of products checked by a consumer with search cost c. We apply Choi and Smith's result to the function γ in (44) and probability measure given by the search cost distribution G. We first check below that the function γ defined in (44) is differentiable, has no flat regions and is quasi-concave in p_i. Two (sufficiently different) values c' and c'' correspond to two terms of the sum in (45), that is, $\gamma_{k(c')}(p_i;p^*)$ and $\gamma_{k(c'')}(p_i;p^*)$. So considering two (sufficiently different) values c' and c'', $c' < c''$, is equivalent to considering two arbitrary terms $\gamma_h(p_i;p^*)$ and $\gamma_j(p_i;p^*)$ such that $h < j$.

Recall that $k(c)$ takes values on the discrete set $\{0, 1, 2, ..., N\}$. Therefore, in the case of our payoff the integration in (43) can be written as a sum taking into account the proportions $\mu_k(p^*)$ of consumers who search $k = k(c)$ times. Using the uniform distribution of match values, the expected payoff in (43) then becomes:

$$\pi_i(p_i;p^*) = \sum_{k=1}^{N} \gamma_k(p_i;p^*) \tag{45}$$

where

$$\gamma_k(p_i;p^*) = \begin{cases} (p_i - r)\dfrac{k\mu_k(p^*)}{N}\left(\dfrac{\overline{\varepsilon}^k - p^{*k}}{k\overline{\varepsilon}^k} + \dfrac{p^* - p_i}{\overline{\varepsilon}}\right) & \text{if } \; p_i < p^*, \\ (p_i - r)\dfrac{\mu_k(p^*)}{N}\dfrac{(\overline{\varepsilon} - p_i + p^*)^k - p^{*k}}{\overline{\varepsilon}^k} & \text{if } \; p_i \geq p^*. \end{cases} \tag{46}$$

We start by showing that γ_k is well-behaved.

Lemma 2 *γ_k is differentiable, has no flat regions and is quasi-concave on $[r, \overline{\varepsilon}]$.*

Proof. For simplicity of notation, let $\gamma_{Lk}(p_i)$ be the part of the function γ_k corresponding to $p_i < p^*$, that is $\gamma_{Lk}(p_i) \equiv \gamma_k(p_i < p^*; p^*)$. Inspection of the expression in (46) immediately reveals that $\gamma_{Lk}(p_i)$ is quadratic and strictly concave on $[r, p^*]$.

Similarly, let $\gamma_{Rk}(p_i)$ be the part of the function γ_k corresponding to $p_i \geq p^*$, that is $\gamma_{Rk}(p_i) \equiv \gamma_k(p_i \geq p^*; p^*)$. The first and second order derivatives of γ_{Rk} with respect to p_i are:

$$\frac{d\gamma_{Rk}(p)}{dp_i} = \frac{\mu_k}{N\overline{\varepsilon}^k}\left\{(\overline{\varepsilon} + p^* - p_i)^{k-1}\left[(\overline{\varepsilon} + p^* - p_i) - k(p_i - r)\right] - p^{*k}\right\} \tag{47}$$

$$\frac{d^2\gamma_{Rk}(p_i)}{dp_i^2} = -k\frac{\mu_k}{N\overline{\varepsilon}^k}(\overline{\varepsilon} + p^* - p_i)^{k-2}\left[2(\overline{\varepsilon} + p^* - p_i) - (k-1)(p_i - r)\right]. \tag{48}$$

Therefore, γ_{Rk} is strictly concave for $p_i \in \left(r, \frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}\right)$ and strictly convex for $p_i > \frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}$.

Since the first order derivative is positive at $p_i = r$ and negative at both $p_i = \frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}$ and $p_i = \overline{\varepsilon}$, that is,

$$\frac{d\gamma_{Rk}(r)}{dp_i} = \frac{\mu_k}{N\overline{\varepsilon}^k}\left[(\overline{\varepsilon} + p^* - r)^k - p^{*k}\right] > 0,$$

$$\frac{d\gamma_{Rk}}{dp_i}\left(\frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}\right) = -\frac{\mu_k}{N\overline{\varepsilon}^k}\left(\left(\frac{k-1}{k+1}\right)^{k-1}(\overline{\varepsilon} + p^* - r)^k + p^{*k}\right) < 0,$$

$$\frac{d\gamma_{Rk}(\overline{\varepsilon})}{dp_i} = -\frac{\mu_k}{N\overline{\varepsilon}^k}kp^{*k-1}(\overline{\varepsilon} - r) < 0,$$

we conclude that γ_{Rk} is increasing in p_i from r up to its maximum and then decreasing up to $\overline{\varepsilon}$.

We also observe that γ_k is differentiable at $p_i = p^*$, that is, its left hand side and right hand side derivatives at p^* are equal, that is,

$$\frac{d\gamma_{Lk}(p^*)}{dp_i} = \frac{d\gamma_{Rk}(p^*)}{dp_i} = \frac{\mu_k}{N\overline{\varepsilon}^k}\left[\overline{\varepsilon}^k - p^{*k} - k(p^* - r)\overline{\varepsilon}^{k-1}\right].$$

Consequently, γ_k is increasing at r, concave for $r < p_i < \max\left\{p^*, \frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}\right\}$ and decreasing for $p_i > \max\left\{p^*, \frac{2(\overline{\varepsilon} + p^*) + (k-1)r}{k+1}\right\}$. This implies that γ_k is quasi-concave in p_i. The proof is now complete. ■

We have shown that γ_k is quasi-concave and has exactly one peak strictly between r and $\overline{\varepsilon}$. Let the prices corresponding to the peaks of γ_k and γ_ℓ, $k > \ell$, be denoted by $\overline{p}_k$ and $\overline{p}_\ell$, respectively. Our next result shows that these prices are ranked.

Lemma 3 *$\overline{p}_k > \overline{p}_\ell$ for $k < \ell$.*

Proof. We can obtain the result by proving $\overline{p}_k > \overline{p}_{k+1}$ for arbitrary k. First we show that γ_{Lk} and γ_{Lk+1} peak at prices $\overline{p}_{Lk}$ and $\overline{p}_{Lk+1}$ with $\overline{p}_{Lk} > \overline{p}_{Lk+1}$. Taking the FOC in expression (46), these prices must satisfy the equality:

$$\overline{p}_{Lj} = \frac{\overline{\varepsilon}^j - p^{*j}}{2j\overline{\varepsilon}^{j-1}} + \frac{p^* + r}{2},\ j = k, k+1.$$

Therefore, $\overline{p}_{Lk} > \overline{p}_{Lk+1}$ is equivalent to $(k+1)\,\varepsilon^k(\varepsilon^k - p^{*k}) > k\varepsilon^{k-1}(\varepsilon^{k+1} - p^{*k+1})$. Dividing by p^{*2k+1} both sides of this inequality and simplifying gives $(k+1)\,x\left(x^k - 1\right) > k\left(x^{k+1} - 1\right)$, where $x = \overline{\varepsilon}/p^* > 1$. This inequality holds because the function $\phi(x) = (k+1)\,x\left(x^k - 1\right) - k\left(x^{k+1} - 1\right) = x^{k+1} - (k+1)\,x + k > 0$ for $x > 1$. Indeed, $\phi(1) = 0$ and since $d\phi/dx = (k+1)\left(x^k - 1\right) > 0$, ϕ is increasing for $x > 1$. Consequently,

$$\overline{p}_{Lk} > \overline{p}_{Lk+1}. \tag{49}$$

This implies that $\overline{p}_k > \overline{p}_{k+1}$ when γ_k and γ_{k+1} peak at prices below p^*.

Suppose now that γ_k and γ_{k+1} peak at prices above p^*. In this case, these prices must satisfy the FOC:

$$\left(\overline{\varepsilon} + p^* - \overline{p}_j\right)^{j-1}\left[\overline{\varepsilon} + p^* - \overline{p}_j - j(\overline{p}_j - r)\right] - p^{*k} = 0,\ j = k, k+1. \tag{50}$$

The statement is true if the FOC corresponding to $\overline{p}_{k+1}$ evaluated at $\overline{p}_k$ is negative. That is, we need to show that:

$$\left(\overline{\varepsilon} + p^* - \overline{p}_k\right)^k\left[\overline{\varepsilon} + p^* - \overline{p}_k - (k+1)(\overline{p}_k - r)\right] < p^{*k+1}.$$

We can use the FOC (50) corresponding to $\overline{p}_k$ to rewrite the previous inequality as:

$$\frac{\left(\overline{\varepsilon} + p^* - \overline{p}_k\right)^k\left[\overline{\varepsilon} + p^* - \overline{p}_k - (k+1)(\overline{p}_k - r)\right]}{\left(\overline{\varepsilon} + p^* - \overline{p}_k\right)^{k-1}\left[\overline{\varepsilon} + p^* - \overline{p}_k - k(\overline{p}_k - r)\right]} < \frac{p^{*k+1}}{p^{*k}}$$

After simplification we get the following simpler condition:

$$(\overline{\varepsilon} + r - 2\overline{p}_k)(\overline{\varepsilon} + p^* - \overline{p}_k) < k(\overline{\varepsilon} - \overline{p}_k)(\overline{p}_k - r). \tag{51}$$

It is immediate to see that the LHS of this inequality decreases in $\overline{p}_k$. The RHS, on the contrary, increases in $\overline{p}_k$ because its derivative $k(\overline{\varepsilon} + r - 2\overline{p}_k)$ is always positive given that $\overline{p}_k$ is always less than or equal to the monopoly price. These two facts imply that if (51) holds for $\overline{p}_k = p^*$ then it holds for any $\overline{p}_k$. So it is sufficient to prove (51) for $\overline{p}_k = p^*$, that is:

$$(\overline{\varepsilon} + r - 2p^*)\overline{\varepsilon} < k(\overline{\varepsilon} - p^*)(p^* - r).$$

The FOC (50) for $j = k$ and $\overline{p}_k = p^*$ becomes

$$k(p^* - r) = \frac{\overline{\varepsilon} - p^{*k}}{\overline{\varepsilon}^{k-1}}. \tag{52}$$

Using (52), the inequality we wish to prove becomes:

$$(\overline{\varepsilon} + r - 2p^*)\overline{\varepsilon} < \frac{\overline{\varepsilon} - p^{*k}}{\overline{\varepsilon}^{k-1}}(\overline{\varepsilon} - p^*).$$

After simplification, we get:

$$\overline{\varepsilon}^k(p^* - r) > p^{*k}(\overline{\varepsilon} - p^*),$$

which, by using again (52), can be written as $\overline{\varepsilon}^{k+1} - (k+1)\overline{\varepsilon}p^{*k} + kp^{*k+1} > 0$. This inequality is always true because the LHS is increasing in $\overline{\varepsilon}$ and it holds for $\overline{\varepsilon} = 0$.

Finally, we mention that in the case where γ_k peaks above p^* and γ_{k+1} peaks below p^* there is nothing to prove. The case where γ_k peaks below p^* and γ_{k+1} peaks above p^* cannot occur because in this case $d\gamma_{k+1}(p^*)/dp_i > 0$, which means that $d\gamma_{Lk+1}(p^*)/dp_i > 0$ is satisfied as well. This latter inequality implies that $\overline{p}_{Lk+1} > p^*$, so $\overline{p}_{Lk} < \overline{p}_{Lk+1}$. This is, however, a contradiction with (49). The proof is now complete. ■

Lemmas 2 and 3 have shown that any two arbitrary summands $\ell > k$ in the expression (45) are quasi-concave in p_i and that their corresponding peaks $\overline{p}_k$ and $\overline{p}_\ell$ are such that $\overline{p}_k > \overline{p}_\ell$. This implies that for prices p_i on the interval $[\overline{p}_k, \overline{p}_\ell]$ the summand γ_k is increasing in p_i while the summand γ_ℓ is decreasing in p_i.

Following Choi and Smith (2017), for quasi-concavity of the payoff (45), it is sufficient to prove that γ_k is more concave than $-\gamma_\ell$ for values of p_i in the interval $[\overline{p}_\ell, \overline{p}_k]$. This is because for prices $p_i < \overline{p}_\ell$, both γ_k and γ_ℓ are strictly concave (see above) in p_i and for prices $p_i > \overline{p}_k$, both γ_k and γ_ℓ are decreasing in p_i.

We then proceed by showing that

$$\frac{d^2\gamma_k(p_i)/dp_i^2}{d\gamma_k(p_i)/dp_i} \leq \frac{-d^2\gamma_\ell(p_i)/dp_i^2}{-d\gamma_\ell(p_i)/dp_i} \tag{53}$$

for any $p_i \in [\overline{p}_\ell, \overline{p}_k]$, $p_i \neq p^*$.

There are three cases to consider. First, suppose that $p^* > \overline{p}_k$. In this case, both functions γ_k and γ_ℓ are strictly concave on $[\overline{p}_\ell, \overline{p}_k]$. Therefore, the above inequality is satisfied. Second, suppose that $\overline{p}_\ell < p^* < \overline{p}_k$. For the very same reason, the inequality also holds. Finally, the nontrivial case, suppose that $p^* < \overline{p}_\ell < \overline{p}_k$. In this case, after simplifying, inequality (53) becomes:

$$\frac{-k\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(k-1)\left(p_{i}-r\right)\right]}{\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{k-1}\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-k\left(p_{i}-r\right)\right]-p^{*k}} \leq \frac{\ell\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-k}\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(\ell-1)\left(p_{i}-r\right)\right]}{-\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-1}\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-\ell\left(p_{i}-r\right)\right]+p^{*\ell}}, \tag{54}$$

where we have used the expressions for the first and second-order derivatives in (47) and (48).

Note that the denominators of this inequality are both positive. Note also that

$$\frac{2\left(\bar{\varepsilon}+p^{*}\right)+(k-1)\,r}{k+1}>\frac{2\left(\bar{\varepsilon}+p^{*}\right)+(\ell-1)\,r}{\ell+1}.$$

Therefore, when

$$p_{i}<\frac{2\left(\bar{\varepsilon}+p^{*}\right)+(\ell-1)\,r}{\ell+1} \tag{55}$$

the inequality (53) is satisfied trivially. Whether (54) holds or not is nontrivial when the numerator of the RHS is negative, that is, when $p_{i} \geq \frac{2\left(\bar{\varepsilon}+p^{*}\right)+(\ell-1)\,r}{\ell+1}$ because the numerator of the LHS is negative.

We make now some useful observations. First, note that

$$\frac{d\gamma_{Rk}}{dp_{i}}\left(\frac{\bar{\varepsilon}+p^{*}+kr}{k+1}\right)=-\frac{\mu_{k}}{N\bar{\varepsilon}^{k}}p^{*k} \leq 0,$$

which means that $\bar{p}_{k} \leq \frac{\bar{\varepsilon}+p^{*}+kr}{k+1}$. Since p_i is restricted to be smaller than $\bar{p}_k$, it is also smaller than $\frac{\bar{\varepsilon}+p^{*}+kr}{k+1}$. So it is sufficient to prove the inequality (53) for

$$\frac{(\ell-1)\,r+2\left(\bar{\varepsilon}+p^{*}\right)}{\ell+1} \leq p_{i} \leq \frac{\bar{\varepsilon}+p^{*}+kr}{k+1}. \tag{56}$$

(Otherwise, when k and ℓ are such that $\frac{\bar{\varepsilon}+p^{*}+kr}{k+1}<\frac{(\ell-1)\,r+2\left(\bar{\varepsilon}+p^{*}\right)}{\ell+1}$ it holds that γ_ℓ is concave up to the peak $\bar{p}_k$ of γ_k, so inequality (53) is satisfied trivially.)

Inequality (54) is equivalent to:

$$\begin{gathered}\ell\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-k}\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(\ell-1)\left(p_{i}-r\right)\right]\left(\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{k-1}\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-k(p_{i}-r)\right]-p^{*k}\right)\\ -k\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(k-1)\left(p_{i}-r\right)\right]\left(\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-1}\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-\ell\left(p_{i}-r\right)\right]-p^{*\ell}\right) \geq 0.\end{gathered}$$

Taking common factors together, this expression can be written as:

$$\begin{gathered}\ell\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-1}\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(\ell-1)\left(p_{i}-r\right)\right]\left(\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-k(p_{i}-r)\right]-p^{*}\left(\tfrac{p^{*}}{\bar{\varepsilon}+p^{*}-p_{i}}\right)^{k-1}\right)\\ -k\left(\bar{\varepsilon}+p^{*}-p_{i}\right)^{\ell-1}\left[2\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-(k-1)\left(p_{i}-r\right)\right]\left(\left[\left(\bar{\varepsilon}+p^{*}-p_{i}\right)-\ell\left(p_{i}-r\right)\right]-p^{*}\left(\tfrac{p^{*}}{\bar{\varepsilon}+p^{*}-p_{i}}\right)^{\ell-1}\right) \geq 0.\end{gathered}$$

and by dropping the factor $(\bar{\varepsilon} + p^* - p_i)^{\ell-1}$ we obtain:

$$\begin{gathered}
-\ell \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (\ell - 1) \left(p_i - r\right)\right] p^* \left(\tfrac{p^*}{\bar{\varepsilon}+p^*-p_i}\right)^{k-1} + \\
\ell \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (\ell - 1) \left(p_i - r\right)\right] \left[\left(\bar{\varepsilon} + p^* - p_i\right) - k(p_i - r)\right] + \\
k \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (k - 1) \left(p_i - r\right)\right] p^* \left(\tfrac{p^*}{\bar{\varepsilon}+p^*-p_i}\right)^{\ell-1} - \\
k \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (k - 1) \left(p_i - r\right)\right] \left[\left(\bar{\varepsilon} + p^* - p_i\right) - \ell \left(p_i - r\right)\right] \geq 0.
\end{gathered}$$

Putting together the second and the third lines of this expression we get:

$$\begin{gathered}
-\ell \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (\ell - 1) \left(p_i - r\right)\right] p^* \left(\tfrac{p^*}{\bar{\varepsilon}+p^*-p_i}\right)^{k-1} + \\
k \left[2 \left(\bar{\varepsilon} + p^* - p_i\right) - (k - 1) \left(p_i - r\right)\right] p^* \left(\tfrac{p^*}{\bar{\varepsilon}+p^*-p_i}\right)^{\ell-1} + \\
(\ell - k) \left[2 \left(\bar{\varepsilon} + p^* - p_i\right)^2 + \ell k (p_i - r)^2 - (\ell + k - 1) \left(\bar{\varepsilon} + p^* - p_i\right) \left(p_i - r\right)\right] \geq 0
\end{gathered} \tag{57}$$

The first term of this inequality is positive because by (56) $p_i \geq \dfrac{2\left(\bar{\varepsilon} + p^*\right) + (\ell - 1) r}{\ell + 1}$. The second term is positive because by (56) $p_i \leq \dfrac{\bar{\varepsilon} + p^* + kr}{k + 1}$, which is less than $\dfrac{2\left(\bar{\varepsilon} + p^*\right) + (k - 1) r}{k + 1}$. The third term divided by $(\ell - k) \left(p_i - r\right)^2$ is equal to the quadratic function

$$A(t) = 2t^2 - (\ell + k - 1) t + \ell k$$

in $t = \dfrac{\bar{\varepsilon} + p^* - p_i}{p_i - r} > 0$. Therefore, $A(t) \geq 0$ for any t, if its discriminant $(\ell + k - 1)^2 - 8\ell k \leq 0$. Solving this inequality gives

$$1 + 3\ell - \sqrt{8\ell(\ell + 1)} \leq k \leq 1 + 3\ell + \sqrt{8\ell(\ell + 1)}. \tag{58}$$

The RHS of this inequality is trivially satisfied because $k < \ell$. The LHS of this inequality is satisfied for $k = 1$ only if $\ell \leq 8$. This is because the expression $1 + 3\ell - \sqrt{8\ell(\ell + 1)}$ is increasing in $\ell \geq 1$ and is equal to 1 for $\ell = 8$. Consequently, (58) is satisfied for any k, ℓ such that $1 \leq k < \ell \leq N$ when $N \leq 8$ and this implies that $A(t) \geq 0$ for any t. This completes the proof that the payoff is quasi-concave when $N \leq 8$.

For $N \geq 9$ there is at least one pair of k, ℓ with $1 \leq k < \ell \leq N$ such that the discriminant of $A(t)$ is positive. Therefore, in this case we follow a different strategy. Recall that when (55) holds the inequality (53) is satisfied trivially. Note that condition (55) is equivalent to $t > \dfrac{\ell - 1}{2}$; so for $t > \dfrac{N - 1}{2}$ (55) is satisfied for any $\ell \leq N$. At the same time we observe that because t is decreasing in p_i, we have

$$t = \frac{\bar{\varepsilon} + p^* - p_i}{p_i - r} \geq \frac{\bar{\varepsilon} + r - p^m}{p^m - r} = \frac{\bar{\varepsilon} + r - (\bar{\varepsilon} + r)/2}{(\bar{\varepsilon} + r)/2 - r} = \frac{\bar{\varepsilon} + r}{\bar{\varepsilon} - r},$$

where $p^m = (\bar{\varepsilon} + r)/2$ is the monopoly price. For given N and $\bar{\varepsilon}$, by choosing the marginal cost r such that $\dfrac{\bar{\varepsilon} + r}{\bar{\varepsilon} - r} > \dfrac{N - 1}{2}$, the inequality (53) will be satisfied trivially for any k, ℓ such that

$1 \leq k < \ell \leq N$. This means that r should satisfy $r > \frac{N-3}{N+1}\bar{c}$; in this case the payoff will be quasi-concave. The proof is now finished. ■

Proof of Proposition 4. For equation (33) to hold, either the first or the second bracket must be equal to zero (or both of course). Suppose that the second bracket is equal to zero. In that case, the first equation (32) simplifies to:

$$1 - 2p_1^* = 0,$$

which implies that $p_1^* = 1/2$. However, this solution is not valid because the equilibrium price cannot be the monopoly price of 1/2. Therefore, it must be the case that the first bracket of equation (33) is equal to zero, that is, $p_1^*(4p_2^* - 2) + p_2^*(4 - 3p_2^*) - 1 = 0$. From this, we obtain an expression for p_1^*:

$$p_1^* = \frac{4p_2^* - 3p_2^{*2} - 1}{2(1 - 2p_2^*)}. \tag{59}$$

Because $p_1^* < 1/2$, it is easy to see that p_2^* has to satisfy $p_2^* < \frac{1}{3}\left(3 - \sqrt{3}\right)$. Moreover, because $p_1^* > p_2^*$, it must be the case that $p_2^* > \sqrt{2} - 1$.

We can also use the relationship in equation (59) to find the interval of admissible $\bar{c}$'s. For this we plug p_1^* into the expression for $c_1(p_1^*, p_2^*)$, which gives:

$$c_1(p_2^*) = \frac{20p_2^{*5} - 53p_2^{*4} + 104p_2^{*3} - 158p_2^{*2} + 92p_2^* - 17}{24(1 - 2p_2^*)^2}.$$

Because $c_1(p_2^*)$ must be lower than $\bar{c}$, and $p_2^* \in [\sqrt{2} - 1, \frac{1}{3}\left(3 - \sqrt{3}\right)]$, the equilibrium does not exist outside the interval of search cost upper bounds $\bar{c} \in [\frac{8\sqrt{2}-11}{3}, \frac{1}{8} + \frac{1}{18\sqrt{3}}]$.

If we plug the value of p_1^* in equation (59) into equation (32) and rearrange it, we obtain an expression involving only p_2^* and $\bar{c}$:

$$\begin{aligned}\mathcal{P}(p_2^*, \bar{c}) \;\equiv\; & 384\bar{c} - 420p_2^9 + 2073p_2^8 - 5328p_2^7 + 10060p_2^6 - 24(192\bar{c} + 545)p_2^5 + 6(2688\bar{c} + 1733)p_2^4 \\ & -48(424\bar{c} + 103)p_2^3 + 108(112\bar{c} + 13)p_2^2 - 12(288\bar{c} + 19)p_2 + 17 = 0. \end{aligned} \tag{60}$$

Note that $\mathcal{P}(p_2, \bar{c})$ computed at $p_2 = \sqrt{2} - 1$ is equal to

$$\mathcal{P}(\sqrt{2} - 1, \bar{c}) = \left(645696 - 456576\sqrt{2}\right)\bar{c} - 3395968\sqrt{2} + 4802624,$$

which computed at $\bar{c} = \frac{8\sqrt{2}-11}{3}$ is

$$\mathcal{P}\left(\sqrt{2} - 1, \frac{8\sqrt{2} - 11}{3}\right) = 0,$$

and when computed at $\overline{c} = \frac{1}{8} + \frac{1}{18\sqrt{3}}$ is

$$\mathcal{P}\left(\sqrt{2}-1, \frac{1}{8} + \frac{1}{18\sqrt{3}}\right) = 1.4988 \times 10^{-3} > 0.$$

Further, note that $\mathcal{P}(p_2, \overline{c})$ computed at $p_2 = \frac{1}{3}\left(3-\sqrt{3}\right)$ is

$$\mathcal{P}\left(\frac{1}{3}\left(3-\sqrt{3}\right), \overline{c}\right) = \frac{1124}{81}\sqrt{3} - \frac{649}{27} = -2.159\,2 \times 10^{-3} < 0.$$

Consequently, for any $\overline{c} \in \left(\frac{8\sqrt{2}-11}{3}, \frac{1}{8} + \frac{1}{18\sqrt{3}}\right)$ we have

$$\begin{aligned}\mathcal{P}(\sqrt{2}-1, \overline{c}) &> 0\\ \mathcal{P}(\frac{1}{3}\left(3-\sqrt{3}\right), \overline{c}) &< 0,\end{aligned}$$

which implies that there is $p_2^* \in [\sqrt{2}-1, \frac{1}{3}\left(3-\sqrt{3}\right)]$ for which $\mathcal{P}(p_2^*, \overline{c}) = 0$. Because the corresponding p_1^* is of course greater than p_2^*, the proof that an equilibrium exists is now complete.

We can also prove that the above established solution is unique by showing that $\mathcal{P}(p_2, \overline{c})$ is strictly decreasing in p_2 for any admissible $\overline{c}$. In order to do this, note that

$$\frac{d\mathcal{P}(p_2, \overline{c})}{dp_2} = \mathcal{H}_1(p_2)\overline{c} + \mathcal{H}_2(p_2) \tag{61}$$

where

$$\mathcal{H}_1(p_2) = -\,3456 + 24\,192p_2 - 61\,056p_2^2 + 64\,512p_2^3 - 23040p_2^4 \tag{62}$$

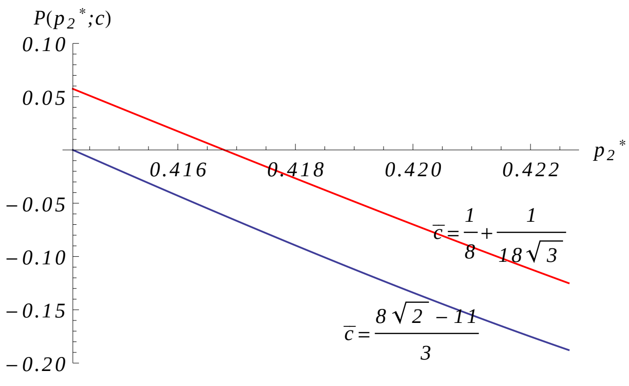
$$\begin{aligned}\mathcal{H}_2(p_2) = &-228 + 2808p_2 - 14\,832p_2^2 + 41\,592p_2^3 - 65\,400p_2^4 + 60\,360p_2^5 - 37\,296p_2^6\\ &+ 16\,584p_2^7 - 3780p_2^8.\end{aligned} \tag{63}$$

We now make the following observations about $\mathcal{H}_1(p_2)$ and $\mathcal{H}_2(p_2)$. First, $\mathcal{H}_1(p_2)$ is increasing in $p_2 \in \left[\sqrt{2}-1, \frac{1}{3}\left(3-\sqrt{3}\right)\right]$ but because $\mathcal{H}_1(\frac{1}{3}\left(3-\sqrt{3}\right)) = -2.4624$, it is always negative. Second, $\mathcal{H}_2(p_2)$ is also increasing in $p_2 \in \left[\sqrt{2}-1, \frac{1}{3}\left(3-\sqrt{3}\right)\right]$.

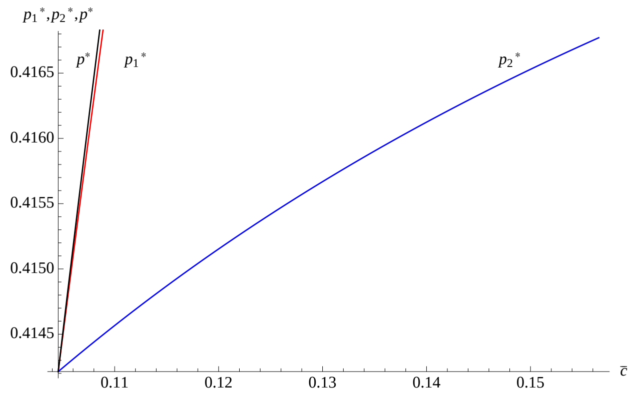
From these remarks we conclude that if the derivative (61) is negative for the lowest possible value for $\overline{c}$ and the highest possible value for p_2, then it is always negative. Substituting $\overline{c} = \frac{8\sqrt{2}-11}{3}$ and $p_2 = \frac{1}{3}\left(3-\sqrt{3}\right)$ in (61) gives

$$\frac{d\mathcal{P}(\frac{1}{3}\left(3-\sqrt{3}\right), \frac{8\sqrt{2}-11}{3})}{dp_2} = -2.4624 \cdot \frac{8\sqrt{2}-11}{3} + 0.14987 = -0.10762.$$

This shows that $\frac{d\mathcal{P}(p_2,\overline{c})}{dp_2}$ is negative for all admissible p_2's and $\overline{c}$'s, so the function $\mathcal{P}(p_2, \overline{c})$ is strictly decreasing in p_2 for any admissible $\overline{c}$, and therefore, there can only be one solution to $\mathcal{P}(p_2, \overline{c}) = 0$.



(a) Polynomial $\mathcal{P}(\cdot)$



(b) Equilibrium prices

Figure 8: Prominence price equilibrium ($\alpha = 1$) vs. symmetric equilibrium price

We illustrate the the above findings using Figure 8. The graph of Figure 8(a) shows the polynomial in equation (60) for the highest (red curve) and lowest (blue curve) admissible upper bound of the search cost distribution. We plot the polynomials for all $p_2^* \in \left(\sqrt{2}-1, 1-\sqrt{3}/3\right)$. In both cases, there is a unique solution in p_2^*. The graph of Figure 8(b) gives the price equilibrium for all admissible $\overline{c}$ values. The price of the prominent firm is in red, the price of the non-prominent firm is in blue and, for comparison purposes, we also plot the symmetric equilibrium price in black. Clearly, irrespective of the value of $\overline{c}$, both prices are lower than the symmetric equilibrium price. As a result, consumers are better off when one firm is prominent. ■

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