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ASSET PRICING VS ASSET EXPECTED RETURNING IN FACTOR-PORTFOLIO MODELS

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Asset Pricing vs Asset Expected Returning in Factor-Portfolio Models

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Abstract

Standard factor-portfolio models focus on returns and leave prices undetermined. This approach ignores information contained in the time-series of asset prices, relevant for long-term investors and for detecting potential mis-pricing. To address this issue, we provide a new (co-)integrated methodology to factor modeling based on both prices and returns. Given a long-run relationship between the value of buy-and-hold portfolios in test assets and factors, we argue that a term—naturally labeled as Equilibrium Correction Term (ECT)—should be included when regressing returns on factors. We also propose to validate factor models by the existence of such a term. Empirically, we show that the ECT predicts equity returns, both in-sample and out-of-sample.

JEL Codes: C38, G11, G17.
Keywords: Long-Horizon Returns, Predictability, Mispricing, Factor Models, Equilibrium Correction Term.

1   Introduction

In his AFA presidential address, John Cochrane (2011) states *"...We have to answer the central question, what is the source of price variation? When did our field stop being "asset pricing" and become "asset expected returning"?..."*.

It is common in the asset pricing literature to parsimoniously characterize the dynamics of returns with factors that can explain the cross-section of average excess stock returns (e.g., Fama and French, 2015; Hou et al., 2015). However, standard models relating returns to factors leave price dynamics undetermined. We argue that such traditional approach to factor modeling prevents the possibility of using prices to learn information about the time-series dynamics of assets that one cannot detect when focusing only on returns.

Why study price dynamics? Think of long-term investors, e.g., a pension fund. To a long-run investor, the relevant quantity of interest is the cumulative returns over the investment horizon. However, the relation between period-returns and factors (as, for example, the five-factors proposed by Fama and French, 2015) has no implications for the ability of factors to explain long-term returns, since cumulative returns over a long-horizon bear no relationship with period-returns. Now, consider constructing a buy-and-hold portfolio in the factors. There is a natural possibility for a relationship between such a portfolio and the cumulative returns of an investment strategy over the long-run. We define risk drivers as the value of buy-and-hold portfolios investing in standard factors (e.g., market, size, value, etc.) and prices as the value of buy-and-hold portfolios investing in any test assets (e.g., single stocks, basket of bonds, etc.). Our analysis starts by noting that prices and risk drivers share a common stochastic long-term trend, but may diverge from one another in the short-run. The existing literature on factor models for returns is not really applicable to this framework because the possibility of determining price dynamics has not been modeled. We propose to fill this void by explicitly modeling the common long-term trend between asset prices and risk drivers.

Only when prices and risk drivers share a common stochastic trend, i.e., they are cointegrated, the value of a buy-and-hold portfolio in any given asset can be related to a set of risk drivers. In this case, an Equilibrium Correction Term (henceforth, ECT)—capturing temporary asset-specific price discrepancies—should be included in standard factor models projecting returns on factors. The ECT could predict returns as expected future returns may

respond to current divergences between prices and risk drivers. Such correction mechanism guarantees that in the long-run prices will converge to their equilibrium value determined by the cointegrating relationship with risk drivers. In this paper, we study the implications of cointegration between asset prices and risk drivers to select the relevant factor model specification, and we investigate the empirical relationship between price deviations and return predictability.[1]

Factor models are widely used to reduce the number of parameters to be estimated for asset allocation and risk measurement of portfolios with many assets (see, for example, Kolanovic and Wei, 2013; Ang, 2014). Despite the great popularity of these models, the literature on the relationship between the choice of factors and the investment horizon has been much less developed. Specifically, the factor-based approach to portfolio allocation and risk management has concentrated almost exclusively on modeling returns with factors while not devoting enough attention to the relationship between the long-run performance of assets and factors. Moreover, the factor models literature has traditionally focused on factor-representation of stationary variables and only very recently the factor framework has been extended to non-stationary cointegrated factors (e.g., Barigozzi et al., 2015; Banerjee et al., 2017).

We argue that the analysis of potential cointegration between risk drivers and prices is useful to assess the validity of factor models to explain long- and short-run dynamics of assets. Risk drivers should explain the long-run performance of any given portfolio. As risk drivers and prices are non-stationary variables, the validity of a given set of risk drivers to explain portfolio prices is naturally investigated by assessing if there exists a stationary linear combination of them (i.e., if they are cointegrated). In fact, the presence of a stationary linear combination of prices and drivers of risk rules out the possibility of an omitted risk driver because the omission of a relevant one would prevent cointegration. Therefore, non-stationary deviation of prices from risk drivers is an argument to discard any factor model. We provide a simple example of our proposal to validate factor models by a simulation exercise.

Cointegration between risk drivers and prices delivers a persistent but stationary Equilibrium Correction Term.[2] The interpretation of this component is not trivial. The ECT

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[1] Hansen et al. (2008) are among the firsts to provide evidence on the importance of understanding long-run dynamics for equity returns and assets valuation.

[2] Pioneer work by Summers (1986) and Fama and French (1988) also find a persistent mean-reverting

can result from time-varying risk premia generated by any given rational pricing model in an efficient market. Alternatively, the ECT may capture agents' over-reaction to news in prices that are subsequently corrected in the dynamics of returns.[3] The incapability to disentangle the two competing economic stories for the existence of transitory price deviations goes back to early works by Poterba and Summers (1988) and Fama and French (1988), and is also in line with recent findings by Brennan and Wang (2010) that state "[...]there is now extensive evidence that common stocks are mispriced[...], although the reasons for the pricing discrepancies remain in dispute", and by Kozak et al. (2018) who argue that the absence of near-arbitrage opportunities is a sufficient condition to prevent the possibility of discriminating between alternative models of investor beliefs (e.g., "rational" vs. "behavioral" asset pricing models) in a reduced-form factor framework.

Independent from its interpretation, the ECT improves the empirical performance of a given factor model under several dimensions, and predicts the distribution of returns. In our empirical analysis, we consider the 25 Portfolios formed on Size and Book-to-Market (5x5) from Kenneth R. French's Data Library from 1963 to 2018 as test assets, and the five-factor model of Fama and French (2015) augmented with a momentum factor, like recently proposed by Fama and French (2018), as the reference factor model (we label this model as six-factor Fama-French model, or FF6). When testing the six-factor Fama-French model on the 25 Portfolios formed on Size and Book-to-Market, the inclusion of the ECT delivers: (a) time-varying intercepts[4] that always significantly include zero; (b) a quasi-diagonal variance-covariance residuals matrix; (c) a multivariate normal variance-covariance residuals matrix. Such results imply that a standard, parsimonious factor model augmented with the ECT can appropriately characterize the distribution of assets for mean-variance analysis.

Finally, the ECT is a predictive variable, specific to each asset. In principle, a return predictor could forecast either the systematic or the idiosyncratic component of returns.[5] Empirically, we find that the Equilibrium Correction Terms across the 25 Portfolios mostly

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component in prices.

[3]The literature on "diagnostic expectations" (e.g., Gennaioli et al., 2015; Bordalo et al., 2017; Gennaioli and Shleifer, 2018) offers a natural framework to understand such dynamics.

[4]As we shall see later, we can rewrite any factor model with the ECT as a standard factor model with a time-varying intercept.

[5]In a recent paper, Engleberg et al. (2019) find that the value-weighted mean of idiosyncratic anomaly variables does not predict aggregate market returns. However, Pesaran and Smith (2019) show that pricing errors can predict risk premia when they (at least partially) covary with the stochastic discount factor.

predict the idiosyncratic component of returns, accounting for 15.4% of yearly idiosyncratic return variance. The ECT contains asset-specific information that do not aggregate to generate factor-return predictability. Also, we consider two alternative return predictors—the dividend-price ratio and a latent predictive variable–and we find that they are unrelated to the ECT, suggesting that the ECT captures an additional, new dimension of return predictability. On average, approximately 1.2% of the annual variation in U.S. stock excess returns between 1963 and 2018 is explained by the ECT.[6] When asset prices are higher (lower) than the long-run equilibrium implied by their relationship with risk drivers, we expect lower (higher) returns in the next period. In our sample, a positive 1% log price deviation is associated to a lower future log return by 38 basis points.[7]

The predicted distribution of returns at year $t + 1$ is centered on the observed ECT at year t. The ability of the ECT to detect shifts in the distribution of returns has relevant implications for risk measurement and long-term asset allocation.[8] For example, think of a situation in which a market crash causes a change in the relative position of prices and risk drivers so that the ECT becomes very negative. This deviation will shift the distribution of future asset returns to the right, decreasing the associated conditional Value-at-Risk. Practically, such evidence could settle the old-fashioned dispute between risk managers and portfolio managers after a crisis, when portfolio managers complain with risk managers who do not allow taking positions geared to exploit the opportunity of a market crash because they are deemed "too risky".

The rest of the paper is organized as follows. We discuss the related literature next. Section 2 lays out the joint model for prices, returns, factors, and risk drivers, and provides some evidence on the relevance of including price dynamics in factor-portfolio models. Section 3 describes our data, documents the existence of the Equilibrium Correction Term, and illustrates the empirical performance of the co-integrated factor model. Section 4 presents several robustness tests, while Section 5 concludes.

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[6]Fama and French (1988) report a similar value for the predictable returns variation.

[7]Bossaerts and Green (1989) derive a dynamic pricing model with time-varying risk premia in which the risk of individual securities and equilibrium risk premia change predictably, featuring the inverse relationship between prices and returns.

[8]In this paper, we illustrate some empirical implications of our approach for risk measurement, while in a companion paper we investigate the potential of a cointegrated factor model for portfolio allocation using single stocks.

Related literature. This paper is related to the literature on the relationship between cointegrated variables and error correction models (see, for example, Hendry, 1986; Engle and Yoo, 1987; Johansen, 1995; Pesaran and Shin, 1998; Liu and Timmermann, 2013). Lettau and Ludvigson (2001) are a first notable example of the use of cointegration analysis in macro-finance. They show that aggregate consumption, asset holdings, and labor income share a common long-term trend and temporary trend-deviations successfully predict short- and medium-term expected stock returns. While their work focuses on economic variables, we are interested in financial markets' dynamics and the relation between asset values and risks. Bansal et al. (2007) find that the cointegrating relationship between dividends and consumption, a measure of long-run consumption risks, is a key determinant of expected equity excess returns across different investment horizons. At long horizons, risks are dominated by long-run consumption risks. Bansal and Kiku (2011) show that economic restrictions of cointegration between dividends and aggregate consumption have important implications for the prediction of return dynamics and the implementation of optimal portfolio rules, particularly at long investment horizons. Our work provides new supportive evidence that cointegration analysis can play a relevant role for financial markets and long-term investors.

This paper also relates to the large empirical literature that studies temporary deviations of asset values from fundamentals. In an early contribution, Poterba and Summers (1988) find positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons which can be explained by persistent, but transitory, divergences between prices and fundamental values. Concurrently, Fama and French (1988) argue that the U-shaped pattern in first-order autocorrelation observed in U.S. stock returns for the 1926-85 period is consistent with the view that prices have a slowly decaying stationary component. Brennan and Wang (2010) show that a premium in average returns is created as a result of Jensen's inequality when stock prices diverge from fundamental values because of stochastic pricing errors, even when the mispricing has an average of zero. Lou (2012) finds that mutual funds flow-induced trading can significantly affects stock returns and in the short-run drives asset prices away from equilibrium. Lewis et al. (2017) use a unique database of U.S. corporate bonds to test empirical implications of a number of mispricing models and show a significant and persistent mispricing in the sample period 2008-2012. Daniel et al. (2017) find that the Fama-French five-factor model (Fama and French, 2015) contains large unpriced components correlated with unpriced risks, and hedging such risks increase the

Sharpe ratio achievable investing only in the five factors. Hendershott et al. (2018) provide structural estimates of the magnitude and duration of pricing errors through a model with inattentive investors. Ehsani and Linnainmaa (2019) argue that factor momentum aggregates the auto-correlations found in other factors and may stem from mispricing. Avramov et al. (2019) show that mispricing occurs across financial distressed firms during periods of high market sentiment because in these times both retail and institutional investors are overly optimistic about the likelihood and consequences of financial distress. The sluggish investors' response to correct overpricing leads to a wide range of anomalies in the cross-section of stocks and bonds. Van Binsbergen and Opp (2019) investigate the implications of financial market mispricing for the real economy and find that pricing errors can cause allocative distortions. Although the literature on mispricing is rapidly growing, most of the papers focus on commonalities in mispricing and their identification (e.g., Stambaugh and Yuan, 2016). Conversely, our methodology allows to detect transitory asset-specific price deviations.

Finally, our work closely relates to recent studies on the relevance of long-horizon returns for asset pricing models. Chernov et al. (2018) find that popular factor pricing models are unable to price their own factors at multiple return horizons. To address this issue, Chernov et al. (2018) provide a novel test in the context of GMM estimation in which they use multi-horizon returns to test overidentifying restrictions of a given model. They introduce conditional SDF sensitivities that must exhibit large and persistent time-variation to match multi-horizon returns. Instead, we model directly the documented persistence in the returns of buy-and-hold factor-portfolios and use stationary deviations of prices from such returns to validate any factor model. Cho and Polk (2020) derive an exact identity relating future abnormal returns to a price measure of mispricing. They find that abnormal payoffs that occur sooner or in more valuable states are associated with larger price deviations. Since Cho and Polk (2020) concentrate on the cross-section of prices and consider stationary variables, they do not address the time-series implications of mispricing, that is the focus of our work.

2 From Factors for Asset Returns to Risk Drivers for Asset Prices

2.1 Traditional Factor Models for Returns

Factor models are commonly used to characterize parsimoniously the predictive distribution of asset returns. To this end multi-factor models, in which k factors characterize in a lower parametric dimension the distribution of n asset returns, have the following general form:

$$
\begin{aligned}
r_{t+1}^i &= \alpha^i + \beta_i' \mathbf{f}_{t+1} + v_{t+1}^i & (1) \\
\mathbf{f}_{t+1} &= E\left(\mathbf{f}_{t+1} \mid I_t\right) + \epsilon_{t+1} & (2) \\
\epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0}, \boldsymbol{\Sigma}\right) \\
Cov\left(v_{t+1}^i, v_{t+1}^j\right) &= 0, \quad for \ \ each \ \ i \neq j
\end{aligned}
$$

in which $\mathbf{f}_{t+1}$ is a k-dimensional vector of factors at time $t+1$, r_{t+1}^i is the return on the i-th of the n assets at time $t+1$, and the vector β_i' contains the loadings for asset i on the k factors. Equation (1) specifies the conditional distribution of returns on factors, while equation (2) specifies the predictive distribution for factors at time $t+1$ conditioning on information available at time t. A baseline specification for this system assumes away factors predictability thus implying that conditional expectations of factors have no variance (i.e., $E\left(\mathbf{f}_{t+1} \mid I_t\right) = \mu$). Returns and factors are stationary variables.[9]

For a factor model to effectively characterize the distribution of n assets it is crucial that v_{t+1}^i represent idiosyncratic risks and therefore they are orthogonal to the factors, serially uncorrelated, and contemporaneously uncorrelated across assets. The diagonality of the variance-covariance matrix of the residuals coming from projecting asset returns on factors is a necessary–and testable–requirement for the validity of any factor model. These assumptions imply that covariances are easy to estimate for mean-variance analysis. Indeed, only when v_{t+1}^i behaves as a proper idiosyncratic shock the factor structure allows to reduce the problem of modeling the n-variate distribution of returns (in which n can be very large) to model the k-variate distribution of factors (in which k is typically small). Testing for the

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[9]The unconditional moments of returns are constant, although the conditional ones might be time varying.

validity of these assumptions is crucial to guarantee that any given factor model produces the valid reduction of dimensionality which is particularly useful for asset allocation, risk measurement, and risk management.[10]

Further validation of factor models for returns is traditionally based on testing restrictions on their coefficients. Consider the time-series and the expected return-beta representation of a factor model (e.g., Cochrane, 2005). Factor loadings can be calculated via n time-series regressions of the n assets on the k factors:

$$r^i_{t+1} = \alpha_1 + \beta_{i,f^1} f^1_{t+1} + \beta_{i,f^2} f^2_{t+1} + \cdots + \beta_{i,f^k} f^k_{t+1} + v_{t+1} \tag{3}$$

The affine expected return-beta cross-sectional regression is:

$$E(\mathbf{r}) = \gamma_0 + \gamma_1 \boldsymbol{\beta}_{f1} + \gamma_2 \boldsymbol{\beta}_{f2} + \cdots + \gamma_k \boldsymbol{\beta}_{fk} \tag{4}$$

where $E(\mathbf{r})$ is a vector containing the n-assets' averages over time, and $\boldsymbol{\beta}_{fi}$, is a vector containing the exposure of all assets to factor i.

In the case in which all factors are excess returns, equation (4) holds also for the factors, and we have $\gamma_i = E(f^i)$ where $E(f^i)$ is the mean over time of the factor i.

A two-step test (e.g., Fama and MacBeth, 1973) for the validity of any factor model of the form (1) can be run by regressing the vector of mean returns $E(\mathbf{r})$ on the cross-sectional loadings with length n previously obtained from the time-series regression (3):

$$E(\mathbf{r}) = \gamma_0 + \gamma_1 \hat{\boldsymbol{\beta}}_{f1} + \gamma_2 \hat{\boldsymbol{\beta}}_{f2} \ldots + \gamma_k \hat{\boldsymbol{\beta}}_{fk}$$

and by testing the following null hypothesis:

$$\hat{\gamma}_0 = \bar{r}^f, \ \hat{\gamma}_i = E\left(f^i\right)$$

If both test assets and factors are excess returns, there is no need to run the cross-sectional regression, as the validity of the model can be simply tested by evaluating the null that all intercepts in the time-series model are zero (see Gibbons et al., 1989). Recently, Barillas and Shanken (2017) have proposed an alternative procedure based on examining the extent

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[10]Using the joint distribution of returns requires the estimation of $n + n(n + 1)/2$ parameters while using a factor structure requires the estimation of only $n(k + 2) + k(k + 1)/2$ parameters.

to which each factor models price the factors in alternative models.

In case factors are not excess returns such that factor-risk premia might be different from the mean value of factors, time-series tests cannot be run without the aid of the cross-sectional regression.

The focus of the literature on these aspects has somewhat overlooked the fact that the standard framework relating asset returns to stationary factors leaves asset prices undetermined. Our proposal to overcome this limitation is to extend the framework defining the mapping of asset returns into factors to include the relationship between asset prices and risk drivers.

2.2 A (Co-)Integrated Approach to Factor Modeling

Consider a test asset i that has a log period return of r_t^i. We define the log price of this asset as:

$$\ln P_t^i = \ln P_{t-1}^i + \mathbf{r}_t^i \tag{5}$$

Prices are cumulative returns of any test assets. The analogous of the (log) price for an asset can be constructed for any given factor. We define as factor-risk drivers the cumulative returns of a portfolio investing in standard factors (e.g., market, size, momentum, etc.) that have a period log return of $\mathbf{f}_t$.[11] The generic risk driver associated to a factor with a log period return of $\mathbf{f}_t$ evolves according to the following process:

$$\ln \mathbf{F}_t = \ln \mathbf{F}_{t-1} + \mathbf{f}_t \tag{6}$$

If test assets returns and factors are stationary, then prices and risk drivers are non-stationary. It is therefore natural to think of the possibility of the determination of prices in the long-run as function of risk drivers. If factor-risk drivers are the non-stationary variables that drive the non-stationary dynamics of prices, then a linear combination of prices and risk drivers should be stationary, in other words prices and risk drivers should be cointegrated.

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[11]Meucci (2011) introduces the concept of risk drivers of any given security as a set of random variables that completely specifies the security price and that follow a stochastic process homogeneous across time. We adopt the terminology here with some adaptation.

The exposure of any given portfolio P_t^i to risk drivers is determined by estimating parameters in the following model:

$$\ln P_t^i = \alpha_0^i + \alpha_1^i t + \beta_i' \ln \mathbf{F}_t + u_t^i \tag{7}$$

We include a linear trend in equation (7) since it allows to recover the standard short-run specification–returns are regressed on factors plus a constant–when taking first-differences.

When the chosen set of risk drivers describes the long-run dynamics of prices, the estimation of equation (7) delivers stationary error terms u_t^i. Only in the case of cointegration between prices and risk drivers the factor model is capable to replicate fluctuations in prices of any given portfolio up to stationary residuals. Since u_t^i captures "disequilibria" in the long-run relationship between prices and factor-risk drivers, we label the residual "Equilibrium Correction Term". The coefficient α_1 measures the systematic long-run component in the relative performance of portfolios and factor-risk drivers; note that a positive α_1 in the long-run generates "alpha" in returns.

We model the joint distribution of portfolio prices, factors, and risk drivers as follows:

$$
\begin{aligned}
\ln P_{t+1}^i &= \alpha_0^i + \alpha_1^i t + \beta_i' \ln \mathbf{F}_{t+1} + u_{t+1}^i \tag{8}\\
u_{t+1}^i &= \rho_i u_t^i + v_{t+1}^i\\
\mathbf{f}_{t+1} &= E\left(\mathbf{f}_{t+1} \mid I_t\right) + \epsilon_{t+1}\\
\ln P_t^i &= \ln P_{t-1}^i + r_t^i\\
\ln \mathbf{F}_t &= \ln \mathbf{F}_{t-1} + \mathbf{f}_t\\
\epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0}, \mathbf{\Sigma}\right)\\
Cov\left(v_{t+1}^i, v_{t+1}^j\right) &= 0
\end{aligned}
$$

In equation (8), u_t^i captures deviations of prices from the long-run values predicted by risk drivers. If u_t^i is stationary, then prices and risk drivers are cointegrated.[12] Stationarity of the u_t^i is achieved if and only if $\mid \rho_i \mid < 1$. In this case, when $0 < \rho_i < 1$ we have positive persistence in the term v_{t+1}^i and the effect of the "innovations" lasts more than one-period, while when $-1 < \rho_i < 0$ "news" ignite an oscillatory, but convergent, effect.

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[12] We model u_t^i as an AR(1) process. Our choice is also supported empirically by statistical tests for serial correlation of this term.

We derive the relationship between returns and factors implied by the model in (8) by taking first differences:

$$r_{t+1}^i = \alpha_1^i + \beta_i' \mathbf{f}_{t+1} + (\rho_i - 1)u_t^i + v_{t+1}^i \qquad (9)$$

where u_t^i is the Equilibrium Correction Term associated with asset i at time t. When risk drivers explain portfolio values, cointegration implies that portfolio returns respond to the Equilibrium Correction Term, so far omitted in the empirical asset pricing literature. The inclusion of the ECT ensures that the specification for returns is consistent with the long-run relationship between risk drivers and portfolio prices. The omission of the ECT leads to a mis-specification of the factor model, that leaves price dynamics undetermined.[13]

Interestingly, a traditional factor model would not be affected by omitting the disequilibrium term only if risk drivers and prices are not cointegrated (i.e., when $\mid \rho_i \mid = 1$). It means that the incapability of a given factor structure of pricing buy-and-hold portfolios might be missed when only the relationship between returns and factors is specified without tracking the implied relationship between prices and risk drivers.

2.3 Evidence From Monte-Carlo Simulation of a Simple DGP

A small simulation exercise can be useful to understand the importance of modeling price dynamics for assessing the validity of factor models to explain the long-run performance of

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[13]Note that cointegration implies an error correction model (ECM) representation as in equation (9), which allows us to rewrite a model in levels, involving non-stationary time-series, as a model involving only stationary variables. Such variables are stationary either because they are first differences of I(1) variables or because they are stationary linear combinations of non-stationary variables (cointegrating vectors).

any given portfolio. Let's assume the following Data Generating Process (DGP):

$$\ln P_{t+1} = \alpha_0 + \alpha_1 t + \beta \ln F_{t+1} + u_{t+1} \tag{10}$$
$$u_{t+1} = \rho u_t + \sigma_1 \sqrt{1 - \rho^2} v_{t+1}$$
$$f_{t+1} = \mu + \sigma_2 \epsilon_{t+1}$$
$$\ln P_t = \ln P_{t-1} + r_t$$
$$\ln F_t = \ln F_{t-1} + f_t$$
$$\begin{pmatrix} u_t \\ \epsilon_t \end{pmatrix} \sim N.I.D. \left[\mathbf{0}, \mathbf{I} \right]$$

In this simplified DGP, we consider only one test asset and one factor. The cointegrating relationship between the price and the risk driver associated respectively to the return and the factor is controlled by the parameter ρ. We assume no predictability for the factor, that follows a Constant Expected Return (CER) specification. We calibrate μ and σ_2 to 0.05 and 0.17 respectively such that f_t features the mean and the standard deviation of U.S. stock market excess returns over the risk free in annual data for the period 1963-2018. Similarly, we calibrate parameters in the long-run regression (10) by considering the values delivered by projecting (log) cumulative returns for Portfolio Small and Value (Port 15) from the 25 Portfolios formed on Size and Book-to-Market (5x5) in Kenneth R. French's Data Library on the risk driver associated with our single-factor. Thus, α_0 is 0.25, α_1 is 0.085, and β is 0.5. Finally, we set $\sigma_1 = 0.28$, that equals the standard deviation of equation (9) for Port 15.

We study the effect of estimating and simulating a traditional factor model and a FECM model given the above DGP. The traditional factor model is specified as follows:

$$r_{t+1} = \gamma_1 + \gamma_2 f_{t+1} + v_{1,t+1}$$
$$f_{t+1} = \gamma_3 + v_{2,t+1}$$
$$\ln P_{t+1} = \ln P_t + r_{t+1}$$
$$\ln F_{t+1} = \ln F_t + f_{t+1}$$

13

While the FECM is:

$$
\begin{aligned}
\ln P_{t+1} &= \delta_1 + \delta_2 t + \delta_3 \ln F_{t+1} + u_{1,t+1} \\
r_{t+1} &= \delta_4 + \delta_5 f_{t+1} + \delta_6 u_{1,t} + \epsilon_{1,t+1} \\
f_{t+1} &= \delta_7 + \epsilon_{2,t+1} \\
\ln P_{t+1} &= \ln P_t + r_{t+1} \\
\ln F_{t+1} &= \ln F_t + f_{t+1}
\end{aligned}
$$

First, consider the case in which cointegration holds, i.e., we simulate the DGP by calibrating $\rho = 0.5$. The omission of the ECT from the traditional model does not generate any inconsistency in the estimation. However, when the model is used to predict future returns, the predicted value has zero variance while some predictability is present in the DGP. Such predictability is exploited by the inclusion of the Equilibrium Correction Term in the specification. Figure 1a illustrates the point by reporting—for one run of the Monte-Carlo simulation—the simulated returns along with the returns predicted respectively by the standard single-factor model (i.e., the CAPM) and the single-factor model with the inclusion of the ECT (i.e., the FECM). The predictability implied by the DGP, which can be relevant for risk measurement, risk management, and asset allocation, is exploited by the FECM and neglected by the CAPM. Interestingly, our approach delivers predictability through the ECT also when a CER specification for factors is assumed. As shown in the second panel of Figure 1a, price deviations from the long-run equilibrium implied by the risk driver are mean-reverting towards zero.

We consider now the case of no-cointegration, obtained by simulating the DGP by setting $\rho = 1$. In this case, the FECM specification collapses to the CAPM, and the ECT does not have power in forecasting returns. Figure 1b illustrates simulated returns, and fitted and predicted returns by the standard single-factor model. As shown in Figure 1b, the model fits well the relation between returns and factors, but price deviations from long-run equilibrium implied by the risk driver are non-stationary. Indeed, the CAPM is not able to track the value of the buy-and-hold portfolio.

FIGURE 1 ABOUT HERE

2.4   On Interpreting the ECT

Several comments are in order to help the interpretation of the Equilibrium Correction specification.

The Equilibrium Correction Term is asset-specific. It cannot be considered as a "factor" in itself since it has no power in pricing the cross-section of returns. Indeed, any cross-sectional asset pricing test (see, for example, Cochrane, 2005) would deliver an insignificant parameter attached to the ECT because its time-series average is zero by construction. This fact may explain why the ECT has gone unrecognized so far.

Every factor structure that generates cointegration between risk drivers and prices naturally leads to an Equilibrium Correction Term. The ECT would not be significant to explain returns only in the case of non-cointegration between risk drivers and prices. The existence of cointegration discards the possibility of an omitted risk driver, as risk drivers are non-stationary variable and the omission of a factor whose associated risk driver is relevant to determine the price dynamics of a given portfolio would prevent cointegration between portfolio prices and any set of risk drivers that omits the relevant one. Cointegration between prices and risk drivers rules out permanent deviations of prices from their projection on risk drivers. Non-stationarity could in fact be taken as an argument to reject any factor model.

Deviations in the relation between prices and risk drivers is temporary and reverts to zero in the long-run. Persistence in the idiosyncratic component of risk would prevent cointegration between prices and risk drivers associated to any set of factors chosen to explain return dynamics. As we shall see in the empirical section, there are some set of factors for which this is not the case observed in the data.

Price divergences could be related both to frictions that prevent rational traders from exerting the price pressure necessary to instantaneously eliminate such deviations and to irrational behaviour of agents. For example, idiosyncratic risk can be interpreted as an holding cost that makes temporary deviations of market prices from equilibrium prices possible (e.g., Pontiff, 1996; Pontiff, 2006).

On the other side, models of belief formation based on representativeness heuristic rationalize temporary discrepancies as a consequence of over-reaction to news caused by an exaggeration of probability of states that are objectively more likely (e.g., Bordalo et al., 2017). The predictability of returns conditional upon the ECT is in line with the evidence

reported in La Porta (1996) and Bordalo et al. (2017) according to which returns on stocks with the most optimistic analysts long-term earning growth forecasts are substantially lower than those for stocks with the most pessimistic forecasts. Accordingly, the ECT may capture agents' over-reaction to news in prices that are subsequently corrected in the dynamics of returns.

A reduced-form factor framework is of little use to identify the sources of deviations of prices from the long-term equilibrium implied by any given set of risk drivers. However, whenever the ECT is significant, the empirical model of asset prices and returns in equation (8) has a distinctive feature with respect to standard factor asset pricing specifications: the Equilibrium Correction Term is a predictive variable such that the predicted distribution of returns at time $t + 1$ is centered on the observed ECT.

3   Empirical Results

3.1   Data

In the empirical analysis, we focus on U.S. annual data–NYSE, AMEX, and Nasdaq stocks from the Center for Research in Security Prices (CRSP) and Compustat data required for sorting–made available from Kenneth R. French's Data Library for the sample 1963–2018.[14] Our empirical model is designed for low-frequency returns so that there is sufficient time in the data to respond to disequilibrium. Specifically, the time horizon at which asset returns are defined must be sufficiently long to allow for a reaction of returns at time $t + 1$ to disequilibrium in the relationship between risk drivers and prices at time t. At yearly frequencies, the exploitable predictability in the mean is usually paired with a constant volatility specification.

We consider annual returns for the 25 Portfolios formed on Size and Book-to-Market (5x5), the Fama-French five factors (EXC MKT, SMB, HML, RMW, CMA), and momentum (Mom). Table 1 reports summary statistics for the variables used. Panel A shows descriptive statistics for percentage log returns of the Fama-French five factors plus Momentum. The U.S. stock market return (i.e., the "market") in the period 1963–2018 has an average of 9.38%

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[14]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

per year and a standard deviation of 16.94, with an associated Sharpe ratio of about 0.55.[15] The Sharpe ratios related to the other factors are 0.19, 0.24, 0.30, 0.33, and 0.19 respectively for size (SMB), value (HML), profitability (RMW), investment (CMA), and momentum (Mom). The factor (different from the market) with the best risk-return profile at an annual frequency is CMA, followed by RMW. Panel B shows Pearson's pairwise correlations for log returns of the Fama-French five factors plus Momentum. Market has a positive correlation with size, and negative correlation with all the other factors. The highest correlation is 0.74 between value and investment. Panel C shows descriptive statistics for log percentage returns for the 25 Fama-French Portfolios formed on Size and Book-to-Market. In the sample period, Port 15 and Port 11 exhibit respectively the best and the worst performances.

In Figure 2 we show the yearly dynamics of (log) prices and risk drivers, that we compute as described in equations (6) and (5). Figure 2a shows the yearly dynamics for the six risk drivers associated with the six-factors of Fama and French. Figure 2b shows the yearly dynamics for the log prices for the 25 Fama-French Portfolios formed on Size and Book-to-Market. The time-series behavior of risk drivers and prices portends a possibility to model a common stochastic trend between the two variables for the 25 Fama-French portfolios.

FIGURE 2 ABOUT HERE

3.2   The Statistical Evidence on the Equilibrium Correction Term

Our statistical investigation on the relevance of the Equilibrium Correction Term starts from the identification of a potential long-run relationship between (log) prices and risk drivers. We estimate by ordinary least squares the following regression:[16]

$$
\begin{aligned}
\ln P_t^i &= \alpha_0^i + \alpha_1^i t + \beta_i' \ln \mathbf{F}_t + u_t^i \quad i = 1, ..., 25 \\
\mathbf{F}_t' &= \begin{bmatrix} F_t^{EXCMKT} & F_t^{SMB} & F_t^{HML} & F_t^{RMW} & F_t^{CMA} & F_t^{Mom} \end{bmatrix}
\end{aligned}
\tag{11}
$$

---

[15]The U.S. stock market return in excess of the risk free rate in the period 1963–2018 has an average of 4.7% and a standard deviation of 17.17, with an associated Sharpe ratio of about 0.28.

[16]The evidence of cointegration and the estimates of the parameters in the cointegrating relationships are substantially unaltered when either a dynamic model specifying simultaneously long-run and short-run dynamics or a dynamic least squares (DLS) technique (e.g., Stock and Watson, 1993) for the long-run specification are considered.

where $\ln P_t^i$ is the annual (log) price for portfolio i in the 25 Fama-French portfolio in year t, $\ln \mathbf{F}_t$ are the annual risk drivers, and t is a time trend.

Including a linear trend in equation (11) allows to recover the standard short-run specification when taking first-differences. Moreover, as discussed by Engle and Yoo (1987) and MacKinnon (2010), the inclusion of the trend is a simple way to avoid the dependence of the distribution of test statistics for residuals on α_1.

We test for cointegration following the methodology proposed by Engle and Granger (1987) and Davidson and Mackinnon (1993). First, we estimate regression (11) for each portfolio i in the 25 Fama-French portfolios using monthly observations, then we run the regression $\Delta \hat{u}_t^i = \gamma^i \hat{u}_{t-12}^i + \varepsilon_t^i$ on the 25 time-series for residuals.[17] If γ is statistically different from zero, we reject the null of non-stationarity for u_t, or, equivalently, we reject the null of non-cointegration among the log prices of the 25 Fama-French portfolios and the six-risk drivers. Figure 3 reports results for the Engle and Granger (1987) cointegration test for the 25 time-series of residuals $\hat{u}$ from equation (11). All the statistics are clearly beyond the simulation-based critical values suggested by MacKinnon (2010), that account for the fact that residuals are estimated rather than directly observed. The test provides a uniform rejection of the null hypothesis of non-cointegration.

FIGURE 3 ABOUT HERE

Given the evidence of cointegration, we proceed to specify a system of 25 equations for the annual returns of the 25 Fama-French portfolios that includes–in addition to the standard Fama-French six factors–the Equilibrium Correction Terms derived from the estimation of

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[17]In our sample, there is evidence of cointegration starting from five months, i.e., using $\hat{u}_{t-5}^i$ in the Engle-Granger regression. While we leave open the question of the economic determinants of the timing of cointegration, we decide to focus on the annual frequency for the empirical analysis.

the long-run cointegrating relationships:

$$
\begin{aligned}
r^i_{t+1} &= \alpha^i_1 + \beta'_i \mathbf{f}_{t+1} + \delta_i \hat{u}^i_t + v^i_{t+1} \\
\ln P^i_t &= \ln P^i_{t-1} + r^i_t \\
\ln \mathbf{F}_t &= \ln \mathbf{F}_{t-1} + \mathbf{f}_t \\
\ln P^i_t &= \alpha^i_0 + \alpha^i_1 t + \beta'_i \ln \mathbf{F}_t + u^i_t \\
\mathbf{F}'_t &= \begin{bmatrix} F^{EXCMKT}_t & F^{SMB}_t & F^{HML}_t & F^{RMW}_t & F^{CMA}_t & F^{Mom}_t \end{bmatrix}
\end{aligned}
\tag{12}
$$

where r^i_{t+1} is the excess return of test asset i at year $t+1$ and $\hat{u}^i_t$ is the ECT for test asset i observed at year t, estimated as in equation (11).

Figures 4 and 5 report all estimated coefficients with their associated 95% confidence intervals showing clear evidence for a consistent rejection of the non-significance of each Equilibrium Correction Term.[18]

FIGURES 4 and 5 ABOUT HERE

The null hypothesis of joint non-significance of all ECT factors is strongly rejected by the data. When the null that all the 25 coefficients on the ECTs is tested after estimating the system of our 25 equations for the test assets using a Wald test, distributed as a χ^2 with 25 degrees of freedom, a value of 293 is obtained with an associated probability well below one per cent. If this null is imposed and a traditional factor model is considered, then the null that all the 25 intercepts are zero is strongly rejected with a value for the relevant Wald test, again distributed as a χ^2 with 25 degrees of freedom, of 73.

When the traditional factor model is augmented allowing for the ECT, the equivalent of the test that the intercept is zero becomes a test that $\hat{\alpha}^i_1 + \hat{\delta}_i \hat{u}^i_t = 0$. We report in Figure 7 the time-series of the 25 terms with their associated 95% confidence intervals. Note that for each portfolio, zero is always included in the confidence interval. This is a strong supportive evidence for the ability of the ECT to improve the standard factor pricing model.

---

[18]In Appendix A, we illustrate coefficients with their associated confidence intervals in case of a standard Fama-French six-factor model, without ECT.

Further statistical evidence on the importance of the ECT is provided by analyzing the properties of the correlation matrix of the residuals of different factor models taking as a benchmark the correlation matrix of the returns on the 25 Fama-French portfolios. We estimate residuals by regressing test assets excess returns on factors, or factors and the ECT, depending on the specification considered. The heatmaps reported in Figure 8 show a progressive success of single-factor (CAPM), traditional six-factor (FF6), and the six-factor models augmented with the ECT (FECM) in modeling the strong correlation of returns for the 25 Fama-French portfolios.

Finally, we gain further insights on the improvement generated by the FECM specifications by evaluating the evidence on the multivariate normality of residuals–as previously determined–of the progressively more articulated specifications. Table 2 shows a test for multivariate normality of residuals that highlights the sizable relative contributions of the ECT in delivering (multivariate-)normally distributed residuals.

To sum up, the evidence in favor of the inclusion of the 25 ECTs in the system relating factors and returns is strong and uniform.

3.3 Long-Run Evidence and the Detection of Mis-Specification in Factor Models

Our co-integrated approach to modeling asset prices and returns is relevant to detect misspecification in factor models. In fact, even a gross misspecification in modeling buy-and-hold returns with risk drivers is not easily detected in a model relating returns to factors. When

we consider only the projection of returns on factors–as in the traditional approach to factor asset pricing models–, the unit root in the residuals from the projection of prices on risk drivers is removed.

Note that our framework cannot be introduced by considering a standard factor model and by showing that residuals are correlated. Indeed, the joint analysis of prices, returns, factors, and risk drivers, shows that a traditional factor model that is not supported by cointegration between risk drivers and (log) prices will show no-autocorrelation in its residuals.

To illustrate this phenomenon consider, as an example, a CAPM specification for the returns of the first Fama-French Portfolio–Small and Growth–along with the associated relationship between log prices and risk drivers:

$$
\begin{aligned}
\left(r_{t+1}^{11} - r_{t+1}^{rf}\right) &= \alpha_1 + \beta_1 \left(r_{t+1}^{m} - r_{t+1}^{rf}\right) + u_{2,t+1}^{11} \\
\ln P_{t+1}^{11} &= \alpha_0 + \alpha_1 t + \beta_1 \ln F_{t+1}^{m} + (1 - \beta_1) \ln F_{t+1}^{rf} + u_{1,t+1}^{11}
\end{aligned}
\tag{13}
$$

The ADF statistics in Table 3 is of particular interest. Table 3 illustrates for the single-factor model that the null hypothesis of a unit root in the residuals from the model for buy-and-hold portfolio (no-cointegration) cannot be rejected though the unit root in the residuals is removed when the standard CAPM is obtained by differencing. As a consequence, misspecification is apparent in the model in levels but much less evident when the standard CAPM specification for asset returns is considered. Figure 6, which shows the time-series of actual and fitted values for the two specifications, highlights this point.

TABLE 3 and FIGURE 6 ABOUT HERE

3.4 ECT and Risk Measurement

Explicitly modeling the relationship between risk drivers and asset prices has important consequences for the predictive distribution of returns, which is helpful both for risk measurement and asset allocation. This happens for two reasons. Firstly, the ECT is a predictive variable that is observed at time t and is related to the distribution of returns at time $t+1$.

Secondly, the equilibrium relationship(s) among risk drivers have predictive power for factors at time $t + 1$ in a VECM specification.

To show the relevance of the ECT specification for risk measurement, we consider the properties of a traditional six-factor and of the FECM specifications for the returns of the Fama-French Portfolio Small and Growth (Port 11). The traditional factor model takes the following specification in which returns are projected on factors and the unconditional distribution of factors is used for simulations:

$$
\begin{aligned}
r_{t+1}^{11} &= \alpha_1^{11} + \beta_{11}' \mathbf{f}_{t+1} + v_{t+1}^{11} \\
\mathbf{f}_{t+1} &= \mu_0 + \epsilon_{t+1} \\
\epsilon_{t+1} &\sim \mathcal{D}(\mathbf{0}, \, \boldsymbol{\Sigma}) \\
\mathbf{f}_t' &= \begin{bmatrix} f_t^{EXCMKT} & f_t^{SMB} & f_t^{HML} & f_t^{RMW} & f_t^{CMA} & f_t^{Mom} \end{bmatrix}
\end{aligned}
\tag{14}
$$

Instead, the FECM approach includes both the predictive Equilibrium Correction Term in the equation for returns and the levels of the risk drivers in the VECM specification for factors:

$$
\begin{aligned}
r_{t+1}^{11} &= \alpha_1^{11} + \beta_{11}' \mathbf{f}_{t+1} + \delta_{11} \hat{u}_t^{11} + v_{t+1}^{11} \\
\mathbf{f}_{t+1} &= \mu_0 + \boldsymbol{\Pi} \ln \mathbf{F}_t + \epsilon_{t+1} \\
\ln P_t^{11} &= \ln P_{t-1}^{11} + r_t^{11} \\
\ln \mathbf{F}_t &= \ln \mathbf{F}_{t-1} + \mathbf{f}_t \\
\ln P_t^{11} &= \alpha_0^{11} + \alpha_1^{11} t + \beta_{11}' \ln \mathbf{F}_t + u_t^{11} \\
\epsilon_{t+1} &\sim \mathcal{D}(\mathbf{0}, \, \boldsymbol{\Sigma}) \\
\mathbf{f}_t' &= \begin{bmatrix} f_t^{EXCMKT} & f_t^{SMB} & f_t^{HML} & f_t^{RMW} & f_t^{CMA} & f_t^{Mom} \end{bmatrix}
\end{aligned}
\tag{15}
$$

The results of the estimation of the FECM specification and of the VECM for factors are reported in Tables 4 and 5.[19]

TABLES 4 and 5 ABOUT HERE

---

[19]In Appendix B, we report results of estimating Port 11 in case of a standard Fama-French six-factor model, without ECT.

Table 4 shows estimation results for both the long-run cointegrating equation and the short-run factor error correction model for Portfolio Small and Growth. The first column reports estimated coefficient for the long-run specification (i.e., variables are cumulative log returns). (Log) prices of Port 11 are positively related to EXC MKT, SMB, and Mom risk drivers, while are negatively associated to HML, RMW, and CMA. The trend coefficient is statistically significant and negative. The constant is statistically indifferent from zero. The second column reports estimated coefficient for the short-run specification (i.e., variables are log returns). (Log) returns of Port 11 are positively related to EXC MKT, SMB, and Mom factors, while are negatively associated to HML, RMW, and CMA. The ECT coefficient is economically and statistically significant, and negative. It means that positive deviation of (log) prices for Port 11 from their long-term relation with the related risk drivers in this period implies a lower expected return for the next period, with an order of magnitude of 0.724 per unit of deviation. Interestingly, the specification of a full model for factors allows to estimate the FECM by Generalized Method of Moments (GMM), using as instruments for the returns the lagged levels of the risk drivers. As the third column of Table 4 illustrates, the GMM estimation delivers similar results compared to the OLS estimation, confirming the significance of the ECT.

Further supporting evidence for the importance of the ECT for understanding the time-series dynamics of returns is provided by the analysis of the semi-partial R^2. The semi-partial R^2 is defined as the difference between the overall regression R^2 and the R^2 of the regression that includes all regressors except the ith-regressor for which the semi-partial R^2 is computed. We show the results for the semi-partial R^2 associated to each factor in the last column of Table 4. The ECT is the fourth most relevant factor, explaining by itself about 2% of the total variance of returns.

Table 5 illustrates the relevance of risk drivers for predicting one-year ahead factors. For each factor–market, size, value, profitability, investment, and momentum–, at least two risk drivers are economically and statistically different from zero. The coefficients attached to the intercepts obtained by regressing the Fama-French six factors on the associated lagged risk drivers are never statistically different from zero. Interestingly, risk drivers predicts on average more than 22% of the variance associated with factors in the sample.

Finally, we use the standard and the FECM specifications to predict the distributions for one-year ahead returns for 2008 and 2009 for Portfolio Small and Growth. The two models

are fitted on the sample up to 2007, then the out-of-sample predicted return distributions are obtained by bootstrapping the correlated residuals ϵ_{t+1} and the idiosyncratic error v_{t+1}^{11}.[20]

The predictive distributions resulting from the traditional factor model are reported in Figure 9 which highlights the immutability of the distributions that do not change despite the crash of 2008. Differently, the predictive distributions from the FECM specification, reported in Figure 10, shows an evident shift to the right after the crisis. This is due to the fact that the crash of 2008 brought the prices from above the long-run equilibrium to below the long-run equilibrium, causing a shift in the mean of the predictive distribution. Importantly, the one-year ahead 10% VaR from the FECM goes from -0.367 for 2008 to -0.067 for 2009, as a consequence of the crash, while the VaR from the standard model–reflecting the unconditional distribution of assets returns and factors–remains mostly unchanged.

FIGURES 9 and 10 ABOUT HERE

3.5   The Information in ECT: Systematic or Idiosyncratic Component?

A predictive variable for returns could predict both the systematic or the idiosyncratic components of returns. Which information does the ECT convey? To address this question, we follow a recent work by Engleberg et al. (2019).

Let's consider the six-factor Fama-French model:

$$
\begin{aligned}
r_{t+1}^i &= \alpha_i + \beta_i' \mathbf{f}_{t+1} + v_{t+1}^i \\
\mathbf{f}_t' &= \begin{bmatrix} f_t^{EXCMKT} & f_t^{SMB} & f_t^{HML} & f_t^{RMW} & f_t^{CMA} & f_t^{Mom} \end{bmatrix}
\end{aligned}
\tag{16}
$$

where r_{t+1}^i is the return of asset i at time $t + 1$. A predictor is systematic if it forecasts the systematic component of the model (16), while it is an idiosyncratic predictor if it forecasts the idiosyncratic portion.

---

[20]A remarkable stability of the parameters emerges for the FECM specification estimated on the full sample and estimated only up to 2007.

Specifically, we define a systematic return predictor the variable X^{syst} if γ_1 is statistically different from zero in the regression:

$$f^i_{t+1} \;\; = \;\; \gamma^i_0 + \gamma^i_1 X^{syst}_t + \epsilon^i_{t+1} \tag{17}$$

where f^i_{t+1} is the factor-portfolio i at time $t+1$.

We consider as potential systematic predictor the average ECT across the 25 Portfolios for each point in time t ($\overline{ECT}_t$). Table 6 reports the ordinary least squares estimate for γ^i_1 and the R^2 for regression (17). $\overline{ECT}$ forecasts significantly only the market, with a t-stat slightly above 2, and the size factor at 10% level of significance, with an associated R^2 respectively of 6.89% and 8.87%. This evidence resembles Engleberg et al. (2019) that find little evidence that cross-sectional predictors are good time-series predictors out-of-sample.

Then, we define an idiosyncratic predictor the variable X^{idio} if π_1 is statistically different from zero in the regression:

$$v^i_{t+1} \;\; = \;\; \pi^i_0 + \pi^i_1 X^{idio}_t + \epsilon^i_{t+1} \tag{18}$$

where v^i_{t+1} is the residual for portfolio i at time $t+1$ from the equation (16).

We consider as potential idiosyncratic predictor the asset-specific ECT. Table 7 reports the ordinary least squares estimate for π^i_1 and the R^2 for regression (18). π_1 is always significant at 1% or at 5% level of significance for each portfolios (except for Port 42); the average R^2 is about 15.4%.

In sum, our results provide evidence that the ECT is mostly related to the idiosyncratic component rather than to the systematic portion of returns. The ECT contains asset-specific information that do not forecast aggregate market returns.

TABLES 6 and 7 ABOUT HERE

4   Robustness

In this section we address four issues. First, we consider a different set of factors to demonstrate that the evidence of cointegration is not specific to the empirical model for returns that

25

one assumes.[21] Second, we ask whether the ECT is related to other return predictors. Then, we test for the significance of the new Equilibrium Correction Term when considering different specifications. Finally, we implement a Principal Component Analysis on time-varying alphas.

4.1 The q-Factor Model and the ECT

We are interested in understanding if the evidence of cointegration and the related misspecification of the standard factor-model for returns that we document when using the Fama and French (2018) six-factor model is specific to the empirical model that we choose.

In two recent papers, Hou et al. (2015) and Hou et al. (2018) derive and test an alternative factor model to summarize the cross-section of mean stock returns. The empirical model–that the authors label as the "q-factor" model–has its theoretical foundation in the neoclassical q-theory of investment (see Zhang (2017)). The q-factor model consists of four factors: the market excess return (MKT), a size factor (ME), an investment factor (IA), and a profitability factor (ROE).[22] Empirically, Hou et al. (2018) find that the q-factor model subsumes the Fama and French (2018) six-factor model in spanning tests.

We replicate the results on the statistical evidence on the ECT in case of the q-factor model. In particular, first we test for cointegration, then we estimate the long- and the short-run specifications in equation (11) and (12), where $\mathbf{f}$ is a vector containing the four q-factors and $\mathbf{F}$ is a vector containing the related risk drivers.

Figure 11 shows results for the Engle and Granger (1987) cointegration test for specification (11) for the 25 Fama-French Portfolios formed on Size and Book-to-Market where the risk drivers are constructed from the Hou et al. (2015) q-factors. The null of non-cointegration is uniformly rejected. Interestingly, the existence of a stationary linear combination of prices and risk drivers indicates that the q-factor model can be a valid factor model to explain the long- and short-run dynamics of the test assets.

Given the evidence of cointegration, the Equilibrium Correction Term should be included when regressing returns on factors. Figure 12 reports the estimated coefficients obtained by

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[21]We also consider a different set of test assets, e.g., the 30 Industry Portfolios by French's Data Library or single stocks from Dow Jones. We report this analysis is a companion paper in which we focus on the potential of our approach for asset allocation.

[22]The q-factors are available at `http://global-q.org/factors.html`.

regressing the 25 Fama-French portfolios excess returns on the Hou et al. (2015) q-factors and the ECT as in equation (12) with respective confidence intervals at 5% level of significance. We find that only Portfolio (2x3) has an insignificant coefficient associated with the ECT, while all the other test assets have a significant ECT coefficient at 5% level of significance.

Overall, these results suggest that the statistical evidence on the ECT is not specific to the empirical model that one decides to use. Every time that cointegration holds, the empirical model chosen is a potentially valid model for describing asset dynamics, and the ECT should be included in the standard factor regression as it conveys additional information not embedded in the factor structure.

<center>*FIGURES 11 and 12 ABOUT HERE*</center>

4.2   The ECT and Other Return Predictors

How is the ECT related to other return predictors? To address this question, we first need to construct some predictors that are comparable with the ECTs for the 25 Portfolios. We take into consideration two predictors. The first predictor that we consider is the log dividend-price ratio for the 25 Portfolios. The dividend-price ratio is one of the most used variable in predictive return regressions (e.g., Cochrane, 2005; Campbell, 2017).[23] We follow Cochrane (2007) to construct the dividend-price ratio for portfolio i at time t as $\frac{D_{t+1}^i}{P_{t+1}^i} = \frac{R_{t+1}^i}{R_{ex,t+1}^i} - 1$ where $R_{ex,t+1}^i$ are returns without dividends for portfolio i at time $t+1$. The log dividend-price ratio for portfolio i at time t is dp_t^i.

As the second predictor, we consider a latent process, following recent works that use latent processes in predictive regressions (e.g., Van Binsbergen and Koijen, 2010; Hendershott and Menkveld, 2014). To construct our latent predictor, we assume the following specification for time-varying expected returns (e.g., Campbell, 2017):

$$
\begin{aligned}
r_{t+1}^i &= \bar{r} + g_t^i + u_{t+1}^i \\
g_{t+1}^i &= \phi g_t^i + \varepsilon_{t+1}^i
\end{aligned}
\tag{19}
$$

---

[23]By using the Campbell-Shiller log-linear present value model, it is easy to show that the dividend-price ratio is a good proxy for x_t^i if dividend growth is unpredictable.

where r_{t+1}^i are returns of asset i at time $t+1$, $\bar{r}$ is the unconditional mean, and g_t^i is a zero mean AR(1) process that can predict returns of asset i. Then, we estimate g_t^i in (19) by using the Kalman filter algorithm, where the first and the second equations are respectively the observation and the transition equation. We calibrate $\phi = .5$, but results are independent on this choice in the range $.1 \leq \phi \leq 1$.

Table 8 reports the ordinary least squares estimate for β_i and the R^2 from the regression $x_t^i = \alpha_i + \beta_i ECT_t^i + \epsilon_t^i$, where x_t^i is one of the two predictors described above for portfolio i in the 25 Portfolios at time t and ECT_t^i is the ECT for portfolio i at time t. The second column in 8 shows the betas for $x_t^i = dp_t^i$, and the fourth column shows the betas for $x_t^i = g_t^i$. Except for two cases, the ECT is uncorrelated with the other two predictors.

The evidence here reported suggests that the ECT conveys new additional information about the time-series dynamics of assets that are not included in the two alternative predictors considered.

<center>*TABLE 8 ABOUT HERE*</center>

4.3 Significance of ECT

According to standard asset pricing theory, two assets exposed to the same sort of systematic risk should have the same expected return. When it is not the case, the difference between the two expected returns is called "anomaly". The proliferation of anomalies is evident: Harvey et al. (2016) document more than 300 cross-sectional stock anomalies proposed as potential factors in asset-pricing models. Furthermore, Asness et al. (2013) report that value and momentum anomalies have pervasive features across different asset classes.

In a recent paper, Stambaugh and Yuan (2016) propose two new measures of mispricing constructed by averaging on stocks' anomaly rankings. The main intuition motivating their approach is that anomalies partially reflect common mispricing components across stocks. The two mispricing factors considered are management (MGMT) and performance (PERF), resulting from clustering anomalies potentially related to firms' management and performance.[24]

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[24]Monthly time series for the two factors are available on Stambaugh's website.

<center>28</center>

We want to test whether our the ECT loses its significance when systematic mispricing variables are included in the specification. We add the two mispricing factors MGMT and PERF to regression (12), and look at the significance of the ECT coefficients across the 25 Fama-French portfolios. Figure 13 shows the results of our test.[25] Each ECT coefficient is statistically significant at 5% level of significance, providing supportive evidence that the Equilibrium Correction Term contributes in explaining asset dynamics also when common mispricing factors are added to the specification.

Most interestingly, results are not surprising. In fact, the ECT is asset-specific, as it measures the deviation of any asset from his long-run equilibrium relation with factor-risk drivers. This finding is also consistent with Daniel and Titman (1997) who points out that when expected returns reflect both compensation for systematic risks and mispricing, some of the mispricing is asset-specific. The ECT conveys such information.

FIGURE 13 ABOUT HERE

Ang et al. (2006) show that aggregate volatility is a priced factor in the cross-section of stock returns. In particular, aggregate volatility represents a systematic factor because leads to changes in the investment opportunity set of the marginal investor. Therefore, stocks that are more exposed to innovations in volatility earns a premium for risk. We include an idiosyncratic volatility (IVOL) factor in the short-run regression (12) to address the issue whether the ECT loses his significance when an asset-specific factor is included.

We compute IVOL for the 25 Fama-French portfolios as the standard deviation of the residual ϵ_{t+1}^i in the regression $r_{t+1}^i = \alpha^i + \beta_i' \mathbf{f}_{t+1} + \epsilon_{t+1}^i$, where r^i are excess returns of the 25 Fama-French portfolios and $\mathbf{f}$ are the six Fama-French factors. Then, we sort the 25 portfolios at time t based on IVOL at time $t-1$. Excess returns for the zero-investment strategy long on the portfolio associated with the lowest IVOL and short on the portfolio associated with the highest IVOL is the IVOL factor.

Figure 14 illustrates the estimated coefficients for IVOL factor and ECT obtained by running regression (12) when we add also the IVOL factor. Notice that the ECT does not lose any significance, i.e., the term remains always statistically significant at 5% level

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[25]When we consider a factor model with only market and size combined with the two mispricing factors–as in the original Stambaugh and Yuan (2016) paper–plus the ECT, results are the same.

of significance. The ECT helps in explaining asset dynamics even when systematic risk associated with asset-specific exposure to innovations in aggregate volatility is included.

Finally, we consider a long-term reversal factor (LRet). LTRev is the average of the returns on two (Big and Small) low prior return portfolios minus the average of the returns on two high prior return portfolios. Big means a firm is above the median market capitalization on the NYSE at the end of the previous month; Small firms are below the median NYSE market capitalization. Prior return is measured from month -60 to -13.[26]

Figure 15 shows the estimated coefficients for LTRev factor and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. Also in this case, the ECT is always statistically significant at 5% level of significance. A long-term reversal factor does not absorb the significance of ECT.

FIGURE 14 ABOUT HERE

4.4   Principal Component Analysis

Let's consider equation (12). We can rewrite the equation as:

$$r_{t+1}^i \quad = \quad \alpha_{t+1}^i + \beta_i' \mathbf{f}_{t+1} + v_{t+1}^i$$

where $\alpha_{t+1}^i = \alpha_1^i + \delta_i \hat{u}_t^i$ is an asset-specific time-varying intercept.

We analyze the principal components across the time-varying alphas. The main goal is to better understand how many relevant orthogonal variables contribute to the total variation of the set of observed conditional alphas.

The analysis is interesting because the ECT is asset-specific, thus the information embedded in the time-varying alphas are mostly specific. It means that we expect a large number of relevant components resulting from the Principal Component Analysis (PCA).

We perform the PCA on the sample composed by the estimated time-varying alphas for each portfolio in the 25 Fama-French portfolios for the period 1964–2018. We find that the

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[26]LRet is from Kenneth R. French's Data Library. For further details, see `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_lt_rev_factor.html`.

first principal component explains about 26% of total variation and at least 11 principal components are needed to explain about 95% of total variation of the set of conditional alphas. This result supports our argument that the ECT conveys mostly information specific to each asset rather than common mispricing elements.

5   Conclusions

This paper has proposed a novel co-integrated approach to modeling asset prices, returns, factors, and risk drivers. We find that focusing on both prices and returns rather than only on returns naturally leads to the identification of an "Equilibrium Correction Term" that conveys new relevant information about the time-series dynamics of assets. The ECT significantly forecasts asset returns, mainly predicting their idiosyncratic component variation. In addition, we argue that the existence of a long-run relationship between prices and associated drivers of risk can be used to validate the ability of any factor models to explain short- and long-run asset performances.

Our empirical analysis is based on modeling returns and prices of the 25 Fama-French portfolios to capture the additional effects of the Equilibrium Correction Term above the Fama-French five-factor model augmented with a momentum factor along several dimensions (e.g., Fama and French, 2018). In particular, we focus on the importance of the ECT for determining the predictive distribution of returns, and its consequences for risk measurement. Our framework is invariant to the choice of factors and test assets. Therefore, our model can be extended by asset pricers and investment professionals to any set of assets and factors.

We have given less emphasis to the importance of the long-run relations among risk drivers to predict factors. This is an issue left for further research that could be particularly interesting in models based on the simultaneous utilization of local and global factors to model asset returns (e.g., Griffin, 2002). Cointegration among local and global risk drivers has an obvious potential for explaining the dynamics of local factors as determined by the response to an Equilibrium Correction Term in which global risk drivers determine the local ones.

References

Andrews, D. (1991). Heteroskedasticity and autocorrelation consistent covariant matrix estimation. *Econometrica 59*(3), 817–858.

Ang, A. (2014). *Asset management: A systematic approach to factor investing.* Oxford University Press.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The Journal of Finance 61*(1), 259–299.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and momentum everywhere. *The Journal of Finance 68*(3), 929–985.

Avramov, D., T. Chordia, G. Jostova, and A. Philipov (2019). Bonds, stocks, and sources of mispricing. Working Paper.

Banerjee, A., M. Marcellino, and I. Masten (2017). Structural fecm: Cointegration in large-scale structural favar models. *Journal of Applied Econometrics 32*(6), 1069–1086.

Bansal, R., R. Dittmar, and D. Kiku (2007). Cointegration and consumption risks in asset returns. *The Review of Financial Studies 22*(3), 1343–1375.

Bansal, R. and D. Kiku (2011). Cointegration and long-run asset allocation. *Journal of Business & Economic Statistics 29*(1), 161–173.

Barigozzi, M., M. Lippi, and M. Luciani (2015). Dynamic factor models, cointegration, and error correction mechanisms. *arXiv preprint arXiv:1510.02399*.

Barillas, F. and J. Shanken (2017). Which alpha? *The Review of Financial Studies 30*(4), 1316–1338.

Bordalo, P., N. Gennaioli, R. L. Porta, and A. Shleifer (2017). Diagnostic expectations and stock returns. Technical report, National Bureau of Economic Research.

Bossaerts, P. and R. C. Green (1989). A general equilibrium model of changing risk premia: Theory and tests. *The Review of Financial Studies 2*(4), 467–493.

Brennan, M. J. and A. W. Wang (2010). The mispricing return premium. *The Review of Financial Studies 23*(9), 3437–3468.

Campbell, J. Y. (2017). *Financial decisions and markets: a course in asset pricing.* Princeton University Press.

Chernov, M., L. A. Lochstoer, and S. R. Lundeby (2018). Conditional dynamics and the multi-horizon risk-return trade-off. NBER Working Paper No. 25361.

Cho, T. and C. Polk (2020). Asset pricing with price levels. Working Paper.

Cochrane, J. H. (2005). *Asset pricing: Revised edition.* Princeton university press.

Cochrane, J. H. (2007). The dog that did not bark: A defense of return predictability. *The Review of Financial Studies 21*(4), 1533–1575.

Cochrane, J. H. (2011). Presidential address: Discount rates. *The Journal of Finance 66*(4), 1047–1108.

Daniel, K., L. Mota, S. Rottke, and T. Santos (2017). The cross-section of risk and return. NBER Working Paper No. 24164.

Daniel, K. and S. Titman (1997). Evidence on the characteristics of cross sectional variation in stock returns. *The Journal of Finance 52*(1), 1–33.

Davidson, R. and J. Mackinnon (1993). *Estimation and inference in econometrics.* Oxford University Press.

Ehsani, S. and J. Linnainmaa (2019). Factor momentum and momentum factor. Working Paper.

Engle, R. F. and C. W. Granger (1987). Co-integration and error correction: Representation, estimation, and testing. *Econometrica 55*(2), 251–276.

Engle, R. F. and B. S. Yoo (1987). Forecasting and testing in co-integrated systems. *Journal of Econometrics 35*(1), 143–159.

Engleberg, J., D. R. Mclean, J. Pontiff, and M. C. Ringgenberg (2019). Are cross-sectional predictors good market-level predictors? Working Paper.

Fama, E. F. and K. R. French (1988). Permanent and temporary components of stock prices. *Journal of Political Economy 96*(2), 246–273.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Fama, E. F. and K. R. French (2018). Choosing factors. *Journal of Financial Economics 128*(2), 234–252.

Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–636.

Gennaioli, N. and A. Shleifer (2018). *A crisis of beliefs: Investor psychology and financial fragility.* Princeton University Press.

Gennaioli, N., A. Shleifer, and R. Vishny (2015). Neglected risks: The psychology of financial crises. *American Economic Review 105*(5), 310–14.

Gibbons, M. R., S. A. Ross, and J. Shanken (1989). A test of the efficiency of a given portfolio. *Econometrica 57*(5), 1121–1152.

Griffin, J. M. (2002). Are the fama and french factors global or country specific? *The Review of Financial Studies 15*(3), 783–803.

Hansen, L. P., J. C. Heaton, and N. Li (2008). Consumption strikes back? measuring long-run risk. *Journal of Political Economy 116*(2), 260–302.

Harvey, C. R., Y. Liu, and H. Zhu (2016). . . . and the cross-section of expected returns. *The Review of Financial Studies 29*(1), 5–68.

Hendershott, T. and A. J. Menkveld (2014). Price pressures. *Journal of Financial Economics 114*(3), 405–423.

Hendershott, T., A. J. Menkveld, R. Praz, and M. S. Seasholes (2018). Asset price dynamics with limited attention. Working Paper.

Hendry, D. F. (1986). Econometric modelling with cointegrated variables: an overview. *Oxford Bulletin of Economics and Statistics 48*(3), 201–212.

Hou, K., H. Mo, C. Xue, and L. Zhang (2018). Which factors? *Review of Finance 23*(1), 1–35.

Hou, K., C. Xue, and L. Zhang (2015). Digesting anomalies: An investment approach. *The Review of Financial Studies 28*(3), 650–705.

Johansen, S. (1995). *Likelihood-based inference in cointegrated vector autoregressive models.* Oxford University Press on Demand.

Kolanovic, M. and Z. Wei (2013). Systematic strategies across asset classes: risk factor approach to investing and portfolio management. *JP Morgan Global Quantitative and Derivate Strategy*, 1–205.

Kozak, S., S. Nagel, and S. Santosh (2018). Intepreting factor models. *The Journal of Finance 73*(3), 1183–2023.

La Porta, R. (1996). Expectations and the cross-section of stock returns. *The Journal of Finance 51*(5), 1715–1742.

Lettau, M. and S. Ludvigson (2001). Consumption, aggregate wealth, and expected stock returns. *The Journal of Finance 56*(3), 815–849.

Lewis, K. F., F. A. Longstaff, and L. Petrasek (2017). Asset mispricing. NBER Working Paper No. 23231.

Liu, J. and A. Timmermann (2013). Optimal convergence trade strategies. *The Review of Financial Studies 26*(4), 1048–1086.

Lou, D. (2012). A flow-based explanation for return predictability. *The Review of Financial Studies 25*(12), 3457–3489.

MacKinnon, J. G. (2010). Critical values for cointegration tests. Technical report, Queen's Economics Department.

Meucci, A. (2011). The prayer ten-step checklist for advanced risk and portfolio management.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Pesaran, M. H. and Y. Shin (1998). An autoregressive distributed-lag modelling approach to cointegration analysis. *Econometric Society Monographs 31*, 371–413.

Pesaran, M. H. and R. P. Smith (2019). The role of factor strength and pricing errors for estimation and inference in asset pricing models. Working Paper.

Pontiff, J. (1996). Costly arbitrage: Evidence from closed-end funds. *The Quarterly Journal of Economics 111*(4), 1135–1151.

Pontiff, J. (2006). Costly arbitrage and the myth of idiosyncratic risk. *Journal of Accounting and Economics 42*(1-2), 35–52.

Poterba, J. M. and L. H. Summers (1988). Mean reversion in stock prices: Evidence and implications. *Journal of Financial Economics 22*(1), 27–59.

Stambaugh, R. F. and Y. Yuan (2016). Mispricing factors. *The Review of Financial Studies 30*(4), 1270–1315.

Stock, J. H. and M. W. Watson (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica*, 783–820.

Summers, L. H. (1986). Does the stock market rationally reflect fundamental values? *The Journal of Finance 41*(3), 591–601.

Székely, G. J. and M. L. Rizzo (2005). A new test for multivariate normality. *Journal of Multivariate Analysis 93* (1), 58–80.

Van Binsbergen, J. H. and R. S. Koijen (2010). Predictive regressions: A present-value approach. *The Journal of Finance 65* (4), 1439–1471.

Van Binsbergen, J. H. and C. C. Opp (2019). Real anomalies. *The Journal of Finance 74* (4), 1659–1706.

Zhang, L. (2017). The investment CAPM. *European Financial Management 23* (4), 545–603.

Tables and Figures

Table 1: Summary statistics

This table reports summary statistics of the variables used in the empirical analysis. Panel A shows descriptive statistics for log returns of the Fama-French five factors plus Momentum. Panel B shows Pearson pairwise correlation for log returns of the Fama-French five factors plus Momentum. Panel C shows descriptive statistics for log returns of the 25 Fama-French Portfolios formed on Size and Book-to-Market. Returns are in percentage. The sample period is 1963 to 2018.

Panel A: Descriptive Statistics

Factor	N	Mean	St. Dev.	Min	Median	Max
MKT	54	9.38	16.94	-45.79	13.30	32.38
SMB	54	2.60	13.36	-33.76	3.68	40.85
HML	54	3.32	13.82	-38.13	5.15	33.43
RMW	54	2.89	9.58	-32.02	2.61	23.44
CMA	54	3.10	9.51	-16.30	3.28	28.26
Mom	54	5.32	27.78	-172.77	8.78	31.58

Panel B: Pairwise Correlations

	MKT	SMB	HML	RMW	CMA	Mom
MKT	1	0.208	-0.193	-0.245	-0.318	-0.154
SMB	0.208	1	0.136	-0.153	0.009	-0.130
HML	-0.193	0.136	1	0.099	0.744	-0.068
RMW	-0.245	-0.153	0.099	1	-0.105	0.042
CMA	-0.318	0.009	0.744	-0.105	1	-0.073
Mom	-0.154	-0.130	-0.068	0.042	-0.073	1

<div align="center">**Panel C**: Descriptive Statistics</div>

Portfolio	N	Mean	St. Dev.	Min	Median	Max
Log Ret Port 11	54	-0.83	33.24	-78.86	0.75	82.31
Log Ret Port 12	54	6.39	27.07	-61.08	10.86	68.20
Log Ret Port 13	54	6.94	24.37	-54.26	14.44	61.36
Log Ret Port 14	54	9.88	22.43	-44.19	12.71	61.83
Log Ret Port 15	54	10.42	25.32	-53.66	15.57	62.75
Log Ret Port 21	54	2.86	27.09	-75.72	3.64	54.78
Log Ret Port 22	54	6.89	21.62	-62.14	10.56	46.44
Log Ret Port 23	54	8.25	20.28	-44.81	10.85	61.36
Log Ret Port 24	54	8.95	19.89	-39.86	11.14	50.44
Log Ret Port 25	54	8.96	21.36	-42.35	11.35	48.23
Log Ret Port 31	54	3.39	24.25	-65.50	5.53	40.65
Log Ret Port 32	54	7.39	19.93	-48.29	10.22	40.56
Log Ret Port 33	54	7.06	17.72	-36.97	12.44	34.41
Log Ret Port 34	54	8.56	19.72	-36.27	9.22	41.32
Log Ret Port 35	54	9.76	20.13	-32.19	10.20	47.04
Log Ret Port 41	54	5.32	21.42	-54.34	5.25	47.39
Log Ret Port 42	54	5.43	18.61	-49.78	10.25	34.95
Log Ret Port 43	54	6.47	19.40	-70.02	9.86	36.94
Log Ret Port 44	54	8.04	18.54	-45.60	11.75	46.44
Log Ret Port 45	54	7.47	21.58	-65.02	11.41	42.42
Log Ret Port 51	54	4.37	18.41	-44.16	5.77	34.47
Log Ret Port 52	54	4.98	15.59	-41.35	8.79	32.60
Log Ret Port 53	54	5.14	15.40	-44.39	6.21	29.12
Log Ret Port 54	54	4.16	18.65	-81.92	9.84	33.78
Log Ret Port 55	54	5.75	21.03	-65.86	11.36	33.75

Table 2: E-Test for Multivariate Normality (Székely and Rizzo, 2005)

This table reports the E-test for multivariate normality proposed by Székely and Rizzo (2005) on CAPM, FF6, and FECM residuals. The null hypothesis is multivariate normality.

CAPM	Test Statistic	2.684
	p-value	0
FF6	Test Statistic	2.618
	p-value	0.08
FECM	Test Statistic	2.611
	p-value	0.22

Table 3: Augmented Dickey-Fuller Tests

This table reports the augmented Dickey-Fuller (ADF) tests for the single-factor specifications in equation (13) where P^{11} and r^{11} are respectively prices and returns for Portfolio Small and Growth in the 25 Fama-French portfolios. The null hypothesis is non-stationarity.

Single-Risk Driver (EXC MKT) Model	Test Statistic	-2.783
	p-value	0.259
Single-Factor (CAPM) Model	Test Statistic	-3.873
	p-value	0.022

Table 4: FECM for Portfolio 11

This table reports the estimated coefficients for the specifications in equation (15) where P^{11} and r^{11} are respectively prices and returns for Portfolio Small and Growth in the 25 Fama-French portfolios. The second column reports results for the long-run specification (log prices and log risk drivers). The third column reports least squares estimates for the short-run specification (log returns and log factors). The fourth column reports GMM estimates for the short-run specification (log returns and log factors).The last column reports the semi-partial R^2 for each regressor in the third column. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

	Long-Run	FECM (OLS)	FECM (GMM)	FECM SpR^2
Trend	−0.059***			
	(0.005)			
EXC MKT	1.050***	1.179***	1.263***	0.262
	(0.038)	(0.041)	(0.245)	
SMB	1.370***	1.404***	1.673***	0.259
	(0.050)	(0.087)	(0.207)	
HML	−0.353***	−0.634***	−1.004***	0.025
	(0.092)	(0.071)	(0.363)	
RMW	−0.544***	−0.273***	−0.038	0.005
	(0.060)	(0.080)	(0.315)	
CMA	−0.396***	−0.049	0.582	0
	(0.120)	(0.092)	(0.721)	
Mom	0.094***	0.070**	0.197*	0.003
	(0.024)	(0.012)	(0.113)	
ECT		−0.724***	−0.959***	0.018
		(0.140)	(0.286)	
Constant	0.008	−0.075***	−0.106***	
	(0.028)	(0.007)	(0.036)	
Observations	55	54	54	
Adjusted R^2	0.980	0.967	0.940	

Table 5: VECM for Factors as Functions of Risk Drivers

This table reports the estimated coefficients for Portfolio Small and Growth in the 25 Fama-French portfolios for the VECM specification $\mathbf{f}_{t+1} = \mu_0 + \mathbf{\Pi} \ln \mathbf{F}_t + \epsilon_{t+1}$, where $\mathbf{f}$ are the five Fama-French factors plus momentum and $\mathbf{F}$ are the associated risk drivers. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

	Factor					
	EXC MKT	SMB	HML	RMW	CMA	Mom
EXC MKT RD(-1)	−0.283***	−0.222**	−0.077	0.211***	−0.097	0.249**
	(0.084)	(0.098)	(0.102)	(0.067)	(0.067)	(0.095)
SMB RD(-1)	0.011	−0.250***	0.116	0.040	0.015	−0.023
	(0.093)	(0.089)	(0.093)	(0.061)	(0.061)	(0.086)
HML RD(-1)	0.196	−0.202	−0.542**	−0.222	−0.032	0.444**
	(0.209)	(0.197)	(0.204)	(0.135)	(0.134)	(0.190)
RMW RD(-1)	0.435***	−0.183	−0.033	−0.584***	0.037	−0.193
	(0.147)	(0.197)	(0.204)	(0.135)	(0.134)	(0.189)
CMA RD(-1)	−0.062	0.242	0.208	0.540***	−0.418**	−0.302
	(0.334)	(0.271)	(0.281)	(0.186)	(0.184)	(0.261)
Mom RD(-1)	−0.063	0.205**	0.219**	−0.029	0.241***	−0.160*
	(0.078)	(0.086)	(0.089)	(0.059)	(0.058)	(0.083)
Constant	0.015	0.115	−0.019	−0.025	−0.072	0.095
	(0.068)	(0.071)	(0.073)	(0.048)	(0.048)	(0.068)
Observations	42	42	42	42	42	42
Adjusted R^2	0.137	0.242	0.191	0.322	0.282	0.172

Table 6: Systematic Component of Returns and ECT

This table reports the ordinary least squares estimate for γ_1^i and the R^2 from the regression: $f_{t+1}^i = \gamma_0^i + \gamma_1^i \overline{ECT}_t + \epsilon_{t+1}^i$, where f_{t+1} is the excess return for each of the six Fama-French factors at time $t+1$ and $\overline{ECT}$ is the cross-sectional average of the ECTs for the 25 Fama-French portfolios at time t. Standard errors are computed according to Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1965 to 2018.

	$\hat{\gamma}_1$	R^2
MKT	-1.719**	6.89
SMB	1.481*	8.86
HML	-0.266	-1.59
RMW	0.227	-1.42
CMA	0.026	-1.91
Mom	-1.700	1.36

Table 7: Idiosyncratic Component of Returns and ECT

This table reports the ordinary least squares estimate for π_1^i and the R^2 from the regression: $v_{t+1}^i = \pi_0^i + \pi_1^i ECT_t^i + \epsilon_{t+1}^i$, where v_{t+1}^i is the residual obtained by regressing excess returns for portfolio i in the 25 Fama-French portfolios at time $t+1$ on the six Fama-French factors, and ECT_t^i is the ECT for portfolio i at time t. Standard errors are computed according to Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1965 to 2018.

	$\hat{\pi}_1$	R^2
Port 11	-0.588***	31.65
Port 12	-0.209***	13.57
Port 13	-0.171***	10.77
Port 14	-0.103*	9.75
Port 15	-0.122**	8.66
Port 21	-0.468***	23.47
Port 22	-0.295***	12.11
Port 23	-0.209***	8.35
Port 24	-0.530***	24.52
Port 25	-0.420***	20.08
Port 31	-0.194*	9.74
Port 32	-0.534***	28.94
Port 33	-0.212*	8.01
Port 34	-0.587***	36.32
Port 35	-0.244***	9.98
Port 41	-0.393***	18.82
Port 42	-0.117*	4.73
Port 43	-0.254***	11.04
Port 44	-0.170**	8.73
Port 45	-0.174**	7.42
Port 51	-0.387***	26.61
Port 52	-0.226***	12.23
Port 53	-0.129	4.75
Port 54	-0.282***	11.50
Port 55	-0.544***	23.04

Table 8: The ECT and Other Returns Predictors

This table reports the ordinary least squares estimate for β_i and the R^2 from the regression: $x_t^i = \alpha_i + \beta_i ECT_t^i + \epsilon_t^i$, where x_t^i is a predictor for portfolio i in the 25 Fama-French portfolios at time t and ECT_t^i is the ECT for portfolio i at time t. As predictors, we consider the log dividend-price ratio dp_t^i and the latent variable g_t^i for portfolio i at time t. Standard errors are computed according to Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1965 to 2018.

	$\hat{\beta}_{dp}$	R^2_{dp}	$\hat{\beta}_{lat}$	R^2_{lat}
Port 11	0.005	0.737	0.025	0.007
Port 12	0.017	2.453	0.154	0.569
Port 13	0.022	3.745	0.112	0.444
Port 14	0.011	2.033	0.124	1.457
Port 15	0.016	4.136	0.195	2.096
Port 21	0.014	1.248	0.612	3.142
Port 22	0.001	0.006	0.235	1.213
Port 23	-0.028	1.194	0.157	0.433
Port 24	-0.044	0.773	-0.966	6.279
Port 25	0.014	0.221	0.239	0.741
Port 31	-0.003	0.131	0.353	3.209
Port 32	-0.023	0.992	0.604	6.180
Port 33	-0.009	0.091	0.520**	5.717
Port 34	-0.002	0.002	0.269	1.201
Port 35	-0.016	0.479	-0.020	0.016
Port 41	-0.017	1.430	0.309	1.683
Port 42	0.014	1.564	0.147	1.895
Port 43	0.014	0.545	0.207	2.364
Port 44	-0.017	0.808	0.162	2.176
Port 45	0.007	0.103	0.171	1.264
Port 51	-0.031	5.038	-0.552**	6.319
Port 52	0.014	0.519	0.127	0.689
Port 53	0.016	0.944	0.128	1.512
Port 54	0.032	1.658	0.196	1.679
Port 55	-0.014	0.197	0.149	0.722
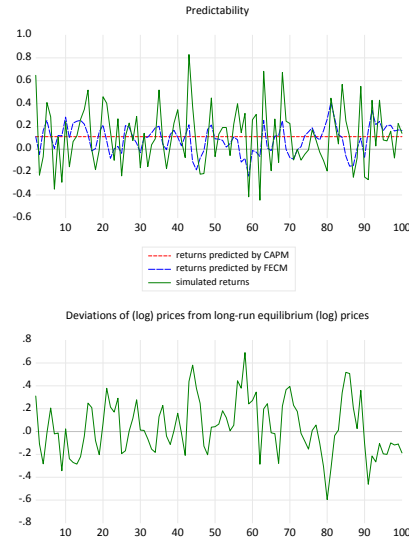
(a) This figure shows simulated data from the DGP in (10) by calibrating $\rho = 0.5$, i.e., with cointegration between prices and risk drivers.
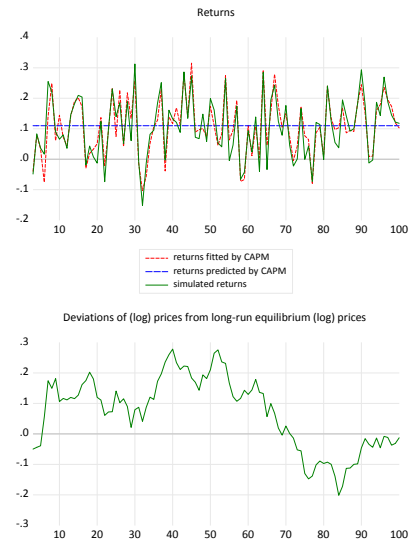
(b) This figure shows simulated data from the DGP in (10) by calibrating $\rho = 1$, i.e., without cointegration between prices and risk drivers.

Figure 1: Monte-Carlo Simulation
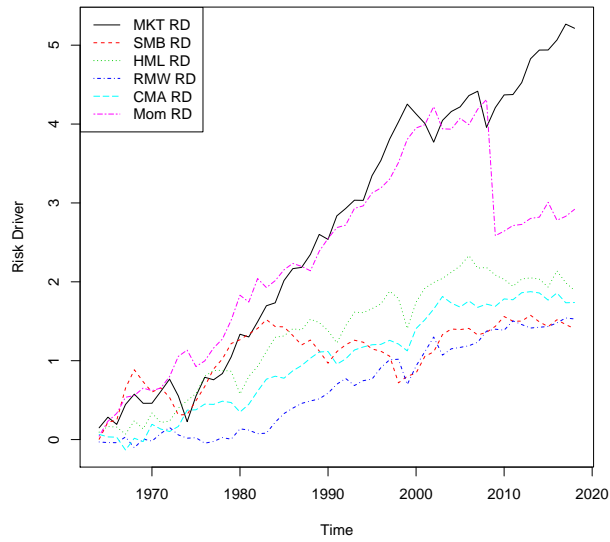
(a) This figure shows the yearly dynamics for the six risk drivers associated with the six Fama-French factors.
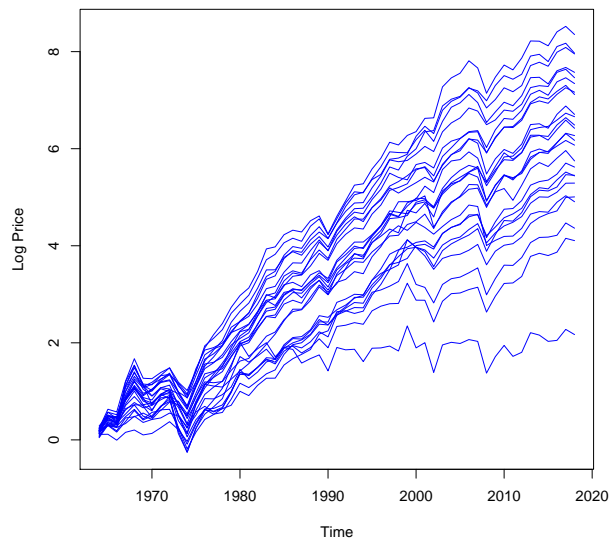
(b) This figure shows the yearly dynamics for the log prices for the 25 Fama-French Portfolios formed on Size and Book-to-Market.

Figure 2: Risk Drivers and Log Prices Dynamics
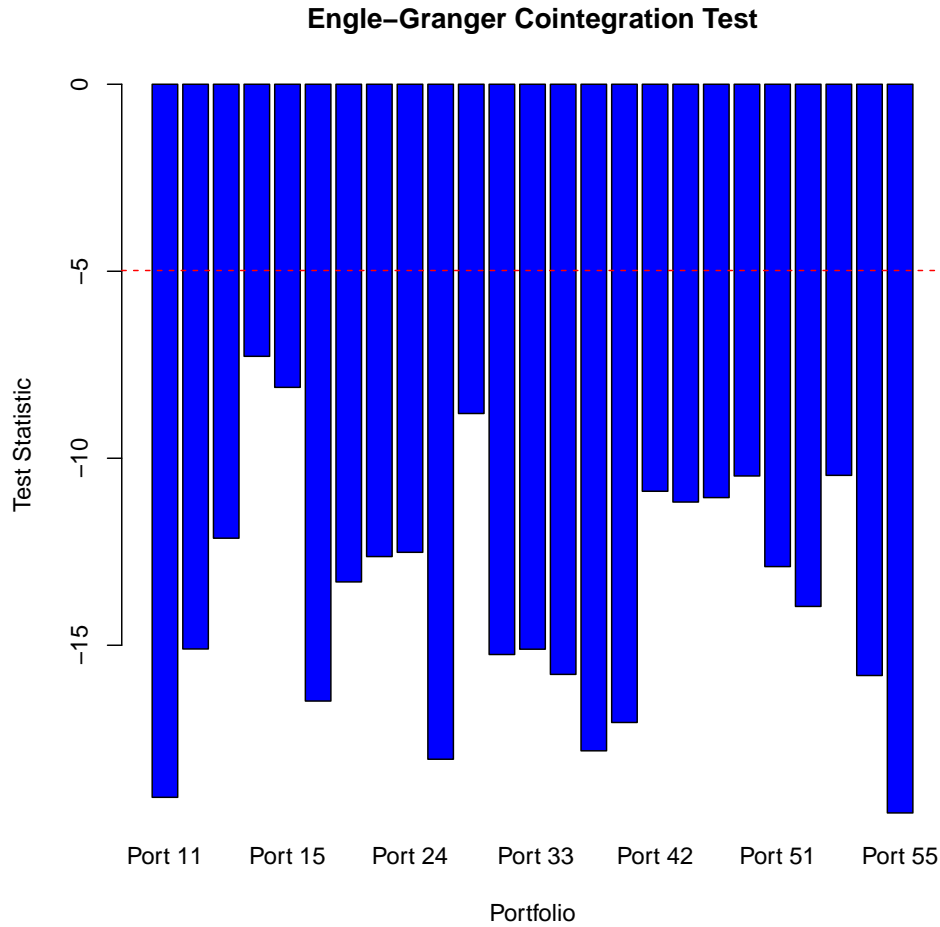
Figure 3: Engle and Granger (1987) Cointegration Test for the Six-Risk Driver Model Residuals

This figure shows results for the Engle and Granger (1987) cointegration test. First, we estimate regression (11) for each portfolio i in the 25 Fama-French Portfolios formed on Size and Book-to-Market on the Fama and French (2018) six-risk drivers using monthly observations, then we run the regression $\Delta \hat{u}_t^i = \gamma^i \hat{u}_{t-12}^i + \varepsilon_t^i$ on the 25 time-series for residuals. We test if γ is statistically different from zero. The null hypothesis for the test is non-cointegration. The (dashed) red line is the critical value at 5% level of significance as suggested by MacKinnon (2010).
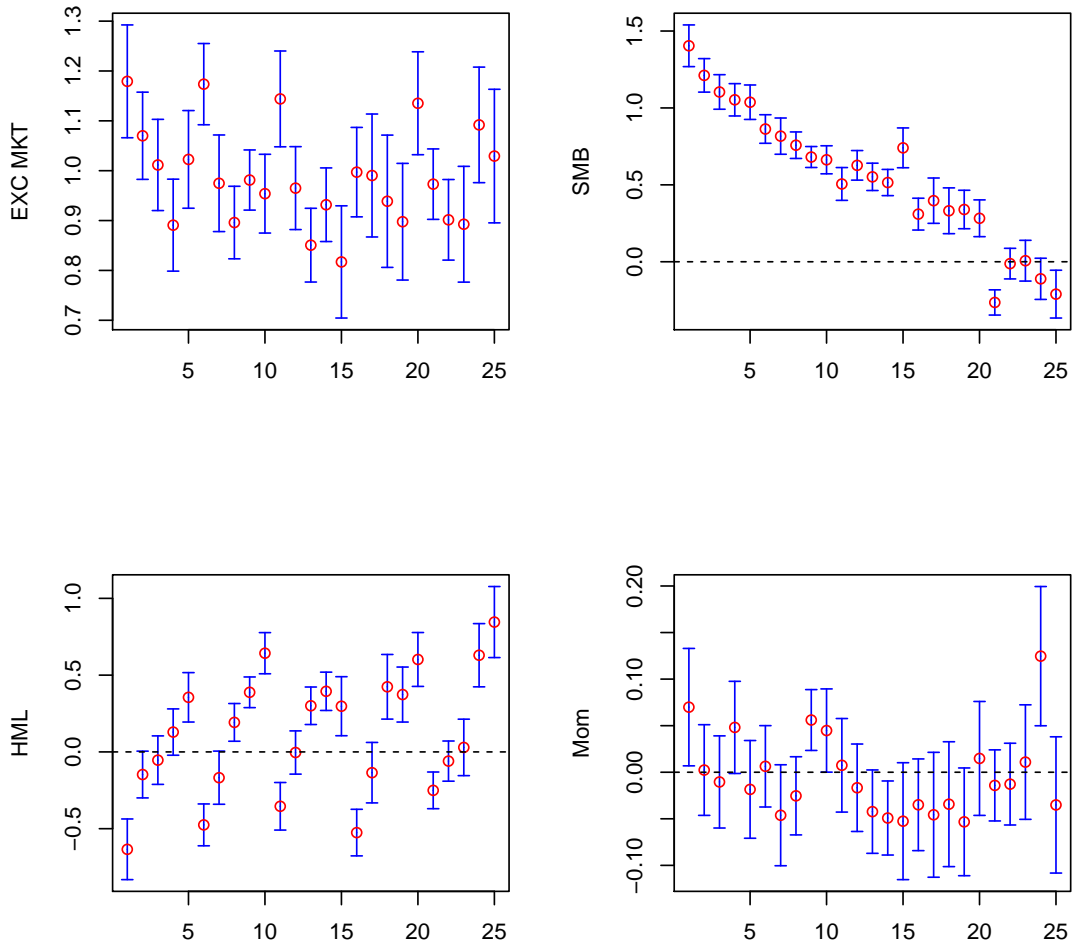
Figure 4: Factors Significance for the 25 Fama-French Portfolios (1)

This figure shows the estimated coefficients for EXC MKT, SMB, HML and Mom obtained by regressing the 25 Fama-French portfolios excess returns on factors plus the ECT as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.
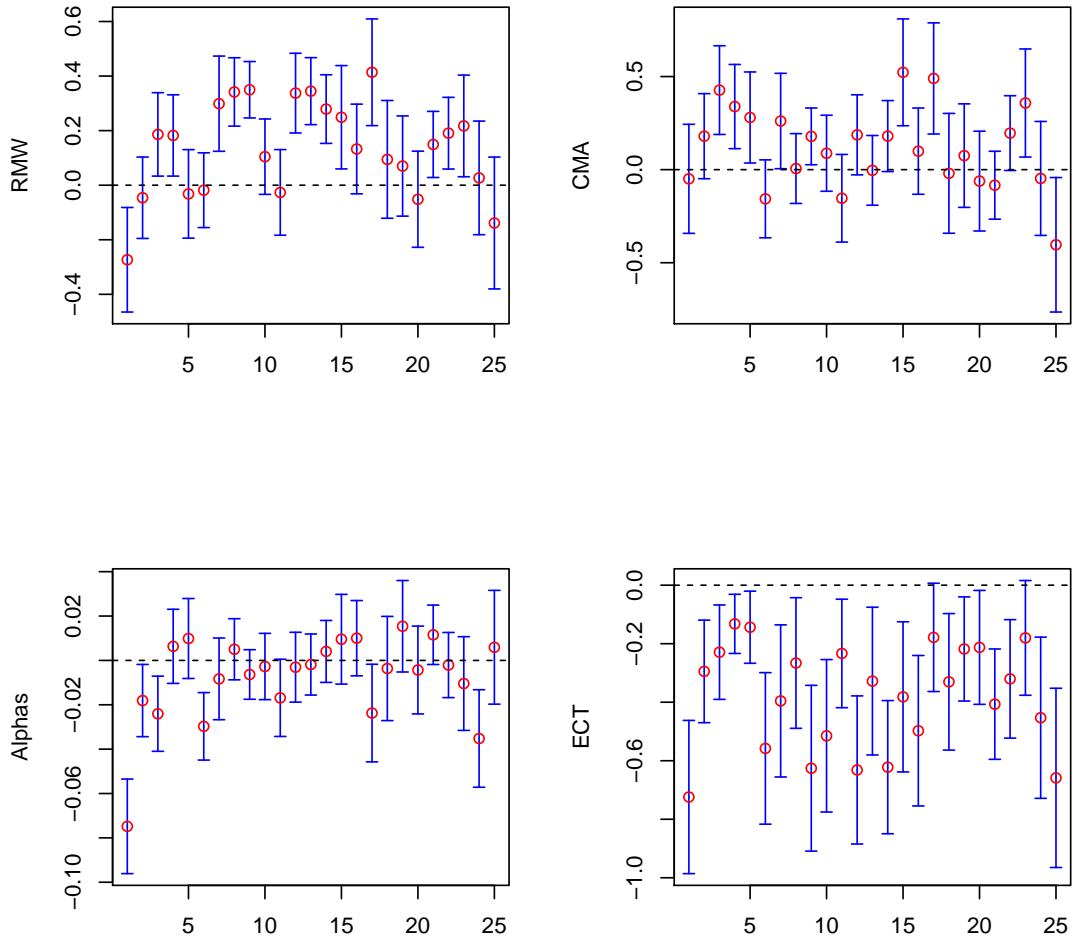
Figure 5: Factors Significance for the 25 Fama-French Portfolios (2)

This figure shows the estimated coefficients for RMW, CMA, alphas and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors plus the ECT as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.
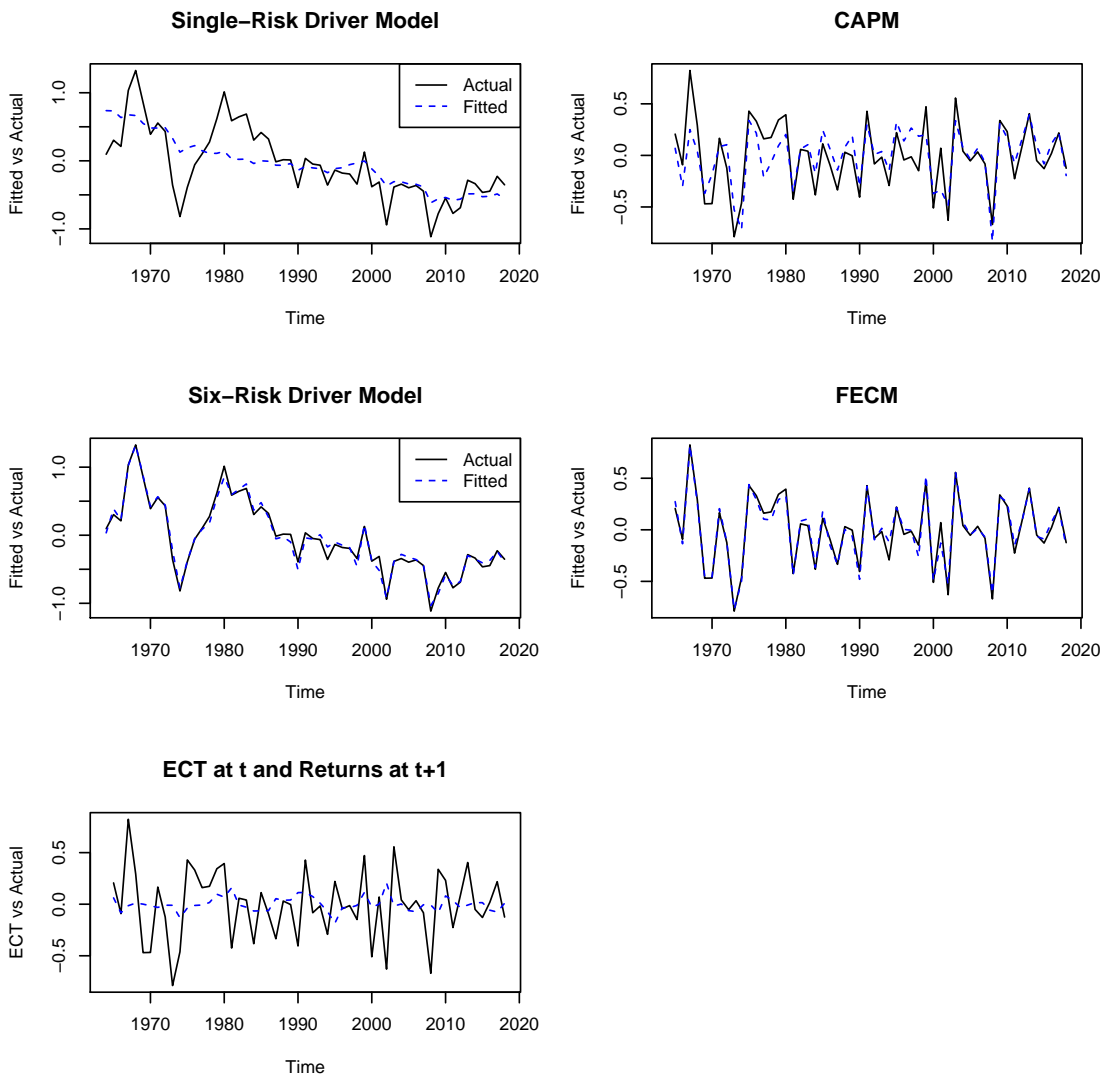
Figure 6: Single- and Six-Factor Specifications for Portfolio Small and Growth

This figure shows observed log prices and log returns for Portfolio Small and Growth in the 25 Fama-French portfolios and fitted values for Portfolio Small and Growth estimated by using respectively the single-risk driver (EXC MKT) (top left), six-risk driver (FF6) (middle left), CAPM (top right) and FECM (middle right) specifications (13) and (12). The black lines are actual values, the dashed blue lines are fitted values. The last figure illustrates the dynamics of the ECT compared to the portfolio Small and Growth observed returns.
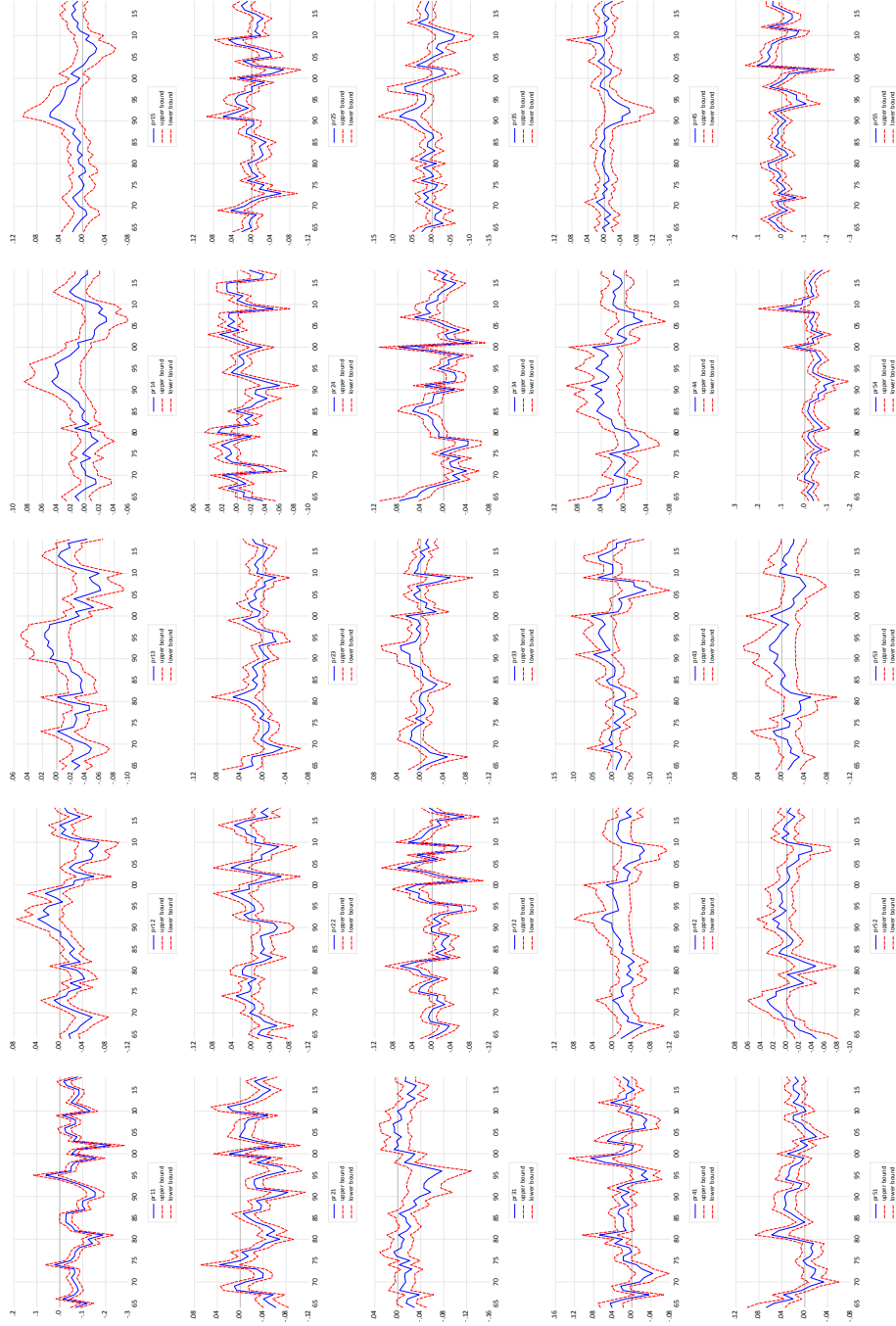
Figure 7: Intercept and ECT

This figure shows the estimated values for the time-varying intercept $\hat{\alpha}^i_{t+1} = \hat{\alpha}^i_1 + \hat{\delta}_i \hat{u}^i_t$ from regression (12) for the 25 Fama-French portfolios with respective confidence intervals at 5% level of significance.
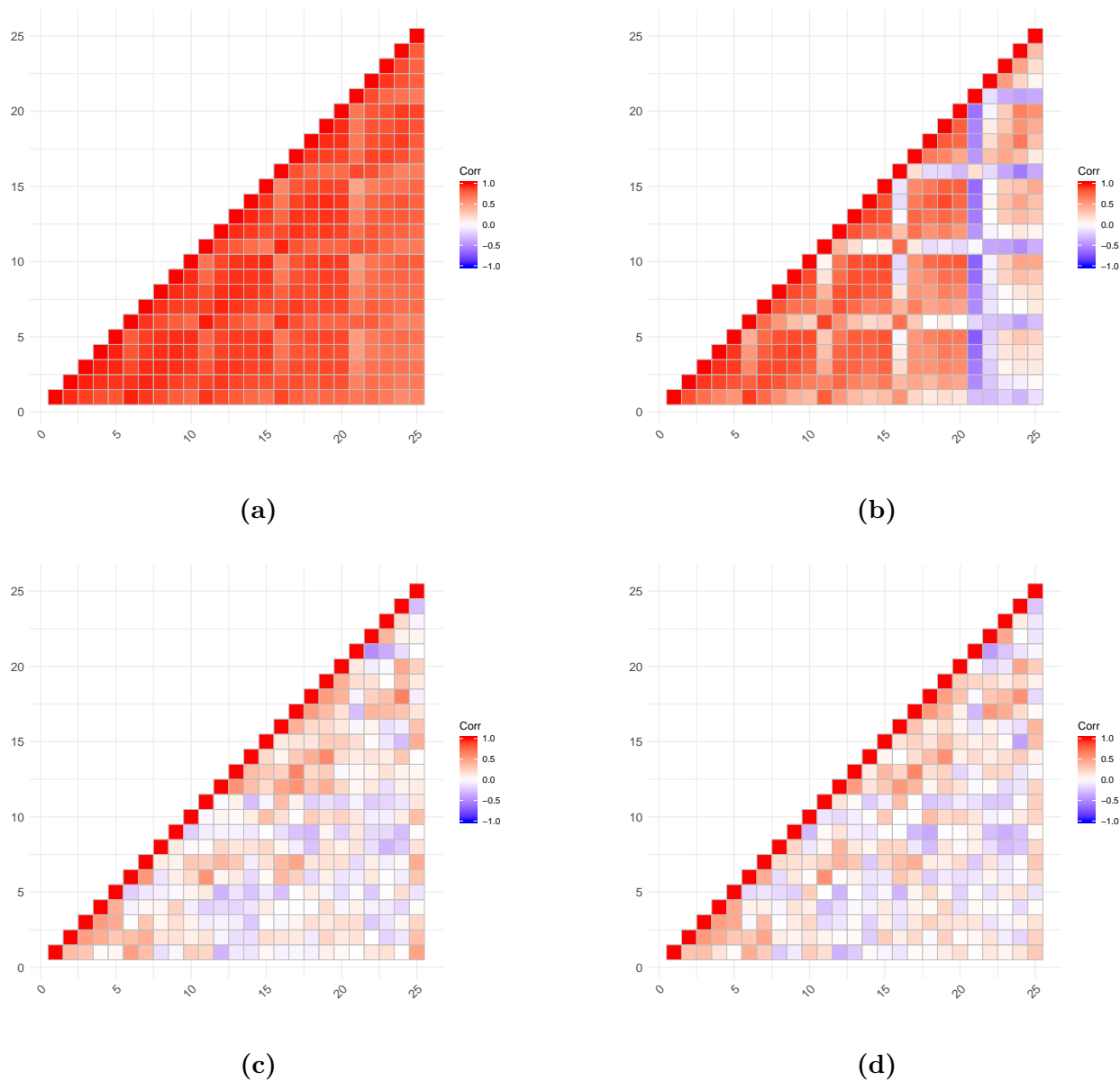
Figure 8: Correlation Matrix Plots

This figure shows heatmaps for the correlation matrix for the 25 Fama-French portfolio excess log returns (a) and the residuals for the 25 portfolios estimated by using respectively the CAPM (b), FF6 (c), and FECM (d) specification. The sample period is 1964 to 2018.
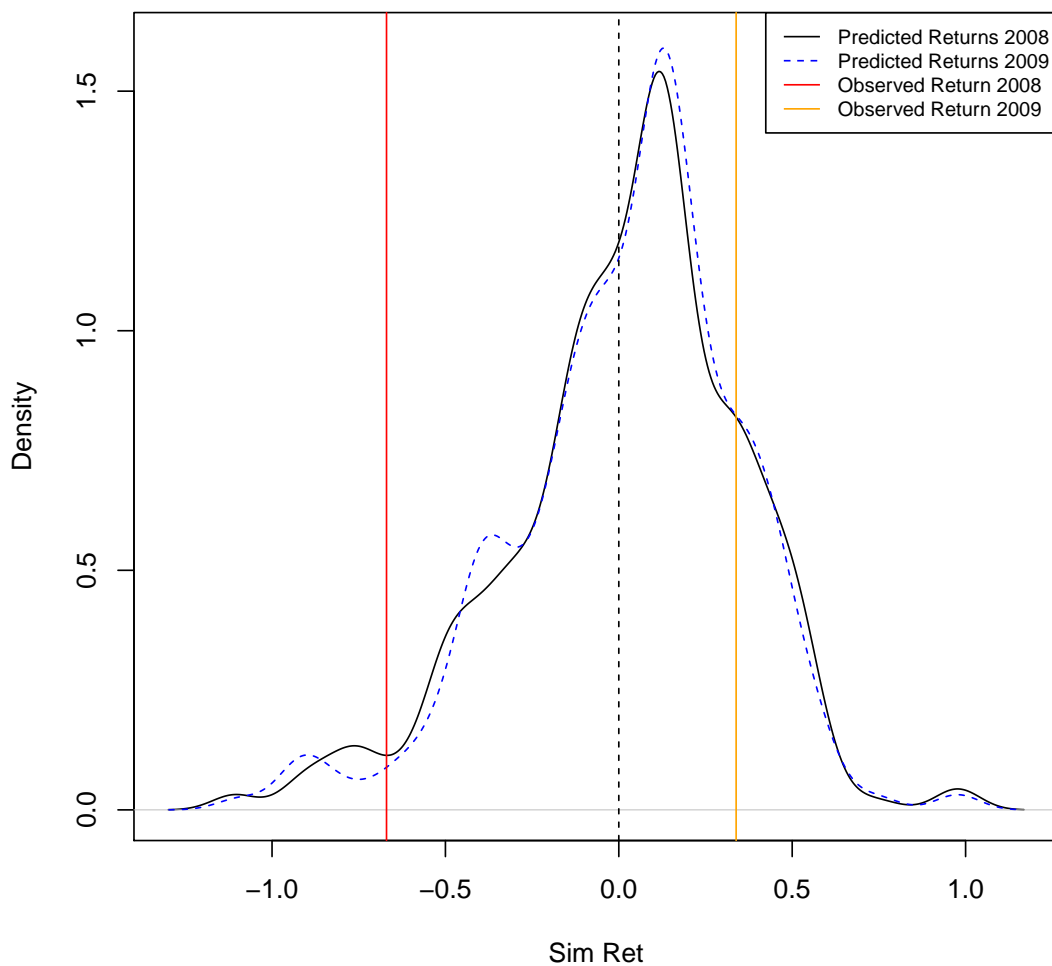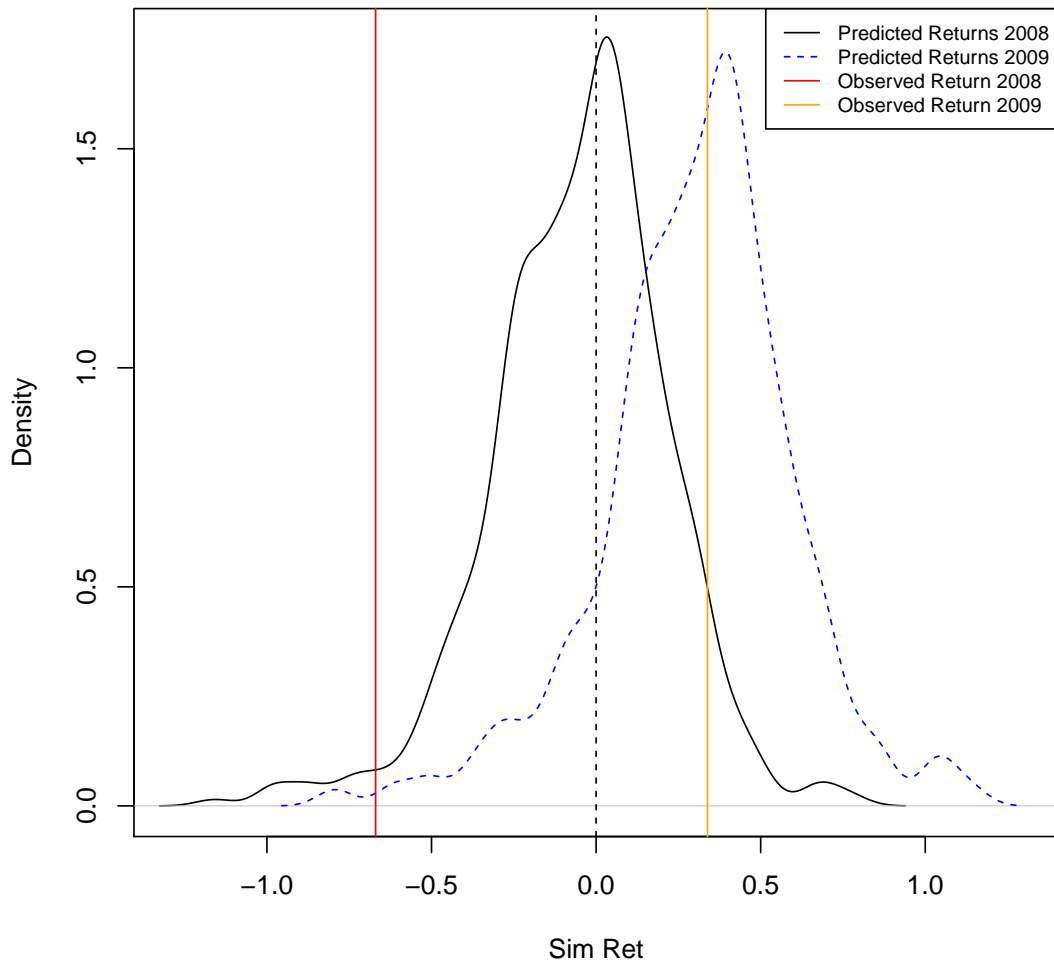
Figure 9: Predicted Returns: the Traditional Approach

This figure shows observed and out-of-sample predicted returns for portfolio Small and Growth in the 25 Fama-French portfolios during the crash 2007–2009 using the traditional specification (14). We estimate the model in the sample period 1964 to 2007 and we predict the distribution of returns by bootstrapping residuals. Bootstrapped 10% VaR takes the value of -0.440 for 2008 and of -0.404 for 2009. The unconditional 10% VaR is -0.469.

Figure 10: Predicted Returns: the EC Approach

This figure shows observed and out-of-sample predicted returns for portfolio Small and Growth in the 25 Fama-French portfolios during the crash 2007–2009 using the FECM specification (15). We estimate the model in the sample period 1964 to 2007 and we predict the distribution of returns by bootstrapping residuals. Bootstrapped 10% VaR takes the value of −0.367 for 2008 and of −0.067 for 2009.
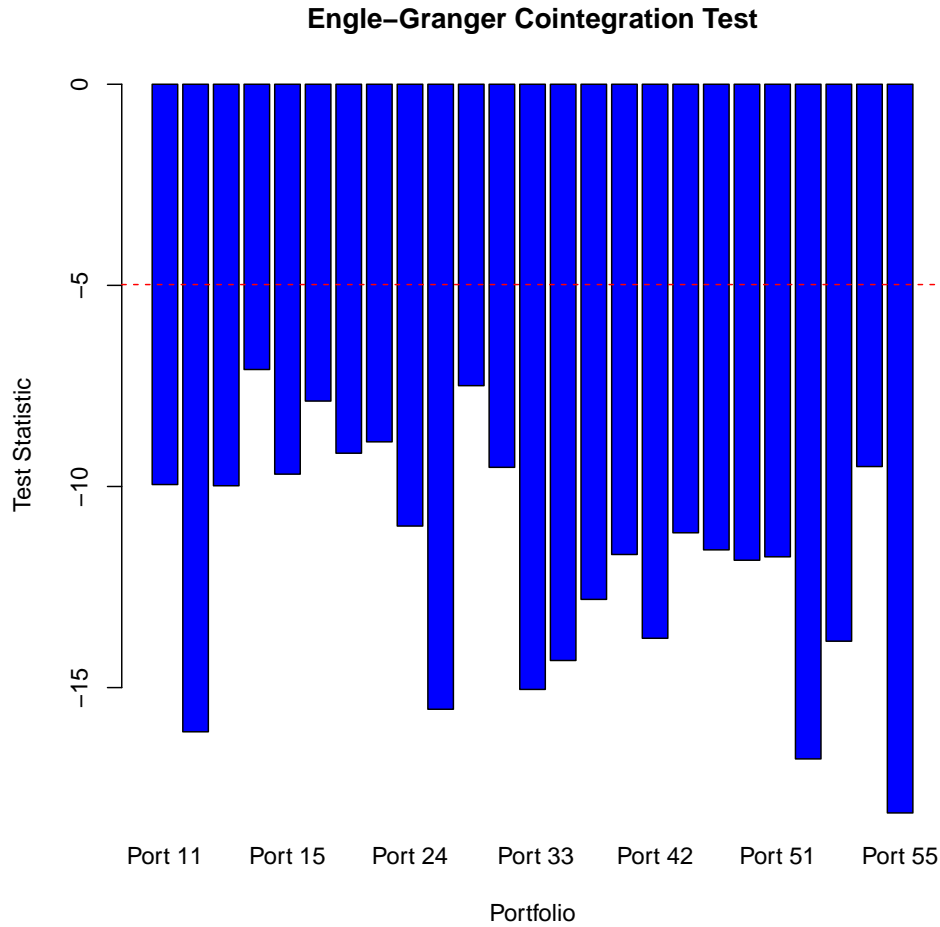
Figure 11: Engle and Granger (1987) Cointegration Test for the q-Factor-Risk Driver Model Residuals

This figure shows results for the Engle and Granger (1987) cointegration test. First, we estimate regression (11) for each portfolio i in the 25 Fama-French Portfolios formed on Size and Book-to-Market on the Hou et al. (2015) q-factor-risk drivers using monthly observations, then we run the regression $\Delta \hat{u}_t^i = \gamma^i \hat{u}_{t-12}^i + \varepsilon_t^i$ on the 25 time-series for residuals. We test if γ is statistically different from zero. The null hypothesis for the test is non-cointegration. The (dashed) red line is the critical value at 5% level of significance as suggested by MacKinnon (2010).
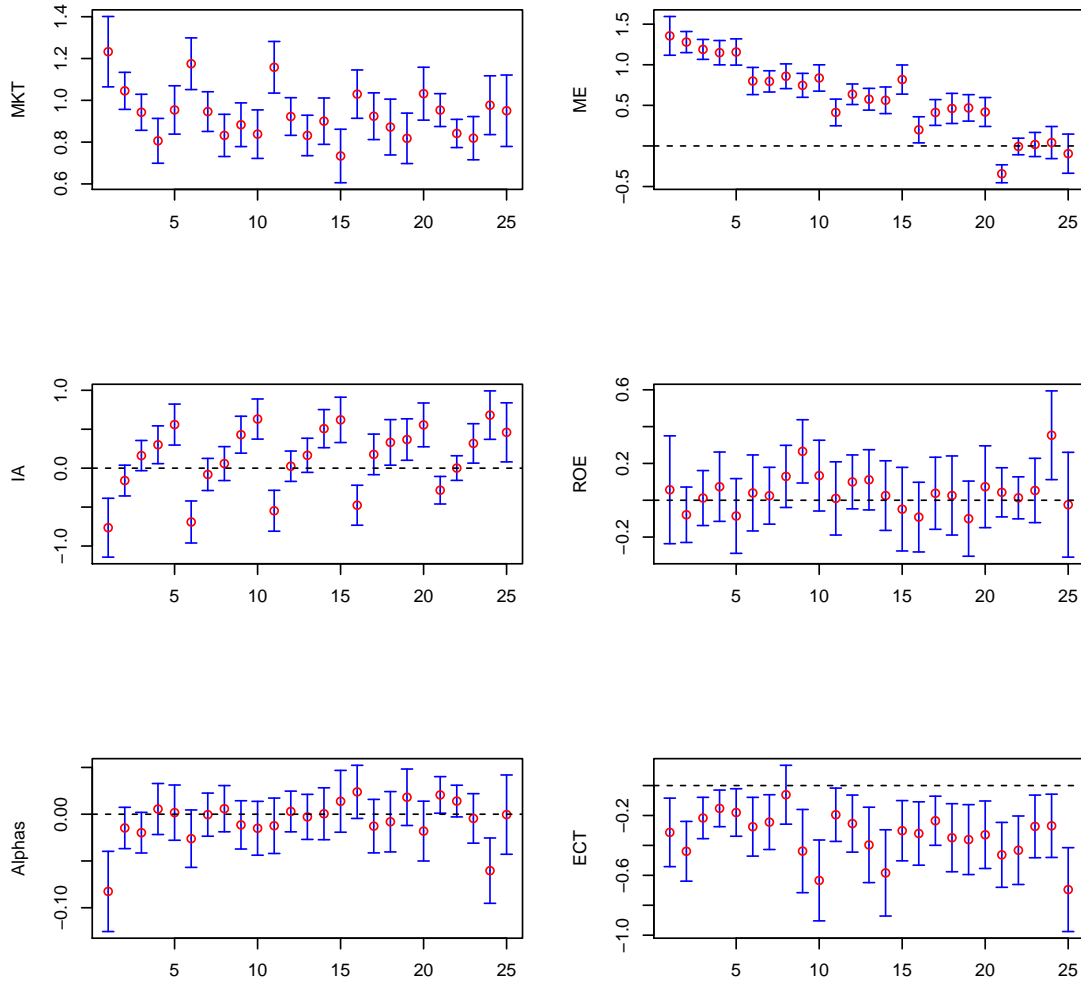
Figure 12: The q-Factor Model and the ECT

This figure shows the estimated coefficients obtained by regressing the 25 Fama-French portfolios excess returns on the Hou et al. (2015) q-factors plus the ECT as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2018.
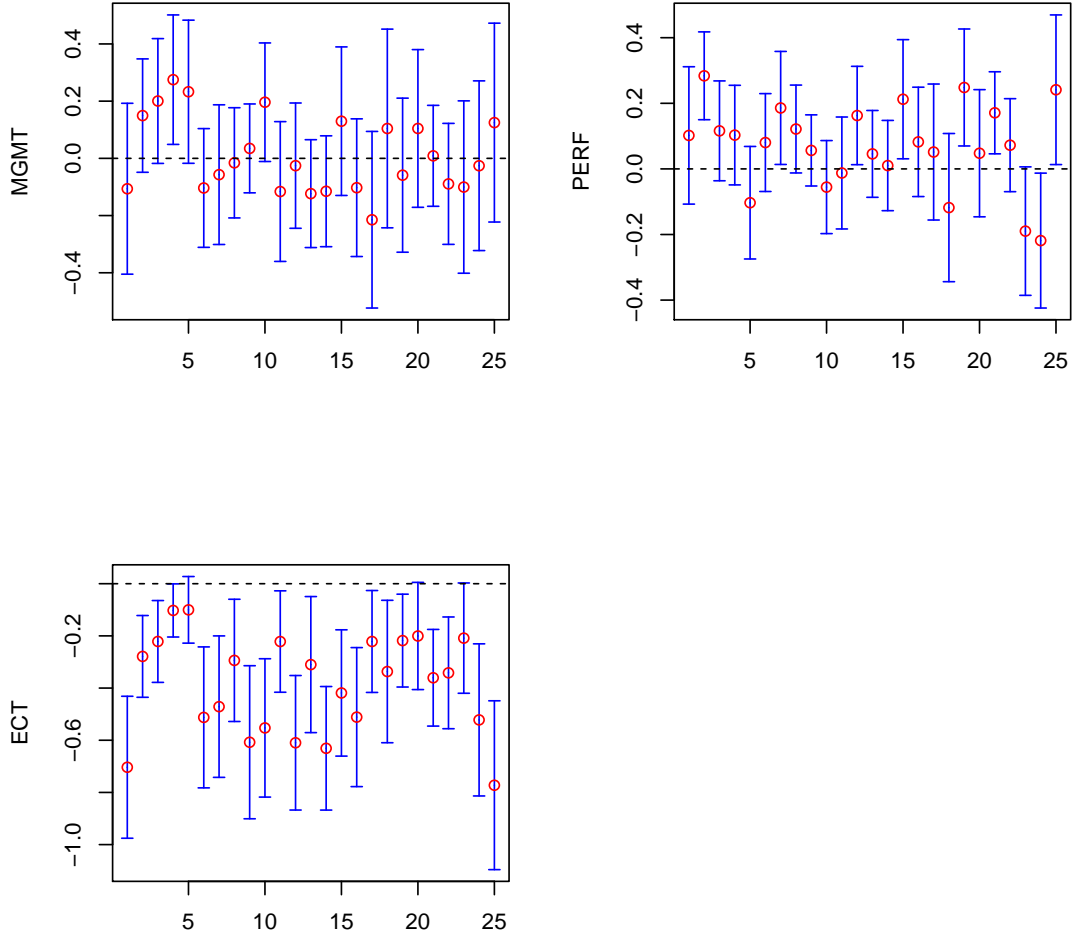
Figure 13: Mispricing Factors and ECT

This figure shows the estimated coefficients for MGMT, PERF, and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2016.
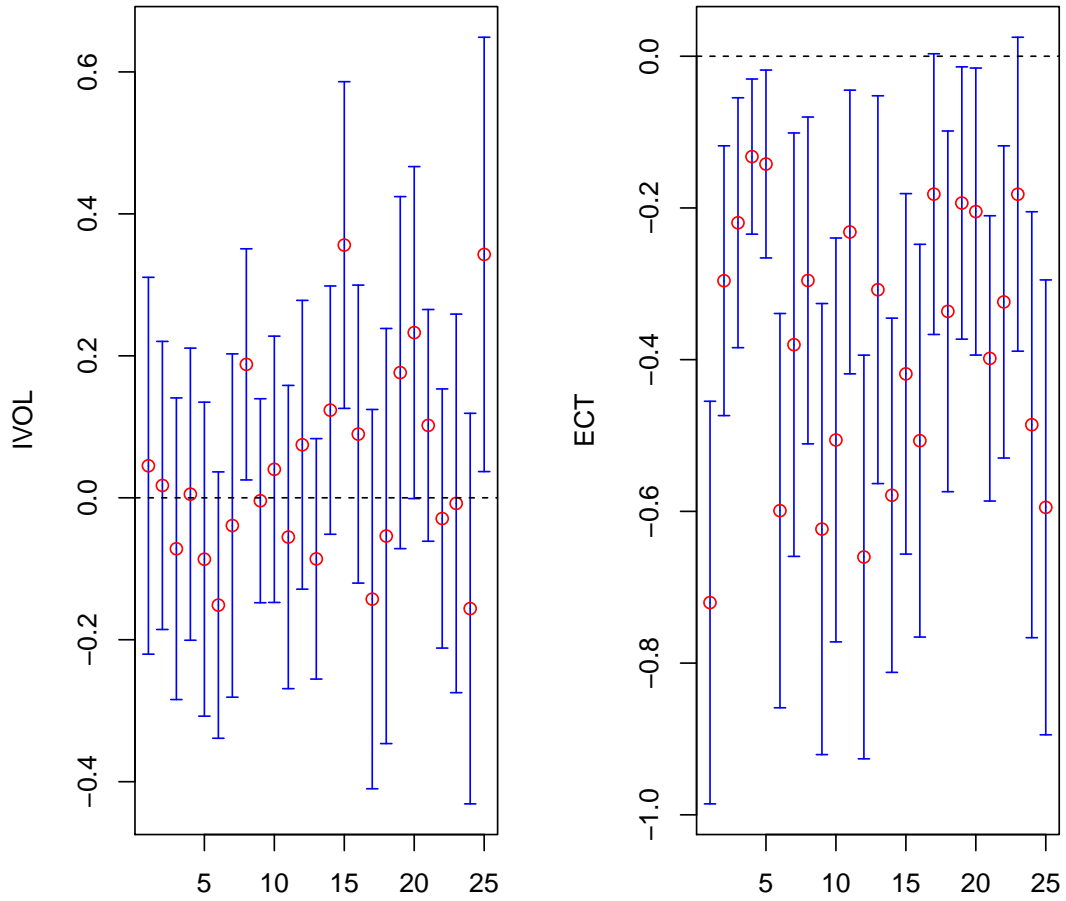
Figure 14: IVOL Factor and ECT

This figure shows the estimated coefficients for IVOL and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2016.
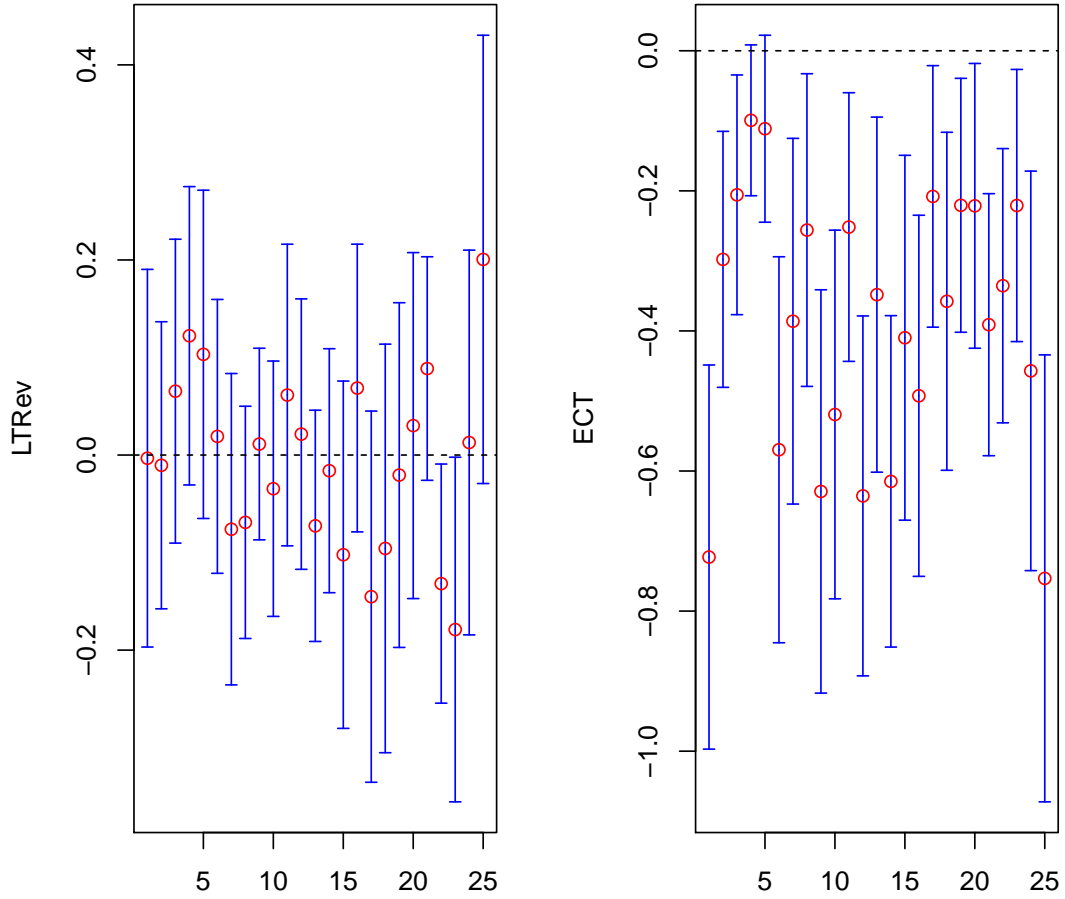
Figure 15: LTRev Factor and ECT

This figure shows the estimated coefficients for LTRev and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2016.

Appendix

A Empirical Analysis for 25 Fama-French Portfolios formed on Size and Book-to-Market without ECT
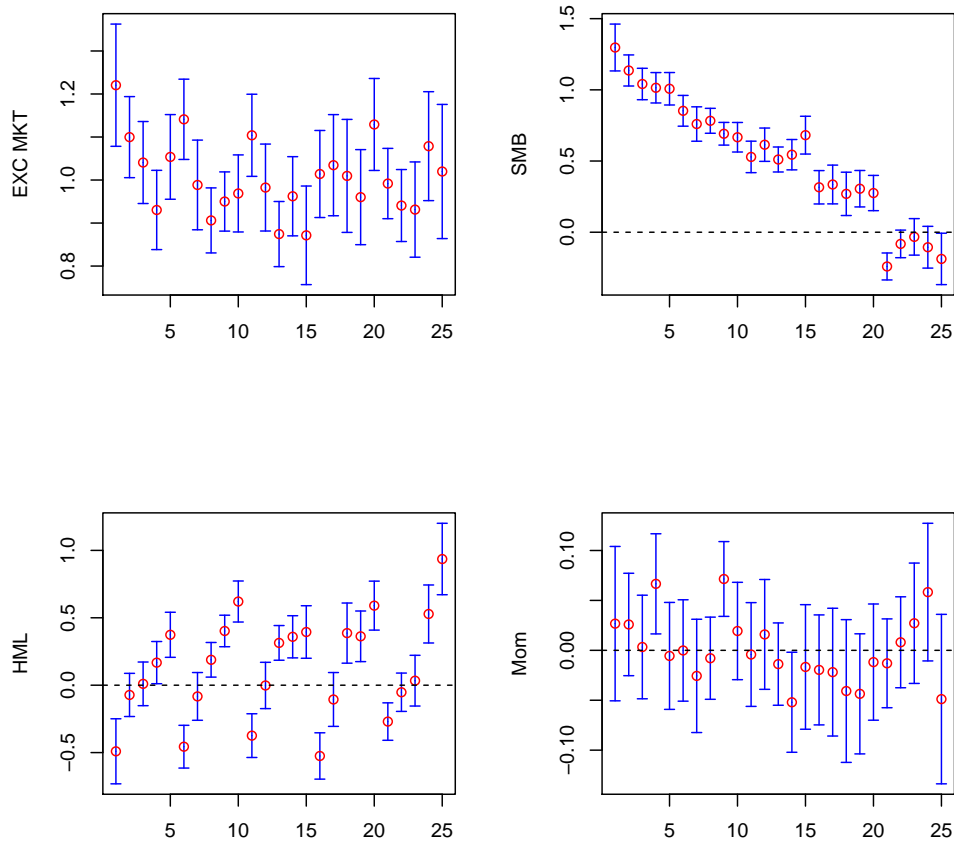

Figure 16: Factors Significance for the 25 Fama-French Portfolios (1)

This figure shows the estimated coefficients for EXC MKT, SMB, HML and Mom obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.

Figure 17: Factors Significance for the 25 Fama-French Portfolios (2)

This figure shows the estimated coefficients for RMW, CMA and alphas obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (12) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.
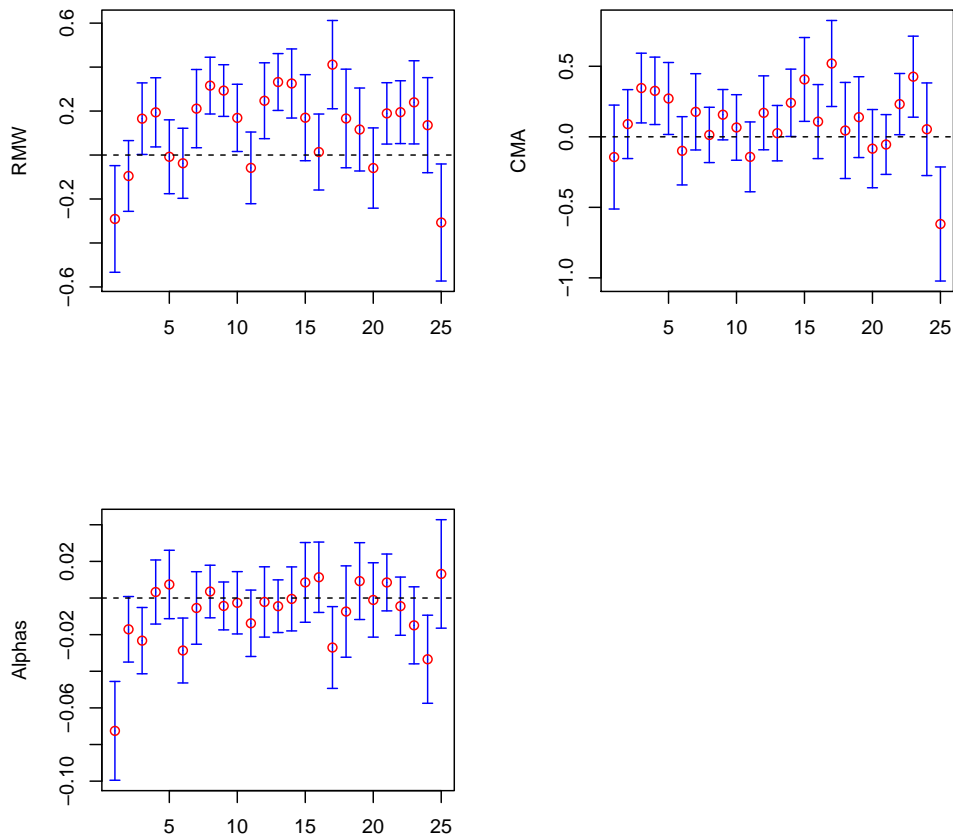
B Empirical Analysis for Portfolio Small and Growth without ECT

FF6 for Portfolio 11

This table reports the estimated coefficients for the six-factor Fama-French specification in equation (15) where r^{11} are log returns for Portfolio Small and Growth in the 25 Fama-French portfolios. The last column reports the semi-partial R^2 for each regressor. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with optimal truncation lag chosen as suggested by Andrews (1991). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

	FF6	SpR^2
EXC MKT	1.220***	0.286
	(0.065)	
SMB	1.297***	0.241
	(0.090)	
HML	−0.490***	0.016
	(0.154)	
RMW	−0.291***	0.006
	(0.064)	
CMA	−0.144	0.001
	(0.154)	
Mom	0.027	0
	(0.019)	
Constant	−0.073***	
	(0.009)	
Observations	54	
Adjusted R^2	0.947	