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Abstract

We study round-robin tournaments with four symmetric players and two identical prizes where players compete against each other in games modeled as an all-pay contest. We demonstrate that in this common structure players may have an incentive to manipulate the results, namely, depending on the outcomes of the first round, a player may have an incentive to lose in the second round in order to maximize his expected payoff in the tournament.

JEL Classification: N/A

Keywords: N/A

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Alex Krumer, Reut Megidish, Aner Sela

Abstract

We study round-robin tournaments with four symmetric players and two identical prizes where players compete against each other in games modeled as an all-pay contest. We demonstrate that in this common structure players may have an incentive to manipulate the results, namely, depending on the outcomes of the first round, a player may have an incentive to lose in the second round in order to maximize his expected payoff in the tournament.

Keywords: Multi-stage contests, all-pay auctions, round-robin tournaments.

JEL classification: D00, L00, D20, Z20, D44, O31.

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1. Introduction

Round-robin tournaments in which each team competes against all the others in sequential games is one of the most common sports tournaments, when the round-robin tournament with four teams in which the best two teams qualify for the next stage being the most frequently used format. In this paper, we point out a meaningful disadvantage of this structure since in one stage a player may have an incentive to lose, and, as such, the round-robin tournament can be exposed to strategic manipulations.

Previous literature has discussed many cases in which the rules of the tournament can cause contestants to lose a game in order to gain some benefit in the post-game. Such a phenomenon is harmful to the reputation of the sports industry, and therefore sports authorities exert efforts to avoid it. In their review, Kendall and Lenten (2017) presented several examples of such scenarios, a prominent one being the 2012 Olympic Badminton Tournament where teams tried to lose in order to face a weaker opponent in the next (playoff) stage.[1]

Probably, the most well-known example of teams having an incentive to lose is the NBA. Taylor and Trogdon (2002) indicated that this incentive is driven by the rule according to which the worst teams of the league have the highest probabilities of gaining a better position in the following season's draft when teams can pick the best new players for the league. Therefore, in the middle of the NBA season, teams that have lost their chance to qualify for the play-off have an incentive to lose in the remaining games of that season in order to increase their chances of a better draft position. Another example is given by Duggan and Levitt (2002) who showed that professional sumo fighters preferred to lose their last fight of the current season when it was not relevant for them in exchange for securing a victory in the next season. Elaad, Krumer, and

[1] It is also worth mentioning the 1994 Caribbean Cup soccer qualification game between Barbados and Grenada, where both teams could benefit from scoring an own goal in regular time because of the bizarre rule of assigning two goals instead of one in extra time (Preston and Szymanski, 2003). Incentives to lose games have also been demonstrated in US college basketball (Balsdon, Fong and Thayer 2007) and European soccer (Csató, 2019).

Kantor (2018) showed that such a trade-off between losing in the last game of a soccer season and winning in the following season occurs significantly more frequently in countries known to be corrupt.

Incentives to lose may also crop up owing to a collision between two parallel tournaments that award the same prize. For example, Dagaev and Sonin (2018) mentioned the last round of the 2011–12 season in the Russian Premier League (soccer). In this case Lokomotiv Moscow was better off losing to Spartak Moscow as this could have transferred the right to participate in the European Cup from Rubin Kazan to Lokomotiv Moscow due to the intersection with the rules of the Russian Cup, which is another tournament that assigns a qualification slot for the European Cups. One last instance worth mentioning is the study of Haugen and Krumer (2019) who showed how two different qualification paths for the 2020 UEFA European Championship could have created incentives to lose in order to increase the chances of qualifying for the playoffs in the UEFA Nations League.

The above-mentioned papers show that losing in a tournament may be advantageous for the next tournaments or for parallel tournaments. In this work, we demonstrate rather how losing in a tournament may already yield a benefit in the same tournament. The reason is that strategic considerations may motivate a contestant to lose in one stage of the tournament in order to get a higher expected payoff. We arrive at this conclusion by studying a theoretical round-robin tournament among four symmetric players. These players behave strategically and exert efforts in order to win more games than at least two of their opponents and consequently to win one of the two equal prizes. We assume that players have the same common knowledge valuation for winning, and model each game between two players as an all-pay contest.[2] As a

[2] Applications of the all-pay contest have been made to rent-seeking and lobbying in organizations, R&D races, political contests, promotions in labor markets, trade wars, military and biological wars of attrition (see, for example, Hillman and Riley 1989, Baye, Kovenock and de Vries 1993, Amman and Leininger 1996, Krishna and Morgan 1997, and Che and Gale 1998).

result, the win probabilities in each game become endogenous in that they result from mixed equilibrium strategies and are positively correlated to win valuations. Moreover, the win probabilities depend on the stage of the tournament in which the game takes place, and on the identity of the future expected opponents. Thus, in order to determine the tournament's outcome, we compute a dynamically intertwined set of pair-wise equilibria for each allocation of the players which enables us to provide analytic solutions. [3]

In addition, we identify a situation where one of the contestants has a higher expected value in case of a loss rather than in case of a win, thereby creating an incentive to lose. The intuition behind this finding is quite simple. Each player wants to maximize his expected payoff which is equal to his expected value of winning minus his expected cost of effort. When there are two prizes for four players, only one win may give a player one of the prizes and therefore after one stage a player who already has one win considers whether it is worthwhile to exert costly efforts in the next stages. In a round-robin tournament with only one prize such a situation is not feasible since one win is not sufficient for winning the prize and the players have to compete in at least one more game in order to ensure that they win the tournament.

In support of this result, Krumer, Megidish and Sela (2017) who studied a round-robin tournament with four symmetric players did not detect such a situation in which one of the players has an incentive to lose. This suggests that the number of prizes in round-robin tournaments explains the strategic manipulations in this structure. The disadvantage of allocating two prizes instead of one prize is also reflected in the findings of Krumer, Megidish and Sela (2019) who studied the optimal design of round-robin tournaments with three

[3] Sahm (2019) analysed round-robin tournaments where each game is modelled as a Tullock contest.

symmetric players. It was shown that in order to maximize the players' expected total effort the designer should allocate only one prize.

Earlier studies have also shown why a designer may prefer an allocation of one over several prizes in contests. For example, Moldovanu, and Sela (2001) showed that in all-pay auctions under incomplete information when cost functions are linear or concave in effort, it is optimal to allocate the entire prize sum to a single first prize, but when cost functions are convex, several positive prizes may be optimal. Later, Moldovanu and Sela (2006) studied a two-stage all-pay auction with multiple prizes under incomplete information and showed that for a contest designer who maximizes the expected total effort, if the cost functions are linear in effort, it is optimal to allocate the entire prize sum to a single first prize. In symmetric all-pay auctions under complete information, Barut and Kovenock (1998) indicated that a revenue maximizing prize structure allows any combination of k-1 prizes, where k is the number of players. In other words, the contest designer is indifferent to whether he should allocate one or several prizes.

We focus on all-pay contests in order to describe players' behavior in real-life round-robin tournaments. Indeed, we can find several evidences that the all-pay contest reflects well players' behavior in other real-life contests. To illustrate, Krumer, Megidish and Sela (2017) studied round-robin tournaments with three players by means of an all-pay model and found that a player who competes in the first and the third rounds has the highest probability to win the tournament. Based on real-world data from wrestling Olympic tournaments, these findings were empirically confirmed by Krumer and Lechner (2017). Moreover, these authors showed that in six out of seven possible cases, the all-pay model correctly predicted the identity of a wrestler with a higher probability of winning. In addition, Krumer (2013) proposed a theoretical explanation for the empirical finding that there is a second-leg home advantage in European soccer cups as presented in Page and Page (2007) by using an all-pay model. Thus, we believe that our finding which demonstrates the difference in the effects of allocating either one or two prizes are relevant to the players' behaviour in contests in general and, in particular, in round-robin tournaments.

2. The round-robin tournament with four symmetric players and two identical prizes

We consider a round-robin all-pay tournament with four symmetric players which are denoted by $i \in \{1,2,3,4\}$. In each round $r, r \in \{1,2,3\}$ there are two different sequential pair-wise games, such that each player competes once in each of the three different rounds. Thus, the number of games is six and each player has three games as depicted in Table 1. The two players with the highest number of wins receive an equal prize. Should more than two players have the same highest number of wins, then each of them wins a prize with the same probability. Each game is modelled as an all-pay auction. In each game, both players exert efforts and the player with the higher effort wins the respective game. Without loss of generality, we assume that player i's value of winning (the value of the prize) is $= \frac{1}{2}$, and his cost function is $c(x_i) = x_i$, where x_i is his effort.

Table 1: The schedule of the round-robin tournament with four players

Round 1	Game 1: Player 1 - Player 2 Game 2: Player 3 - Player 4
Round 2	Game 3: Player 1 - Player 3 Game 4: Player 2 - Player 4
Round 3	Game 5: Player 1 - Player 4 Game 6: Player 2 - Player 3

We begin the analysis by explaining how the players' strategies are calculated in each game of the tournament. Suppose that players *i* and *j* compete in game $g, g \in \{1,2,3,4,5,6\}$. We denote by p_{ig} the probability that player i wins the game g against player j and E_{ig} and E_{jg} being the expected payoffs of players i and j in game g, respectively. The mixed strategies of the players in each game are denoted by $F_{kg}(x), k \in \{i,j\}$. In addition, we assume that player i's continuation value if he wins in game g is w_{ig} given the previous and possible future outcomes. Similarly, we assume that player i's continuation value if he loses game g is l_{ig}, given the previous and possible future outcomes. Without loss of generality, we assume that $w_{ig} - l_{ig} > w_{jg} - l_{jg}$.

Then, according to Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996), there is always a unique mixed-strategy equilibrium in which players *i* and *j* randomize on the interval $[0, w_{jg} - l_{jg}]$, according to their effort cumulative distribution functions, which are given by

$$E_{ig} = w_{ig}F_{jg}(x) + l_{ig}\left(1 - F_{jg}(x)\right) - x = l_{jg} + w_{ig} - w_{jg}$$

$$E_{jg} = w_{jg}F_{ig}(x) + l_{jg}\left(1 - F_{ig}(x)\right) - x = l_{jg}$$

Thus, player *i*'s equilibrium effort in game g is uniformly distributed; that is

$$F_{ig}(x) = \frac{x}{w_{jg} - l_{jg}}$$

while player *j*'s equilibrium effort in game g is distributed according to the cumulative distribution function

$$F_{jg}(x) = \frac{l_{jg} - l_{ig} + w_{ig} - w_{jg} + x}{w_{ig} - l_{ig}}$$

Player *i*'s probability of winning game g against player *j* is then

$$p_{ig} = 1 - \frac{w_{jg} - l_{jg}}{2\left(w_{ig} - l_{ig}\right)} > \frac{1}{2}$$

When $w_{ig} - l_{ig} = 0$, player *i* is indifferent between winning or losing and then he has no incentive to exert a positive effort and therefore we actually have no equilibrium. To overcome this complication, we can assume, similarly to Groh et al. (2012), that in every game each player has an additional value of winning a single game of $m > 0$ where $m \to 0$. This assumption does not affect the players' behavior, but ensures the existence of equilibrium. However, when $w_{ig} - l_{ig}$ is strictly negative, player *i* has incentive to lose. Below we describe such a strategic manipulation.

3. Strategic manipulations

Here we show that in round 2-game 3 of our round-robin tournament where players 1 and 3 compete against each other (see Table 1), player 1 prefers to lose. For this purpose, we analyse the subgame perfect equilibrium of the relevant part of this tournament. We begin with the last stage and go backwards until the game between players 1 and 3 in game 3 of round 2. Figure 1 presents this part of the game tree of our round-robin tournament.

Round 1: Player 1 vs. Player 2 / Player 3 vs. Player 4

Round 2: Player 1 vs. Player 3 / Player 2 vs. Player 4

Round 3: Player 1 vs. Player 4 / Player 2 vs. Player 3

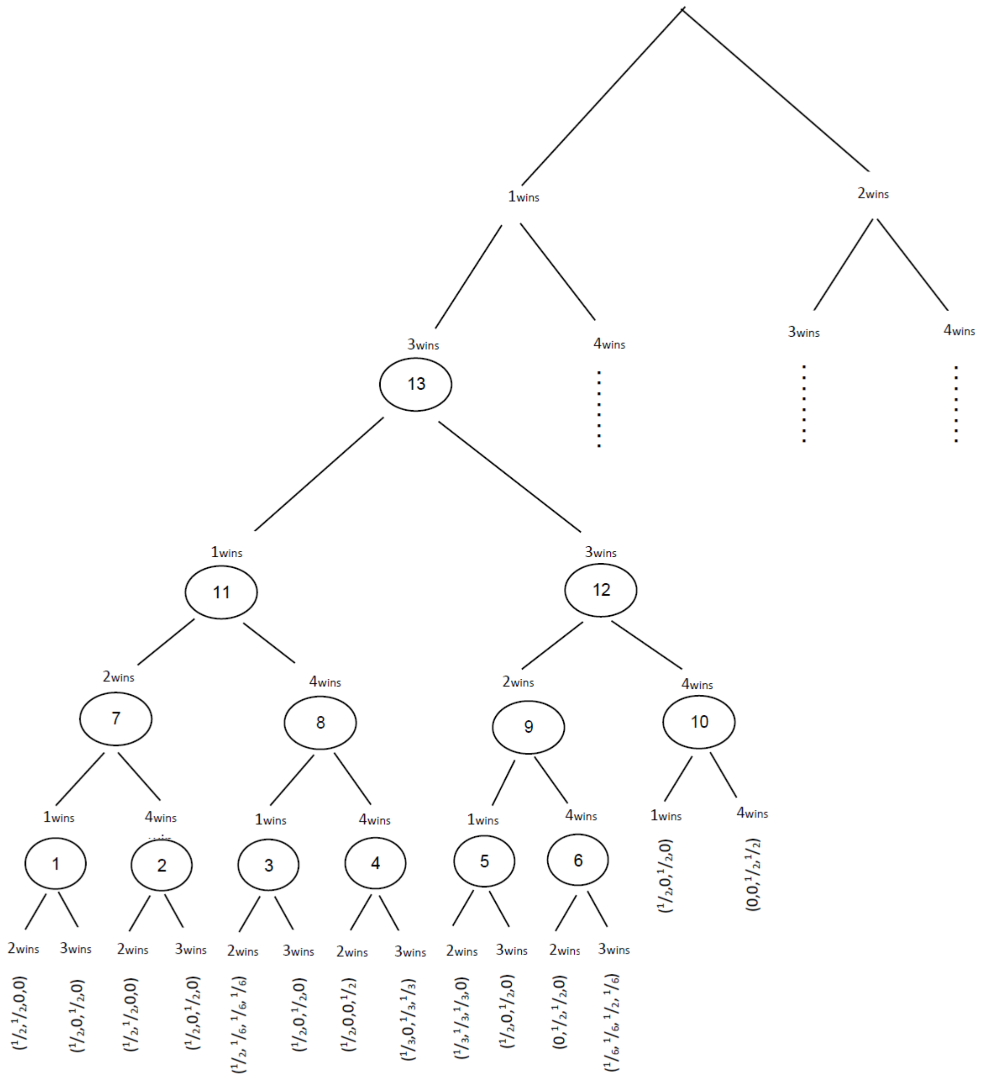

Figure 1: Part of the game tree of the round-robin tournament.

The players' mixed equilibrium strategies, their expected payoffs, and their probabilities of winning in the above Vertexes 1-12 are summarized in the following Table 2.

Table 2: Summary of the players' mixed equilibrium strategies, their expected payoffs, and their probabilities of winning.

<u>Vertex 1:</u> $E_{26} = \frac{1}{2}F_{36}(x) - x = 0$ $E_{36} = \frac{1}{2}F_{26}(x) - x = 0$ $p_{26} = \frac{1}{2}$, $p_{36} = \frac{1}{2}$	<u>Vertex 7:</u> $E_{15} = \frac{1}{2}F_{45}(x) + \frac{1}{2}(1 - F_{45}(x)) - x = \frac{1}{2}$ $E_{45} = 0 \cdot F_{15}(x) - x = 0$ $p_{15} = \frac{1}{2}$, $p_{45} = \frac{1}{2}$
<u>Vertex 2:</u> $E_{26} = \frac{1}{2}F_{36}(x) - x = 0$ $E_{36} = \frac{1}{2}F_{26}(x) - x = 0$ $p_{26} = \frac{1}{2}$, $p_{36} = \frac{1}{2}$	<u>Vertex 8:</u> $E_{15} = \frac{1}{2}F_{45}(x) + \frac{1}{3}(1 - F_{45}(x)) - x = \frac{1}{3}$ $E_{45} = \frac{1}{3}F_{15}(x) + \frac{1}{24}(1 - F_{15}(x)) - x = \frac{1}{6}$ $p_{15} = \frac{2}{7}$, $p_{45} = \frac{5}{7}$
<u>Vertex 3:</u> $E_{26} = \frac{1}{6}F_{36}(x) - x = 0$ $E_{36} = \frac{1}{2}F_{26}(x) + \frac{1}{6}(1 - F_{26}(x)) - x = \frac{1}{3}$ $p_{26} = \frac{1}{4}$, $p_{36} = \frac{3}{4}$	<u>Vertex 9:</u> $E_{15} = \frac{3}{8}F_{45}(x) - x = \frac{3}{8}$ $E_{45} = 0 \cdot F_{15}(x) - x = 0$ $p_{15} = 1$, $p_{45} = 0$
<u>Vertex 4:</u> $E_{26} = 0 \cdot F_{36}(x) - x = 0$ $E_{36} = \frac{1}{3}F_{26}(x) - x = \frac{1}{3}$ $p_{26} = 0$, $p_{36} = 1$	<u>Vertex 10:</u> $E_{15} = \frac{1}{2}F_{45}(x) - x = 0$ $E_{45} = \frac{1}{2}F_{15}(x) - x = 0$ $p_{15} = \frac{1}{2}$, $p_{45} = \frac{1}{2}$
<u>Vertex 5:</u> $E_{26} = \frac{1}{3}F_{36}(x) - x = \frac{1}{6}$ $E_{36} = \frac{1}{2}F_{26}(x) + \frac{1}{3}(1 - F_{26}(x)) - x = \frac{1}{3}$ $p_{26} = \frac{3}{4}$, $p_{36} = \frac{1}{4}$	<u>Vertex 11:</u> $E_{24} = 0 \cdot F_{44}(x) - x = 0$ $E_{44} = \frac{1}{6}F_{24}(x) - x = \frac{1}{6}$ $p_{24} = 0$, $p_{44} = 1$
<u>Vertex 6:</u> $E_{26} = \frac{1}{2}F_{36}(x) + \frac{1}{6}(1 - F_{36}(x)) - x = \frac{1}{2}$ $E_{36} = \frac{1}{2}F_{26}(x) + \frac{1}{2}(1 - F_{26}(x)) - x = \frac{1}{2}$ $p_{26} = 1$, $p_{36} = 0$	<u>Vertex 12:</u> $E_{24} = \frac{1}{6}F_{44}(x) - x = \frac{1}{6}$ $E_{44} = 0 \cdot F_{24}(x) - x = 0$ $p_{24} = 1$, $p_{44} = 0$

A complete analysis of the players' mixed strategies, their expected payoffs, and their probabilities of winning in Vertexes 1-12 are described in detail in the Appendix.

In the following we focus on the competition between players 1 and 3 in Vertex 13 who compete in there only if player 1 won the first game and player 3 won the second game. If player 1 wins, he will reach Vertex 8 via Vertex 11 with certainty (see the Appendix) where his expected payoff is $\frac{1}{3}$ ($w_{13} = \frac{1}{3}$). If, on the other hand, player 1 loses in Vertex 13, he reaches Vertex 9 via Vertex 12 with certainty (see the Appendix) where his expected payoff is $\frac{3}{8}$ ($l_{13} = \frac{3}{8}$). Thus, we can see that in Vertex 13, player 1's value of losing ($l_{13} = \frac{3}{8}$) is strictly higher than his value of winning ($w_{13} = \frac{1}{3}$), and therefore player 1 has an incentive to lose that game.

If player 3, on the other hand, wins in Vertex 13, he will reach Vertex 5 via Vertexes 12 and 9 with certainty (see the Appendix) where his expected payoff is $\frac{1}{3}$ ($w_{33} = \frac{1}{3}$). If, on the other hand, player 3 loses in Vertex 13, he reaches Vertex 8 via Vertex 11 with certainty. From there, he has a probability of $\frac{2}{7}$ to reach Vertex 3, where his expected payoff is $\frac{1}{3}$, and a probability of $\frac{5}{7}$ to reach Vertex 4, where his expected payoff is $\frac{1}{3}$. Thus, player 3's value of losing is $l_{33} = \frac{1}{3}$ which is equal to his value of winning. Given, the additional value of winning a single game of $m > 0$ where $m \to 0$ player 3 in contrast to player 1 has an incentive to win that game.

The intuition behind the result according to which player 1 has an incentive to lose is that by winning of player 1, player 4, his next opponent, has a chance to win one of the prizes, and as such, player 4 exerts effort when he meets player 1 in the last round. On the other hand, if player 1 loses in Vertex 13, player 4's chance to win a prize (or to make a positive expected payoff) vanishes and, as such, in the next round when he competes against player 1, he does not have an incentive to exert a positive effort and as such player 1 wins one of the prizes with almost certainty.

In fact the intuition for the manipulation of player 1 in Vertex 13 is more concrete and the reason that a player has an incentive to lose after one game in which he won is that this player has already a significant probability to win one of the two prizes, and since he maximizes his expected payoff over all the tournament, he might prefer to minimize his efforts in the next rounds of the round-robin tournament.

4. Conclusion

We analyzed the subgame perfect equilibrium of round-robin all-pay tournaments with four symmetric players and two identical prizes. We showed that one of the winners of the first round maximizes his expected payoff by losing in the second round. By comparing our results with the findings in Krumer, Megidish and Sela (2017) who studied a similar tournament, but did not detect such a situation in which players have an incentive to lose, we conclude that the number of prizes in round-robin tournaments explains the strategic manipulations in this structure.

In order to avoid strategic manipulations where players might have an incentive to lose as we showed, the schedule of the tournament has to be contingent on the results, such that the schedule in each round has to be decided according to the outcomes in the previous rounds. In particular, the game between the winners of the first round should be delayed from the third game to the fourth one, and then the game between the winners of the first round will not affect the other game in the second round of the round-robin tournament.

Appendix

Game 6: Player 2 vs. Player 3

Vertex 1

As presented in Figure 1, players 2 and 3 compete in the last sixth game only if the identity of the two winners has not determined after the fifth game. This game in Vertex 1 occurs only if player 1 won all of his three games, player 3 won the second game, and player 2 won the fourth game. Thus, player 1 wins a prize, and each of the players 2 and 3 who wins the last game, also wins the other prize ($w_{26} = w_{36} = \frac{1}{2}$). In the case of a loss, each of players 2 and 3, gets zero ($l_{26} = l_{36} = 0$). Thus, there is always a unique mixed-strategy equilibrium in which players 2 and 3 randomize on the interval $[\,0, \frac{1}{2}]$, according to their effort cumulative distribution functions, which are given by

$$E_{26} = \frac{1}{2}F_{36}(x) - x = 0$$

$$E_{36} = \frac{1}{2}F_{26}(x) - x = 0$$

Then, player 2's probability of winning game 6 against player 3 is $p_{26} = \frac{1}{2}$.

Vertex 2

This game in Vertex 2 occurs only if player 1 won the first game, player 3 won the second game, player 1 won the third game, player 2 won the fourth game, and player 4 won the fifth one. Then if each of the players 2 and 3 wins the last game, he also wins the prize ($w_{26} = w_{36} = \frac{1}{2}$). In the case of a loss, each of the players gets nothing ($l_{26} = l_{36} = 0$). Thus, there is always a unique mixed-strategy equilibrium in which players 2 and 3 randomize on the interval $[\,0, \frac{1}{2}]$, according to their effort cumulative distribution functions, which are given by

$$E_{26} = \frac{1}{2}F_{36}(x) - x = 0$$

$$E_{36} = \frac{1}{2}F_{26}(x) - x = 0$$

Then, player 2's probability of winning game 6 against player 3 is $p_{26} = \frac{1}{2}$.

Vertex 3

Players 2 and 3 compete in Vertex 3 only if player 1 won all of his previous games (implying that he won first prize), player 3 won the second game, and player 4 won the fourth game. Then, if player 2 wins, he will have the same number of wins as the three other players and therefore, he gets a prize of $\frac{1}{2}$ with a probability of $\frac{1}{3}$. Thus, player 2's value in case of a win is $w_{26} = \frac{1}{6}$, but in case of a loss, he gets nothing ($l_{26} = 0$).

In that case, if player 3 wins, he wins the prize ($w_{36} = \frac{1}{2}$). On the other hand, if he loses, there will be three players with the same number of wins, and therefore, he gets a prize of $\frac{1}{2}$ with a probability of $\frac{1}{3}$. Thus, player 3's value of losing is $l_{36} = \frac{1}{6}$. Therefore, there is always a unique mixed-strategy equilibrium in which players 2 and 3 randomize on the interval $[0, \frac{1}{6}]$, according to their effort cumulative distribution functions, which are given by

$$E_{26} = \frac{1}{6}F_{36}(x) - x = 0$$

$$E_{36} = \frac{1}{2}F_{26}(x) + \frac{1}{6}(1 - F_{26}(x)) - x = \frac{1}{3}$$

Then, player 2's probability of winning game 6 against player 3 is $p_{26} = \frac{1}{4}$.

Vertex 4

Players 2 and 3 compete in Vertex 4 only if player 1 won the first and the third games, player 3 won the second game, and player 4 won the fourth and the fifth games. Then, even if player 2 wins, he will still get nothing, because two other players (players 1 and 4) have more wins. In that case, player 2's expected payoff is zero, and therefore he has no incentive to exert a positive effort. However, as we previously mentioned, we assume that each player obtains an additional prize for winning a single game of $m > 0$ where $m \to 0$. This assumption does not affect the players' behavior, but ensures the existence of equilibrium. Consequently, player 3 wins with certainty and will have the same number of wins as the two other players. Thus, he and each of these players get a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$, which implies that player 3's value of winning is $w_{36} = \frac{1}{3}$, but in case of a loss, he gets nothing ($l_{36} = 0$). Thus, we have

$$E_{26} = 0 \cdot F_{36}(x) - x = 0$$

$$E_{36} = \frac{1}{3} F_{26}(x) - x = \frac{1}{3}$$

Then, player 2's probability of winning in game 6 against player 3 is $p_{26} = 0$.

Vertex 5

Players 2 and 3 compete in Vertex 5 only if player 1 won the first and the fifth games, player 3 won the second and the third games, and player 2 won the fourth one. Then, if player 2 wins, he will have the same number of wins as the two other players and therefore, he as well as the other two players get a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$. This implies that player 2's value of winning is $w_{26} = \frac{1}{3}$, but in case of a loss, he gets nothing ($l_{26} = 0$).

If player 3 wins this game, he wins the prize ($w_{36} = \frac{1}{2}$). On the other hand, if he loses, there will be three players with the same number of wins, and therefore, he and each of the other two players get a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$. This implies that player 3's value of losing is $l_{36} = \frac{1}{3}$. Thus, there is always a unique mixed-strategy equilibrium in which players 2 and 3 randomize on the interval $[0, \frac{1}{6}]$, according to their effort cumulative distribution functions, which are given by

$$E_{26} = \frac{1}{3} F_{36}(x) - x = \frac{1}{6}$$

$$E_{36} = \frac{1}{2} F_{26}(x) + \frac{1}{3}(1 - F_{26}(x)) - x = \frac{1}{3}$$

Then player 2's probability of winning game 6 against player 3 is $p_{26} = \frac{3}{4}$.

Vertex 6

Players 2 and 3 compete in Vertex 6 only if player 1 won the first game, player 3 won the second and the third games, player 2 won the fourth game, and player 4 won the fifth one. Then, player 3 gets a prize regardless of whether or not he wins, whereas player 2 gets a prize only if he wins ($w_{26} = \frac{1}{2}$). Given our assumption that there is an additional prize for winning a single game of $m > 0$ where $m \to 0$ there exists an equilibrium. If player 2 loses, he will have the same number of wins as two other players. Therefore, he gets a prize of $\frac{1}{2}$ with a probability of $\frac{1}{3}$. Thus, player 2's value of losing is $l_{26} = \frac{1}{6}$, and players 2 and 3's cumulative distribution functions are given by

$$E_{26} = \frac{1}{2} F_{36}(x) + \frac{1}{6}(1 - F_{36}(x)) - x = \frac{1}{2}$$

$$E_{36} = \frac{1}{2} F_{26}(x) + \frac{1}{2}(1 - F_{26}(x)) - x = \frac{1}{2}$$

Then, player 2's probability of winning game 6 against player 3 is $p_{26} = 1$.

Game 5: Player 1 vs. Player 4

Vertex 7

As presented in Figure 1, players 1 and 4 compete in Vertex 7 only if player 1 won the first and the third games, player 3 won the second game, and player 2 won the fourth game. Then, both players have no incentive to exert efforts since even if player 1 loses he still gets a prize, since it is not possible that there will be a player with more wins than player 1. On the other hand, if player 4 wins, he will still not get any prize, because there will be two players with more wins than him. Thus, based on our assumption that there is an additional prize for winning a single game of $m > 0$ where $m \to 0$ we have an equilibrium where players 1 and 4's cumulative distribution functions are given by

$$E_{15} = \frac{1}{2}F_{45}(x) + \frac{1}{2}(1 - F_{45}(x)) - x = \frac{1}{2}$$

$$E_{45} = 0 \cdot F_{15}(x) - x = 0$$

In such a case, we assume an equal probability of winning for both players.

<u>Vertex 8</u>

Players 1 and 4 compete in Vertex 8 only if player 1 won the first and the third games, player 3 won the second game, and player 4 won the fourth one. Then, if player 1 wins, he will get a prize ($w_{15} = \frac{1}{2}$). If he loses, then his expected payoff depends on the result in the sixth game depicted in Vertex 4, and, as we have shown, player 3 wins that game with certainty. Thus, if player 1 loses in Vertex 8, he will have the same number of wins as the other two players, and therefore, he gets a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$. Thus, player 1's value of losing is $l_{15} = \frac{1}{3}$.

If player 4 wins, then his expected payoff depends on the result in the sixth game depicted in Vertex 4. As previously, player 4 will have the same number of wins as the two other players and therefore, he will also get a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$. Thus, player 4's value of winning is $w_{45} = \frac{1}{3}$. If, on the other hand, player 4 loses, his expected payoff depends on the result in the sixth game depicted in Vertex 3, and as we have shown, player 2 wins that game with a probability of $\frac{1}{4}$. Thus, if player 4 loses, he will have the same number of wins as the two other players and therefore he gets a prize of $\frac{1}{2}$ with a probability of $\frac{1}{3}$. This implies that player 4's value of losing is $l_{45} = \frac{1}{24}$. Should player 3 wins the sixth game, which happens with a probability of $\frac{3}{4}$, player 4 gets nothing.

Thus, there is always a unique mixed-strategy equilibrium in which players 1 and 4 randomize on the interval $[0, \frac{1}{6}]$, according to their effort cumulative distribution functions, which are given by

$$E_{15} = \frac{1}{2}F_{45}(x) + \frac{1}{3}(1 - F_{45}(x)) - x = \frac{1}{3}$$

$$E_{45} = \frac{1}{3}F_{15}(x) + \frac{1}{24}(1 - F_{15}(x)) - x = \frac{1}{6}$$

Then player 1's probability of winning game 5 against player 4 is $p_{15} = \frac{2}{7}$.

Vertex 9

Players 1 and 4 compete in Vertex 9 only if player 1 won the first game, player 3 won the second and the third games, and player 2 won the fourth one. In such a case, player 4 has no incentive to compete. The reason is that even if player 4 wins, his expected payoff depends on the outcome of the last game depicted in Vertex 6, and as we have shown, in that vertex, player 2 wins with certainty. Thus players 2 and 3 have more wins than player 4, and therefore even if he wins he gets nothing. Although player 4 has an expected payoff of zero, given our assumption that there is an additional prize for winning a single game of $m > 0$ where $m \to 0$ there exists an equilibrium.

Thus, since player 4 has no incentive to win, player 1 wins with certainty. In such a case, his expected payoff depends on the outcome of the sixth game depicted in Vertex 5. More specifically, if player 2 wins that game, which happens with a probability of $\frac{3}{4}$, there will be three players with two wins, such that each player gets a prize of $\frac{1}{2}$ with a probability of $\frac{2}{3}$. On the other hand, if player 3 wins the sixth game, which happens with a probability of $\frac{1}{4}$, player 1 wins the prize. Thus, player 1's value of winning is $w_{15} = \frac{3}{8}$ while his value of losing is $l_{15} = 0$. Therefore, players 1 and 4's cumulative distribution functions are given by

$$E_{15} = \frac{3}{8} F_{45}(x) - x = \frac{3}{8}$$

$$E_{45} = 0 \cdot F_{15}(x) - x = 0$$

Then, player 1's probability of winning game 5 against player 4 is $p_{15} = 1$.

Vertex 10

Players 1 and 4 compete in Vertex 10 only if player 1 won the first game, player 3 won the second and the third games, and player 4 won the fourth one. Then, if each of the players wins the last game, he also wins a prize together with player 3 ($w_{15} = w_{45} = \frac{1}{2}$). In the case of a loss, each of the players gets nothing ($l_{15} = l_{45} = 0$). Thus, there is always a unique mixed-strategy equilibrium in which players 1 and 4 randomize on the interval $[0, \frac{1}{2}]$, according to their effort cumulative distribution functions, which are given by

$$E_{15} = \frac{1}{2} F_{45}(x) - x = 0$$

$$E_{45} = \frac{1}{2} F_{15}(x) - x = 0$$

Then, player 1's probability of winning game 5 against player 4 is $p_{15} = \frac{1}{2}$.

Note that there will be no sixth game after Vertex 10. The reason is that the identity of the two winners (players 1 or 4 together with player 3) have already been determined.

Game 4: Player 2 vs. Player 4

Vertex 11

Players 2 and 4 compete in Vertex 11 only if player 1 won the first and the third games, and player 3 won the second one. In such a case, player 2 has no incentives to compete. The reason is that even if player 2 wins, his expected payoff would be zero, since by winning he will reach either Vertex 1 or Vertex 2 via Vertex 7 with the same probability. As we have shown, in Vertexes 1 and 2, player 2's expected payoff is zero, and therefore he has no incentive to exert any effort, but we have an equilibrium according to the assumption that each player obtains an additional prize for winning a single game of $m > 0$ where $m \to 0$. As such, player 2 loses with certainty, since player 4's expected payoff in Vertex 11 in the case of winning is positive as was depicted in Vertex 8.

Player 4's expected payoff depends on the outcome of the fifth game depicted in Vertex 8. More specifically, by winning the fourth game, which happens with certainty, player 4's expected payoff in Vertex 8 is $\frac{1}{6}$. Thus, player 4's value of winning in Vertex 11 is $w_{44} = \frac{1}{6}$. Then, Players 2 and 4's cumulative distribution functions are given by

$$E_{24} = 0 \cdot F_{44}(x) - x = 0$$

$$E_{44} = \frac{1}{6} F_{24}(x) - x = \frac{1}{6}$$

Thus, player 2's probability of winning game 4 against player 4 is $p_{24} = 0$.

Vertex 12

Players 2 and 4 compete in Vertex 12 only if player 1 won the first game, and player 3 won the second and the third ones. In such a case, player 4 has no incentive to compete. The reason is that even if player 4 wins, his expected payoff would be zero, since by winning he will reach Vertex 10, and as we have shown, player 4's expected payoff in Vertex 10 is zero. Therefore, player 4 has no incentive to exert any effort, but we have an equilibrium according to the assumption that each player obtains an additional prize for winning a single game of $m > 0$ where $m \to 0$. Thus, player 4 loses with certainty, since player 2's expected payoff is positive if he wins the following game as was depicted in Vertex 5 for which player 2 reaches with certainty via Vertex 9. Thus, player 2's expected payoff depends on the outcome in the sixth game depicted in Vertex 5. More specifically, by winning the fourth game which happens with certainty, player 2's expected payoff in Vertex 5 (which he also reaches with certainty) is $\frac{1}{6}$. Thus, player 2's value of winning is $w_{24} = \frac{1}{6}$ and his value of losing is $l_{24} = 0$.

Then, players 2 and 4's cumulative distribution functions are given by

$$E_{24} = \frac{1}{6} F_{44}(x) - x = \frac{1}{6}$$

$$E_{44} = 0 \cdot F_{24}(x) - x = 0$$

Thus, player 2's probability of winning in game 4 against player 4 is $p_{24} = 1$.

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