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## Abstract

We study best-of-three all-pay auctions with two players who compete in three stages with a single match per stage. The first player to win two matches wins the contest. We assume that a prize sum is given, and show that if players are symmetric, the allocation of prizes does not have any effect on the players' expected total effort. On the other hand, if players are asymmetric, in order to maximize the players' expected total effort, independent of the players' types, it is not optimal to allocate a single final prize to the winner. Instead, it is optimal to allocate intermediate prizes in the first stage or/and in the second stage in addition to the final prize. When the asymmetry of the players' types is sufficiently high, it is optimal to allocate intermediate prizes in both two first stages and a final prize to the winner.

JEL Classification: N/A

Keywords: N/A

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# On the Optimal Allocation of Prizes in Best-of-Three All-Pay Auctions

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February 13, 2020

**Abstract**

We study best-of-three all-pay auctions with two players who compete in three stages with a single match per stage. The first player to win two matches wins the contest. We assume that a prize sum is given, and show that if players are symmetric, the allocation of prizes does not have any effect on the players' expected total effort. On the other hand, if players are asymmetric, in order to maximize the players' expected total effort, independent of the players' types, it is not optimal to allocate a single final prize to the winner. Instead, it is optimal to allocate intermediate prizes in the first stage or/and in the second stage in addition to the final prize. When the asymmetry of the players' types is sufficiently high, it is optimal to allocate intermediate prizes in both two first stages and a final prize to the winner.

**JEL Classification Numbers** D72, D82, D44

**Keywords** Best-of-three contests, intermediate prizes, all-pay auctions

## 1 Introduction

In the literature on contest theory it has been shown that when players are risk neutral it is optimal for the designer who wishes to maximize the players' total effort to allocate the entire prize sum to a single first prize. Examples include the all-pay auction under complete information for which Barut and Kovenock

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(1998) found that if there are $n$ players who are symmetric, then any allocation of the entire prize sum into $k$ prizes, $k < n$, yields the same expected total effort, and that is also optimal to allocate one single prize. In the all-pay auction under incomplete information, Moldovanu and Sela (2001) showed that when cost functions are linear or concave in effort it is optimal to allocate the entire prize sum to a single first prize. Later (2006) these authors studied a two-stage all-pay auction with multiple prizes under incomplete information and showed that if the cost functions are linear in effort, it is optimal for a contest designer who wishes to maximize the expected total effort to allocate a single first prize in the last (second) stage. Similarly, in symmetric Tullock contests, Clark and Riis (1996, 1998) found that the contestants' total effort is maximized when only one prize is awarded, and Fu and Lu (2012) who studied multi-stage sequential elimination Tullock contests demonstrated that the optimal contest eliminates one contestant at each stage until the final one. Then, the winner of the final takes the entire prize sum. Our goal in this paper is to analyze the optimal allocation of prizes in best-of-three contests and, in particular, to examine whether or not it is optimal to allocate the entire prize sum to a single first prize.

The best-of-$k$ contest consists of a sequence of $k$ matches ($k$ is an odd integer) where the player who is first to win the majority of matches ($\frac{k+1}{2}$matches) wins the overall contest. Such contests can be found especially in sports (see Szymanski 2003 and Malueg and Yates 2009), but may also be observed in political races (see Klumpp and Polborn 2006) and also in the context of R&D (see Fudenberg et al. 1983 and Harris and Vickers 1985, 1987). Best-of-$k$ contests are played between either two teams where in each stage different players compete, or between two players who compete in all the stages until the winner is decided. We consider the version of best-of-three contests between two players. Such contests have been analyzed when each match is modeled as either an all-pay auction (Konrad and Kovenock 2009, Sela 2011, and Krumer 2015) or a Tullock contest (Klumpp and Polborn 2006, Malueg and Yates 2006, and Mago et al. 2013), or a rank order tournament (Ferall and Smith 1999).

How to optimally allocate prizes in best-of-$k$ contests is a challenging task since in each of the stages except the first, one of the players may have an advantage over his opponent if he performed better previously (see, for example, Klumpp and Polborn 2006, and Malueg and Yates 2010). This advantage could be so considerable that in one of the stages, a player might find himself with an expected payoff of zero, in which case he will have no incentive to keep on playing. To deal with this problem, allocating intermediate prizes might encourage such a player to continue playing until the end. Indeed, Konrad and Kovenock (2009), characterized the unique equilibrium in best-of-$k$ contests with two players ($k \geq 3$) where players are matched in an all-pay auction, assuming that players are given an additional identical intermediate prize in each of the matches and as such each player has an incentive to exert effort in each match. Sela (2011), furthermore, showed that allocating prizes to the losers in each stage may increase the players' total effort, and Mago et al. (2013) by an experimental analysis of a best-of three Tullock contest, demonstrated that intermediate prizes lead to higher efforts. Fu et al. (2015) investigated a contest between two teams that compete in a best-of-$k$ contest and found that when allocating intermediate prizes, the probability of winning in every single match depends on the players' types and not on the outcome of the previous matches.

Several real-life best-of-$k$ contests with intermediate prizes can be mentioned. Iqbal and Krumer (2019) used data from tennis matches in a Davis Cup tournament which is a best-of-five contest. Based on the fact that since 2009, the Association of Tennis Professionals (ATP) decided to assign intermediate prizes in the form of ranking points to the winner of a single match in the Davis Cup, they examined the performance of players in tournaments with (after 2009) and without (before 2009) intermediate prizes. They found that the intermediate prizes have a significant effect on performance. Likewise, since 2010 in the best-of-five contests of the Federation Internationale de Volleyball (FIVB) World League (for men) and World Grand Prix (for women) the winning team wins 3 match points and the losing team wins 0, if the final set score is either 3-0 or 3-1. Otherwise, if it is 3-2, the winning team wins 2 match points and the losing team wins 1 (see Jiang 2018).

The fact that allocating intermediate prizes increases the players' effort in best-of-$k$ contests is not surprising since in almost any form of contest, additional prizes increase the players' efforts. The interesting question, however, is whether or not intermediate prizes should be allocated in multi-stage contests when the prize sum is fixed such that the main prize for the winner of the contest is automatically reduced. For this purpose, Feng and Lu (2018) characterized the effort-maximizing prize allocation in multi-battle Tullock contests when the prize sum is fixed and the players' prizes contingent on the number of wins. As we mentioned, we also study the optimal allocation of prizes in a best-of-three contest with two players in which the players' types (abilities or alternatively values of winning) are common knowledge but each match is modelled as an all-pay auction. Feng and Lu assume that the players are symmetric while we assume that the players might be asymmetric, namely, they might have different abilities (cost functions) or, alternatively, different values for winning the entire contest or each of the matches in the contest. In addition, Feng and Lu assume that each player may win only one prize, while in our model an intermediate prize is allocated in each of the first two matches and a final prize is allocated for the winner. As such, each player can win more than one prize. Actually the maximal number of prizes that each player can win is three and that happens if he wins the first two matches. If a player wins two matches where one of them is in the last third stage, then he wins two prizes, and if he wins one match only he wins a single prize. Note that the values of the prizes in our model depend on the stage such that if a player wins only in the first stage he might get a different prize than if he wins only in the second stage.

Our results show that when the entire prize sum is fixed, allocating intermediate prizes in our contest with two symmetric players does not have any effect on the players' expected total effort. In other words, the players' expected total effort is the same whether or not intermediate prizes are allocated. If, on the other hand, the players are asymmetric, it is quite intuitive that the intermediate prizes are effective since they encourage the weaker player to keep on playing. But when the players' asymmetry is not sharp and none of the players stop playing in any stage, the effect of the intermediate prizes on the players' expected total effort is not clear at all. Nonetheless, we show that a best-of-three all-pay auction with a single final prize is not optimal for a designer who wishes to maximize the players' expected total effort. In particular, we show that when the prize sum is fixed, compared to the best-of-three all-pay auction with a single final prize, allocating an intermediate prize in the first stage increases the players' expected total effort, and similarly allocating an intermediate prize in the second stage increases the players' expected total effort as well. Furthermore, compared to the best-of-three all-pay auction with a single final prize, allocating intermediate prizes in the first stage and the second stage together also increases the players' expected total effort. In addition, when the prize sum is fixed and asymmetry between the players is sufficiently high, we prove that in order to maximize the players' expected total effort, allocating intermediate prizes in both of the first stages is better than allocating an intermediate prize in only one of these stages. These results demonstrate that in any best-of-three all-pay auction, independent of the asymmetry of the players, it is optimal to allocate intermediate prizes. From this perspective, the best-of three all-pay auction differs from the one-stage all-pay auction and even from other multi-stage contest forms in which it is optimal to allocate the entire prize sum to a single one.

The rest of the paper is organized as follows: Section 2 introduces our best-of-three all-pay auction. In Section 3 and 4 we analyze the allocation of prizes in this contest with symmetric players and asymmetric players, respectively. Section 5 concludes. The proofs appear in the Appendix.

## 2 The model

Consider two players (or teams) $i = 1, 2$ who compete in a best-of-three all-pay auction such that they compete in sequential matches, and the first to win two matches wins the contest. We model each match as an all-pay auction: both players exert efforts, and the one exerting the higher effort wins. Participating in the contest generates a (sunk) cost $\frac{x_i}{v_i}$ for player $i$, where $x_i$ is the the effort of player $i$ and $v_i$ is his type (ability). Player $i$'s type $v_i$ is common knowledge. We assume that $v_1 \geq v_2$, namely, the ability of player 1 is larger than that of player 2. The prize sum is normalized to be 1. The designer awards a prize for the winner of the contest $\gamma > 0$; he could also award an intermediate prize for the winner in the first stage $\alpha \geq 0$ and an intermediate prize for the winner of the second stage $\beta \geq 0$ such that $\alpha + \beta + \gamma = 1$. Note that this model is strategically equivalent to a model where all the players have the same marginal cost of 1, but they have different values of the prizes. Then, if player $i$ wins in the first stage he has a value of $\alpha v_i$; if he wins in the second stage he has a value of $\beta v_i$; and if he wins the entire contest he has a value of $\gamma v_i = (1 - \alpha - \beta) v_i$. The designer's goal is to choose the prize allocation that maximizes the players' expected total effort.

## 3 The best-of-three all-pay auction with symmetric players

In order to analyze the subgame-perfect equilibrium of the best-of-three all-pay auction, we begin with the last stage and go backwards to the previous stages. We assume first that players are symmetric and we denote the players' type by $v = v_1 = v_2$.

### 3.1 Stage 3

The players compete in the last stage only if each player won one of the previous matches. Therefore, the expected value of player $i$ if he wins the match in stage 3 is $v(1-\alpha-\beta)$ and if he loses, it is zero. Thus, based on the analysis of the one-stage all-pay auction (Hillman and Reily 1989 and Baye et al. 1996), players 1 and 2 randomize on the interval $[0, v(1-\alpha-\beta)]$ according to their cumulative distribution function $F^{(3)}$which is given by

$$v(1-\alpha-\beta)F^{(3)}(x) - x = 0 \tag{1}$$

The players' probabilities of winning in this stage are $p_1^{(3)} = p_2^{(3)} = 0.5$ and their expected total effort is

$$TE^{(3)} = v(1-\alpha-\beta)$$

### 3.2 Stage 2

Without loss of generality, assume that player 1 won the first match in stage 1. Then, if player 2 wins in this stage, by (1), his payoff is $\beta v$, but if he loses his payoff is zero. On the other hand, if player 1 wins in this stage, he wins the contest, and his payoff is $v(1-\alpha)$, but if he loses, by (1), his expected payoff in the next stage is zero. Thus, since $1-\alpha \geq \beta$, we obtain that players 1 and 2 randomize on the interval $[0, \beta v]$ according to their effort cumulative distribution functions $F_i^{(2)}, i = 1, 2$ which are given by

$$\begin{aligned} v(1-\alpha)F_2^{(2)}(x) - x &= v(1-\alpha-\beta) \\ \beta v F_1^{(2)}(x) - x &= 0 \end{aligned} \tag{2}$$

The players' probabilities of winning in this stage are

$$p_1^{(2)} = 1 - \frac{\beta}{2(1-\alpha)}, p_2^{(2)} = 1 - p_1^2$$

and their expected total effort is

$$TE^{(2)} = \frac{\beta v}{2}(1 + \frac{\beta v}{v(1-\alpha)}) = \frac{v\beta(1-\alpha+\beta)}{2(1-\alpha)}$$

### 3.3 Stage 1

If player 1 or 2 wins, by (2), his payoff is $v(1-\beta)$, but if he loses his expected payoff in the next stage is zero. Thus, we obtain that players 1 and 2 randomize on the interval $[0, v(1-\beta)]$ according to their effort cumulative distribution function $F^{(1)}, i = 1, 2$, which is given by

$$v(1-\beta)F^{(1)}(x) - x = 0$$

The players' probabilities of winning in this stage are $p_1^{(1)} = p_2^{(1)} = 0.5$ and their expected total effort is

$$\begin{aligned} TE^{(1)} &= v(1-\beta) + \frac{v\beta(1-\alpha+\beta)}{2(1-\alpha)} \\ &\quad +2(1-\frac{\beta}{2(1-\alpha)}) \end{aligned}$$

### 3.4 Results

The expected total effort in all the stages of the best-of-three contest is

$$\begin{aligned} TE &= TE^{(1)} + TE^{(2)} + 2p_2^{(1)}p_1^{(2)}TE^{(3)} \\ &= v(1-\beta) + \frac{v\beta(1-\alpha+\beta)}{2(1-\alpha)} \\ &\quad +2\frac{1}{2}\frac{\beta}{2(1-\alpha)}v(1-\alpha-\beta) = v \end{aligned}$$

Thus, we can conclude that

**Proposition 1** *The expected total effort in the best-of-three all-pay auction with symmetric players does not depend on the allocation of the prize sum among the stages, and as such, allocation of intermediate prizes do not change the players' expected total effort.*

In contrast, in the next section when the players are asymmetric, we show that the intermediate prizes have a meaningful effect on the players' expected total effort.

# 4 The best-of-three all-pay auction with asymmetric players

In order to analyze the subgame-perfect equilibrium of the best-of-three all-pay auction with asymmetric players, we begin with the last stage and go backwards to the previous stages. Usually, if players are asymmetric, they also have different expected values in all the stages, and then according to Baye et al. (1996), the subgame-perfect equilibrium of the best-of-three all-pay auction that we present is unique.

### 4.1 Stage 3

The players compete in the last stage only if each player won one of the previous matches. Therefore, the expected value of player $i$ if he wins the match in stage 3 is $v_i(1-\alpha-\beta)$ and if he loses, it is zero. Thus, based on the analysis of the one-stage all-pay auction, since $v_1 \geq v_2$, players 1 and 2 randomize on the interval $[0, v_2(1-\alpha-\beta)]$ according to their cumulative distribution functions $F_i^{(3)}, i = 1, 2$ which are given by

$$\begin{aligned} v_1(1-\alpha-\beta)F_2^{(3)}(x) - x &= (v_1 - v_2)(1-\alpha-\beta) \\ v_2(1-\alpha-\beta)F_1^{(3)}(x) - x &= 0 \end{aligned} \tag{3}$$

The players' probabilities of winning in this stage are

$$p_1^{(3)} = 1 - \frac{v_2}{2v_1}, p_2^{(3)} = 1 - p_1^{(3)}$$

and their expected total effort is

$$TE^{(3)} = \frac{v_2(1-\alpha-\beta)}{2}(1 + \frac{v_2}{v_1}) = \frac{v_2(1-\alpha-\beta)(v_1+v_2)}{2v_1}$$

### 4.2 Stage 2

**Case A:** Assume first that player 1 won the first match in stage 1. Then, if player 2 wins in this stage, by (1), his payoff is $\beta v_2$, but if he loses his payoff is zero. Similarly, if player 1 wins in this stage, he wins the contest, and his payoff is $v_1(1-\alpha)$, but if he loses, by (1), his expected payoff in the next stage is $(v_1 - v_2)(1-\alpha-\beta)$. As such, we obtain that player's 1 expected value (the difference of his payoff when he wins and when he loses) is larger than player 2's expected value; that is,

$$\begin{aligned} & [v_1(1-\alpha) - (v_1 - v_2)(1-\alpha-\beta)] - [\beta v_2] \\ = \quad & \beta(v_1 - v_2) + v_2(1-\alpha-\beta) \geq 0 \end{aligned}$$

The last inequality holds since $v_1 \geq v_2$ and $\alpha + \beta < 1$. Thus, we obtain that players 1 and 2 randomize on the interval $[0, \beta v_2]$ according to their effort cumulative distribution functions $F_i^{(2)}, i = 1, 2$ which are given by

$$\begin{aligned} v_1(1-\alpha)F_2^{(2)}(x) + (v_1 - v_2)(1-\alpha-\beta)(1 - F_2^{(2)}(x)) - x &= v_1(1-\alpha) - \beta v_2 \\ \beta v_2 F_1^{(2)}(x) - x &= 0 \end{aligned} \tag{4}$$

The players' probabilities of winning in this stage are

$$\begin{aligned} p_1^{(2)} &= 1 - \frac{\beta v_2}{2(v_1(1-\alpha) - (v_1 - v_2)(1-\alpha-\beta))} = 1 - \frac{\beta v_2}{2(\beta v_1 + v_2(1-\alpha-\beta))} \\ p_2^{(2)} &= 1 - p_1^{(2)} \end{aligned}$$

and their expected total effort is

$$TE^{(2)} = \frac{\beta v_2}{2}(1 + \frac{\beta v_2}{v_1(1-\alpha) - (v_1 - v_2)(1-\alpha-\beta)})$$

**Case 2B:** Assume now that player 2 won the first match in stage 1. Then, if player 1 wins in this stage, by (1), his expected payoff is $(v_1 - v_2)(1-\alpha-\beta) + \beta v_1 = v_1(1-\alpha) - v_2(1-\alpha-\beta)$, but if he loses, his payoff is zero. Similarly, if player 2 wins in this stage, he wins the contest, and then his payoff is $v_2(1-\alpha)$, but if he loses, by (1), his expected payoff in the next stage is zero. Now we have two subcases. In the first one, player 1's expected value (the difference between his expected payoffs when he wins and loses) is larger than that of player 2, and in the second, player1' expected value is smaller than that of player 2.

**Case 2B1:** Player 1's expected value in the second stage is larger than that of player 2, namely, $v_1(1-\alpha) - v_2(1-\alpha-\beta) > v_2(1-\alpha)$ or equivalently,

$$v_1(1-\alpha) - v_2(2 - 2\alpha - \beta) \geq 0 \tag{5}$$

Then, we obtain that players 1 and 2 randomize on the interval $[0, v_2(1-\alpha)]$ according to their effort cumulative distribution functions $F_i^{(2B1)}, i = 1, 2$ which are given by

$$\begin{aligned} [v_1(1-\alpha) - v_2(1-\alpha-\beta)]\, F_2^{(2B1)}(x) - x &= v_1(1-\alpha) - v_2(2 - 2\alpha - \beta) \\ v_2(1-\alpha)F_1^{(2B1)}(x) - x &= 0 \end{aligned} \tag{6}$$

The players' probabilities of winning in this case are

$$\begin{aligned} p_1^{(2B1)} &= 1 - \frac{v_2(1-\alpha)}{2(v_1(1-\alpha) - v_2(1-\alpha-\beta))} \\ p_2^{(2B1)} &= 1 - p_1^{(2B1)} \end{aligned}$$

and their expected total effort is

$$TE^{(2B1)} = \frac{v_2(1-\alpha)}{2}(1 + \frac{v_2(1-\alpha)}{v_1(1-\alpha) - v_2(1-\alpha-\beta)})$$

**Case 2B2**: Player 2's expected value in the second stage is larger than that of player 1, namely,

$$v_1(1-\alpha) - v_2(2-2\alpha-\beta) < 0 \tag{7}$$

Then, we obtain that players 1 and 2 randomize on the interval $[0, v_1(1-\alpha) - v_2(1-\alpha-\beta)]$ according to their effort cumulative distribution functions $F_i^{(2B2)}, i = 1, 2$ which are given by

$$\begin{aligned} [v_1(1-\alpha) - v_2(1-\alpha-\beta)]\, F_2^{(2B2)}(x) - x &= 0 \\ v_2(1-\alpha)F_1^{(2B2)}(x) - x &= v_2(2-2\alpha-\beta) - v_1(1-\alpha) \end{aligned} \tag{8}$$

The players' probabilities of winning in this case are

$$\begin{aligned} p_1^{(2B2)} &= \frac{v_1(1-\alpha) - v_2(1-\alpha-\beta)}{2v_2(1-\alpha)} \\ p_2^{(2B2)} &= 1 - p_1^{(2B2)} \end{aligned}$$

and their expected total effort is

$$TE^{(2B2)} = \frac{v_1(1-\alpha) - v_2(1-\alpha-\beta)}{2}(1 + \frac{v_1(1-\alpha) - v_2(1-\alpha-\beta}{v_2(1-\alpha)})$$

## 4.3 Stage 1

If player 1 wins, by (4), his expected payoff is $(v_1(1-\alpha) - \beta v_2) + \alpha v_1 = v_1 - \beta v_2$. But if he loses, given that condition 5 holds, by (6), his expected payoff is $v_1(1-\alpha) - v_2(2-2\alpha-\beta)$, and if condition 7 holds, by (8), his expected payoff is zero.

Similarly, if player 2 wins, given that condition 5 holds, by (6), his expected payoff is $\alpha v_2$ , and if condition 7 holds, by (8), his expected payoff is $v_2(2-2\alpha-\beta) - v_1(1-\alpha) + \alpha v_2 = v_2(2-\alpha-\beta) - v_1(1-\alpha)$. If player 2 loses, by (4), his expected payoff is zero. Thus, if condition 5 holds, we obtain that players 1 and 2 randomize on the interval $[0, \alpha v_2]$ according to their effort cumulative distribution functions $F_i^{(1B1)}, i = 1, 2$, which are given by

$$\begin{aligned} (v_1 - \beta v_2)F_2^{(1B1)}(x) + (v_1(1-\alpha) - v_2(2-2\alpha-\beta))(1 - F_2^{(1B1)}(x)) - x &= v_1 - v_2(\alpha+\beta) \\ \alpha v_2 F_1^{(1B1)}(x) - x &= 0 \end{aligned} \tag{9}$$

The players' probabilities of winning in this case are

$$\begin{aligned} p_1^{(1B1)} &= 1 - \frac{\alpha v_2}{2((v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta)))} \\ p_2^{(1B1)} &= 1 - p_1^{(1B1)} \end{aligned}$$

and their expected total effort is

$$TE^{(1B1)} = \frac{\alpha v_2}{2}\left(1 + \frac{\alpha v_2}{(v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta))}\right)$$

If condition 7 holds, we obtain that players 1 and 2 randomize on the interval $[0, v_2(2-\alpha-\beta) - v_1(1-\alpha)]$ according to their effort cumulative distribution functions $F_i^{(1B2)}, i = 1, 2$, which are given by

$$\begin{aligned} (v_1 - \beta v_2)F_2^{(1B2)}(x) - x &= (v_1 - v_2)(2-\alpha) \\ (v_2(2-\alpha-\beta) - v_1(1-\alpha))F_1^{(1B2)}(x) - x &= 0 \end{aligned} \tag{10}$$

The players' probabilities of winning in this case are

$$\begin{aligned} p_1^{(1B2)} &= 1 - \frac{v_2(2-\alpha-\beta) - v_1(1-\alpha)}{2(v_1 - \beta v_2)} \\ p_2^{(1B2)} &= 1 - p_1^{(1B2)} \end{aligned}$$

and their expected total effort is

$$TE^{(1B2)} = \frac{v_2(2-\alpha-\beta) - v_1(1-\alpha)}{2}\left(1 + \frac{v_2(2-\alpha-\beta) - v_1(1-\alpha)}{v_1 - \beta v_2}\right)$$

When condition 5 holds, the players' expected total effort in all the stages is

$$TE^{(B1)} = TE^{(1B1)} + p_1^{(1B1)}TE^{(2)} + p_2^{(1B1)}TE^{(2B1)} + (p_1^{(1B1)}p_2^{(2)} + p_2^{(1B1)}p_1^{(2B1)})TE^{(3)} \tag{11}$$

and when condition 7 holds, their expected total effort in all the stages is

$$TE^{(B2)} = TE^{(1B2)} + p_1^{(1B2)}TE^{(2)} + p_2^{(1B2)}TE^{(2B2)} + (p_1^{(1B2)}p_2^{(2)} + p_2^{(1B2)}p_1^{(2B2)})TE^{(3)} \tag{12}$$

### 4.4 Results

Without loss of generality we assume in this section that the players' types satisfy $v = v_1 \geq v_2 = 1$. Let $(\alpha, \beta, \gamma)$ be the prizes, where $\alpha$ is the intermediate prize for the winner in the first stage, $\beta$ is the prize for the winner in the second stage , and $\gamma = 1 - (\alpha + \beta)$ is the prize for the winner of the contest. When an intermediate prize is allocated only in the first stage, we have

**Proposition 2** *There is a value* $1 > \alpha > 0$ *for which the players' expected total effort in the best-of-three all-pay auctions under the allocation of prizes* $(\alpha, 0, 1-\alpha)$ *is larger than under the allocation of prizes* $(0, 0, 1)$. *Thus, optimally allocating an intermediate prize in the first stage increases the players' expected total effort compared to the best-of-three all-pay auction with a single final prize.*

**Proof.** See Appendix. ■

When an intermediate prize is allocated only in the second stage, we have

**Proposition 3** *There is a value* $1 > \beta > 0$ *for which the players' expected total effort in the best-of-three all-pay auctions under the allocation of prizes* $(0, \beta, 1-\beta)$ *is larger than under the allocation of prizes* $(0, 0, 1)$. *Thus, optimally allocating an intermediate prize in the second stage increases the players' expected total effort compared to the best-of-three all-pay auction with a single final prize.*

**Proof.** See Appendix. ■

When intermediate prizes are allocated in both the first and the second stages, we have

**Proposition 4** *There is a value* $1 > \alpha > 0$ *for which the players' expected total effort in the best-of-three all-pay auctions under the allocation of prizes* $(\alpha, \alpha, 1 - 2\alpha)$ *is larger than under the allocation of prizes* $(0, 0, 1)$. *Thus, optimally allocating intermediate prizes in the first and second stages of the best-of-three all-pay auction increases the players' expected total effort compared to the best-of-three all-pay auction with a single final prize.*

**Proof.** See Appendix. ■

The following result shows that if the asymmetry between the players is sufficiently large, namely $v_1 >> v_2$, then allocating intermediate prizes in both first stages is more efficient than allocating an intermediate prize in only one of the first stages.

**Proposition 5** *When the prize sum is fixed and asymmetry between the players is sufficiently high, in order to maximize the players' expected total effort, it is optimal to allocate intermediate prizes in both stages of the best-of-three all-pay auction.*

# 5 Concluding remarks

In several forms of contests, including multi-stage contests, when the prize sum is fixed it is optimal to allocate the entire prize sum to a single first prize that is awarded to the winner in the last stage of the contest. We show rather, that in the best-of-three all-pay auction with asymmetric players, independent of the players' types, if the goal of the designer is to maximize the players' expected total effort, allocating a single prize to the winner is not optimal. In that case, it is always optimal to allocate an intermediate prize either in the first stage and/or the second one. We also show that when the asymmetry between the players is sufficiently high, in order to maximize the players' expected total effort, it is optimal to allocate intermediate prizes in both first stages in addition to the final prize. It is important to note that we do not find the optimal relation between the final prize for the winner of the contest and these intermediate prizes since this beyond of the scope of the paper. According to our results as well as those of Sela (2011) that the expected total effort in the best-of-three all-pay auction is smaller than in the one-stage all-pay auction when the entire prize sum is awarded as a single prize, it would be interesting to verify whether or not the expected total effort in the best-of-three all-pay auction with intermediate prizes is still smaller than in the one-stage all-pay auction. This challenging goal requires that the optimal values of the intermediate prizes be known which according to the complexity of the present analysis is not a simple task.

# 6 Appendix

## 6.1 Proof of Proposition 2

1) If condition 5 holds, the expected total effort given by (11) is

$$\begin{aligned}
TE^{(B1)} &= TE^{(1B1)} + p_1^{(1B1)} TE^{(2)} + p_2^{(1B1)} TE^{(2B1)} + p_1^{(1B1)} p_2^{(2)} TE^{(3)} + p_2^{(1B1)} p_1^{(2B1)} TE^{(3)} \qquad (13)\\
&= \frac{\alpha v_2}{2}(1 + \frac{\alpha v_2}{(v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta))})\\
&+(1 - \frac{\alpha v_2}{2((v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta)))})\frac{\beta v_2}{2}(1 + \frac{\beta v_2}{v_1(1-\alpha) - (v_1 - v_2)(1-\alpha-\beta)})\\
&+\frac{\alpha v_2}{2((v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta)))}\frac{v_2(1-\alpha)}{2}(1 + \frac{v_2(1-\alpha)}{v_1(1-\alpha) - v_2(1-\alpha-\beta)})\\
&+(1 - \frac{\alpha v_2}{2((v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta)))})\frac{\beta v_2}{2(\beta v_1 + v_2(1-\alpha-\beta))}\frac{v_2(1-\alpha-\beta)(v_1+v_2)}{2v_1}\\
&+\frac{\alpha v_2}{2((v_1 - \beta v_2) - (v_1(1-\alpha) - v_2(2-2\alpha-\beta)))}(1 - \frac{v_2(1-\alpha)}{2(v_1(1-\alpha) - v_2(1-\alpha-\beta))})\frac{v_2(1-\alpha-\beta)(v_1+v_2)}{2v_1}
\end{aligned}$$

Let $v_1 = v, v_2 = 1$ and $\beta = 0$. Then we have

$$\begin{aligned}
TE_\alpha^{(B1)} &= \frac{\alpha}{2} + \frac{\alpha^2}{2\alpha v + 4 - 4\alpha} + \frac{\alpha - \alpha^2}{2(2\alpha v + 4 - 4\alpha)} + (\frac{\alpha}{2\alpha v + 4 - 4\alpha})(\frac{(1-\alpha)^2}{2v - 2\alpha v - 2 + 2\alpha})\\
&+(\frac{v + 1 - \alpha v - \alpha}{2v})(\frac{\alpha}{2(\alpha v + 2 - 2\alpha})(1 - \frac{1-\alpha}{2v - 2\alpha v - 2 + 2\alpha})
\end{aligned}$$

The marginal effect of $\alpha$ on the expected total effort is

$$\frac{dTE_\alpha^{(B1)}}{d\alpha} = \frac{4\alpha^2 v^3 + 16(\alpha - \alpha^2)v^2 + (13\alpha^2 - 32\alpha + 24)v + 6\alpha^2 - 12\alpha + 6}{8\alpha^2 v^3 + 32(\alpha - \alpha^2)v^2 + 32(\alpha^2 - 2\alpha + 1)v} \qquad (14)$$

when $\alpha$ approaches zero we obtain that

$$\lim_{\alpha \to 0} \frac{dTE_\alpha^{(B1)}}{d\alpha} = \frac{24v + 6}{32v}$$

Thus, if condition 5 holds, allocating a sufficiently small prize of $\alpha$ in the first stage, increases the players' expected total effort.

2) If condition 7 holds, the expected total effort given by (12) is

$$
\begin{aligned}
TE_{\alpha}^{(B2)} \quad = \quad & \frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{2}(1+\frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{v_1-\beta v_2}) \\
& +(1-\frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{2(v_1-\beta v_2)})\frac{\beta v_2}{2}(1+\frac{\beta v_2}{v_1(1-\alpha)-(v_1-v_2)(1-\alpha-\beta)}) \\
& +\frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{2(v_1-\beta v_2)}\frac{v_1(1-\alpha)-v_2(1-\alpha-\beta)}{2}(1+\frac{v_1(1-\alpha)-v_2(1-\alpha-\beta}{v_2(1-\alpha)}) \\
& +(1-\frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{2(v_1-\beta v_2)})\frac{\beta v_2}{2(\beta v_1+v_2(1-\alpha-\beta))}\frac{v_2(1-\alpha-\beta)(v_1+v_2)}{2v_1} \\
& +\frac{v_2(2-\alpha-\beta)-v_1(1-\alpha)}{2(v_1-\beta v_2)}\frac{v_1(1-\alpha)-v_2(1-\alpha-\beta)}{2v_2(1-\alpha)}\frac{v_2(1-\alpha-\beta)(v_1+v_2)}{2v_1}
\end{aligned}
\tag{15}
$$

Let $v_1 = v, v_2 = 1$ and $\beta = 0$. Then we have

$$
\begin{aligned}
TE_{\alpha}^{(B2)} \quad = \quad & \frac{2-\alpha-v(1-\alpha)}{2}+\frac{(2-\alpha-v(1-\alpha))^2}{2v} \\
& +\frac{(2-\alpha-v(1-\alpha))}{2v}(\frac{v(1-\alpha)-1+\alpha}{2}+\frac{(v(1-\alpha)-1+\alpha)^2}{2(1-\alpha)}) \\
& +\frac{(2-\alpha-v(1-\alpha))}{2v}(\frac{v(1-\alpha)-1+\alpha}{2(1-\alpha)})(\frac{v-\alpha v+1-\alpha}{2v})
\end{aligned}
$$

The marginal effect of $\alpha$ on the expected total effort is

$$
\frac{dTE_{\alpha}^{(B2)}}{d\alpha}=\frac{4(\alpha-1)v^4+(12-14\alpha)v^3+(18-23)v^2+(18-10\alpha)v+2\alpha-3}{8v^2} \tag{16}
$$

When $\alpha$ approaches zero we obtain that

$$
\lim_{\alpha\to 0}\frac{dTE_{\alpha}^{(B2)}}{d\alpha}=-\frac{-4v^4+12v^3-23v^2+18v-3}{8v^2}
$$

It can be verified that the last term is positive for all $v > 1$. Thus, if condition 7 holds, compared to the best-of-three all-pay auction with a single final prize, allocating a sufficiently small prize of $\alpha$ in the first stage increases the players' expected total effort.

## 6.2 Proof of Proposition 3

1) Let $v_1 = v, v_2 = 1$, and $\alpha = 0$. Then, if condition 5 holds, the expected total effort given by (13) is

$$
TE_{\beta}^{(B1)}=\frac{\beta}{2}+\frac{\beta^2}{2\beta v+2-2\beta}+\frac{(1-\beta)(v+1)}{2v}\frac{\beta}{2\beta v+2-2\beta}
$$

The marginal effect of $\beta$ on the expected total effort is

$$
\frac{dTE_{\beta}^{(B1)}}{d\beta}=\frac{2\beta^2v^3+(4\beta-3\beta^2)v^2+(3-2\beta)v+\beta^2-2\beta+1}{4\beta^2v^3+8(\beta-\beta^2)v^2+4(\beta^2-2\beta+1)v} \tag{17}
$$

when $\beta$ approaches zero we obtain that

$$\lim_{\beta \to 0} \frac{dTE_{\beta}^{(B1)}}{d\beta} = \frac{3v+1}{4}$$

Thus, if condition 5 holds, allocating a sufficiently small prize of $\beta$ in the second stage increases the players' expected total effort.

2) Let $v_1 = v, v_2 = 1$, and $\alpha = 0$. Then if condition 7 holds, the expected total effort given by (15) is

$$\begin{aligned} TE_{\beta}^{(B2)} &= \frac{2-\beta-v}{2} + \frac{(2-\beta-v)^2}{2(v-\beta)} \\ &+ \frac{3v-\beta-2}{2(v-\beta)}\left(\frac{\beta}{2} + \frac{\beta^2}{2(\beta v+1-\beta)}\right) \\ &+ \frac{2-\beta-v}{2(v-\beta)}\left(\frac{v-1+\beta}{2} + \frac{(v-1+\beta)^2}{2}\right) \\ &+ \frac{3v-\beta-2}{2(v-\beta)} \frac{\beta}{2(\beta v+1-\beta)} \frac{v+1-\beta v-\beta}{2v} \\ &+ \frac{2-\beta-v}{2(v-\beta)} \frac{v-1+\beta}{2} \frac{v+1-\beta v-\beta}{2v} \end{aligned}$$

The marginal effect of $\beta$ on the expected total effort is

$$\frac{dTE_{\beta}^{(B2)}}{d\beta} = \frac{A}{B} \tag{18}$$

where

$$\begin{aligned} A &= -7\beta^2 v^6 - (8\beta^3 - 41\beta^2 + 14\beta)v^5 - (-\beta^4 - 36\beta^3 + 116\beta^2 - 68\beta + 7)v^4 \\ &\quad -(-2\beta^5 + 9\beta^4 + 56\beta^3 - 18\beta^2 + 140\beta - 30)v^3 - (6\beta^5 - 24\beta^4 - 24\beta^3 + 144\beta^2 - 136\beta + 41)v^2 \\ &\quad -(-6\beta^5 + 25\beta^4 - 16\beta^3 - 40\beta^2 + 54\beta - 20)v - (2\beta^5 - 9\beta^4 + 12\beta^3 - 3\beta^2 - 4\beta + 2) \end{aligned} \tag{19}$$

and

$$\begin{aligned} B &= \beta^2 v^5 + 16(-\beta^3 - \beta^2 + \beta)v^4 + 8(\beta^4 + 4\beta^3 - 3\beta^2 - 2\beta + 1)v^3 \\ &\quad +16(-\beta^4 + 2\beta^2 - \beta)v^2 + 8(\beta^4 - 2\beta^3 + \beta^2)v \end{aligned} \tag{20}$$

When $\beta$ approaches zero we obtain that

$$\lim_{\beta \to 0} \frac{dTE_{\beta}^{(B2)}}{d\beta} = \frac{-7v^4 + 30v^3 - 41v^2 + 20v - 2}{8v^3}$$

It can be verified that the last term is positive for all $1 < v \leq 2$. Since condition 7 holds if $1 \leq v < 2 - \beta$, we obtain that compared to the best-of-three all-pay auction with a single final prize, allocating a sufficiently small prize of $\beta$ in the second stage increases the players' expected total effort.

## 6.3 Proof of Proposition 4

1) Let $v_1 = v, v_2 = 1$, and $\alpha = \beta$. Then, if condition 5 holds, the expected total effort given by (13) is

$$\begin{aligned} TE^{(B1)}_{\alpha\beta} &= \frac{\alpha}{2} + \frac{\alpha^2}{2(\alpha v + 2 - 4\alpha)} + (1 - \frac{\alpha}{2(\alpha v + 2 - 4\alpha)}(\frac{\alpha}{2} + \frac{\alpha^2}{2(\alpha v + 1 - 2\alpha)} \\ &+ \frac{\alpha}{2(\alpha v + 2 - 4\alpha)}(\frac{1-\alpha}{2} + \frac{(1-\alpha)^2}{2(v - \alpha v - 1 + 2\alpha)}) \\ &+ \frac{v + 1 - 2\alpha v - 2\alpha}{2v}(1 - \frac{\alpha}{2(\alpha v + 2 - 4\alpha)})\frac{\alpha}{2(\alpha v + 1 - 2\alpha)} \\ &+ \frac{v + 1 - 2\alpha v - 2\alpha}{2v}\frac{\alpha}{2(\alpha v + 2 - 4\alpha)}(1 - \frac{1-\alpha}{2(v - \alpha v - 1 + 2\alpha)}) \end{aligned}$$

The marginal effect of $\alpha$ on the expected total effort is

$$\frac{dTE^{(B1)}_{\alpha\beta}}{d\alpha} = \frac{C}{D} \tag{21}$$

where

$$\begin{aligned} C &= (8\alpha^6 - 16\alpha^5 + 8\alpha^4)v^7 + (-132\alpha^6 + 29\alpha^5 - 212\alpha^4 + 48\alpha^3)v^6 \\ &+ (872\alpha^6 - 2144\alpha^5 + 1930\alpha^4 - 764\alpha^3 + 114\alpha^2)v^5 \\ &+ (-2932\alpha^6 + 7752\alpha^5 - 8128\alpha^4 + 4258\alpha^3 - 1123\alpha^2 + 120\alpha)v^4 \\ &+ (5192\alpha^6 - 14560\alpha + 16974\alpha^4 - 10506\alpha^3 + 3625\alpha^2 - 656\alpha + 48)v^3 \\ &+ (-4240\alpha^6 + 12544\alpha^5 - 15820\alpha^4 + 10868\alpha^3 - 4278\alpha^2 + 912\alpha - 82)v^2 \\ &+ (480\alpha^6 - 1664\alpha^5 + 2480\alpha^4 - 1984\alpha^3 + 886\alpha^2 - 208\alpha + 20)v \\ &+ 896\alpha^6 - 2688\alpha^5 + 3360\alpha^4 - 2240\alpha^3 + 840\alpha^2 - 168\alpha + 14 \end{aligned} \tag{22}$$

and

$$\begin{aligned} D &= (8\alpha^6 - 16\alpha^5 + 8\alpha^4)v^7 + (-128\alpha^6 + 288\alpha^5 - 208\alpha^4 + 48\alpha^3)v^6 \\ &+ (832\alpha^6 - 2048\alpha^5 + 1840\alpha^4 - 720\alpha^3 + 104\alpha^2)v^5 \\ &+ (-2816\alpha^6 + 7424\alpha^5 - 7680\alpha^4 + 3904\alpha^3 - 976\alpha^2 + 96\alpha)v^4 \\ &+ (5248\alpha^6 - 14592\alpha^5 + 16576\alpha^4 - 9792\alpha^3 + 3144\alpha^2 - 512\alpha + 32)v^3 \\ &+ (-5120\alpha^6 + 14848\alpha^5 - 17920\alpha^4 + 11520\alpha^3 - 4160\alpha^2 + 800\alpha - 64)v^2 \\ &+ (2048\alpha^6 - 6144\alpha^5 + 7680\alpha^4 - 5120\alpha^3 + 1920\alpha^2 - 384\alpha + 32)v \end{aligned} \tag{23}$$

When $\alpha$ approaches zero we obtain that

$$\lim_{\alpha\to 0}\frac{dTE^{(B1)}_{\alpha\beta}}{d\alpha}=\frac{48v^3-82v^2+20v+14}{32v^3-64v^2+32v}$$

It can be verified that the last term is positive for all $v \geq 1$. Thus, if condition 5 holds, compared to the best-of-three all-pay auction with a single final prize, allocating a sufficiently small prize of $\alpha$ in both first stages increases the players' expected total effort.

2) If condition 7 holds, $v_1 = v, v_2 = 1$, and $\alpha = 0$, then the expected total effort given by (15) is

$$\begin{aligned}TE^{(B2)}_{\alpha\beta} &= \frac{2-\beta-v}{2}+\frac{(2-\beta-v)^2}{2(v-\beta)}\\ &+\frac{3v-\beta-2}{2(v-\beta)}\left(\frac{\beta}{2}+\frac{\beta^2}{2(\beta v+1-\beta)}\right)\\ &+\frac{2-\beta-v}{2(v-\beta)}\left(\frac{v-1+\beta}{2}+\frac{(v-1+\beta)^2}{2}\right)\\ &+\frac{3v-\beta-2}{2(v-\beta)}\frac{\beta}{2(\beta v+1-\beta)}\frac{v+1-\beta v-\beta}{2v}\\ &+\frac{2-\beta-v}{2(v-\beta)}\frac{v-1+\beta}{2}\frac{v+1-\beta v-\beta}{2v}\end{aligned}$$

The marginal effect of $\beta$ on the expected total effort is

$$\frac{dTE^{(B2)}_{\alpha\beta}}{d\beta}=\frac{E}{F} \tag{24}$$

where

$$\begin{aligned}E &= (-4\alpha^3+4\alpha^2)v^7+(2\alpha^4+40\alpha^3-43\alpha^2+8\alpha)v^6\\ &+(-20\alpha^4-17\alpha^3+206\alpha^2-58\alpha+4)v^5+(88\alpha^4+384\alpha^3-559\alpha^2+200\alpha-19)v^4\\ &+(-208\alpha^4-312\alpha^3+835\alpha^2-386\alpha+53)v^3+(262\alpha^4+120\alpha^3-578\alpha^2+340\alpha-59)v^2\\ &+(-156\alpha^4+76\alpha^3+135\alpha^2-112\alpha+23)v+(32\alpha^4-32\alpha^3+8\alpha-2)\end{aligned} \tag{25}$$

and

$$\begin{aligned}F &= 8\alpha^2v^5+(-16\alpha^3-32\alpha^2+16\alpha)v^4+(8\alpha^4+64\alpha^3-32\alpha+8)v^3\\ &+(-32\alpha^4-48\alpha^3+64\alpha^2-16\alpha)v^2+(32\alpha^4-32\alpha^3+8\alpha^2)v\end{aligned} \tag{26}$$

When $\beta$ approaches zero we obtain that

$$\lim_{\alpha\to 0}\frac{dTE^{(B2)}_{\alpha\beta}}{d\alpha}=\frac{4v^5-19v^4+53v^3-59v^2+23v-2}{8v^3}$$

It can be verified that the last term is positive for all $v>1$. Since condition 7 holds if $1\leq v<2-\beta$, we obtain that compared to the best0of-three all-pay auction with a single final prize, allocating a sufficiently small prize of $\beta$ in the second stage increases the players' expected total effort.

### 6.4 Proof of Proposition 5

When players are sufficiently asymmetric, namely, $v_1=v>>v_2=1$, condition 5 holds. In that case, if we allocate a prize of $\alpha$ in the first stage, by (14), we obtain that the marginal effect of the prize on the expected total effort when $v$ approaches infinity is

$$\lim_{v\to\infty}\frac{dTE^{(B1)}_{\alpha}}{d\alpha}=\frac{4\alpha^2}{8\alpha^2}=\frac{1}{2}$$

If we allocate a prize of $\beta$ in the second stage, by (17), we obtain that the marginal effect of the prize on the expected total effort when $v$ approaches infinity is

$$\lim_{v\to\infty}\frac{dTE^{(B1)}_{\beta}}{d\beta}=\frac{2\beta^2}{4\beta^2}=\frac{1}{2}$$

and if we allocate a prize of $\alpha$ in both the first stages, by (21), (22) and (23), we obtain that the marginal effect of the prizes on the expected total effort when $v$ approaches infinity is

$$\lim_{v\to\infty}\frac{dTE^{(B1)}_{\alpha\beta}}{d\alpha}=\frac{8\alpha^6-16\alpha^5+8\alpha^4}{8\alpha^6-16\alpha^5+8\alpha^4}=1$$

Thus, we obtain that

$$\lim_{v\to\infty}\frac{dTE^{(B1)}_{\alpha\beta}}{d\alpha}>\lim_{v\to\infty}\frac{dTE^{(B1)}_{\alpha}}{d\alpha}=\lim_{v\to\infty}\frac{dTE^{(B1)}_{\beta}}{d\beta}$$

In other words, in order to maximize the players' expected total effort. it is better to award intermediate prizes in both first stages than in only one of them .

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