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AVERAGE INFLATION TARGETING AND THE INTEREST RATE LOWER BOUND

Abstract

A discretionary central bank with a mandate to stabilize an average inflation rate---rather than period-by-period inflation---increases welfare of a sticky-price economy in which nominal interest rates are occasionally constrained by a lower bound. The welfare gain is driven by two monetary policy motives that arise in the presence of an average inflation objective: the history-dependence motive makes expected future inflation an increasing function of current inflation shortfalls, and vice versa, acting as an automatic stabilizer; and the lower bound risk motive induces the central bank to raise inflation when the risk of hitting the lower bound constraint increases. Under rational expectations, the optimal averaging window is infinitely long, so that the optimal average inflation targeting framework is tantamount to price level targeting. Most of the welfare improvement can, however, be attained by a framework with a finite, but sufficiently long, averaging window. Under boundedly-rational expectations, if cognitive limitations are sufficiently strong, the optimal averaging window is finite, and the welfare gain of adopting an average inflation target can be small.

JEL Classification: E31, E52, E58, E61

Keywords: Monetary Policy Objectives, Makeup Strategies, Average Inflation Targeting, liquidity trap, Deflationary Bias

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1 Introduction

For most of the last decade, monetary policy in major parts of the industrialized world has been constrained by a lower bound on nominal interest rates, and inflation rates have been hovering around levels below central banks' targets. Against this backdrop, current monetary policy frameworks, which were typically instituted when the possibility of being constrained by the lower bound seemed small, have come under increased scrutiny. Some central banks are currently reviewing their monetary policy strategies and discussing whether some modifications are warranted in light of the challenges associated with the lower bound.

This paper contributes to this discussion by analyzing the effects of average inflation targeting (AIT) on macroeconomic stabilization and society's welfare in the presence of a lower bound on nominal interest rates. AIT has recently attracted increasing attention as a possible alternative to currently prevailing inflation targeting frameworks (e.g. Svensson (2019); Brainard (2019)), notably because of its 'makeup' feature whereby past inflation shortfalls are made up for by temporarily higher future inflation and vice versa.

In the spirit of the policy delegation literature (e.g. Rogoff (1985)), we consider the optimization problem of a central bank that acts under discretion and whose objective function features the volatility of average inflation rates over a pre-specified time period, as opposed to the volatility of the current inflation rate. The analysis is based on two variants of the standard New Keynesian model with a lower bound on nominal interest rates. In one variant, agents form expectations rationally. In the other variant, agents have boundedly-rational expectations, as in Gabaix (2019). We include a model with boundedly-rational expectations in our analysis because some have questioned the suitability of the standard rational expectations model for analysis of monetary policy strategies on the ground that it can give rise to implausible large effects of forward guidance—a promised interest rate change in the future—on current inflation and economic activity. Bounded rationality is one way to attenuate this so-called forward guidance puzzle because it implies that the private sector's consumption and pricing decisions are less dependent on future economic conditions than in the standard model. We specify the central bank's AIT objective in the form of an exponential moving average, which allows us to solve both model variants nonlinearly using global methods.[1]

We find that AIT improves welfare considerably when agents form expectations rationally. Following a large recessionary shock that drives the policy rate to the lower bound, a central bank with an AIT objective keeps the policy rate low for longer than a central bank with a standard inflation targeting objective, thereby engineering a temporary overshooting in future inflation that helps to mitigate the decline of output and inflation at the lower bound via the expectations channel. This 'history dependence motive' of monetary policy under AIT is complemented by a 'lower bound risk motive' that makes current inflation under AIT an increasing function of a model-consistent

[1] Specifying the AIT objective in the form of an arithmetic moving average would be prohibitively expensive from a computational point of view when considering averaging windows that are sufficiently long. For instance, if the averaging window is 4 years (16 quarters), there are 16 endogenous state variables, making it nearly infeasible to solve the model accurately using global methods in a reasonable amount of time.

measure of the risk of hitting the lower bound in the future. The lower bound risk motive contributes to society's welfare by counteracting the so-called deflationary bias of discretionary monetary policy, i.e. the phenomenon that the mere possibility that the constraint binds in the future—as opposed to the constraint being actually binding—results in a systematic inflation shortfall when the policy rate is away from the lower bound (Nakov, 2008; Nakata and Schmidt, 2019a; Hills et al., 2019). We find that the optimal averaging window is infinitely long, and the finding is robust to various alternative parameterizations of the model. The AIT objective with an infinitely long averaging window—the optimal AIT in the rational expectations model—coincides with a price-level-targeting (PLT) objective. However, we find that most of the welfare improvement associated with the optimal AIT can be attained by an AIT objective with a finite, but sufficiently long, averaging window. In our baseline calibration, AIT with an averaging window capturing a few years can attain most of the welfare gain associated with the optimal AIT.

AIT also improves welfare in the model with boundedly-rational expectations, but the welfare gain from AIT in this model is smaller than in the rational-expectations model. For a range of values of the cognitive discounting parameters in the model's Euler equation and Philips curve used in the literature the optimal averaging window remains infinite. That is, the results from the rational-expectations model are robust to including plausible degrees of bounded rationality. However, if the degree of cognitive discounting is sufficiently large, marginal increases in the discounting parameters lower the optimal averaging window. In such cases, welfare under PLT may be even lower than that under standard inflation targeting.

In both models, welfare can be further increased by assigning a relative weight on output gap stabilization that is smaller than the one in society's objective function ('inflation conservatism'). When the only source of uncertainty is a natural real rate shock, it is optimal to assign zero weight on the output gap. This holds true independently of whether the central bank's nominal target variable is period-by-period inflation, an average inflation rate or the price level. Strict (average) inflation targeting eliminates the deflationary bias away from the lower bound which raises inflation expectations in all states and thereby improves stabilization outcomes at the lower bound. The gains from correcting the deflationary bias by adjusting the central bank's relative weight on output gap stabilization are particularly large under standard inflation targeting—the monetary policy regime under which the deflationary bias is most pronounced to start with. When people have boundedly-rational expectations and their cognitive abilities are sufficiently low, adjusting the central bank's relative weight on output gap stabilization can lead to a larger welfare gain than changing the inflation objective.

Our paper is related to two strands of the literature on monetary policy and the interest rate lower bound. First, Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006), Nakov (2008) and Bilbiie (2019) characterize optimal monetary policy under commitment and show that the central bank uses communication about its future interest rate policy to steer the economy when the contemporaneous policy rate cannot be lowered any further.[2] Nakata et al.

[2]Several other papers study models where the central bank is committed to follow an interest-rate feedback rule

(2019) and Levin and Sinha (2019) analyze the optimal commitment policy in a model similar to the model with boundedly-rational agents used here, and find that it is typically optimal for the central bank to partially compensate for the reduced effect of a future rate cut by keeping the policy rate at the lower bound for longer than in the benchmark rational-expectations setup. Our paper differs from these papers in that we assume that the central bank, while committed to its assigned objective(s), sets its policy instruments with discretion. Within that framework, we show that assigning an average inflation target to a central bank is a practical way to reap most of the benefits of the optimal commitment policy without requiring the central bank to engage in time-inconsistent policies.

Second, in the spirit of the policy delegation literature, some papers have proposed modifications to the central bank objective function in order to improve welfare in models with a lower bound when policymakers act under discretion. For instance, Nakata and Schmidt (2019a,b) show that the discretionary equilibrium in models with an occasionally binding lower bound constraint can be improved by the appointment of an inflation conservative central banker and by the assignment of an interest-rate smoothing term to the central bank objective function, respectively. Similarly, Billi (2017) compares price level targeting and nominal GDP targeting to standard inflation targeting in a model with an interest rate lower bound.[3] To our knowledge, we are the first to formally assess how assigning an average inflation objective to a discretionary central bank affects stabilization outcomes and welfare in models with an interest rate lower bound. In so doing, we build on Nessen and Vestin (2005) who assess the desirability of (arithmetic) average inflation targeting using the policy delegation approach in a standard New Keynesian model without a lower bound on nominal interest rates.[4]

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the results for the rational-expectations variant of the model, and Section 4 for the variant of the model with boundedly-rational expectations. Section 5 concludes.

2 Model

We use an infinite-horizon New Keynesian model formulated in discrete time. The economy is inhabited by identical households who consume and work, goods-producing firms that act under monopolistic competition and are subject to price rigidities, and a central bank. We consider two alternative expectations formation mechanisms. In the benchmark setup, agents have rational expectations (see Galí, 2015), and in the alternative setup agents have boundedly-rational expectations as in Gabaix (2019). In the latter setup, agents do not fully understand the world as

with 'makeup' features that implement policies akin to the optimal commitment policy, e.g. Reifschneider and Williams (2000) and, more recently Bernanke et al. (2019), Mertens and Williams (2019), and Coenen et al. (2019).

[3]See also Bodenstein and Zhao (2019) and Nakata et al. (2018) for analyses of monetary policy strategies with a 'speed limit policy' objective in an economy with short-lived and long-lasting lower bound episodes, respectively.

[4]Our paper is also related to Vestin (2006) who compares standard inflation targeting to price level targeting in a standard New Keynesian model without a lower bound constraint. Walsh (2019) evaluates price level targeting and average inflation targeting using the policy delegation framework, but in a model without an interest rate lower bound.

represented by the model. When they contemplate the future, their expectations are geared to the steady state of the economy, which serves as a simple benchmark.

2.1 Private sector behavior and welfare

Aggregate private sector behavior is described by a Phillips curve and a Euler equation

$$\pi_t = \kappa y_t + \beta(1-\alpha_{PC})\mathrm{E}_t\pi_{t+1} \tag{1}$$
$$y_t = (1-\alpha_{EE})\mathrm{E}_t y_{t+1} - \sigma\left(i_t - \mathrm{E}_t\pi_{t+1} - r_t^n\right) \tag{2}$$

The private sector behavioral constraints have been (semi) log-linearized around the intended zero-inflation steady state.[5] π_t is the inflation rate between periods $t-1$ and t, y_t denotes the output gap, i_t is the *level* of the riskless nominal interest rate between periods t and $t+1$, r_t^n is the exogenous natural real rate of interest, and E_t is the rational expectations operator conditional on information available in period t. Parameters α_{PC} and α_{EE} capture the degree of cognitive discounting by firms and households, respectively. In the rational expectations benchmark, $\alpha_{PC} = \alpha_{EE} = 0$. Under boundedly rational expectations, α_{PC} and α_{EE} are functions of the cognitive discounting parameter denoted $\bar{m}$ in Gabaix (2019)

$$1-\alpha_{EE} = \bar{m} \tag{3}$$
$$1-\alpha_{PC} = \bar{m}\left[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)\right], \tag{4}$$

where $\beta \in (0,1)$ is the pure rate of time preference and $\varphi \in (0,1)$ denotes the share of firms that cannot reoptimize their price in a given period.

The other parameters in the private sector aggregate behavioral constraints are defined as follows: $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, and κ represents the slope of the Phillips curve.[6]

Households' welfare at time t is given by the expected discounted sum of current and future utility flows. Following Gabaix (2019), boundedly-rational agents are assumed to experience utility from consumption and leisure like rational agents. Hence, the welfare criterion is invariant to the expectations formation mechanism. A second-order approximation to household preferences leads to

$$V_t = -\frac{1}{2}\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\left[\pi_{t+j}^2 + \lambda y_{t+j}^2\right], \tag{5}$$

[5] See Nakata (2016, 2017) for analyses of optimal policy in fully nonlinear New Keynesian models. Key insights on optimal policy do not depend on whether private sector behavioral equations are put in nonlinear or in log-linearized form.

[6] κ is itself a function of several structural parameters of the economy: $\kappa = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi(1+\eta\theta)}(\sigma^{-1}+\eta)$, where $\eta > 0$ is the inverse of the labor-supply elasticity, and $\theta > 1$ denotes the price elasticity of demand for differentiated goods.

where $\lambda = \kappa/\theta$.[7]

2.2 Central bank objective, monetary policy strategies and equilibrium

The central bank controls the one-period nominal interest rate i_t, which we will also refer to as the policy rate. It does not have a commitment technology, that is, it acts under discretion. The monetary policy objective function in some generic period t is given by

$$V_t^{CB} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\hat{\pi}_{t+j}^2 + \lambda^{CB}(\omega y_{t+j})^2\right], \tag{6}$$

where

$$\hat{\pi}_{t+j} = \omega \pi_{t+j} + (1-\omega)\hat{\pi}_{t+j-1}, \tag{7}$$

$\omega \in [0,1]$ and $\lambda^{CB} \geq 0$. For most of our analysis, and unless stated otherwise, we assume $\lambda^{CB} = \lambda$.

This central bank objective function nests three monetary policy strategies:

- *Standard inflation targeting (IT)*: When $\omega = 1$, monetary policy follows a standard flexible inflation targeting strategy whereby the central bank aims to stabilize the period-by-period inflation rate π_t and the output gap y_t. For $\lambda^{CB} = \lambda$, the central bank's objective function coincides with society's objective function (5).

- *Average inflation targeting (AIT)*: When $\omega \in (0,1)$, the central bank aims to stabilize an exponential moving average inflation rate $\hat{\pi}$, as defined in equation (7).[8] Weighting the output gap term in the central bank's objective function with the moving-average parameter ω ensures that variations in ω do not affect the weight on y_t^2 relative to the weight on π_t^2, the two terms that also show up in society's objective function.[9]

- *Price level targeting (PLT)*: When $\omega \to 0$, the central bank aims to stabilize the price level $p_t \equiv \pi_t + p_{t-1}$. To see this, note that

$$\frac{\hat{\pi}_t}{\omega} = \pi_t + (1-\omega)\frac{\hat{\pi}_{t-1}}{\omega} \tag{8}$$

$$= \sum_{j=0}^{t}(1-\omega)^j \pi_{t-j} + (1-\omega)^{t+1}\frac{\hat{\pi}_{-1}}{\omega} \tag{9}$$

Assuming $\hat{\pi}_{-1} = 0$, and re-scaling by $1/\omega^2$, the central bank's objective function can be

[7] We assume that the steady state distortions arising from monopolistic competition are offset by a wage subsidy so that the steady state is first best.

[8] As briefly mentioned in the Introduction, defining average inflation in terms of an exponential moving average rather than an arithmetic moving average economizes on the number of state variables and thereby facilitates the solution of the model using global methods.

[9] To see this, note that $\hat{\pi}_t^2 + \lambda^{CB}(\omega y_t)^2 = \omega^2\left[\pi_t^2 + \lambda^{CB} y_t^2\right] + (1-\omega)\left((1-\omega)\hat{\pi}_{t-1}^2 + 2\omega\pi_t\hat{\pi}_{t-1}\right)$.

written as

$$-\frac{1}{2}\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\left[\left(\sum_{k=0}^{t+j}(1-\omega)^k\pi_{t+j-k}\right)^2+\lambda^{CB}y_{t+j}^2\right], \tag{10}$$

which, for $\omega \to 0$, becomes

$$-\frac{1}{2}\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\left[(p_{t+j}-p_{-1})^2+\lambda^{CB}y_{t+j}^2\right], \tag{11}$$

where p_{-1} is the log of the initial price level, which can be normalized to zero.

The policy problem of a generic central bank is as follows. Each period t, it chooses the inflation rate π_t, the average inflation rate $\hat{\pi}_t$, the output gap, and the nominal interest rate to maximize its objective function (6) subject to the behavioral constraints of the private sector (1)–(2), the definition of the inflation average (7), and the lower bound constraint $i_t \geq 0$, with the value and policy functions at time $t+1$ taken as given. Formally,

$$\begin{aligned} V^{CB}(\hat{\pi}_{t-1}, r_t^n) = \max_{\pi_t, y_t, i_t, \hat{\pi}_t} & -\frac{1}{2}\left[\hat{\pi}_t^2+\lambda^{CB}(\omega y_t)^2\right]+\beta\mathrm{E}_t V^{CB}(\hat{\pi}_t, r_{t+1}^n) \\ & + \phi_t^{PC}\left[\pi_t-\beta(1-\alpha_{PC})\mathrm{E}_t\pi(\hat{\pi}_t, r_{t+1}^n)-\kappa y_t\right] \\ & + \omega^2\phi_t^{EE}\left[y_t-(1-\alpha_{EE})\mathrm{E}_t y(\hat{\pi}_t, r_{t+1}^n)+\sigma(i_t-\mathrm{E}_t\pi(\hat{\pi}_t, r_{t+1}^n)-r_t^n)\right] \\ & + \omega^2\phi_t^{LB} i_t \\ & + \phi_t^{AI}\left[\hat{\pi}_t-\omega\pi_t-(1-\omega)\hat{\pi}_{t-1}\right], \end{aligned} \tag{12}$$

where $\phi_t^{PC}, \omega^2\phi_t^{EE}, \omega^2\phi_t^{LB} \geq 0, \phi_t^{AI}$ are Lagrange multipliers and $\pi(\hat{\pi}_t, r_{t+1}^n), y(\hat{\pi}_t, r_{t+1}^n)$ characterize the equilibrium that the central bank expects to occur in period $t+1$, conditional upon the level of the natural real rate r_{t+1}^n.

A *Markov-Perfect equilibrium* is defined as a set of time-invariant value and policy functions $\{V^{CB}(\cdot), \pi(\cdot), \hat{\pi}(\cdot), y(\cdot), i(\cdot)\}$ that solves the central bank's problem (12).

We report social welfare of an economy for a particular monetary policy strategy ω in terms of the perpetual consumption transfer–expressed as a share of its steady state–that would make households in the hypothetical economy without any shocks indifferent to living in the actual stochastic economy

$$W := (1-\beta)\,\frac{\theta}{\kappa}\left(\sigma^{-1}+\eta\right)\mathrm{E}[V], \tag{13}$$

where the mathematical expectation is taken with respect to the unconditional distribution of r_t^n, and V is defined in equation (5).

2.3 Analytical insights

Before turning to the numerical analysis, it is useful to explore analytically some of the key properties of AIT. Solving the central bank's optimization problem gives rise to the following first-order condition[10]

$$\pi_t = -(1-\omega)\frac{\hat{\pi}_{t-1}}{\omega} + \frac{\beta(1-\omega)}{\kappa\sigma}\left(\mathrm{E}_t\phi_{t+1}^{LB} + \lambda^{CB}\sigma\mathrm{E}_t y_{t+1}\right) + A_{LB}(\hat{\pi}_t)\phi_t^{LB} + A_y(\hat{\pi}_t)y_t \tag{14}$$

where

$$A_y(\hat{\pi}_t) \equiv \left(\beta(1-\alpha_{PC})\frac{\partial \mathrm{E}_t\pi_{t+1}}{\partial(\hat{\pi}_t/\omega)} - 1\right)\frac{\lambda^{CB}}{\kappa} \tag{15}$$

$$A_{LB}(\hat{\pi}_t) \equiv \left(\frac{\beta}{\kappa}(1-\alpha_{PC})\frac{\partial \mathrm{E}_t\pi_{t+1}}{\partial(\hat{\pi}_t/\omega)} - \frac{1}{\kappa} + (1-\alpha_{EE})\frac{\partial \mathrm{E}_t y_{t+1}}{\partial(\hat{\pi}_t/\omega)} + \sigma\frac{\partial \mathrm{E}_t\pi_{t+1}}{\partial(\hat{\pi}_t/\omega)}\right)\sigma^{-1}. \tag{16}$$

For $\omega = 1$, (14) collapses to $\pi_t = -(\lambda^{CB}/\kappa)y_t - (\sigma^{-1}/\kappa)\phi_t^{LB}$, the well-known first-order condition under standard inflation targeting. When $\omega < 1$, monetary policy is influenced by two motives that are absent under standard inflation targeting. The *history dependence motive* is represented by the first term on the right-hand side of equation (14), and makes the inflation rate in period t an increasing function of past inflation shortfalls, as memorized by the previous period's average inflation rate $\hat{\pi}_{t-1}$. The *lower bound risk motive* is represented by the second term on the right-hand side of equation (14) and makes the inflation rate an increasing function of the expected lower-bound multiplier in the next period, $\mathrm{E}_t\phi_{t+1}^{LB}$, a measure of the risk of hitting the lower bound in the next period.[11]

To better understand these two channels, let us temporarily assume that $\lambda^{CB} = 0$ in (14).[12] In a period in which the lower bound constraint is not binding, $\phi_t^{LB} = 0$, the central bank's first-order condition reads

$$\pi_t = -(1-\omega)\frac{\hat{\pi}_{t-1}}{\omega} + \frac{\beta(1-\omega)}{\kappa\sigma}\mathrm{E}_t\phi_{t+1}^{LB}. \tag{17}$$

Suppose, $\hat{\pi}_{t-1} < 0$, perhaps because the lower bound constraint was binding in period $t-1$. Then, all else equal, the central bank aims for a higher inflation rate π_t than it would do if average inflation had been at target in the previous period. When agents expect higher period-t inflation in period $t-1$, this has a stimulative effect on inflation in period $t-1$, and mitigates the average inflation shortfall in period $t-1$.

Now suppose $\hat{\pi}_{t-1} \approx 0$, commensurate with a situation where the lower bound constraint has

[10] Under price level targeting, i.e. when $\omega = 0$, $\hat{\pi}_t/\omega$ in (14)-(16) has to be replaced with p_t.

[11] Another motive, which is not the focus of our analysis, is related to the occurrence of the expected output gap term on the right-hand side of (14). The Phillips curve describes a potential trade-off between inflation and output gap stabilization which, in turn, can give rise to a trade-off between average inflation and the output gap. When the central bank expects a trade-off between average inflation and the output gap to materialize in the next period, it can improve this trade-off by adjusting the current inflation rate in the opposite direction of expected next period's inflation rate.

[12] In the parlance of the policy delegation literature, $\lambda^{CB} = 0$ describes an inflation-conservative central bank.

not been binding for a long time. Then, current period-by-period inflation depends only on the expected Lagrange multiplier associated with the lower bound constraint. When there is a positive probability that the lower bound constraint becomes binding in the future, $\mathrm{E}_t\phi^{LB}_{t+1} > 0$, the central bank aims for a higher inflation rate than it would do in the absence of such lower bound risk. By implementing a strictly positive inflation rate when the lower bound is not binding, the central bank mitigates the decline in next period's average inflation rate in case the lower bound constraint becomes binding. These two motives will also be at play in the numerical analysis presented in the next section.

3 Results under rational expectations

We now turn to the numerical analysis of how the assignment of an average inflation targeting objective affects society's welfare and economic dynamics when the lower bound on nominal interest rates is an occasionally binding constraint. This section presents results for the model under rational expectations. Results for the model under boundedly-rational expectations are presented in the next section.

Table 1 reports our baseline parameterization, which is based on Nakata and Schmidt (2019b), for both variants of the model. The natural real rate shock r^n_t is governed by a stationary AR(1) process, which has been estimated using U.S. data.[13]

Table 1: **Parameterization**

Parameter	Value	Economic interpretation
β	0.99	Subjective discount factor
σ	2	Intertemporal elasticity of substitution in consumption
η	0.47	Inverse labor supply elasticity
θ	10	Price elasticity of demand
φ	0.8106	Share of firms per period keeping prices unchanged
ρ_r	0.85	AR coefficient natural real rate process
σ_r	$\frac{0.4}{100}$	Standard deviation natural real rate shock

Note: These parameter values imply $\kappa = 0.0079$ and $\lambda = 0.00079$.

We solve the model using the collocation method. The algorithm is explained in the Appendix.

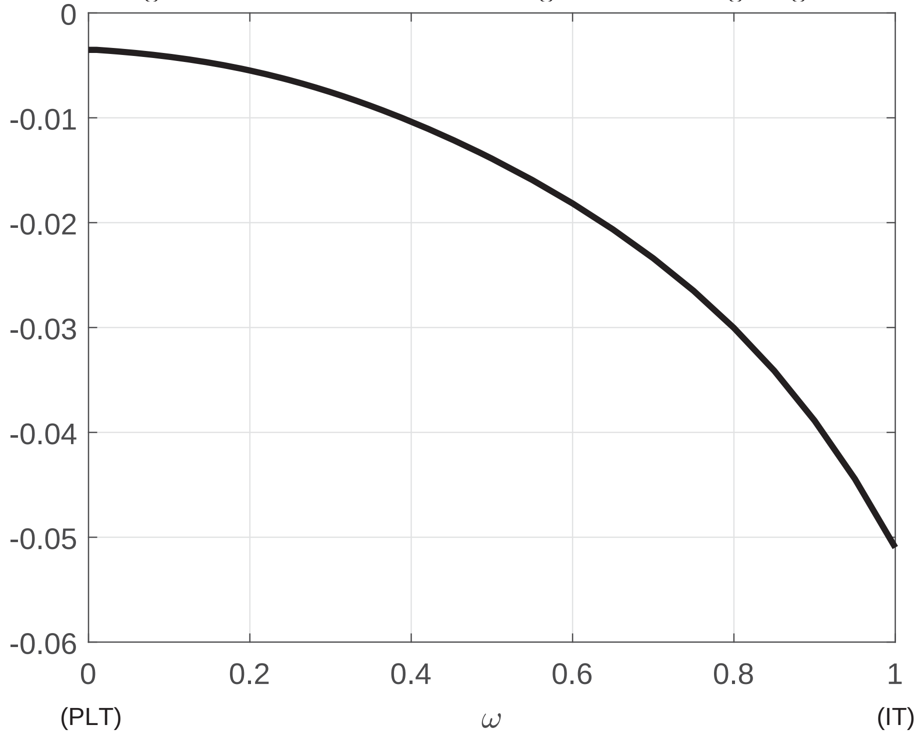
3.1 Optimal averaging window

Figure 1 shows how society's welfare (13) varies with the inflation-averaging parameter ω in the model with rational expectations ($\alpha_{EE}, \alpha_{PC} = 0$).

According to the figure, welfare increases monotonically as ω declines from 1 (the IT case) to 0 (the PLT case). Hence, PLT is the optimal monetary policy strategy in the class of considered frameworks. Interestingly, a rather modest averaging window for the inflation objective $\hat{\pi}$ can lead

[13]See Appendix B in Nakata and Schmidt (2019b) for the details of the estimation.

Figure 1: Welfare effects of average inflation targeting



Note: Welfare is defined in equation (13).

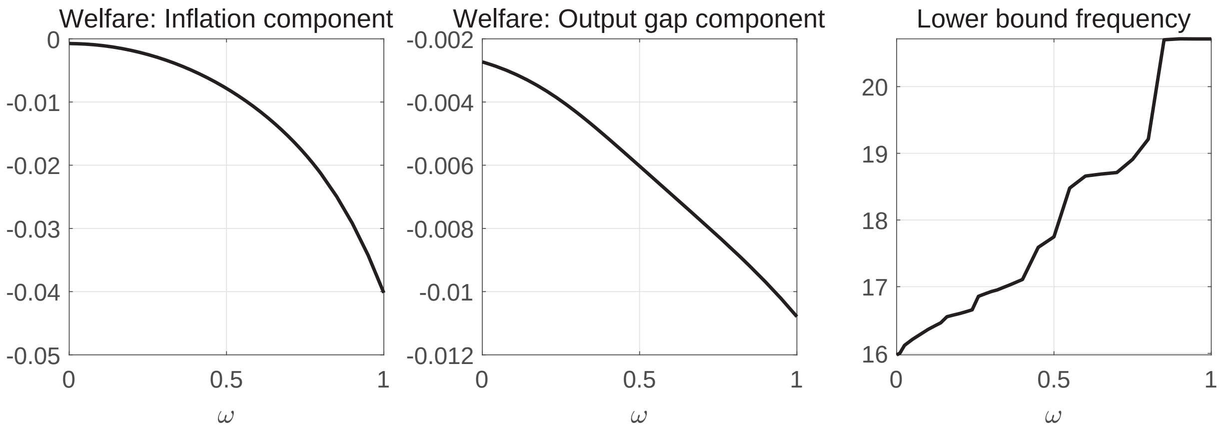
to substantial welfare gains when compared to standard IT. For instance, for $\omega = 0.7$, the welfare costs are only half as large as under IT. In our model, these welfare gains from choosing $\omega < 1$ arise solely because of the presence of the lower bound on nominal interest rates. Indeed, in the absence of the lower bound constraint, optimal monetary policy would replicate the efficient equilibrium for any value of ω.

Figure 2 decomposes society's overall welfare into the inflation component (left panel) and the output gap component (middle panel). Both welfare components are monotonically decreasing in ω. Hence, there is no trade-off in the choice of ω between inflation and output gap stabilization. Finally, the right panel of Figure 2 shows that the frequency of a binding lower bound is either unchanged or declines as we lower ω.

3.2 Why average inflation targeting is welfare-improving

To better understand the benefits of a monetary policy strategy that entails $\omega < 1$, consider the following liquidity trap scenario. The economy is initially in the risky steady state. In period 0, the economy is hit by a shock that drives the natural real rate into negative territory where it stays for six quarters. Thereafter, it jumps back to its steady state level. At each point in time, agents are unaware of the future path of the natural real rate, expecting it to gradually return to its steady state according to the AR(1) process. This exemplary natural real rate path is of course rather extreme, but is useful in demonstrating the implications of $\omega < 1$ for output, inflation and interest rate dynamics in a transparent way. Figure 3 plots the evolution of key variables in this

Figure 2: Welfare decomposition and lower bound frequency



Note: Lower bound frequency is reported as the percentage share of simulated periods in which the lower bound constraint was binding.

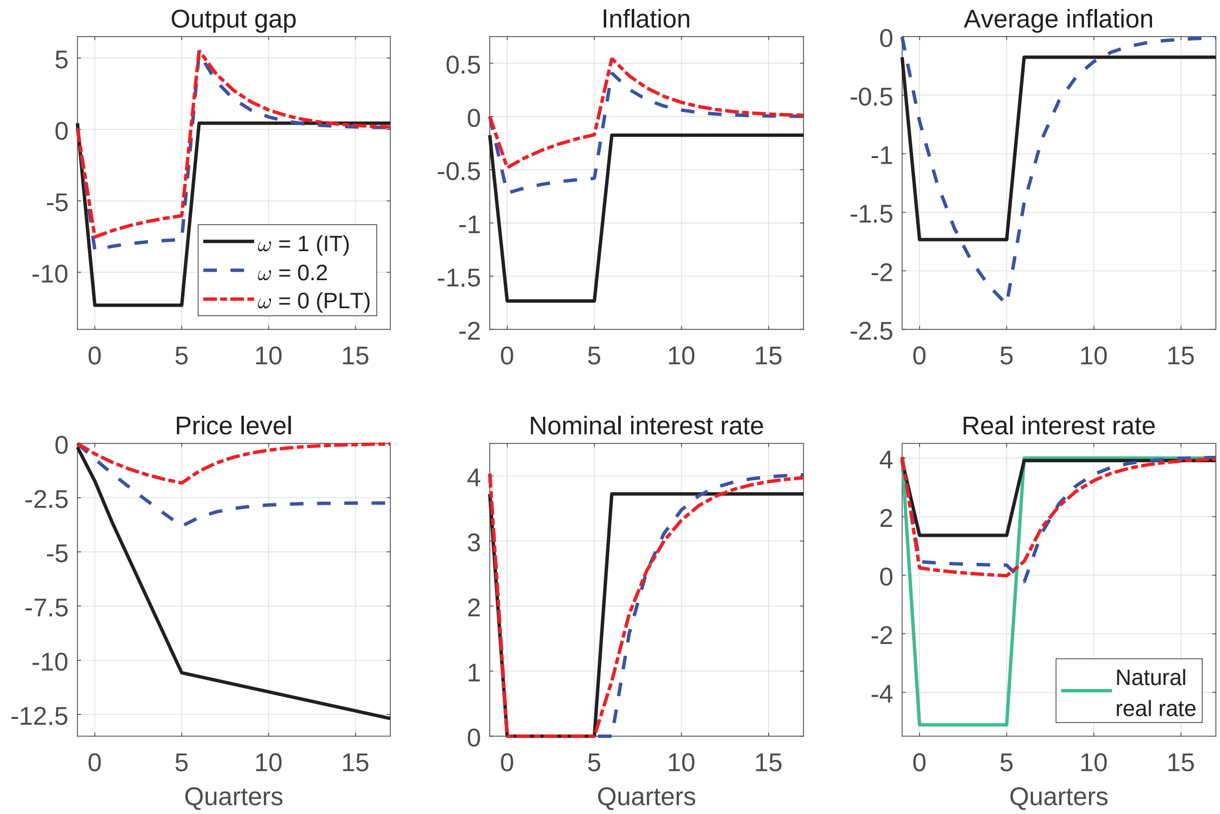
scenario for standard inflation targeting ($\omega = 1$), average inflation targeting (here for $\omega = 0.2$) and price-level targeting ($\omega = 0$).

Under IT, the central bank lowers the policy rate to zero when the shock occurs, but the real interest rate only falls to a level somewhat below two percent, reflecting a large decline in inflation expectations. The gap between the natural real rate and the actual real interest rate, in turn, leads to large declines in output and inflation. When the natural real rate finally jumps back to its steady state, the central bank immediately raises the policy rate back to its pre-shock level. The inflation rate increases but remains negative. This so-called 'deflationary bias' arises because agents are aware that the lower bound might be binding again in the future, which puts downward pressure on conditional inflation expectations. When $\lambda^{CB} > 0$, as is the case under our assumption that $\lambda^{CB} = \lambda$, subdued inflation expectations result in a trade-off for the central bank between inflation and output gap stabilization. In equilibrium, the inflation rate is negative and the output gap is positive whenever the lower bound constraint is slack.[14] Finally, because of the deflationary bias, the price level remains on a downward trajectory after the liquidity trap episode.

Under AIT, the central bank also lowers the policy rate to zero when the shock occurs, but when the shock recedes after 6 quarters the central bank raises the policy rate only gradually. Interest rate gradualism arises because average inflation is still negative when the natural real rate jumps back to its steady state level, and to stabilize average inflation, the central bank has to set the policy rate so that period-by-period inflation temporarily overshoots its long-run level. That is the history dependence motive in operation. Forward-looking households' and firms' inflation expectations at the lower bound are thus higher than under standard inflation targeting, and hence the real interest rate gap is smaller than under standard inflation targeting, resulting in improved outcomes for output and inflation at the lower bound. After the temporary overshooting, inflation is stabilized close to zero. Improved stabilization outcomes at the lower bound mitigate the downward pressure

[14] This trade-off between inflation and output gap stabilization arising from lower bound risk is analyzed in more detail in Nakata and Schmidt (2019a) and Hills et al. (2019).

Figure 3: Liquidity trap scenario



Note: All variables except for output have been annualized. Average inflation is normalized by $1/\omega$.

on conditional inflation expectations away from the lower bound, and the remaining deflationary pressures from these subdued inflation expectations are fully offset by the central bank's lower bound risk motive.[15] Therefore, the price level is eventually stabilized, although at a lower level than prior to the shock.

Finally, dynamics under PLT are similar to those under AIT (with $\omega = 0.2$), but now the price level is stabilized at its pre-shock level after the liquidity trap episode. Any previous shortfall in period-by-period inflation is thus made up for one-for-one, whereas under AIT previous inflation shortfalls are made up for less than one-for-one. The stronger form of history dependence under PLT further mitigates the decline in the output gap and inflation at the lower bound.

3.3 Optimal relative weight on output gap stabilization

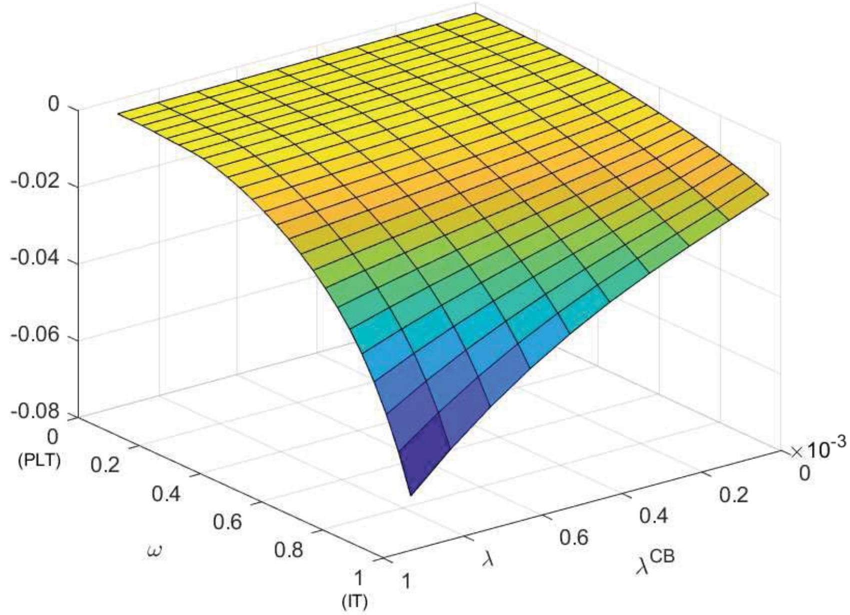
So far, we have presented results for the case where the central bank's objective function puts the same weight on output gap stabilization relative to period-by-period inflation stabilization as society's objective function, $\lambda^{CB} = \lambda$. In principle, however, the value assigned to λ^{CB} does not

[15] Elimination of the deflationary bias is not a generic feature of average inflation targeting. Numerically, we find that the deflationary bias vanishes under AIT if the value of ω is sufficiently small. In the Appendix, we show that the lower bound risk motive contributes to welfare under AIT.

have to coincide with the value of λ. Therefore, we now relax the assumption that $\lambda^{CB} = \lambda$.

Figure 4 plots society's welfare as a function of both ω and λ^{CB}. We can make several obser-

Figure 4: Welfare effects of average inflation targeting and inflation conservatism

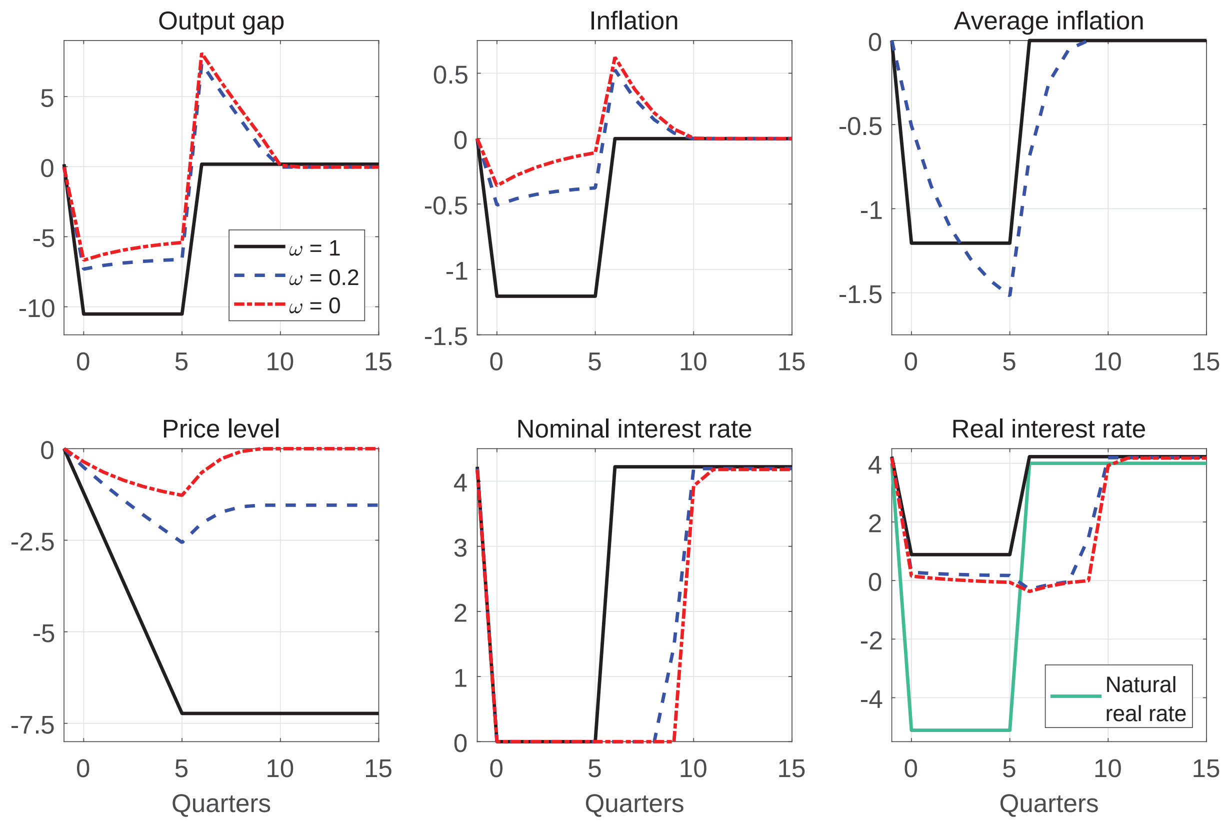


Note: Welfare is defined in equation (13). Here, $\lambda = 0.00079$.

vations. First, when maximizing society's welfare over both $\omega \in [0, 1]$ and $\lambda^{CB} \geq 0$, strict PLT ($\omega = 0, \lambda^{CB} = 0$) is optimal. Second, for any $\omega \in [0, 1]$ welfare increases monotonically as λ^{CB} declines. This numerical finding extends the analytical result in Nakata and Schmidt (2019a) on the desirability of 'inflation conservatism' ($\lambda^{CB} < \lambda$) under standard IT to the case where $\omega < 1$. Third, for any $\lambda^{CB} \geq 0$, welfare increases monotonically as ω declines, which means that our result from the previous section on the desirability of AIT/PLT also holds when λ^{CB} differs from λ. Finally, optimizing one of the two policy parameters alone—either ω or λ^{CB}—can improve welfare quite a bit. In the current model with rational expectations, welfare is higher in the PLT regime with $\lambda^{CB} = \lambda$ than in the IT regime with $\lambda^{CB} = 0$, but as we will see shortly this does not have to be the case in the model with boundedly-rational expectations.

To understand how the assignment of a smaller relative weight on output gap stabilization in the central bank's objective function affects stabilization outcomes and, thereby, welfare, we go back to the liquidity trap scenario considered before, but we now assume that $\lambda^{CB} = 0$. Figure 5 shows the evolution of key model variables under IT, AIT and PLT. Under all three monetary policy strategies, the decline in the output gap and inflation in response to the shock is smaller than is the case when $\lambda^{CB} = \lambda$ (see Figure 3). Furthermore, the deflationary bias away from the lower bound—previously observed under standard inflation targeting with $\lambda^{CB} > 0$—disappears. When the central bank cares only about inflation stabilization, the subdued inflation expectations arising from lower bound risk do no longer create a central bank trade-off between stabilization of

Figure 5: Liquidity trap scenario ($\lambda^{CB} = 0$)



Note: All variables except for output have been annualized. Average inflation is normalized by $1/\omega$.

the nominal target variable and economic activity. Finally, the absence of such a trade-off implies that when $\omega < 1$, i.e. when the central bank pursues AIT or PLT, the central bank not only raises interest rates more gradually after the liquidity trap episode than under IT—as is the case when $\lambda^{CB} = \lambda$—it now keeps the nominal interest rate at the lower bound for longer. This more accommodative interest rate policy heightens the overshooting in the output gap relative to the case where $\lambda^{CB} = \lambda$ and speeds up the return of the average inflation rate/price level to its target.

4 Results under boundedly-rational expectations

We now turn to the analysis of AIT in the model with boundedly-rational expectations.

As pointed out by Del Negro et al. (2015) and Carlstrom et al. (2015), the effects on inflation and output of an interest rate cut in the future are implausibly large in the standard New Keynesian model (so-called "forward guidance puzzle").[16] Many papers recently have proposed modifications

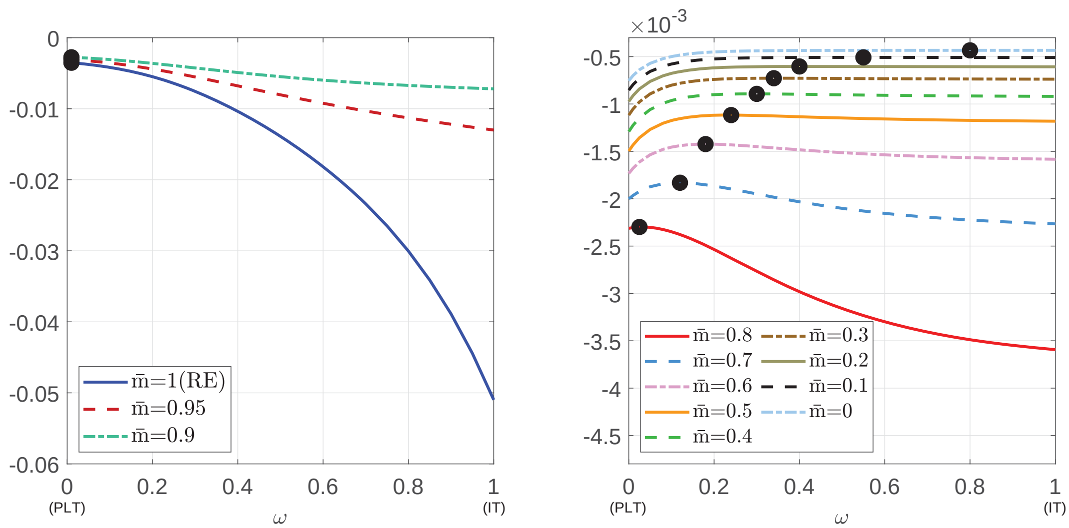
[16] In evaluating the effects of forward guidance shocks, it is typically assumed that the promise of an interest-rate cut far into the future is perfectly credible, but this assumption might not be realistic. For example, Haberis et al. (2019) show that, once the assumption of perfect credibility is relaxed, the effects of forward guidance shocks become smaller.

to the standard New Keynesian model to attenuate this forward guidance puzzle, with the introduction of boundedly-rational expectations being one of them. Because AIT improves allocations by promising interest rate cuts in the future, it is important to investigate the effectiveness of AIT using a model with moderated forward guidance effects.

4.1 Optimal averaging window

Figure 6 shows how welfare varies with the value of ω for various values of $\bar{m}$, the cognitive discounting parameter.[17] According to the figure, the optimal value of ω remains zero as long as $\bar{m}$ is sufficiently close to one. However, when the value of $\bar{m}$ is sufficiently below one, a further decline in $\bar{m}$ reduces the optimal ω. In an extreme case in which $\bar{m} = 0$, the decisions of households and firms are almost static, and the optimal omega is close to one.

Figure 6: Welfare effects of average inflation targeting with boundedly-rational expectations



Note: Welfare is defined in equation (13).

Another feature of Figure 6 is that the welfare gain from adopting the optimally calibrated AIT policy declines as $\bar{m}$ declines. This feature makes sense because AIT improves allocations relative to standard IT by taking advantage of the forward-looking behavior of households and firms. When $\bar{m}$ is less than 0.5, welfare under the optimally calibrated AIT is about the same as welfare under standard IT. Finally, it is also notable that, when $\bar{m}$ is low, welfare under PLT (that is, $\omega = 0$) is lower than welfare under standard IT. In our numerical example, this occurs when $\bar{m}$ is below 0.7.

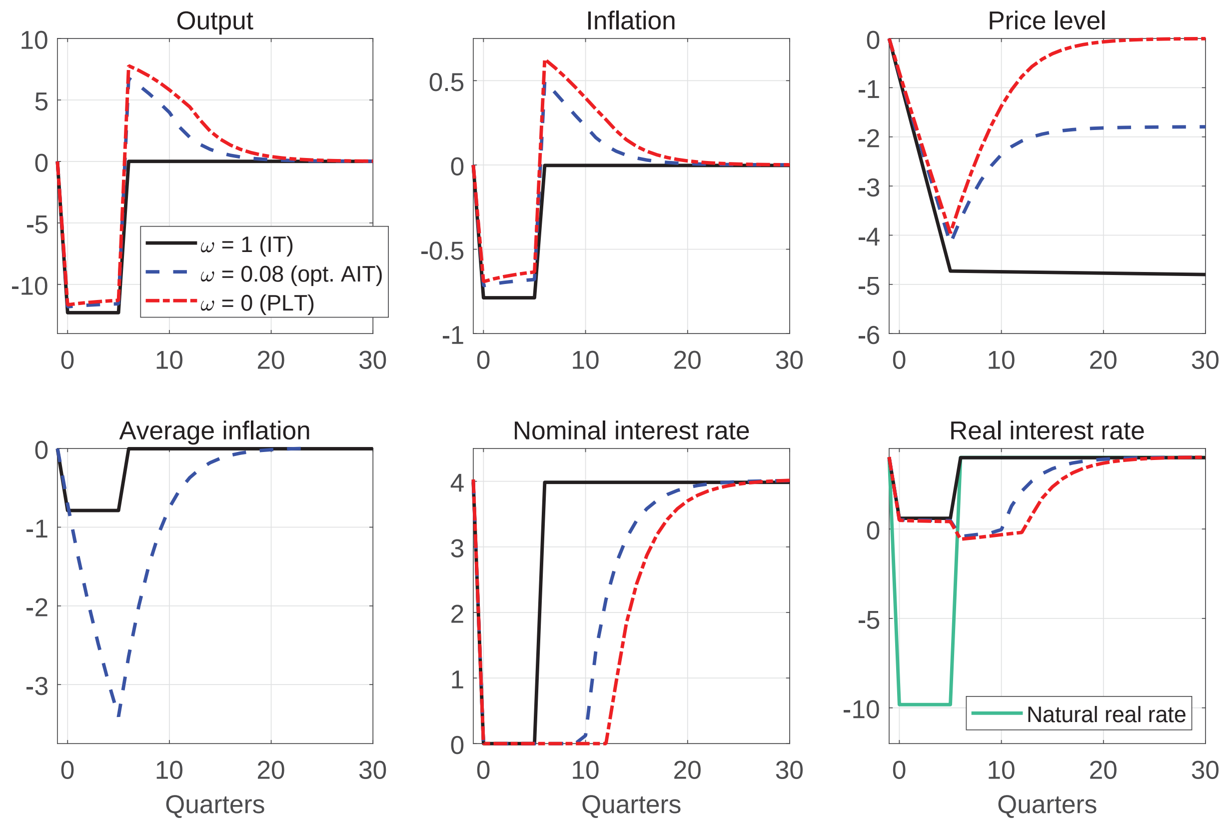
4.2 Why bounded rationality diminishes the welfare gain from AIT

To understand why the optimal ω declines with $\bar{m}$, we show in Figure 7 how the economy with $\bar{m} = 0.75$ evolves in a recession scenario comparable to the one in Figure 3, under a few different

[17] Recall that a higher $\bar{m}$ means a higher cognitive ability. When $\bar{m} = 1$, the model corresponds to the rational expectations model. As $\bar{m}$ becomes lower, agents' decisions today depend less on expected future inflation and output.

values of ω.[18] The size of the shock is chosen so that, when $\omega = 1$, the decline in output is the same as that in the model with rational expectations.[19]

Figure 7: Liquidity trap scenario with boundedly-rational expectations



Note: All variables except for output have been annualized. Average inflation is normalized by $1/\omega$.

Consider first the case with PLT (that is, the case with $\omega = 0$)—shown by dash-dotted red lines. Under PLT, the policy rate is kept at the lower bound for several quarters after the natural real rate turns positive, creating a temporary overheating of the economy. However, relative to the case with $\bar{m} = 1$, the benefit of the temporary overheating on economic activity when the natural rate is negative are small, as future economic conditions have a smaller impact on today's allocations when $\bar{m} = 0.75$. As a result, both inflation and output decline by more during the crisis in the model with boundedly-rational expectations than in the model with rational expectations. At the same time, a lower inflation path during the crisis implies that the inflation overshoot—and as a result the output overshoot—in the aftermath of the crisis is larger than in the rational-expectations model.

Next, consider an AIT regime with $\omega = 0.08$. In this case—shown by dashed blue lines—the policy rate is at the lower bound for a shorter duration and the inflation and output overshoots are

[18] Gabaix (2019) chooses the value of 0.8 for $\bar{m}$ in his numerical exercises, informed by some reduced-form estimates of the IS curve and the Phillips curve in the literature.

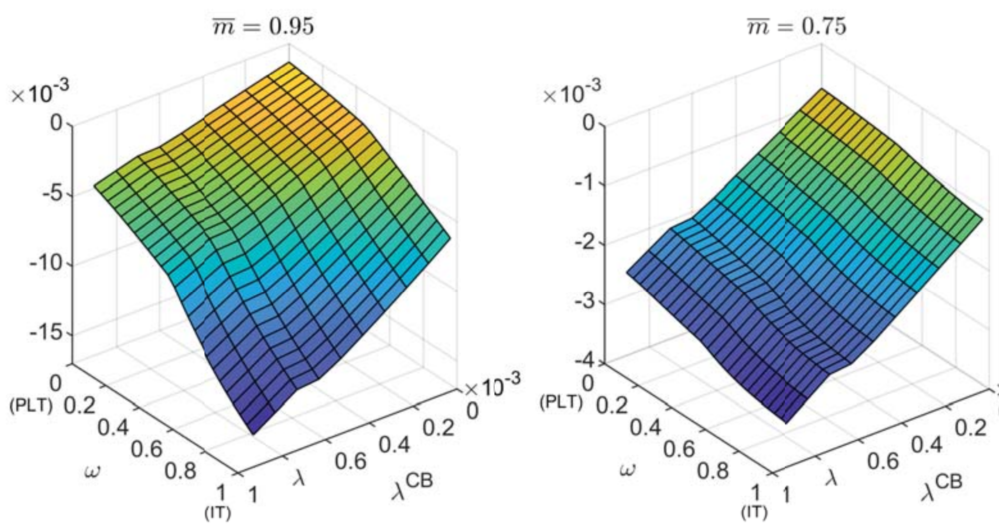
[19] If we kept the shock size unchanged, the declines in output and inflation would be much smaller in the model with boundedly-rational expectations for any value of ω than in the model with rational expectations, and it is difficult to see the effects of adopting the AIT.

smaller than under $\omega = 0$. While inflation and output decline by more in the crisis under $\omega = 0.08$ than under $\omega = 0$, the differences are small. That is, an increase of ω from 0 to 0.08 lowers the cost of AIT nontrivially, while lowering the benefit of AIT only by a small amount, increasing the overall welfare. For our model calibration with $\bar{m} = 0.75$, $\omega = 0.08$ is indeed the optimal value.

4.3 Optimal relative weight on output gap stabilization

We now relax again the assumption that $\lambda^{CB} = \lambda$. Figure 8 plots society's welfare as a function of both ω and λ^{CB}. The left panel shows results when the cognitive discounting parameter $\bar{m}$ equals 0.95, and the right panel shows results when $\bar{m} = 0.75$, as in the liquidity trap scenario considered in the previous subsection.

Figure 8: Welfare effects of AIT and inflation conservatism with boundedly-rational expectations



Note: Welfare is defined in equation (13).

When cognitive limitations are moderate, as in the left panel of Figure 8, strict PLT ($\omega = 0, \lambda^{CB} = 0$) is optimal, as in the rational-expectations model. Optimizing one of the two policy parameters alone—either ω or λ^{CB}—can improve welfare quite a bit, which is also in line with the results obtained for the rational-expectations model.

When cognitive limitations are more severe, as in the right panel, strict PLT is no longer optimal.

Specifically, for $\bar{m} = 0.75$, the optimal values for the policy parameters are $\omega = 0.1$ and $\lambda^{CB} = 0$.[20] Unlike in both the rational-expectations model and the boundedly-rational expectations model with values of $\bar{m}$ close to one, the welfare gain from optimizing over λ^{CB} is much larger than the gain from optimizing over ω. The reason is as follows. With boundedly-rational expectations, policies that improve macroeconomic outcomes at the lower bound by raising expectations about future output gap and inflation are less effective than in the case of rational expectations. The benefits from both AIT and inflation conservatism—i.e. lowering the weight on output gap stabilization—are thus smaller under boundedly-rational expectations. AIT, unlike inflation conservatism, also comes at a cost, which is the temporary output gap and inflation rate overshooting in the future. Due to an adverse feedback loop, this cost is increasing in the degree of agents' cognitive limitations. A given increase in expected future output gap and inflation at the lower bound has a smaller stabilizing effect on current output gap and inflation, which, because of the history dependence motive, requires a larger future overshooting. Inflation conservatism, instead, does not induce a history dependence motive and is thus not prone to the same adverse feedback loop.

5 Conclusion

We study the effect of average inflation targeting—a monetary policy strategy that aims to stabilize an average inflation rate as opposed to a period-by-period inflation rate—on macroeconomic outcomes. The analysis is based on a New Keynesian model with a lower bound on nominal interest rates. We consider two variants of the model, one with rational expectations and one with boundedly-rational expectations. Following the policy delegation approach, we consider the optimization problem of a central bank that takes the assigned objective function as given and sets the short-term nominal interest rate under discretion.

Under rational expectations, assigning an average inflation targeting objective to the central bank improves macroeconomic outcomes and increases people's welfare when compared to standard inflation targeting. While the optimal averaging window is infinitely long, most of the welfare gain can be attained by a finite, but sufficiently long, averaging window.

The results from the rational-expectations model continue to hold true in the model with boundedly-rational expectations as long as cognitive limitations remain small. If, on the other hand, cognitive limitations are sufficiently strong, the optimal averaging window is finite, and the welfare improvement from abandoning standard inflation targeting in favor of average inflation targeting can be small.

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Appendix

A Numerical algorithm

We approximate the policy functions for the inflation rate, output, the policy rate and the average inflation rate with a finite elements method using collocation. For the basis functions we use cubic splines. The algorithm uses fixed-point iteration and proceeds in the following steps:

1. Construct the collocation nodes. Use a Gaussian quadrature scheme to discretize the normally distributed innovation to the natural real rate shock.

2. Start with a guess for the basis coefficients.

3. Use the current guess for the basis coefficients to approximate the expectation terms.

4. Solve the system of equilibrium conditions for inflation, output, the policy rate and average inflation at the collocation nodes, assuming that the zero lower bound is not binding. For those nodes where the zero bound constraint is violated solve the system of equilibrium conditions associated with a binding zero bound.

5. Update the guess for the basis coefficients. If the new guess is sufficiently close to the old one, the algorithm has converged. Otherwise, go back to step 3.

B Welfare effects of lower bound risk motive

To assess the role of the lower bound risk motive for welfare, we solve the benchmark rational-expectations model under a version of monetary policy equation (14) without the term capturing the lower bound risk motive[21]

$$\pi_t = -(1-\omega)\frac{\hat{\pi}_{t-1}}{\omega} + \frac{\beta(1-\omega)\lambda^{CB}}{\kappa}\mathrm{E}_t y_{t+1} + A_{LB}(\hat{\pi}_t)\phi_t^{LB} + A_y(\hat{\pi}_t)y_t, \tag{B.1}$$

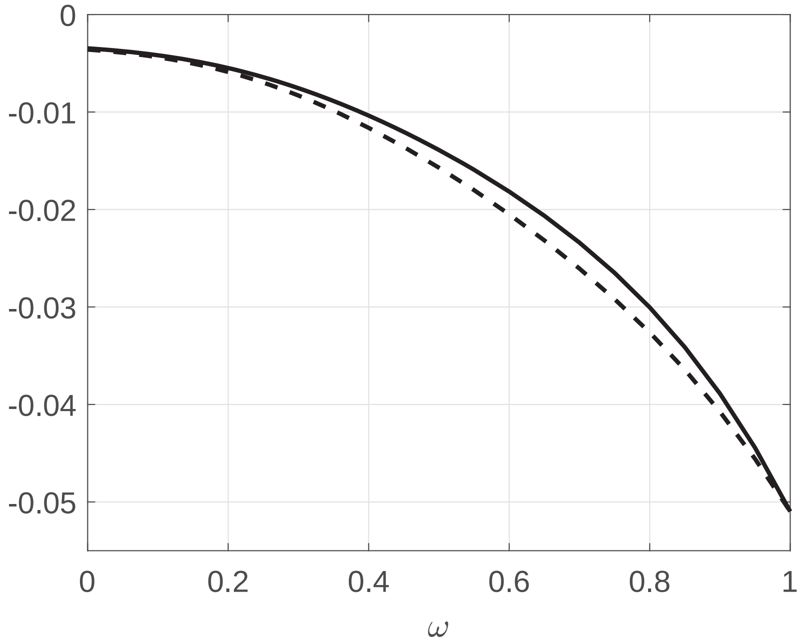
where $A_{LB}(\hat{\pi}_t)$ and $A_y(\hat{\pi}_t)$ are defined in equations (15) and (16).

Figure B.1 shows how society's welfare (13) varies with the inflation-averaging parameter ω in the rational-expectations model ($\alpha_{EE}, \alpha_{PC} = 0$) when monetary policy is characterized by (i) equation (14) (solid line), and (ii) equation (B.1) (dashed line).

Welfare is lower when monetary policy is not guided by the lower bound risk motive than under the optimal discretionary policy. Quantitatively, however, the difference between the two welfare curves is relatively small. One explanation could be that for values of ω close to one, the coefficient on the Lagrange multiplier associated with the lower bound constraint ϕ_t^{LB} is very small, because it entails the term $(1-\omega)$. For values of ω further below one, the coefficient on the Lagrange multiplier becomes larger but lower bound risk itself—i.e. the size of the Lagrange multiplier—is

[21] The lower bound risk motive in (14) is captured by the term $\frac{\beta(1-\omega)}{\kappa\sigma}\mathrm{E}_t\phi_{t+1}^{LB}$.

Figure B.1: Welfare with and w/o lower bound risk motive



Note: Welfare is defined in equation (13).

mitigated because when ω is small, the decline in the output gap and inflation at the lower bound is less severe, which itself is a result of the history dependence motive.