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## ABSTRACT

### R&D Alliances as Non-cooperative Supergames\*

R&D alliances (Research Joint Ventures or other institutional forms) normally involve repeated, non-contractible actions (investments in R&D), and uncertainty regarding both success and the termination date. Accordingly, we model these agreements as equilibria of infinite-period supergames.

Our approach is normative, namely that of finding optimal equilibria from the perspective of the firms involved in the agreement. The results show that repeated interaction allows for important gains in equilibrium pay-offs. The optimal solutions are still inefficient from the firms' perspective, however. The sources of inefficiency include delay in investment outlays, suboptimal levels of investment, and abandonment of profitable projects.

Lastly, we consider R&D cooperation between firms that also interact in the product market. In some cases, product market interaction is irrelevant from the perspective of optimal R&D agreements. In other cases, optimal agreements imply that firms behave more aggressively in the product market.

JEL Classification: C72, L1, O3

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# R&D ALLIANCES AS NON-COOPERATIVE SUPERGAMES

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## NON-TECHNICAL SUMMARY

The number of formal alliances between firms has grown dramatically in recent years. In the United States, for example, this number has increased from about 750 in the 1970s to about 20,000 in 1987–92 (*The Economist*, 2 September 1995). Although the scope of these alliances cannot always be easily ascertained, R&D cooperation seems to play an important role in most. The actual mechanism underlying each agreement can take various forms, from strategic alliances without equity commitments to mergers or joint ventures that involve a separate shared entity.

The increase in cooperative R&D activity results, to a large extent, from the tolerance – or even encouragement – that competition policy authorities have increasingly shown towards these kind of agreements. Despite this freedom from regulatory constraint, however, it remains a fact that formal, complete interfirm contracts pertaining to R&D activities are extremely difficult to write down and enforce. This may explain why agreements are frequently very simple (e.g. cross licensing of newly developed technologies), and also why formal joint ventures turn out to be very unstable.

In other words, R&D agreements normally involve non-contractible (or non-contracted) actions, namely the investments in R&D. This is certainly true of 'strategic alliances' and other simple agreements (like cross licensing) that do not directly specify investment levels. But even the more complex forms (joint ventures, for example) frequently imply sharing information and expertise, something which is very difficult to contract upon.

Another important characteristic of R&D agreements is that they apply to a dynamic world in which firms make sequential investment decisions in a context of uncertainty. Sequentiality is a direct consequence of the nature of the R&D activity: information from stage 1 investment is necessary before proceeding to stage 2. Uncertainty, in turn, has various components. In particular, the possibility of a pre-emptive discovery or development by a rival outside the agreement implies that the termination date for any given agreement is uncertain.

Following these considerations, we model R&D agreements as non-cooperative equilibria of infinite-period supergames. The concept of a non-cooperative equilibrium produces the idea that it is not possible to write enforceable contracts: any actions that firms agree upon must be, at any point, in each firm's unilateral best interest. The concept of an infinite-period game,

in turn, formalizes the idea that there is an uncertain terminal date: each firm considers the future as a potentially infinite sequence of periods, even though the agreement ends in finite time for sure; and part of each firm's future discount factor consists of the probability that the agreement will end (for exogenous reasons) at the end of each period.

We derive optimal non-cooperative equilibria in different contexts (observable or non-observable actions; gradual or drastic success in R&D). We find that profits in these equilibria are lower than, but possibly very close to, profits from an agreement based on an enforceable contract. In other words, dynamic interaction between firms can, in some circumstances, almost perfectly substitute for an enforceable contract.

When actions by each firm are observable by the other firm, the optimal equilibrium consists of a series of gradual investments by each partner. Even though these investments are not in each firm's immediate interest (they would prefer to free ride on the other firm's investment), the incentives to invest are sustained by the implicit threat of abandoning the project in case some firm deviates from the prescribed plan. In equilibrium, however, the project is not abandoned; rather, the stock of R&D investments converges to the efficient level, the one that would be implemented if contracts could be written and enforced. The optimal non-cooperative solution is therefore 'almost' efficient, the difference resulting from the delay in the investments.

When investments in R&D are not observable and success is uncertain, the optimal agreement dictates that the project be abandoned at each stage with some probability conditional on no success having been attained by then. This suggests that the abandonment of a joint project may reflect factors other than the arrival of new information on its merits. Just as in the perfect-observability case, the implicit threat of abandoning a project is required to give firms an incentive to invest in R&D. But, since success is uncertain, abandonment of a profitable project will take place with positive probability; the optimal agreement is thus inefficient.

In the last section of the paper, we consider the case where firms engaging in an R&D agreement also interact in the product market. Previous authors have stressed the possibility of R&D agreements facilitating collusion in the product market. We show that, conversely, product market interaction can facilitate R&D cooperation when R&D contracts cannot be enforced.

# 1 Introduction

The number of formal alliances between firms has grown dramatically in recent years. In the U.S., for example, this number has increased from 750 in the 70s to 20,000 in 1987–92 (*The Economist*, September 2nd, 1995). Although the scope of these alliances cannot always be easily ascertained, R&D cooperation seems to play an important role in most of them. The actual mechanism underlying each agreement can take various forms, from strategic alliances without equity commitments to mergers or joint ventures that involve a separate shared entity.

The increase in cooperative R&D activity results, to a large extent, from the tolerance—or even encouragement—that competition policy authorities have increasingly shown towards these kind of agreements.<sup>1</sup> However, despite this freedom from regulatory constraint, it remains a fact that formal, complete interfirm contracts pertaining to R&D activities are extremely difficult to write down and enforce. This may explain why, frequently, agreements are very simple (e.g., cross licensing of newly developed technologies), and also why formal joint ventures turn out to be very unstable.<sup>2</sup>

In other words, R&D agreements normally involve non-contractible (or non-contracted) actions, namely the investments in R&D. This is certainly true of “strategic alliances” and other simple agreements (like cross licensing) that do not directly specify investment levels. But even the more complex forms (joint ventures, for example) frequently imply sharing information and expertise, something which is very difficult to contract upon.

Another important characteristic of R&D agreements is that they apply to a dynamic world in which firms make sequential investment decisions in a context of uncertainty. Sequentiality is a direct consequence of the nature of the R&D activity: information from stage 1 investment is necessary before proceeding on to stage 2. Uncertainty, in turn, has various components. In particular, the possibility of a

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<sup>1</sup>Cf Jacquemin (1988), Geroski (1993), Martin (1993).

<sup>2</sup>For discussions on the issue of stability of joint ventures, see Harrigan (1985), Kogut (1989) and Minchart (1993).

preemptive discovery or development by a rival outside of the agreement implies that the termination date for any given agreement is uncertain.

Following these considerations, we model R&D agreements as equilibria of infinite-period supergames. In these games, firms make R&D investments (possibly zero investments) in each of an infinite series of periods. Firms' actions are not contractible, but repeated interaction allows for Nash equilibria that are mutually beneficial with respect to the myopic noncooperative equilibrium. Uncertain terminal date is modeled as a component of the firms' discount factor, so the idea that the agreement lasts for ever should not be taken literally.<sup>3</sup> These supergames are not repeated games in a fundamental way, since R&D introduces stock variables and nonstationary processes. However, as we will see, several insights of the repeated-game literature apply in this context.

Our approach is normative, namely that of finding optimal equilibria from the perspective of the firms involved in the agreement. We first consider the case of an incremental-success project. In this case, firms make repeated investments and enjoy a benefit flow which is a function  $\pi$  of their cumulative investment. We show that, in the optimal equilibrium, provided firms discount little the future, the stock of R&D comes arbitrarily close to the efficient level, although with real time delay.

Next we consider an all-or-nothing innovation project. Success is an uncertain, zero-one event, and the probability of success in each period is a function  $f$  of investment in that period. Moreover, R&D investments are assumed not to be observable (whereas in the incremental-success research model it is assumed they are observable but non-contractible). We show that the optimal equilibrium can be implemented by investing a constant amount in each period and abandoning the project with some constant conditional probability. In addition, we show that the equilibrium is inefficient for *any* value of the discount factor.

Finally, we consider the case when the firms forming an agreement on a drastic-

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<sup>3</sup>In other words, the discount factor is given by  $\delta = e^{-r\Delta}(1 - \rho)$ , where  $r$  is the continuous time discount rate,  $\Delta$  the length of each period, and  $\rho$  the conditional probability that a rival firm will preempt the R&D agreement in each period. We assume for simplicity that  $\rho$  is constant in time. This is obviously not a very realistic assumption, but not a crucial one either.

that relates to our paper. The first model we present, where we assume perfect observability, is related to models of private provision of a public good (Admati and Perry (1991)), joint exploitation of a common resource (Lewis and Cowans (1984), Caves (1987)), environmental protection (Barrett (1994)), oligopoly with durable goods (Ausubel and Deneckere (1987)), and models of firm regulation with capital investment (Salant and Woroch (1993)). In Section 2, the relation of our results to this literature is made more precise.

Dutta (1995) and others have developed Folk theorems for dynamic (non-repeated) games with perfect observability. However, these models typically assume asymptotic state independence, an assumption that is violated in the context of R&D agreements.<sup>4</sup> In addition, most of the results in our paper pertain to the case of imperfect observability. In this case, the relevant related literature is the one on repeated games with imperfect public information: Porter (1983), Green and Porter (1984), Radner (1986), Radner, Myerson and Maskin (1986), Abreu, Pearce and Stacchetti (1986, 1990), and Fudenberg, Levine and Maskin (1994). Although the games we consider are not repeated games, we will show that optimal equilibria have a flavor that is similar to repeated games.

## 2 Incremental-success research with perfect observability

We first consider a model of an agreement on an incremental-success research project. There are two firms and a complete-spillover project yielding  $\pi(X_t)$  (payoff flow) in period  $t$  to each firm, where  $X_t$  is the level of cumulative investment by both firms up to time  $t$ :

$$X_t = \sum_{\tau=1}^t x_{1\tau} + x_{2\tau},$$

where  $x_{i\tau}$  is investment by firm  $i$  at time  $\tau = 1, \dots$  and  $X_0$  is given. The values  $x_{i\tau}$  are perfectly observable, although not contractible. We assume that  $\pi(X)$  is a smooth

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<sup>4</sup>Specifically, it is required that the state of long run feasible payoffs be state independent.

innovation project are also competitors in the product market. In some cases, an irrelevance result applies: product market interaction has no impact on the optimal R&D agreement. In other cases, however, the optimal solution involves competitive pricing as a means of supporting a more efficient R&D agreement.

■ **Related literature.** Most of the literature, starting with a seminal papers by Katz (1986) and d'Aspremont and Jacquemin (1988), has looked at atemporal models of R&D cooperation and product market competition. The literature is now quite extensive and includes De Bondt, Wu and Lievens (1992), Henriques (1992), Kamien, Muller and Zang (1992), Suzumura (1992), Kamien and Zang (1993), and Martin (1994). In these models, contrary to our paper, contractibility or R&D investments is not an issue.

A few papers look at explicitly dynamic models of R&D cooperation. Kesteloot and Veugelers (1993) analyze a repeated two-stage game in which firms make R&D and product market decisions. Their paper looks at optimal trigger-strategy equilibria. It is shown that collusion becomes easier as the degree of spillovers increases. In Veugelers and Kesteloot (1994), the same analysis is extended to the case of asymmetric firms. These papers differ from ours in that they assume R&D investments which depreciate in one period and actions which are ex-post perfectly observable. Sandonis (n.d.) considers the case when there is uncertainty about the payoffs from R&D cooperation and each firm's contribution is unobservable. In this context, firms invest gradually as they gain more information; and, with positive probability, the R&D project is discontinued before completion. Martin (1993, 1995) and van Wegberg (1995) show that agreements in the market for innovation have implications for product market performance. In fact, he argues, the threat to break up an R&D joint venture will help sustain tacit collusion in product markets. Both Sandonis (n.d.), Martin's (1993, 1995), and van Wegberg's (1995) results relate to some of ours, although there are important differences both in the assumptions and in the results, as shown in Sections 3 and 4.

There is also an extensive literature in Game Theory and Applied Game Theory

increasing concave function. More specifically, we make the following assumption regarding the payoff function.

**Assumption 1**  $\pi''(X) \leq -\xi < 0$  for all  $X$  such that  $\pi'(X) > 0$ .

Since R&D does not depreciate, the efficient solution, or cooperative equilibrium, consists of a one-time investment in period 1. This is the value that maximizes  $2\pi(X)/(1 - \delta) - X$ , which yields  $\pi'(X^*) = (1 - \delta)/2$ .

The purpose of our analysis is to see how dynamic considerations may help firms attain, in a non-cooperative equilibrium, a payoff that is close to the cooperative solution. In the context of repeated games, the way to put this question is: how much better a payoff can we obtain in the equilibrium of a repeated game relative to the equilibrium of the stage game. Now, the game we consider here is not a repeated game, so the question has to be put differently. A natural extension could be the following: can history-dependent equilibria yield a better payoff than Markov equilibria, that is, equilibria which only depend on the level of  $X$  and are otherwise history-independent?<sup>5</sup> Unfortunately, the restriction to Markov equilibria is not sufficient to rule out a number of equilibria that are very similar to history dependent equilibria (see the remarks below Theorem 2). However, if we additionally require that the subgame value function  $V(X)$  be a differentiable function of  $X$  then we obtain an equilibrium which constitutes a useful benchmark of comparison.

**Lemma 1** *There exists a unique symmetric Markov subgame perfect Nash equilibrium such that the value function  $V(X)$  of a subgame starting with a stock  $X$  is differentiable in  $X$ . In this equilibrium,  $X_t \rightarrow \hat{X}$ , where  $\pi'(\hat{X}) = 1 - \delta$ .*

(Proofs are contained in the appendix.) Clearly,  $\hat{X} < X^*$ , so the equilibrium in Lemma 1 is inefficient. In fact, as the next result shows, inefficiency is a feature of any Nash equilibrium.

**Theorem 1** *There exists no Nash equilibrium such that  $X_t = X^*$  for some  $t$ .*

<sup>5</sup>Notice that, for repeated games, the two questions yield the same answer if we assume that there is only one Markov state.

However, as the next result shows, any level arbitrarily close to  $X^*$  can be attained in finite time provided the discount factor is large enough.

**Theorem 2** *For every  $\epsilon > 0$ , there exists a  $\delta'$  such that: for  $\delta > \delta'$ , there exists a subgame perfect Nash equilibrium such that  $X^* - \epsilon$  is reached in finite time.<sup>5</sup>*

Notice that the equilibrium proposed in the proof of Theorem 2 can be implemented as a Markov equilibrium if  $X_0 \geq \hat{X}$ . In fact, it suffices to consider the following Markov strategies: at state  $X = X_{t-1}$ , invest  $x_t$ ; at all other states, invest zero. (If  $X_0 < \hat{X}$ , implementation by Markov equilibrium can be attained if the initial investments are asymmetric.) Therefore, a restriction like the one considered in Lemma 1 seems necessary. Another important remark is that the equilibrium proposed in the proof of Theorem 2 is not necessarily optimal. The optimal equilibrium will be one in which the no-deviation constraints are just binding in each period.

In summary, our results show that, with perfect observability, the efficient level of R&D can be arbitrarily approached in finite time provided the discount factor is sufficiently close to one. However, even with very little discounting the optimal non-cooperative agreement is less efficient than the optimal cooperative agreement. The source of inefficiency is the delay in getting to the optimal level of R&D.

### 3 All-or-nothing research agreements with imperfect observability

In the previous section, we have considered a model that is perhaps appropriate to describe a joint *development* project. In that model, success is an incremental

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<sup>5</sup>This result is similar to the one which appears in Salant and Woroch (1993). In fact, we believe it generalizes their result. In their model, a regulator plays against a regulated firm that makes investments in capital which do not depreciate. The optimal capacity can be approximated by a trigger-strategy equilibrium in which the firm invests gradually and the regulator progressively reduces the allowed rates.

Our model and the present result also bear some resemblance to Admati and Perry's (1991) analysis of joint projects without commitment. Their model differs from ours in different respects; in particular, they assume alternate moves by each player, a step  $\pi$  function ( $\pi(X) = 0$  if  $X < K$ ,  $\pi(X) = 1$ ,  $X \geq K$ ), and a convex cost function.

process, and effort is perfectly observable. The model considered in this section differs in both these aspects. First, we assume that success in R&D is a zero-one event: in each period, there is an i.i.d. smooth probability function of success,  $f(x_1, x_2)$ , success being worth  $\Pi$  to each firm. Moreover, we assume that research effort  $x_i$  is not observable by firm  $j$  at any time.<sup>7</sup> Given the nature of these assumptions, the model of this section better describes agreements on *basic research*.

We begin by making a two-part assumption on the function of probability of success. Denote by  $f'(\cdot, \cdot)$  the first partial derivative with respect to the first argument.

**Assumption 2** (i)  $f'(0, 0)\Pi > 1$ ; (ii)  $f(0, x) = 0$ .

The first part is an assumption of non-triviality: if  $f'(0, 0)\Pi < 1$ , then the only equilibrium will be for firms never to invest. The second part can be interpreted to imply that there is some sort of complementarity between the research efforts of each firm. Its importance for the results that follow stems from the implication that not investing in any period constitutes an equilibrium.<sup>8</sup>

Our first result characterizes the optimal symmetric equilibrium.<sup>9</sup>

**Lemma 2** *The best symmetric equilibrium payoff can be implemented by a designated investment level in each period ( $x$ ) and a probability of continuation in case of no success ( $y$ ).*

By probability of continuation  $y$  we mean that in each period, conditional on investment  $x$  not being successful, firms will stop investing ( $x = 0$  in all future periods) with probability  $1 - y$ , and, with probability  $y$ , invest  $x$  again in the following period.

Several remarks are in order. First,  $y < 1$  implies that a profitable project is abandoned with probability one in finite time. Sandonis Díez (n.d.) also presents a

<sup>7</sup>The assumption that the signal firm  $i$  receives about firm  $j$ 's action is common knowledge is not innocuous; cf Compte (1996).

<sup>8</sup>The existence of this subgame equilibrium may allow for history-dependent equilibria yielding each firm a higher payoff than in any of the Markov equilibria. Notice the analogy with finitely repeated games; cf Benoit and Krishna (1985).

<sup>9</sup>The game at hand is one of incomplete information, and the equilibrium concept is that of Sequential Equilibrium (cf Kreps and Wilson, 1982).

model in which projects are abandoned, with positive probability, along the equilibrium path. Our result differs from his in several respects. In particular, in our model the decision to abandon a project does not depend on new information being revealed; rather, abandonment has the sole purpose of providing firms with incentives to invest in R&D.

Second, it should be stressed that Lemma 2 does *not* imply that the optimal equilibrium *must* be implemented in the indicated way. In fact, as the proof of the lemma shows, only the continuation payoff  $U$  matters in terms of sustaining a given equilibrium. Any two continuation equilibria yielding the same expected payoff are therefore equivalently good.

There are at least two “natural” alternatives for implementing an optimal solution. The first one is to continue investing during  $T_1$  periods in case of no success during the first period; and abandoning the project (i.e., investing zero forever) if no success is attained by then. The second one is to stop investing during  $T_2$  periods and then resume play as in the first period.<sup>10</sup>

The third remark is that, apparently, no further characterization of the optimal solution is possible. The natural next step would be to characterize the optimal value of  $y$ . However, we can show by example that either  $y = 0$ ,  $y = 1$ , or  $0 < y < 1$  can be optimal for reasonable functional forms and parameter values. In fact, if  $f(x_1, x_2) = \frac{x_1 + x_2}{\alpha + x_1 + x_2}$  (with  $\alpha = 1$ ,  $\delta = .9$ ,  $\Pi = 2$ ), then  $y^* = 1$ ; if  $f(x_1, x_2) = \frac{\alpha}{\alpha + \exp(-x_1 - x_2)}$  (with  $\alpha = .05$ ,  $\delta = .9$ ,  $\Pi = 10$ ), then  $0 < y^* < 1$ ; finally, if  $f(x_1, x_2) = (x_1 + x_2)^\alpha$  (with  $\alpha = .5$ ,  $\delta = .8$ ,  $\Pi = 2$ ), then  $y^* = 0$ .<sup>11</sup>

Although a complete general characterization of the optimal solution is not possible, it can be shown that the optimal solution is always inefficient.

**Theorem 3** *For all  $\delta < 1$ , the optimal equilibrium is inefficient.*

Notice that our inefficiency result holds *for all*  $\delta$ , which contrasts with results

<sup>10</sup>This latter possibility resembles more closely the equilibrium structure proposed by Green and Porter (1984).

<sup>11</sup> $y = 0$  implies that firms invest only in the first period. In a non-strategic context, Dutta (1992) presents necessary and sufficient conditions for the optimality of a “bold play”—spending all the remaining budget on the current stage of a sequential R&D project.

from the repeated-game literature. In particular, Fudenberg, Levine and Maskin (1994) have established a folk theorem for repeated games with imperfect observability. Although the game we consider is not a repeated game, its structure is very similar to that of a repeated game. In fact, the difference in results between us and Fudenberg, Levine and Maskin (1994) lies on the latter's assumption regarding observability: they require "that there exist action profiles with property that ... no two deviations—one by each player— give rise to the same probability distribution over public outcomes." (p. 997) an assumption that is clearly violated in our model.

Finally, we confirm, as in the previous section, that dynamic interaction can help firms attain a better payoff than in history-independent equilibria. In the present context, a history-independent equilibrium is one with  $\gamma = 1$ . But, as we have seen, the optimal equilibrium may very well imply  $\gamma < 1$ , in which case the optimal equilibrium is strictly better than the best history-independent equilibrium. Moreover, as Theorem 3 establishes, the optimal equilibrium is strictly worse than the efficient solution, so the second inequality presented in the previous section also holds when actions are not observable.

## 4 Product market competition

In a recent policy paper, Martin (1993) argues that cooperative R&D may not be a good thing, the idea being that "joint R&D makes it more likely that firms will be able to sustain tacit collusion on output markets" (pp. 1-2) (see also Martin (1995) and van Wegberg (1995)). This idea is reminiscent of Bernheim and Whinston's (1990) theory of multimarket contact: firms that interact in more than one market are able to sustain collusion more easily than firms interacting in one market only. Although an R&D agreement is not isomorphic to a product market, Martin (1993) shows that Bernheim and Whinston's (1990) intuition extends to the R&D cooperation-cum-product-market-competition case. He assumes that R&D investments are contractible and shows that the scope for collusion in the product market is extended when the breakdown of the R&D agreement is taken into account as a possible punishment for deviations in the output market.

Our assumptions regarding R&D cooperation differ from Martin's (1990). Throughout the paper, we assume that R&D investments cannot be contracted upon. In addition, in the previous and in this section we also assume that R&D investments are not observable. As we will see, these alternative assumptions imply significant differences in the nature of the optimal equilibria.

As in the previous section, we consider a model of all-or-nothing success in research. Specifically, we assume that  $x_i \in \{0, 1\}$  and that the success function is as follows:  $f(1, 1) = \alpha$ ,  $f(1, 0) = f(0, 1) = \beta$ ,  $f(0, 0) = 0$ . Moreover, we assume that  $\beta$  is small.<sup>12</sup>

Finally, we also assume that, in addition to setting R&D expenditures, firms simultaneously set prices in a homogeneous product market where they sell at constant marginal cost. The value of the innovation is now measured in terms of profits in the product market. Specifically, before success in R&D, monopoly profits are given by  $\underline{\pi}$ , whereas after success in R&D monopoly profits are  $\bar{\pi}$ , where  $\underline{\pi} < \bar{\pi}$ .

Specifically, the timing of the game is the following: at the beginning of each period, firms determine R&D expenditures and conditional prices:  $p$  if no success is achieved and  $q$  if success is achieved. The outcome of the R&D process is known and payoffs are received (at the beginning of the period).

As before, we restrict attention to symmetric equilibria. In addition, we follow Lemma 2 and consider equilibria stipulating levels of  $x$  (investment in R&D) and  $y$  (probability of continuing investing).<sup>13</sup> A symmetric equilibrium can therefore be denoted by (i) a vector  $(x, y, p, q)$ , where  $x$  and  $y$  are defined as before, and  $p$  and  $q$  are the conditional prices defined above; (ii) a system of punishment equilibria following observable deviations from the equilibrium path (i.e., from the price path).<sup>14</sup> For

<sup>12</sup>Specifically, the assumption that is required is that

$$\beta < \frac{1}{\delta} \left( \sqrt{1 + \delta^2(1 - \alpha) - \delta(2 - \alpha)} - (1 - \delta) \right).$$

This assumption guarantees that there always exists a Markov equilibrium in pure strategies. When the assumption does not hold, the analysis becomes more complicated with no significant gain in insight.

<sup>13</sup>It is straightforward to show that the intuition of Lemma 2 extends to the present context.

<sup>14</sup>Note that, for simplicity, this structure makes the additional (reasonable) assumption that prices are constant along the equilibrium path before and after success.

simplicity in notation, we measure  $p$  and  $q$  on a scale between zero (price equal to marginal cost) to one (monopoly price).

In order to consider the relevance of interaction in more than one strategic dimension (R&D expenditures and prices), we introduce the concept of separate optimal equilibria.

**Definition 1** *A Symmetric Separate Optimal Equilibrium (SOE), is a vector  $(x, y, p, q)$  such that  $(x, y)$  is a symmetric optimal equilibrium given  $(p, q)$  and  $(p, q)$  is a symmetric optimal equilibrium given  $(x, y)$ .*

Our main result is that, depending on parameter values, a SOE may or may not be an OE. The negative case implies that “multimarket” contact is relevant, as initially conjectured.

**Proposition 1** *Depending on parameter values, a SOE may or may not be an OE.*

1. If  $\delta > \frac{1}{2}$  and

$$1 \leq \frac{\alpha - \beta}{1 - \delta}(\bar{\pi} - \underline{\pi}) \leq \frac{1 - \delta(1 - \beta)}{1 - \delta}, \quad (1)$$

then  $(1, y, 1, 1)$ , for a given  $y \in (0, 1)$ , is the unique SOE; it is also one of multiple OE;

2. If  $\delta > \frac{1}{2}$  and

$$1 - \beta < \frac{\alpha - \beta}{1 - \delta}(\bar{\pi} - \underline{\pi}) < 1. \quad (2)$$

then  $(0, 0, 1, 1)$  is the unique SOE, whereas the unique OE is given by  $(1, 1, p, 1)$ , for a given  $p \in (0, 1)$ ;

3. In all other cases, there exists a unique SOE and a unique OE, and these are identical.

The first part of Proposition 1 states that, under (1), a SOE is an OE, that is, product market contact is irrelevant for the purpose of determining the optimal R&D agreement. Bernheim and Whinston (1990) also present an irrelevance result for

firms interacting in several markets. Our model of R&D cooperation-cum-product-market-competition bears some resemblance to multimarket contact. However, our irrelevance result is of very different nature. The crucial element in our result is that one of the strategic variables is unobservable. Bernheim and Whinston (1990), in turn, consider repeated games with perfectly observable outcomes.

Specifically, the intuition for our irrelevance result stems from the fact the optimization problem to be solved in determining the OE has the form

$$\begin{aligned} \max_{p_1, U} \quad & \alpha \frac{\pi}{1-\delta} + (1-\alpha)(\pi_1(p_1) + \delta U) - 1 \\ \text{s.t.} \quad & (\alpha - \beta) \left( \frac{\pi}{1-\delta} - \pi_1(p_1) - \delta U \right) \geq 1 \end{aligned}$$

where  $p_1$  is price in the first period and  $U$  the continuation value in case of no success. The crucial point to notice is that both  $V$  and the no-deviation constraint depend equally on the value of  $\pi_1(p_1) - \delta U$ . Therefore, any combination of  $p_1$  and  $U$  (or  $p_1$  and  $y$ ) yielding the same value of  $\pi_1(p_1) - \delta U$  yields the same equilibrium payoff. In particular, the combination implicit in the SOE is optimal.

The intuition for the second part of Proposition 1 is based on Arrow's (1962) idea of the *replacement effect*: a firm's incentive to invest in R&D is inversely related to its current profit.<sup>15</sup> By setting low prices, the firms engaged in the R&D agreement create an additional incentive for them to invest. In some cases, this may be what is necessary to turn an unstable agreement into a stable one; and, if the value of the innovation is sufficiently high, then the short-run sacrifice in profits is worth making.<sup>16</sup>

<sup>15</sup>I am grateful to Vincenzo Denicolo for pointing this out to me.

<sup>16</sup>Lin (1995) presents a model in which the replacement effect implies that firms behave *less* competitively. His model differs from ours in that he assumes firms commit to future prices (or quantities) before engaging in an R&D *race* (not an agreement). The intuition for his result is that, by behaving less aggressively, firms are able to soften up their rivals in the R&D race.

## 5 Extensions

The analysis of the previous sections can be extended in several ways, including the following.

■ **Lumps in  $x$  and in  $\pi$ .** The equilibrium in the proof of Theorem 2 calls for an exponentially decreasing sequence of investments in R&D. Clearly, there must be minimum units of R&D (money units if we are talking about monetary investments). Likewise, it is reasonable to assume that there are lumps in the payoff function  $\pi$ . For example, in developing a new, faster chip, fractional increases in speed may be *payoff* irrelevant as they are not marketable as such. How important is this in terms of Theorem 2?

We conjecture that, if  $\pi$  is a smooth strictly concave function and  $X$  has to move on a fine but finite grid, then the maximum equilibrium level of  $X$  is given by the Markov equilibrium level of Lemma 1.<sup>17</sup> If, however, there are also large lumps in  $\pi$ , then the result of Theorem 2 holds again, in fact it is strengthened. Specifically, assume that the  $X$  grid is finer than the grid of jumps in  $\pi$ . Then, we conjecture, the efficient level can be achieved in finite time.

■ **Incremental-success research agreements with imperfect observability.**

The model of incremental-success research agreements considered in Section 2 assumes perfect observability. What if R&D investments cannot be observed, as in the all-or-nothing model? Specifically, consider the same model as in Section 2 with the following changes: (a) R&D expenditures are not observable; (b) in each period, the increase in the stock of knowledge,  $z_t = X_t - X_{t-1}$ , is randomly distributed according to a twice differentiable cdf  $F(z_t | x_{1t}, x_{2t}, X_{t-1})$  with support  $\mathbb{R}_0^+$  and density  $f(z_t | x_{1t}, x_{2t}, X_{t-1})$ .

Full characterization of an optimal equilibrium in this context is rather difficult. However, partial characterization seems possible. In particular, we strongly con-

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<sup>17</sup>For a similar result, see Section 5.4 in Salant and Woroch (1993).

ture that optimal equilibria will have a “bang-bang” feature similar to what appears in Abreu, Pearce and Stacchetti (1990) (but, notice these authors model a repeated game, which leads to a rather different problem than ours).

In particular, suppose that the following assumptions hold: (a)  $x_{it} = 0 \Rightarrow z_t = 0$  (cf Assumption 2(ii)); (b)  $\frac{1}{f} \cdot \frac{\partial f(z|x_1, x_2, X)}{\partial z}$  is increasing in  $z$ .<sup>18</sup> Then, we conjecture an optimal equilibrium can be implemented by a simple rule stating an investment level  $x(X)$  and a “minimum progress threshold”  $\bar{z}(X)$  such the project is abandoned if  $z < \bar{z}(X)$ .

## 6 Final remarks

Based on the assumption that R&D investments are difficult to contract upon, we have considered R&D agreements as equilibria of infinite-period supergames.

Optimal equilibria are typically time-dependent. When actions are perfectly observable, firms invest gradually up to (very close) the efficient level. When actions are not observable, optimal agreements stipulate investment levels and a probability of abandonment of the project.

Repeated interaction allows for important gains in equilibrium payoffs. However, the optimal solutions are still inefficient from the firms’ perspective. The sources of inefficiency include delay in investment outlays, suboptimal levels of investment, abandonment of profitable projects, and low prices in the product market.

The possibility that firms interact in the product market in addition to R&D may, in some cases, be irrelevant: separate optimization of the R&D agreement and of product market collusion yields a global optimum. In other cases, however, accounting for multi-dimensional interaction allows for significant improvements in equilibrium payoffs. When this is the case, firms set lower prices prior to success in R&D than they would in a separate optimum.

Care must be taken when deriving policy implications from our results. Bertrand

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<sup>18</sup>This is like a monotone likelihood ratio assumption. It is stronger than monotonicity in the sense of first-order stochastic dominance, that is, assumption (b) implies that  $\frac{\partial}{\partial z} F(z; x_1, x_2, X) < 0$ , but not the reverse. Assumption (b) is satisfied by the Exponential, Truncated Normal, Gamma and Weibull distributions where the mean is a function of  $x_1 x_2$ .

competition typically yields drastic outcomes, and our model is no exception. In particular, it is crucial to our results that, when the discount factor is low, no collusion in prices is ever possible, and as a consequence no investment in R&D is ever desirable. For other models of product market competition, the result may well be that a global optimum implies higher prices than separate optimization, as in Martin (1993, 1995) and van Wegberg (1995).

Finally, we note that, while the models we have considered are rather stylized, we believe the qualitative results do extend to more general contexts. Nonetheless, such extension would constitute a worthwhile line for future research.

## Appendix

**Proof of Lemma 1:** Since  $V(X)$  is differentiable and the equilibrium is symmetric by assumption, equilibrium investment in each visited state must satisfy

$$x \in \arg \max_x \pi(X - x + x') + \delta V(X - x + x') - x'. \quad (3)$$

This implies that

$$\pi'(X) + \delta V'(X) = 1,$$

and, in fact, that, for all states  $X_t$  visited along the equilibrium path,

$$\pi(X_t) + \delta V(X_t) = a + bX_t. \quad (4)$$

By the assumption of strict concavity of  $\pi$  (Assumption 1), equilibrium values of  $X$  must be bounded. Since  $X_t$  is increasing, it follows that  $X_t$  must converge in equilibrium. In the limit as  $t \rightarrow \infty$ ,  $x_{it} \rightarrow 0$  and  $X_t \rightarrow \bar{X}$ , whence  $V(X) \rightarrow V(\bar{X}) = \pi(\bar{X})/(1 - \delta)$ . Equation (3) implies that, in the limit,

$$0 \in \max_x \pi(\bar{X} + x)/(1 - \delta) - x',$$

which in turn implies  $\pi'(\bar{X}) = 1 - \delta$  and thus  $\bar{X} = \hat{X}$ .

Substituting in (4) and solving for  $V(X)$ , we get the unique solution

$$V(X) = \frac{\pi(\hat{X})}{1 - \delta} + \frac{\pi(\hat{X}) - \pi(X)}{\delta} - \frac{\hat{X} - X}{\delta}.$$

Equilibrium investment  $x(X)$  is then determined by

$$V(X) = \pi(X + 2x) - x + \delta V(X + 2x),$$

which can be simplified into

$$x = V(X) - X - \left( \frac{\pi(\hat{X})}{1 - \delta} - \hat{X} \right).$$

(Notice that, since  $V(X) \downarrow \frac{\pi(\hat{X})}{1 - \delta}$  and  $V'(X) \uparrow 1$  as  $X \rightarrow \hat{X}$ , this defines a positive sequence  $x_t$  that converges to zero.) ■

**Proof of Theorem 1:** By contradiction. Let  $\bar{t}$  be the lowest  $t$  such that  $X_t = X^*$ . Player  $i$ 's equilibrium payoff starting from period  $\bar{t} - 1$  is given by  $\pi(X^*)/(1 - \delta) - x_{i,\bar{t}-1}$ . Alternatively, he could choose  $x < x_{i,\bar{t}-1}$  and receive a payoff of at least  $\pi(x) = \pi(X^* - x_{i,\bar{t}-1} + x)/(1 - \delta) - x$ . Since the derivative of  $\pi(x)$  with respect to  $x$  is negative and  $\pi(x_{i,\bar{t}-1}) = \pi(X^*)/(1 - \delta) - x_{i,\bar{t}-1}$ , the result follows. ■

**Proof of Theorem 2:** Suppose that  $X_0 > \hat{X}$ , so that  $x_{it} = 0$  for all  $t$  is a Markov equilibrium (proof: suppose all subgames call for  $x(X) = 0$ ). Consider an equilibrium given by the following trigger strategies. In each period invest

$$x_{it} = x_t = \frac{1}{2}\alpha(\hat{X} - X_{t-1}), \quad (5)$$

unless someone has deviated in the past, in which case invest zero.

(Step 1) Payoff along the equilibrium path can be written as a series of “marginal” gains

$$\sum_{t=1}^{\infty} \delta^{t-1} (\pi(X_t) - x_t) = \frac{1}{1-\delta} \left( \pi(X_0) + \sum_{t=1}^{\infty} \delta^{t-1} \left( (\pi(X_t) - \pi(X_{t-1})) - (1-\delta)x_t \right) \right).$$

Since  $X_0 > \bar{X}$ , and given the nature of punishments, every deviation with  $x > 0$  is dominated by  $x = 0$ . Maximum payoff upon deviation in the first period is thus given by

$$\frac{1}{1-\delta} \pi(X_0 + x_1).$$

The no-deviation constraint is therefore

$$\sum_{t=1}^{\infty} \delta^{t-1} (\pi(X_t) - x_t) \geq \frac{1}{1-\delta} \pi(X_0 + x_1),$$

or simply

$$\begin{aligned} \frac{1}{1-\delta} \left( \pi(X_0) + \sum_{t=1}^{\infty} \delta^{t-1} \left( (\pi(X_t) - \pi(X_{t-1})) - (1-\delta)x_t \right) \right) &\geq \\ \frac{1}{1-\delta} \left( \pi(X_0) + (\pi(X_0 + x_1) - \pi(X_0)) \right). & \end{aligned}$$

Since  $\pi(X_0 + x_1) < \pi(X_1)$ , a sufficient condition is

$$\sum_{t=2}^{\infty} \delta^{t-1} \left( (\pi(X_t) - \pi(X_{t-1})) - (1-\delta)x_t \right) \geq (1-\delta)x_1. \quad (6)$$

(Step 2) Since  $\pi'(X^*) = (1-\delta)/2$  and  $\pi''(X) \leq -\xi$  for all  $X < X^*$  (by Assumption 1), a lower bound on the value of  $\pi'(X_t)$  is given by  $\frac{1-\delta}{2} + \xi(X^* - X_t)$ . It thus follows that

$$\pi(X_t) - \pi(X_{t-1}) > (X_t - X_{t-1})\pi'(X_t) \geq \alpha(\bar{X} - X_{t-1}) \left( \frac{1-\delta}{2} + \xi(X^* - X_t) \right),$$

where the first inequality follows from (strict) concavity of  $\pi(X)$ . We conclude that

$$\begin{aligned} \pi(X_t) - \pi(X_{t-1}) - (1-\delta)x_t &= \pi(X_t) - \pi(X_{t-1}) - (1-\delta)(\bar{X} - X_{t-1}) \\ &> \alpha(\bar{X} - X_{t-1})\xi(X^* - X_t) \\ &> \alpha(\bar{X} - X_{t-1})\xi(X^* - \bar{X}) \\ &= \alpha(1-\alpha)^{t-1}(\bar{X} - X_0)\xi(X^* - \bar{X}), \end{aligned}$$

where the equality follows from (5).

(Step 3) Substituting the lower bound in (6), we get the following sequence of sufficient conditions:

$$\begin{aligned} \sum_{t=2}^{\infty} \delta^{t-1} \alpha(1-\alpha)^{t-1} (\bar{X} - X_0) \xi(X^* - \bar{X}) &\geq (1-\delta)x_1 \\ \frac{\alpha\delta(1-\alpha)}{1-\delta(1-\alpha)} (\bar{X} - X_0) \xi(X^* - \bar{X}) &\geq (1-\delta) \frac{1}{2} \alpha (\bar{X} - X_0) \\ \xi(X^* - \bar{X}) &\geq \frac{(1-\delta)(1-\delta(1-\alpha))}{2\delta(1-\alpha)}. \end{aligned} \quad (7)$$

Notice that (7) does not depend on  $X_0$ . Therefore, (7) implies the no-deviation constraint holds for all subgames along the equilibrium path.

Setting  $\alpha = 0$ , (7) reduces to

$$\xi(X^* - \bar{X}) \geq \frac{(1 - \delta)^2}{2\delta}.$$

If this condition holds, then we can find an  $\bar{X}'$  greater to but close to  $\bar{X}$  and a positive and small  $\alpha$  such that the proposed strategies do indeed constitute an equilibrium. Since  $X_t$  converges to  $\bar{X}'$  it reaches  $\bar{X}$  in finite time. Finally,  $\bar{X}$  can be made arbitrarily close to  $X^*$  whenever  $\delta$  is arbitrarily close to one.

(Step 4) If  $X_0 < \bar{X}$ , then let equilibrium strategies be  $x_{t1} = x_1 = \frac{1}{2}(\bar{X} - X_0)$ . In case of deviation, let the subgame call for the innocent player to invest zero and the guilty one  $\frac{1}{2}(X - X_0)$ . ■

**Proof of Lemma 2:** An optimal symmetric equilibrium solves the problem

$$\begin{aligned} \max_{x,U} \quad & V = f(x, x)\Pi + (1 - f(x, x))\delta U - x \\ \text{s.t.} \quad & f'(x, x)(\Pi - \delta U) = 1 \\ & x \geq 0 \\ & U \in W \end{aligned}$$

where  $V$  is the value of the game,  $U$  is the payoff of the continuation equilibrium in case of no success, and  $W$  is the set of all symmetric equilibrium payoffs. But since only the payoff of the continuation equilibrium matters, any continuation equilibrium can be substituted by a combination of the best and the worst continuation equilibria, resulting in the same value of  $U$ . ■

**Proof of Proposition 3:** Consider a small increase in  $x = x_1 = x_2$  starting from the level of an optimal symmetric equilibrium. The increase in the value of the game would be given by

$$\begin{aligned} \frac{\partial V}{\partial x} &= 2f'(x, x)(\Pi - \delta U) - 1 \\ &> f'(x, x)(\Pi - \delta U) - 1 = 0, \end{aligned}$$

which implies an improvement can be made on the optimal equilibrium. ■

**Proof of Proposition 1:** Suppose that  $p = q = 1$ . For the purpose of deriving the no-deviation constraint for  $x = y = 1$  to be a SOE, it is useful to think of  $\bar{\pi}$  as the "sure" payoff in each period and  $\bar{\pi} - \underline{\pi}$  as the gain from innovation. The condition is then

$$\alpha \frac{\bar{\pi} - \underline{\pi}}{1 - \delta} + (1 - \alpha)\delta V - 1 \geq \beta \frac{\bar{\pi} - \underline{\pi}}{1 - \delta} + (1 - \beta)\delta V,$$

where  $V$  is the (redefined) equilibrium value. Simplifying, we get

$$(\alpha - \beta) \left( \frac{\bar{\pi} - \underline{\pi}}{1 - \delta} - \delta V \right) \geq 1.$$

Since  $V = \alpha \frac{\bar{\pi} - \pi}{1 - \delta} + (1 - \alpha)\delta V - 1$ , we have

$$V = (1 - (1 - \alpha)\delta)^{-1} \left( \alpha \frac{\bar{\pi} - \pi}{1 - \delta} - 1 \right).$$

and the no-deviation constraint becomes

$$(\alpha - \beta) \left( \frac{\bar{\pi} - \pi}{1 - \delta} - \delta(1 - (1 - \alpha)\delta)^{-1} \left( \alpha \frac{\bar{\pi} - \pi}{1 - \delta} - 1 \right) \right) \geq 1.$$

Simplifying, we get

$$\bar{\pi} - \pi \geq \frac{1 - \delta(1 - \beta)}{\alpha - \beta}.$$

The condition for  $x = 1$  not to be a SOE (still assuming  $p = q = 1$ ) is that the no-deviation constraint is violated even when  $y$  is set to zero. We then have

$$\alpha \frac{\bar{\pi} - \pi}{1 - \delta} - 1 \leq \beta \frac{\bar{\pi} - \pi}{1 - \delta},$$

or simply

$$\bar{\pi} - \pi \leq \frac{1 - \delta}{\alpha - \beta}.$$

Finally, if (1) holds, then the SOE will consist of choosing  $y$  such that the no-deviation constraint holds as an equality.

We will now show that, if (1) holds, then the SOE is an OE. Clearly,  $q = 1$  is optimal for not only are future profits as high as possible but the incentives for investing in R&D are higher than otherwise. The problem for finding the OE is therefore

$$\begin{aligned} \max_{p_1, U} \quad & \alpha \frac{\bar{\pi}}{1 - \delta} + (1 - \alpha)(\pi_1(p_1) + \delta U) - 1 \\ \text{s.t.} \quad & (\alpha - \beta) \left( \frac{\bar{\pi}}{1 - \delta} - \pi_1(p_1) - \delta U \right) \geq 1 \end{aligned}$$

where  $p_1$  is price in the first period and  $U$  the continuation value in case of no success. The crucial point to notice is that both  $V$  and the no-deviation constraint depend equally on the value of  $\pi_1(p_1) - \delta U$ . Therefore, any combination of  $p_1$  and  $U$  yielding the same value of  $\pi_1(p_1) - \delta U$  yields the same equilibrium payoff. In particular, the combination implicit in the SOE is optimal.

Now suppose that

$$\bar{\pi} - \pi \leq \frac{1 - \delta}{\alpha - \beta}$$

(the second inequality in (2)), so that the best SOE calls for  $x = 0$ . Consider the following (non-separate-optimal) equilibrium strategies: set  $x = y = q = 1$  and  $p < 1$ . (Clearly, this is not a SOE since, given  $x = y = 1$ , it would be optimal to set  $p = 1$ .) Since  $\delta > \frac{1}{2}$ , the no-deviation constraint for  $p$  is satisfied for  $p < 1$  just as for any other value of  $p$ . As to the no-deviation constraint for the value of  $x$ , we have, by analogy with the expression derived before,

$$\bar{\pi} - \pi(p) \geq \frac{1 - \delta(1 - \beta)}{\alpha - \beta}.$$

Clearly,  $p$  should be chosen as high as possible, consistent with the no-deviation constraint. We thus have

$$\pi(p) = \bar{\pi} - \frac{1 - \delta(1 - \beta)}{\alpha - \beta}.$$

(Notice that the second inequality in (2) implies, as expected, that  $\pi(p) \leq \bar{\pi}$ .) The value of this solution is

$$V = \alpha \frac{\bar{\pi}}{1 - \delta} + (1 - \alpha)(\pi(p) + \delta V) - 1.$$

Substituting and solving for  $V$ , we get

$$V = \frac{\bar{\pi}}{1 - \delta} - \frac{1 - \beta}{\alpha - \beta}.$$

This solution is superior to the SOE  $(0, 0, 1, 1)$  if and only if

$$\frac{\bar{\pi}}{1 - \delta} - \frac{1 - \beta}{\alpha - \beta} > \frac{\pi}{1 - \delta},$$

where the right-hand side is the value of the SOE, an inequality equivalent to the first inequality in (2).

An argument similar to the proof of the first part of the proposition shows that other combinations of  $p$  and  $y$  would yield the same equilibrium value; the OE derived is therefore not unique.

Similar arguments to the ones above show that, if  $\delta > \frac{1}{2}$  and neither (1) nor (2) hold, then both the SOE and the OE are unique and identical. Specifically, if the second inequality in (1) does not hold, then the OE is  $(1, 1, 1, 1)$ ; if the first inequality in (2) does not hold, then the OE is  $(0, 0, 1, 1)$ .

Finally, if  $\delta < \frac{1}{2}$ , then the unique SOE and OE is  $(0, 0, 0, 0)$ . In fact, by subgame perfection, it must be  $q = 0$ . Given this, the value of the innovation is zero, and  $x = 0$ . Finally,  $\delta < \frac{1}{2}$  also implies that  $p = 0$ . ■

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