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COMMON AGENCY AND COORDINATION: GENERAL THEORY AND APPLICATION TO TAX POLICY

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ABSTRACT

Common Agency and Coordination: General Theory and Application to Tax Policy*

We develop a model of common agency with complete information and general preferences with non-transferable utility, and prove that the principals' Nash equilibrium in truthful strategies implements an efficient action. We apply this theory to construct a positive model of public finance, where organized special interests can lobby the government for consumer and producer taxes or subsidies and targeted lump-sum taxes or transfers. The lobbies use only the non-distorting transfers in their non-cooperative equilibrium, but their inter-group competitition for transfers turns into a prisoners' dilemma in which the government captures all the gain that is potentially available to the parties. Therefore, we suggest that pressure groups capable of sustaining an *ex-ante* agreement will make a commitment to forgo direct transfers and to confine their lobbying to distorting taxes and subsidies.

JEL Classification: D0, H2
Keywords: common agency, tax policy

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NON-TECHNICAL SUMMARY

Common agency is a multilateral relationship where several principals simultaneously try to influence the actions of an agent. Such situations occur very frequently, particularly in the political processes that generate economic policies.

Information asymmetries are important in a common agency just as in an agency with a single principal. Even with complete information, however, the existence of multiple principals introduces the new issues of whether they can achieve an outcome that is efficient for the group of players (the principals and the agent together), and of how the available surplus gets distributed among players. Bernheim and Whinston (1986) show that a non-cooperative menu auction among the principals does have an efficient equilibrium. Their model has found many applications, including the study of lobbying for tariffs and for consumer and producer taxes and subsidies.

The Bernheim-Whinston model assumes quasi-linear preferences, however, so monetary transfers are equivalent to transferable utility among the principals and their common agent. This is usual and acceptable in the partial equilibrium analysis of industrial organization for which the Bernheim-Whinston model was originally designed, but it is generally inappropriate in most other economic settings, which require a more general equilibrium analysis. In models of tax and transfer policies, whether normative or positive, the most important drawback of quasi-linearity is that it gives incomplete or implausible answers to distributional questions. For example, consider policy-makers with Benthamite (additive) social welfare functions as part of their objectives. Then, since quasi-linearity implies constant marginal utilities of income, they can have no concern for distribution *per se*. In reality, leaders do often care about income inequality. Next, in the common agency framework of recent political economy models, where the politically organized interest groups are the principals and the government is the agent, the government's implied objective is a weighted sum of utilities where favoured or organized groups get a higher weight. Then a government that has access to efficient means of transfer will drive the less-favoured or unorganized groups down to their minimum subsistence utility levels, while distribution among the favoured or organized groups will be indeterminate; both features are unrealistic. Finally, quasi-linearity makes the agent's actions independent of the distribution of pay-offs among the principals. To sum up, the assumption of quasi-linearity makes the model unsuitable for analysing distribution and transfer policies which are of the essence in public finance and political economy.

In most economic applications, money is indeed transferable, but the players' pay-offs are not linear in money. The strict concavity of utilities in money incomes makes the levels of transfers in the political equilibrium determinate and non-extreme. In this paper we generalize the theory of common agency to handle such situations. We thereby hope to enlarge the scope of applicability of the theory.

We begin by characterizing equilibria for the general common agency problem We proceed to show that, even when utility is not transferable across players, the agent's actions in equilibrium achieve an efficient outcome for the group of players (principals and agent). Of course, the actions are no longer independent of the distribution of pay-offs among the players, and in equilibrium the two sets of magnitudes must be determined simultaneously.

We then consider a political process of economic policy-making in the common agency framework. A subset of all individuals are allowed to lobby the government, and promise contributions in return for policy favours. The government cares for social welfare defined over the utilities of all individuals (lobbying or not) and for its receipts from the lobbyists. The efficiency theorem then says that the government uses the available policy instruments in a Pareto efficient manner.

We develop this in greater detail in a positive model of the formation of tax policy. Our model is rich enough to become a counterpart to the familiar normative model of Diamond and Mirrlees. The policy instruments we allow are commodity tax or subsidy policies and individualized lump-sum transfers, and the political process admits lobbying of the sort described above.

Here the efficiency result implies that only the non-distorting lump-sum transfers are used in the political equilibrium, not consumption or production taxes or subsidies. This should not be interpreted as general proof of the efficiency of politics, however. The game of lobbying for transfers turns into a prisoners' dilemma for the lobbyists. Indeed, under mild additional assumptions, we find that the government captures all of the gains that exist in the common agency relationship. This suggests that if the lobbies could commit *ex ante* to a 'constitution' for lobbying, they would all agree not to lobby for lump-sum transfers. This opens the way for the use of economically inferior instruments such as production subsidies, with an attendant violation of production efficiency in the political equilibrium, contrary to an important general feature of the normative optimum.

1 Introduction

Common agency is a multilateral relationship where several principals simultaneously try to influence the actions of an agent. Such situations occur very frequently, particularly in the political processes that generate economic policies. For example, legislatures respond to many diverse pressures, including those from voters, contributors, and party officials. Administrative agencies, formally responsible to either the legislative or the executive branches, are in practice influenced by the courts, the media, and various interest and advocacy groups. In the European Union, several sovereign governments deal with a common policy-making apparatus in Brussels. In the United States, growing decentralization of economic power to the states and localities may give governments at these levels the standing of principals in relation to the federal government.

Information asymmetries are important in a common agency just as in an agency with a single principal. However, even with complete information, the existence of multiple principals introduces the new issues of whether they can achieve an outcome that is efficient for the group of players (the principals and the agent together), and of how the available surplus gets distributed among players. Bernheim and Whinston (1986) show that a non-cooperative menu auction among the principals does have an efficient equilibrium. Their model has found many applications, including in the study of lobbying for tariffs (Grossman and Helpman, 1994) and for consumer and producer taxes and subsidies (Dixit, 1995).

However, the Bernheim-Whinston model assumes quasi-linear preferences, so monetary transfers are equivalent to transferable utility among the principals and their common agent. This is usual and acceptable in the partial equilibrium analysis of industrial organization for which the Bernheim-Whinston model was originally designed, but it is generally inappropriate in most other economic settings, which require a more general equilibrium analysis. In models of tax and transfer policies, whether normative or positive, the most important drawback of quasi-linearity is that it gives incomplete or implausible answers to distributional questions. For example, consider a policymaker who has a Benthamite (additive) social welfare function as part of his objective. Then, since quasi-linearity implies constant marginal utilities of income, the policymaker can have no concern for distribution per se. In reality leaders do often care about income inequality. Next, in the common agency framework of recent political economy models, where the politically organized interest groups are the principals and the government is the agent, the government's implied objective is a weighted sum of utilities where favored or organized groups get a higher weight. Then a government that has access to efficient means of transfer will drive the less-favored or unorganized groups down to their minimum subsistence utility levels, while distribution among the favored or organized groups will be indeterminate; both features are unrealistic. (See Dixit 1995 for further discussion of this.) Finally, quasi-linearity makes the agent's actions independent of the distribution of payoffs among the principals.[1] To sum up, the assumption of quasi-linearity makes the model unsuitable for analyzing distribution and transfer policies which are of the essence in public finance and political economy.

In most economic applications, money is indeed transferable, but the players' payoffs are not linear in money. The strict concavity of utilities in money incomes makes the levels of transfers in the political equilibrium determinate and non-extreme. In this paper we generalize the theory of common agency to handle such situations. We thereby hope to enlarge the scope of applicability of the theory.

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[1] Note the parallel with the Coase theorem, where under quasi-linear utility (no income effects on the activities in question), resource allocation is independent of distribution.

[2] The parallel with the Coase theorem or the core with non-transferable utility should again be apparent. However, we should stress that ours is an equilibrium of a non-cooperative game, not a cooperative solution concept.

and individualized lump-sum transfers, and the political process admits lobbying of the sort described above.

Here the efficiency result implies that only the non-distorting lump-sum transfers are used in the political equilibrium, not consumption or production taxes or subsidies. However, this should not be interpreted as a general proof of the efficiency of politics. The game of lobbying for transfers turns into a prisoners' dilemma for the lobbyists. Indeed, under mild additional assumptions, we find that the government captures all of the gains that exist in the common agency relationship.[3] This suggests that if the lobbies could commit ex ante to a "constitution" for lobbying, they would all agree not to lobby for lump-sum transfers. This opens the way for the use of economically inferior instruments such as production subsidies, with an attendant violation of production efficiency in the political equilibrium, contrary to an important general feature of the normative optimum (Diamond and Mirrlees, 1971).

2 General Theory

Consider the following problem. There is a set L of principals. For each $i \in L$, principal i has continuous preferences $U^i(\mathbf{a}, c_i)$, where the vector $\mathbf{a}$ denotes the agent's action and the scalar c_i denotes principal i's payment to the agent. Each principal's preference function is declining in his payment to the agent. The agent's continuous preference function is $G(\mathbf{a}, \mathbf{c})$, where $\mathbf{c}$ is the vector of the principals' payments. The function G is increasing in each component of $\mathbf{c}$. Thus, for any given action, each principal dislikes making contributions and the agent likes receiving them; their preferences with regard to actions are not restricted in general, but we will place some specific restrictions for particular results below. We refer to the values of the functions $U^i(\mathbf{a}, c_i)$ and $G(\mathbf{a}, \mathbf{c})$ as the utility levels of the principals and the agent respectively.

Principal i can choose a payment function $C_i(\mathbf{a})$ from a set $\mathcal{C}_i$ and the agent can choose $\mathbf{a}$ from a set $\mathcal{A}$. The sets $\mathcal{C}_i$ and $\mathcal{A}$ describe feasibility and institutional constraints. For example, from feasibility considerations $\mathcal{C}_i$ may consist only of functions that provide principal i with a non-negative income. Or it may consist only of non-negative functions, implying that

[3] Grossman and Helpman (1994) similarly argued that lobbies would get more surplus using tariffs instead of the economically more efficient production subsidies. Our result for lump-sum transfers is even more dramatic and extreme: the lobbies get zero surplus with the most efficient instruments.

the principal can pay the agent but not the reverse. This would describe an institutional constraint. And it may contain only functions with an upper bound on payments, thereby describing another institutional constraint. Similarly, $\mathcal{A}$ may describe institutional or feasibility constraints on the actions of the agent. If, for example, an element of $\mathbf{a}$ equals one plus an ad-valorem tax rate, then feasibility requires $\mathcal{A}$ to contain only non-negative vectors.

Throughout, we maintain the following assumption on the sets of feasible payment functions:

Assumption 1: Let $C_i \in \mathcal{C}_i$. Then $C_i(\mathbf{a}) \geq 0$ for all $\mathbf{a} \in \mathcal{A}$; and every payment function C_i^* that satisfies: (i) $C_i^*(\mathbf{a}) \geq 0$ for all $\mathbf{a} \in \mathcal{A}$, and (ii) $C_i^*(\mathbf{a}) \leq C_i(\mathbf{a})$ for all $\mathbf{a} \in \mathcal{A}$, also belongs to $\mathcal{C}_i$.

Explanation: Payments from the principals to the agent have to be nonnegative, and if a payment function is feasible, all "smaller" payment functions are also feasible. This conforms to the requirements of most relevant economic applications.

Equilibrium

Our aim is to construct and study a concept of equilibrium for a two-stage game. In the second stage, the agent chooses an action optimally, given the payment functions of all the principals. In the first stage, each principal chooses a payment schedule, knowing that all the other principals are simultaneously and non-cooperatively choosing their own payment schedules, and looking ahead to the response of the agent in the second stage.

We will denote magnitudes pertaining to an equilibrium by the superscript $\circ$. Since the game is non-cooperative, we will have to start with a "candidate" for such an equilibrium, and study the consequences of allowing the strategies to deviate from this, one player at a time. For this purpose we establish the following notation: $\mathbf{C}^\circ(\mathbf{a})$ will denote the vector of contribution functions with component functions $C_j^\circ(\mathbf{a})$, for all $j \in L$; while $(\{C_j^\circ(\mathbf{a})\}_{j \neq i}, c)$ will denote the vector where the i-th component is replaced by c, and all the other components $j \neq i$ are held fixed at $C_j^\circ(\mathbf{a})$. Sometimes c itself may be the value of another payment function $C_i(\mathbf{a})$ for principal i.

We begin by defining the principals' best response strategies.

Definition 1: A payment function $C_i^\circ \in \mathcal{C}_i$ is a best response of principal i to the payment functions $\{C_j^\circ\}_{j \in L, j \neq i}$ of the other principals if there does not exist a payment function $C_i \in \mathcal{C}_i$ such that

(i) $U^i[\mathbf{a}_i, C_i(\mathbf{a}_i)] > U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$; where

(ii) $\mathbf{a}_i = \arg\max_{\mathbf{a} \in \mathcal{A}} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, C_i(\mathbf{a}))]$, and

(iii) $\mathbf{a}^\circ = \arg\max_{\mathbf{a} \in \mathcal{A}} G[\mathbf{a}, \mathbf{C}^\circ(\mathbf{a})]$.

Explanation: The best response calculation of principal i holds fixed the simultaneously chosen strategies (payment functions) of all the other principals at their candidate equilibrium positions, but recognizes that in the second stage the agent will optimize with respect to these payment functions along with any deviated function proposed by principal i. If principal i cannot find another feasible payment function that yields a better outcome for him, taking into account the agent's anticipated response, then the original candidate payment function C_i° is a best response for principal i to the candidate functions C_j° of all the other principals.

Next we define equilibrium. This is the standard definition of a subgame perfect Nash equilibrium for this two-stage game; it is stated explicitly only so that we can refer to the specific conditions (a)-(c) later.

Definition 2: An equilibrium of the common agency problem consists of a vector of payment functions $\mathbf{C}^\circ = \{C_i^\circ\}_{i \in L}$ and a policy vector $\mathbf{a}^\circ$ such that:

(a) $C_i^\circ \in \mathcal{C}_i$ for all $i \in L$;

(b) $\mathbf{a}^\circ = \arg\max_{\mathbf{a} \in \mathcal{A}} G[\mathbf{a}, \mathbf{C}^\circ(\mathbf{a})]$; and

(c) for every $i \in L$ the payment function C_i° is a best response of principal i to the payment functions $\{C_j^\circ\}_{j \in L, j \neq i}$ of the other principals.

The following result provides a characterization of an equilibrium:

Proposition 1: A vector of payment functions $\mathbf{C}^\circ = \{C_i^\circ\}_{i \in L}$ and a policy vector $\mathbf{a}^\circ$ constitute an equilibrium if and only if:

(a) $C_i^\circ \in \mathcal{C}_i$ for all $i \in L$;

(b) $\mathbf{a}^\circ = \arg\max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, \mathbf{C}^\circ(\mathbf{a})]$; and

(c) for every $i \in L$:

$$[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)] = \arg\max_{(\mathbf{a},c)} U^i(\mathbf{a}, c)\,, \tag{2.1}$$

subject to $\mathbf{a} \in \mathcal{A}$, $c = C_i(\mathbf{a})$ for some $C_i \in \mathcal{C}_i$, and

$$G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j\neq i}, c)] \geq \max_{\mathbf{a}'\in\mathcal{A}} G[\mathbf{a}', (\{C_j^\circ(\mathbf{a}')\}_{j\neq i}, 0)]\,. \tag{2.2}$$

Explanation:[4] Observe that (a) and (b) are mere restatements of the corresponding parts of Definition 2. The reformulation of (c) is the key aspect of Proposition 1; it focuses on the relationship between the agent and one of the principals, and helps determine how the potential gains from this relationship get allocated between them in equilibrium.

Examine the situation from the perspective of principal i. He takes as given the strategies of all other principals $j \neq i$, and contemplates his own choice. He must provide the agent at least the level of utility which the agent could get from his outside option, namely by choosing his best response to the payment functions offered by all the other principals when principal i offers nothing. This is what constraint (2.2) expresses. Subject to this constraint, principal i can propose to the agent an action and a feasible payment that maximizes his own utility. That is the content of equation (2.1). Then Proposition 1 says that such constrained maximization by each principal is equivalent to equilibrium as previously defined.

The intuition behind our result can be appreciated with the aid of Figure 1. Suppose for the sake of illustration that the action is a scalar. Curve G_iG_i depicts combinations of the action a (on the horizontal axis) and payments c by principal i (on the vertical axis) that give the agent a fixed level of utility when the contribution functions of the other principals are given. The particular indifference curve shown in the figure describes the highest welfare the agent can attain when principal i makes no contribution whatsoever (his payment function coincides with the horizontal axis): the agent then chooses the action associated with the point labelled A_{-i}. The shaded rectangle depicts the combinations of feasible actions and feasible payment levels (there is an upper bound on payments, payments

[4] In the text we give a verbal and intuitive explanation of the propositions; more formal proofs are in the Appendix.

have to be nonnegative, and the action is bounded below and above). Considering the agent's option to take action A_{-i}, the best the principal i can do is to design a payment schedule that induces the agent to choose a point in the shaded area that lies above or on the indifference curve G_iG_i. Suppose the principal's welfare is increasing in the action. Then his indifference curves are upwards sloping. In the event, he will choose a point on the rising portion of G_iG_i that is both feasible and gives him the highest welfare level, namely the tangency point A between his indifference curve U_iU_i and G_iG_i. It is easy to see from the figure how the principal can construct a payment schedule that induces the agent to choose point A. For example, he might offer a schedule that coincides with the horizontal axis until some point to the right of A_{-i}, and then rises to a tangency with G_iG_i at A without ever crossing that curve.

Corollary to Proposition 1: Let $(\mathbf{C}^\circ, \mathbf{a}^\circ)$ be an equilibrium. Then, for each $i \in L$.

$$G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)] = \max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j\neq i}, 0)]\,.$$

Explanation: This says that the utility level of the agent in equilibrium is the same as what he would get if any one of the principals were to contribute zero while all the others maintained their equilibrium payment *functions*. and the agent then were to choose his optimum action in response to this deviation. The intuition is implicit in our discussion of condition (c) of the proposition. Each principal must ensure that the agent gets a utility equal to his outside opportunity; it is not in the principal's interest to give the agent any more.

Truthful Equilibria

The above model can have multiple subgame perfect Nash equilibria. some of which can be inefficient. As in Bernheim and Whinston (1986), we now develop a refinement that selects equilibria that implement Pareto efficient actions (the concept of Pareto efficiency is of course constrained by the set of available actions). We first establish a closely related property that is necessary for all equilibria given some additional restrictions, and then develop the property that is sufficient for efficient equilibria.

Definition 3: A payment function $C_i^\circ \in \mathcal{C}_i$ is said to be *locally truthful* if

$$\nabla_{\mathbf{a}} C_i^\circ(\mathbf{a}^\circ) = -\left(\frac{\partial U^i[\mathbf{a}^\circ. C_i^\circ(\mathbf{a}^\circ)]}{\partial c_i}\right)^{-1} \nabla_{\mathbf{a}} U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)] \text{ for all } i \in L, \qquad (2.3)$$

where the operator ∇ applied to a function denotes the gradient vector of the partial derivatives of the function with respect to the vector argument which appears as the subscript of the operator.

Explanation: By the implicit function theorem, the right hand side is the vector of the marginal rates of substitution between actions and contributions along an indifference surface for individual i, evaluated at the equilibrium point. In equilibrium, the slopes of a locally truthful payment function are equal to these marginal rates of substitution. In other words, the principal's marginal payment for each component of action equals his valuation of that component. This property is similar to the truthful revelation of people's valuation of public goods, externalities etc. under suitably designed mechanisms; hence the name. It holds for any interior equilibrium supported by differentiable payment schedules.

Proposition 2: Let the preference functions $(\{U^i\}_{i\in L}, G)$ be differentiable and let the sets of feasible payment schedules be restricted to functions that are differentiable where positive. Then whenever the equilibrium is interior, in the sense that $\mathbf{a}^\circ$ belongs to the interior of $\mathcal{A}$ and $C_i^\circ(\mathbf{a}^\circ) > 0$ for all $i \in L$, then the equilibrium payment functions are locally truthful at the equilibrium point.

Next we consider a stronger property of payment functions, namely *global truthfulness*. A globally truthful payment function for principal i rewards the agent for every change in the action exactly the amount of change in the principal's welfare, provided that the payment both before and after the change is strictly positive. In other words, the shape of the payment schedule mirrors the shape of the principal's indifference surface not only near the equilibrium point (as with local truthfulness) but everywhere the payments are positive. Then the principal gets the same utility for all actions $\mathbf{a}$ that induce positive payments $C_i(\mathbf{a}) > 0$; the payment is just the compensating variation. We show that the common agency game has an equilibrium in which all the principals follow globally truthful strategies, and that such an equilibrium is Pareto efficient. We call such an equilibrium a *truthful equilibrium*.

Focus on truthful equilibria may seem restrictive, but can be justified in several different ways. First, for any set of feasible strategies of the $(L-1)$ principals other than i, the set of best response strategies for principal i contains a truthful payment function. Thus each principal bears essentially no cost from playing a truthful strategy, no matter what he expects from the other players. Then the result that an equilibrium in truthful strategies implements a Pareto efficient action may make such strategies focal for the group of principals. Second, since the setting has no incomplete information, the players have "nothing to hide" and truthful strategies provide a simple device to achieve efficiency without any player conceding his right to grab as much as he can for himself.

Notice too that we do not restrict the space of feasible payment functions to truthful ones at the outset; in a truthful equilibrium each principal's truthful strategy is a best response to his rivals even when the space of feasible payment functions is the larger one of Assumption 1. Thus we have an equilibrium in the full sense, where the strategies happen to be truthful.

We now proceed to formalize the idea and the results.

Definition 4: A payment function $C_i^T(\mathbf{a}, u_i^*)$ for principal i is *globally truthful* relative to the constant u_i^* if

$$C_i^T(\mathbf{a}, u_i^*) \equiv \min[\bar{C}_i(\mathbf{a}), \max[0, \varphi_i(\mathbf{a}, u_i^*)]] \quad \text{for all } \mathbf{a} \in \mathcal{A}, \tag{2.4}$$

where φ_i is implicitly defined by $U^i[\mathbf{a}, \varphi_i(\mathbf{a}, u_i^*)] = u_i^*$ for all $\mathbf{a} \in \mathcal{A}$, and $\bar{C}_i(\mathbf{a}) = \sup\{C_i(\mathbf{a}) \mid C_i \in \mathcal{C}_i\}$ for all $\mathbf{a} \in \mathcal{A}$.

Explanation: The definition of φ_i is the basic concept of the compensating variation. Equation (2.4) merely serves to ensure that the truthful payment function also satisfies the upper and lower bounds on feasible payments. Note that a competition in truthful strategies boils down to non-cooperative choices of the constants $\{u_j^*\}_{j\in L}$, which determine the equilibrium payoffs of the principals.

Proposition 3: The best response set of principal i to payment functions $\{C_j^\circ(\mathbf{a})\}_{j\in L, j\neq i}$ of the other principals contains a globally truthful payment function.

Explanation: The result can be illustrated in the aforementioned Figure 1. The principals other than i induce in the agent the indifference curve G_iG_i with their payment offers. These offers might be truthful or not. In any event, the best response set for principal i includes all payment functions that induce the action and contribution associated with point A. A truthful strategy in this set is the payment function that coincides with the horizontal axis from the origin until its intersection with U_iU_i, and that coincides with U_iU_i thereafter.

Definition 5: A *truthful equilibrium* is an equilibrium in which all payment functions are globally truthful relative to the equilibrium welfare levels.

Proposition 4: Let $(\{C_i^T\}_{i\in L}, \mathbf{a}^\circ)$ be a truthful equilibrium in which u_i° is the equilibrium utility level of principal i, for all $i \in L$. Then $(\{u_i^\circ\}_{i\in L}, \mathbf{a}^\circ)$ is characterized by:

(a) $\mathbf{a}^\circ = \arg\max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, \{C_i^T(\mathbf{a}, u_i^\circ)\}_{i\in L}]$;

(b) for every $i \in L$,

$$G[\mathbf{a}^\circ, \{C_i^T(\mathbf{a}^\circ, u_i^\circ)\}_{i\in L}] = \max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, (\{C_j^T(\mathbf{a}, u_i^\circ)\}_{j\neq i}, 0)] .$$

Explanation: This is just a restatement of the Corollary to Proposition 1, for the case of truthful equilibria, and the explanation given above applies. The added advantage is in actual use. If we tried to use that corollary to determine equilibria, we would have to solve the conditions simultaneously for all the payment *functions*, which is a complicated fixed point problem and has a large multiplicity of solutions. The corresponding set of equations in Proposition 4 involves the equilibrium utility *numbers*; therefore they constitute a simpler simultaneous equation problem with solutions that are in general locally determinate, and in applications often unique. We will consider one such application in the next section.

Now we establish that an equilibrium in globally truthful strategies implements an efficient action.

Proposition 5: Let a policy vector $\mathbf{a}^\circ$ and a vector of payment functions $\mathbf{C}^\circ$ that are globally truthful with respect to the utility levels $u_i^\circ = U^i(\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ))$ constitute a truthful equilibrium. Then there do not exist an action $\mathbf{a}^*$ and a payment vector $\mathbf{c}^*$ such that

(i) feasibility:

$$\mathbf{a}^* \in \mathcal{A}; \qquad 0 \leq c_i^* \leq \bar{C}_i(\mathbf{a}^*) \text{ for all } i \in L ;$$

(ii) Pareto superiority:

$$G(\mathbf{a}^*, \mathbf{c}^*) \geq G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)] ,$$

$$U^i(\mathbf{a}^*, c_i^*) \geq U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)] \qquad \text{for all } i \in L .$$

with at least one strict inequality.

Explanation: The efficiency of truthful equilibria extends a similar result proved by Bernheim and Whinston (1986, Proposition 2) for the case of transferable utility. We can provide a familiar interpretation by invoking only local truthfulness. This is easiest to see in the case where the agent's preferences depend only on the sum of the total payments, $c = \Sigma_{j\in L} c_j$. Then the first-order conditions from the proof in the Appendix, namely (A1), (A3), (A4), combine to yield

$$\frac{\partial G/\partial a_k}{\partial G/\partial c} = \sum_{j\in L} \frac{\partial U^j/\partial a_k}{\partial U^j/\partial c_j},$$

or

$$\left(\frac{dc}{da_k}\right)_{G \text{ constant}} = \sum_{j\in L} \left(\frac{dc}{da_k}\right)_{U^j \text{ constant}}.$$

This says that the marginal payment the agent requires for supplying an additional unit of action k (the marginal cost) equals the sum of all the principals' marginal willingness to pay for this unit. That is just the Samuelson optimality condition for the provision of a public good, which is the appropriate interpretation here, because the action is a public good affecting all players. The agent's maximization ensures equality between the agent's marginal cost and the sum of the slopes of the payment schedules, whereas the truthfulness property ensures equality between the slopes of the schedules and the principals' marginal utilities. We leave it to the readers to develop a similar interpretation in the more general case.

Quasi-Linear Preferences

The above equilibrium can be pinned down in greater detail when all the players' preferences are linear in the payments. Specifically, the action is independent of the distribution in this case.

Corollary 1 to Proposition 5: Let the preference functions $(\{U^i\}_{i\in L}, G)$ be of the quasi-linear form

$$U^i(\mathbf{a}, c_i) = \omega^i(\mathbf{a}) - \kappa_i c_i \qquad \text{for all } i \in L,$$

and

$$G(\mathbf{a}, \mathbf{c}) = \Gamma(\mathbf{a}) + \gamma \sum_{i\in L} c_i .$$

Consider a truthful equilibrium where the action is $\mathbf{a}^\circ$ and all payments are in the interior: $0 < C_i^\circ(\mathbf{a}^\circ) < \bar{C}_i(\mathbf{a}^\circ)$. Then

$$\mathbf{a}^\circ = \arg\max_{\mathbf{a}\in\mathcal{A}} \Gamma(\mathbf{a})/\gamma + \sum_{i\in L} \omega_i(\mathbf{a})/\kappa_i\,.$$

Explanation: With quasi-linear preferences, the equilibrium action maximizes a weighted sum of gross welfare levels of the principals and the agent. This result has been useful in applications to political economy, such as in Grossman and Helpman (1994). There, the agent is a government that sets a vector of tariff policies, while the principals are interest groups representing owners of sector-specific factors of production. The government's objective is assumed to be linear in the aggregate welfare of voters and the total of campaign contributions collected from special interests. The corollary predicts a structure of protection that maximizes a simple weighted sum of the welfare of voters and interest-group members.

Government Policy-Making

As we noted in the introduction, common agency arises frequently in the political processes that generate economic policies. The policymaker often can be viewed as an agent and some or all of their constituents as principals. Principals can "lobby" the policy makers by promising payments in return for policies, within some prescribed limits on available policies and feasible donations. The payments may take the form of illicit bribes or, more typically, implicit (and therefore legal) offers of campaign support. The Grossman-Helpman application to tariff-setting is but one example of this. In such settings, it may be natural to think of the government as having an objective function with social welfare and the total of contribution receipts as arguments. The government might care about social welfare for ethical reasons, or it may want to provide a high standard of living to enhance its re-election prospects, or to keep the populace sufficiently happy to prevent riots, etc. Contributions likewise might enter the government's objective because they affect its re-election chances, or merely as utility of the governing elites' private consumption. Accordingly, we suppose $G(\mathbf{a}, \mathbf{c}) = g(\mathbf{u}, c)$, where $\mathbf{u}$ is the vector of all the individuals' utilities, and $c = \sum_{i=1}^{n} c_i$ is the aggregate contribution received by the government. We assume that g is strictly increasing in all its arguments.[5]

Let L be the set of individuals who can lobby the government for special favors. We leave L exogenous – some individuals may have special connections, or some groups of individuals may be able to solve the free rider problem of organized action while others cannot. (In the latter case we combine all the members of one such group into one of our "individuals.") Then $C_i(\mathbf{a}) \equiv 0$ for $i \notin L$. For $i \in L$, the upper limit on feasible contributions, $\bar{C}_i(\mathbf{a})$, is implicitly defined by $U^i[\mathbf{a}, \bar{C}_i(\mathbf{a})] = \underline{u}_i$, where $\underline{u}_i$ is the lowest or subsistence utility level for individual i.

Proposition 5 has strong implications for the outcome of this lobbying game.

Corollary 2 to Proposition 5: Let the agent's preferences be given by $G(\mathbf{a}, \mathbf{c}) = g(\mathbf{u}, c)$, where $c = \sum_{i=1}^{n} c_i$. Let a set $L \subset \{1, 2, \ldots, n\}$ of individuals offer payment schedules $\{C_i(\mathbf{a})\}_{i \in L}$, while $C_i(\mathbf{a}) \equiv 0$ for $i \notin L$. Finally, let a policy vector $\mathbf{a}^\circ$ and a vector of payment functions $\mathbf{C}^\circ$ which are globally truthful with respect to the utility levels $u_i^\circ = u_i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$ for $i \in L$ constitute a truthful equilibrium in which $u_i^\circ = u_i(\mathbf{a}^\circ, 0)$ for $i \notin L$. Then there exists no other policy vector $\mathbf{a}' \in \mathcal{A}$ such that $u_i(\mathbf{a}', c_i^\circ) \geq u_i^\circ$ for all $i \in \{1, 2, \ldots, n\}$ with strict inequality holding for some i.

Explanation: The corollary says that, even under the pressure of lobbying from a subset of organized special interests, a government that has some concern for social welfare will make Pareto efficient choices from the set of feasible policies. With truthful payment schedules, the government has incentive to collect its tributes efficiently. If the government's objective weighs positively the well-being of all members in society, then efficiency for the government and lobbies translates into Pareto efficiency for the polity as a whole. We will see an application of this in the next section.[6]

It is important to distinguish between efficiency in the sense of the earlier Proposition 5 and that of its Corollary 2 above. In the former, only the welfare of the active players in

[5] A special case frequent in economic models is where the individual utilities are channelled through a social welfare function of the Bergson-Samuelson type:

$$w = W[u_1(\mathbf{a}, c_1), u_2(\mathbf{a}, c_2) \ldots u_n(\mathbf{a}, c_n)]$$

and $G(\mathbf{a}, \mathbf{c}) = g(w, c)$. But the more general form $g(\mathbf{u}, c)$ will suffice for our purpose.

[6] It also follows from Proposition 5 that there exists no vector of policies $\mathbf{a}$ and total contributions c which would leave the government and all lobbyers and non-lobbyers at least as well off as in the political equilibrium, and some individual or the government strictly better off. In this sense, the political outcome achieves second-best efficiency given the set of available policy instruments.

the game (the lobbies and the government) is considered. This leaves open the possibility that when there are other individuals in the background but they are not strategic players (principals in the lobbying game), inefficiencies in their welfares can remain. In Corollary 2 to Proposition 5, the government's objective function gives some weight to the welfare levels of such individuals, and therefore, for the given level of its receipts from the lobbies, it implements an action that is efficient for all individuals, whether lobbying or not. The distribution of welfare levels across these groups remains to be determined, however.

Our result seems similar to Becker (1983), especially his Corollary on p. 384 and his Proposition 4 on p. 386. But our mechanism is different and leads to very different distributional implications. We discuss this later.

We should also clarify that although the available instruments are used efficiently in our equilibrium, the result need not be the economists' familiar first-best if the set of available instruments is restricted. For example, if production subsidies are available but pure profit taxes or subsidies are not, then the equilibrium will not have aggregate production efficiency in the sense of Diamond and Mirrlees (1971).

To summarize, our general theory preserves the flavor of results in Bernheim and Whinston (1986). In any subgame perfect Nash equilibrium, the agent's action and the payment from principal i maximize the joint welfare of the agent and that principal, given the equilibrium payment functions of the other principals. This is similar to their Lemma 2. For any set of offers by principals $j \neq i$, the best response set for principal i contains a truthful strategy, just as in their Proposition 1. And the truthful equilibrium that results when all principals announce truthful payment functions is Pareto efficient for the group of principals and the agent. By extending their results, we have significantly expanded the domain of their theory.

In the next section we show the usefulness of the general theory in a specific application. We study a positive analog to the normative theory of tax setting à la Diamond and Mirrlees (1971). We extend their analysis to the case of a government that cares not only about aggregate welfare, but also about the contributions it can amass. The endogenous taxes and transfers are those that arise as an equilibrium in a common-agency game where the special interests bid for favored treatment.

3 Application to Tax and Transfer Policies

We closely follow Diamond and Mirrlees (1971), but introduce lobbying for taxes and transfers. Let there be n consumers, labelled $i \in N = \{1, 2, \ldots, n\}$. We will continue to refer to these as "individuals", but in reality most lobbying is undertaken by special interest groups. If such groups have access to optimal internal transfer schemes, each of them can be regarded as a Samuelsonian aggregated individual in our model.

Only the subset L of these "individuals" can lobby the government. These may be players with the largest stake in the policies, or with personal connections to the politicians, or groups who have overcome the free rider problem of collective political action. As before, lobbying takes the form of contingent contributions. The lobbies hope to influence the government's policy choices.

To simplify the exposition we consider a small open economy.[7] Let $\mathbf{p}^w$ denote the exogenous vector of world prices, and $\mathbf{q}$, $\mathbf{p}$ the price vectors faced by the domestic consumers and producers respectively. Then $\mathbf{q} - \mathbf{p}^w$ is the implied vector of consumer tax rates (negative components are subsidies), and $\mathbf{p} - \mathbf{p}^w$ the implied vector of producer subsidy rates (negative components are taxes). The government's tax and subsidy policies are therefore equivalent to choosing $\mathbf{q}$ and $\mathbf{p}$. The government can also make lump-sum transfers or levy lump-sum taxes on any or all individuals; let $\mathbf{t}$, with components t_i for $i \in N$, denote the vector of such transfers (negative components are taxes). We leave out any other government activities for simplicity.

There are several firms labelled $f \in M = \{1, 2, \ldots m\}$ with profit functions $\psi^f(\mathbf{p})$, and by Hotelling's Lemma, supply functions $\nabla_\mathbf{p} \psi^f(\mathbf{p})$. Individual i owns an exogenous share ω_{if} of firm f, and therefore gets profit income

$$\pi^i(\mathbf{p}) = \sum_{f \in M} \omega_{if} \, \psi^f(\mathbf{p}) .$$

Purely for notational convenience, we define

$$S^i(\mathbf{p}) \equiv \nabla_\mathbf{p} \, \pi^i(\mathbf{p}) = \sum_{f \in M} \omega_{if} \; \nabla_\mathbf{p} \, \psi^f(\mathbf{p}) .$$

[7] All the results concerning efficiency have identical counterparts when some or all commodities are non-tradeable; all that is needed is some additional algebra for the domestic market-clearing conditions.

the supply attributable to the fractional firms owned by individual i. We refer to this for brevity as the supply "from" individual i, although individuals as such do not do any production or supplying. The total supply is

$$S(\mathbf{p}) = \sum_{i \in N} S^i(\mathbf{p}) .$$

Let c_i denote the lobbying payment of individual i to the government, for $i \in L$. Set $c_i \equiv 0$ for $i \notin L$. Then individual i's income is

$$I_i \equiv \pi^i(\mathbf{p}) + t_i - c_i . \tag{3.1}$$

We write his resulting indirect utility function as

$$u_i = V^i(\mathbf{q}, I_i) .$$

We assume that each V^i is strictly increasing and strictly concave in I_i. These lump-sum incomes I_i do not have to be non-negative, because individuals have additional incomes from sales of factor services. There is some other lower bound to the I_i.[8] However, we assume an "Inada condition" that the marginal utility of income $V^i_I(\mathbf{q}, I_i)$ goes to infinity as this lower bound is approached; therefore the bound is never hit and we ignore it in what follows.

The individual's demands are given by Roy's Identity:

$$D^i(\mathbf{q}, I_i) = -\nabla_{\mathbf{q}} V^i(\mathbf{q}, I_i) \,/\, V^i_I(\mathbf{q}, I_i) .$$

We should emphasize that the payments made by the lobbies do not enter into the government's tax and transfer budget. This budget reflects the "public" or policy part of the government's activities. The lobbies' payments go into a separate "private" or political kitty. They might be used by the governing party for its re-election campaign, or by a governing dictator for his own consumption. Write $\mathbf{b}$ for the vector of such purchases. Thus there are two budget constraints, one for the policy budget

$$(\mathbf{q} - \mathbf{p}^w)' \sum_{i \in N} D^i(\mathbf{q}, I_i) - (\mathbf{p} - \mathbf{p}^w)' S(\mathbf{p}) - \sum_{i \in N} t_i = 0$$

[8] This limit may depend on the price vector $\mathbf{q}$, and is defined by the condition $V^i(\mathbf{q}, I_i) = \underline{u}_i$, where $\underline{u}_i$ is the lowest or subsistence utility level.

and the other for the political budget

$$\mathbf{p}^{\mathbf{w}\prime}\mathbf{b} = \sum_{i \in L} c_i \, .$$

These constitute only one independent equation, because one can be derived from the other using the individuals' and the producers' budget identities and the economy's trade balance condition. We will use the policy budget condition, and aggregate the political purchases at the world prices so only the sum of the contributions enters the government's objective function.[9]

The government's objective is $g(\mathbf{u}, \bar{c})$, where $\mathbf{u} = (u_1, \ldots u_n)$ is the vector of all individuals' utility levels, and $\bar{c}$ is the aggregate of the contributions received by the government from the individuals in L. We assume that g is strictly increasing and strictly quasi-concave in its arguments.

We can now regard the government as choosing $\mathbf{a} = (\mathbf{q}, \mathbf{p}, \mathbf{t})$ subject to a budget constraint. This puts the problem in the framework of the subsection on Government Policy-Making above. Corollary 2 to Proposition 5 there immediately gives a strong result concerning the choice of action in a truthful equilibrium of the policy game.

3.1 Efficiency

Corollary 2 to Proposition 5, from the previous section, tells us that the equilibrium action achieves a Pareto efficient outcome in an auxiliary problem where the lobbies' payments are held fixed at their equilibrium levels. In the auxiliary problem, the government's choice is the standard normative optimal tax and transfer problem, where we know that if lump-sum transfers are available, distorting commodity taxes and subsidies will not be used. Therefore we have shown that the political equilibrium will also preserve $\mathbf{q} = \mathbf{p} = \mathbf{p}^w$ and use only the lump-sum transfers $\mathbf{t}$ for the two purposes of eliciting contributions from the lobbies and of meeting the government's concern for the welfare of the non-lobbying individuals.

Before the readers form the belief that we have established the efficiency of the political process of tax policy, however, we should warn that the story is not yet complete. It remains to examine the distribution of gains between the lobbies and the government in the political

[9]We have assumed that the government's political purchases do not pay the tax; the alternative of making them subject to tax would complicate the algebra somewhat but would not alter the conclusions.

equilibrium; that analysis will cast doubt on the efficient equilibrium as a description of political reality. We do this in the next subsection.

Here we clarify the efficiency property further, by using familiar revealed preference methods (see Dixit and Norman, 1980, chapter 3) to show how any tax structure that has price distortions cannot be a truthful equilibrium.

Consider a truthful equilibrium with endogenous policies $(\mathbf{q}^\circ, \mathbf{p}^\circ, \mathbf{t}^\circ)$, utility levels $\{u_i^\circ\}_{i \in N}$, and truthful payment schedules $\{C_i^\circ(\mathbf{q}, \mathbf{p}, t_i, u_i^\circ)\}_{i \in L}$. The compensating variations φ_i relative to this equilibrium are defined by

$$V^i(\mathbf{q}, \pi^i(\mathbf{p}) + t_i - \varphi_i) = u_i^\circ$$

or

$$\varphi_i(\mathbf{q}, \mathbf{p}, t_i, u_i^\circ) = \pi^i(\mathbf{p}) + t_i - E^i(\mathbf{q}, u_i^\circ)\,,$$

where E^i is the expenditure function dual to the indirect utility function. The truthful schedules relative to the equilibrium are

$$C_i^\circ(\mathbf{q}, \mathbf{p}, t_i, u_i^\circ) = \max\left[0, \varphi_i(\mathbf{q}, \mathbf{p}, t_i, u_i^\circ)\right].$$

Suppose that the equilibrium contribution levels

$$c_i^\circ = C_i^\circ(\mathbf{q}^\circ, \mathbf{p}^\circ, t_i^\circ, u_i^\circ)$$

are zero for a set P of individuals i (the "passive" lobbies), and are positive (and therefore equal to the compensating variations) for a set $A = L - P$ (the "active" lobbies). Of course P and A are determined endogenously in the equilibrium. We characterize the equilibrium using Proposition 4 of the previous section. Its condition (a) determines the government's policy choice, and condition (b) determines the contributions of the lobbies. We examine these in turn, under the headings of efficiency and distribution. While efficiency has already been proved, we now show more explicitly how an inefficient set of policies can be improved upon.

Therefore suppose that domestic prices in equilibrium differ from world prices.[10] We prove that, given the equilibrium payment schedules of the individuals in L, the government

[10] The world prices $\mathbf{p}^w$ are exogenously normalized, but even when that is done, there is one degree of freedom: the scale of the whole vector $(\mathbf{p}, \mathbf{q}, \mathbf{t})$ of domestic prices and transfers can be changed without changing any real allocations. As usual, proportionality of $(\mathbf{q}, \mathbf{p})$ and $\mathbf{p}^w$ suffices for efficiency, but equality can be chosen without loss of generality.

can do at least as well (and generally better) with a non-distorting policy $(\mathbf{p}^w, \mathbf{p}^w, \mathbf{t}^1)$ which keeps all domestic prices equal to the corresponding world prices, and chooses transfers suitably. This violates Condition (a) of Proposition 4.

We define the alternative transfers by

$$V^i(\mathbf{p}^w, \pi^i(\mathbf{p}^w) + t_i^1 - c_i^\circ) = u_i^\circ$$

for all $i \in N$. Here we have $c_i^\circ \equiv 0$ for the individuals $i \in N - L$ who are simply not allowed to lobby by the rules of the game; for $i \in L$ the c_i° are as stated above.

Given this alternative policy of the government, all individuals have the same compensating variations as before, and therefore all those in L will contribute the same amounts as before, zero for those in P whose compensating variations are non-positive, and given by the compensating variations $c_i^1 = c_i^\circ > 0$, for the $i \in A$. All get the same utilities as before, so the government's objective function takes the same value as before. It remains to show that the policy satisfies the government's budget constraint.

Rewrite the definition of the transfers as

$$t_i^1 = E^i(\mathbf{p}^w, u_i^\circ) - \pi^i(\mathbf{p}^w) + c_i^\circ .$$

Now note that, by convexity of the profit functions,

$$\begin{aligned} \pi^i(\mathbf{p}^w) &\geq \pi^i(\mathbf{p}^\circ) + (\mathbf{p}^w - \mathbf{p}^\circ)' S^i(\mathbf{p}^\circ) \\ &= \pi^i(\mathbf{p}^\circ) - (\mathbf{p}^\circ - \mathbf{p}^w)' S^i(\mathbf{p}^\circ) . \end{aligned}$$

Similarly, by concavity of the expenditure functions,

$$\begin{aligned} E^i(\mathbf{p}^w, u_i^\circ) &\leq E^i(\mathbf{q}^\circ, u_i^\circ) + (\mathbf{p}^w - \mathbf{q}^\circ)' \tilde{D}^i(\mathbf{q}^\circ, u_i^\circ) \\ &= E^i(\mathbf{q}^\circ, u_i^\circ) - (\mathbf{q}^\circ - \mathbf{p}^w)' D^i(\mathbf{q}^\circ, I_i^\circ) , \end{aligned}$$

because the levels of Hicksian compensated demands $\tilde{D}^i$ and the uncompensated demands D^i are equal at the original equilibrium $\circ$. If i is either in the non-lobbying set $(N - L)$ or in the passive lobbying set P, we have $c_i^\circ = 0$ so

$$E^i(\mathbf{q}^\circ, u_i^\circ) = I_i = \pi^i(\mathbf{p}^\circ) + t_i^\circ .$$

Combining all these equations and inequalities, we get

$$t_i^1 \leq t_i^\circ - (\mathbf{q}^\circ - \mathbf{p}^w)' D^i(\mathbf{q}^\circ, I_i^\circ) + (\mathbf{p}^\circ - \mathbf{p}^w)' S^i(\mathbf{p}^\circ) .$$

If i is in the active lobbying set A, then c_i° is given by the compensating variation above. Again we combine the relevant equations, and the result is the same inequality for t_i^1 as the one above.

Finally, adding these inequalities for all $i \in N$,

$$\sum_{i \in N} t_i^1 \leq \sum_{i \in N} t_i^\circ - (\mathbf{q}^\circ - \mathbf{p}^w)' \sum_{i \in N} D^i(\mathbf{q}^\circ, I_i^\circ) + (\mathbf{p}^\circ - \mathbf{p}^w)' \sum_{i \in N} S^i(\mathbf{p}^\circ) \leq 0 ,$$

using the budget constraint which is satisfied by the equilibrium $\circ$. This proves the feasibility of the alternative – no-distortions, transfers only – policy.

If $\mathbf{q}^\circ$ and $\mathbf{p}^\circ$ are not both equal to $\mathbf{p}^w$, and there is some substitution possibility in either consumption or production for at least some i, then at least one of the inequalities on the profit and expenditure functions will be strict. Then the alternative policy can be turned into a strictly superior response for the government. It can increase t_i^1 for any $i \in N - L$ or $i \in P$ slightly, which increases u_i^1, or increase t_i^1 for any $i \in A$, which increases c_i^1, in each case leaving all the other arguments of the government's objective function unchanged.

3.2 Distribution

Condition (b) of Proposition 4 helps us to calculate the individuals' utility levels u° and the government's receipts from the lobbies. The condition says that the government's utility in equilibrium should equal what it would get by responding optimally to the equilibrium payment schedules of all the lobbies except any one, when that one pays nothing. The set of equations this generates are to be solved simultaneously.

Using the efficiency result of the previous subsection, we set $\mathbf{p} = \mathbf{q} = \mathbf{p}^w$, and omit these arguments from the various functions. Let $U^i(I_i) = V^i(\mathbf{p}^w, I_i)$. Define $\pi_i = \pi^i(\mathbf{p}^w)$, and think of them as the individuals' endowments. Finally, write $\pi \equiv \sum_{i \in N} \pi_i$ for the total endowment in the economy, and assume that it is positive. Then the non-lobbyers' incomes are $\pi_i + t_i$, and the lobbies' truthful contribution schedules are $C_i(t_i, u_i^\circ) = \max\,[\pi_i + t_i - E^i(u_i^\circ), 0]$.

We will find that when there are two or more lobbies, any one of them has no economic power in its agency relationship with the government. Indeed, the effect is exactly as if the government could rob the official budget directly for its political kitty, without having to rely on any lobbies or their contributions at all. If given this much power, the only reason the government would give anything to any group is that it cares directly about social welfare as well as about its own consumption. In other words, such a "partially benevolent dictator" government would solve the following maximization problem:

Problem A

$$\max_{\mathbf{I},c} \; g(U^1(I_1), \ldots U^n(I_n), c)$$

subject to $c \geq 0$, and

$$\sum_{i \in N} I_i + c \leq \pi .$$

Given our assumptions, namely that all the functions $U^i(I_i)$ are strictly increasing and strictly concave, and that the function $g(\mathbf{u}, c)$ is strictly increasing and quasi-concave, this problem has a unique solution. Because we have assumed the "Inada condition" that the marginal utility of each individual goes to infinity as the utility goes to its subsistence level, we do not need to impose any lower bounds on the I_i.

We state the equivalence between the political equilibrium and the choice of this "partially benevolent dictator" in the following result:

Proposition 6: Assume that the set L has at least two members. Then the unique solution of Problem A yields a truthful equilibrium. Moreover, if all the functions $U^i(\cdot)$ and $g(\cdot)$ are differentiable, then any truthful equilibrium solves Problem A (and is therefore unique).

Explanation: We see that the government achieves the same utility in the truthful equilibrium as it would if it had access to all of the resources in the economy and could allocate them freely to its own political kitty or to any of its constituents. Clearly, this is the best possible outcome for the government. The lobbies, on the other hand, fare no better than they would if they were non-lobbyers but some others were active in lobbying. This is a terrible outcome for the lobbies, because each one could achieve the same result *unilaterally* by renouncing its lobbying activities. However, such unilateral renunciation by *all* lobbies would not be an equilibrium, because starting from such a position each one would want to lobby; that is the essence of a prisoners' dilemma!

The government's power in the agency relationship derives from its credible threat to cut any lobby out of the bargain and to deal only with the others. Suppose that some deviant lobby were to demand positive surplus from the government in exchange for its contribution. The government could always cut the transfer to the lobby by the amount of its promised gift and redirect the funds to some other lobbies. Since these others all have truthful payment schedules, the government would receive back the entire amount of the redirected transfers as additional contributions from them. All of the lobbies are perfect substitutes in the eyes of the government as sources of funding and so no one of them can bring harm to the government by threatening to withhold its tribute.

Note that our result requires that there be at least two lobbies. The second lobby is necessary so that, were one to deviate, the government could find a substitute with which to "work a deal". If there is only one lobby, then were that lobby to deviate, the government could not bestow its transfer on another contributor and get it back dollar for dollar. The best the government could do would be to spread its transfer around to maximize social welfare, which cannot be better or else it would already be doing so in the equilibrium. In short, a single lobby captures all the surplus inherent in its relationship with the government.[11] But as soon as there are two or more lobbies, each one loses all power and the government captures the enitre surplus in the form of contributions.

We can also see that the existence of lump-sum transfers is essential for this argument. If the available redistributive instruments all were distortionary, then to compensate fully for the contributions lost when one lobby deviates, the government must increase the levels of the instruments favoring the other lobbies. This causes greater and greater marginal distortion, and so is costly to the government. The extra cost is the power that each potential deviator has in its dealings with the government, and the equivalent variation of this extra distortion equals the amount of surplus that the lobby can extract in equilibrium. This is illustrated by Grossman and Helpman (1994) for tariffs and by Dixit (1995) for production subsidies.

Our result concerning distribution also brings out the contrast between our efficiency result and that of Becker (1983); see his discussion on pp. 385–6. In his model, the replacement of a less efficient by a more efficient instrument generally allows the lobbies to achieve the same or better results using less resources in exerting political pressure. Therefore they unanimously favor the more efficient instrument. In our model, the government's

[11] The single lobby derives its power from its assumed ability to make take-it-or-leave-it offers. If the lobby and the government instead were to negotiate over the size of the tribute, then each would share in the surplus from their bilateral relationship.

choice of action achieves efficiency, because the government attaches some weight to social welfare. The lobbying groups actually fare poorly in their competition when more efficient instruments are used; they would favor a prior agreement to use less efficient instruments.

Finally, our findings cast the efficiency result in a very different light. Even though the polity as a whole may exhibit greater efficiency when the lobbies have access to, and therefore obtain in equilibrium, non-distorting lump-sum instruments, the consumers fare poorly as a result. If they can look ahead, and write ex ante a constitution for lobbying so that they become committed to those rules of the game, they will unanimously agree to a rule which prohibits lobbying for direct transfers, and instead restricts lobbying to distorting policies. Far from justifying an efficient outcome, the result suggests another reason why we might expect inefficient policies to emerge in a political equilibrium.[12]

[12] Hammond (1979) argues that individualized lump-sum transfers are infeasible for informational reasons. See also Coate and Morris (1995) for an informational reason and Dixit and Londregan (1995) for a commitment reason why the political process uses inefficient instruments.

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Appendix: Proofs of the Propositions

Proof of Proposition 1: Conditions (a) and (b) of the proposition just restate those in the definition of an equilibrium. It therefore remains to prove necessity and sufficiency of condition (c) given that (a) and (b) hold. We prove both parts "negatively," that is, start by assuming that condition (c) in one place (definition or theorem) is violated and proving that this implies condition (c) must be violated in the other place as well.

Necessity: Suppose that condition (c) of the proposition does not hold for some $i \in L$. Then there exists a vector $(\mathbf{a}^*, c^*)$ that solves the maximization problem on the right hand side of equation (2.1) when applied to that i, and yields $U^i(\mathbf{a}^*, c^*) > U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a})]$. Since $(\mathbf{a}^*, c^*)$ satisfies the constraints of the maximization problem, there exists a payment function $\hat{C}_i \in \mathcal{C}_i$ with $\hat{C}_i(\mathbf{a}^*) = c^*$. Now define the function $\varphi_i(\mathbf{a})$ that satisfies

$$G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, \varphi_i(\mathbf{a}))] = G[\mathbf{a}^*, (\{C_j^\circ(\mathbf{a}^*)\}_{j \neq i}, c^*)] \qquad \text{for all } \mathbf{a} \in \mathcal{A}.$$

Since the function $G(\mathbf{a}, \mathbf{c})$ is assumed to be increasing in each component of $\mathbf{c}$, φ_i is a well defined function and $\varphi_i(\mathbf{a}^*) = c^*$. Moreover, $(\mathbf{a}^*, c^*)$ satisfies (2.2). Combining this with the definition of φ_i, we have, for all $\mathbf{a} \in \mathcal{A}$,

$$\begin{aligned} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, \varphi_i(\mathbf{a}))] &= G[\mathbf{a}^*, (\{C_j^\circ(\mathbf{a}^*)\}_{j \neq i}, c^*)] \\ &\geq \max_{\mathbf{a}' \in \mathcal{A}} G[\mathbf{a}', (\{C_j^\circ(\mathbf{a}')\}_{j \neq i}, 0)] \\ &\geq G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, 0)]. \end{aligned}$$

The first step is the definition of φ_i, the second is the constraint (2.2) satisfied by $(\mathbf{a}^*, c^*)$, and the third follows from just a particular choice of $\mathbf{a}'$ in this constraint.

Since $G(\mathbf{a}, \mathbf{c})$ is increasing in each component of $\mathbf{c}$, this implies that $\varphi_i(\mathbf{a}) \geq 0$ for all $\mathbf{a} \in \mathcal{A}$. It follows from Assumption 1 that the payment function C_i^* defined by $C_i^*(\mathbf{a}) = \min[\hat{C}_i(\mathbf{a}), \varphi_i(\mathbf{a})]$ for all $\mathbf{a} \in \mathcal{A}$ is feasible for principal i, and that $C_i^*(\mathbf{a}^*) = c_i^*$. Therefore, for all $\mathbf{a} \in \mathcal{A}$,

$$\begin{aligned} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, C_i^*(\mathbf{a}))] &\leq G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, \varphi_i(\mathbf{a}))] \\ &= G[\mathbf{a}^*, (\{C_j^\circ(\mathbf{a}^*)\}_{j \neq i}, c_i^*)] \\ &= G[\mathbf{a}^*, (\{C_j^\circ(\mathbf{a}^*)\}_{j \neq i}, C_i^*(\mathbf{a}^*))]. \end{aligned}$$

The first step follows from $C_i^*(\mathbf{a}) \leq \varphi_i(\mathbf{a})$, the second is the definition of φ_i, and the third follows from $C_i^*(\mathbf{a}^*) = c_i^*$. This chain proves that

$$\mathbf{a}^* = \arg\max_{\mathbf{a} \in \mathcal{A}} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j \neq i}, C_i^*(\mathbf{a}))].$$

But $U^i[\mathbf{a}^*, C_i^*(\mathbf{a}^*)] > U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$; which violates condition (c) in the definition of an equilibrium.

Sufficiency: Suppose that condition (c) in the definition of an equilibrium does not hold for some i. Namely, suppose that there exists a feasible payment function C_i and a feasible action $\mathbf{a}_i$ such that:

(i) $\mathbf{a}_i = \arg\max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j\neq i}, C_i(\mathbf{a}))]$ and

(ii) $U^i[\mathbf{a}_i, C_i(\mathbf{a}_i)] > U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$.

Then it follows from Assumption 1 that $C_i(\mathbf{a}_i) \geq 0$, which together with (i) above implies that $[\mathbf{a}_i, C_i(\mathbf{a}_i)]$ is feasible in the maximization problem in condition (c) of Proposition 1. Together with (ii) above, this contradicts condition (c) of the proposition; namely, $[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$ does not solve the maximum problem in condition (c) of the proposition. □

Proof of Corollary to Proposition 1: This result follows directly from conditions (b) and (c) of Proposition 1. There are two possibilities: $C_i^\circ(\mathbf{a}^\circ)$ is either $= 0$ or > 0. In the former case the corollary is obvious. In the latter case if the inequality in the constraint (2.2) of the proposition's condition (c) is strict, then c can be further reduced without altering $\mathbf{a}$ and still preserve feasibility. This raises $U^i(\mathbf{a}, c)$, thereby violating this condition. □

Proof of Proposition 2: Given differentiability and the fact that the equilibrium is interior, we have the first-order conditions for the maximization in (b) of Proposition 1:

$$\frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial a_k} + \sum_{j\in L} \frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial c_j} \frac{\partial C_j^\circ(\mathbf{a}^\circ)}{\partial a_k} = 0 \tag{A1}$$

for all components k of the action vector. Similarly, the first-order conditions for the maximization in (c) of Proposition 1 are

$$\frac{\partial U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]}{\partial a_k} + \lambda \left[\frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial a_k} + \sum_{j\in L, j\neq i} \frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial c_j} \frac{\partial C_j^\circ(\mathbf{a}^\circ)}{\partial a_k} \right] = 0 \tag{A2}$$

for all i, k, and

$$\frac{\partial U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]}{\partial c_i} + \lambda \left[\frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial c_i} \right] = 0 \tag{A3}$$

for all i, where λ is a non-negative Lagrange multiplier for the non-negativity constraint in condition (c). Combining (3.1) and (3.1) yields

$$\frac{\partial U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]}{\partial a_k} = \lambda \frac{\partial G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]}{\partial c_i} \frac{\partial C_i^\circ(\mathbf{a}^\circ)}{\partial a_k}. \tag{A4}$$

Combining this with (3.1) and stacking up the components into gradient vectors, we get the local truthfulness condition (2.3). □

Proof of Proposition 3: Let C_i° be some best response of principal i to payment functions $\{C_j^\circ(\mathbf{a})\}_{j\in L, j\neq i}$ of the other principals, let $\mathbf{a}^\circ$ be the agent's best response to the whole set of payment functions $\{C_j^\circ(\mathbf{a})\}_{j\in L}$, and let $U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)] = u_i^\circ$ be the resulting utility levels of the principals. Define the globally truthful payment function $C_i^T(\mathbf{a}, u_i^\circ)$ relative to u_i°. We claim that it is also a best response to the same given payment functions of all the others.

To see this, let $\mathbf{a}'$ denote the agent's choice of action

$$\mathbf{a}' = \arg\max_{\mathbf{a}\in\mathcal{A}} G[\mathbf{a}, (\{C_j^\circ(\mathbf{a})\}_{j\neq i}, C_i^T(\mathbf{a}, u_i^\circ))] .$$

If $\mathbf{a}' = \mathbf{a}^\circ$, then the truthful strategy trivially yields to principal i the same utility level u_i° as does the strategy C_i° which is a best response, so the truthful strategy must itself be a best response. If $\mathbf{a}' \neq \mathbf{a}^\circ$, this must be because, other things equal, the truthful strategy elicits a larger payment. For if $C_i^\circ(\mathbf{a}') \geq C_i^T(\mathbf{a}', u_i^\circ)$, we have

$$\begin{aligned} G[\mathbf{a}', \mathbf{C}^\circ(\mathbf{a}')] &\geq G[\mathbf{a}', (\{C_j^\circ(\mathbf{a}')\}_{j\neq i}, C_i^T(\mathbf{a}', u_i^\circ))] \\ &> G[\mathbf{a}^\circ, (\{C_j^\circ(\mathbf{a}^\circ)\}_{j\neq i}, C_i^T(\mathbf{a}^\circ, u_i^\circ))] \\ &= G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)] . \end{aligned}$$

The first step follows from the fact that G is increasing in payments, the second because $\mathbf{a}'$ is a maximizer of G while $\mathbf{a}^\circ$ is not, and the third because $C_i^T(\mathbf{a}^\circ, u_i^\circ) = C_i^\circ(\mathbf{a}^\circ)$. Then $\mathbf{a}^\circ$ would not have been the agent's best choice in equilibrium.

We have shown $C_i^T(\mathbf{a}', u_i^\circ) > C_i^\circ(\mathbf{a}') \geq 0$, and there remain two logical cases to consider: $C_i^T(\mathbf{a}', u_i^\circ)$ can be equal to $\varphi_i(\mathbf{a}', u_i^\circ)$, or at the upper limit $\bar{C}_i(\mathbf{a}')$. In the first of these, principal i gets utility u_i°, which is the same as he gets with C_i°, so C_i^T is also a best response. In the second, principal i would have been willing to pay even more while still getting at least u_i°; but then C_i° could not have been the best response and this case cannot arise. □

Proof of Proposition 4: This follows directly from condition (b) in the definition of an equilibrium, from the Corollary to Proposition 1, and from the definition of globally truthful payment functions. □

Proof of Proposition 5: Suppose there do exist $\mathbf{a}^*$ and $\mathbf{c}^*$ as stipulated in the statement. For all i, we have

$$U^i(\mathbf{a}^*, c_i^*) \geq U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)] = U^i[\mathbf{a}^*, \varphi_i(\mathbf{a}^*, u_i^\circ)],$$

by the definition of φ, so $c_i^* \leq \varphi_i(\mathbf{a}^*, u_i^\circ)$. This also ensures $\varphi_i(\mathbf{a}^*, u_i^\circ) \geq 0$, and we have assumed $c_i^* \leq \bar{C}_i(\mathbf{a}^*)$, so

$$c_i^* \leq \min[\bar{C}_i(\mathbf{a}^*), \max[0, \varphi_i(\mathbf{a}^*, u_i^\circ)]] = C_i^\circ(\mathbf{a}^*)$$

using the definition of a truthful schedule.

Since G is increasing in payments, this shows $G[\mathbf{a}^*, \mathbf{C}^\circ(\mathbf{a}^*)] \geq G(\mathbf{a}^*, \mathbf{c}^*)$. Also, since $\mathbf{a}^\circ$ is the agent's best response to $\mathbf{C}^\circ$, we have $G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)] \geq G[\mathbf{a}^*, \mathbf{C}^\circ(\mathbf{a}^*)]$. If the agent's utility inequality is strict in (ii) of the statement of the proposition, we already have a contradictory chain of inequalities. So consider the case where

$$G(\mathbf{a}^*, \mathbf{c}^*) = G[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)] = G[\mathbf{a}^*, \mathbf{C}^\circ(\mathbf{a}^*)] .$$

We prove that this leads to another contradiction if the utility inequality in (ii) is strict for any one principal i.

Consider the constraint (2.2) in Condition (c) of Proposition 1. It is satisfied by $[\mathbf{a}^\circ, \mathbf{C}^\circ(\mathbf{a}^\circ)]$ and therefore by $(\mathbf{a}^*, \mathbf{c}^*)$ in the case we are now considering. But for all principals $j \neq i$, we have already established $c_j^* \leq C_j^\circ(\mathbf{a}^*)$, and G is increasing in every component of the payment vector. Therefore the constraint is also satisfied by $[\mathbf{a}^*, (\{C_j^\circ(\mathbf{a}^*)\}_{j\neq i}, c_i^*)]$. Thus $(\mathbf{a}^*, c_i^*)$ is feasible in the maximization problem in Condition (c) of Proposition 1. But we are supposing that it gives strictly more utility than $U^i[\mathbf{a}^\circ, C_i^\circ(\mathbf{a}^\circ)]$. This contradicts Condition (c) of Proposition 1, which o being an equilibrium must satisfy. □

Proof of Corollary 1 to Proposition 5: If not, a better neighboring feasible action, plus a suitable rearrangement of payments, can achieve a Pareto superior outcome, contradicting Proposition 5. □

Proof of Corollary 2 to Proposition 5: Suppose, to the contrary, that there exists a vector $\mathbf{a}'$ such that $u_i(\mathbf{a}', c_i^\circ) \geq u_i^\circ$ for all $i \in \{1, 2, \ldots, n\}$, with strict inequality for some i. Since the contributions c_i° were feasible when the action was $\mathbf{a}^\circ$, we know $u_i(\mathbf{a}^\circ, c_i^\circ) \geq \underline{u}_i$. A fortiori, $u_i(\mathbf{a}', c_i^\circ) \geq \underline{u}_i$; thus the contributions c_i° satisfy $c_i^\circ \leq \bar{C}_i(\mathbf{a})$, and are therefore feasible.

Since the function g is increasing in all of its arguments, the individual utility inequalities imply that the government also gets greater utility using $\mathbf{a}'$:

$$g[u_1(\mathbf{a}', c_1^\circ), u_2(\mathbf{a}', c_2^\circ) \ldots u_n(\mathbf{a}', c_n^\circ), \mathbf{c}^\circ] > g(u_1^\circ, u_2^\circ, \ldots, u_n^\circ), \mathbf{c}^\circ) \,.$$

We have found a feasible and Pareto superior alternative pair of action and contribution vectors, which violates part (ii) of Proposition 5 for a truthful equilibrium. □

Proof of Proposition 6:

We first formally characterize the government's action in a truthful equilibrium:

Problem G Given $\{u_i^\circ \mid i \in L\}$,

$$\max_{\mathbf{t}} \; g\left[\left(u_i^\circ \mid i \in L, U^i(\pi_i + t_i) \mid i \notin L\right), \sum_{i \in L} \max\left[\pi_i + t_i - E^i(u_i^\circ), 0\right]\right]$$

subject to

$$\pi_i + t_i \geq E^i(u_i^\circ) \qquad \text{for all } i \in L,$$

and

$$\sum_{i \in N} t_i \leq 0$$

The first set of constraints ensures that the members of L actually get the equilibrium utility levels u_i°. Either the constraint is slack, when the excess comes back to the government in the form of the truthful contribution, or the constraint is tight, when the contribution is zero and the endowment plus the transfer only just suffices to deliver the required utility level. In view of this constraint, we can omit the max operator in the truthful contribution schedules in the objective function.

Now we reformulate this in a way that takes it closer to the Problem A solved by the "partially benevolent dictator" in the text:

Problem B

Given $\{u_i^\circ \mid i \in L\}$,

$$\max_{\mathbf{I},c} \; g(U^1(I_1), \ldots U^n(I_n), c)$$

subject to $c \geq 0$,

$$I_i \geq E^i(u_i^\circ) \qquad \text{for all } i \in L$$

and

$$\sum_{i \in N} I_i + c \leq \pi$$

We prove the

Lemma: Given any feasible action in Problem G, there is a feasible choice in Problem B that yields the same value of the objective, and vice versa.

Proof: The given $\{u_i^\circ \mid i \in L\}$ in either problem must satisfy $\sum_{i\in L} E^i(u_i^\circ) \leq \pi$ if the feasible choice set is to be non-empty.

Now take any feasible $\mathbf{t}$ for Problem G, and define

$$I_i = \begin{cases} E^i(u_i^\circ) & \text{for } i \in L \\ \pi_i + t_i & \text{for } i \notin L\,. \end{cases}$$

These satisfy the constraints $I_i \geq E^i(u_i^\circ)$ for $i \in L$ in Problem B with equality. Also, using the constraints of Problem G, we get

$$\begin{aligned} \sum_{i \in N} I_i &= \sum_{i \in L} E^i(u_i^\circ) + \sum_{i \notin L} (\pi_i + t_i) \\ &\leq \sum_{i \in N} (\pi_i + t_i) \leq \sum_{i \in N} \pi_i + 0 \leq \pi\,. \end{aligned}$$

Then we can define

$$c = \sum_{i \in N} \pi_i - \sum_{i \in L} E^i(u_i^\circ) \geq 0\,,$$

and verify that the $(\mathbf{I}, c)$ so constructed is a feasible choice for Problem B yielding the same value of the objective as the starting point, namely the feasible $\mathbf{t}$ of Problem G.

Conversely, given a feasible $(\mathbf{I}, c)$ for Problem B, construct any non-negative c_i for $i \in L$ adding up to the given non-negative c. Then define

$$t_i = \begin{cases} I_i - \pi_i + c_i & \text{for } i \in L \\ I_i - \pi_i & \text{for } i \notin L\,. \end{cases}$$

For all $i \in L$, we have $I_i \geq E^i(u_i^\circ)$ for feasibility in Problem G, and $c_i \geq 0$ by construction, therefore $\pi + t_i \geq E^i(u_i^\circ)$. Also

$$\begin{aligned} \sum_{i \in N} t_i &= \sum_{i \in N} I_i - \sum_{i \in N} \pi_i + \sum_{i \in L} c_i \\ &= \sum_{i \in N} I_i + c - \sum_{i \in N} \pi_i \leq 0 \end{aligned}$$

for feasibility in Problem B. Thus the constructed $\mathbf{t}$ is feasible for Problem G, and is easily seen to yield the same value of the objective.

Explanation: The point is that the extra non-negative c_i can be constructed and allocated to members of L entirely arbitrarily; they all come right back to the government as contributions along the truthful schedules. This is the reason why no lobby has any power in its relationship with the government; the latter can get exactly the same contribution by switching the tranfer to another lobby.

Now we turn to the proof of the proposition itself. First we take the unique solution of Problem A, and show that it satisfies all the requirements of a truthful equilibrium.

Let $(\mathbf{I}^\circ, c^\circ)$ solve Problem A. Let $u_i^\circ = U^i(I_i^\circ)$ for $i \in L$. Given these utility levels, the constraints $I_i \geq E^i(u_i^\circ)$ of Problem B are fulfilled as equalities. Then $(\mathbf{I}^\circ, c^\circ)$ must also solve Problem B, else something else feasible for Problem A and yielding a higher value of the same objective function could have been obtained. Thus condition (a) of Proposition 4 characterizing a truthful equilibrium is met.

Next, take any $j \in L$. When its contribution is set equal to zero, the government solves a problem exactly like Problem G but with the set of lobbies L replaced by $L - \{j\}$. So long as the set L has at least two members, we see from the Lemma that this is equivalent to a corresponding version of Problem B with the constraint $I_j \geq E^j(u_j^\circ)$ deleted; call it Problem B-j. The proof immediately above applies, and shows that the solution of Problem A solves Problem B-j. Moreover, the maximum values of Problem B and Problem B-j are obviously equal, thus fulfilling condition (b) of Proposition 4. This shows that the solution of Problem A yields a truthful equilibrium.

Now we turn to the converse. Consider a truthful equilibrium where the members of L get utilities u_i° and the government's transfers form the vector $\mathbf{t}^\circ$. Recall that we are now assuming all functions to be differentiable. Then the government's maximization problems, B when all members of L participate, and B-j when some particular $j \in L$ does not lobby, have differentiable strictly quasi-concave objectives and linear constraints, so their solutions can be characterized by the Lagrangian conditions.

Consider Problem B-j, which is non-vacuous when L has at least two members. The conditions for j, and for $i \notin L$, and c are

$$\begin{aligned} (\partial g/\partial u_j)\, U_I^j(I_j) &= \lambda_j\,, \\ (\partial g/\partial u_i)\, U_I^i(I_i) &= \lambda_j \quad \text{for all } i \notin L\,, \\ \partial g/\partial c &\leq \lambda_j \quad \text{complementary to } c \geq 0\,, \end{aligned}$$

where λ_j is the Lagrange multiplier for the constraint on the total available resources, $\sum_{i \in N} I_i + c \leq \pi$. Note that the incomes I_i are interior and the first-order conditions are equalities because we have assumed the "Inada condition" that the marginal utilities approach infinity at the lower limit of the incomes.

These are satisfied for all j by the same $(\mathbf{I}^\circ, c^\circ)$ of the truthful equilibrium. Therefore all the λ_j must be equal. Writing λ_0 for their common value, we have

$$\begin{aligned} (\partial g/\partial u_i)\, U_I^i(I_i) &= \lambda_0 \text{ for all } i \in N \\ \partial g/\partial c &\leq \lambda_0 \quad \text{complementary to } c \geq 0\,. \end{aligned}$$

But these are the first-order conditions for Problem A, which are sufficient because of that problem has a quasi-concave objective and linear constraints. □

Explanation: In Problem B, the choice of I_j for each j is constrained by $I_j \geq E^j(u_j^\circ)$. In Problem B-j this constraint is removed for a particular j. Condition (b) of Proposition 4 says that the government's objective function is the same in the two situations. Given differentiability, if there is no local improvement on relaxing constraints one at a time, there is no local improvement on relaxing them all together either. And given quasi-concavity, if there is no local improvement, there is no global improvement either.

Without differentiability, the government's objective function can have "ridges" such that its value can be increased by lowering I_j for two or more $j \in L$ simultaneously, even though no such increase is possible by lowering any one of the I_j.

This can be seen from the following Figure 2. There are two individuals, both of them in the lobbying set L. The figure shows their incomes I_1, I_2 and the government's consumption c along the three axes. The simplex ABC is the feasible set. The government's indifference surfaces $g(U^1(I_1), U^2(I_2), c) =$ constant are not shown.

Let point Q be the truthful equilibrium. Then I_1 is constant along FG and I_2 is constant along DE, both lines in the plane ABC. The shaded area is the set of feasible points that are better than Q for the government. If the government is allowed to change only I_1 and c it cannot reach a point that is better than Q; likewise for I_2 and c. Therefore Q satisfies condition (b) of Proposition 4 which characterizes the equilibrium. However, the government can do better if it can change the incomes of both individuals simultaneously.

This shaded region is possible only if the government's preference function is not differentiable; if it were, the shaded area could not have a corner or kink at Q as shown in the figure, but would have to have a smooth curve at Q, and therefore would have to cut through at least one of the lines DE and FG.

Figure 1

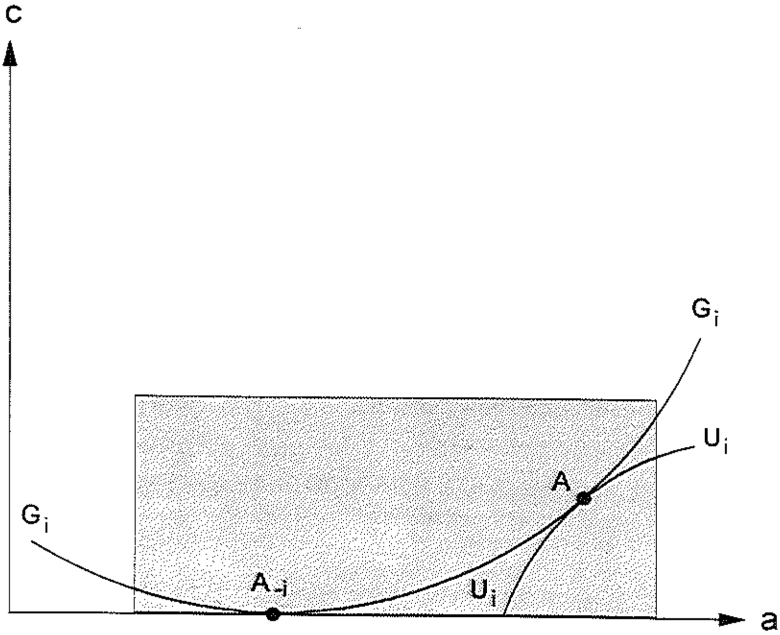
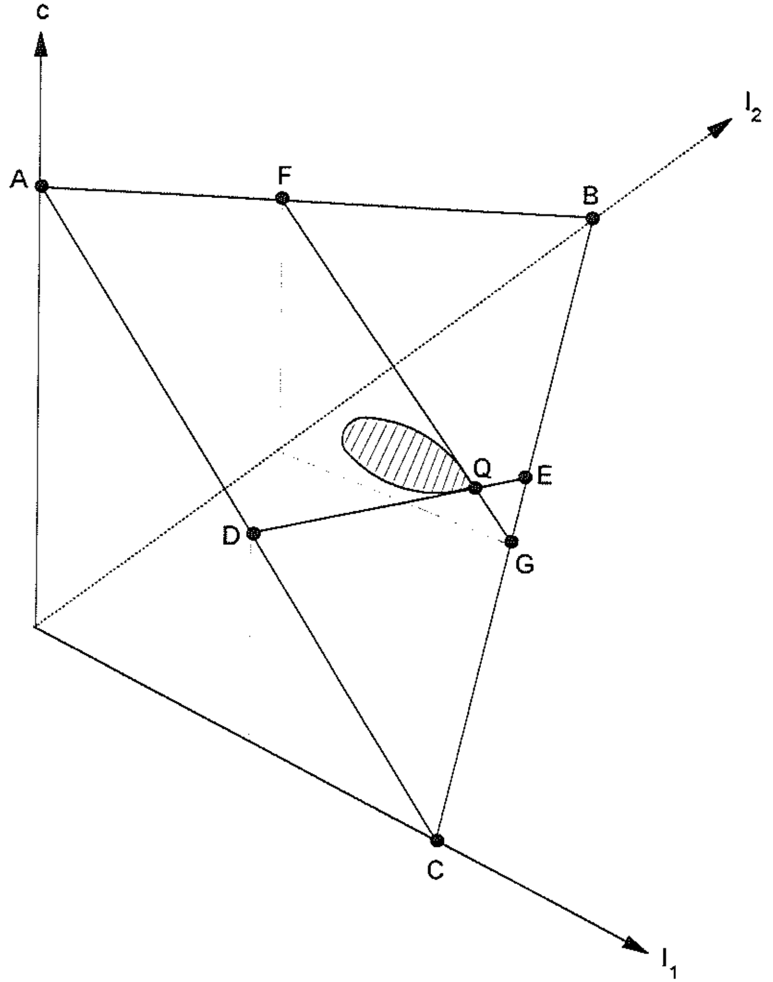


Figure 2



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